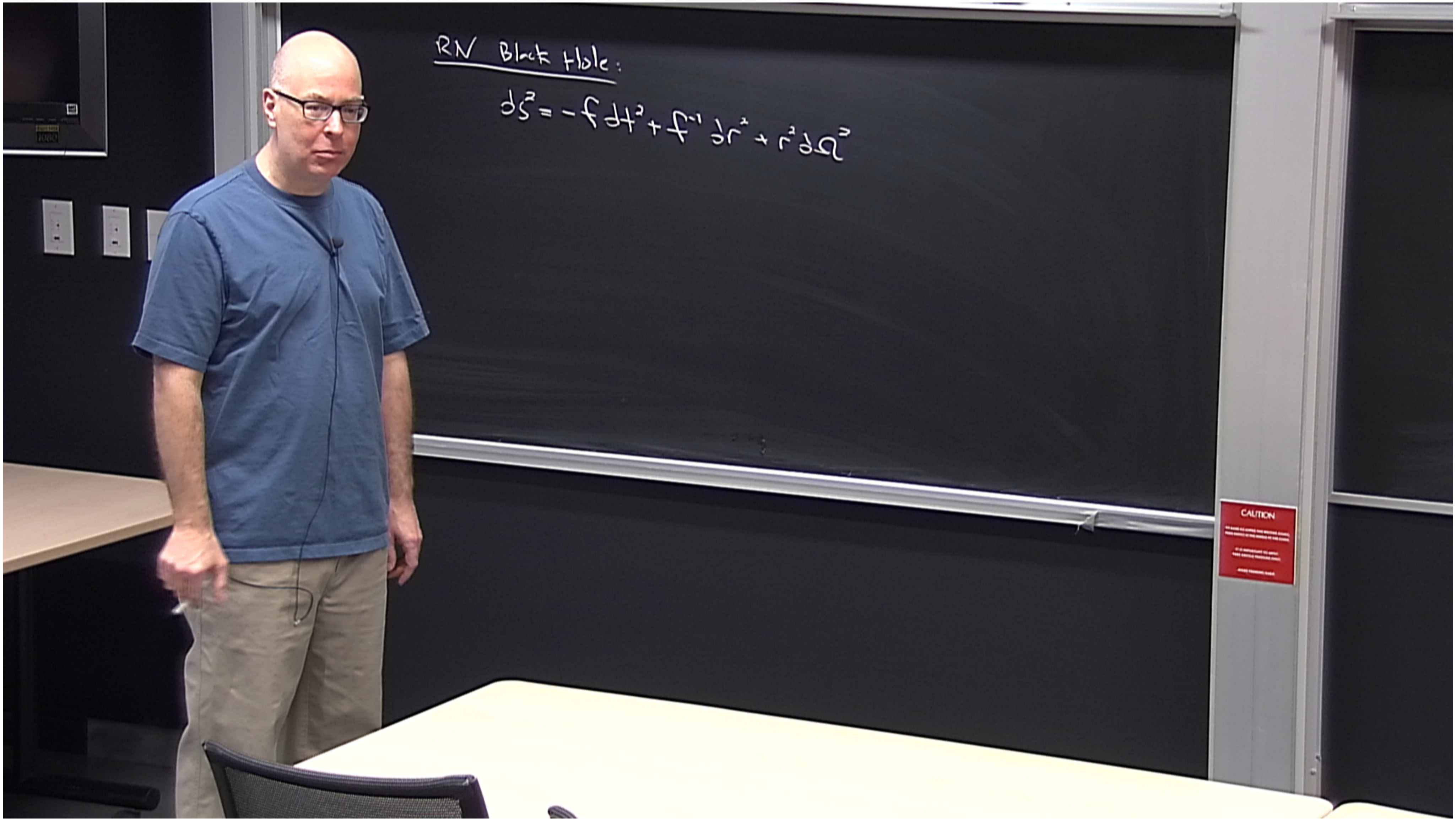
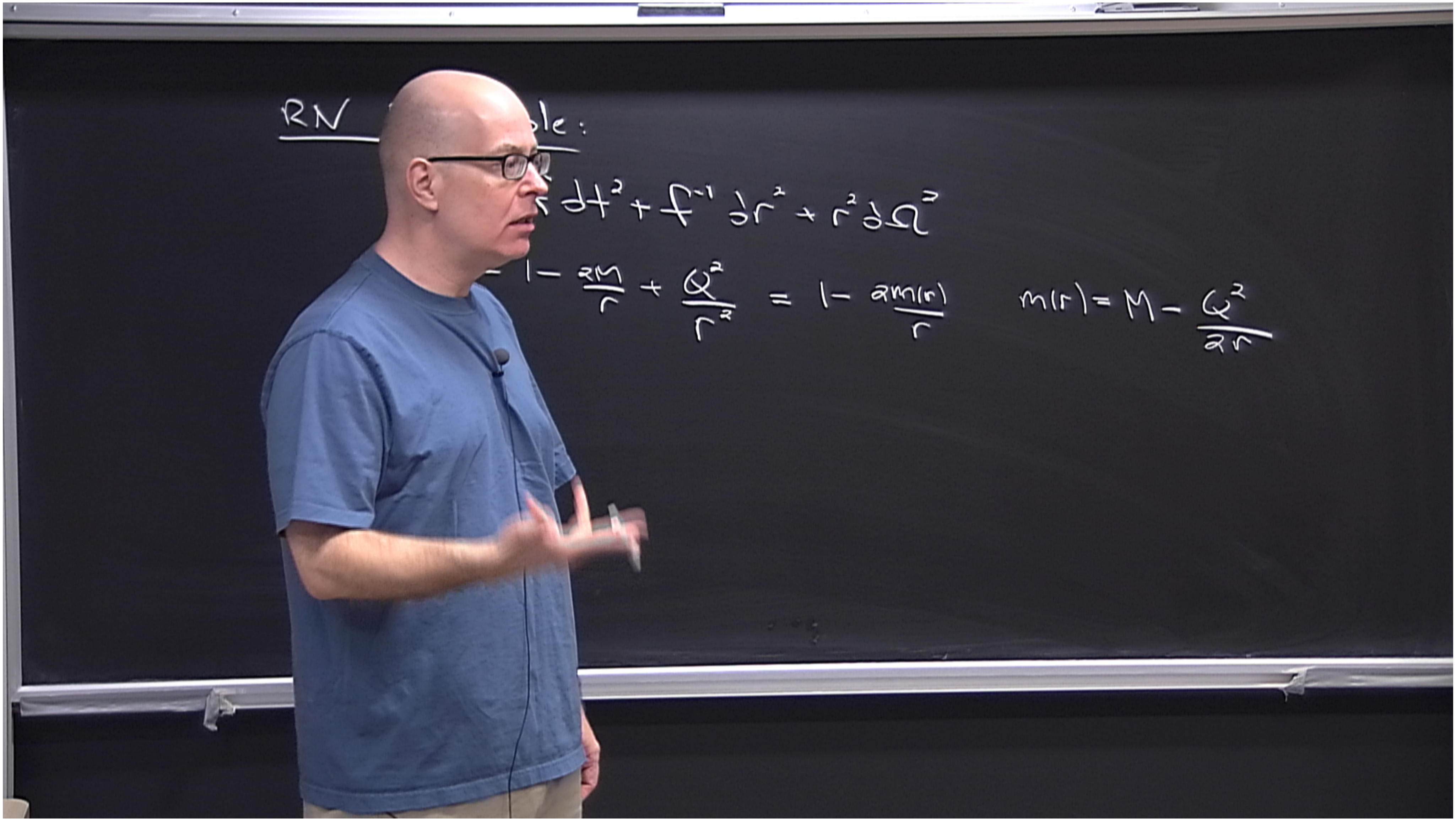
Title: Advanced General Relativity - Lecture 21

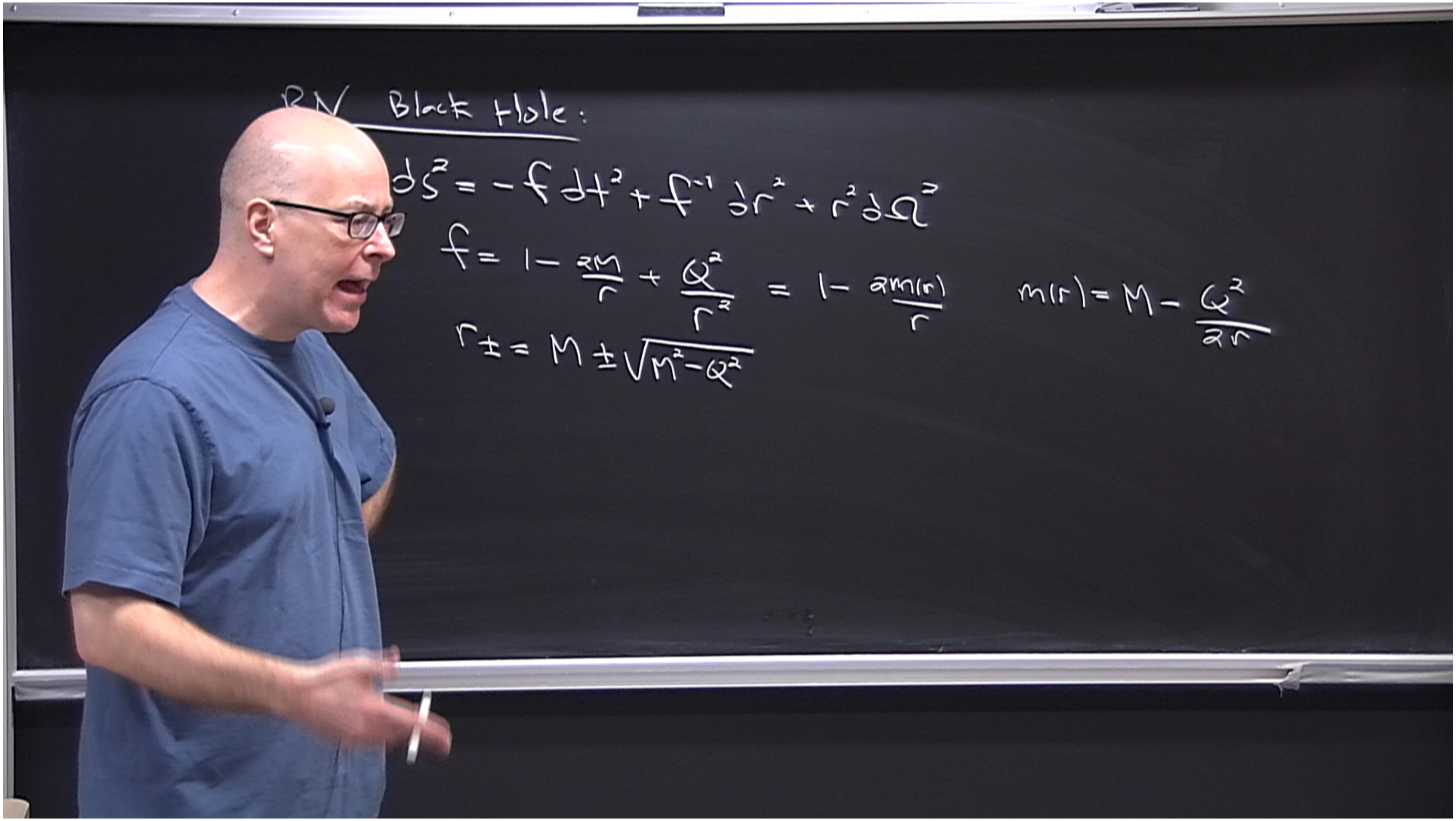
Date: Mar 28, 2012 10:00 AM

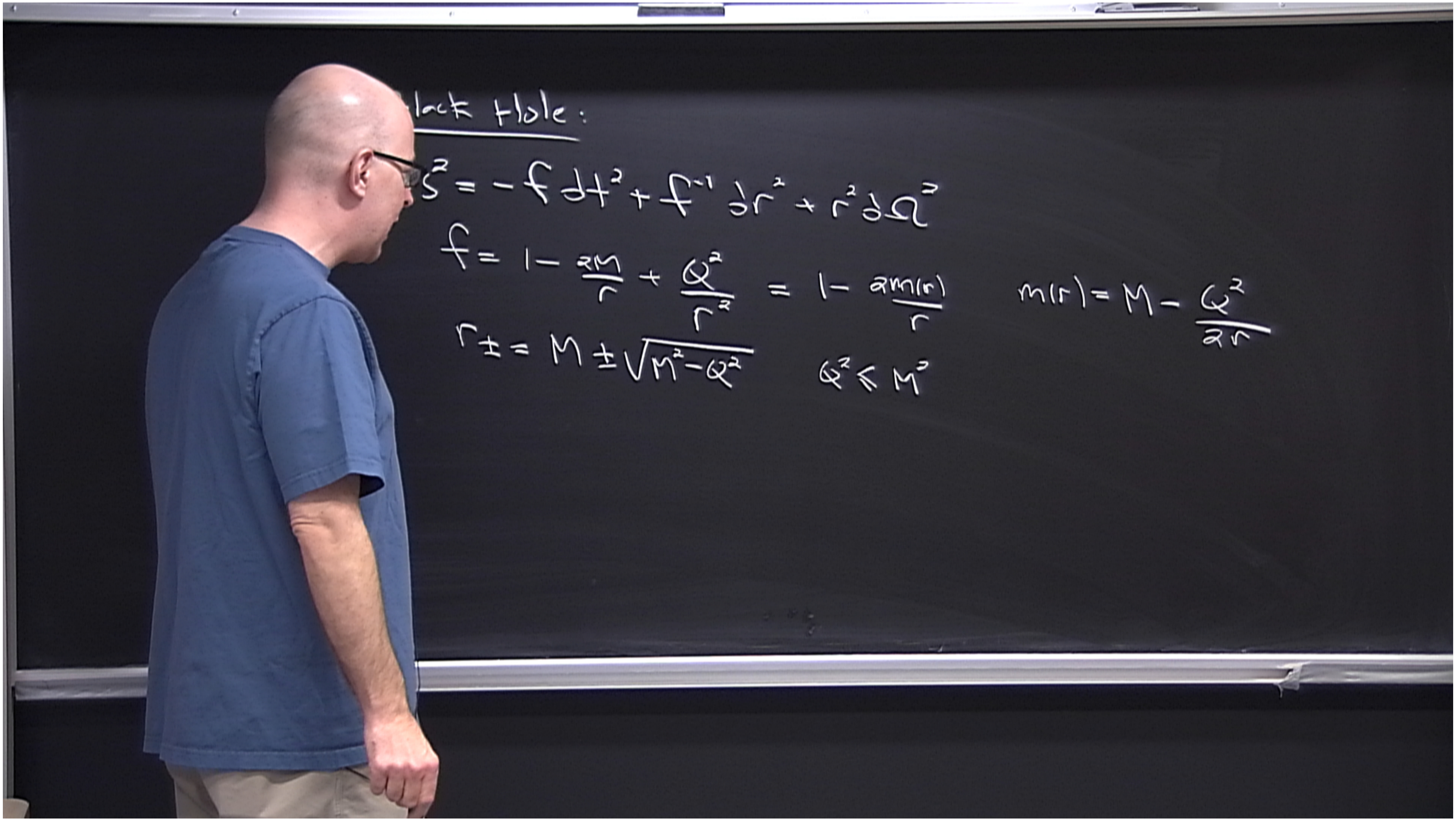
URL: http://pirsa.org/12030071

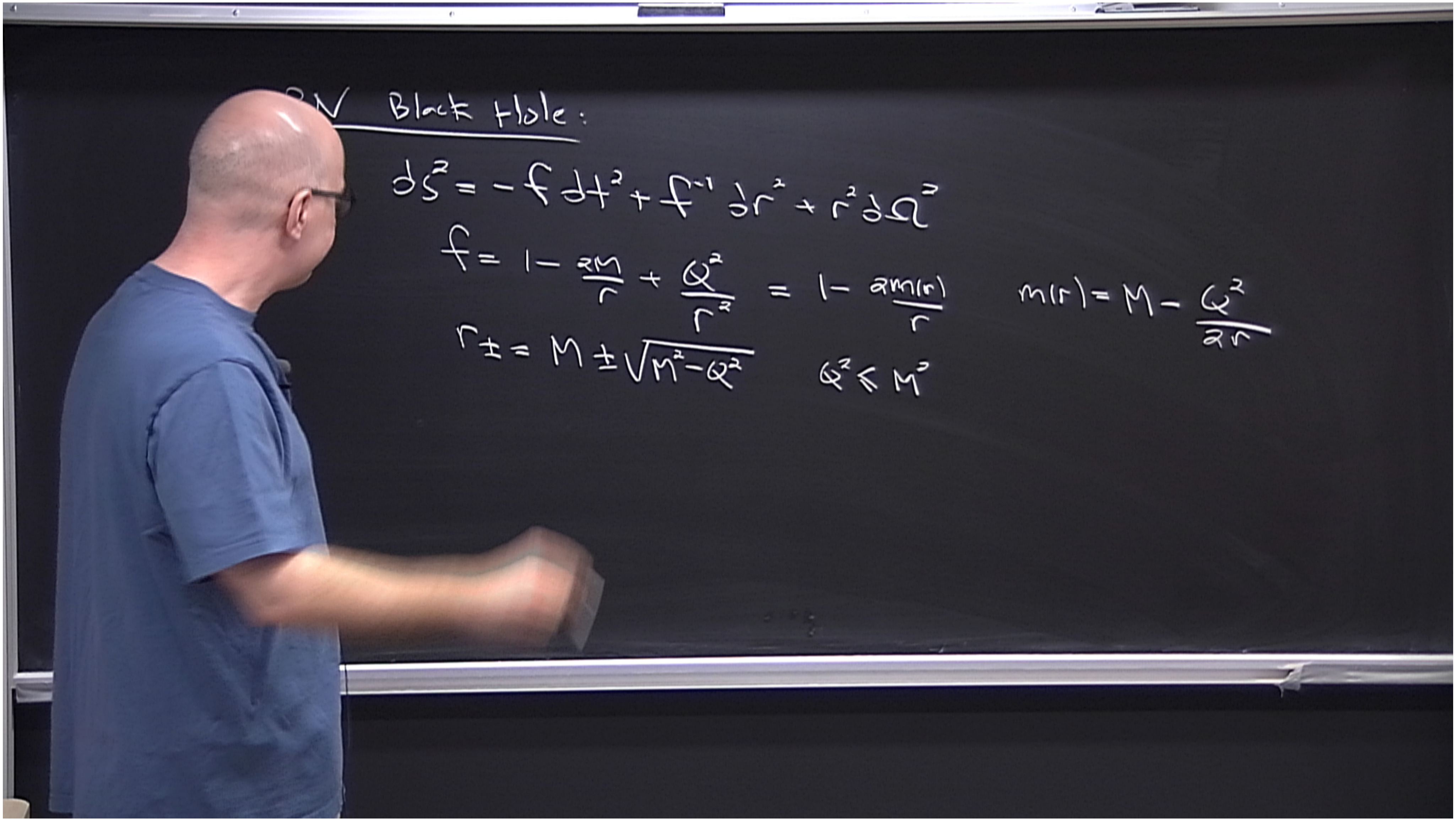
Abstract:


RN Black Hole:
ds² = −f dt² + f⁻¹ dr² + r² dΩ²
CAUTION

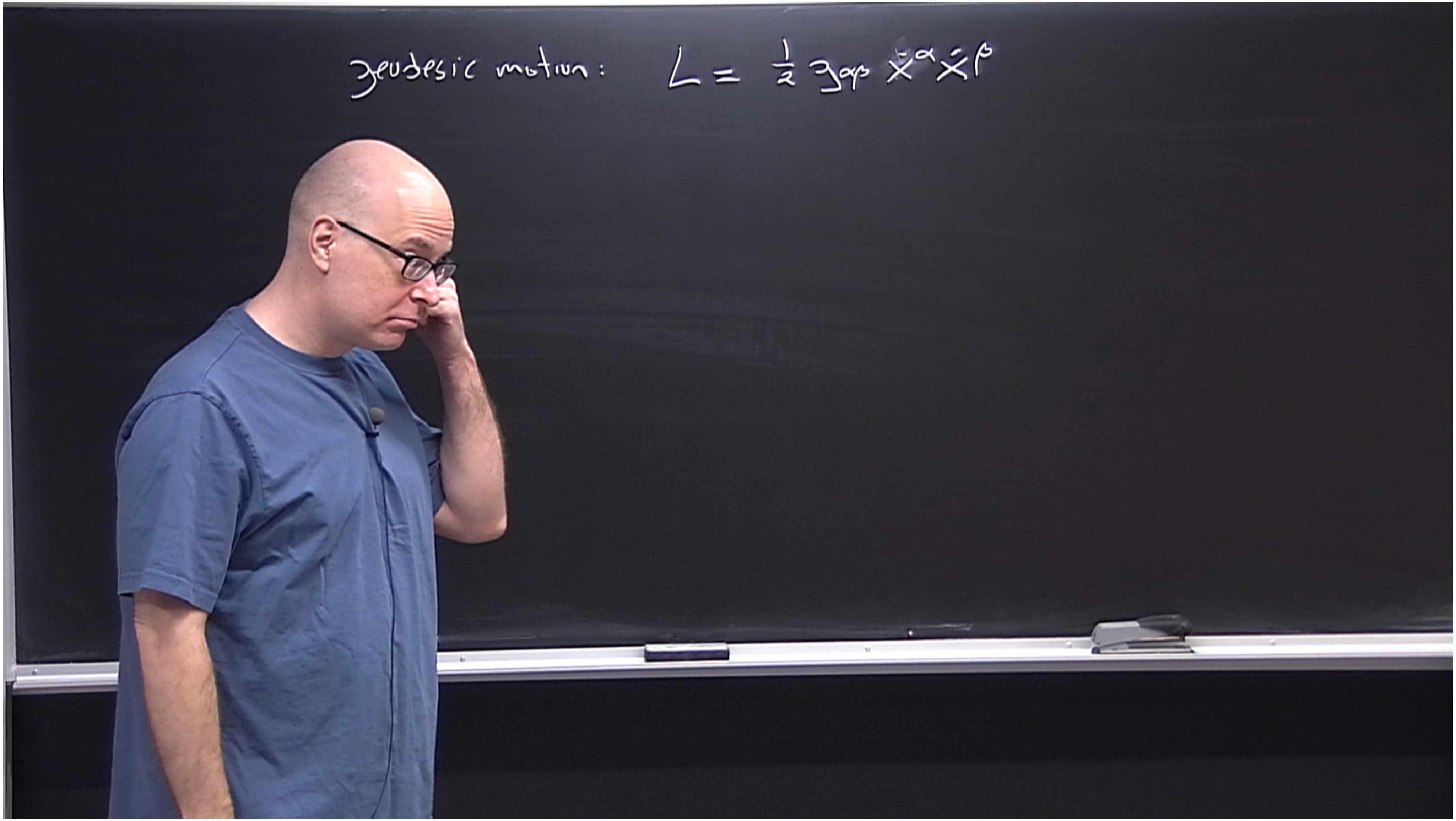

RN
le:
dt² + f⁻¹ dr² + r² dΩ²
1 − 2M/r + Q²/r² = 1 − 2m(r)/r
m(r) = M − Q²/2r

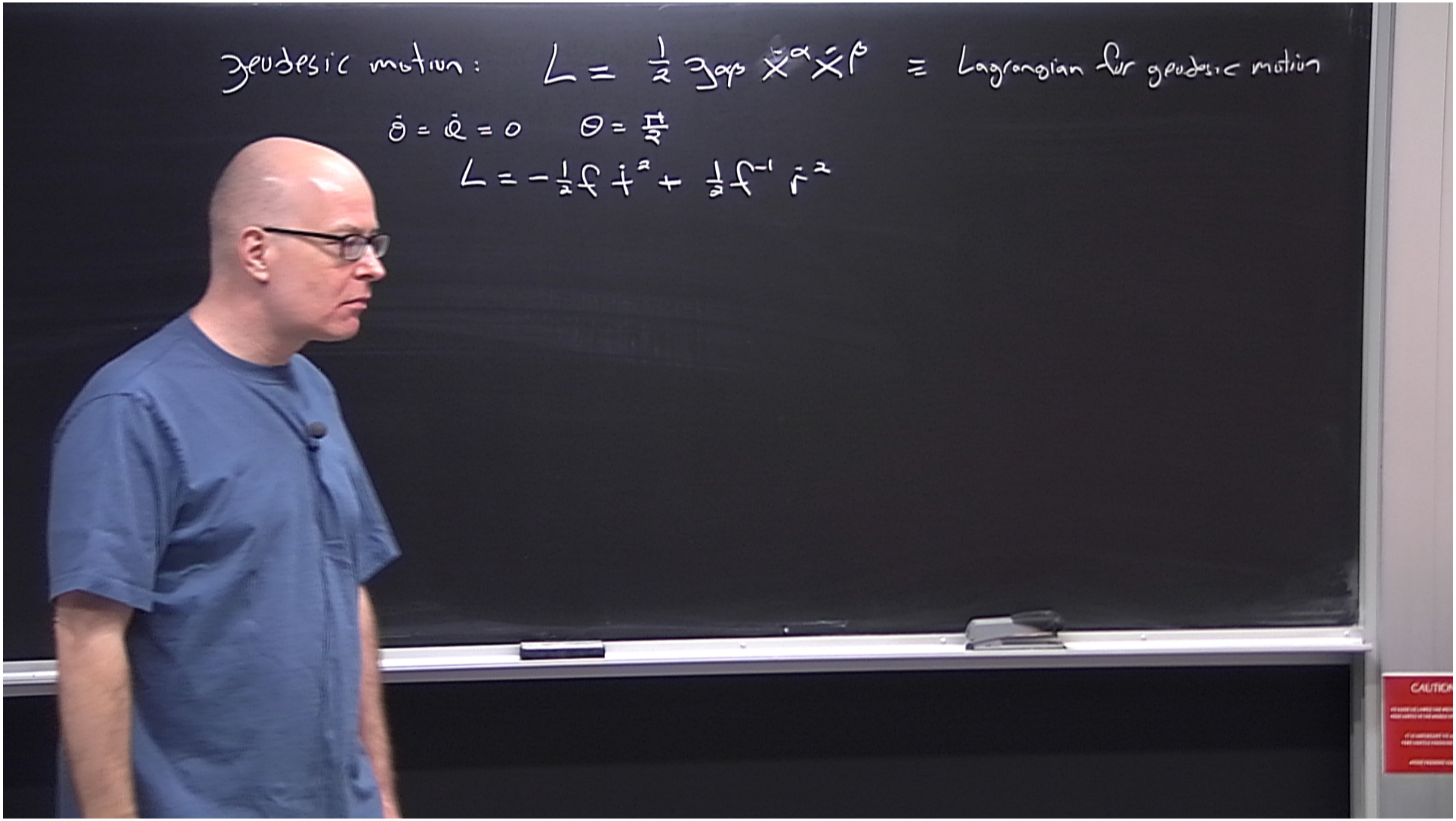

RN Black Hole:
ds² = −f dt² + f⁻¹ dr² + r² dΩ²
f = 1 − 2M/r + Q²/r² = 1 − 2m(r)/r
m(r) = M − Q²/2r
r± = M ± √(M² − Q²)

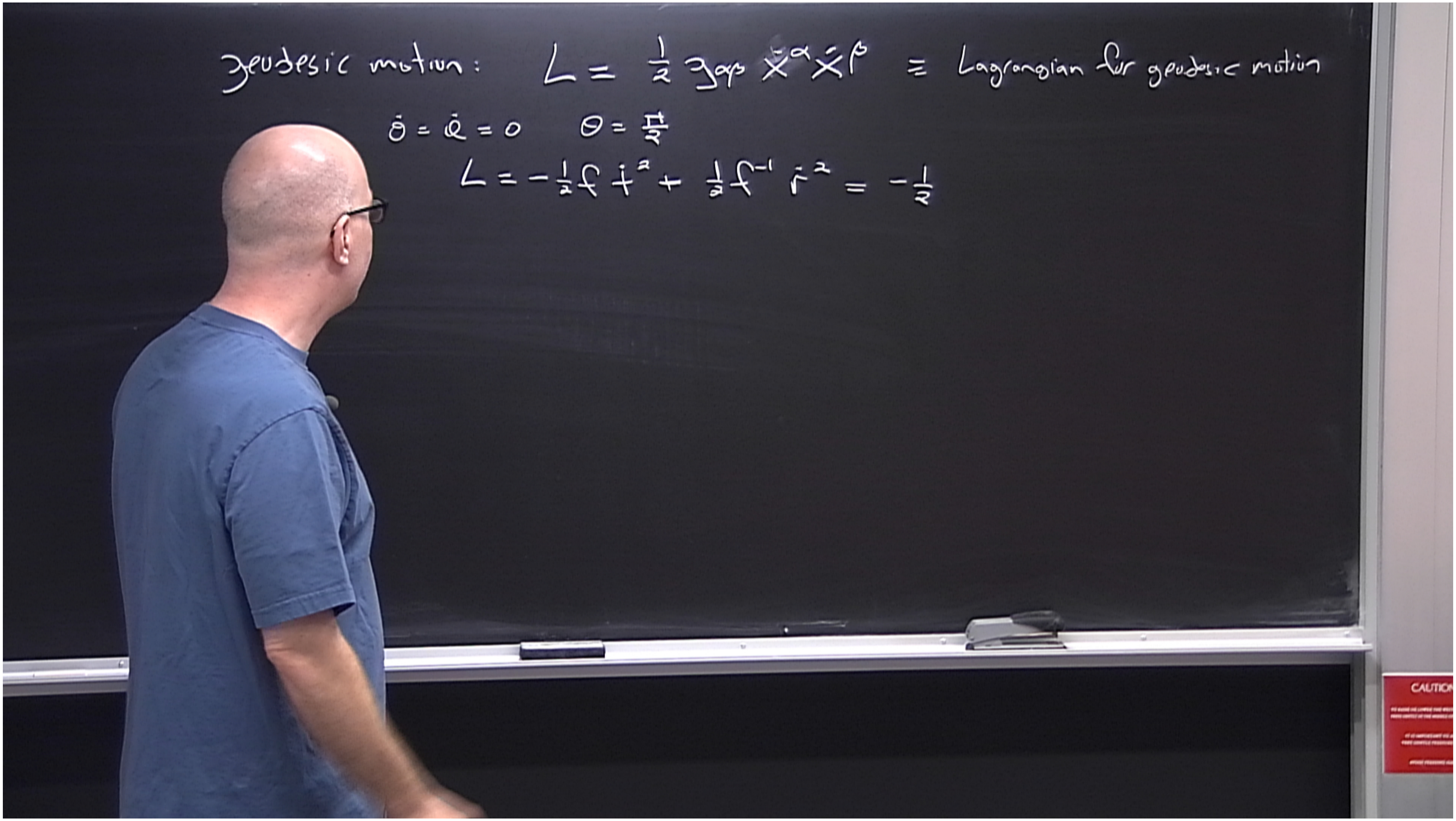

lack Hole:
$s^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2 d\Omega^2$
$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 1 - \frac{2m(r)}{r}$
$m(r) = M - \frac{Q^2}{2r}$
$r_\pm = M \pm \sqrt{M^2 - Q^2}$
$Q^2 \leq M^2$

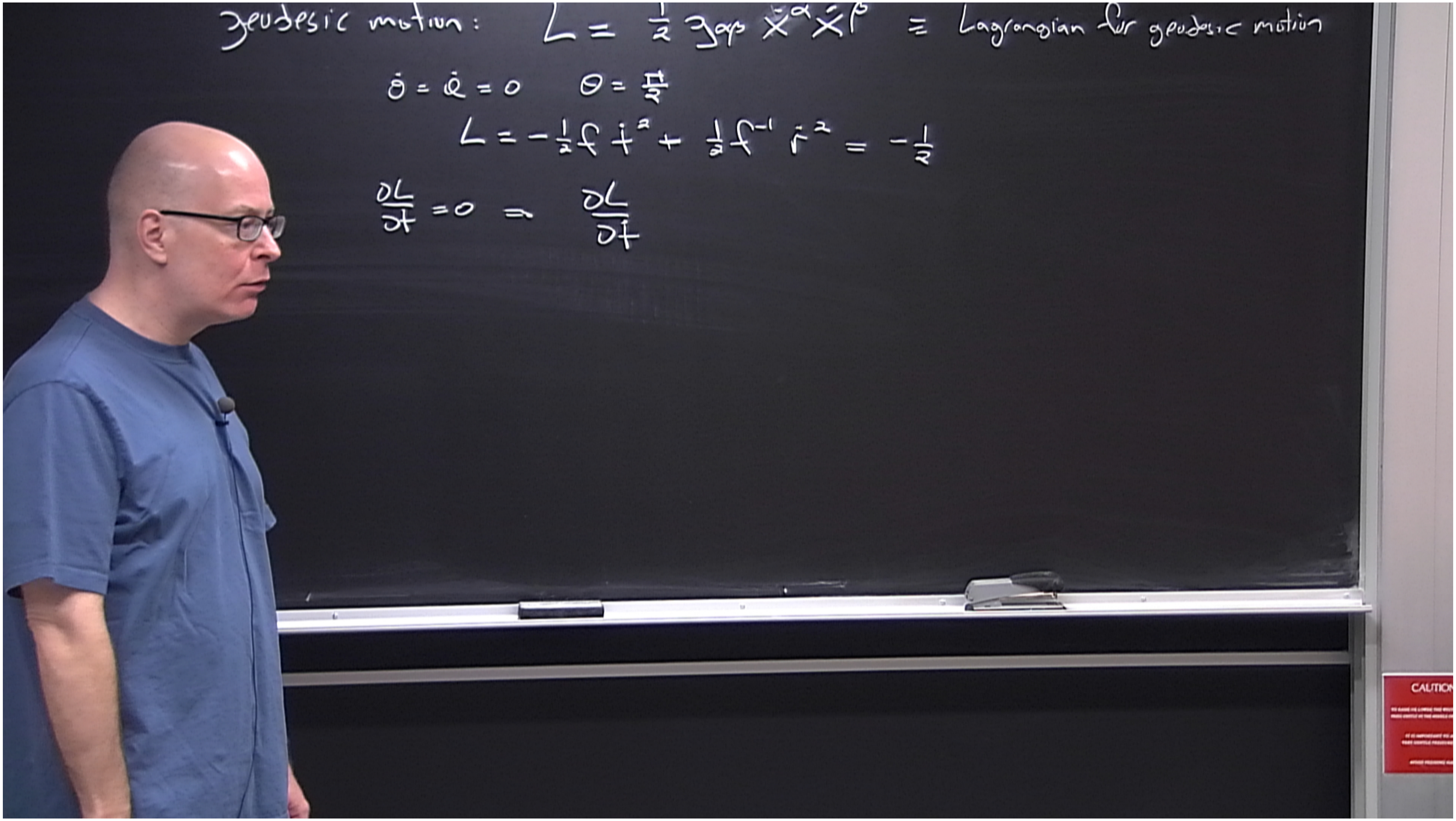

RN Black Hole:
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 1 - \frac{2m(r)}{r}$
$m(r) = M - \frac{Q^2}{2r}$
$r_\pm = M \pm \sqrt{M^2 - Q^2}$
$Q^2 \leq M^2$

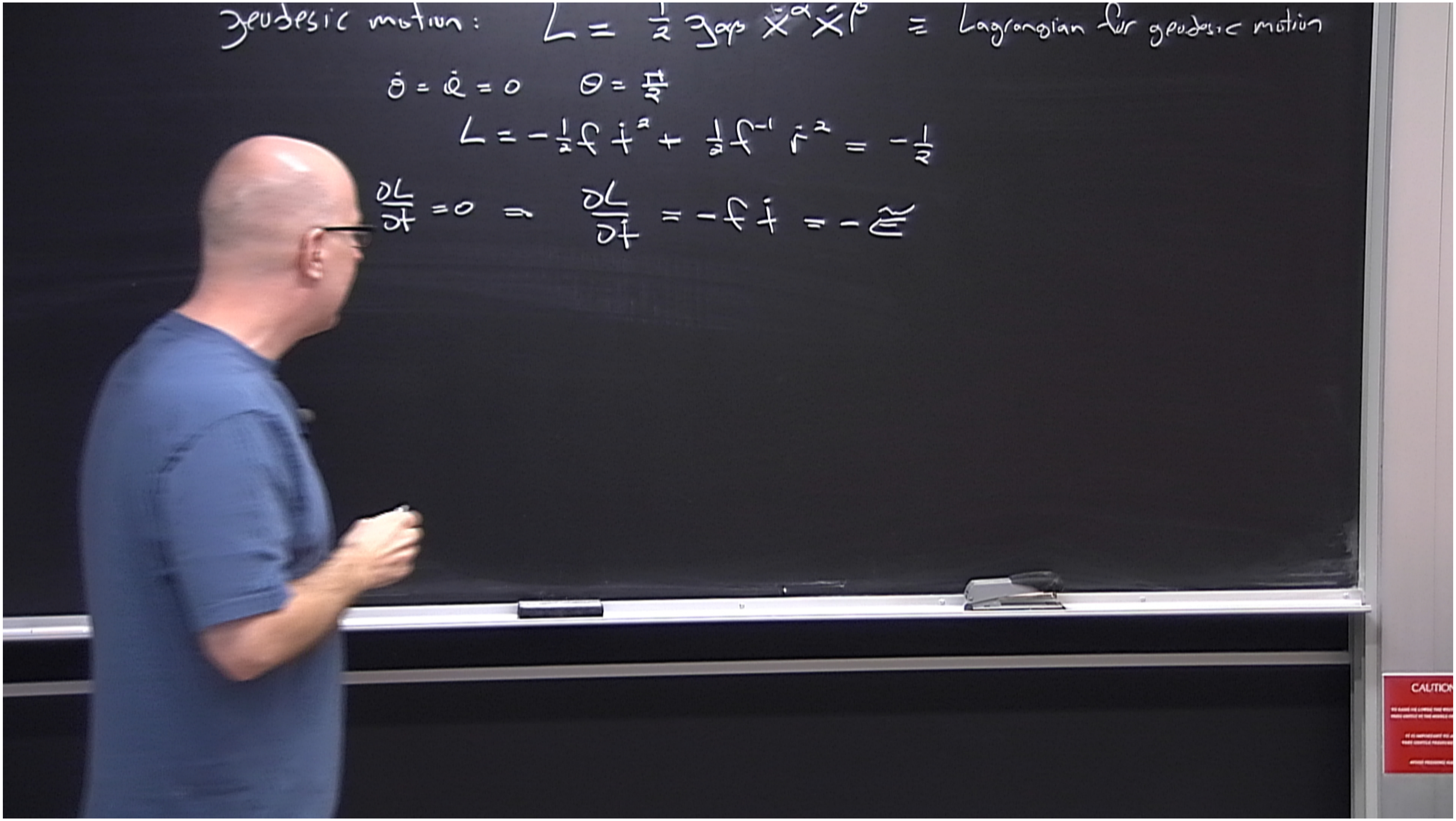

geodesic motion:

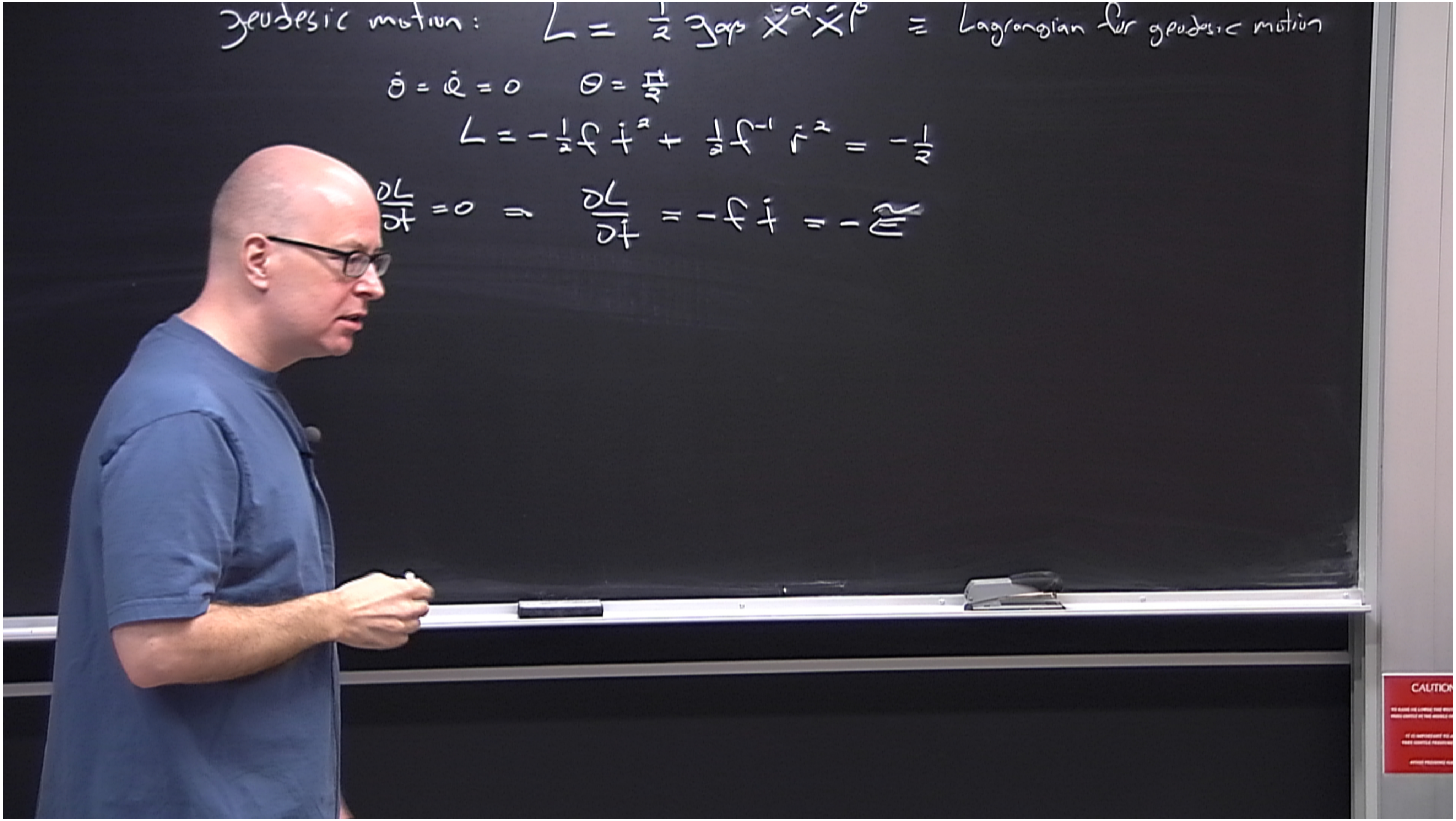

geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$

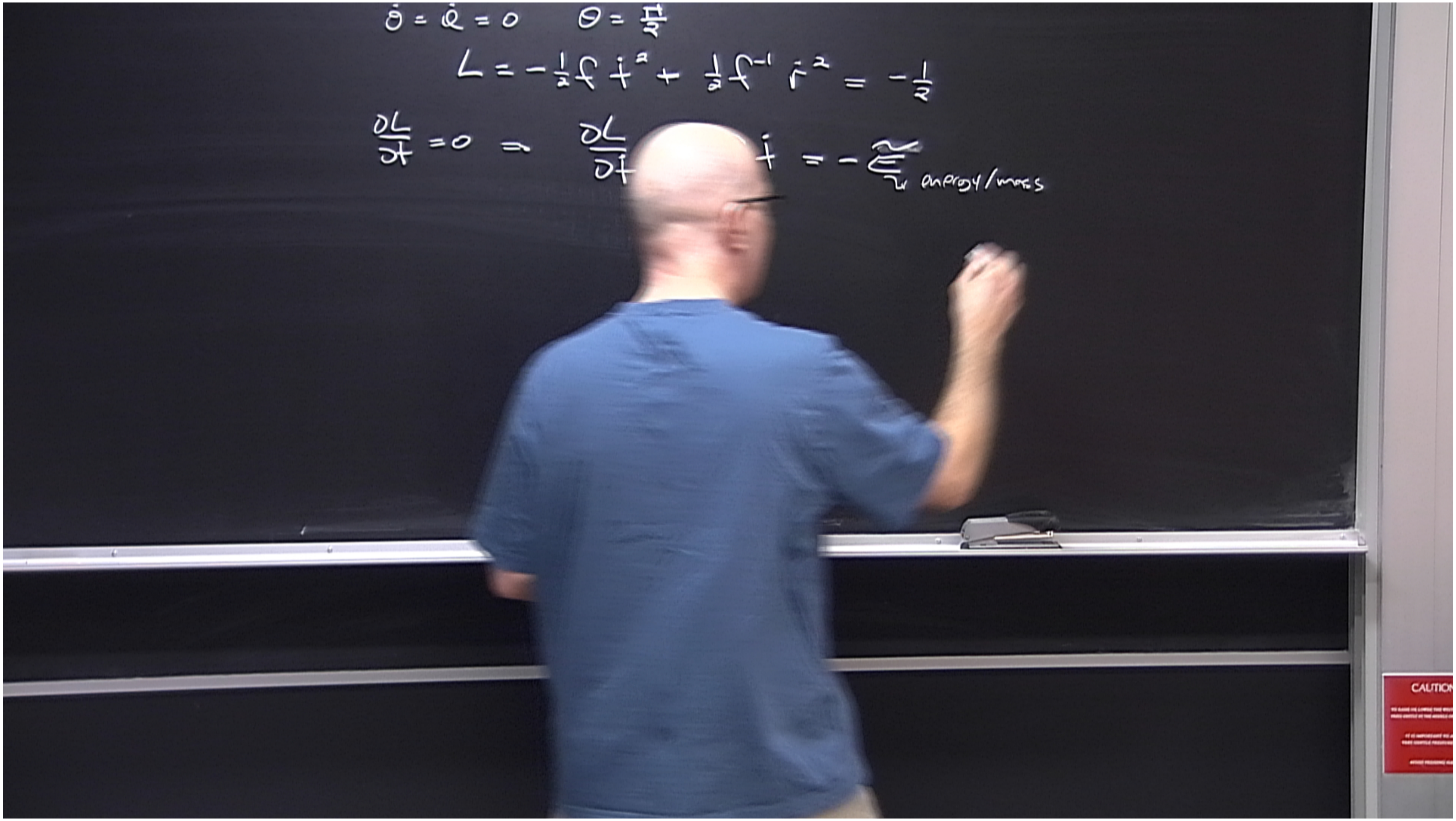

geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ = Lagrangian for geodesic motion
$\dot{\theta} = \dot{\varphi} = 0$ $\theta = \frac{\pi}{2}$
$L = -\frac{1}{2} f \dot{t}^2 + \frac{1}{2} f^{-1} \dot{r}^2$
CAUTION

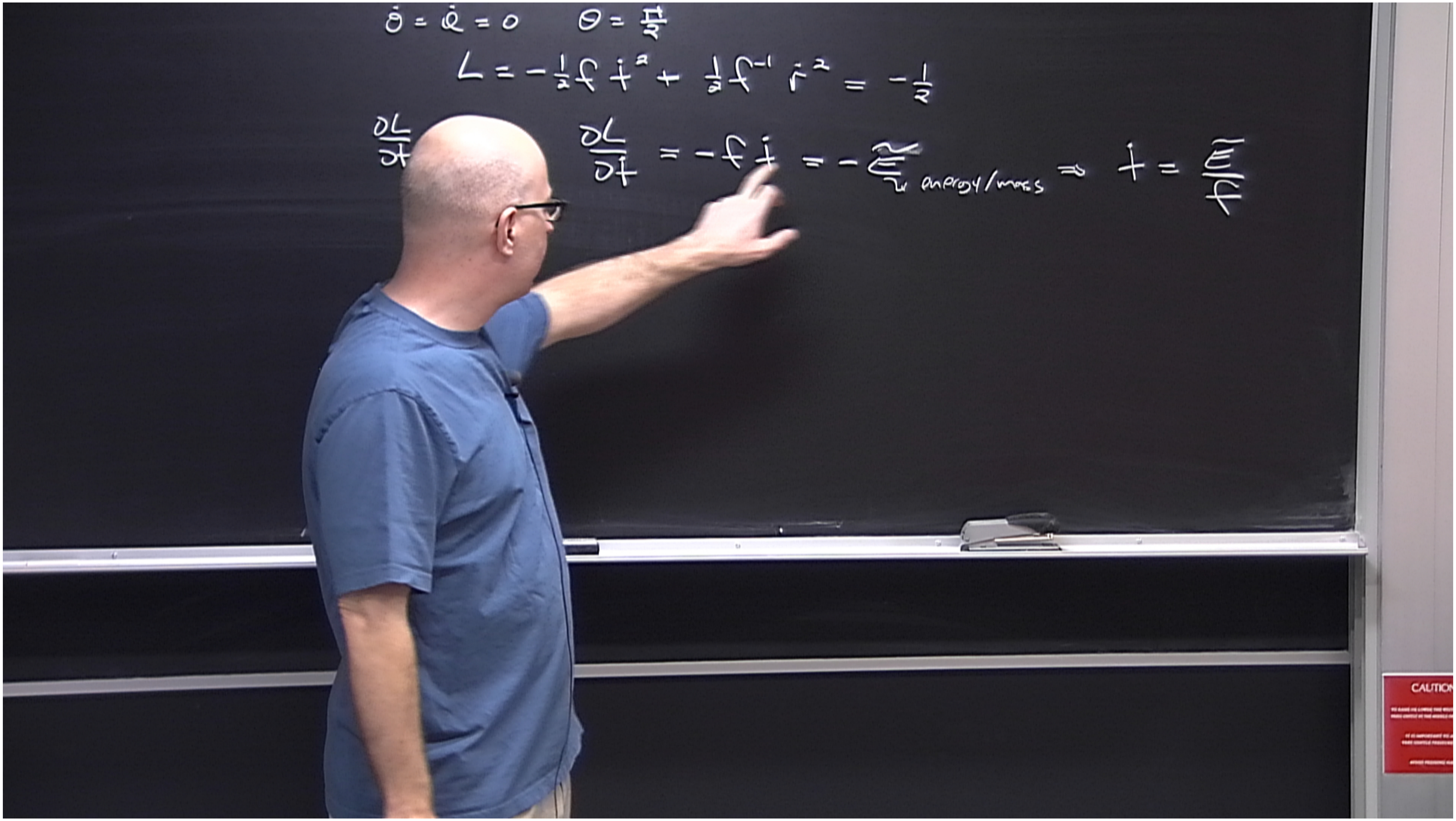

geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ ≡ Lagrangian for geodesic motion
$\dot{\theta} = \dot{\varphi} = 0 \quad \theta = \frac{\pi}{2}$
$L = -\frac{1}{2} f \dot{t}^2 + \frac{1}{2} f^{-1} \dot{r}^2 = -\frac{1}{2}$
CAUTION

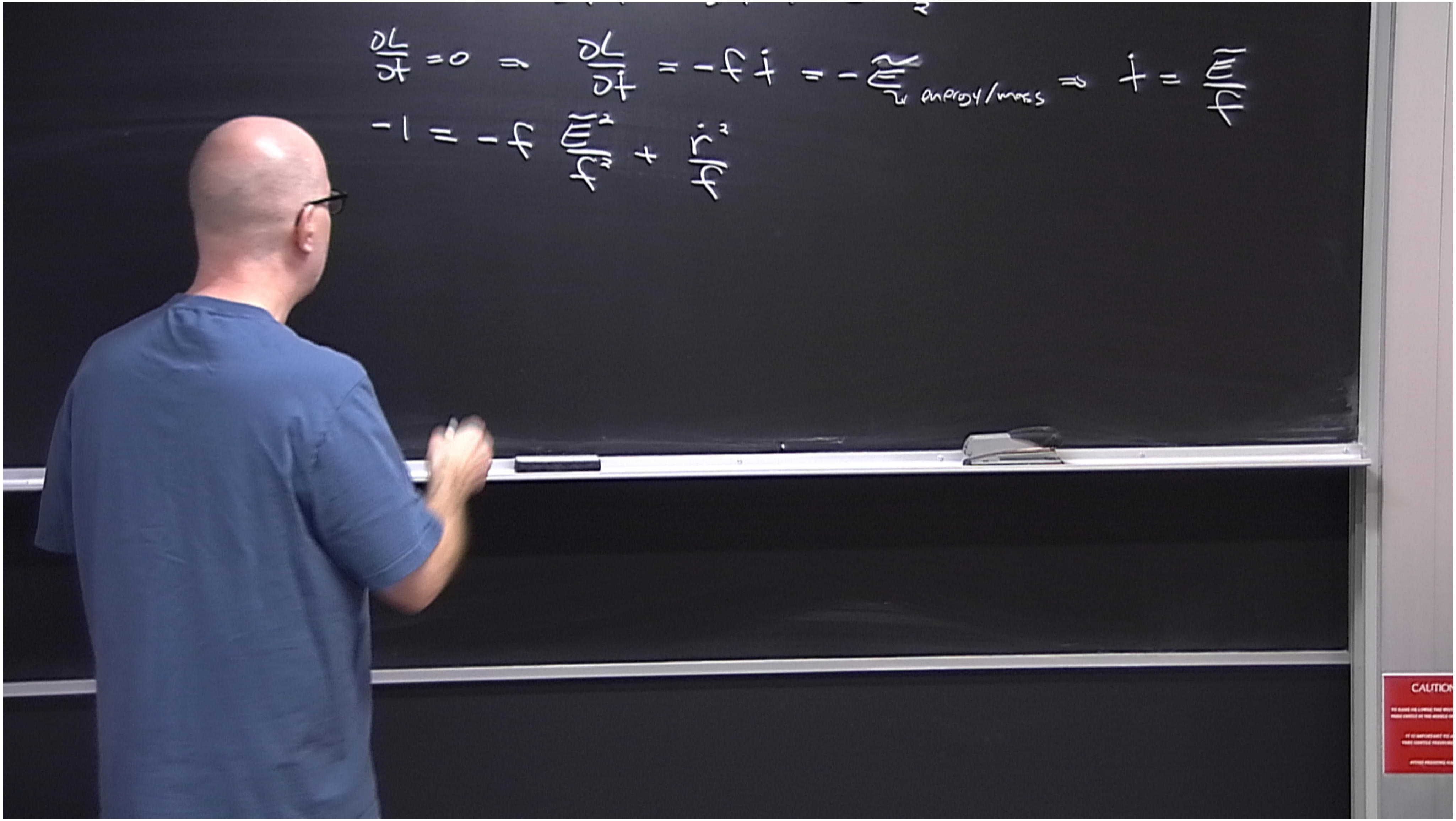

geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ ≡ Lagrangian for geodesic motion
$\dot{\theta} = \dot{\varphi} = 0 \quad \theta = \frac{\pi}{2}$
$L = -\frac{1}{2} f \dot{t}^2 + \frac{1}{2} f^{-1} \dot{r}^2 = -\frac{1}{2}$
$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}}$
CAUTION


geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ ≡ Lagrangian for geodesic motion
$\dot{\theta} = \dot{\varphi} = 0 \quad \theta = \frac{\pi}{2}$
$L = -\frac{1}{2} f \dot{t}^2 + \frac{1}{2} f^{-1} \dot{r}^2 = -\frac{1}{2}$
$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = -f \dot{t} = -\tilde{E}$

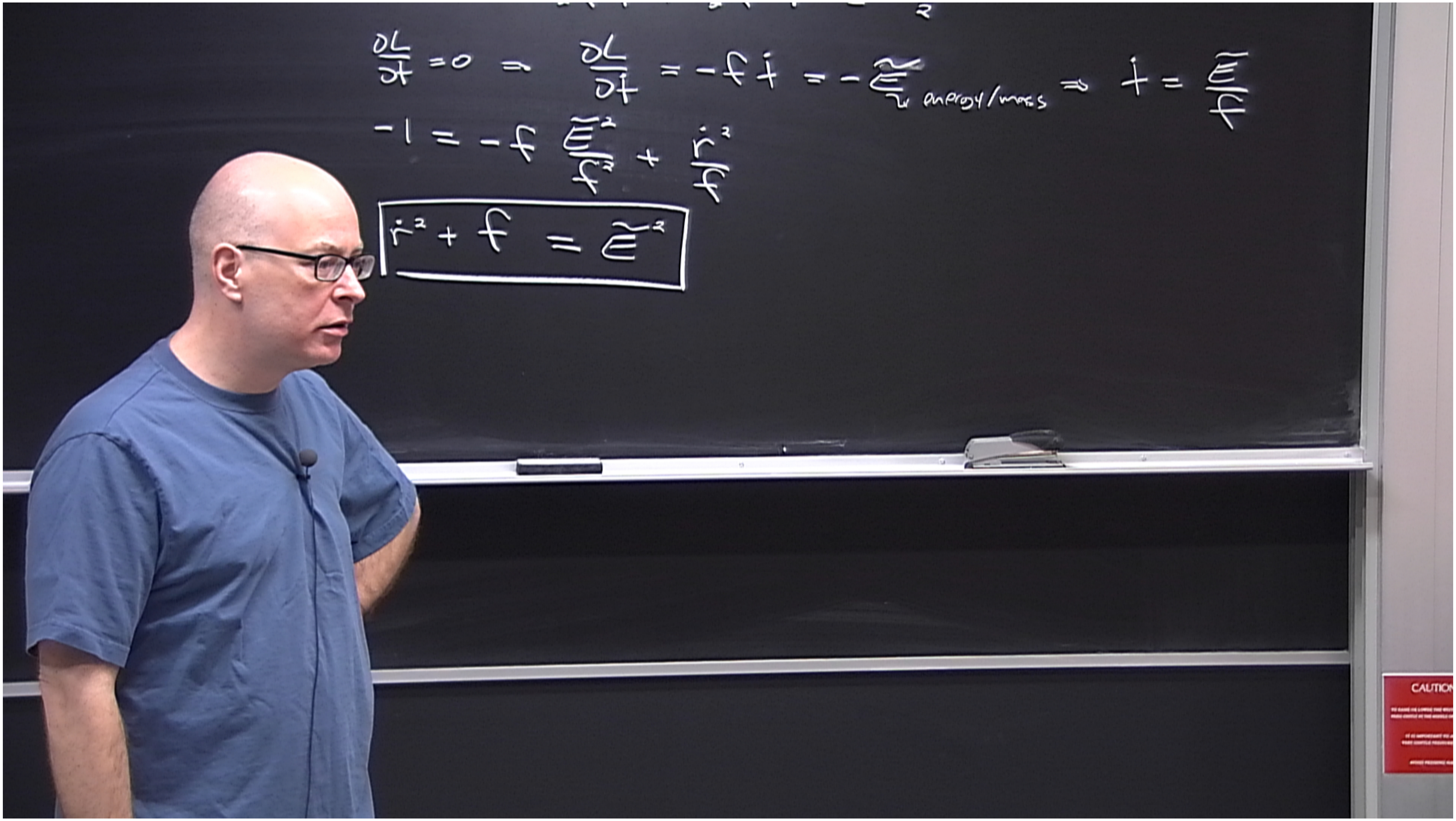

geodesic motion: $L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ ≡ Lagrangian for geodesic motion
$\dot{\theta} = \dot{\varphi} = 0 \quad \theta = \frac{\pi}{2}$
$L = -\frac{1}{2} f \dot{t}^2 + \frac{1}{2} f^{-1} \dot{r}^2 = -\frac{1}{2}$
$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = -f\dot{t} = -\tilde{E}$
CAUTION

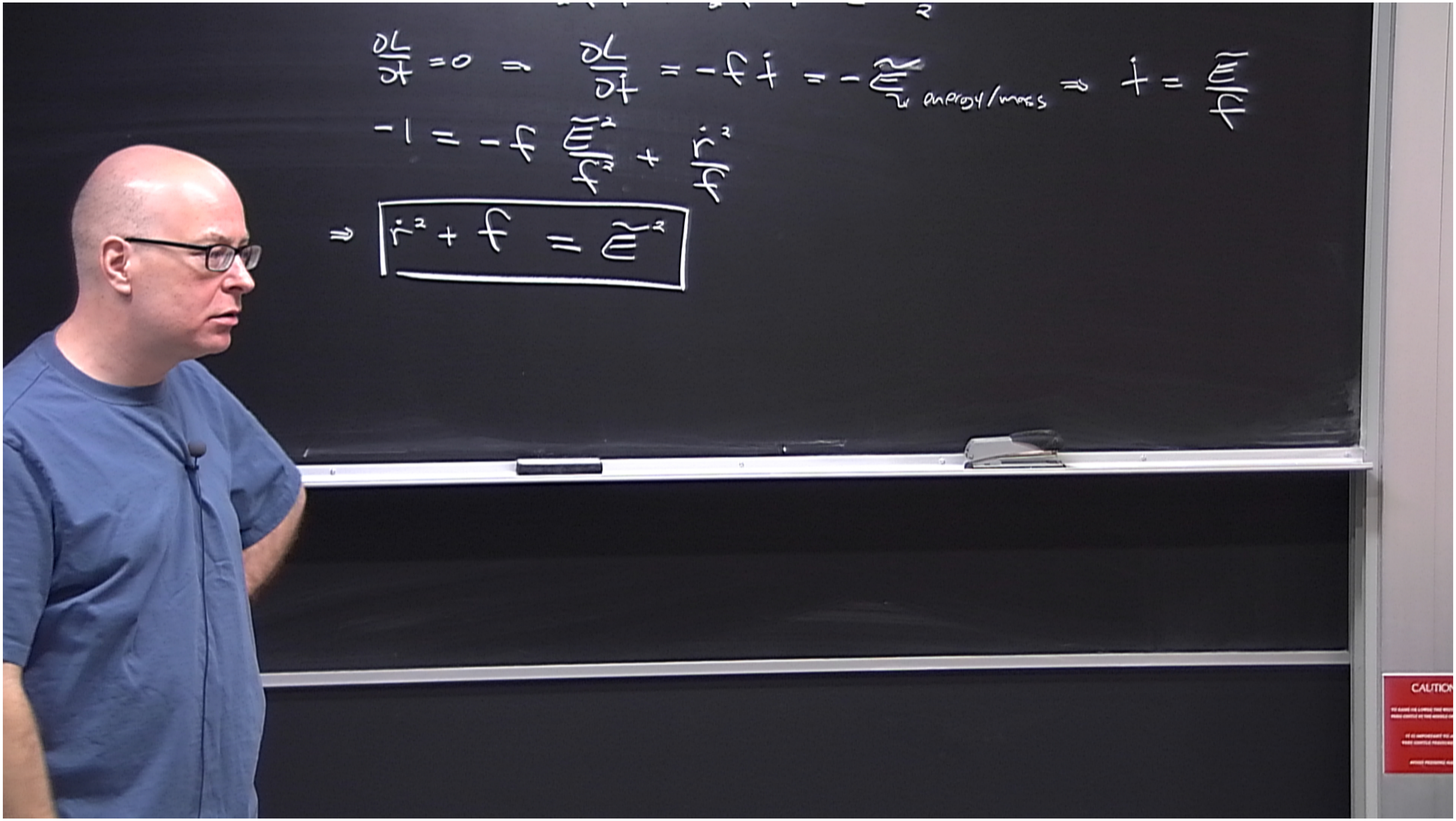

θ̇ = φ̇ = 0   θ = π/2
L = −½ f ṫ² + ½ f⁻¹ ṙ² = −½
∂L/∂t = 0 ⇒ ∂L/∂ṫ
ṫ = −E
energy/mass

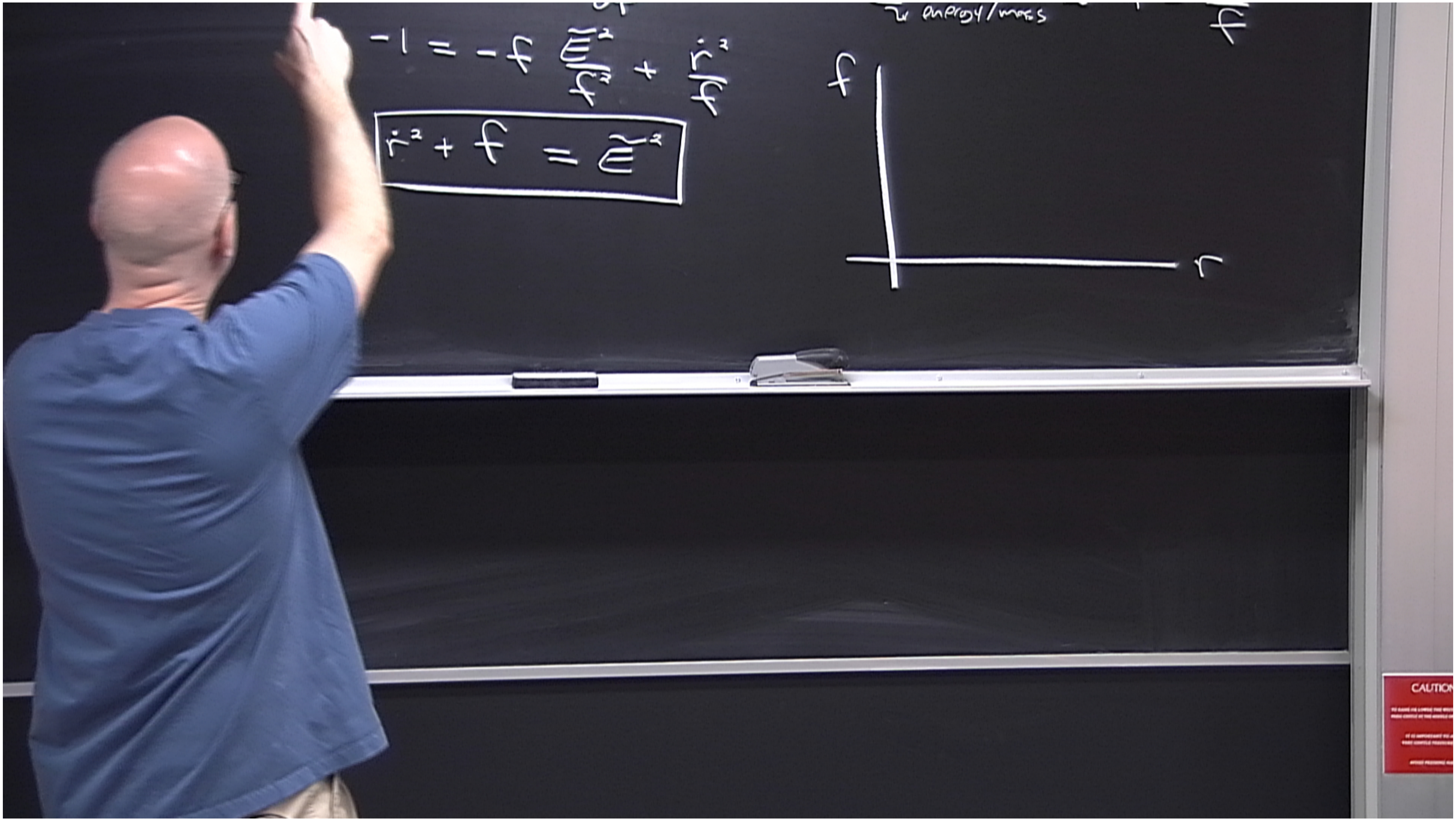

θ̇ = φ̇ = 0   θ = π/2
L = −½ f ṫ² + ½ f⁻¹ ṙ² = −½
∂L/∂ṫ = −f ṫ = −Ẽ energy/mass ⇒ ṫ = Ẽ/f
CAUTION

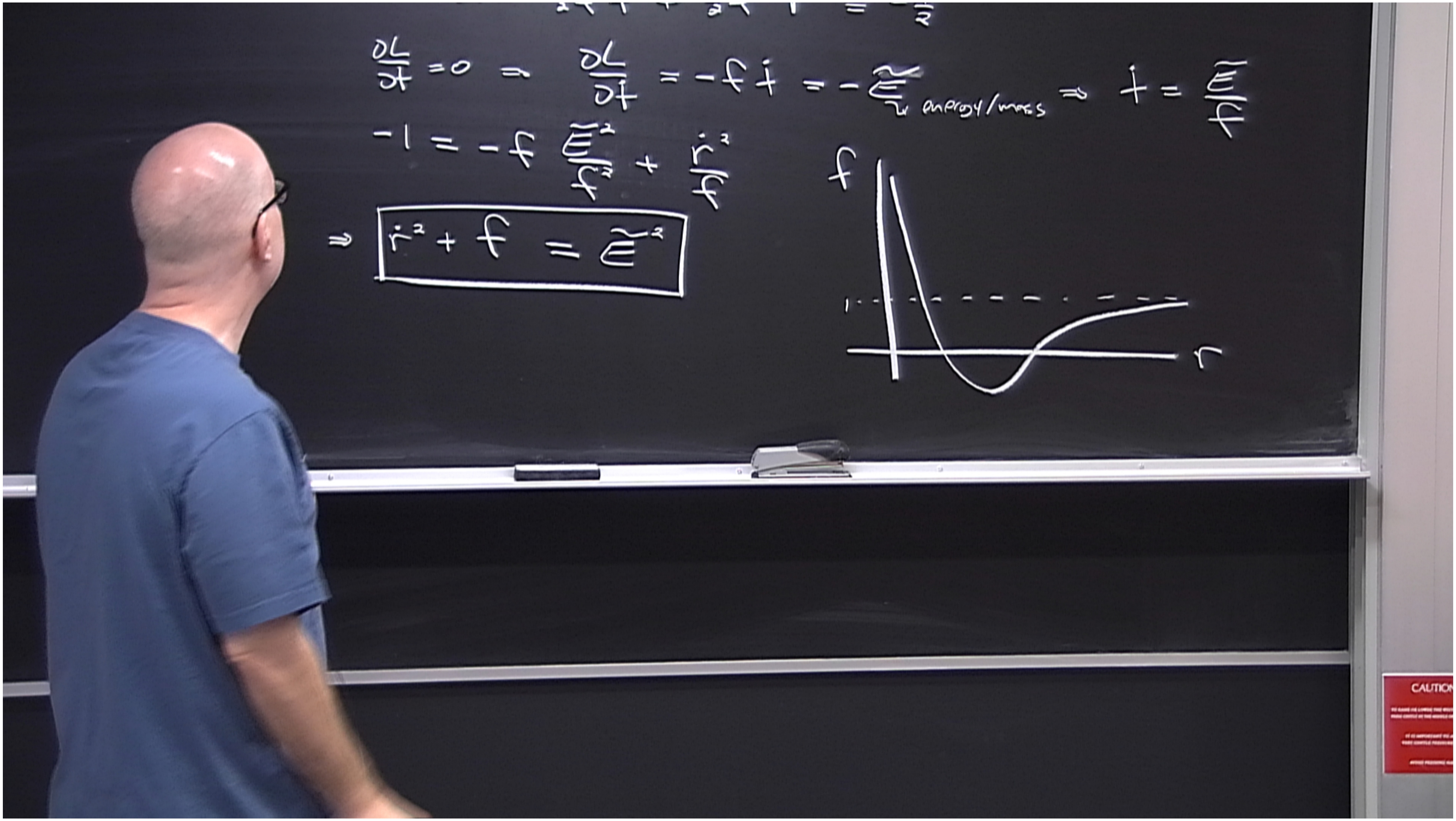


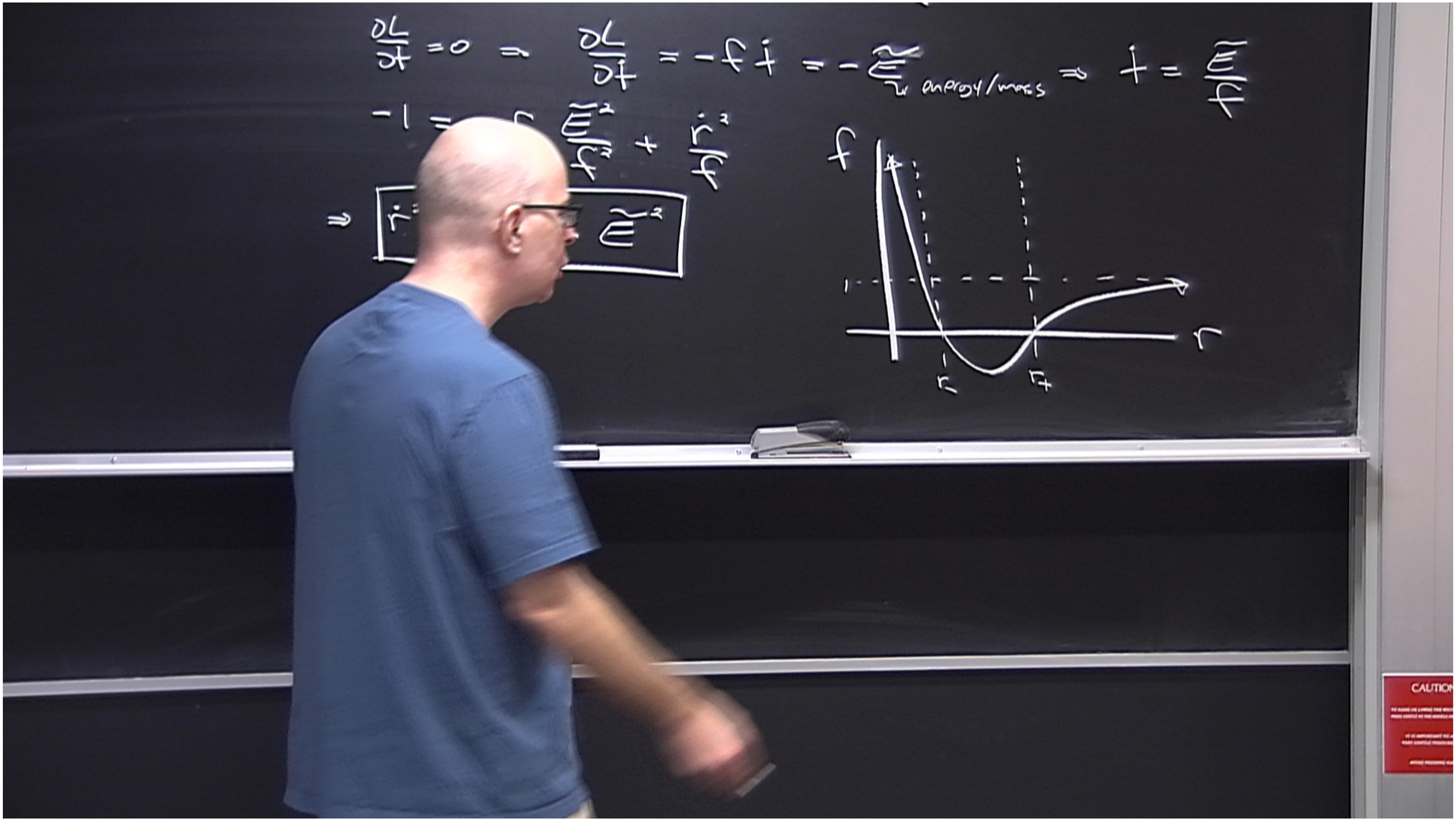
$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = -f\dot{t} = -\tilde{E}$ enerqy/mass $\Rightarrow \dot{t} = \frac{\tilde{E}}{f}$
$-1 = -f\frac{\tilde{E}^2}{f^2} + \frac{\dot{r}^2}{f}$

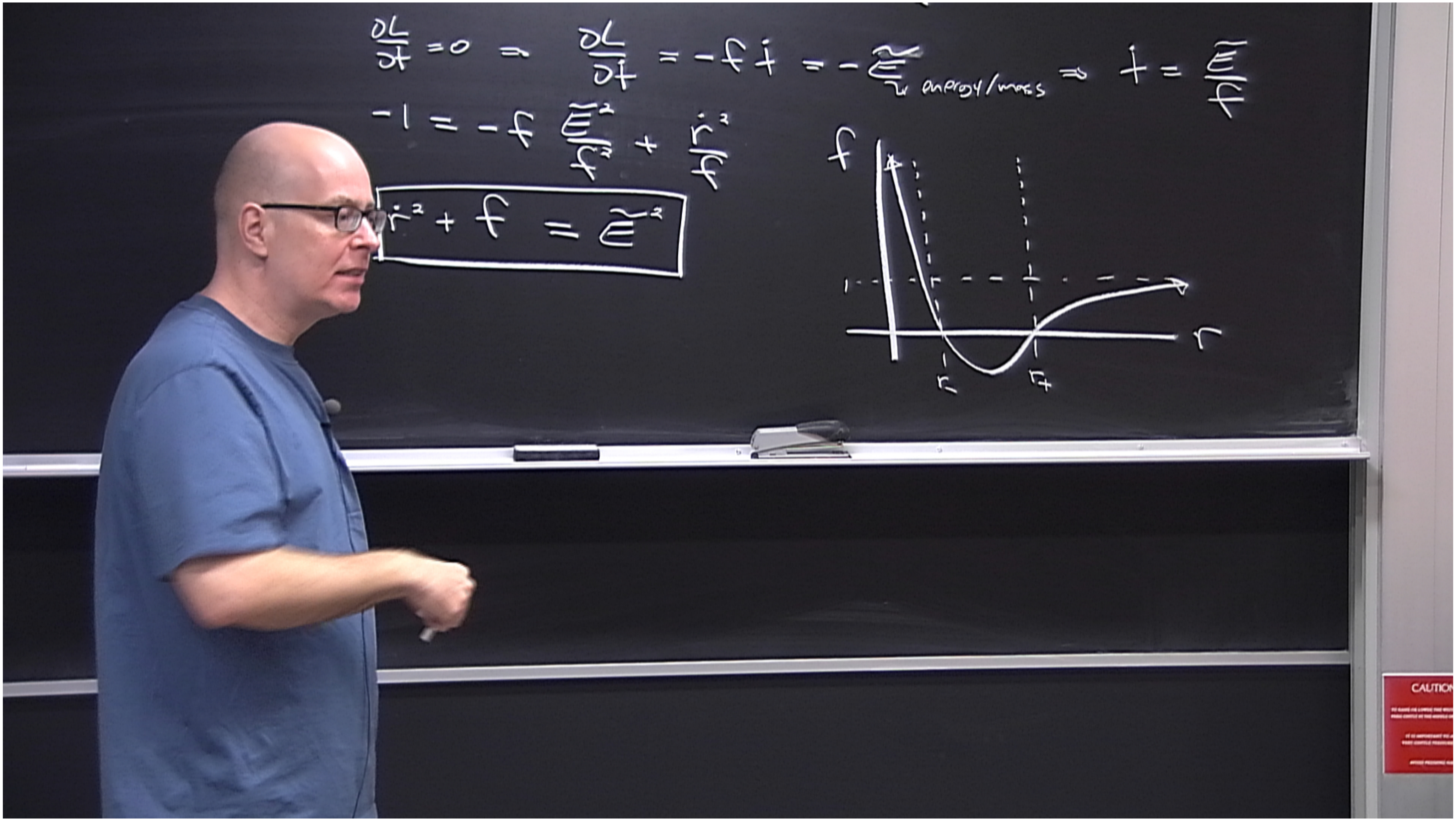

$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot t} = -f\dot t = -\tilde E$ energy/mass $\Rightarrow \dot t = \frac{\tilde E}{f}$
$-1 = -f\frac{\tilde E^2}{f^2} + \frac{\dot r^2}{f}$
$\dot r^2 + f = \tilde E^2$
CAUTION

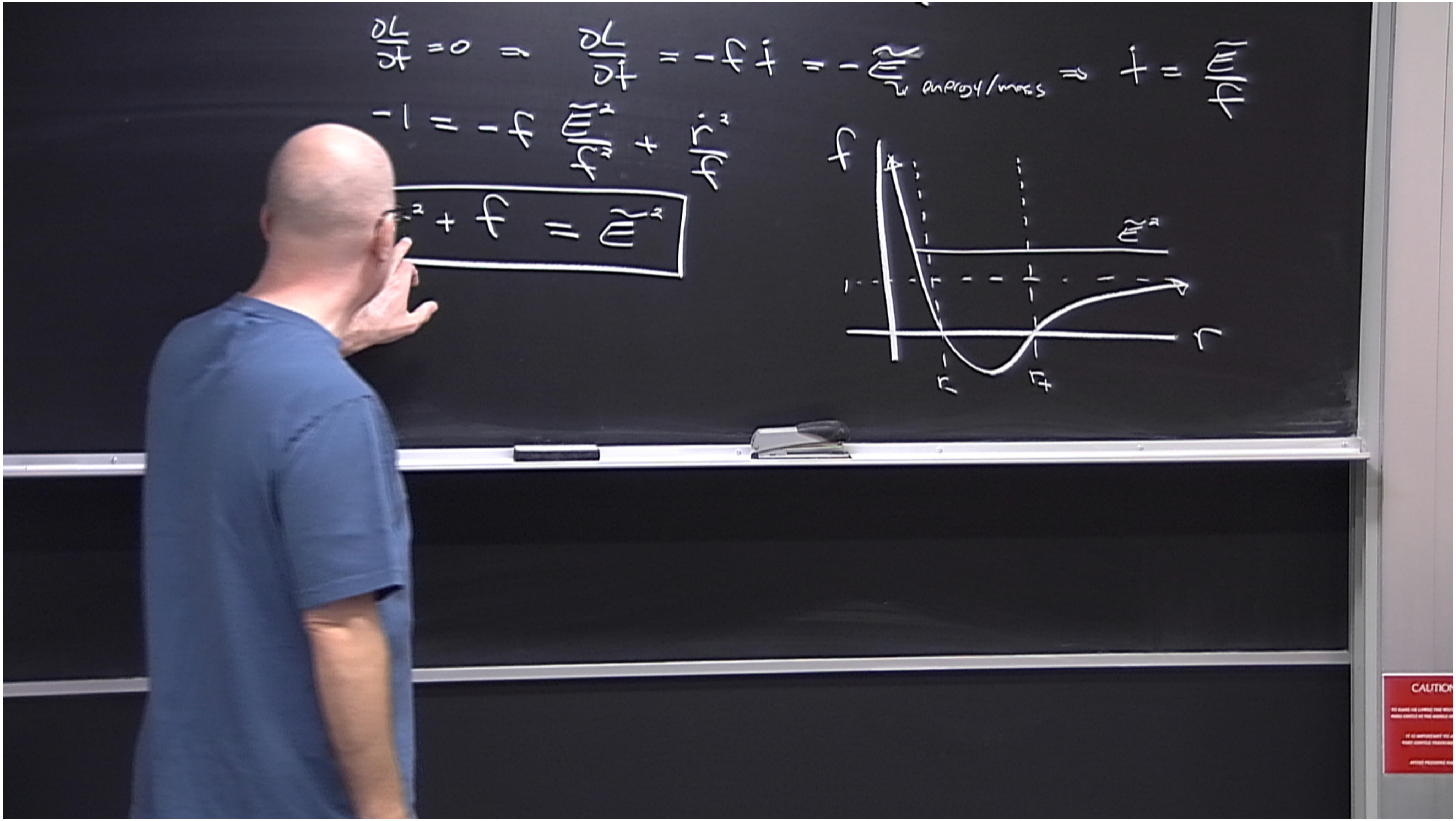

$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = -f\dot{t} = -\tilde{E}$ energy/mass $\Rightarrow \dot{t} = \frac{\tilde{E}}{f}$
$-1 = -f\frac{\tilde{E}^2}{f^2} + \frac{\dot{r}^2}{f}$
$\Rightarrow \dot{r}^2 + f = \tilde{E}^2$
CAUTION


