Title: Lifshitz Solutions in String Theory

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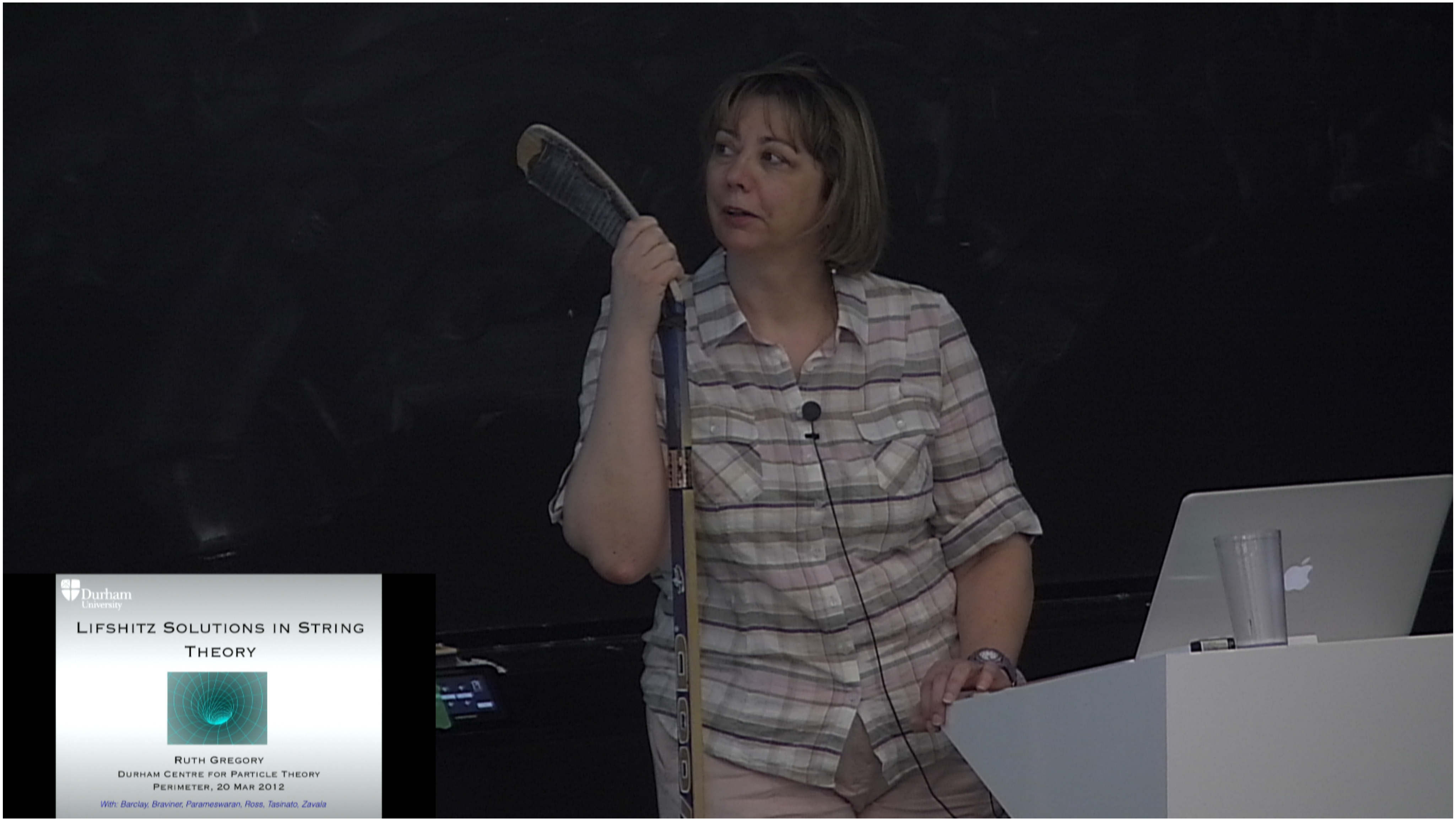
Abstract: I'll discuss solutions with Lifshitz scaling within string supergravity for an arbitrary scaling component z. After showing how to get exact Lifshitz spacetimes, I'll then look at more general solutions, including black holes and flows between Lifshitz and adS spacetimes.


Durham
University
LIFSHITZ SOLUTIONS IN STRING THEORY
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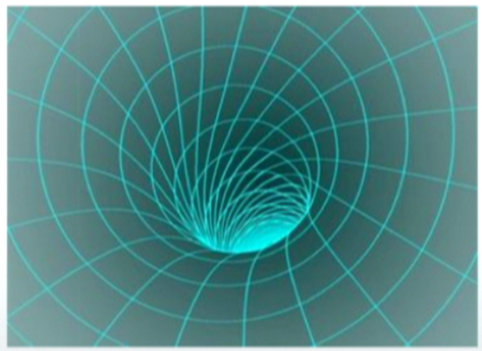
DURHAM CENTRE FOR PARTICLE THEORY
PERIMETER, 20 MAR 2012
With: Barclay, Braviner, Parameswaran, Ross, Tasinato, Zavala

Durham University

Lifshitz Solutions in String Theory



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OUTLINE

- Lifshitz spaces
- Embedding Lifshitz into string theory
- Panorama of solutions: flows
- Black hole solutions
- Thermodynamics
- Summary

BACKGROUND

By its nature, the AdS/CFT correspondence concerns scale invariant systems, but often we want to study more general systems.

An interesting scaling is Lifshitz, in which there is a dynamical exponent:

$$t \to \lambda^z t \quad , \quad x \to \lambda x \quad , \quad r \to r/\lambda$$

Can we have such spacetimes within string theory?

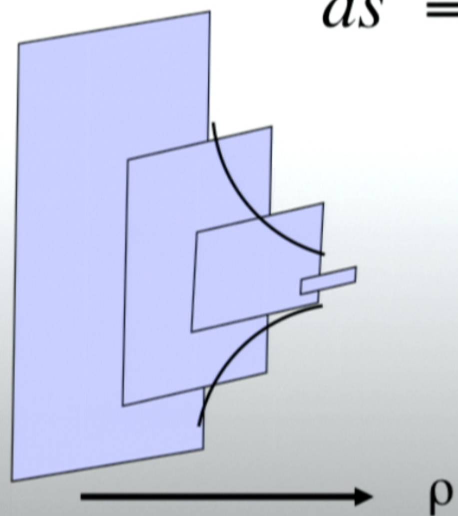
Can we build black holes?

Lifshitz Spacetime

The Lifshitz spacetime is an anisotropic generalization of ads, where time and space warp differently across the bulk:

$$ds^2 = e^{-2\rho}\,[e^{-2(z-1)\rho}\,dt^2 - d\underline{x}^2] - d\rho^2$$

(Poincare-style coords)



Requires background matter

$$R_t^t = z(z+2) \quad , \quad R_x^x = z+2 \quad , \quad R_\rho^\rho = z^2 + 2$$

Bottom up models

First achieved in 4D by having an empirical model with coupled 1 and 2-form gauge fields. *(Kachru, Liu, Mulligan)* Dual to a massive vector theory:

$$L = -R - 2\Lambda - \frac{1}{4}F^2 + \frac{m^2}{2}A^2$$

Where the scaling is supported by a massive vector flux, with very specific values for **q, m,** Λ

$$ds^2 = r^{2z}dt^2 - \frac{dr^2}{r^2} - r^2 dx^2$$

$$A = qr^z dt$$

$$m^2 = (D-2)\frac{z}{L^2}$$

$$\Lambda = -\frac{1}{2L^2}\left(z^2 + (D-3)z + (D-2)^2\right)$$

$$q = L\sqrt{\frac{2(z-1)}{z}}$$

- Bottom up models very useful to explore possibilities with Lifshitz scaling, e.g. black holes & superconductors.

- The Lifshitz scaling as r -> 0 is problematic.

- In spite of the field content, no string theory model found with the specific values of m and Λ, although some models with specific z later found.

- Would like to embed generic Lifshitz in string theory, and explore the range of geometries possible. (Ideally analytically!)

Tops Down!

In spite of the apparently simple field content, string theory embeddings were not simple to find.

The key feature of the Lifshitz spacetime is the matter source which is anisotropic in space and time, and has a strong asymptotic presence.

For string motivated spacetimes, we will have to compactify in such a way as to preserve the Chern-Simons structure of the prototype model, but Λ can be replaced by a false vacuum.

Can achieve this via consistent truncations of IIA and IIB supergravity: 6D and 5D Romans SUGRA, together with a flux compactification on H_2.

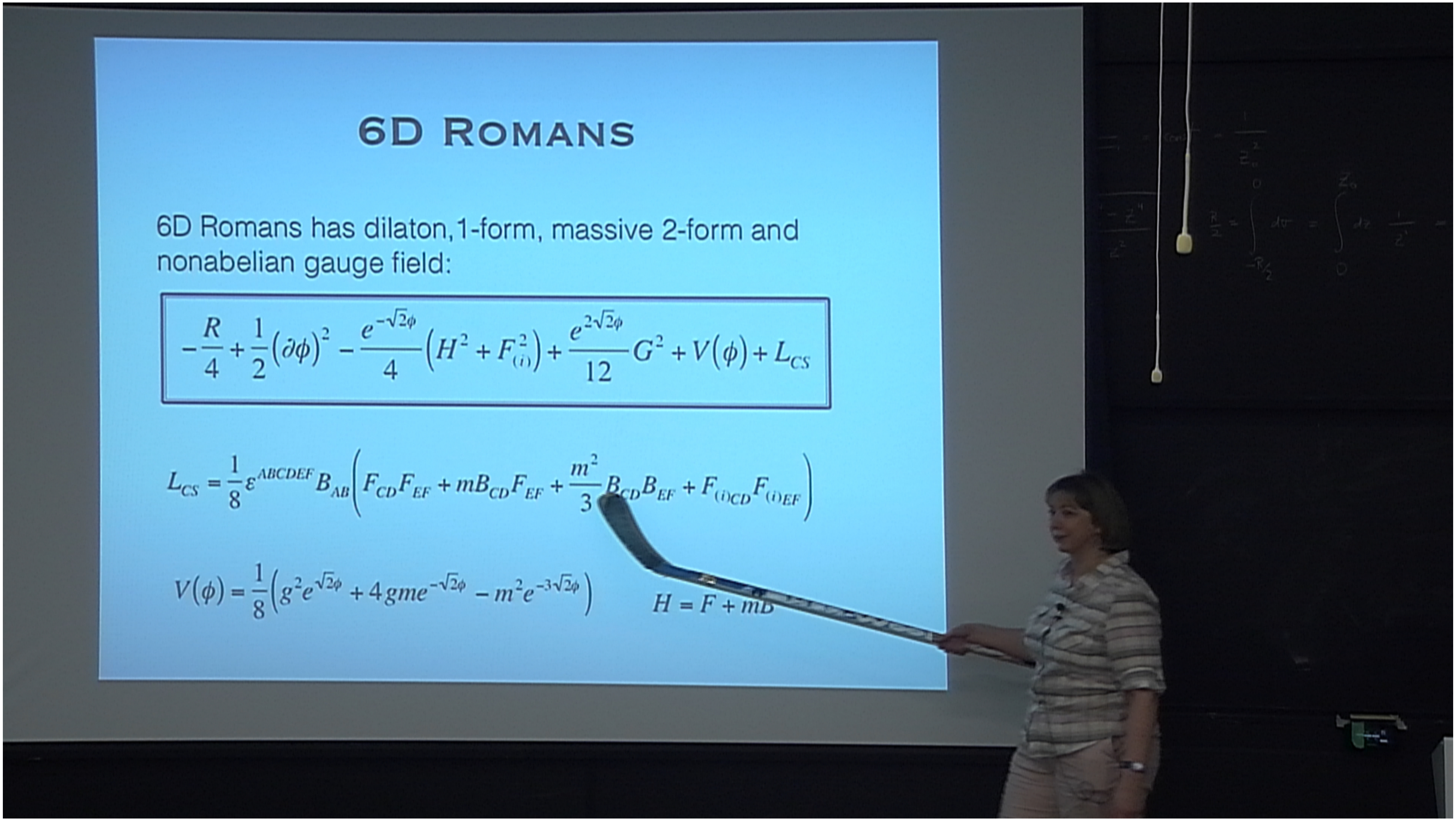
6D Romans

6D Romans has dilaton, 1-form, massive 2-form and nonabelian gauge field:

