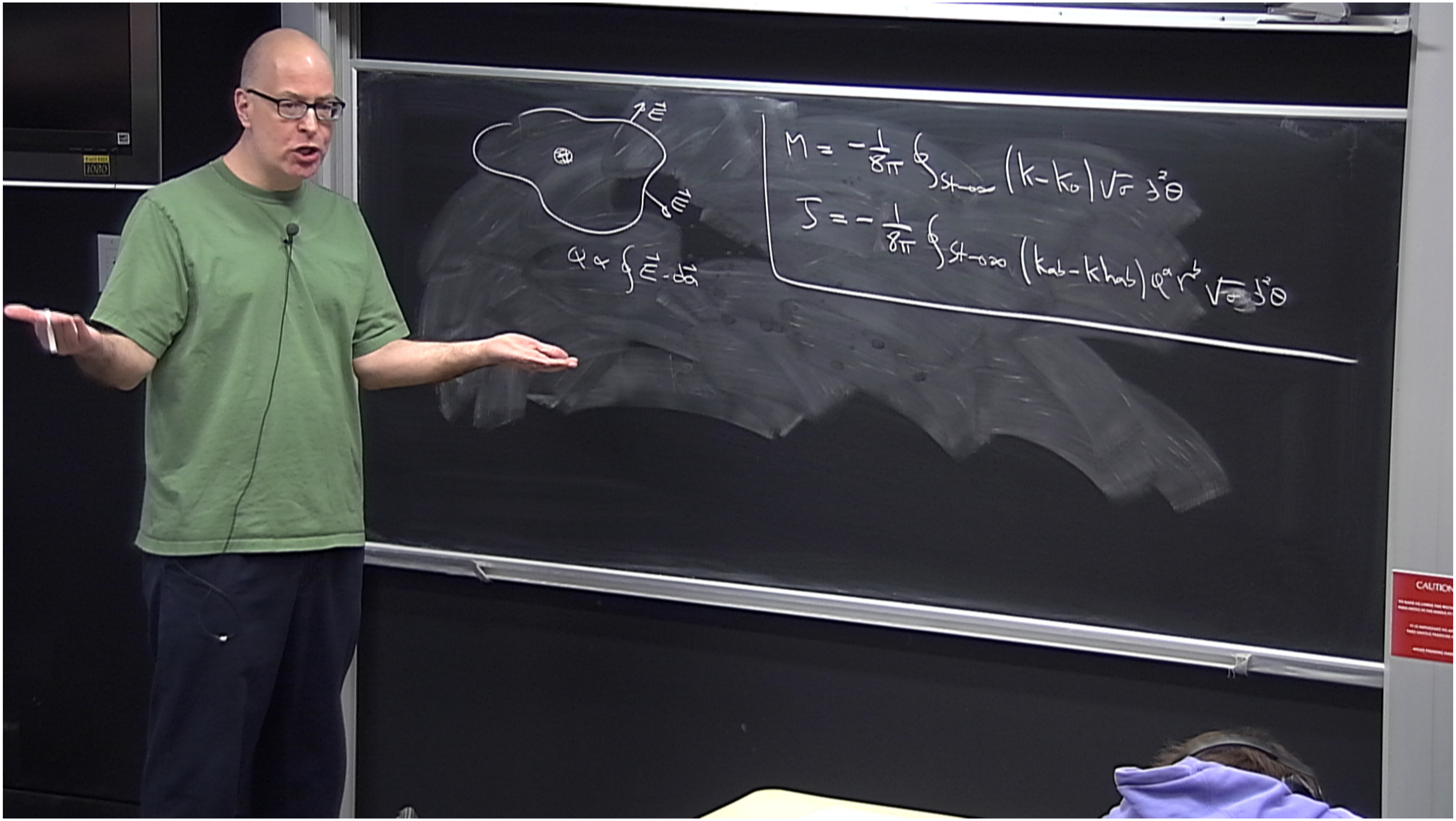
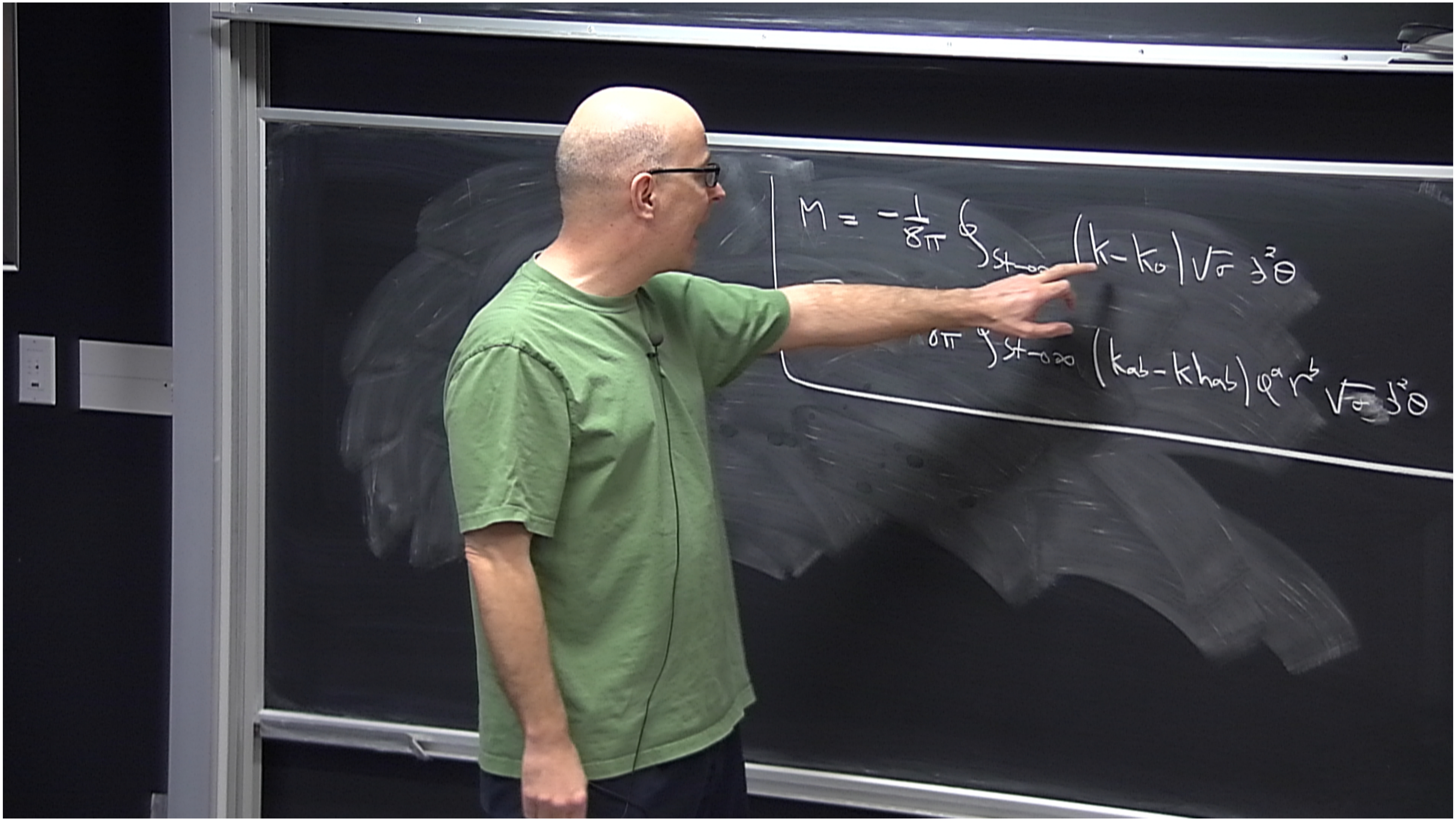
Title: Advanced General Relativity - Lecture 18

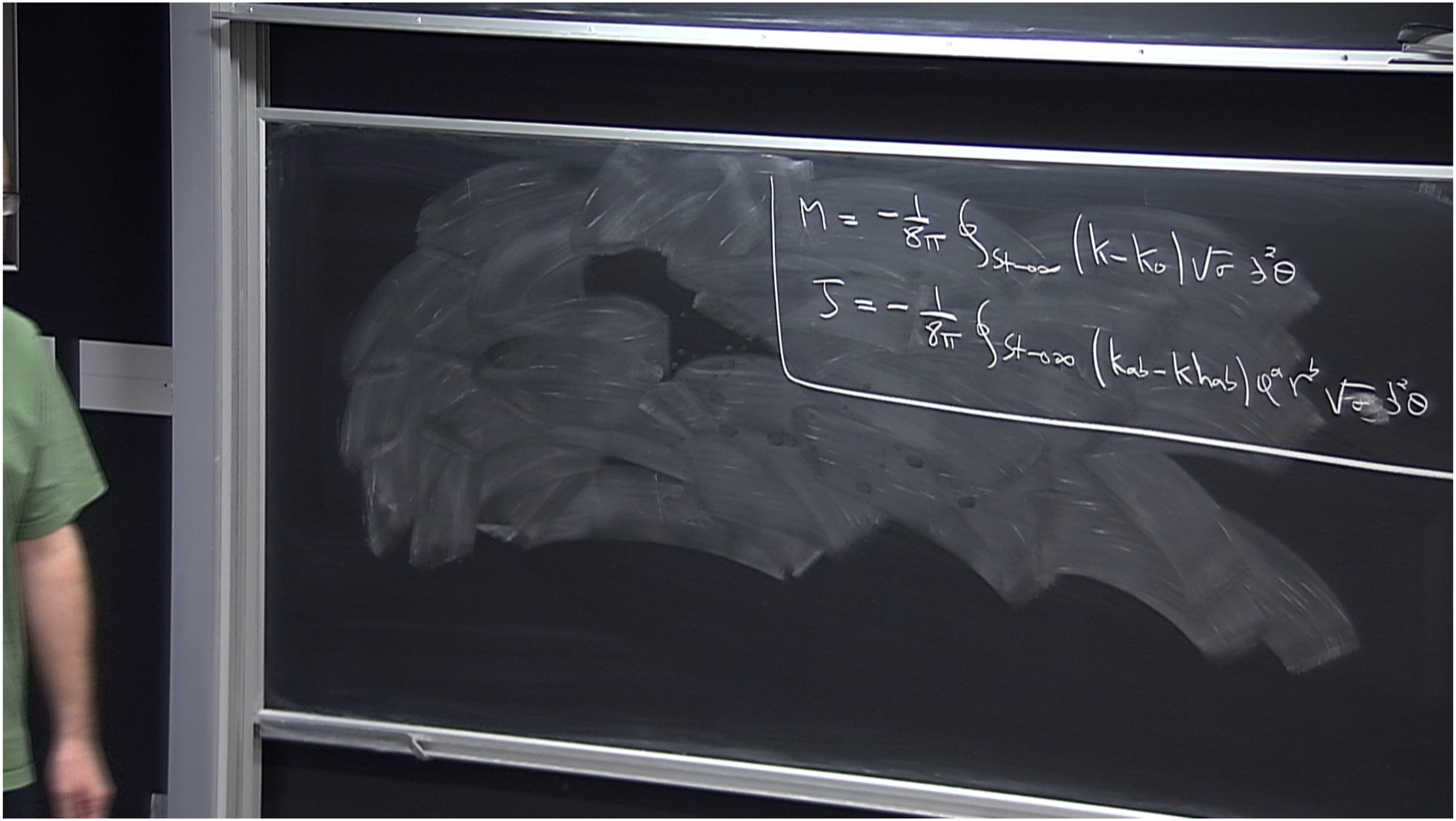
Date: Mar 14, 2012 03:30 PM

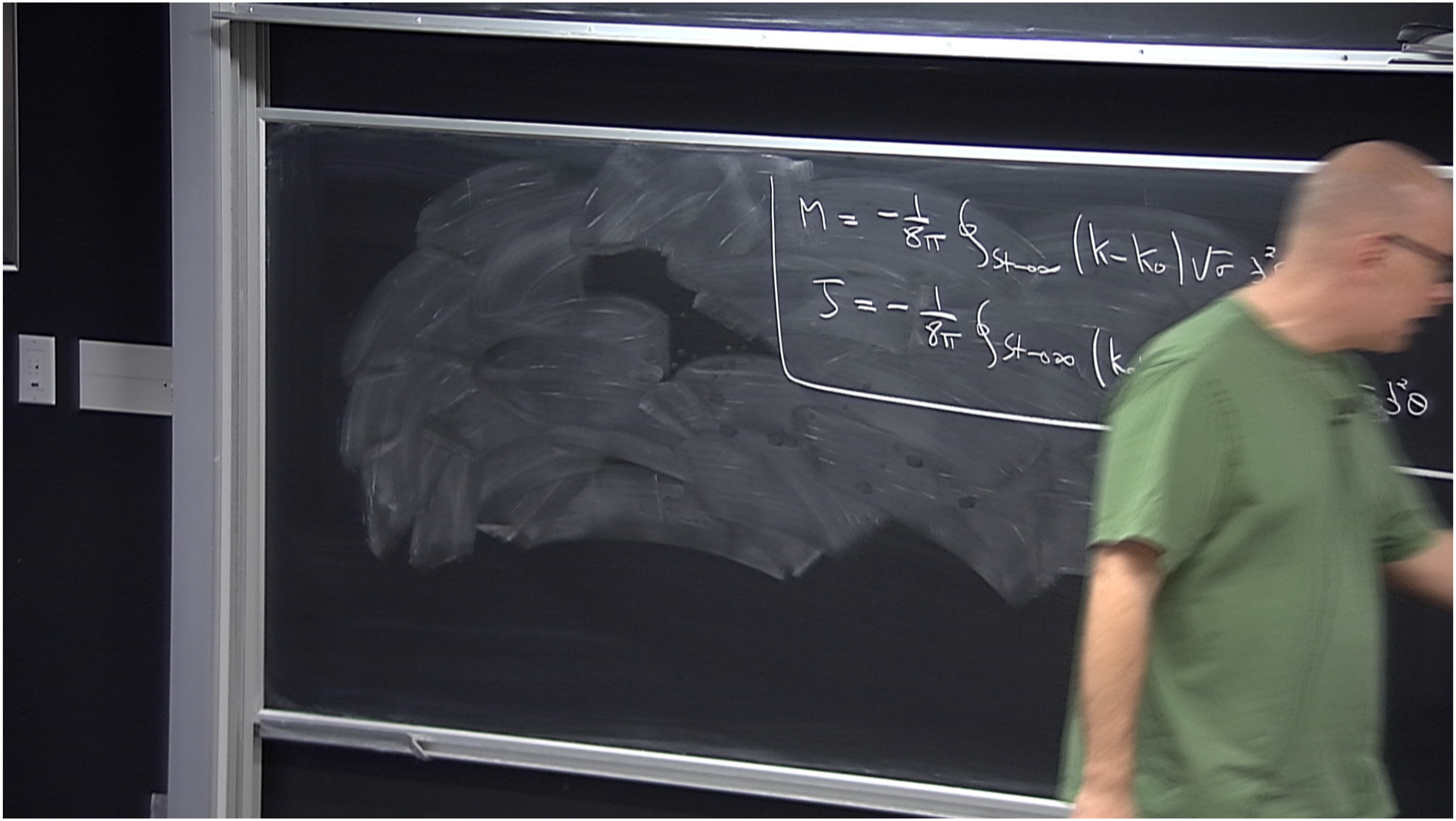
URL: http://pirsa.org/12030073

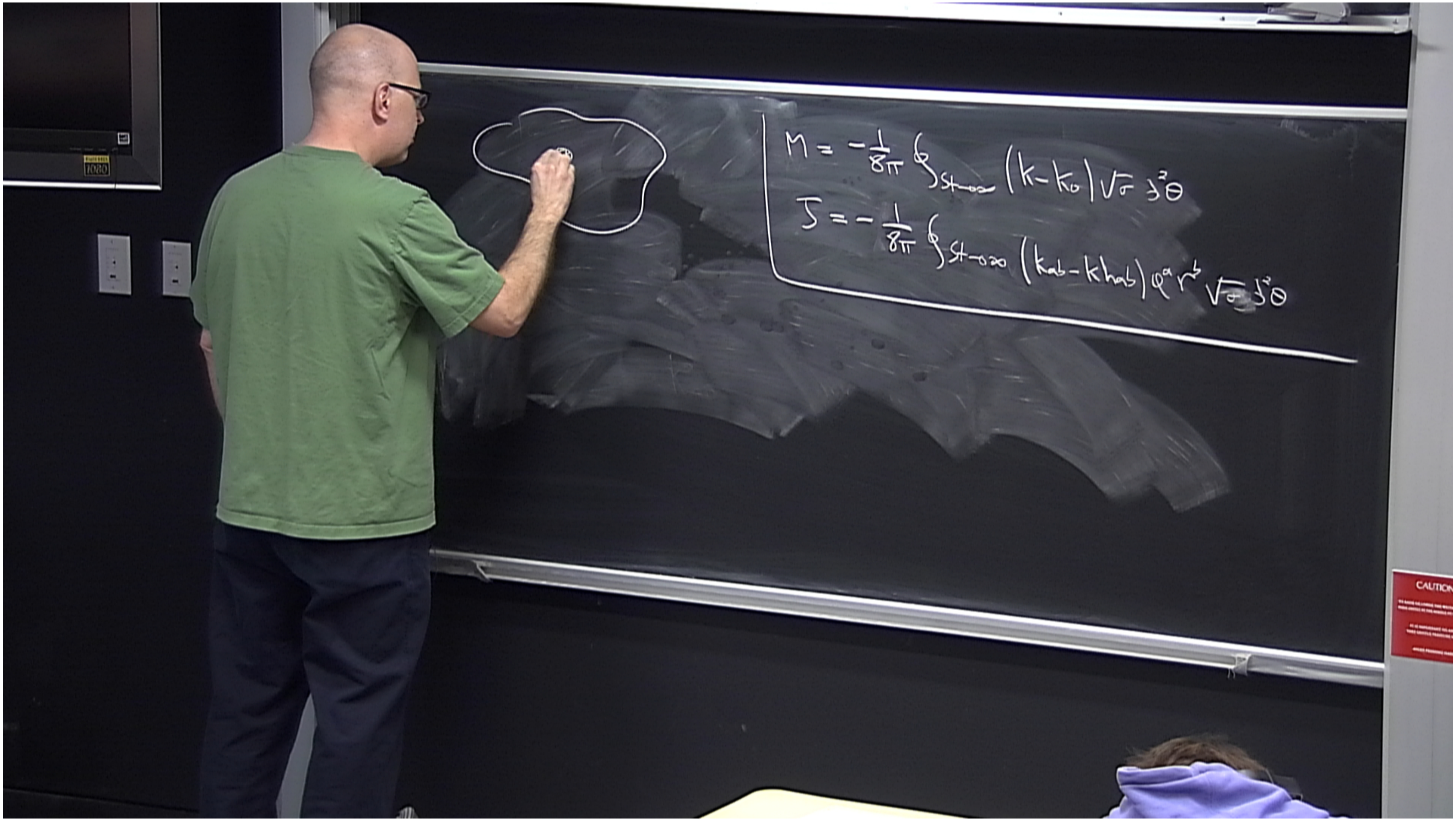
Abstract:


M = - 1/8π ∮_{St→∞} (k - k_0) √σ d²θ
J = - - 1/8π ∮_{St→∞} (k_ab - k h_ab) φ^a r^b √σ d²θ
φ ∝ ∮ E · da
CAUTION


M = − 1/8π ∮St→∞ (k − k0) √σ d²θ
8π ∮St→∞ (kab − khab) φa rb √σ d²θ


$M = -\frac{1}{8\pi} \oint_{S_t \to \infty} (k - k_0) \sqrt{\sigma}\, d^2\theta$
$J = -\frac{1}{8\pi} \oint_{S_t \to \infty} (k_{ab} - k h_{ab}) \varphi^a r^b \sqrt{\sigma}\, d^2\theta$


M = −1/8π ∮_{St→∞} (k − k₀) √σ d²
J = − 1/8π ∮_{St→∞} (k


$M = -\frac{1}{8\pi} \oint_{S_{t\to\infty}} (k - k_0) \sqrt{\sigma}\, d^2\theta$
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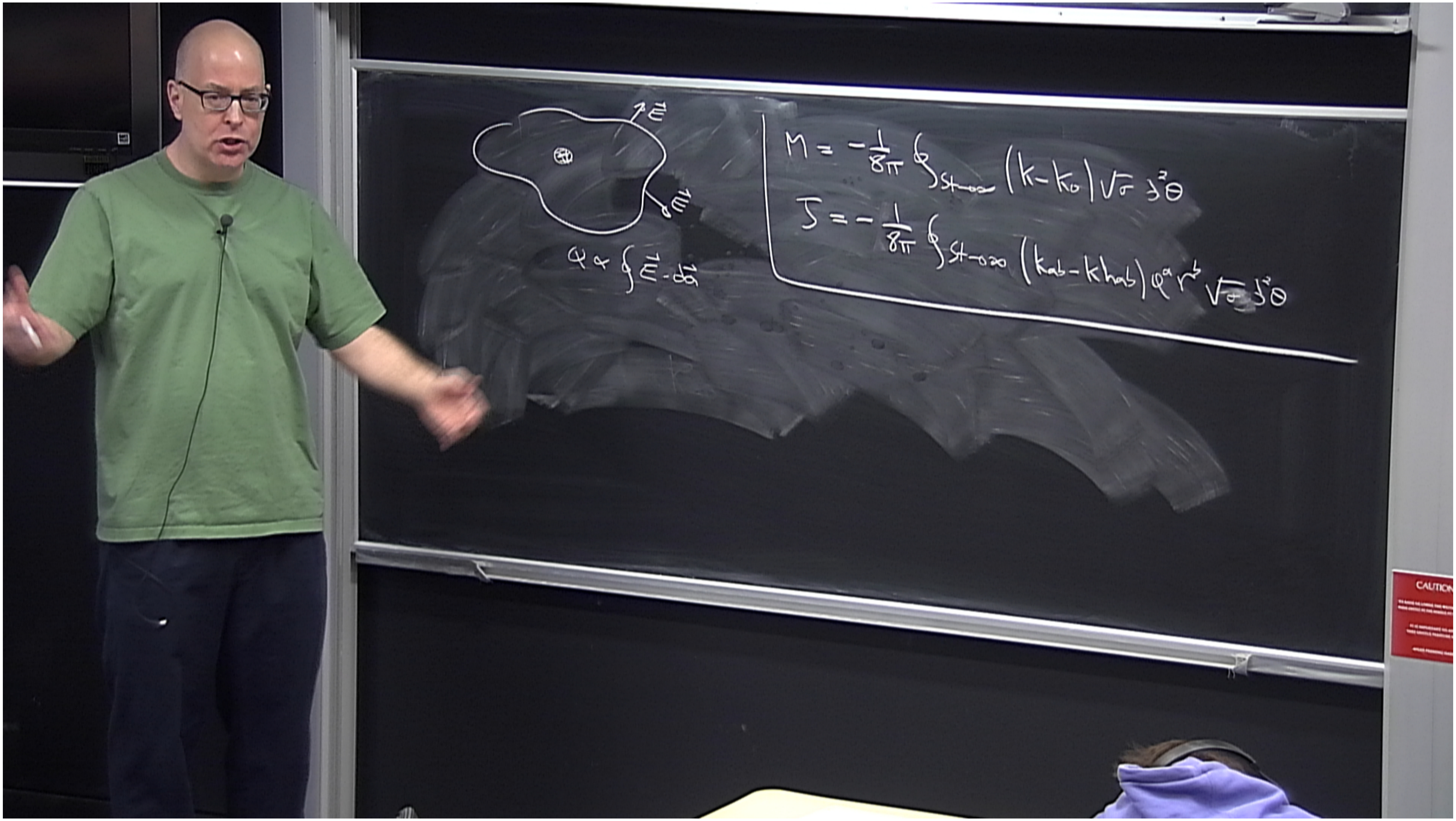

