Title: Dark Matter Axions

Date: Mar 20, 2012 01:00 PM

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Abstract: The axion provides a solution to the strong CP problem and is a cold dark matter candidate. I'll briefly review the limits on the axion from particle physics, stellar evolution and cosmology. The various constraints suggest that the axion mass is in the micro-eV to milli-eV range. In this window, axions contribute significantly to the energy density of the universe in the form of cold dark matter. It was recently found that dark matter axions thermalize and form a Bose-Einstein condensate (BEC). As a result, it may be possible to distinguish axions from other forms of dark matter, such as weakly interacting massive particles (WIMPs), on observational grounds. Axions accreting onto a galactic halo fall in with net overall rotation because almost all go to the lowest energy available state for given angular momentum. In contrast, WIMPs accrete onto galactic halos with an irrotational velocity field. The inner caustics are different in the two cases. I'll argue that the dark matter is axions because there is observational evidence for the type of inner caustic produced by, and only by, an axion BEC.

Outline

Introduction

Bose-Einstein condensation of dark matter axions (axions are different)

The inner caustics of galactic halos (axions are better)

Axions and cosmological parameters

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The Strong CP Problem

$$L_{\mathrm{QCD}} = \;\ldots + \theta \frac{g^2}{32\pi^2} G^a{}_{\mu\nu} \tilde{G}^{a\mu\nu}$$

Because the strong interactions conserve P and CP, $\theta \leq 10^{-10}$.

The Standard Model does not provide a reason for θ to be so tiny,

but a relatively small modification of the model does provide a reason …

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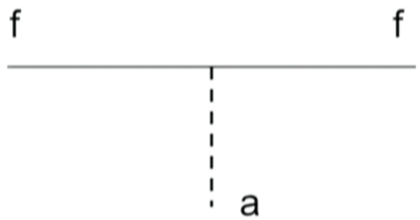
If a $\mathrm{U_{PQ}(1)}$ symmetry is assumed,

$$L = \ldots + \frac{a}{f_a}\frac{g^2}{32\pi^2} G^a{}_{\mu\nu}\,\tilde{G}^{a\mu\nu} + \frac{1}{2}\partial_\mu a\,\partial^\mu a + \ldots$$

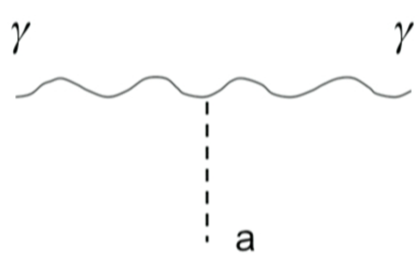
$\theta = \dfrac{a}{f_a}$ relaxes to zero,

and a light neutral pseudoscalar particle is predicted: the axion.

$$m_a \simeq 6 \text{ eV} \frac{10^6 \text{ GeV}}{f_a}$$



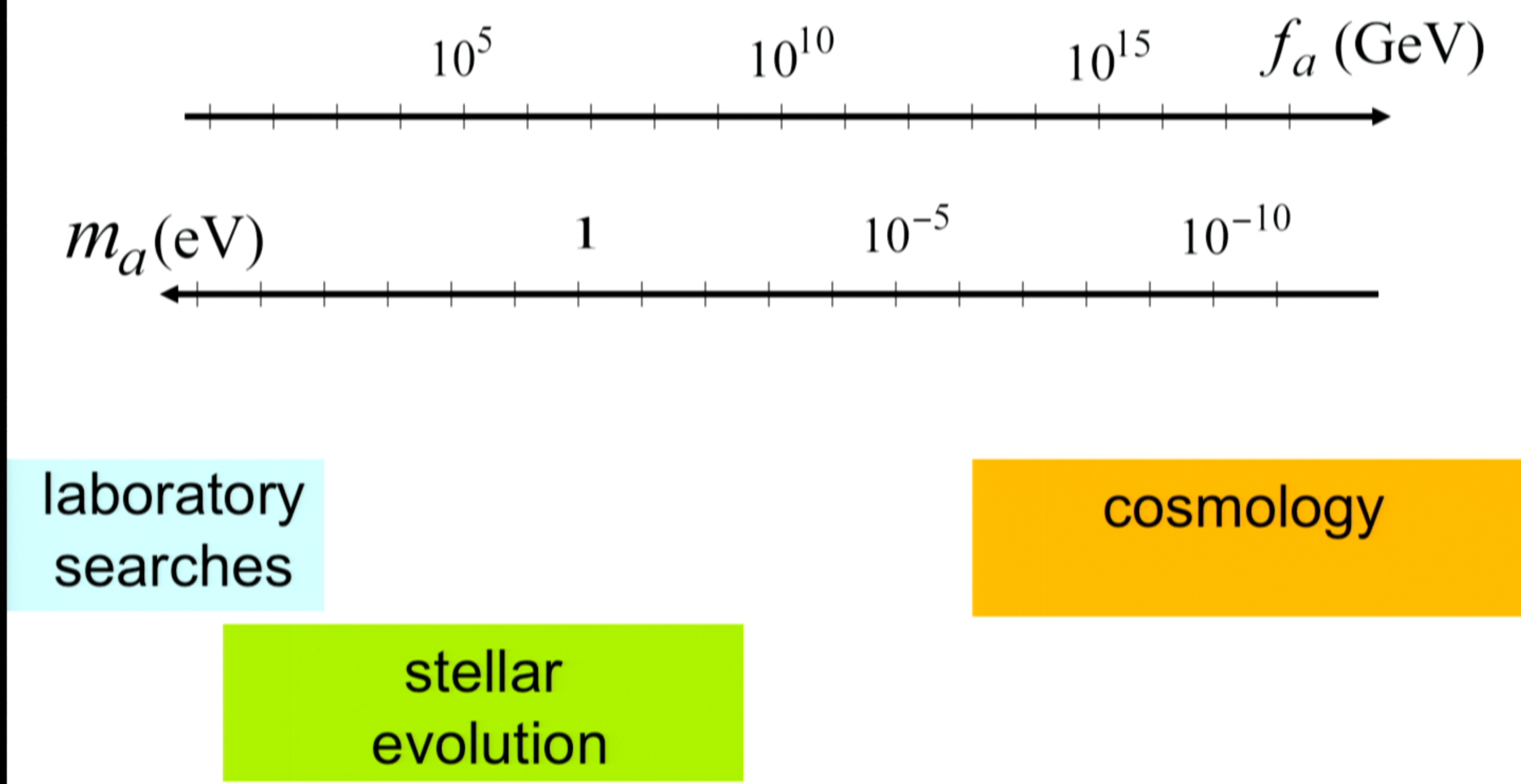
$$L_{a\bar{f}f} = i g_f \frac{a}{f_a} \bar{f} \gamma_5 f$$



$$L_{a\gamma\gamma} = g_\gamma \frac{\alpha}{\pi} \frac{a}{f_a} \vec{E} \cdot \vec{B}$$

g_γ = 0.97 in KSVZ model
0.36 in DFSZ model

The remaining axion window



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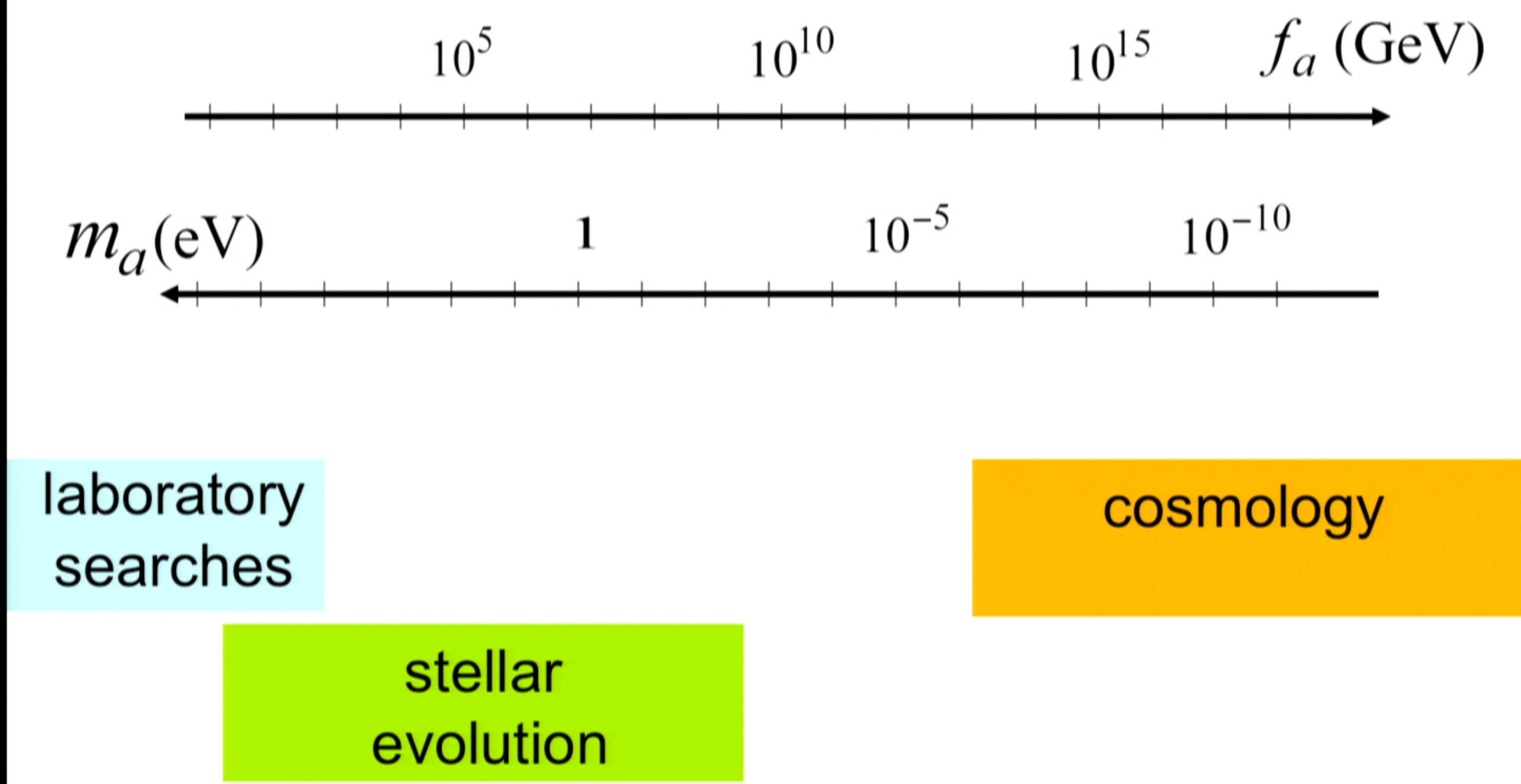
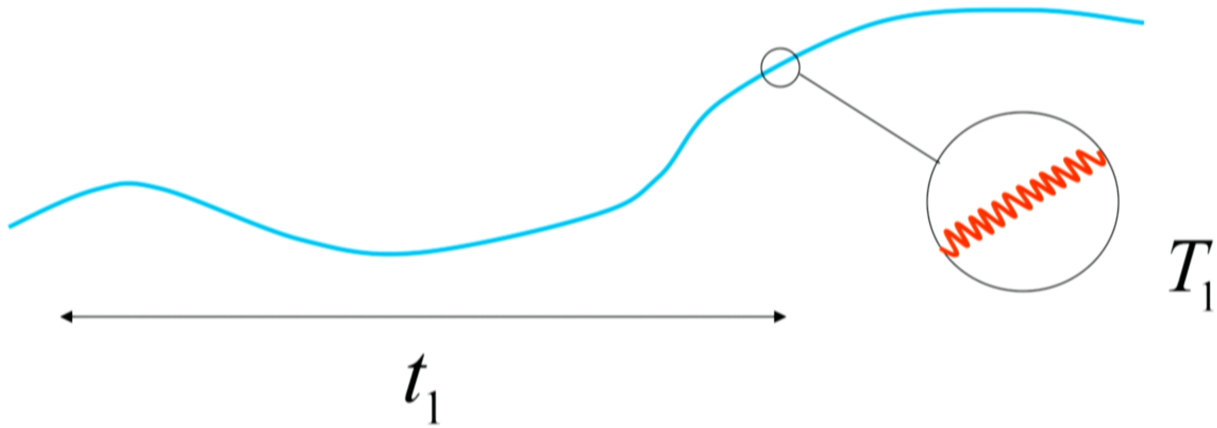


There are two cosmic axion populations: hot and cold.



When the axion mass turns on, at QCD time,

$$T_1 \simeq 1 \text{ GeV}$$

$$t_1 \simeq 2 \cdot 10^{-7} \text{ sec}$$

$$p_a(t_1) = \frac{1}{t_1} \simeq 3 \cdot 10^{-9} \text{ eV}$$

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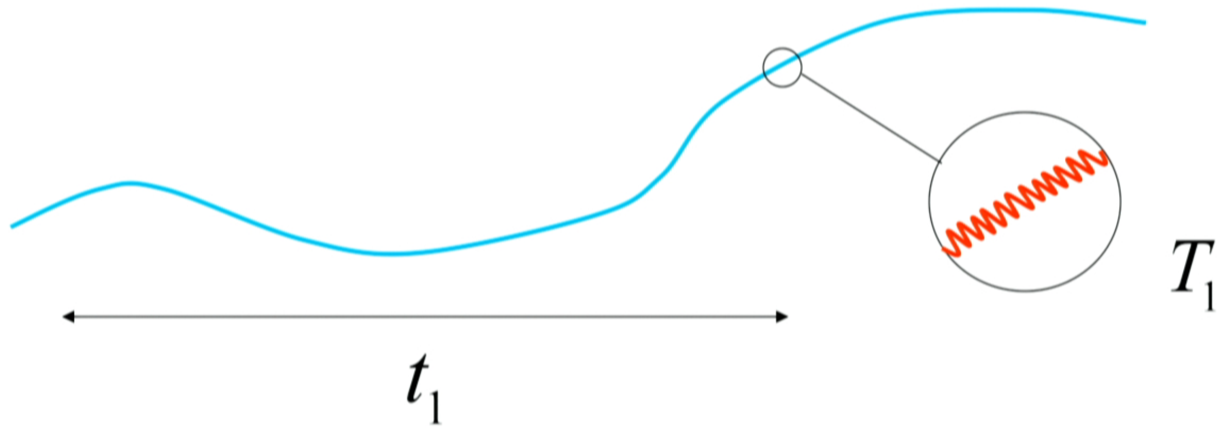

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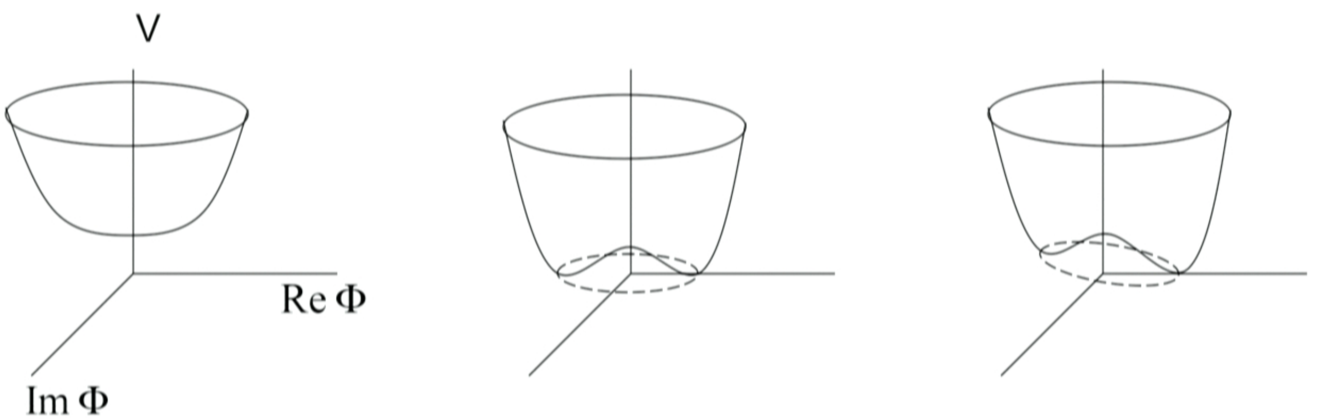
Effective potential V(T,Φ)



