Title: Demonstration of Self-correcting Quantum Memory in Three Dimensions

Date: Mar 07, 2012 04:00 PM

URL: http://pirsa.org/12030109

Abstract: Based on the joint work with Sergey Bravyi, IBM Watson.   We show that any topologically ordered local stabilizer model of spins in three dimensional lattices that lacks string logical operators can be used as a reliable quantum memory against thermal noise. It is shown that any local process creating a topologically charged particle separated from other particles by distance R, must cross an energy barrier of height $c \log R$. This property makes the model glassy. We devise an efficient decoding algorithm that should be used at the final read-out, and prove a lower bound on the memory time until which the fidelity between the outcome of the decoder and the initial state is close to 1. The memory time increases as L^{β} where L is the system size and β the inverse temperature, as long as $L < L^\star \sim e^\beta$. Hence, the optimal memory time scales as e^{β^2}. Our bound applies when the system interacts with thermal bath via a Markovian master equation. We give an example of 3D local stabilizer codes that satisfies all of our assumptions. We numerically verify for this example that our bound is tight up to constants.

Demonstration of Self-correcting Quantum Memory in Three Dimensions

Jeongwan Haah

Institute for Quantum Information and Matter
California Institute of Technology

7 March 2012
Perimeter Institute

Memory - Storage medium

Repeat a symbol many times.

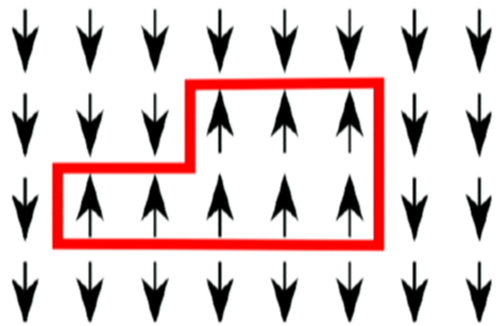
$$0 \Longleftrightarrow 000001000000000000000010000$$
$$1 \Longleftrightarrow 111111111111101111111101$$

Physical model

- Classical Ising model : $H = -\sum_{\langle ij \rangle} Z_i Z_j$
- 2-fold degenerate ground state

Memory in 2D

Good at $T > 0$!



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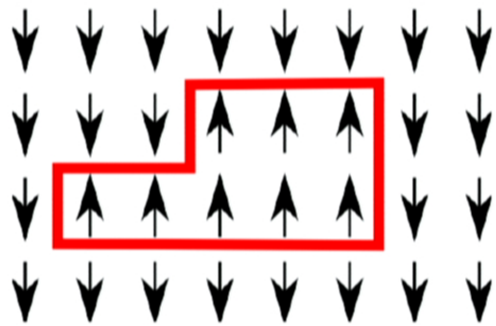

But,

$$\frac{|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle}{\sqrt{2}} \overset{Z}{\leftrightsquigarrow} \frac{|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle}{\sqrt{2}}$$

Bad as quantum memory

Memory in 2D

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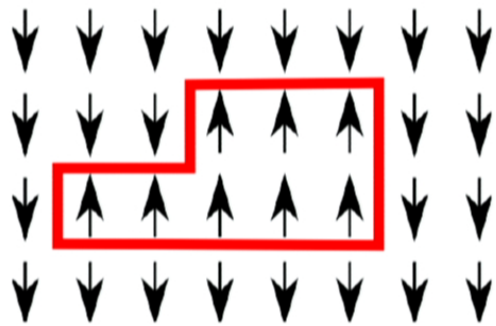

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Bad as quantum memory

States must be **locally indistinguishable**.

Memory in 2D

Toric code

- Qubits on the edges

$$H = -\sum_s \begin{array}{ccc} & | & \\ & X & \\ & | & \\ -X- & \cdot & -X- \\ & | & \\ & X & \\ & | & \end{array} \quad -\sum_p \begin{array}{ccc} & -Z- & \\ | & & | \\ Z & & Z \\ | & & | \\ & -Z- & \end{array}$$

- **Locally indistinguishable** 4-fold degenerate ground state.

Memory in 2D

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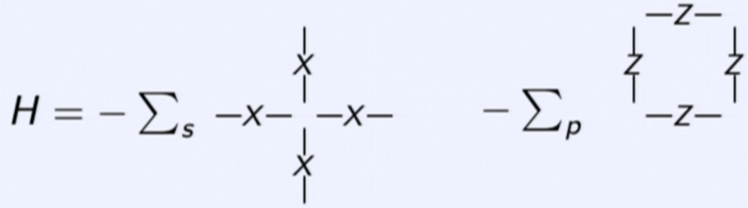

- **Locally indistinguishable** 4-fold degenerate ground state.
- But, $|\psi_0\rangle \Longleftrightarrow |\psi_1\rangle$ by dragging a quasi-particle across.

• −Z− -Z- · · · -Z- •

Bad at $T > 0$.

Memory

1 – Write

Initialize the memory → Spins at Ground state

Locally indistinguishable degenerate ground state.

2 – Store

Wait for long → Interact with thermal bath

Error accumulation energetically suppressed.

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Access the memory → e.g., Measure **average** spin direction

Error correction at read-out step.

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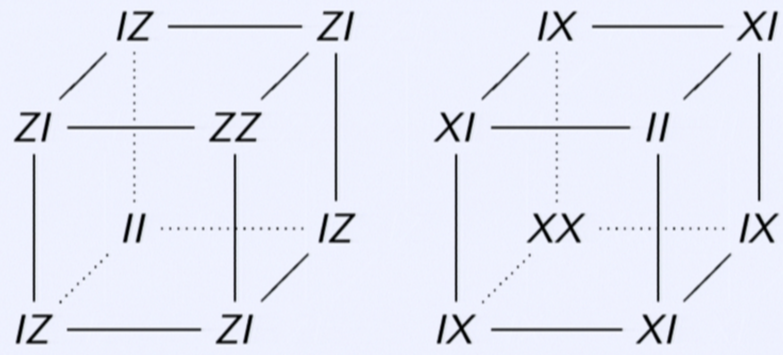
Possible in **4D** Dennis,Kitaev,Landahl,Preskill, J. Math. Phys. 43, 4452(2002)

Model[1]

Cubic code 1

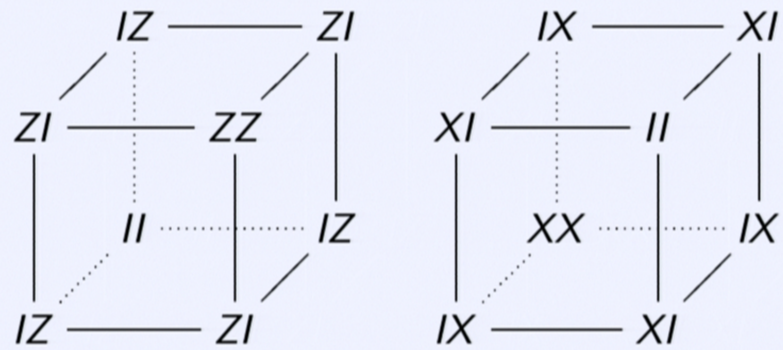


- $H = -J \sum_c Q_c^Z - J \sum_c Q_c^X$

[1] JH, Phys. Rev. A 83,042330 (2011)

Model[1]

Cubic code 1



- $H = -J \sum_c Q_c^Z - J \sum_c Q_c^X$
- All terms commute.
- Degenerate ground state
- Topologically ordered.

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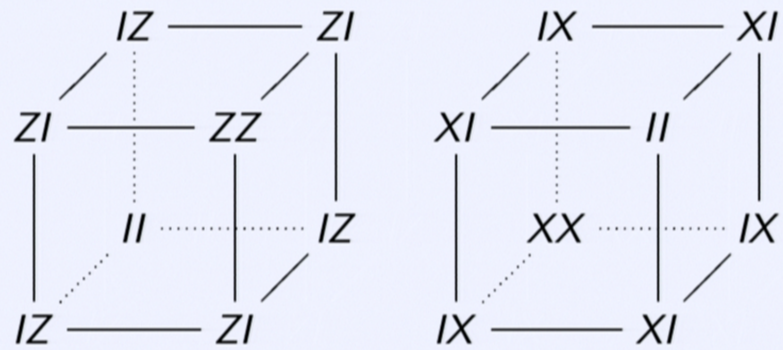

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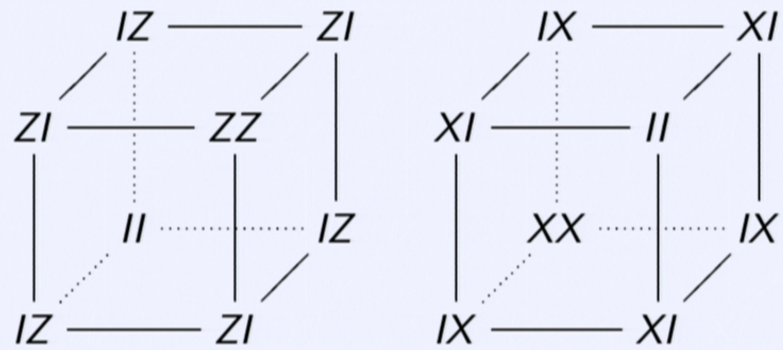

- $H = -J \sum_c Q_c^Z - J \sum_c Q_c^X$
- All terms commute.
- Degenerate ground state
- Topologically ordered.
- Defects **cannot** propagate.

Although quantum, the analysis is classical.

