Title: Simple Scattering Amplitudes in Higher Dimensions

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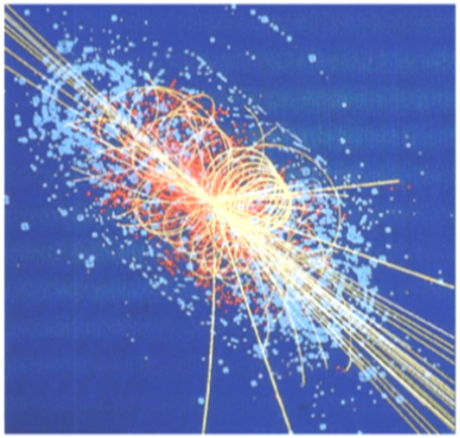
Abstract: TBA



Simple scattering amplitudes in higher dimensions

Rutger Boels
University of Hamburg

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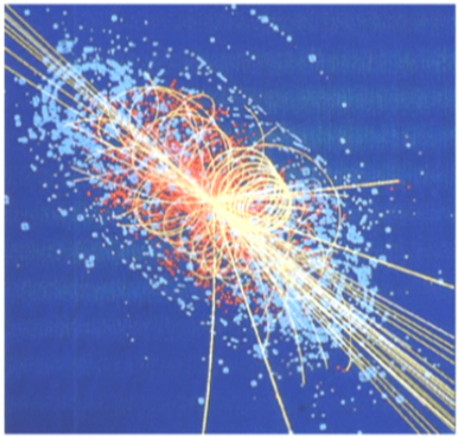

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main physical motivation:

$$\boxed{\text{simplicity}} \longleftrightarrow \boxed{\text{symmetry}}$$

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• in four dimensions massive progress at tree and loop level inspired by [Witten,03], guided by simplicity

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higher dimensions?
- dimensional regularisation in four D
- string theory
- inherent interest: also simple?

main idea today:

"How symmetries determine simple scattering amplitudes in higher dimensions."

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based on:

arXiv:1201.2653 [hep-th] Donal O'Connell & RB

arXiv:1201.2655 [hep-th] RB

arXiv:1204.xxxx [hep-th] RB

Bringing out simplicity: spinor helicity

(vectors as spinors)

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(vectors as spinors)

- Poincare quantum numbers for multiple plane waves

$$K\,|k\rangle = k\,|k\rangle \qquad K_\mu \qquad K_{[\mu}\Sigma_{\nu\rho]}$$

- on-shell states: little group
  - SO(D-2) $\quad K^2 = 0$
  - SO(D-1) $\quad K^2 \neq 0$

- massless, 4D: Abelian little group → helicity

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e.g. in Yang-Mills [Parke-Taylor, 87]:

$$\boxed{A_n(\mathrm{MHV}) = \frac{\langle i,j\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}}$$

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Spinor helicity in higher D

(LiE software online)

- technical complication: non-Abelian little groups

$(m^2 = 0)$

- previous formulations in higher D: 3,4,5 pt amplitudes
  - spinor helicity in D=6 [Cheung-O'Connell, 09]
  - spinor helicity in D≥4 [RB, 09] including supersymmetry
  - see also D=10, [Caron-Huot-O'Connell, 10], D=6 [Dennen-Huang-Siegel, 09]

3,4,5 pt supersymmetric amplitudes

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but:

- maximal susy "lives" in D=10/D=11
- superstring / M-theory

Spinor helicity in higher D

chiral representation of Gamma matrix algebra

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^{\mu,BA'} \\ \bar{\sigma}^\mu_{B'A} & 0 \end{pmatrix} \qquad \psi = \begin{pmatrix} \lambda^A \\ \tilde{\lambda}_{A'} \end{pmatrix}$$

Weyl spinors $1, 2, \ldots, \mathcal{D}$

$\exists$ charge conjugation matrix: $\quad C\Gamma^\mu C^{-1} = -\left(\Gamma^\mu\right)^T$

$$C = \begin{pmatrix} \Omega_{BA} & 0 \\ 0 & \Omega^{B'A'} \end{pmatrix}, \qquad D = 4k+4$$

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→ spinor products: $\lambda_{A}\psi^{A} \equiv [\lambda\psi] \qquad \lambda^{A'}\psi_{A'} \equiv \langle\lambda\psi\rangle$

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symmetries: from $\{\, [\psi\lambda]\,,\ [\lambda\psi]\,,\ \langle\lambda\psi\rangle\,,\ \langle\psi\lambda\rangle \,\}$ 2 independent

Vectors and spinors

solve massles chiral Dirac equation

$$k_\mu \sigma^{\mu,BA'} \lambda_{A',a'} = 0 \qquad k_\mu \bar{\sigma}^\mu_{A'A} \lambda^{A,a} = 0 \qquad k^2 = 0$$

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- can see little group as separate or as subgroup of SO(D)