M = - 1/8π ∮ St→∞ (k - k0) √σ d²θ
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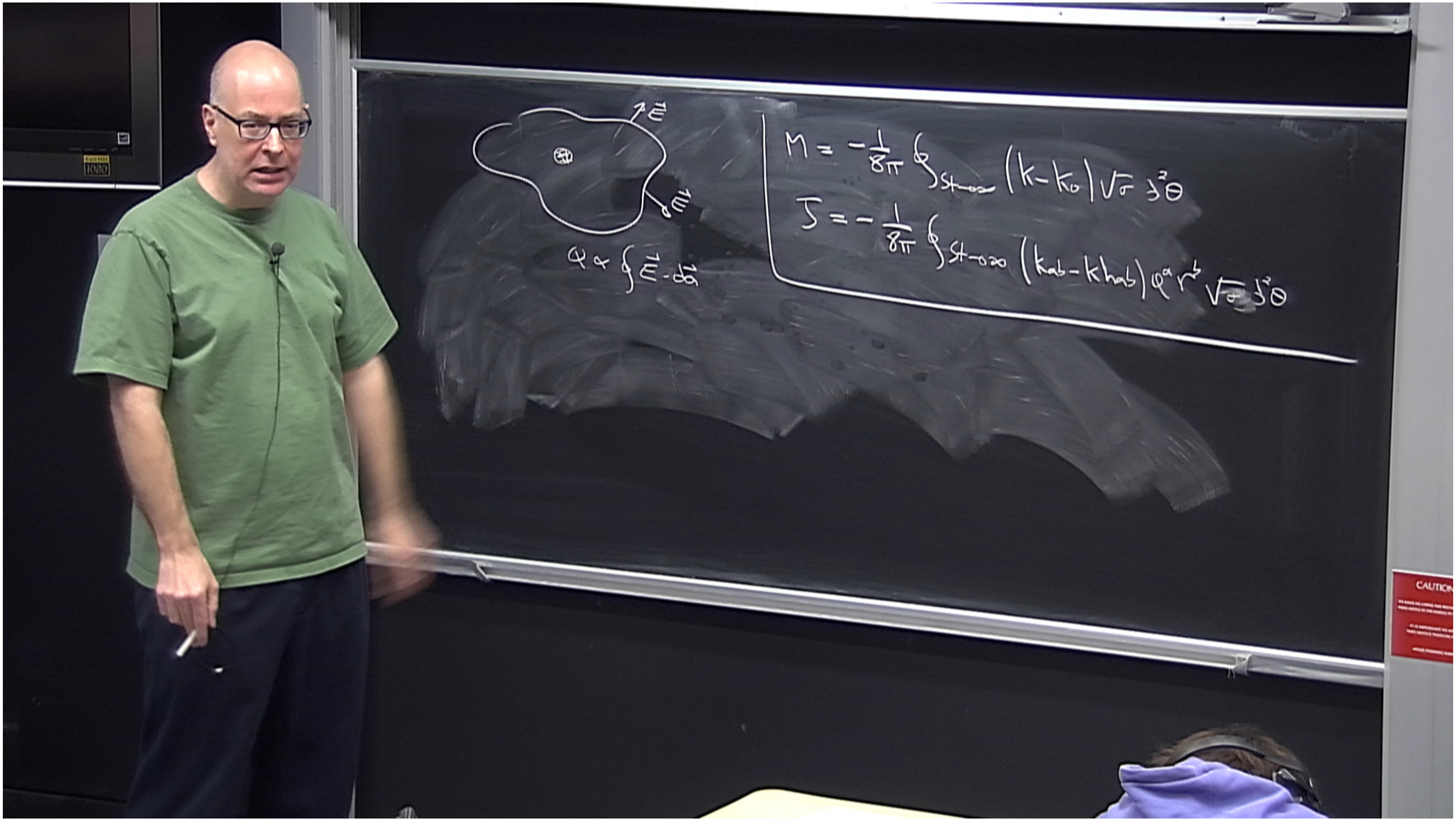
Φ ∝ ∮ E · da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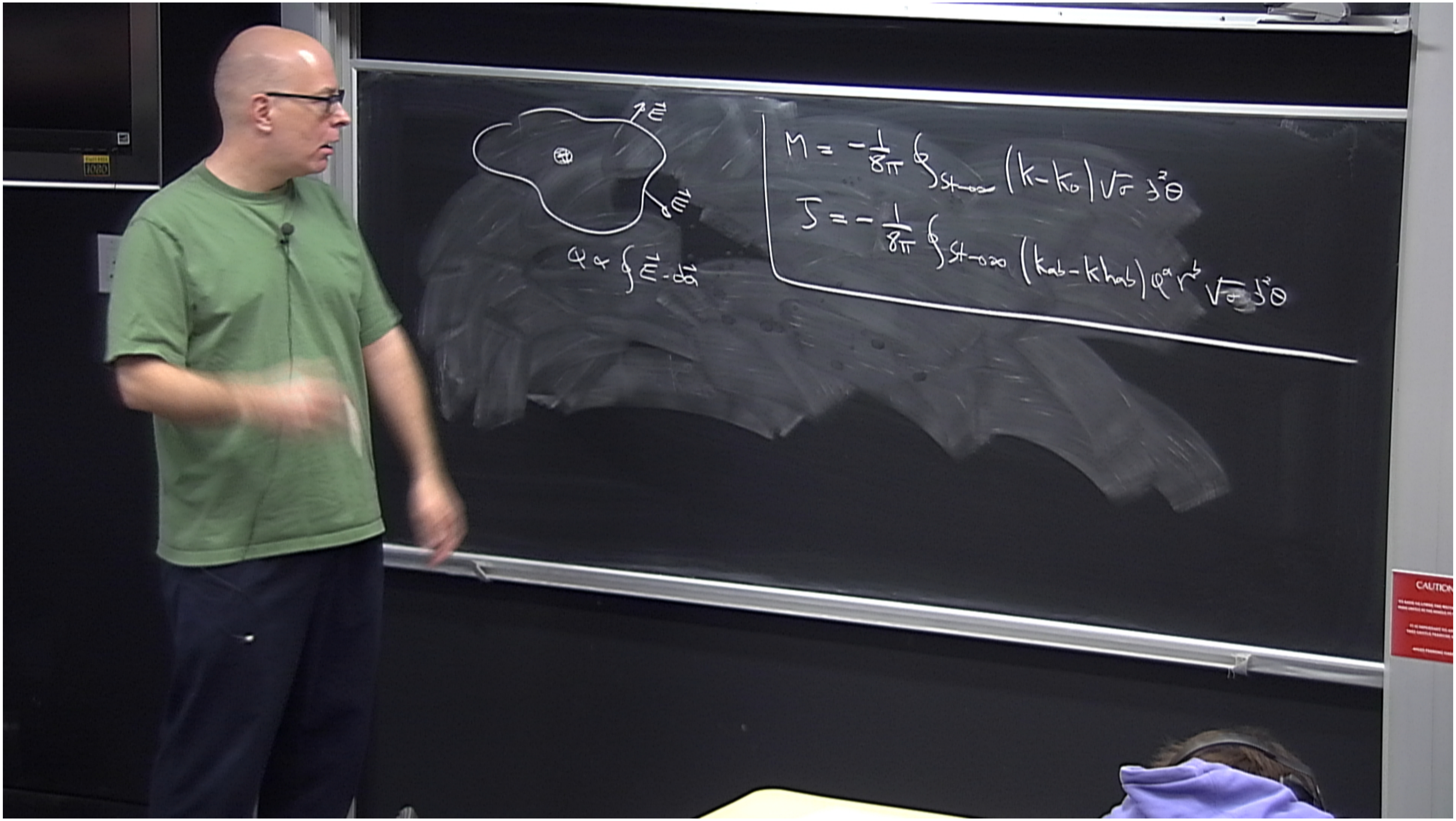
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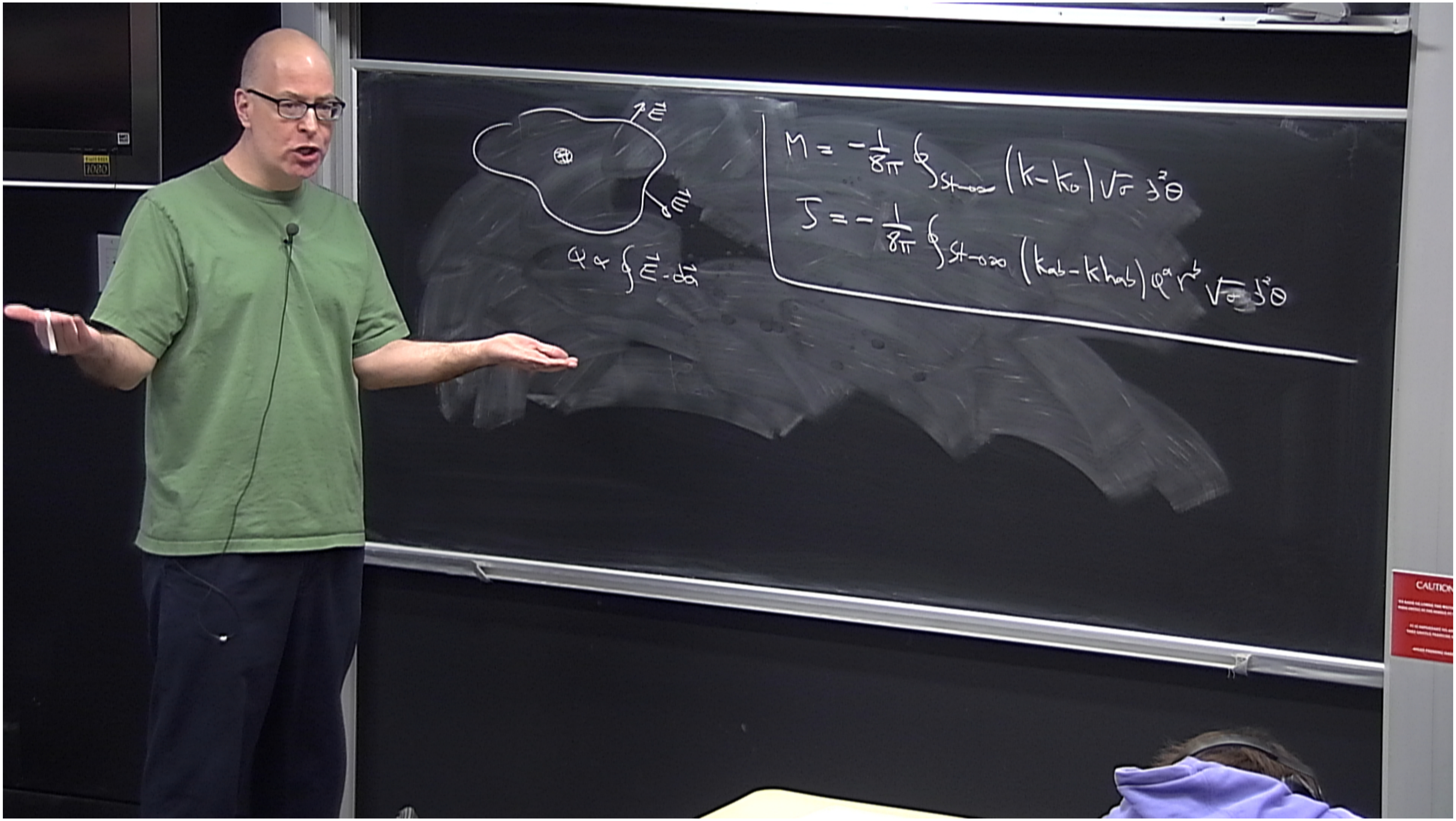
$\vec{E}$
$\vec{E}$
$\Phi \propto \oint \vec{E} \cdot d\vec{a}$
$M = -\frac{1}{8\pi} \oint_{St\to\infty} (k - k_0) \sqrt{\sigma}\, d^2\theta$
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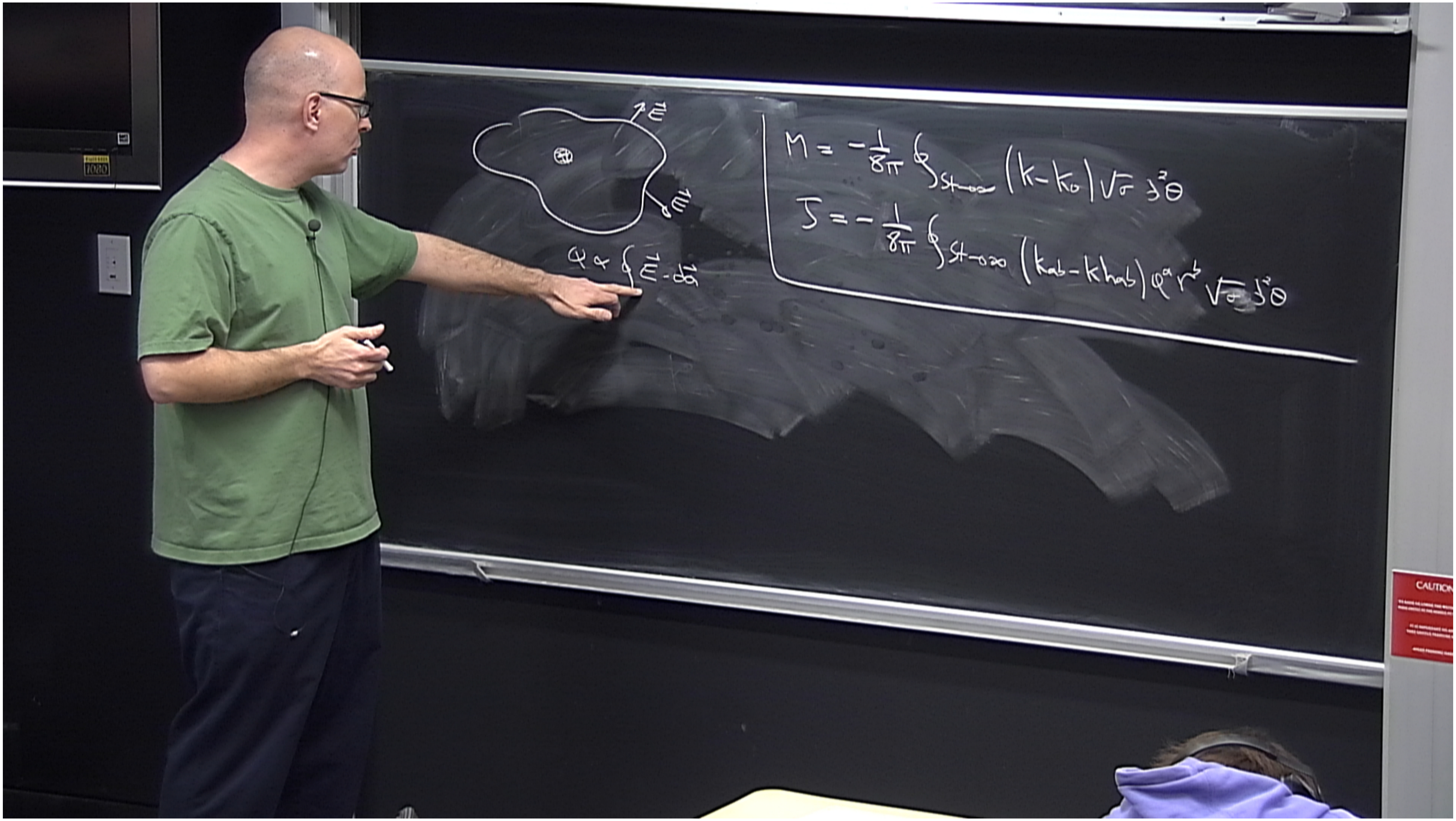
E
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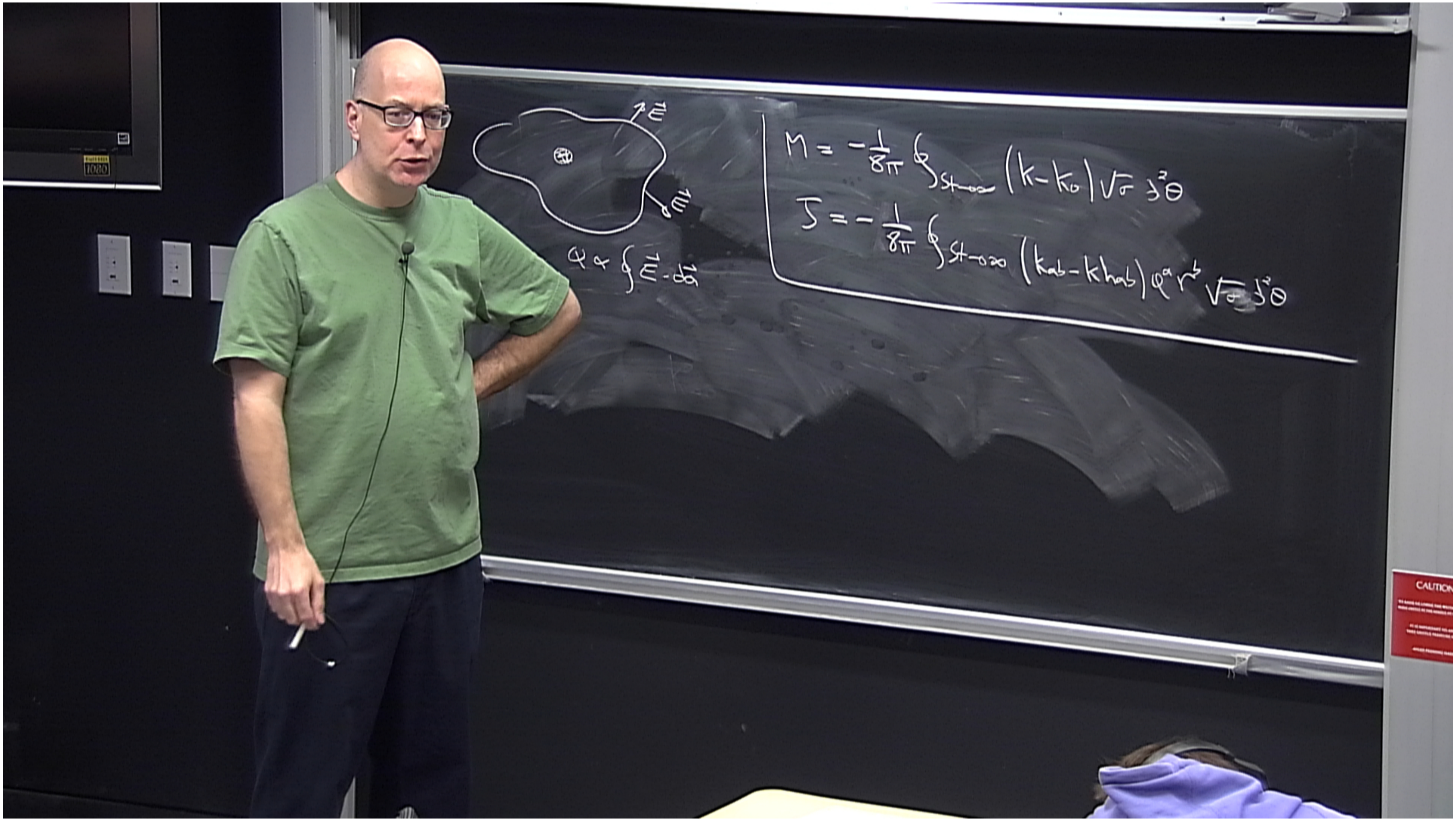
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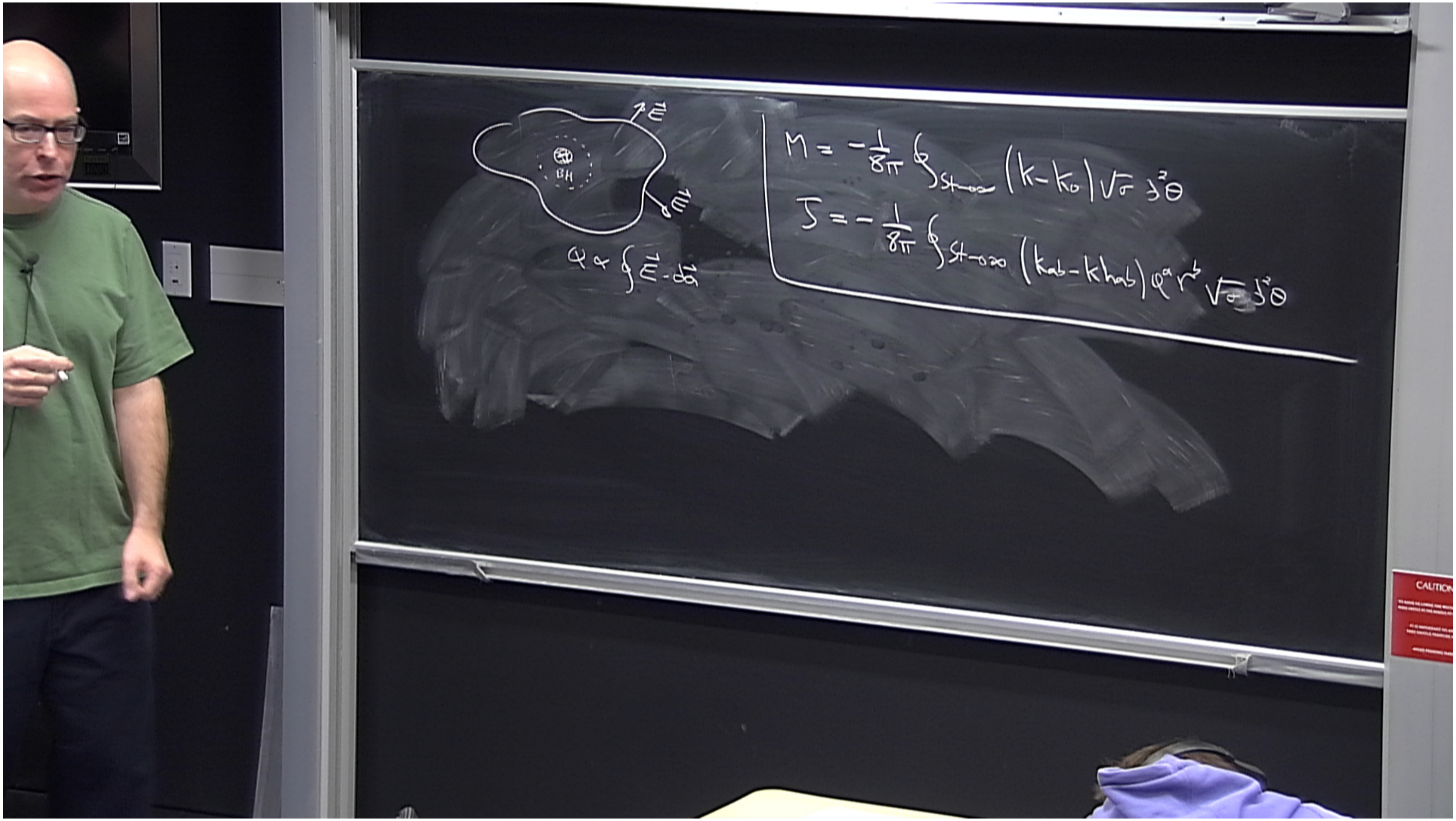

