

Title: Advanced Quantum Field Theory - Lecture 14

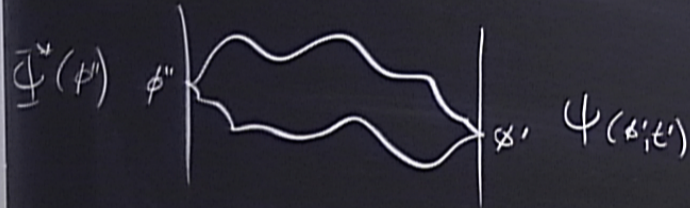
Date: Mar 02, 2012 09:30 AM

URL: <http://pirsa.org/12030076>

Abstract:

$$\langle 0, t'' | 0, t' \rangle = N^2 \int_{\text{un}}$$

$$\langle 0, t'' | 0, t' \rangle = N^2 \int_{\text{unconstrained}}$$



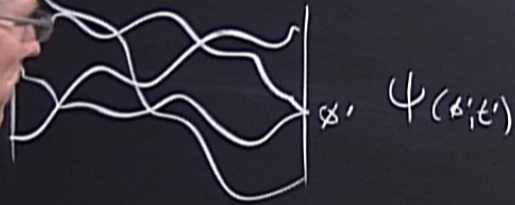
$$\langle 0, t'' | 0, t' \rangle = N^2 \int_{\text{unconstrained}} \mathcal{D}\phi \mathcal{D}\pi \exp \left\{ i \int d^3x dt \left[\pi \dot{\phi} - \mathcal{L}(\pi, \phi) \right] \right.$$

$$\left. \begin{aligned} & - \frac{1}{2} \int d^3x d^3y \phi(x, t'') \epsilon(x-y) \phi(y, t'') \\ & - \frac{1}{2} \int d^3x d^3y \phi(x, t') \epsilon(x-y) \phi(y, t') \end{aligned} \right\}$$

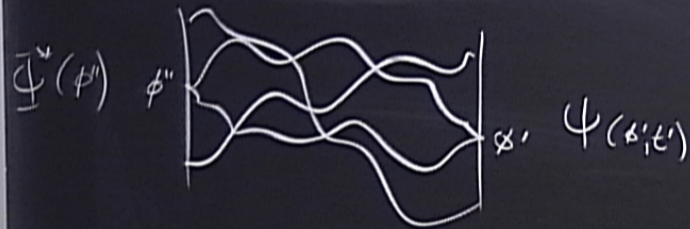
$$\langle 0, t'' | 0, t' \rangle = N^2 \int_{\text{unconstrained}} \mathcal{D}\phi \mathcal{D}\pi \exp \left\{ i \int d^3x dt \left[\pi \dot{\phi} - \mathcal{H}(\pi, \phi) \right] \right.$$

$$\left. - \frac{1}{2} \int d^3x d^3y \phi(x, t'') \varepsilon(x-y) \phi(y, t'') \right. \\ \left. - \frac{1}{2} \int d^3x d^3y \phi(x, t') \varepsilon(x-y) \phi(y, t') \right\}$$

$$\varepsilon(x-y) = \int \frac{d^3k}{(2\pi)^3} E_k e^{i\mathbf{k} \cdot (\mathbf{x}-\mathbf{y})} \quad E_k = \sqrt{k^2 + m^2} \geq 0$$



$$\langle 0, t'' | 0, t' \rangle = N^2 \int_{\text{unconstrained}} \mathcal{D}\phi \mathcal{D}\pi \exp \left\{ i \int d^3x dt \left[\pi \dot{\phi} - \mathcal{H}(\pi, \phi) \right] \right.$$



$$- \frac{1}{2} \int d^3x d^3y \phi(x, t'') \varepsilon(x-y) \phi(y, t'')$$

$$- \frac{1}{2} \int d^3x d^3y \phi(x, t') \varepsilon(x-y) \phi(y, t')$$

$$\varepsilon(x-y) = \int \frac{d^3k}{(2\pi)^3} E_k e^{i\mathbf{k} \cdot (\mathbf{x}-\mathbf{y})} \quad E_k = \sqrt{k^2 + m^2} \geq 0$$