→ proof of:

$$k_\mu \sigma^{\mu,BA'} = \lambda^{B,a} \lambda_a^{A'}$$

$$[\lambda^a \lambda^b] = 0$$

$$\epsilon^{\mu,n} \gamma_n^{a'a} \propto \frac{\lambda^{a'} \sigma^\mu \not{v} \lambda^a}{2k \cdot v}$$

UHH

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- complete dictionairy between vectors and spinors

- little group basis choice through a set of fixed spinors:

$$\lambda^{A,a} \propto k^{AA'} \xi^a_{A'} \qquad \lambda_{A',a'} \propto k_{A'A} \xi^A_{a'} \,,$$

(leads to complete basis, numerical convenience)

Superpoincare → on-shell superspaces

covariant representation of on-shell supersymmetry algebra

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- other reps by fermionic fourier transform
- BPS reps from higher D massless

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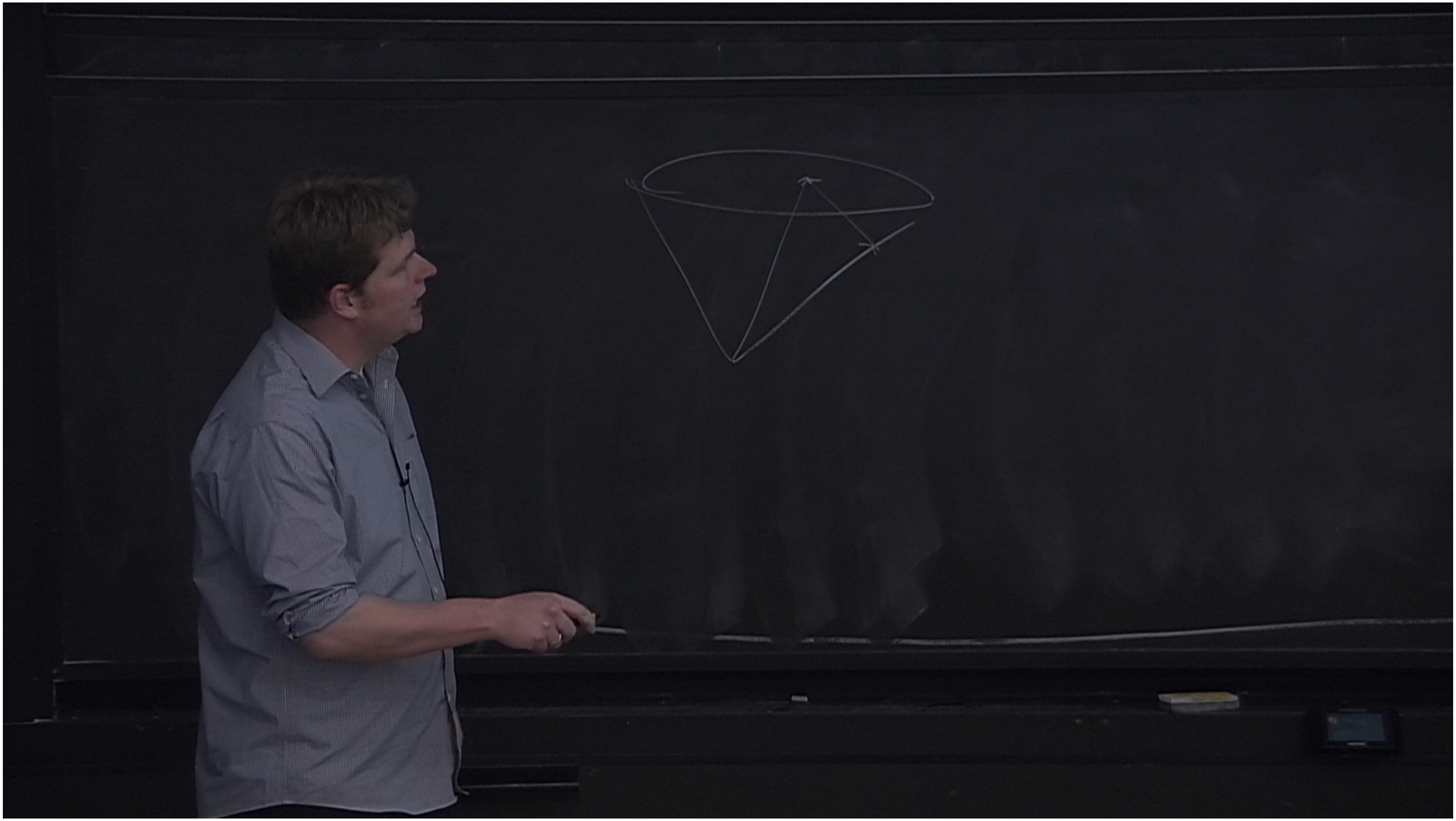
- BPS reps from higher D massless $\quad k = k^\flat + \frac{m^2}{2q\cdot k}q$
- extension to massive case (red)