BH
$\vec{E}$
$\vec{E}$
$\Phi \propto \oint \vec{E} \cdot d\vec{a}$
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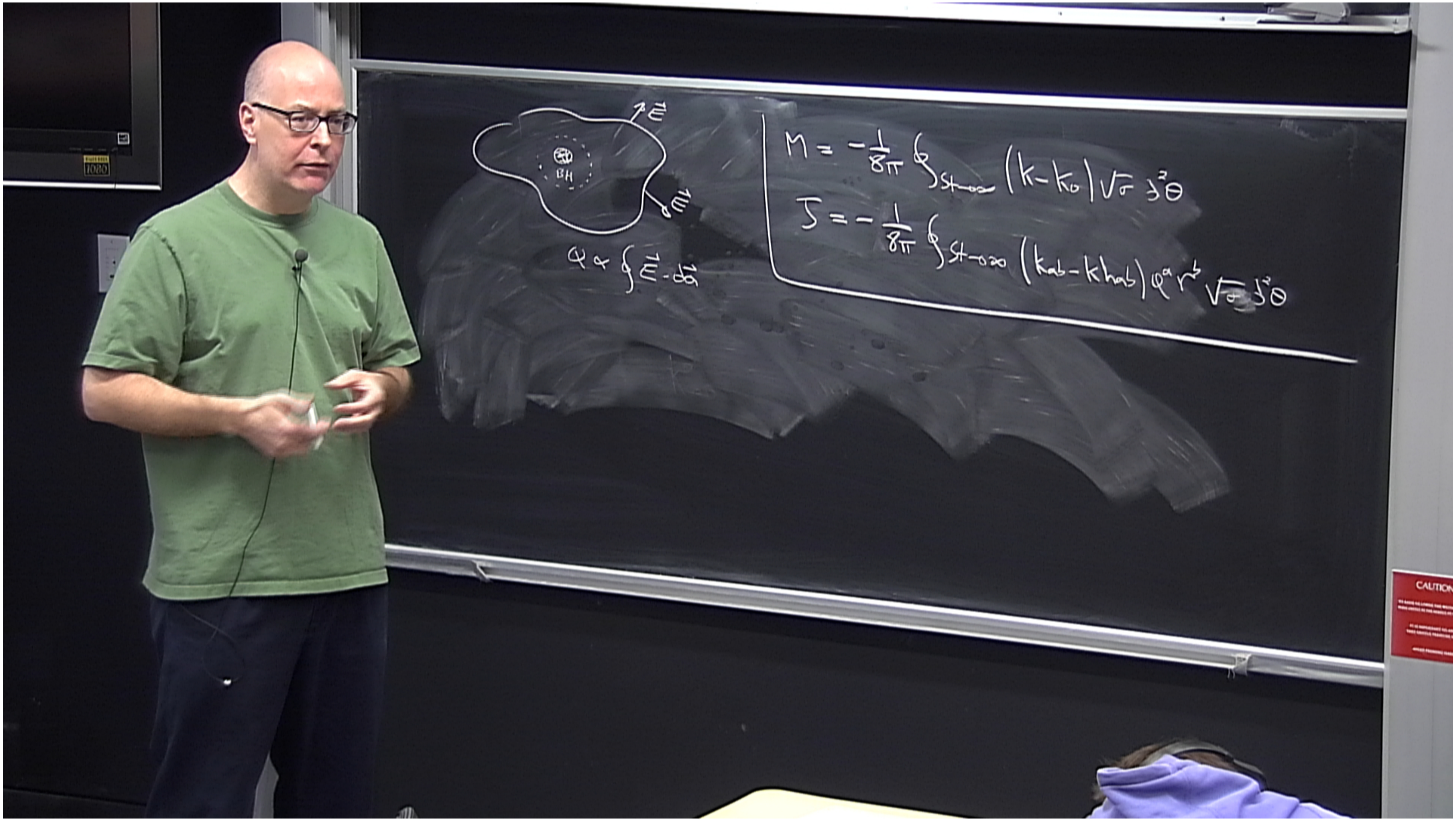
BH
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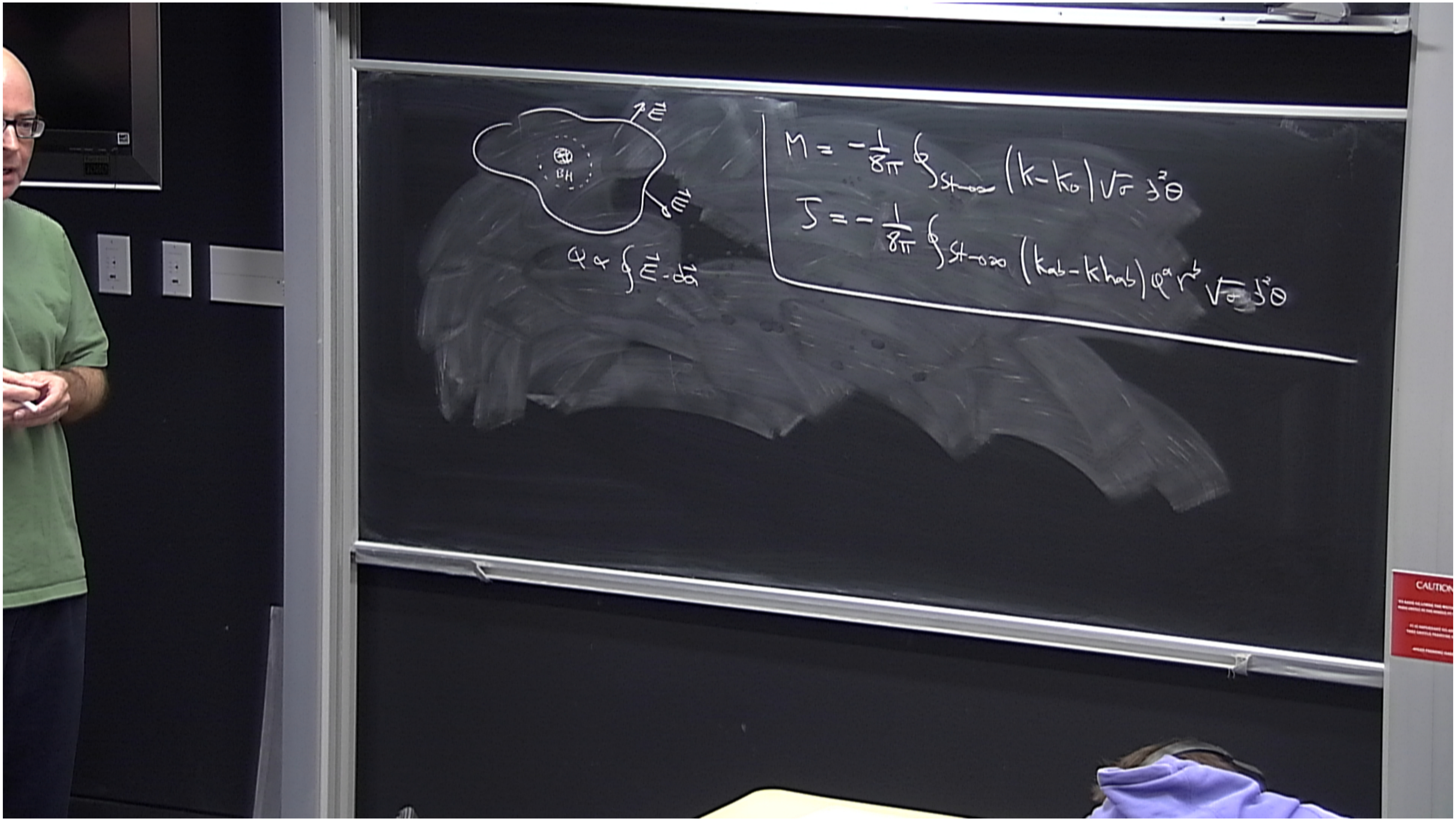
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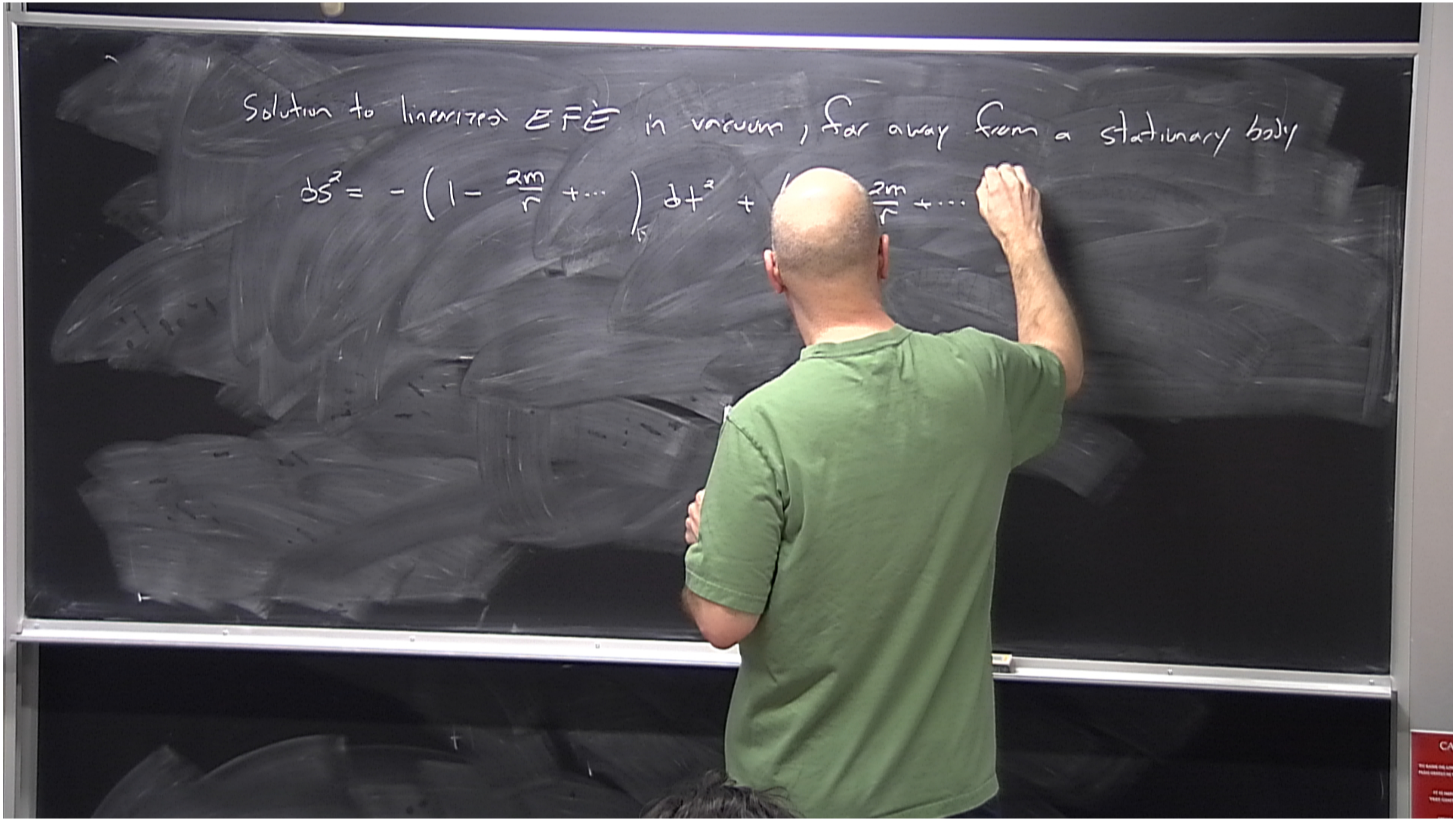
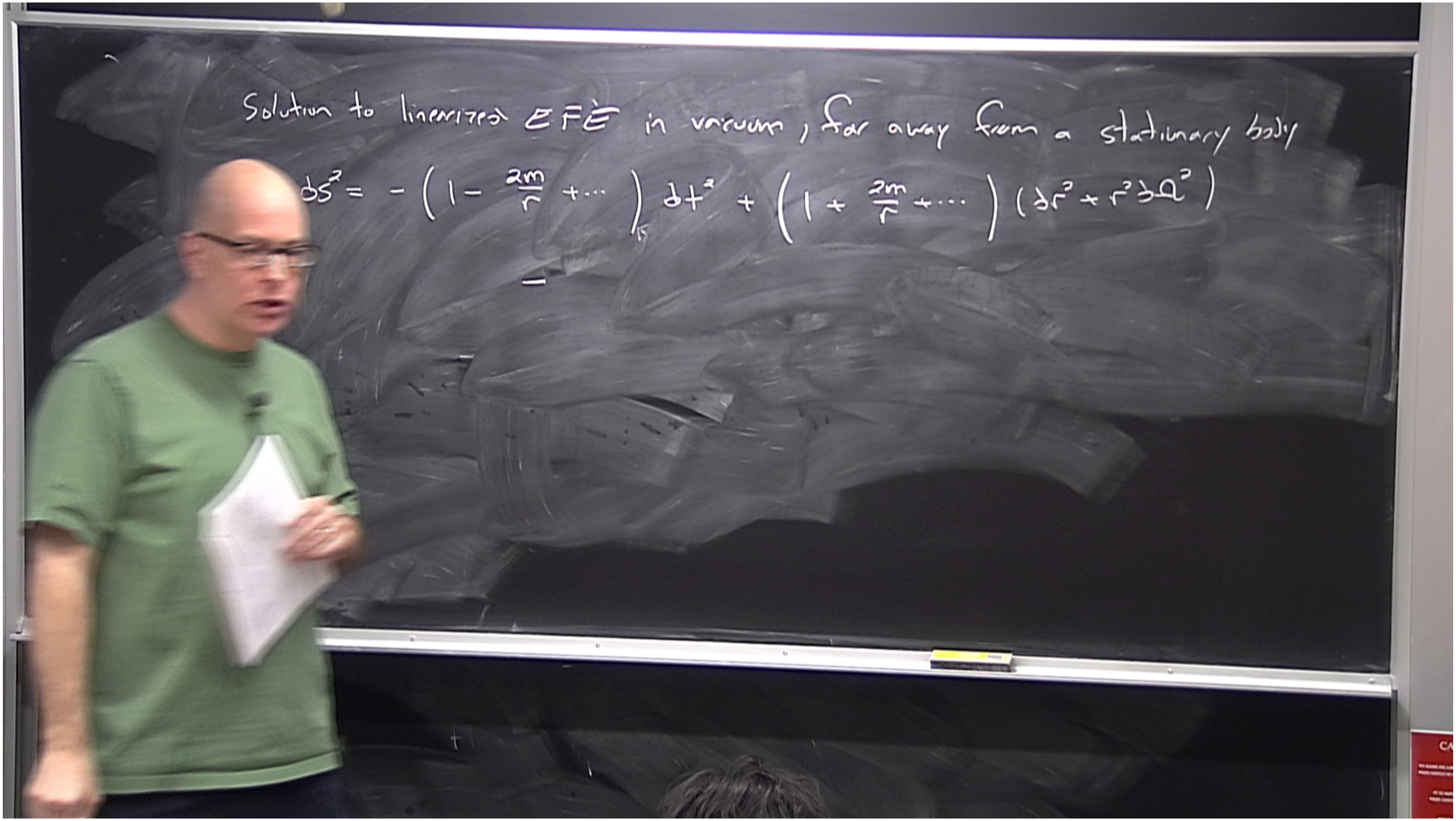

Solution to linearized EFE in vacuum, far away from a stationary body
$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \quad \frac{2m}{r} + \cdots$


Solution to linearized EFE in vacuum, far away from a stationary body
ds² = − (1 − 2m/r + ···) dt² + (1 + 2m/r + ···) (dr² + r² dΩ²)

Solution to linearized EFE in vacuum, far away from a stationary body

$$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right)dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)\left(dr^2 + r^2 d\Omega^2\right)$$

$$- \left(\frac{4j\sin^2\theta}{r} + \cdots\right)dt\,d\varphi$$

Solution to linearized EFE in vacuum, far away from a stationary body

$$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)\left(dr^2 + r^2 d\Omega^2\right)$$

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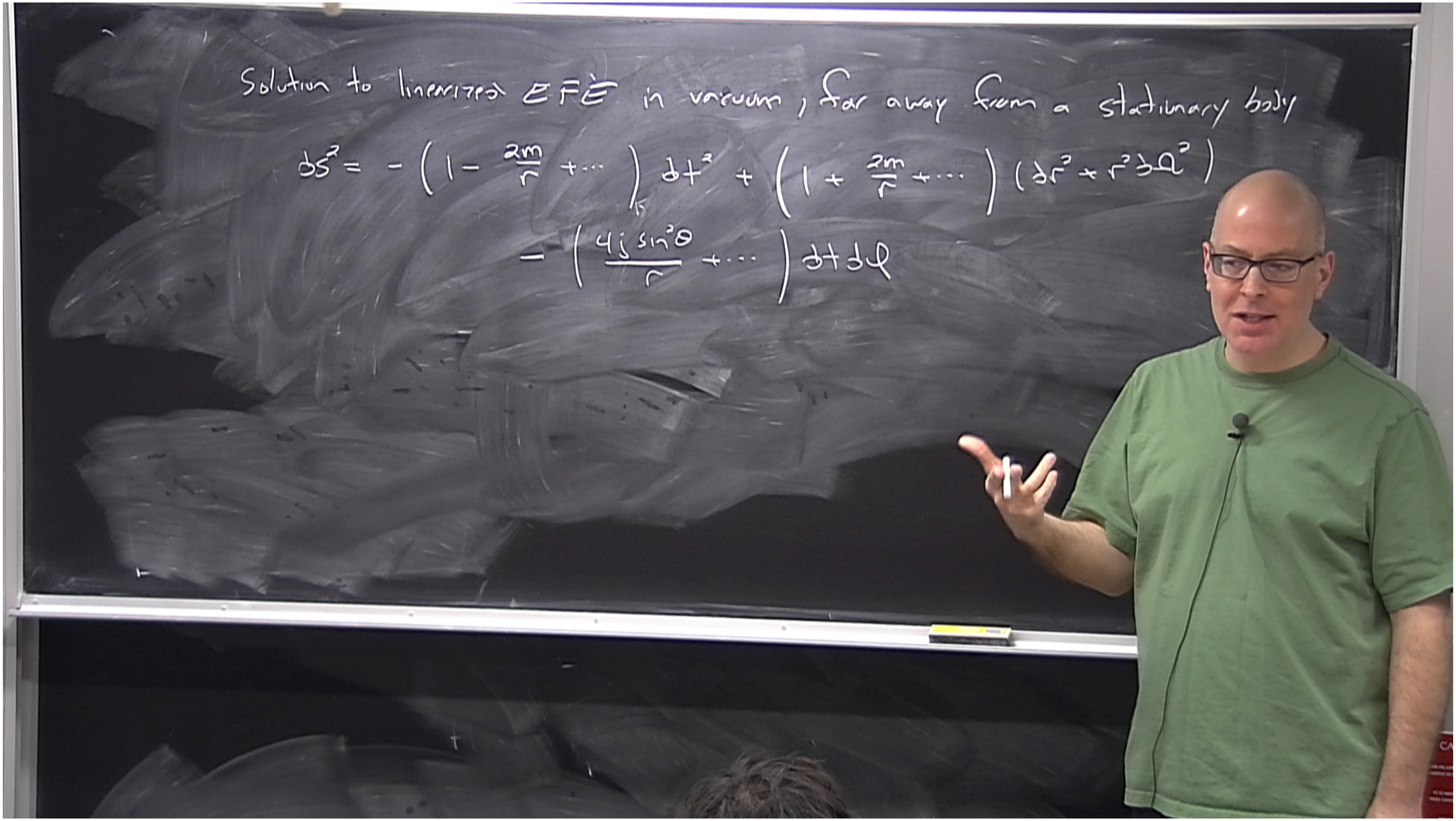
Solution to linearized EFE in vacuum, far away from a stationary body
$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)(dr^2 + r^2 d\Omega^2)$
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Solution to linearized EFE in vacuum, far away from a stationary body

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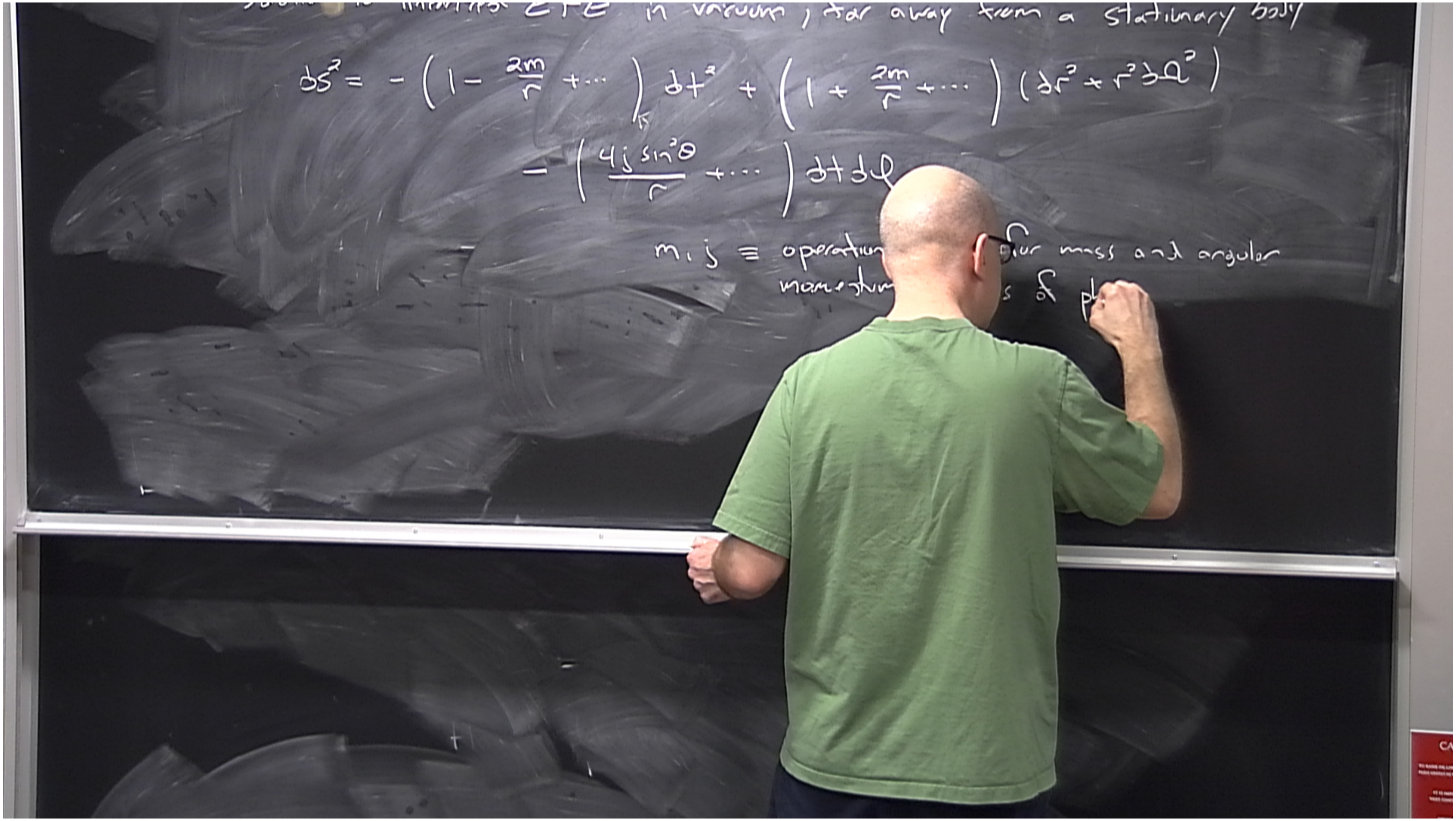
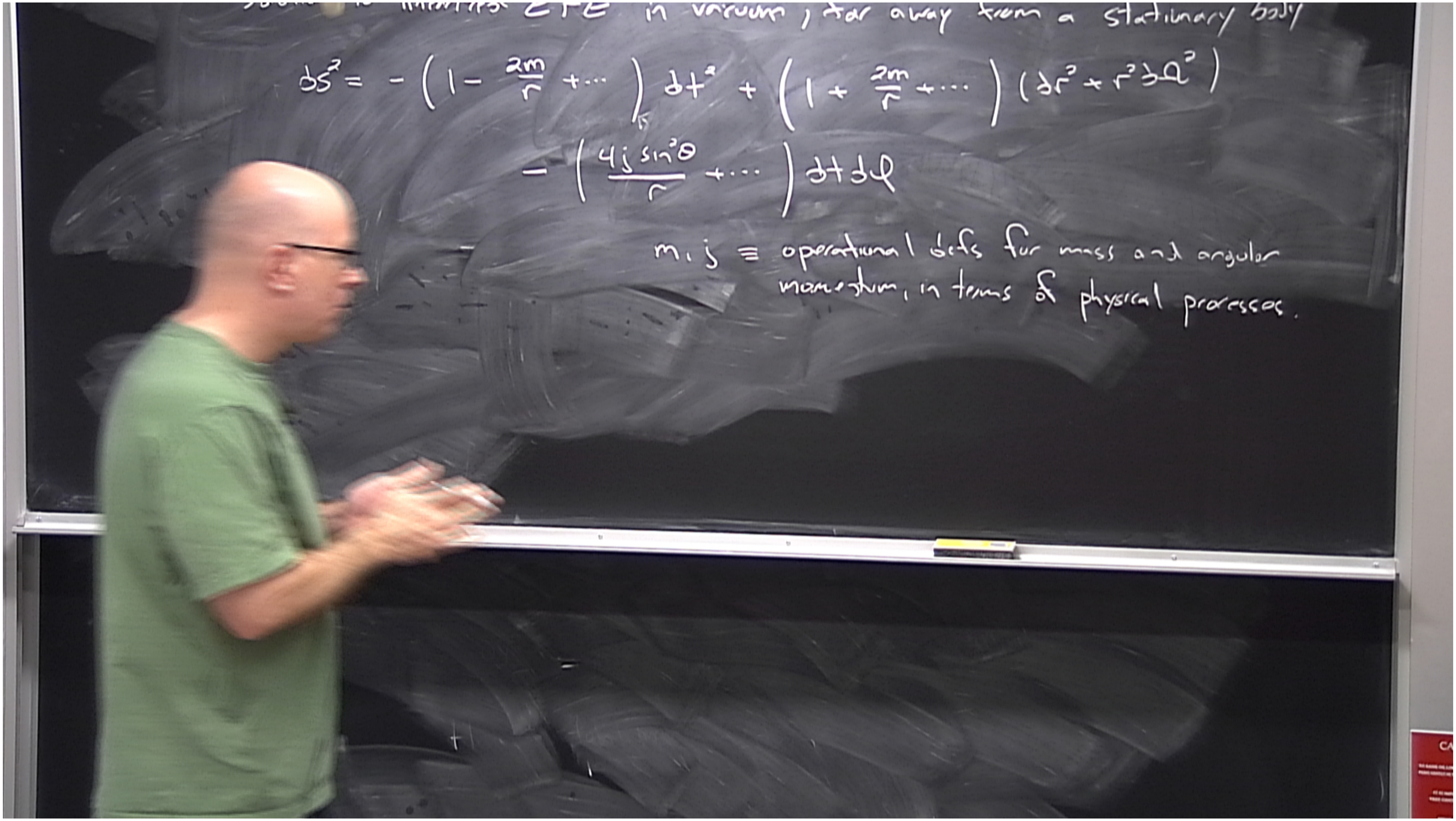

in vacuum, far away from a stationary body
$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)(dr^2 + r^2 d\Omega^2)$
$- \left(\frac{4j \sin^2\theta}{r} + \cdots\right) dt\, d\varphi$
m, j ≡ operation
mass and angular
momentum


in vacuum, far away from a stationary body
$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)(dr^2 + r^2 d\Omega^2)$
$- \left(\frac{4 j \sin^2\theta}{r} + \cdots\right) dt\, d\phi$
m, j ≡ operational defs for mass and angular momentum, in terms of physical processes.

