Title: G-Bounce

Date: Mar 19, 2012 02:00 PM

URL: http://pirsa.org/12030103

Abstract: I will discuss a wide class of models which realise a bounce in a spatially flat Friedmann universe in standard General Relativity. The key ingredient is a noncanonical, minimally coupled scalar field
belonging to the class of theories with Kinetic Gravity Braiding/Galileon-like self-couplings. In these models, the universe smoothly volves from contraction to expansion, suffering neither from ghosts
nor gradient instabilities around the turning point. The end-point of he evolution can be a standard radiation-domination era or an nflationary phase.
The talk is based on arXiv:1109.1047v2 [hep-th], JCAP11(2011)021.

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

Alexander Vikman

(CERN)

19.03.2012

THIS TALK IS BASED ON

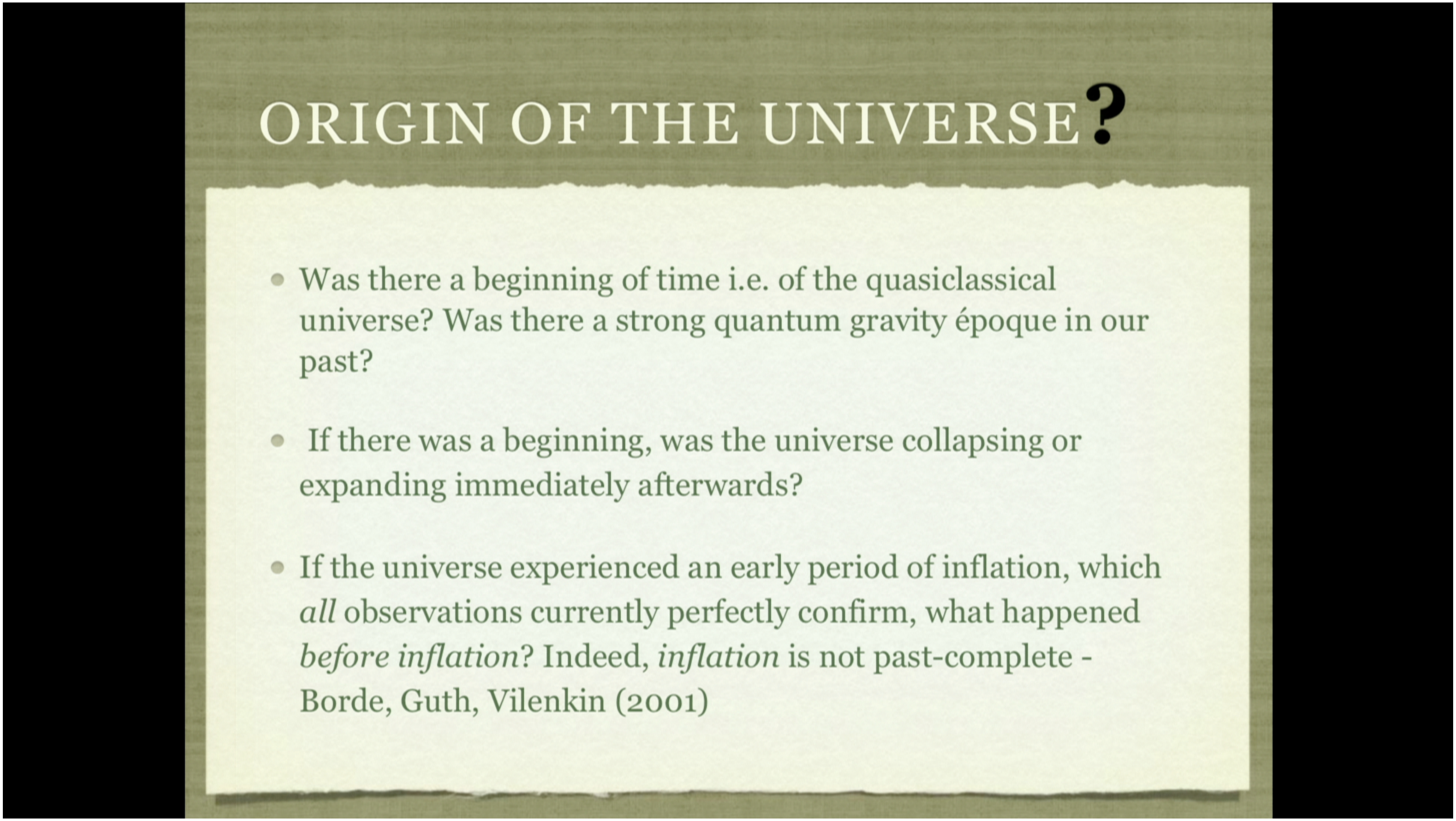
- ***Imperfect Dark Energy from Kinetic Gravity Braiding*** [1]
  arXiv:1008.0048, JCAP 1010:026, 2010
- ***The Imperfect Fluid behind Kinetic Gravity Braiding*** [2]
  arXiv:1103.5360, JHEP 1111 (2011) 156
- ***G-Bounce,*** [3] **arXiv:1109.1047,** JCAP 1111:021, 2011