Trick: $A(t)$ consider $\underline{I} = \epsilon \int_{-\infty}^0 dt e^{\epsilon t} A(t)$, $\epsilon \rightarrow 0$

$$\underline{I} = \int_{-\infty}^0 \frac{d}{dt} (e^{\epsilon t}) A(t) dt = \left[e^{\epsilon t} A(t) \right]_{-\infty}^0 - \int_{-\infty}^0 dt e^{\epsilon t} \frac{dA}{dt}$$

$$= \cancel{A(0)} - \left[A \right]_{-\infty}^{\cancel{0}} + o(\epsilon)$$

$$= A(-\infty) + o(\epsilon)$$

$$\phi(x, t') \phi(y, t') = \epsilon \int_{-\infty}^0 dt e^{\epsilon t} \phi(x, t) \phi(y, t) \quad t' \rightarrow -\infty$$

$$\phi(x, t'') \phi(y, t'') = \epsilon \int_0^{\infty} dt e^{-\epsilon t} \phi(x, t) \phi(y, t) \quad t'' \rightarrow \infty$$

$$\int \frac{d^3k}{(2\pi)^3} E_k e^{ik \cdot (x-y)} \quad E_k = \sqrt{k^2 + m^2} \geq 0$$

use $\phi(x) = \int \frac{d^3p}{(2\pi)^3} \phi(p) e^{ip \cdot x}$

$$\varepsilon(x-y) = \int \frac{d^3k}{(2\pi)^3} \varepsilon(k) e^{ik \cdot (x-y)}$$

$$\begin{aligned} \int d^3x d^3y \phi(x) \varepsilon(x-y) \phi(y) &= \int d^3x d^3y \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \phi(p) \varepsilon(k) \phi(q) e^{ip \cdot x} e^{ik \cdot (x-y)} e^{iq \cdot y} \\ \int d^3x e^{i(p+k) \cdot x} &= (2\pi)^3 \delta^3(p+k) \quad \int d^3y e^{i(q-k) \cdot y} = (2\pi)^3 \delta^3(q-k) \end{aligned}$$

$$\int d^3x d^3y \phi(x) \varepsilon(x-y) \phi(y) = \int \frac{d^3k}{(2\pi)^3} \phi(k) \varepsilon(k) \phi(-k)$$

$$\int d^3x \mathcal{L} = \int d^3x \left[\frac{1}{2} \nabla^2 \phi - m^2 \phi^2 \right] = \int \frac{d^3k}{(2\pi)^3} \phi(-k) \left[-\vec{k}^2 - m^2 \right] \phi(k)$$

$$\int d^3x d^3y \phi(x) \varepsilon(x-y) \phi(y) = \int \frac{d^3k}{(2\pi)^3} \phi(k) \varepsilon(k) \phi(-k)$$

$$\int d^3x \mathcal{L} = \int d^3x \left[\phi \nabla^2 \phi - m^2 \phi^2 \right] = \int \frac{d^3k}{(2\pi)^3} \phi(-k) \left[-\vec{k}^2 - m^2 \right] \phi(k)$$

CAUTION

DO NOT LEAVE THE SYSTEM UNATTENDED
OR LEAVE THE ROOM OF THE BOARD

IT IS IMPORTANT TO HAVE
YOUR BOARD PROTECTED WHEN

PLEASE DO NOT

$$i \int \frac{d^3k}{(2\pi)^3} dt \left\{ + \frac{1}{2} \dot{\phi}(\vec{k}) \dot{\phi}(-\vec{k}) - \phi(\vec{k}) \phi(-\vec{k}) [\vec{k}^2 + m^2] + \frac{i}{2} e^{\epsilon|\vec{k}|} \phi(\vec{k}) \phi(-\vec{k}) \epsilon(\vec{k}) \right\}$$

$$i \int \frac{d^3k}{(2\pi)^3} dt \left\{ + \frac{1}{2} \bar{\phi}(\vec{k}) \dot{\phi}(-\vec{k}) - \phi(\vec{k}) \phi(-\vec{k}) [\vec{k}^2 + m^2] + \frac{i}{2} \epsilon e^{\epsilon|\vec{k}|} \phi(\vec{k}) \phi(-\vec{k}) \epsilon(\vec{k}) \right\}$$