$$-\frac{R}{4}+\frac{1}{2}(\partial\phi)^2-\frac{e^{-\sqrt{2}\phi}}{4}\left(H^2+F_{(i)}^2\right)+\frac{e^{2\sqrt{2}\phi}}{12}G^2+V(\phi)+L_{CS}$$

$$L_{CS}=\frac{1}{8}\varepsilon^{ABCDEF}B_{AB}\left(F_{CD}F_{EF}+mB_{CD}F_{EF}+\frac{m^2}{3}B_{CD}B_{EF}+F_{(i)CD}F_{(i)EF}\right)$$

$$V(\phi)=\frac{1}{8}\left(g^2e^{\sqrt{2}\phi}+4gme^{-\sqrt{2}\phi}-m^2e^{-3\sqrt{2}\phi}\right) \qquad H=F+mB$$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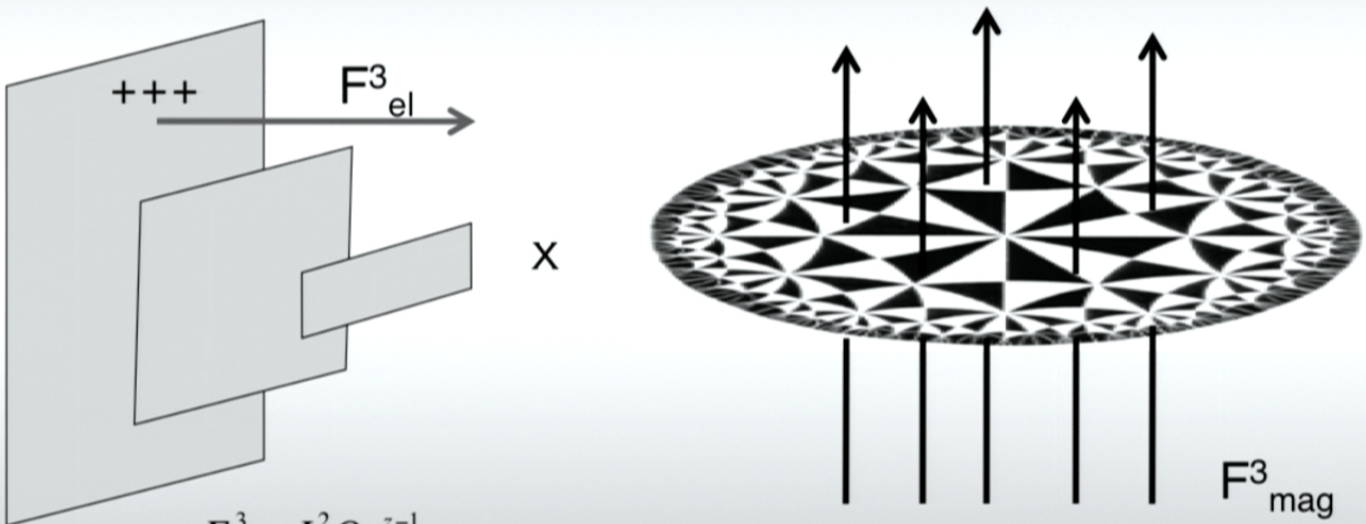
$H=F+mB$

THE GEOMETRY

Lifshitz solution has compact H^2 carrying flux. A1-parameter family of analytic solutions determined by z for ANY z.



$$F^3_{tr} = L^2 Q r^{z-1}$$

$$F^3_{y_1 y_2} = q\varepsilon_{12}$$

$$B_{x_1 x_2} = L^2 P r^2 / 2$$

$$ds^2 = L^2\left[r^{2z}dt^2 - r^2\left(dx_1^2 + dx_2^2\right) - \frac{dr^2}{r^2}\right] - a^2 dH_2^2$$

(RG, Parameswaran, Tasinato, Zavala: 1009.3445)

In more detail.....

With the assumption of flux threading the H_2, the gauge eqns reduce to just one function, which then gives the requisite structure for the "energy-momentum", and Einstein eqns become algebraic relations.

$$d * e^{-\sqrt{2}\phi} F^{(3)} = qG$$

$$d * e^{2\sqrt{2}\phi} G = m^2 e^{-\sqrt{2}\phi} * B + qF^{(3)}$$

$$\Rightarrow F^{(3)} = -qe^{\sqrt{2}\phi} * B$$

$$R^t_t = V + e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} + 6q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} + \frac{m^2}{2} B_{12} B^{12} e^{\sqrt{2}\phi}$$

$$R^r_r = V - e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} + 6q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} + \frac{m^2}{2} B_{12} B^{12} e^{\sqrt{2}\phi}$$

$$R^1_1 = V - e^{2\sqrt{2}\phi} G_{r12} G^{r12} + \frac{q^2}{2a^4} e^{-\sqrt{2}\phi} - 2q^2 B_{12} B^{12} \frac{e^{\sqrt{2}\phi}}{a^4} - \frac{3}{2} m^2 B_{12} B^{12} e^{\sqrt{2}\phi}$$

SOLUTION SPACE

Given the SUGRA parameters g and m, L and the dilaton can be tuned to give any value of z. There is also a 1-parameter family of ads solutions.

LIF

$$\hat{b}^2 = z - 1$$

$$\hat{g}^2 = 2z(4+z)$$

$$\hat{m}^2 = \frac{2}{z}\left[6 + z \mp 2\sqrt{2(z+4)}\right]$$

$$\hat{q}^2 = \frac{1}{2z}\left[(z+2)(z-3) \pm 2\sqrt{2(z+4)}\right]$$

$$\frac{1}{\hat{a}^2} = \left[6 + 3z \mp 2\sqrt{2(z+4)}\right]$$

ADS

$$\hat{g} \in \left[\sqrt{6},\ 3\sqrt{\frac{6}{5}}\right]$$

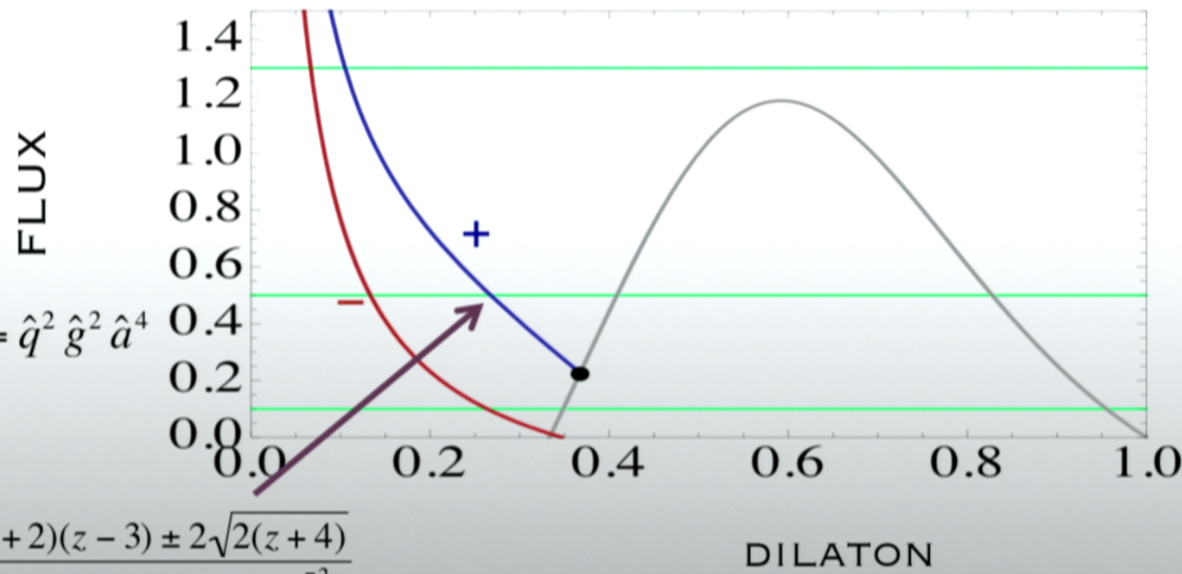
$$\hat{m} = \hat{g} - \sqrt{\hat{g}^2 - 6}$$

$$\hat{q}^2 = -\frac{3}{4}(\hat{g}^2 - 6) + \frac{\hat{g}}{2}\sqrt{\hat{g}^2 - 6}$$

$$\hat{a}^{-2} = \frac{3}{2}\hat{g}^2 - 6 - \hat{g}\sqrt{\hat{g}^2 - 6}$$

General Lifshitz Spacetimes

The flux on the internal H_2 is a constant of the system, but in principle the dilaton and L can take more than one value.



$$Q = \hat{q}^2 \, \hat{g}^2 \, \hat{a}^4$$

$$(z+4)\frac{(z+2)(z-3) \pm 2\sqrt{2(z+4)}}{\left[3(z+2) \mp 2\sqrt{2(z+4)}\right]^2}$$

For example, for Q = 0.5, we have 4 solutions: two ads (below) and two lifshitz: z = 1.48 and z = 10.3

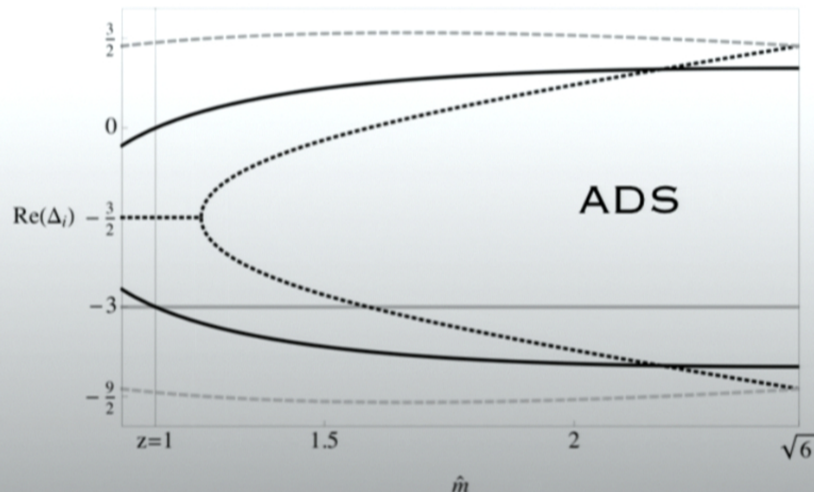
$$e^{2\sqrt{2}\phi_0} = 2.45\frac{m}{g} \quad ; \quad L^4 = \frac{34.8}{mg^3} \qquad\qquad e^{2\sqrt{2}\phi_0} = 1.21\frac{m}{g} \quad ; \quad L^4 = \frac{31.6}{mg^3}$$

For Q > 1.2, there are only Lifshitz solutions, and for Q<0.227, there is only one Lifshitz (larger z) solution. For Q = 0.227, the Lifshitz (smaller z) and ads branches join.