$T > f_a$

$f_a > T > 1\,\mathrm{GeV}$ → axion strings

$1\,\mathrm{GeV} > T$ → axion domain walls

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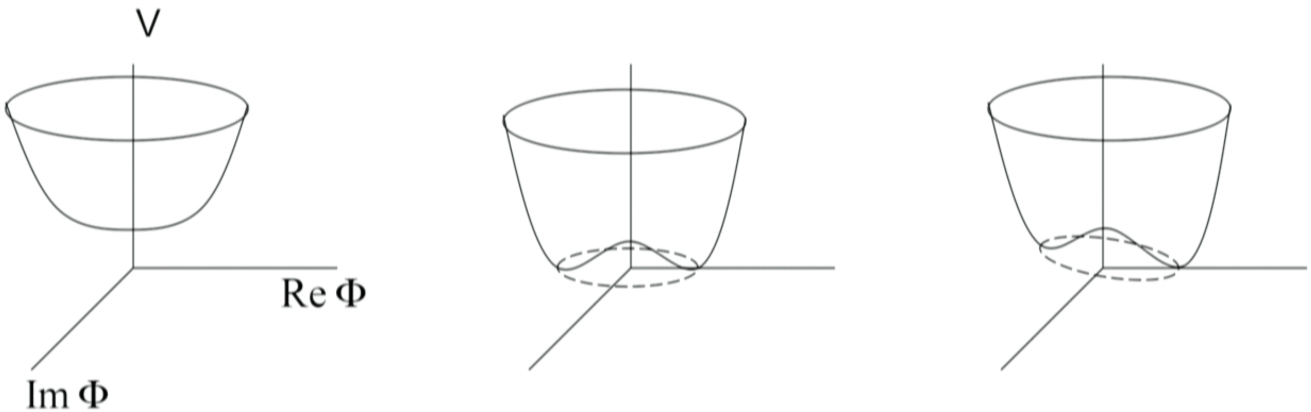

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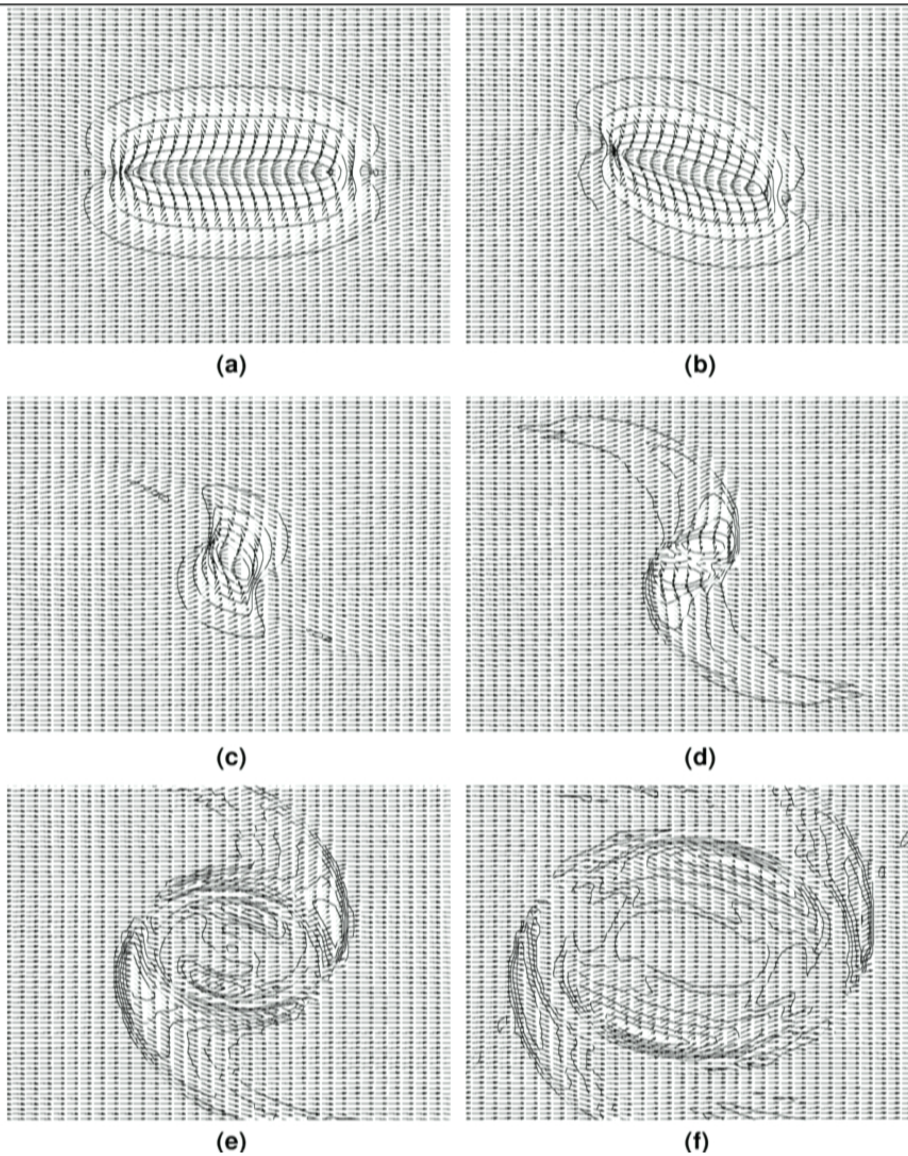
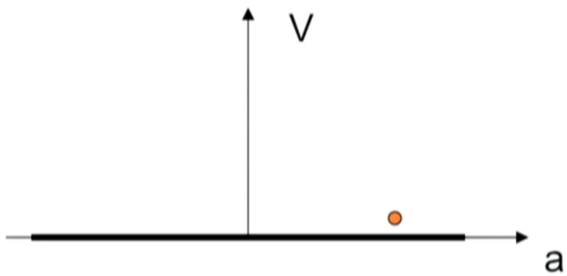


Domain wall bounded by string decaying into axion radiation

Axion production by vacuum realignment



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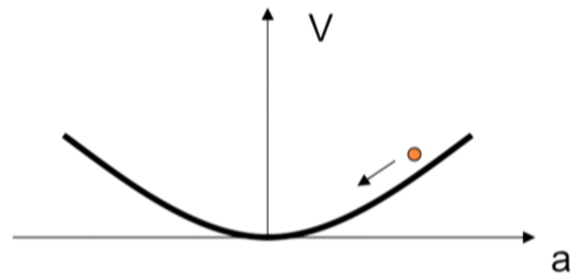

$T \leq 1\,\mathrm{GeV}$

$$n_a(t_1) \simeq \frac{1}{2} m_a(t_1)\, a(t_1)^2 \simeq \frac{1}{2t_1} f_a^{\,2}\, \alpha(t_1)^2$$

initial misalignment angle

$$\rho_a(t_0) \simeq m_a\, n_a(t_1) \left(\frac{R_1}{R_0}\right)^3 \propto m_a^{-\frac{7}{6}}$$

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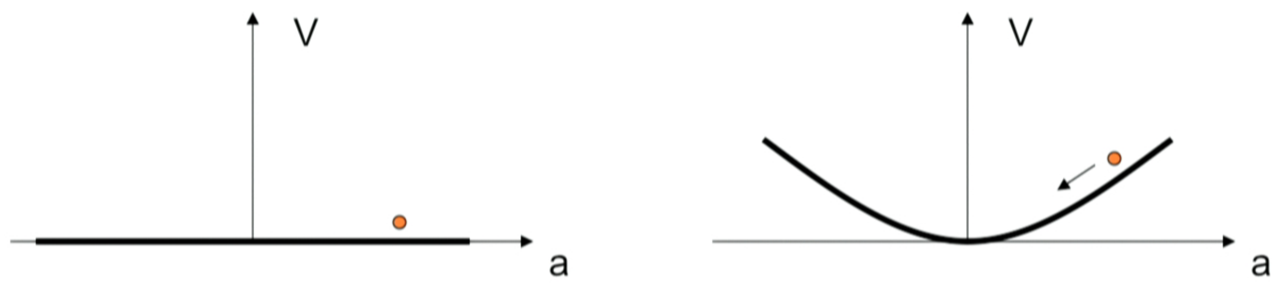

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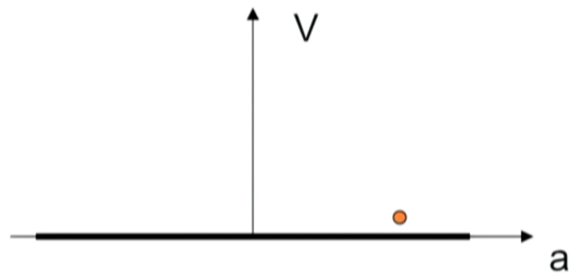

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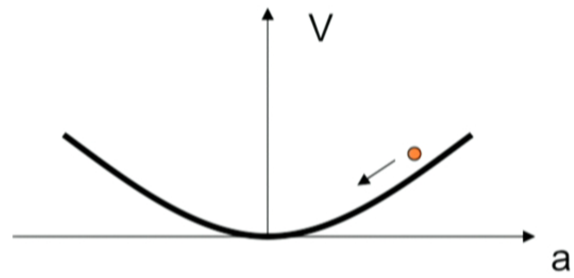

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Cold axion properties

- number density

$$n(t) \sim \frac{4\cdot 10^{47}}{\text{cm}^3}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{\frac{5}{3}}\left(\frac{a(t_1)}{a(t)}\right)^3$$

- velocity dispersion

$$\delta \text{v}(t) \sim \frac{1}{m_a\, t_1}\,\frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\text{N} \sim n(t)\,\frac{(2\pi)^3}{\frac{4\pi}{3}(m_a\,\delta \text{v})^3} \sim 10^{61}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{\frac{8}{3}}$$

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why do they do that?

by yielding their energy to the non-condensed particles, the total entropy is increased.

preBEC BEC

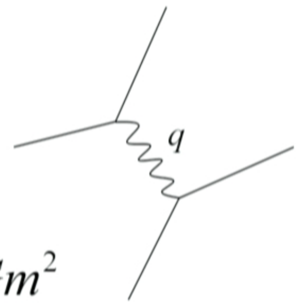
A critical aspect of axion BEC phenomenology is whether the BEC continues to thermalize after it has formed.

Axion BEC means that almost all axions go to one state.

However, only if the BEC continually rethermalizes does the axion state track the lowest energy available state.

Thermalization occurs due to gravitational interactions

PS + Q. Yang, PRL 103 (2009) 111301



$$\frac{Gm^2}{q^2}$$

$$\Gamma_g \sim 4\pi G n m^2 l^2 \quad \text{with } l = (m\,\delta \mathrm{v})^{-1}$$

$$\sim 5 \cdot 10^{-7} H(t_1) \left(\frac{f}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}}$$

at time t_1

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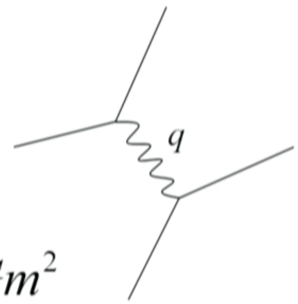

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Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \sim 500 \text{ eV} \left(\frac{f}{10^{12} \text{ GeV}}\right)^{\frac{1}{2}}$$

After that

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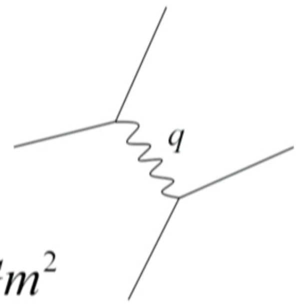

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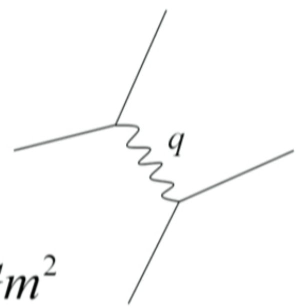