IN COLLABORATION WITH

Cédric Deffayet[1]**, Damien Easson**[3]**,**
Oriol Pujolàs[1,2]**, Ignacy Sawicki**[1,2,3]

PLAN

- Motivation
- General class of models - perfect imperfect fluids
- G - Bounce
- Open problems


ORIGIN OF THE UNIVERSE?
Was there a beginning of time i.e. of the quasiclassical universe? Was there a strong quantum gravity époque in our past?
If there was a beginning, was the universe collapsing or expanding immediately afterwards?
If the universe experienced an early period of inflation, which all observations currently perfectly confirm, what happened before inflation? Indeed, inflation is not past-complete - Borde, Guth, Vilenkin (2001)

CAN ONE CONSTRUCT
A **STABLE**
(WITH REAL SOUND SPEED AND WITHOUT GHOSTS)
CLASSICAL MODEL
WHERE A
SPATIALLY FLAT
FRIEDMANN UNIVERSE
BOUNCES:
GOES FROM COLLAPSE TO EXPANSION **???**

OBSTACLE:

TO BOUNCE ONE HAS TO VIOLATE
NULL ENERGY CONDITION (NEC),
BUT NORMAL FIELDS AND PERFECT
FLUIDS CANNOT DO IT!

go to imperfect fluids and to even less canonical fields!

GO *G*!

G STANDS FOR GALILEON

Nicolis, Rattazzi , Trincherini, (2008)

In current literature: ***Galileons*** **=** scalar-tensor theories with

***higher derivatives* in the action**

but with **equations of motion** which are ***all*** of the

second order* - *NO *Ostrogradsky's* ***ghosts***

the most general theory of this type was derived

by *Horndeski (1974), and rederived by Deffayet, Gao, Steer, Zahariade (2011)*

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 *July* 1973

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$$\mathcal{L} = K + \mathcal{G}\phi^{;\mu}_{;\mu} +$$

$$+ \mathcal{G}_2 R + \mathcal{G}_2' \left[\left(\phi^{;\mu}_{;\mu}\right)^2 - \phi^{;\nu}_{;\mu}\phi^{;\mu}_{;\nu} \right] +$$

$$+ \mathcal{G}_3 G^{\mu}_{\nu} \phi^{;\nu}_{;\mu} - \frac{1}{6}\mathcal{G}_3' \left[\left(\phi^{;\mu}_{;\mu}\right)^3 - 3\phi^{;\lambda}_{;\lambda}\phi^{;\nu}_{;\mu}\phi^{;\mu}_{;\nu} + 2\phi^{;\mu}_{;\alpha}\phi^{;\alpha}_{;\beta}\phi^{;\beta}_{;\mu} \right]$$

Kobayashi, Yamaguchi, Yokoyama (2011)

where we have 4 free functions!

$K(\phi, X)$ and $\mathcal{G}_i(\phi, X)$

$$\mathcal{G}_i' = \frac{\partial \mathcal{G}_i}{\partial X}$$

$$X \equiv \tfrac{1}{2} g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

WHY IS THIS *G* INTERESTING?

One can **stably violate** the most basic of the classical energy conditions-Null Energy Condition (NEC): $T_{\mu\nu}n^{\mu}n^{\nu} \geq 0$ which in cosmology reduces to: $p + \varepsilon \geq 0$ or $\dot{H} \leq 0$

Pandora's box

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Pandora's box

- *Healthy* and *testable* Phantom Dark Energy,

  Deffayet, Pujolas, Sawicki, AV, 2010
- *Bouncing* Cosmology for a *spatially flat* Friedmann universe

  Creminelli, Nicolis, Trincherini 2010;
  Easson, Sawicki, AV;
  Taotao Qiu, Evslin, Cai, Li, Zhang 2011
- *Super*inflation with *blue* spectra of gravity waves

  Kobayashi, Yamaguchi, Yokoyama 2010

WHY IS THIS *G* STILL INTERESTING?

The scalar field can model ***imperfect superfluid*** and should not be necessarily considered more fundamental as a velocity potential or a "phase"

Pujolas, Sawicki, AV 2011

- New linear and nonlinear effects in cosmological perturbations - theoretical laboratory to test **DE** / Λ

Similarly to gravity, these theories are highly nonlinear and a UV completion they can admit should be by the ***classicalization*** or something more exotic - nonlocal, etc..