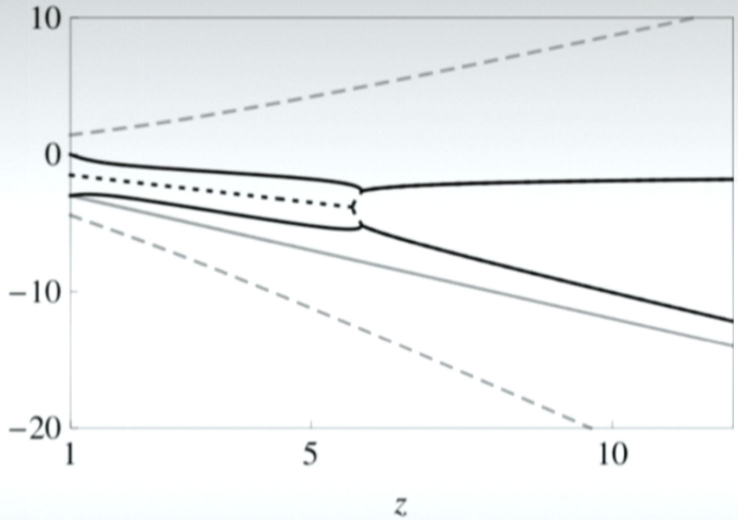
More General Solutions

Studying the general radial eqns of motion shows the solution space is 7 dimensional. Critical points in this space are the ADS/LIF solutions. Perturbing around the critical points gives (ir)relevant operators in the dual field theory. It also reveals flows and asymptotic black hole solutions.

$$\delta\Phi_i = V_{ij}\ r^{-\Delta_j}$$

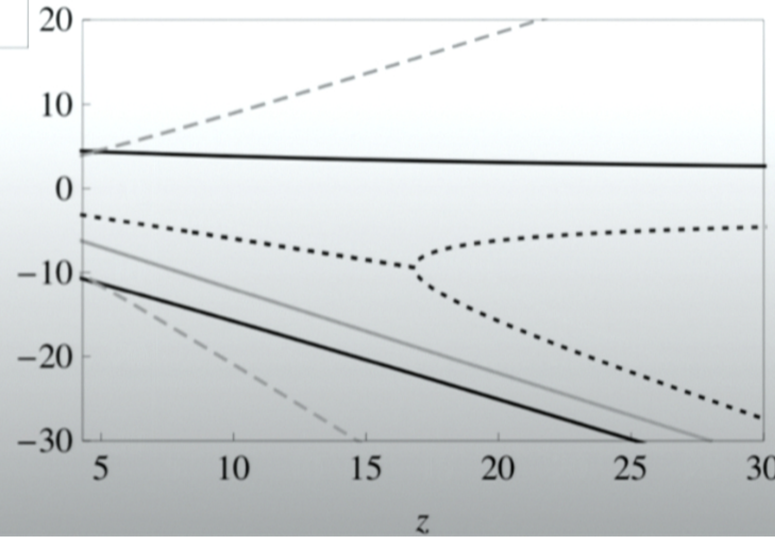


Lifshitz Spectrum



$$\hat{q}^2 = \frac{1}{2z}\left[(z+2)(z-3)+2\sqrt{2(z+4)}\right]$$

Upper sign

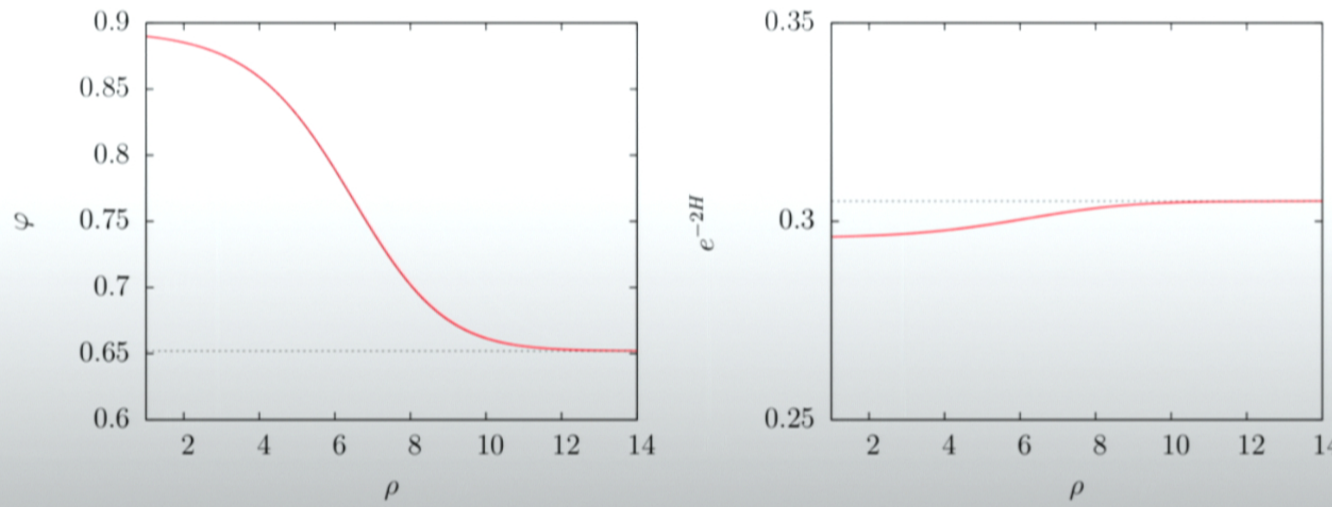


$$\hat{q}^2 = \frac{1}{2z}\left[(z+2)(z-3)-2\sqrt{2(z+4)}\right]$$

Lower sign

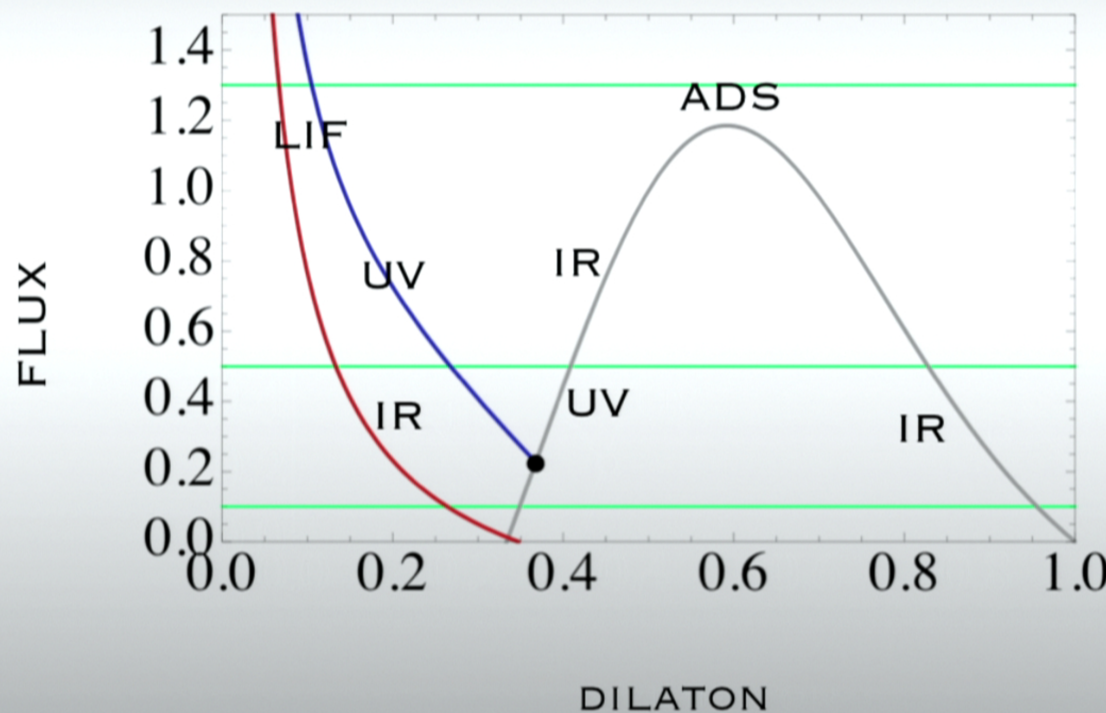
Flows

Analyzing the perturbations around the critical points shows that we can flow between LIF and ADS solutions. These will correspond to a field theory with different dynamical scaling at different scales