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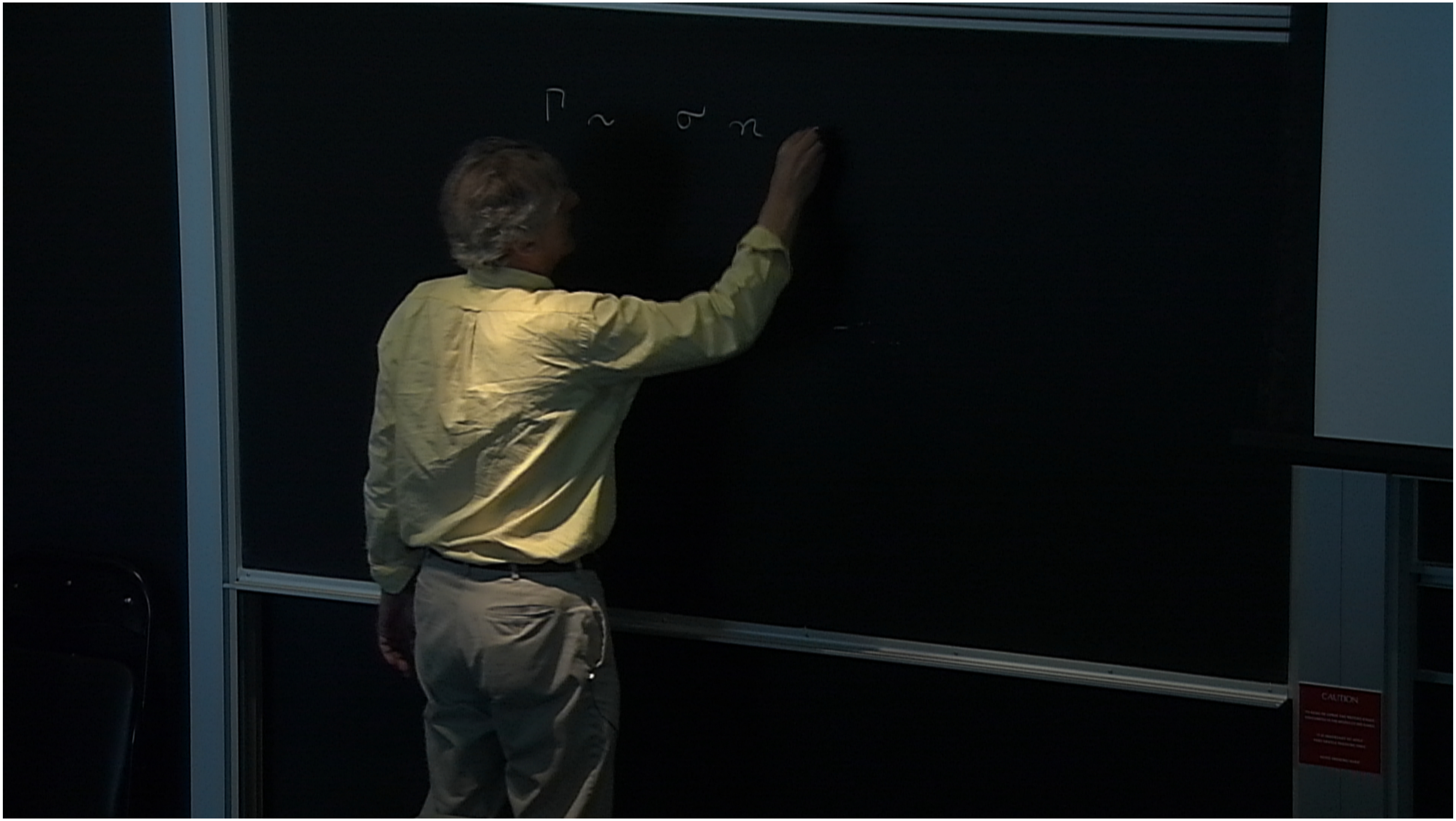
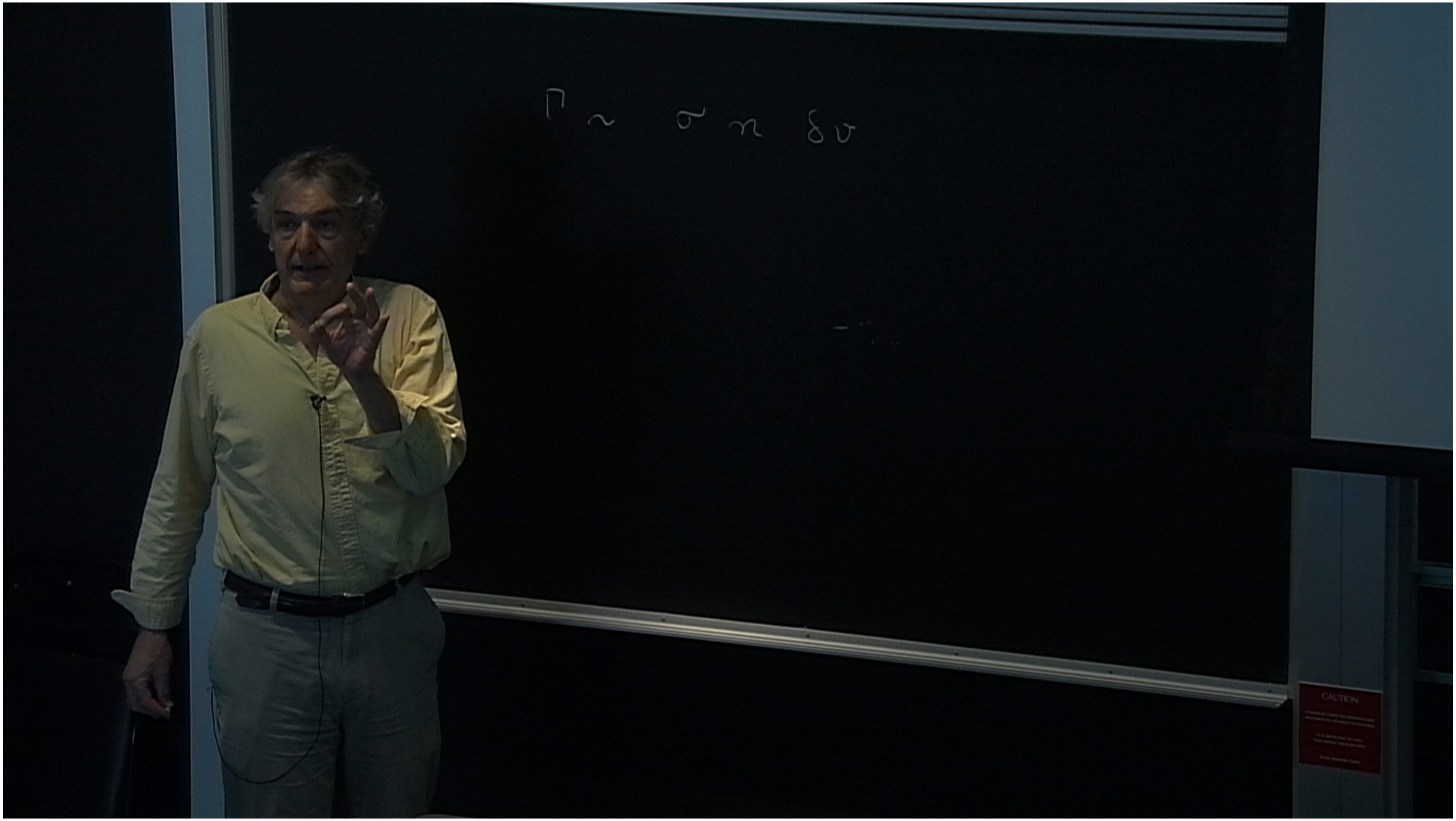
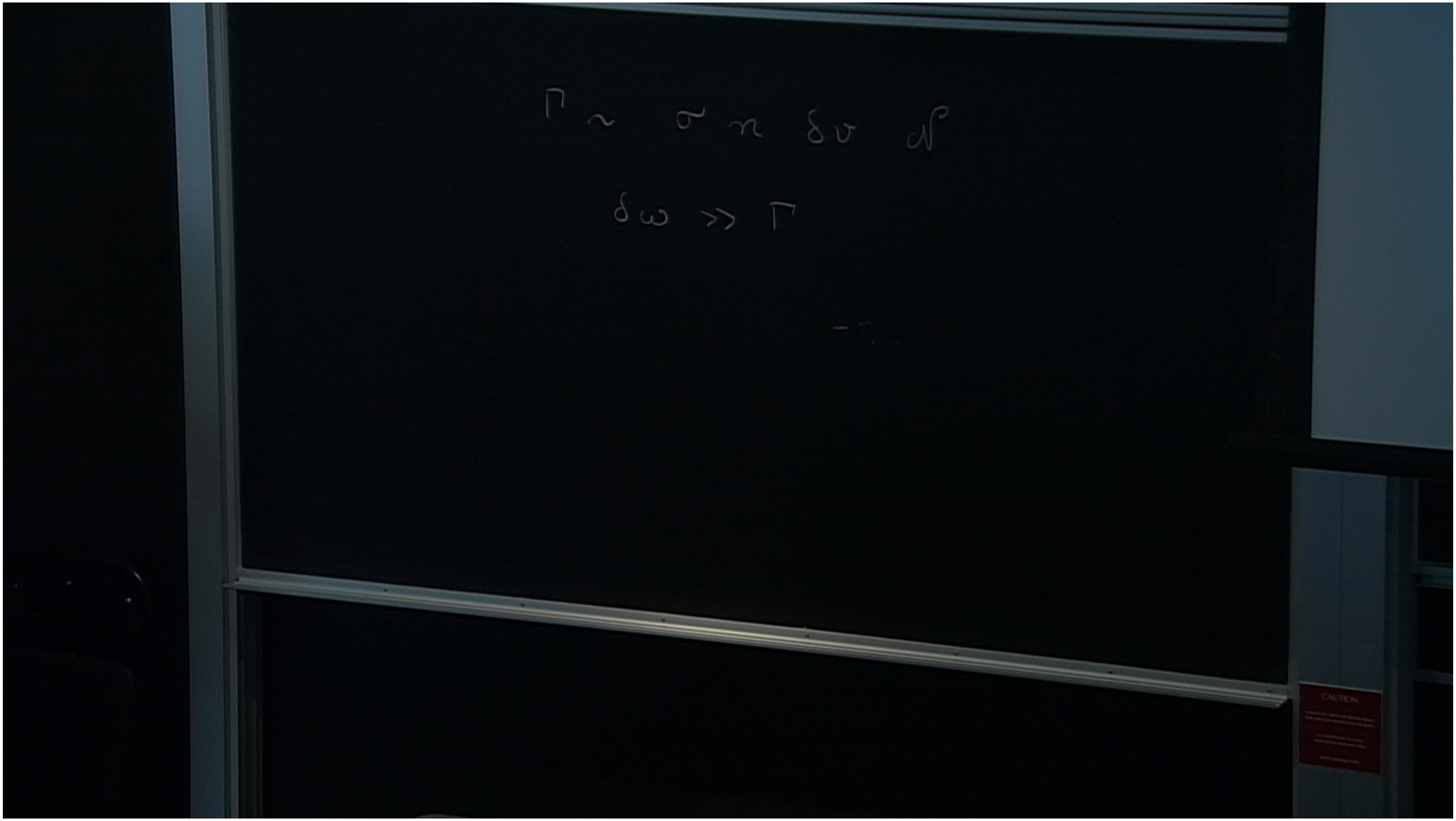
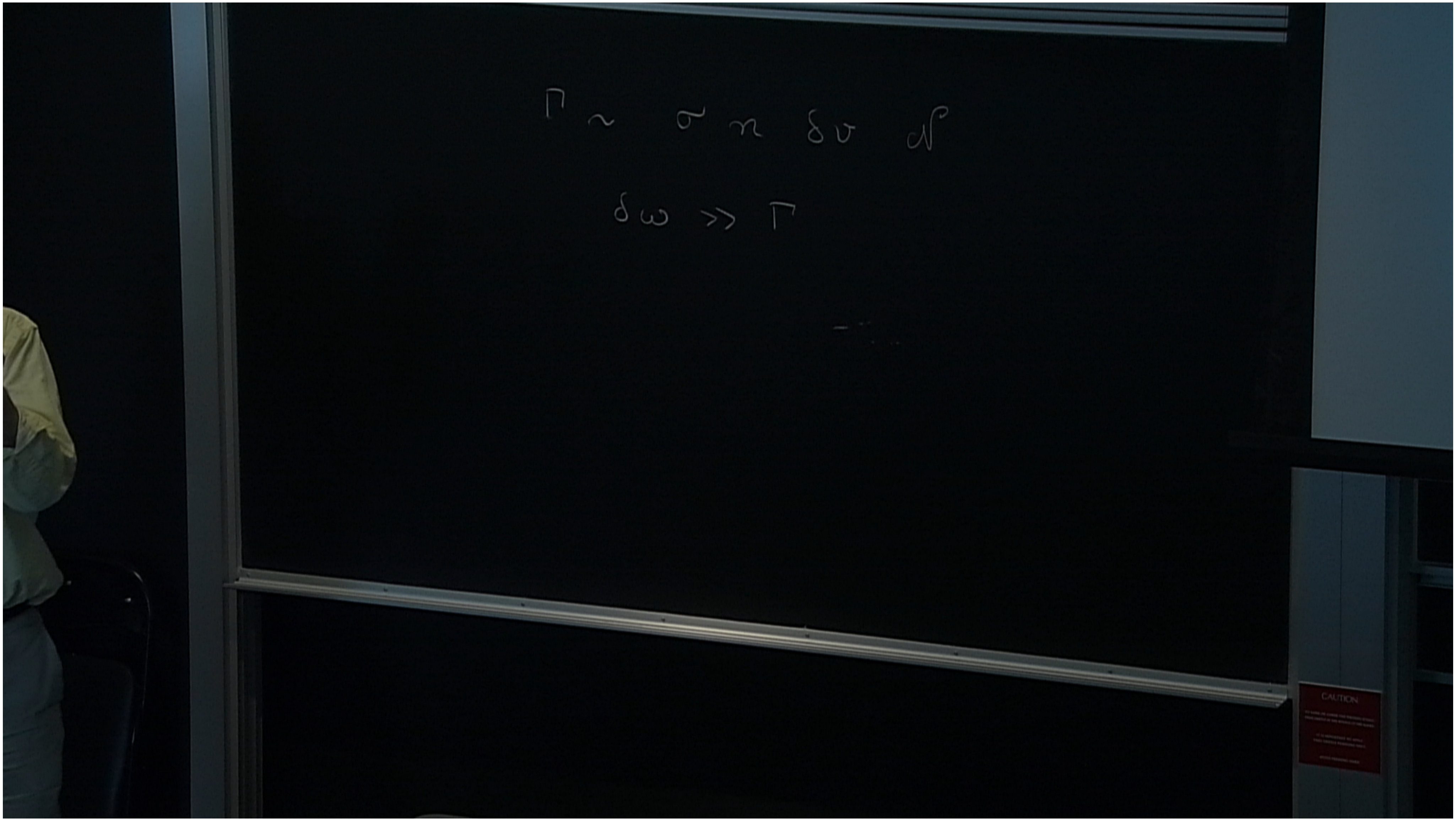


Γ ~ σ n


Γ ~ σ n δv


Γ ~ σ n δv N
δω >> Γ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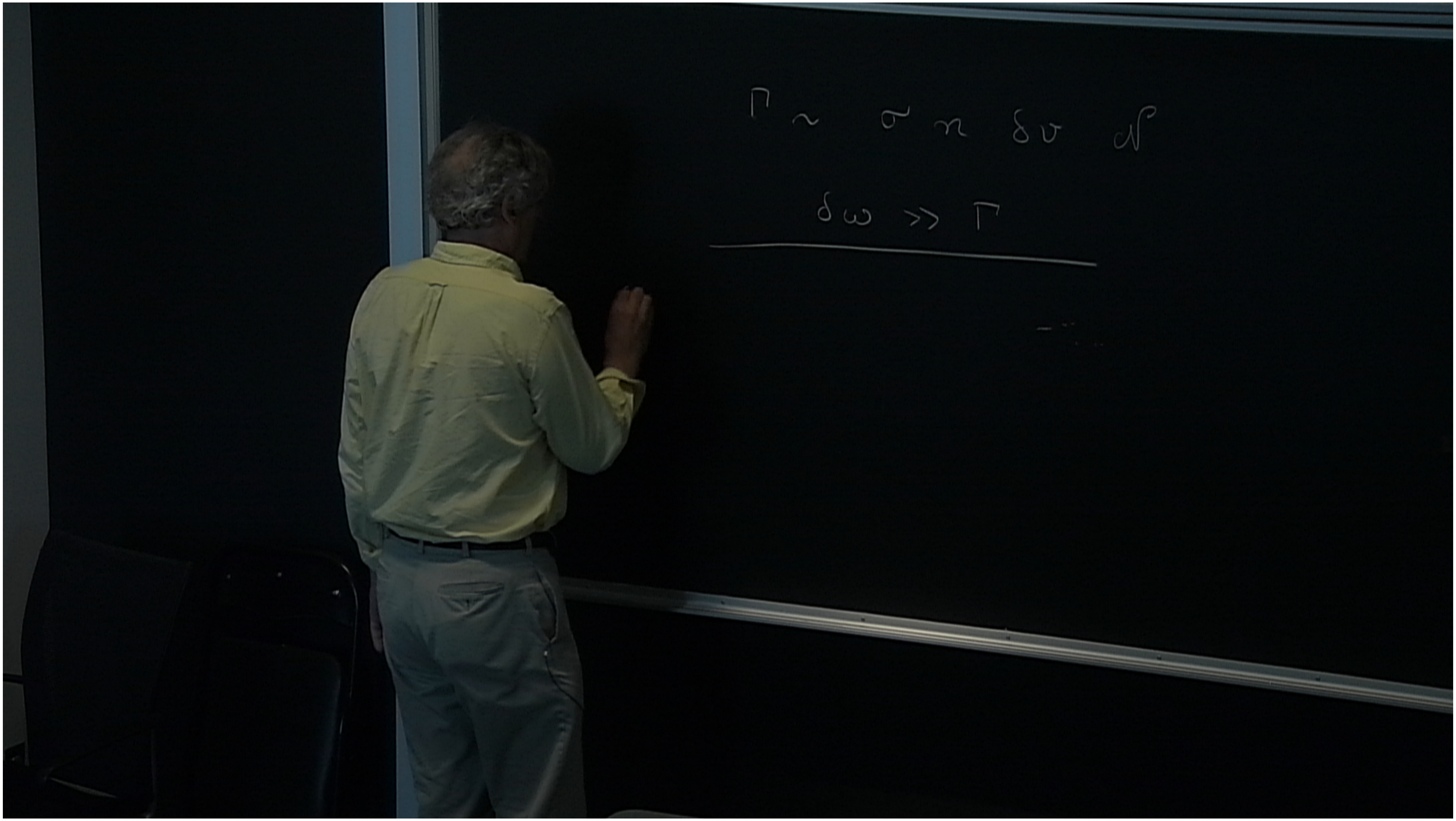
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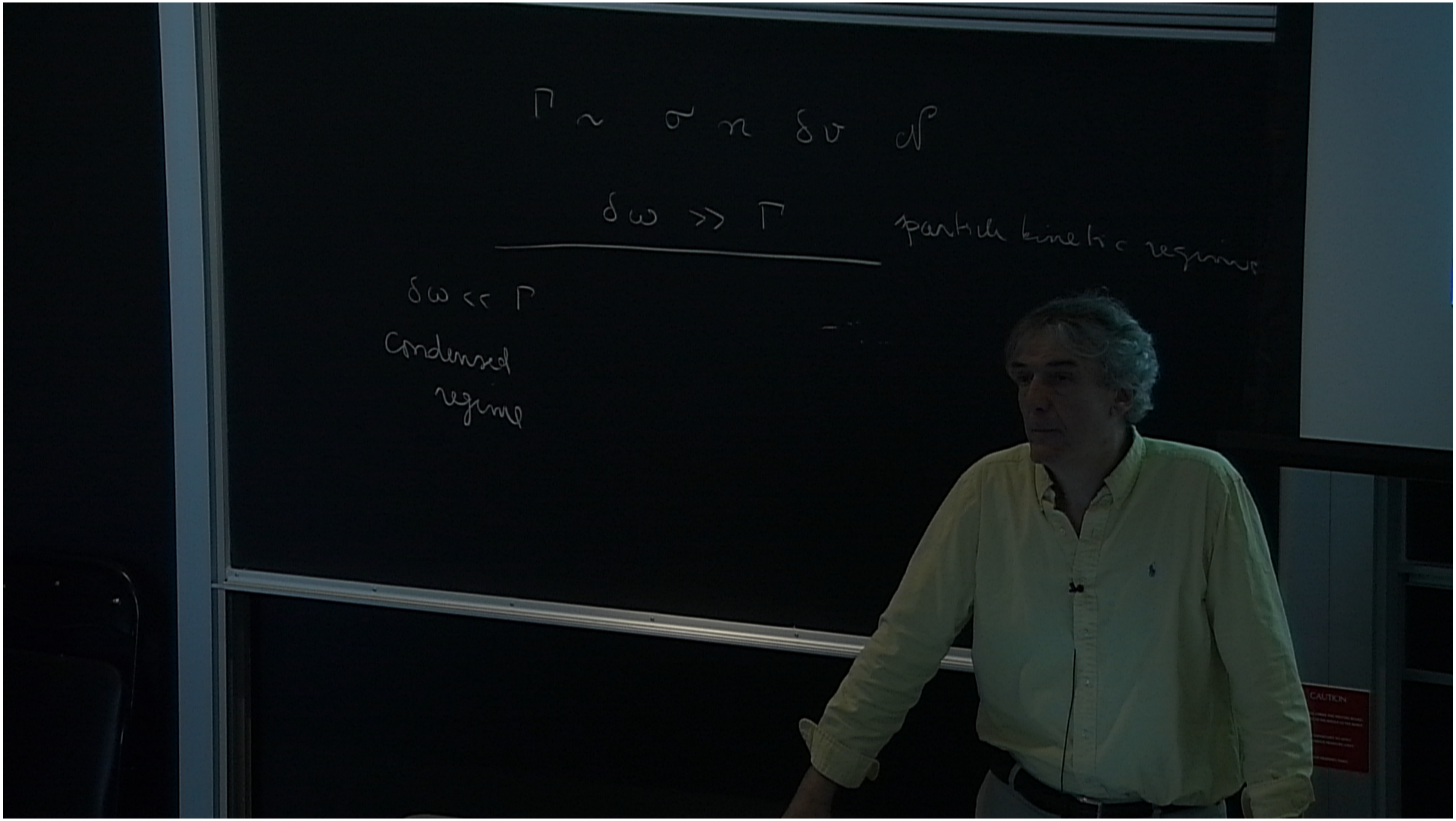
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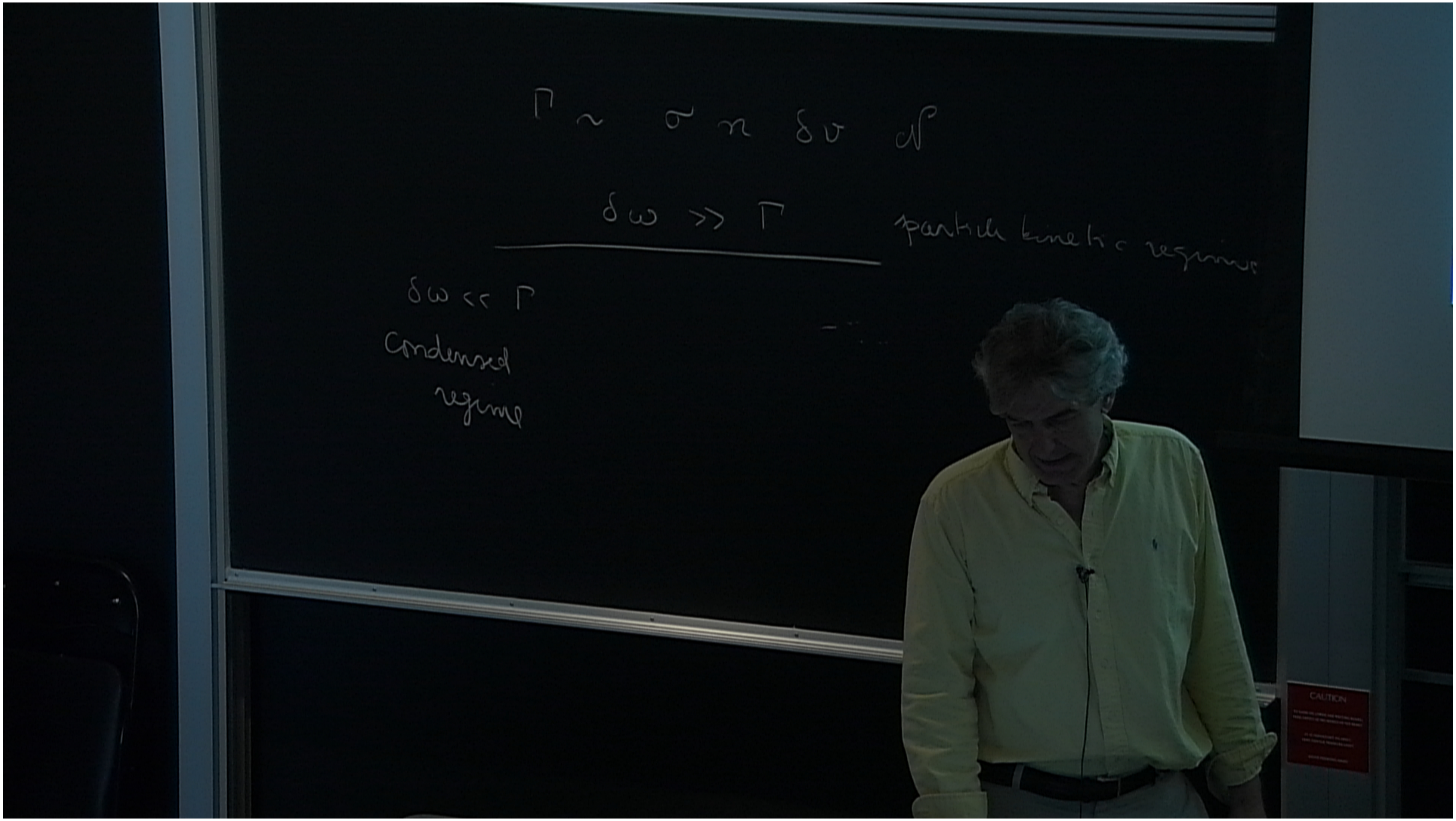
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In the linear regime, within the horizon, the axion BEC remains in the same state

