

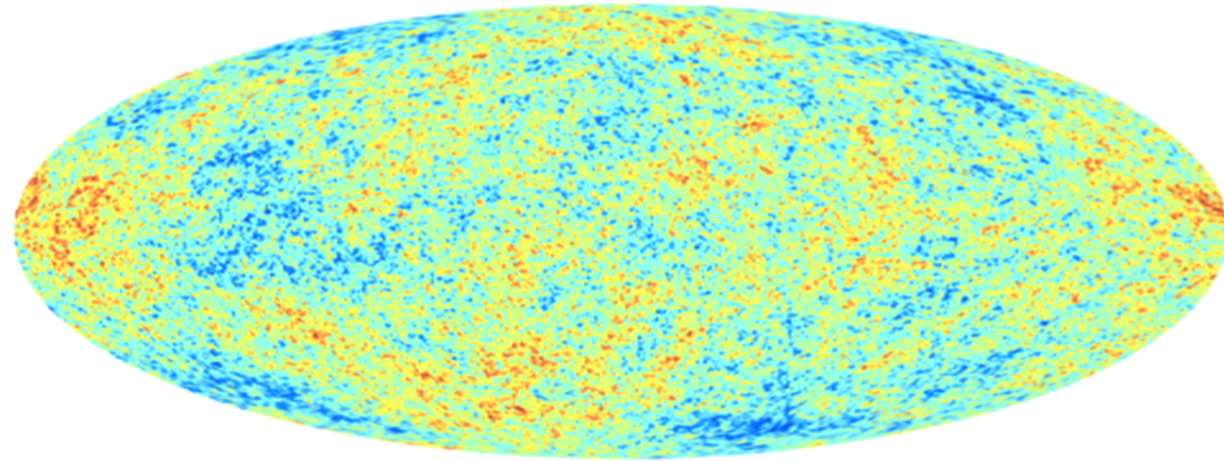
Title: Some new things to do with the CMB and galaxy surveys

Date: Mar 20, 2012 11:00 AM

URL: <http://pirsa.org/12030090>

Abstract: Over the past several decades we have obtained increasingly precise data on the distribution of galaxies in the Universe and on the distribution of primordial perturbations via CMB measurements. This trend is likely to continue for the foreseeable future. In this talk I will discuss some new things to do with data from the CMB, galaxy surveys, and future 21-cm surveys look for new physics in the early and late Universe. Topics will include cosmic birefringence, new tests for parity violation, gravitational lensing, gravitational waves, and new inflationary physics.



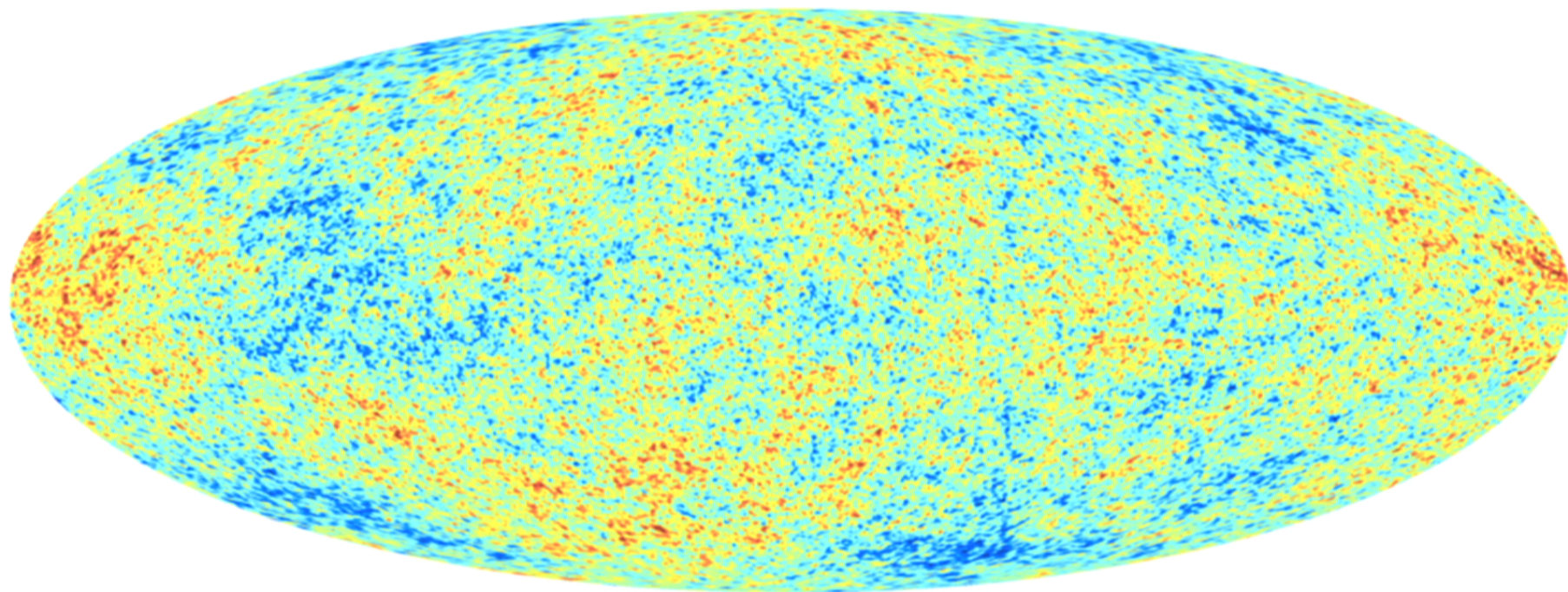


The CMB, Galaxy Surveys (and 21-cm fluctuations): Beyond Power Spectra

Marc Kamionkowski

(Johns Hopkins University)

Perimeter Institute, 20 March 2012



$$T(\hat{n})$$

Here's how we quantify it:

$$a_{lm} = \int d^2\hat{n} T(\hat{n}) Y_{lm}^*(\hat{n})$$

depends on coordinate system.

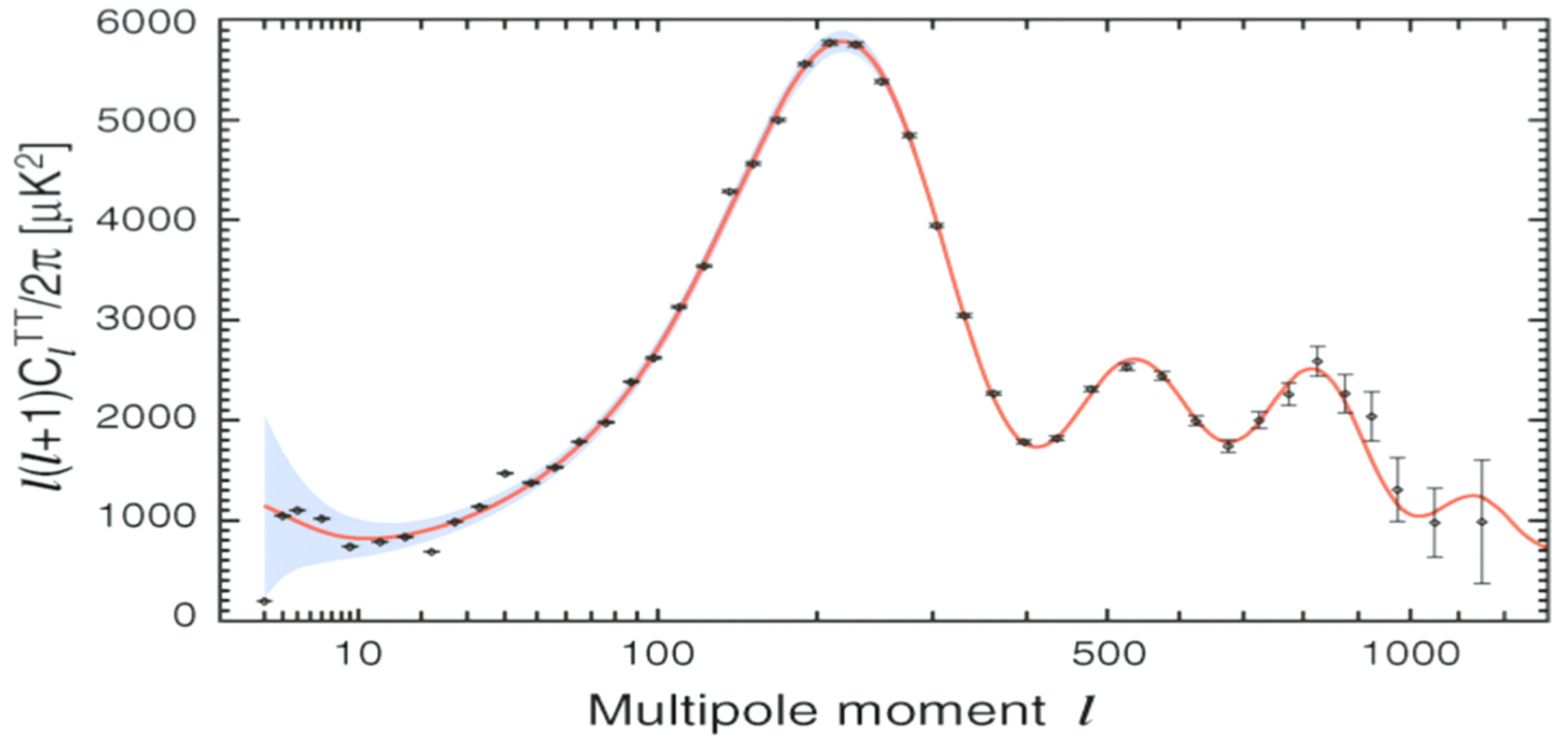
Power spectrum

$$C_l = \sum_m \frac{|a_{lm}|^2}{2l + 1}$$

is rotational invariant.

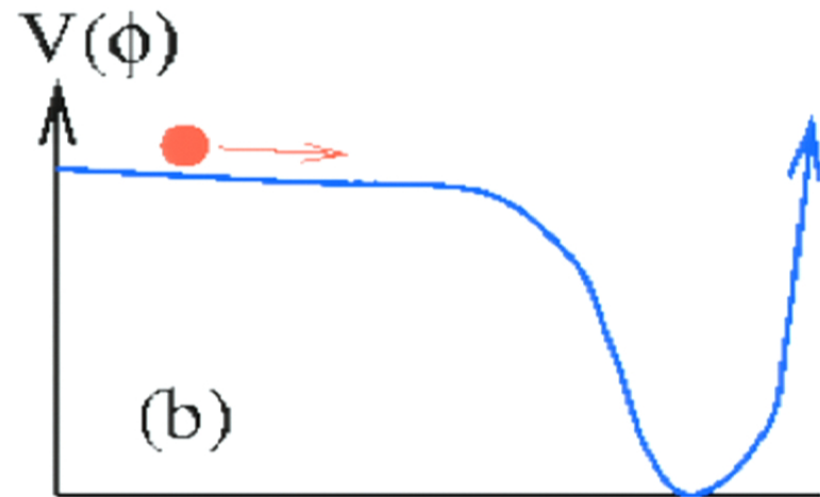
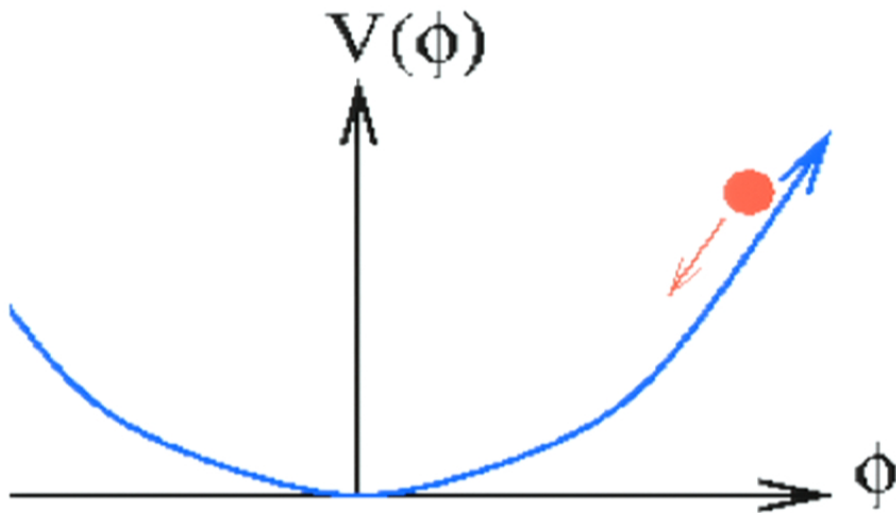
Variance of temperature distribution is

$$\langle (\Delta T)^2 \rangle = \sum_l \frac{2l + 1}{4\pi} C_l$$



Interpretation: Temperature fluctuations
from primordial density perturbations
from inflation

The mechanism: Vacuum energy associated with new ultra-high-energy physics (e.g., grand unification, strings, supersymmetry, extra dimensions....)



Every Fourier mode of inflaton field satisfies SHO-like equation of motion:

$$\ddot{\phi}_{\vec{k}} + k^2 \phi_{\vec{k}} = 0$$

Then imprinted as primordial density perturbations

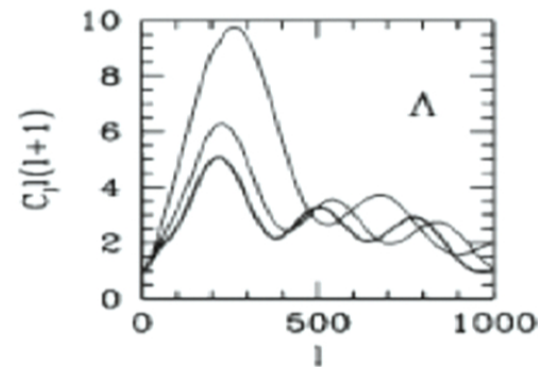
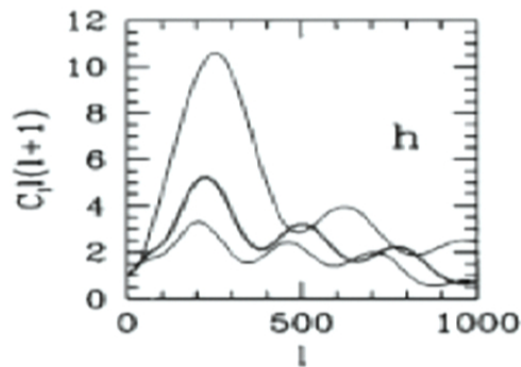
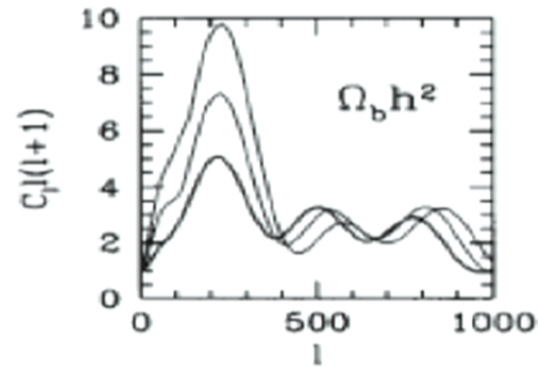
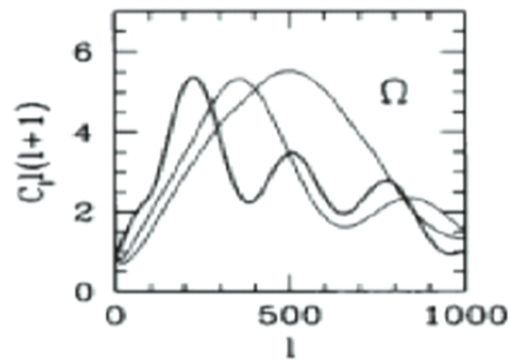
→ Amplitude of each Fourier mode selected from Gaussian random distribution (the ground-state SHO wave function)

Inflationary prediction is that CMB is realization of statistically isotropic Gaussian random field: each a_{lm} is statistically independent and there is no m dependence

$$\langle a_{lm}^* a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

Power spectra depend on cosmological parameters, so measurement allows their determination (Jungman, MK, et al. 1996)

$$\{ \Omega_m, \Omega_{de}, w, \Omega_b, \Omega_\nu, H_0, \tau_{reion}, n_s, \dots \}$$



As has now been done.....

FIVE-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE* OBSERVATIONS: LIKELIHOODS AND PARAMETERS FROM THE WMAP DATA

J. DUNKLEY^{1,2,3}, E. KOMATSU⁴, M. R. NOLTA⁵, D. N. SPERGEL^{2,6}, D. LARSON⁷, G. HINSHAW⁸, L. PAGE¹, C. L. BENNETT⁷, B. GOLD⁷, N. JAROSIK¹, J. L. WEILAND⁹, M. HALPERN¹⁰, R. S. HILL⁹, A. KOGUT⁸, M. LIMON¹¹, S. S. MEYER¹², G. S. TUCKER¹³, E. WOLLACK⁸, AND E. L. WRIGHT¹⁴

This paper focuses on cosmological constraints derived from analysis of *WMAP* data alone. A simple Λ CDM cosmological model fits the five-year *WMAP* temperature and polarization data. The basic parameters of the model are consistent with the three-year data and now better constrained: $\Omega_b h^2 = 0.02273 \pm 0.00062$, $\Omega_c h^2 = 0.1099 \pm 0.0062$, $\Omega_\Lambda = 0.742 \pm 0.030$, $n_s = 0.963^{+0.014}_{-0.015}$, $\tau = 0.087 \pm 0.017$, and $\sigma_8 = 0.796 \pm 0.036$, with $h = 0.719^{+0.026}_{-0.027}$. With five years of polarization data, we have measured the optical depth to reionization, $\tau > 0$, at 5σ significance. The redshift of an instantaneous reionization is constrained to be $z_{\text{reion}} = 11.0 \pm 1.4$ with 68% confidence. The 2σ lower limit is $z_{\text{reion}} > 8.2$, and the 3σ limit is $z_{\text{reion}} > 6.7$. This excludes a sudden reionization of the universe at $z = 6$ at more than 3.5σ significance, suggesting that reionization was an extended process. Using two methods for polarized foreground cleaning we get consistent estimates for the optical depth, indicating an error due to the foreground treatment of $\tau \sim 0.01$. This cosmological model also fits small-scale cosmic microwave background (CMB) data, and a range of astronomical data measuring the expansion rate and clustering of matter in the universe. We find evidence for the first time in the CMB power spectrum for a nonzero cosmic neutrino background, or a background of relativistic species, with the standard three light neutrino species preferred over the best-fit Λ CDM model with $N_{\text{eff}} = 0$ at $> 99.5\%$ confidence, and $N_{\text{eff}} > 2.3$ (95% confidence limit (CL)) when varied. The five-year *WMAP* data improve the upper limit on the tensor-to-scalar ratio, $r < 0.43$ (95% CL), for power-law models, and halve the limit on r for models with a running index, $r < 0.58$ (95% CL). With longer integration we find no evidence for a running spectral index, with $dn_s/d \ln k = -0.037 \pm 0.028$, and find improved limits on isocurvature fluctuations. The current *WMAP*-only limit on the sum of the neutrino masses is $\sum m_\nu < 1.3$ eV (95% CL), which is robust, to within 10%, to a varying tensor amplitude, running spectral index, or dark energy equation of state.