Solution to linearized EFE in vacuum, far away from a stationary body

$$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right)dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)\left(dr^2 + r^2 d\Omega^2\right)$$

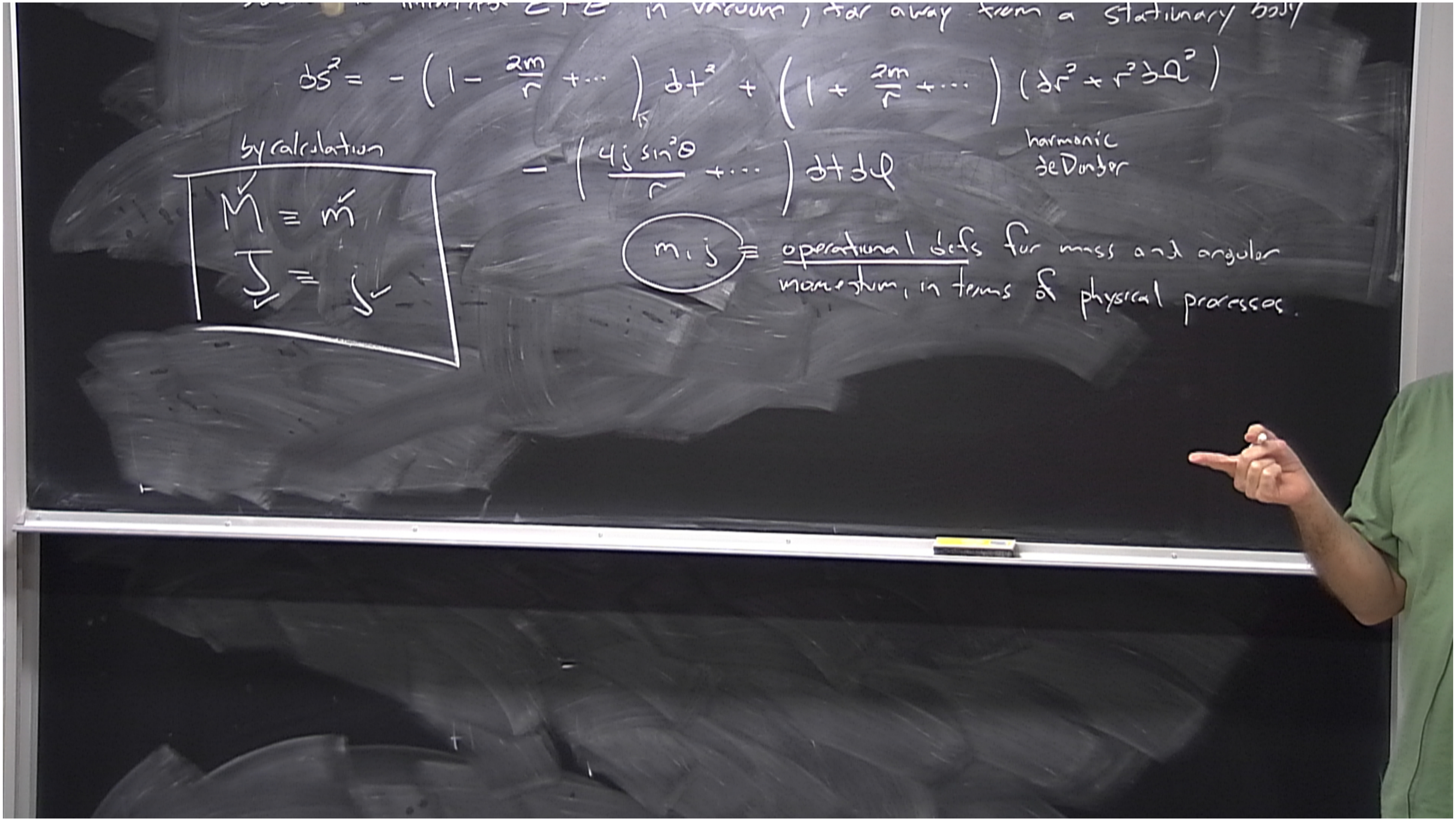
$$- \left(\frac{4j\sin^2\theta}{r} + \cdots\right)dt\,d\varphi$$

by calculation

$$M = m$$

$$J = j$$

$m, j \equiv$ operational defs for mass and angular momentum, in terms of physical processes.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$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right) dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)(dr^2 + r^2 d\Omega^2)$
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harmonic
deDonder
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$M \equiv m$
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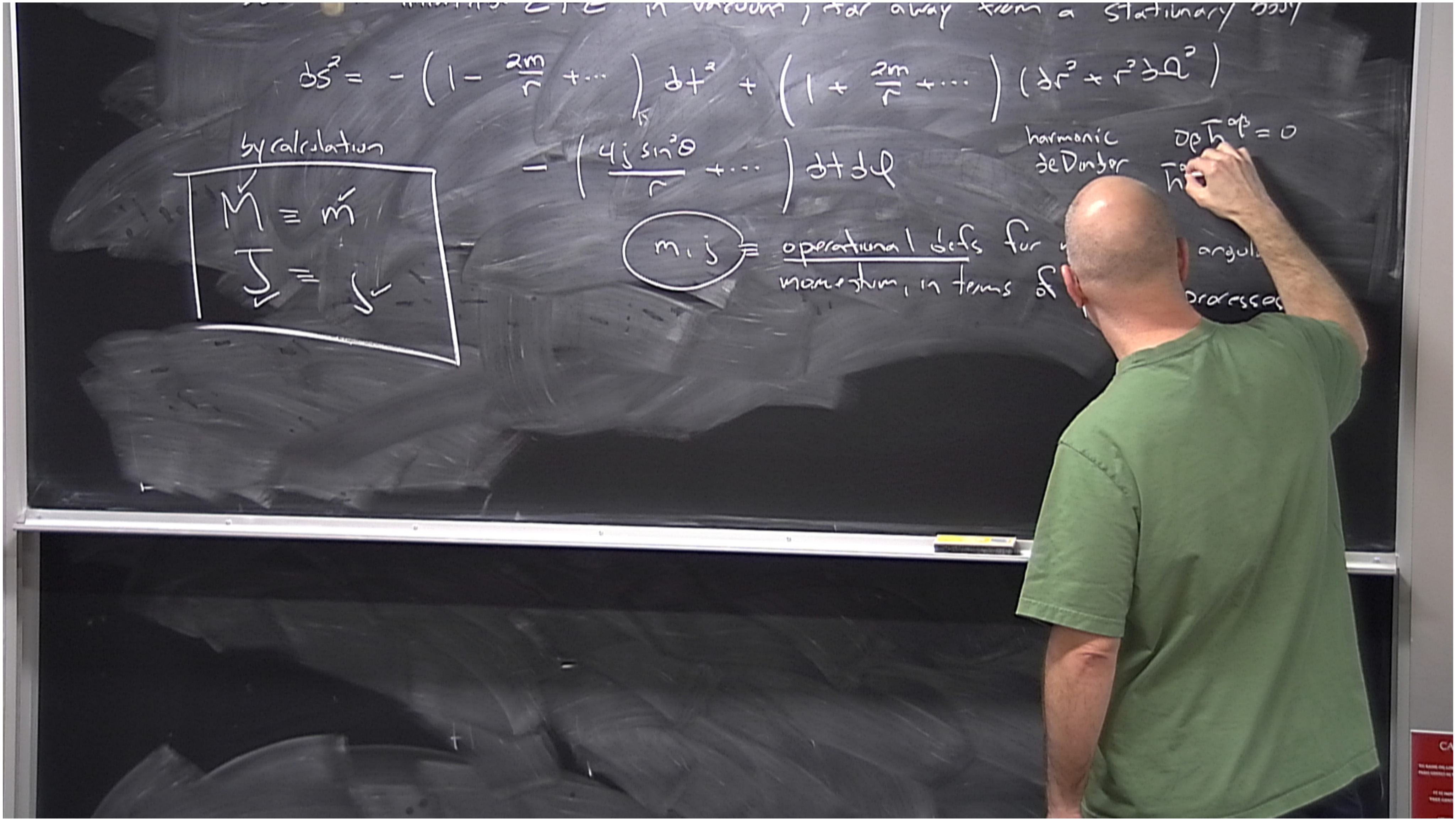
in vacuum, far away from a stationary body
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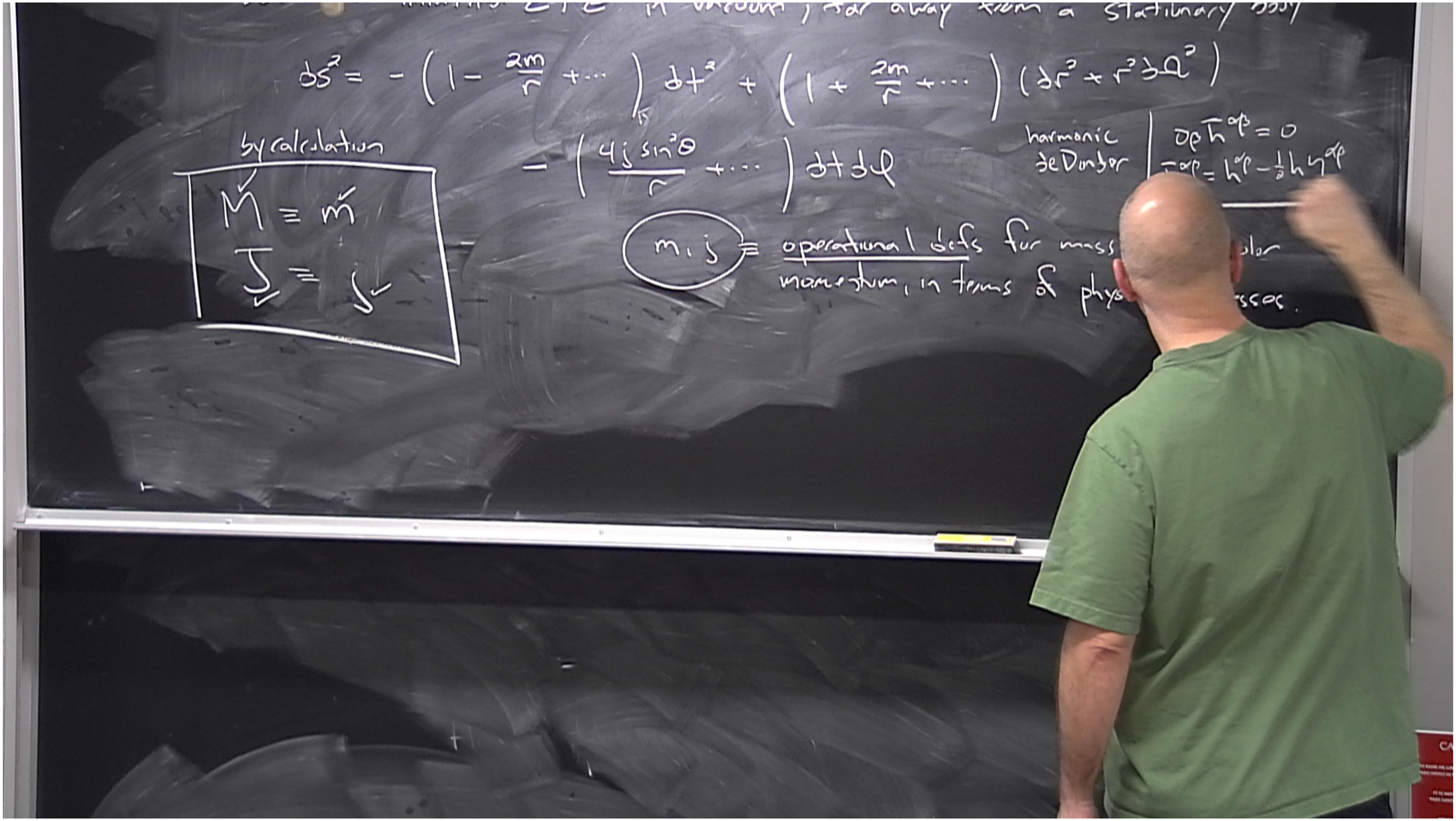
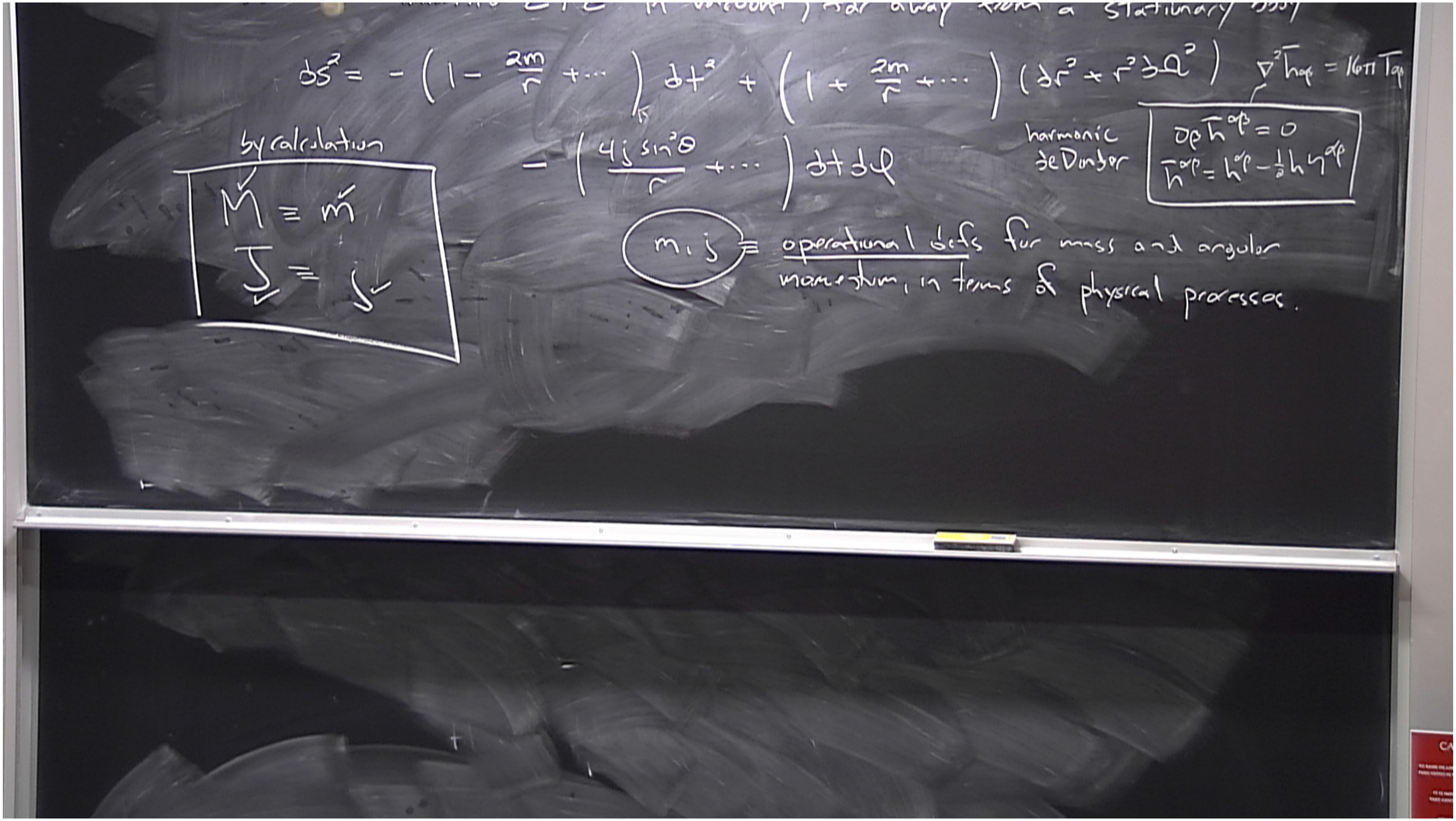
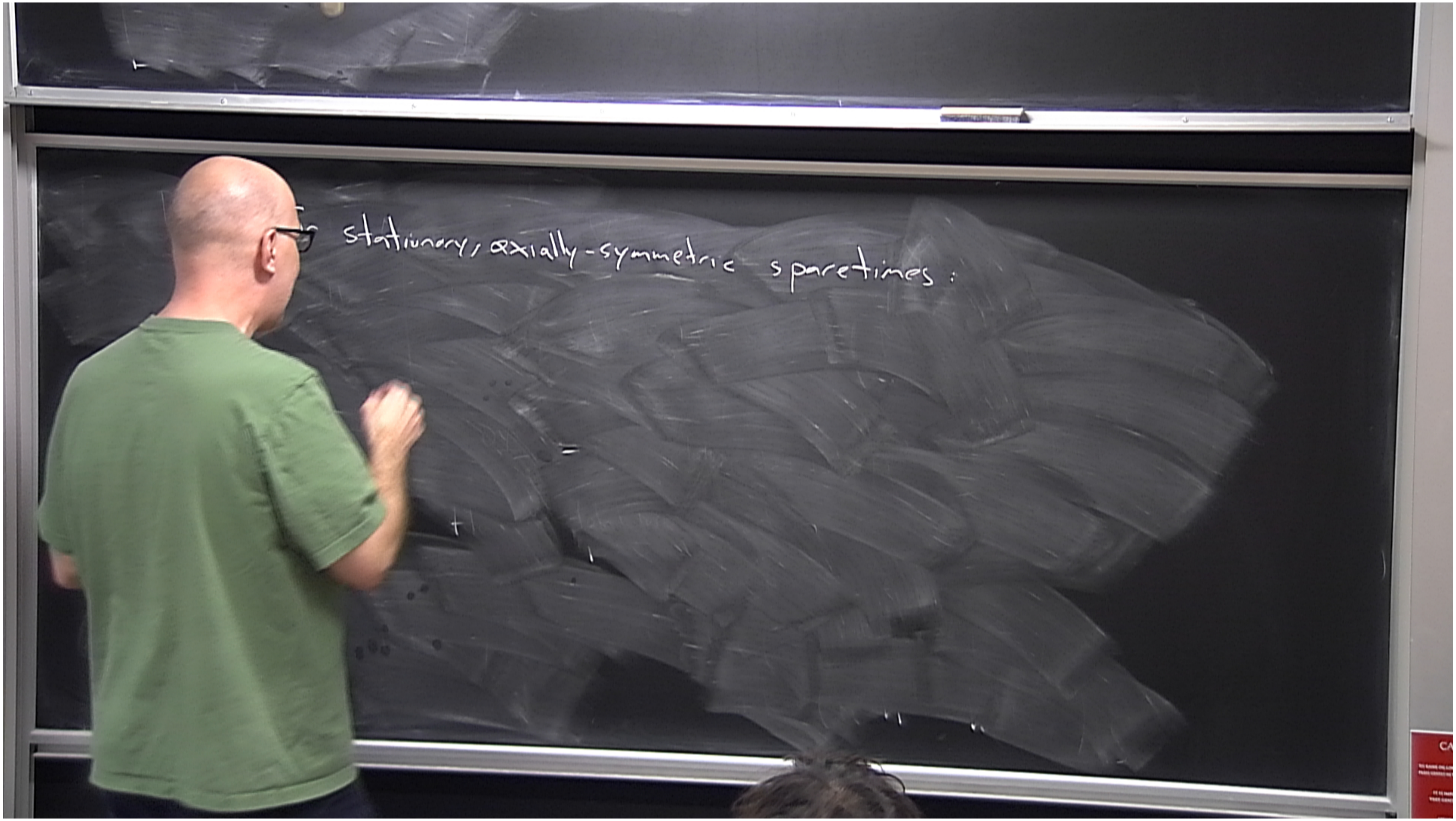
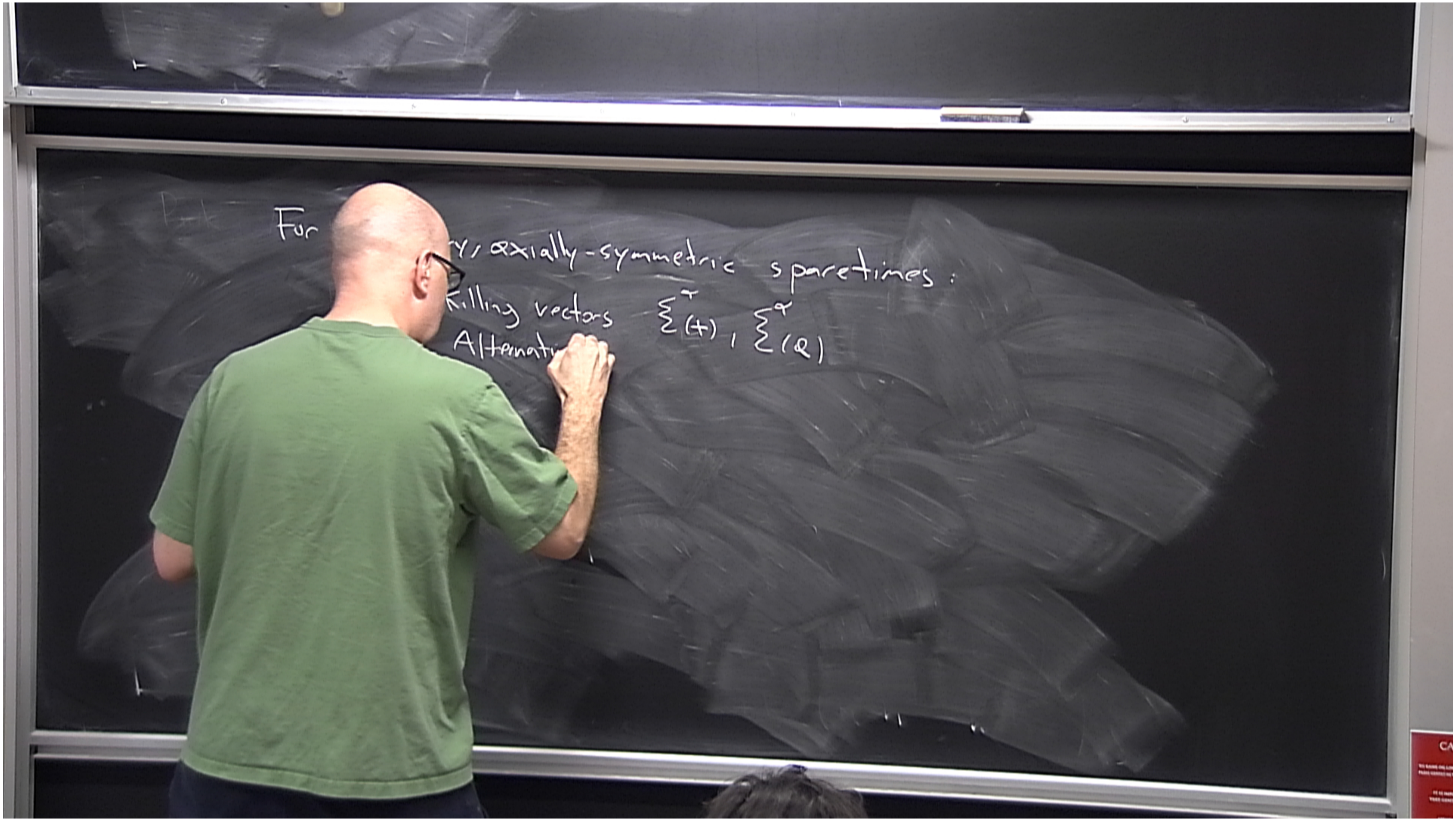
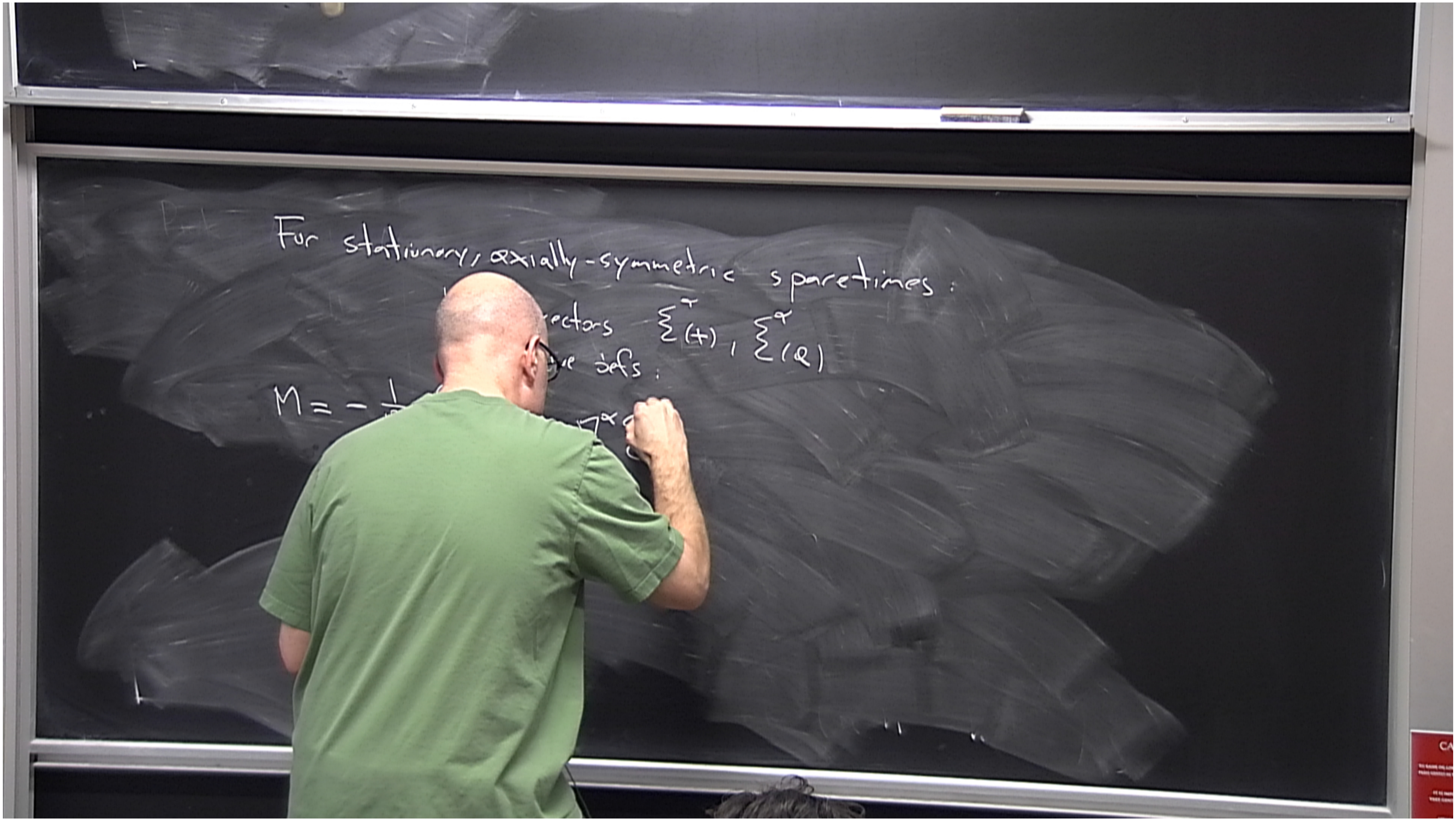
by calculation
$M \equiv m$
$J \equiv j$
harmonic
deDonder
$m, j \equiv$ operational defs for
momentum, in terms of