axion BEC density perturbations obey

$$\partial_t^{\,2}\,\delta(\vec{k},t) + 2H\partial_t\delta(\vec{k},t) - \left(4\pi G\rho_0 - \frac{k^4}{4m^2a^4}\right)\delta(\vec{k},t) = 0$$

Jeans' length

$$\ell_J = \left(16\pi G\rho m^2\right)^{-\frac{1}{4}} = 1.02\cdot 10^{14}\ \mathrm{cm}\left(\frac{10^{-5}\ \mathrm{eV}}{m}\right)^{\frac{1}{2}}\left(\frac{10^{-29}\ \mathrm{g/cc}}{\rho}\right)^{\frac{1}{4}}$$

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In the linear regime within the horizon, axion BEC and CDM are indistinguishable on all scales of observational interest,

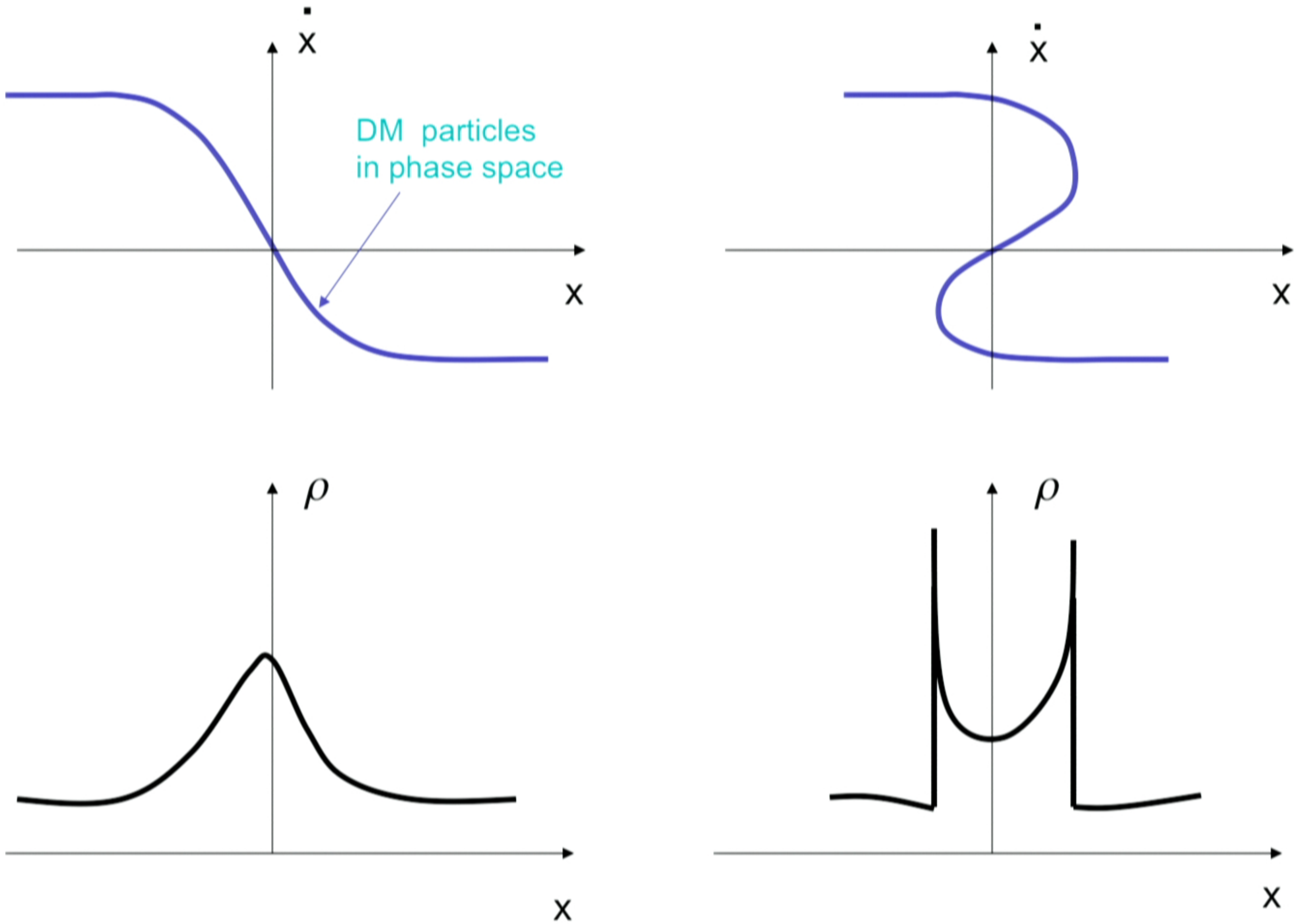
but

axion BEC differs from CDM when it rethermalizes

in the non-linear regime &

upon entering the horizon

DM forms caustics in the non-linear regime



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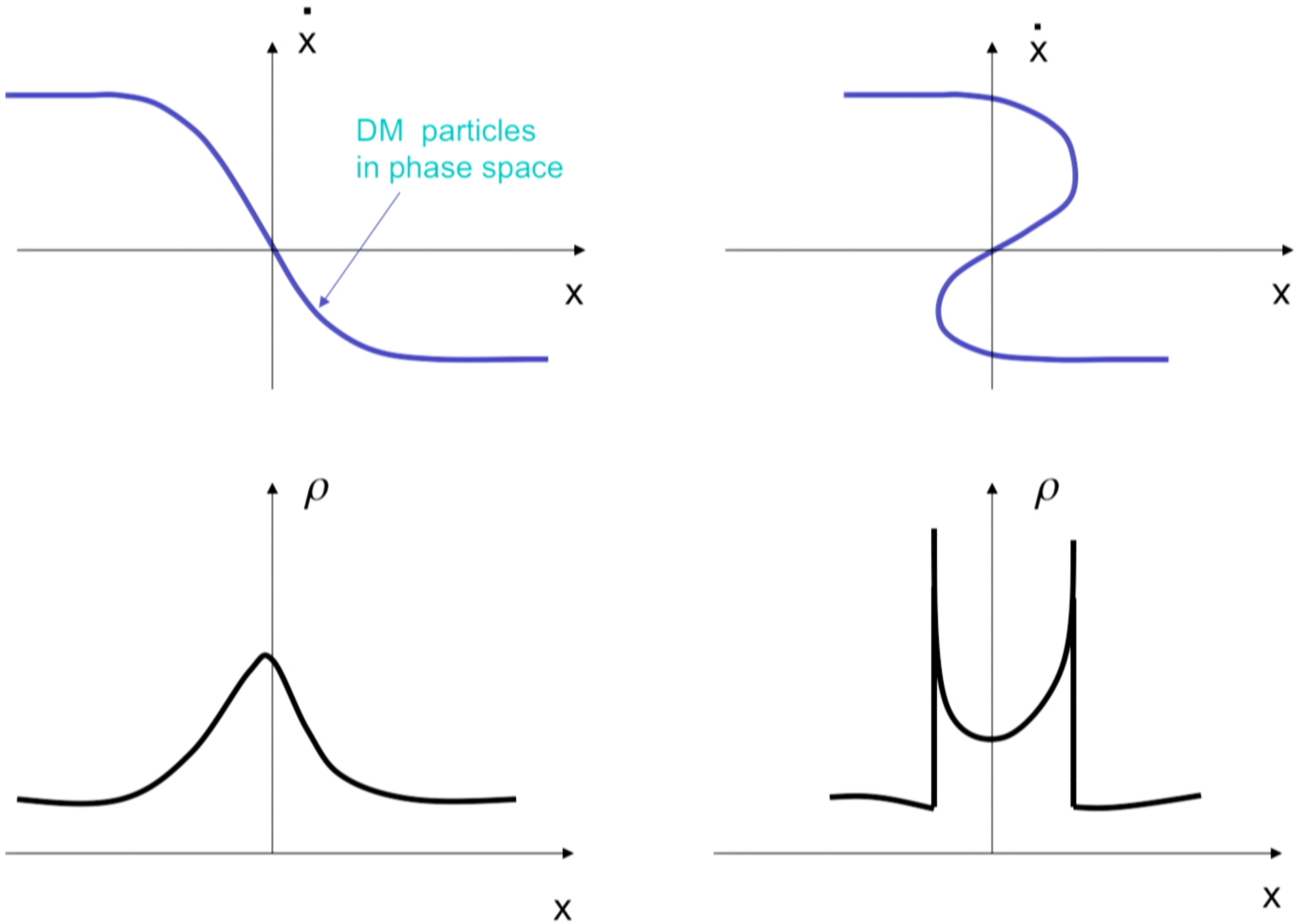
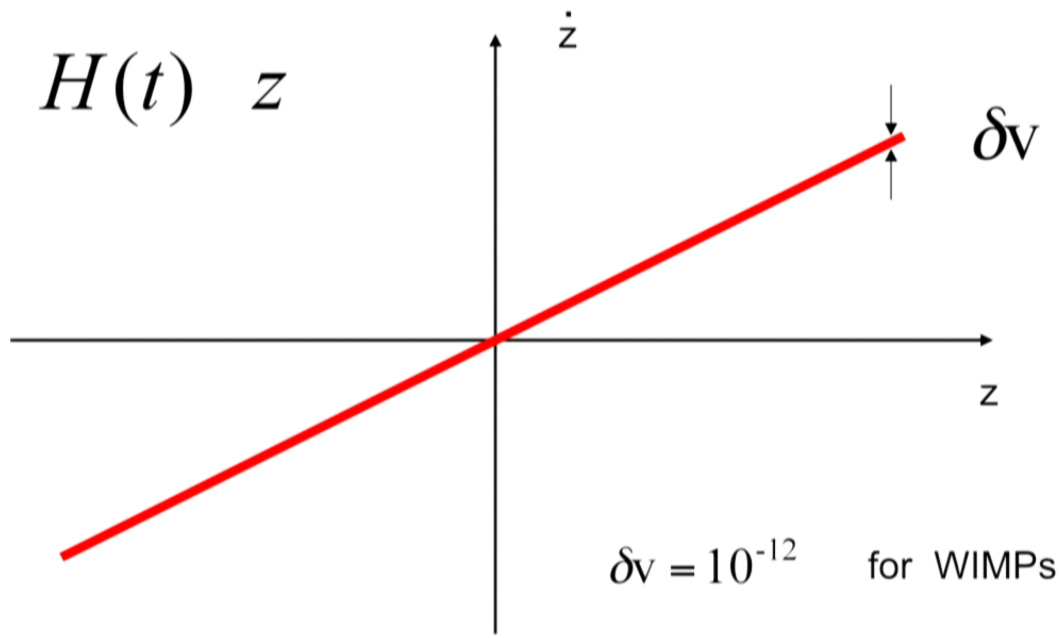