~ energy/mass
-1 = -f Ẽ²/f² + ṙ²/f
ṙ² + f = Ẽ²
f
r
CAUTION

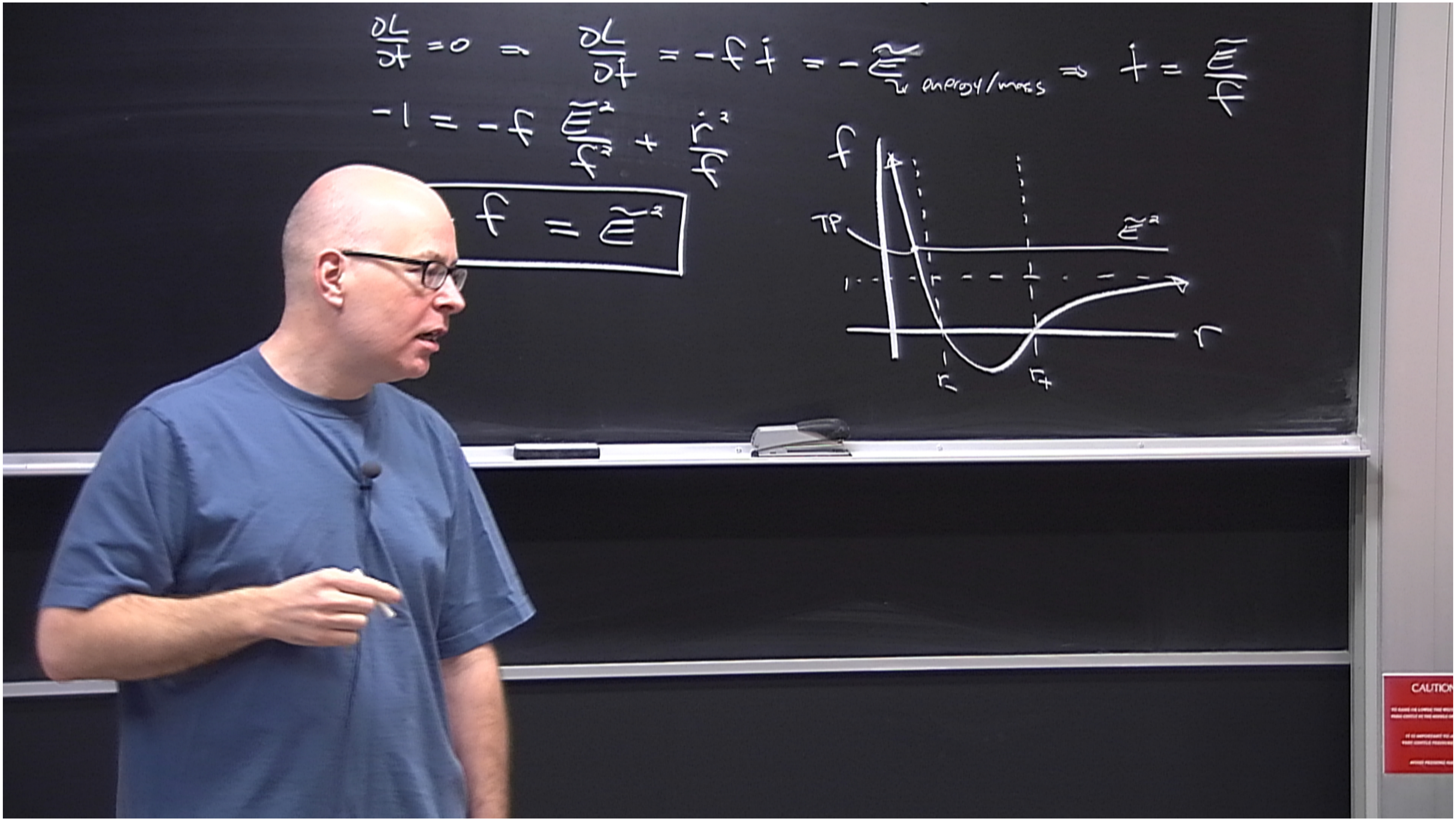

∂L/∂t = 0 ⇒ ∂L/∂ṫ = −f ṫ = −Ẽ energy/mass ⇒ ṫ = Ẽ/f
−1 = −f Ẽ²/f² + ṙ²/f
⇒ ṙ² + f = Ẽ²
f
1
r

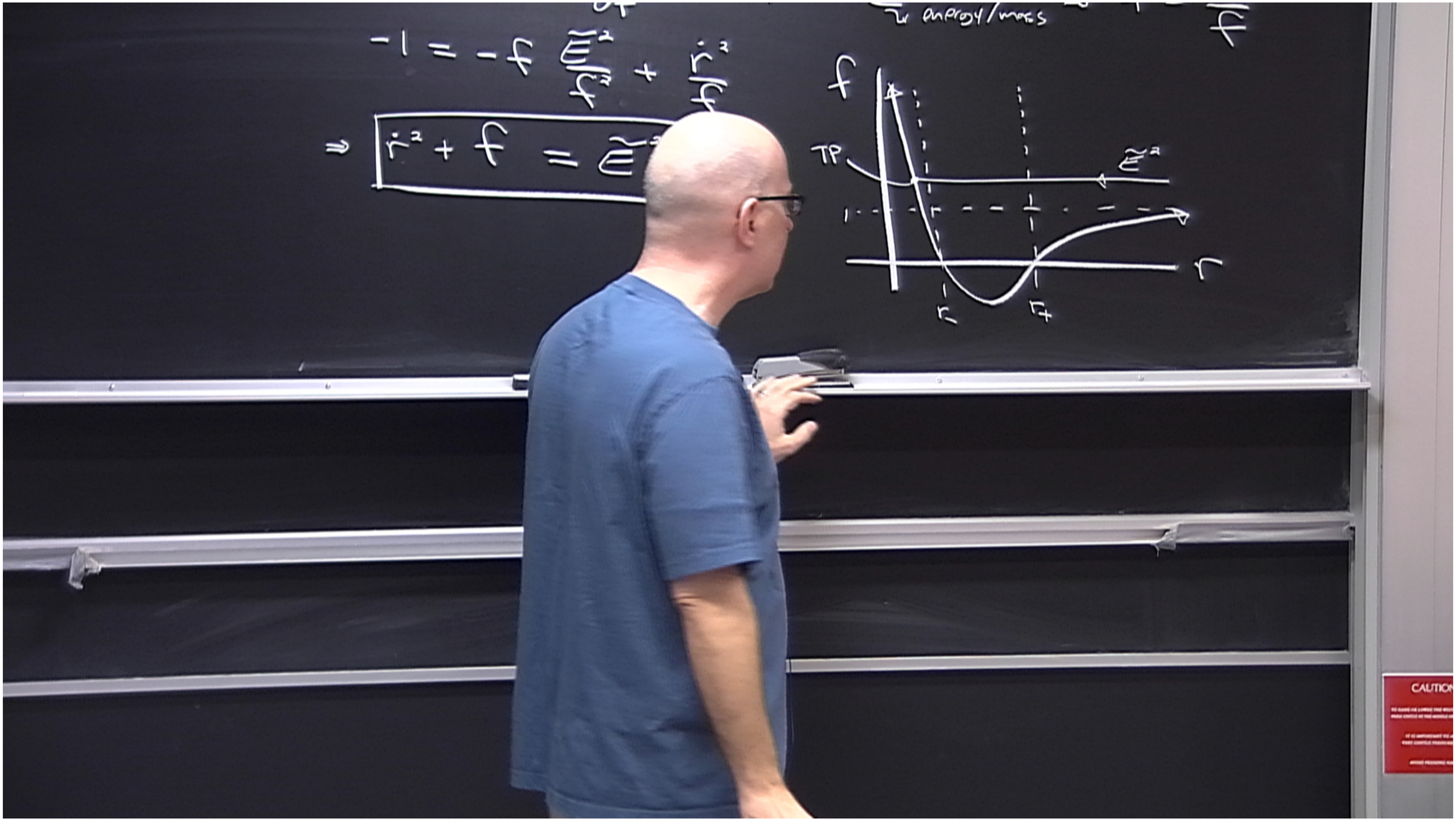

energy/mass
f
1
r

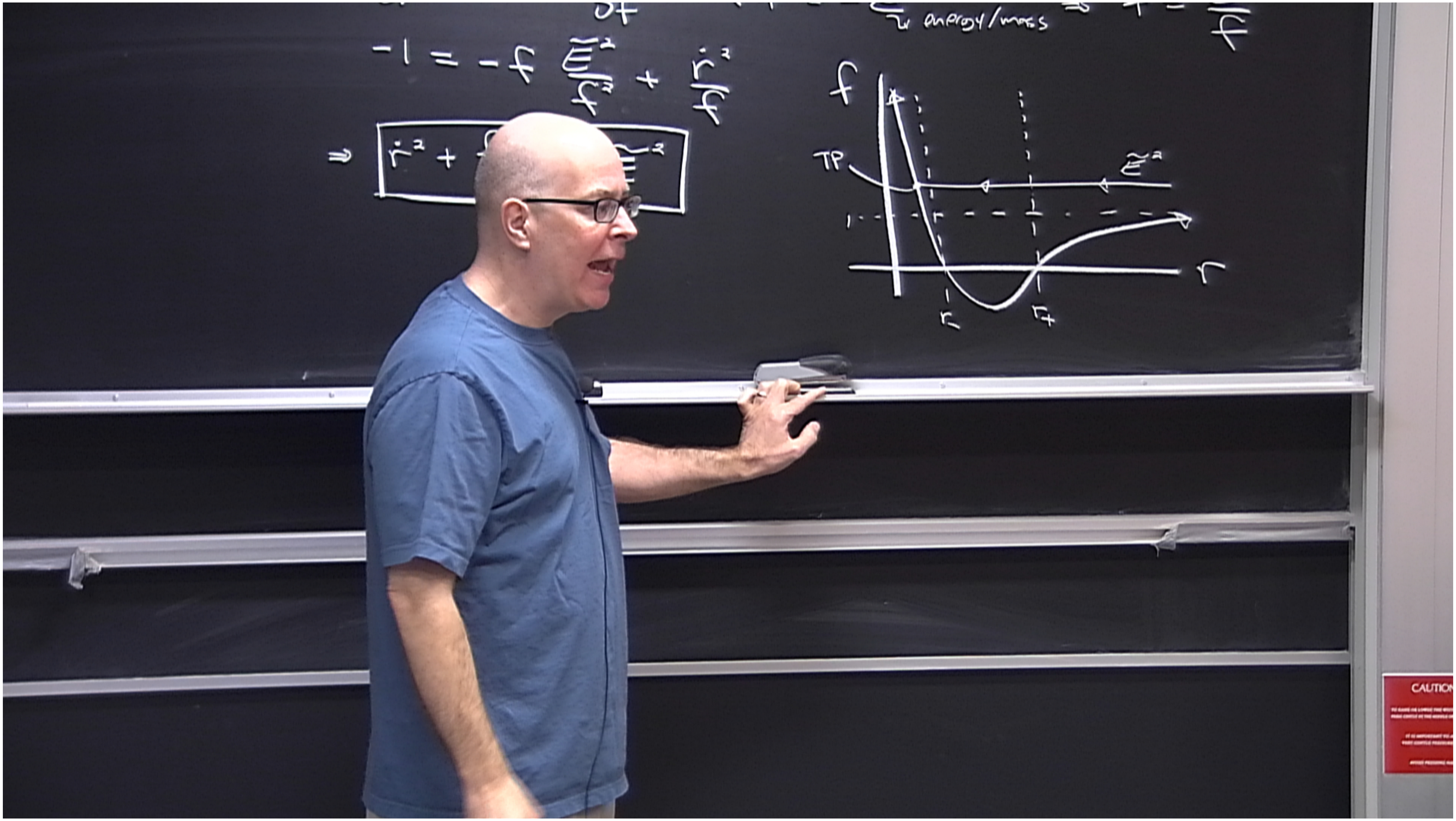

$\frac{\partial L}{\partial t} = 0 \Rightarrow \frac{\partial L}{\partial \dot{t}} = -f\dot{t} = -\tilde{E}$ energy/mass $\Rightarrow \dot{t} = \frac{\tilde{E}}{f}$
$-1 = -f\frac{\tilde{E}^2}{f^2} + \frac{\dot{r}^2}{f}$
$\dot{r}^2 + f = \tilde{E}^2$
f
1
r
r_-
r_+
CAUTION



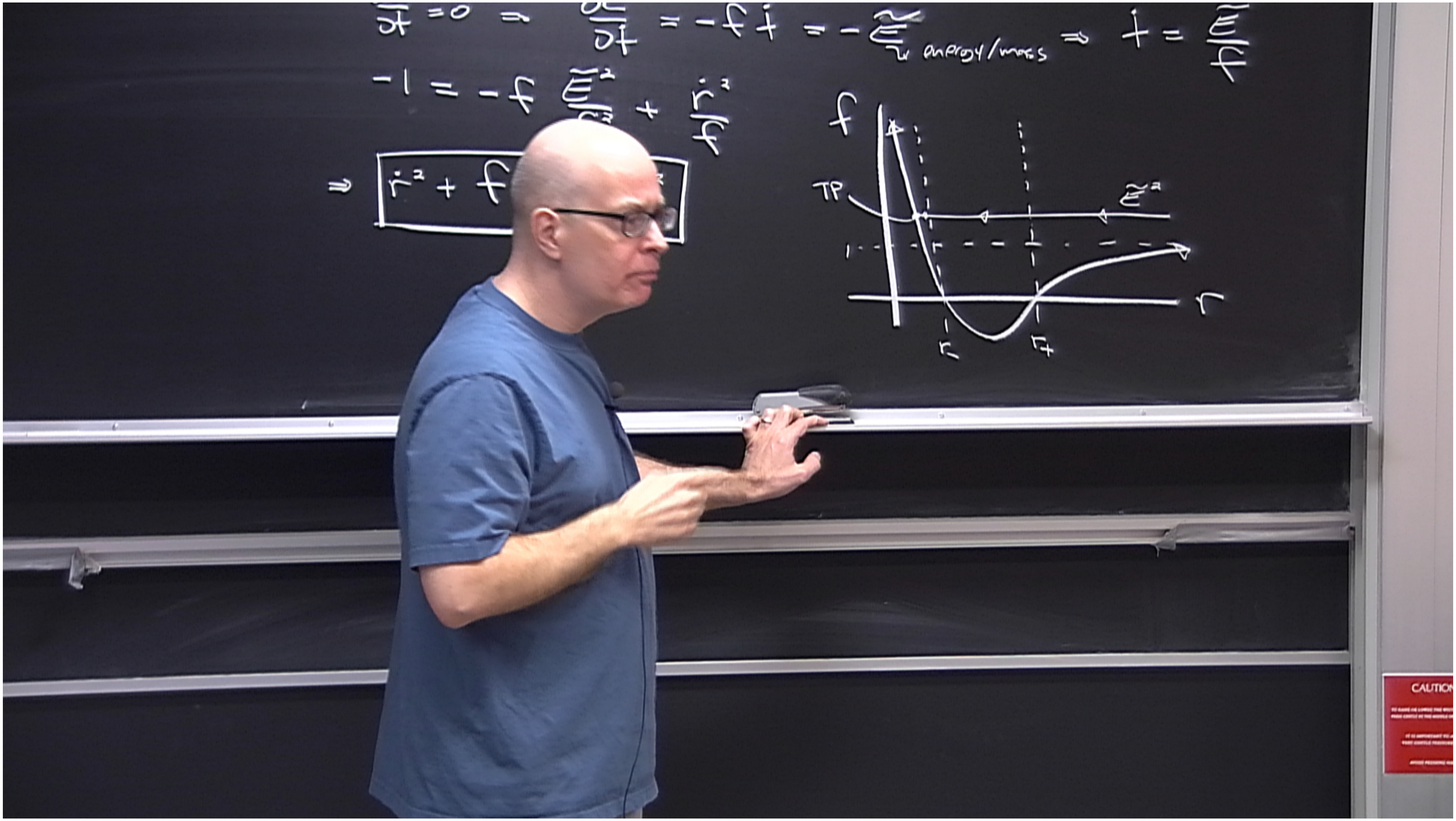
energy/mass
f
r

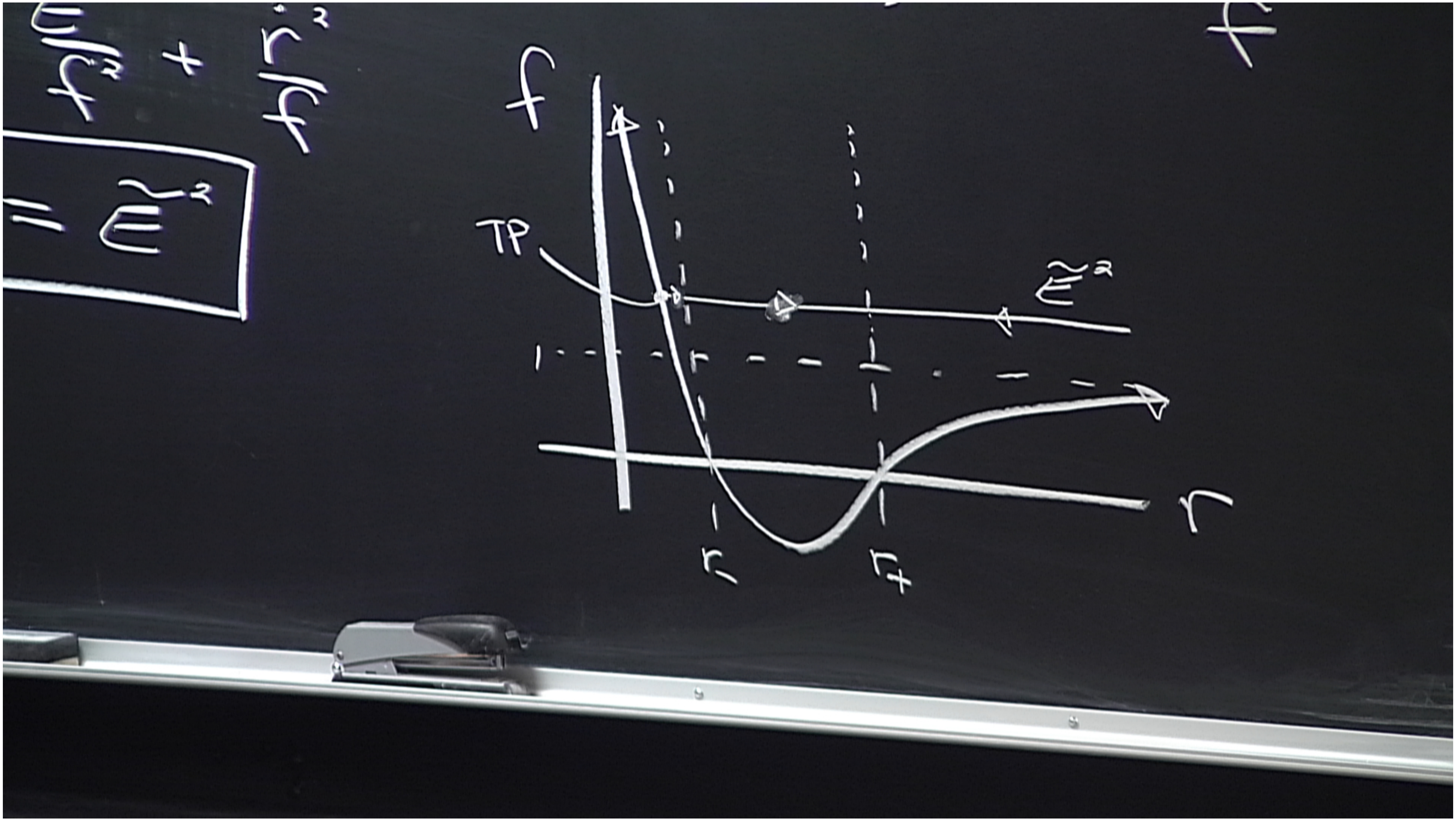

energy/mass
f
TP
r

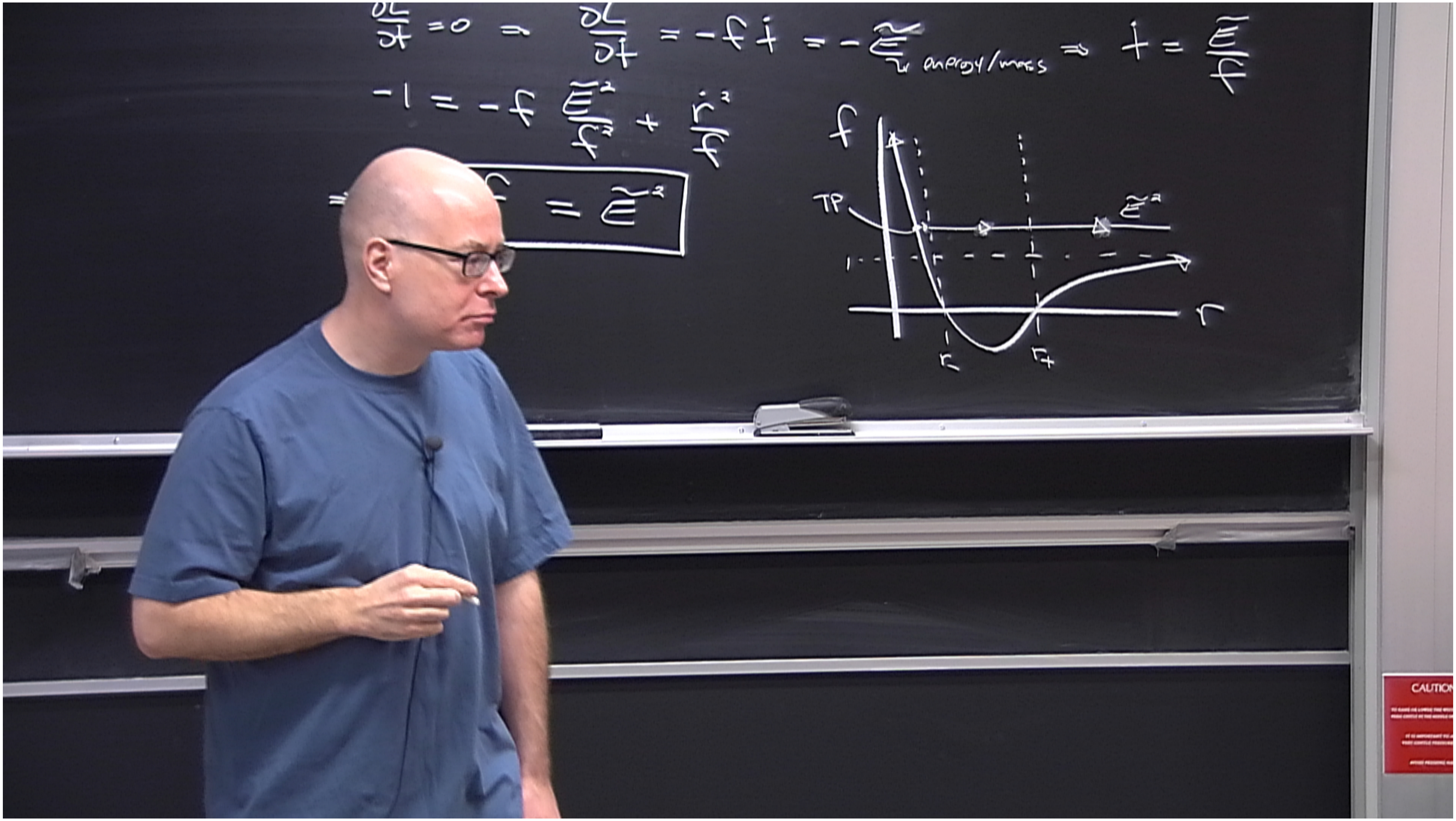

energy/mass
-1 = -f Ẽ²/f² + ṙ²/f
⇒ ṙ² + f = Ẽ²
f
TP
Ẽ²
1
r
r₋
r₊
CAUTION

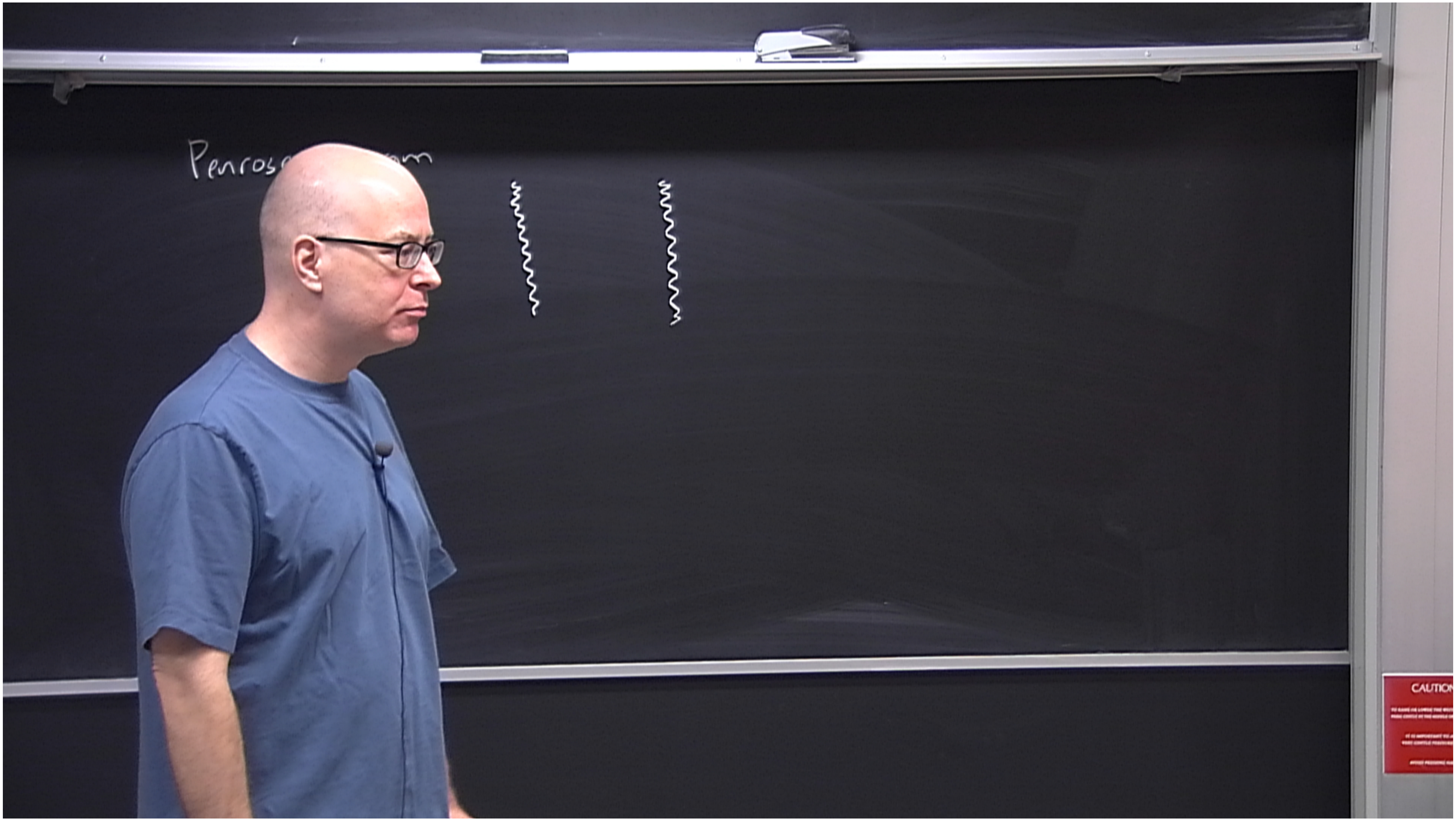


energy/mass
-1 = -f Ẽ²/f² + ṙ²/f
f
TP
Ẽ²
r
CAUTION


energy/mass
TP
r
r_-
r_+
CAUTION


f
TP
$\tilde{E}^2$
1
r
r_-
r_+


energy/mass
TP
CAUTION


Penros
am
CAUTION

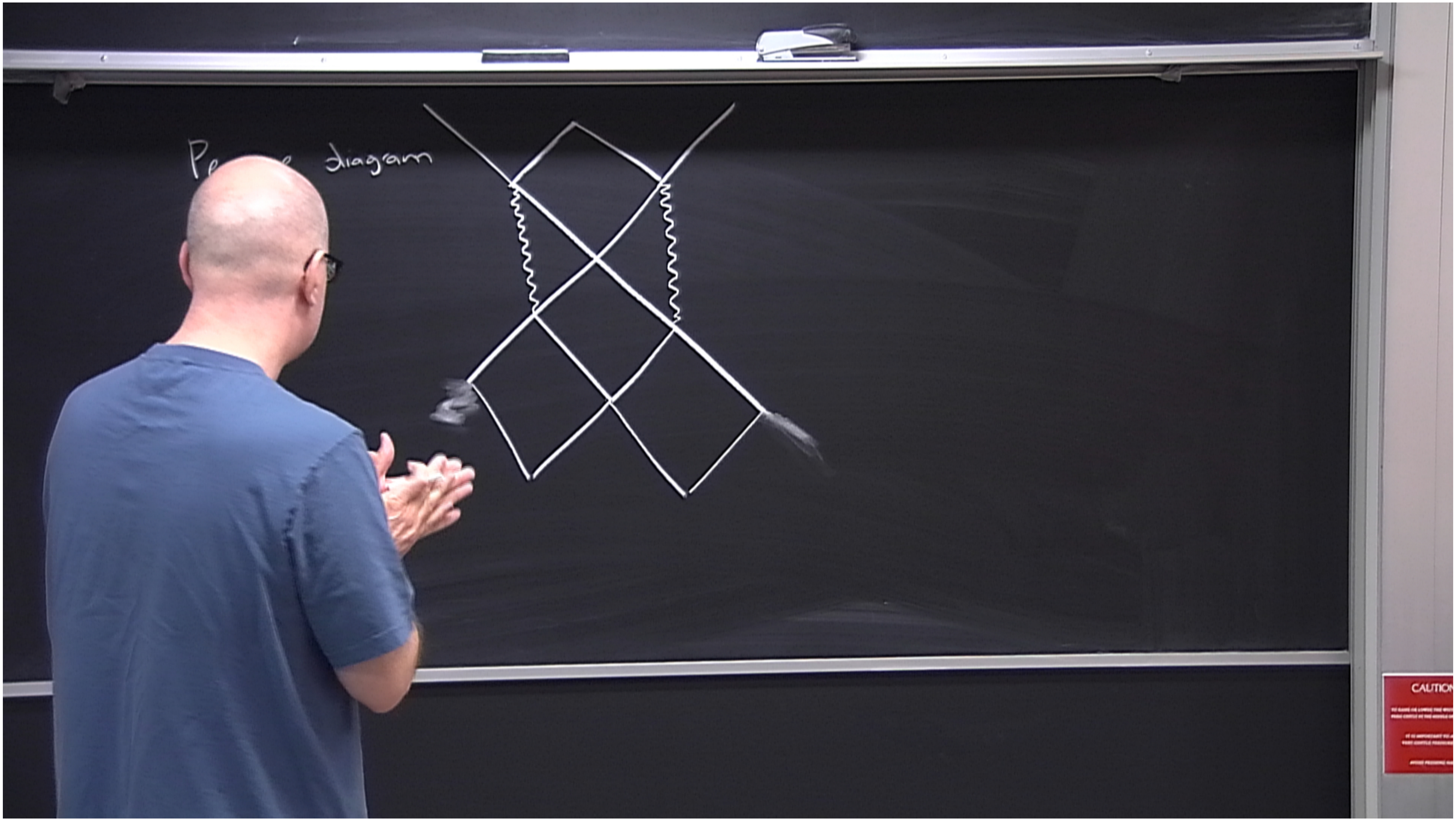
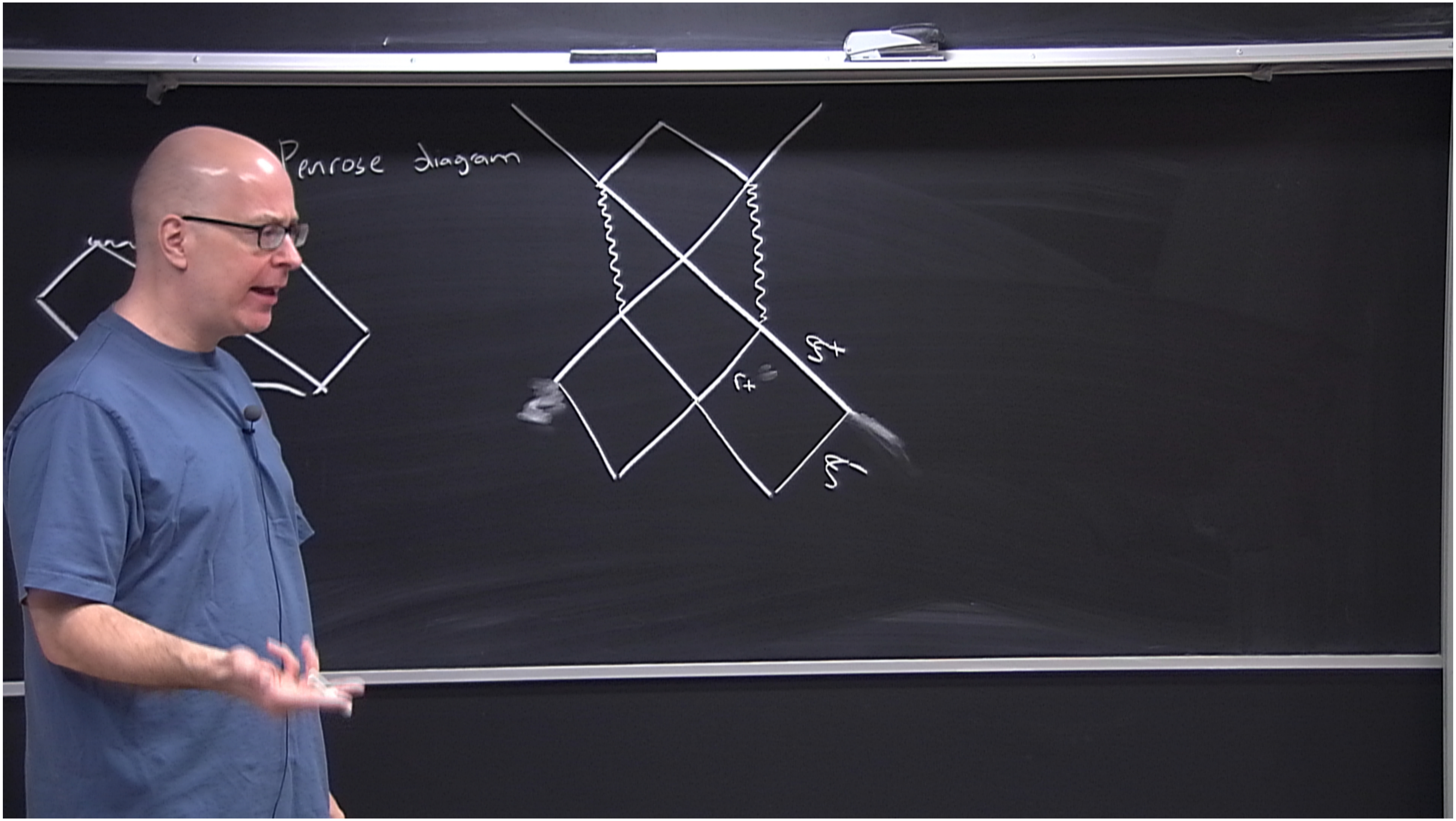
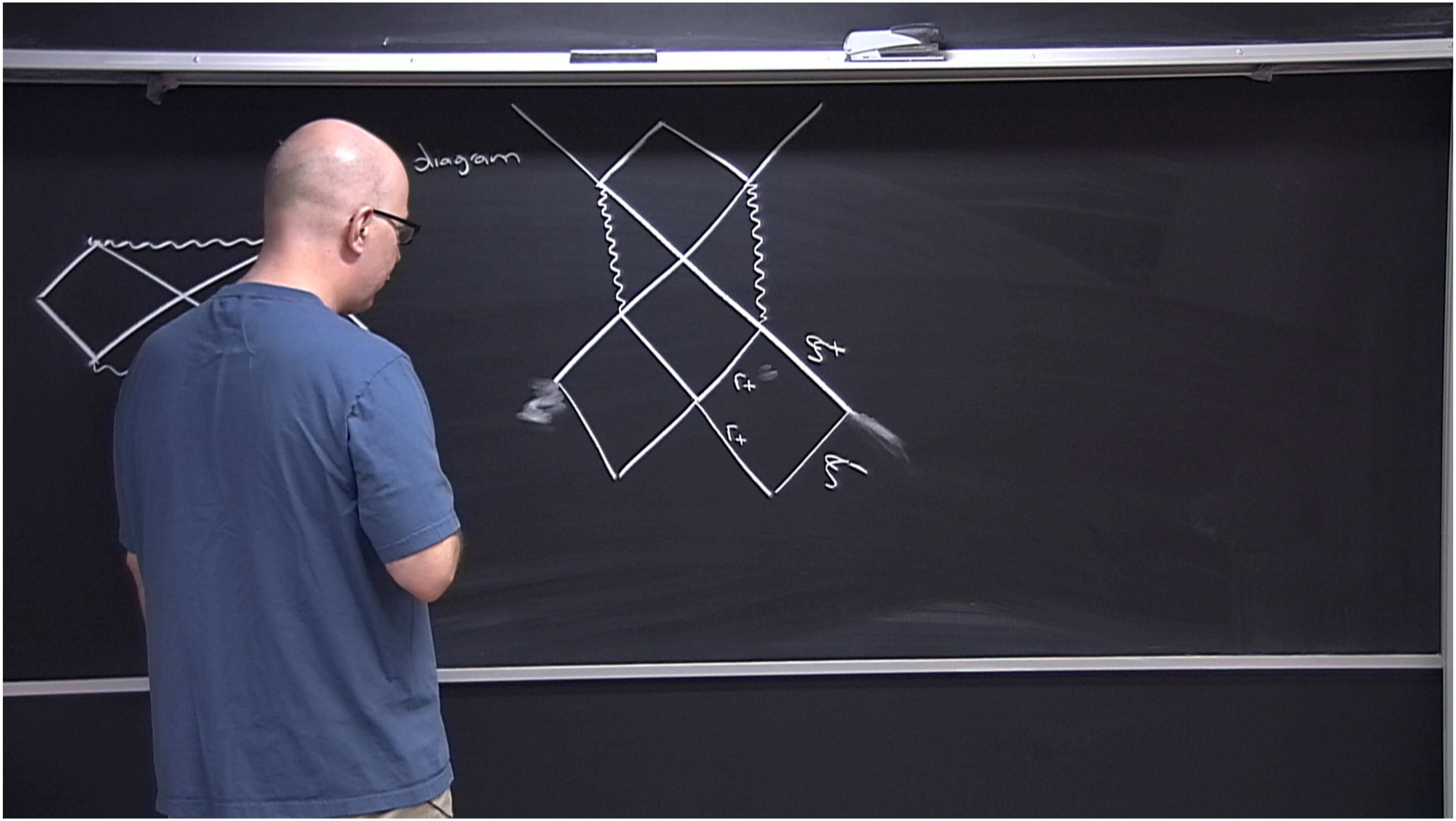


diagram
CAUTION


Penrose diagram



diagram

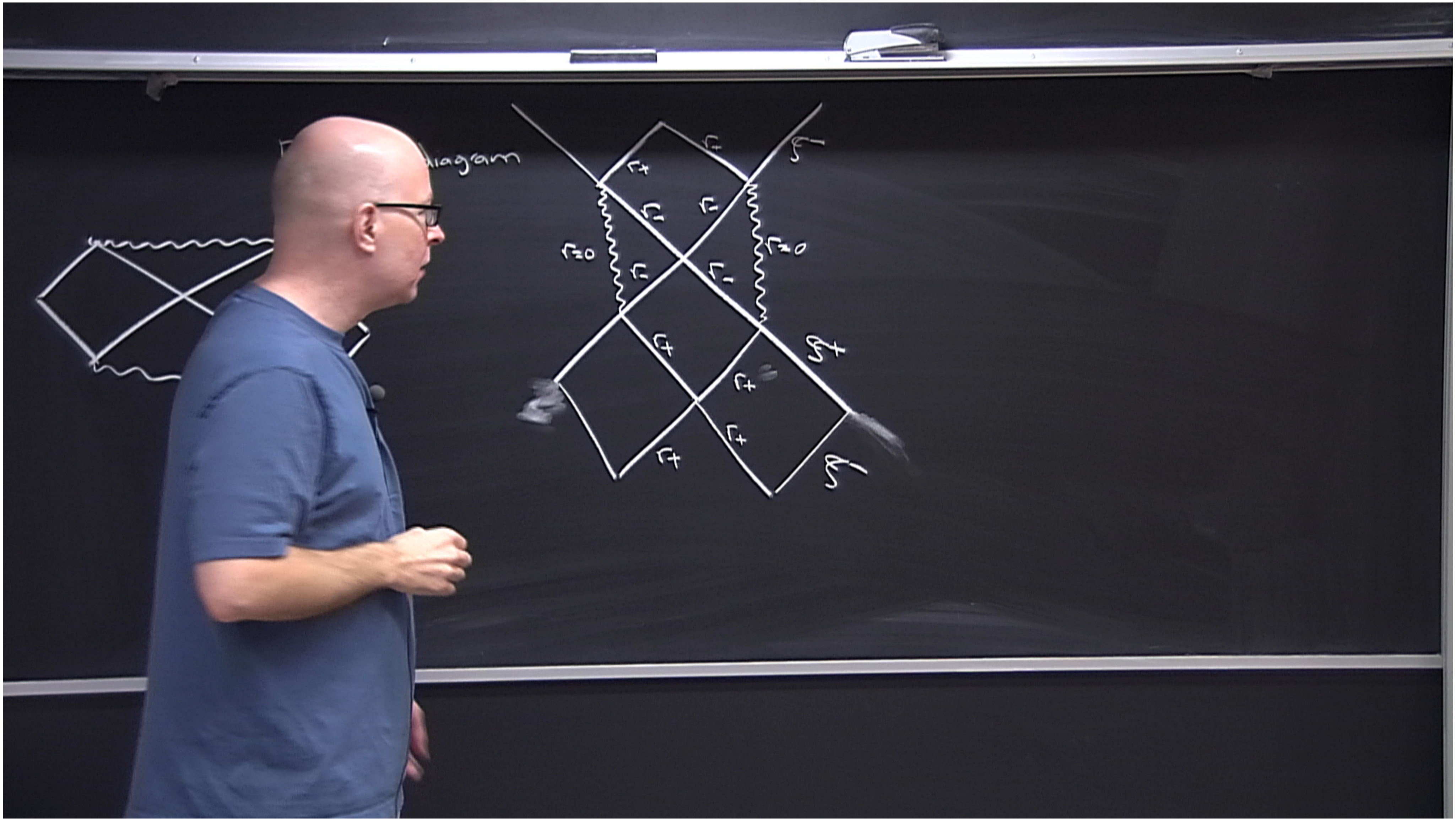
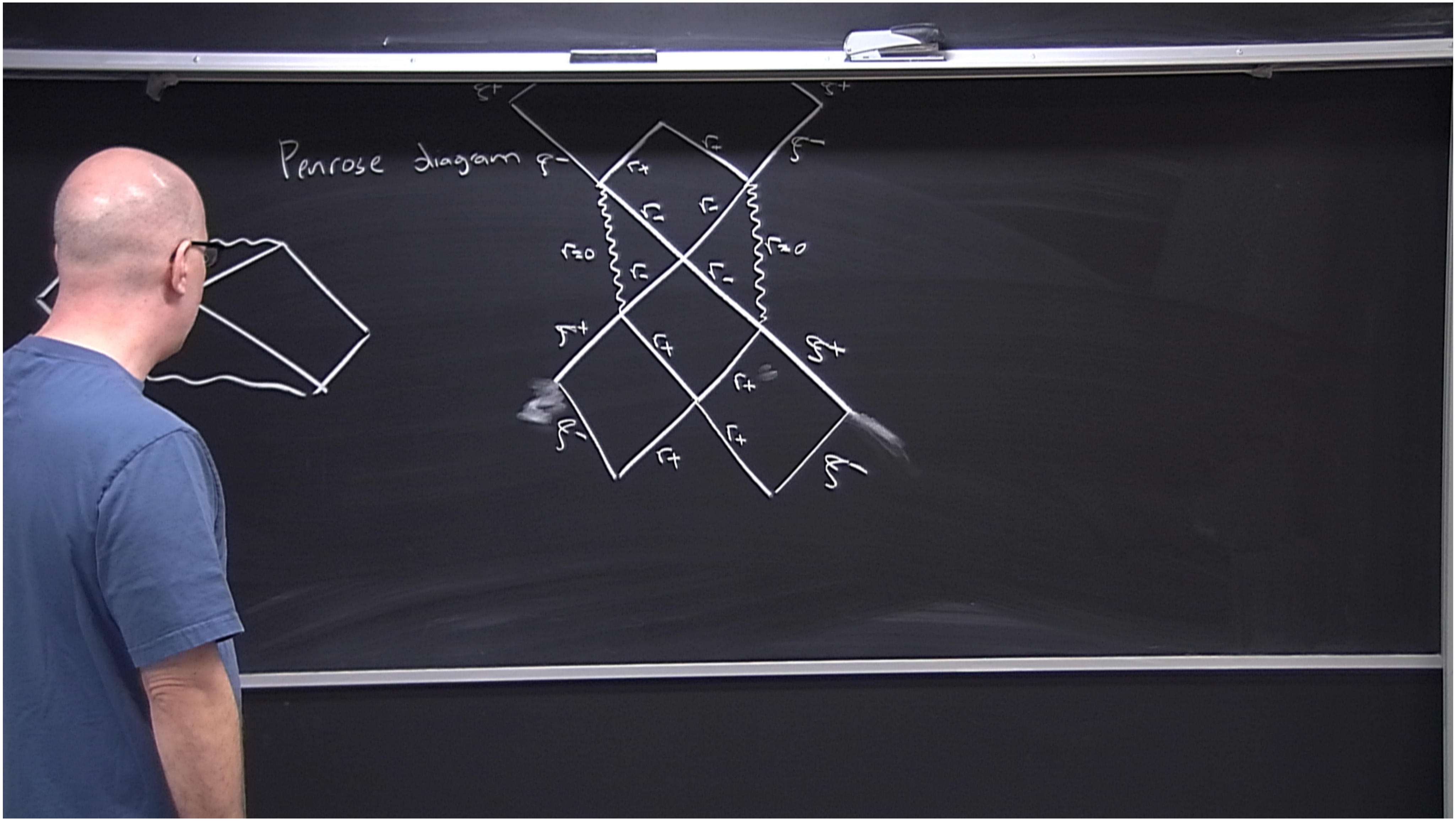