What else can we do?

What else can we do?

1. Polarization

Detection of gravitational waves with CMB polarization

Temperature map: $T(\hat{n})$ “E modes”

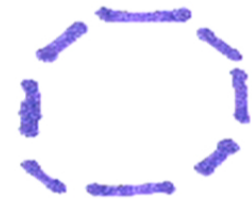
Polarization Map: $\vec{P}(\hat{n}) = \vec{\nabla} A + \vec{\nabla} \times \vec{B}$ “B modes”

Density perturbations have no handedness”
so they *cannot* produce a polarization with a curl

Gravitational waves do have a handedness, so they
can (and do) produce a curl

(MK, Kosowsky, Stebbins 1996; Seljak, Zaldarriaga 1996)

E modes



No
handedness

B modes



Handedness

Now **SIX** CMB power spectra

$$C_l^{TT}$$

$$C_l^{EE}$$

$$C_l^{BB}$$

$$C_l^{TE}$$

$$C_l^{TB}$$

$$C_l^{EB}$$

(parity breaking)

From three sets of temperature/polarization coefficients:

$$a_{lm}^T$$

$$a_{lm}^E$$

$$a_{lm}^B$$

Cosmological birefringence

- If some $\Phi(t)$ couples to E&M through:

$$\mathcal{L} = \frac{1}{2} (\vec{E}^2 - \vec{B}^2) + \frac{\phi(t)}{M_*} \vec{E} \cdot \vec{B}$$

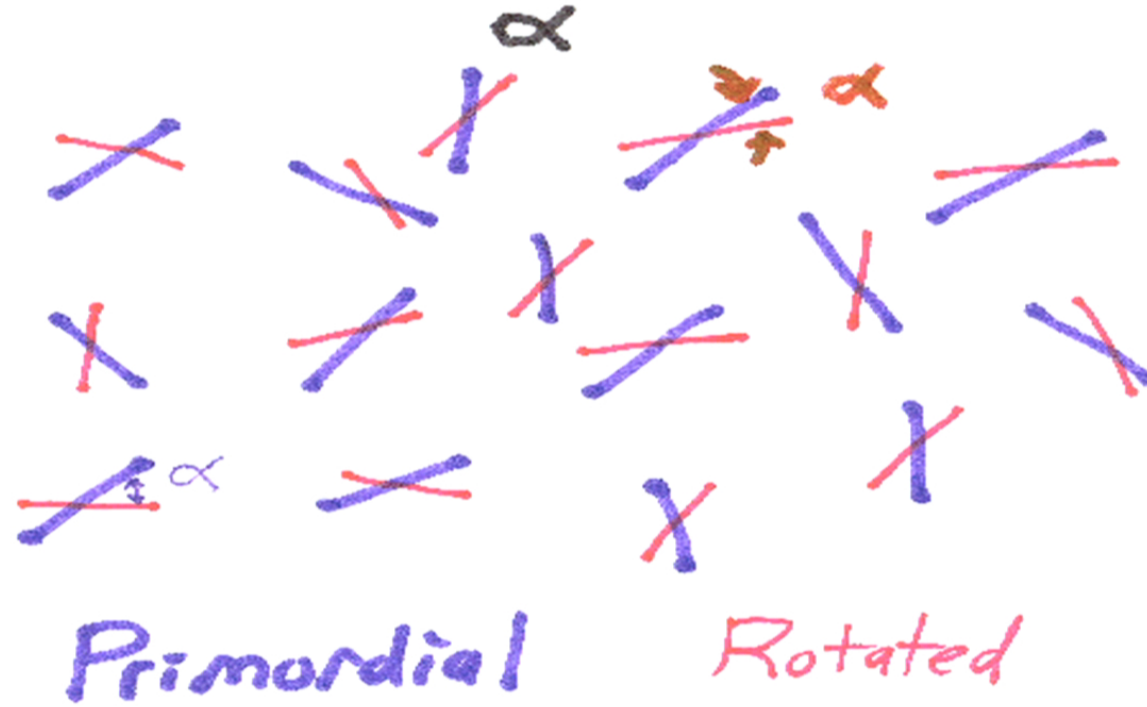
$\Phi(t)$ leads to rotation by $\alpha = \Delta\Phi/M_*$, of polarization as photons propagate (Carroll, Field, Jackiw 1998)

Rotation induces EB cross-correlation (Lue, Wang, MK 1999)

CMB searches: $\alpha < \text{few degrees}$ (Feng et al., astro-ph/0601095; Komatsu et al. 2008; Wu et al. 2008.....)

Right- and left-circularly polarized EM waves propagate with same speed to 10^{-30} !!!

Uniform Rotation



What else can we do?

2. Departures from Gaussianity and Statistical Isotropy

Inflationary prediction,

$$\langle a_{lm}^* a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

can be modified by

- ◆ late-time effects (e.g., gravitational lensing)
- ◆ Inflationary physics beyond vanilla inflation
- ◆ Exotica

Any such effects can be parametrized as

$$\langle a_{lm}^* a_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'} + \sum_{L \geq 0} \sum_{M=-L}^L C_{lm l' m'}^{LM} A_{ll'}^{LM}$$

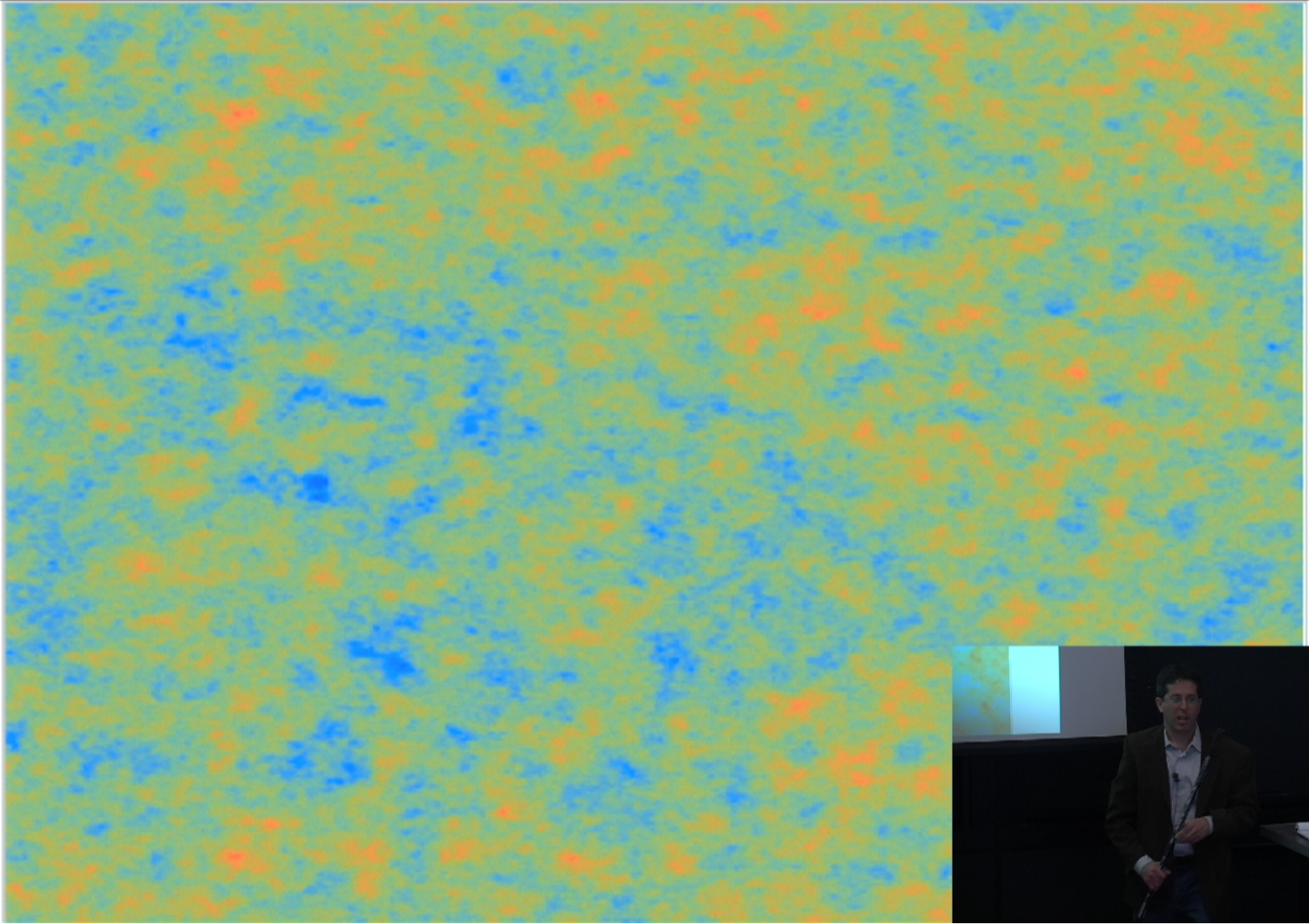
Clebsch-Gordan
coefficients

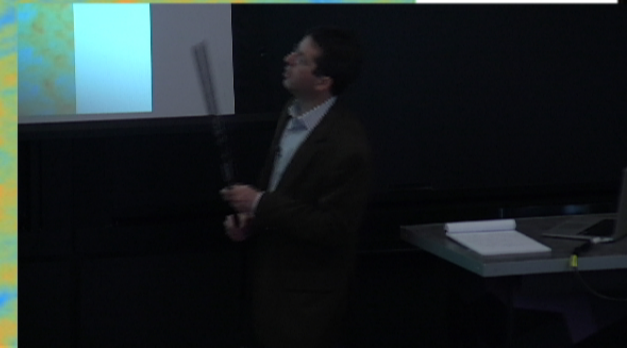
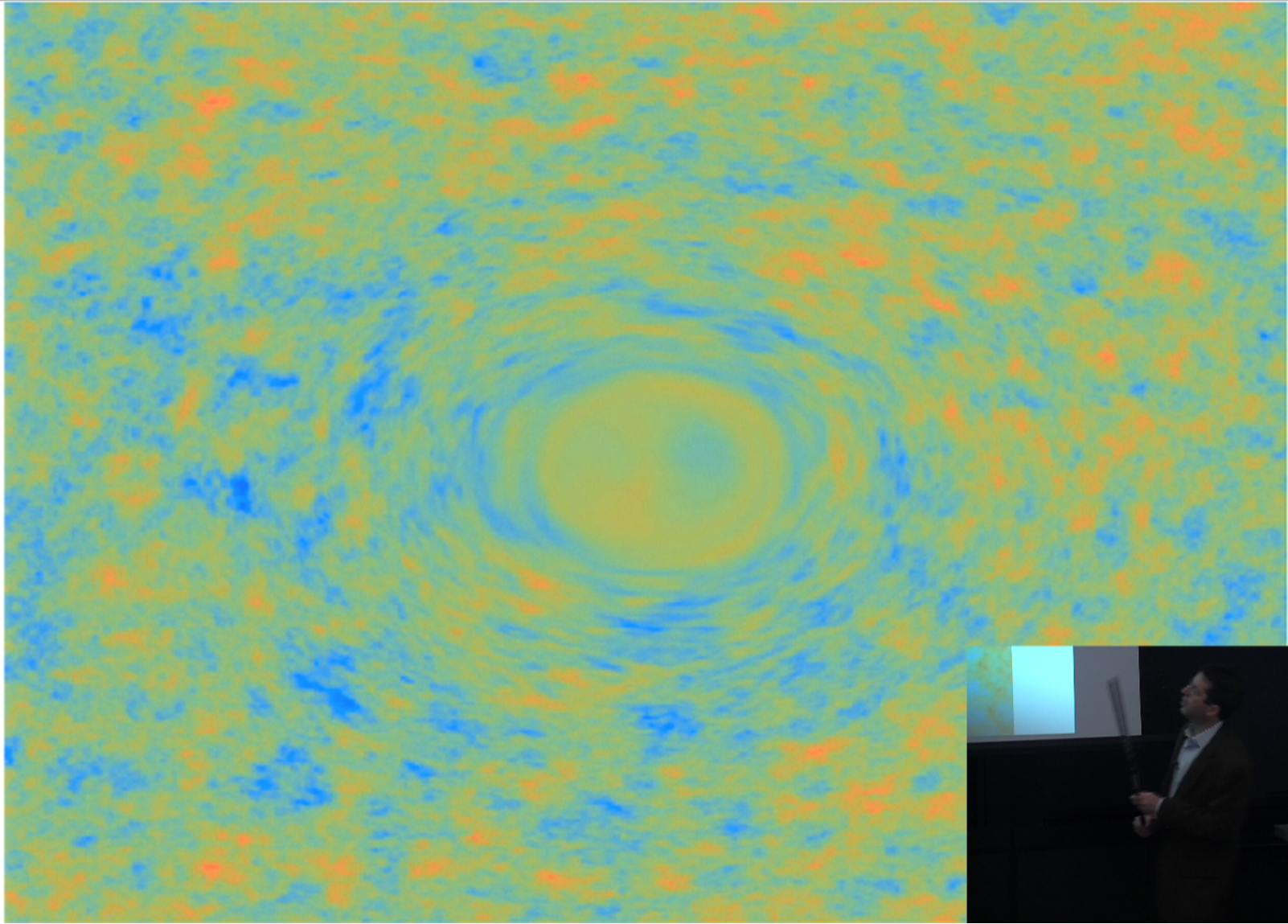
Bipolar spherical
harmonics (BiPoSHs)
(Hajian & Souradeep 2004;
Book, MK, Souradeep 2011)

Recipe for measuring BiPoSHs (generalizations of power spectrum to direction-dependent fluctuations):

$$\widehat{A}_{ll'}^{LM} = \sum_{lml'm'} C_{lml'm'}^{LM} a_{lm}^* a_{l'm'}$$

Example 1: Weak gravitational lensing of CMB





$$A_{ll'}^{\oplus LM} = \frac{\phi_{LM}}{\sqrt{2L+1}} \left[\frac{C_l G_{l'l}^L}{\sqrt{l'(l'+1)}} + \frac{C_{l'} G_{ll'}^L}{\sqrt{l(l+1)}} \right] = Q_{ll'}^{\oplus L} \phi_{LM}$$

where deflection angle is $\vec{\alpha} = \vec{\nabla} \phi$

Lensing of CMB recently detected!
through cross-correlation with galaxy surveys

(Smith, Zahn, Dore 2007; Hirata et al. 2008)

And now without galaxies (Das et al. (ACT) 2010)

Example II (Exotica): Departures from Statistical Isotropy (Pullen, MK, 2007)

- Inflation: Universe statistically isotropic and homogeneous
- Statistical isotropy: Power spectrum does not depend on direction

To test....

