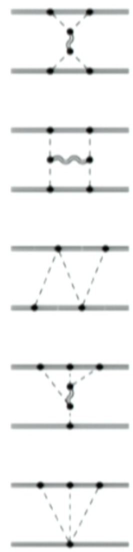
Title: The Effective Field Theorist's Approach to Gravitational Dynamics

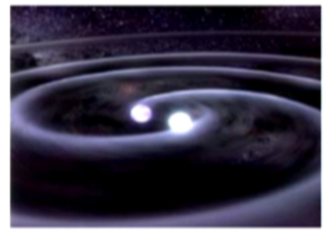
Date: Mar 01, 2012 01:00 PM

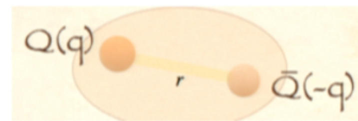
URL: http://pirsa.org/12030088

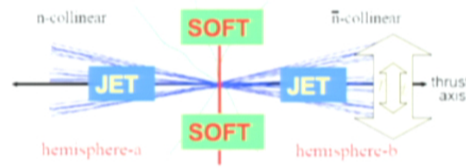
Abstract: I review the uses of effective field theory (EFT) techniques, originally developed in particle physics, to study gravitational dynamics. I will focus on the EFT approach to gravitational wave (GW) radiation, aka NRGR, and show how it has succeeded in producing the most accurate description of spinning binary systems to date, opening the door to a new era of precise astrophysical & cosmological measurements and tests of General Relativity via GW interferometry. I will also briefly discuss EFT applications for black hole dissipation/absorption, inflationary dynamics and high energy gravitational scattering.

The Effective field theorist's approach to gravitational dynamics: From Black Holes to Cosmology on the shoulders of Particle Physics...









Rafael A. Porto
Institute for Advanced Study

Perimeter Institute - 03/2012

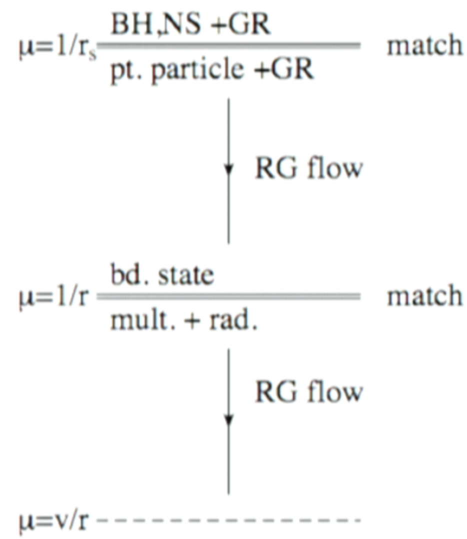
An (incomplete) list of EFT applications in gravity...

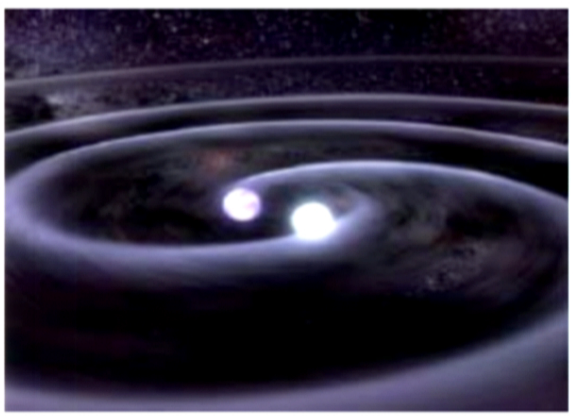
- NRGR - EFT for Gravitational Wave radiation
- EFT/BH duality - Dissipation / Absorption
- EFT of multifield-inflation / dissipative effects
- EFT for high energy gravitational Scattering
- EFT for cosmological PT & dissipation in hydrodynamics

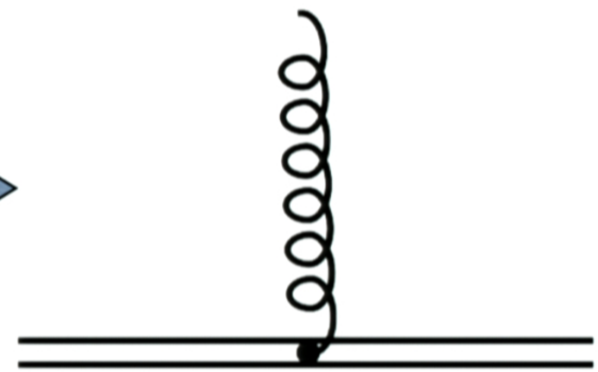
but...

I will not cover all of this in detail (some still in progress.)

Chapter I: NRGR
An EFT approach to GW radiation from binary BHs/NSs



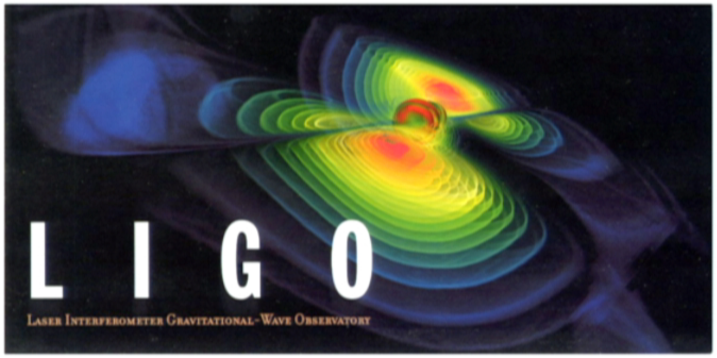


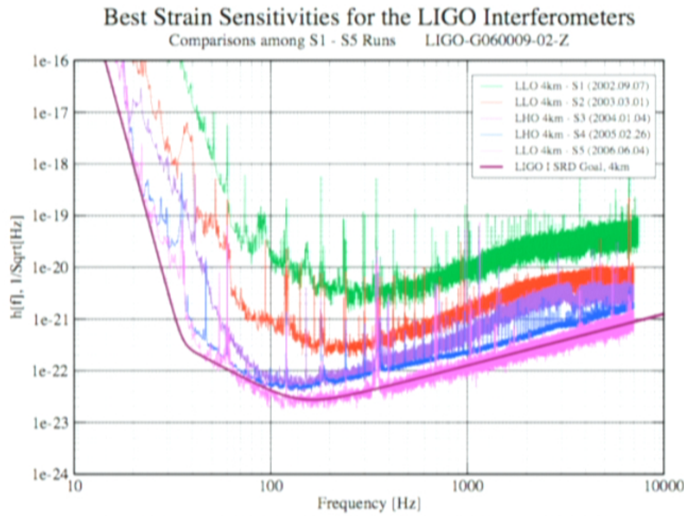


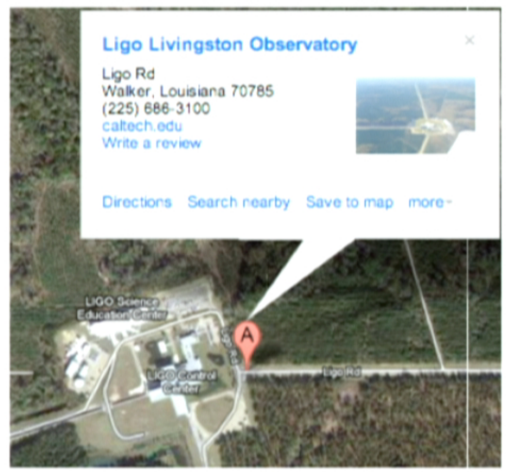
Motivation

We've learned a great deal about BHs
Imaging how much can we learn about gravity
smashing them into each other!

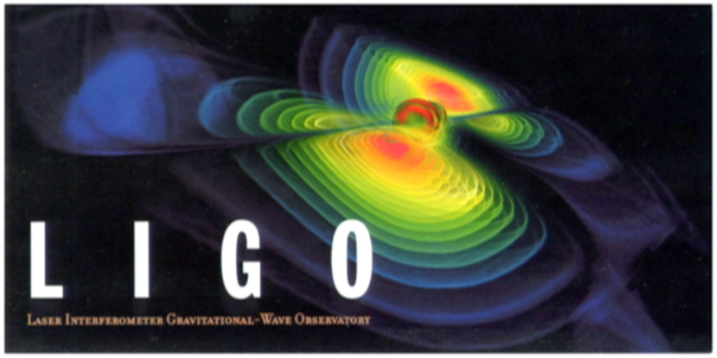
‘The L(B)HC’: AdvLIGO (2014-ish)

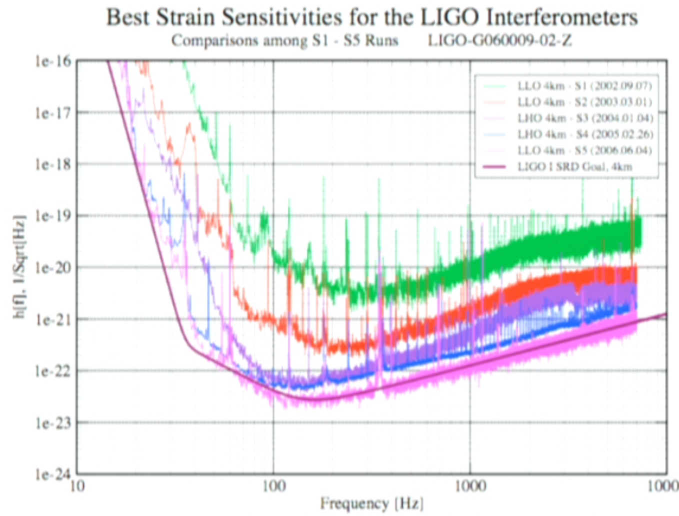




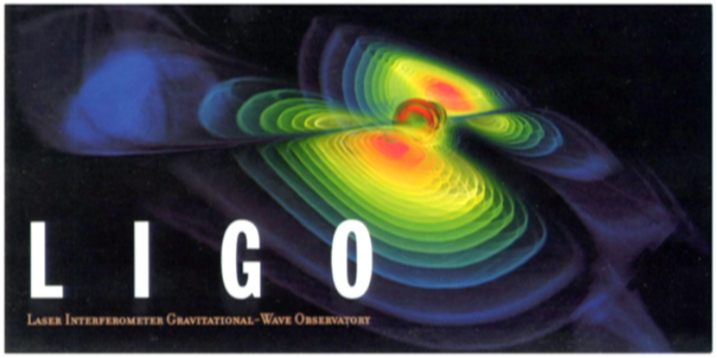


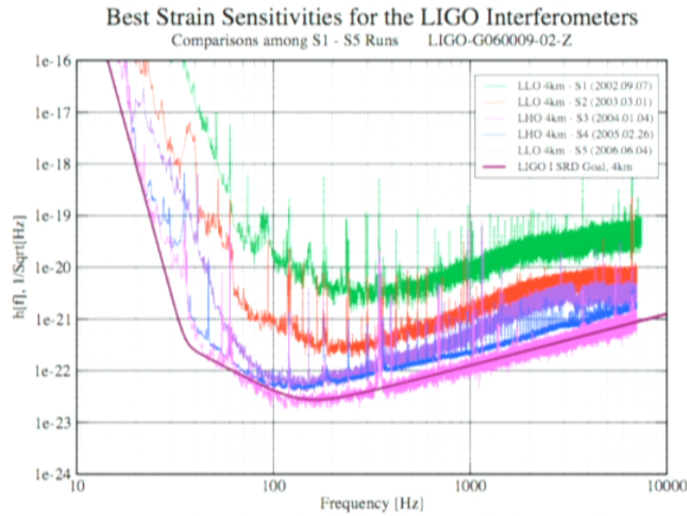
‘The L(B)HC’: AdvLIGO (2014-ish)



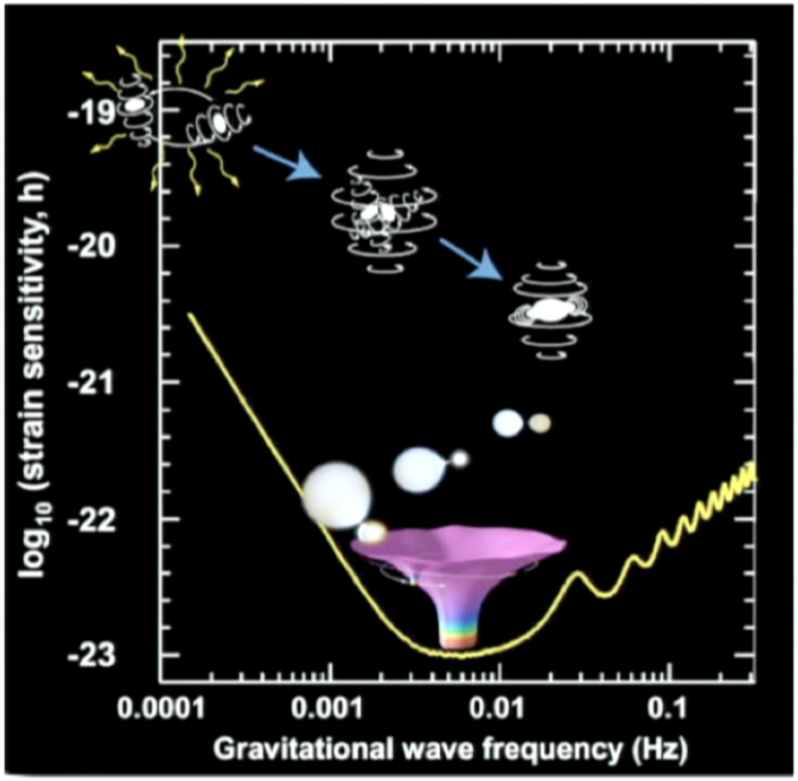


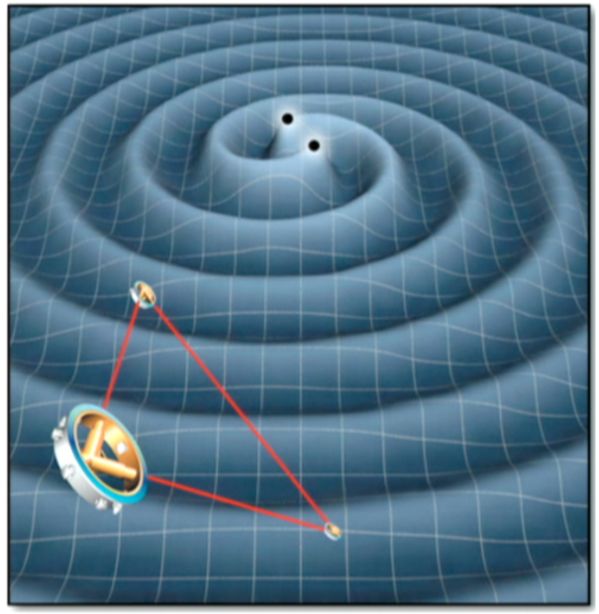
‘The L(B)HC’: AdvLIGO (2014-ish)





'The ILC': Space-based - eLISA





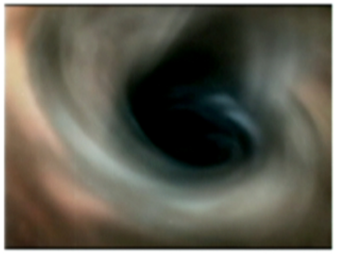
Super-massive binary BHs (10^5-10^6), Galactic binaries, EMRIs...