diagram
r_+
r_+
r_-
r_-
$r=0$
$r=0$
r_-
r_-
r_+
r_+
r_+
r_+


Penrose diagram
r=0
r=0

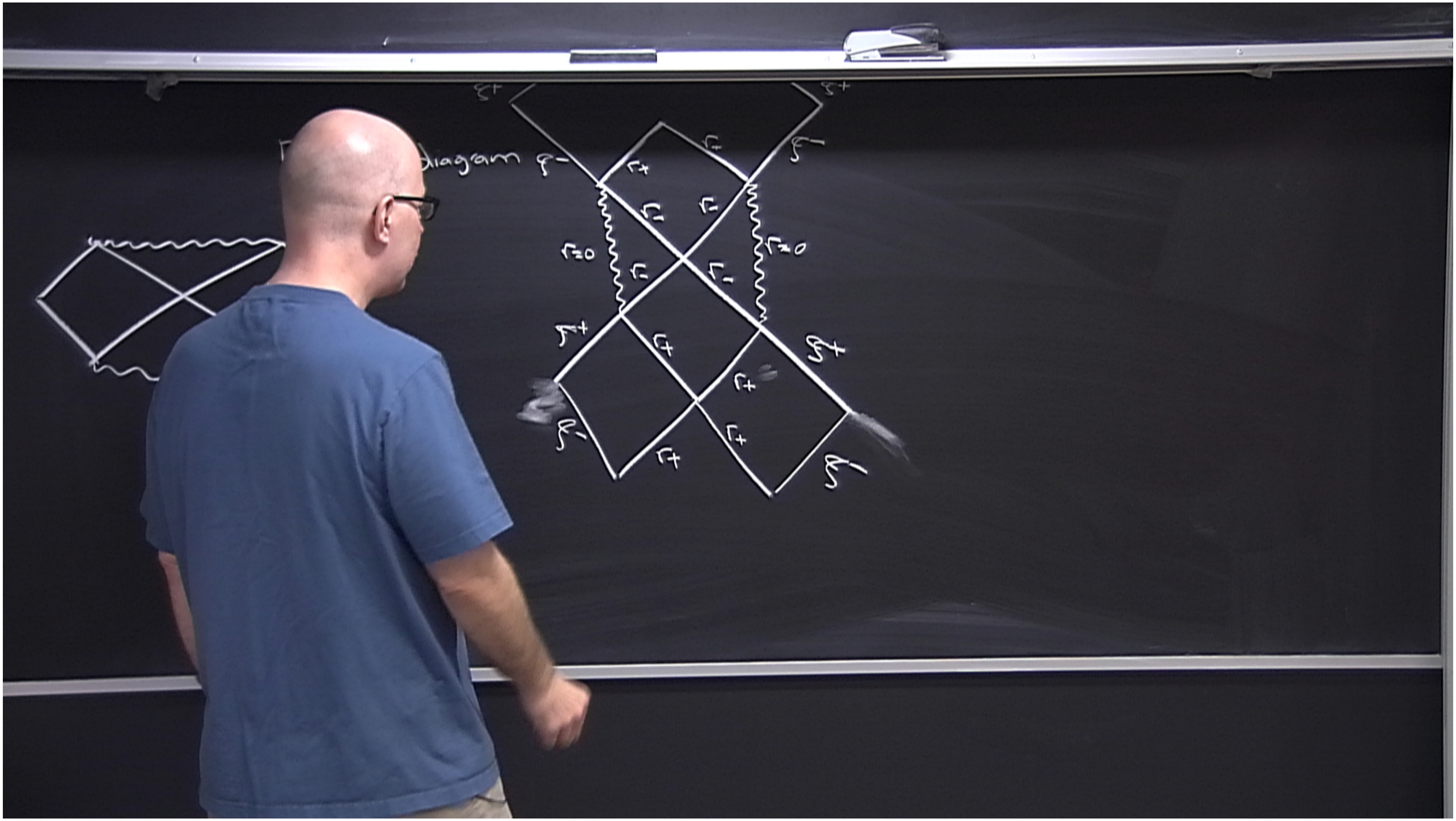
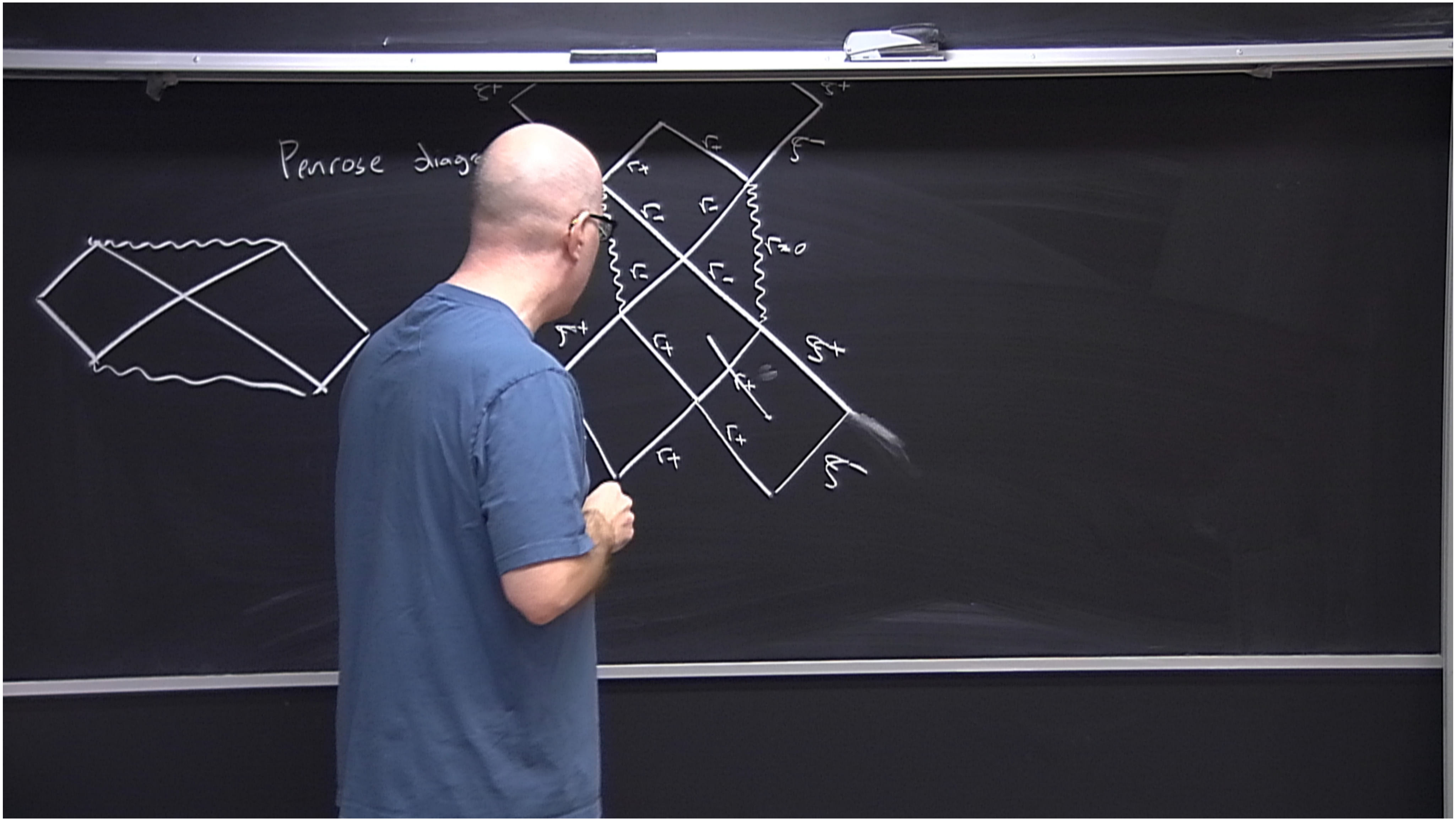


diagram
r=0
r=0
r+
r-


Penrose
r=0

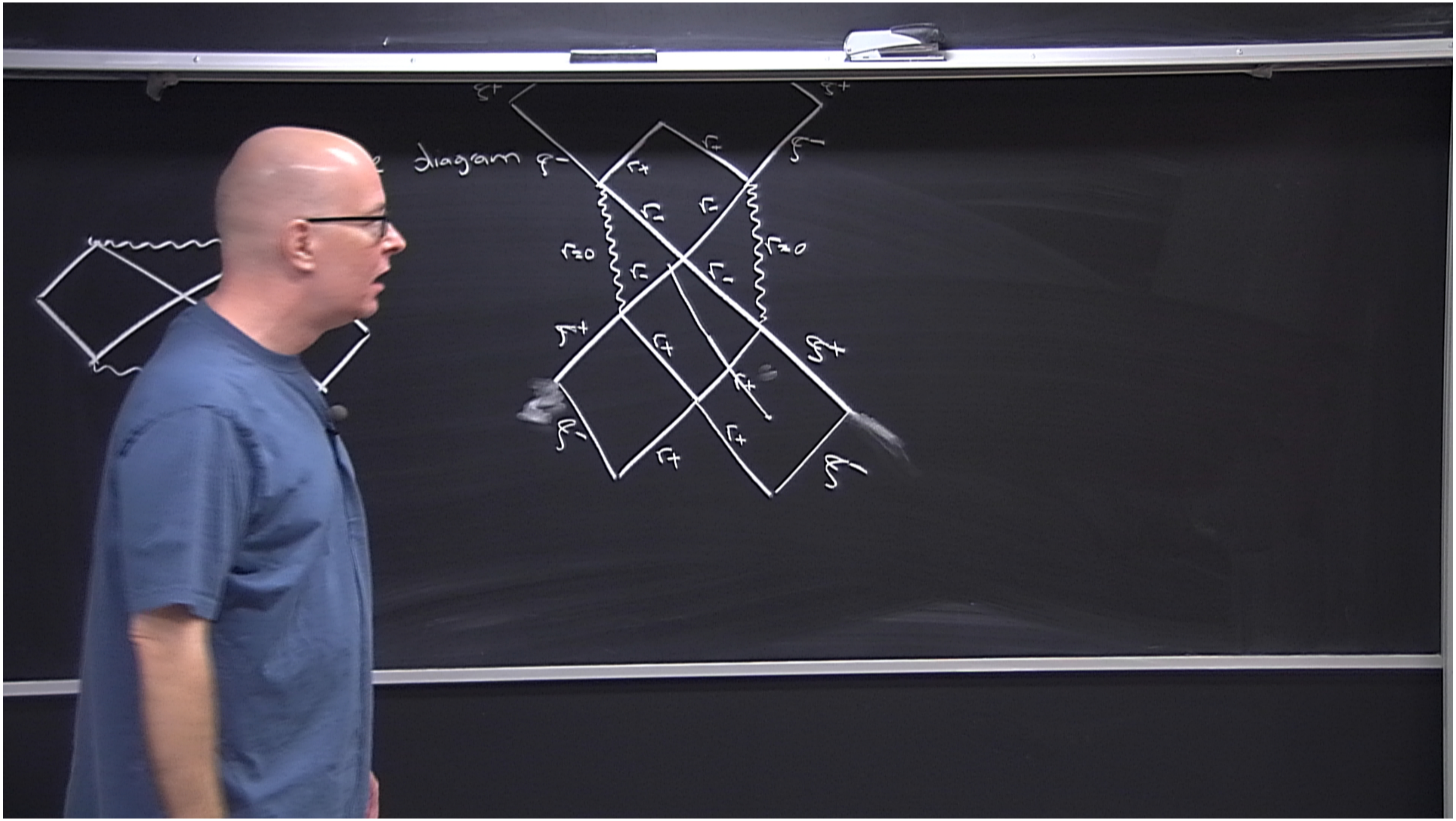


diagram
r=0
r=0

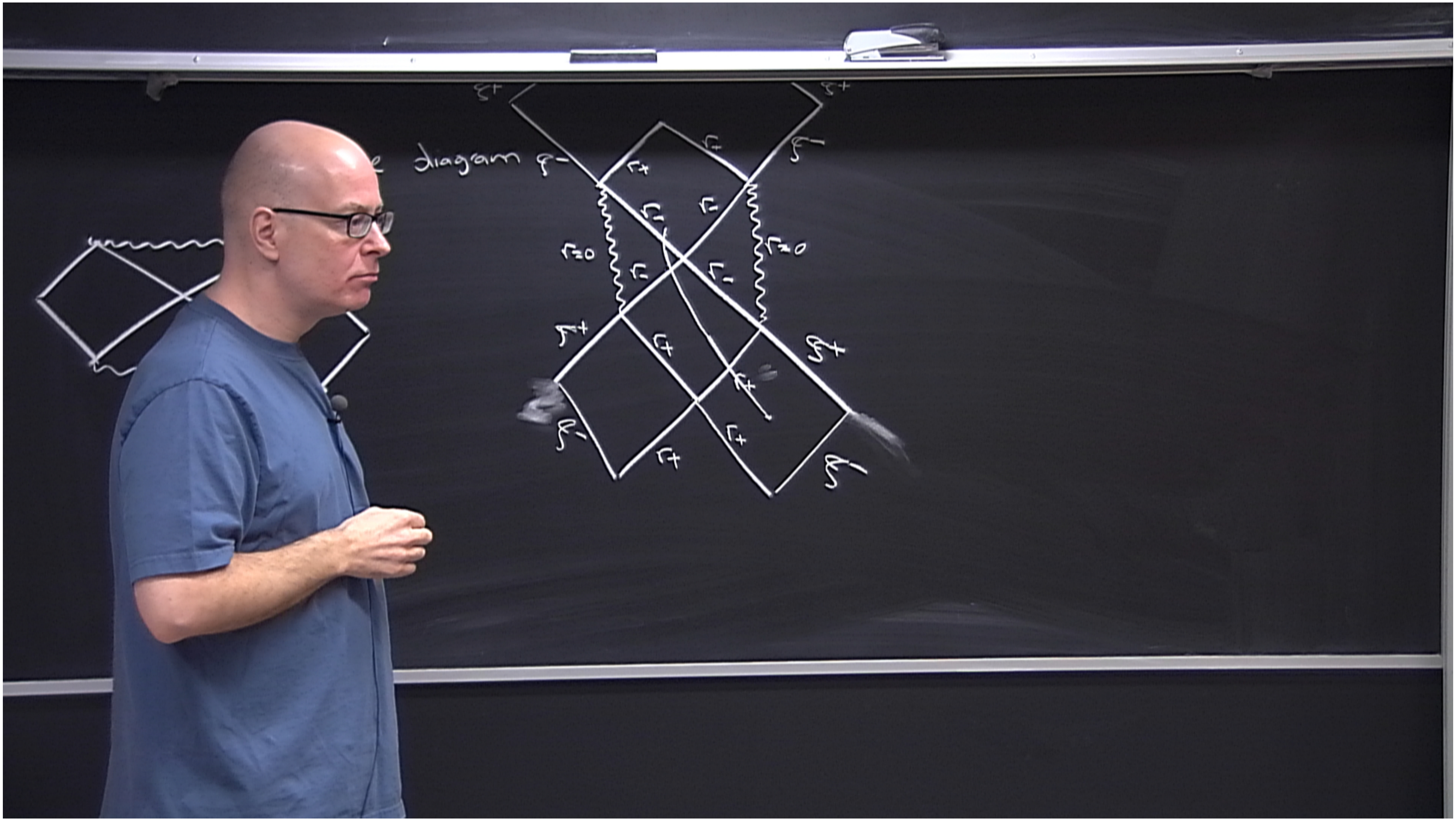


diagram
r=0
r=0

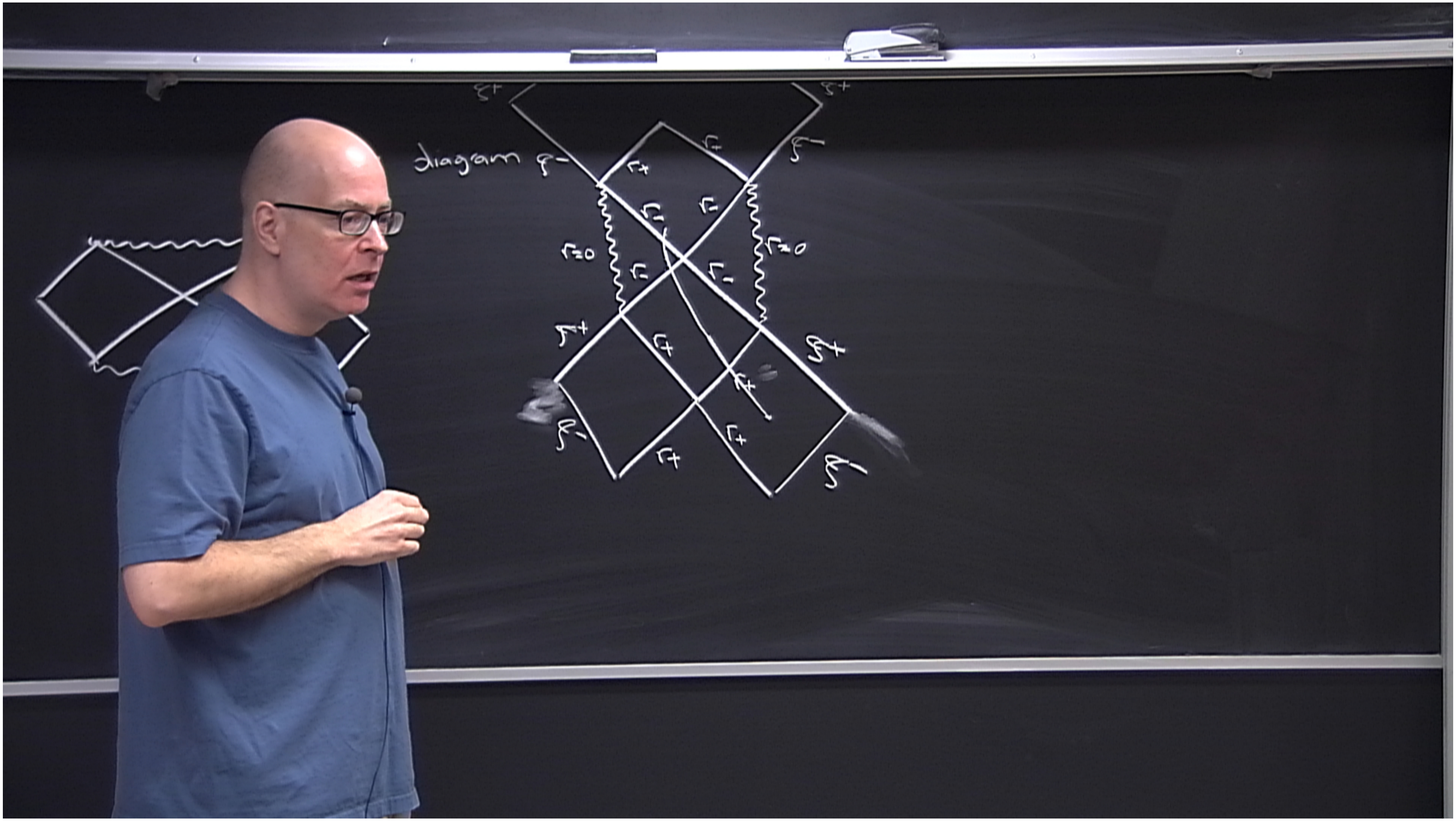
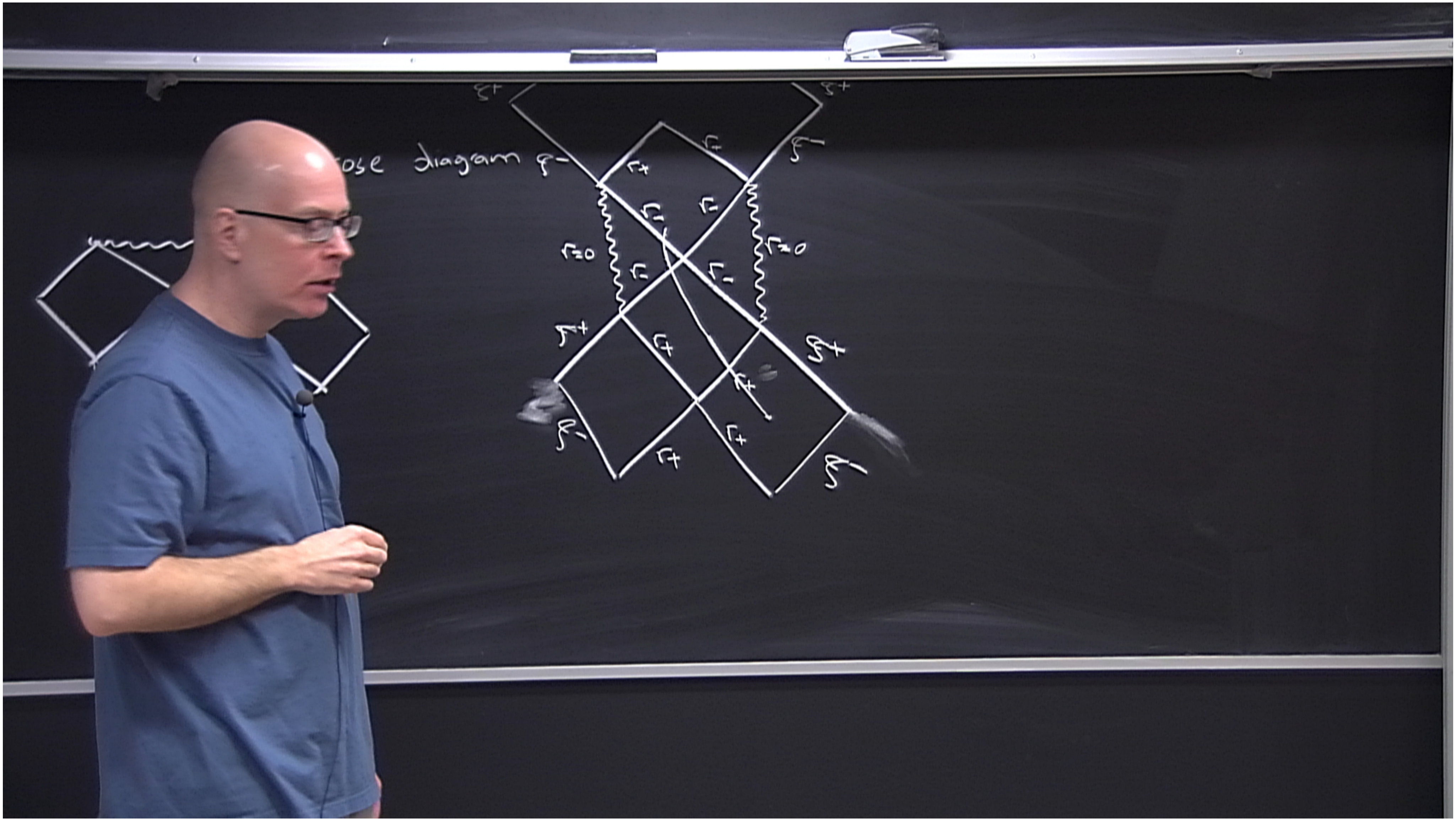
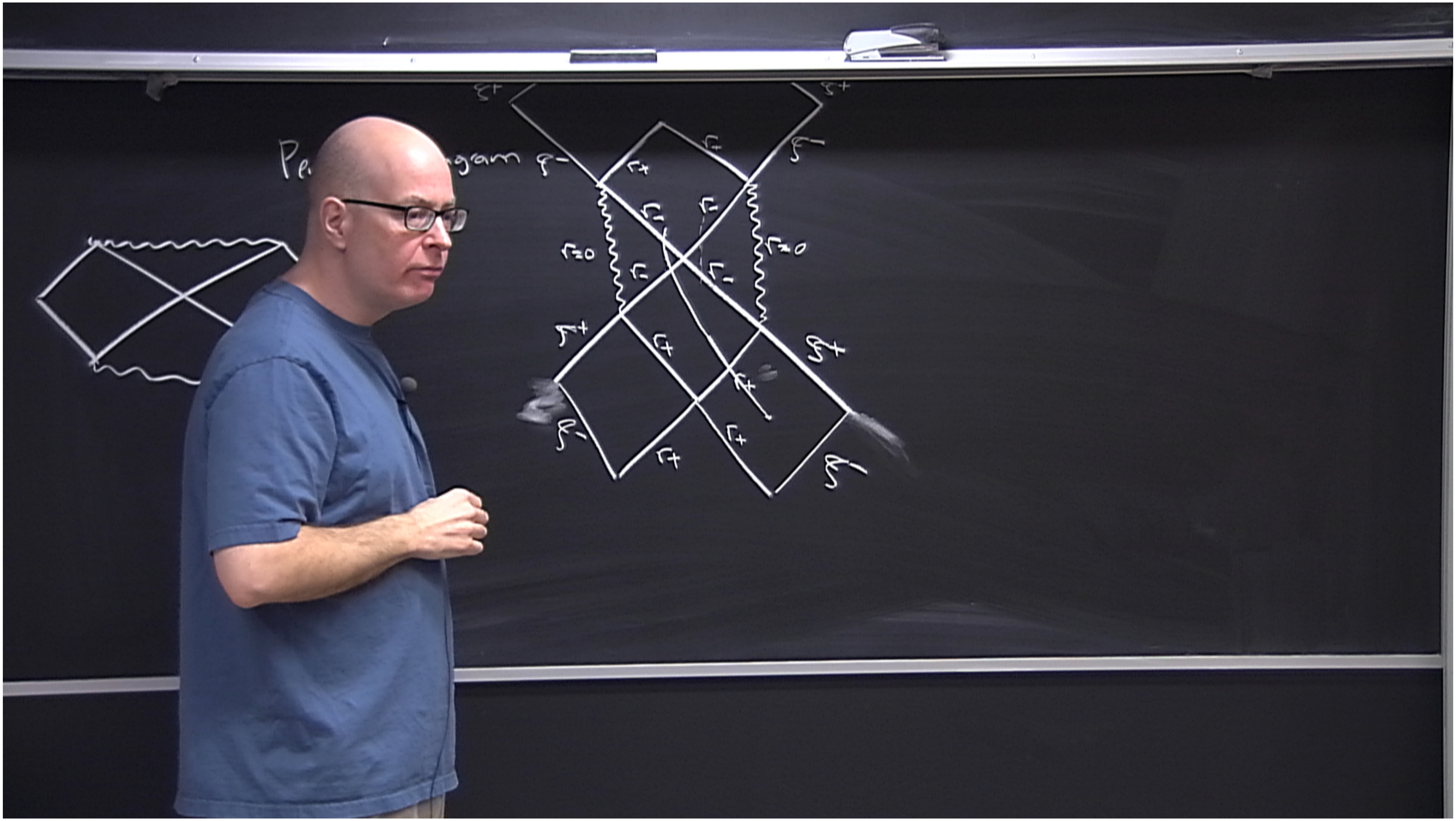
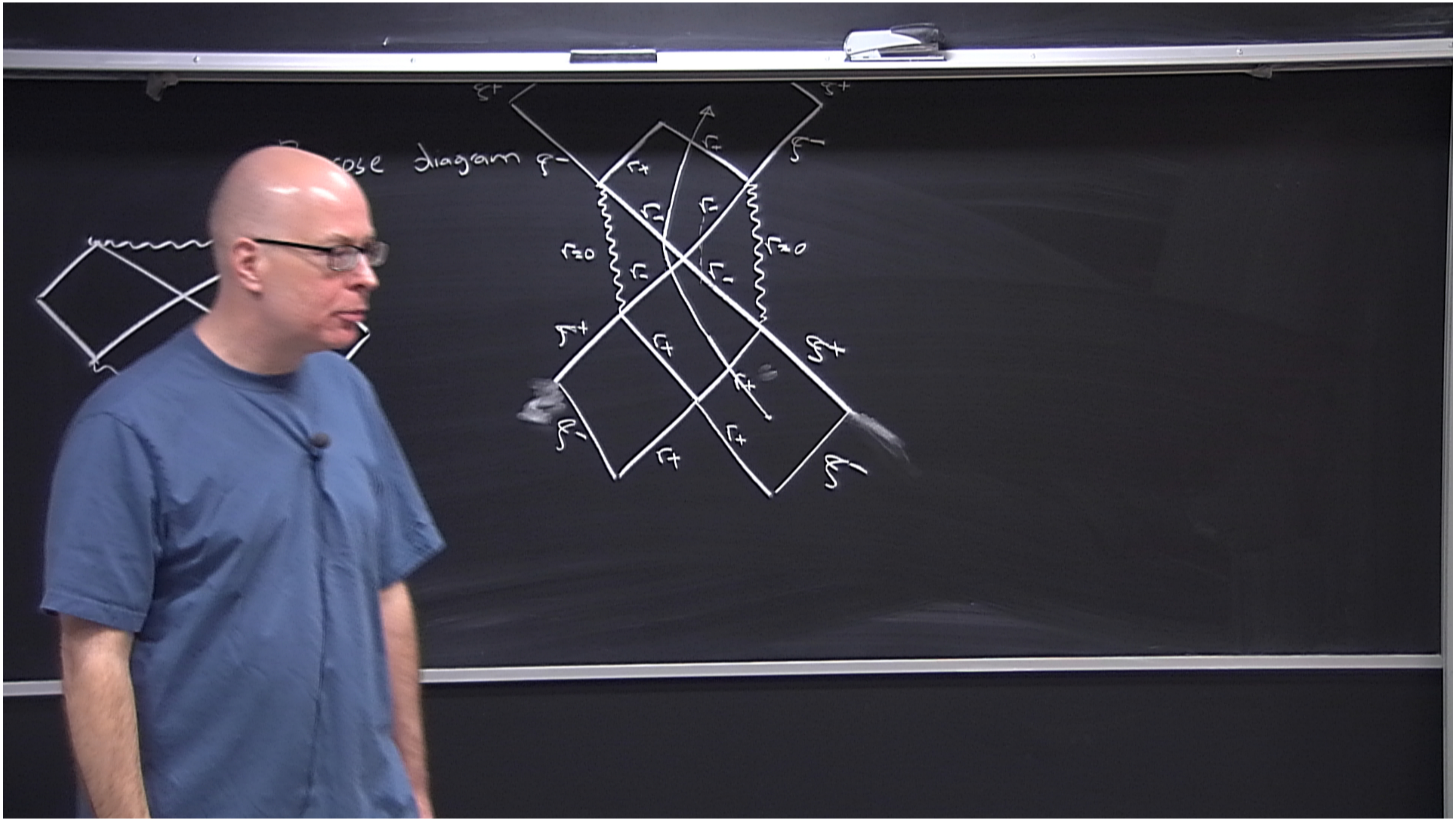


diagram
$r=0$
$r=0$
r_+
r_-
$\mathcal{I}^+$
$\mathcal{I}^-$


ose diagram
ℐ⁺
ℐ⁻
r₊
r₋
r=0


r=0
r=0


Penrose diagram

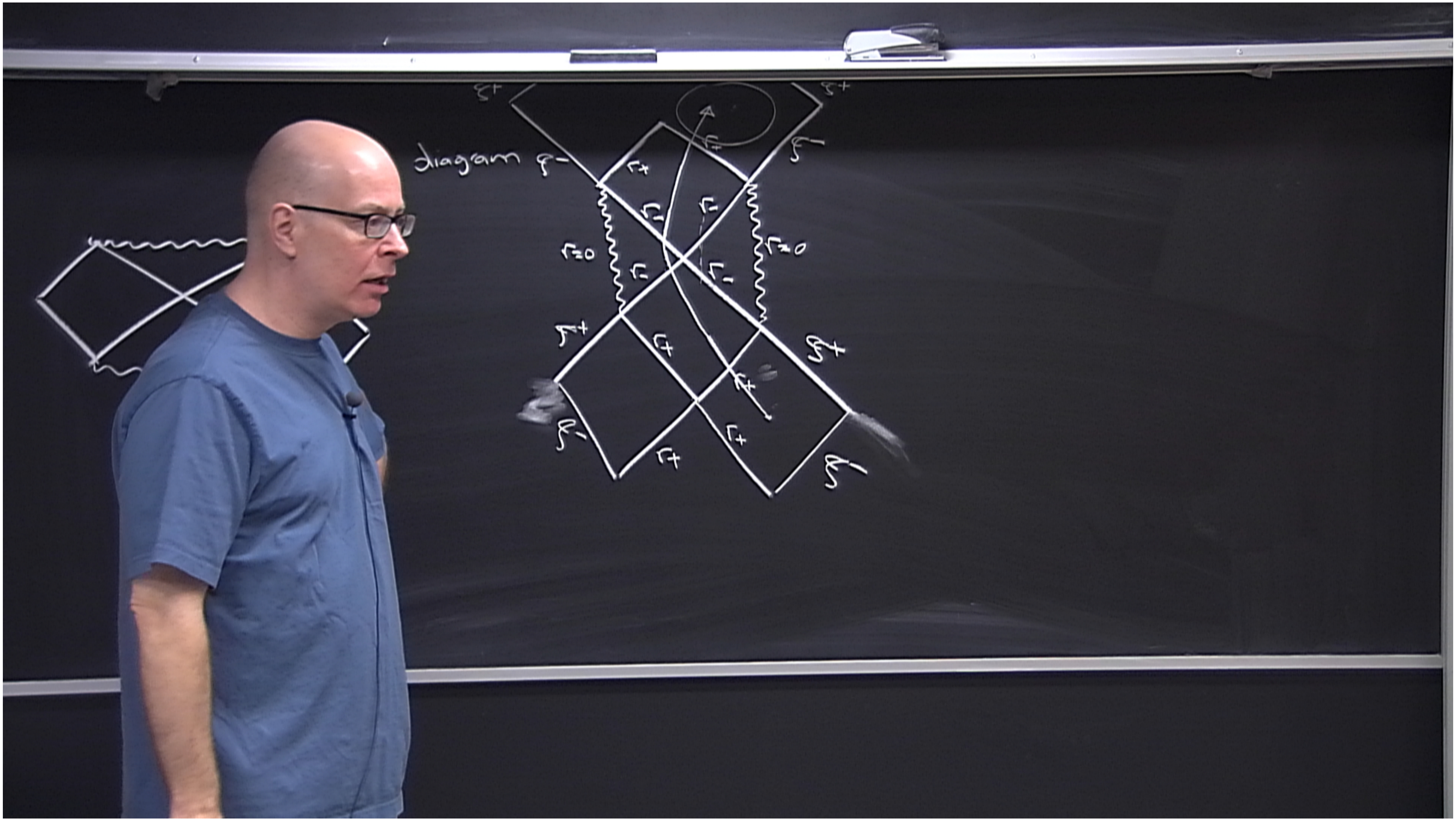
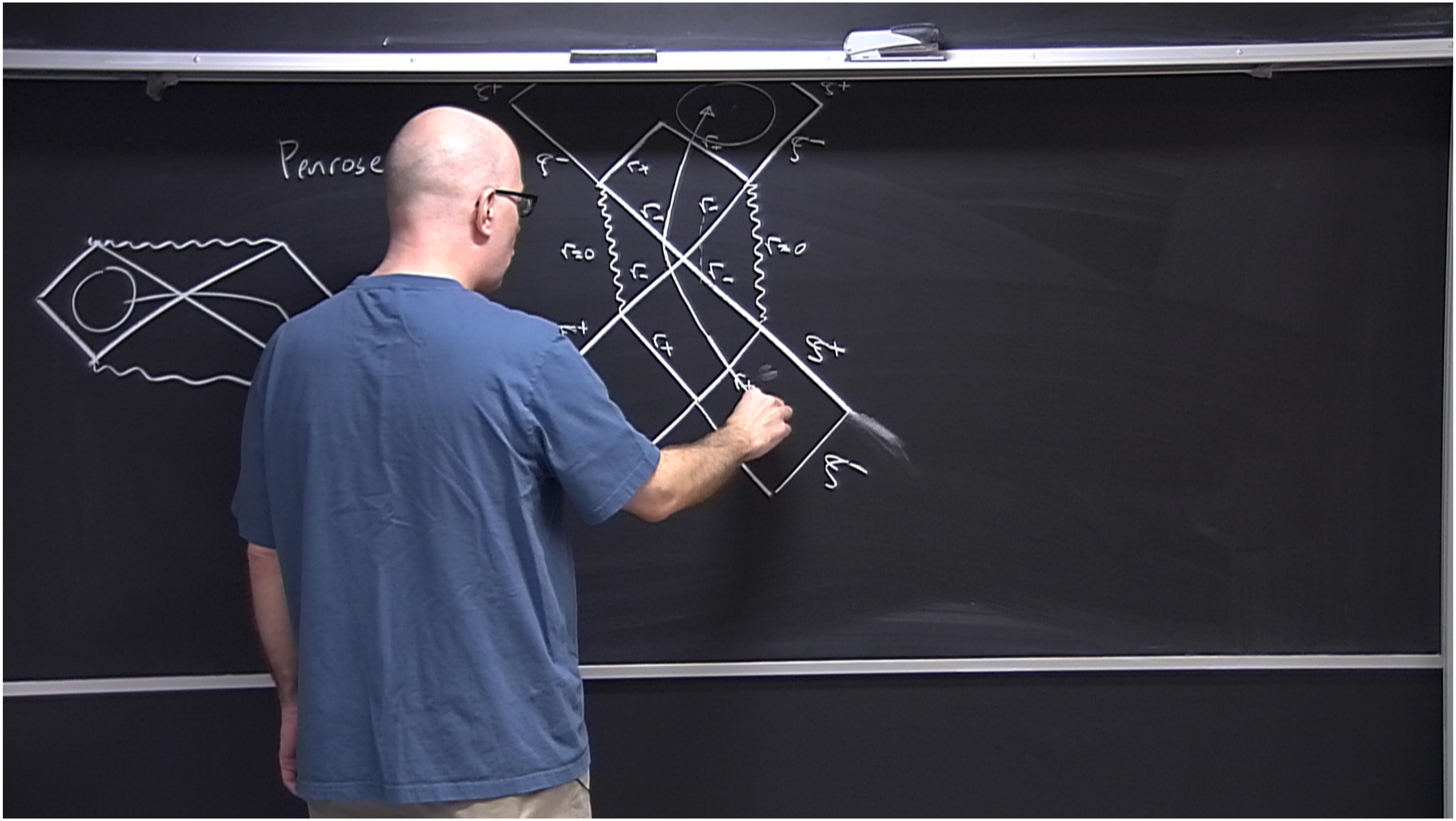
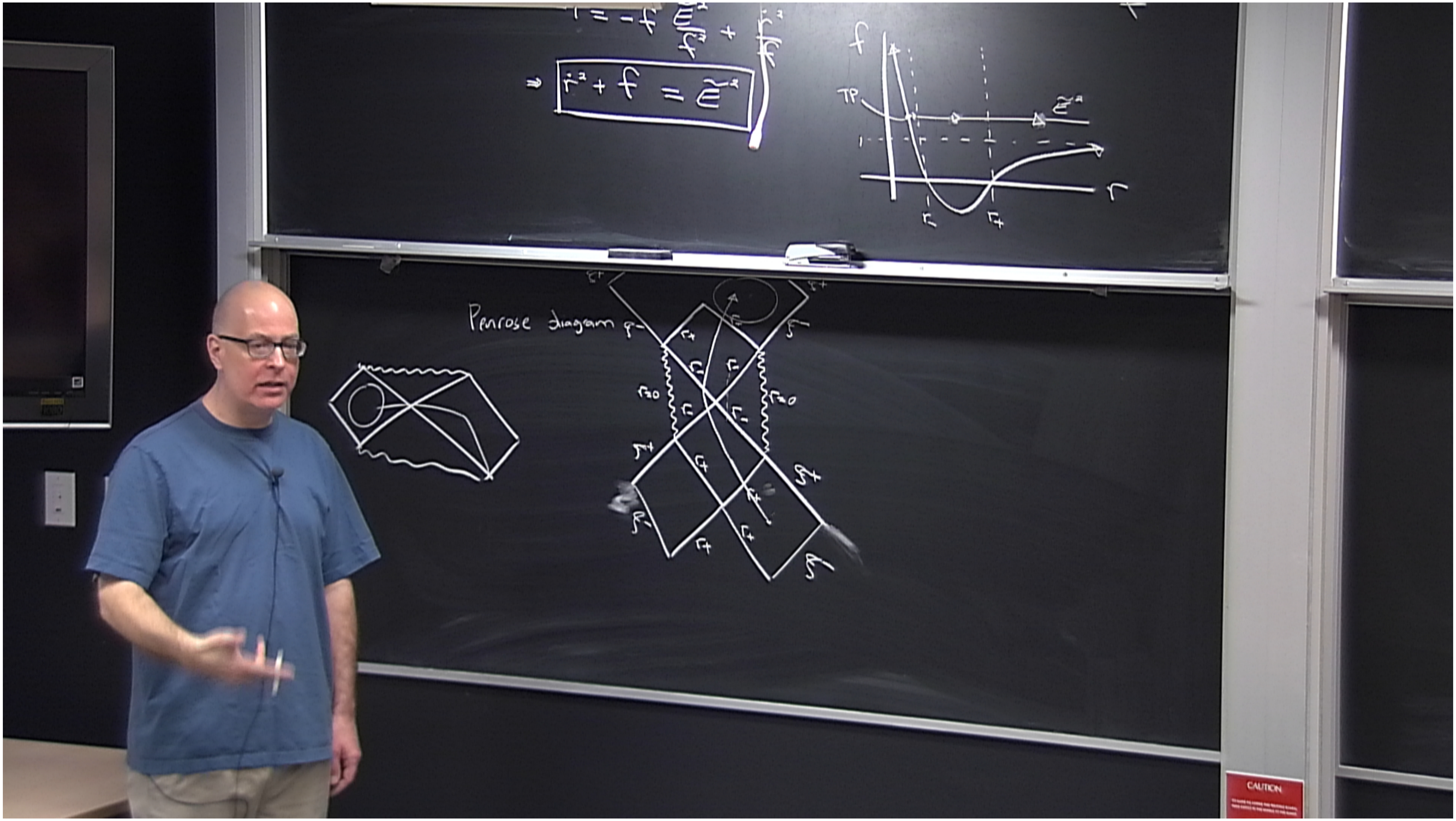
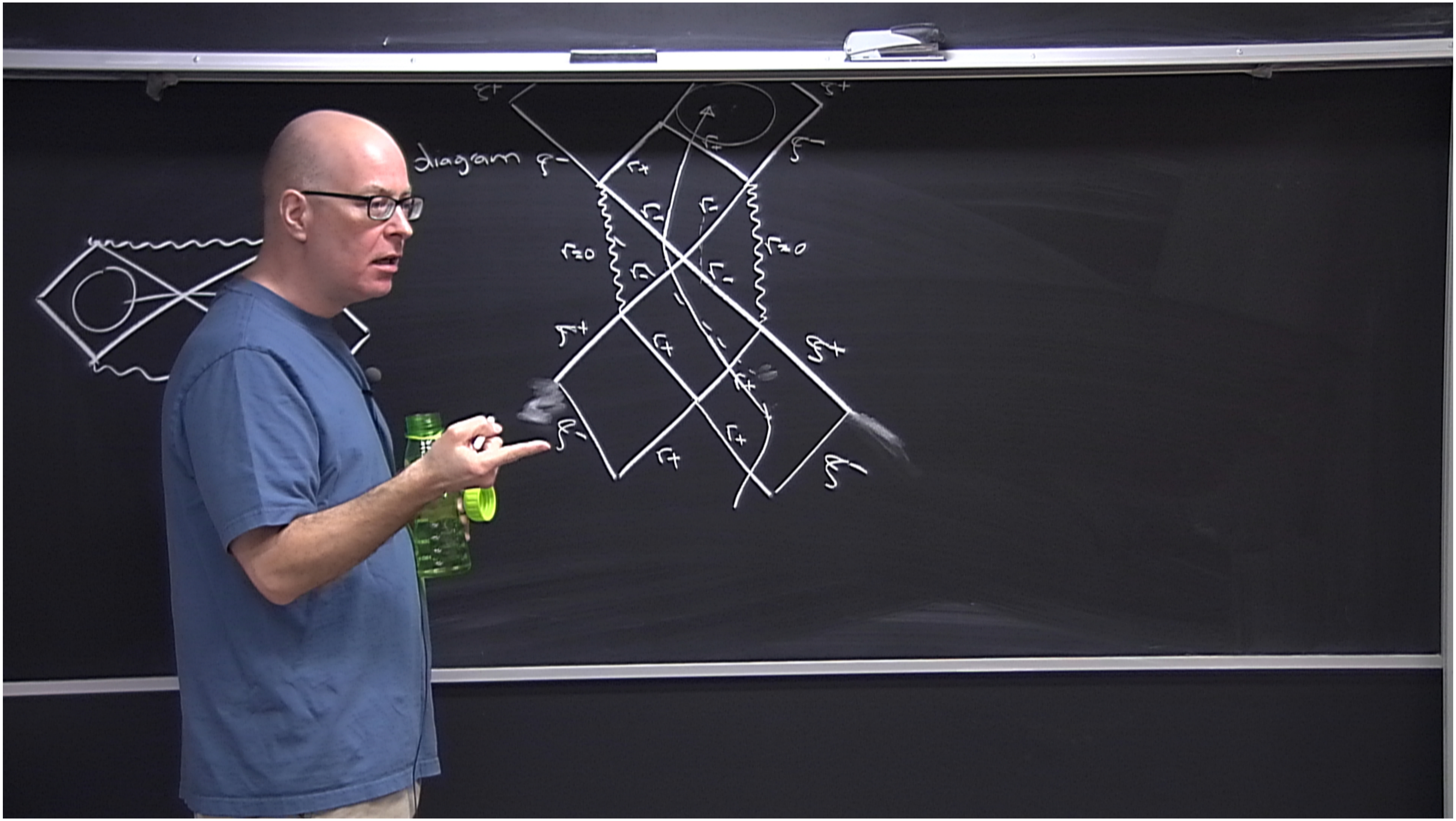