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No local observables distinguish ground states

If O has small support, then $\Pi_{GS} O \Pi_{GS} = c(O) \Pi_{GS}$.

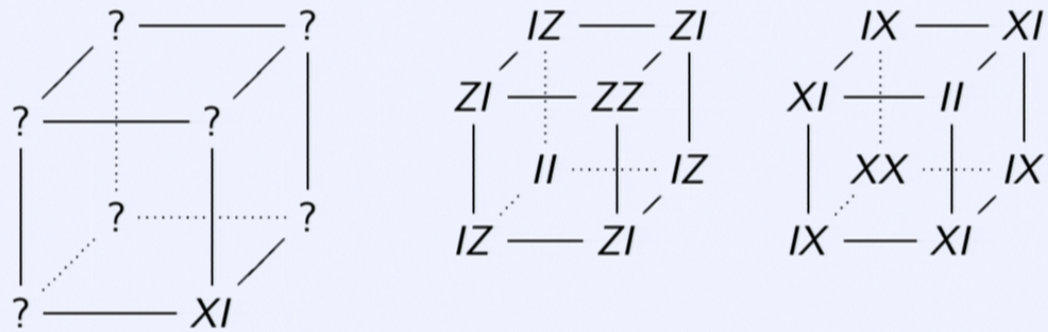
No local observables distinguish ground states

If O has small support, then $\Pi_{GS} O \Pi_{GS} = c(O) \Pi_{GS}$.

- Claim: Any Pauli operator of bounded support commuting with all stabilizers is a stabilizer.
- Any operator is a linear combination of Pauli operators.
- Because $Q_X \leftrightarrow Q_Z$, it suffices to consider X-type Pauli operator.

No local observables distinguish ground states

Proof



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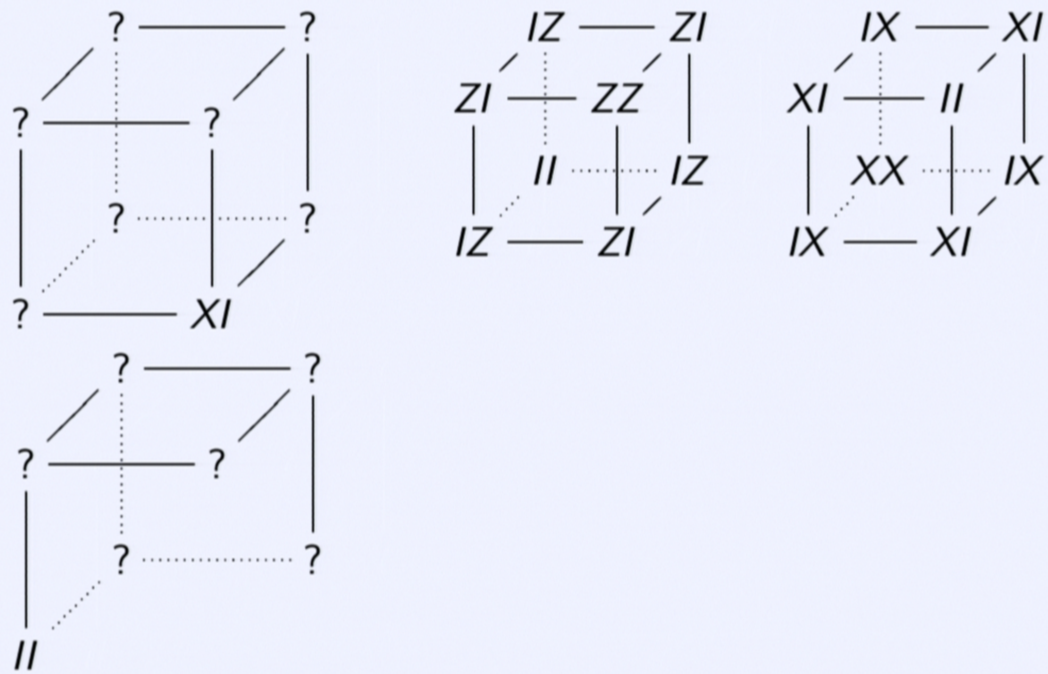

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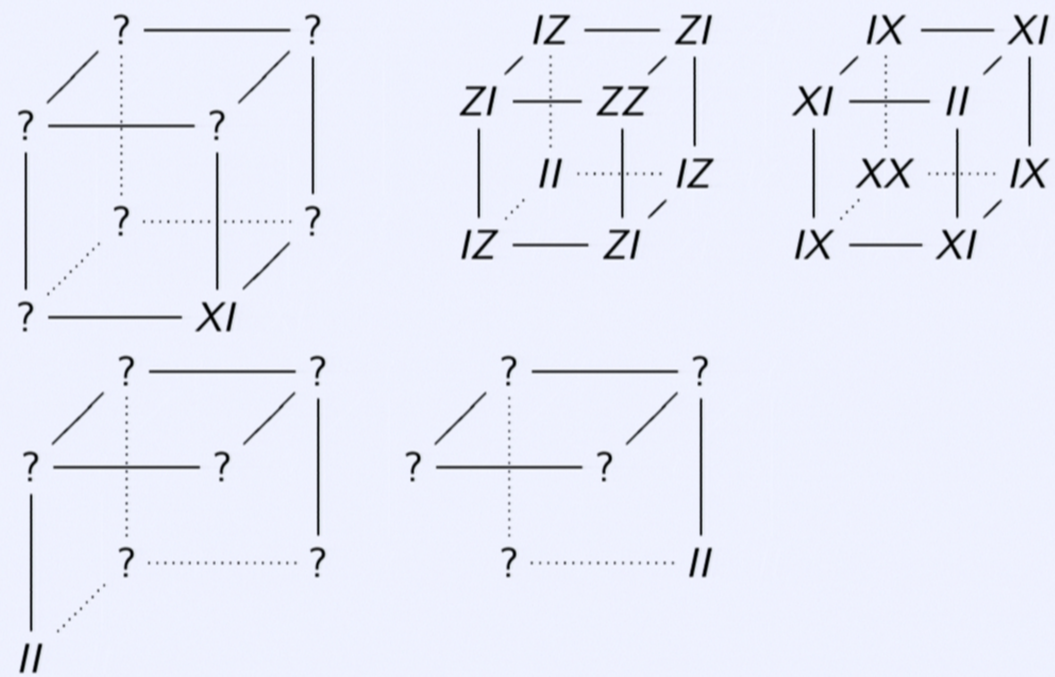

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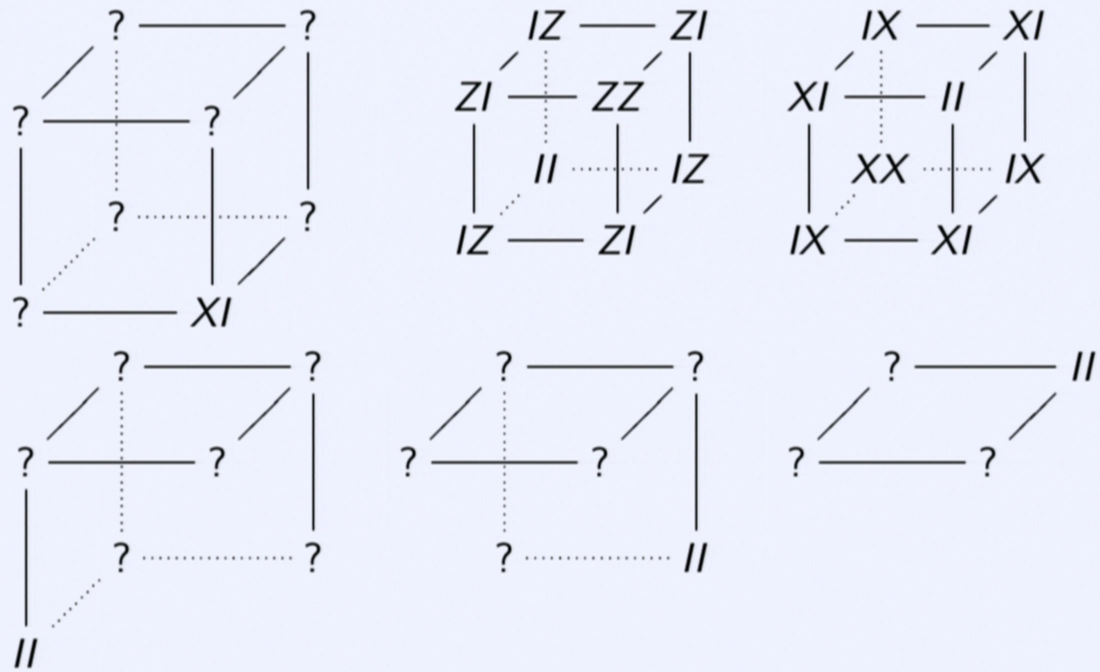

Q.E.D.