Dvali et al. 2010

SIMPLEST INTERESTING SUBSECTOR OF GALILEONS /HORNDESKI'S THEORIES -

Kinetic Gravity Braiding

kinetic mixing / braiding

$\partial\phi \quad \partial g$

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left[K(\phi, X) + G(\phi, X)\,\Box\phi\right]$$

Armendariz-Picon, Damour, Mukhanov, Steinhardt 1999/2000

Pujolàs, *Deffayet, Sawicki, AV*, 2010

where $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

Minimal coupling to gravity $S_{\mathrm{tot}} = S_\phi + S_{\mathrm{EH}}$

However, derivatives of the metric are coupled to the derivatives of the scalar, provided $G_X \neq 0$

No Galilean symmetry! ~~$\partial_\mu\phi \rightarrow \partial_\mu\phi + c_\mu$~~

EQUATION OF MOTION I

$$L^{\mu\nu}\nabla_\mu\nabla_\nu\phi + (\nabla_\alpha\nabla_\beta\phi)\,Q^{\alpha\beta\mu\nu}\,(\nabla_\mu\nabla_\nu\phi) +$$
$$+Z - G_X R^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = 0$$

Braiding

EOM is of the second order: $L_{\mu\nu}$, $Q^{\alpha\beta\mu\nu}$, Z

constructed from field and it's first derivatives

$Q^{\alpha\beta\mu\nu}$ **is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation, always *linear* in** $\ddot{\phi}$

SHOULD ϕ BE FUNDAMENTAL?

NO NOT AT ALL

ϕ CAN MODEL SOME HYDRODYNAMICS !

$K(X)$ for equation of state, $G(X)$: transport coefficient

EQUATION OF MOTION II

- **Shift-Charge Noether Current: - interpret as "particle" current** J_μ
- **New Equivalent Lagrangian:** $\mathcal{P}$ **pressure!**
- **Equation of motion is a "conservation law":**

$$\nabla_\mu J^\mu = \mathcal{P}_\phi$$

IMPERFECT FLUID FOR TIMELIKE GRADIENTS

- **Four velocity :** $u_\mu \equiv \dfrac{\nabla_\mu \phi}{\sqrt{2X}}$ → ϕ is an ***internal clock***
- **projector:** $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$
- **Time derivative:** $(\dot{\ }) \equiv \dfrac{\mathrm{d}}{\mathrm{d}\tau} \equiv u^\lambda \nabla_\lambda$
- **Expansion :** $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$

comoving volume

Shift-symmetry
$\phi \to \phi + c$
violates
$\phi \to -\phi$
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arrow of time

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EFFECTIVE MASS & CHEMICAL POTENTIAL

$$\kappa \equiv 2XG_X$$

- charge density: $n \equiv J^{\mu}u_{\mu} = n_0 + \kappa\theta$

"Braiding"

- energy density: $\mathcal{E} \equiv T^{\mu\nu}u_{\mu}u_{\nu} = \mathcal{E}_0 + \theta\dot{\phi}\kappa$
- effective mass per shift-charge / chemical potential:

$$m \equiv \left(\frac{\partial \mathcal{E}}{\partial n}\right)_{V,\phi} = \sqrt{2X} = \dot{\phi}$$

SHIFT-CURRENT AND "DIFFUSION"

"Diffusion"

$$J_\mu = n u_\mu - \frac{\kappa}{m} \perp_\mu^\lambda \nabla_\lambda m$$

§ 59, L&L, vol. 6

$$\kappa \equiv 2 X G_X$$

Is a "diffusivity"/ transport coefficient

Particle / charge current is not parallel to energy flow!

IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure** $\mathcal{P} = P_0 - \kappa \dot{m}$

- **Energy Flow**

No Heat Flux!

$$q_\mu = -\kappa \perp_\mu^\lambda \nabla_\lambda m = m \perp_\mu^\lambda J_\lambda$$

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2 u_{(\mu} q_{\nu)}$$

Solving for $\dot{m}$ *for small gradients or small* κ *one obtains "bulk viscosity" but change of frame results in a prefect fluid with vorticity up to* $\mathcal{O}\left(\kappa^2\right)$

DIFFUSION OF CHARGE?