diagram
r=0
r=0


Penrose
r=0
r=0


Penrose diagram



diagram

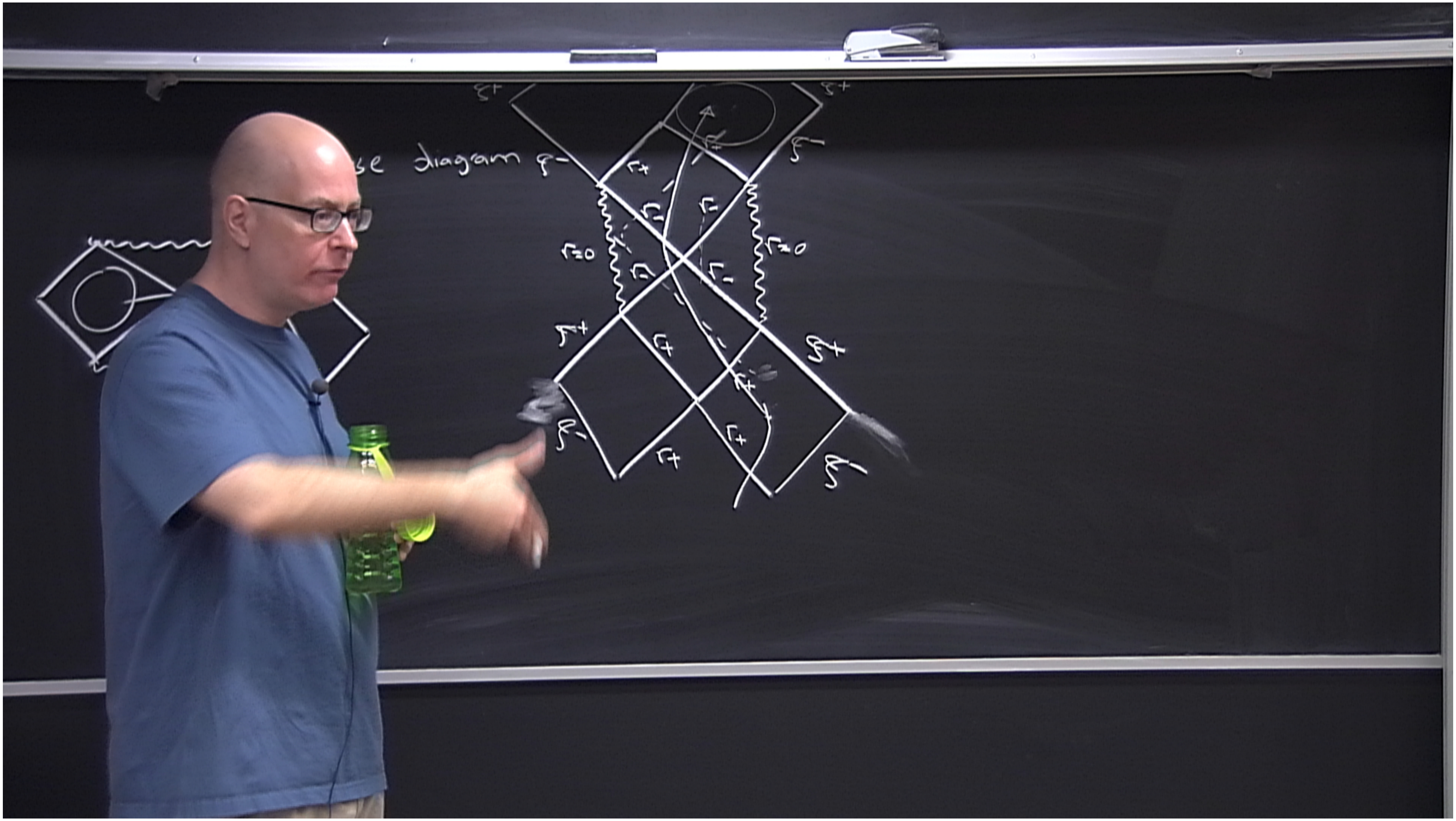
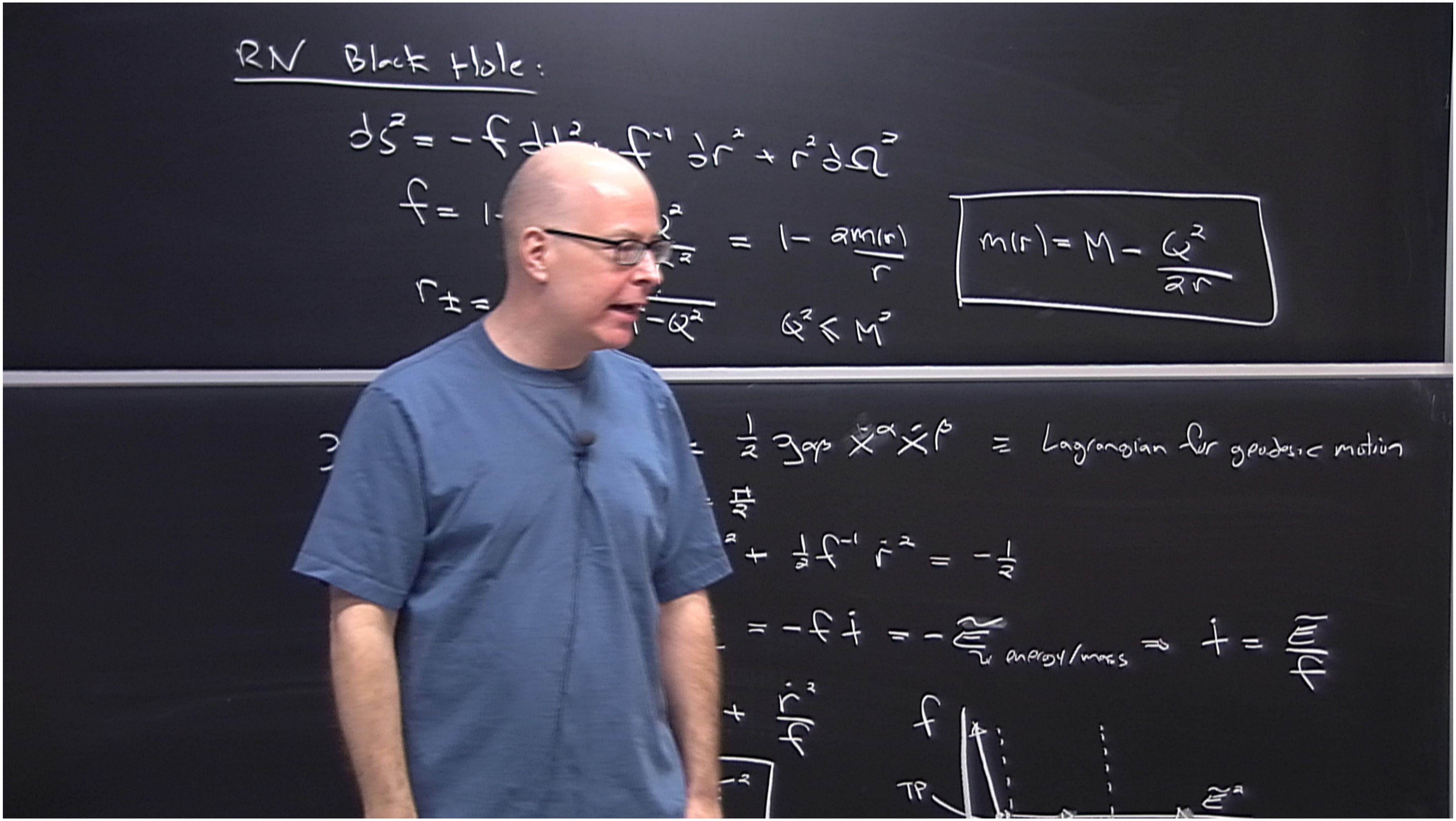
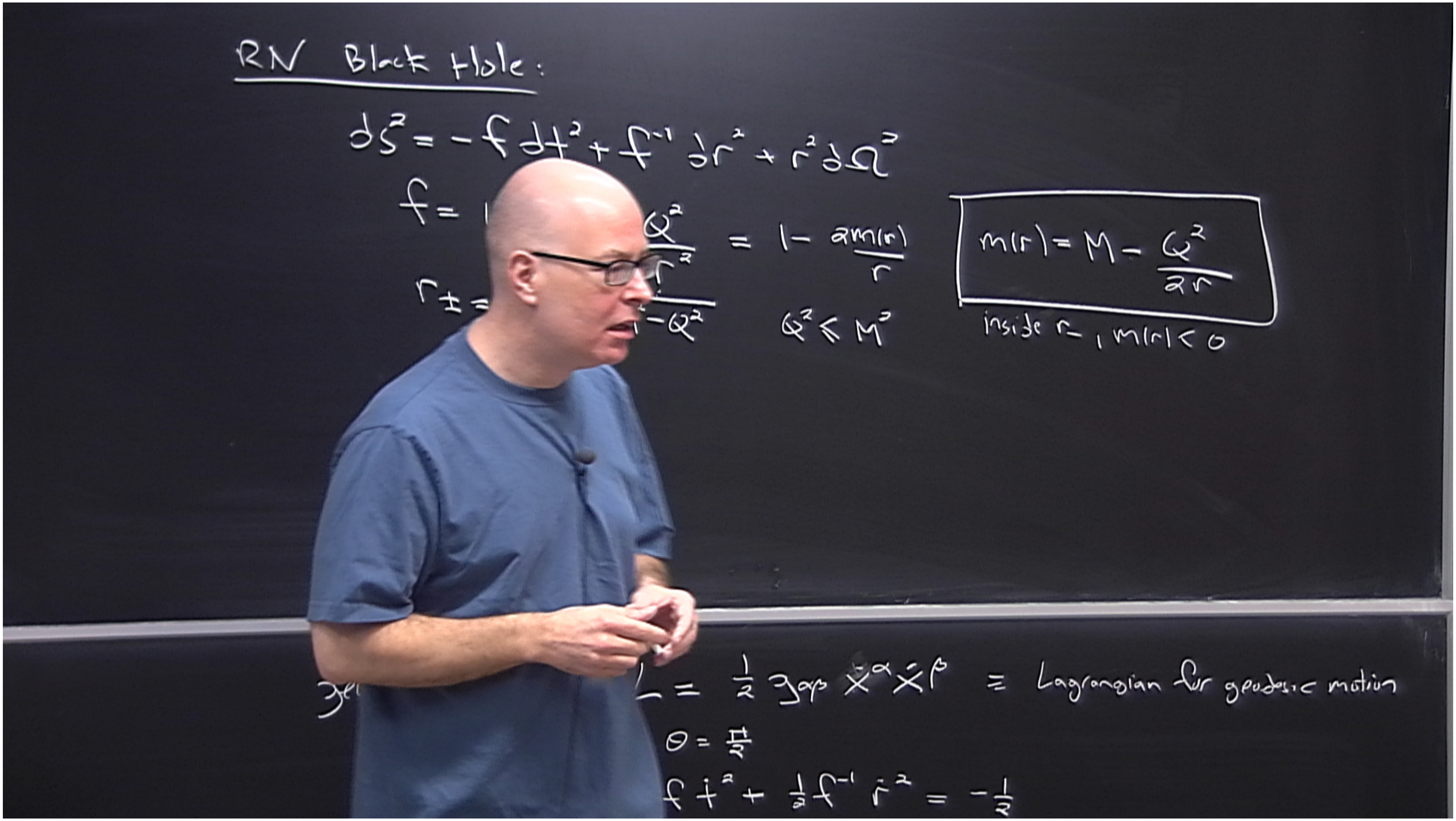
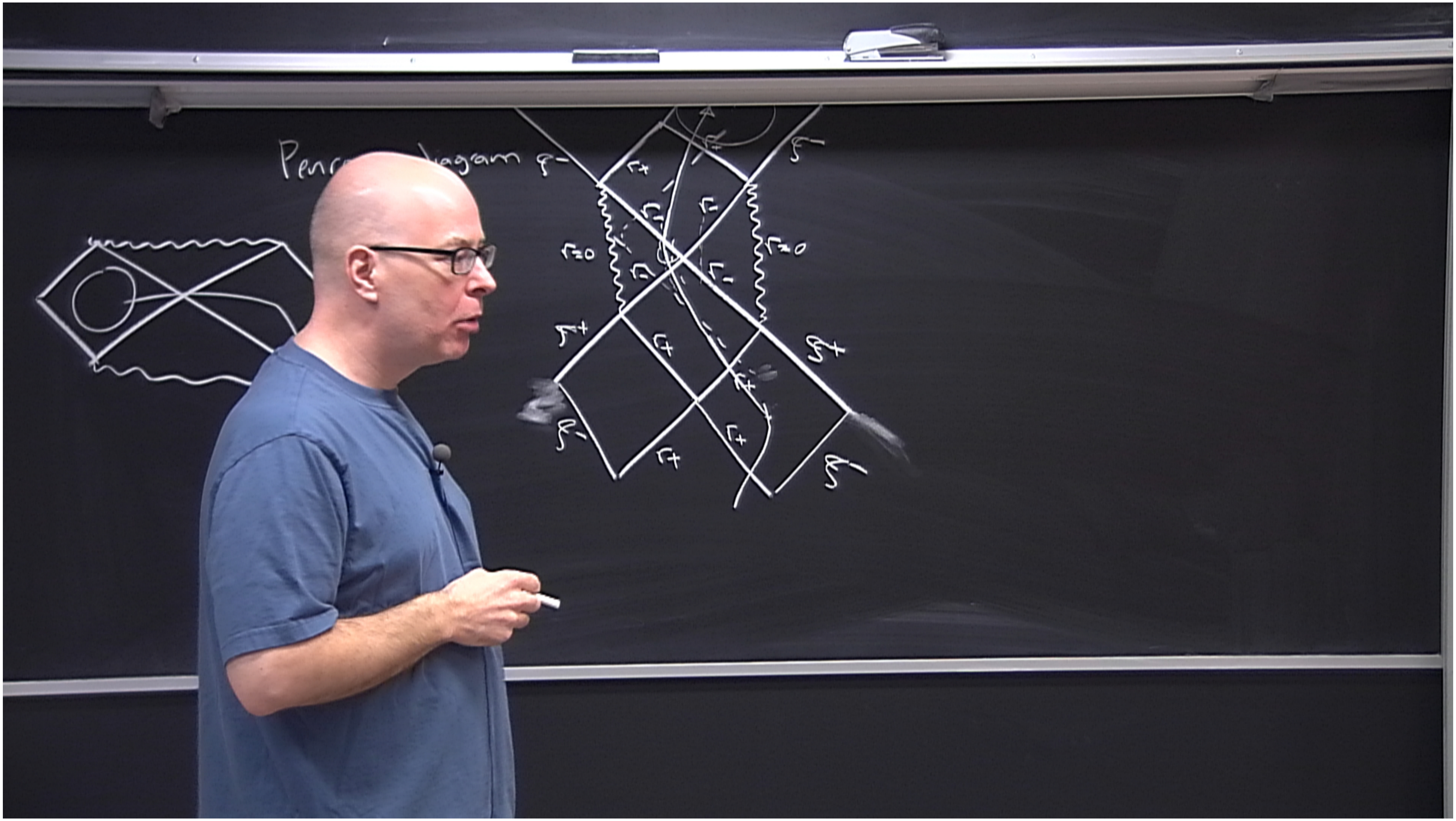
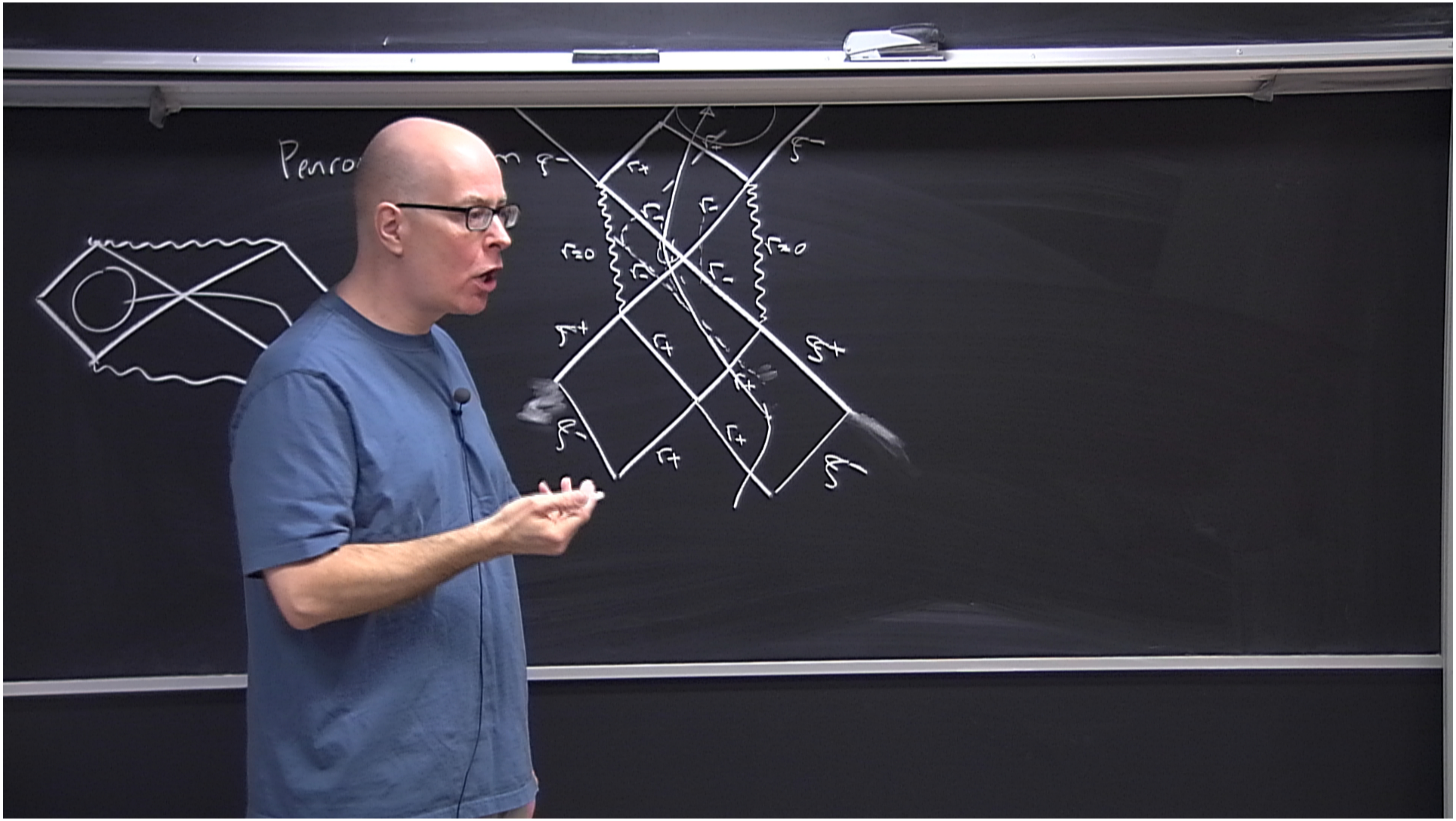


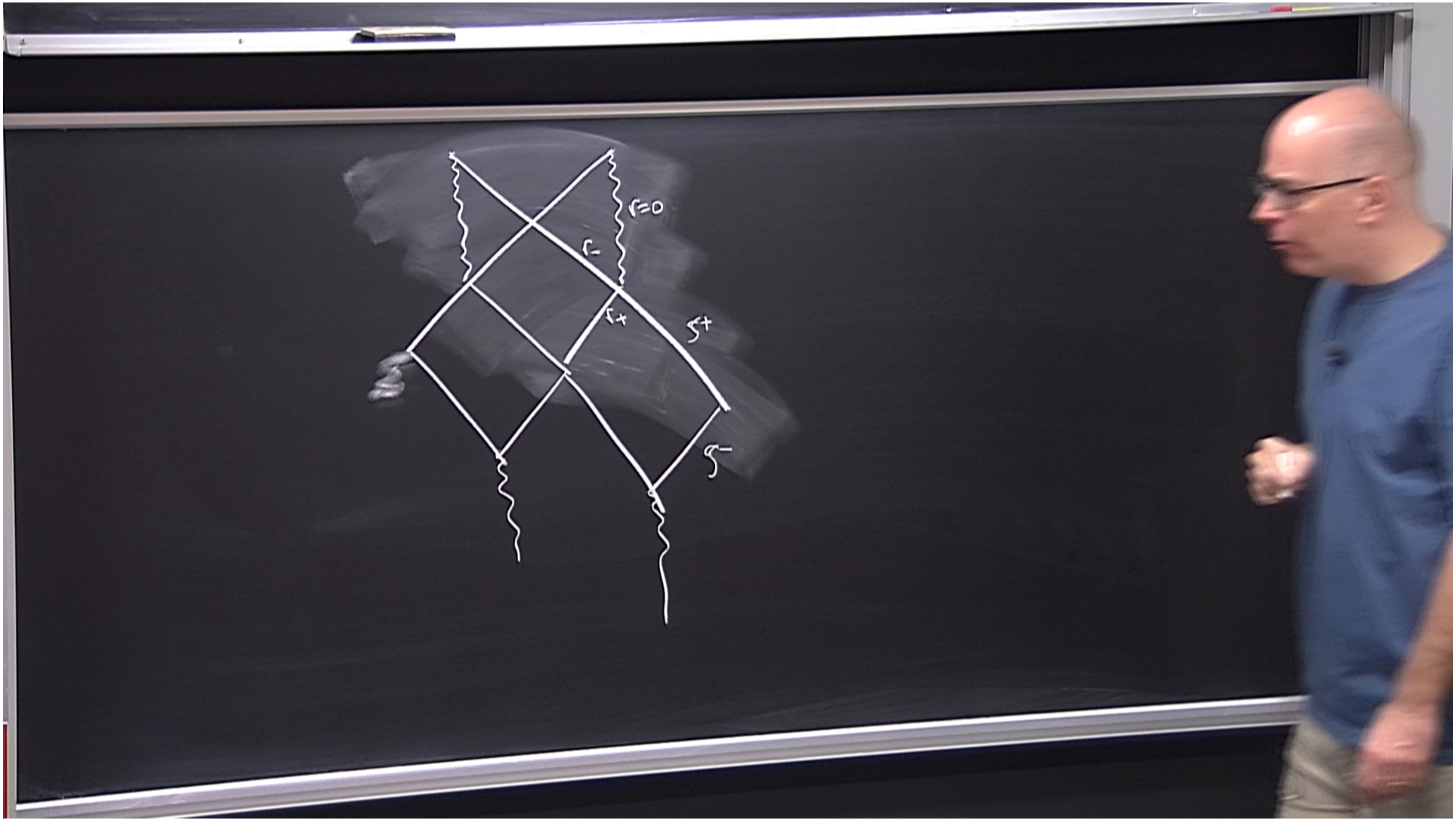
diagram
r=0
r=0
r+
r-
r-
r-
r+
r+
r+
r+

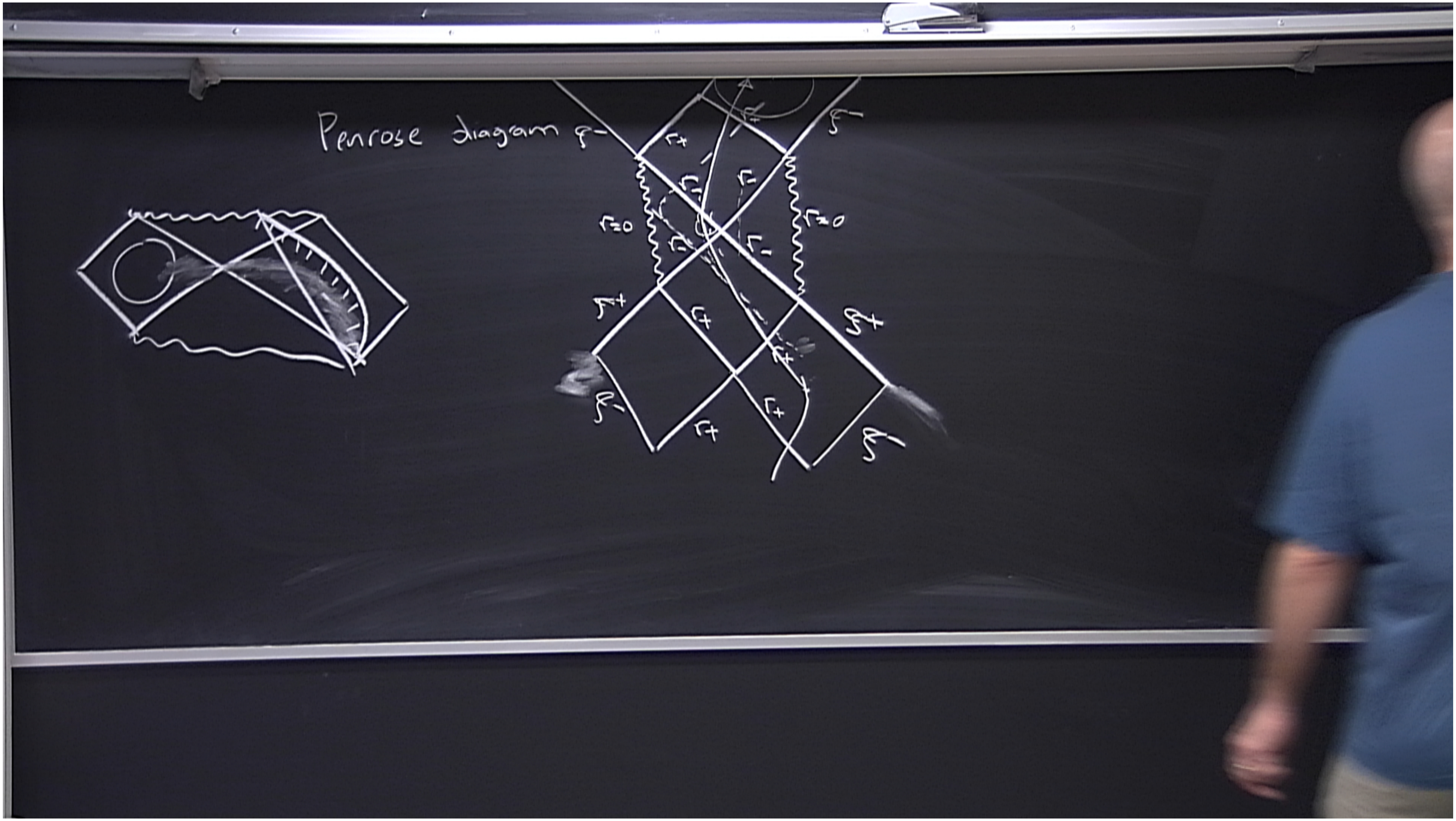

RN Black Hole:
m(r) = M − Q²/2r
= 1 − 2m(r)/r
Q² ≤ M²
Lagrangian for geodesic motion
energy/mass
TP
f

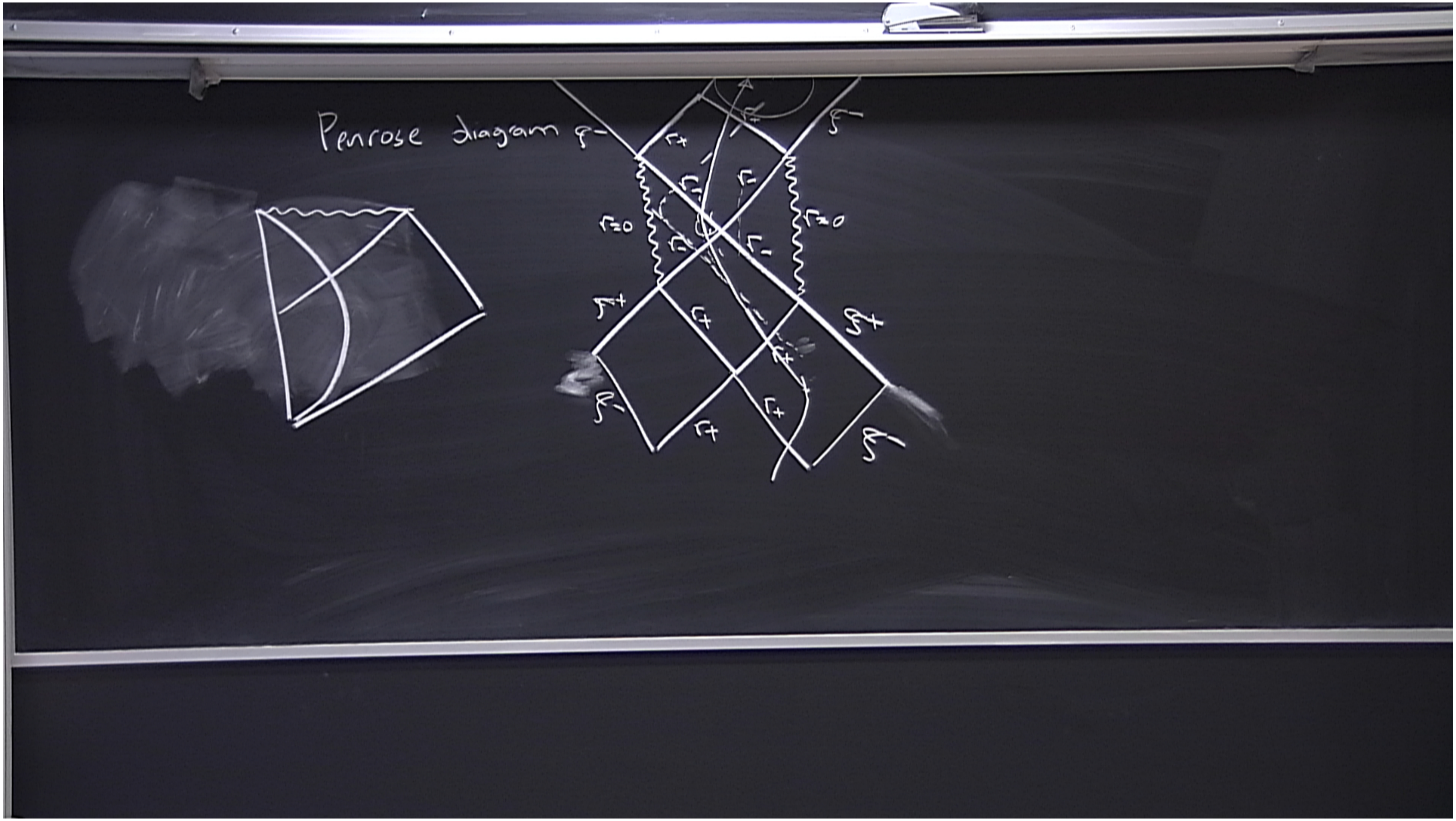

RN Black Hole:
$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega^2$
$= 1 - \frac{2m(r)}{r}$
$m(r) = M - \frac{Q^2}{2r}$
inside r_-, $m(r) < 0$
$Q^2 \leq M^2$
$L = \frac{1}{2} g_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta$ = Lagrangian for geodesic motion
$\theta = \frac{\pi}{2}$
$= -\frac{1}{2}$