$\epsilon = \frac{1}{2} \epsilon e^{\epsilon|\vec{k}|} \sqrt{\vec{k}^2 + m^2}$
 > 0

$$i \int \frac{d^3k}{(2\pi)^3} dt \left\{ + \frac{1}{2} \dot{\phi}(\vec{k}) \dot{\phi}(-\vec{k}) - \phi(\vec{k}) \phi(-\vec{k}) [\vec{k}^2 + m^2] + \frac{i}{2} \epsilon e^{\epsilon|\vec{k}|} \phi(\vec{k}) \phi(-\vec{k}) \epsilon(\vec{k}) \right\}$$

$\epsilon_{\text{new}} = \frac{1}{2} \epsilon e^{\epsilon|\vec{k}|}$
 > 0

$$i \int \frac{d^3k}{(2\pi)^3} dt \left[\phi(-\vec{k}) \left[-\vec{k}^2 - m^2 + i \epsilon_{\text{new}} \right] \phi(\vec{k}) + \dot{\phi}^2 \text{ terms} \right]$$

$$i \int \frac{d^3k}{(2\pi)^3} \left\{ + \frac{1}{2} \bar{\phi}(\vec{k}) \dot{\phi}(-\vec{k}) - \phi(\vec{k}) \phi(-\vec{k}) [\vec{k}^2 + m^2] + \frac{i}{2} e^{i\epsilon|\vec{k}|} \phi(\vec{k}) \phi(-\vec{k}) \epsilon(\vec{k}) \right\}$$

$$\epsilon_{\text{new}} = \frac{1}{2} e^{i\epsilon|\vec{k}|} \sqrt{\vec{k}^2 + m^2} > 0$$

$$\phi(-\vec{k}) \left[-\underbrace{\vec{k}^2 + m^2}_{\text{}} + i\epsilon_{\text{new}} \right] \phi(\vec{k}) + \text{other terms}$$

$$i \int \frac{d^3k}{(2\pi)^3} dt \left\{ + \frac{1}{2} \dot{\phi}(\vec{k}) \dot{\phi}(-\vec{k}) - \phi(\vec{k}) \phi(-\vec{k}) [\vec{k}^2 + m^2] + \frac{i}{2} e^{i\epsilon t} \phi(\vec{k}) \phi(-\vec{k}) \epsilon(\vec{k}) \right\}$$

$\epsilon_{\text{new}} = \frac{1}{2} \epsilon e^{i\epsilon t} \sqrt{\vec{k}^2 + m^2}$
 > 0

$$\left[\vec{k}^2 - m^2 + i\epsilon_{\text{new}} \right] \phi(\vec{k}) + \text{c.c. terms}$$

$$\int d^3x d^3y \phi(x) \varepsilon(x-y) \phi(y)$$

$$\frac{1}{(2\pi)^3} \phi(k) \varepsilon(k) \phi(-k)$$

$\downarrow \sqrt{k^2 + m^2}$

$$\int d^3x \mathcal{L} = \int d^3x \left[\frac{1}{2} \nabla^2 \phi - m^2 \phi \right] \phi(-k) \left[-\frac{1}{2} k^2 - m^2 \right] \phi(k)$$