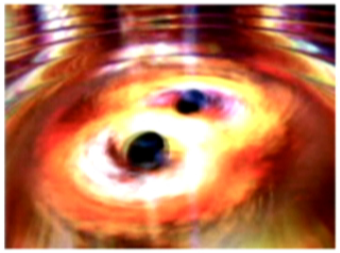
Illustrations from 'Lisa - Probing the Universe with GWs'

(...or perhaps atom interferometers - Stanford group)

Testing the warped side of the Universe

(illustration courtesy K. Thorne)

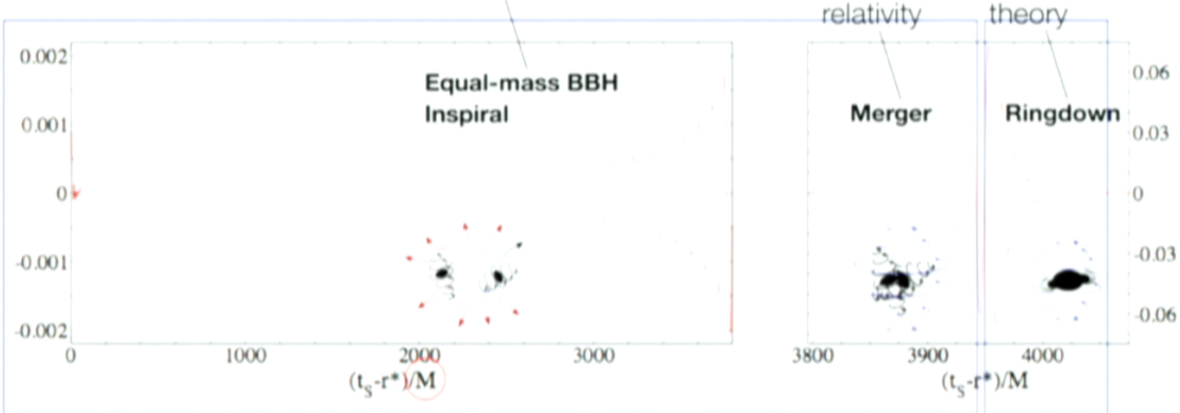




- High(est) precision tests of General Relativity from binary black hole systems. (The most accurate description of relativistic systems in strong gravitational fields)
- Measurements of spins and masses of neutron stars and black holes to the percent level (history of galaxy formation, super massive black holes in the center of the galaxy...)
- Bounds on light particles (axions) from GW observations (from superradiant effects for spinning black holes or finite size effects, e.g. Axiverse)
- Test of matter at high pressure/density with neutron stars binaries, e.g. equation of state (QCD) (from imprint of finite size effects in the waveforms)
- GWs from primordial fluctuations...
- .New physics! Precise understanding of known background sources (survey population) (Like QCD background at the LHC)

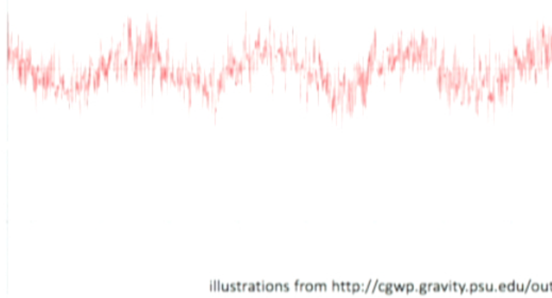
GW science in a nutshell: what's in a waveform?

(illustration courtesy Kip Thorne, Laura Cadonati, M. Vallisneri)



Depends on binary masses, spins, orientations

Depends on mass, spin, and orientation of final BH



illustrations from http://cgwp.gravity.psu.edu/outreach/

To compute the phase we need:

$$v^3 = \dot{\phi}_{\rm GW}\,\frac{M}{2}\,, \qquad \frac{dE}{dt}(v) = -\mathcal{F}(v)$$

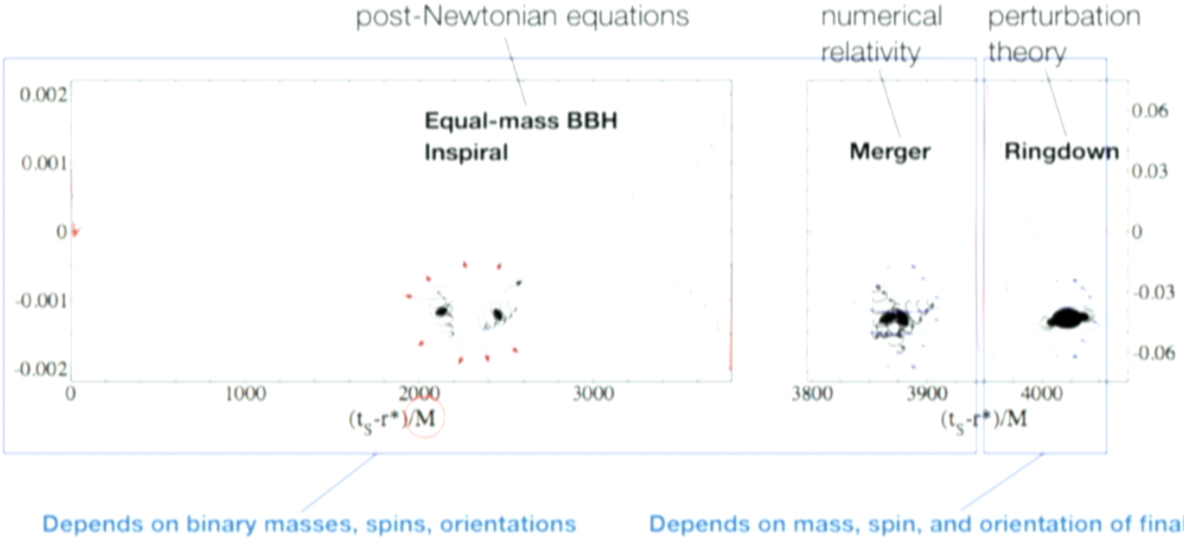
$$\phi_{\rm GW}(v) \;=\; \phi_c + 2\int_v^{v_c} dv\, v^3\, \frac{E'(v)}{\mathcal{F}(v)}$$

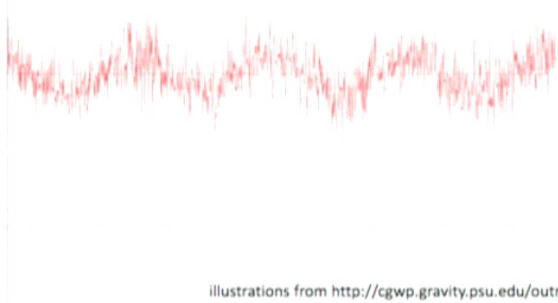
- phase needs to match during all cycles observed

The payoff of GW science relies on accurate templates

GW science in a nutshell: what's in a waveform?

(illustration courtesy
Kip Thorne,
Laura Cadonati,
M. Vallisneri)





illustrations from http://cgwp.gravity.psu.edu/outreach/

To compute the phase we need:

$$v^3 = \dot{\phi}_{\rm GW}\,\frac{M}{2}\,, \qquad \frac{dE}{dt}(v) = -\mathcal{F}(v)$$

$$\phi_{\rm GW}(v) \;=\; \phi_c + 2\int_v^{v_c} dv\, v^3\, \frac{E'(v)}{\mathcal{F}(v)}$$

- phase needs to match during all cycles observed

The payoff of GW science relies on accurate templates

General Relativity 101

Solve Einstein's Equations $\sqrt{g}\; g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$G^{\alpha\beta}[g, \partial g, \partial^2 g] = \frac{8\pi G}{c^4} T^{\alpha\beta}[g] \quad \longrightarrow \quad \Box h^{\alpha\beta} = \frac{16\pi G}{c^4} \tau^{\alpha\beta}.$$

$$\tau^{\alpha\beta} = |g| T^{\alpha\beta} + \frac{c^4}{16\pi G} \Lambda^{\alpha\beta}. \quad T^{\alpha\beta}(x) \sim m \int v^\alpha(\tau) v^\beta(\tau) \delta^4(x(\tau) - x) d\tau$$

$$\Lambda^{\alpha\beta} = N^{\alpha\beta}[h,h] + M^{\alpha\beta}[h,h,h] + L^{\alpha\beta}[h,h,h,h] + \mathcal{O}(h^5).$$

This is an expansion in powers of $\frac{GM}{c^2 r} \sim \frac{v^2}{c^2}$

For a bound state:
Potential Energy ~ Kinetic Energy

Post-Newtonian expansion

$$\text{nPN} \rightarrow \mathcal{O}\left(v^{2n}/c^{2n}\right)$$

$$\text{1PN} \rightarrow \mathcal{O}\left(v^2/c^2\right), \text{ etc}$$

General Relativity 101

Solve Einstein's Equations $\sqrt{g}\; g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$G^{\alpha\beta}[g, \partial g, \partial^2 g] = \frac{8\pi G}{c^4} T^{\alpha\beta}[g] \quad \longrightarrow \quad \Box h^{\alpha\beta} = \frac{16\pi G}{c^4}\tau^{\alpha\beta}.$$

$$\tau^{\alpha\beta} = |g| T^{\alpha\beta} + \frac{c^4}{16\pi G}\Lambda^{\alpha\beta}. \qquad T^{\alpha\beta}(x) \sim m \int v^{\alpha}(\tau) v^{\beta}(\tau) \delta^4(x(\tau) - x) d\tau$$

$$\Lambda^{\alpha\beta} = N^{\alpha\beta}[h,h] + M^{\alpha\beta}[h,h,h] + L^{\alpha\beta}[h,h,h,h] + \mathcal{O}(h^5).$$

This is an expansion in powers of $\frac{GM}{c^2 r} \sim \frac{v^2}{c^2}$

For a bound state:
Potential Energy ~ Kinetic Energy

Post-Newtonian expansion

$$\mathrm{nPN} \rightarrow \mathcal{O}\left(v^{2n}/c^{2n}\right)$$

$$\mathrm{1PN} \rightarrow \mathcal{O}\left(v^2/c^2\right), \ \mathrm{etc}$$

Advanced Calculus: *Traditional* approach

(L. Blanchet, T. Damour, G. Schafer, C. Will...)

Finally, at the 3PN order, we have

$$\hat{T} = \Box_{\mathcal{R}}^{-1}\Big[-4\pi G\Big(\frac{1}{4}\sigma_{ij}\hat{W}_{ij} + \frac{1}{2}V^2\sigma_{ii} + \sigma V_i V_i\Big) + \hat{Z}_{ij}\partial_{ij}V + \hat{R}_i\partial_t\partial_i V - 2\partial_i V_j \partial_j \hat{R}_i$$
$$-\partial_i V_j \partial_t \hat{W}_{ij} + V V_i \partial_t \partial_i V + 2V_i \partial_j V_i \partial_j V + \frac{3}{2}V_i \partial_t V \partial_i V + \frac{1}{2}V^2 \partial_t^2 V$$
$$+\frac{3}{2}V(\partial_t V)^2 - \frac{1}{2}(\partial_t V_i)^2\Big] + \delta_{\mathrm{Leibniz}}\hat{T}\,,$$

← regularization piece(s)

$$\hat{Y}_i = \Box_{\mathcal{R}}^{-1}\Big[-4\pi G\Big(-\sigma\hat{R}_i - \sigma V V_i + \frac{1}{2}\sigma_k \hat{W}_{ik} + \frac{1}{2}\sigma_{ik}V_k + \frac{1}{2}\sigma_{kk}V_i\Big) + \hat{W}_{kl}\partial_{kl}V_i - \partial_t \hat{W}$$
$$+\partial_i \hat{W}_{kl}\partial_k V_l - \partial_k \hat{W}_{il}\partial_l V_k - 2\partial_k V \partial_i \hat{R}_k - \frac{3}{2}V_k \partial_i V \partial_k V - \frac{3}{2}V \partial_t V \partial_i V$$
$$-2V\partial_k V \partial_k V_i + V\partial_t^2 V_i + 2V_k \partial_k \partial_t V_i\Big] + \delta_{\mathrm{Leibniz}}\hat{Y}_i\,,$$

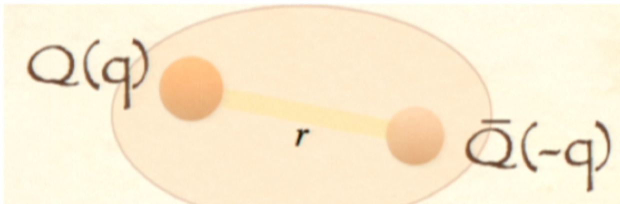
$$\hat{Z}_{ij} = \Box_{\mathcal{R}}^{-1}\Big[-4\pi G V(\sigma_{ij} - \delta_{ij}\sigma_{kk}) - 2\partial_{(i}V\partial_t V_{j)} + \partial_i V_k \partial_j V_k + \partial_k V_i \partial_k V_j - 2\partial_{(i}V_k \partial_k V_{j)}$$
$$-\delta_{ij}\partial_k V_m(\partial_k V_m - \partial_m V_k) - \frac{3}{4}\delta_{ij}(\partial_t V)^2\Big] + \delta_{\mathrm{Leibniz}}\hat{Z}_{ij}\,.$$

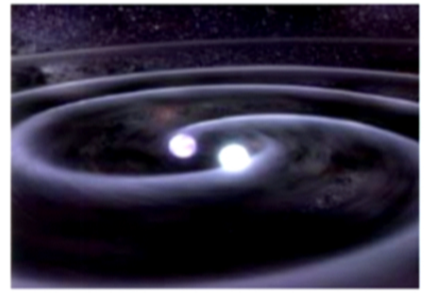
(don't worry, just to scare you away...this goes on for pages and pages)

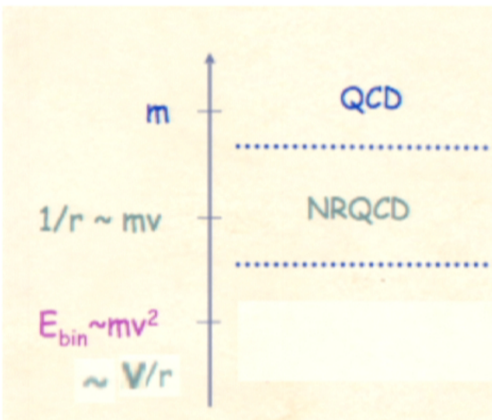
Issues due to lack of systematics:

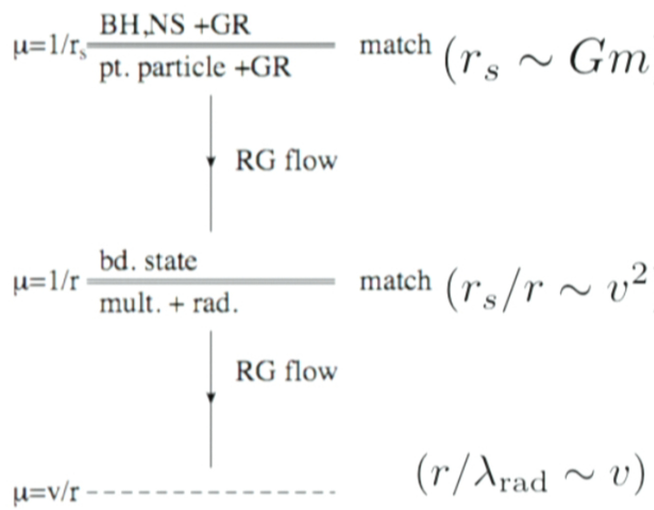
- Ambiguous regularization.
- It doesn't exploit power of symmetries (many regularization pieces derive from a single counter-term!)
- Not clear how to systematically include spin degrees of freedom or finite size effects.