r=0
r=0
r+
r+
r+
r+
r−
r−

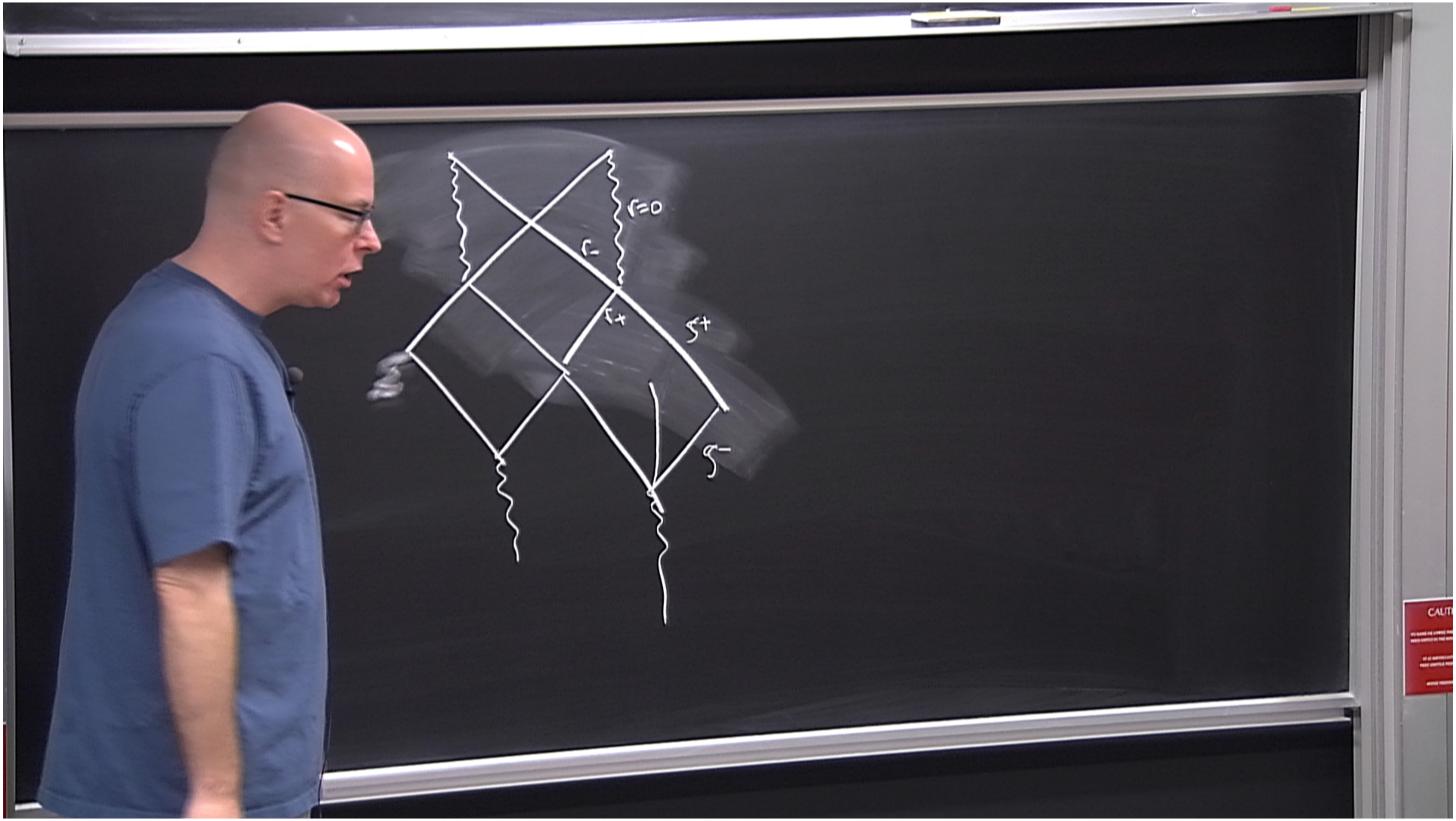

Penro
m ℐ−
r+
r−
r=0
ℐ−
ℐ+
r+
r−
ℐ+
ℐ−

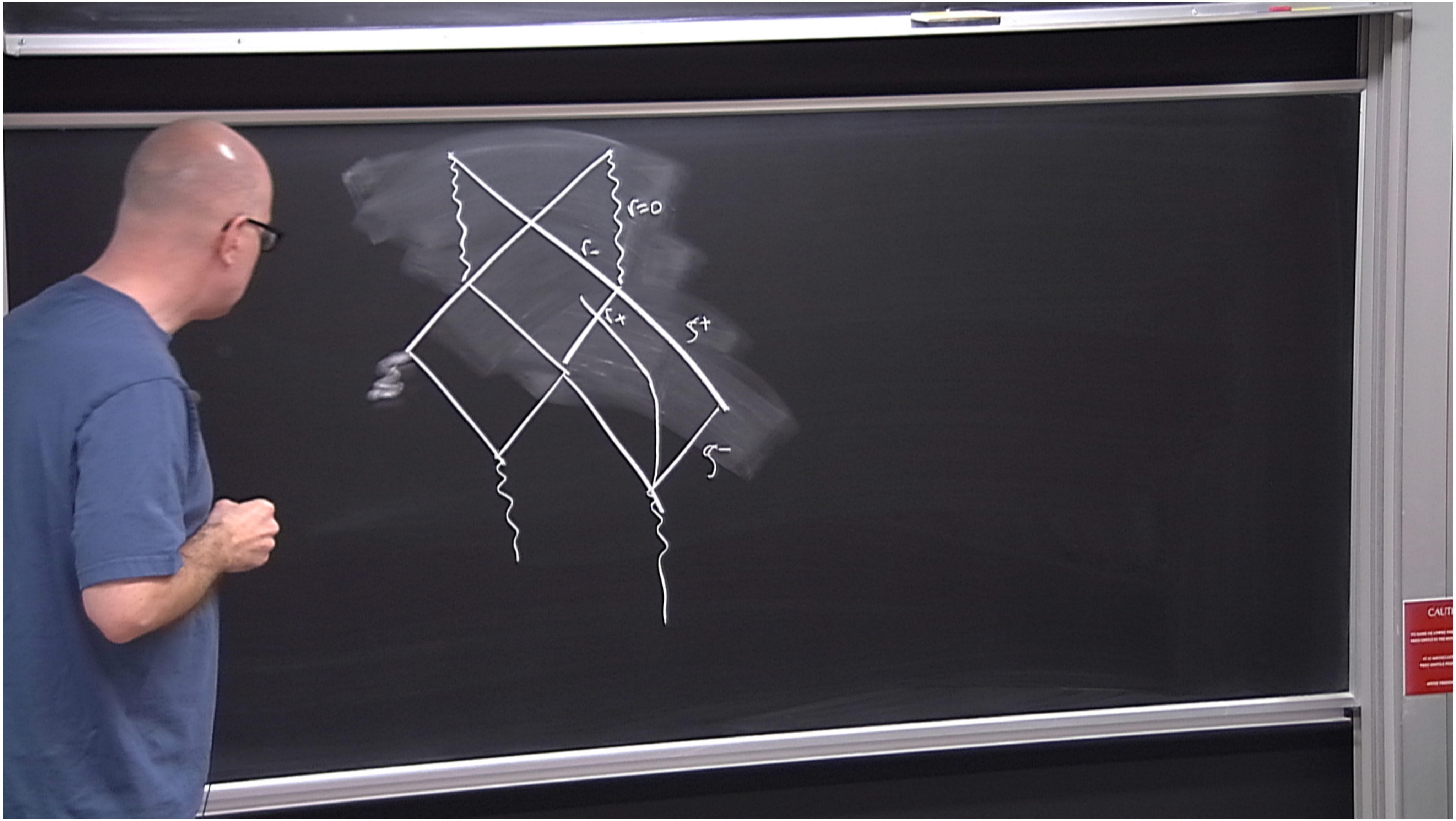

r=0
r-
r+
𝒥+
𝒥-

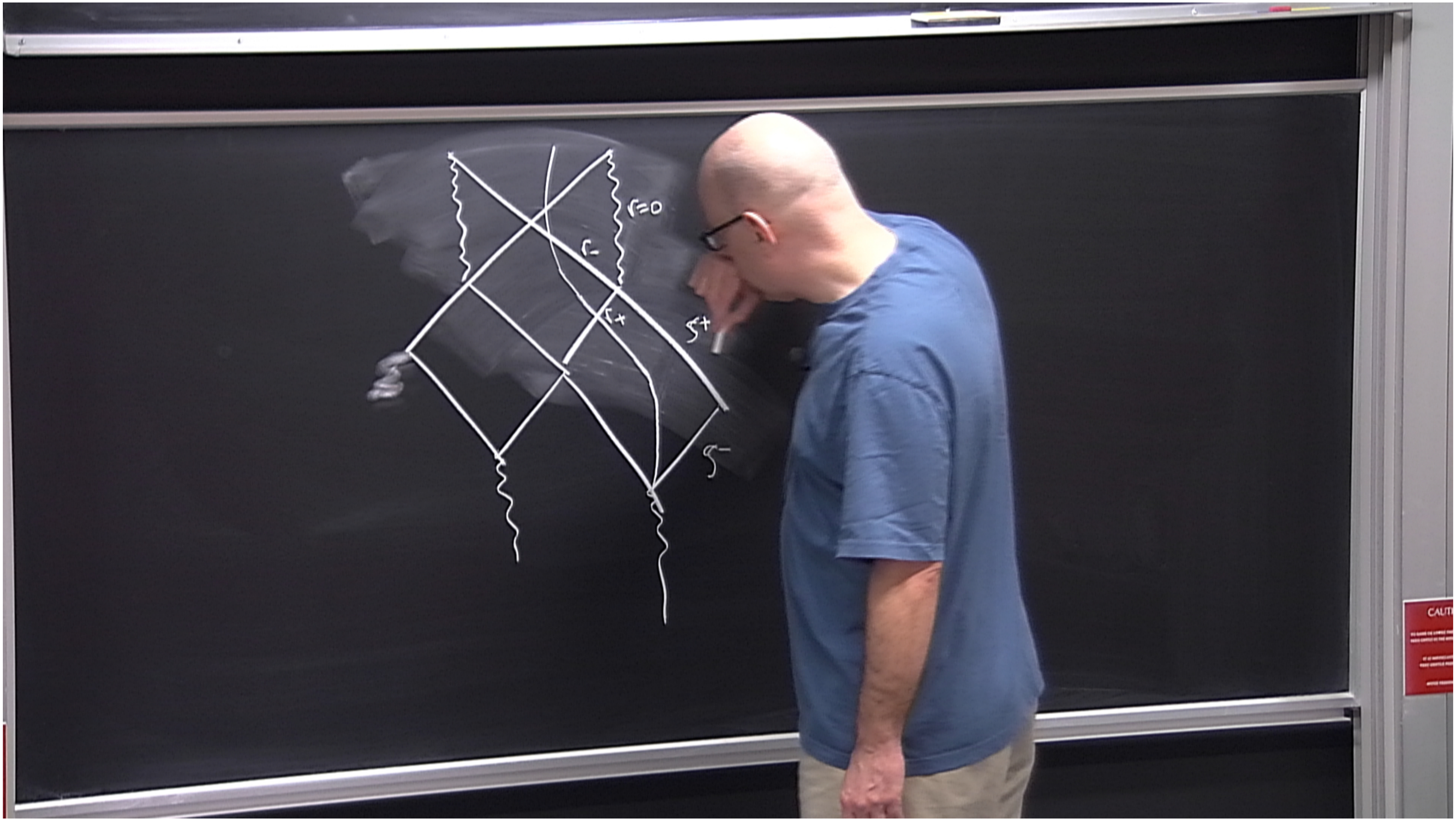

Penrose diagram
r=0
r=0

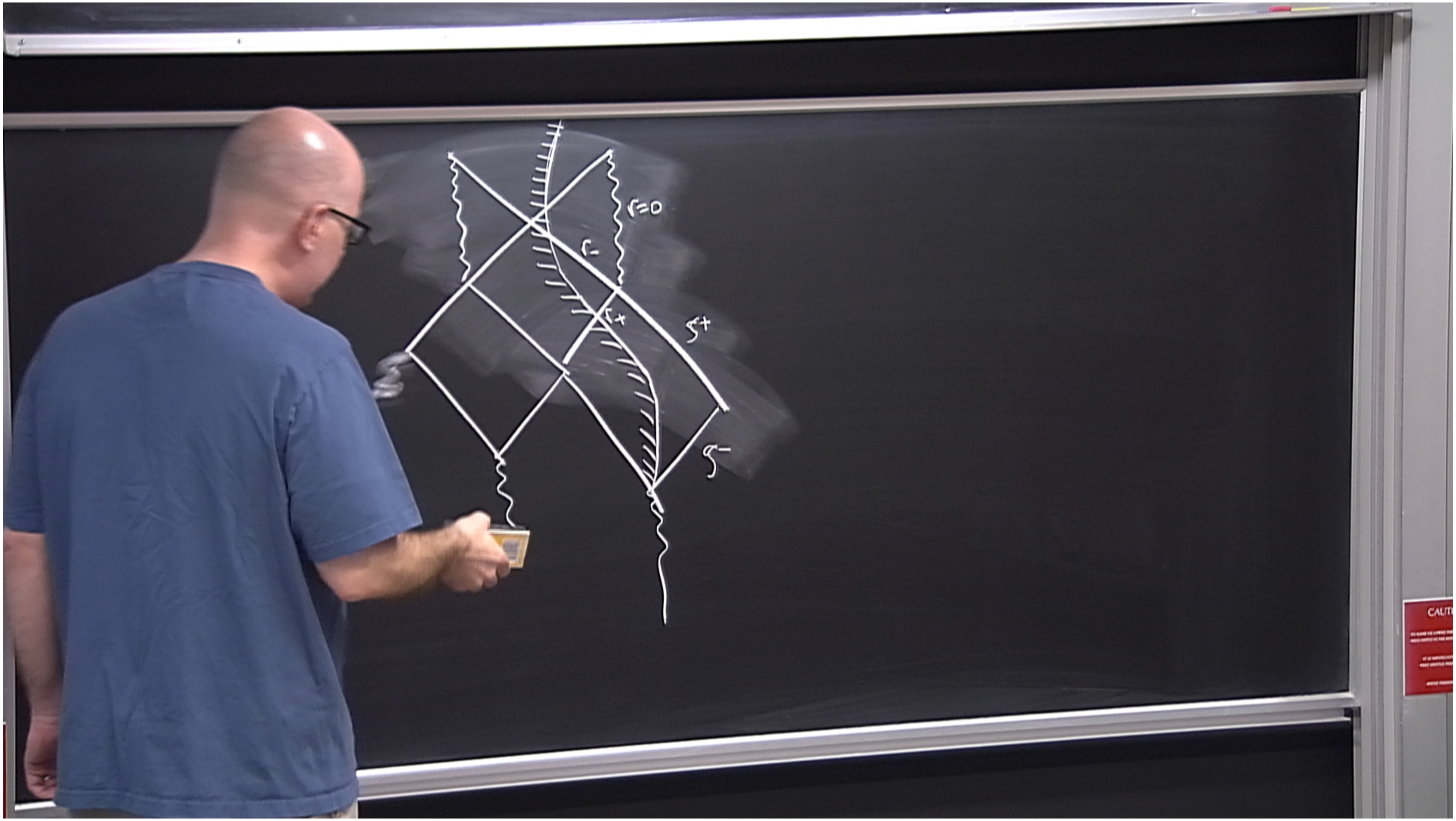


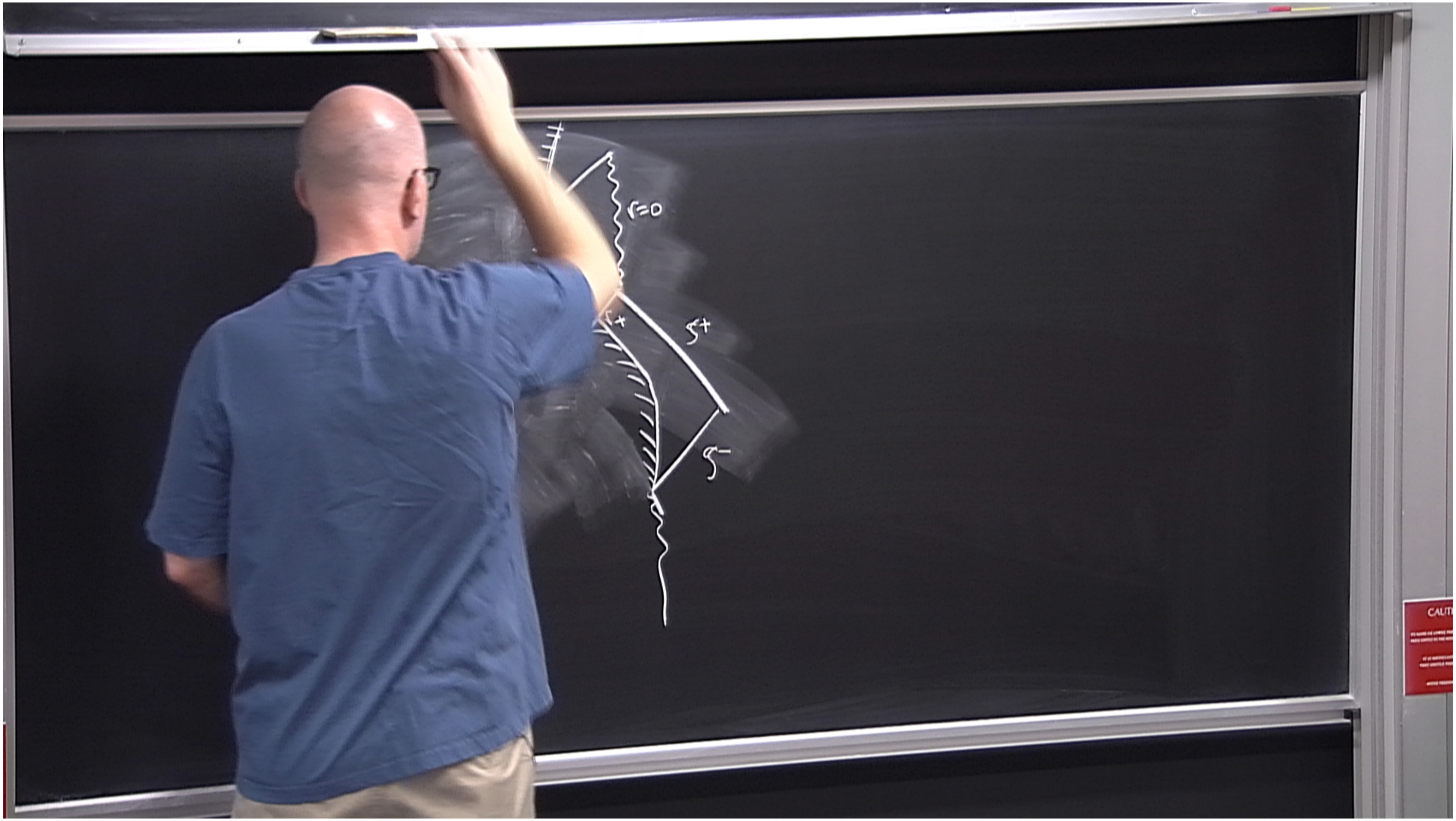
Penrose diagram
r=0
r=0

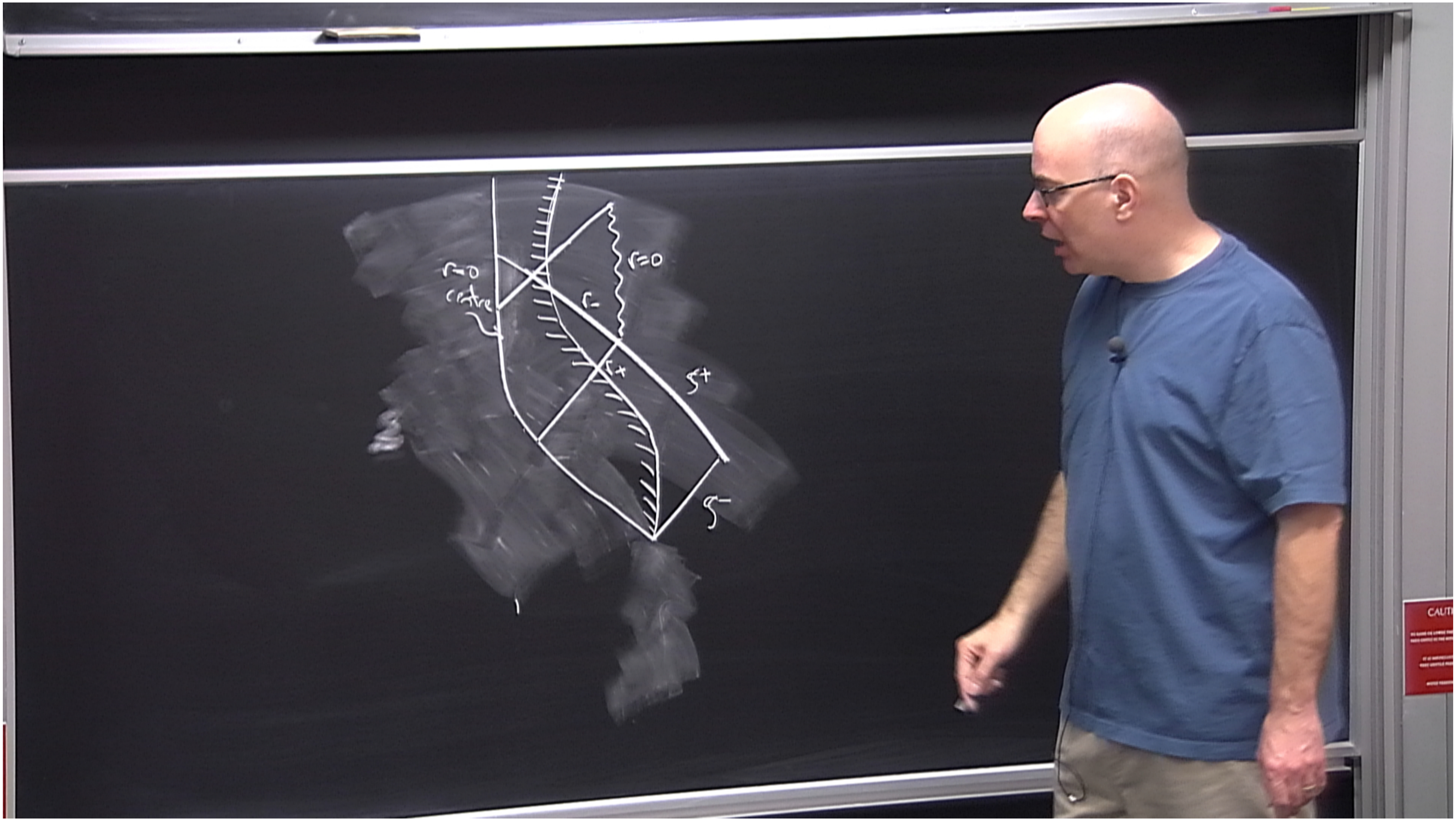

r=0
r₋
r₊
𝒥⁺
𝒥⁻


r=0
r₋
r₊
J⁺
J⁻

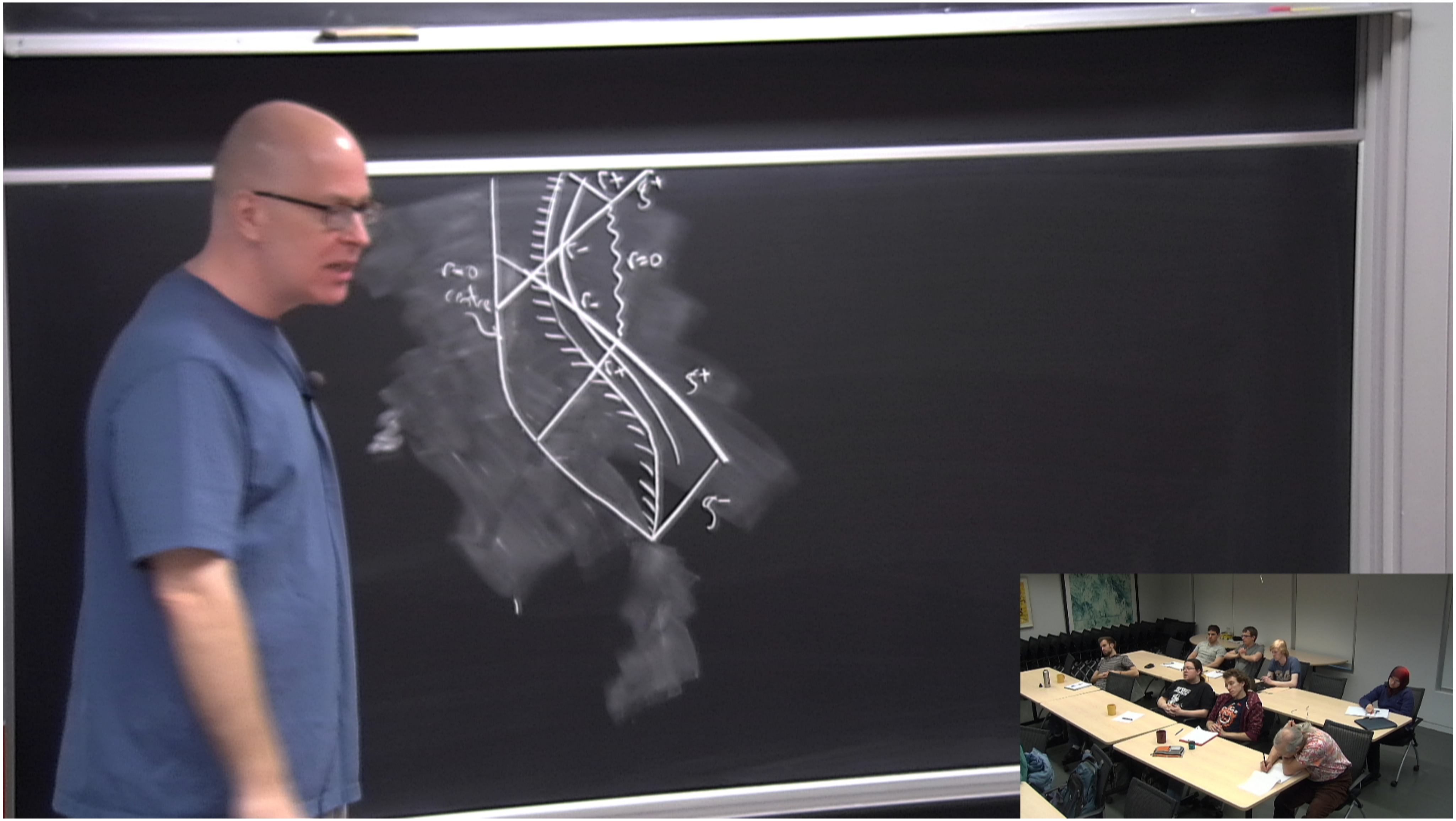

r=0
r₋
r₊
ℐ⁺
ℐ⁻

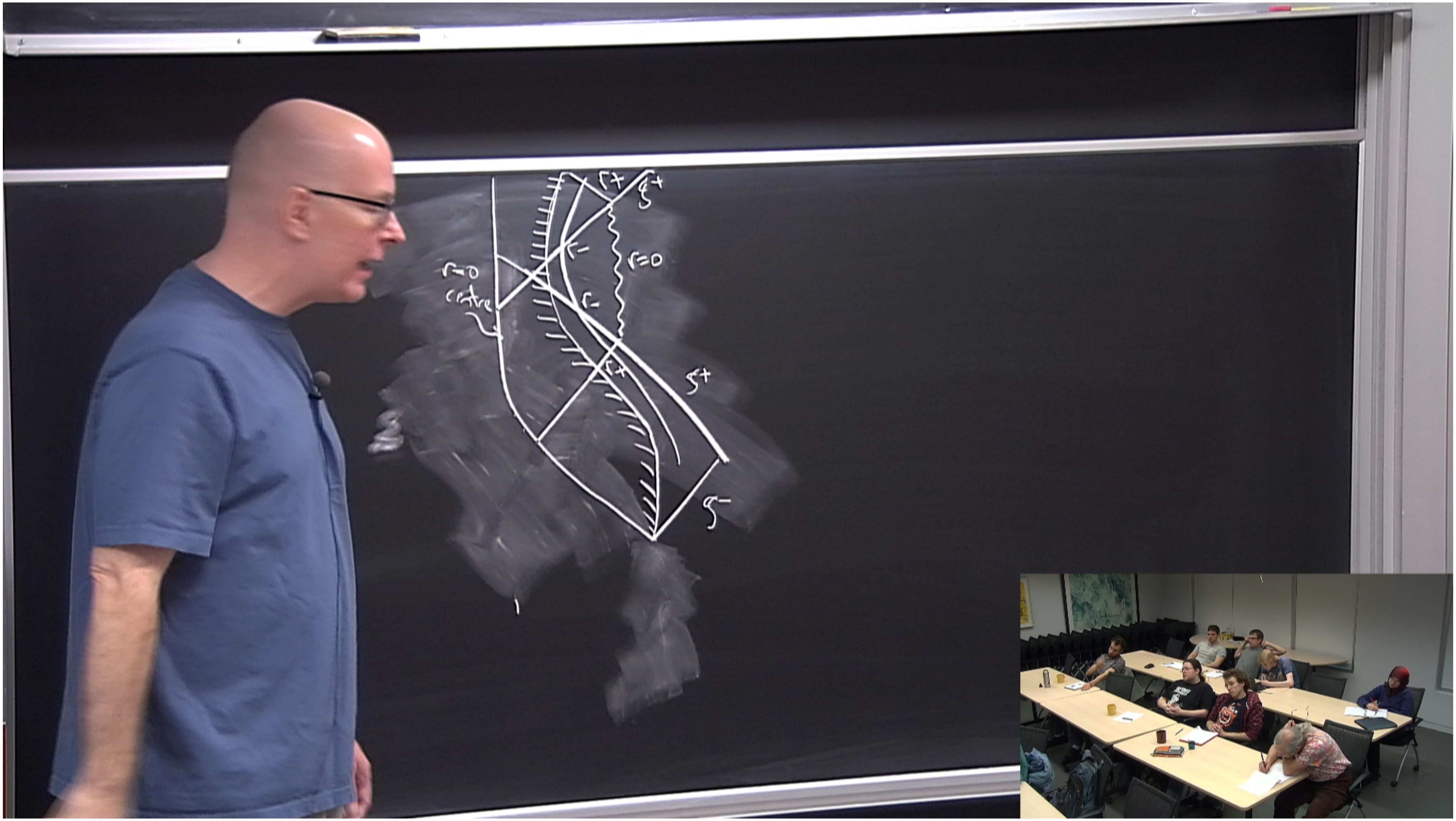

r=0
r₋
r₊
ℐ⁺
ℐ⁻

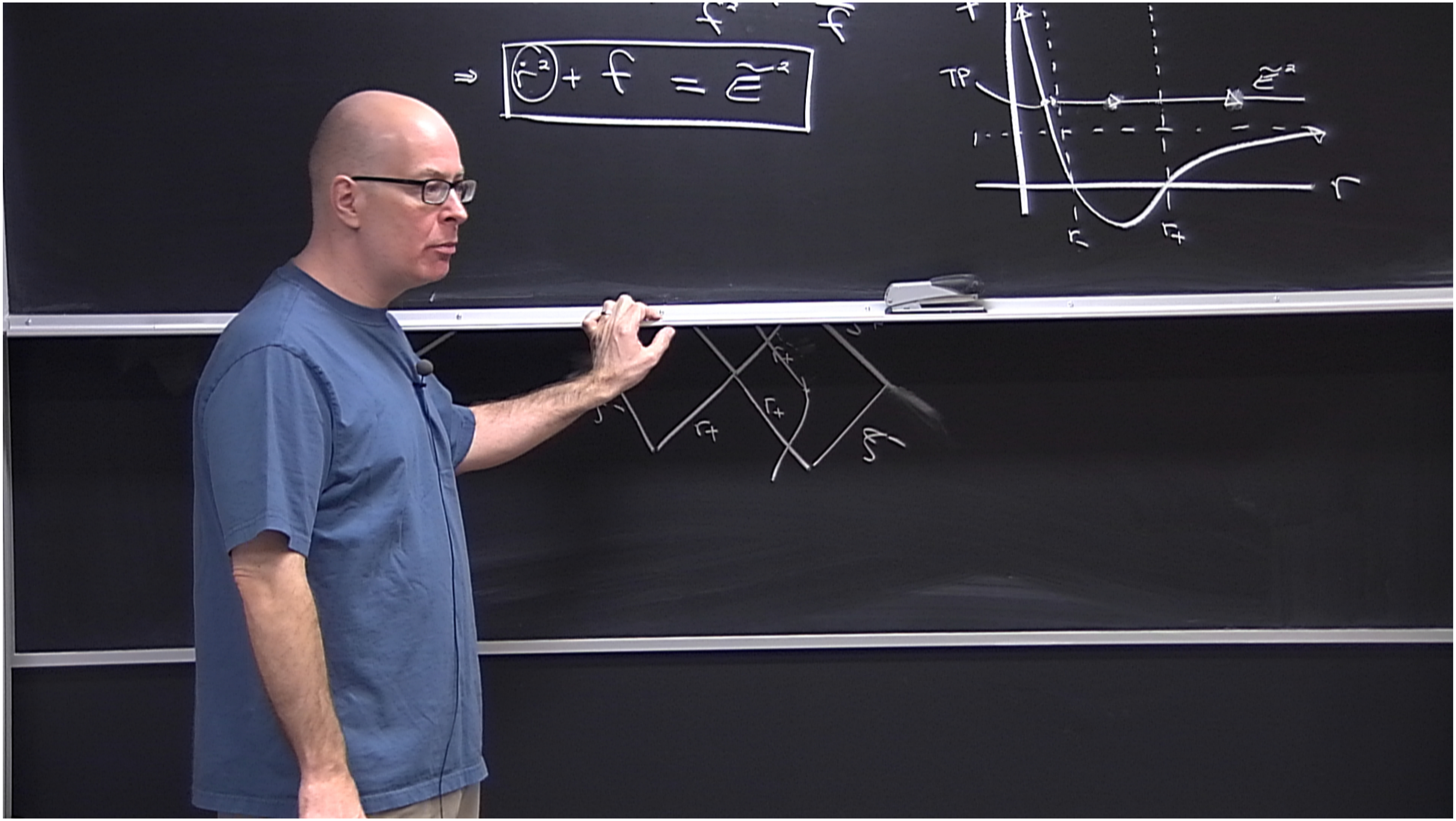

r=0

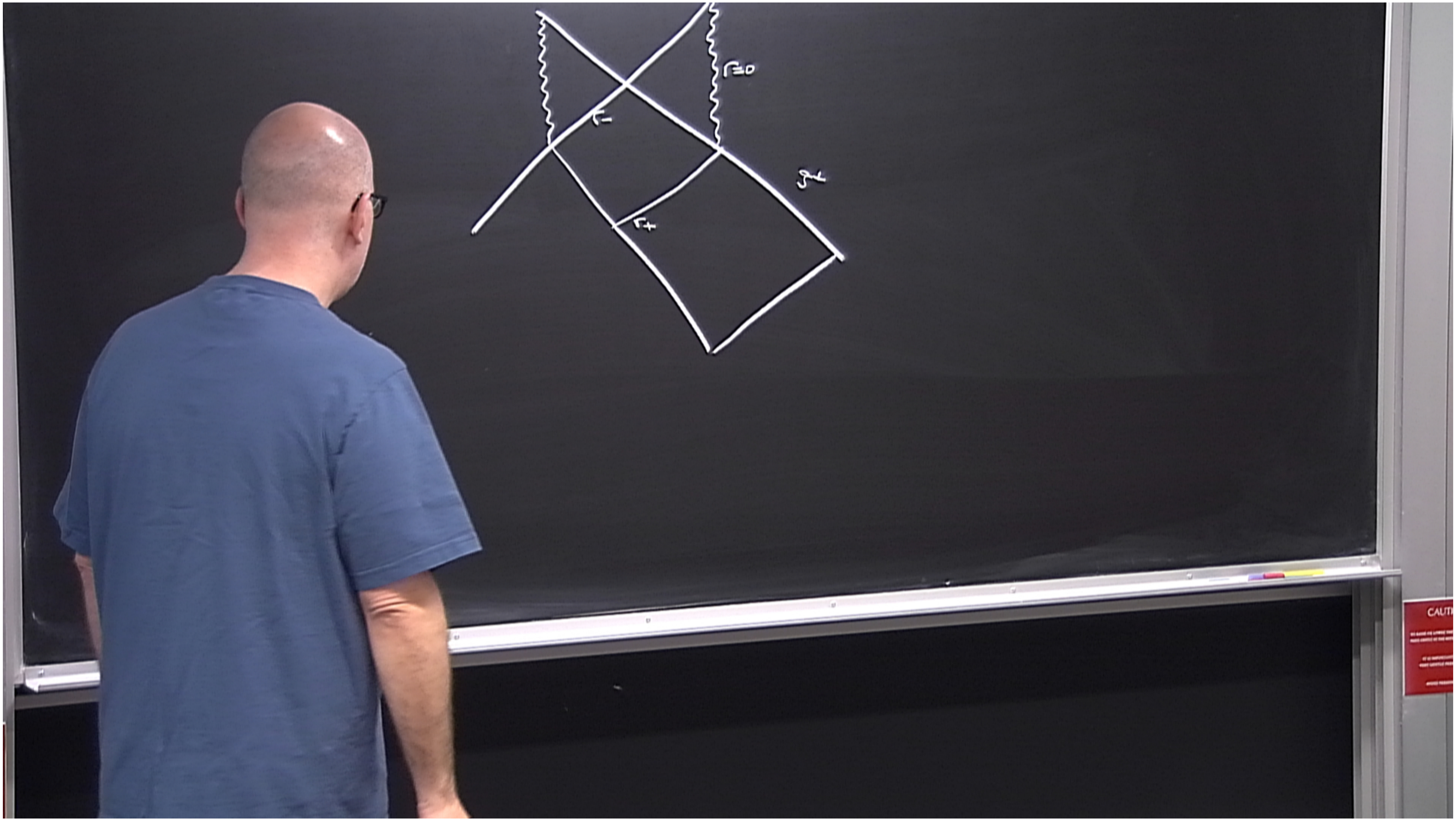

r=0
centre
r=0
r-
r+
$\mathscr{I}^+$
$\mathscr{I}^-$

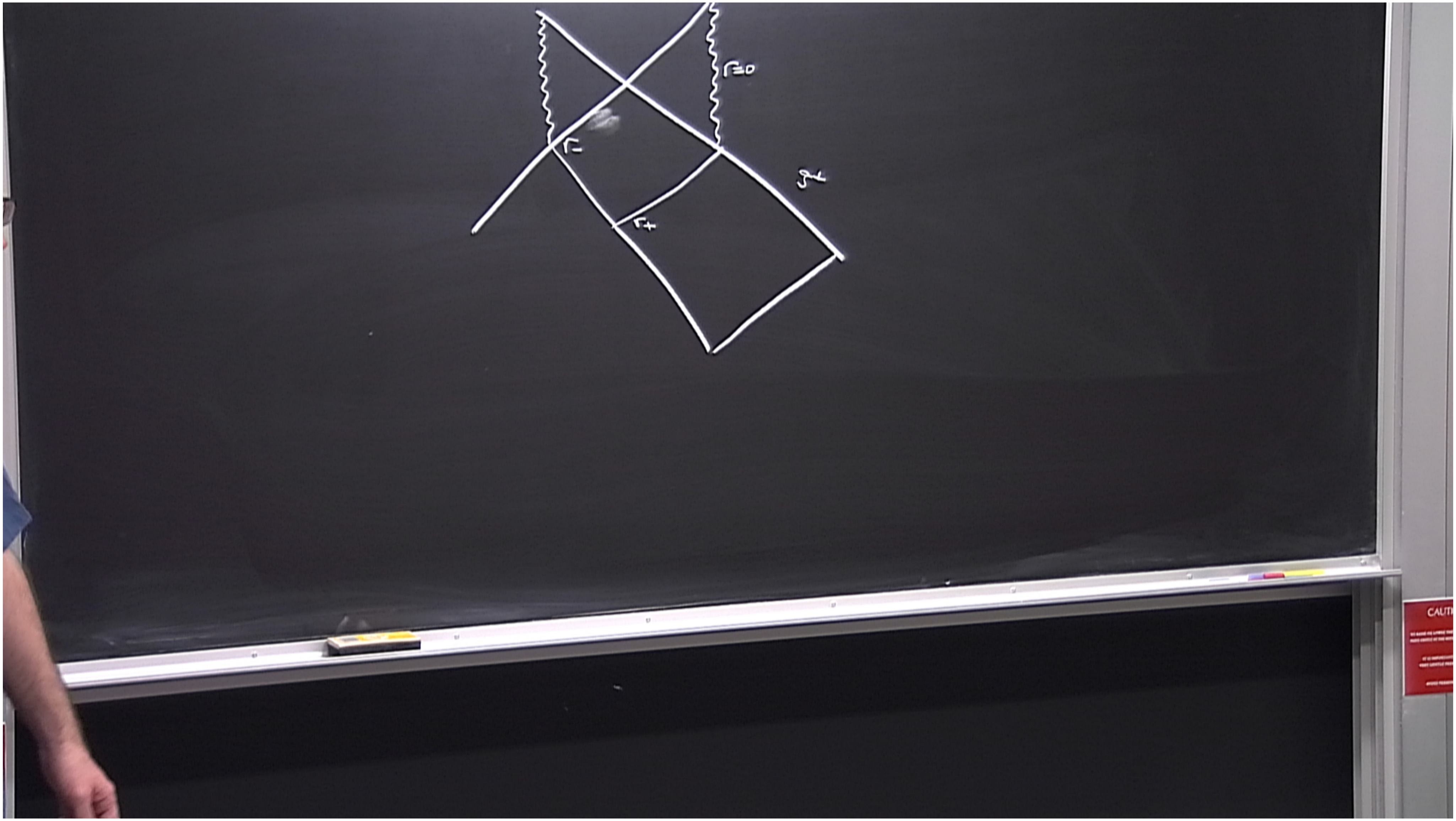

r+
$\mathcal{J}^+$
r−
r=0
r=0
centre
r−
r+
$\mathcal{J}^+$
$\mathcal{J}^-$

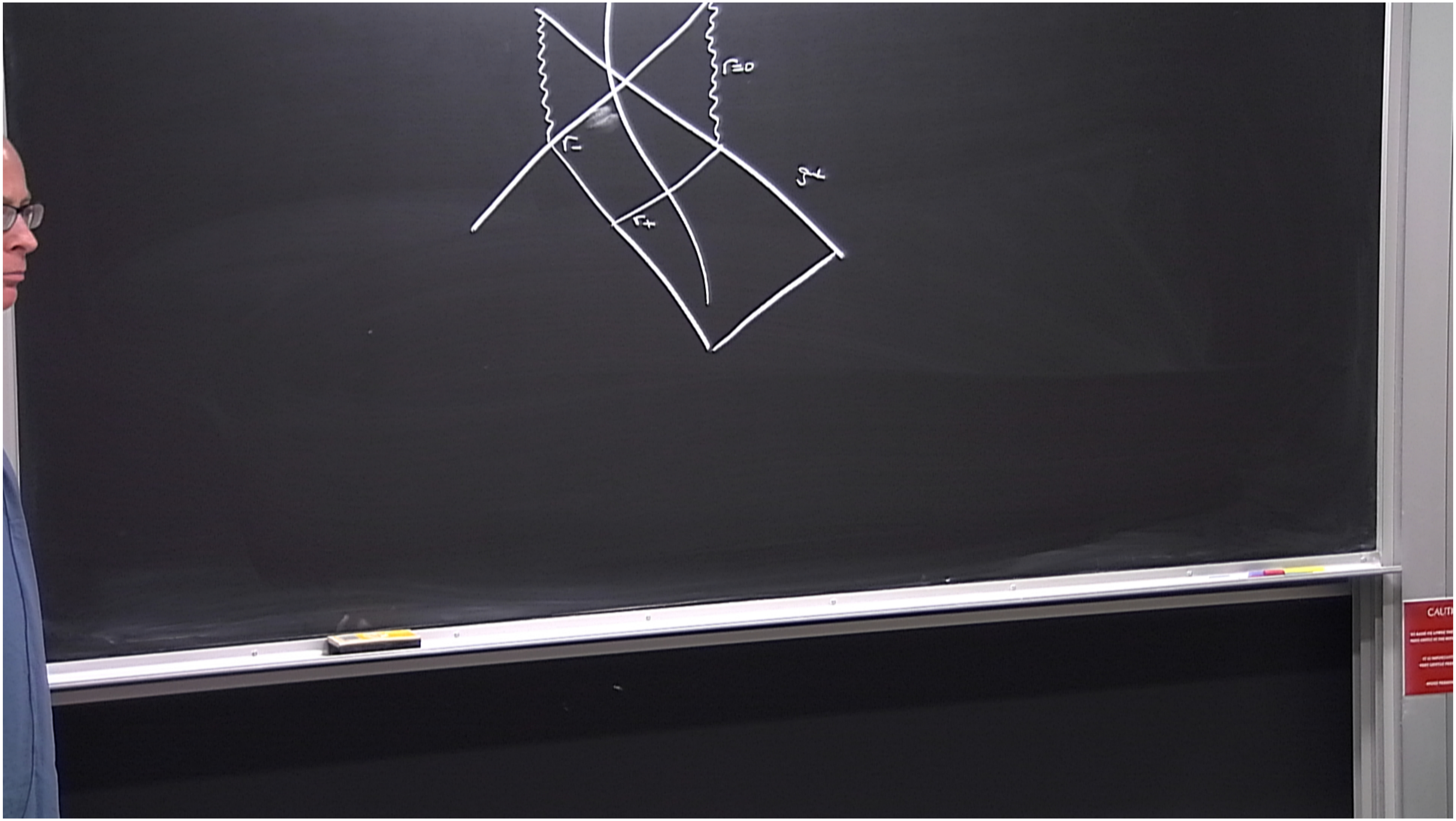

r=0
centre
r=0

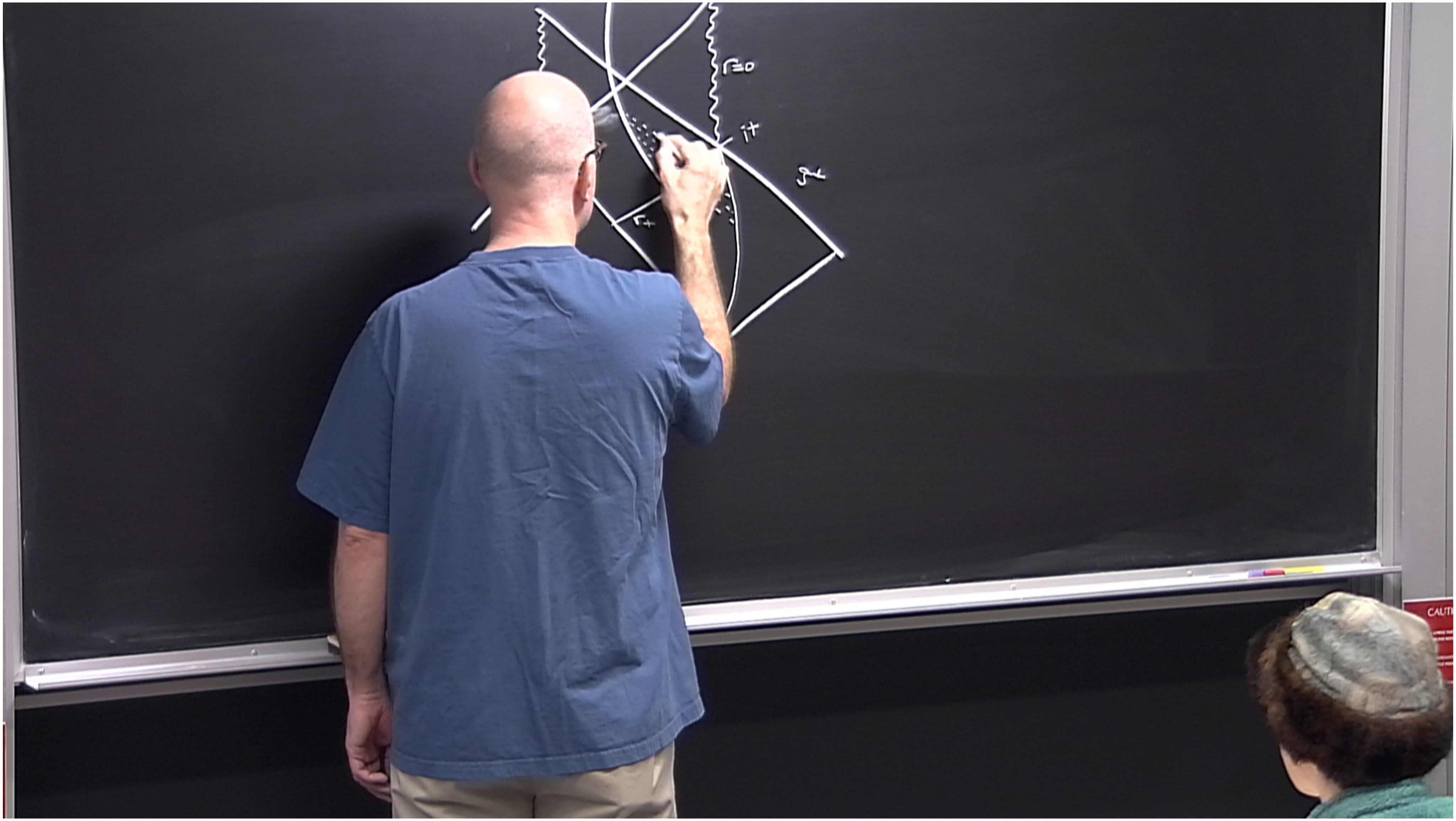

r+
$\mathscr{I}^+$
r−
r=0
r=0
centre