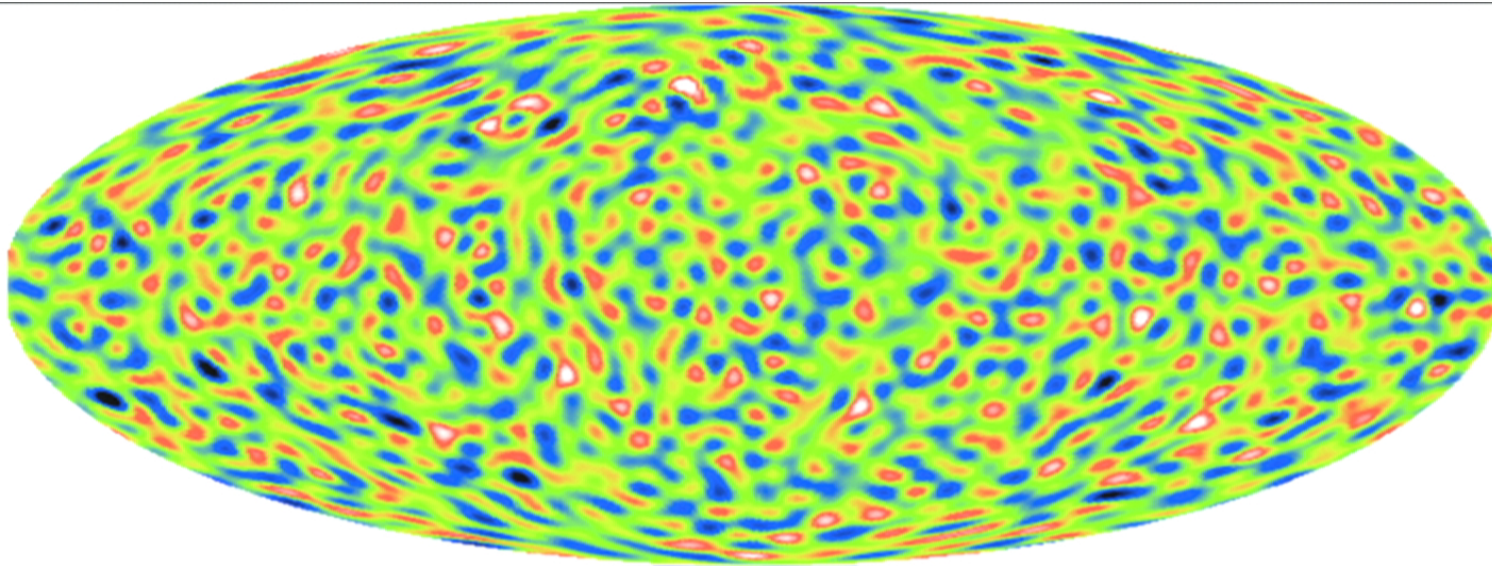
- Require family of models that violate statistical isotropy and/or homogeneity, with departure from SI/SH parametrized by quantity that can be dialed to zero (e.g., inflationary models of Ackerman, Carroll, Wise predict power quadrupole)
- Develop estimators that measure SI/SH-violating parameters from observables

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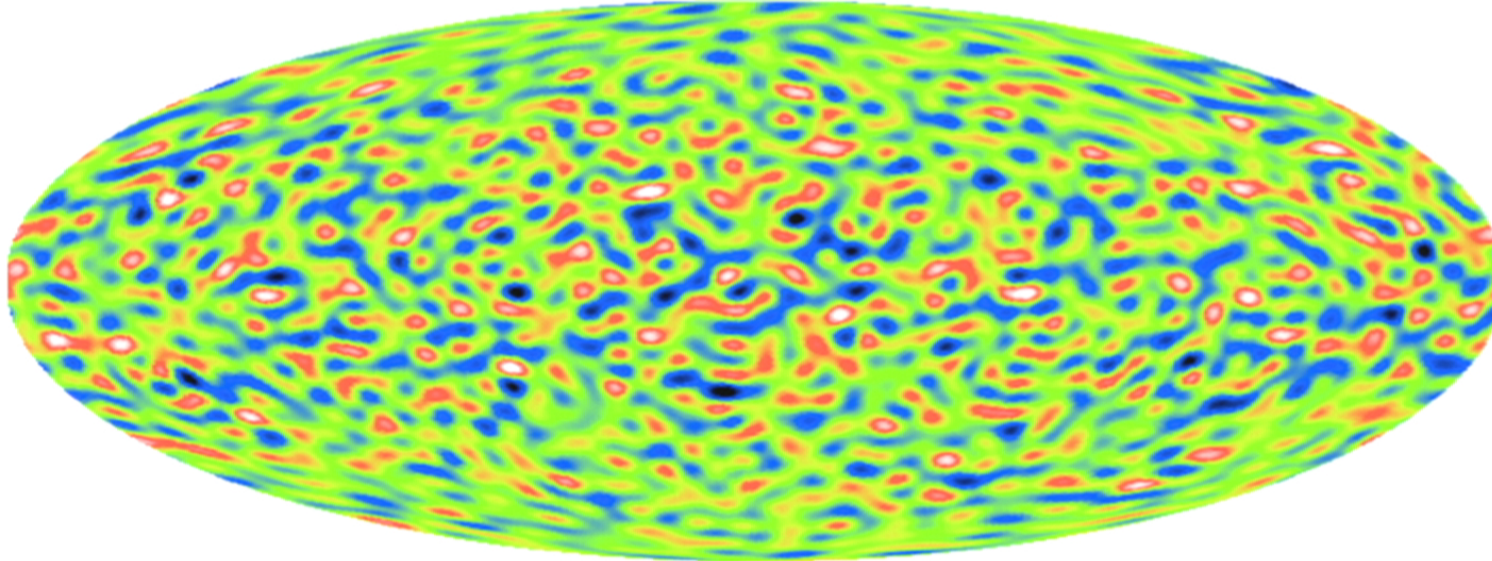
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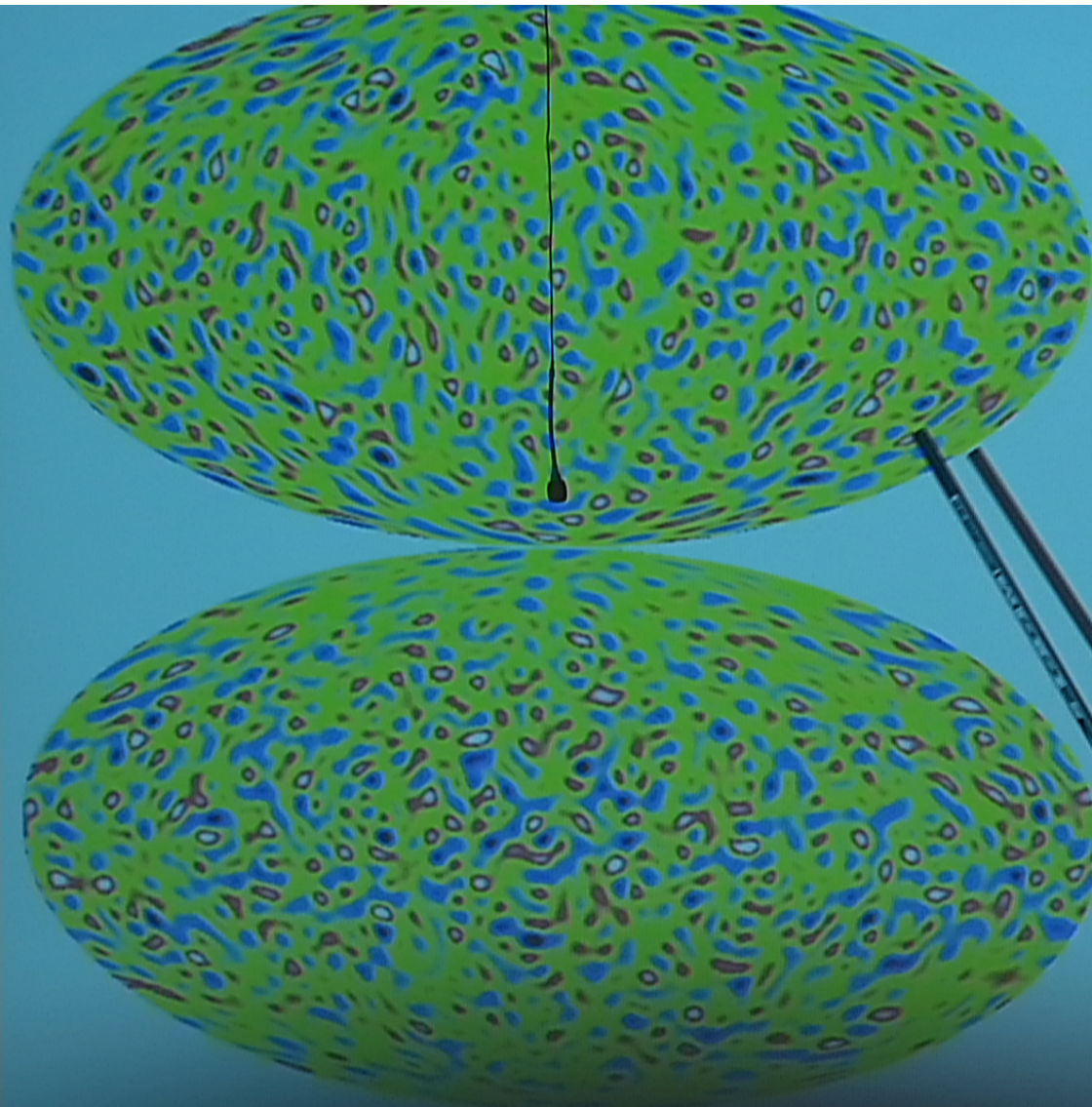
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Statistically
isotropic



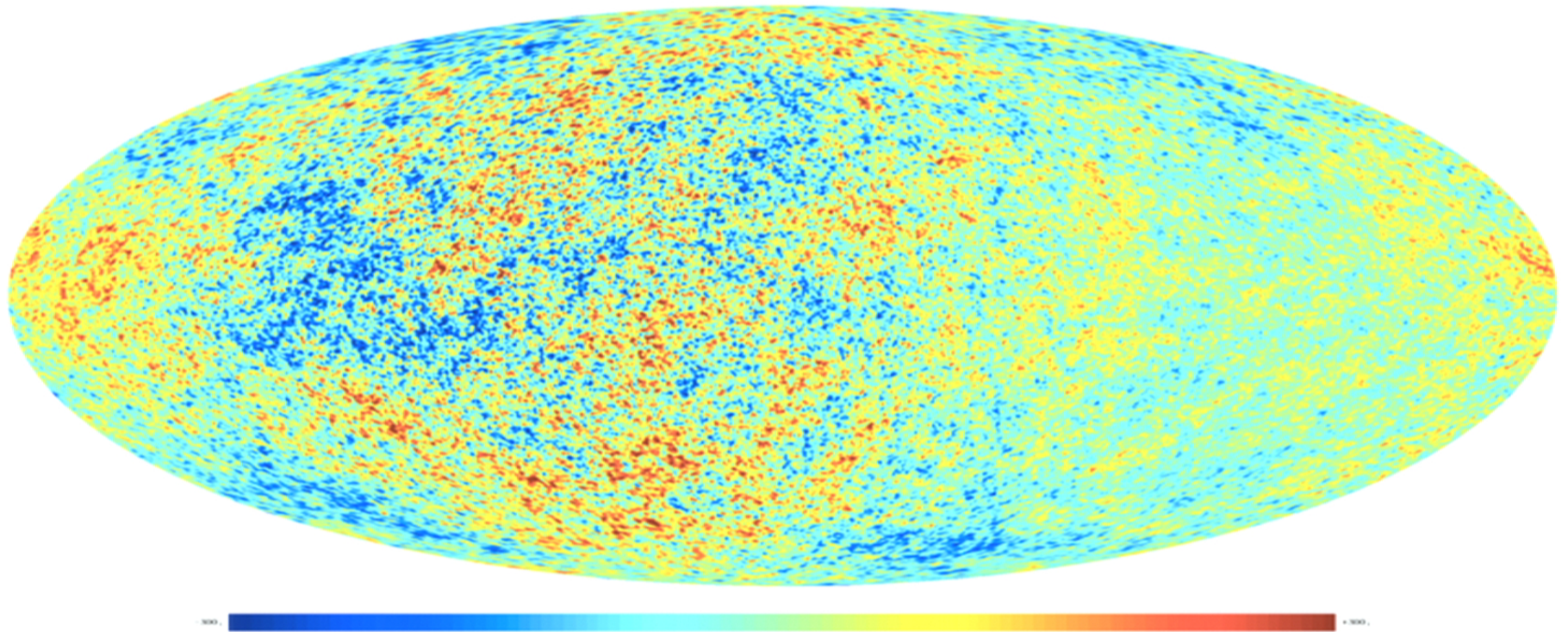
A power
quadrupole



Statistically isotropic

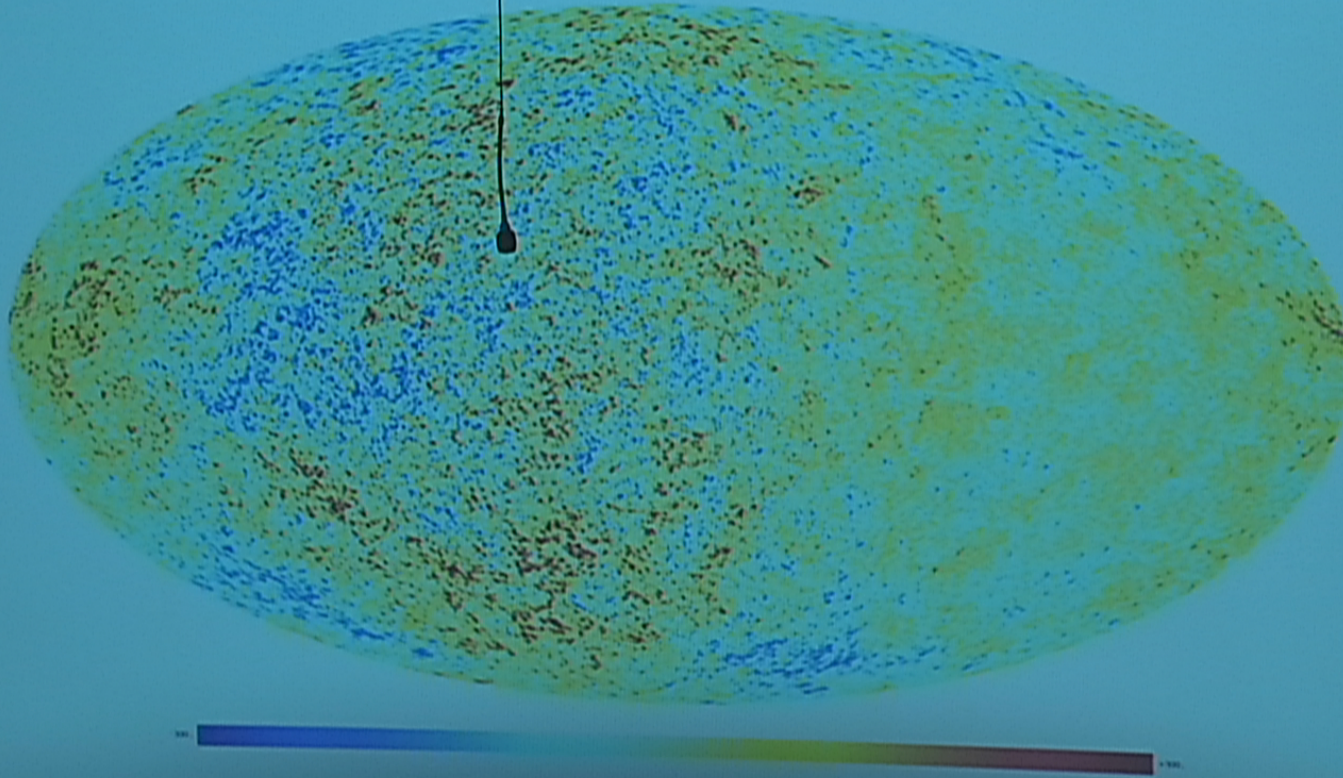
A power quadrupole

A power dipole



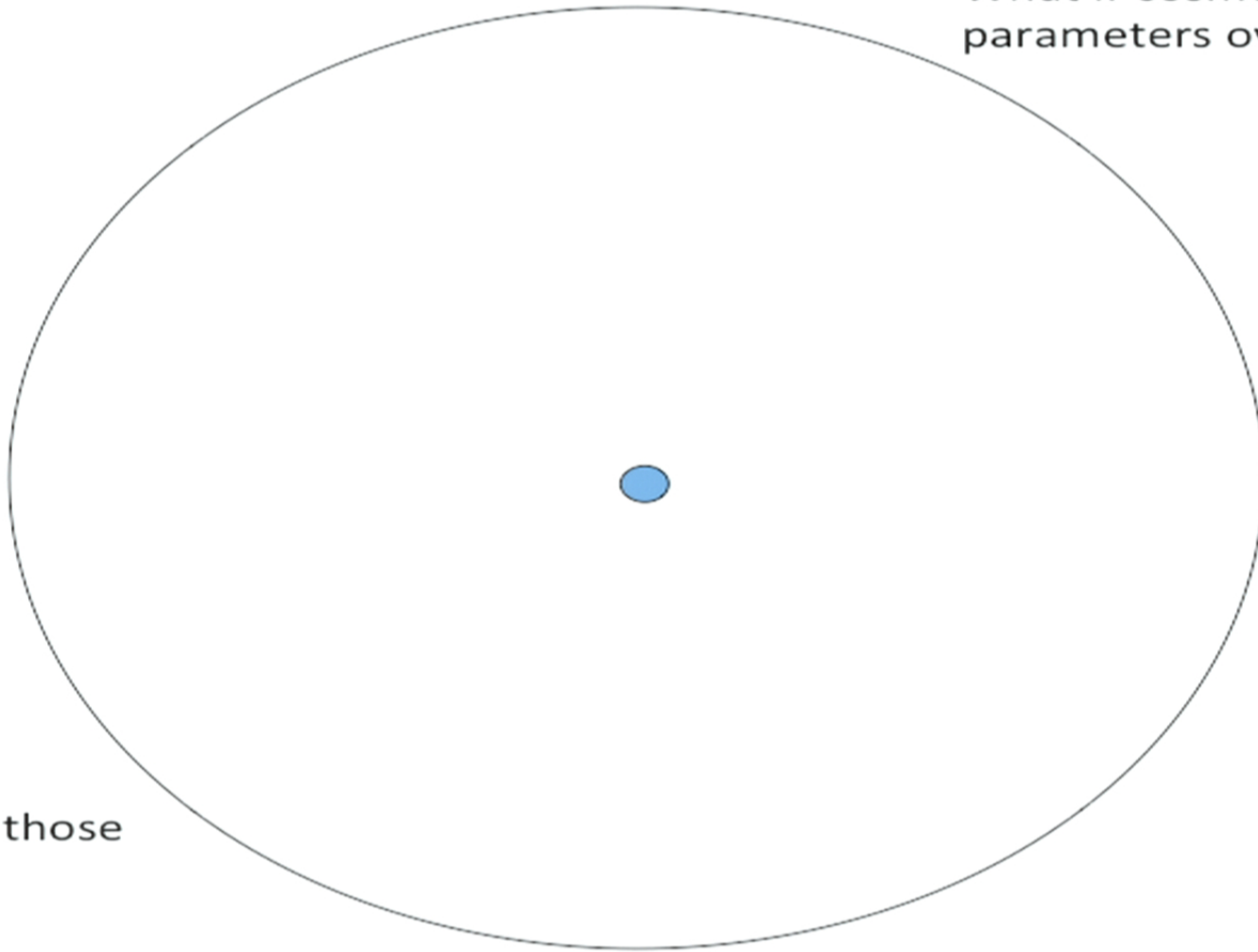
Erickcek, MK, Carroll 2008; Erickcek, Carroll, MK 2008; Erickcek, Hirata, MK 2009

A power dipole



Erickcek, MK, Carroll 2008; Erickcek, Carroll, MK 2008; Erickcek, Hirata, MK 2009

What if cosmological parameters over here.....