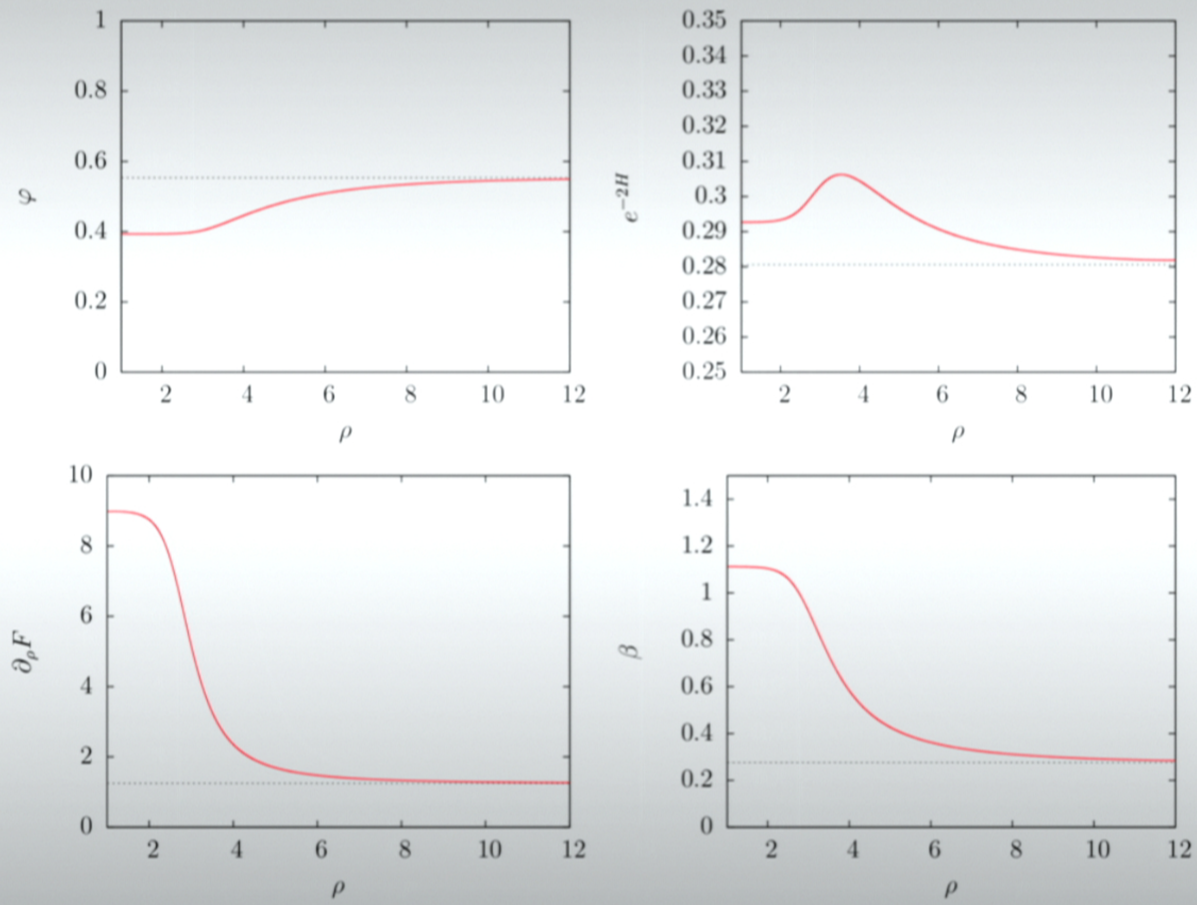


(Braviner, RG, Ross:1108.3067)