Phase space distribution of DM in a homogeneous universe

$$\dot{z} = H(t)\ z$$



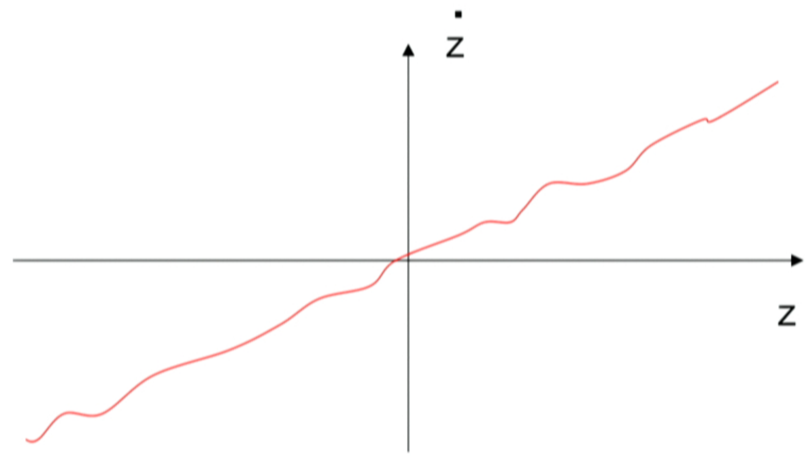
$\delta v = 10^{-12}$ for WIMPs

$\delta v = 10^{-17}$ for axions (preBEC)

$\delta v = 10^{-8}$ for sterile neutrinos

The dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase space

the physical density is the projection of the phase space sheet onto position space



$$\vec{v}(\vec{r},t) = H(t)\vec{r} + \Delta\vec{v}(\vec{r},t)$$

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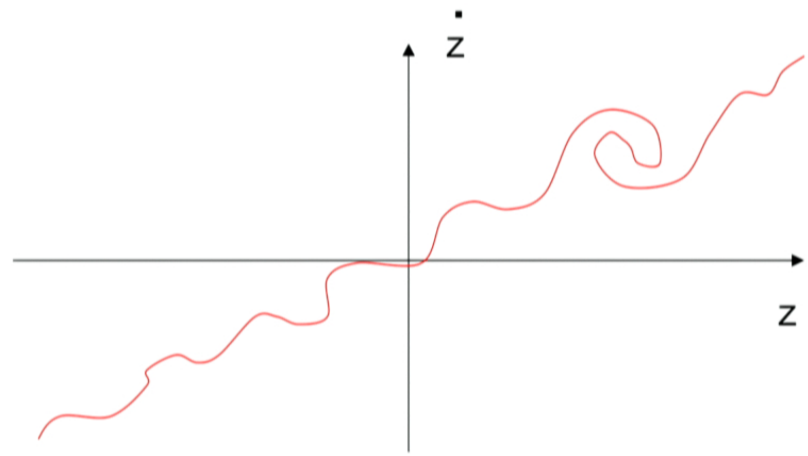
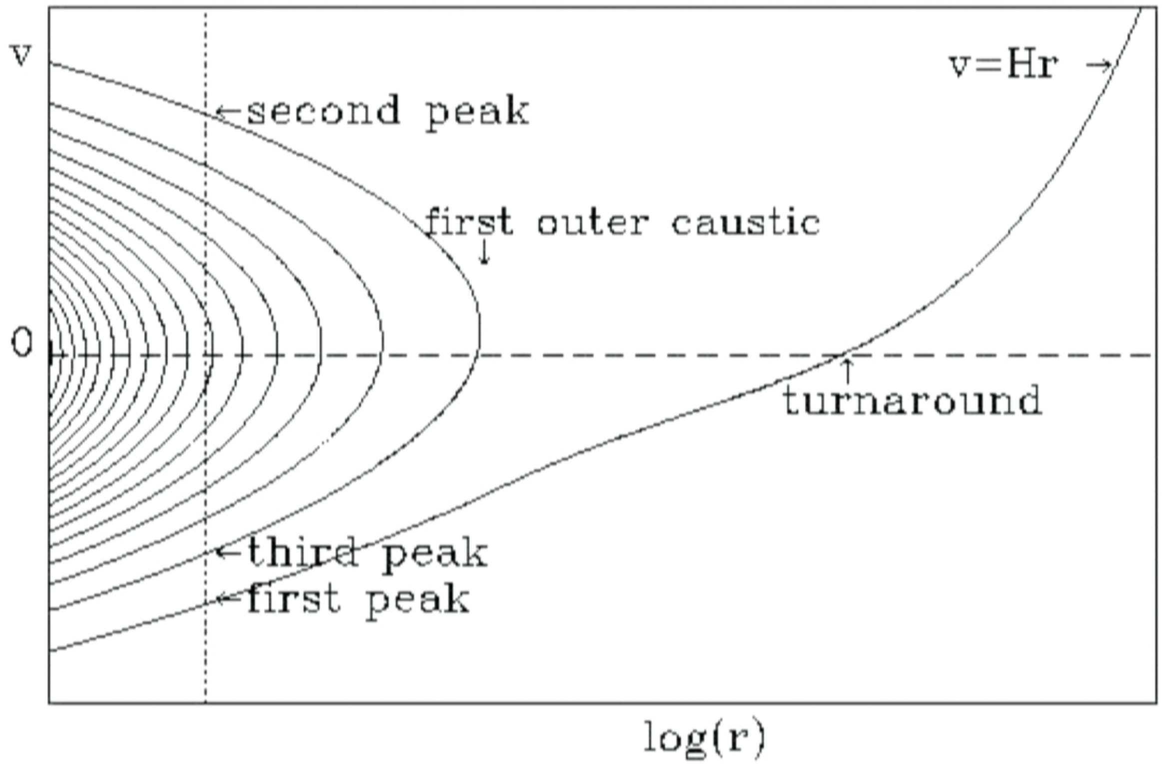


Multiple flows and caustics occur in the non linear regime

Phase space structure of spherically symmetric halos



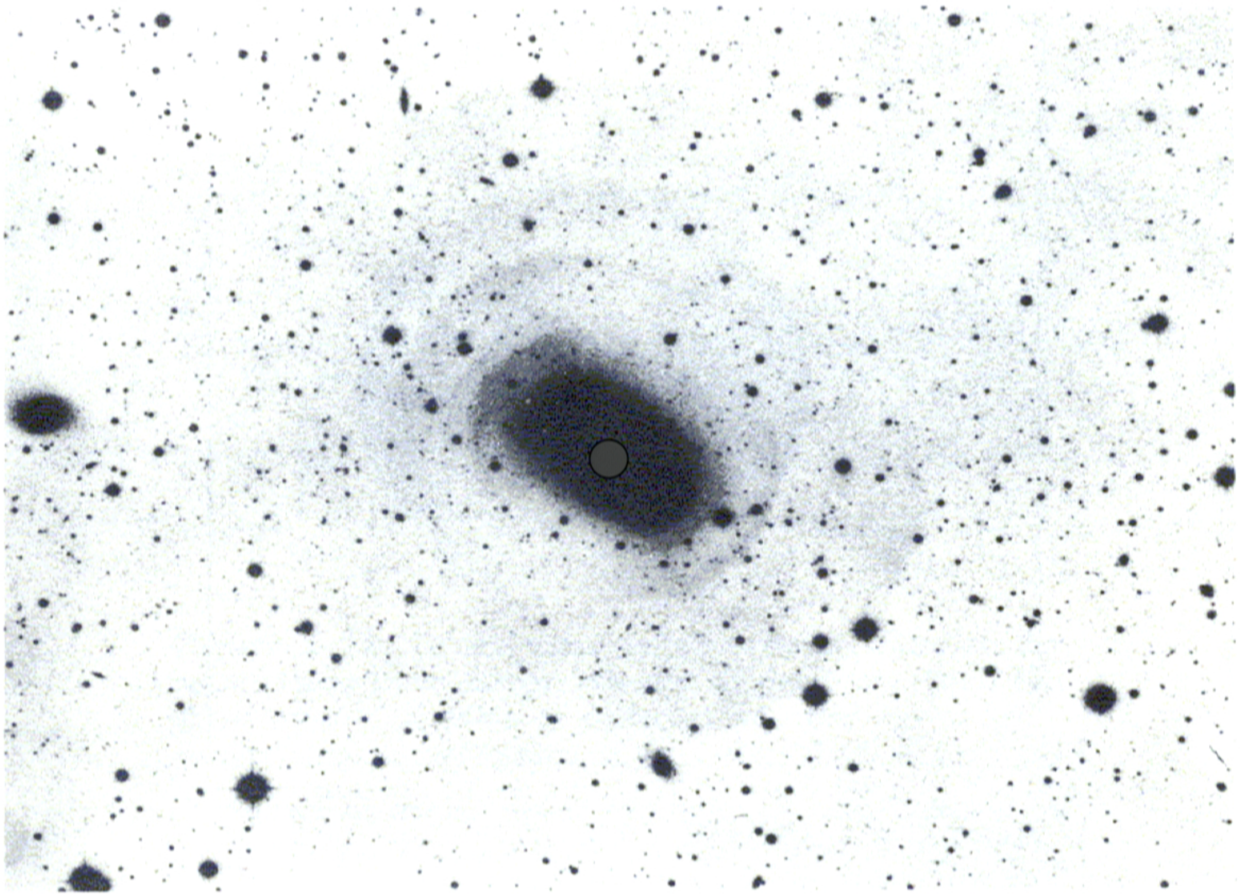


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)

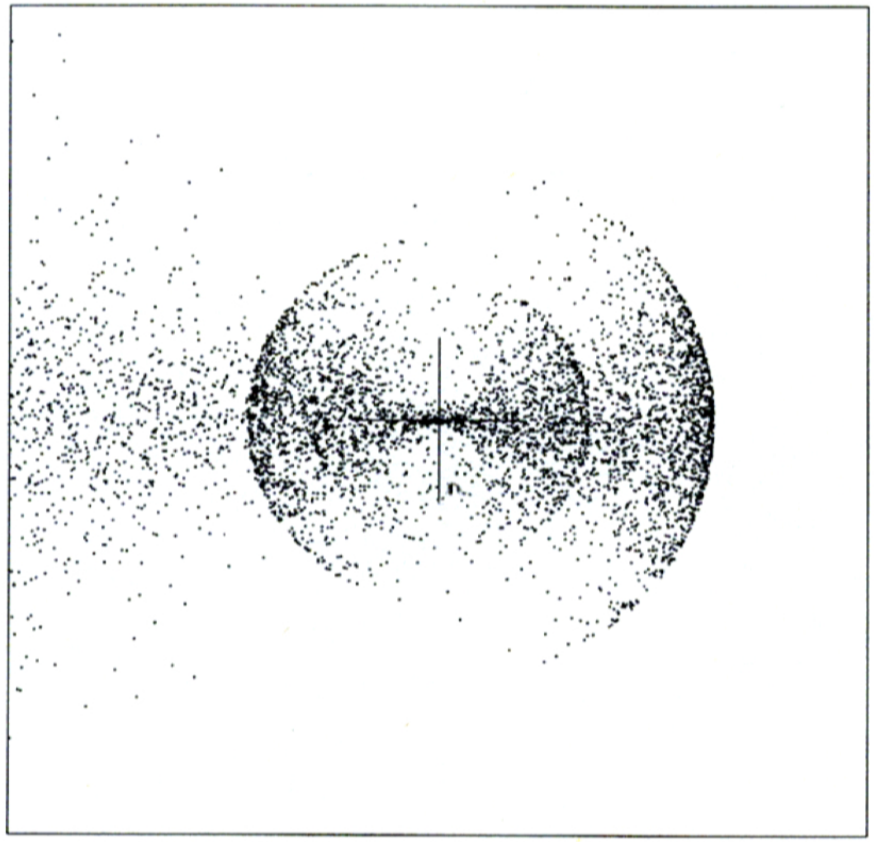


Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)

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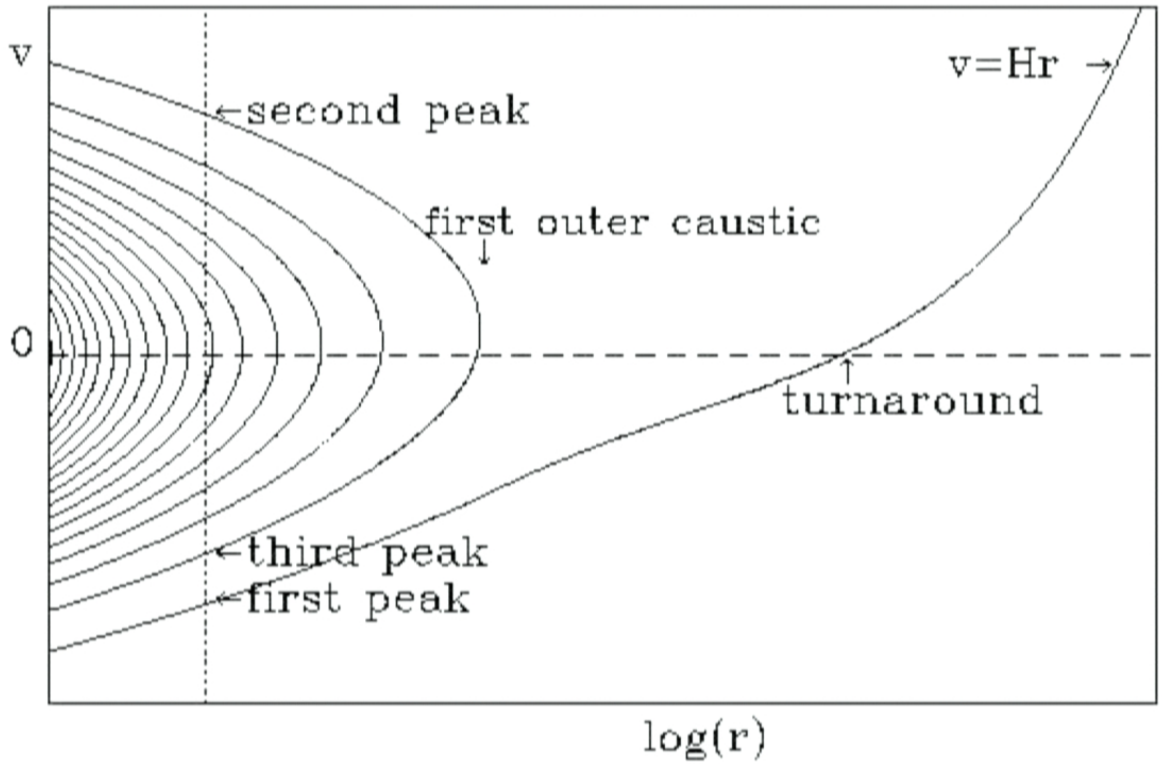


Galactic halos have inner caustics as well as outer caustics.