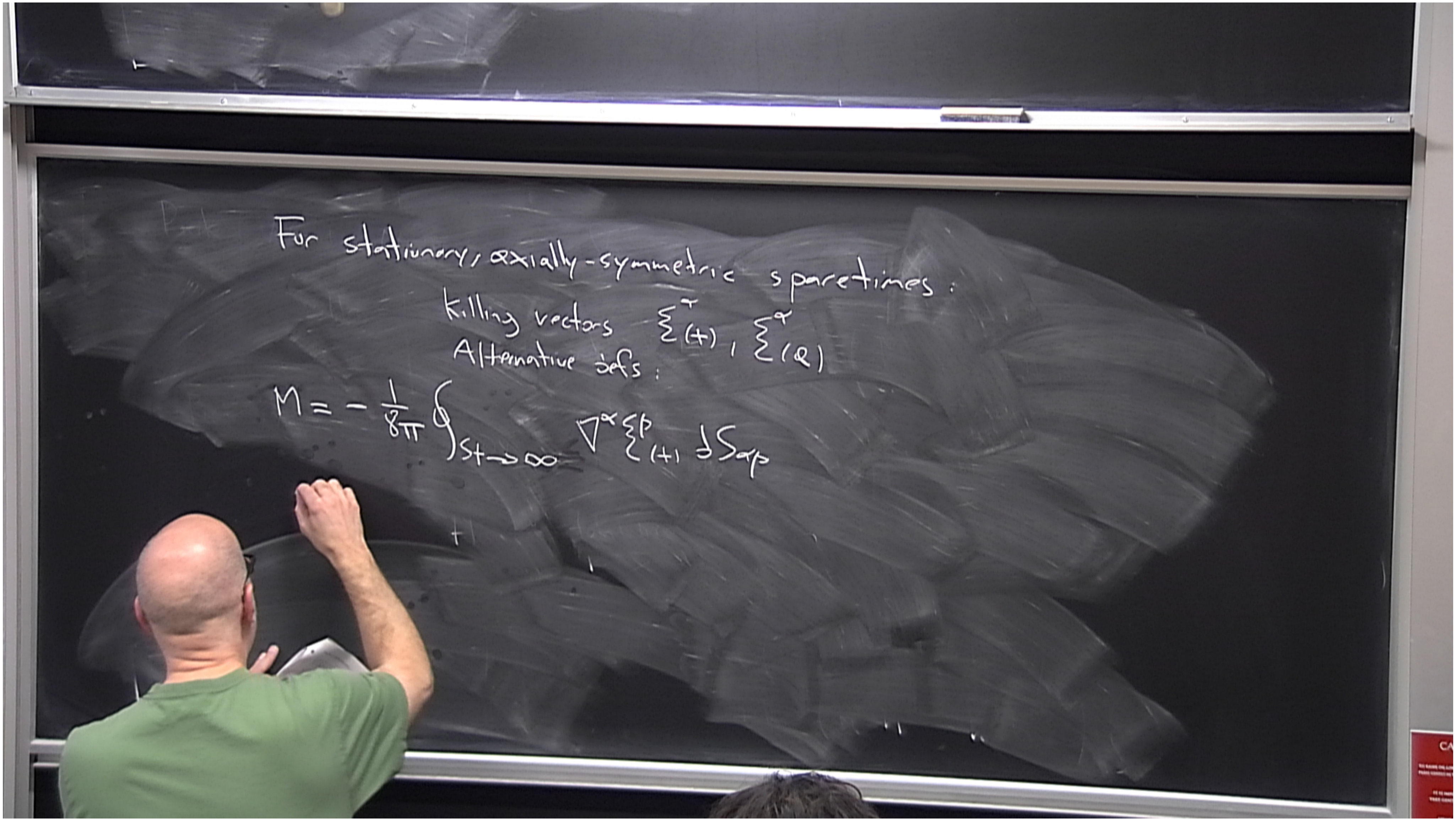

far away from a stationary body
by calculation
harmonic
deDonder
operational defs for mass
momentum, in terms of phys

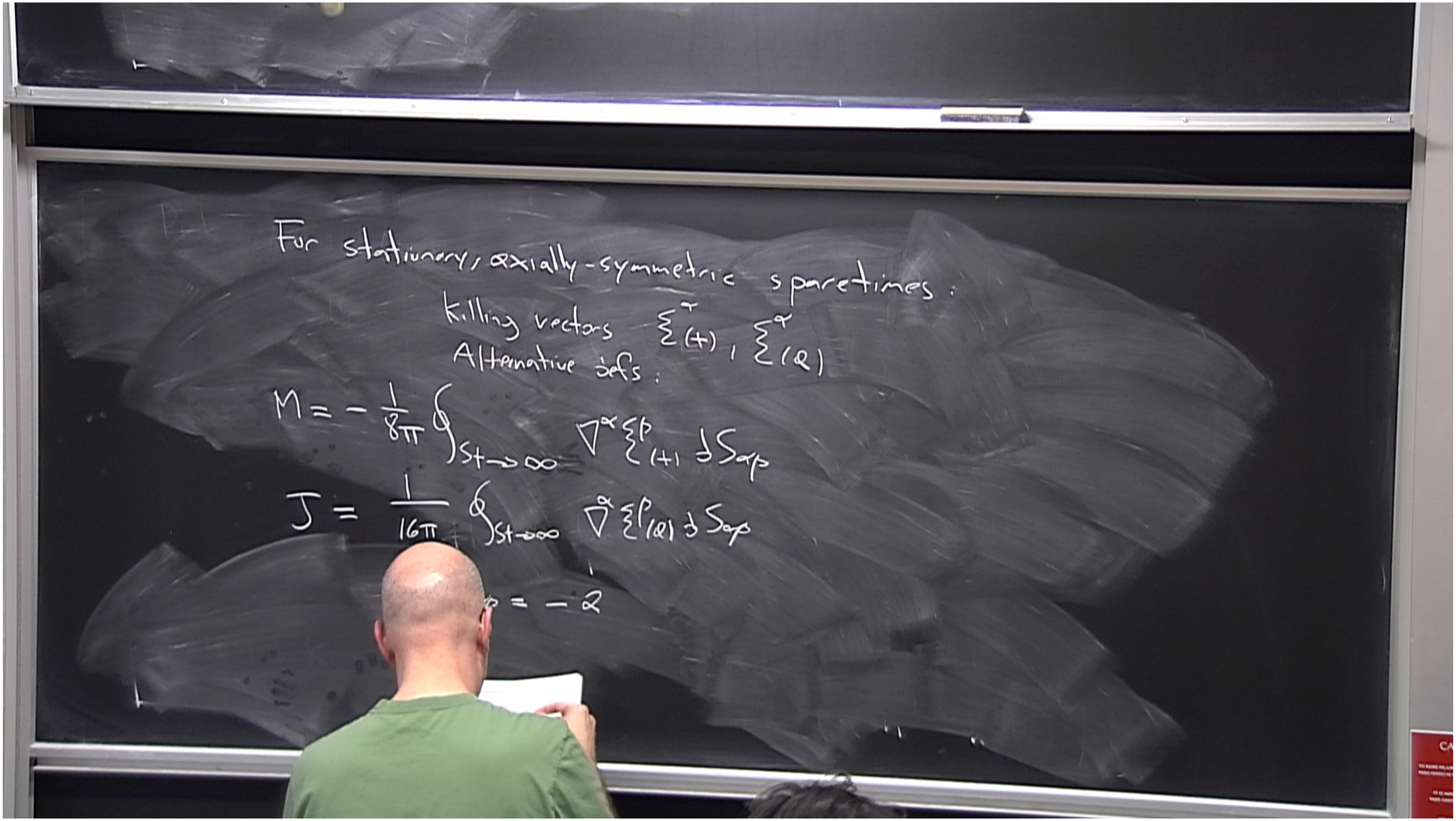

far away from a stationary body
$ds^2 = -\left(1 - \frac{2m}{r} + \cdots\right)dt^2 + \left(1 + \frac{2m}{r} + \cdots\right)(dr^2 + r^2 d\Omega^2)$
$- \left(\frac{4j\sin^2\theta}{r} + \cdots\right) dt\, d\varphi$
$\nabla^2 \bar{h}_{\alpha\beta} = 16\pi T_{\alpha\beta}$
harmonic
deDonder
$\partial_\beta \bar{h}^{\alpha\beta} = 0$
$\bar{h}^{\alpha\beta} = h^{\alpha\beta} - \frac{1}{2} h \eta^{\alpha\beta}$
by calculation
$M = m$
$J = j$
$m, j \equiv$ operational defs for mass and angular momentum, in terms of physical processes.

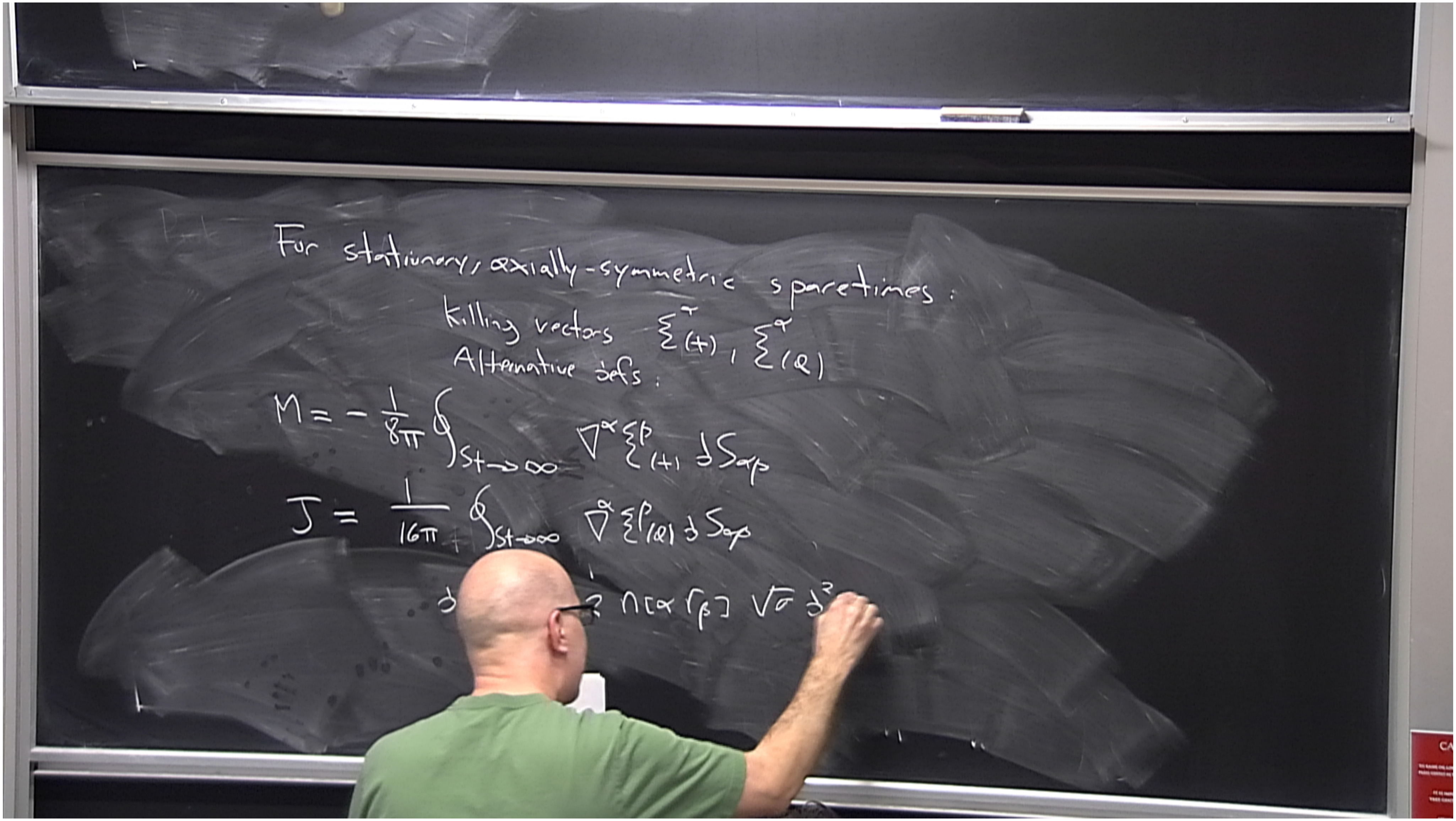

stationary, axially-symmetric spacetimes :


For
ry, axially-symmetric spacetimes :
Killing vectors
Alternati


For stationary, axially-symmetric spacetimes:
ectors ξ(t), ξ(φ)
defs:
M = −


For stationary, axially-symmetric spacetimes:
Killing vectors $\xi^\alpha_{(t)}$, $\xi^\alpha_{(\phi)}$
Alternative defs:
$M = -\frac{1}{8\pi}\oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(t)} \, dS_{\alpha\beta}$


For stationary, axially-symmetric spacetimes:
Killing vectors $\xi^{\alpha}_{(t)}, \xi^{\alpha}_{(\phi)}$
Alternative defs:
$M = -\frac{1}{8\pi} \oint_{S_t \to \infty} \nabla^{\alpha} \xi^{\beta}_{(t)} \, dS_{\alpha\beta}$
$J = \frac{1}{16\pi} \oint_{S_t \to \infty} \nabla^{\alpha} \xi^{\beta}_{(\phi)} \, dS_{\alpha\beta}$
$= -2$


For stationary, axially-symmetric spacetimes:
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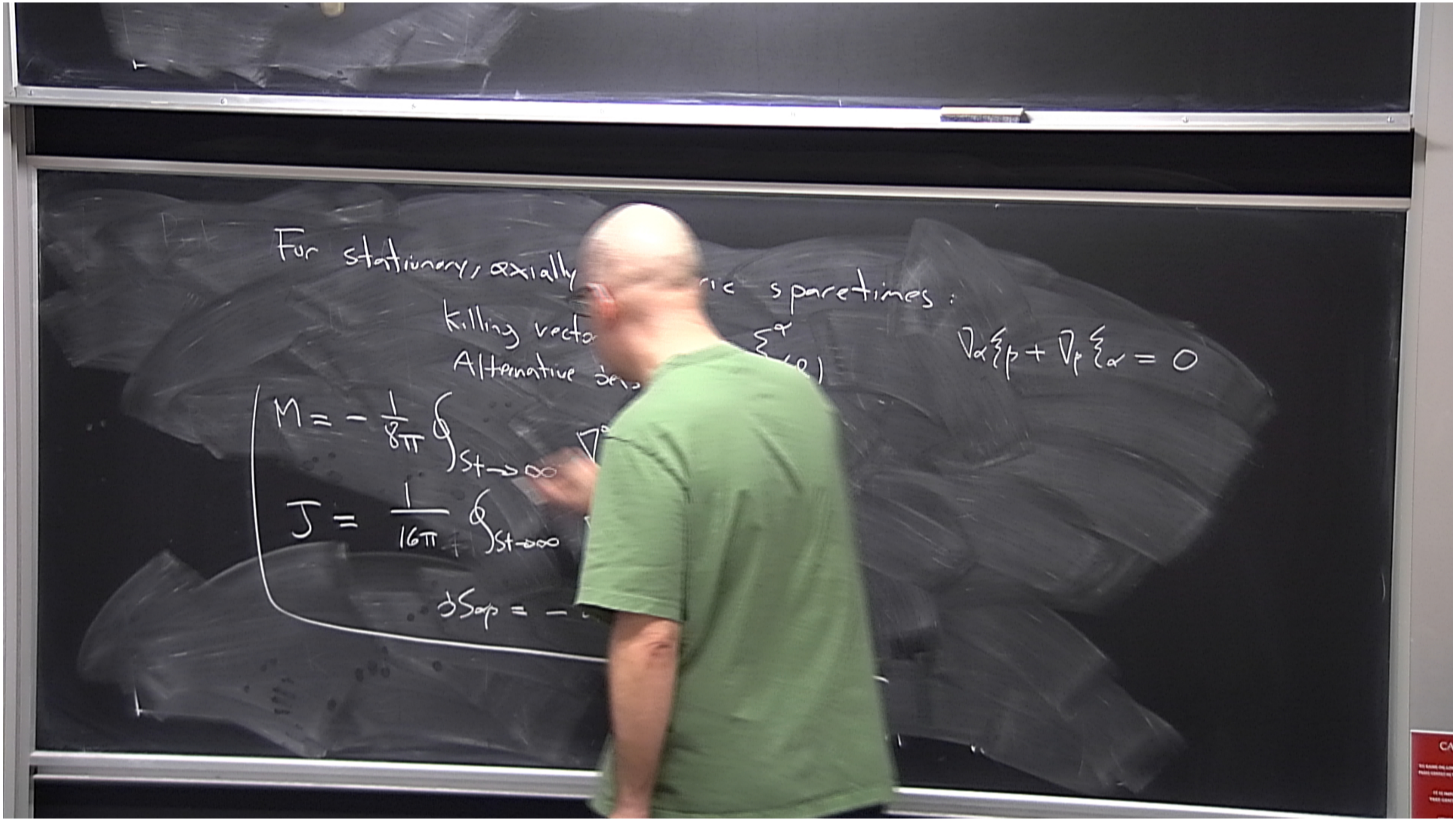

For stationary, axially
spacetimes :
Killing vecto
Alternative
∇α ξβ + ∇β ξα = 0
M = − 1/8π ∮St→∞
J = 1/16π ∮St→∞

For stationary, axially-symmetric spacetimes:

Killing vectors $\xi^{\alpha}_{(t)}, \xi^{\alpha}_{(\phi)}$ $\qquad \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha = 0$

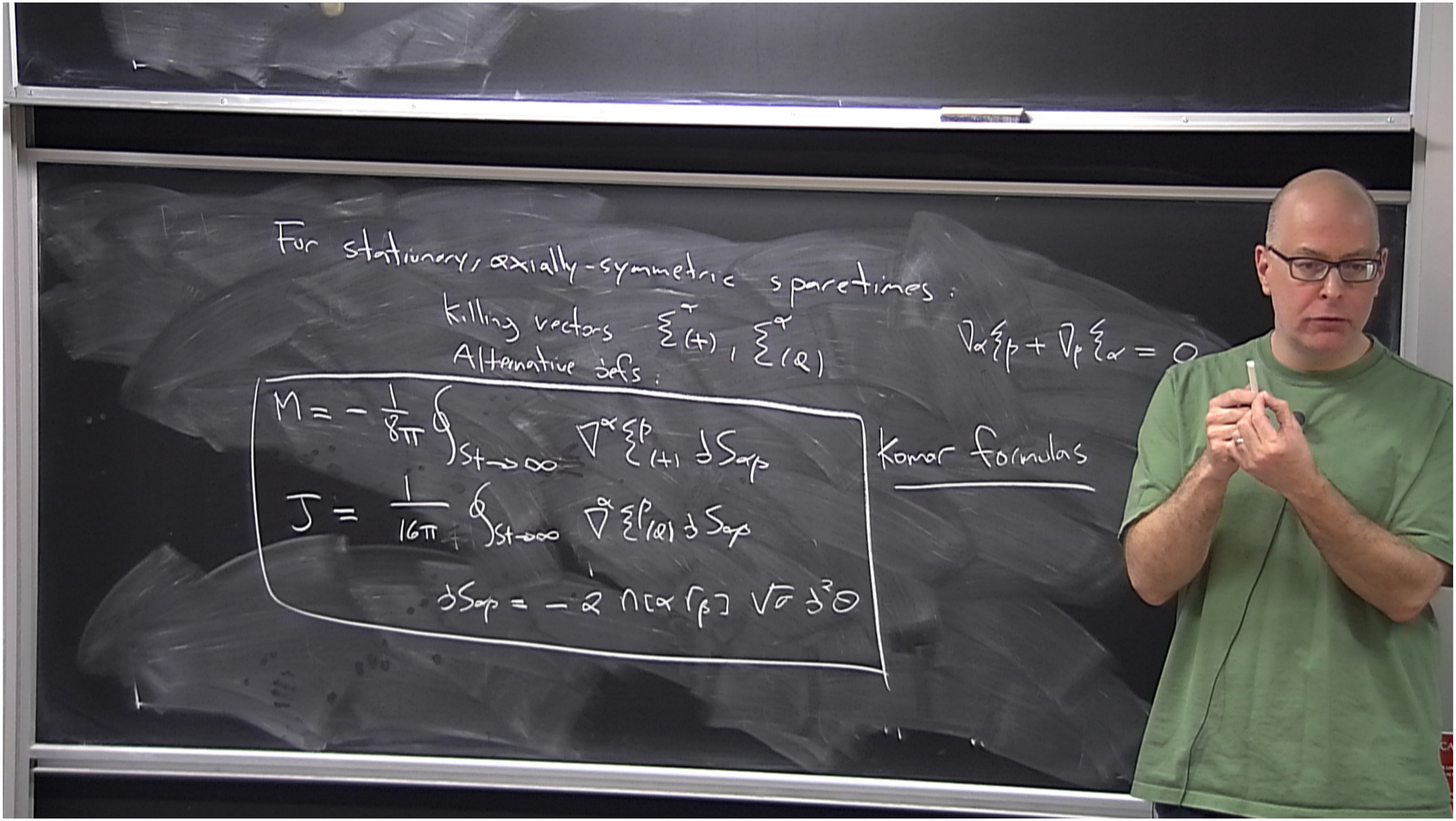
Alternative defs:

$$M = -\frac{1}{8\pi} \oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(t)} \, dS_{\alpha\beta}$$

$$J = \frac{1}{16\pi} \oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(\phi)} \, dS_{\alpha\beta}$$

$$dS_{\alpha\beta} = -2\, n_{[\alpha} r_{\beta]} \sqrt{\sigma} \, d^2\theta$$

Komar formulas


For stationary, axially-symmetric spacetimes:
Killing vectors $\xi^\alpha_{(t)}$, $\xi^\alpha_{(\phi)}$
$\nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha = 0$
Alternative defs:
$M = -\frac{1}{8\pi} \oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(t)} \, dS_{\alpha\beta}$
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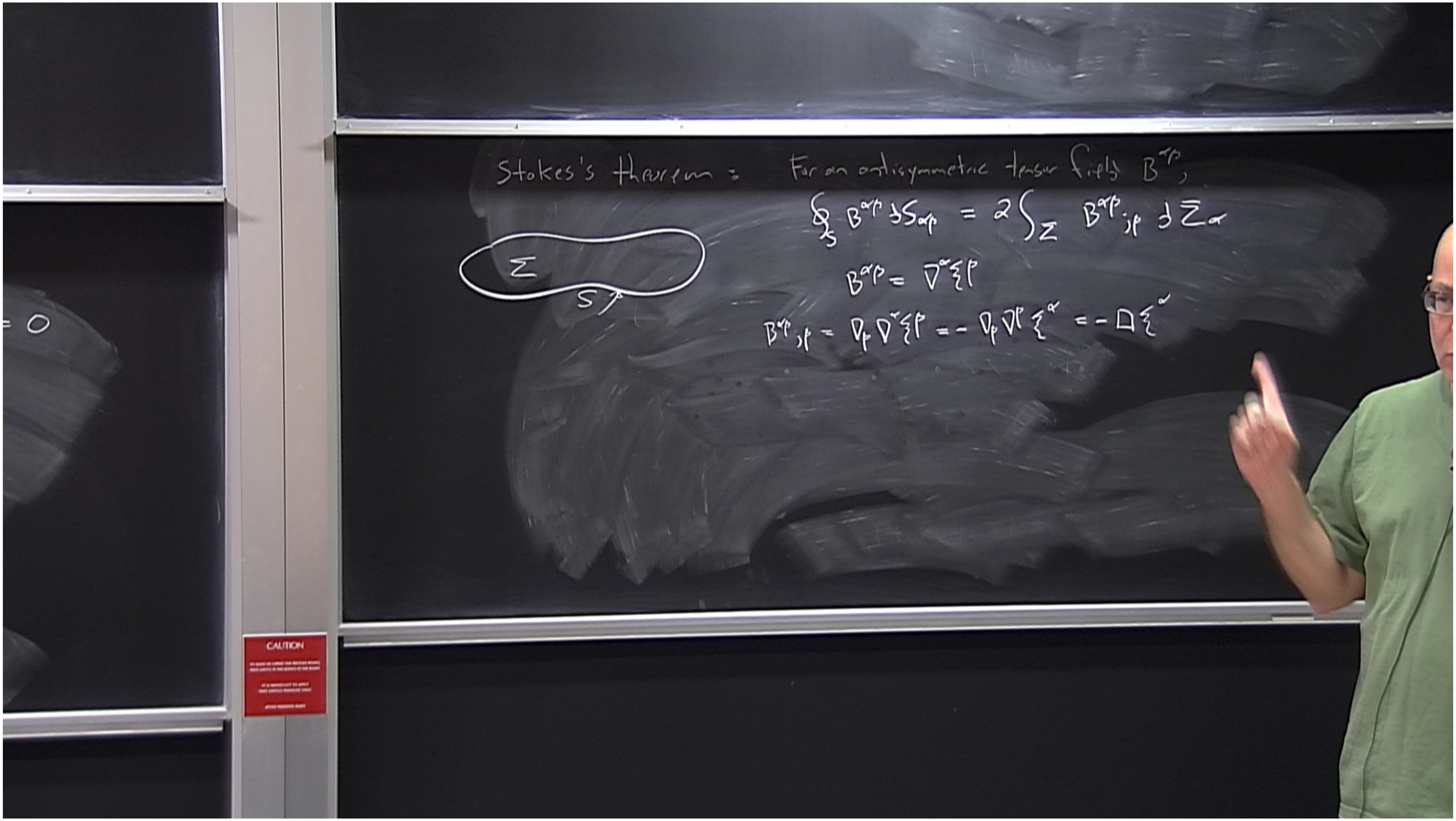
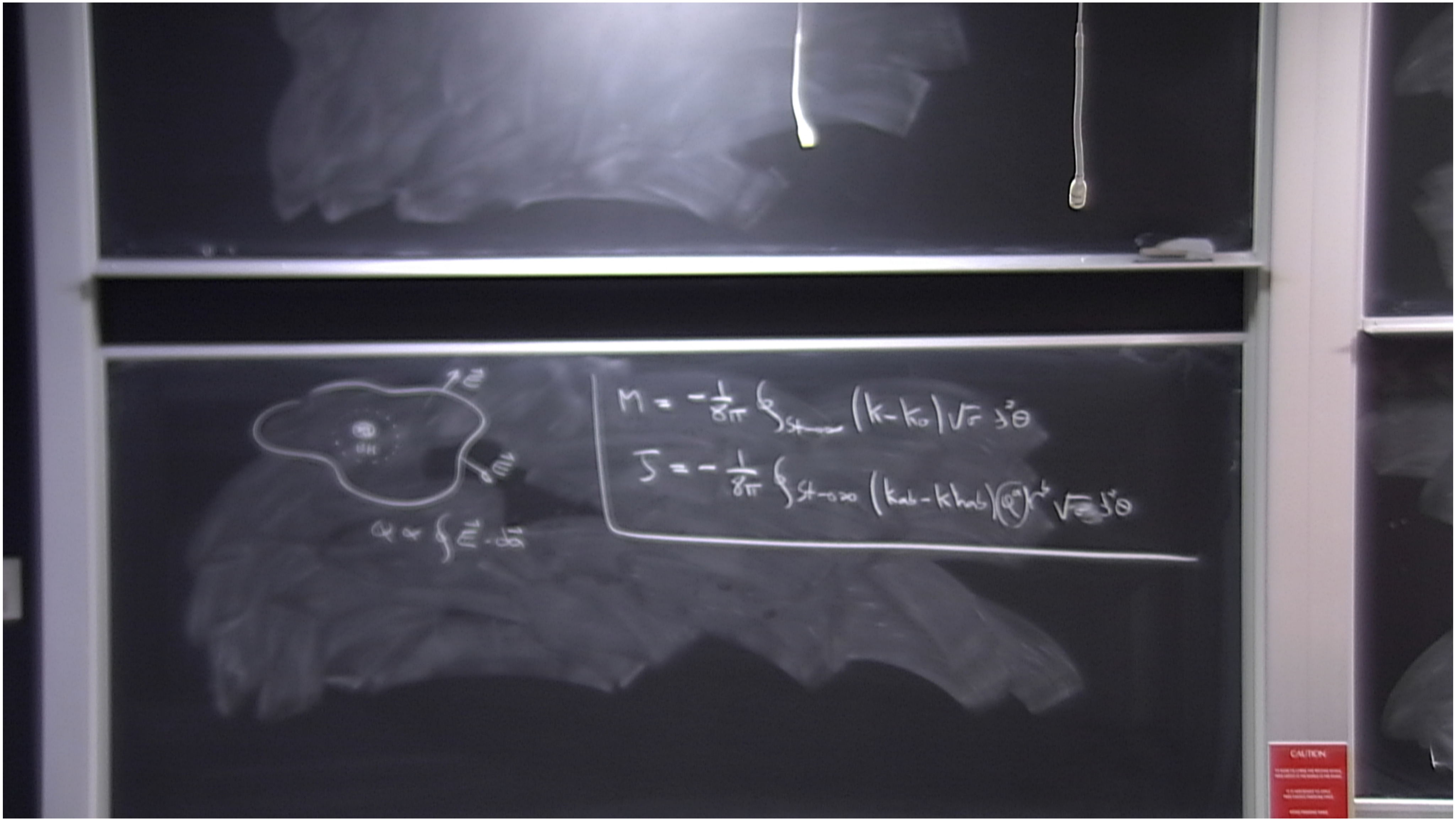
$dS_{\alpha\beta} = -2 \, n_{[\alpha} r_{\beta]} \sqrt{\sigma} \, d^2\theta$
Komar formulas