Trick: $A(t)$ consider $\underline{I} = \epsilon \int_{-\infty}^0 dt e^{\epsilon t} A(t)$, $\epsilon \rightarrow 0$

$$I = \int_{-\infty}^0 \frac{d}{dt} (e^{\epsilon t}) A(t) dt = \left[e^{\epsilon t} A(t) \right]_{-\infty}^0 - \int_{-\infty}^0 dt e^{\epsilon t} \frac{dA}{dt}$$

$$= A(0) - \left[A \right]_{-\infty} + o(\epsilon)$$

Trick: $A(t)$: consider $\underline{I} = \epsilon \int_{-\infty}^0 dt e^{\epsilon t} A(t)$, $\epsilon \rightarrow 0$

$$\underline{I} = \int_{-\infty}^0 \frac{d}{dt} (e^{\epsilon t}) A(t) dt = \left[e^{\epsilon t} A(t) \right]_{-\infty}^0 - \int_{-\infty}^0 dt e^{\epsilon t} \frac{dA}{dt}$$

$$= [A]_{-\infty}^0 + o(\epsilon)$$

$$= A(0) + o(\epsilon)$$

Trick: $A(t)$ consider $\underline{I} = \epsilon \int_{-\infty}^0 dt e^{\epsilon t} A(t)$, $\epsilon \rightarrow 0$

$$\begin{aligned} \underline{I} &= \int_{-\infty}^0 \frac{d}{dt} (e^{\epsilon t}) A(t) dt = \left[e^{\epsilon t} A(t) \right]_{-\infty}^0 - \int_{-\infty}^0 dt e^{\epsilon t} \frac{dA}{dt} \\ &= A(0) - \left[A \right]_{-\infty}^0 + o(\epsilon) \\ &= A(0) + o(\epsilon) \end{aligned}$$

$$\frac{d}{dx} \theta(x) = \delta(x) \quad \theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\langle 0, \infty | T [A(x_1) B(x_2)] | 0, -\infty \rangle = \langle 0, \infty | A(x_1) B(x_2) | 0, -\infty \rangle \theta(x_1^0 - x_2^0) \\ + \langle 0, \infty | B(x_2) A(x_1) | 0, -\infty \rangle \theta(x_2^0 - x_1^0)$$

$$\frac{\partial}{\partial x_1^\mu} \langle 0, \infty | T [A(x_1), B(x_2)] | 0, -\infty \rangle = \langle 0, \infty | T [\partial_\mu A(x_1) B(x_2)] | 0, -\infty \rangle \\ + \delta(x_1^0 - x_2^0) \langle 0, \infty | [A(x_1), B(x_2)] | 0, -\infty \rangle$$

CAUTION

ALWAYS USE PROPER HANDLING
OF THE BOARD

DO NOT WRITE ON THE BOARD
WHILE PRESENTING

$$\int \frac{d^3k}{(2\pi)^3} E_k e^{i\mathbf{k}\cdot\mathbf{r}} \quad E_k = \sqrt{k^2 + m^2} \geq 0$$

$$\langle 0, \infty | 0, -\infty \rangle = N^2 \int \mathcal{D}\phi \, e^{iS(\phi)} \quad \leftarrow \text{incl. i.e. prescr.}$$

$$\langle 0, \infty | T^*[\phi(x_1) \dots \phi(x_n)] | 0, -\infty \rangle = N^2 \int \mathcal{D}\phi \, \phi(x_1) \dots \phi(x_n) e^{iS(\phi)}$$