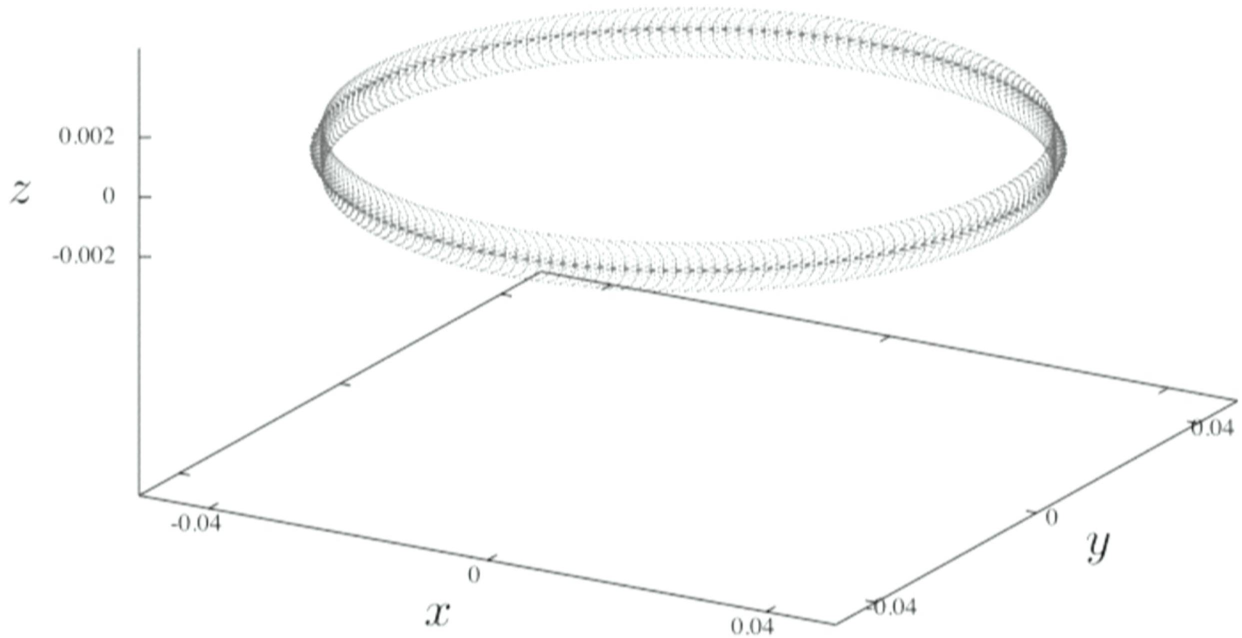
If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

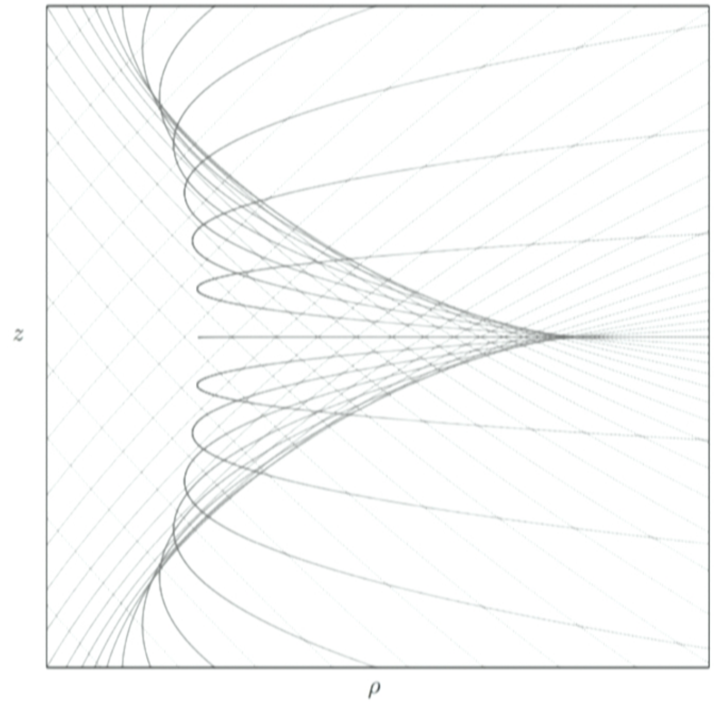
(Arvind Natarajan and PS, 2005).

simulations by Arvind Natarajan

in case of net overall rotation



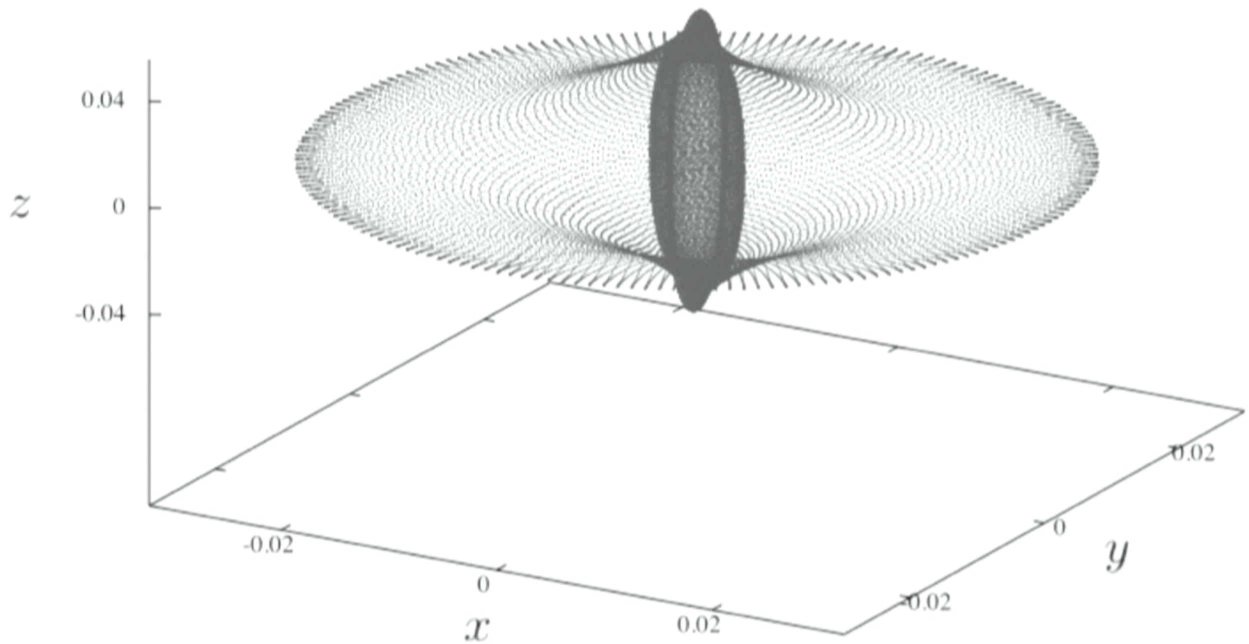
The caustic ring cross-section



D_{-4}

an elliptic umbilic catastrophe

in case of irrotational flow



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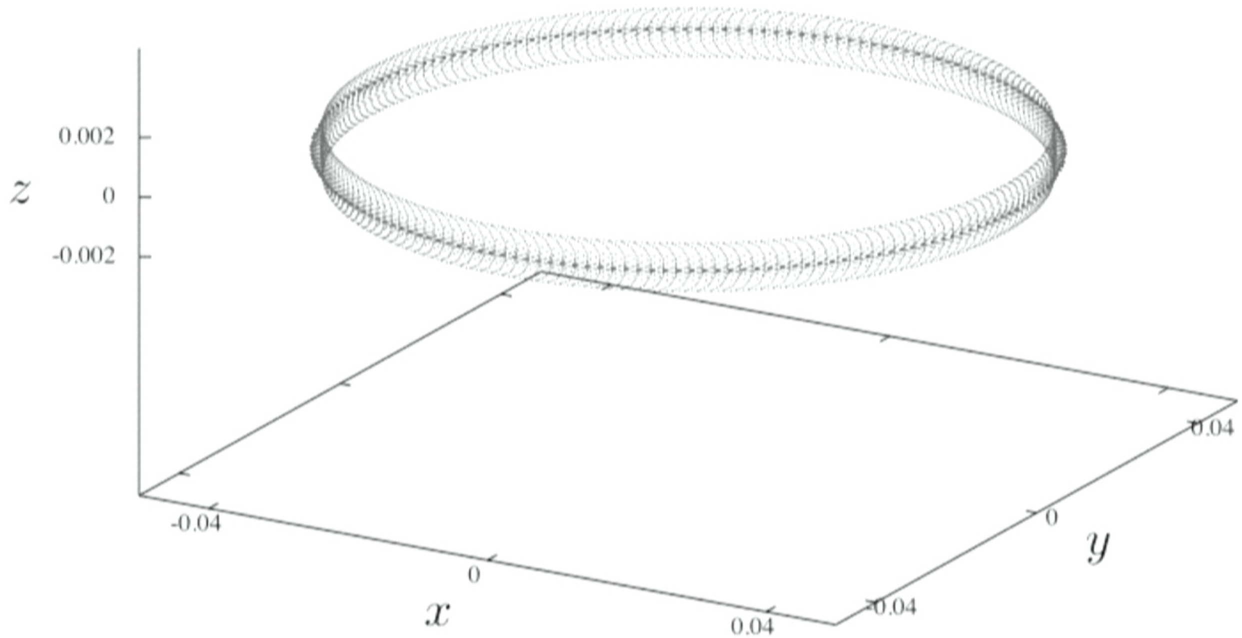

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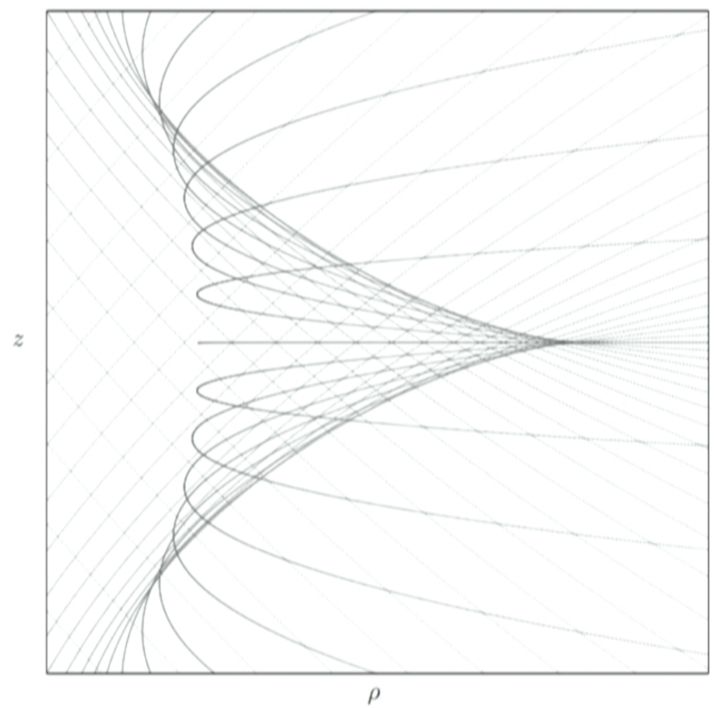

D_{-4}

an elliptic umbilic catastrophe

On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

in the galactic plane

with radii $(n = 1, 2, 3 \ldots)$

$$a_n = \frac{40\text{kpc}}{n}\left(\frac{v_{rot}}{220\text{km/s}}\right)\left(\frac{j_{max}}{0.18}\right)$$

$j_{max} \cong 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

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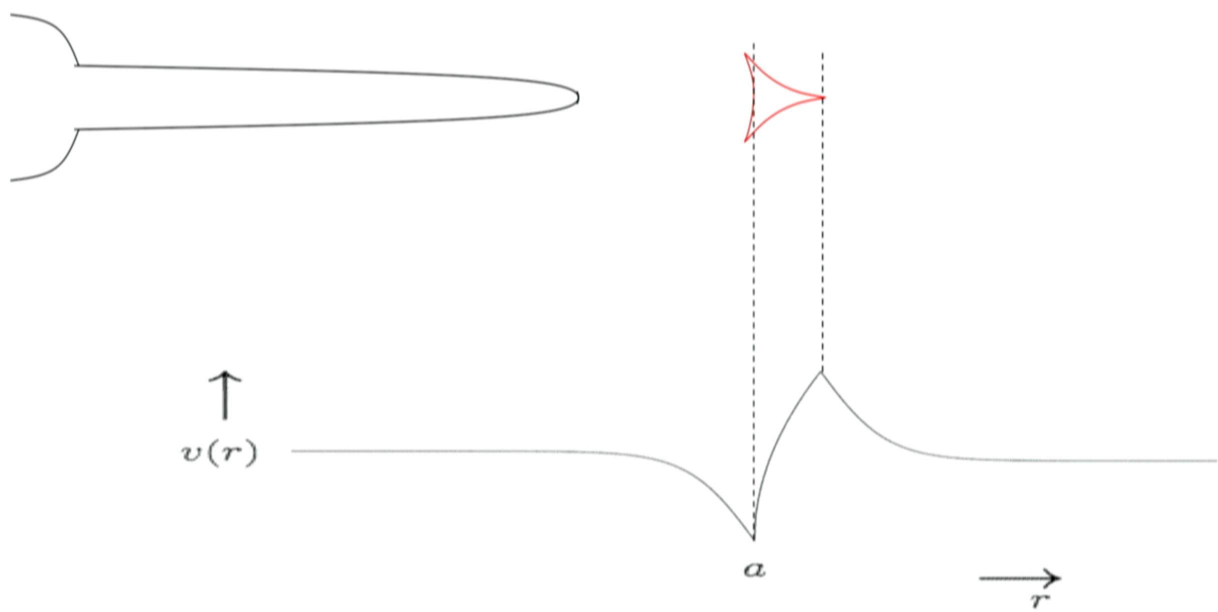
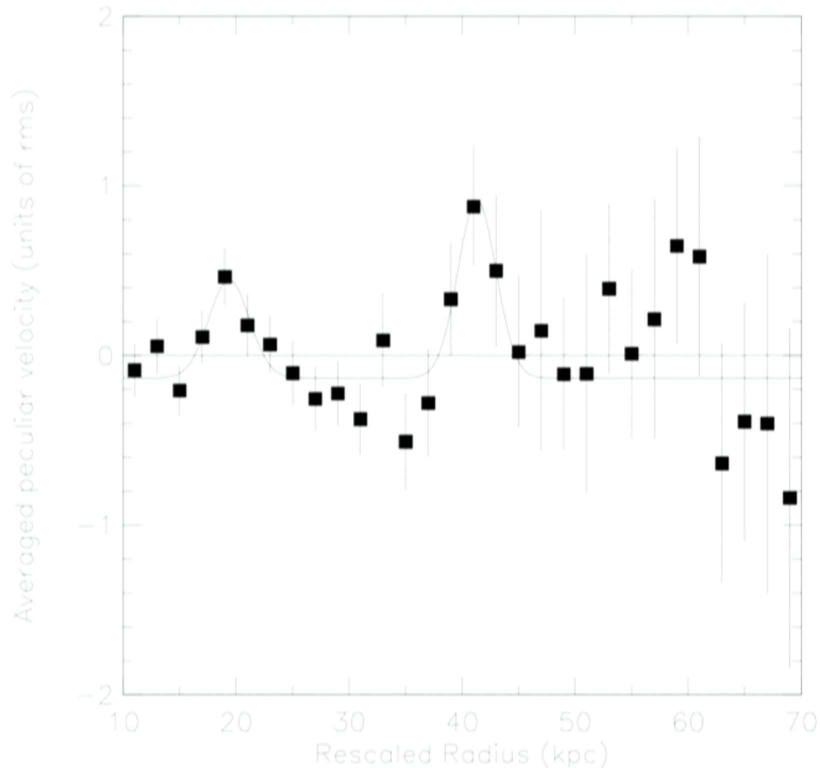
Effect of a caustic ring of dark matter upon the galactic rotation curve