Superfields for rep: $Q^A = \lambda^{A,a}\eta_a \qquad \bar{Q}^{A'} = \lambda_a^{A'}\frac{\delta}{\delta\eta_a}$

identify massless field content:

little group spinor rep

$$\phi(\eta) = \phi_0 + \phi^a\eta_a + \ldots + \overline{\phi}_0(\eta)^{\text{"}\mathcal{D}-2\text{"}}$$

dim	$\mathcal{D}$	"$\mathcal{D}-2$"
4	2	1
6	4	2
8	8	4
10	16	8

16 states: span max sYM multiplet

256 states: span max sugra multiplet

- general: ϕ_0 transforms in some representation of little group
- fundamental multiplet: ϕ_0 is a scalar

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- general: ϕ_0 transforms in some representation of little group
- fundamental multiplet: ϕ_0 is a scalar
- other states antisymmetrized tensor products of ϕ_0 with chiral spinor of SO(D-2)
- can calculate their Dynkin labels

Massless on-shell superspace in D=10

D=10: 256 states in the fundamental multiplet

$$\phi(\eta) = \phi_0 + \phi^a \eta_a + \ldots + \overline{\phi}_0 (\eta)^8$$

field content as SO(8) representations:

	bosonic		**fermionic**
0	$\underline{1}_4$	1	$\underline{8}_3$
2	$\underline{28}_2$	3	$\underline{56}_1$
4	$\underline{35}_0 + \underline{35}'_0$	5	$\underline{56}_{-1}$
6	$\underline{28}_{-2}$	7	$\underline{8}_{-3}$
8	$\underline{1}_{-4}$		

IIB supergravity

Massive on-shell superspace in D=10

D=10: 65.536 states in the fundamental multiplet

$$\phi(\eta,\iota) = \phi_0 + \phi^a \eta_a + \phi_{a'} \iota^{a'} + \ldots + \bar{\phi}\left((\eta)^8 \iota^8\right)$$

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can calculate it's SO(9) Dynkin labels:

0	$(0,0,0,0)_1$
1	$(0,0,0,1)_{16}$
2	$(0,1,0,0)_{36} + (0,0,1,0)_{84}$
3	$(1,0,0,1)_{128} + (0,1,0,1)_{432}$
4	$(2,0,0,0)_{44} + (0,0,0,2)_{126} + (1,1,0,0)_{231} + (0,2,0,0)_{495} + (1,0,0,2)_{924}$
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embed in bigger group?

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2	$(0,1,0,0,0,0,0,0)_{120}$
3	$(0,0,1,0,0,0,0,0)_{560}$
4	$(0,0,0,1,0,0,0,0)_{1820}$
5	$(0,0,0,0,1,0,0,0)_{4368}$
6	$(0,0,0,0,0,1,0,0)_{8008}$
7	$(0,0,0,0,0,0,1,1)_{11440}$
8	$(0,0,0,0,0,0,2,0)_{6435} + (0,0,0,0,0,0,0,2)_{6435}$

Various other groups in paper

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Superamplitudes $Q^A = \lambda^{A,a}\eta_a \qquad \bar{Q}^{A'} = \lambda_a^{A'}\frac{\delta}{\delta\eta_a}$

- promote each leg of an amplitude $A(k_i) \to A(\{k_i, \eta_i\})$
- component amplitudes by fermionic integration

simple formulation of the on-shell susy Ward identities

$$Q = \sum_i Q_i$$

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- factors four point Veneziano amplitude, up to "Hagedorn"
- Ward identities solved for more massive legs at three points

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Massless on-shell superspace in D=10, type IIB

D=10: 256 states in the fundamental multiplet

$$\phi(\eta) = \phi_0 + \phi^a \eta_a + \ldots + \overline{\phi}_0 (\eta)^8$$

field content:

	bosonic		**fermionic**
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- → no massless poles in field theory limit

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

from general properties at string tree level:

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determining remaining constants? → soft limits

supersymmetric soft limit: $k_i, \eta_i \to 0$

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from general properties at string tree level:

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- diff operator annihilates gravitational coupling

→ relates c_i for various multiplicities, up to kinematics

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how many points for which order?

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include more stringy symmetries?

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

more stringy symmetry in IIB: SL(2,Z)

[Green-Gutperle, 97], [Green et.al., 97-12]

- results for effective action, $R^4, D^4R^4, D^6R^6, \lambda^{16}$ couplings

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e.g.: $$f^k_\beta(\tau_b, \bar{\tau}_b) = \sum_{(l,m)\neq(0,0)} (l + m\tau_b)^{2k-\beta}(l + m\bar{\tau}_b)^{-\beta}$$

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- "k" $\leftrightarrow$ U(1)$_R$ non-conservation(!)

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- weak string coupling expansion:

$$\lim_{\tau_b \to i\infty} f^k_{\frac{3}{2}}(\tau_b, \overline{\tau}_b) \propto \zeta(3) + (\text{1-loop}) + \text{instanton}$$

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Your Question Here?