r−
r+
$\mathscr{I}^+$
$\mathscr{I}^-$

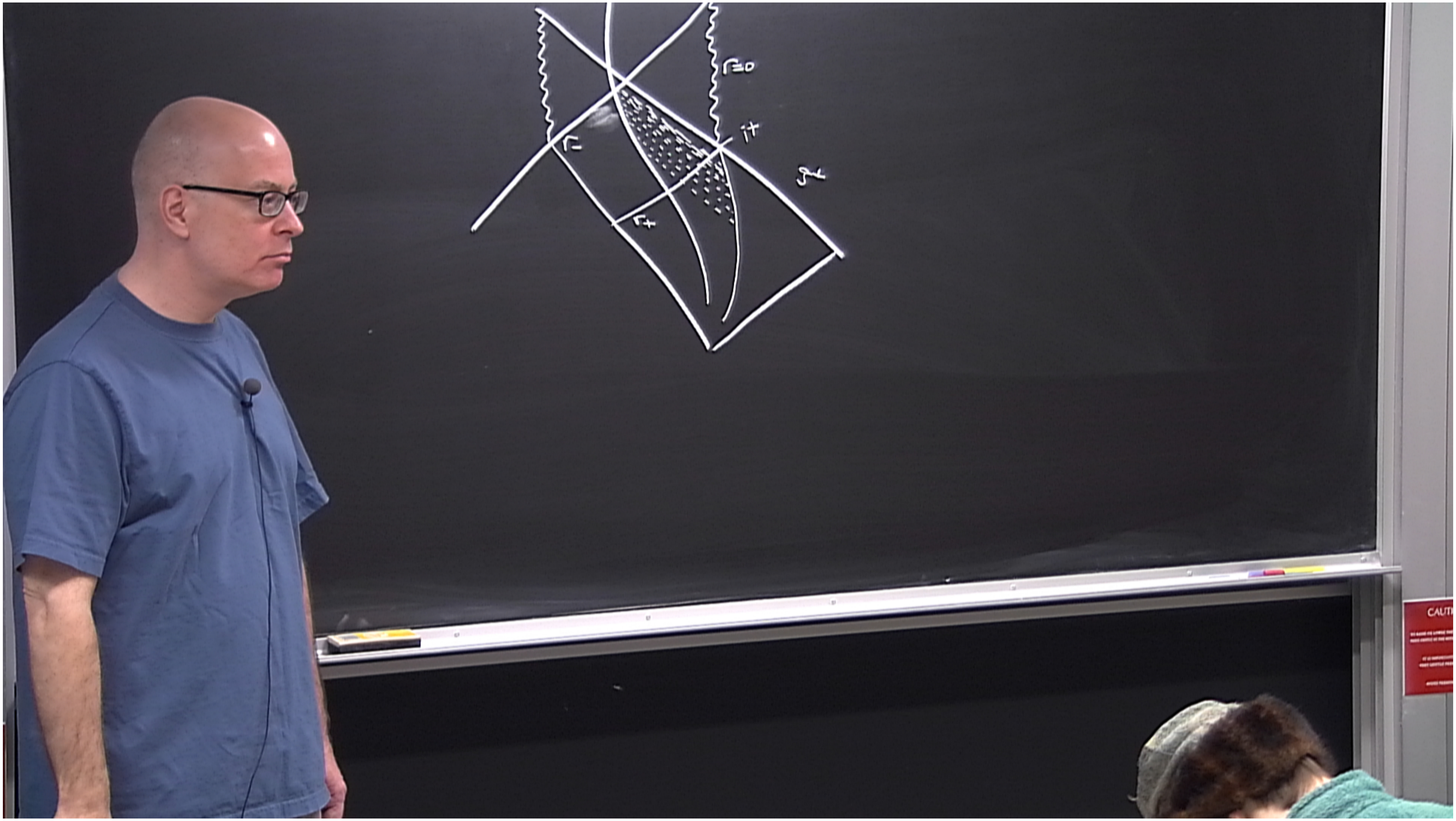

$\Rightarrow \dot{r}^2 + f = \tilde{E}^2$
TP
$\tilde{E}^2$
1
r
r_-
r_+

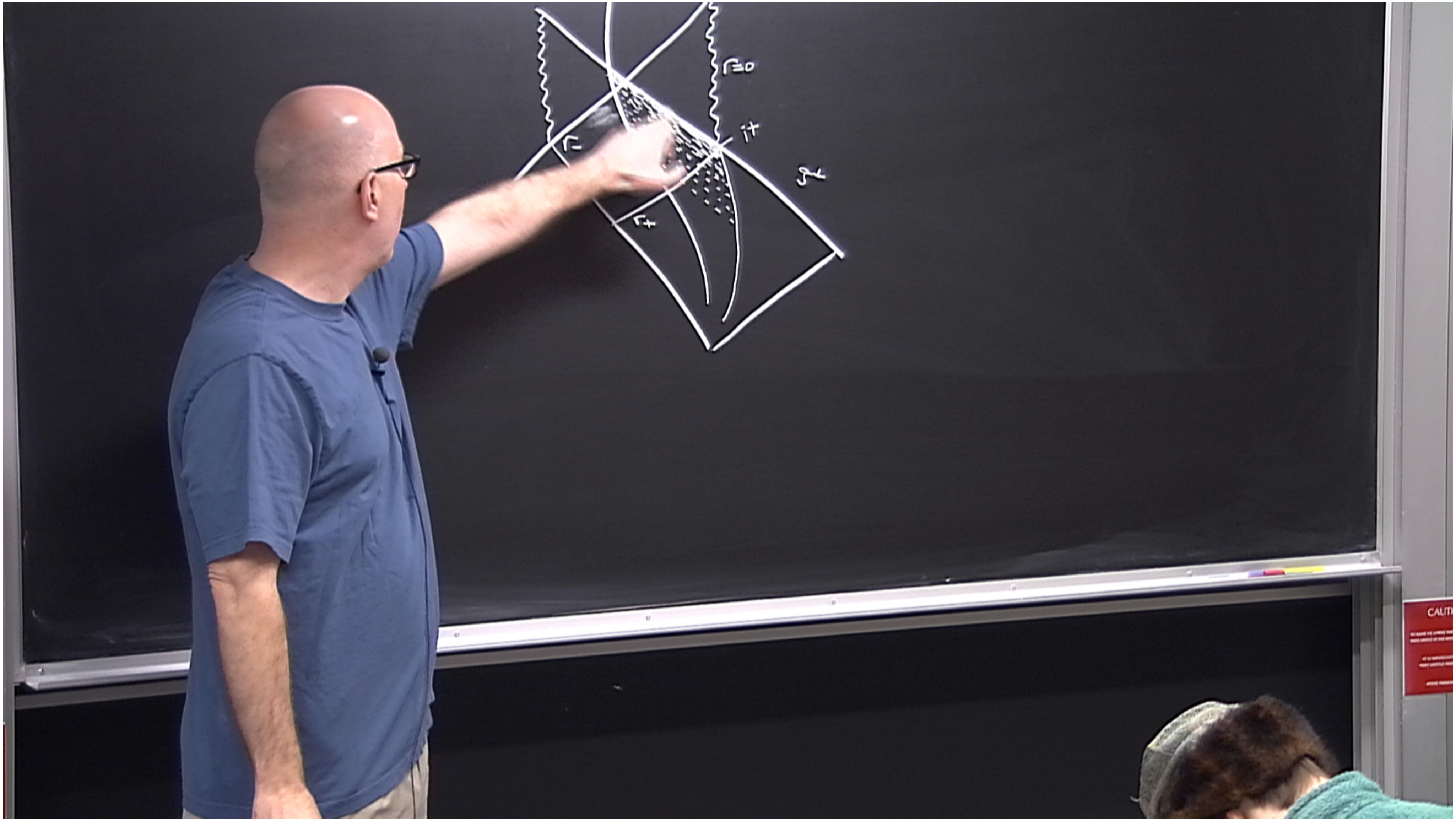

r=0
r₋
r₊

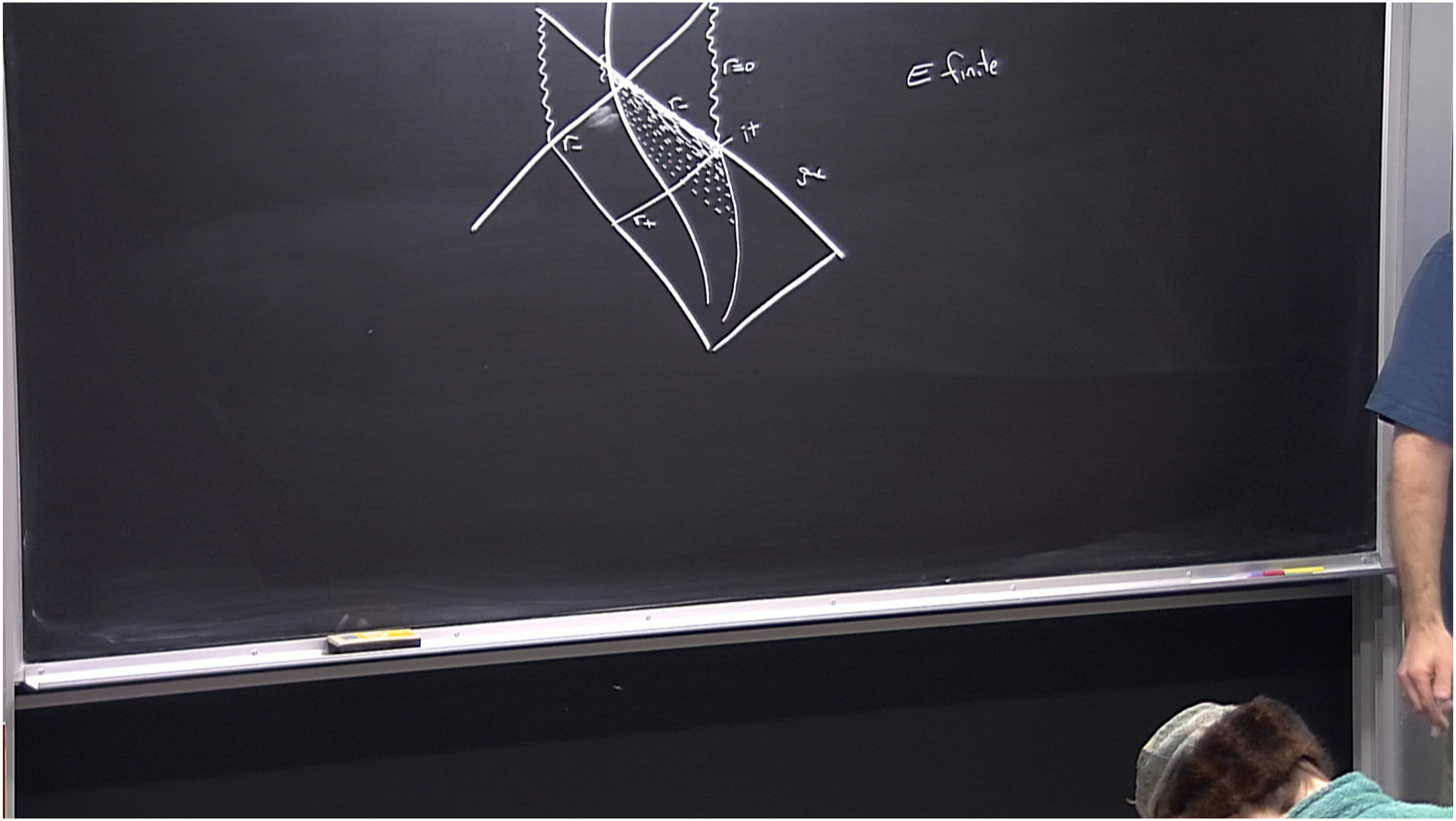

r=0
r_
r+

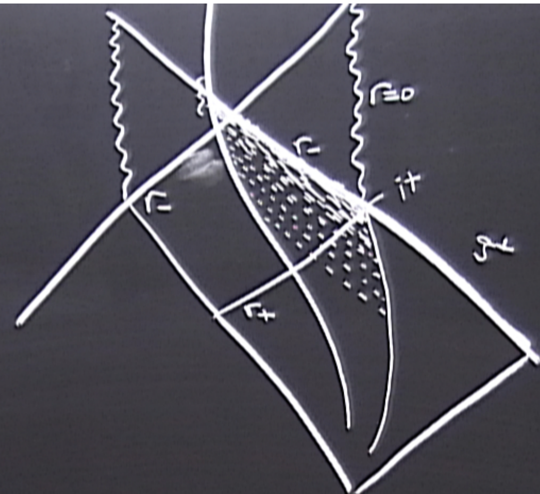

r=0
r₋
r₊


r=0
i+
r_+

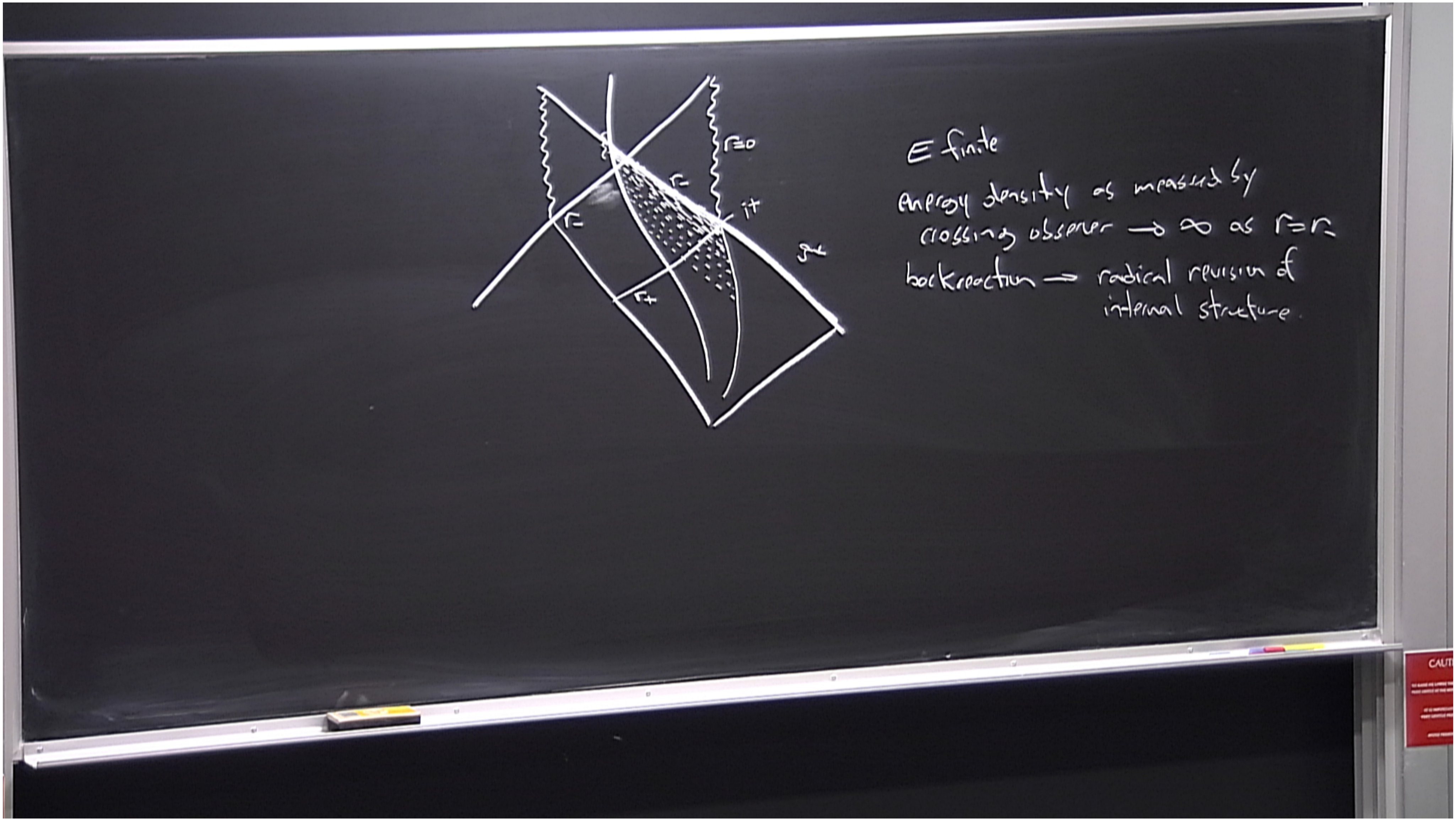

r=0
i+
r-
r+

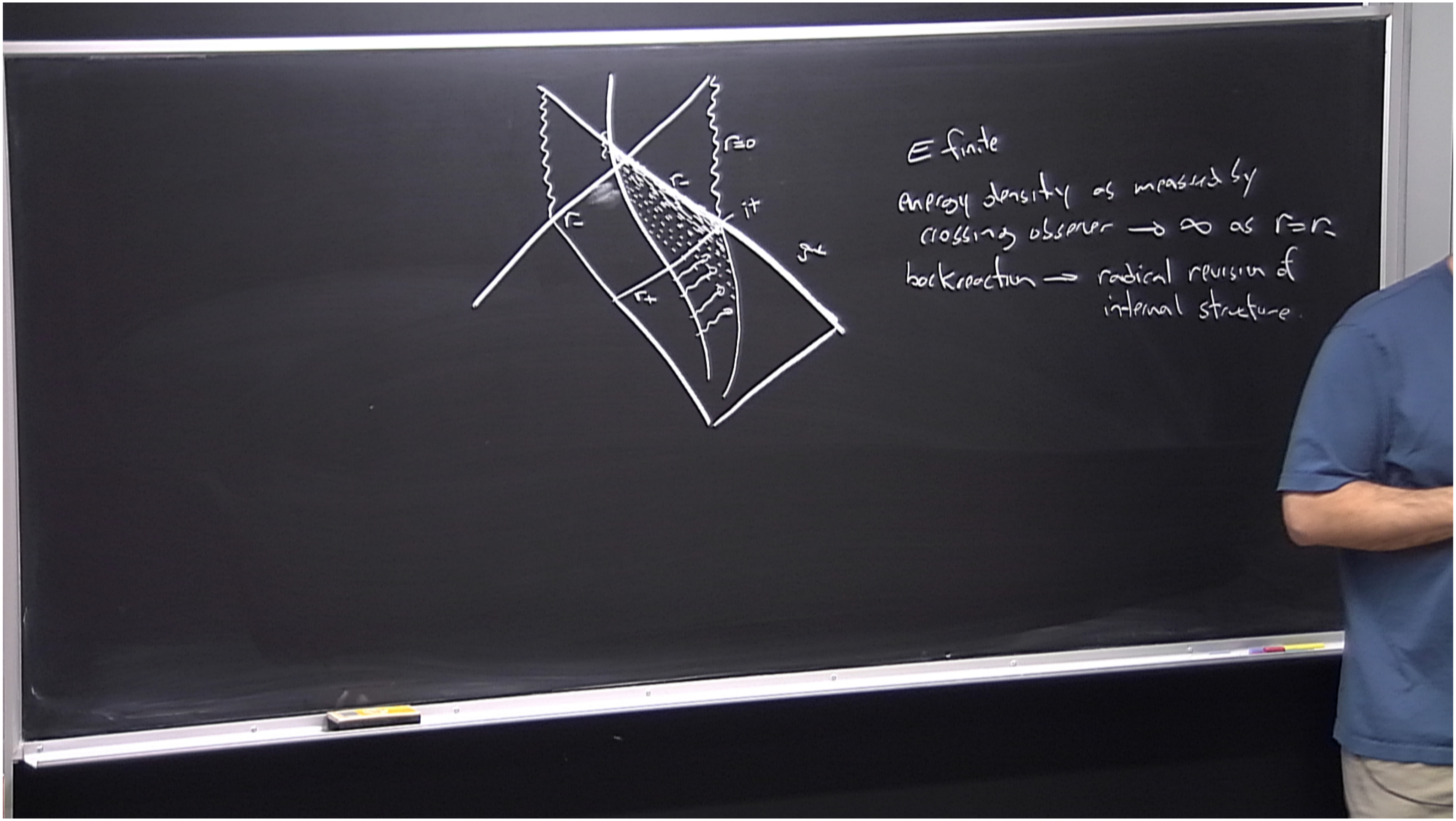

r=0

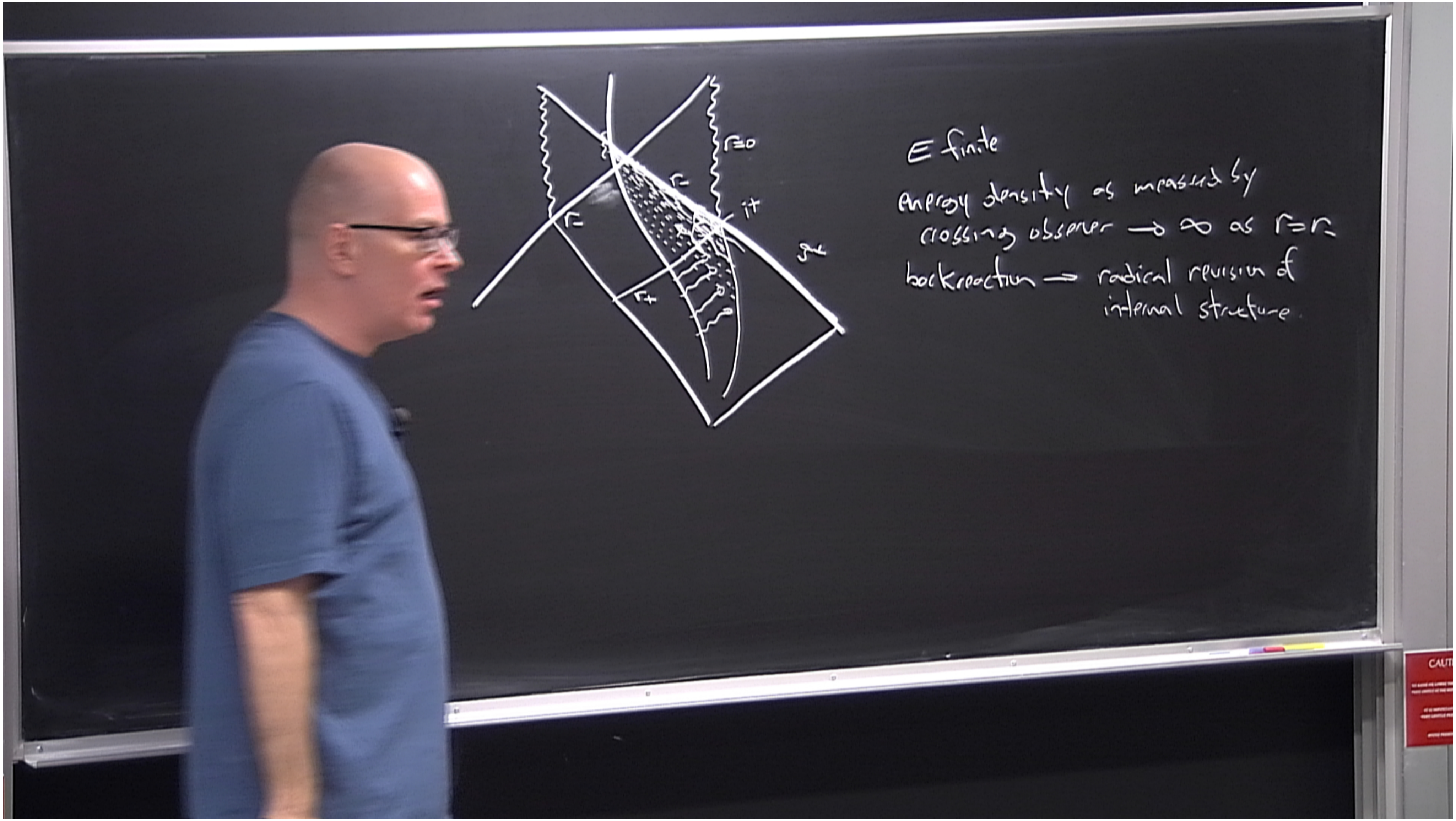

r=0
ε finite

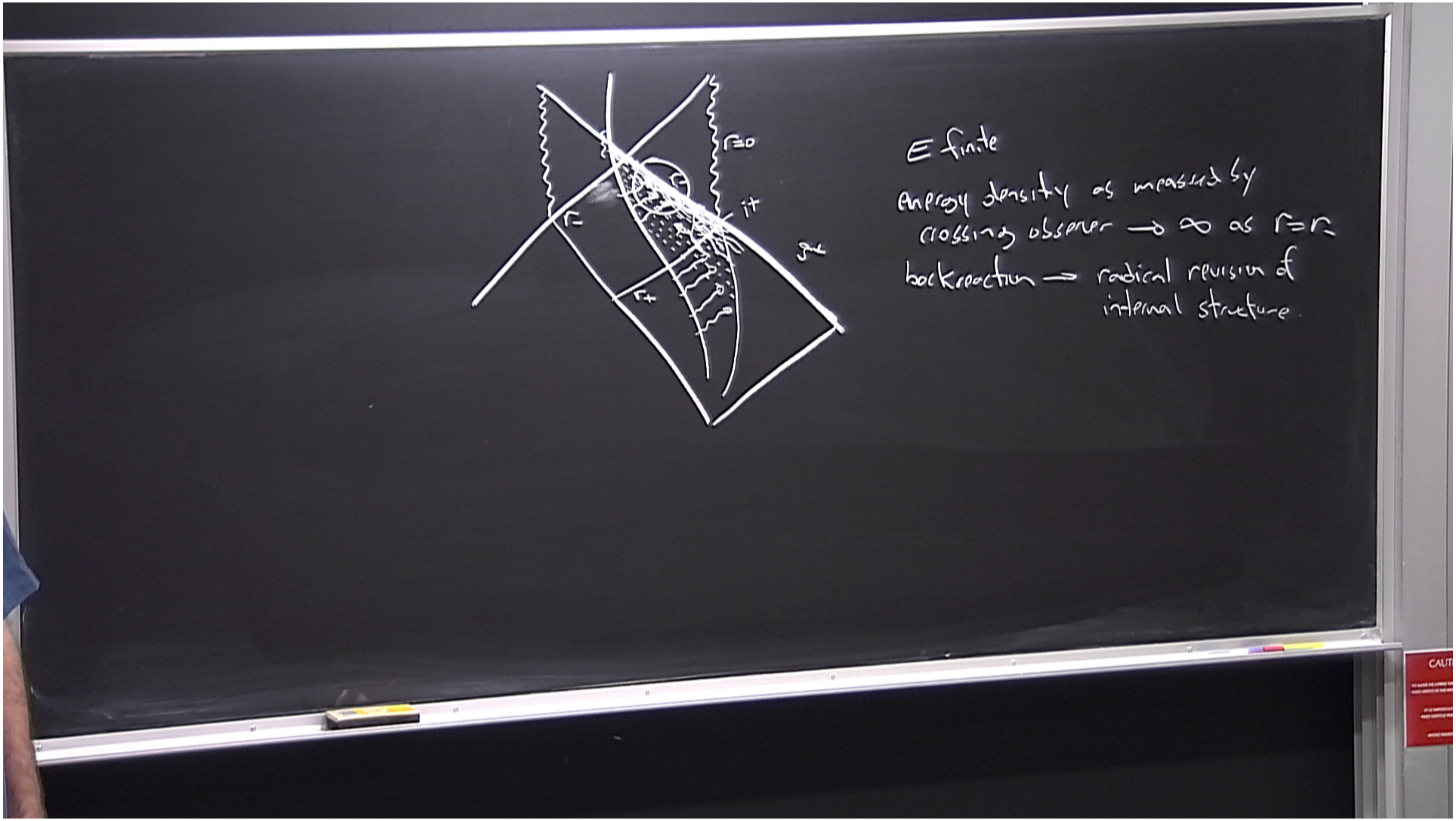


E finite

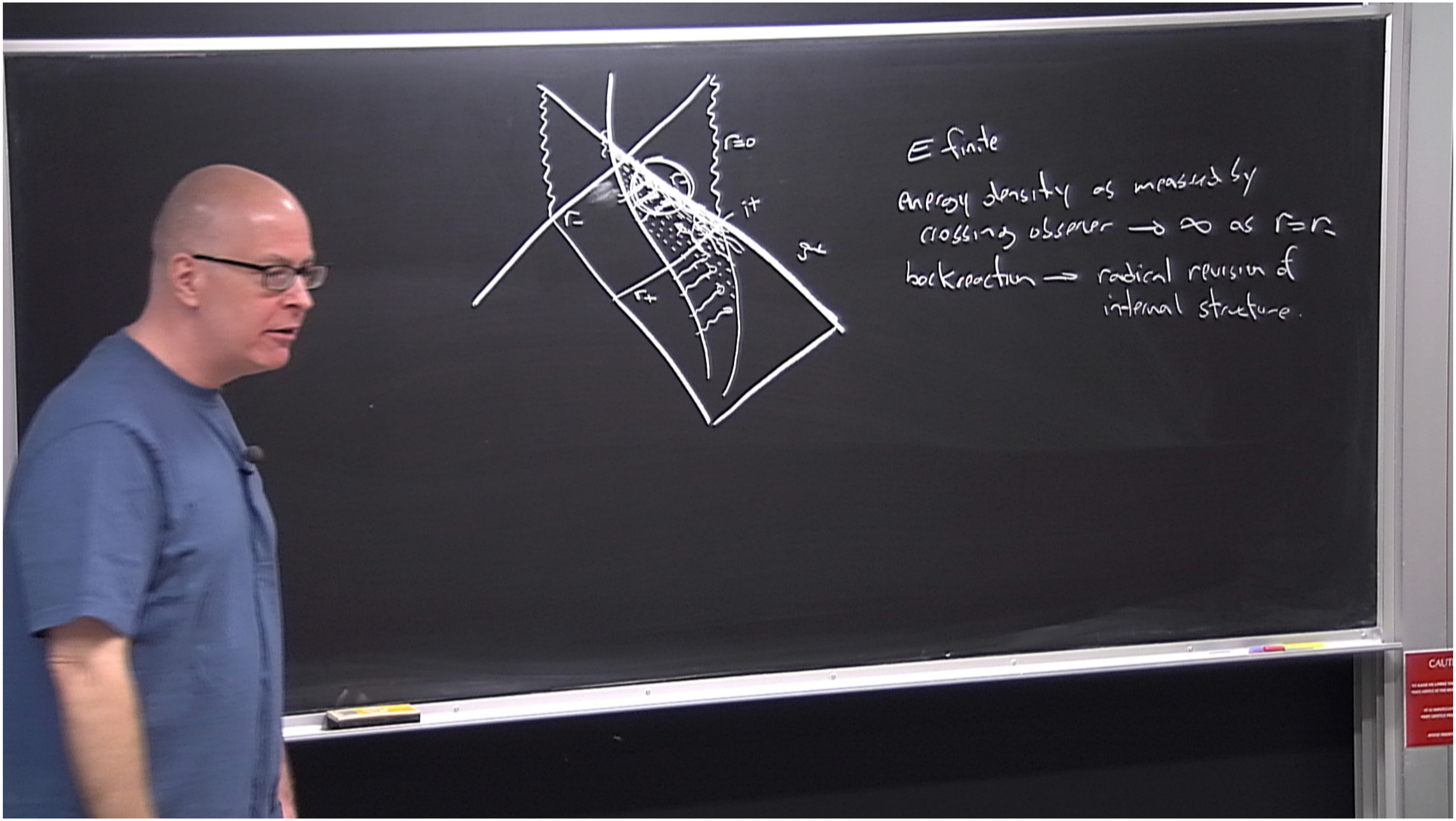

r=0
i+
ε finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.

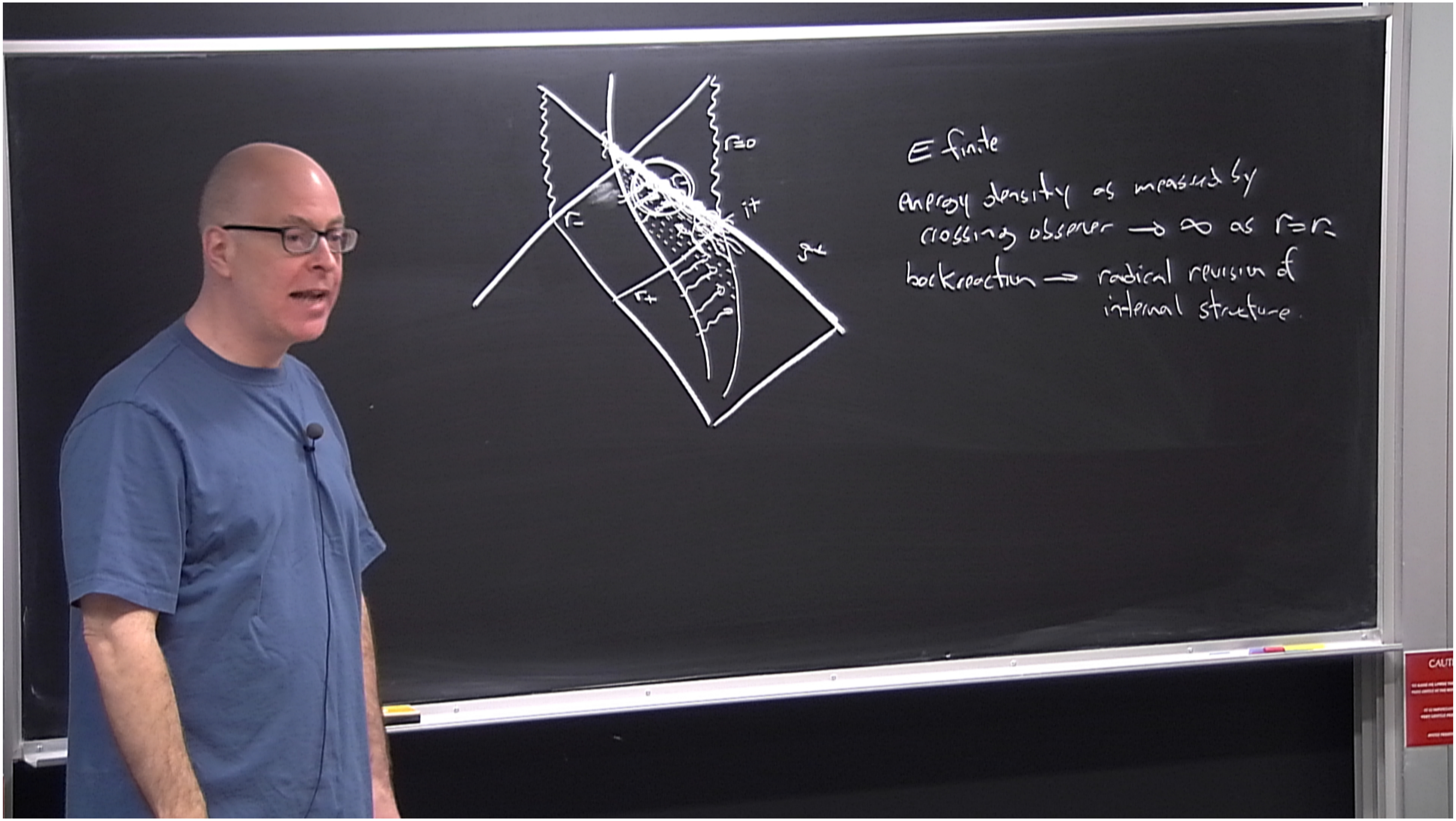

r=0
i^+
r_-
r_+
ε finite
energy density as measured by
crossing observer → ∞ as $r=r_-$
backreaction → radical revision of
internal structure.

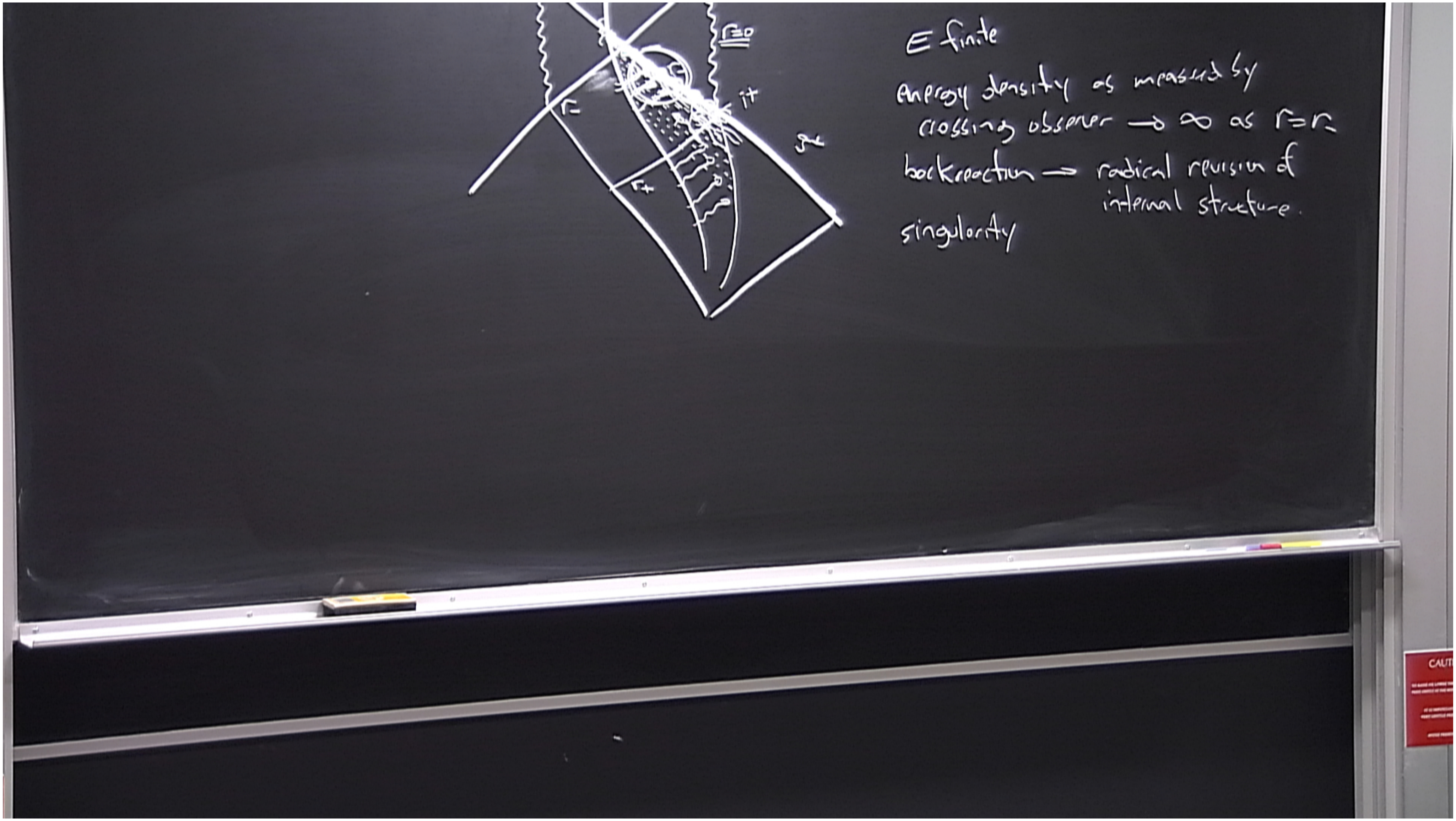

r=0
ε finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.