NRGR: Binary systems as heavy quark bound states

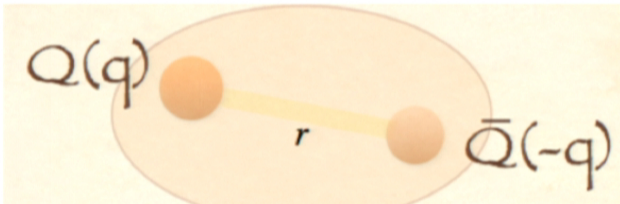


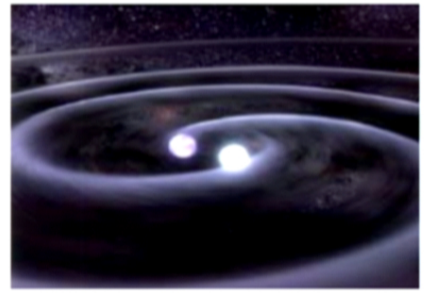


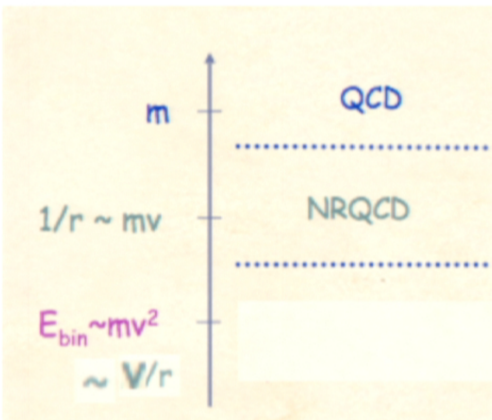


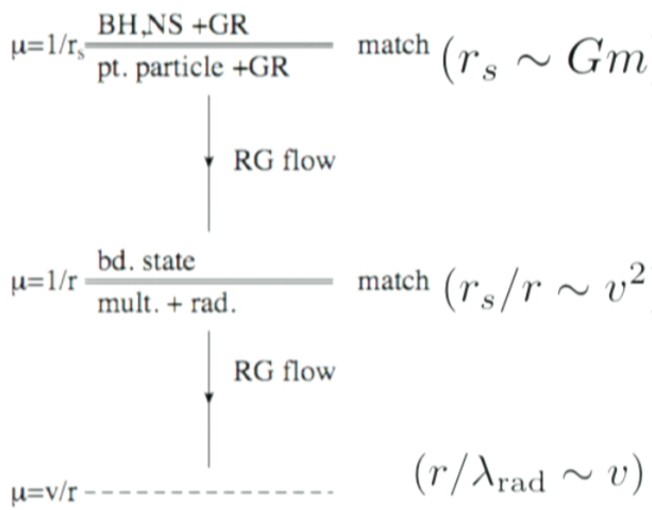


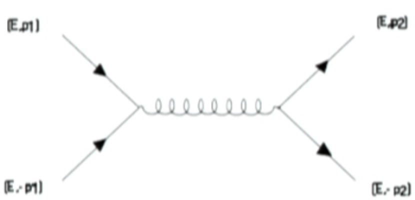
NRGR: Binary systems as heavy quark bound states

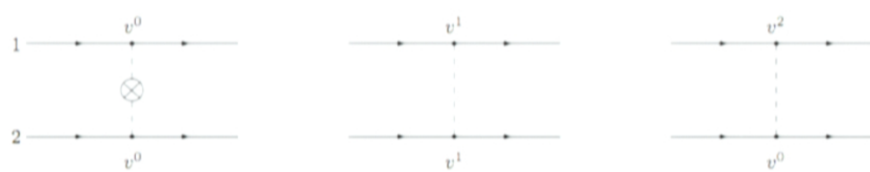




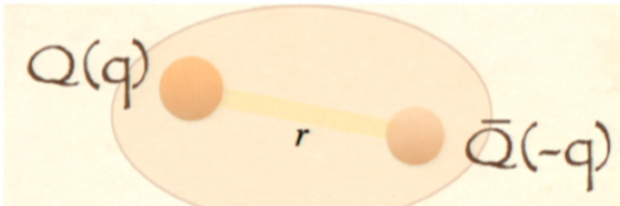


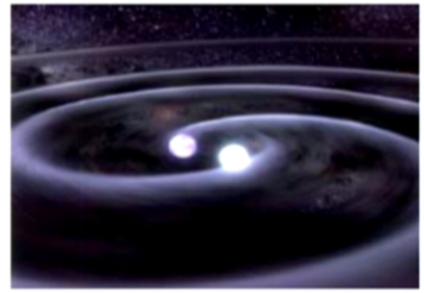


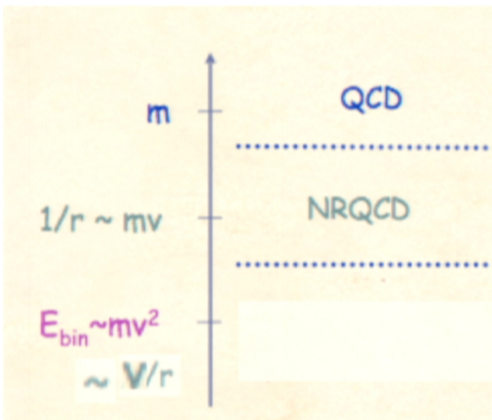


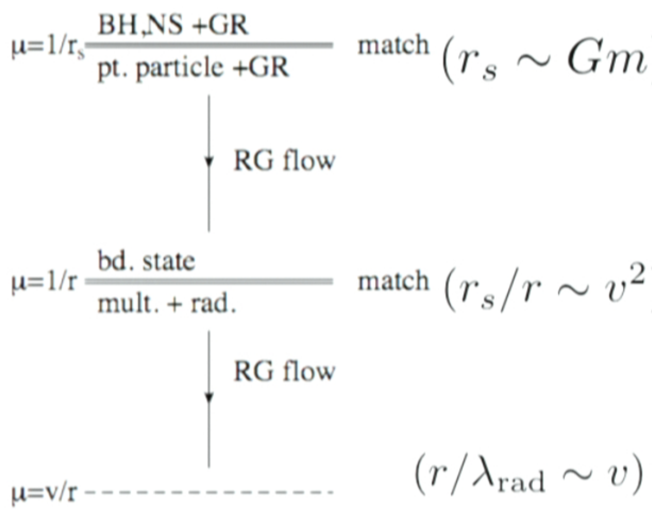


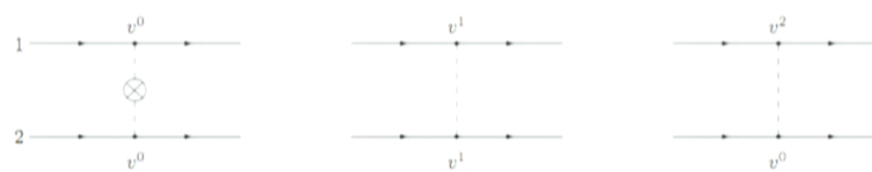
NRGR: Binary systems as heavy quark bound states











EFT methodology

(Goldberger & Rothstein, hepth/0409156)

Perform the Classical path integral one scale at a time

$$\mathcal{A} \sim \int \mathcal{D}x\mathcal{D}g e^{iS[g,x]}, \quad \mathcal{A} \sim e^{iS_{\text{eff}}[x_{\text{cl}}]} \equiv e^{iS_{\text{cl}}}$$

To integrate out the 'short distance' dynamics we need to consistently implement the point particle approximation.

$$S[g,x] = S_{\text{EH}} + S_{pp}$$ ⟵ Einstein-Hilbert + worldline action

All the dynamics is then encoded in Re and Im of the effective action

$$\langle in, 0 \mid out, 0 \rangle_J \sim e^{(iV(J) - \Gamma/2)T}$$

Potential Energy → $V(J)$

Power Loss → Γ

The BHs (or NSs) play the role of 'external sources' of 'gravitons'

We work at zeroth order in $\hbar$ (Saddle point approximation)

EFT methodology

(Goldberger & Rothstein, hepth/0409156)

Perform the Classical path integral one scale at a time

$$\mathcal{A} \sim \int \mathcal{D}x\mathcal{D}g e^{iS[g,x]}, \; \mathcal{A} \sim e^{iS_{\rm eff}[x_{\rm cl}]} \equiv e^{iS_{\rm cl}}$$

To integrate out the 'short distance' dynamics we need to consistently implement the point particle approximation.

$$S[g,x] = S_{\rm EH} + S_{pp}$$ ⟵ Einstein-Hilbert + worldline action

All the dynamics is then encoded in Re and Im of the effective action

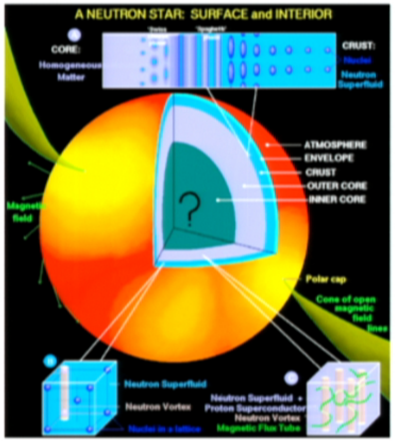
$$\langle in, 0 \mid out, 0 \rangle_J \sim e^{(iV(J) - \Gamma/2)T}$$

Potential Energy

Power Loss

The BHs (or NSs) play the role of 'external sources' of 'gravitons'

We work at zeroth order in $\hbar$ (Saddle point approximation)



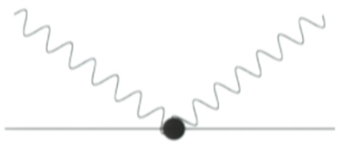
NSs and BHs as point particles

$$S_{pp} = \int \left(m + c_E E_{ab}E^{ab} + c_B B_{ab}B^{ab}\right) d\tau + \ldots$$

→ encode effects from 'short distance' physics

CE and CB are so called 'matching' coefficients.
They can be read off by comparing with computations in the 'full theory', e.g. GW scattering cross section:

$$\sigma_{full}(\omega) = r_s^2 F(\omega r_s) \sim r_s^2(\ldots + \alpha r_s^8 \omega^8 + \ldots)$$



$$\sigma_{EFT}(\omega) = \ldots + \frac{C_{E,B}^2}{m_{pl}^4}\omega^8 + \ldots$$

$$C_{E,B} \sim r_s^5 m_{pl}^2$$

Effacement Theorem:

Finite-size effects first appear at 5PN order for non-rotating compact objects.
Enhanced for NSs (Love number). Finite size effects due to SPIN! appear at lower orders

$$r_s \sim \frac{GM}{c^2} \rightarrow \frac{r_s}{r} \sim \frac{1}{c^2}\frac{GM}{r} \sim \frac{v^2}{c^2}$$

(Note: For Black Holes C_E vanishes in D=4
Damour&Nagar, Binnington&Poisson, Kol&Smolkin)

An EFT for spinning compact objects

(RAP gr-qc/0511061, gr-qc/0701106, PhD Thesis 07')

(Let's spare the technicalities of dealing with spin in GR,
it would take a whole other talk: Routhians, second class constraints, SSCs, Tmn)

$$S_{pp}(x_i, g, S_i^{ab}) = \int d\tau \left(-m - \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + C_{ES^2} E_{ab} S^{bc} S_c^a + \dots\right)$$

spin coupling

short distance physics
(self-induced finite size effect)

(slightly more complicated/
technical stuff. But waaaaay
simpler than ADM
formalism!)

Matching with rotating BHs:

Compute the one-point function and 'match'. For example with the Kerr metric, e.g. we read off the quadrupole moment of the space-time:

$$C^i_{ES^2} = 1/2m_i$$

Using standard power counting rules we can show there is one and *only one* coefficient to NLO (3PN).

(Finite size effects are highly arbitrary/complicated in traditional ADM approach)

An EFT for spinning compact objects

(RAP gr-qc/0511061, gr-qc/0701106, PhD Thesis 07')

(Let's spare the technicalities of dealing with spin in GR,
it would take a whole other talk: Routhians, second class constraints, SSCs, Tmn)

$$S_{pp}(x_i, g, S_i^{ab}) = \int d\tau \left(-m - \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + C_{ES^2} E_{ab} S^{bc} S_c^a + \ldots \right)$$

spin coupling

short distance physics
(self-induced finite size effect)

(slightly more complicated/
technical stuff. But waaaaay
simpler than ADM
formalism!)

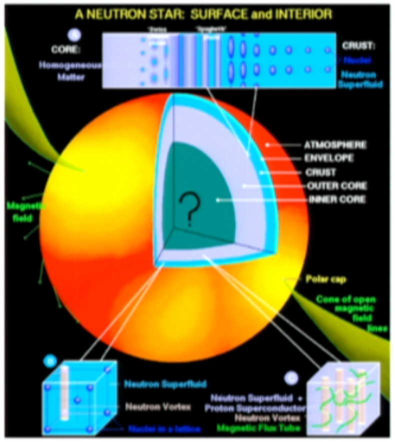
Matching with rotating BHs:

Compute the one-point function and 'match'. For example with
the Kerr metric, e.g. we read off the quadrupole moment of the space-time:

$$C^i_{ES^2} = 1/2m_i$$

Using standard power counting rules we can show there is one and *only one*
coefficient to NLO (3PN).

(Finite size effects are highly arbitrary/complicated in traditional ADM approach)



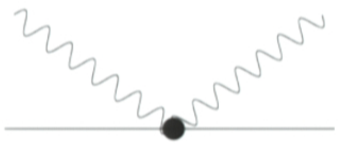
NSs and BHs as point particles

$$S_{pp} = \int \left(m + c_E E_{ab}E^{ab} + c_B B_{ab}B^{ab}\right) d\tau + \ldots$$

→ encode effects from 'short distance' physics

CE and CB are so called 'matching' coefficients.
They can be read off by comparing with computations in the 'full theory', e.g. GW scattering cross section:

$$\sigma_{full}(\omega) = r_s^2 F(\omega r_s) \sim r_s^2(\ldots + \alpha r_s^8 \omega^8 + \ldots)$$



$$\sigma_{EFT}(\omega) = \ldots + \frac{C_{E,B}^2}{m_{pl}^4}\omega^8 + \ldots$$

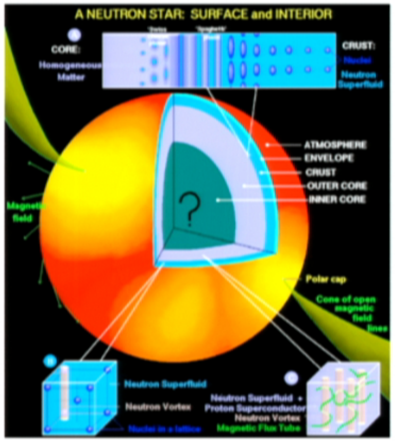
$$C_{E,B} \sim r_s^5 m_{pl}^2$$

Effacement Theorem:

Finite-size effects first appear at 5PN order for non-rotating compact objects.
Enhanced for NSs (Love number). Finite size effects due to SPIN! appear at lower orders

$$r_s \sim \frac{GM}{c^2} \to \frac{r_s}{r} \sim \frac{1}{c^2}\frac{GM}{r} \sim \frac{v^2}{c^2}$$

(Note: For Black Holes C_E vanishes in D=4
Damour&Nagar, Binnington&Poisson, Kol&Smolkin)



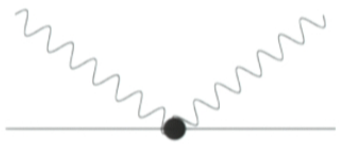
NSs and BHs as point particles

$$S_{pp} = \int \left(m + c_E E_{ab} E^{ab} + c_B B_{ab} B^{ab}\right) d\tau + \ldots$$

→ encode effects from 'short distance' physics

CE and CB are so called 'matching' coefficients.
They can be read off by comparing with computations
in the 'full theory', e.g. GW scattering cross section:

$$\sigma_{full}(\omega) = r_s^2 F(\omega r_s) \sim r_s^2(\ldots + \alpha r_s^8 \omega^8 + \ldots)$$



$$\sigma_{EFT}(\omega) = \ldots + \frac{C_{E,B}^2}{m_{pl}^4}\omega^8 + \ldots$$

$$C_{E,B} \sim r_s^5 m_{pl}^2$$

Effacement Theorem:

Finite-size effects first appear at 5PN order for non-rotating compact objects.
Enhanced for NSs (Love number). Finite size effects due to SPIN! appear at lower orders

$$r_s \sim \frac{GM}{c^2} \rightarrow \frac{r_s}{r} \sim \frac{1}{c^2}\frac{GM}{r} \sim \frac{v^2}{c^2}$$

(Note: For Black Holes C_E vanishes in D=4
Damour&Nagar, Binnington&Poisson, Kol&Smolkin)

An EFT for spinning compact objects

(RAP gr-qc/0511061, gr-qc/0701106, PhD Thesis 07')

(Let's spare the technicalities of dealing with spin in GR, it would take a whole other talk: Routhians, second class constraints, SSCs, Tmn)

$$S_{pp}(x_i, g, S_i^{ab}) = \int d\tau \left(-m - \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + C_{ES^2} E_{ab} S^{bc} S_c^a + \dots \right)$$

spin coupling

short distance physics (self-induced finite size effect)

(slightly more complicated/ technical stuff. But waaaaay simpler than ADM formalism!)