Stokes's theorem: For an antisymmetric tensor field $B^{\alpha\beta}$,
$\oint_S B^{\alpha\beta} dS_{\alpha\beta} = 2\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha$
Σ
S
$B^{\alpha\beta} = \nabla^\alpha \xi^\beta$
$B^{\alpha\beta}{}_{;\beta} = \nabla_\beta \nabla^\alpha \xi^\beta = -\nabla_\beta \nabla^\beta \xi^\alpha = -\Box \xi^\alpha$
= 0
CAUTION



For stationary, axially-symmetric spacetimes:

Killing vectors $\xi^\alpha_{(t)}$, $\xi^\alpha_{(\varphi)}$

$$\nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha = 0$$

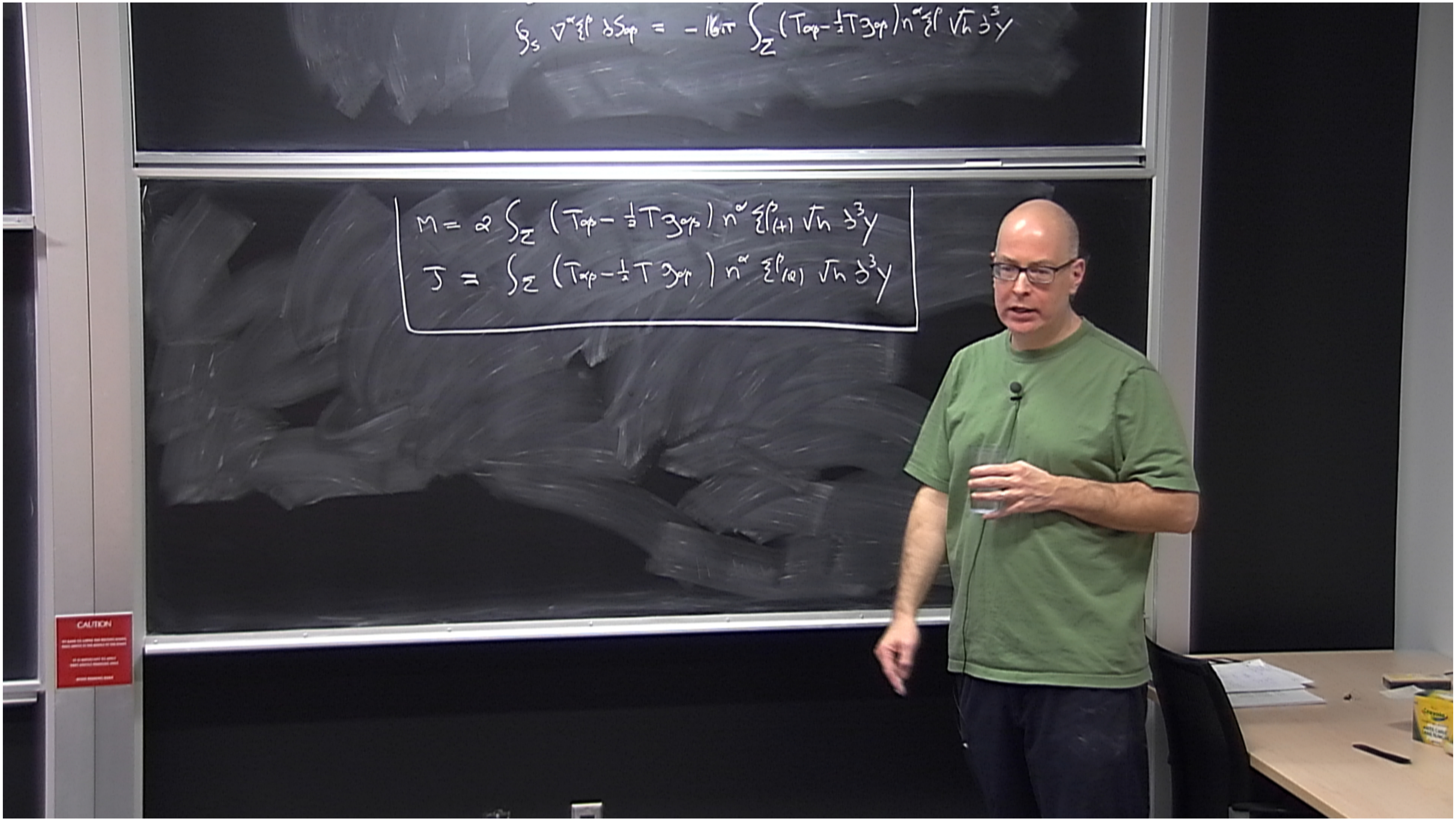
Alternative defs:

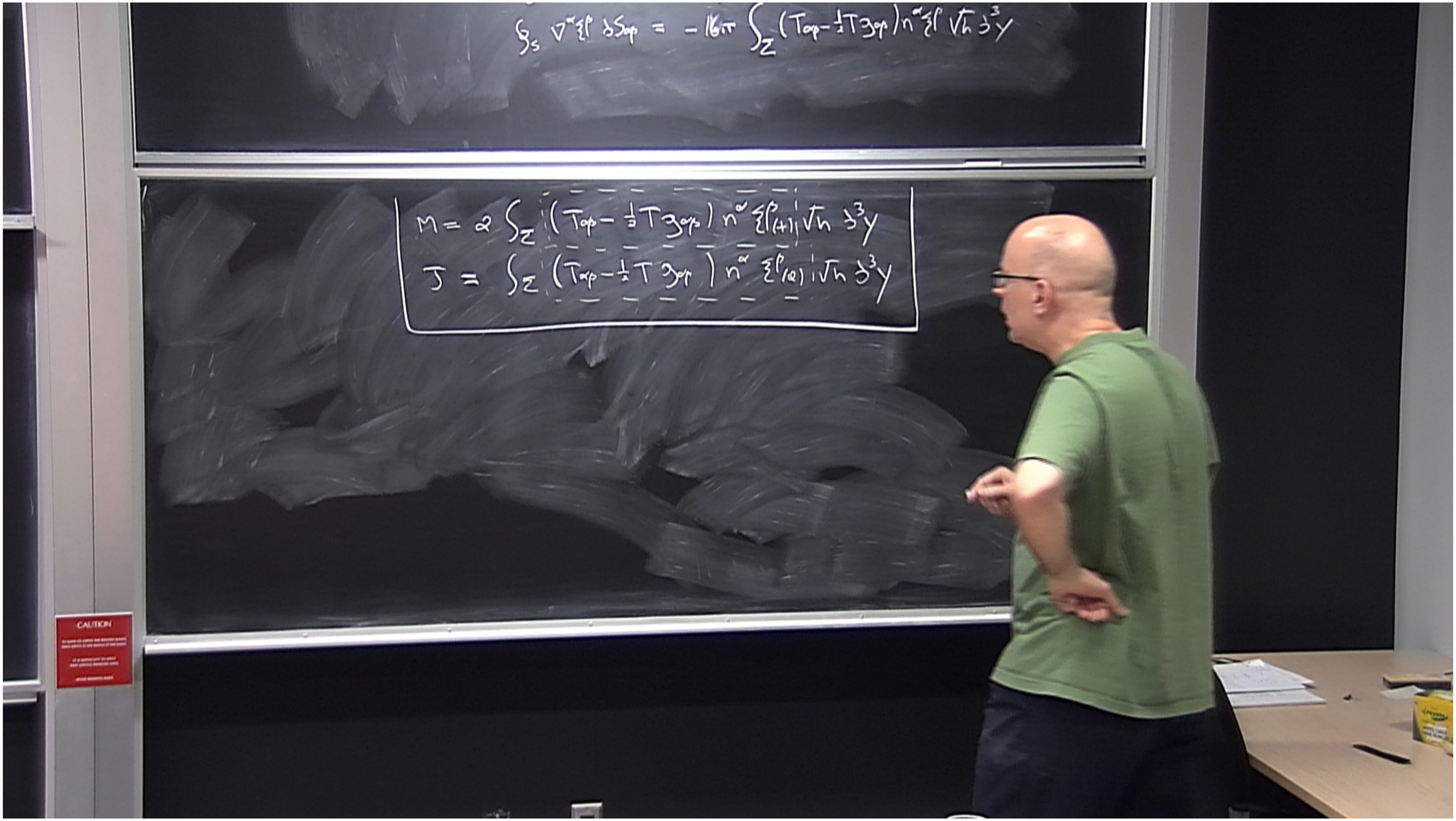
Komar formulas

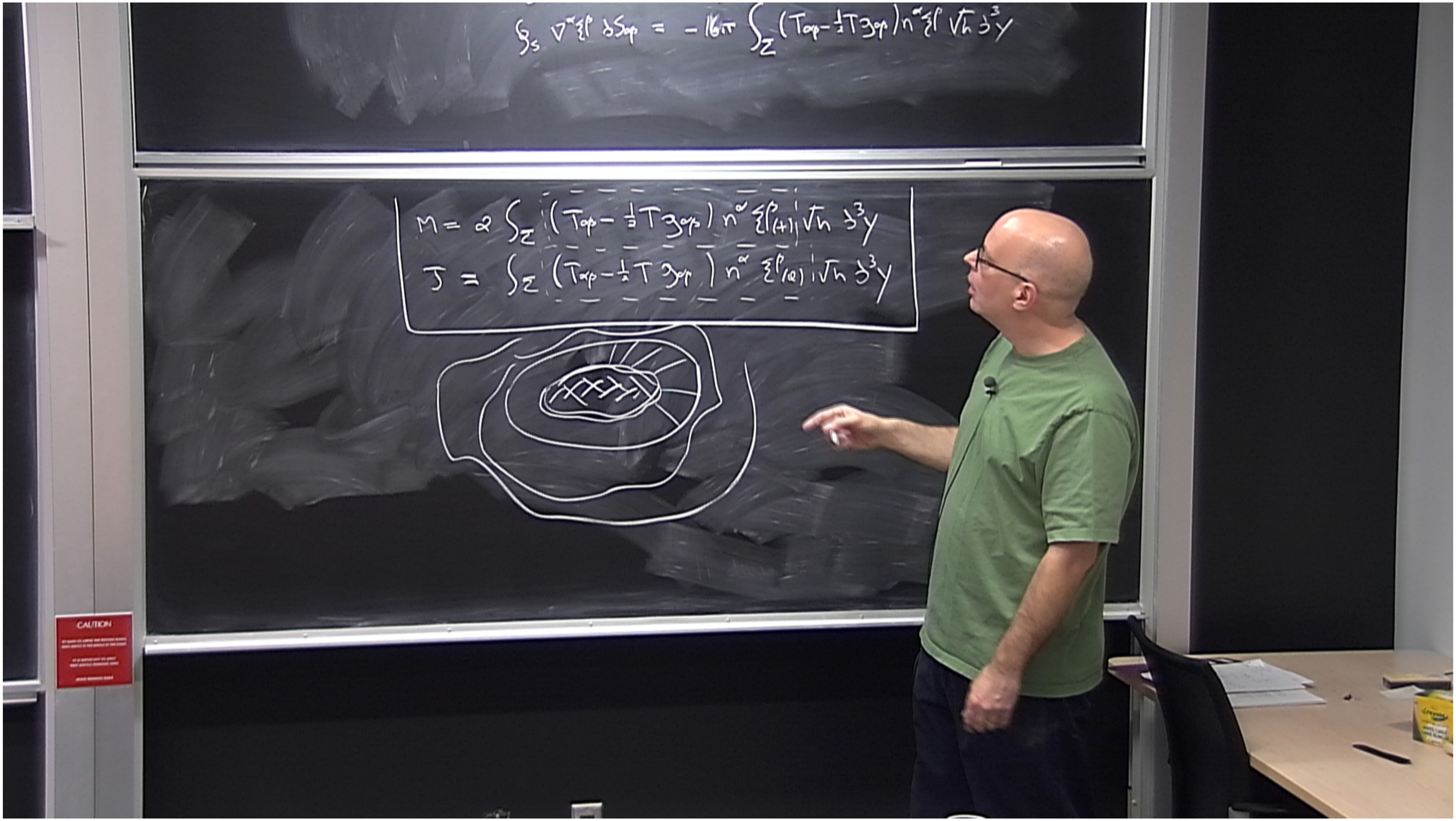
$$M = -\frac{1}{8\pi}\oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(t)}\, dS_{\alpha\beta}$$

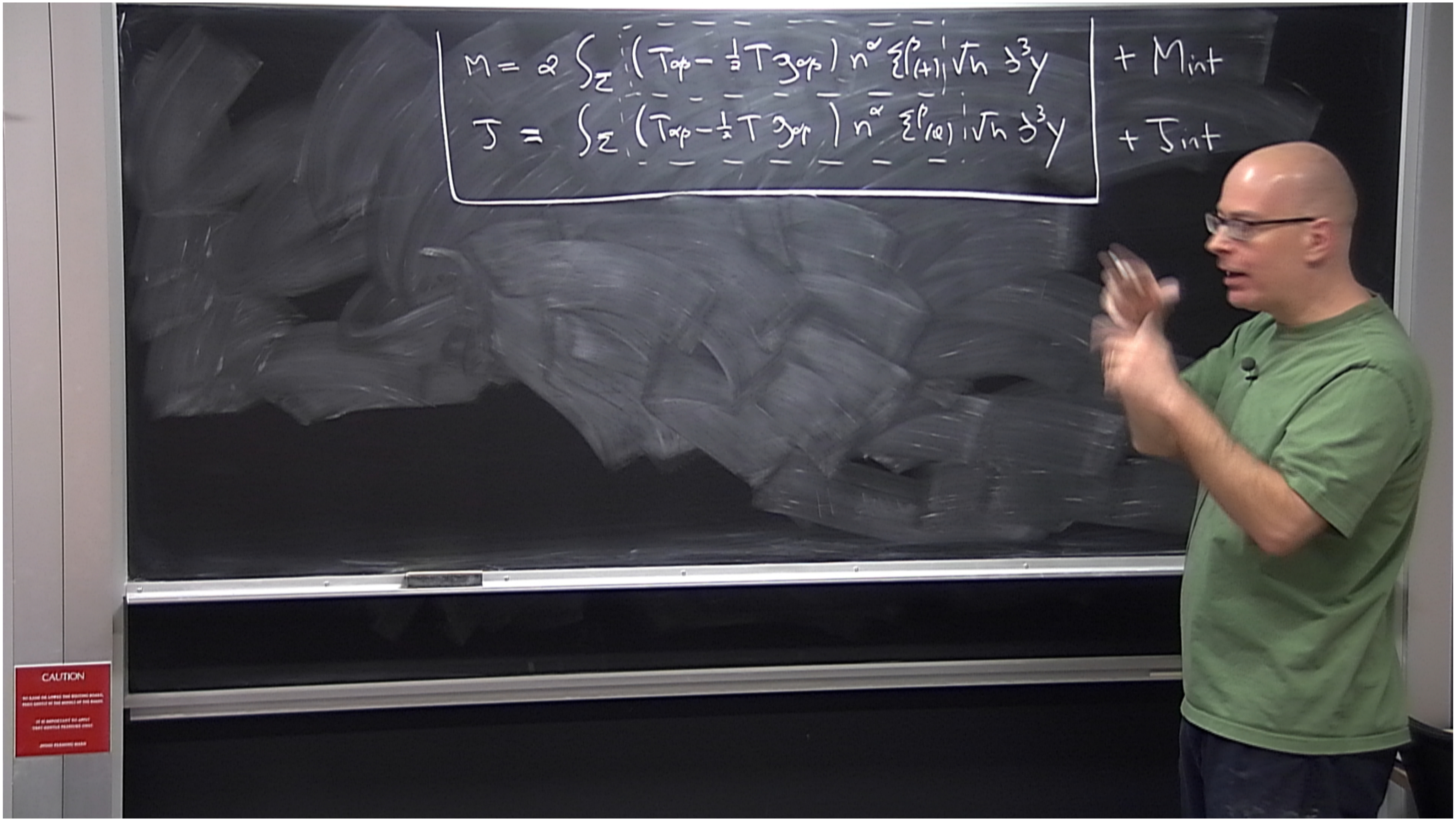
$$J = \frac{1}{16\pi}\oint_{S_t \to \infty} \nabla^\alpha \xi^\beta_{(\varphi)}\, dS_{\alpha\beta}$$

$$dS_{\alpha\beta} = -2\, n_{[\alpha} r_{\beta]} \sqrt{\sigma}\, d^2\theta$$

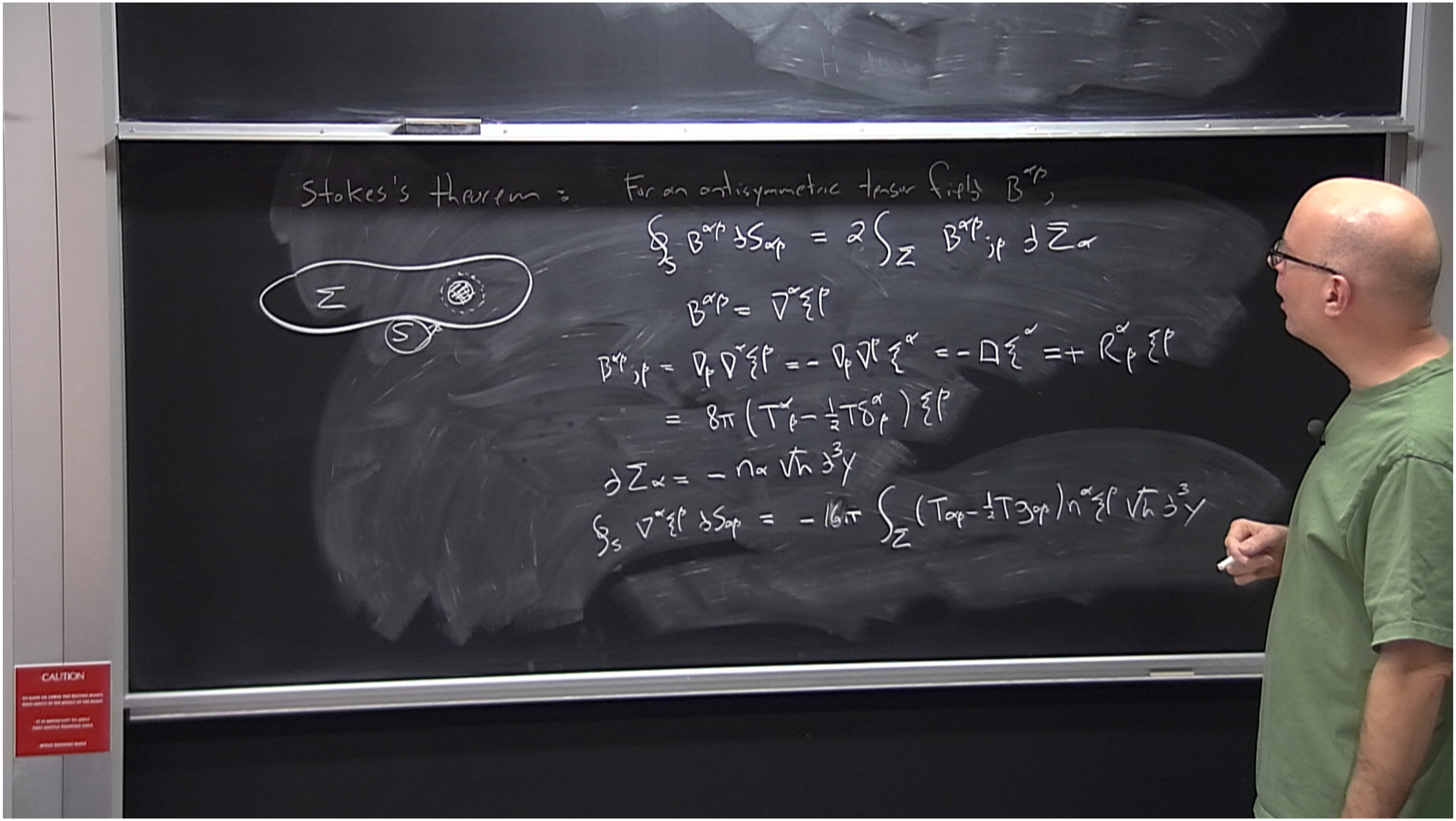

$\oint_S \nabla^\alpha \xi^\beta dS_{\alpha\beta} = -16\pi \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta \sqrt{h}\, d^3y$
$M = 2 \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta_{(t)} \sqrt{h}\, d^3y$
$J = \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta_{(\phi)} \sqrt{h}\, d^3y$


$\oint_S \nabla^\alpha \xi^\beta dS_{\alpha\beta} = -16\pi \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta \sqrt{h}\, d^3y$
$M = 2 \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta_{(t)} \sqrt{h}\, d^3y$
$J = \int_\Sigma (T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}) n^\alpha \xi^\beta_{(\phi)} \sqrt{h}\, d^3y$
CAUTION


∮_S ∇^α ξ^β dS_αβ = −16π ∫_Σ (T_αβ − ½ T g_αβ) n^α ξ^β √h d³y
M = 2 ∫_Σ (T_αβ − ½ T g_αβ) n^α ξ^β_(t) √h d³y
J = ∫_Σ (T_αβ − ½ T g_αβ) n^α ξ^β_(φ) √h d³y



$M = 2\int_\Sigma \left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right) n^\alpha \xi^\beta_{(t)} \sqrt{h}\, d^3y + M_{int}$
$J = \int_\Sigma \left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right) n^\alpha \xi^\beta_{(\phi)} \sqrt{h}\, d^3y + J_{int}$
CAUTION