For incompressible motion $\theta \equiv 0$
equation of motion is:

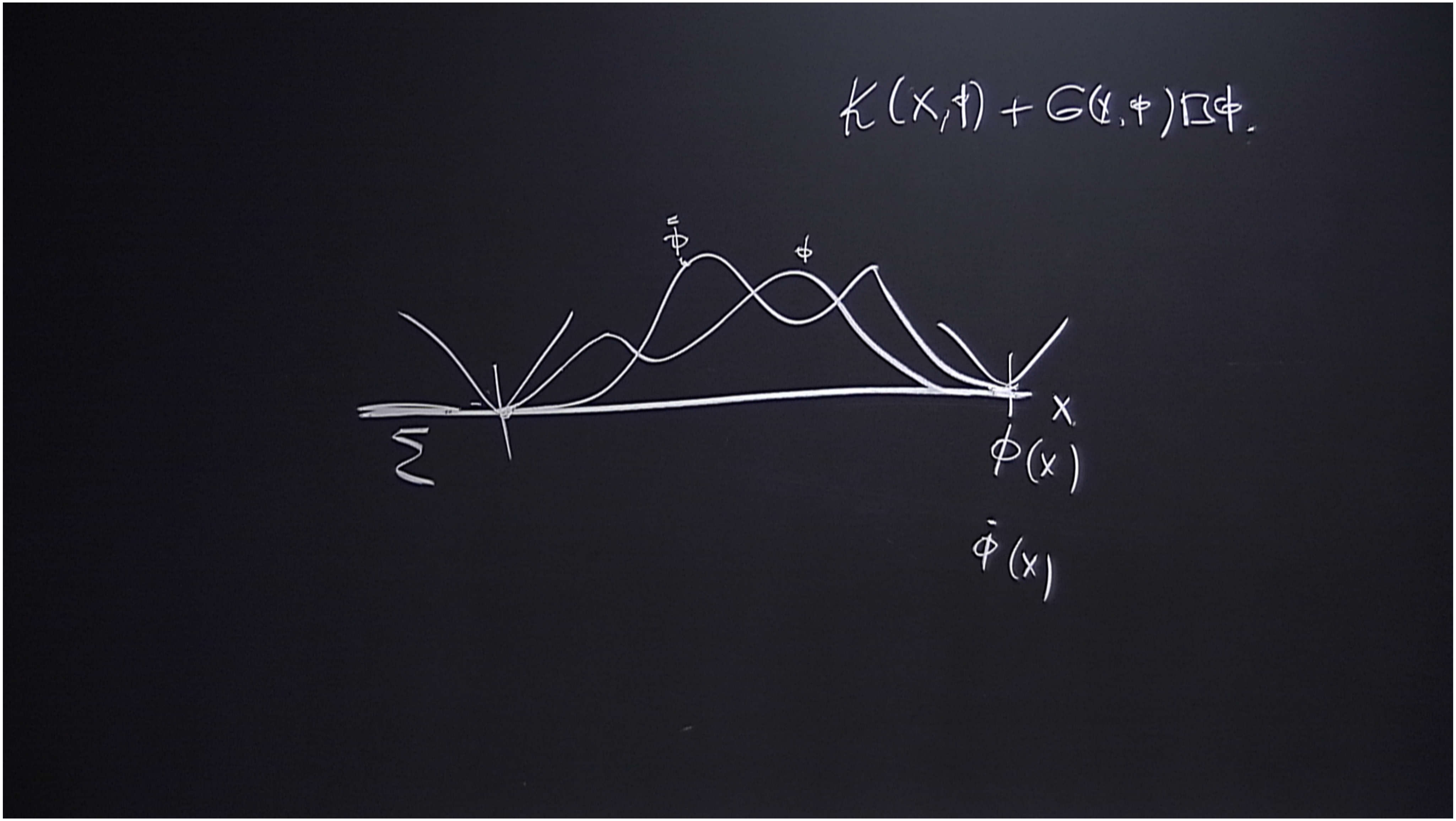
$$\dot{n} = -\overline{\boldsymbol{\nabla}}_\mu \left(\mathfrak{D} \overline{\boldsymbol{\nabla}}^\mu n \right) + \mathfrak{D} a^\mu \overline{\boldsymbol{\nabla}}_\mu n$$

where the diffusion constant: $\mathfrak{D} \equiv -\dfrac{\kappa}{n_m m}$

c.f. § *59, L&L, vol. 6, p 232*

4-acceleration: $a^\mu \equiv \dot{u}^\mu$

spatial gradient: $\overline{\boldsymbol{\nabla}}_\mu \equiv \perp^\nu_\mu \nabla_\nu$


K(x,φ) + G(x,φ)□φ.
φ̄
φ
Σ
x
φ(x)
φ̇(x)

ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation: $u_\nu \nabla_\mu T^{\mu\nu} = 0$

$$\mathrm{d}E = -\mathcal{P}\mathrm{d}V + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