Matching with rotating BHs:

Compute the one-point function and 'match'. For example with the Kerr metric, e.g. we read off the quadrupole moment of the space-time:

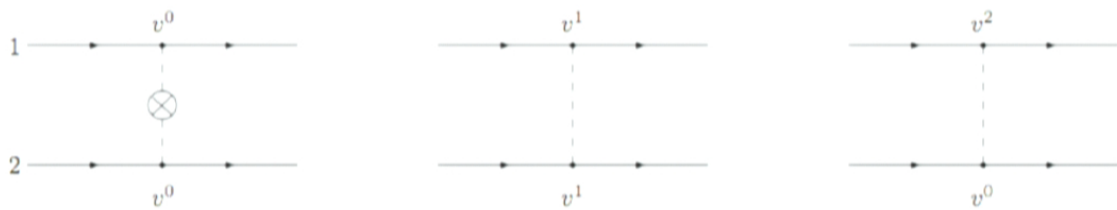
$$C^i_{ES^2} = 1/2m_i$$

Using standard power counting rules we can show there is one and *only one* coefficient to NLO (3PN).

(Finite size effects are highly arbitrary/complicated in traditional ADM approach)

NRGR Diagrammatics I: Orbit scale - potential mode

$$k_{\rm pot} \simeq (v/r, 1/r)$$



Potential modes are 'instantaneous'

$$\frac{1}{p_0^2 - p^2} \sim -\frac{1}{p^2}\left(1 + \frac{p_0^2}{p^2} + \ldots\right)$$

Recall for the heavy quarks

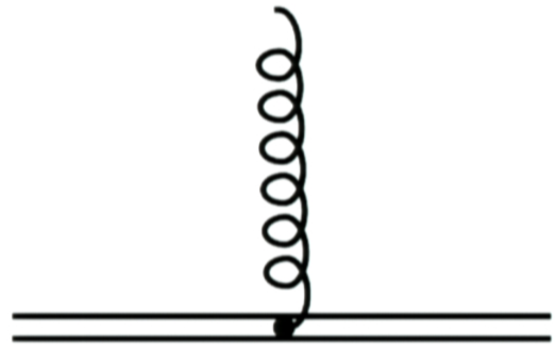
$(E \sim mv^2, p \sim mv)$

Einstein-Infeld-Hoffmann

$$L_{EIH} = \frac{1}{8}\sum_a m_a \mathbf{v}_a^4 + \frac{G_N m_1 m_2}{2|\mathbf{x}_1 - \mathbf{x}_2|}\left[3(\mathbf{v}_1^2 + \mathbf{v}_2^2) - 7(\mathbf{v}_1 \cdot \mathbf{v}_2) - \frac{(\mathbf{v}_1 \cdot \mathbf{x}_{12})(\mathbf{v}_2 \cdot \mathbf{x}_{12})}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right] - \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}$$

Diagrammatics II:
The radiation sector of NRGR

Work in the long-wavelength EFT



on shell:

$$k_{\rm rad} \simeq (v/r, v/r)$$

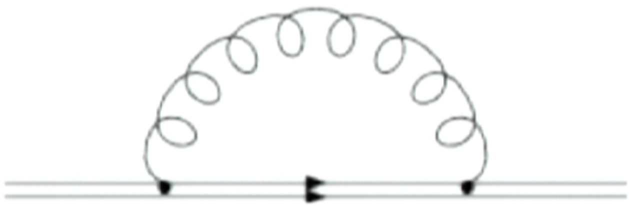
The effective action takes the form of a multipole expansion:

$$S_{\rm rad}[\bar{h}] = -\int dt \left(m\sqrt{g_{00}} + \frac{1}{2} L^{ij}\omega_{ij0} \right) \longleftarrow \text{Total mass/energy and angular momentum}$$

$$+\frac{1}{2}\int dt \left(I^{ij}E_{ij} - \frac{4}{3}J^{ij}B_{ij} + \frac{1}{3}I^{ijk}\nabla_k E_{ij} - \frac{1}{2}J^{ijk}\nabla_k B_{ij} + \cdots \right).$$

Quadrupole (I^{ij}) Current Quadrupole (J^{ij}) Octupole terms... (I^{ijk}, J^{ijk})

We get the power from the optical theorem:



$$\langle in, 0 \mid out, 0 \rangle_J \sim e^{(iV(J)-\Gamma/2)T}$$

Power Loss

$$\frac{1}{T}\mathrm{Im}S_{eff} = \frac{1}{2}\int dE d\Omega \frac{d^2\Gamma}{dE d\Omega}$$

Leading order quadrupole radiation:

$$\mathrm{Im}S_{eff} = -\frac{1}{80m_{pl}^2}\int_{\mathbf{k}} \frac{1}{2|\mathbf{k}|}\mathbf{k}^4|Q_{ij}|^2 \rightarrow \frac{dE}{dt} = -\frac{G_N}{5}\langle \frac{d^3}{dt^3}Q_{ij}\frac{d^3}{dt^3}Q^{ij}\rangle$$

To all orders:

$$\dot{E} = \frac{G}{5}\left\langle\left(\frac{d^3}{dt^3}I^{ij}(t)\right)^2\right\rangle + \frac{16G}{45}\left\langle\left(\frac{d^3}{dt^3}J^{ij}(t)\right)^2\right\rangle + \frac{G}{189}\left\langle\left(\frac{d^4}{dt^4}I^{ijk}(t)\right)^2\right\rangle + \frac{G}{84}\left\langle\left(\frac{d^4}{dt^4}J^{ijk}(t)\right)^2\right\rangle + \cdots$$

We organize the radiation action in terms of moments of the (pseudo) stress tensor using reps of SO(3) and STF tensors.

Sample moments:

$$I_0^{ij} = \int d^3\mathbf{x}\, T^{00}[\mathbf{x}^i\mathbf{x}^j]_{\rm TF},$$

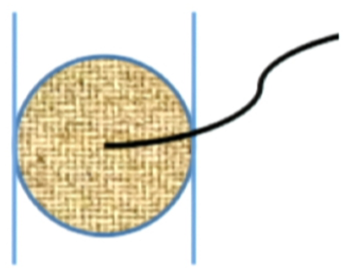
$$I_1^{ij} = \int d^3\mathbf{x}\left(T^{ll} - \frac{4}{3}\dot{T}^{0l}\mathbf{x}^l + \frac{11}{42}\ddot{T}^{00}\mathbf{x}^2\right)[\mathbf{x}^i\mathbf{x}^j]_{\rm TF},$$

$$I_2^{ij} = \int d^3\mathbf{x}\left(\frac{2}{21}\ddot{T}^{ll}\mathbf{x}^2 + \frac{1}{6}\ddot{T}^{lm}\mathbf{x}^l\mathbf{x}^m - \frac{1}{7}\dddot{T}^{0l}\mathbf{x}^l\mathbf{x}^2 + \frac{23}{1512}\ddddot{T}^{00}\mathbf{x}^4\right)[\mathbf{x}^i\mathbf{x}^j]_{\rm TF}$$

$$r/\lambda_{\rm rad} \sim v$$

To obtain the multipoles we compute the one-graviton amplitude and extract T^mn (i.e. match from the theory of potentials):

$$S_{\rm rad}[\bar{h}] = S_0[\bar{h}=0] - \frac{1}{2m_p}\int d^4x T^{\mu\nu}(x)\bar{h}_{\mu\nu}(x)$$



(Goldberger & Ross, 0912.4254)

(RAP, Ross & Rothstein; 1007.1312)

(Ross, 1202.4750)

But the quadrupole formula and EIH potential have been known for 75 years!

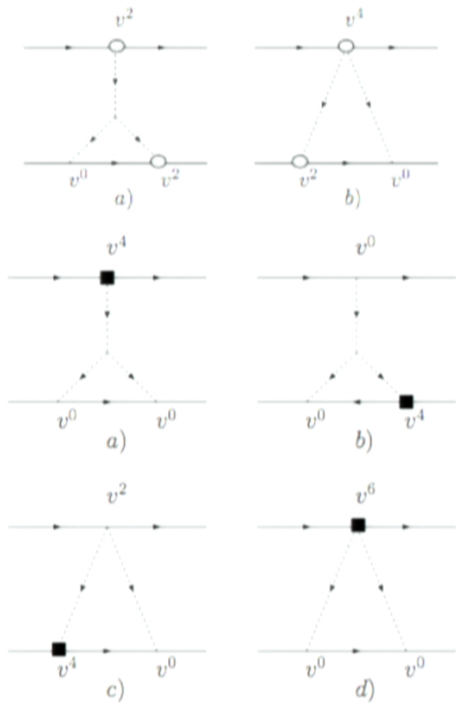
If Feynman were here he'd say:

– "That's nice, but what can you do with it?"

New results I: Dynamics of spinning compact objects to 3PN

(RAP & Rothstein; grqc/0604099, 0712.2032, 0802.0720, 0804.0260)

Sample (non-linear) diagrams:



Same as EIH (not as intricate as *traditional* approach)

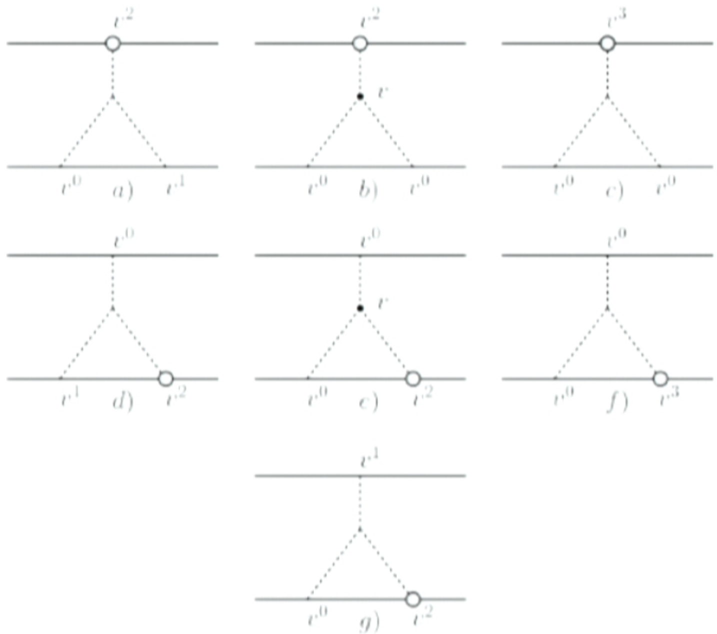
$$
\begin{aligned}
V^{spin} = & \frac{G_N m_2}{r^2} n^j \left(S_1^{j0} + S_1^{jk}(v_1^k - 2v_2^k)\right) - \frac{G_N m_1}{r^2} n^j \left(S_2^{j0} + S_2^{jk}(v_2^k - 2v_1^k)\right) \\
& - \frac{G_N}{r^3}\Big[(\delta^{ij} - 3n^i n^j)\Big(S_1^{i0} S_2^{j0} + \frac{1}{2}\mathbf{v}_1\cdot\mathbf{v}_2 S_1^{ik} S_2^{jk} + v_1^m v_2^k S_1^{ik} S_2^{jm} - v_1^k v_2^m S_1^{ik} S_2^{jm} \\
& + S_1^{i0} S_2^{jk}(v_2^k - v_1^k) + S_1^{ik} S_2^{j0}(v_1^k - v_2^k)\Big) + \frac{1}{2} S_1^{ki} S_2^{kj}\left(3\mathbf{v}_1\cdot\mathbf{n}\mathbf{v}_2\cdot\mathbf{n}(\delta^{ij} - 5n^i n^j)\right. \\
& \left. + 3\mathbf{v}_1\cdot\mathbf{n}(v_2^j n^i + v_2^i n^j) + 3\mathbf{v}_2\cdot\mathbf{n}(v_1^j n^i + v_1^i n^j) - v_1^i v_2^j - v_2^i v_1^j\right) \\
& + (3n^l \mathbf{v}_2\cdot\mathbf{n} - v_2^l) S_1^{0k} S_2^{kl} + (3n^l \mathbf{v}_1\cdot\mathbf{n} - v_1^l) S_2^{0k} S_1^{kl}\Big] \\
& + \left(\frac{G_N}{r^3} - \frac{3MG_N^2}{r^4}\right) S_1^{jk} S_2^{ji}(\delta^{ki} - 3n^k n^i) \\
& + \Big\{ C_{ES^2}^{(1)} \frac{G_N m_2}{2m_1 r^3}\left[S_1^{j0} S_1^{i0}(3n^i n^j - \delta^{ij}) - 2S_1^{k0}\left((\mathbf{v}_1\times\mathbf{S}_1)^k - 3(\mathbf{n}\cdot\mathbf{v}_1)(\mathbf{n}\times\mathbf{S}_1)^k\right)\right] \\
& + C_{ES^2}^{(1)} \frac{G_N m_2}{2m_1 r^3}\Big[\mathbf{S}_1^2\left(6(\mathbf{n}\cdot\mathbf{v}_1)^2 - \frac{15}{2}\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2 + \frac{13}{2}\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{2}\mathbf{v}_2^2 - \frac{7}{2}\mathbf{v}_1^2 - 2\mathbf{a}_1\cdot\mathbf{r}\right) \\
& + (\mathbf{S}_1\cdot\mathbf{n})^2\left(\frac{9}{2}(\mathbf{v}_1^2 + \mathbf{v}_2^2) - \frac{21}{2}\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{15}{2}\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{v}_2\right) + 2\mathbf{v}_1\cdot\mathbf{S}_1\mathbf{v}_1\cdot\mathbf{S}_1 \\
& - 3\mathbf{v}_1\cdot\mathbf{S}_1\mathbf{v}_2\cdot\mathbf{S}_1 - 6\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{S}_1\mathbf{v}_1\cdot\mathbf{S}_1 + 9\mathbf{n}\cdot\mathbf{v}_2\mathbf{n}\cdot\mathbf{S}_1\mathbf{v}_1\cdot\mathbf{S}_1 + 3\mathbf{n}\cdot\mathbf{v}_1\mathbf{n}\cdot\mathbf{S}_1\mathbf{v}_2\cdot\mathbf{S}_1\Big] \\
& - C_{ES^2}^{(1)} \frac{m_2 G_N}{2m_1 r^3}\left(\mathbf{S}_1^2 - 3(\mathbf{S}_1\cdot\mathbf{n})^2\right) + C_{ES^2}^{(1)} \frac{m_2 G_N^2}{2r^4}\left(1 + \frac{4m_2}{m_1}\right)\left(\mathbf{S}_1^2 - 3(\mathbf{S}_1\cdot\mathbf{n})^2\right) \\
& - \frac{G_N^2 m_2}{r^4}(\mathbf{S}_1\cdot\mathbf{n})^2 + \left(\tilde{\mathbf{a}}_{1(1)}^{so}\right)^l S_1^{0l} + \mathbf{v}_1\times\mathbf{S}_1\cdot\tilde{\mathbf{a}}_{1(1)}^{so} + 1\leftrightarrow 2\Big\},
\end{aligned}
$$

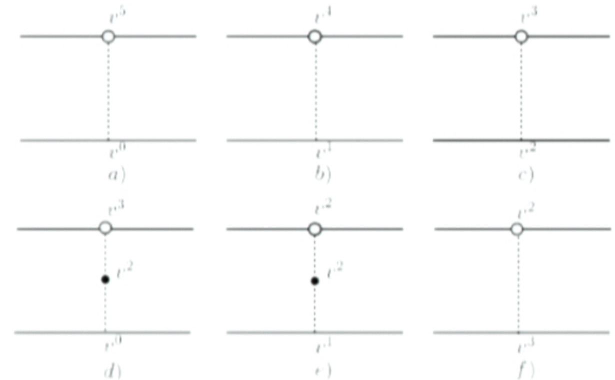
Spin(1)Spin(2) and Spin(1)Spin(1) (finite size) potentials to NLO.