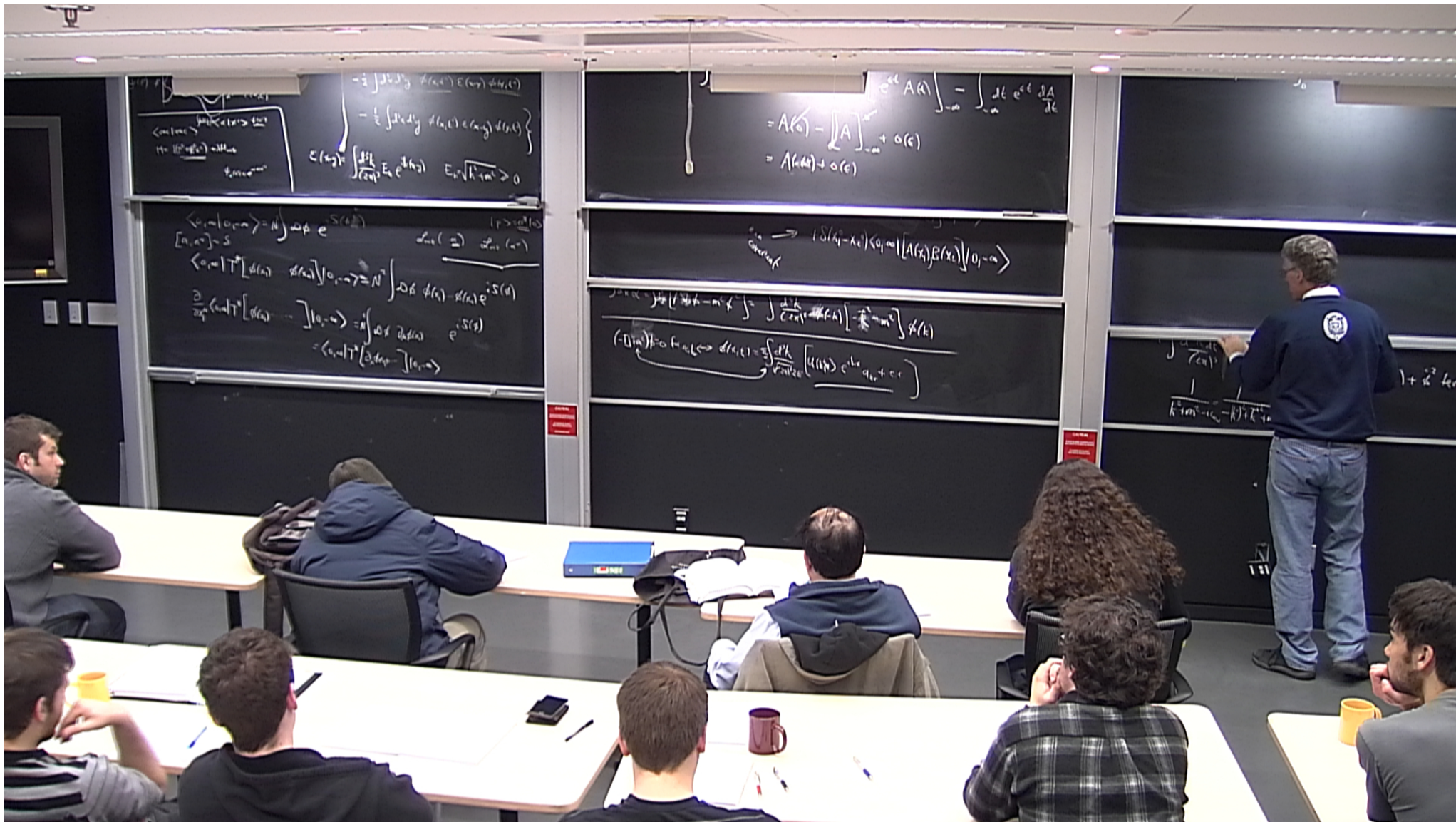
$$\begin{aligned} \langle 0, \infty | T^*[\partial_\mu \phi(x_1) \dots] | 0, -\infty \rangle &= N^2 \int \mathcal{D}\phi \, \partial_\mu \phi(x_1) \dots e^{iS(\phi)} \\ &= \langle 0, \infty | T^*[\partial_\mu \phi(x_1) \dots] | 0, -\infty \rangle \end{aligned}$$

$$\int \frac{d^3k}{(2\pi)^3} E_k e^{i\mathbf{k}\cdot\mathbf{r}} \quad E_k = \sqrt{k^2 + m^2} \geq 0$$

$$\langle 0, \infty | 0, -\infty \rangle = N^2 \int \mathcal{D}\phi \, e^{iS(\phi)} \quad \leftarrow \text{incl. i.e. prescr.}$$