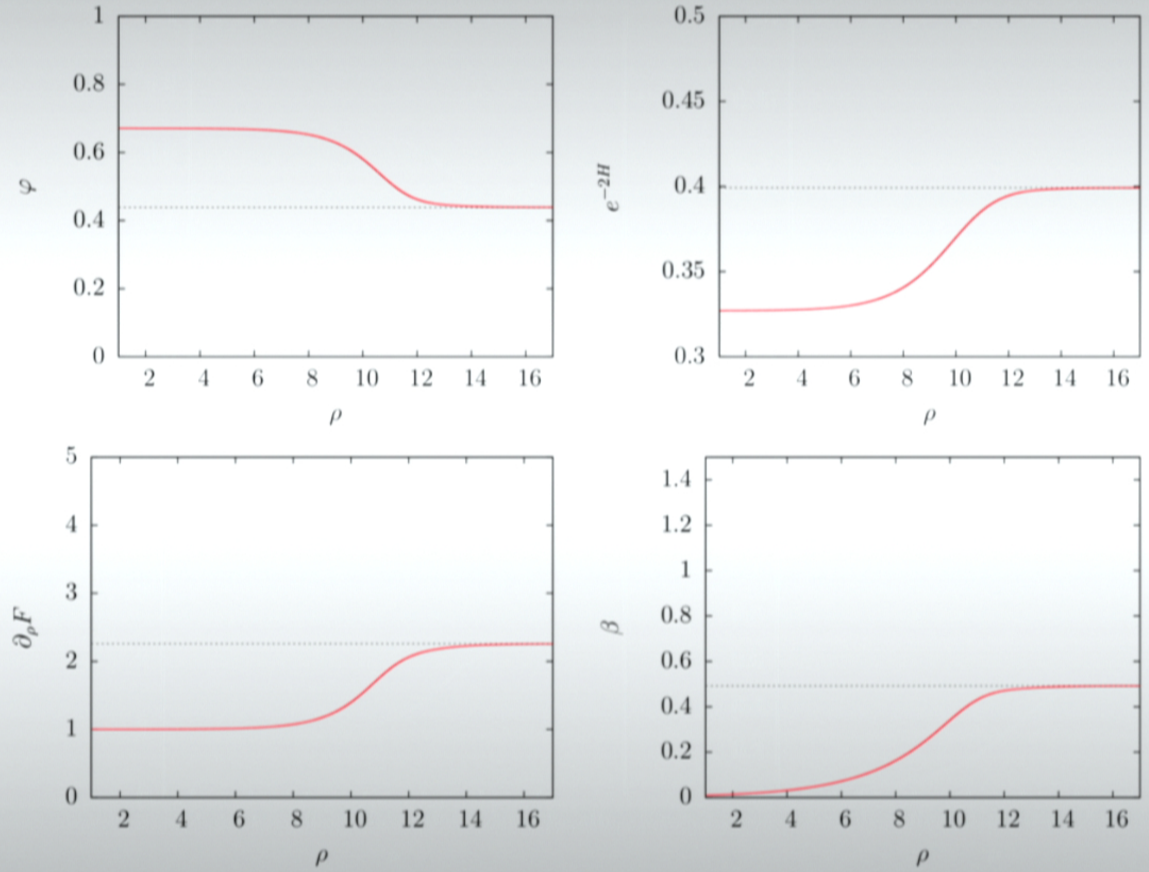
General Flow Picture



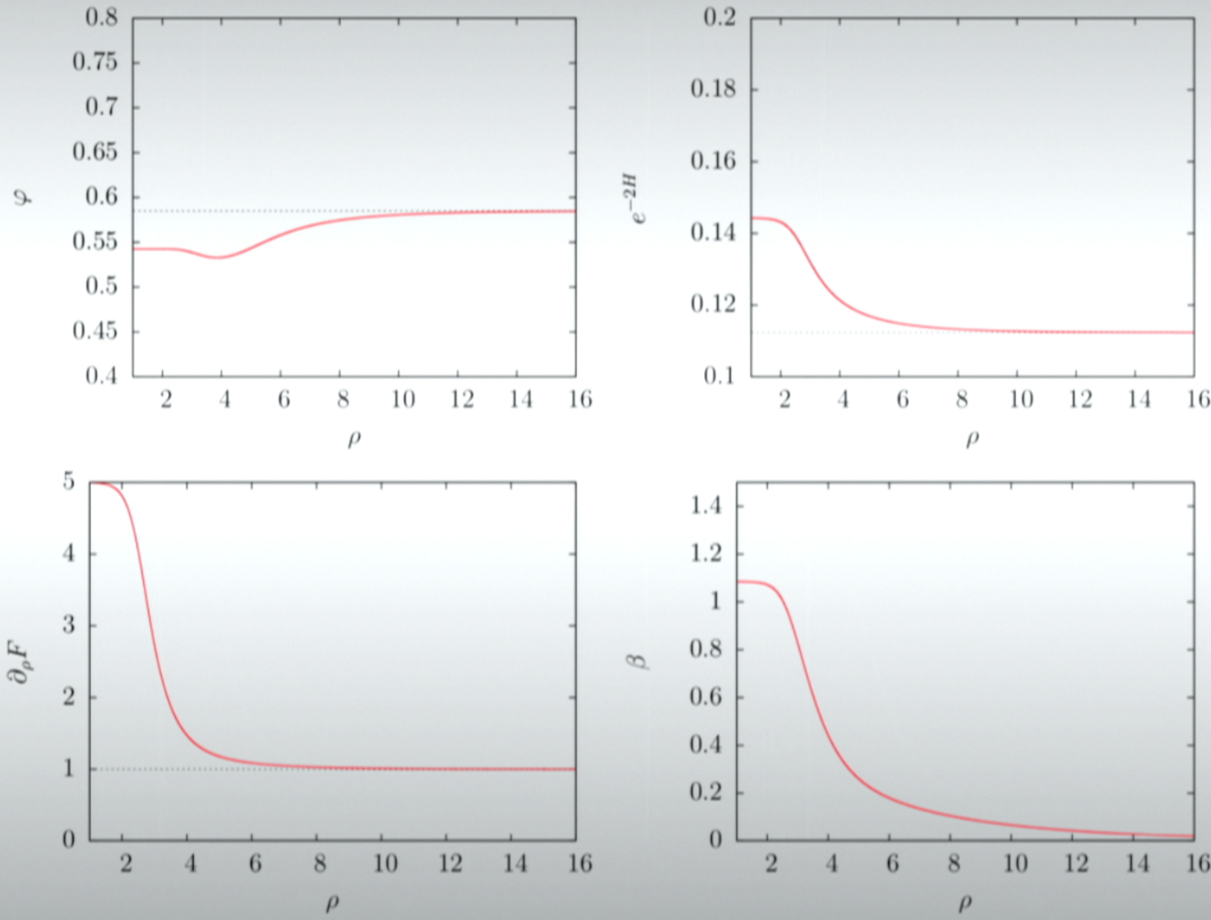
LIF - LIF

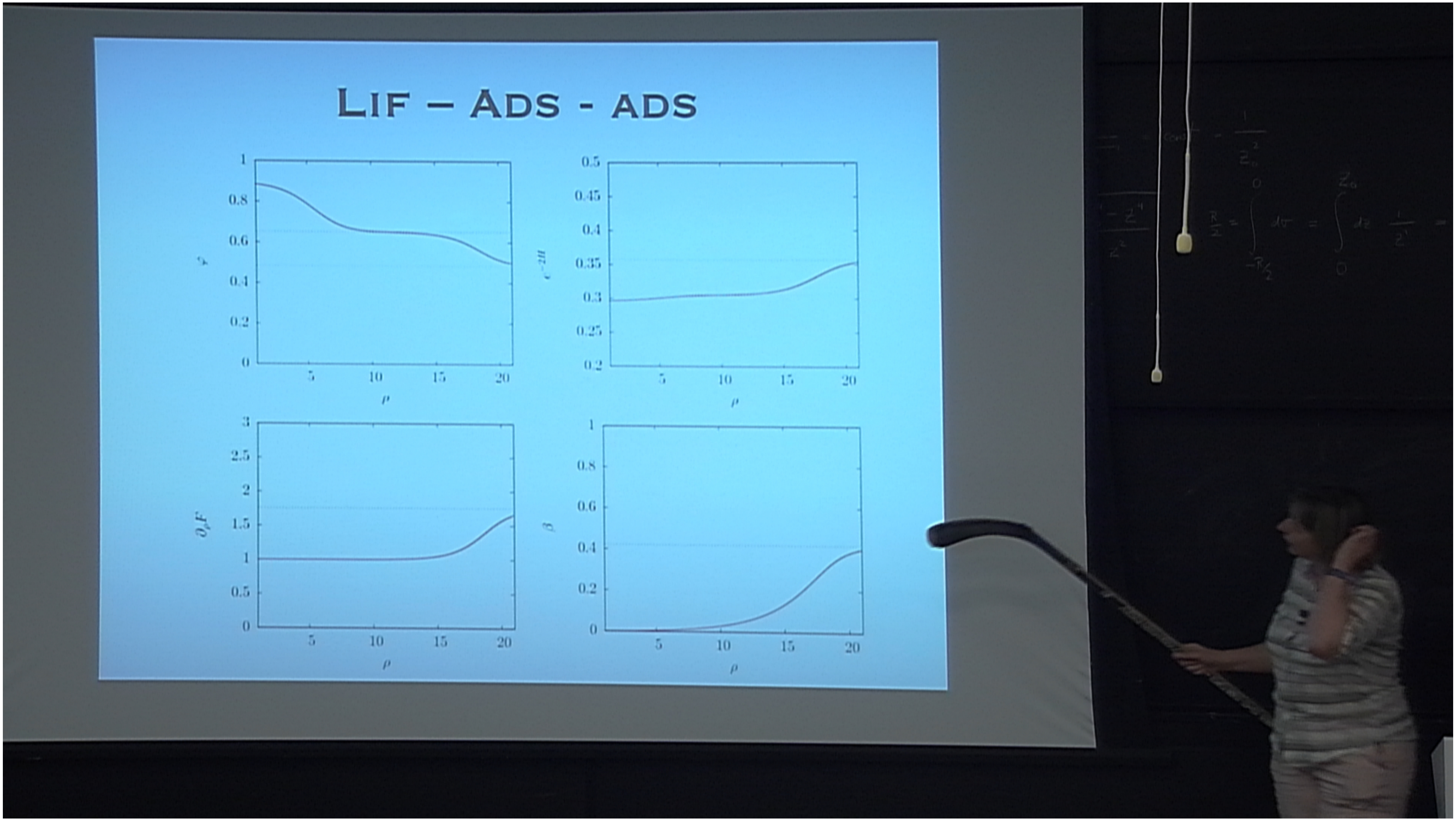


Lif - Ads

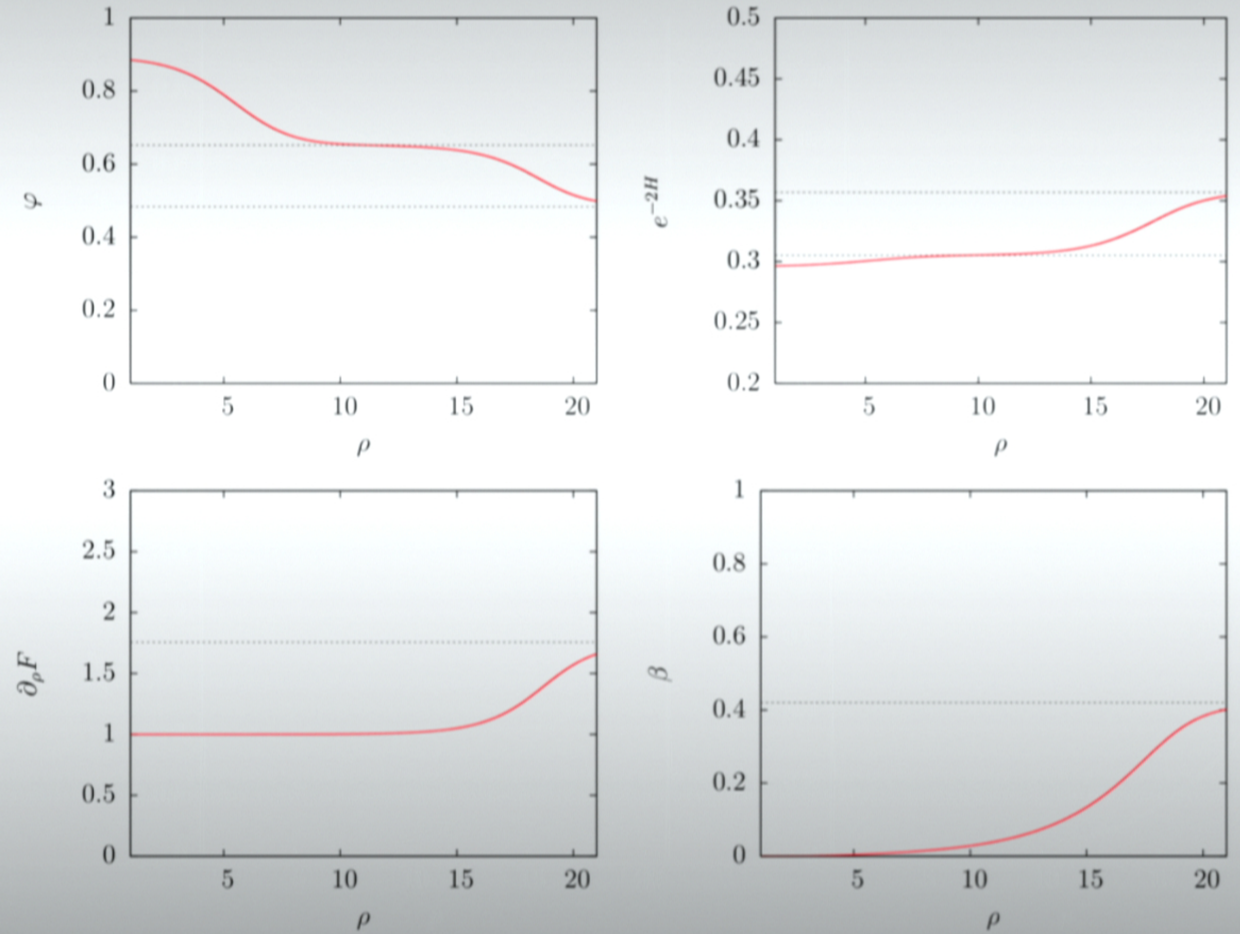


ADS - LIF




Lif – Ads - ads
ρ
ρ
ρ
ρ
$\partial_\rho F$
β

LIF – ADS - ADS



Black Holes

To find a black hole, we must solve the radial equations with a horizon. Know from eigenvalue analysis that all fields are involved.

$$ds^2 = L^2\left[r^{2z}F(r)dt^2 - r^2\left(dx_1^2 + dx_2^2\right) - \frac{dr^2}{r^2 D(r)}\right] - a^2 H(r) dH_2^2$$

$$B_{x_1 x_2} = L^2 P(r) r^2 / 2$$

$$F^3_{tr} = L^2 e^{\sqrt{2}\phi} r^{z-1} \frac{\sqrt{F}}{\sqrt{DH}} P(r)$$

$$\phi = \phi(r)$$

$\infty \quad F = D = H = P = 1$

(Barclay, RG, Parameswaran, Tasinato,

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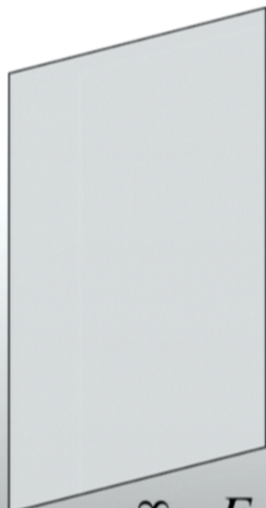

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$$F^3_{tr} = L^2 e^{\sqrt{2}\phi}\, r^{z-1}\frac{\sqrt{F}}{\sqrt{DH}}P(r)$$

$$\phi = \phi(r)$$

$$F \sim f_1(r - r_+) + \ldots$$

$$D \sim d_1(r - r_+) + \ldots$$

$$P \sim P_0 + \ldots$$

$$H \sim H_0 + \ldots$$

$$\varphi \sim \varphi_0 + \ldots$$

∞ $F = D = H = P = 1$

(Barclay, RG, Parameswaran, Tasinato, Zavala: 1203.0576)

AdS example

Can we use our knowledge of ads to help? $F = D = 1 - \left(\frac{r_+}{r}\right)^3$

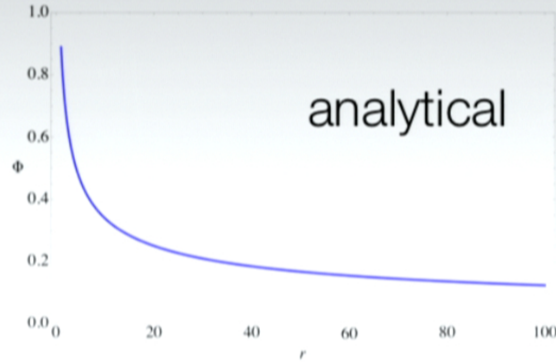
To leading order we can solve the scalar and gauge equations in this background, to get hairy or charged black holes.

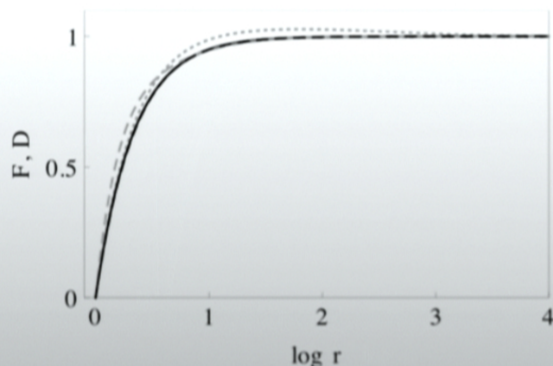
$$\frac{1}{r^2}\frac{d}{dr}\left[r^4\left(1-\frac{r_+^3}{r^3}\right)\frac{d}{dr}\begin{pmatrix}\sqrt{2}\phi \\ H\end{pmatrix}\right] = \begin{bmatrix}(3\hat{m}^2-\hat{g}^2)/2 & 2\hat{q}^2 \\ 2\hat{q}^2 & 2(\hat{q}^2+3)\end{bmatrix}\begin{pmatrix}\sqrt{2}\phi \\ H\end{pmatrix}$$

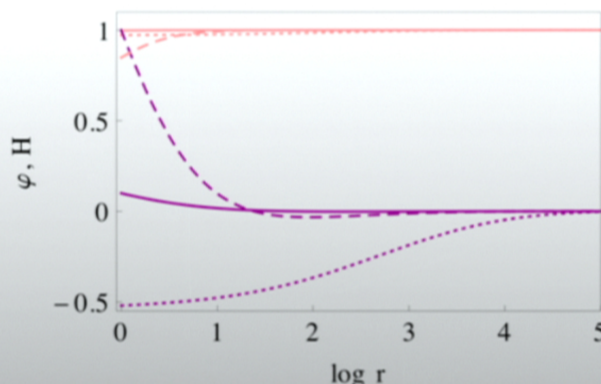
A bit of work gives the eigenvectors and eigenvalues of this operator, and a solution in terms of hypergeometric functions.