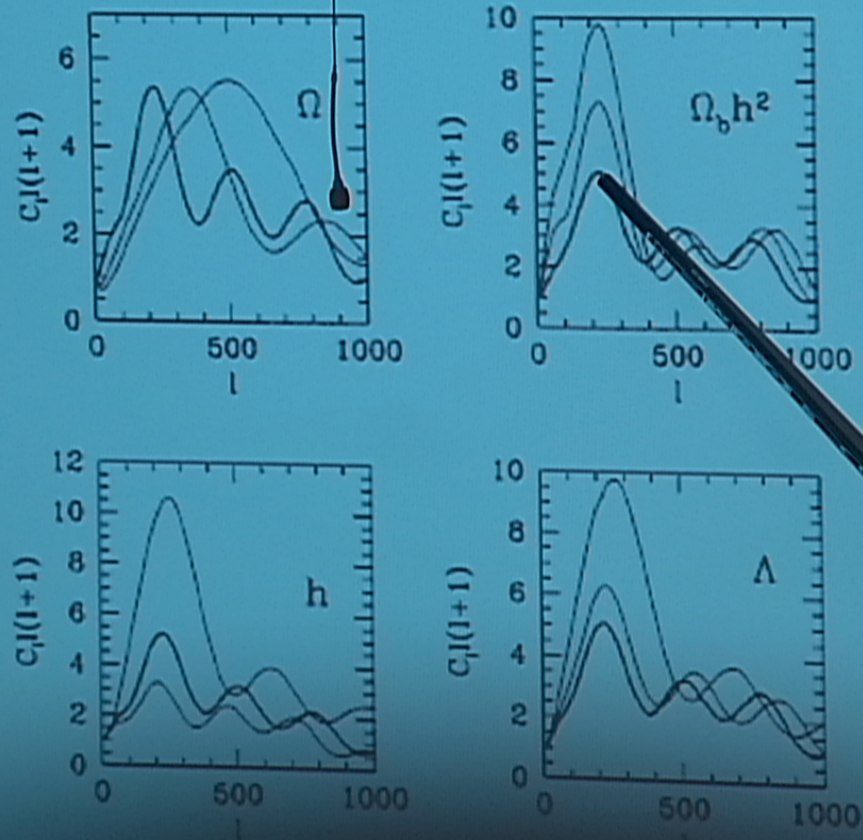


differ from those over here?

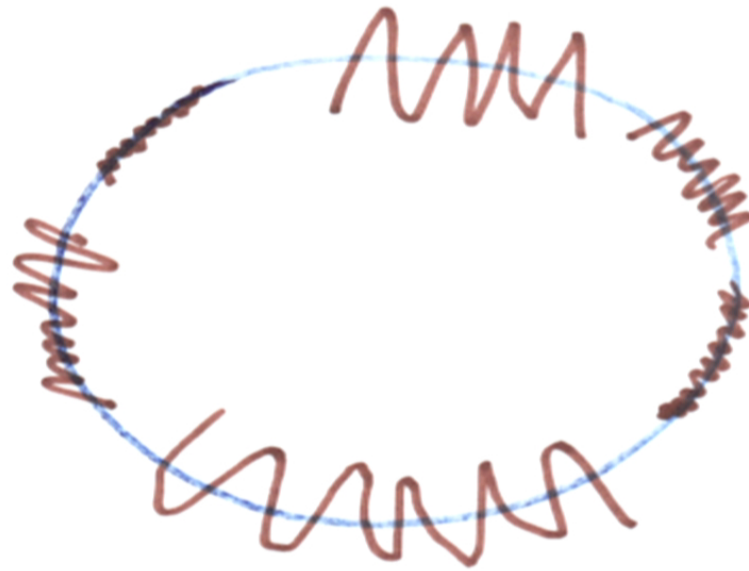
What if cosmological parameters over here.....

differ from those over here?

Then power spectra will differ from one point on sky to another:

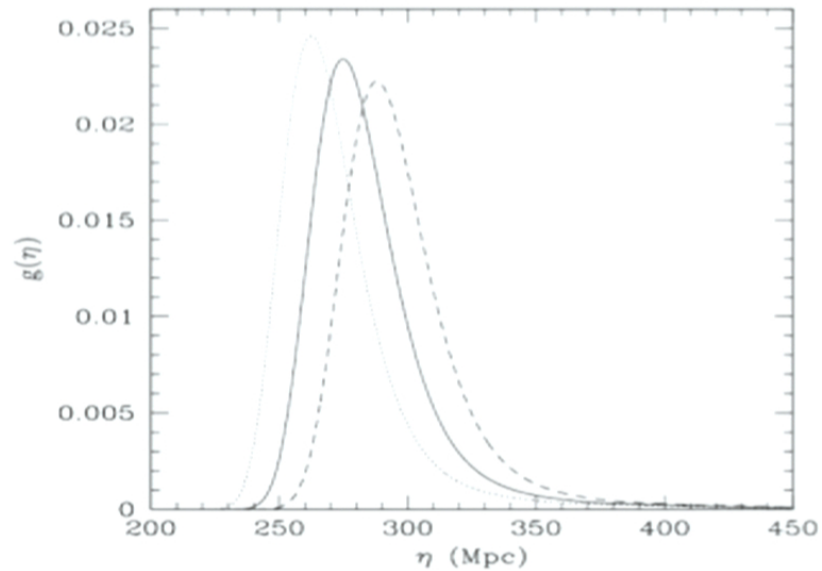


So T fluctuations may vary across sky.....

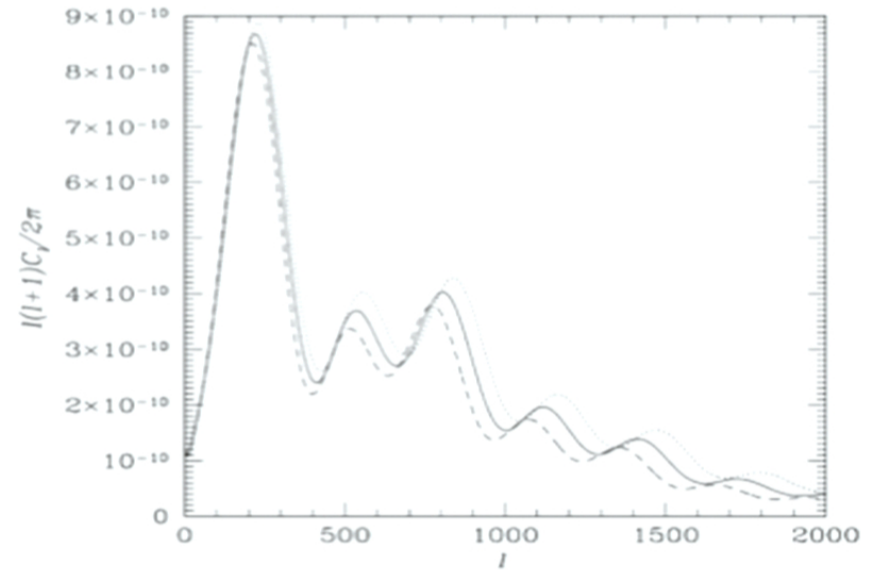


This is described by BiPoSHs

E.g., suppose fine-structure constant varied



Visibility function



Power spectrum

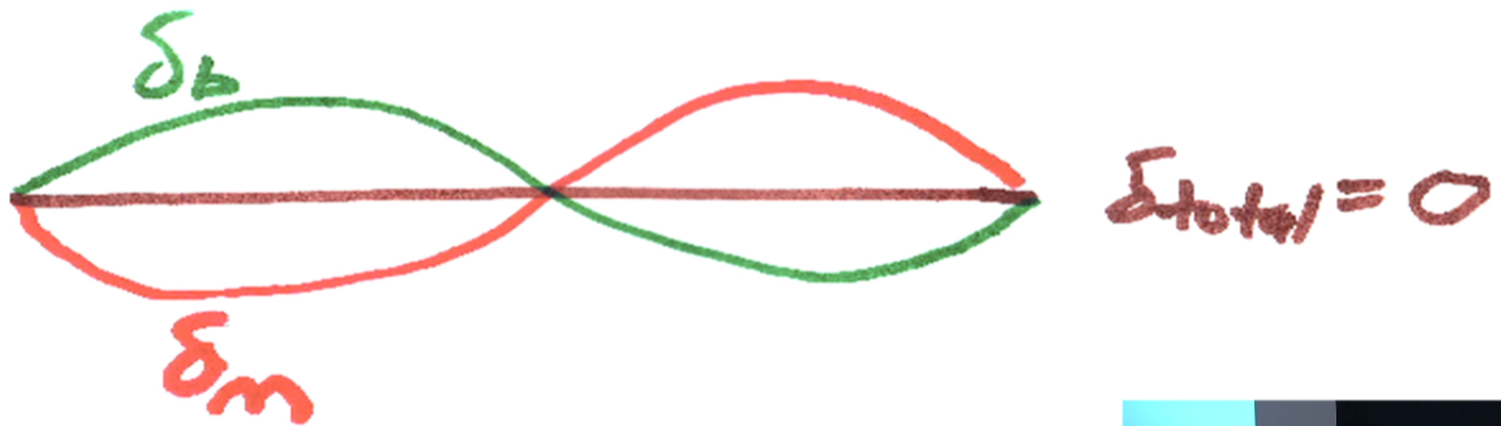
Can be sought with BiPoSHs (Sigurdson, MK, Kurylov 2003)

Another example: Compensated isocurvature perturbations

Baryon-density fluctuation compensated by dark-matter fluctuation of equal magnitude (Pritchard and Gordon 2009; Holder, Nollett, van Engelen 2009)

Strongest current constraint, from baryon-DM ratios measured in galaxy clusters: $\lesssim 10\%$

Compensated isocurvature perturbation



Polarization BiPoSHs: Spatially-varying cosmological birefringence

(MK, 2008; Gluscevic, MK, Cooray 2009; Yadav et al. 2009; MK 2010; Gluscevic, MK, Caldwell 2011)

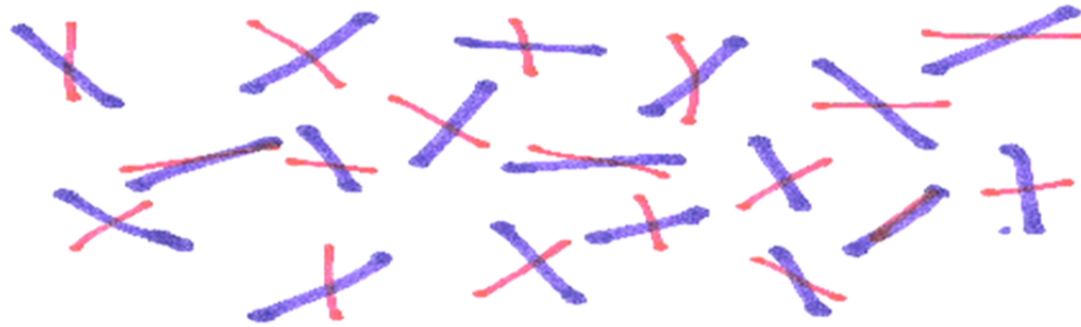
- What if CB rotation angle varies from one point on sky to another?? (e.g., Pospelov, Ritz, Skordis 2008; Caldwell et al., in preparation)
- Then observed polarization has nothing to do with primordial polarization!!!
- Develop technique to measure rotation as function of angle, and thus to infer primordial polarization pattern

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Rotation $\alpha(\hat{n})$ depends
on position \hat{n}



Primordial Rotated

? ? ?



Odd-Parity BiPoSHs (Book, MK, Souradeep 2011)

- Lensing, SI violations, patchy reionization, variable cosmological parms all induce only BiPoSHs with $L+l+l'=\text{even}$.
- But BiPoSHs can have $L+l+l'=\text{odd}$
- Odd/even split \sim E/B decomposition for CMB polarization
- Odd-parity BiPoSHs may be induced by lensing by GWs or by pointing errors, asymmetric beams, etc.

New probes of parity violation

- Cross-correlation between even- and odd-parity BiPoSHs are parity breaking. May be induced by chiral GW background



Odd-parity bispectrum (MK, Souradeep 2010)

- Three-pt functions from inflation have

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \neq 0 \quad \text{for} \quad l_1 + l_2 + l_3 = \text{even}$$

but odd-parity bispectrum, with

$$l_1 + l_2 + l_3 = \text{odd}$$

is also allowed mathematically and can be measured from data.

Allows probe of novel parity-breaking early-Universe physics and new set of null tests for systematics in experiments

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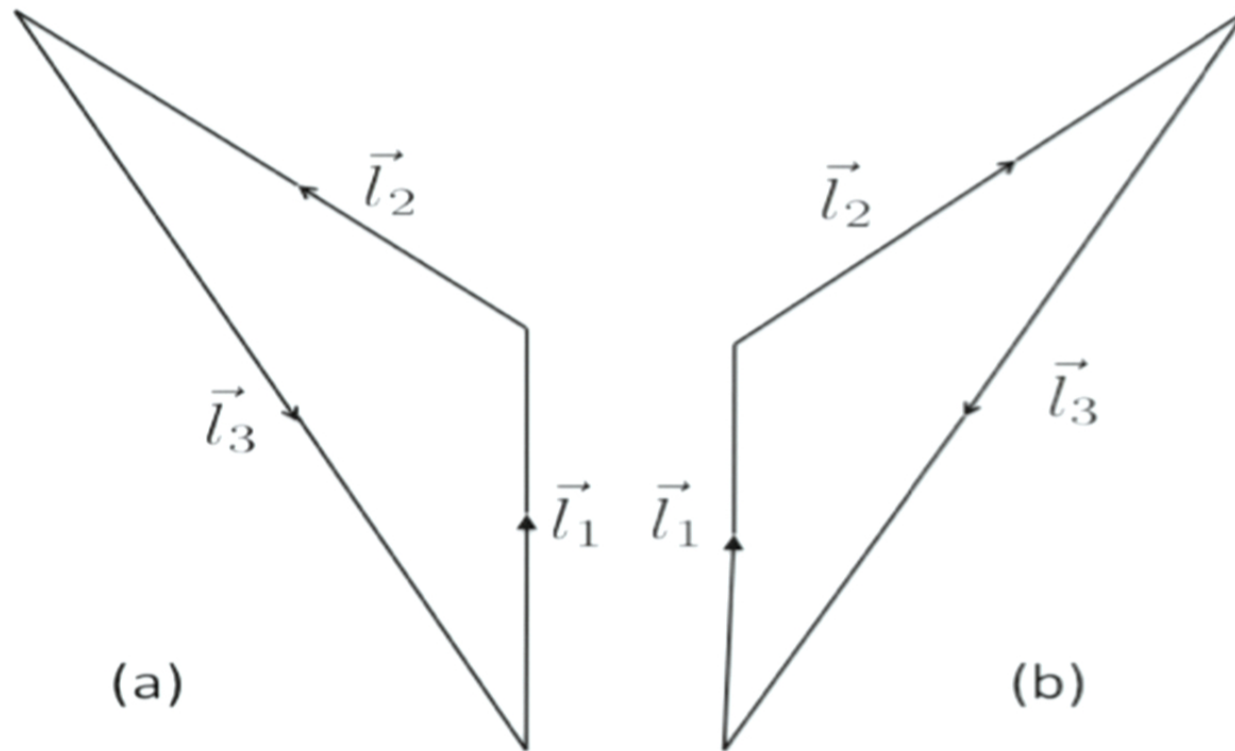
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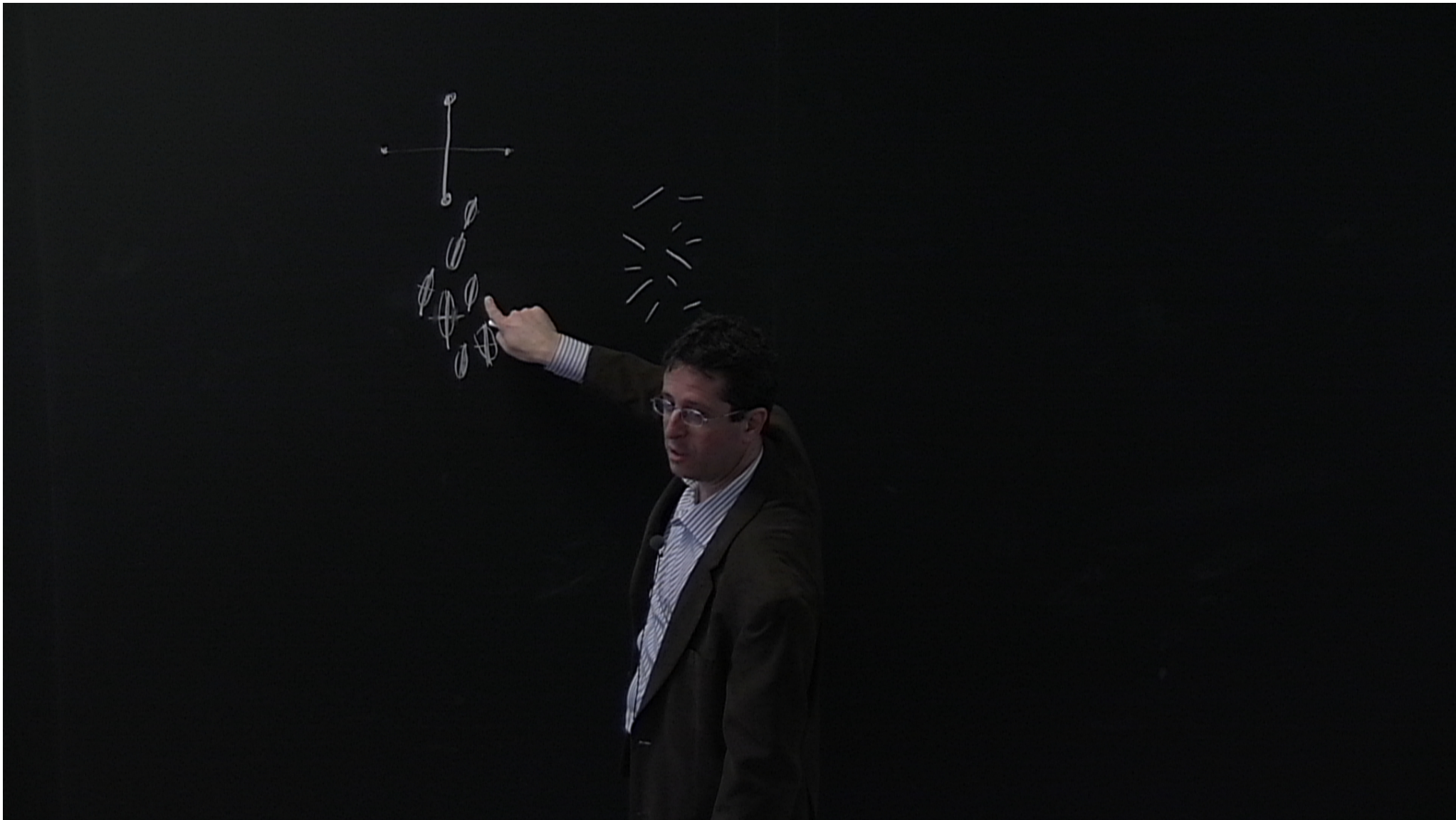
$$l_1 + l_2 + l_3 = \text{odd}$$