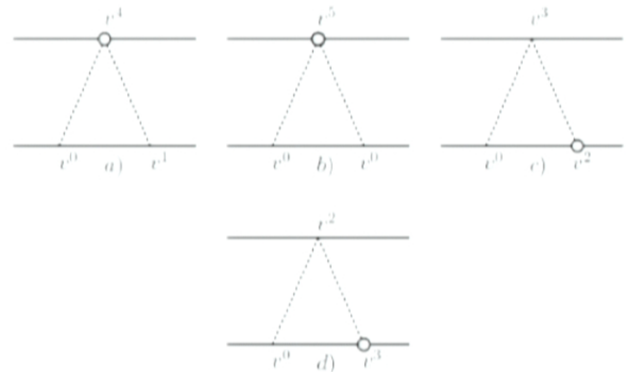
Reproduced in the traditional formalism (Schafer et al., 1110.2094)

NNLO Spin(1)Spin(2) dynamics also computed in the EFT and ADM (Levi, Steinhoff)

NLO spin-orbit potential at 2.5PN. (BBF 06')
More Feynman diagrams
(RAP, 1005.5730)

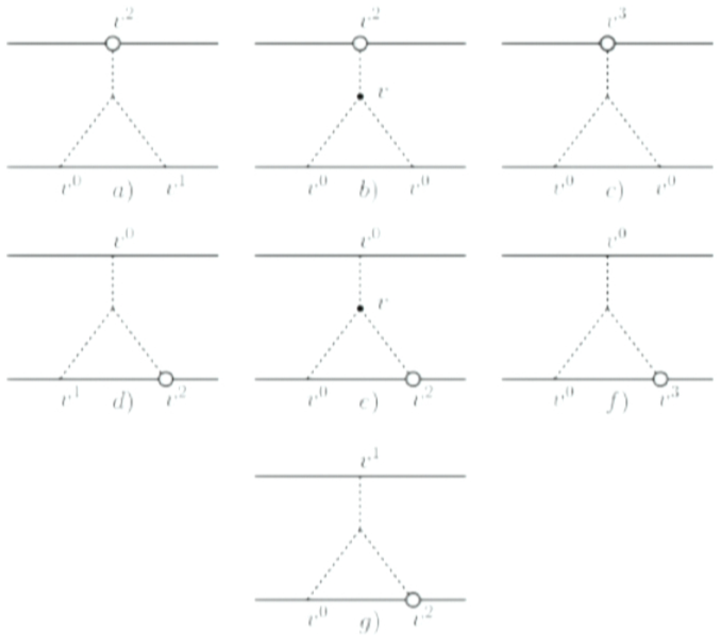


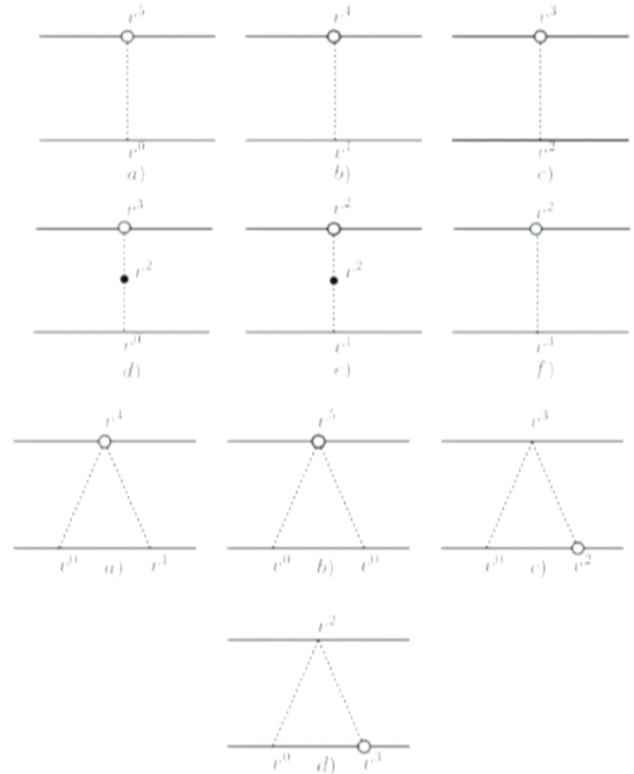




$$V^{so} = \frac{Gm_2}{r^3}\left[\left\{S_1^{i0}\left(1+2\mathbf{v}_2^2-2\mathbf{v}_1\cdot\mathbf{v}_2-\frac{3}{2r^2}(\mathbf{v}_2\cdot\mathbf{r})^2-\frac{G}{r}(3m_1+2m_2)\right)+\right.\right.$$
$$\left(1-\frac{3}{2r^2}(\mathbf{v}_2\cdot\mathbf{r})^2+\frac{G}{2r}(4m_1-m_2)\right)S_1^{ij}\mathbf{v}_1^j-\left(2-2\mathbf{v}_1\cdot\mathbf{v}_2-\frac{3(\mathbf{v}_2\cdot\mathbf{r})^2}{r^2}+2\mathbf{v}_2^2-\frac{G}{2r}(2m_1+5m_2)\right)S_1^{ij}\mathbf{v}_2^j\Bigg\}\mathbf{r}^i$$
$$\left.+S_1^{i0}(\mathbf{v}_1-\mathbf{v}_2)^i\mathbf{v}_2\cdot\mathbf{r}+S_1^{ij}\mathbf{v}_1^j\mathbf{v}_2^i\mathbf{v}_2\cdot\mathbf{r}\right]+1\leftrightarrow 2,$$

NLO spin-orbit potential at 2.5PN. (BBF 06')
More Feynman diagrams
(RAP, 1005.5730)

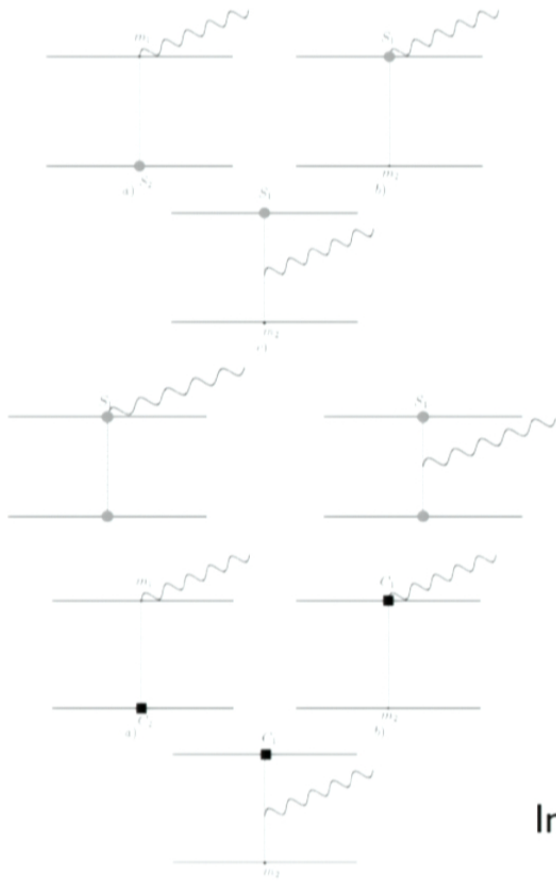




$$V^{\rm so} = \frac{Gm_2}{r^3}\left[\left\{S_1^{i0}\left(1 + 2\mathbf{v}_2^2 - 2\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3}{2r^2}(\mathbf{v}_2\cdot\mathbf{r})^2 - \frac{G}{r}(3m_1+2m_2)\right) + \left(1 - \frac{3}{2r^2}(\mathbf{v}_2\cdot\mathbf{r})^2 + \frac{G}{2r}(4m_1 - m_2)\right)S_1^{ij}\mathbf{v}_1^j - \left(2 - 2\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{3(\mathbf{v}_2\cdot\mathbf{r})^2}{r^2} + 2\mathbf{v}_2^2 - \frac{G}{2r}(2m_1+5m_2)\right)S_1^{ij}\mathbf{v}_2^j\right\}\mathbf{r}^i + S_1^{i0}(\mathbf{v}_1-\mathbf{v}_2)^i\mathbf{v}_2\cdot\mathbf{r} + S_1^{ij}\mathbf{v}_1^j\mathbf{v}_2^i\mathbf{v}_2\cdot\mathbf{r}\right] + 1\leftrightarrow 2,$$

New results II: GW power to 3PN for spinning bodies

(RAP, Ross & Rothstein; 1007.1312)



Spin dependent quadrupole to NLO order:

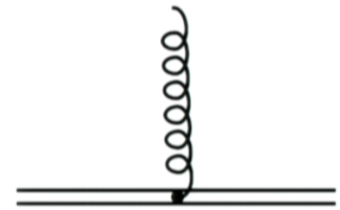
$$
\begin{aligned}
I^{ij}_{\mathbf{S}_A,\mathbf{S}^2_A,\mathbf{S}_A\mathbf{S}_B} = \sum_A \Bigg[& \frac{8}{3}(\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}^j_A - \frac{4}{3}(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{v}^j_A - \frac{4}{3}(\mathbf{x}_A \times \dot{\mathbf{S}}_A)^i \mathbf{x}^j_A \\
& - \frac{4}{3}\frac{d}{dt}\left\{\mathbf{v}_A \cdot \mathbf{x}_A (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}^j_A\right\} + \frac{1}{7}\frac{d^2}{dt^2}\Bigg\{\frac{1}{3}\mathbf{x}_A \cdot \mathbf{v}_A (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}^j_A \\
& + 4\mathbf{x}^2_A (\mathbf{v}_A \times \mathbf{S}_A)^i \mathbf{x}^j_A + \mathbf{x}^2_A (\mathbf{S}_A \times \mathbf{x}_A)^i \mathbf{v}^j_A - \frac{5}{6}(\mathbf{v}_A \times \mathbf{S}_A)\cdot \mathbf{x}_A\, \mathbf{x}^i_A \mathbf{x}^j_A \Bigg\}\Bigg]_{\rm STF} \\
& + \sum_{A,B} \frac{2Gm_B}{r^3}\Big[(\mathbf{v}_B \times \mathbf{S}_A)\cdot \mathbf{r}(\mathbf{x}^i_B \mathbf{x}^j_B - 2\mathbf{x}^i_A \mathbf{x}^j_A) + (\mathbf{v}_A \times \mathbf{S}_A)\cdot \mathbf{r}(\mathbf{x}^i_A \mathbf{x}^j_A + \mathbf{x}^i_B \mathbf{x}^j_B) \\
& + 2r^2 \left\{(\mathbf{v}_B \times \mathbf{S}_A)^i (\mathbf{x}^j_B - \mathbf{x}^j_A) + (\mathbf{r} \times \mathbf{S}_A)^i \left(\mathbf{v}^j_B - \mathbf{v}^j_A - \frac{\mathbf{v}_B \cdot \mathbf{r}}{r^2}(\mathbf{x}^j_A + \mathbf{x}^j_B)\right)\right\}\Big]_{\rm STF} \\
& - \frac{2}{3}\sum_{A,B}\frac{d}{dt}\Big[\frac{Gm_B}{r^3}\Big\{r^2\Big((\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}^j_A - 3(\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}^j_A + 3(\mathbf{x}_B \times \mathbf{S}_A)^i \mathbf{x}^j_B - (\mathbf{x}_A \times \mathbf{S}_A)^i \mathbf{x}^j_B\Big) \\
& - 2\mathbf{r}\cdot \mathbf{x}_B\, (\mathbf{r} \times \mathbf{S}_A)^i (\mathbf{x}^j_A + \mathbf{x}^j_B) + (\mathbf{x}_A \times \mathbf{S}_A)\cdot \mathbf{x}_B (\mathbf{x}^i_A \mathbf{x}^j_A - 2\mathbf{x}^i_B \mathbf{x}^j_B)\Big\}\Big]_{\rm STF} \\
& + \sum_A \frac{C^{(A)}_{ES^2}}{m_A}\Big[\mathbf{S}^i_A \mathbf{S}^j_A \left(-1 + \frac{13}{42}\mathbf{v}^2_A + \frac{17}{21}\mathbf{a}_A \cdot \mathbf{x}_A\right) + \mathbf{S}^2_A \left(-\frac{11}{21}\mathbf{v}^i_A \mathbf{v}^j_A + \frac{10}{21}\mathbf{a}^i_A \mathbf{x}^j_A\right) \\
& - \frac{8}{21}\mathbf{x}^i_A \mathbf{S}^j_A \mathbf{a}_A \cdot \mathbf{S}_A + \frac{4}{7}\mathbf{v}^i_A \mathbf{S}^j_A \mathbf{S}_A \cdot \mathbf{v}_A - \frac{22}{21}\mathbf{a}^i_A \mathbf{S}^j_A \mathbf{S}_A \cdot \mathbf{x}_A\Big]_{\rm STF} \\
& + \sum_{A,B}\frac{G}{2r^3}\Big[\frac{C^{(B)}_{ES^2} m_A}{m_B}\left(\mathbf{S}^2_B + 9(\mathbf{S}_B \cdot \mathbf{n})^2\right)\mathbf{x}^i_B \mathbf{x}^j_B + 6\frac{C^{(B)}_{ES^2} m_A}{m_B} r^2 \mathbf{S}^i_B \mathbf{S}^j_B \\
& + \left(\frac{C^{(B)}_{ES^2} m_A}{m_B}\left(3(\mathbf{S}_B \cdot \mathbf{n})^2 - \mathbf{S}^2_B\right) + 12\mathbf{S}_A \cdot \mathbf{n}\mathbf{S}_B \cdot \mathbf{n} - 4\mathbf{S}_A \cdot \mathbf{S}_B\right)\mathbf{x}^i_A \mathbf{x}^j_A \\
& - 4\frac{C^{(B)}_{ES^2} m_A}{m_B}\mathbf{S}^2_B \mathbf{x}^i_A \mathbf{x}^j_B + 4\left(3\frac{C^{(B)}_{ES^2} m_A}{m_B}\mathbf{S}_B \cdot \mathbf{r} + 2\mathbf{S}_A \cdot \mathbf{r}\right)\mathbf{S}^i_B \mathbf{x}^j_B\Big]_{\rm STF}
\end{aligned}
$$