No local observables distinguish ground states

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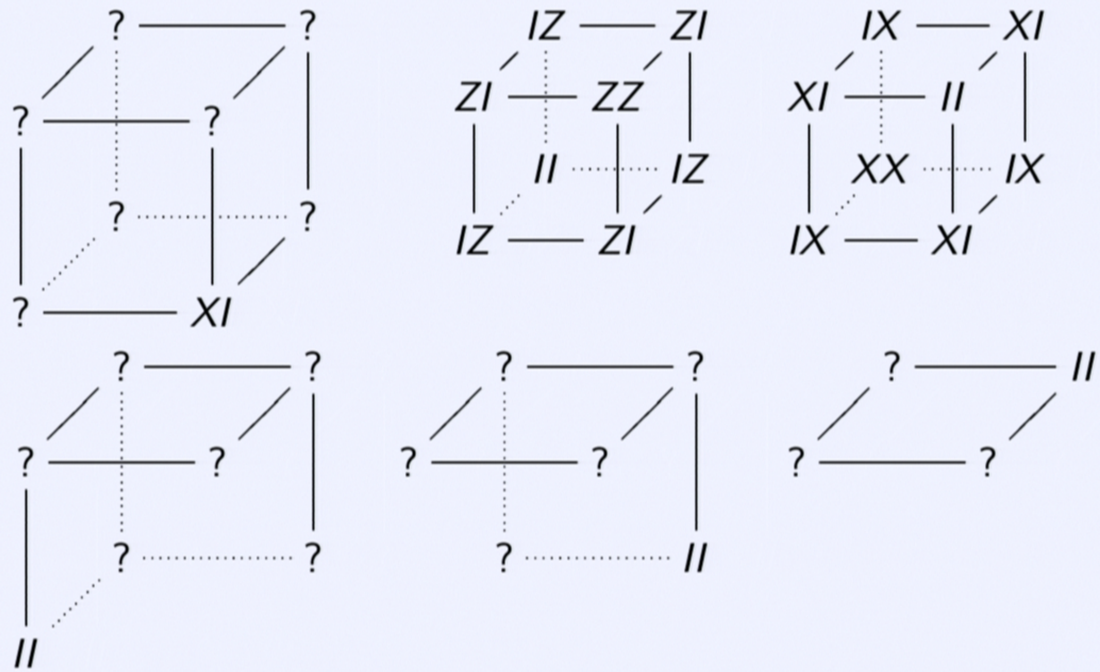

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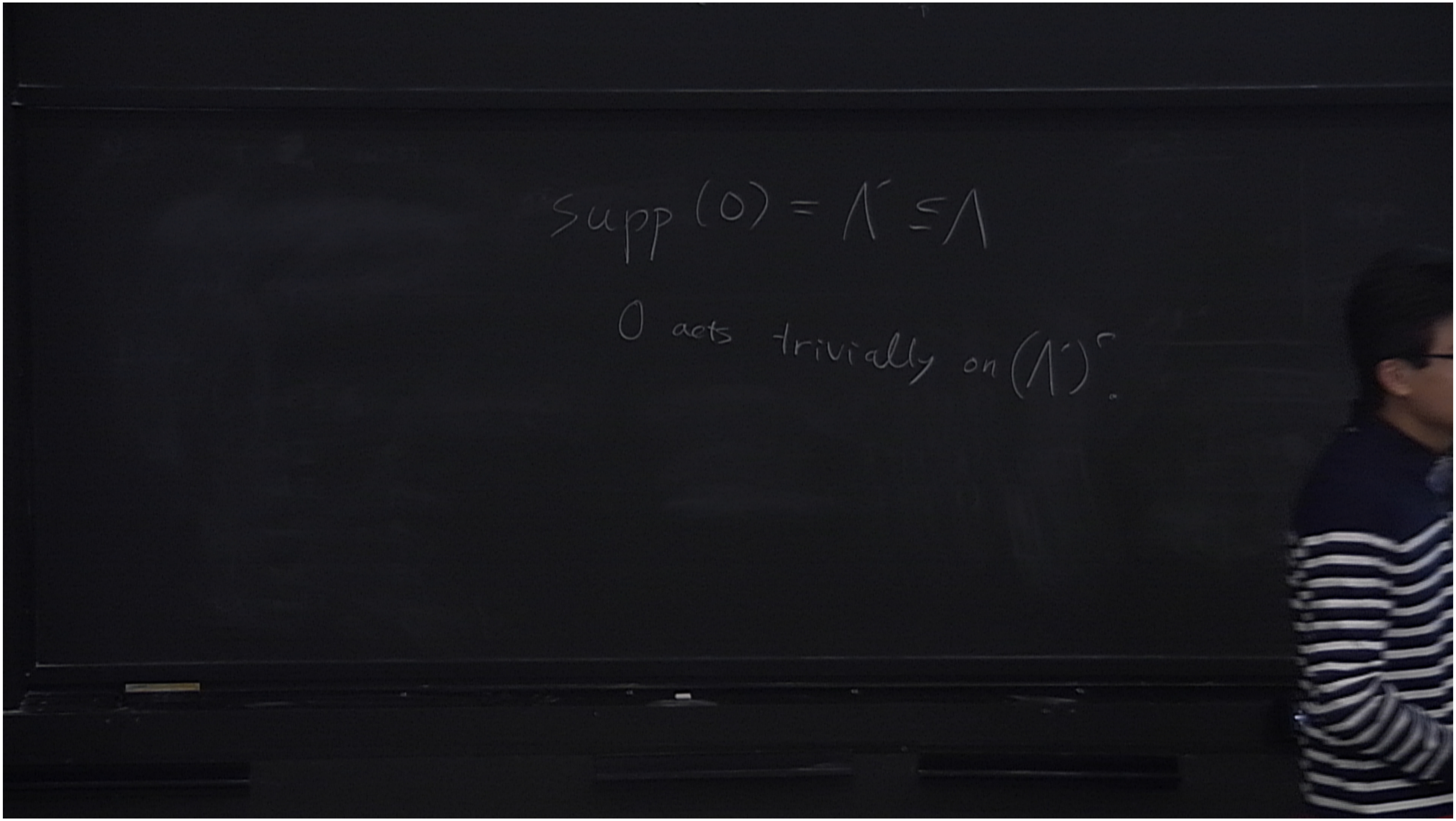
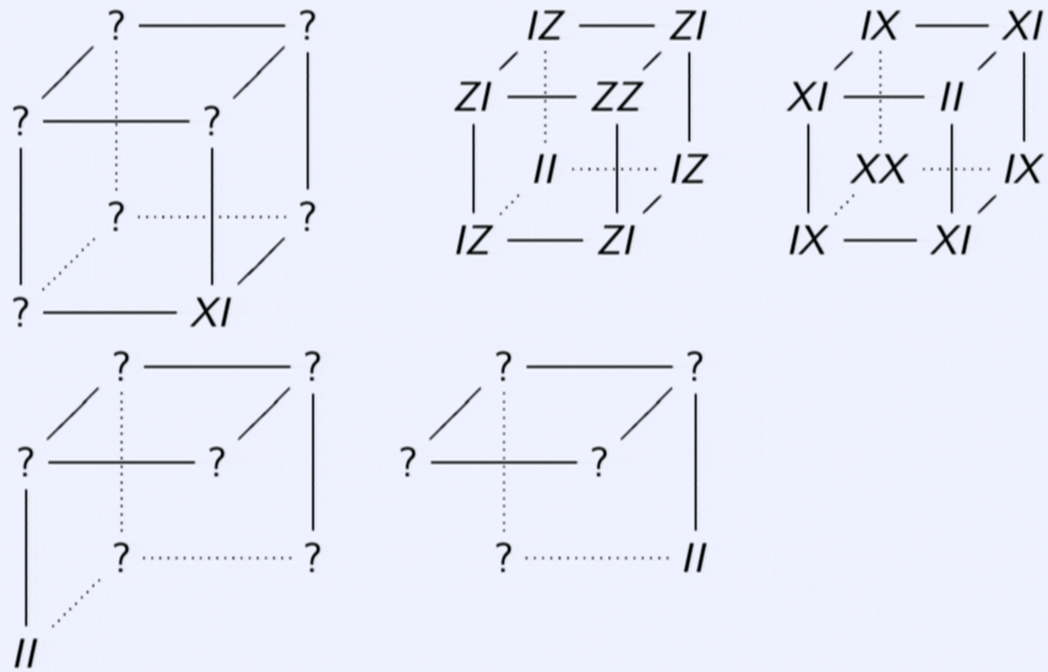

Supp(O) = Λ' ⊆ Λ
O acts trivially on (Λ')^c.

No local observables distinguish ground states

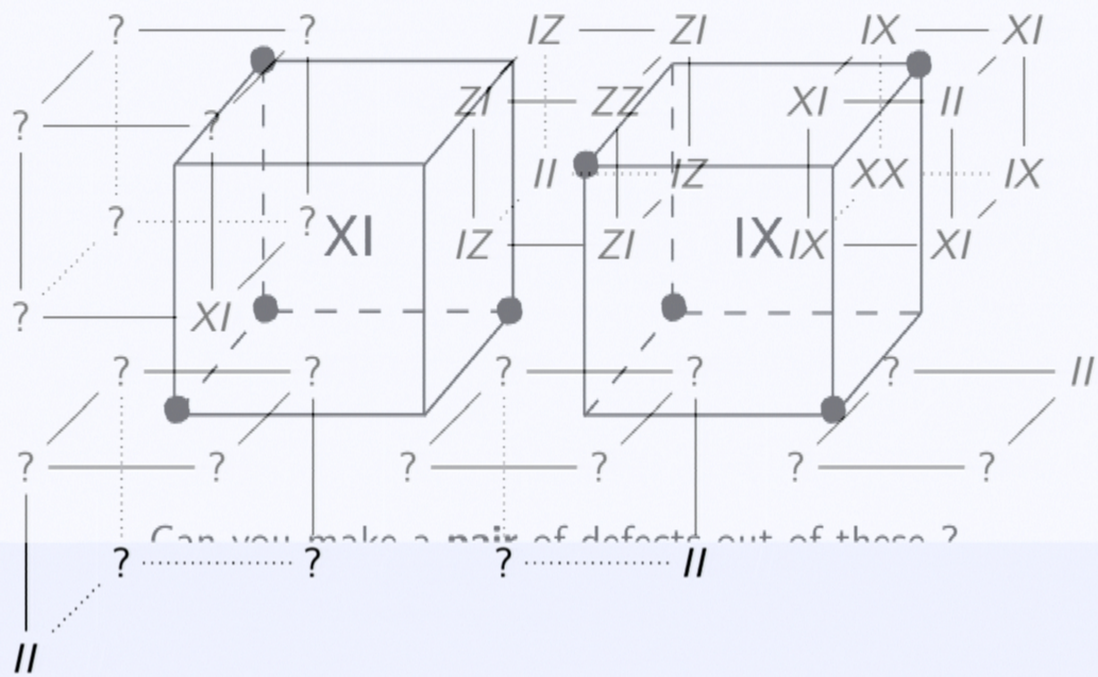
Proof



Elementary Excitations (Syndromes)

No local observable distinguishes ground states

Proof



Q.E.D.