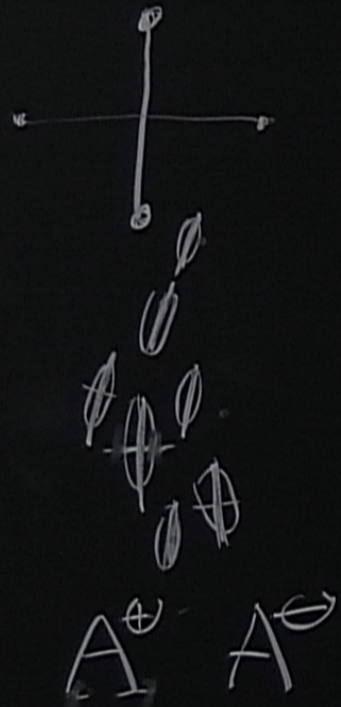
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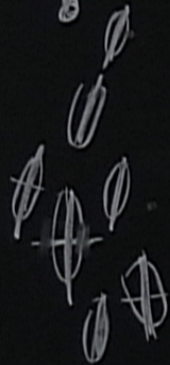
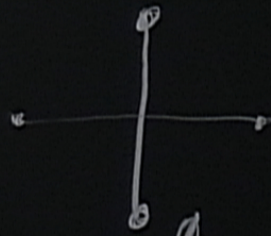
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Odd-parity bispectrum differences, rather than sums, contributions from mirror-image triangles







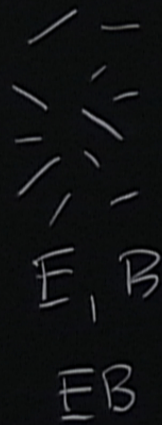
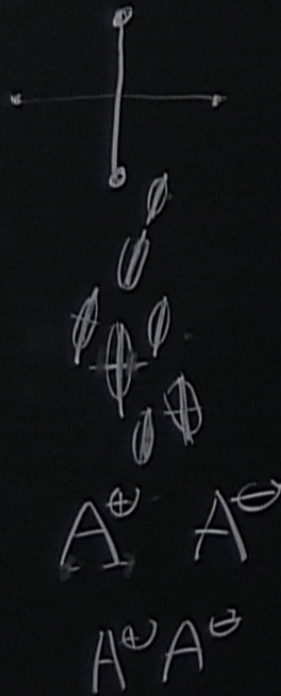
$A^{\oplus} \quad A^{\ominus}$

$A^{\oplus} A^{\ominus}$

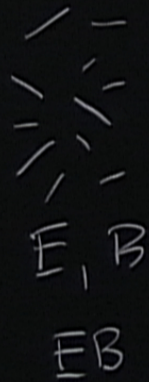
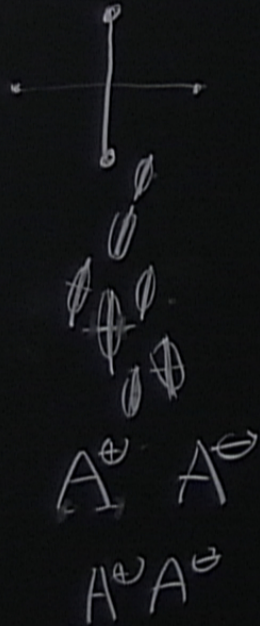


E, B

EB



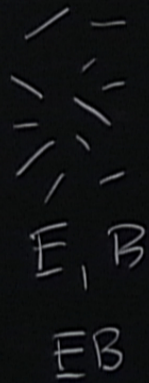
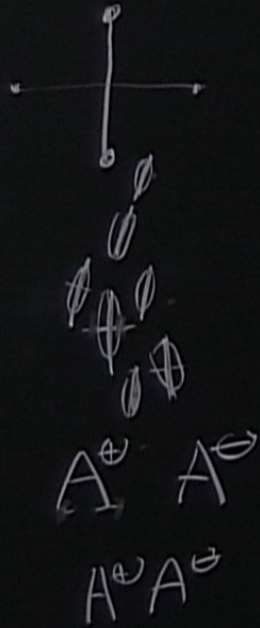
lensing by $\delta \Rightarrow A^{\oplus}$
 lensing by GWs $\Rightarrow A^{\oplus} A^{\ominus}$



lensing by $\delta \Rightarrow A^+$

lensing by GWs $\Rightarrow A^+ A^-$

" " chiral GWs $\Rightarrow \langle A^+ A^- \rangle \neq 0$

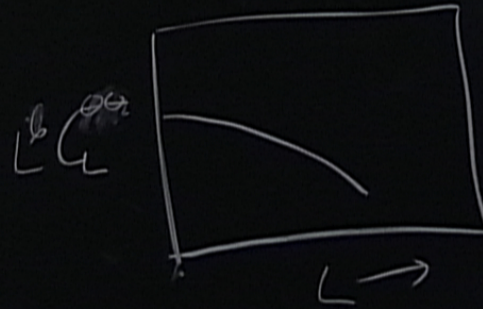


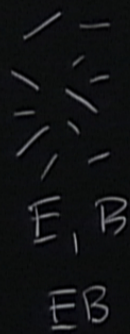
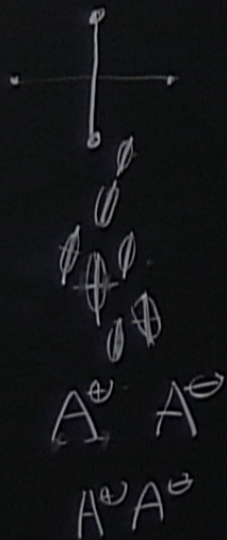
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Book, MK, Sourdeep



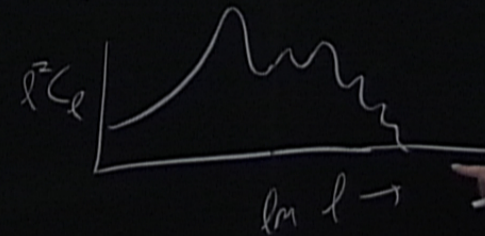
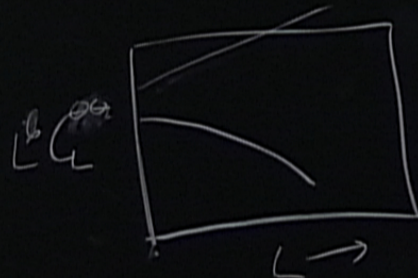


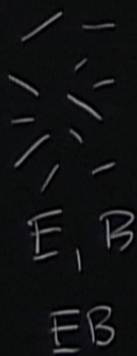
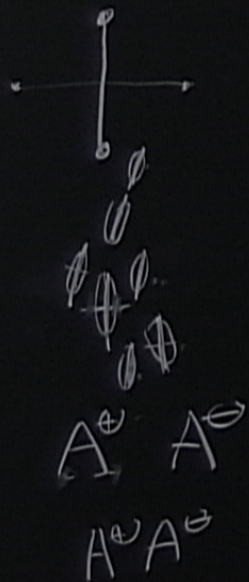
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Book, MK, Sourdeup



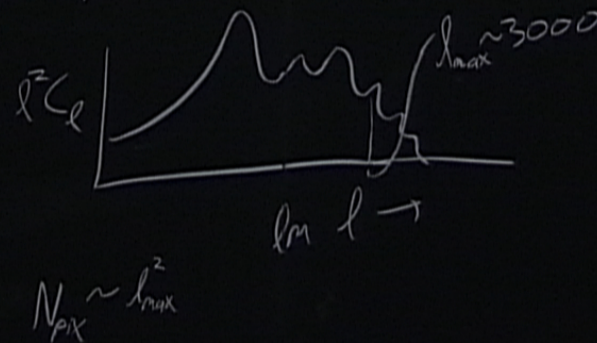
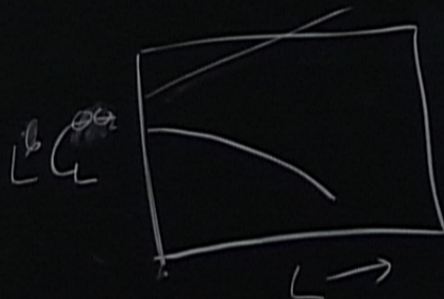


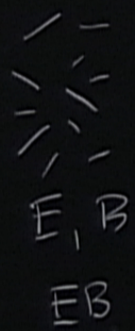
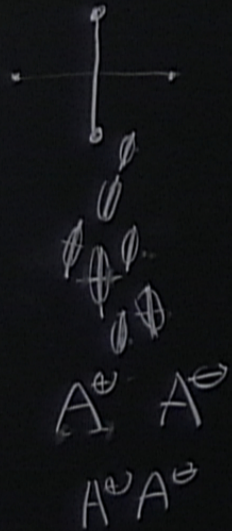
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|| || chiral GWs $\Rightarrow \langle A^{\oplus} A^{\ominus} \rangle \neq 0$

Book, MK, Sourdeep



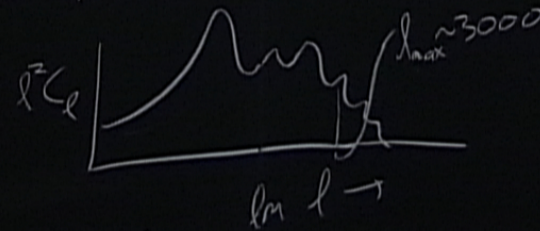
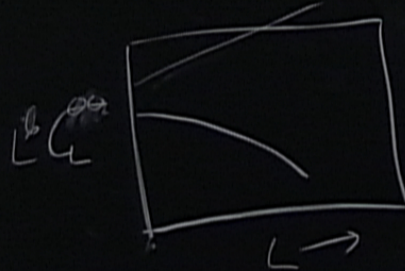


lensing by $\delta \Rightarrow A^\oplus$

lensing by GWs $\Rightarrow A^\oplus A^\ominus$

|| || chiral GWs $\Rightarrow \langle A^\oplus A^\ominus \rangle \neq 0$

Book, MK, Sourdeup



$N_{\text{pix}} \sim l_{\max}^2$

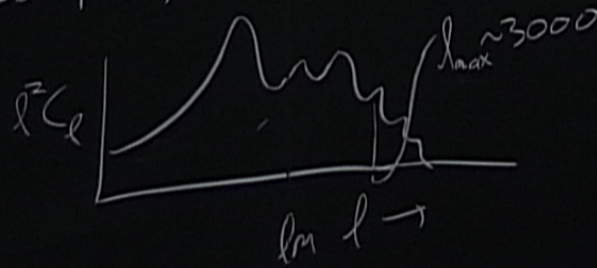
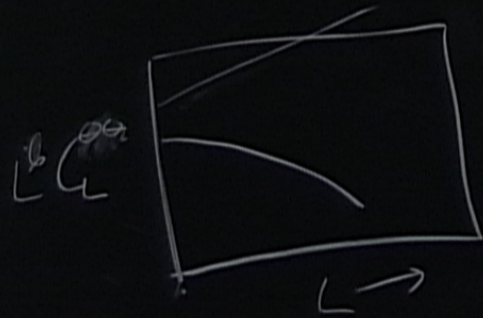
$\left(\frac{\Sigma}{N}\right) \sim 4.5 \times 10^{-6} \left(l_{\max}/10^3\right)^2$

lensing by $\delta \Rightarrow A^\oplus$

lensing by GWs $\Rightarrow A^\oplus A^\ominus$

|| || chiral GWs $\Rightarrow \langle A^\oplus A^\ominus \rangle \neq 0$

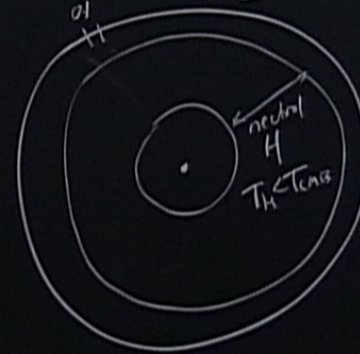
Book, MK, Sourdeep



$$N_{PIX} \sim l_{max}^2$$

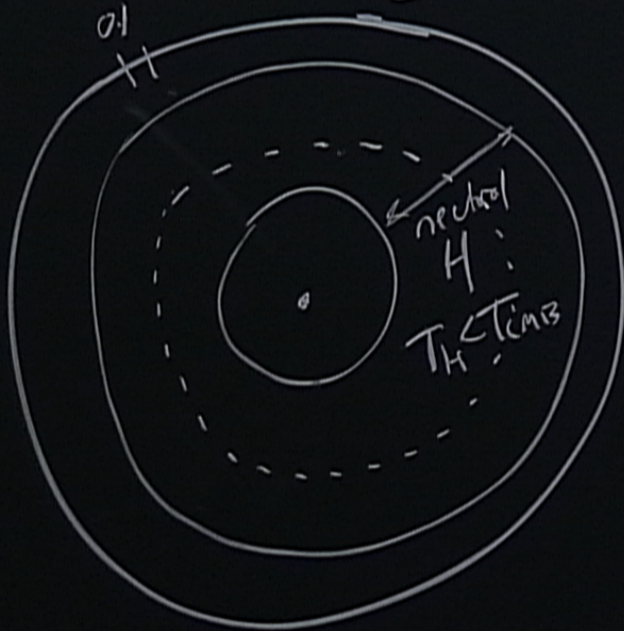
$$\left(\frac{S}{N}\right) \sim 4.5 \times 10^{-6} \left(\frac{l_{max}}{10^3}\right)^2$$

$z \sim 1100$



$\Rightarrow H$ detected by 21-cm absorption

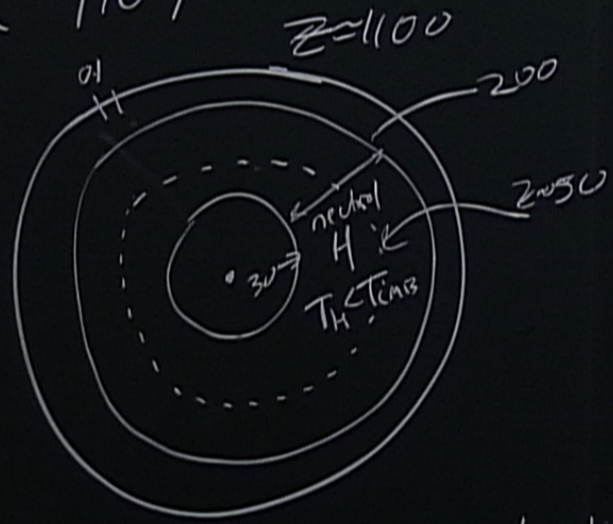
$$\left(\frac{S}{N}\right) \sim 4.5 \times 10^{-6} \left(\frac{l_{\max}}{10^3}\right)^2 \quad z \approx 1100$$



0
deep

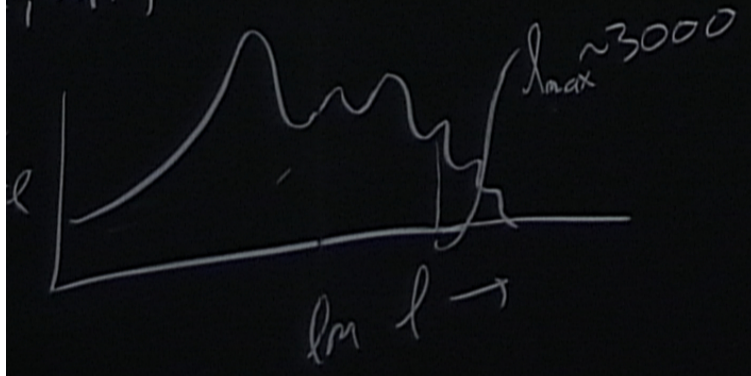
⇒ H detected by 21-cm absorption

$$\left(\frac{S}{N}\right) \sim 4.5 \times 10^{-6} \left(\frac{\lambda_{\max}}{10^3}\right)^2$$