Scalar charged AdS

$$\Gamma[2\mu_-]\Gamma[\mu_+]^2\left(\frac{r_+}{r}\right)^{3\mu_+}\ {}_2F_1\left[\mu_+,\mu_+,2\mu_+;\left(\frac{r_+}{r}\right)^3\right]$$
$$-\Gamma[2\mu_+]\Gamma[\mu_-]^2\left(\frac{r_+}{r}\right)^{3\mu_+}\ {}_2F_1\left[\mu_-,\mu_-,2\mu_-;\left(\frac{r_+}{r}\right)^3\right]$$





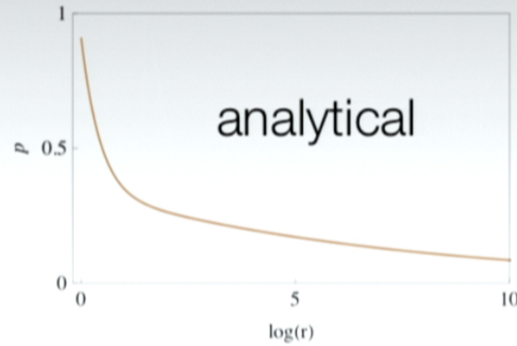


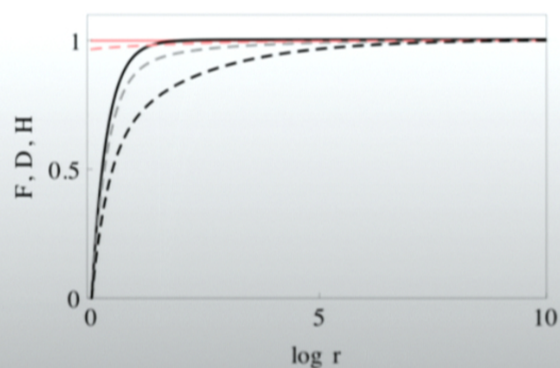
numerical

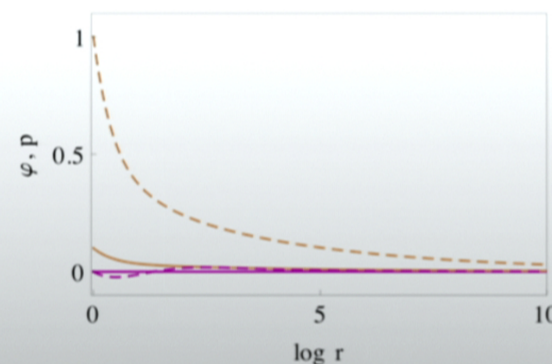
GAUGE CHARGED ADS

$$\Gamma[2\nu_- + 4/3]\Gamma[\nu_+ + 4/3]\Gamma[\nu_+]\left(\frac{r_+}{r}\right)^{3\nu_+}\ {}_2F_1\left[\nu_+, \nu_+ + 4/3, 2\nu_+ + 4/3; \left(\frac{r_+}{r}\right)^3\right]$$

$$-\Gamma[2\nu_+ + 4/3]\Gamma[\nu_- + 4/3]\Gamma[\nu_-]\left(\frac{r_+}{r}\right)^{3\nu_+}\ {}_2F_1\left[\nu_-, \nu_- + 4/3, 2\nu_- + 4/3; \left(\frac{r_+}{r}\right)^3\right]$$

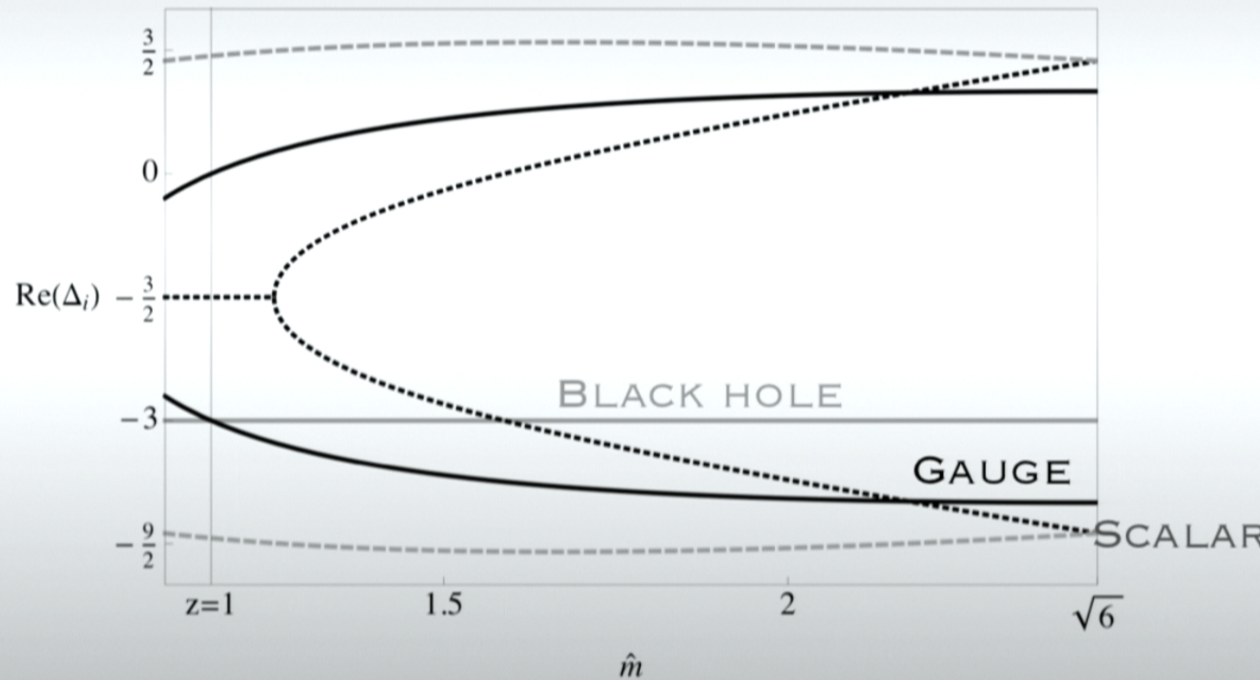






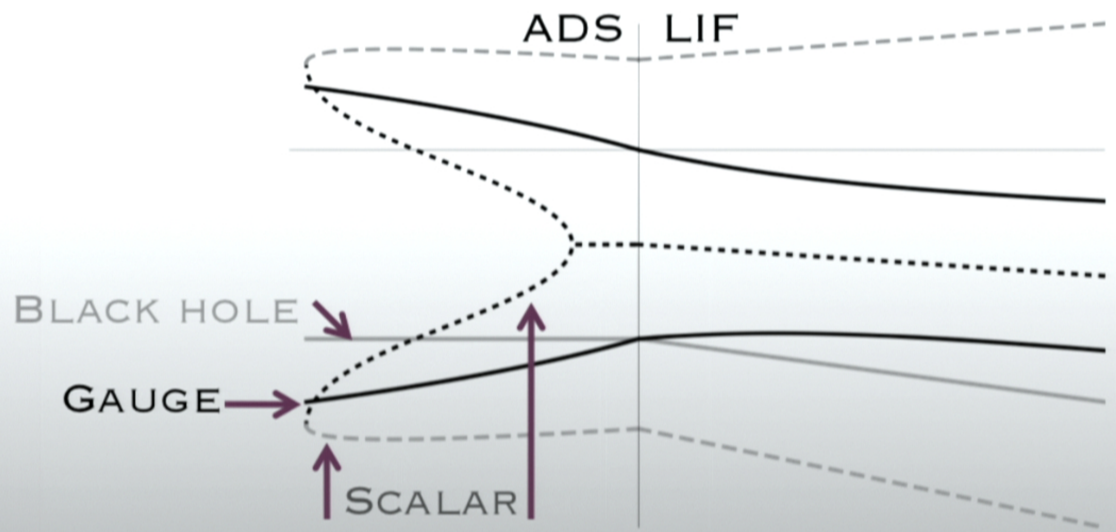
numerical

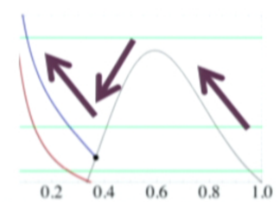
Identifying Eigenvectors



Lifshitz structure

Although can identify the ads black hole in the phase diagram, the Lifshitz solution is much more complicated. The eigenvectors degenerate at the crossing point, and the solution is distinct from ads.





Near ADS Solutions

Expanding near z=1, focussing on the gauge and pure black hole degrees of freedom on ads side:

$$\Delta_1 = -3 - (z-1)$$

$$\Delta_2 = -3 + \left(260\sqrt{10} - 701\right)\frac{z-1}{189}$$

$$\delta\varphi_1 = \frac{\mu\sqrt{z-1}}{126 r^{\Delta_1}}\left(31 - 40\sqrt{10}\right) \qquad \delta\varphi_2 = \frac{\mu\sqrt{z-1}}{126 r^{\Delta_2}}\left(31 - 40\sqrt{10}\right)$$

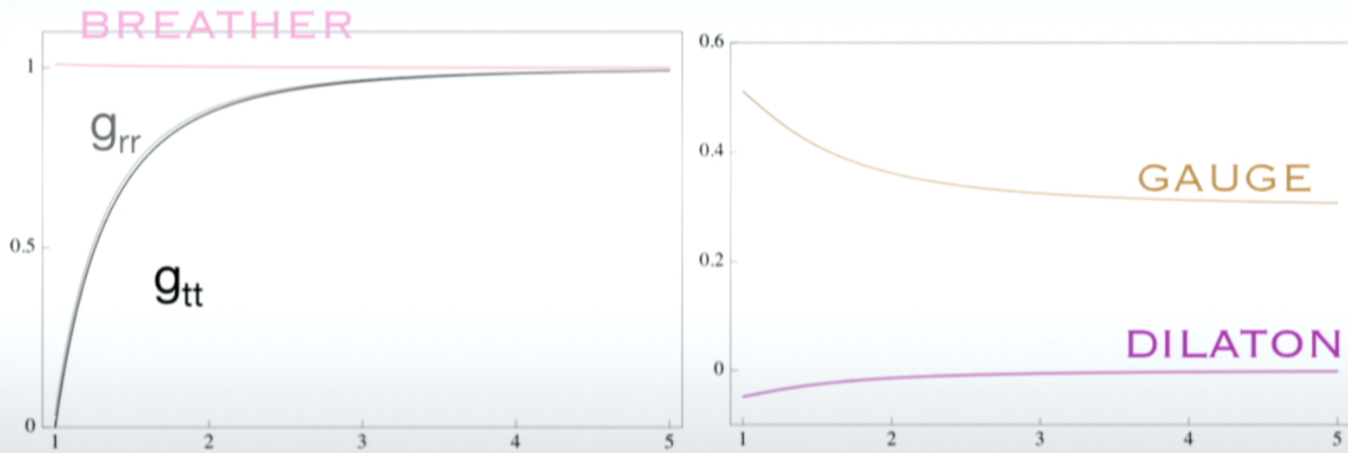
$$\delta F_1 = 1 - \frac{\mu\sqrt{z-1}}{63 r^{\Delta_1}}\left(65\sqrt{10} - 149\right) \qquad \delta F_2 = 1 - \frac{\mu\sqrt{z-1}}{3 r^{\Delta_2}}$$

$$\delta D_1 = 1 + \frac{\mu\sqrt{z-1}}{63 r^{\Delta_1}}\left(11 + 25\sqrt{10}\right) \qquad \delta D_2 = 1 + \frac{\mu\sqrt{z-1}}{63 r^{\Delta_2}}\left(139 - 40\sqrt{10}\right)$$

$$\delta H_1 = 1 + \frac{\mu\sqrt{z-1}}{126 r^{\Delta_1}}\left(101 - 20\sqrt{10}\right) \qquad \delta H_2 = 1 + \frac{\mu\sqrt{z-1}}{126 r^{\Delta_2}}\left(101 - 20\sqrt{10}\right)$$