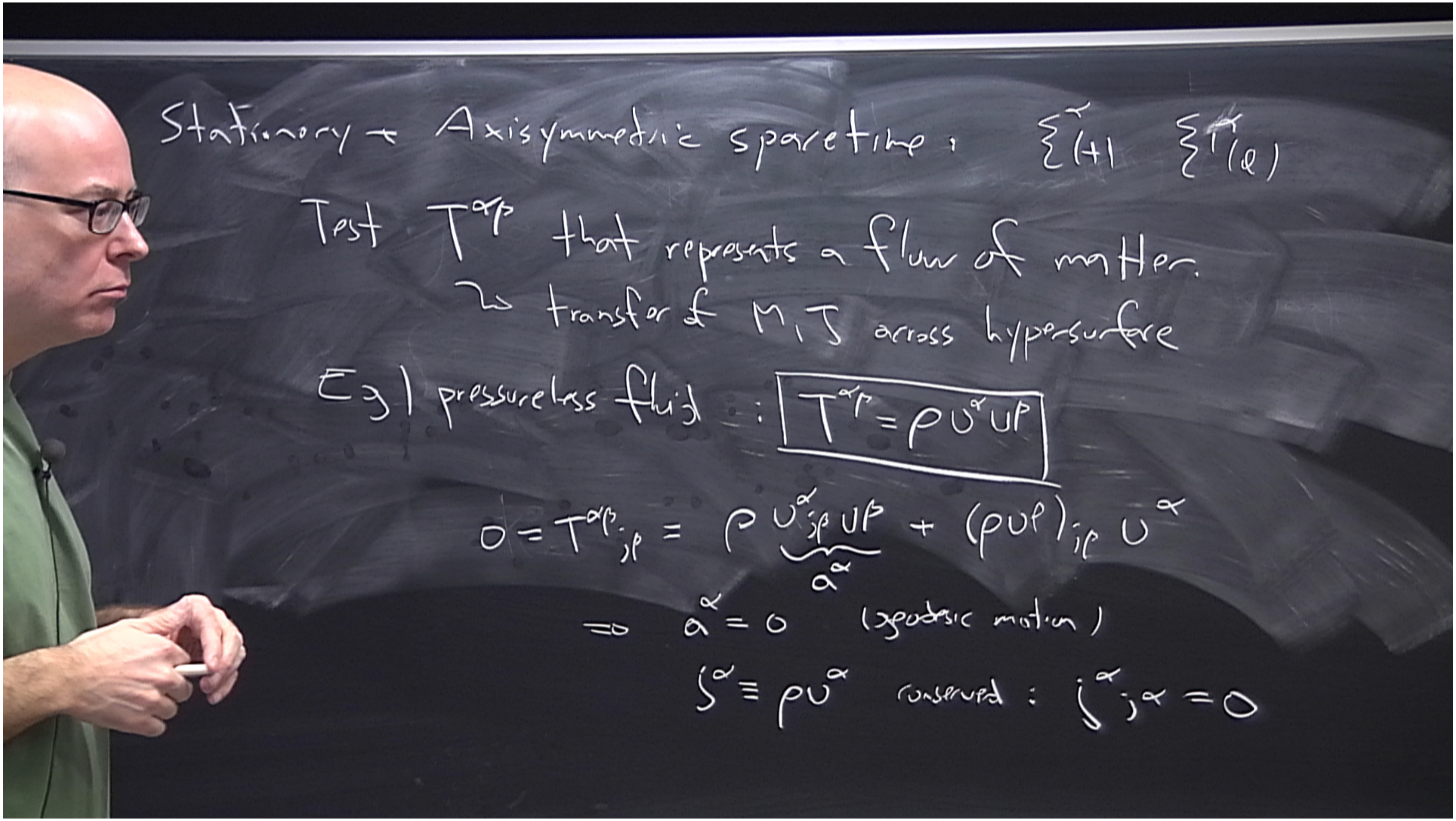

Stokes's theorem: For an antisymmetric tensor field $B^{\alpha\beta}$,
$\oint_S B^{\alpha\beta}\, dS_{\alpha\beta} = 2\int_\Sigma B^{\alpha\beta}{}_{;\beta}\, d\Sigma_\alpha$
Σ
S
$B^{\alpha\beta} = \nabla^\alpha \xi^\beta$
$B^{\alpha\beta}{}_{;\beta} = \nabla_\beta \nabla^\alpha \xi^\beta = -\nabla_\beta \nabla^\beta \xi^\alpha = -\Box \xi^\alpha = + R^\alpha{}_\beta \xi^\beta$
$= 8\pi \left(T^\alpha{}_\beta - \frac{1}{2} T \delta^\alpha{}_\beta\right) \xi^\beta$
$d\Sigma_\alpha = -n_\alpha \sqrt{h}\, d^3y$
$\oint_S \nabla^\alpha \xi^\beta\, dS_{\alpha\beta} = -16\pi \int_\Sigma \left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right) n^\alpha \xi^\beta \sqrt{h}\, d^3y$

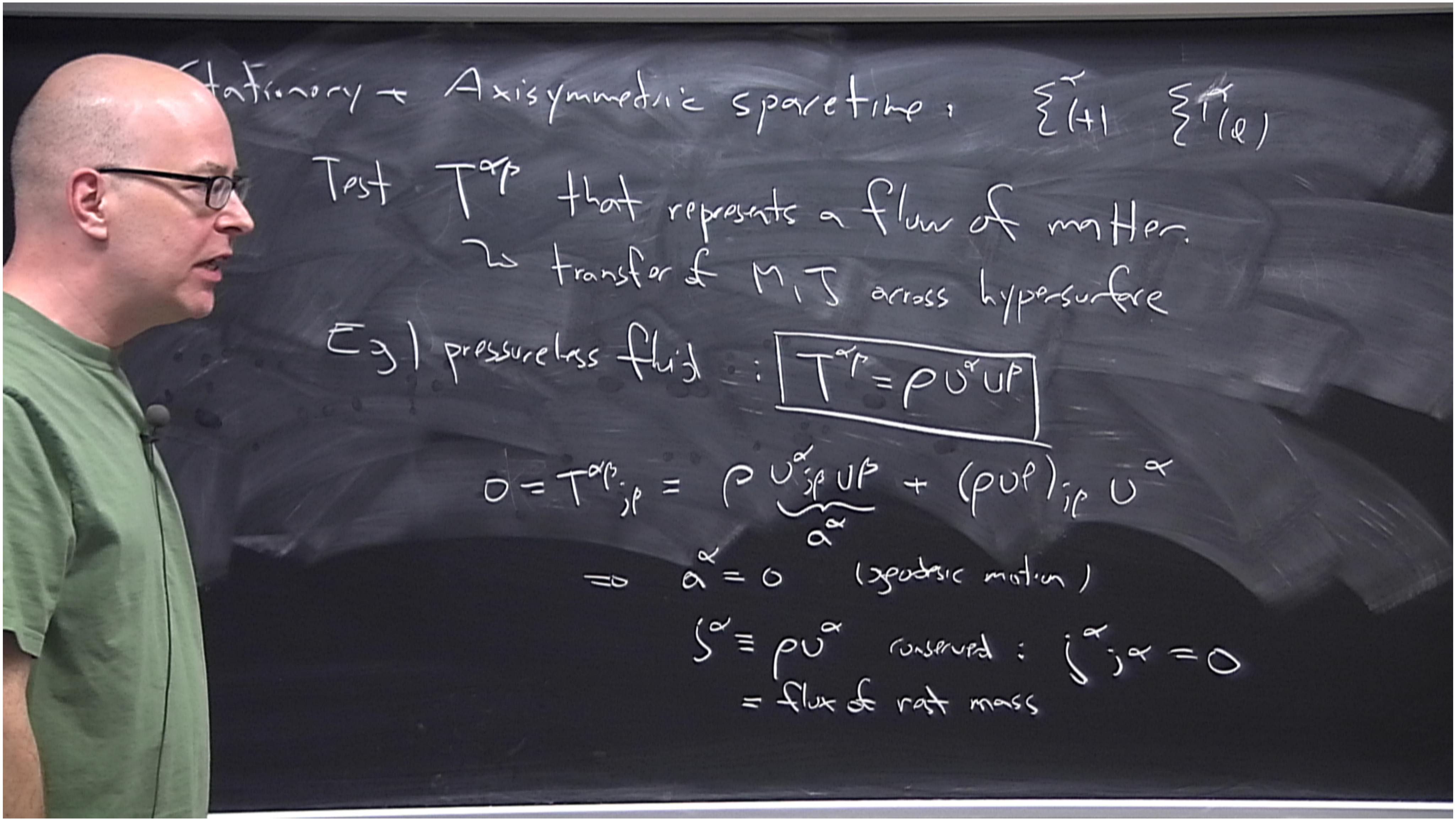
Stationary + Axisymmetric spacetime: $\xi^{\alpha}_{(t)}$ $\xi^{\alpha}_{(\phi)}$

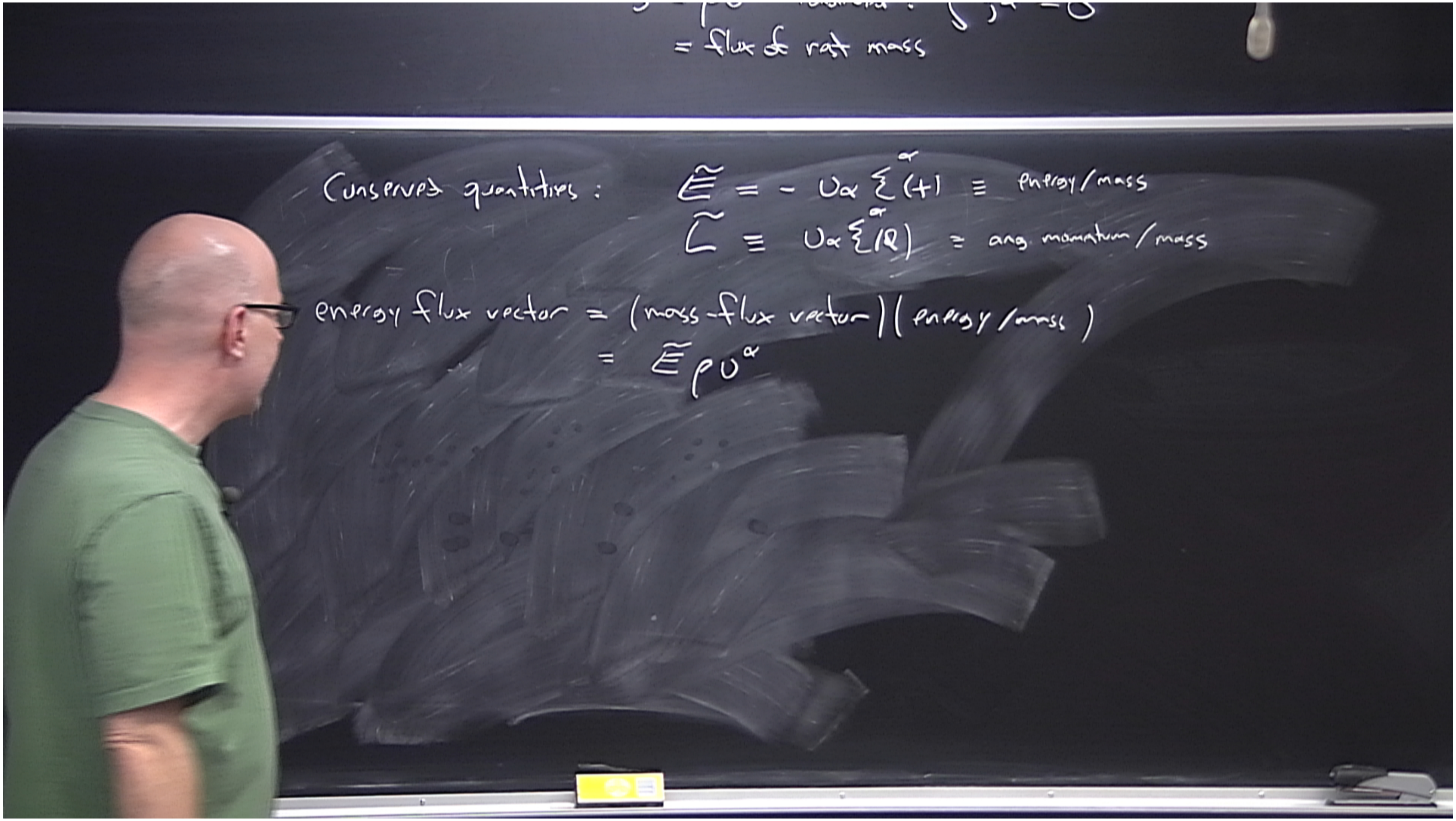
Test $T^{\alpha\beta}$ that represents a flow of matter.

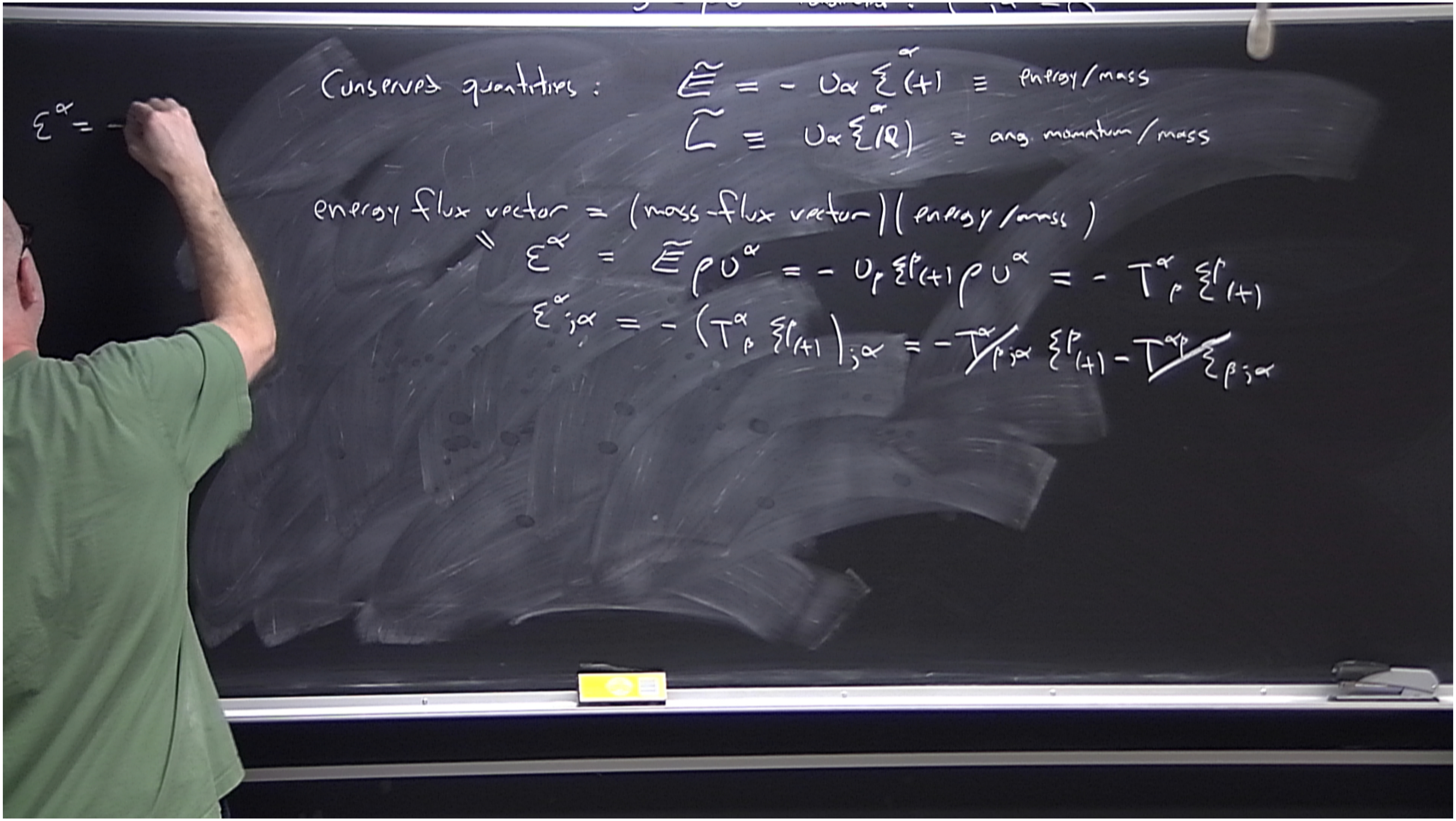
$\rightsquigarrow$ transfer of M, J across hypersurface

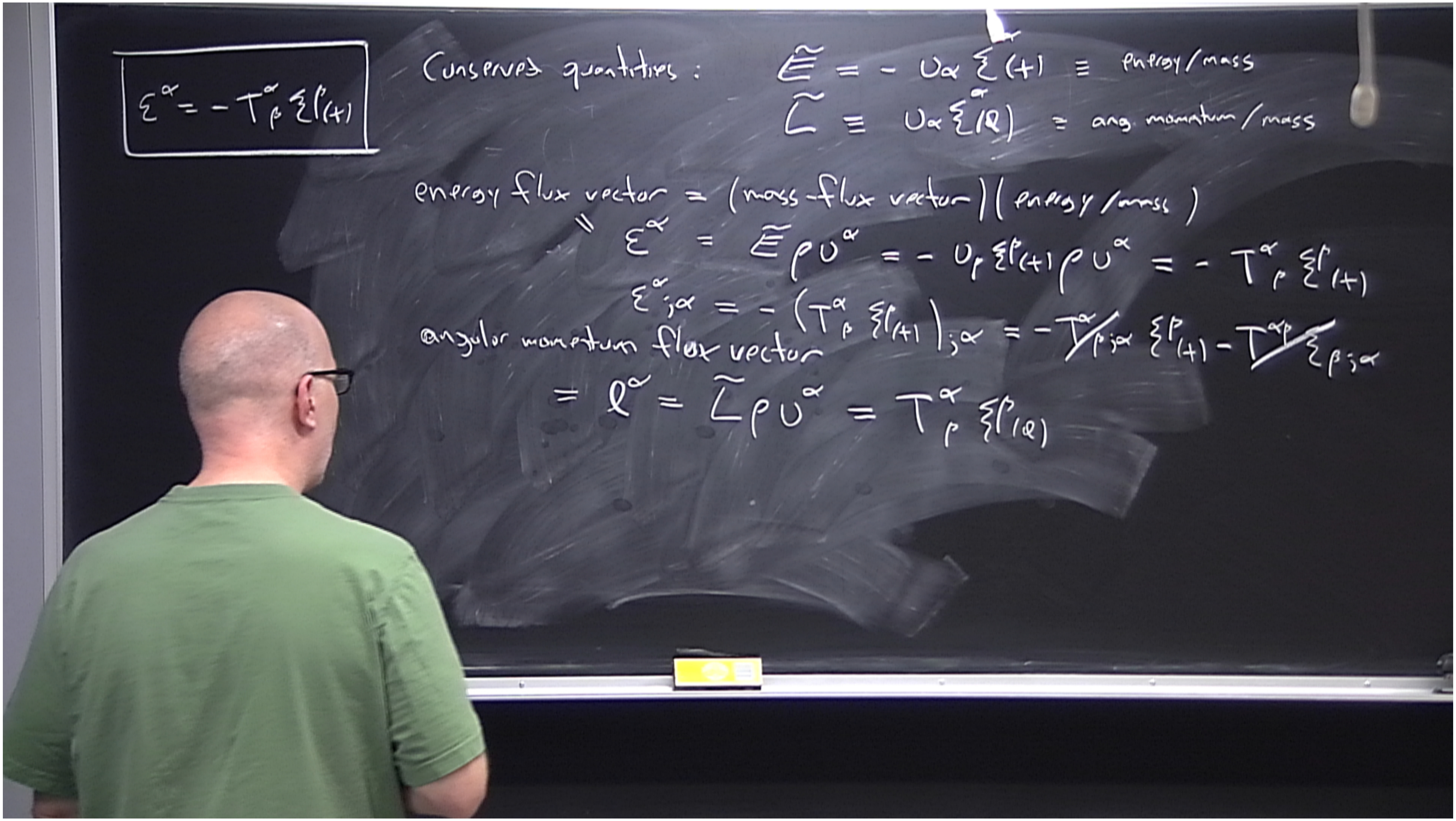
E.g.) pressureless fluid: $T^{\alpha\beta} = \rho u^{\alpha} u^{\beta}$


Stationary + Axisymmetric spacetime: $\xi^\alpha_{(t)}$ $\xi^\alpha_{(\phi)}$
Test $T^{\alpha\beta}$ that represents a flow of matter.
⇝ transfer of M, J across hypersurface
Eg 1 pressureless fluid: $T^{\alpha\beta} = \rho u^\alpha u^\beta$
$0 = T^{\alpha\beta}{}_{;\beta} = \rho \underbrace{u^\alpha{}_{;\beta} u^\beta}_{a^\alpha} + (\rho u^\beta)_{;\beta} u^\alpha$
$\Rightarrow a^\alpha = 0$ (geodesic motion)
$j^\alpha \equiv \rho u^\alpha$ conserved: $j^\alpha{}_{;\alpha} = 0$


Stationary + Axisymmetric spacetime: ξ^α(t) ξ^α(φ)
Test T^αβ that represents a flow of matter.
⇝ transfer of M, J across hypersurface
Eg) pressureless fluid : T^αβ = ρ u^α u^β
0 = T^αβ;β = ρ u^α;β u^β + (ρu^β);β u^α
a^α
⇒ a^α = 0 (geodesic motion)
j^α ≡ ρu^α conserved : j^α;α = 0
= flux of rest mass


= flux of rest mass
Conserved quantities: $\tilde{E} = -u_\alpha \xi^\alpha_{(t)}$ ≡ energy/mass
$\tilde{L} \equiv u_\alpha \xi^\alpha_{(\phi)}$ = ang. momentum/mass
energy flux vector = (mass-flux vector)(energy/mass)
$= \tilde{E} \rho u^\alpha$


Conserved quantities :
$\tilde{E} = - u_\alpha \xi^\alpha_{(t)} \equiv$ energy/mass
$\tilde{L} \equiv u_\alpha \xi^\alpha_{(\phi)} \equiv$ ang. momentum / mass
energy flux vector = (mass-flux vector)(energy/mass)
$\varepsilon^\alpha = \tilde{E}\rho u^\alpha = - u_\beta \xi^\beta_{(t)} \rho u^\alpha = - T^\alpha{}_\beta \xi^\beta_{(t)}$
$\varepsilon^\alpha{}_{;\alpha} = - (T^\alpha{}_\beta \xi^\beta_{(t)})_{;\alpha} = - T^\alpha{}_{\beta;\alpha} \xi^\beta_{(t)} - T^{\alpha\beta} \xi_{\beta;\alpha}$
$\varepsilon^\alpha = -$


$\mathcal{E}^\alpha = -T^\alpha_\beta \xi^\beta_{(t)}$
Conserved quantities: $\tilde{E} = -u_\alpha \xi^\alpha_{(t)} \equiv$ energy/mass
$\tilde{L} \equiv u_\alpha \xi^\alpha_{(\phi)}$ = ang. momentum/mass
energy flux vector = (mass-flux vector)(energy/mass)
$\mathcal{E}^\alpha = \tilde{E}\rho u^\alpha = -u_\beta \xi^\beta_{(t)} \rho u^\alpha = -T^\alpha_\beta \xi^\beta_{(t)}$
$\mathcal{E}^\alpha{}_{;\alpha} = -(T^\alpha_\beta \xi^\beta_{(t)})_{;\alpha} = -T^\alpha_{\beta;\alpha} \xi^\beta_{(t)} - T^{\alpha\beta} \xi_{\beta;\alpha}$
angular momentum flux vector
$= \ell^\alpha = \tilde{L}\rho u^\alpha = T^\alpha_\beta \xi^\beta_{(\phi)}$

$\Rightarrow a^\alpha = 0$ (geodesic motion)

$J^\alpha \equiv \rho U^\alpha$ conserved : $J^\alpha{}_{;\alpha} = 0$

$$\boxed{\varepsilon^\alpha = -T^\alpha{}_\beta \, \xi^\beta{}_{(t)}}$$

energy flux

$$\boxed{\ell^\alpha = T^\alpha{}_\beta \, \xi^\beta{}_{(\varphi)}}$$

ang. momentum flux

$$\varepsilon^\alpha{}_{;\alpha} = 0$$

$$\ell^\alpha{}_{;\alpha} = 0$$

Conserved quantities : $\tilde{E} = -U_\alpha \xi^\alpha{}_{(t)} \equiv$ energy/mass

$\tilde{L} \equiv U_\alpha \xi^\alpha{}_{(\varphi)} =$ ang. momentum/mass

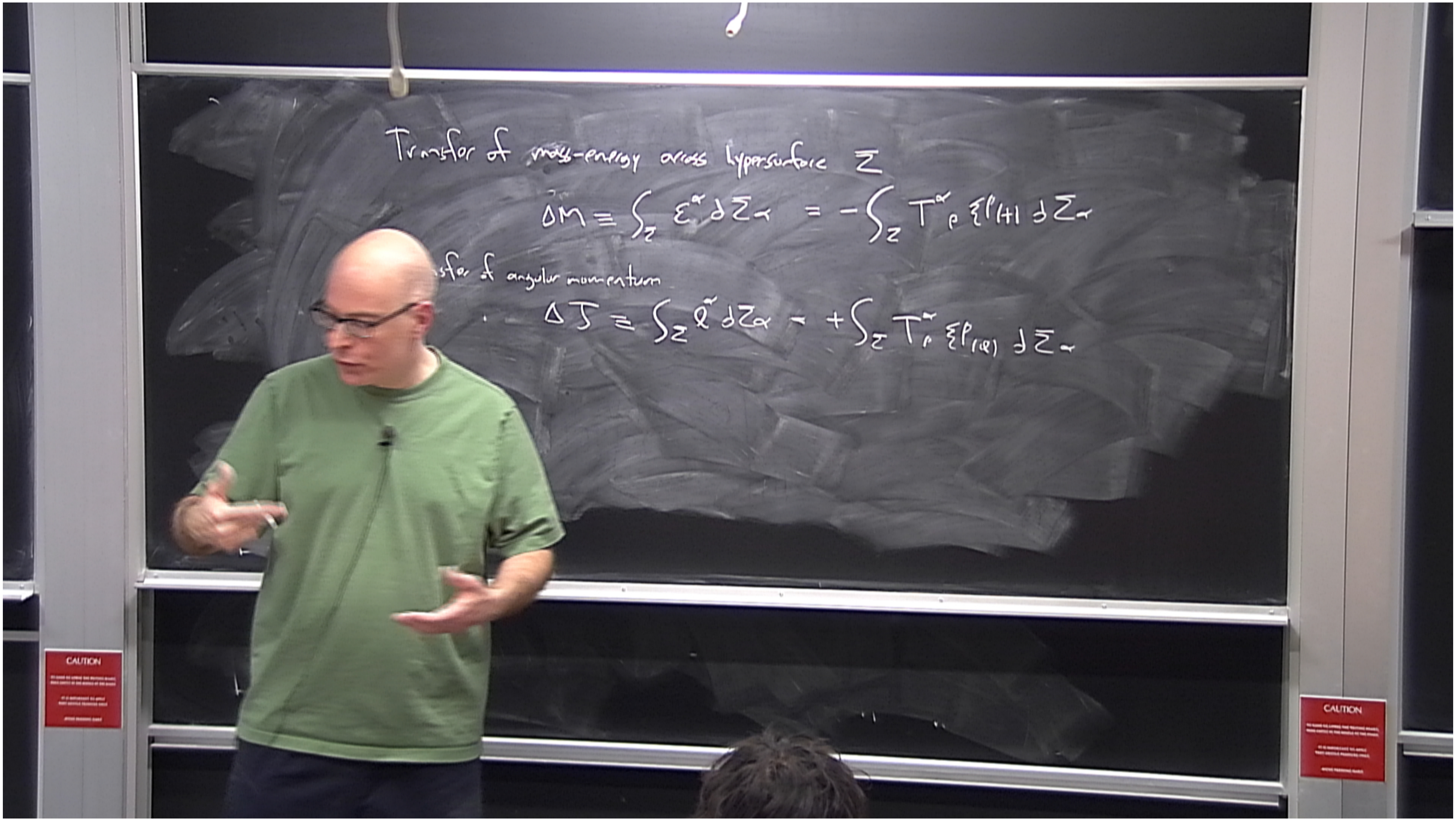
energy flux vector = (mass-flux vector)(energy/mass)