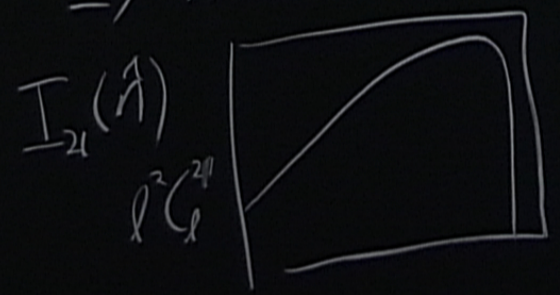


$A_{\nu} > 0$

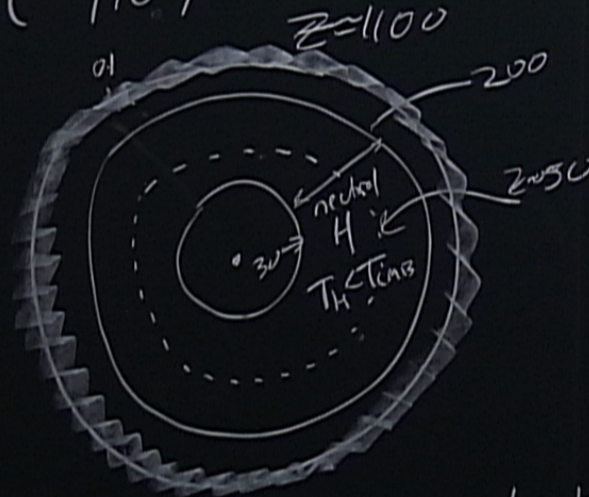
MK, Sovraddeep



\Rightarrow H detected by 21-cm absorption

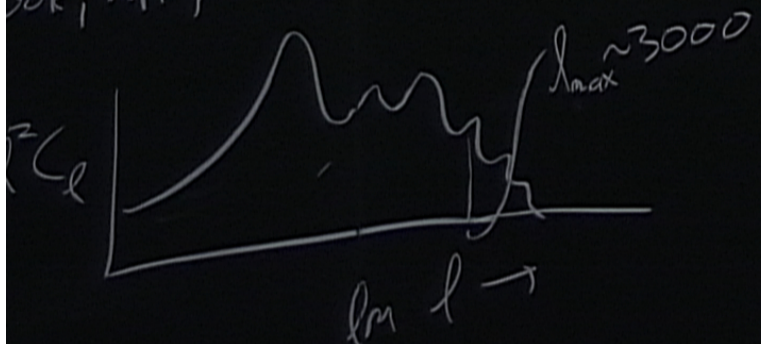


$$\left(\frac{S}{N}\right) \sim 4.5 \times 10^6 \left(\frac{l_{\max}}{10^7}\right)^2$$



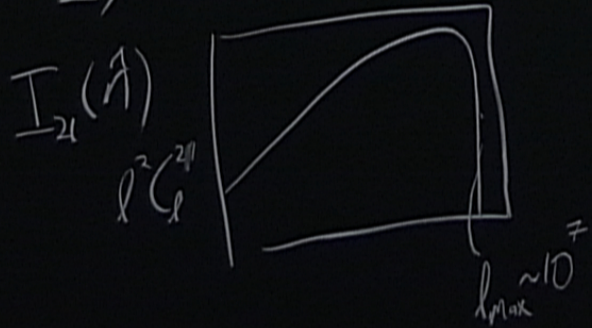
$$\langle \Phi A^e \rangle \neq 0$$

ook, MK, Sourdeep



$$N_{\text{pix}} \sim l_{\max}^2$$

\Rightarrow H detected by 21-cm absorption



$\left(\frac{S}{N}\right) \sim 4.5 \times 10^8 \left(\frac{l_{\max}}{10^7}\right)^2$
 $\left(\frac{S}{N}\right) \sim 6.8 \times 10^8 \left(\frac{l_{\max}}{10^7}\right)^2$

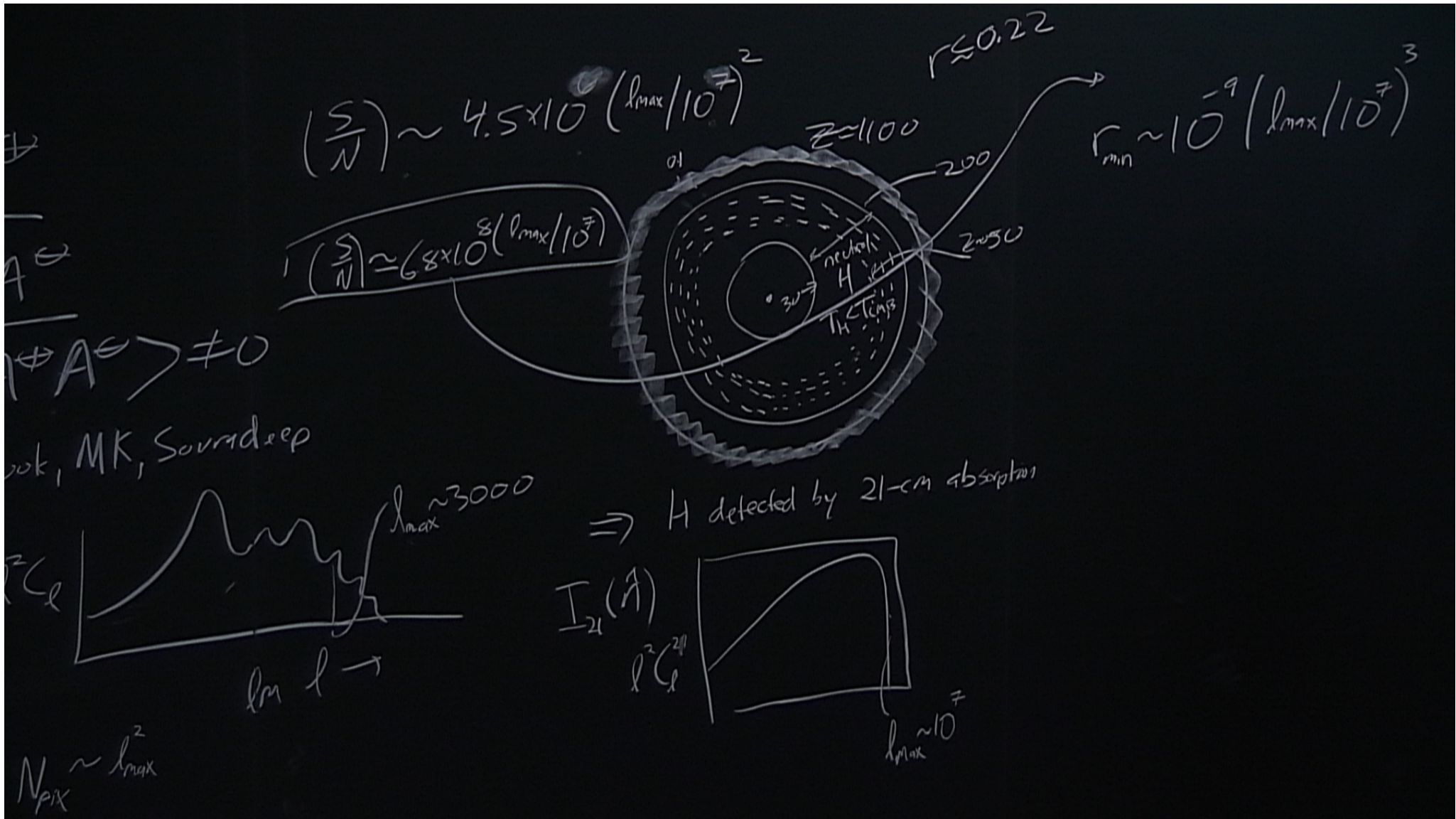
$\Rightarrow H$ detected by 21-cm absorption

$I_{21}(A)$
 l_{\max}^2

$l_{\max} \sim 3000$
 $l_{\max} \sim 10^7$

$N_{\text{pix}} \sim l_{\max}^2$

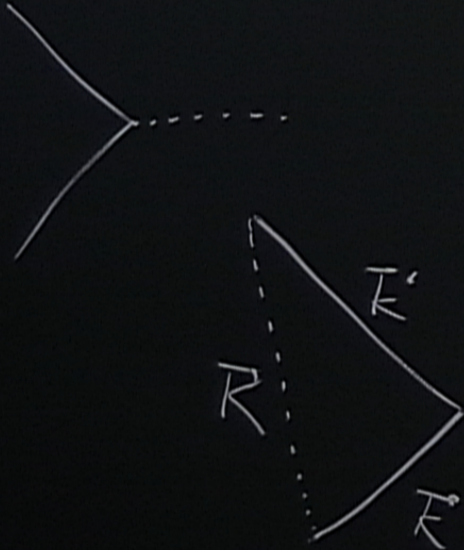
$\Phi A^e > \neq 0$
 ok, MK, Sourdeep



Jeong-MK

$$\delta(\vec{x}) = \frac{\delta P(\vec{x})}{P}$$

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = \delta_{\vec{k}, \vec{k}'}^D P(\vec{k})$$

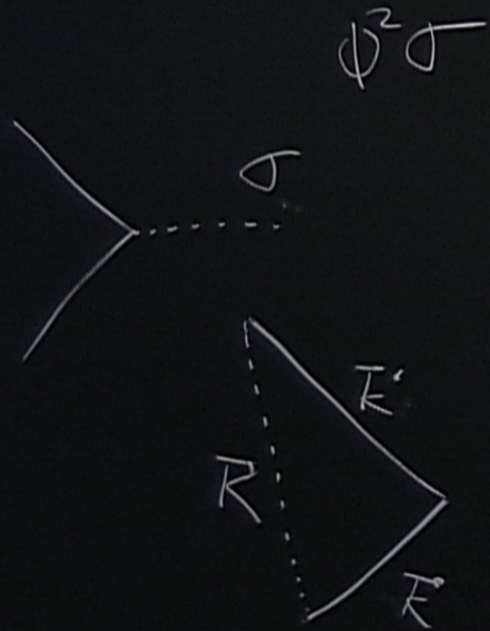


Jeong-MK

$$\delta(x) = \frac{\delta P(x)}{P}$$

$$\langle \delta(K) \delta(K') \rangle = \delta_{KK'}^D P(K)$$

$$\delta(K) \delta(K')$$

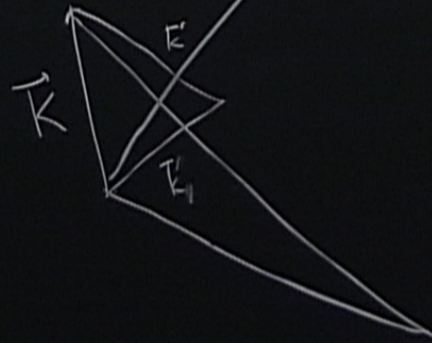


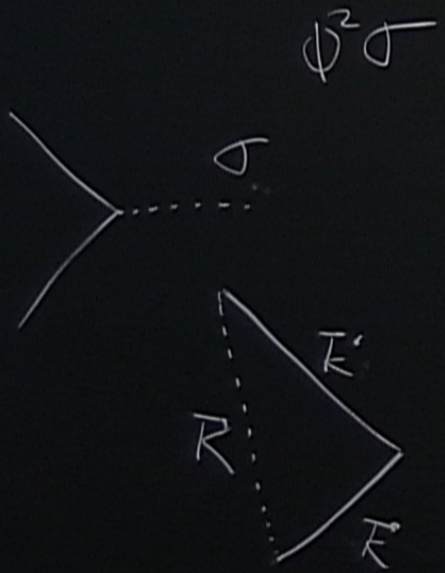
Jeong-MK

$$\delta(\vec{x}) = \frac{\delta P(\vec{x})}{P}$$

$$\langle \delta(\vec{E}) \delta(\vec{E}') \rangle = \int_{\vec{k}, \vec{k}'}^D P(\vec{k})$$

$$\langle \delta(\vec{E}) \delta(\vec{E}') \rangle = \int_{\vec{k}, \vec{k}', \vec{K}=0}^D f(\vec{E}, \vec{k}') \delta(\vec{K})$$





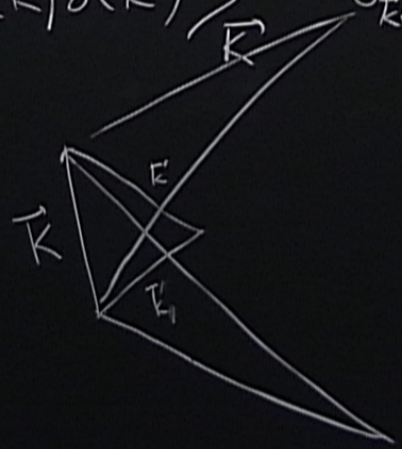
Jeong-MK

$$S(x) = \frac{\delta P(x)}{P}$$

$$\langle S(K) S(K') \rangle = \int_{K, K'}^D P(k)$$

$$\langle S(K) S(K') \rangle = \int_{K, K', K=0}^D f(K, k) \sigma(K)$$

$$\Rightarrow \hat{\sigma}(K)$$



$$(\partial^\mu \phi)(\partial^\mu \phi) \partial^\mu \psi$$

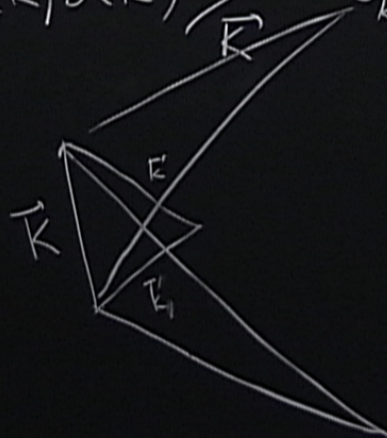
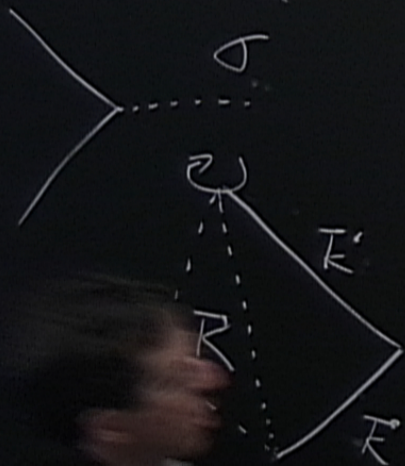
Jeong-MK

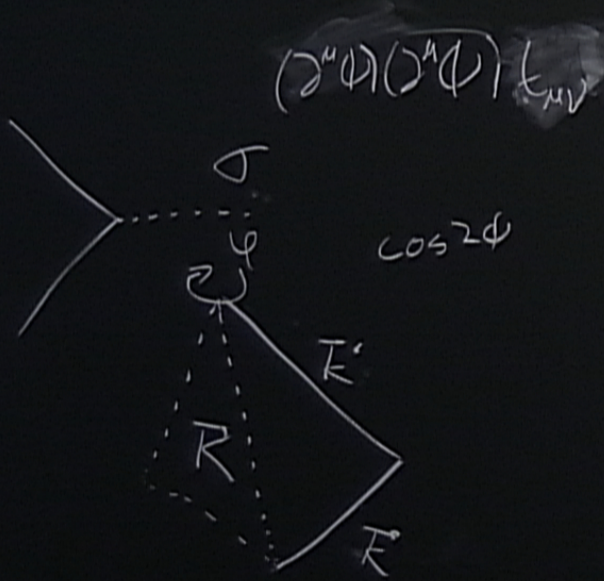
$$\delta(x) = \frac{\delta P(x)}{P}$$

$$\langle \delta(K) \delta(K') \rangle = \int_{K, K'}^D P(K)$$

$$\langle \delta(K) \delta(K') \rangle = \int_{K, K', K=0}^D f(K, K') \delta(K)$$

$$\Rightarrow \hat{\sigma}(K)$$



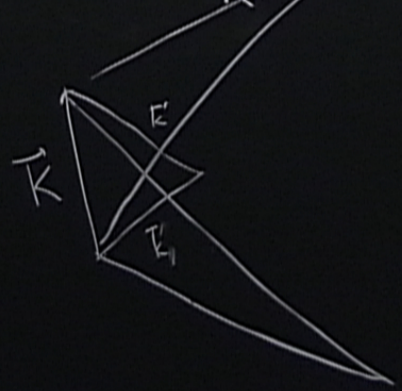


Jeong-MK

$$\delta(x) = \frac{\delta^p(x)}{p}$$

$$\langle \delta(K) \delta(K') \rangle = \int_{K, K'}^D P(k)$$

$$\langle \delta(K) \delta(K') \rangle = \int_{K, K'=0}^D \frac{f(K, k')}{p} \delta(K) \epsilon_{ij}^p k_i k_j$$



$$\Rightarrow \hat{\sigma}(K)$$

- $p = \text{scalar}$
- $p = 2 \text{ vector}$
- $p = 1 \text{ tensor} + x$

$$(\partial^\mu \phi)(\partial^\mu \phi) \epsilon_{\mu\nu}$$

Jeong-MK

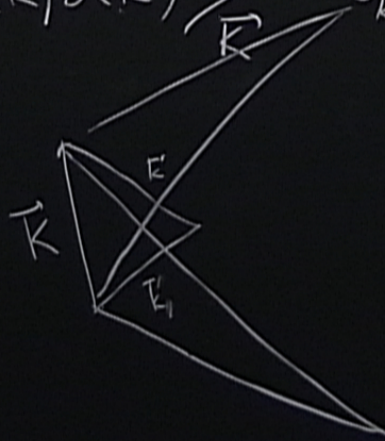
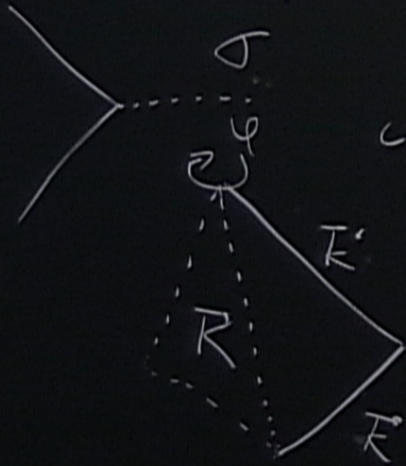
$$\delta(\vec{x}) = \frac{\delta p}{p}(\vec{x})$$