String segment

String?

- 1D object..... – Not well-defined; only for special models.

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- Need to deal with a family of Hamiltonians.....

Definition

A finite Pauli operator that creates excitations at most two locations.

String segment

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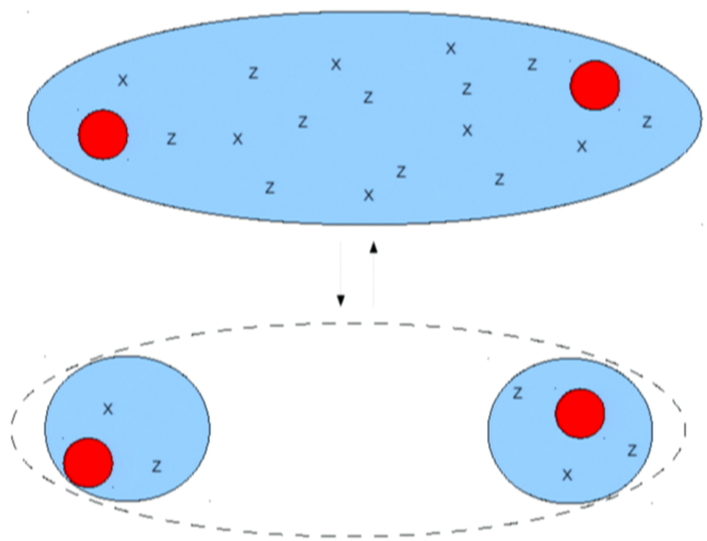
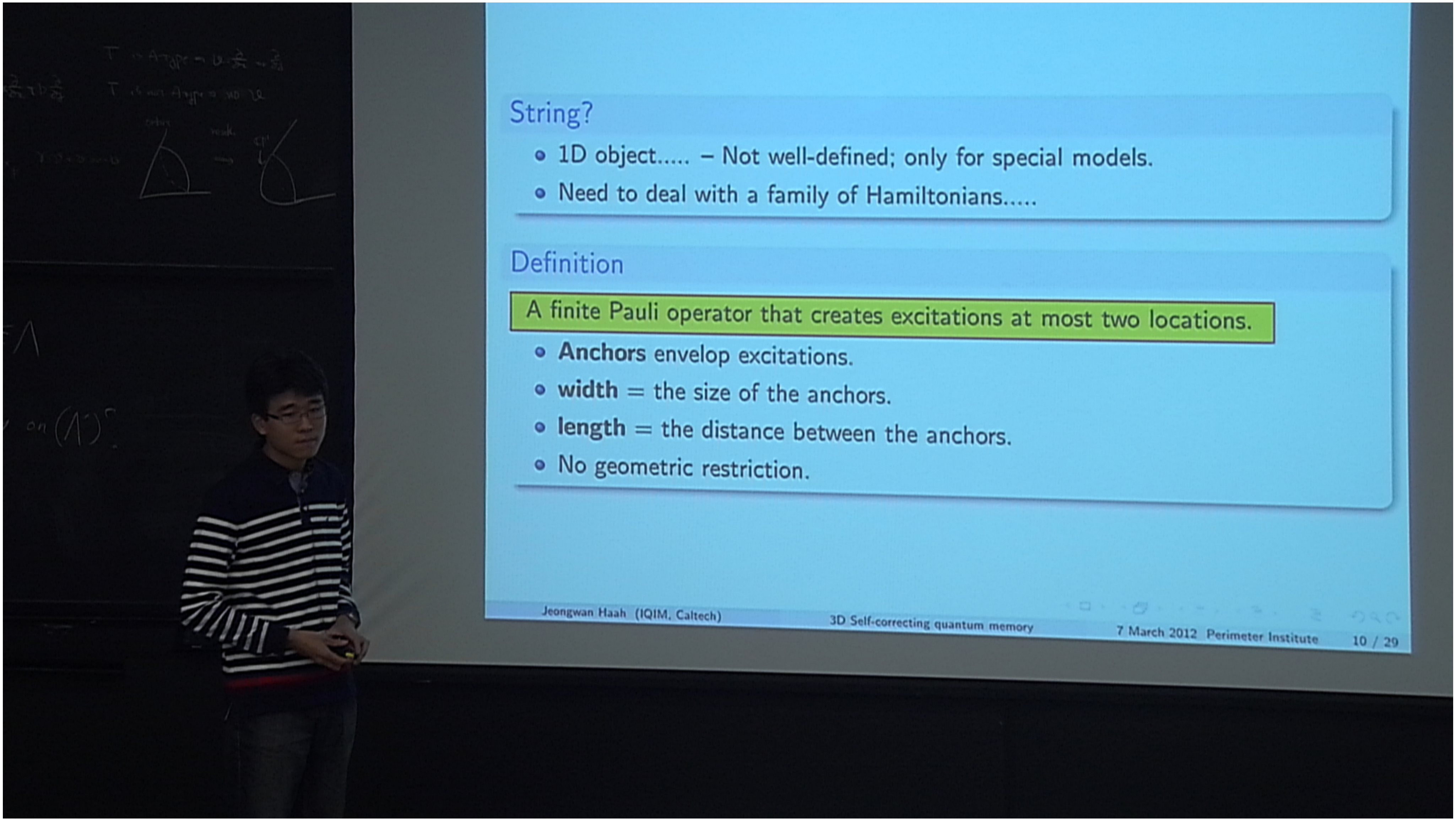
Definition

A finite Pauli operator that creates excitations at most two locations.

- **Anchors** envelop excitations.
- **width** = the size of the anchors.
- **length** = the distance between the anchors.

Trivial string segments

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Definition
A finite Pauli operator that creates excitations at most two locations.
Anchors envelop excitations.
width = the size of the anchors.
length = the distance between the anchors.
No geometric restriction.
Jeongwan Haah (IQIM, Caltech)
3D Self-correcting quantum memory
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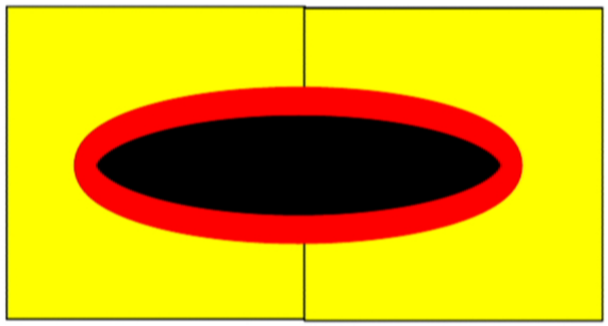
String segment: Example

In 2D toric code:

$\bullet$ $-Z-$ $-Z-$ $-Z-$ $-Z-$ $\bullet$

String segment: Example

In 2D Ising model: $H = -\sum_{\langle ij \rangle} Z_i Z_j$



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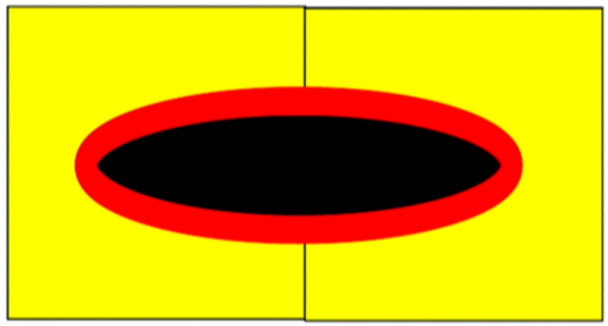
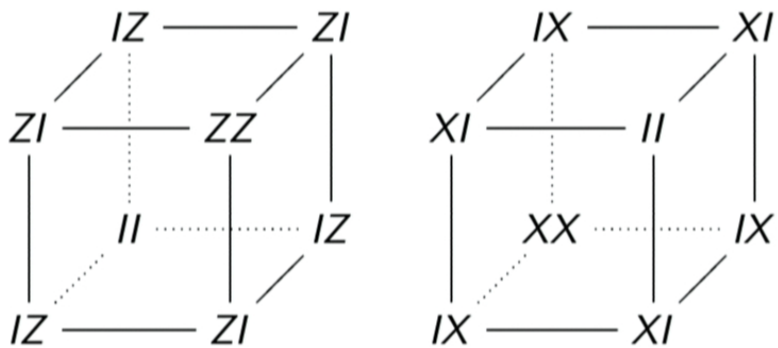


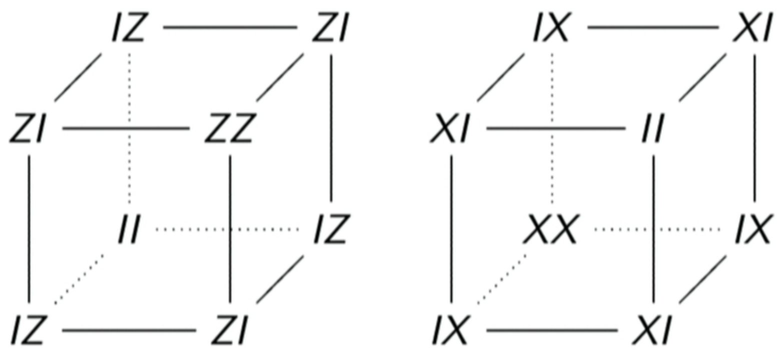
- Non-trivial X-type string segment.
- Two anchors must be adjacent.
- length $= 0$

No-strings rule



A string segment of width w and length $\geq 15w$ is trivial.