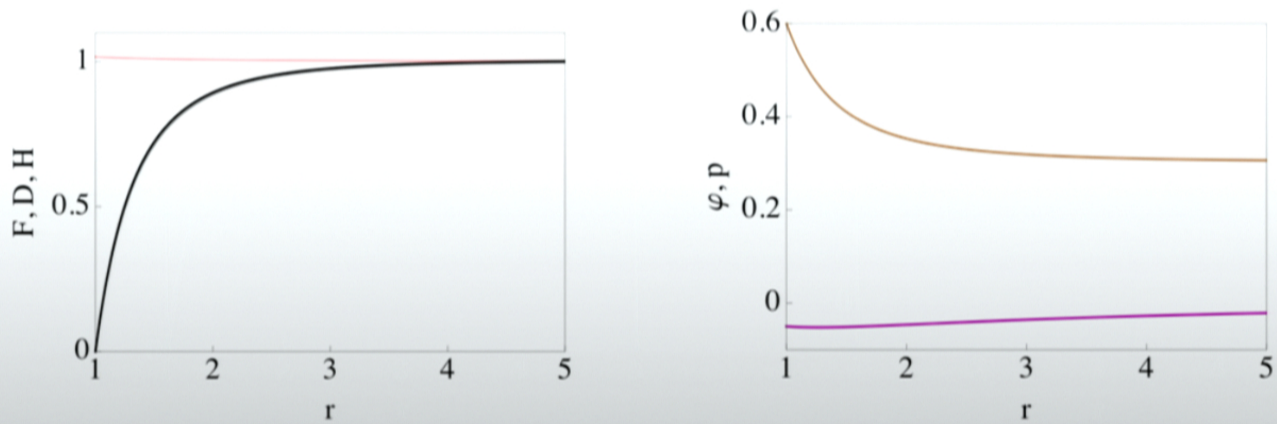
$$\delta p_1 = \sqrt{z-1} + \frac{\mu}{r^{\Delta_1}} \qquad \delta p_2 = \sqrt{z-1} + \frac{\mu}{r^{\Delta_2}}$$

PURE "BLACK HOLE" LINEARIZED



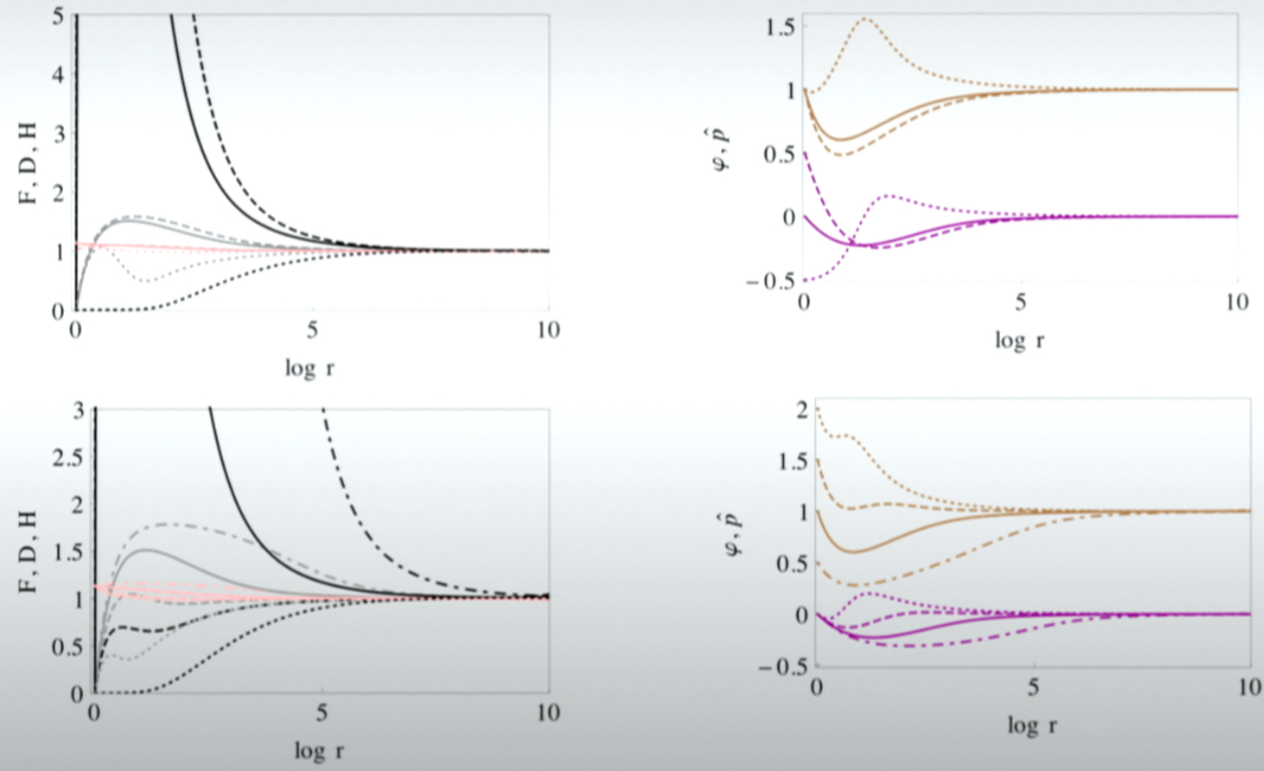
General Solutions

The general solutions are found numerically, integrating out from horizon and exploring possible parameter space. Solutions characterised by two parameters (fix r_+=1), the scalar and vector initial data at the horizon.

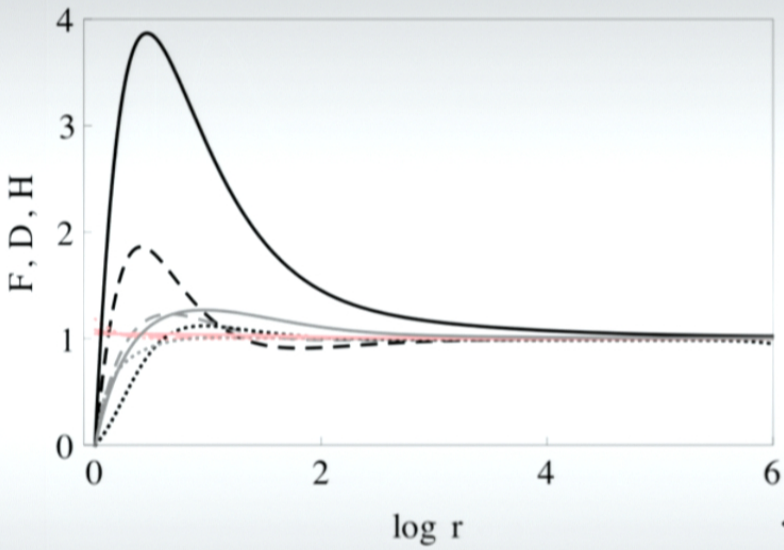


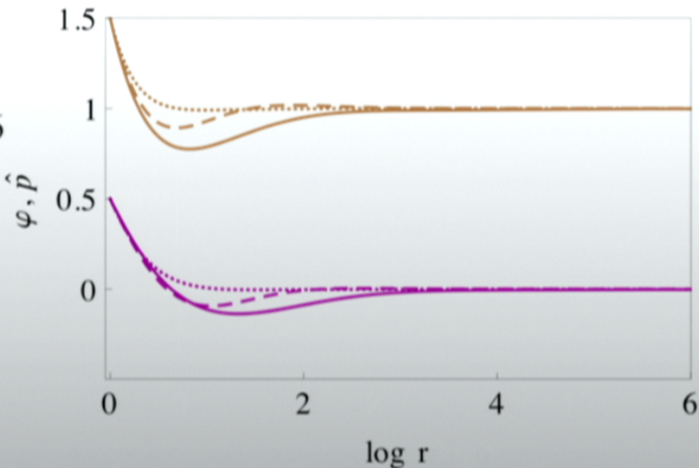
(T = 0.243, vs 0.238 for ads-sch)

Numerical Solutions (z=2)



Z-DEPENDENCE





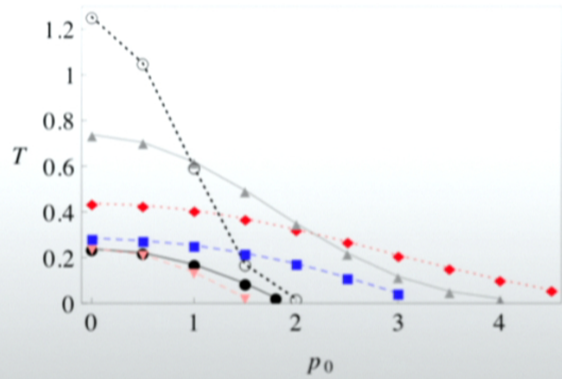
Z=2, 3, 5.75

Generic features

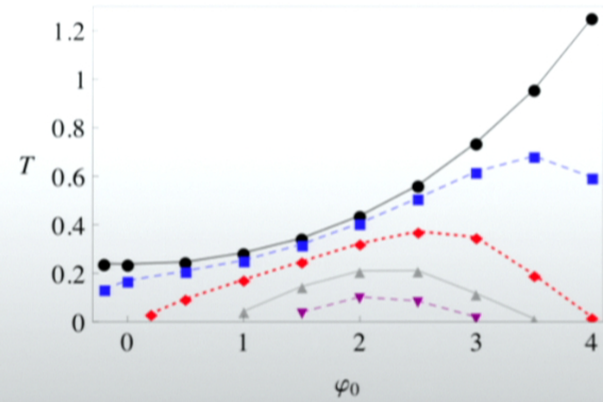
- The generic Lifshitz black hole tends to have a sharp peak in the Newtonian potential – this can sometimes be extremely high (O(100)). By contrast, the radial metric function is relatively well behaved.
- This suggests the area gauge is not the most natural for these black holes, and numerics are possibly missing wormhole type features.
- All the fields have strong modulation in this region, again suggesting instability.