$$\langle 0, \infty | T^*[\phi(x_1) \dots \phi(x_n)] | 0, -\infty \rangle = N^2 \int \mathcal{D}\phi \, \phi(x_1) \dots \phi(x_n) e^{iS(\phi)}$$

$$\begin{aligned} \frac{\partial}{\partial x_1^\mu} \langle 0, \infty | T^*[\phi(x_1) \dots] | 0, -\infty \rangle &= N^2 \int \mathcal{D}\phi \, \partial_\mu \phi(x_1) \dots e^{iS(\phi)} \\ &= \langle 0, \infty | T^*[\partial_\mu \phi(x_1) \dots] | 0, -\infty \rangle \end{aligned}$$



General Properties of correlation functions.

General Properties of correlation functions.

Reminder: Schrodinger Picture
Heisenberg " of time evolution in QM.
Interaction "

Schrodinger picture:

Operators are t -indep but
states are t -dep.

$$|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$$

$$\langle \psi_1(t) | O_1 \dots O_n | \psi_2(t) \rangle = \langle \psi_1(0) | e^{iHt} O_1 \dots O_n e^{-iHt} | \psi_2(0) \rangle$$

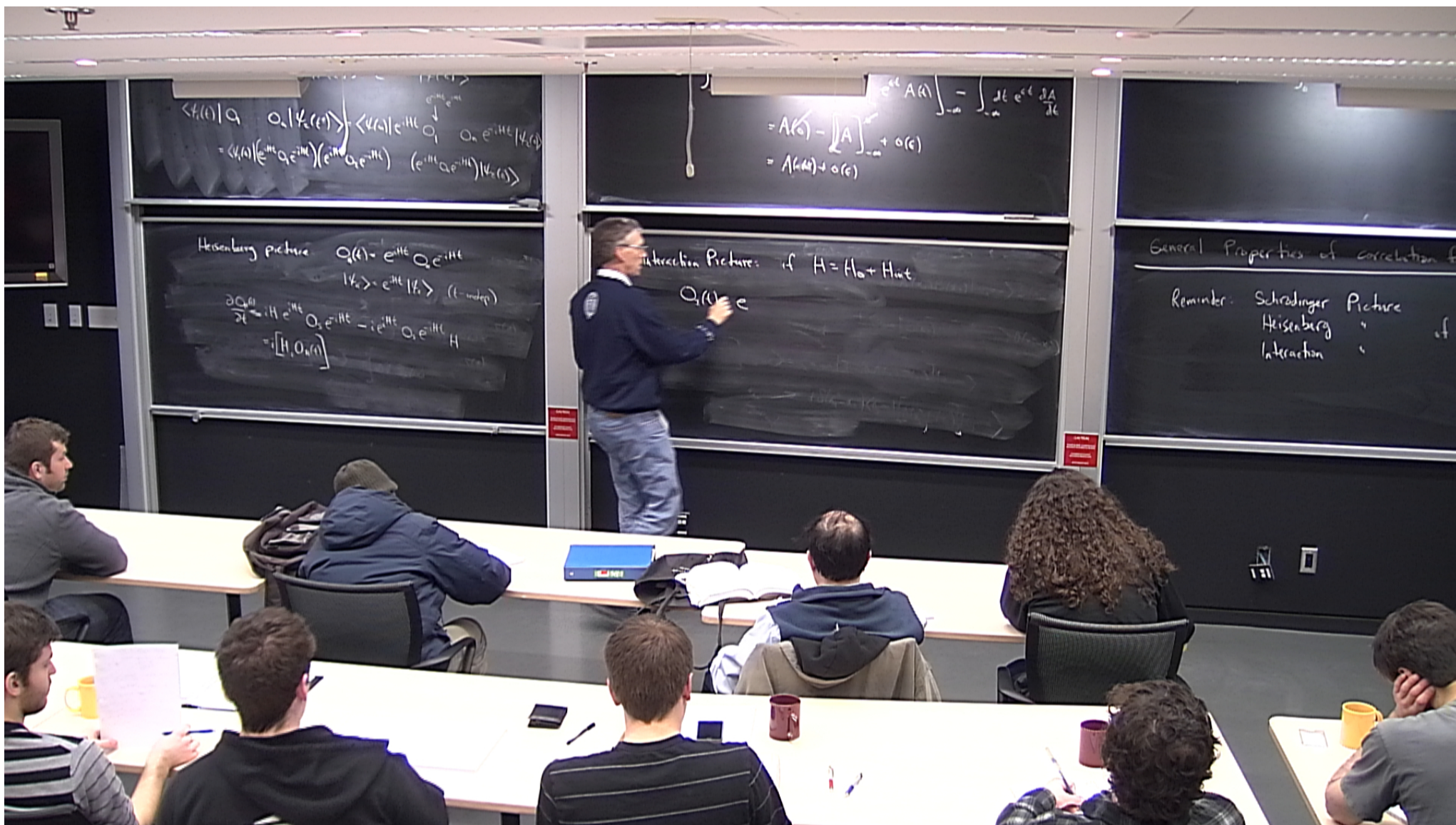
$$\begin{aligned}
 & \textcircled{0} \quad |\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle \\
 & \langle \psi_1 | \dots \textcircled{0}_n |\psi_2(t)\rangle = \langle \psi(0) | e^{iHt} \textcircled{0}_1 \dots \textcircled{0}_n e^{-iHt} |\psi_2(0)\rangle \\
 & \quad \quad \quad \downarrow e^{-iHt} e^{iHt} \\
 & \langle \psi_1(0) | (e^{iHt} \textcircled{0}_1 e^{-iHt}) (e^{iHt} \textcircled{0}_2 e^{-iHt}) \dots (e^{iHt} \textcircled{0}_n e^{-iHt}) |\psi_2(0)\rangle
 \end{aligned}$$