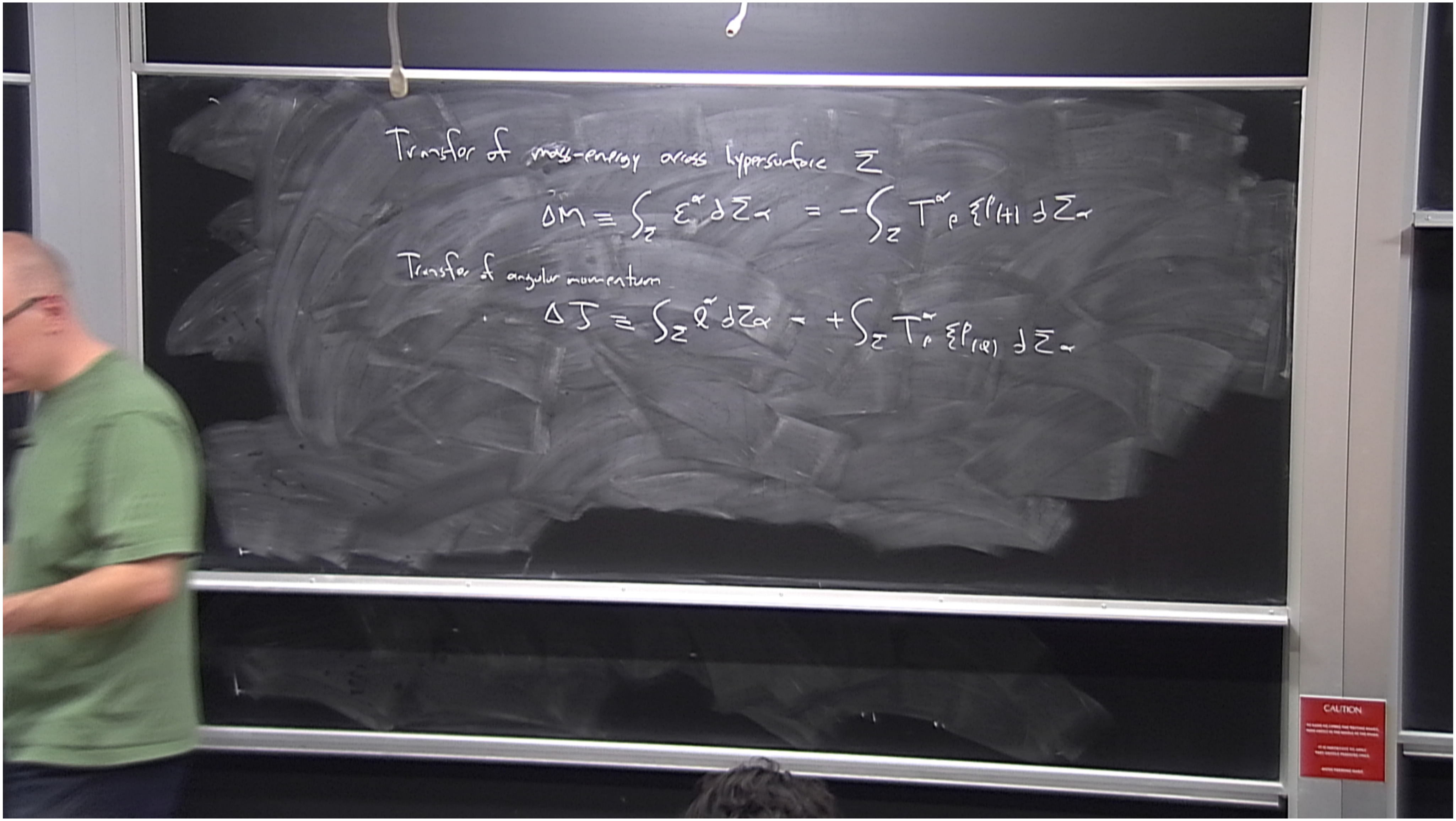
$$= \varepsilon^\alpha = \tilde{E} \rho U^\alpha = -U_\beta \xi^\beta{}_{(t)} \rho U^\alpha = -T^\alpha{}_\beta \xi^\beta{}_{(t)}$$

$$\varepsilon^\alpha{}_{;\alpha} = -\left(T^\alpha{}_\beta \xi^\beta{}_{(t)}\right)_{;\alpha} = -T^\alpha{}_{\beta;\alpha} \xi^\beta{}_{(t)} - T^{\alpha\beta} \xi_{\beta;\alpha}$$

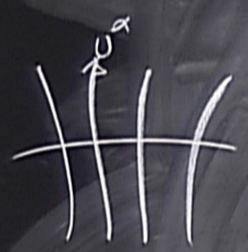
angular momentum flux vector

$$= \ell^\alpha = \tilde{L} \rho U^\alpha = T^\alpha{}_\beta \xi^\beta{}_{(\varphi)}$$


Transfer of mass-energy across hypersurface Σ
$\Delta M \equiv \int_\Sigma \varepsilon^\alpha d\Sigma_\alpha = -\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(t)} d\Sigma_\alpha$
sfer of angular momentum
$\Delta J \equiv \int_\Sigma \ell^\alpha d\Sigma_\alpha = +\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(\varphi)} d\Sigma_\alpha$



Transfer of mass-energy across hypersurface Σ
$\Delta M \equiv \int_\Sigma \varepsilon^\alpha d\Sigma_\alpha = -\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(t)} d\Sigma_\alpha$
Transfer of angular momentum
$\Delta J \equiv \int_\Sigma \ell^\alpha d\Sigma_\alpha = +\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(\varphi)} d\Sigma_\alpha$



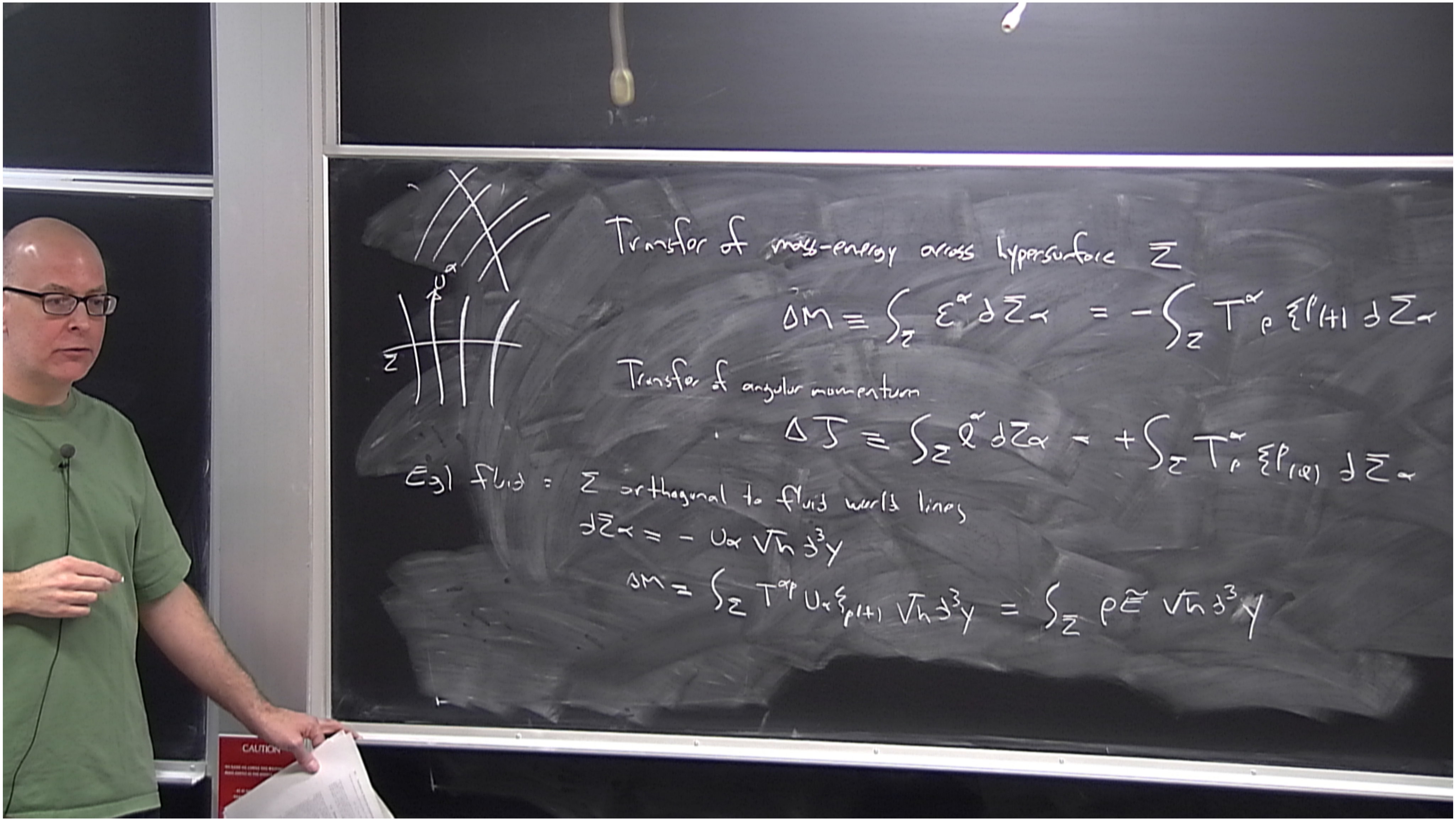
Transfer of mass-energy across hypersurface Σ

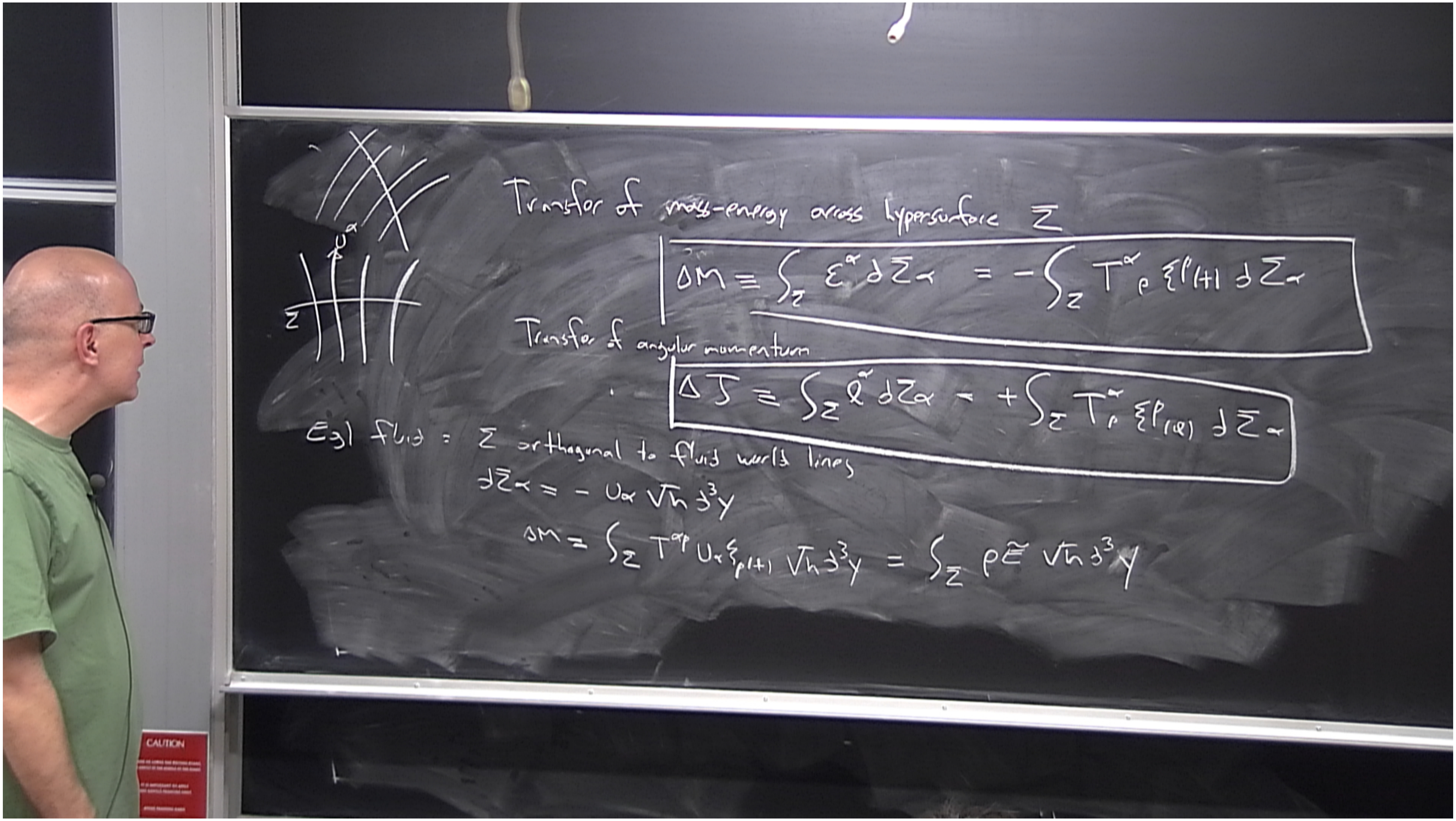
$$\Delta M \equiv \int_\Sigma \varepsilon^\alpha \, d\Sigma_\alpha = -\int_\Sigma T^\alpha{}_\beta \, \xi^\beta_{(t)} \, d\Sigma_\alpha$$

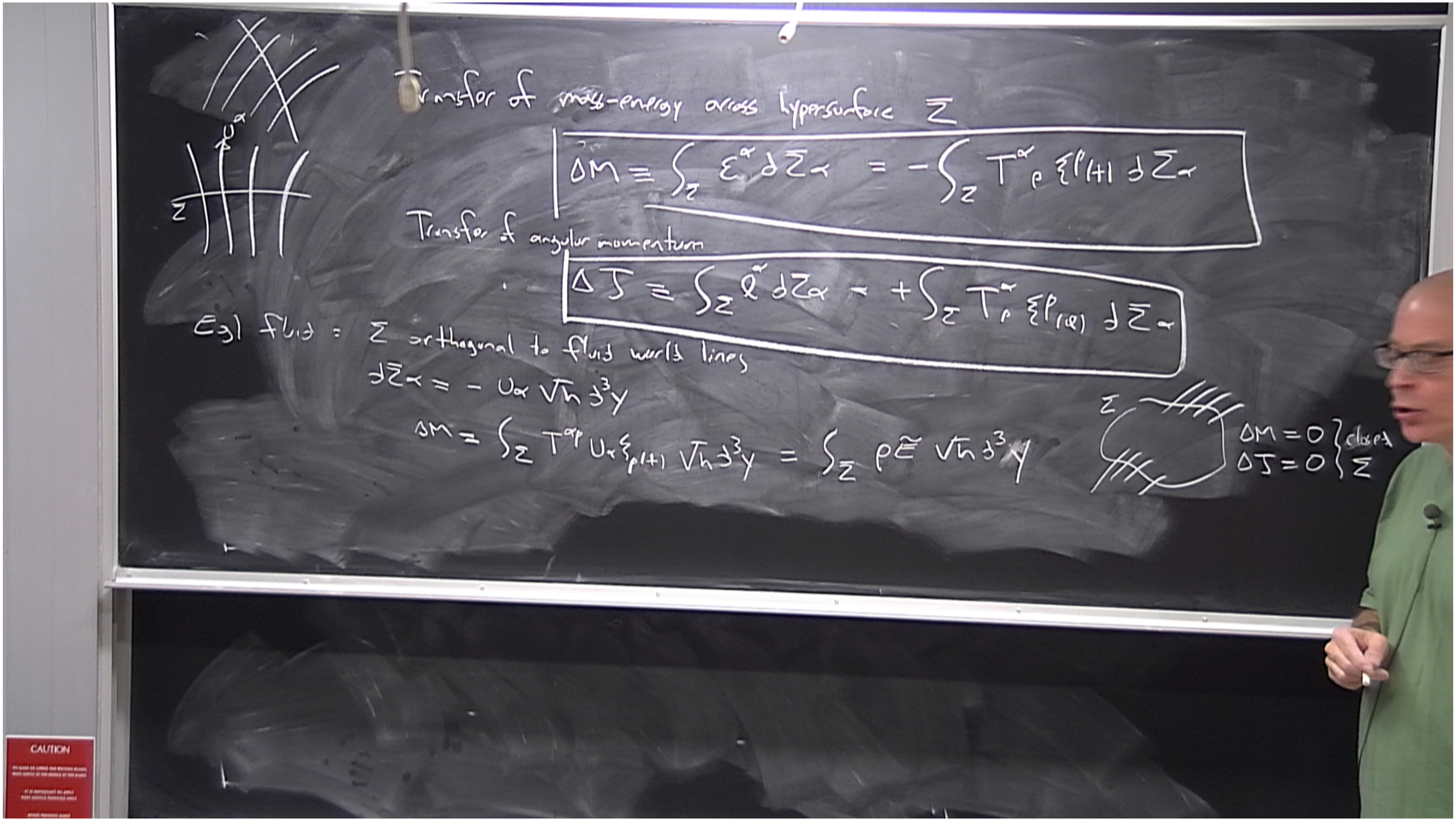
Transfer of angular momentum

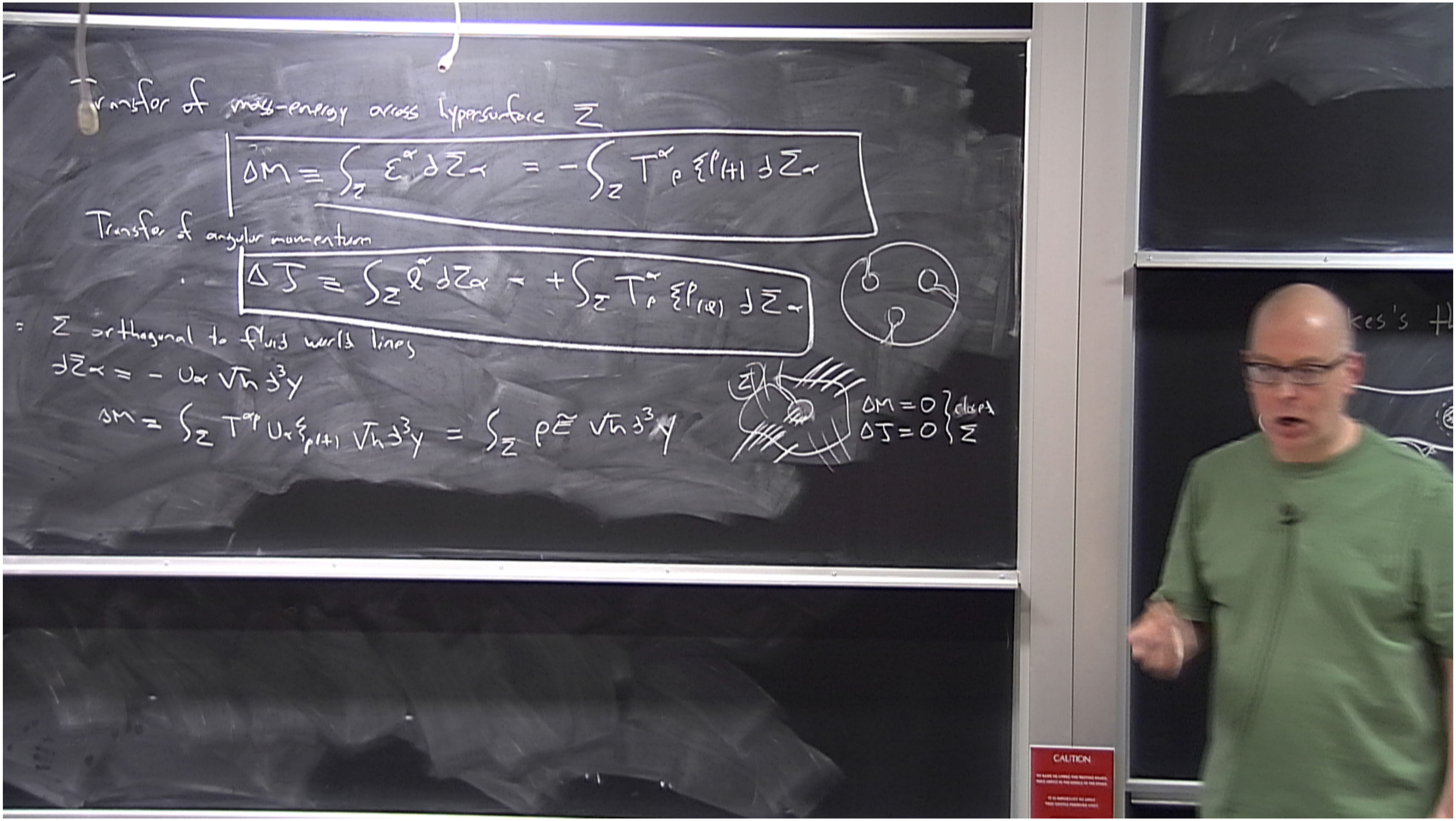
$$\Delta J \equiv \int_\Sigma \ell^\alpha \, d\Sigma_\alpha = +\int_\Sigma T^\alpha{}_\beta \, \xi^\beta_{(\varphi)} \, d\Sigma_\alpha$$

E.g.) fluid: Σ orthogonal to fluid world lines


Transfer of mass-energy across hypersurface Σ
ΔM ≡ ∫_Σ ε^α dΣ_α = −∫_Σ T^α_β ξ^β_(t) dΣ_α
Transfer of angular momentum
ΔJ ≡ ∫_Σ ℓ^α dΣ_α = +∫_Σ T^α_β ξ^β_(φ) dΣ_α
Eg) fluid : Σ orthogonal to fluid world lines
dΣ_α = − U_α √h d³y
ΔM = ∫_Σ T^αβ U_α ξ_β(t) √h d³y = ∫_Σ ρẼ √h d³y
Σ
U^α


Transfer of mass-energy across hypersurface Σ
ΔM ≡ ∫_Σ ε^α dΣ_α = −∫_Σ T^α_β ξ^β_(t) dΣ_α
Transfer of angular momentum
ΔJ ≡ ∫_Σ ℓ^α dΣ_α = +∫_Σ T^α_β ξ^β_(φ) dΣ_α
E.g. fluid: Σ orthogonal to fluid world lines
dΣ_α = − u_α √h d³y
ΔM = ∫_Σ T^αβ u_α ξ_β(t) √h d³y = ∫_Σ ρ E √h d³y
u^α
Σ
CAUTION


ransfer of mass-energy across hypersurface Σ
$\Delta M \equiv \int_\Sigma \varepsilon^\alpha d\Sigma_\alpha = -\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(t)} d\Sigma_\alpha$
Transfer of angular momentum
$\Delta J \equiv \int_\Sigma \ell^\alpha d\Sigma_\alpha = +\int_\Sigma T^\alpha{}_\beta \xi^\beta_{(\phi)} d\Sigma_\alpha$
Eg) fluid: Σ orthogonal to fluid world lines
$d\Sigma_\alpha = -u_\alpha \sqrt{h}\, d^3y$
$\Delta M = \int_\Sigma T^{\alpha\beta} u_\alpha \xi_{\beta(t)} \sqrt{h}\, d^3y = \int_\Sigma \rho\tilde{\varepsilon} \sqrt{h}\, d^3y$
u^α
Σ
$\Delta M = 0$ } closed
$\Delta J = 0$ } Σ
CAUTION


Transfer of mass-energy across hypersurface Σ
ΔM ≡ ∫_Σ ε^α dΣ_α = −∫_Σ T^α_β ξ^β_(t) dΣ_α
Transfer of angular momentum
ΔJ ≡ ∫_Σ ℓ^α dΣ_α = +∫_Σ T^α_β ξ^β_(φ) dΣ_α
Σ orthogonal to fluid world lines
dΣ_α = − u_α √h d³y
ΔM = ∫_Σ T^αβ u_α ξ_β(t) √h d³y = ∫_Σ ρ ε̃ √h d³y
ΔM = 0
ΔJ = 0
closed Σ
CAUTION