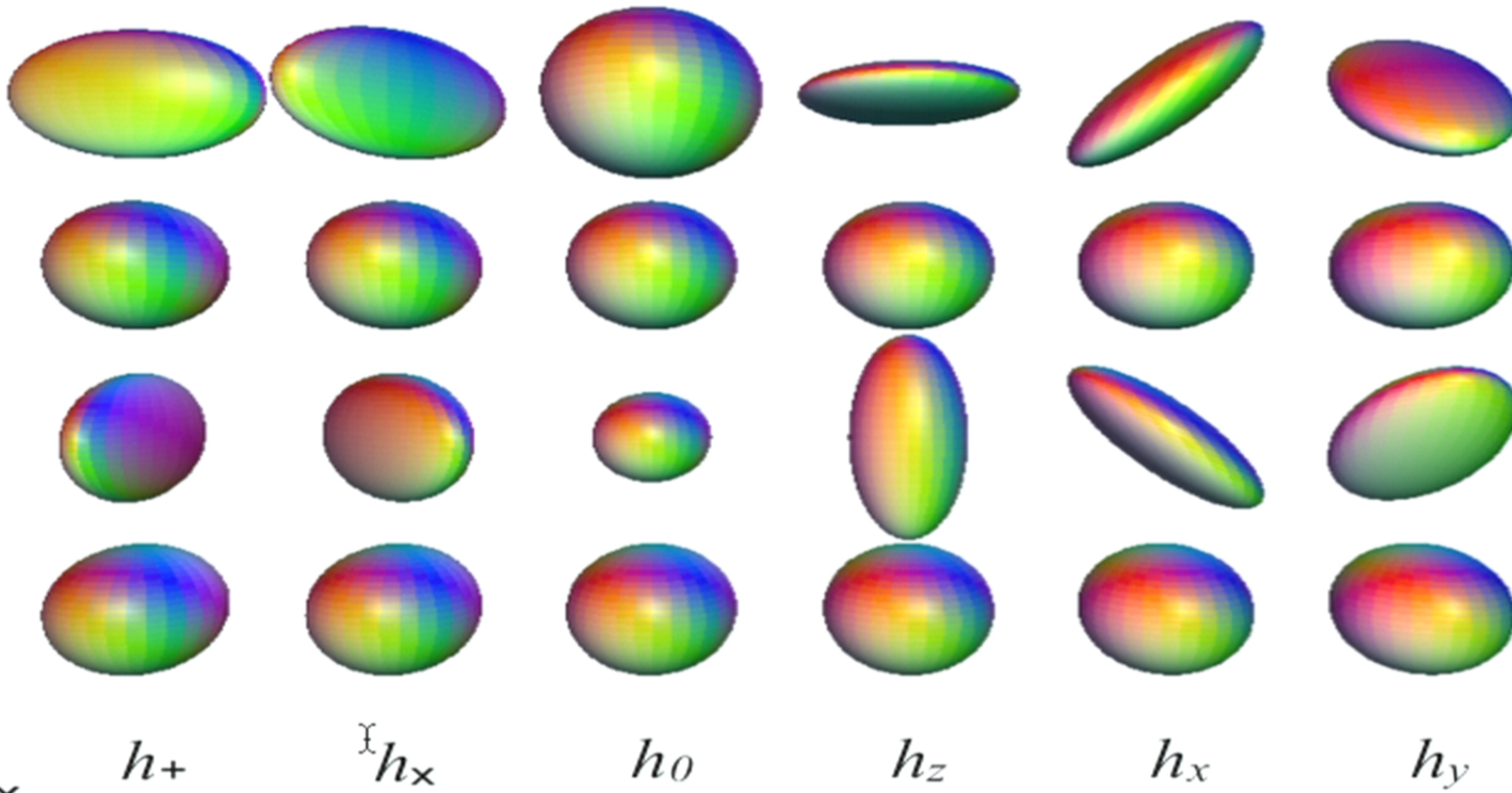
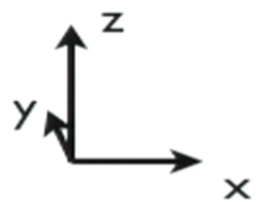
$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = \int_{\vec{k}, \vec{k}'}^D P(k)$$

$$\langle \delta(\vec{k}) \delta(\vec{k}') \rangle = \int_{\vec{k}, \vec{k}', \vec{k}=0}^D \frac{f(\vec{k}, \vec{k}')}{p} \delta(\vec{k}) \epsilon_{ij}^p k_i^i k_j^i$$

$$\Rightarrow \hat{\sigma}(\vec{k})$$

- $p = \text{scalar, scalar}$
- $p = 2 \text{ vector } \times \gamma$
- $p = 1 \text{ tensor } + \chi$



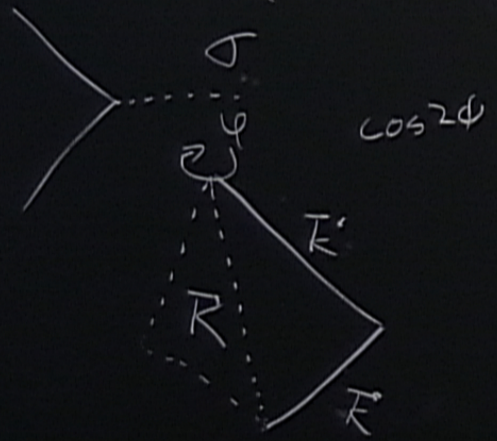


$$(\partial^\mu \phi)(\partial^\nu \phi) \epsilon_{\mu\nu}$$

Jeong-MK

$$\delta(\vec{x}) = \frac{\delta P(\vec{x})}{P}$$

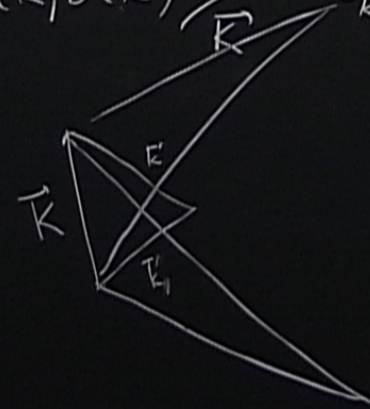
$$\epsilon_{\pm h} = h_{\pm} \pm i h_x$$



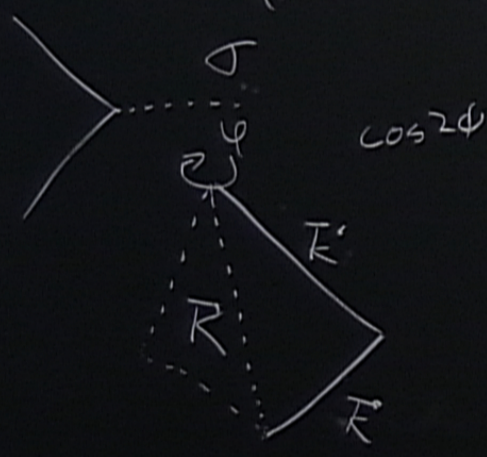
$$\langle \delta(\vec{K}) \delta(\vec{K}') \rangle = \int_{\vec{K}, \vec{K}'}^D P(\vec{k})$$

$$\langle \delta(\vec{K}) \delta(\vec{K}') \rangle = \int_{\vec{K}, \vec{K}'=0}^D \frac{f(\vec{K}, \vec{k})}{P} \delta(\vec{K}) \epsilon_{ij}^p k_i k_j$$

$$\Rightarrow \hat{\sigma}(\vec{K})$$



- $p = \text{scalar, scalar}$
- $p = 2 \text{ vector } \times \gamma$
- $p = 2 \text{ tensor } + \times$



$$(\partial^\mu \phi)(\partial^\nu \phi) \epsilon_{\mu\nu}$$

Jeong-MK

$$\delta(x) = \frac{\delta P(x)}{P(x)}$$

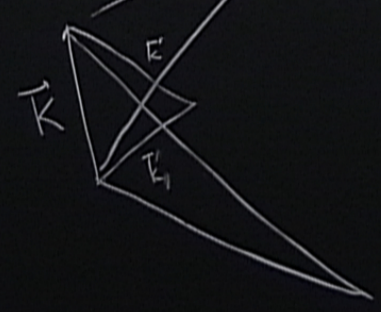
$$\epsilon_{\pm x} = h_{\pm} \pm i h_x$$

$$\epsilon_{\pm y} = h_{\pm} \pm i h_y$$

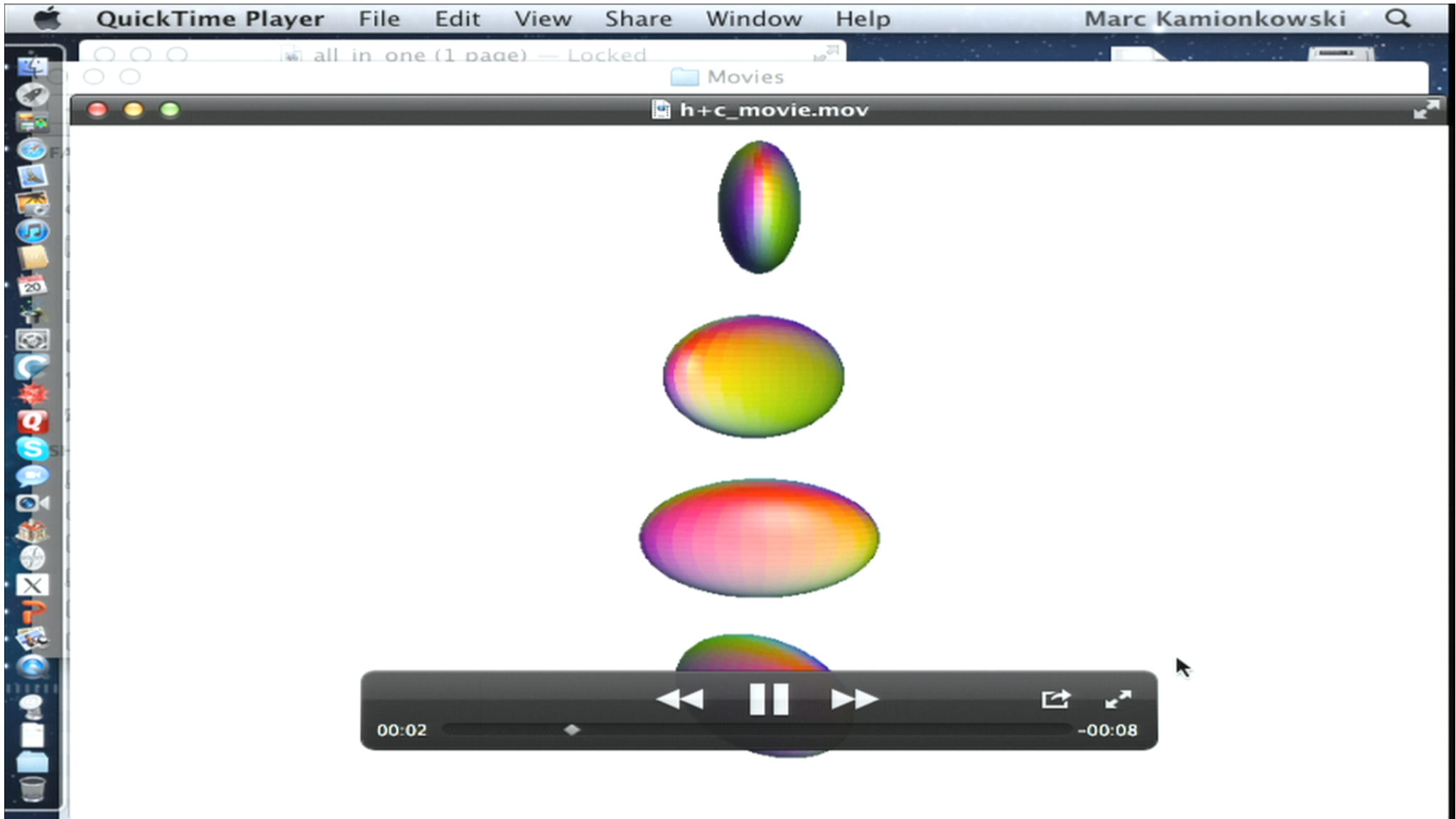
$$\langle \delta(K) \delta(K') \rangle = \int_{K, K'}^D P(k)$$

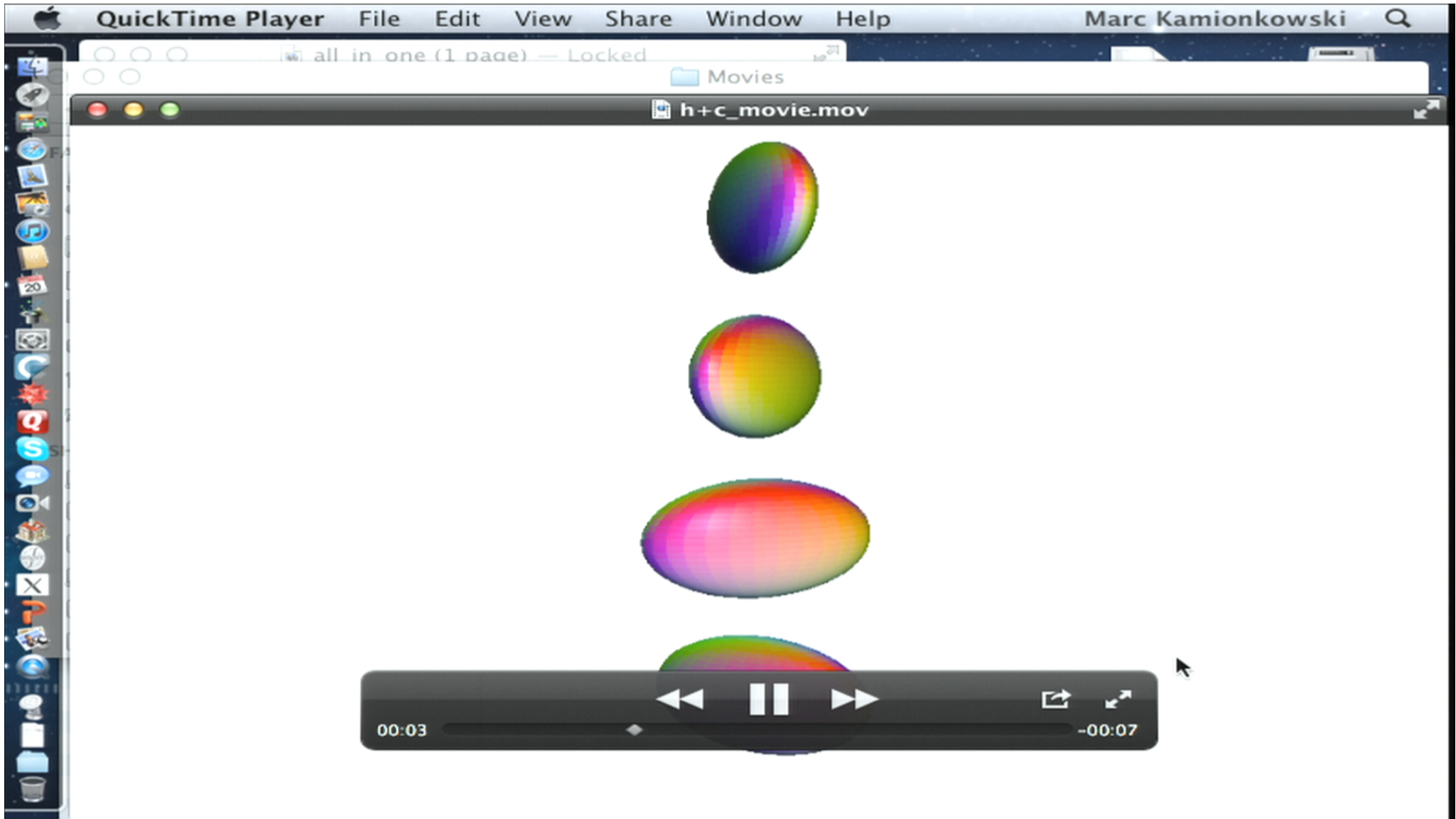
$$\langle \delta(K) \delta(K') \rangle = \int_{K, K'=0}^D \frac{f(K, k)}{P} \hat{\sigma}(K) \epsilon_{ij}^p k_i k_j$$