Euler relation: $\mathcal{E} = mn - P_0$

Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$

COSMOLOGY

$$q_\mu = 0 \quad \textbf{and} \quad \theta = 3H$$

Friedmann Equation:

$$H^2 = \kappa m H + \frac{1}{3}\left(\mathcal{E}_0 + \rho_{\rm ext}\right)$$

$$r_{\rm c}^{-1} = \kappa m$$ **"crossover" scale in DGP**

EQUATION OF MOTION IN COSMOLOGY

(CHARGE CONSERVATION)

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

If there is shift-symmetry then

$$\mathcal{P}_\phi = 0$$

$$\Downarrow$$

$$n \propto a^{-3}$$

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$$n \propto a^{-3}$$

ACTION FOR THE COSMOLOGICAL PERTURBATIONS

$$S_2 = \int \mathrm{d}^3x\,\mathrm{d}t\,A\left[\dot{\zeta}^2 - \frac{c_\mathrm{s}^2}{a^2}\left(\partial_i\zeta\right)^2\right]$$

where $$A = \frac{Xa^3}{(H - m\kappa/2)^2} D$$

$$D = \frac{\mathcal{E}_m - 3H\kappa}{m} + \frac{3}{2}\kappa^2$$

Controls “ghosts” $D > 0$ No ghosts!

SOUND SPEED

$$c_\mathrm{s}^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa\left(4H - \kappa m/2\right)}{\mathcal{E}_m - 3\kappa\left(H - \kappa m/2\right)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$

The relation between the equation of state, the sound speed and the presence of ghosts is very different from the k-*essence* & perfect fluid.

A manifestly stable *Phantom* ($w_X < -1$) is possible even with a *single* degree of freedom and *minimal* coupling to gravity

ACTION FOR THE COSMOLOGICAL PERTURBATIONS

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G BOUNCE IDEA I

- **Consider matter with constant equation of state - radiation, dust, spatial curvature etc...** $p_{\text{ext}} = w\rho_{\text{ext}}$ **with** $w = \text{const}$
- **Shift-symmetric Lagrangian for the scalar field** $K(\cancel{\phi}, X)$ **and** $G(\cancel{\phi}, X)$
- **Phase space is two dimensional** (m, ρ_{ext})
- **Go to new coordinates** (m, H) **by solving the Friedmann equation**
- **Integral of motion 1st integral**

$$I(m, H) = \frac{n^{1+w}}{\rho_{\text{ext}}}$$

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G BOUNCE IDEA II

- **In new coordinates** (m, H) **at** $H = 0$ **pose conditions**

$$\rho_{\rm ext} > 0,\ D > 0,\ \dot{H} > 0,\ c_{\rm s}^2 > 0$$

- **These conditions are on the range of chemical potential** Δm **and on the equation of state** $K(m)$ **along with the transport coefficient** $\kappa(m)$ **or on** $K(X)$ **and** $G(X)$

⇓

"Healthy" Bounce!

IS IT POSSIBLE TO SATISFY ALL THESE CONDITIONS ?


YES WE CAN
We Can Do It!
WAR PRODUCTION CO-ORDINATING COMMITTEE

SIMPLE HIERARCHY

if $K(m)$ and $\kappa(m)$ satisfy the hierarchy:

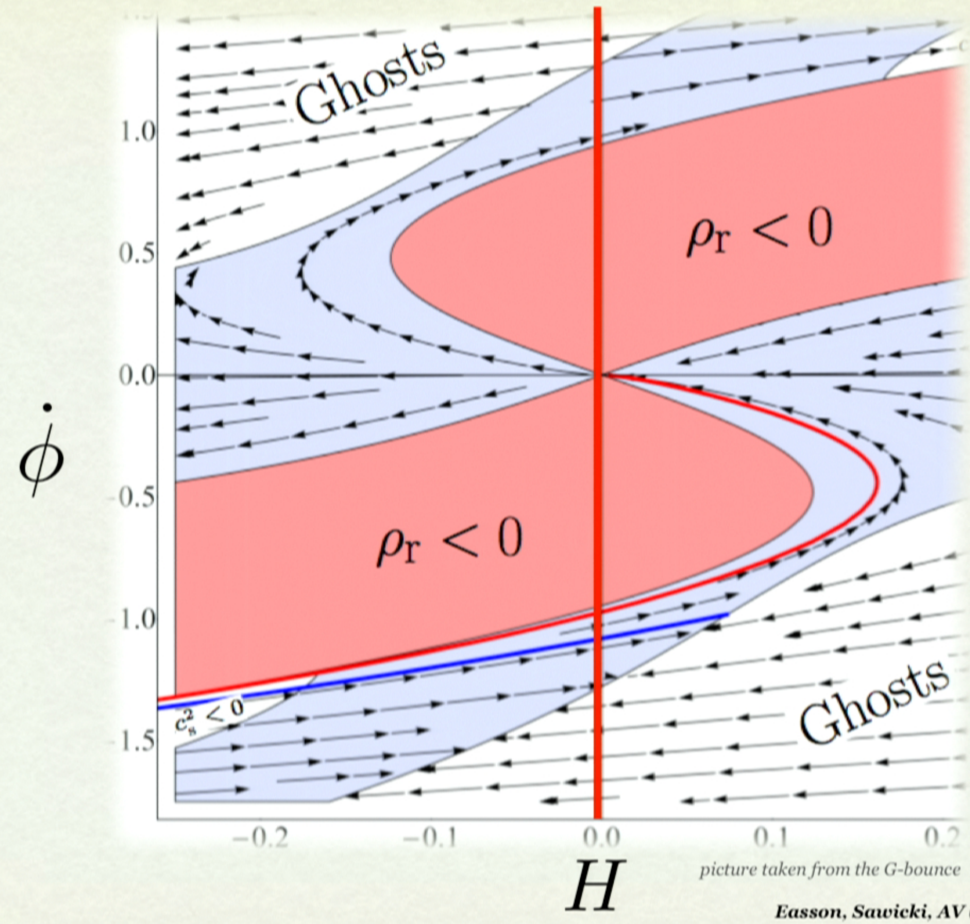
$$K > mK_m > \frac{1}{2}(m\kappa)^2 > 0 > \frac{1}{2}m^2 K_{mm} > -\frac{3}{4}(m\kappa)^2$$