No-strings rule



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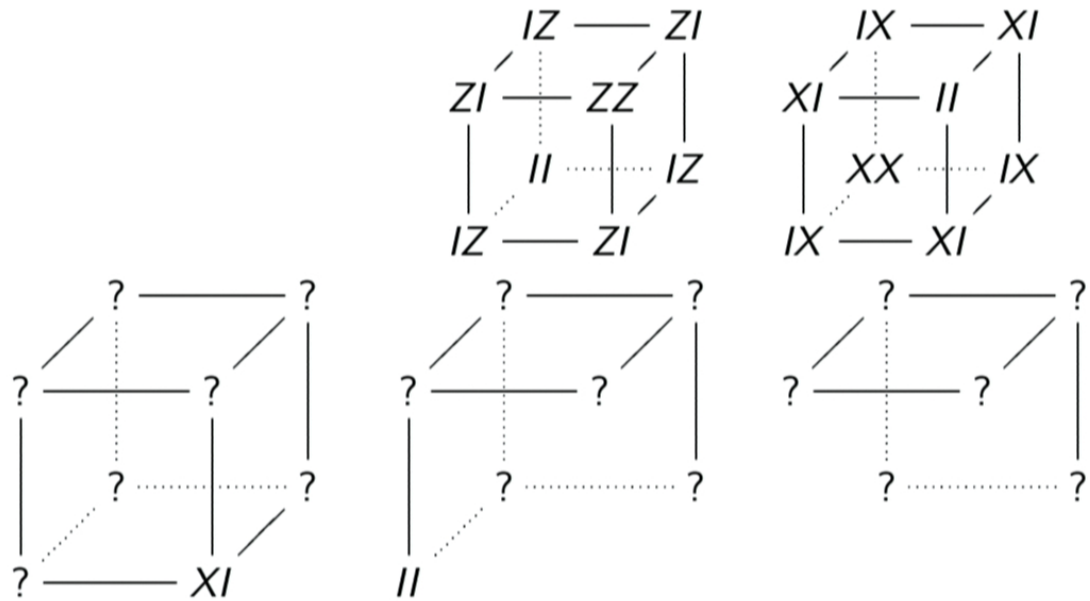
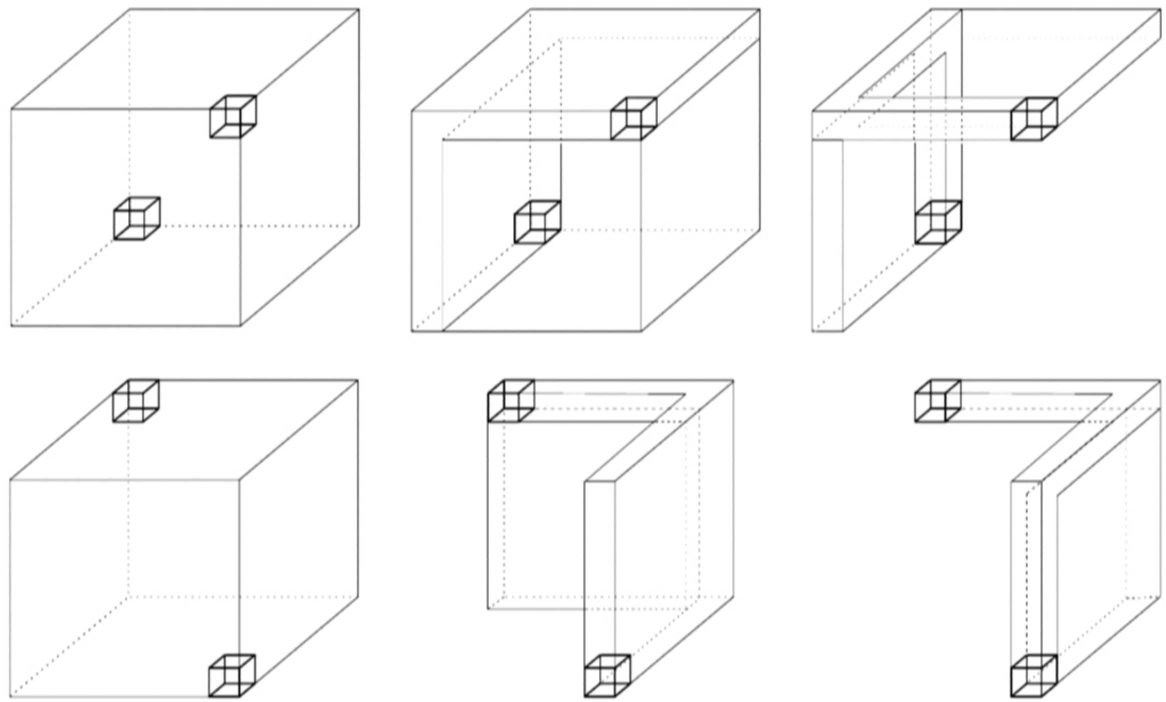
No-strings rule : Proof

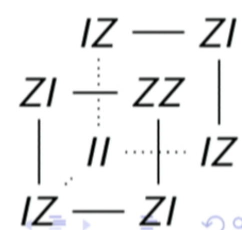
Recall the Eraser:

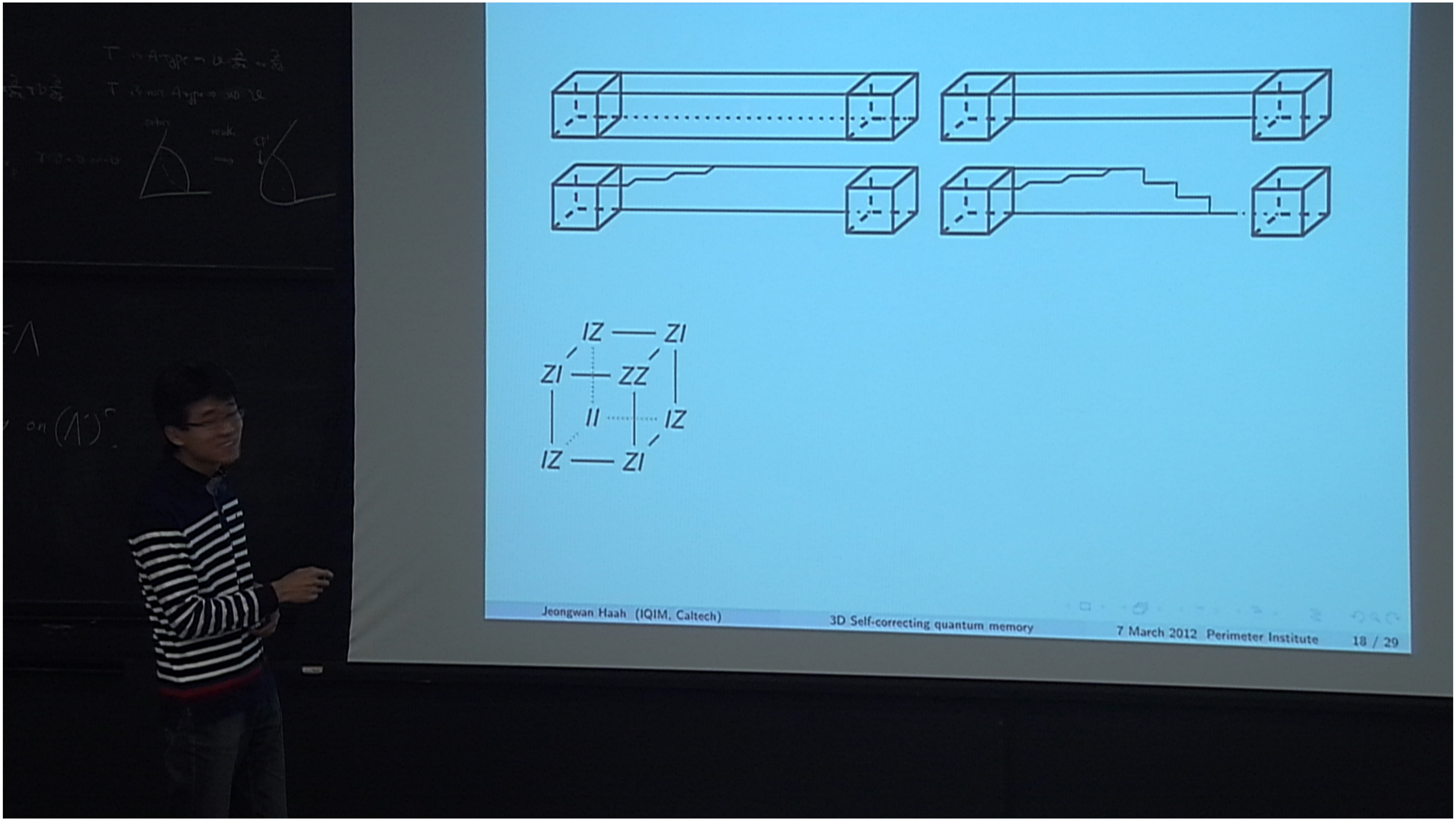


- IZ–ZI has two independent ends.