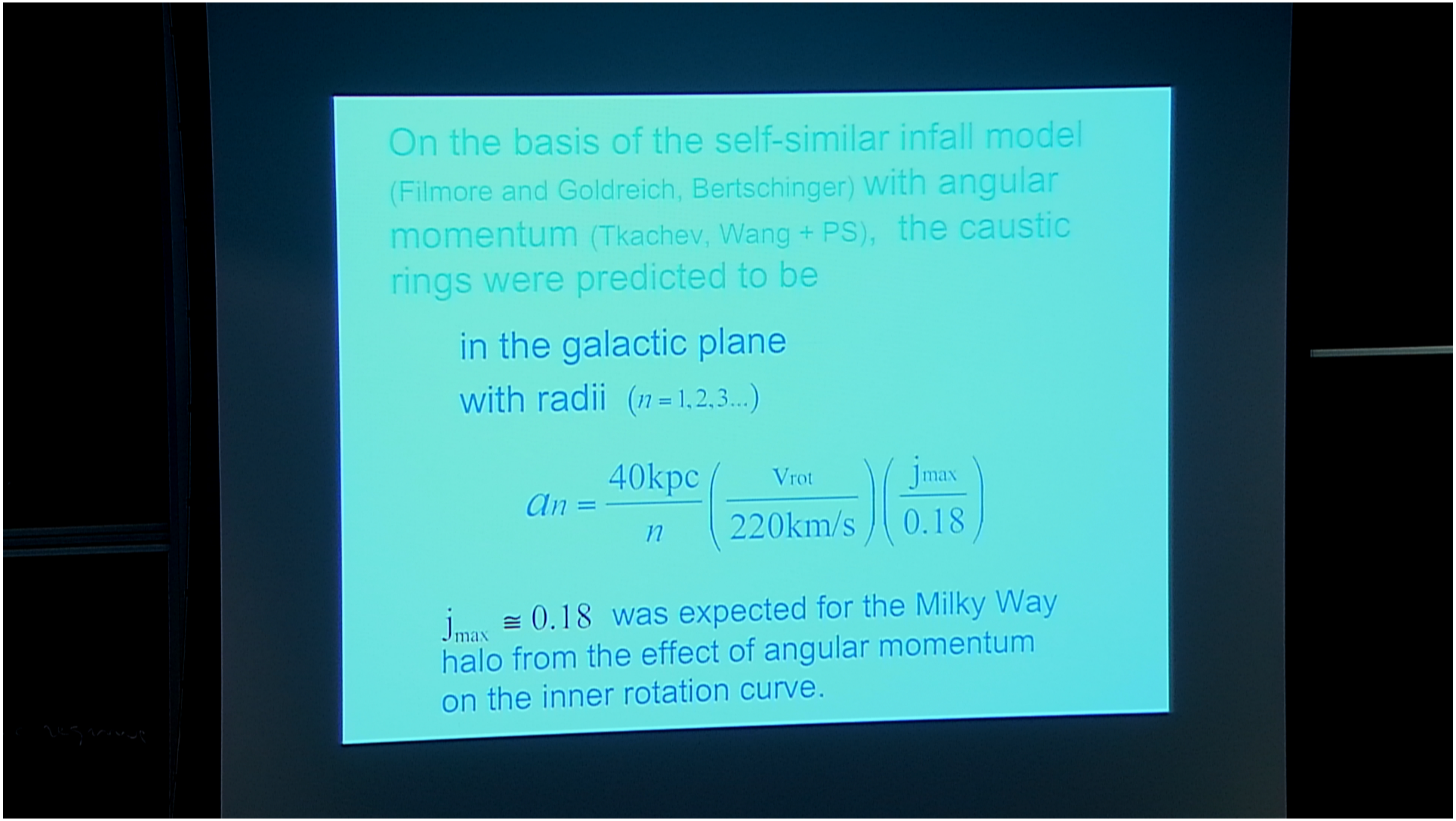


Composite rotation curve

(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy

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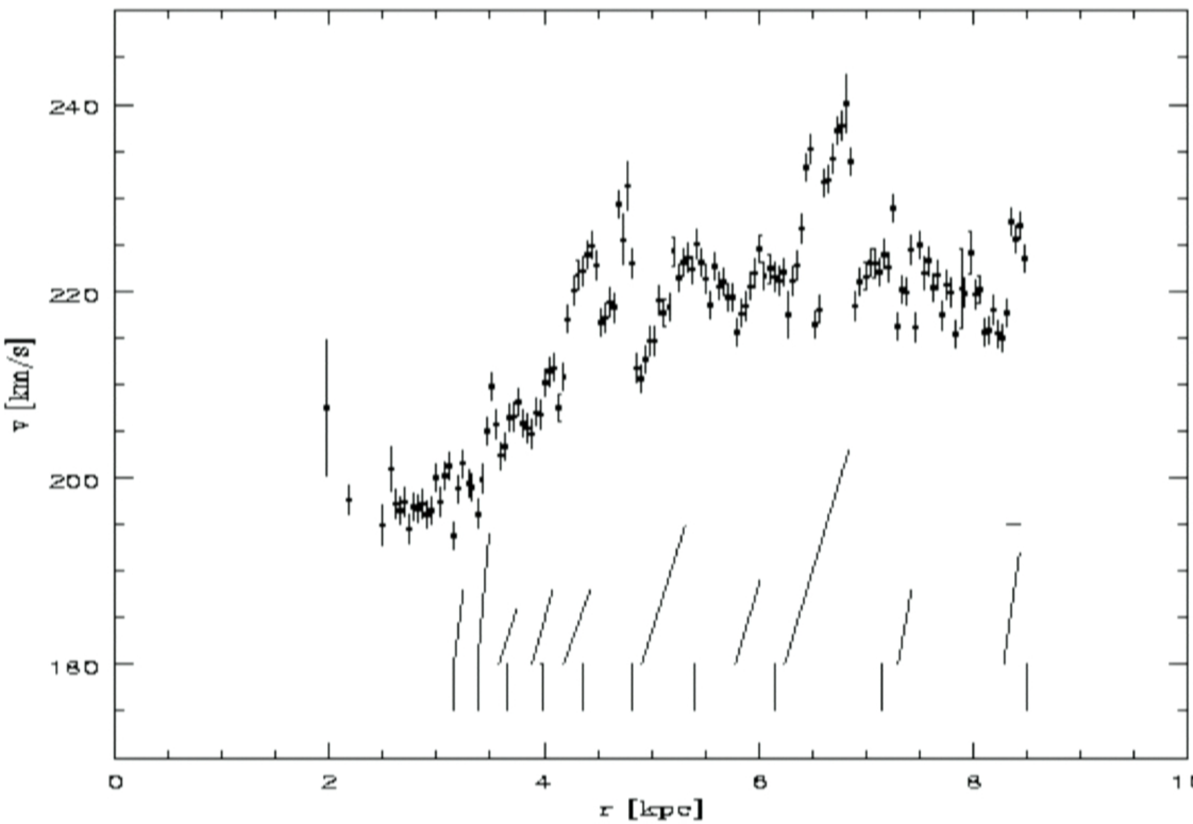
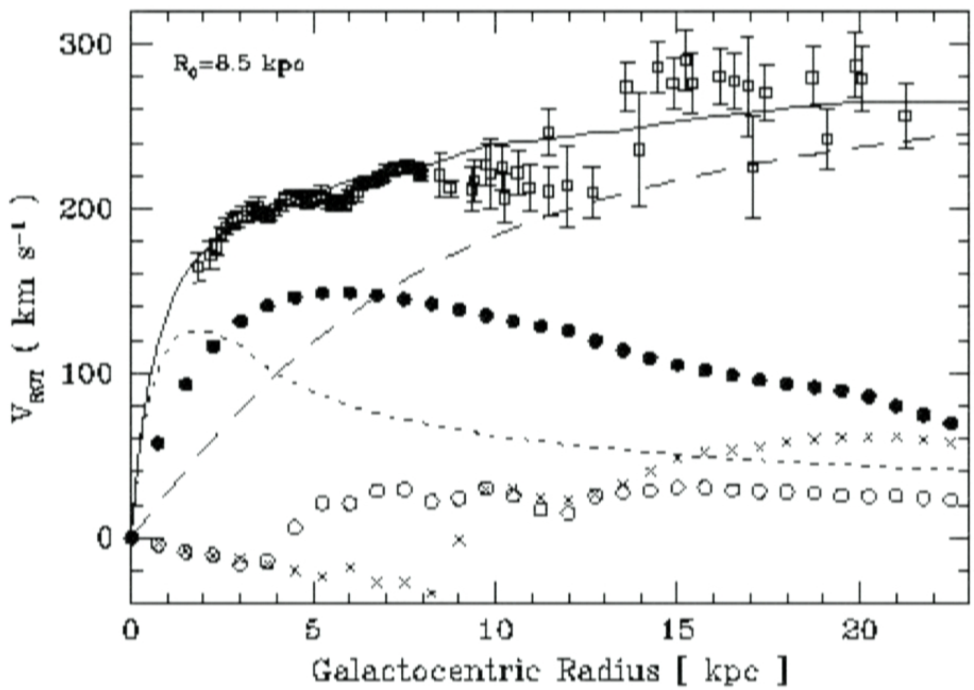
Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)

Outer Galactic rotation curve



R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Monoceros Ring of stars

H. Newberg et al. 2002; B. Yanny et al., 2003; R.A. Ibata et al., 2003;
H.J. Rocha-Pinto et al, 2003; J.D. Crane et al., 2003; N.F. Martin et al., 2005

in the Galactic plane
at galactocentric distance $r \simeq 20\,\text{kpc}$
appears circular, actually seen for $100^0 < l < 270^0$
scale height of order 1 kpc
velocity dispersion of order 20 km/s

may be caused by the n = 2 caustic ring of dark matter (A. Natarajan and P.S. '07)

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Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846

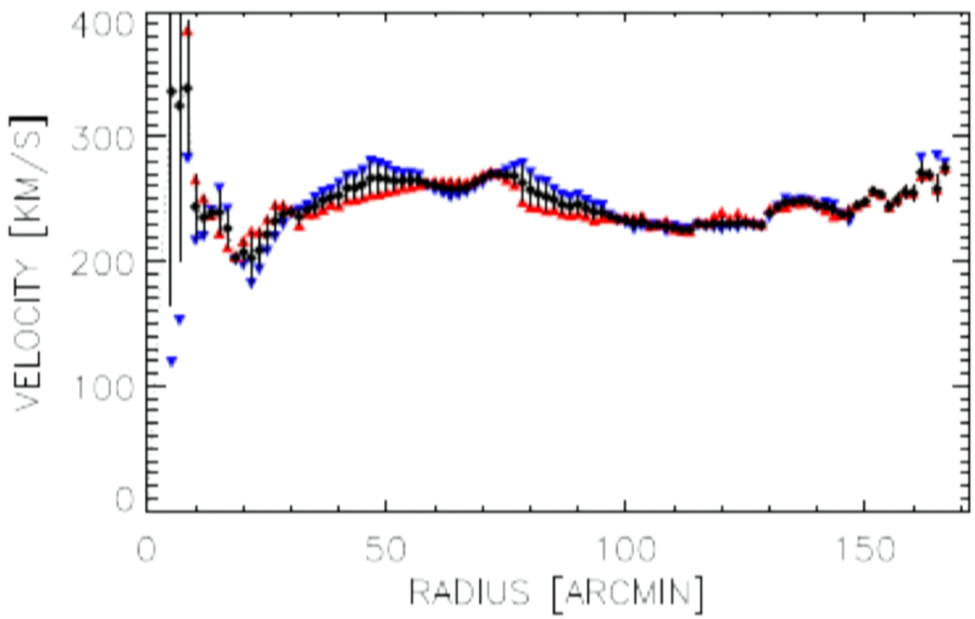
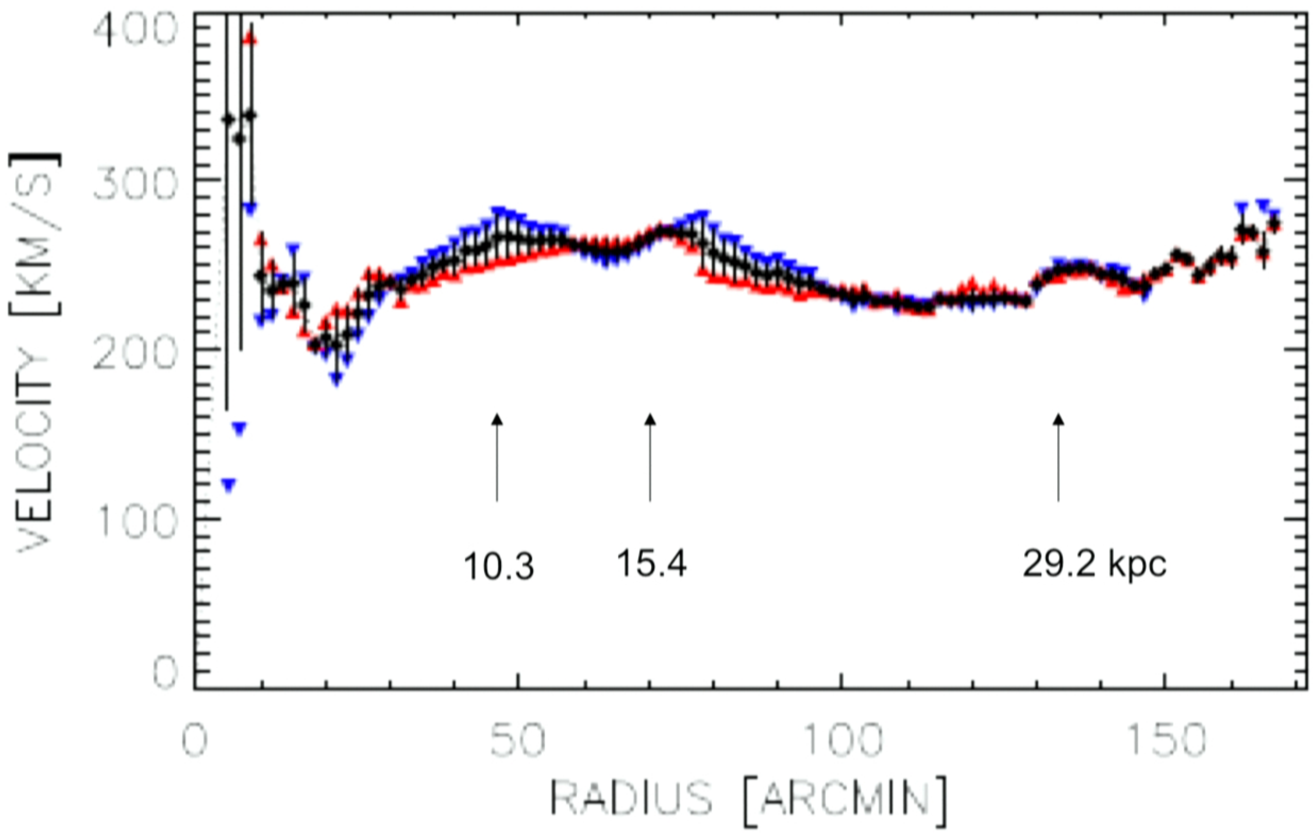
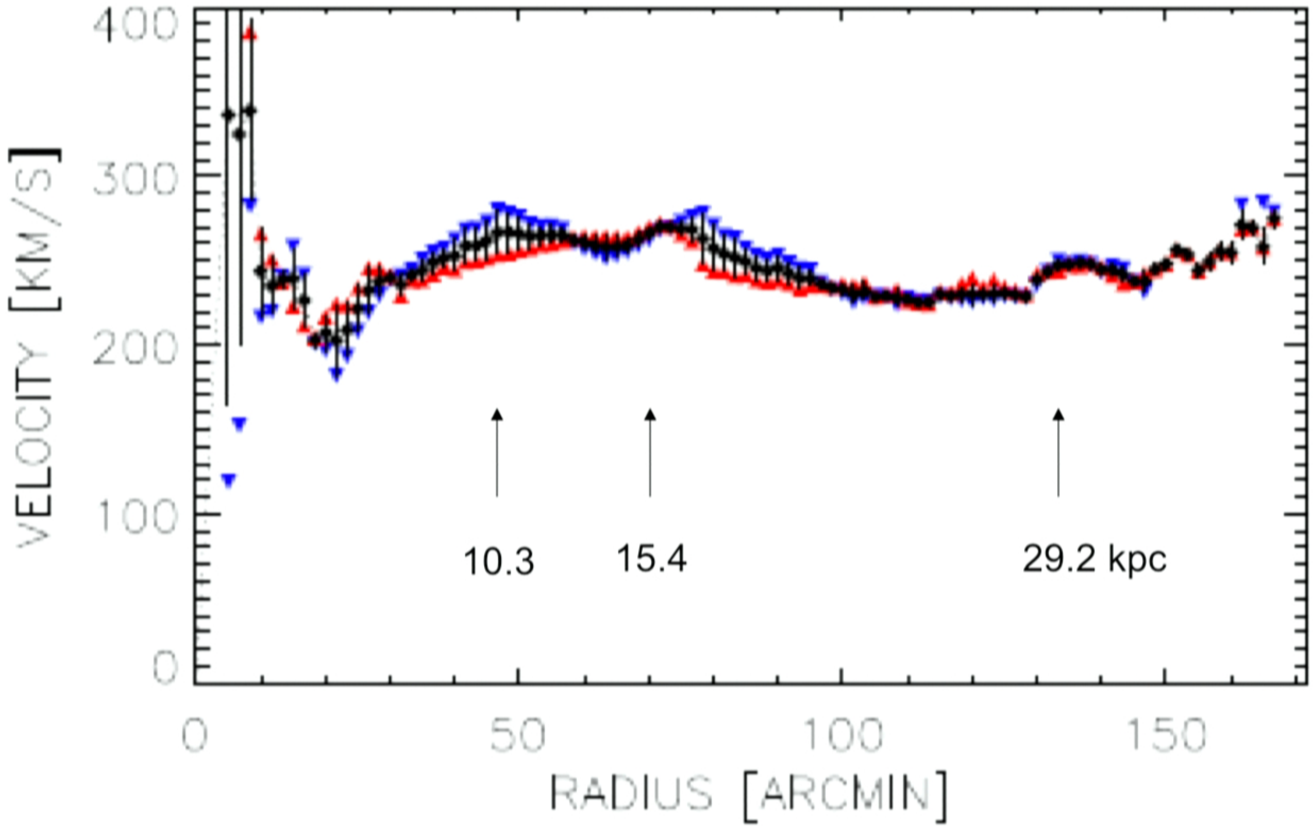


FIG. 10.— HI rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



10 arcmin = 2.2 kpc



10 arcmin = 2.2 kpc

The caustic ring halo model assumes

- net overall rotation
- axial symmetry
- self-similarity

The specific angular momentum distribution on the turnaround sphere

$$\vec{\ell}(\hat{n},t) = j_{\max} \; \hat{n}\times(\hat{z}\times\hat{n}) \; \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

$$0.25 < \varepsilon < 0.35$$

Is it plausible in the context of tidal torque theory?