does not depend on external equation of state w !

then at $H = 0$:

$$\rho_{\rm ext} > 0,\ D > 0,\ \dot{H} > 0,\ c_{\rm s}^2 > 0$$

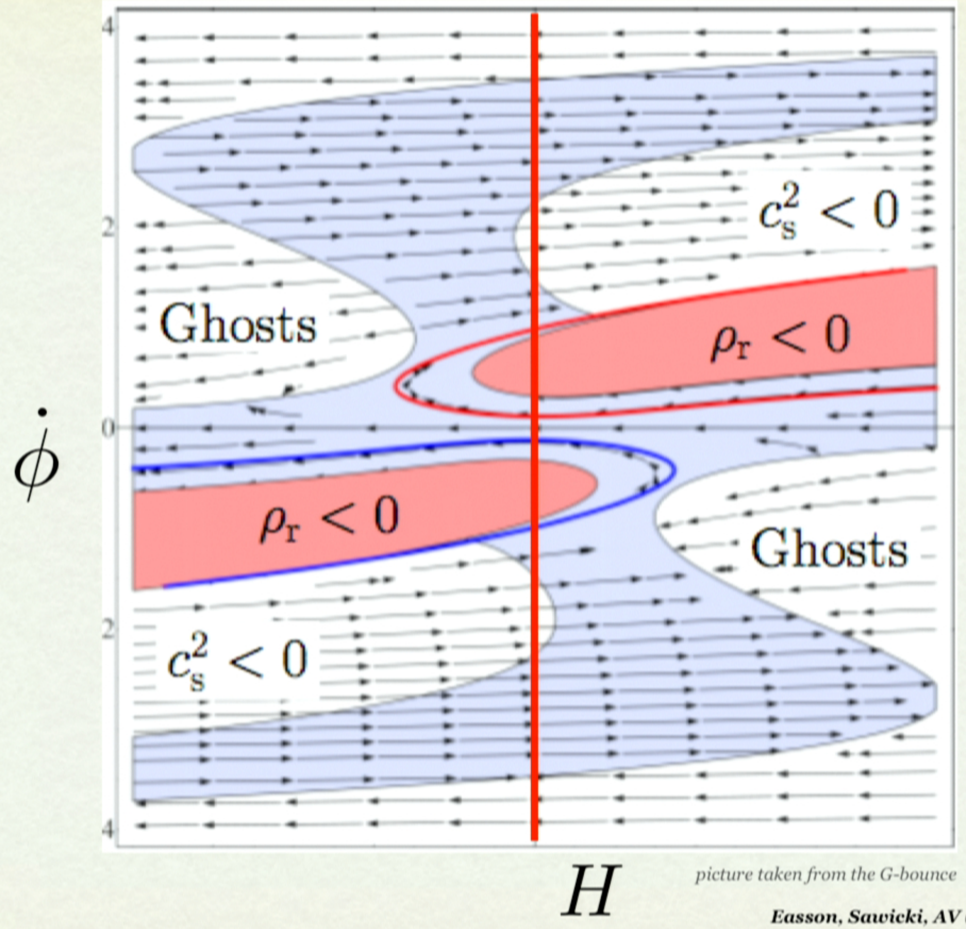
"Healthy" Bounce!