$$= \langle 0, \infty | T^* \left[\partial_\mu \phi(x_1) \dots \right] | 0, -\infty \rangle$$

Heisenberg picture: $O_H(t) = e^{iHt} O_S e^{-iHt}$

$$|\psi_H\rangle = e^{iHt} |\psi_S\rangle \quad (t\text{-indep})$$

$$\begin{aligned} \frac{\partial O_H(t)}{\partial t} &= iH e^{iHt} O_S e^{-iHt} - i e^{iHt} O_S e^{-iHt} H \\ &= i[H, O_H(t)] \end{aligned}$$



Interaction Picture: if $H = H_0 + H_{int}$

$$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$$

$$|\psi_I(t)\rangle = e^{iH_0 t} |\psi_S(t)\rangle$$

$$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$$

$$|\psi_I(t)\rangle = e^{iH_0 t} |\psi_S(t)\rangle$$

$$\frac{\partial O_I(t)}{\partial t} = i [H_0, O_I(t)] \leftarrow \text{free eq. of motion.}$$

Interaction Picture: if $H = H_0 + H_{int}$

$$O_I(t) = e^{iH_0 t} O_S e^{-iH_0 t}$$

$$[\pi(x), \phi(y)] = i\delta(x-y) \quad [a_k, a_q^\dagger] = \delta^3(k-q)$$

$$|\psi_I(t)\rangle = e^{iH_0 t} |\psi_S(t)\rangle$$

$$\frac{\partial O_I(t)}{\partial t} = i[H_0, O_I(t)] \leftarrow \text{free eq of motion.}$$

$$\frac{\partial^2 \phi_I(x,t)}{\partial t^2} = i[H_0, \frac{\partial \phi_I(x,t)}{\partial t}] = i^2 [H_0, [H_0, \phi_I(x,t)]] \Leftrightarrow (\square - m^2)\phi_I = 0.$$

$$\begin{aligned} \langle \psi_1(t) | O_1 \dots O_n | \psi_2(t) \rangle &= \langle \psi_1(0) | e^{iHt} O_1 \dots O_n e^{-iHt} | \psi_2(0) \rangle \\ &= \langle \psi_1(0) | (e^{iHt} O_1 e^{-iHt}) \dots (e^{iHt} O_n e^{-iHt}) | \psi_2(0) \rangle \end{aligned}$$