No-strings rule : Proof






IZ — ZI
ZI — ZZ
II
IZ
IZ — ZI
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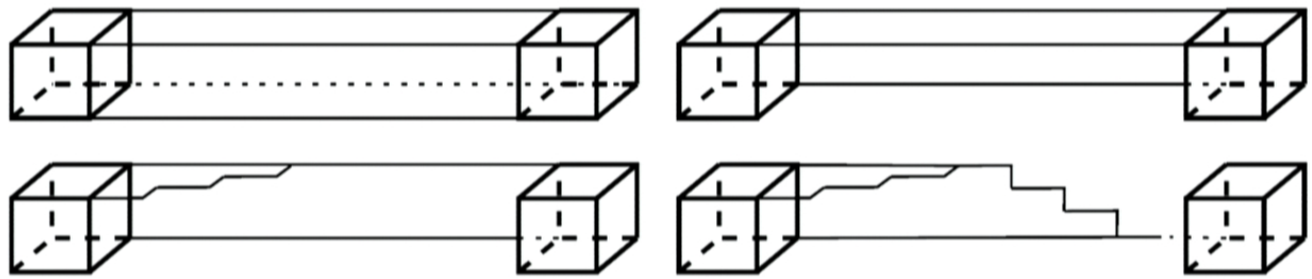
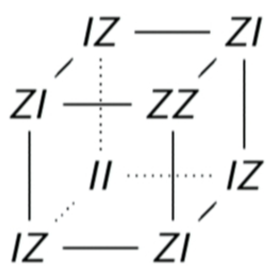
No-strings rule : Proof





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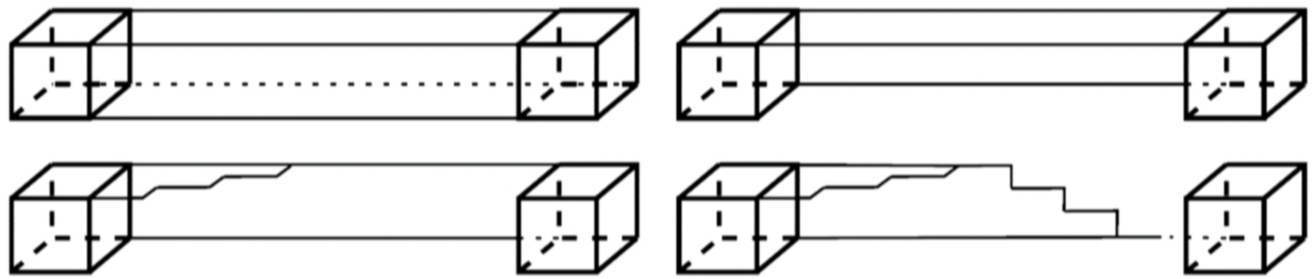

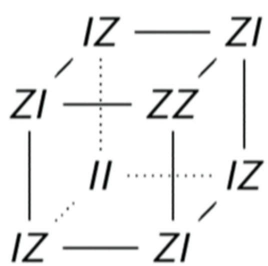

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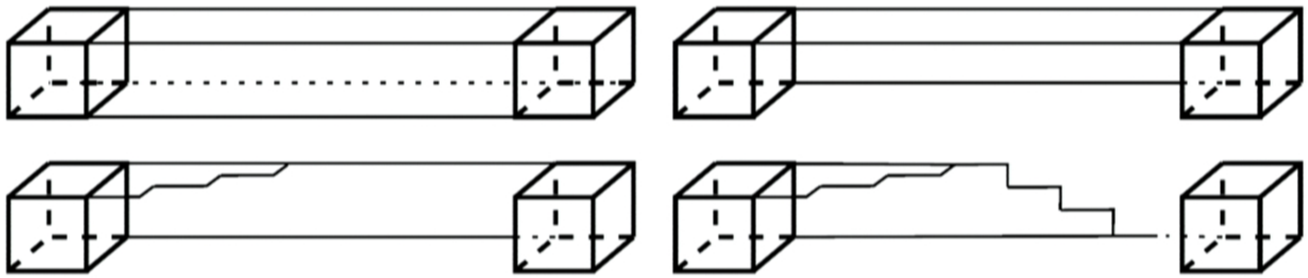
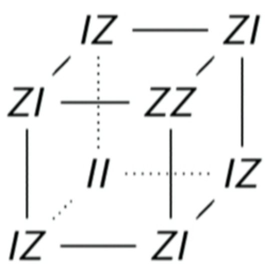
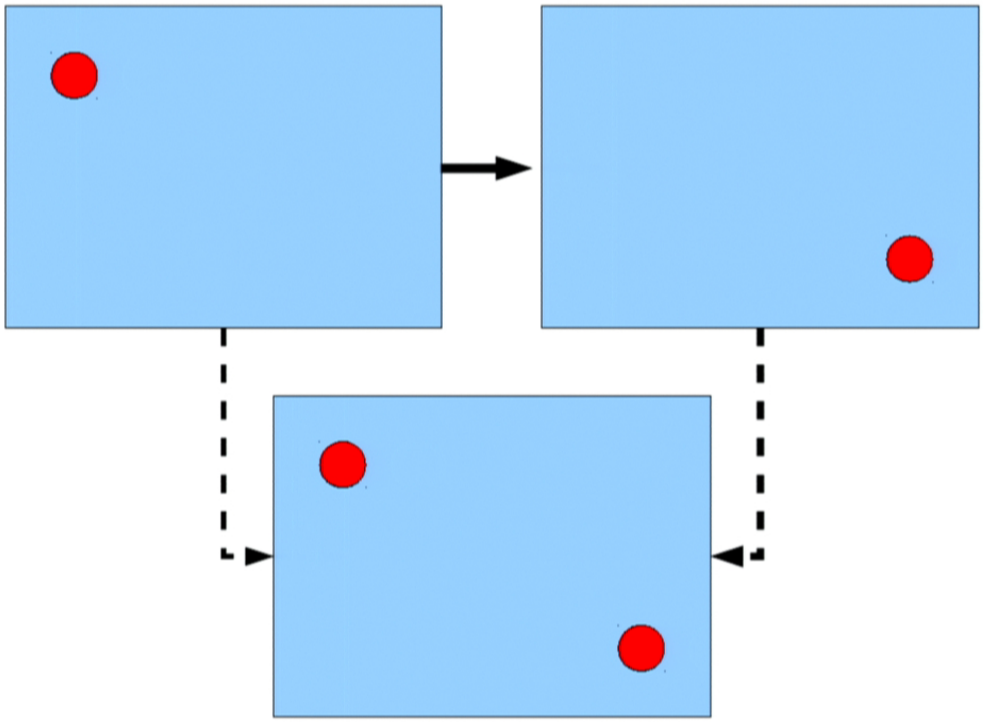




$$[O_1 - O_2, IZ - ZI] = 0$$
$$[O_1 - O_2, II - IZ] = 0 \iff O_1 - O_2 = \begin{cases} II - II \\ XI - II \\ IX - XI \\ XX - XI \end{cases}$$

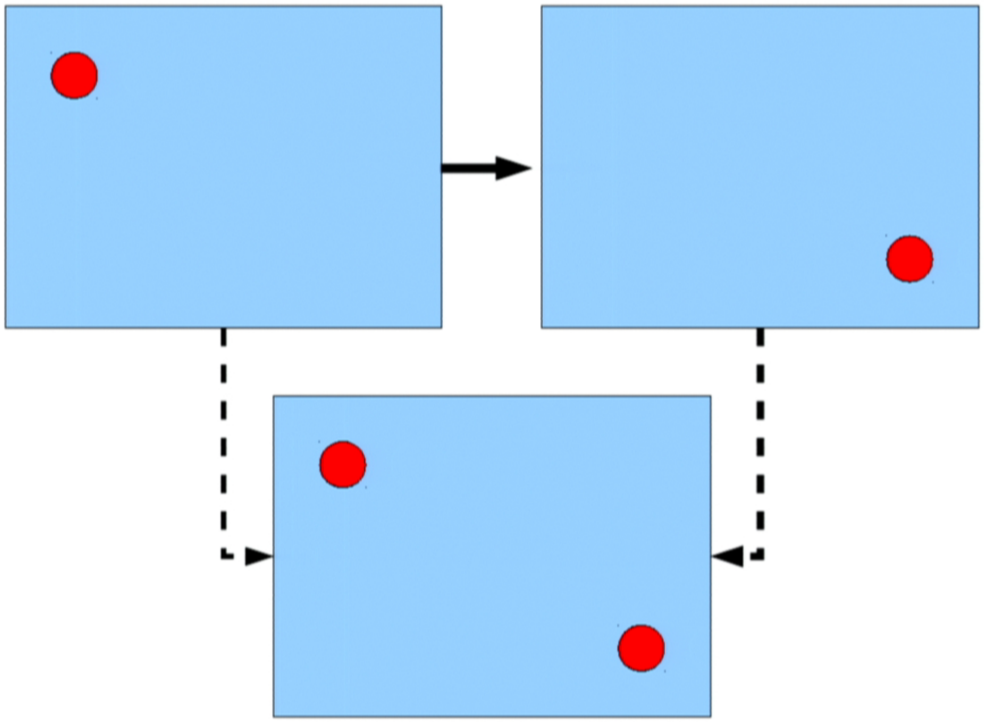
$$\begin{cases} II - II - II - \cdots \\ XX - XI - II - \cdots \\ IX - XI - II - \cdots \\ XI - II - II - \cdots \end{cases}$$

No-strings rule



- You can't drag the defect.
- Annihilate it, and then create it.