picture taken from the G-bounce

Easson, Sawicki, AV (2011)

G-Bounce: $\mathcal{L} = X - \alpha X^3 + \varkappa X \Box \phi$ **+ Radiation** ρ_r



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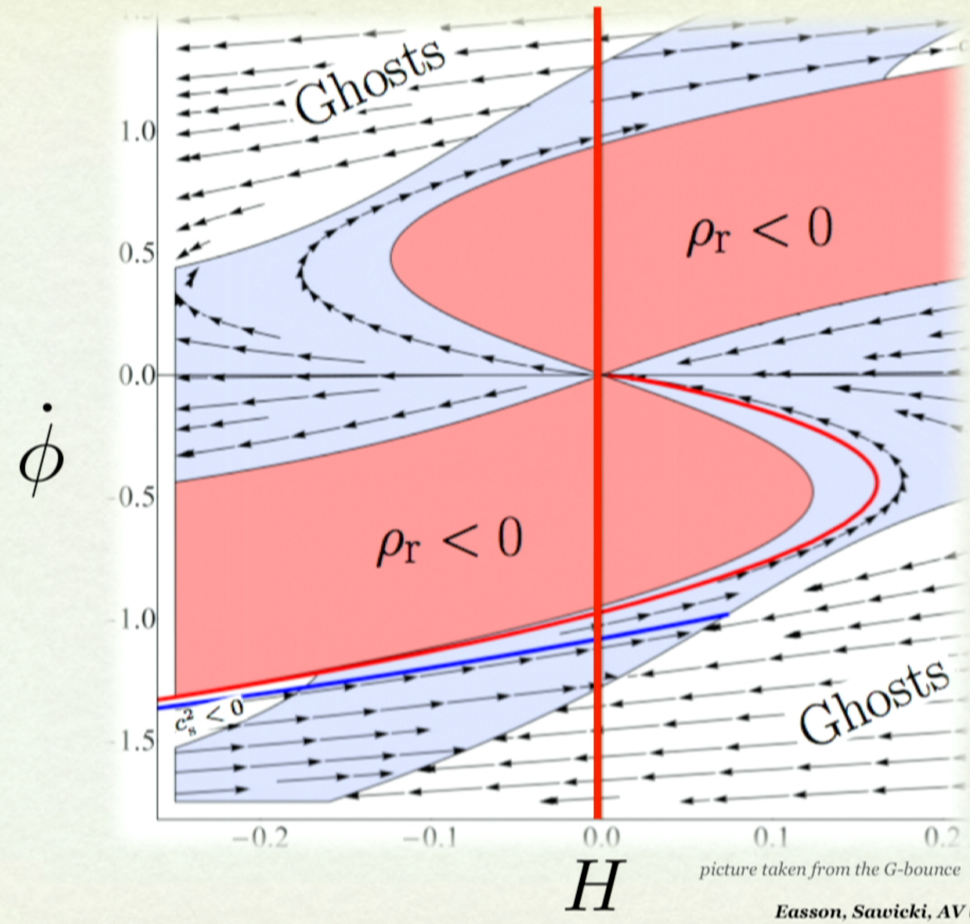
$\kappa = 2\beta X + \gamma X^2 \quad K = -\Lambda + X - X^3$ **+ Radiation** ρ_r