THERMODYNAMICS - ADS

The temperature of ads black holes shows some similarity with RN solution – T drops as p_0 is increased, falling to zero. The scalar 'charge' is more interesting: scalar charge initially increases the range of p-charge, but then dramatically decreases it.



Varying scalar 'charge'
-0.2, 0, 1, 2, 3, 4

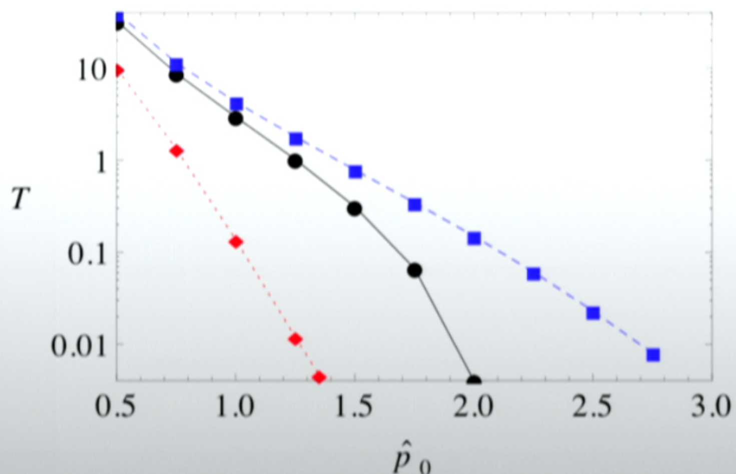


Varying gauge 'charge'
0, 1, 2, 3, 4

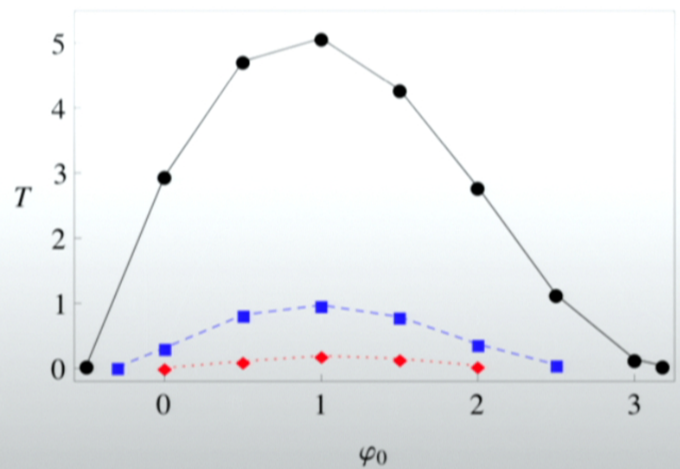
THERMODYNAMICS - LIF

Temperature scales as r_+^z

$$T = \frac{r_+^{z+1}}{4\pi}\sqrt{D'(r_+)F'(r_+)}$$

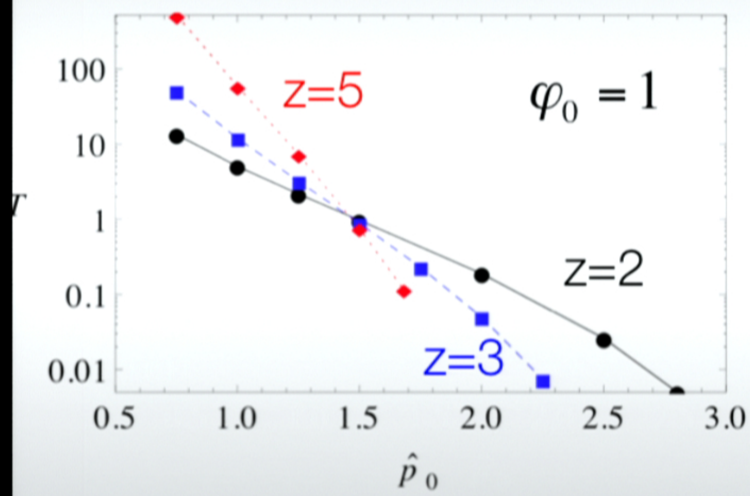


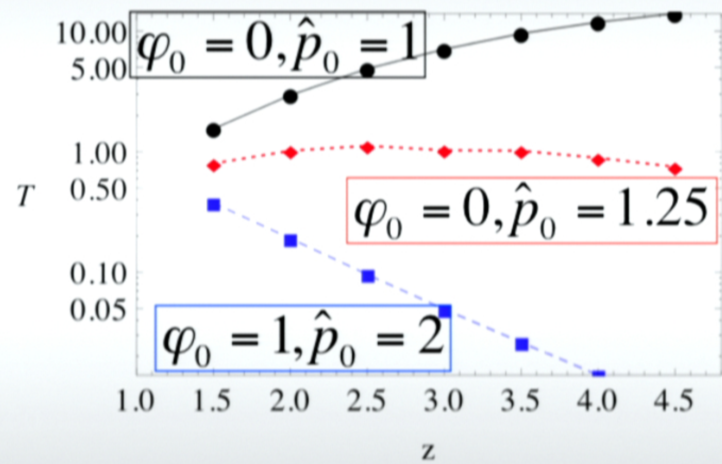
Varying scalar ‘charge’: 0, 1.5, 3



Varying gauge ‘charge’:0, 1, 2

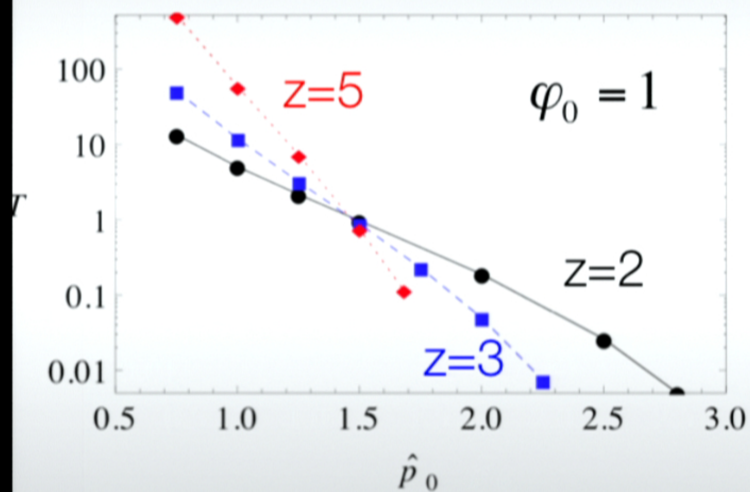
Temperature – z dependence

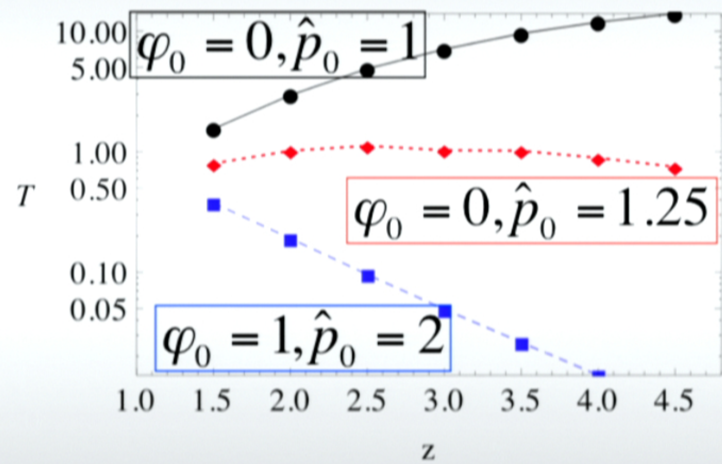




Changing z typically scales the temperature away from T=1: although the geometries seem smoother for higher z, the range of charge becomes smaller.

Temperature – z dependence





Changing z typically scales the temperature away from T=1: although the geometries seem smoother for higher z, the range of charge becomes smaller.

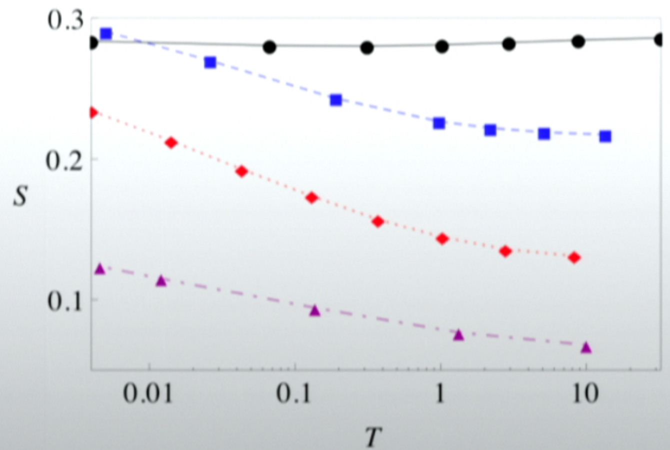
- Temperature decreases as we increase the gauge field near the horizon. This would seem to correspond to ‘charging up’ a black hole.

- The scalar field has a different effect, as in ads, and first increases, then decreases the temperature.

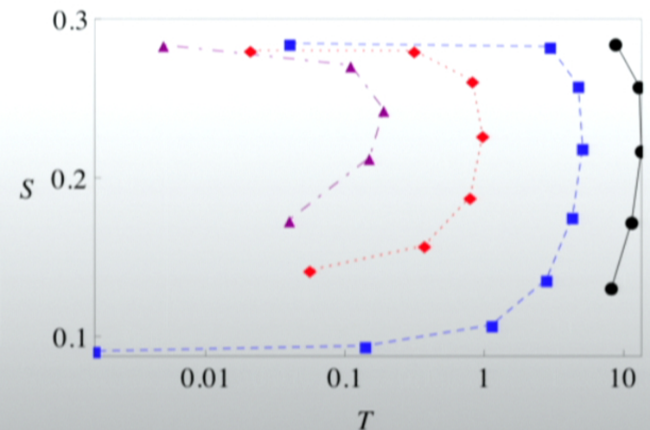
- With the gauge used, there is no extremal limit with $r_+=1$. As T drops, the radial metric potential becomes sharper, which can be ameliorated by dropping r_+, suggesting zero temperature black holes have zero entropy.

Entropy

Typically, increasing scalar or gauge initial data lowers the entropy, though the response to changing the scalar is much more dramatic.



Varying scalar: 0, 1, 2, 3



Varying gauge: 0.75, 1, 1.5, 2

- Entropy density is directly proportional to the value of the breather at the horizon. Since the breather and dilaton are coupled strongly in the eigenvalue equations, we expect this stronger response to scalar initial data.

- The presence of two equal temperature solutions with different entropy suggests that the black hole will shed scalar charge to increase its entropy.

- Clear indication of black hole instability.

SUMMARY

- ❑ Have developed a prescription for embedding Lifshitz into string theory – probably for any z, though issues of quantization arising from compactification of H_2
- ❑ Rich structure of flows and black holes, though mostly have to be found numerically
- ❑ Black holes are generally rather involved solutions, with all of the fields switched on, and rather strongly distorted geometries near the horizon.
- ❑ Stability next!