No-strings rule



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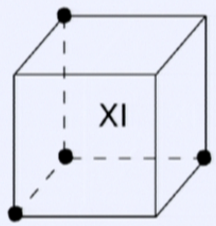
Energy Barrier[2]

Consider a Pauli walk creating an isolated defect.

Pyramid



[2] S. Bravyi, JH, Phys. Rev. Lett. 107,150504 (2011)

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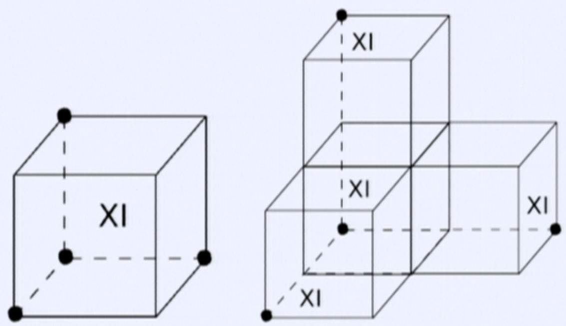

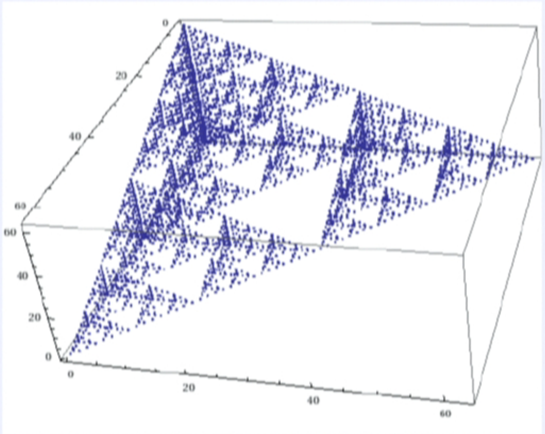

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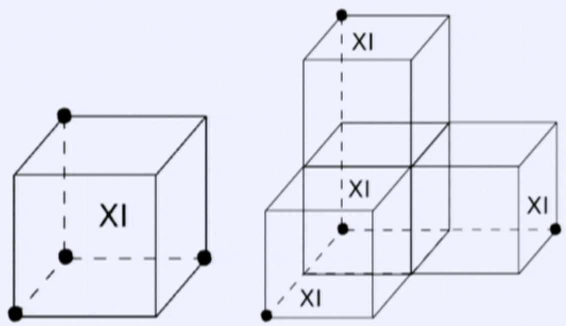

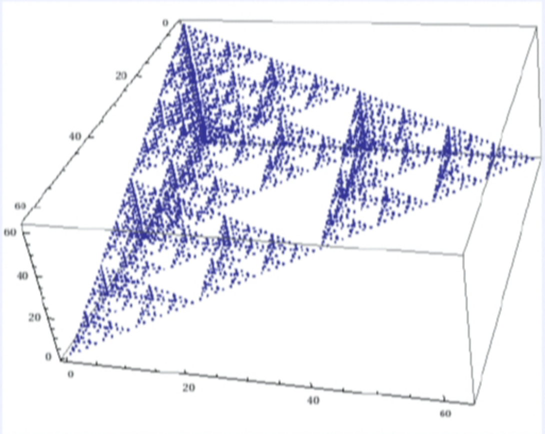

- Isolating a defect from the other by $R = 2^p$ at cost $E = 4p + 4 \simeq 4 \log_2 R$.

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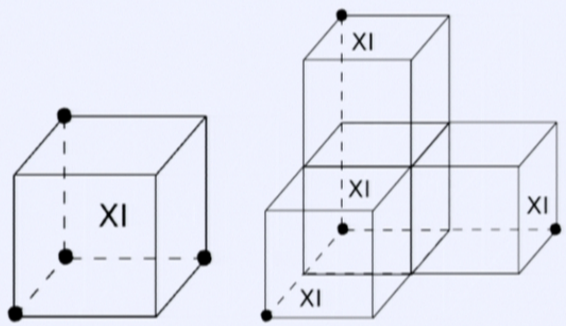

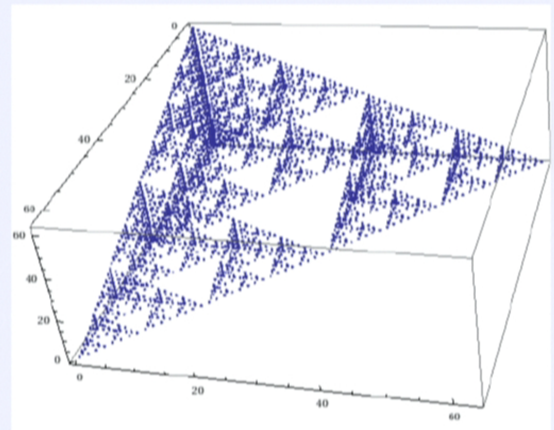

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Why logarithmic barrier to isolate a defect?

No-strings rule

Two charged clusters (overall neutral) of diameter r are at most αr apart.

implies:

Q) Coarsest cluster of defects?

Why logarithmic barrier to isolate a defect?

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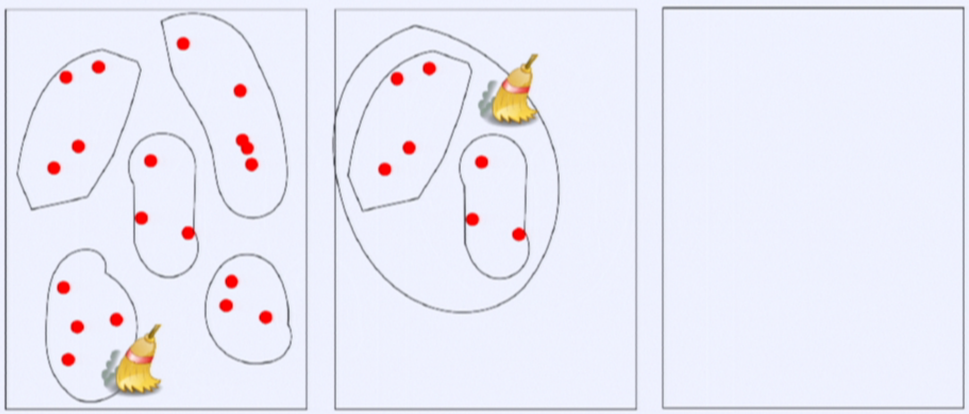
- How far can a defects move given a number of background defects? $R(n)$
- A defect may move α-away from another. $R(2) = \alpha$
- A defect may move $\alpha R(2)$-away from $R(2)$-sized cluster. $R(3) = \alpha^2$.
- $\Rightarrow \#(C) \sim \log R$.

Decoder[3] – Restoring the excited to the ground

Broom algorithm

Record and return the trajectory of the broom.



- Compute the r^p-connected components of the defects.
- Sweep the defects into one corner for each cluster.

[3] S. Bravyi, JH, arXiv:1112.3252

Failure Probability

Performance of the decoder

- The decoder succeeds, if the system never visits a state of energy $\geq \log L$.

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Probability

The system starts from a ground state.

- $\Pr[\text{System visits } E] = e^{-\beta E}$.
- $\Pr[\text{System visits } E \text{ during } t \text{ trials}] \leq te^{-\beta E}$
- $\Pr[\text{System visits } E \geq B \text{ during } t \text{ trials}] \leq \sum_{E \geq B} \binom{V}{E} te^{-\beta E}$.