Including high dim corrections that account for finite size effects (encode inner structure of compact objects)

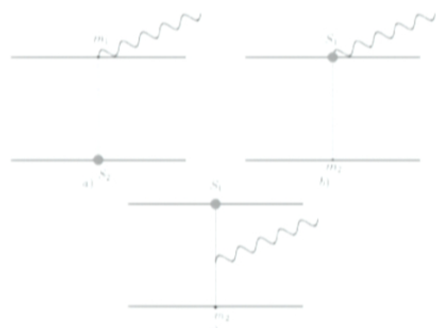
New results III: GW amplitude to 2.5PN for spinning binaries

(RAP, Ross & Rothstein; 1203.XXXX)

$$h_{ij}^{TT}(t,\mathbf{x}) = -\frac{4G}{|\mathbf{x}|}\Lambda_{ij,kl}\left[\left(\frac{1}{2}\frac{d^2 I^{kl}}{dt^2} + \frac{1}{6}\frac{d^3 I^{klm}}{dt^3}n_m + \frac{1}{24}\frac{d^4 I^{klmn}}{dt^4}n_n n_m + \cdots\right)\right.$$
$$- \epsilon^{ab(k}\left(\frac{2}{3}\frac{d^2 J^{l)b}}{dt^2}n_a + \frac{1}{4}\frac{d^3 J^{l)bm}}{dt^3}n_a n_m + \frac{1}{15}\frac{d^4 J^{l)bmn}}{dt^4}n_a n_m n_n\right.$$
$$\left.\left. + \frac{1}{72}\frac{d^5 J^{l)bmnp}}{dt^5}n_a n_m n_n n_p + \cdots\right)\right]$$

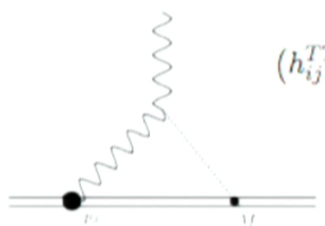


Spin dependent current octupole to NLO order:



$$J^{ijk} = \sum_A 2\left[\mathbf{S}_A^i \mathbf{x}_A^j \mathbf{x}_A^k\right]_{\rm STF} + \sum_A\left[-\frac{2}{3}\left(\mathbf{v}_A^i\mathbf{x}_A^j\mathbf{x}_A^k(\mathbf{S}_A\cdot\mathbf{v}_A) - \mathbf{v}_A^2\mathbf{S}_A^i\mathbf{x}_A^j\mathbf{x}_A^k - 2\mathbf{v}_A^i\mathbf{v}_A^j\mathbf{x}_A^k(\mathbf{S}_A\cdot\mathbf{x}_A)\right.\right.$$
$$\left. + 2(\mathbf{x}_A\cdot\mathbf{v}_A)\mathbf{S}_A^i\mathbf{v}_A^j\mathbf{x}_A^k\right) + \frac{1}{6}\mathbf{a}_A^i\mathbf{x}_A^j\mathbf{x}_A^k(\mathbf{S}_A\cdot\mathbf{x}_A) - \frac{5}{6}\mathbf{S}_A^i\mathbf{x}_A^j\mathbf{x}_A^k(\mathbf{a}_A\cdot\mathbf{x}_A)$$
$$\left. + \frac{2}{9}(\mathbf{a}_A\cdot\mathbf{S}_A)\mathbf{x}_A^i\mathbf{x}_A^j\mathbf{x}_A^k + \frac{2}{3}\mathbf{x}_A^2\left(\mathbf{v}_A^i\mathbf{v}_A^j\mathbf{S}_A^k + \mathbf{x}_A^i\mathbf{a}_A^j\mathbf{S}_A^k\right)\right]_{\rm STF}$$
$$+ \sum_{A,B}\frac{Gm_B}{r^3}\left[\frac{1}{2}(\mathbf{r}\cdot\mathbf{x}_A - 8r^2)\mathbf{S}_A^i\mathbf{x}_A^j\mathbf{x}_A^k + \frac{4}{3}\left(\mathbf{x}_A^i\mathbf{x}_A^j\mathbf{x}_A^k - \mathbf{x}_B^i\mathbf{x}_B^j\mathbf{x}_B^k\right)\mathbf{S}_A\cdot\mathbf{r} - \frac{1}{2}\mathbf{x}_A^i\mathbf{x}_A^j\mathbf{r}^k\mathbf{S}_A\cdot\mathbf{x}_A\right]_{\rm STF}$$

We also computed the tail-effect (scattering off the geometry)



$$\left(h_{ij}^{TT}\right)_{J^{ij}+J^{ij}M}(\tilde{t},\mathbf{x}) = -\frac{2G_N}{|\mathbf{x}|}\Lambda_{ij,kl}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\left[-\omega^2 e^{i\omega\tilde{t}_{\rm ret}(\mu)}e^{i\varphi_{\rm tail}(\omega)}\left(1 + G_N M|\omega|\pi\right)\right]J_0^{ij}(\omega)$$

$$\varphi_{\rm tail}(\omega,\mu) \equiv G_N M\omega\left(\log\frac{\omega^2}{\mu^2} + \gamma_E - \frac{7}{3}\right) \qquad J_0^{ij}(\mathbf{S}) = \frac{3}{2}\sum_A \mathbf{S}_A^i\mathbf{x}_A^j + \ldots$$

The dawn of precision Gravity

Table 1 Post-Newtonian contributions to the number of GW cycles accumulated from $\omega_{\text{in}} = \pi \times 10\,\text{Hz}$ to $\omega_{\text{fin}} = \omega^{\text{ISCO}} = 1/(6^{3/2}\,M)$ for binaries detectable by LIGO and VIRGO. We denote $\kappa_i = \widehat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{\ell}}$ and $\xi = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$.

	$(10+10)M_\odot$	$(1.4+1.4)M_\odot$
Newtonian	601	16034
1PN	$+59.3$	$+441$
1.5PN	$-51.4 + 16.0\,\kappa_1\,\chi_1 + 16.0\,\kappa_2\,\chi_2$	$-211 + 65.7\,\kappa_1\,\chi_1 + 65.7\,\kappa_2\,\chi_2$
2PN	$+4.1 - 3.3\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 1.1\,\xi\,\chi_1\,\chi_2$	$+9.9 - 8.0\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 2.8\,\xi\,\chi_1\,\chi_2$
2.5PN	$-7.1 + 5.5\,\kappa_1\,\chi_1 + 5.5\,\kappa_2\,\chi_2$	$-11.7 + 9.0\,\kappa_1\,\chi_1 + 9.0\,\kappa_2\,\chi_2$
3PN	$+2.2$	$+2.6$
3.5PN	-0.8	-0.9

- Our methods/results reproduces known computation and provided the (new) ingredients to obtain the GW amplitude and phase to 2.5PN & 3PN order.
- Spin and Masses to the percent level (history of BH formation and galaxies, constraints on light particles, axions(?))
- Most accurate test of BH dynamics in GR (strong gravity)
- The templates depend upon the inner structure of the NS (finite size effects, test matter at high pressure/densities with GWs)
- Background for new physics, Cosmology.

The dawn of precision Gravity

Table 1 Post-Newtonian contributions to the number of GW cycles accumulated from $\omega_{\rm in} = \pi \times 10\,{\rm Hz}$ to $\omega_{\rm fin} = \omega^{\rm ISCO} = 1/(6^{3/2}\,M)$ for binaries detectable by LIGO and VIRGO. We denote $\kappa_i = \widehat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{\ell}}$ and $\xi = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$.

	$(10+10)M_\odot$	$(1.4+1.4)M_\odot$
Newtonian	601	16034
1PN	$+59.3$	$+441$
1.5PN	$-51.4 + 16.0\,\kappa_1\,\chi_1 + 16.0\,\kappa_2\,\chi_2$	$-211 + 65.7\,\kappa_1\,\chi_1 + 65.7\,\kappa_2\,\chi_2$
2PN	$+4.1 - 3.3\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 1.1\,\xi\,\chi_1\,\chi_2$	$+9.9 - 8.0\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 2.8\,\xi\,\chi_1\,\chi_2$
2.5PN	$-7.1 + 5.5\,\kappa_1\,\chi_1 + 5.5\,\kappa_2\,\chi_2$	$-11.7 + 9.0\,\kappa_1\,\chi_1 + 9.0\,\kappa_2\,\chi_2$
3PN	$+2.2$	$+2.6$
3.5PN	-0.8	-0.9

- Our methods/results reproduces known computation and provided the (new) ingredients to obtain the GW amplitude and phase to 2.5PN & 3PN order.
- Spin and Masses to the percent level (history of BH formation and galaxies, constraints on light particles, axions(?))
- Most accurate test of BH dynamics in GR (strong gravity)
- The templates depend upon the inner structure of the NS (finite size effects, test matter at high pressure/densities with GWs)
- Background for new physics, Cosmology.

The dawn of precision Gravity

Table 1 Post-Newtonian contributions to the number of GW cycles accumulated from $\omega_{\rm in} = \pi \times 10\,{\rm Hz}$ to $\omega_{\rm fin} = \omega^{\rm ISCO} = 1/(6^{3/2}\,M)$ for binaries detectable by LIGO and VIRGO. We denote $\kappa_i = \widehat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{\ell}}$ and $\xi = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2$.

	$(10+10)M_\odot$	$(1.4+1.4)M_\odot$
Newtonian	601	16034
1PN	$+59.3$	$+441$
1.5PN	$-51.4 + 16.0\,\kappa_1\,\chi_1 + 16.0\,\kappa_2\,\chi_2$	$-211 + 65.7\,\kappa_1\,\chi_1 + 65.7\,\kappa_2\,\chi_2$
2PN	$+4.1 - 3.3\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 1.1\,\xi\,\chi_1\,\chi_2$	$+9.9 - 8.0\,\kappa_1\,\kappa_2\,\chi_1\,\chi_2 + 2.8\,\xi\,\chi_1\,\chi_2$
2.5PN	$-7.1 + 5.5\,\kappa_1\,\chi_1 + 5.5\,\kappa_2\,\chi_2$	$-11.7 + 9.0\,\kappa_1\,\chi_1 + 9.0\,\kappa_2\,\chi_2$
3PN	$+2.2$	$+2.6$
3.5PN	-0.8	-0.9

- Our methods/results reproduces known computation and provided the (new) ingredients to obtain the GW amplitude and phase to 2.5PN & 3PN order.
- Spin and Masses to the percent level (history of BH formation and galaxies, constraints on light particles, axions(?))
- Most accurate test of BH dynamics in GR (strong gravity)
- The templates depend upon the inner structure of the NS (finite size effects, test matter at high pressure/densities with GWs)
- Background for new physics, Cosmology.

The problem of motion is *hard*. It entails several different scales playing at once. The internal structure (r_s), the orbit scale (r) and the radiation scale (r/v). Here is Einstein and Infeld on the 1PN spinless case dealing *only* with the orbit scale!!

ON THE MOTION OF PARTICLES IN GENERAL RELATIVITY THEORY

A. EINSTEIN and L. INFELD

1. Introduction. The gravitational field manifests itself in the motion of bodies. Therefore the problem of determining the motion of such bodies from the field equations alone is of fundamental importance. This problem was solved for the first time some ten years ago and the equations of motion for two particles were then deduced [1]. A more general and simplified version of this problem was given shortly thereafter [2].

Mr. Lewison pointed out to us, that from our approximation procedure, it does not follow that the field equations can be solved up to an arbitrarily high approximation. This is indeed true. We believe that the present work not only removes this difficulty, but that it gives a new and deeper insight into the problem of motion. From the logical point of view the present theory is considerably simpler and clearer than the old one. But as always, we must pay for these logical simplifications by prolonging the chain of technical argument.

The solution takes over several pages of *traditional* GR techniques in terms of surface integrals.
For example:

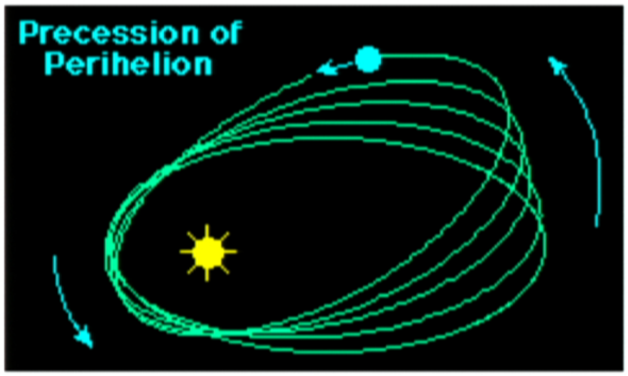
TABLE OF SURFACE INTEGRALS FOR $\int_0^1 \Lambda_{ms} n_s \, dS$

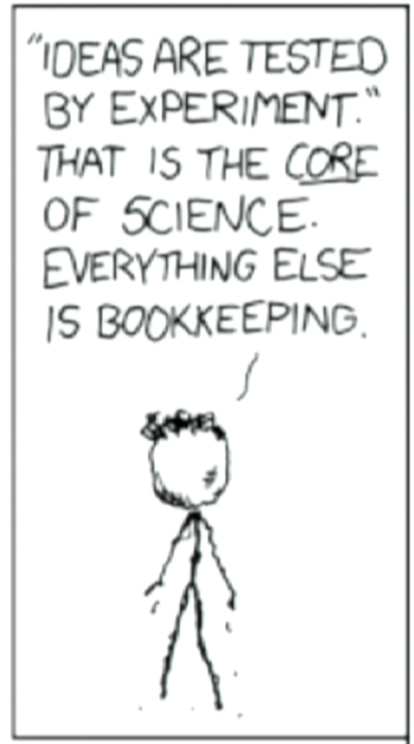
No.	Expression	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	Result	Remarks
1	$\overset{1}{m}\xi_{,s}\eta^s\eta^m$	$-\frac{16}{3}$				$-\frac{4}{3}$		$-\frac{8}{3}$	$-\frac{4}{15}$		$\frac{8}{15}$	$\frac{4}{15}$				$\frac{4}{5}$			-8	$\bar{\xi}_{,s} = -2\overset{2}{m}\frac{\partial \overset{1}{r}}{\partial \eta^s}$
2	$\overset{1}{m}\xi\ddot{\eta}^m$	-2						-4	$-\frac{4}{3}$			$-\frac{29}{3}$	3	$\frac{11}{3}$	$-\frac{5}{3}$	$\frac{2}{3}$	$\frac{32}{3}$	$-\frac{22}{3}$	-8	$\ddot{\xi} = -\frac{2\overset{2}{m}}{r}$; $\ddot{\eta}^m = -\frac{1}{2}\xi_{,m}$
3	$\overset{1}{m}\ddot{\xi}_{,m}\dot{\eta}^s\dot{\eta}^s$	1					$-\frac{4}{3}$	$-\frac{4}{5}$			$\frac{8}{5}$	$\frac{4}{5}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{15}$			2	$\ddot{\xi}_{,m} = -2\overset{1}{m}\ddot{\eta}^m$
4	$\overset{1}{m}\ddot{\xi}_{,m}\dot{\zeta}^s\dot{\zeta}^s$											2	$\frac{1}{3}$	1	$-\frac{1}{3}$				3	
5	$\overset{1}{m}\ddot{\xi}_{,m}\ddot{f}$	$\frac{4}{3}$				2	$\frac{2}{3}$					$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$				5	$\ddot{\xi}_{,m}\ddot{f} = -\ddot{\xi}\ddot{f}_{,m}$; $\ddot{f} = -\frac{2\overset{1}{m}}{r}$
6	$\overset{1}{m}\overset{2}{m}\ddot{r}_{,ssm}$											-2							-2	$\ddot{r}_{,ssm} = (\overset{2}{r}_{,ssm})$ for $x^s = \eta^s$ *
7	$\overset{1}{m}\ddot{\xi}_{,s}\dot{\zeta}^m\dot{\eta}^s$	$\frac{16}{5}$				$\frac{8}{3}$	$\frac{4}{5}$	$\frac{4}{3}$											8	
8	$\overset{1}{m}\ddot{\xi}_{,s}\dot{\zeta}^s\ddot{\eta}^m$	$\frac{16}{5}$					$-\frac{8}{15}$	4	$\frac{4}{3}$	-2									6	
9	$\overset{1}{m}\ddot{\xi}_{,m}\dot{\eta}^s\dot{\zeta}^s$	$-\frac{32}{15}$		$-\frac{16}{3}$	$-\frac{8}{3}$		$\frac{4}{5}$		$\frac{4}{3}$										-8	
10	$\overset{1}{m}\ddot{\xi}_{,s}\dot{\zeta}^s\dot{\zeta}^m$	$-\frac{8}{3}$				-4	$-\frac{4}{3}$												-8	

* $\ddot{r}_{,ssm} = \frac{\partial^3 r}{\partial\eta^s\partial\eta^s\partial\eta^m}\dot{\zeta}^s\dot{\zeta}^s$, as $\frac{\partial^2 r}{\partial\eta^s\partial\eta^s}\frac{\partial \overset{1}{r}}{\partial\eta^s} = 0$.