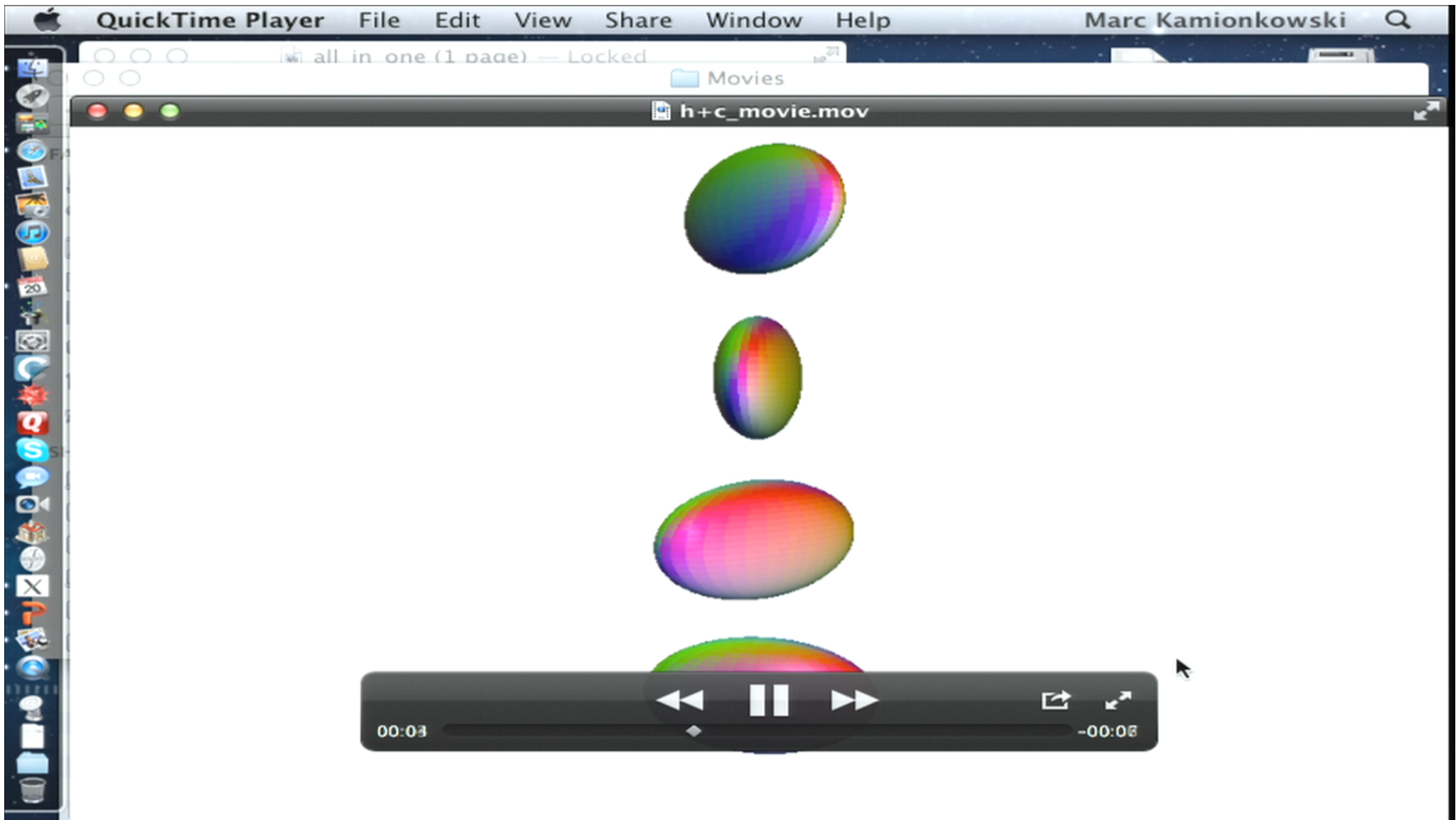
$$\Rightarrow \hat{\sigma}(K)$$

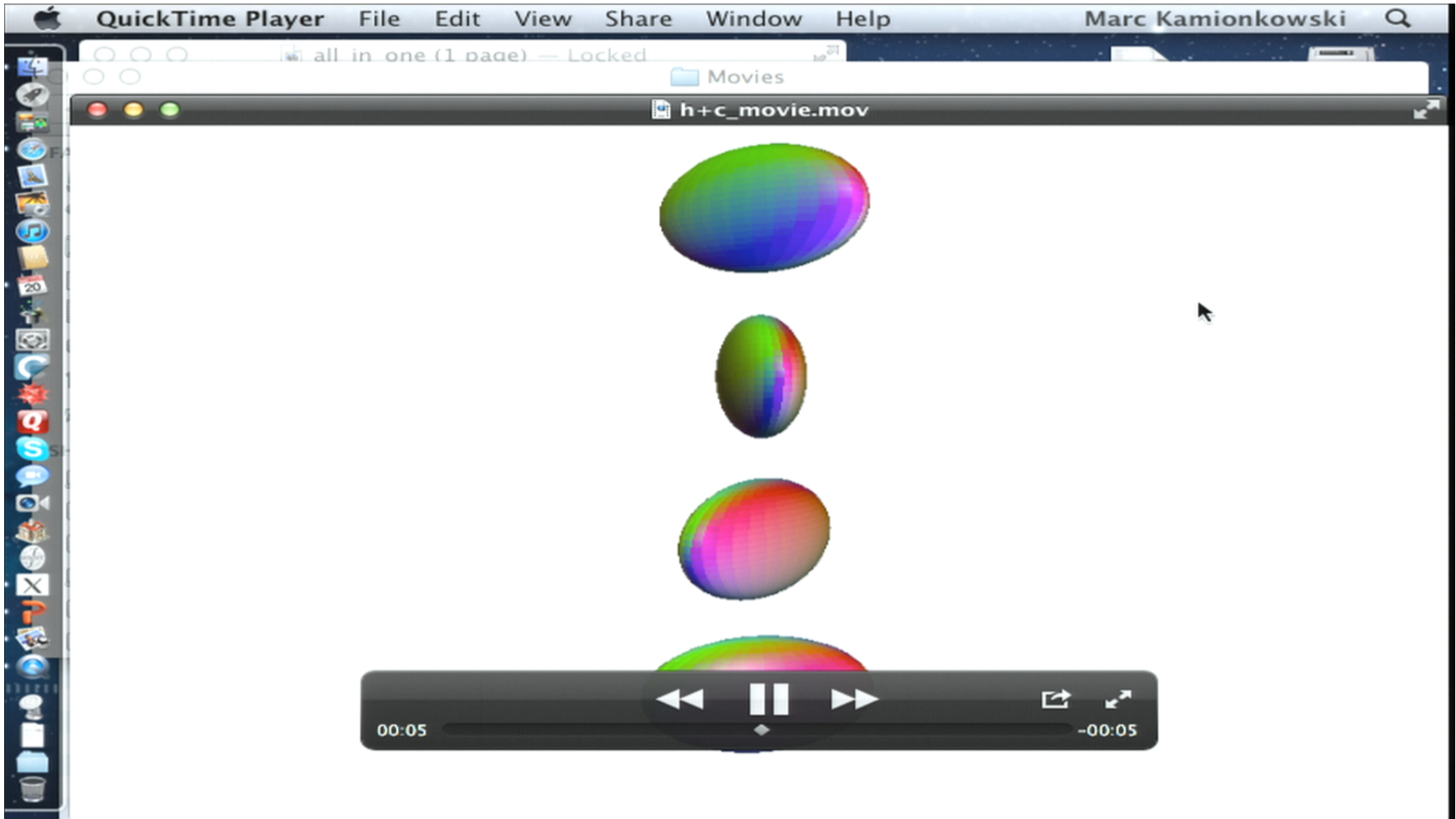


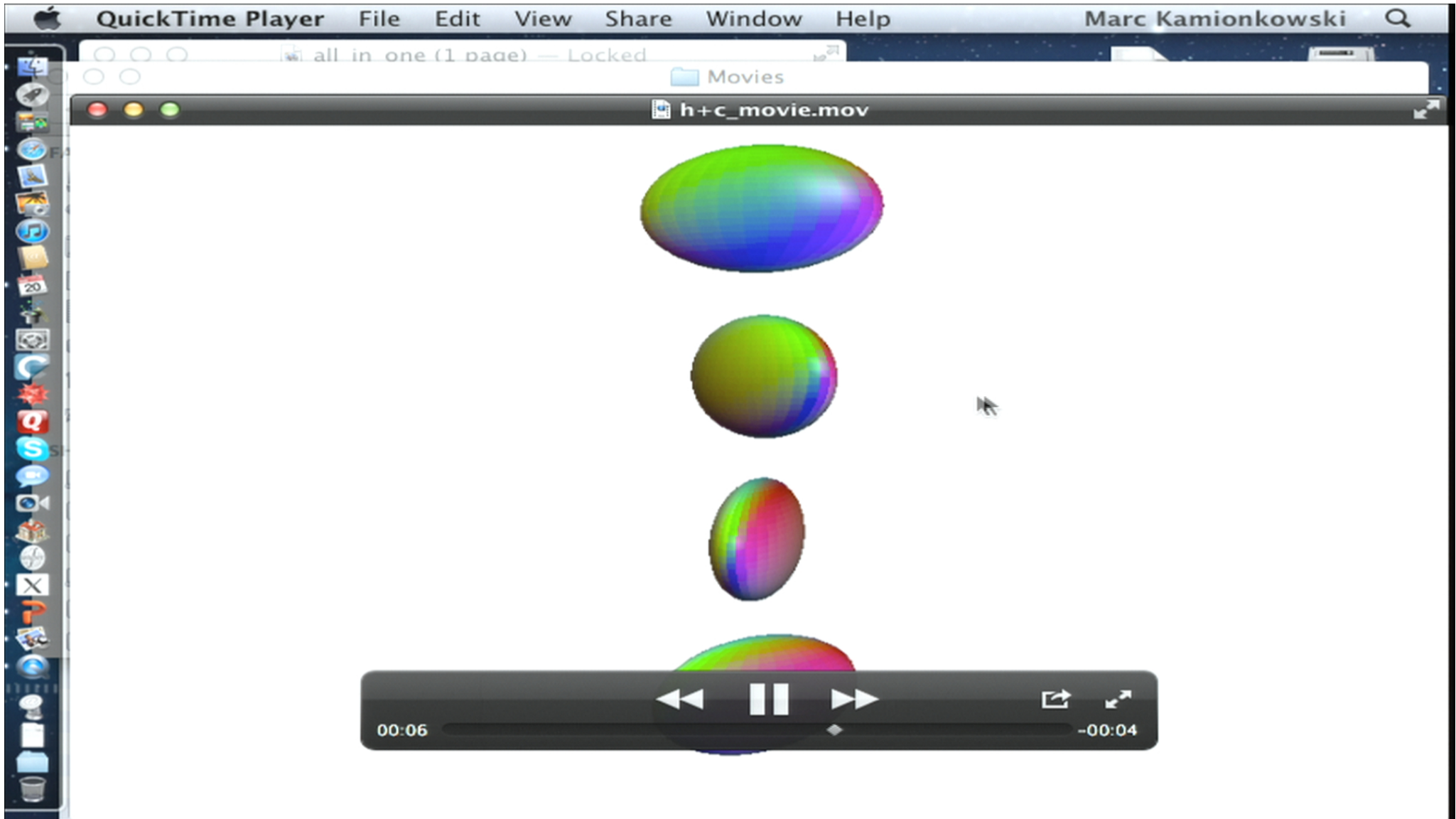
- $p = \text{scalar, scalar}$
- $p = 2 \text{ vector } \times \gamma$
- $p = 2 \text{ tensor } + \times$

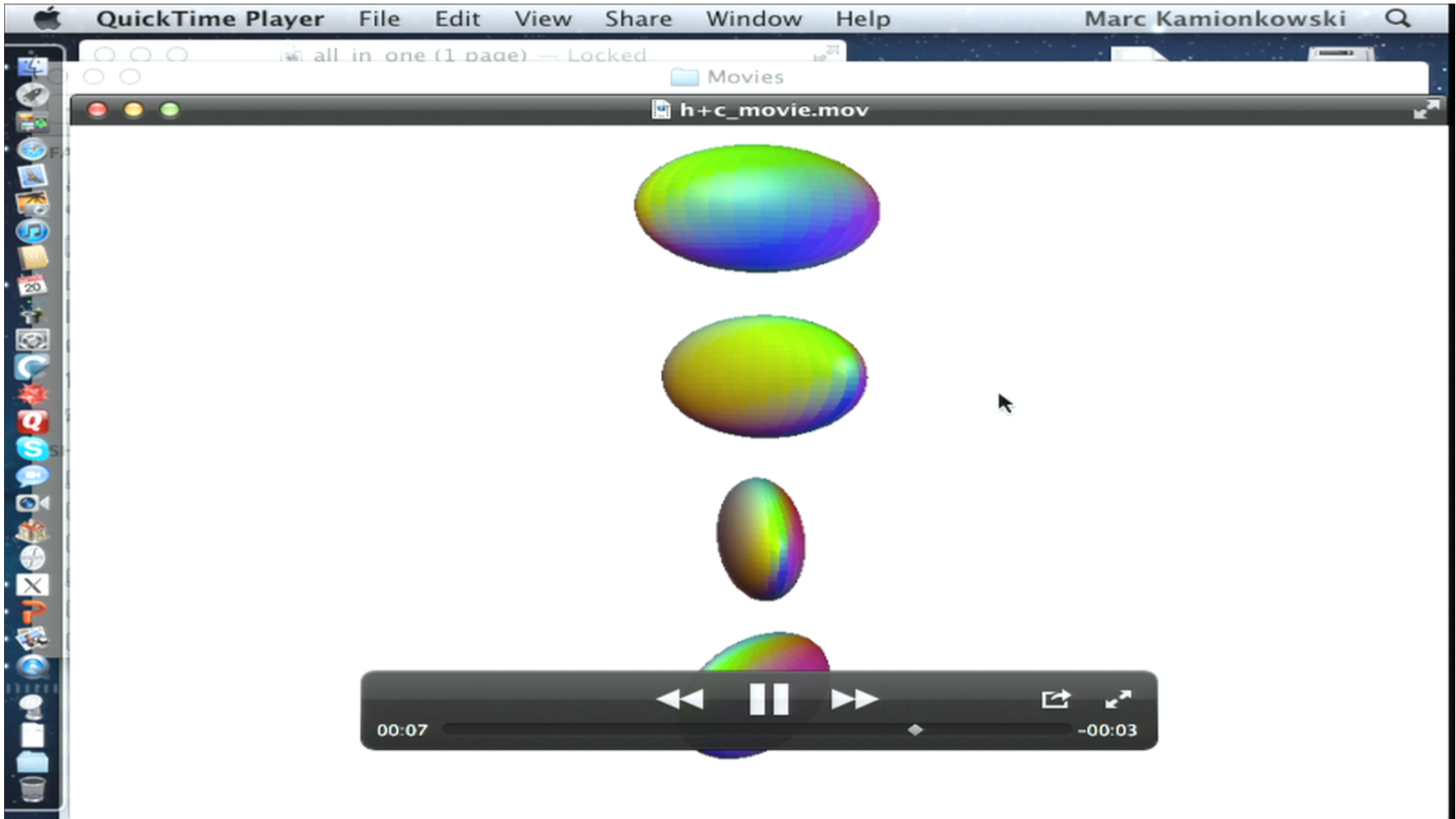


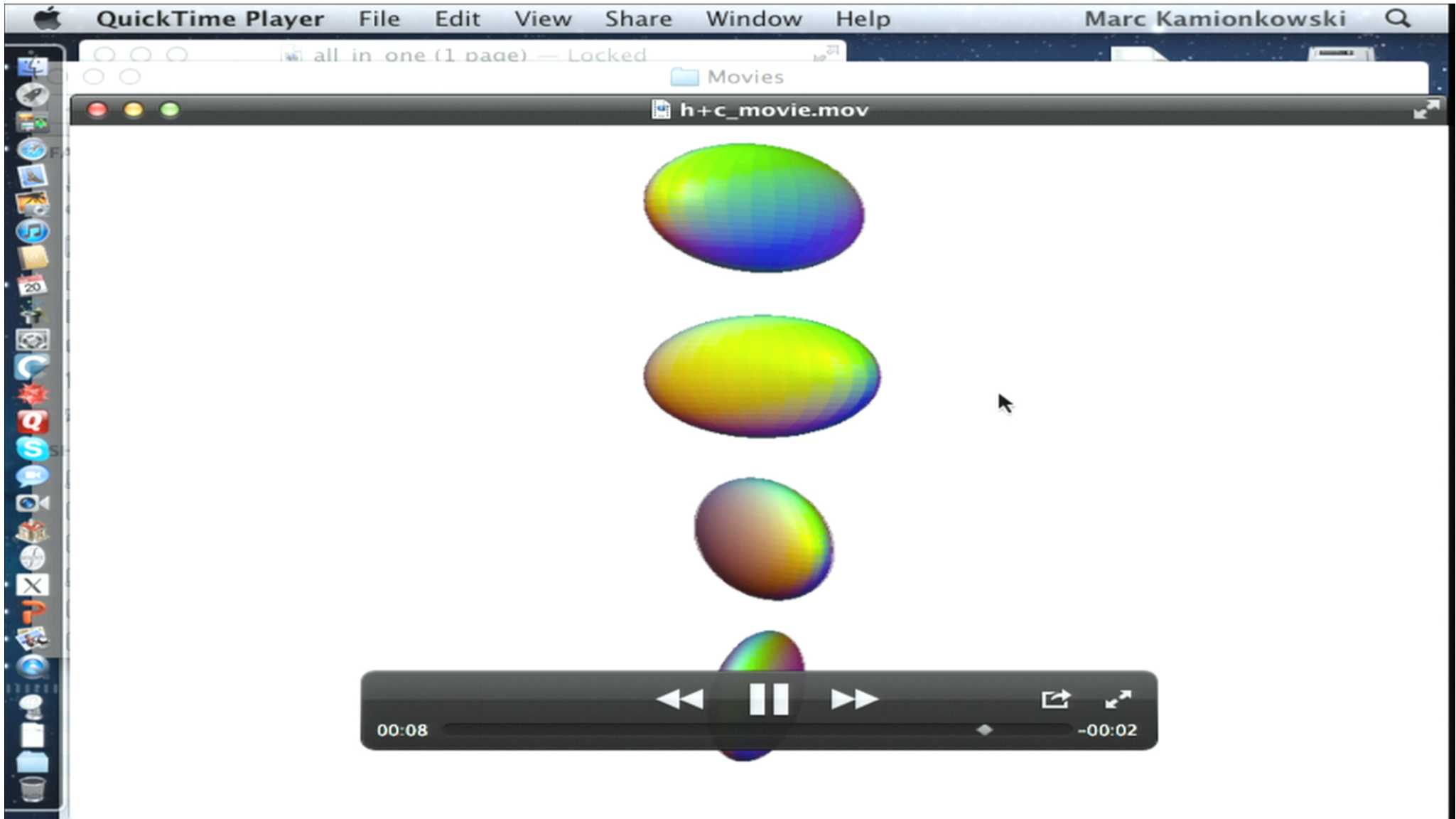












Conclusions

- Current CMB measurements provide precise determination of cosmological parameters and provide strong evidence in support of inflation
- Planck will make these measurements even more precise, and active suborbital search for inflationary gravitational waves under way
- But there is more that can be done with these and future experiments

$\sim 4.5 \times 10^8 (l_{\text{max}}/10^7)^2$

$\sim 8 \times 10^8 (l_{\text{max}}/10^7)$

$r_{\text{min}} \sim 10^{-9} (l_{\text{max}}/10^7)^3$

$\langle |A_{ss}^{\oplus LM}|^2 \rangle = C_{L,ss}^{\oplus}$

$l_{\text{max}} \sim 3000 \Rightarrow \text{H detected by 21-cm absorption}$

$I_{21}(l)$

$l_{\text{max}} \sim 10^7$