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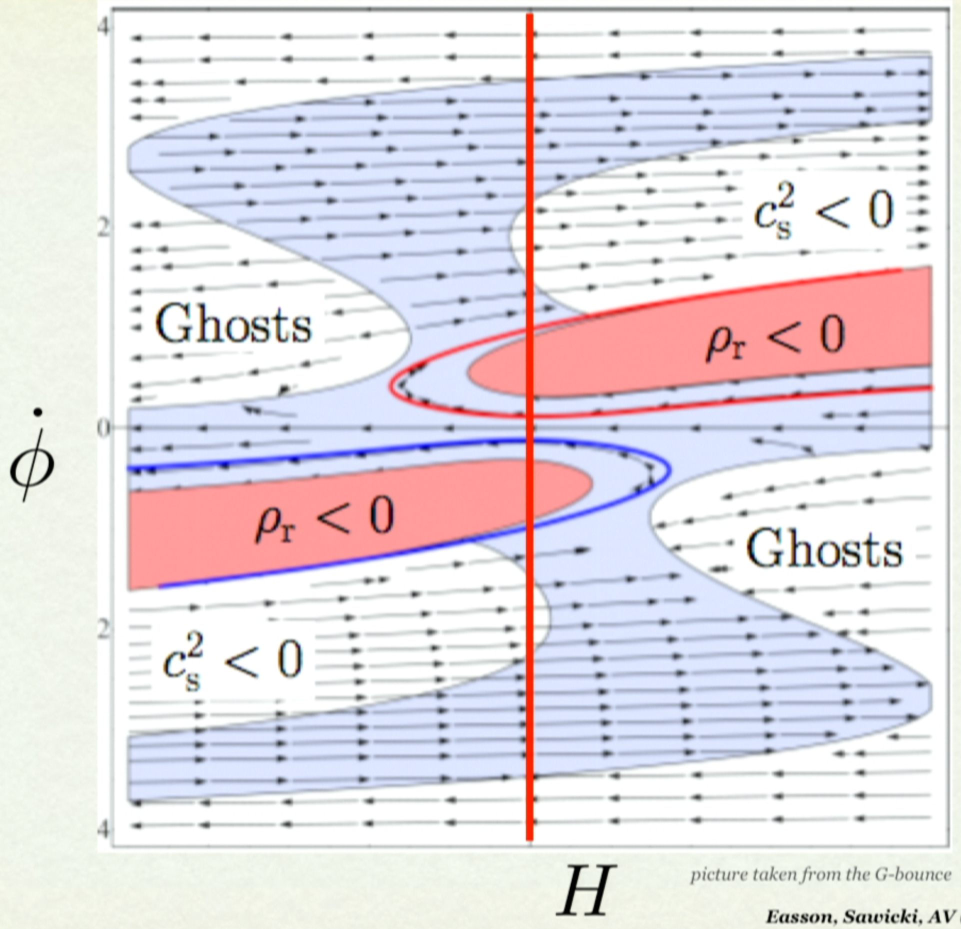

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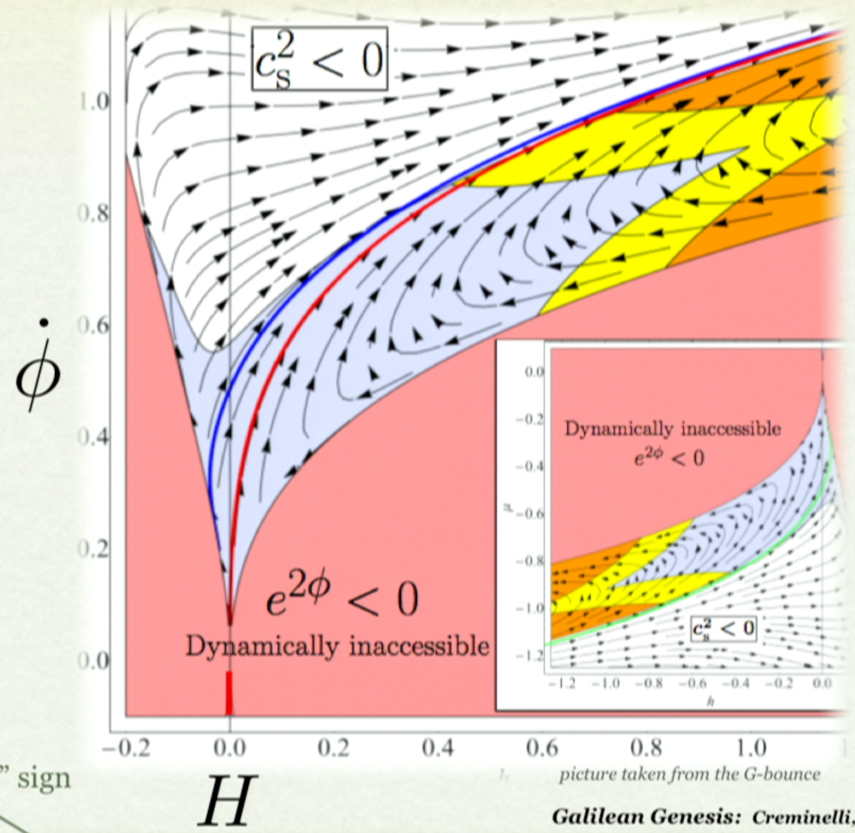

wrong “ghosty” sign

picture taken from the G-bounce

Galilean Genesis: ***Creminelli, Nicolis, Trincherini (2010)***

“Conformal Galileon”:

$$\mathcal{L} = -2f^2 e^{2\phi} X + \frac{2f^3}{\Lambda^3} X^2 + \frac{2f^3}{\Lambda^3} X \Box \phi$$

where: $X \equiv \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$ and Λ, f are constants

OPEN QUESTIONS

- Strong coupling?
- Possible anisotropy? Too strong tachyonic / Jeans instabilities?
- Can one arrange a cyclic i.e. oscillating evolution?
- Can one avoid all singularities and troubles for the past?
- Perturbations?
- Any *realistic* scenarios? Smooth transition to standard cosmology?

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THANKS A LOT FOR YOUR ATTENTION!