The specific angular momentum distribution on the turnaround sphere

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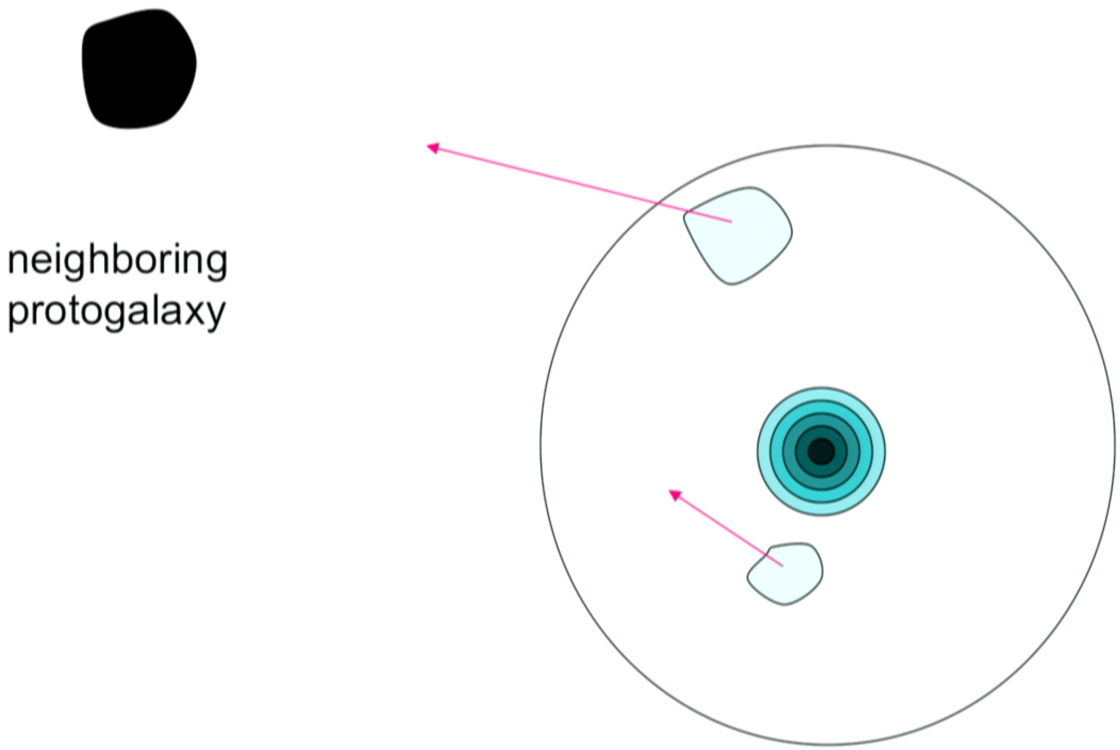
Tidal torque theory



Stromberg 1934; Hoyle 1947; Peebles 1969, 1971

For collisionless particles

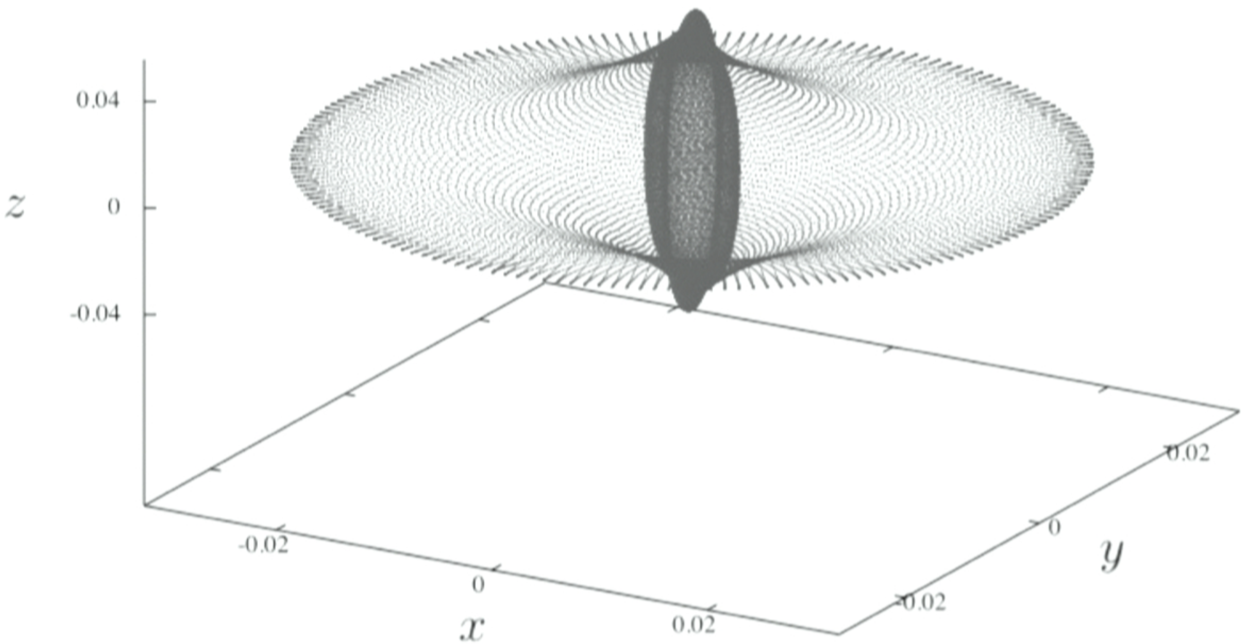
$$\frac{d\,\vec{\mathrm{v}}}{dt}(\vec{r},t) = \frac{\partial\,\vec{\mathrm{v}}}{\partial t}(\vec{r},t) + \left(\vec{\mathrm{v}}(\vec{r},t)\cdot\vec{\nabla}\right)\vec{\mathrm{v}}(\vec{r},t)$$

$$= -\vec{\nabla}\Phi(\vec{r},t)$$

If $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ initially,

then $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ for ever after.

in case of irrotational flow



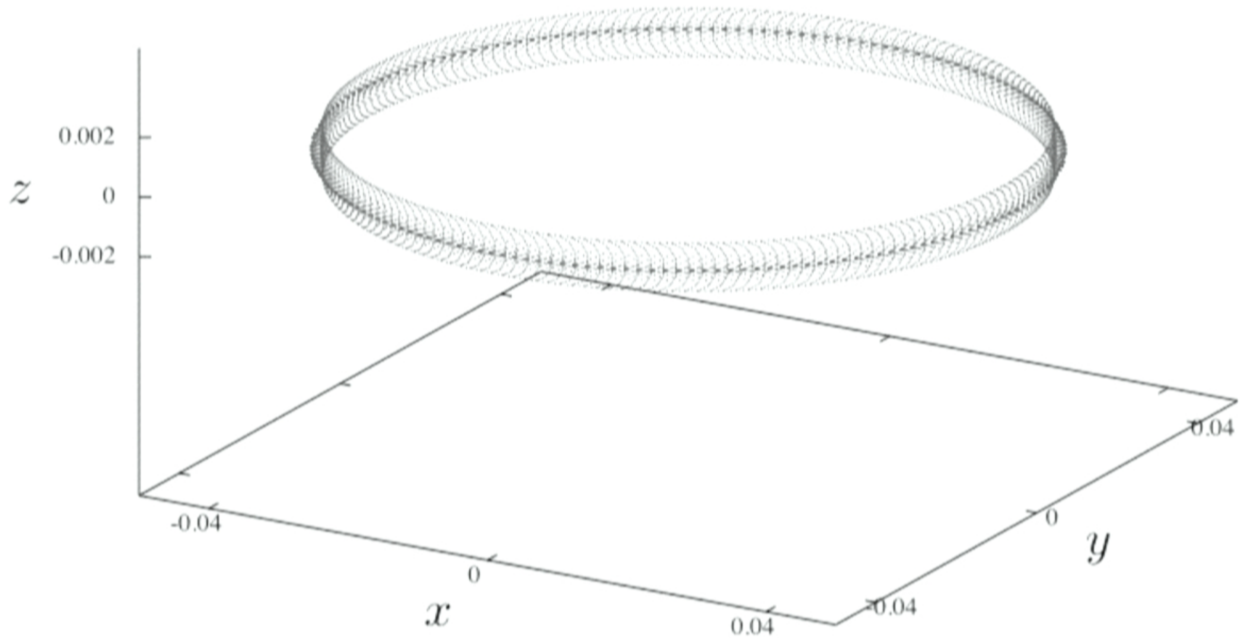
For axion BEC

$$E = \sum_{i=1}^{N} \frac{L_i^{\ 2}}{2I}$$

is minimized for given $L = \sum_{i=1}^{N} L_i$

when $L_1 = L_2 = L_3 = \ \dots \ = L_N$.

in case of net overall rotation



The specific angular momentum distribution on the turnaround sphere

$$\vec{\ell}(\hat{n},t) = j_{\max}\ \hat{n}\times(\hat{z}\times\hat{n})\ \frac{R(t)^2}{t}$$

$$R(t) \propto t^{\frac{2}{3}+\frac{2}{9\varepsilon}}$$

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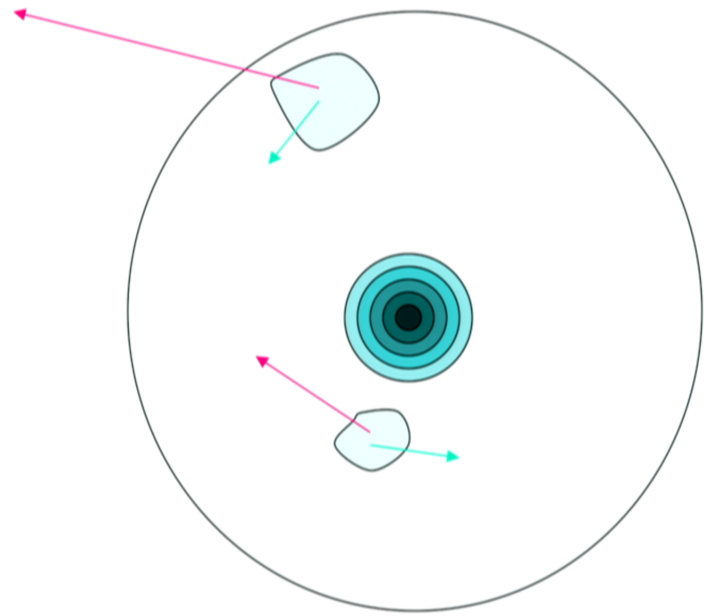
Is it plausible in the context of tidal torque theory?

Tidal torque theory with axion BEC



$$\vec{\nabla} \times \vec{v} \neq 0$$

net overall rotation is obtained because, in the lowest energy state, all axions fall with the same angular momentum

Magnitude of angular momentum

$$\lambda = \frac{L\,|E|^{\frac{1}{2}}}{G\,M^{\frac{5}{2}}} = \sqrt{\frac{6}{5-3\varepsilon}}\;\frac{8}{10+3\varepsilon}\;\frac{1}{\pi}\;j_{\max}$$

$\lambda \approx 0.05$

G. Efstathiou et al. 1979, 1987

$j_{\max} \simeq 0.18$

from caustic rings

fits perfectly ($0.25 < \varepsilon < 0.35$)

Self-Similarity

$$\vec{\tau}(t) = \int_{V(t)} d^3 r \;\; \delta\rho(\vec{r},t) \;\; \vec{r} \times (-\vec{\nabla}\phi(\vec{r},t))$$

a comoving volume

$$\vec{r} = a(t)\,\vec{x} \qquad\qquad \phi(\vec{r} = a(t)\,\vec{x},\; t) = \phi(\vec{x})$$

$$\delta(\vec{r},t) \equiv \frac{\delta\rho(\vec{r},t)}{\rho_0(t)} \qquad\qquad \delta(\vec{r} = a(t)\,\vec{x},\; t) = a(t)\,\delta(\vec{x})$$

$$\vec{\tau}(t) = \rho_0(t)\,a(t)^4 \int_V d^3x \;\; \delta(\vec{x}) \;\; \vec{x} \times (-\vec{\nabla}_x\,\phi(\vec{x}))$$

Conclusion:

The dark matter looks like axions

For axion BEC

$$E = \sum_{i=1}^{N} \frac{L_i^{\,2}}{2I}$$

is minimized for given $L = \sum_{i=1}^{N} L_i$

when $L_1 = L_2 = L_3 = \ \ldots \ = L_N$.

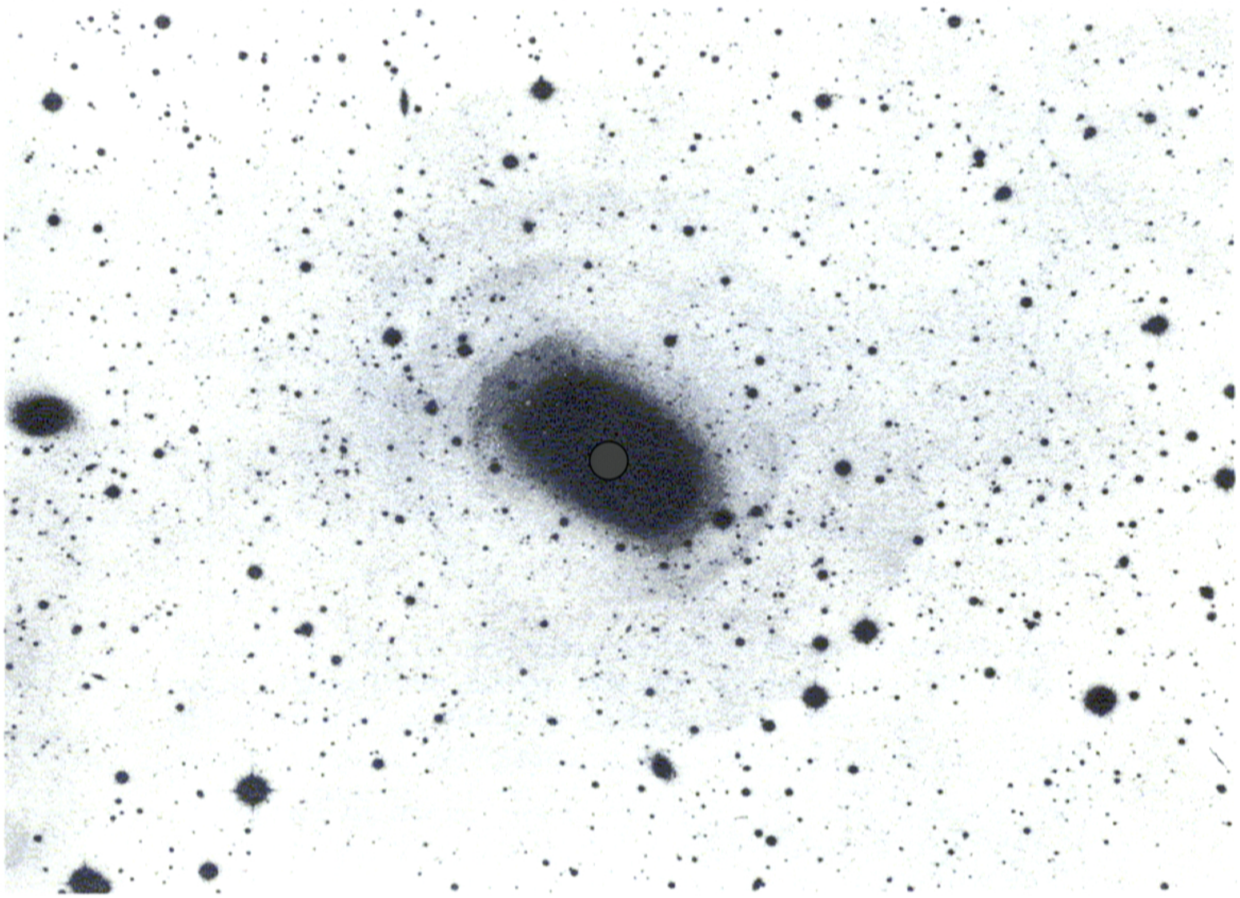


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)