Let me remind you the computation in NRGR

This calculation is one of the first test of GR! *





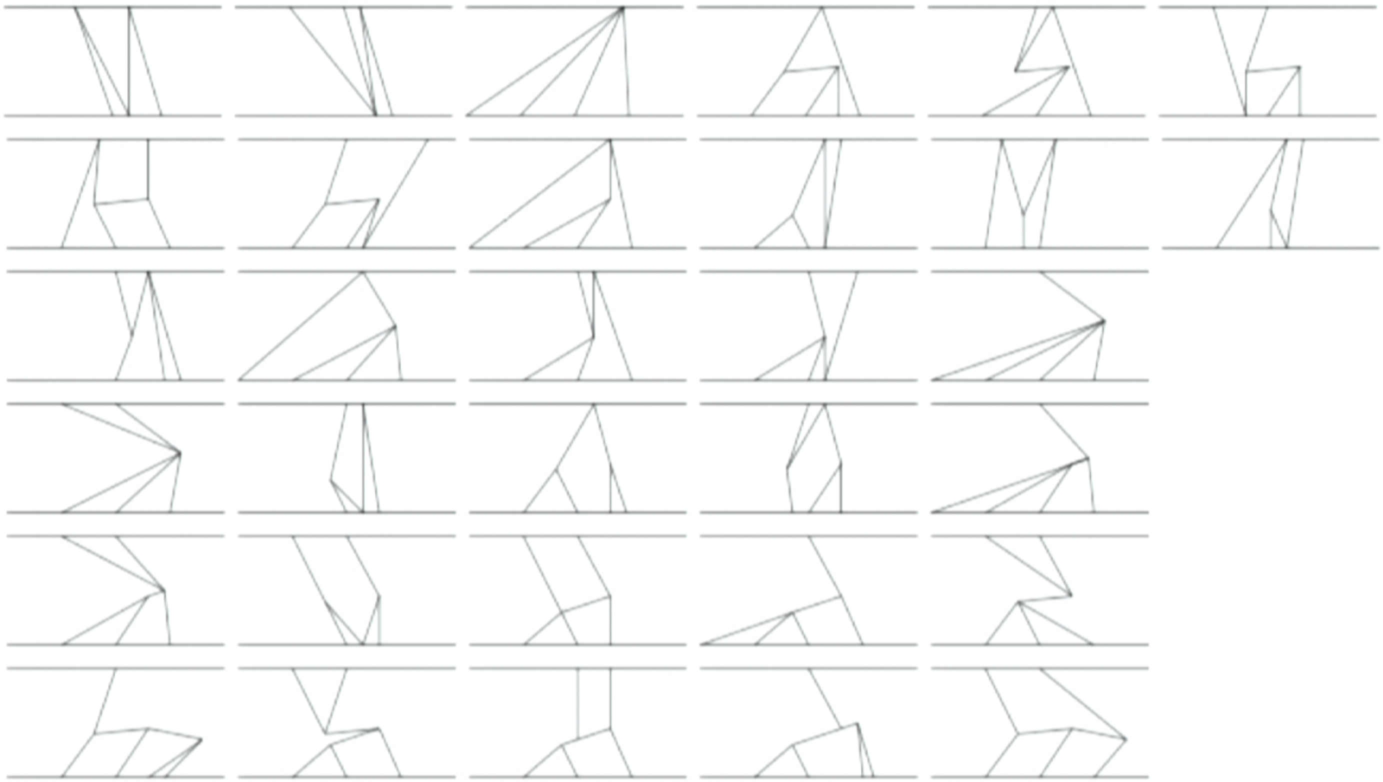
Zombie Feynman (xkcd)

I hope I convinced you that Feynman's bookkeeping *ala* EFT is really efficient/useful!

* Actually GR accounts for the 'anomalous' precession once Jupiter is taken into account (Precise measurements were the key!)

Higher order (spinless) computations
(2PN Gilmore & Ross 0810.1328, 3PN Foffa & Sturani 1104.1122)

Sample of topologies at 3PN (ma'ca):

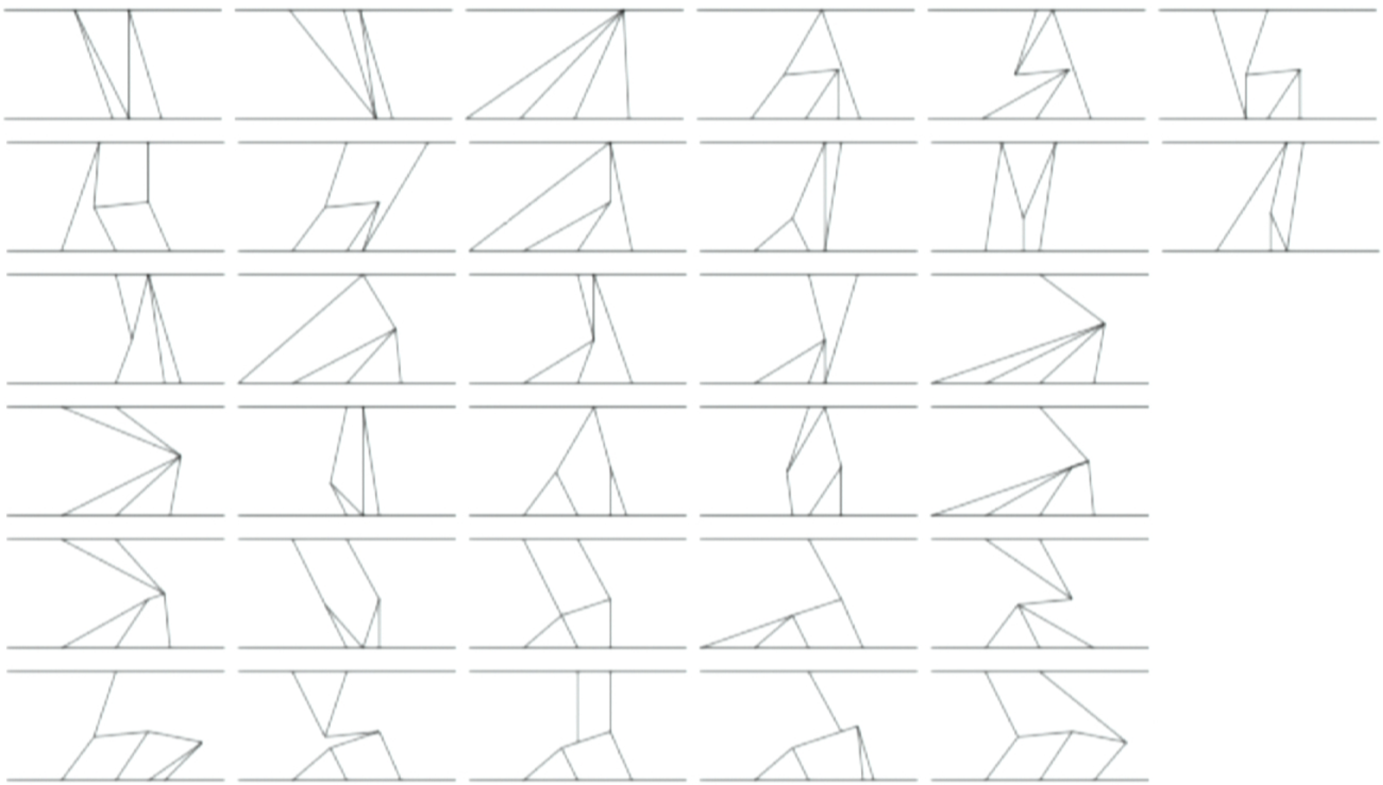


NNNNLO conservative dynamics at 4PN (spinless) in progress... (relevant for eLISA and matching with numGR)

Higher order (spinless) computations

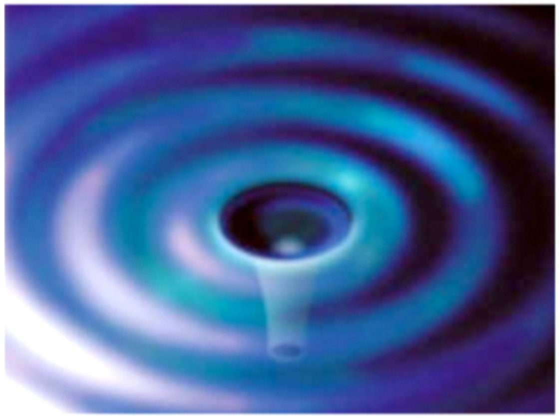
(2PN Gilmore & Ross 0810.1328, 3PN Foffa & Sturani 1104.1122)

Sample of topologies at 3PN (ma'ca):



NNNNLO conservative dynamics at 4PN (spinless) in progress... (relevant for eLISA and matching with numGR)

Chapter II: EFT/BH duality
Dissipative effects in the worldline approach



$\longrightarrow$

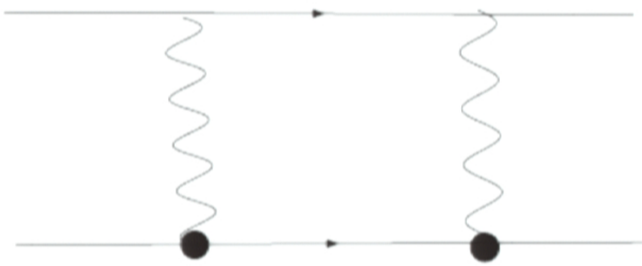
$$S = -\int d\tau Q^E_{ab} E^{ab} + Q^B_{ab} B^{ab} + \ldots$$

The absorption cross section depends on the correlator for the Q's (ala AdS/CFT)

$$\int dx^0 e^{-i\omega x^0} \langle 0 | T Q^E_{ab}(0) Q^E_{cd}(x^0) | 0 \rangle = -\frac{i}{2} Q_{abcd} F(\omega)$$

(Goldberger & Rothstein, hep-th/051133)

Dissipation due to Spin
(trickier due to superradiance)
(RAP, 0710.5150)



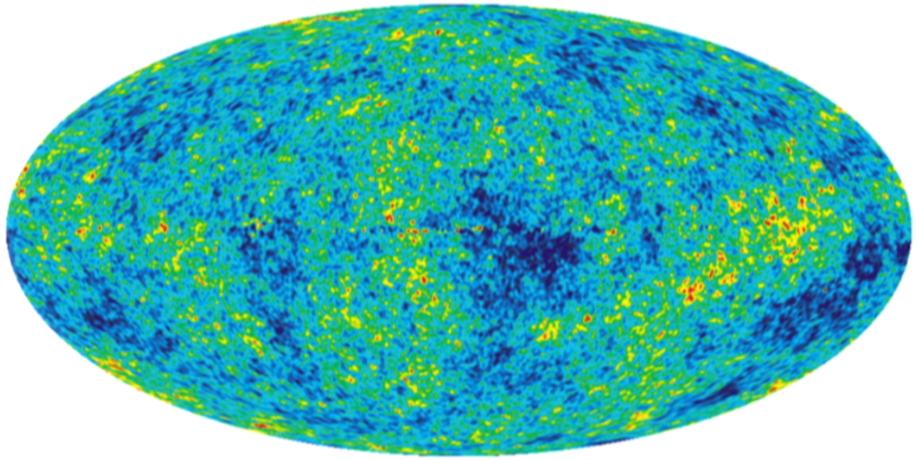
We computed the power of absorption for spinning BH-BH binary systems (RAP, 0710.5150)

$$P_{abs}^{spin} = -\frac{8}{5} G_N^6 m_1^2 m_2^2 \left\langle \frac{1 \cdot \xi}{r^8} (a_* + 3a_*^3) \right\rangle$$

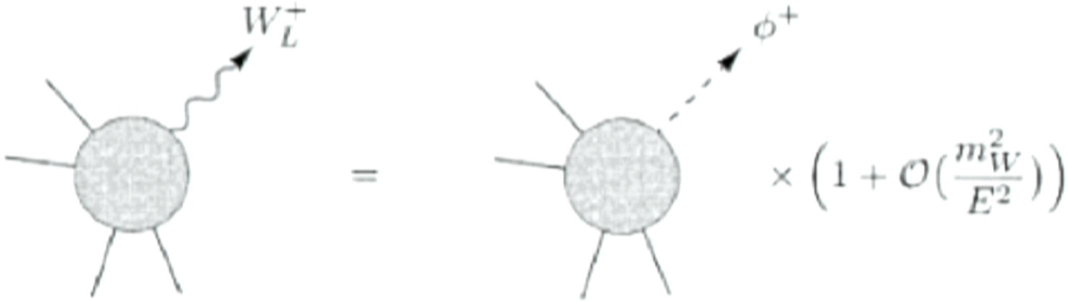
One can generalize to NS absorption cross section depends on inner structure (same as finite size)

$$\frac{dP_{abs}}{d\omega} = \frac{1}{T} \frac{G_N}{32\pi^2} \left\langle \sum_{a \neq b} \frac{\sigma_{abs}^a(\omega)}{\omega^2} m_b^2 |q_{ij}^a(\omega)|^2 \right\rangle$$

Chapter IIIa: EFT of Inflation
density perturbations & Goldstone bosons

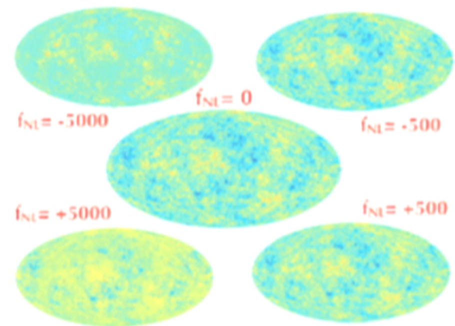


EQUIVALENCE THEOREM:



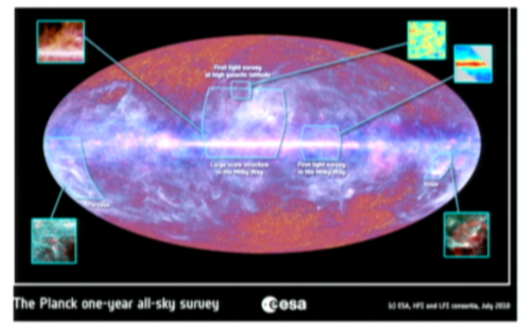
More information: Non-Gaussianities (NG)

Non-linearities can potentially teach us about
the mechanism of inflation



$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \quad = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

- Single field inflation with canonical kinetic term (cs=1) leads to negligible non-gaussianities!
- Some models (with non-standard kinetic term) can enhance non-gaussianities (e.g. DBI, P(X),...)
- Can we make model-independent predictions?