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Memory time

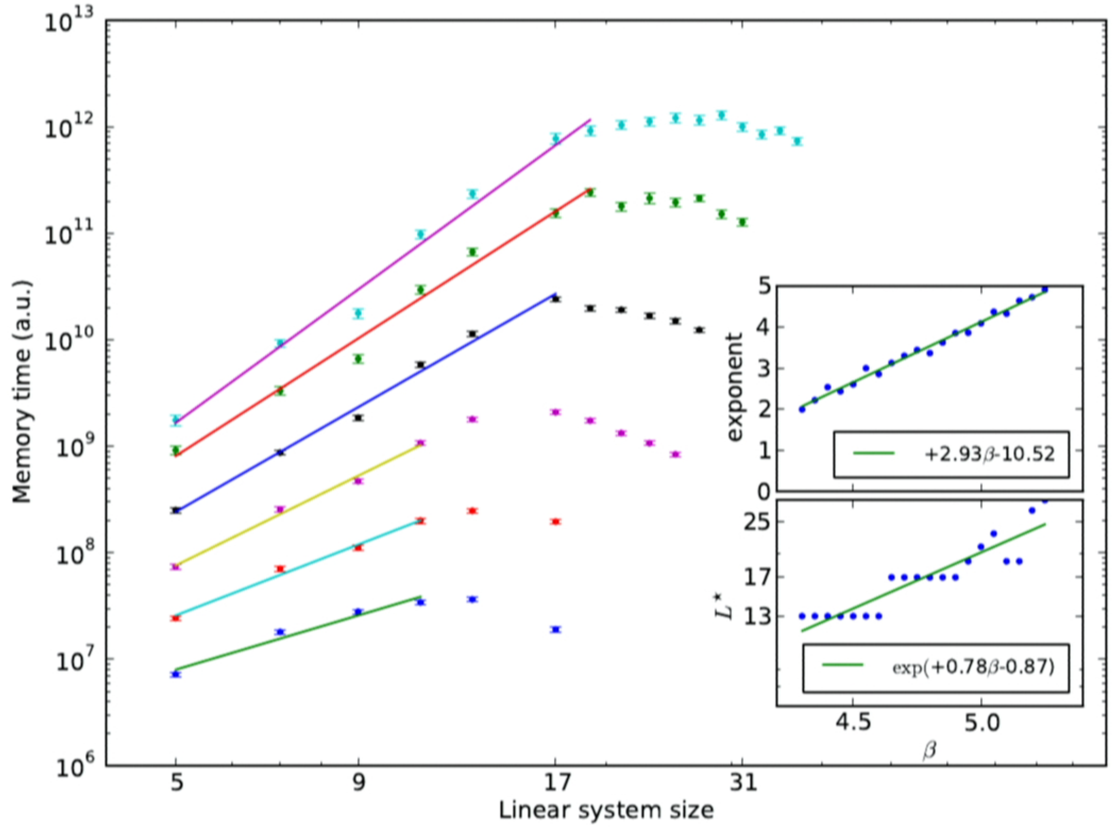
$$t_{mem} \geq \left(\sum_{E \geq c \log V} \binom{V}{E} e^{-\beta E} \right)^{-1} \geq \frac{e^{c\beta \log V}}{(1 + e^{-\beta})^V}$$

- Rigorous with respect to **local Markovian bath**.

Memory time[4]

For small $L \ll e^{\beta/3}$,

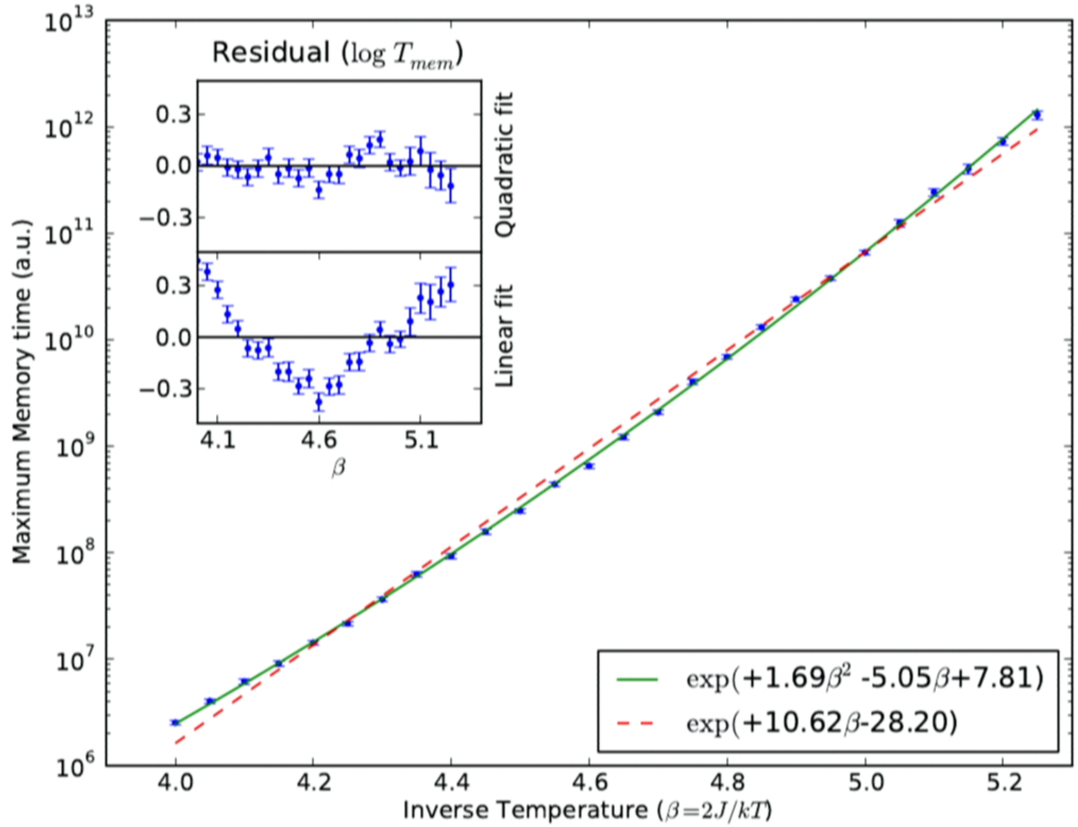
$$t_{mem} \geq t_0 L^{c\beta}$$



Memory time

If $L \sim e^{\beta/3}$,

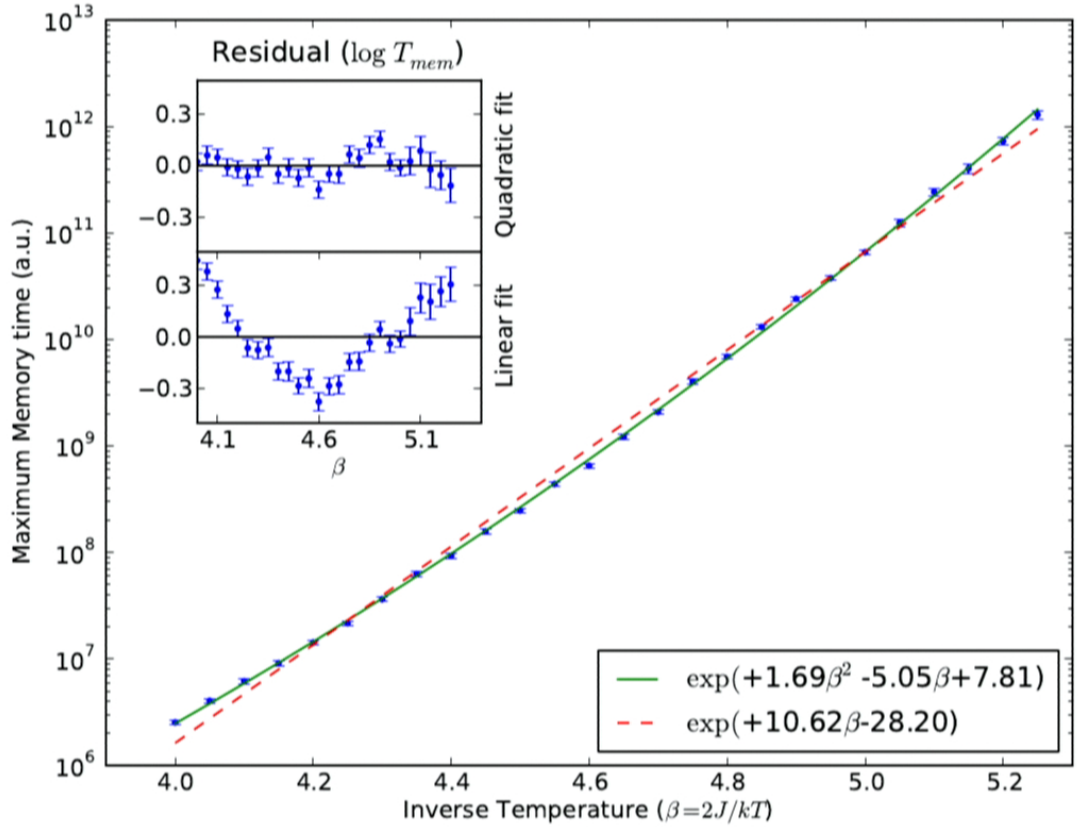
$$t_{mem} \geq t_0 e^{c'' \beta^2}$$



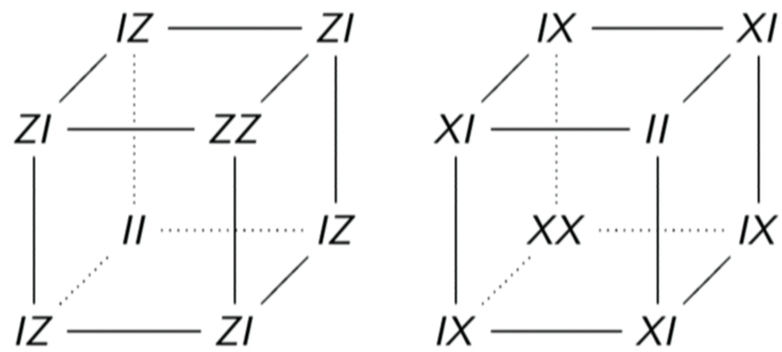
Memory time

If $L \sim e^{\beta/3}$,

$$t_{mem} \geq t_0 e^{c'' \beta^2}$$



Summary



Cubic code 1

- Locally indistinguishable ground states
- Logarithmic energy barrier to isolate a defect.
- Quantum glass without quenched disorder.

Weakly self-correcting quantum memory at $T > 0$

- $t_{mem} \sim L^{c\beta}$ when $L \leq e^{\beta J}$.
- Simple and efficient RG decoder.