$e^{-iHt} e^{iHt}$
 $\downarrow$

$$\begin{aligned} \frac{\partial O_i(t)}{\partial t} &= iH e^{iHt} O_i e^{-iHt} - i e^{iHt} O_i e^{-iHt} H \\ &= i[H, O_i(t)] \end{aligned}$$

$|\psi_s(t)\rangle = e^{-iHt} |\psi_s(0)\rangle$ (t-indep)
 H

When we $\phi_I(x,t) = \sum_{\sigma} \int \frac{d^3k}{\sqrt{(2\pi)^3 2E}} \left[u_{k\sigma} e^{ikx} a_{k\sigma} + c.c. \right]$

$$[a_{k\sigma}, a_{q\sigma'}^\dagger] = \delta_{\sigma\sigma'} \delta^3(k-q)$$

Assumed ϕ_I satisfies free eq of motion.

The S-matrix in perturbation

$$\langle \beta, \text{out} | \alpha, \text{in} \rangle =$$

The S-matrix in perturbation theory:

$$\langle \beta, \text{out} | \alpha, \text{in} \rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \langle \beta | T [V(t_1) \dots V(t_n)] | \alpha \rangle$$

where $H = H_0 + V$

$$V(t) = \int d^3x \mathcal{H}_{\text{int}}(\phi_1(x), \dots, \phi_n(x, t))$$

$$V(t) = e^{iH_0 t} V e^{-iH_0 t} = \int d^3x \mathcal{H}_{\text{int}}(\phi_j)$$

The S-matrix in perturbation theory:

$$\langle \beta, \text{out} | \alpha, \text{in} \rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \langle \beta | T [V(t_1) \dots V(t_n)] | \alpha \rangle$$

where $H = H_0 + V$ and $V(t) = e^{iH_0 t} V e^{-iH_0 t}$

$$V = \int d^3x \mathcal{H}_{\text{int}}(\phi_1(x) \dots \phi_n(x))$$

$$V(t) = e^{iH_0 t} V e^{-iH_0 t} = \int d^3x \mathcal{H}_{\text{int}}(\phi_{J_1}(x,t) \dots \phi_{J_n}(x,t))$$

The S-matrix in perturbation theory:

$$|\alpha\rangle = a^\dagger a^\dagger |0\rangle$$

$$\langle \beta, \text{out} | \alpha, \text{in} \rangle = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \langle \beta | T [V(t_1) \dots V(t_n)] | \alpha \rangle$$

where $H = H_0 + V$ and $V(t) = e^{iH_0 t} V e^{-iH_0 t}$

$$V(t) = \int d^3x \mathcal{H}_{\text{int}}(\phi_1(x,t) \dots \phi_n(x,t))$$

$$V(t) = e^{iH_0 t} V e^{-iH_0 t} = \int d^3x \mathcal{H}_{\text{int}}(\phi_1(x,t) \dots \phi_n(x,t))$$

The perturbative expansion of t -ordered correlations have a very similar Feynman expansion:

$$\langle B, \text{out} | T[A_1(x_1) \dots A_n(x_n)] V(t)$$

when we use $\phi_I(x,t) = \sum_{\sigma} \int \frac{d^3k}{\sqrt{(2\pi)^3 2E}} \left[\underline{u_{k\sigma}} e^{ikx} a_{k\sigma} + c.c. \right]$

where $[a_{k\sigma}, a_{q\sigma'}^\dagger] = \delta_{\sigma\sigma'} \delta^3(k-q)$

then we have assumed ϕ_I satisfies free eq of motion,
 ie the int'l rep.