Key observation of EFT approach:

To end inflation we need a clock. Time translations are (spontaneously) broken => Goldstone boson (GB) eaten by scalar 'mode in the graviton', i.e. g^00.

At 'high energies' (decoupling limit) we can compute correlations functions using GBs (Eq. theorem). Action/interactions set by (non-linearly realized) symmetries

Beyond the *Standard Model*

- What happens if we add more fields?
- Could the spectrum of fluctuations be produced by physics unrelated to the BD vacuum? For example, thermal fluctuations, noise?
- What kind of observational signatures would this new paradigm predict?
- Can we construct an EFT approach ?!

We analyzed a setup with new degrees of freedom
but only *one clock* (*i.e.* do not contribute to curvature perturbations)
(Nacir, RAP, Senatore & Zaldarriga, 1109.4192)

Note: We cannot integrate them out. Even though they are not necessarily 'light', they are produced during inflation (not in vacuum) and act upon the clock at low(er) frequencies (w~H)

Paradigmatic examples include:
Warm inflation (Berera et al.) and Trapped Inflation (Green et al.)
(Adiabaticity is violated during particle production)

Chapter IIIb:
Dissipative effects in the EFT of Inflation

(Nacir, RAP, Senatore & Zaldarriga, 1109.4192)

Couple the goldstone boson to dissipative sector

$$\partial_\mu(t+\pi)\partial^\mu(t+\pi)\tilde{\mathcal{O}} \rightarrow \qquad \tilde{S}_{\rm int} = \int d^4x \tilde{\mathcal{O}}(x)\dot{\pi}(x)$$

matching for the O's: $\mathrm{Im}G^{\mathcal{O}}_{\rm ret}(\omega, \mathbf{q}) \simeq \mathrm{Im}G^{\mathcal{O}}_{\rm ret}(\omega, \mathbf{0}) \simeq \gamma\omega,$

EOM: $\ddot{\pi}_k(t) + (3H+\gamma)\dot{\pi}_k(t) + \frac{c_s^2\mathbf{k}^2}{a^2}\pi_k = -\frac{1}{N}\delta\mathcal{O}_S(t,\mathbf{k})$

but we also generate non-linear terms! $-\frac{1}{2}\tilde{\mathcal{O}}(\partial_i\pi)^2$

Recall the binary case:

$$S = -\int d\tau Q^E_{ab}E^{ab} + Q^B_{ab}B^{ab} + \ldots$$

Take home I: power spectrum

(Nacir, RAP, Senatore & Zaldarriga, 1109.4192)

$$k^3 \langle \zeta\zeta \rangle_{\mathcal{O}} \simeq \nu_{\mathcal{O}\star} \sqrt{\pi H_\star / \gamma_\star} \frac{H_\star^2}{2c_s^\star \left(c_s^\star N_c\right)^2}$$

with $\langle \delta\mathcal{O}_S(t, \mathbf{k}) \delta\mathcal{O}_S(t', \mathbf{q}) \rangle \simeq \frac{\nu_{\mathcal{O}} \delta(t - t')}{a^3(t)} (2\pi)^3 \delta^{(3)}(\mathbf{q} + \mathbf{k}).$

which can be written also as (if FD applies, e.g. Warm inflation)

$$k^3 \langle \zeta\zeta \rangle_T \simeq \sqrt{\pi \gamma_\star H_\star} \frac{T_{\mathcal{O}} H_\star^2}{2c_s^\star \left(c_s^{\star 2} N_c\right)} \simeq \frac{c_s k_\star T_{\mathcal{O}} H_\star^2}{\Lambda_c^4}$$

The power spectrum is thus dominated by physics UNRELATED to Bunch-Davies state!

Take home II: Non-gaussianities

(Nacir, RAP, Senatore & Zaldarriga, 1109.4192)

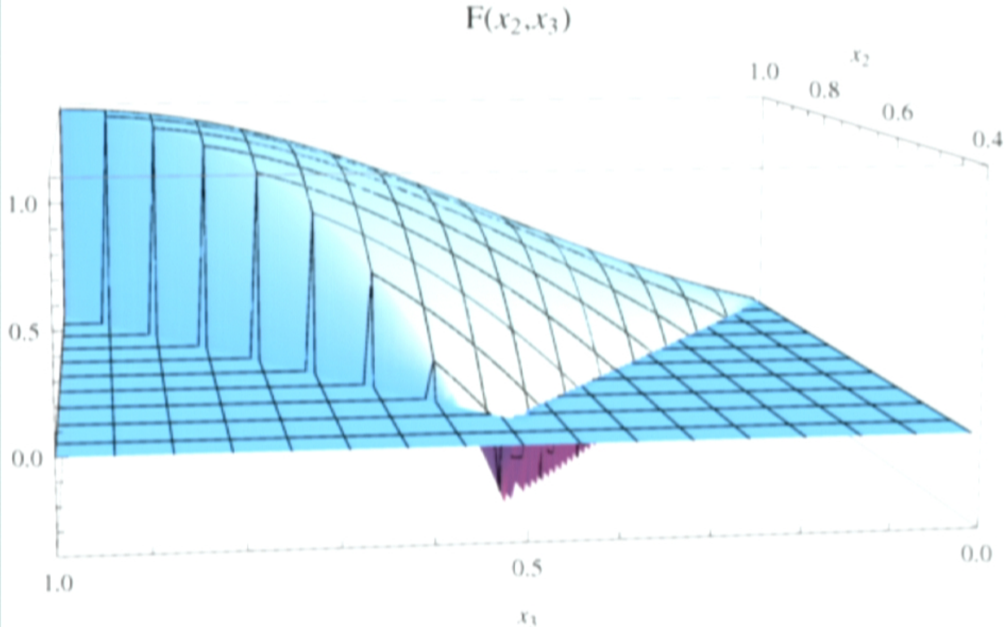
$$\frac{\mathcal{O}(\partial_i \pi)^2}{\mathcal{O}\dot{\pi}}\Bigg|_{k_\star \sim \sqrt{\gamma H/c_s^2},\ \omega_\star \sim H} \sim \frac{k_\star^2 \zeta^2}{H^2 \zeta} \sim \frac{\gamma}{c_s^2 H}\zeta \to |f_{\rm NL}| \sim \frac{\gamma}{c_s^2 H}$$

All the un-known parameters canceled out! (also power spectrum is known)
Only depends on \gamma (and speed of sound)

OUR ANALYSIS IS COMPLETELY MODEL INDEPENDENT!

Observational consequences:

(Nacir, RAP, Senatore & Zaldarriga, 1109.4192)



$$F(x_2, x_3) = x_2^2 x_3^2 \frac{F(1,x_2,x_3)}{F(1,1,1)}$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \quad = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$

Shape is enhanced at folded configurations

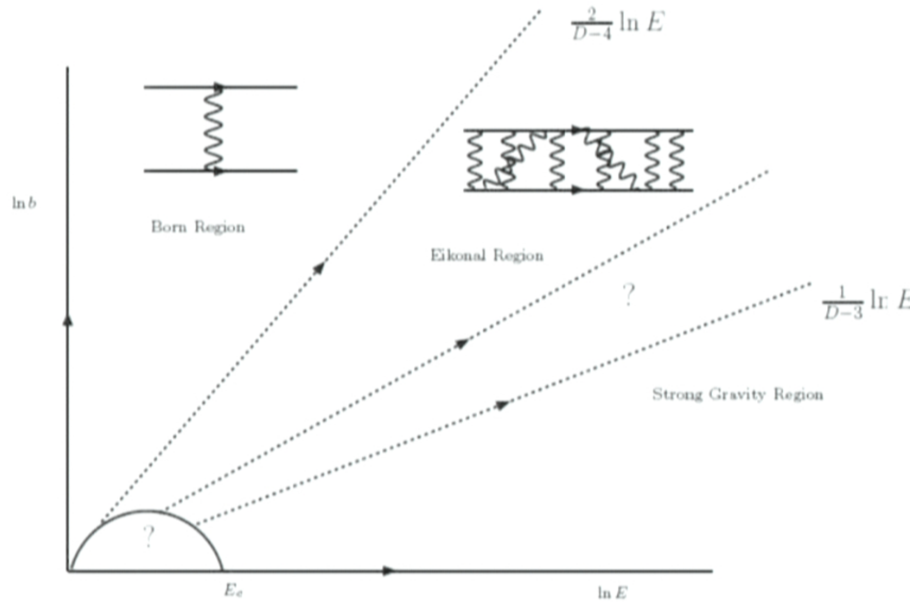
There is no such pick in BD state. This is the smoking gun

We can probe the nature of the primordial seed!

(R. Flauger, RAP & M. Zaldarriga, in progress)

Chapter IV:
EFT for ultra-Planckian high energy scattering

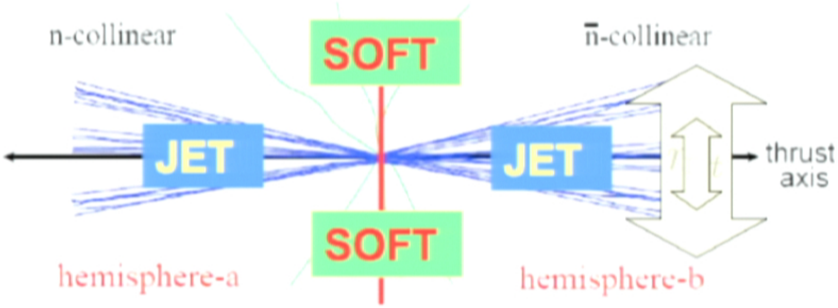
(Giddings, RAP & Schmidt-Sommerfeld in progress)

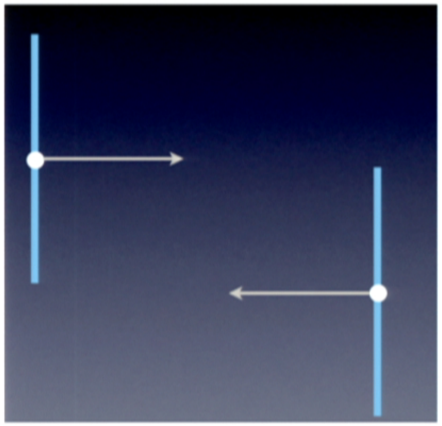


'Phase diagram' for HE gravitational scattering

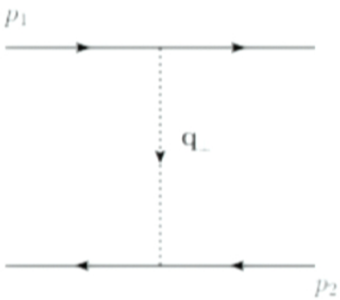
(Giddings & RAP 0908.0004)

a/a SCET



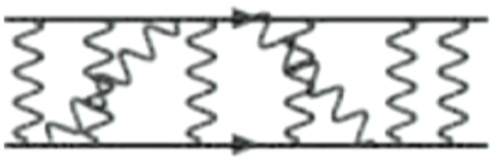


Potential mode (Glauber)

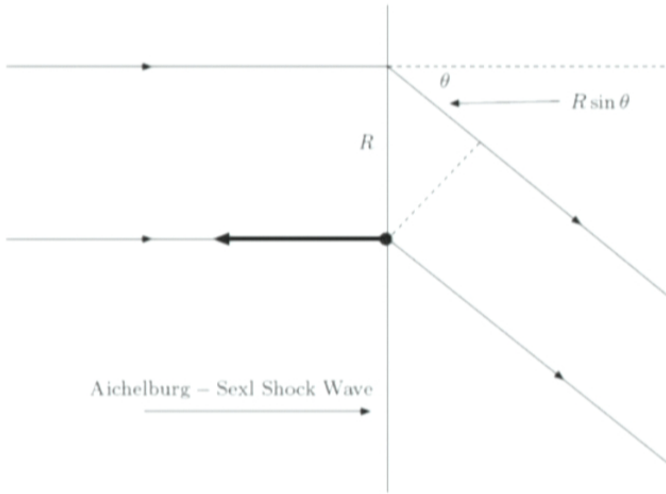


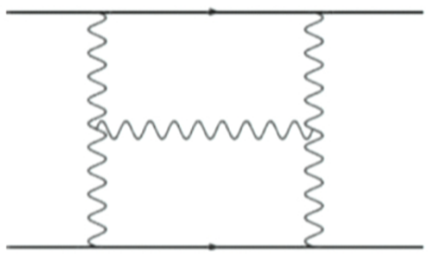
$$\chi_{\rm LO}(b,E) = \frac{4\pi G_D p_1^+ p_2^-}{\Omega_{D-3}(D-4)}\frac{1}{|\mathbf{b}|^{D-4}} = \frac{4\pi G_D s}{\Omega_{D-3}(D-4)}\frac{1}{|\mathbf{b}|^{D-4}},$$

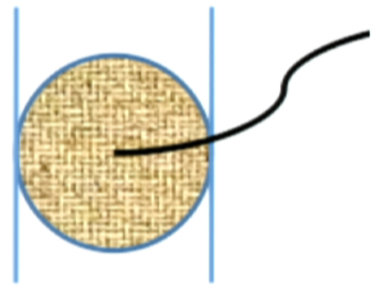
$$\Delta \mathbf{p}_1 = \mathbf{p}_{1\perp} = \frac{\partial S}{\partial \mathbf{x}_{1\perp}} \to \sin\theta = \frac{1}{E}\frac{\partial \chi}{\partial \mathbf{x}_{1\perp}}$$



Radiation: Soft & Collinear







$$\frac{dE}{d\omega} = \omega^2 \int d\Omega P_{ij,kl}(\mathbf{n}) T_{ij}(\omega, \omega\mathbf{n}) T^{\star}_{lk}(\omega, \omega\mathbf{n})$$

$$\Gamma \sim GE^2\theta_c^2 \sim \frac{GE}{b}(Eb)\theta_c^2 \sim \theta_c^2(Eb\theta_c) \sim \theta_c^2 \chi_{\rm LO}$$

Collinear: Matching into 'jets'

Summary

- EFT techniques provide a powerful organizational principle to separate the relevant physics from different scales. Systematic regularization.
- GW Science: New methods reproduced old stuff in a simpler and systematic fashion and rapidly led to new results (most accurate description of spinning binary systems to date, e.g. 3PN)
- EFT/BH duality: New insight into the field theory treatment of BH dynamics including dissipation.
- Inflation: EFT approach allows us to describe all models of inflation in a unified framework and parameterize new physics ala EWPD. Additional d.o.f. using EFT/BH duality ideas. Probing the Bunch-Davies vacuum!
- Gravitational S-matrix: Systematic EFT approach to study the large impact parameter regime of gravitational scattering (BH onset, AdS/QCD). Hybrid approach between NumGR and analytic methods.
- Future applications & In progress: EFT for cosmological perturbations, and dissipative effects in hydrodynamics

"Harris, when I said 'any questions' I was using only a figure of speech."

Any questions?

Thank you...