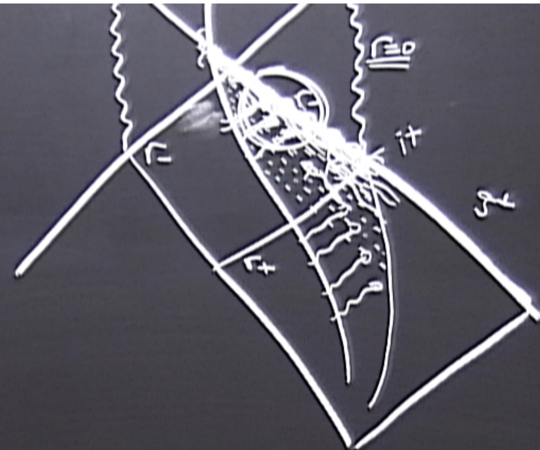


r=0
r_
i+
r+
E finite
energy density as measured by
crossing observer → ∞ as r=r_
backreaction → radical revision of
internal structure.


r=0
i+
E finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.


r=0
r₋
r₊
i+
E finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.


r=0
i+
E finite
energy density as measured by
crossing observer → ∞ as r=r-
backreaction → radical revision of
internal structure.
singularity



E finite

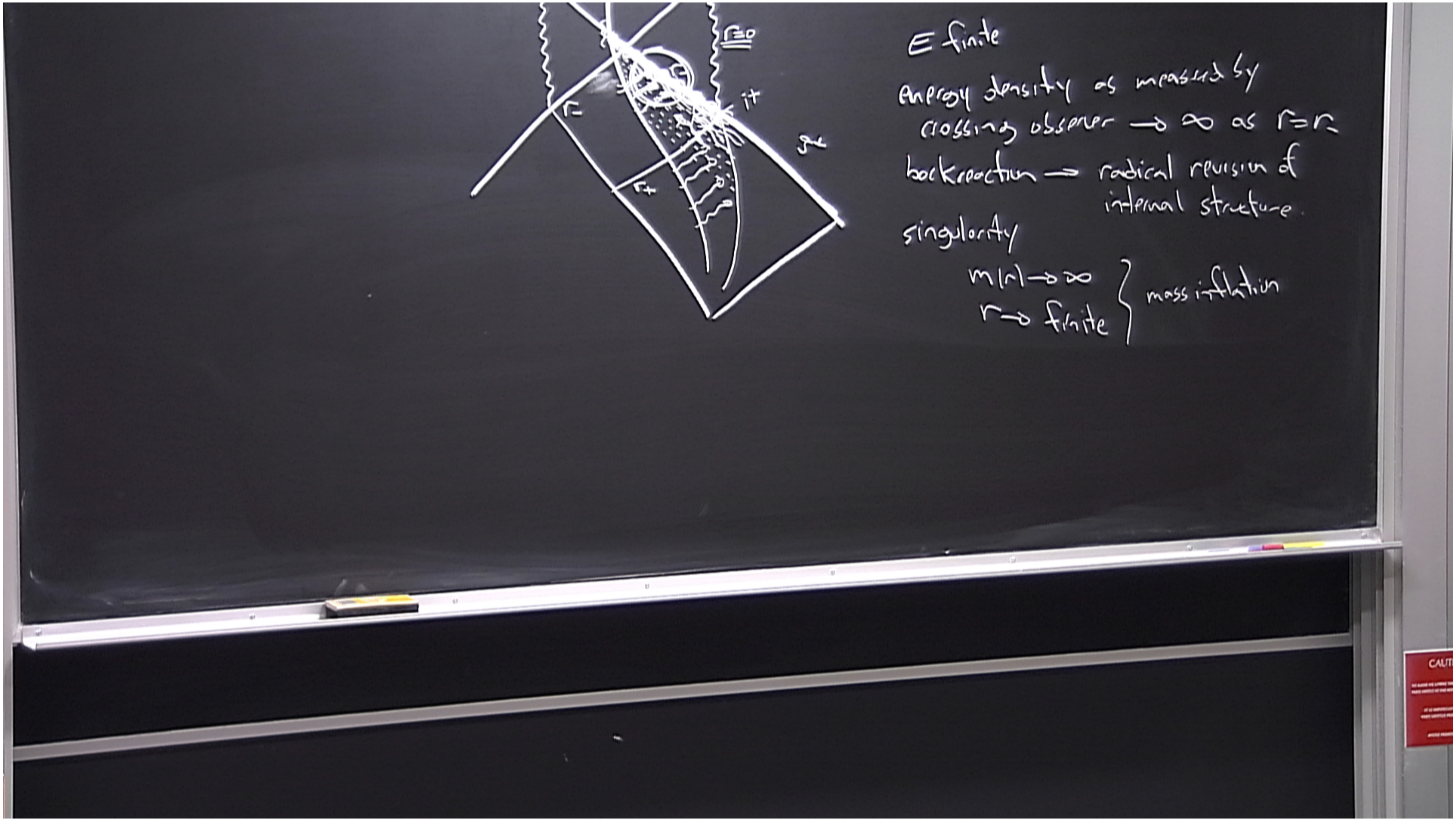
energy density as measured by crossing observer → ∞ as $r = r_-$

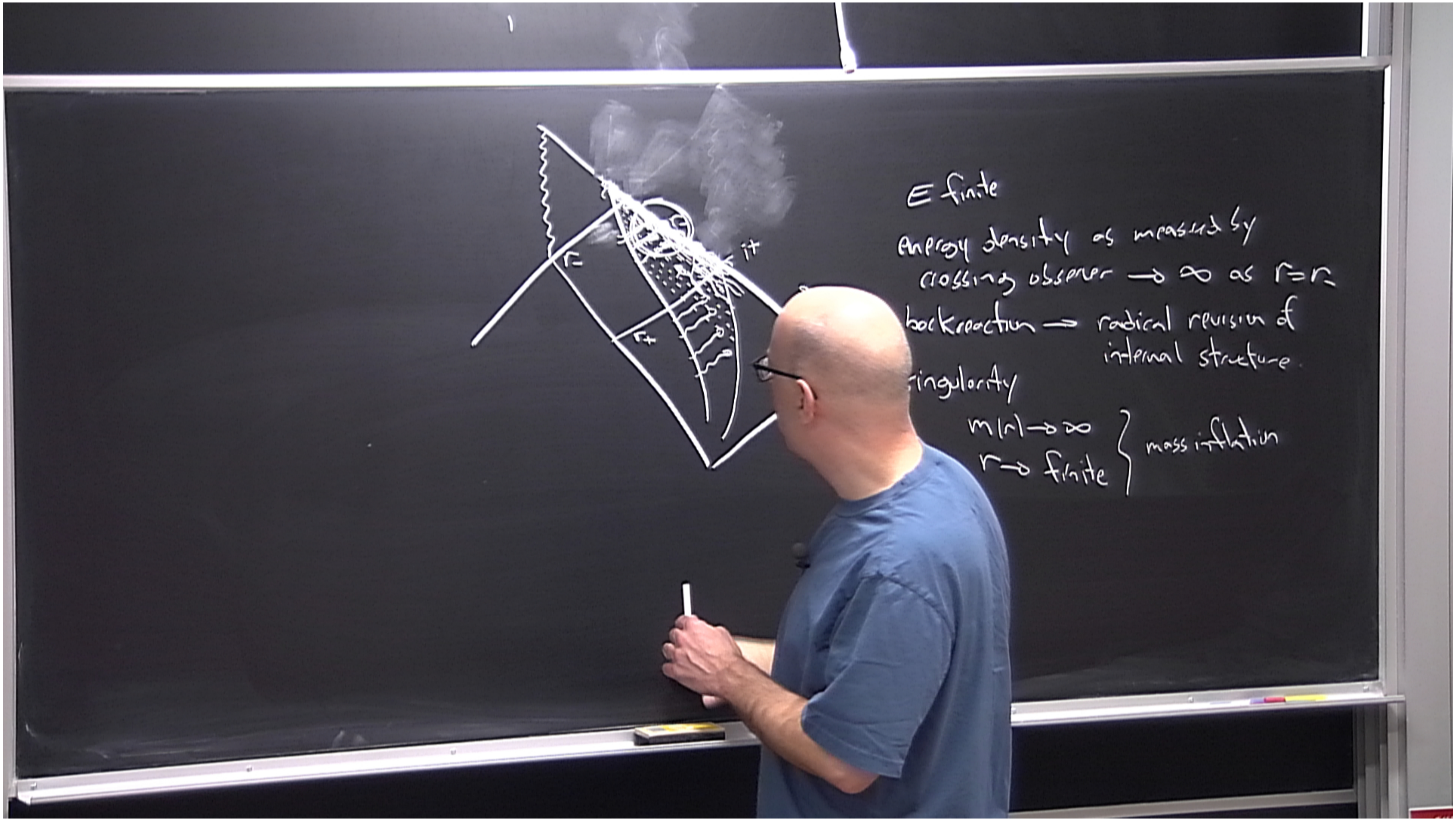
backreaction → radical revision of internal structure.

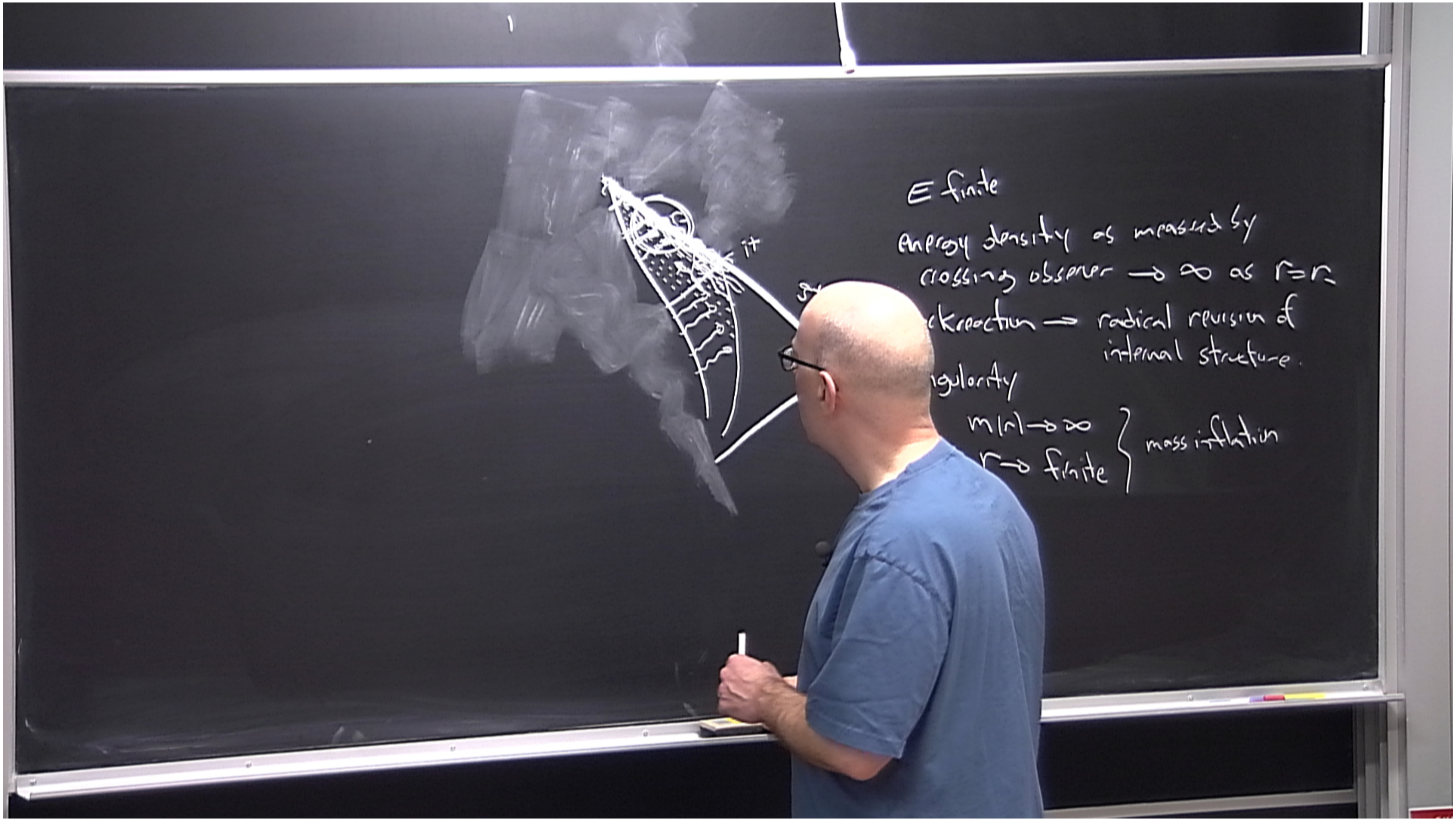
singularity

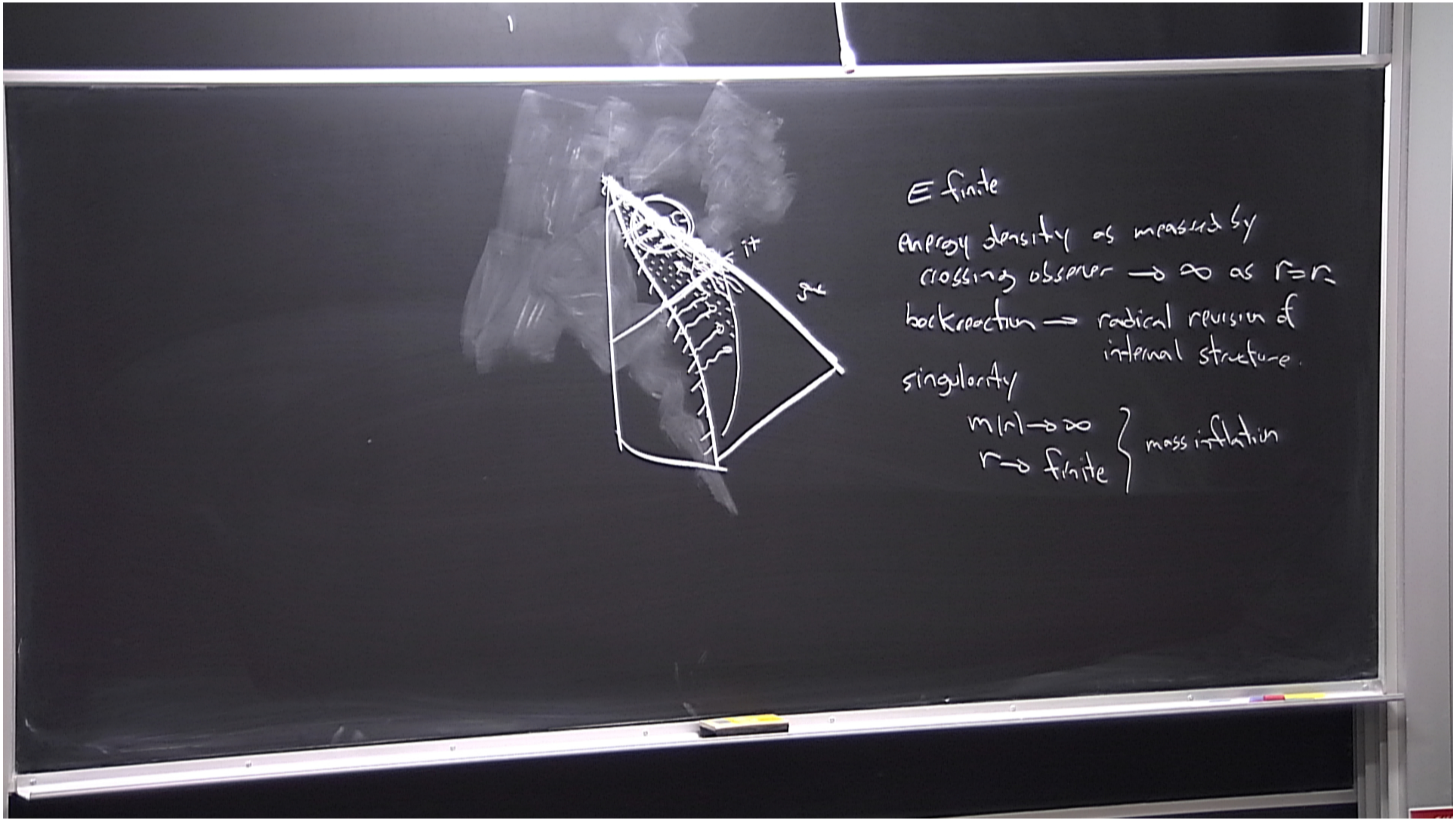
$m(r) \to \infty$

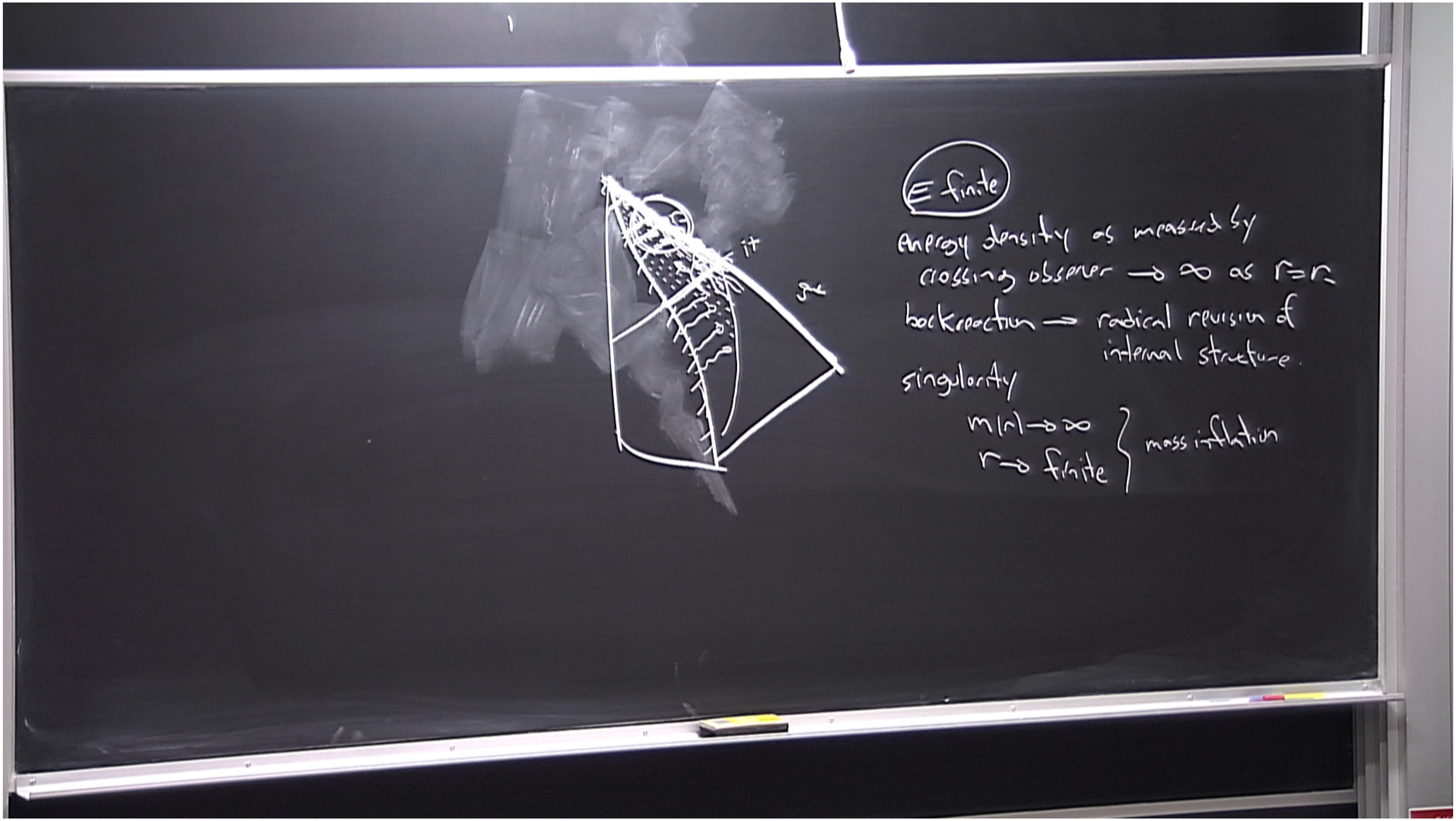
$r \to$ finite

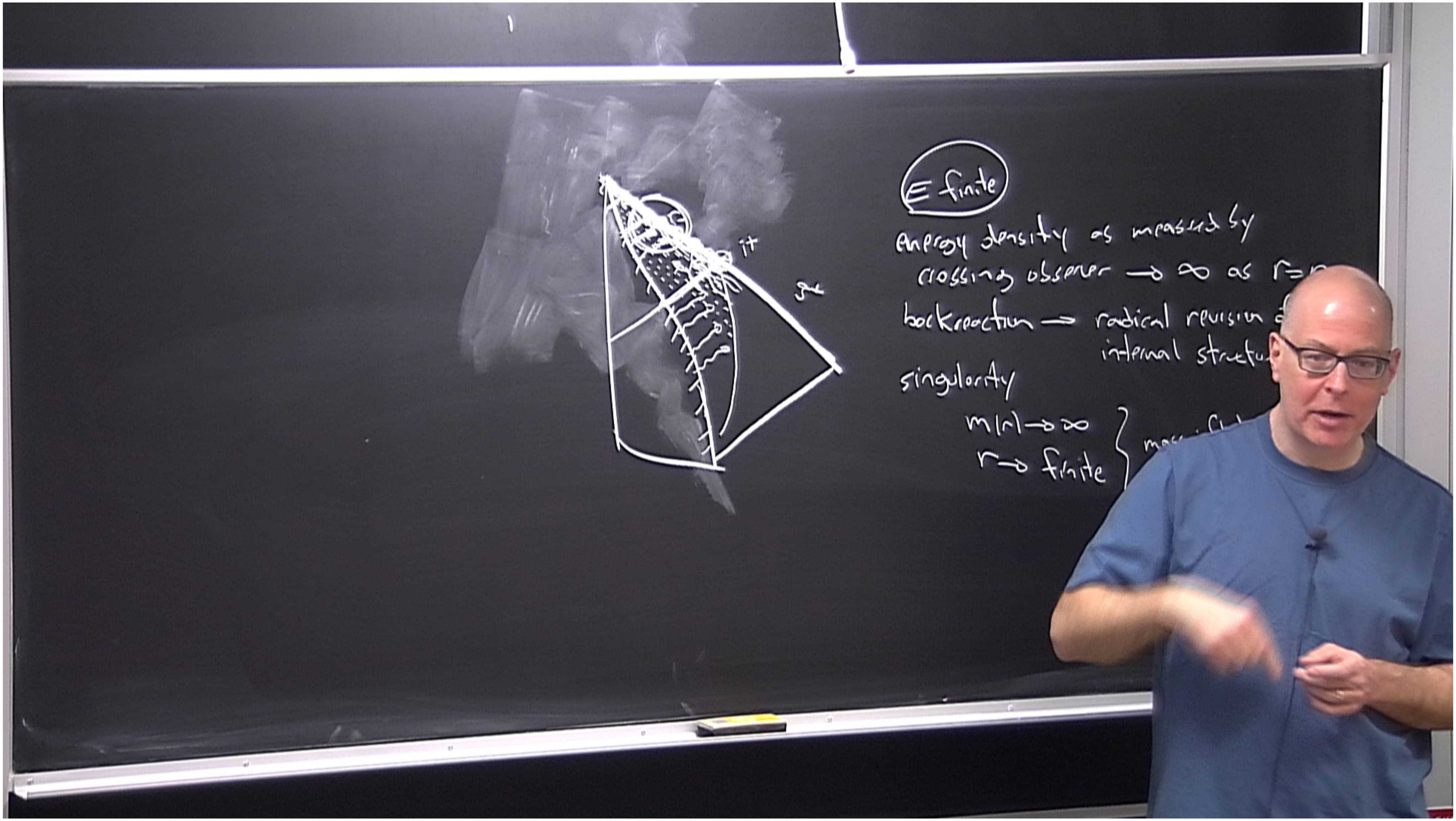

r=0
r-
r+
i+
ε finite
energy density as measured by
crossing observer → ∞ as r=r-
backreaction → radical revision of
internal structure.
singularity
m(r) → ∞
r → finite
mass inflation

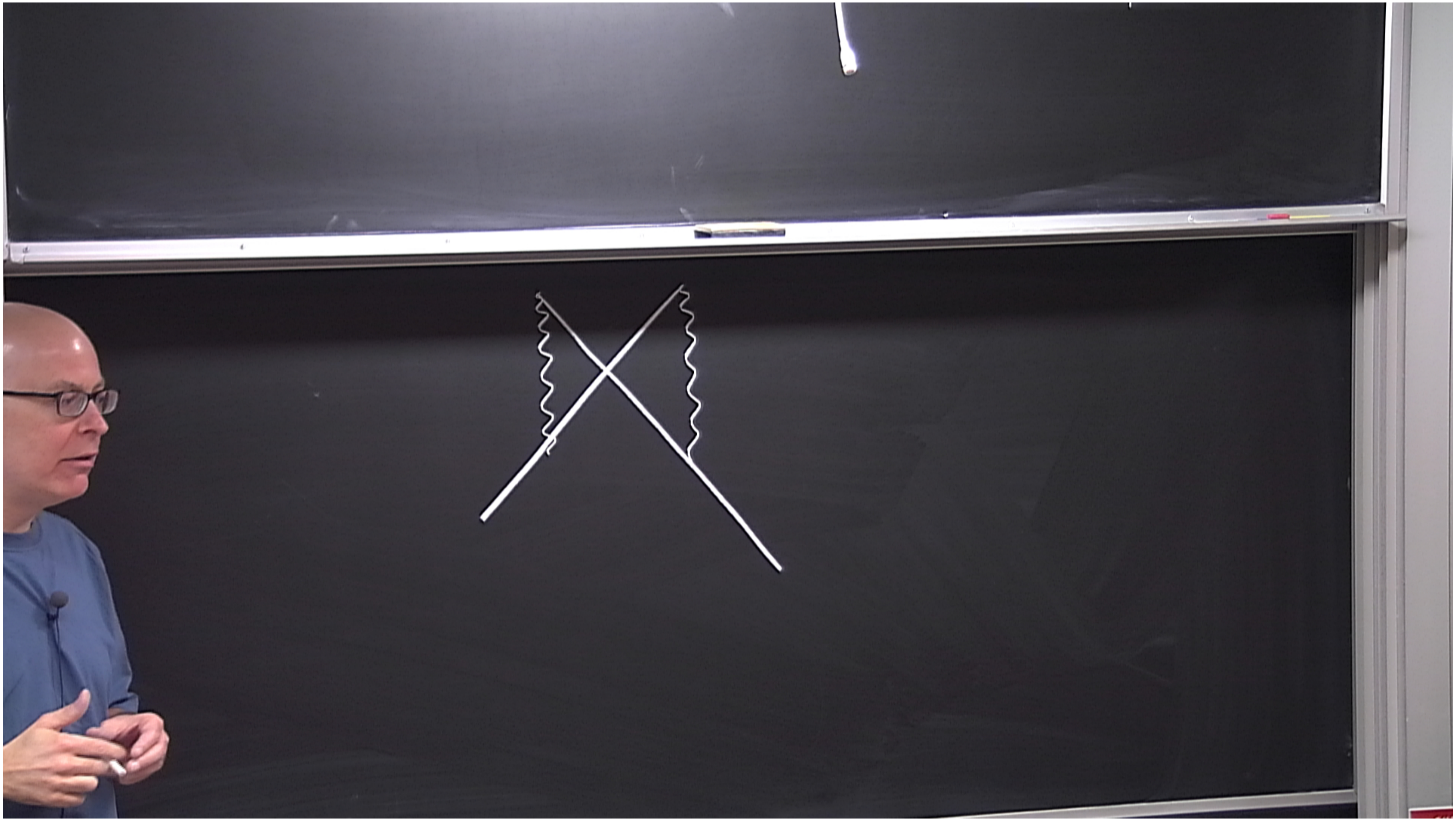

ε finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.
ingularity
m(r) → ∞
r → finite
mass inflation
r₋
r₊

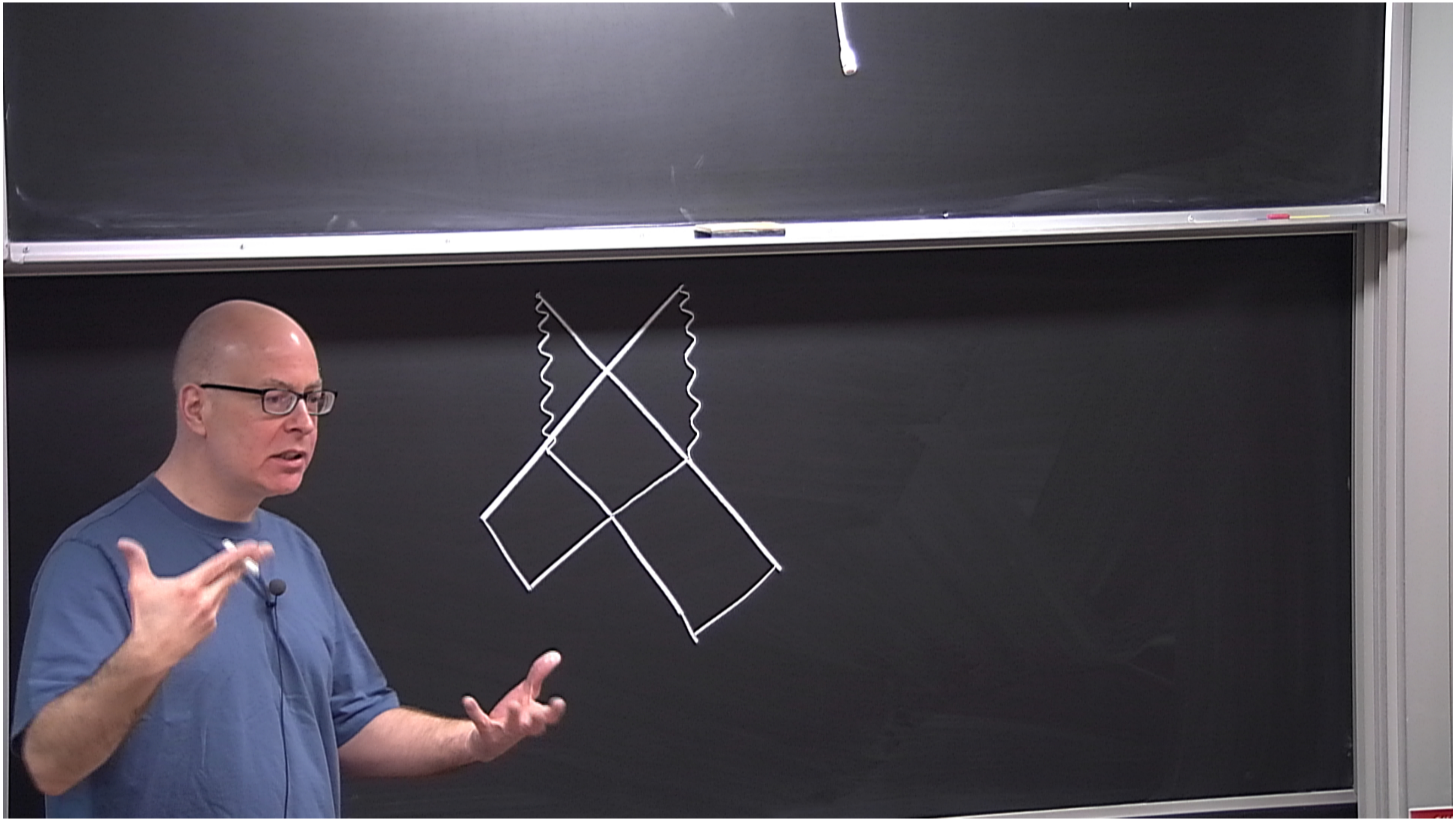

E finite
energy density as measured by
crossing observer → ∞ as r=r
radical revision of
internal structure.
m(r) → ∞
r → finite
mass inflation

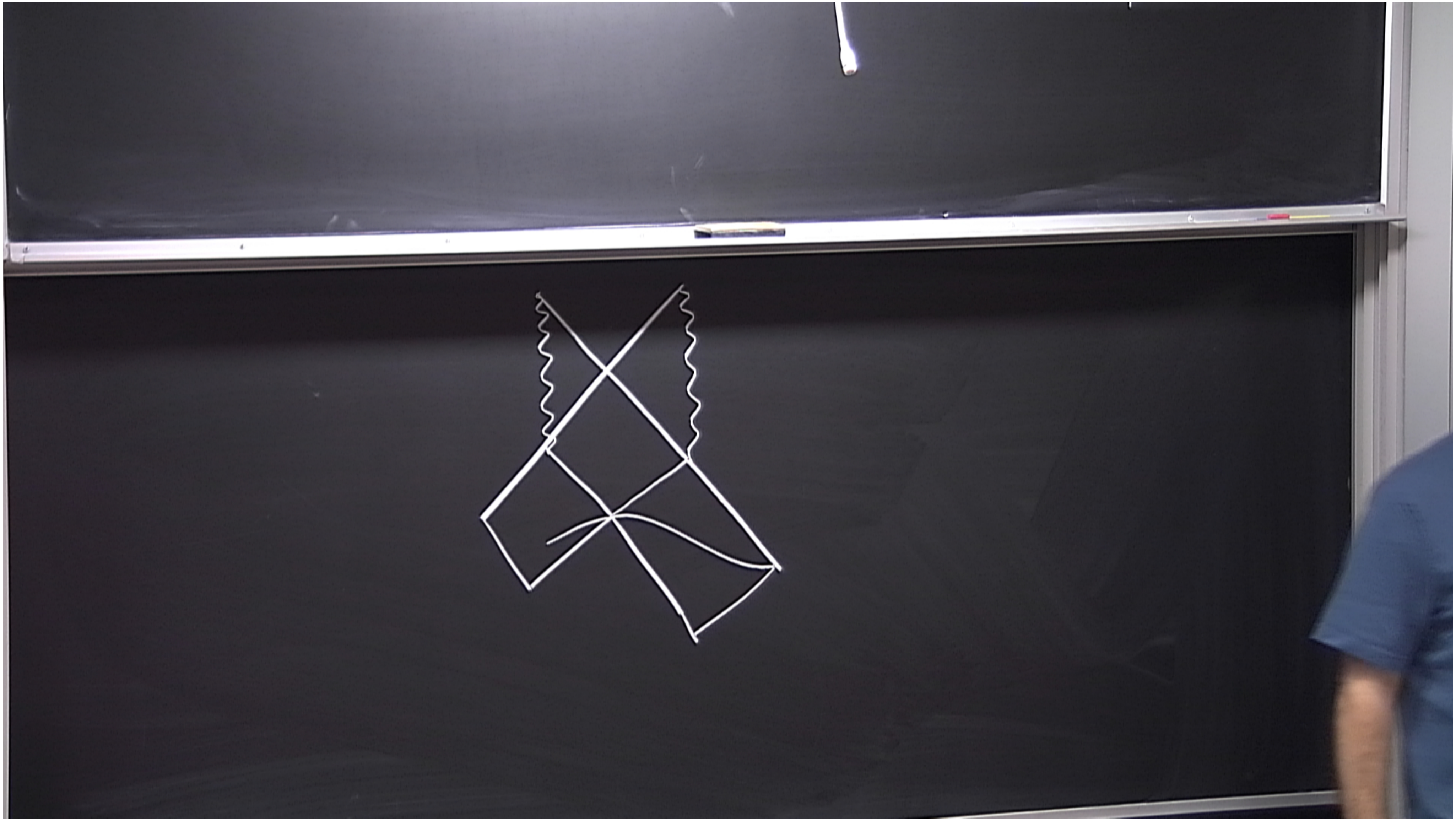

E finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.
singularity
m(r) → ∞
r → finite
mass inflation

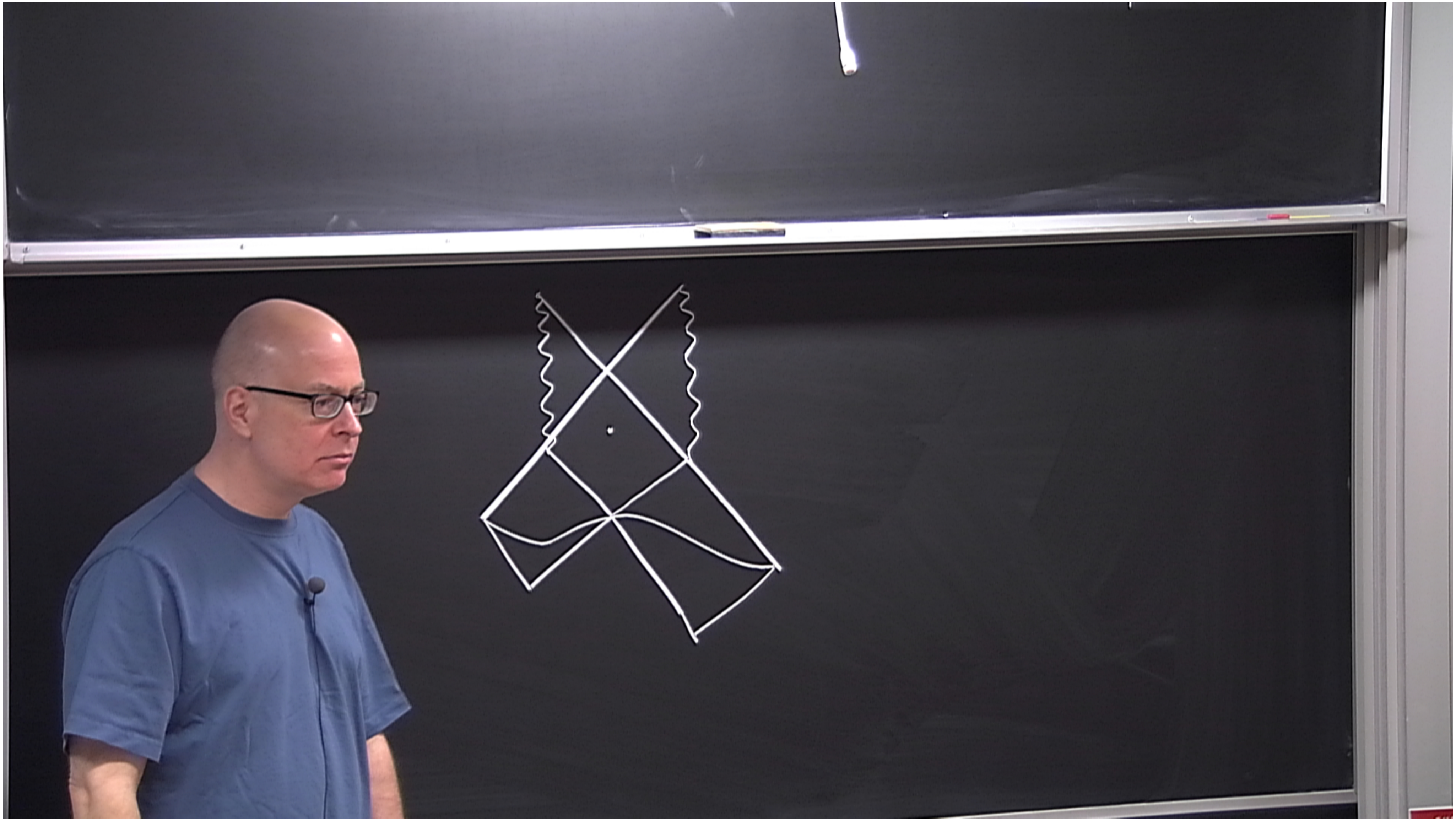

ε finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.
singularity
m(r) → ∞
r → finite
mass inflation

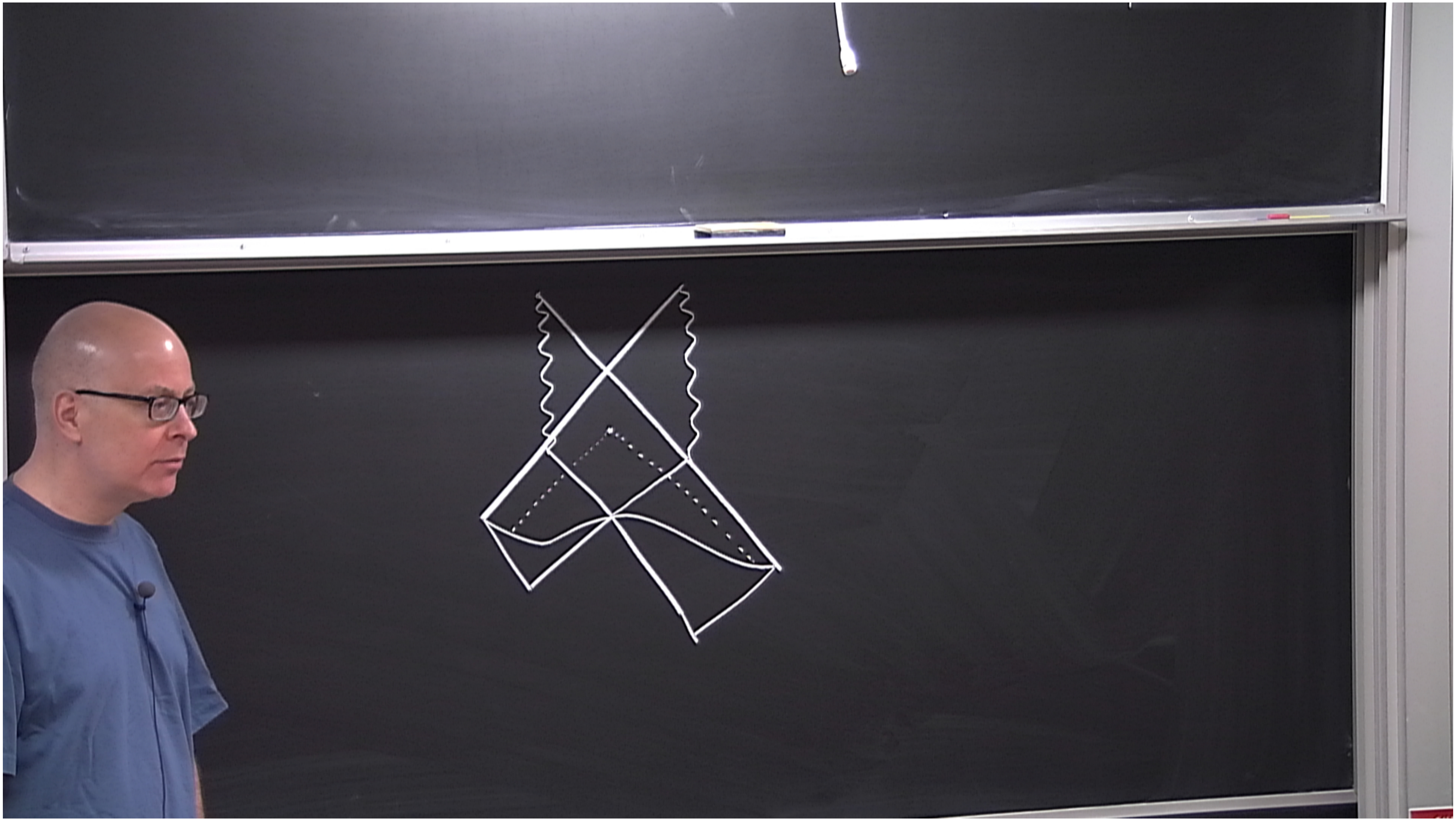

E finite
energy density as measured by
crossing observer → ∞ as r→
backreaction → radical revision of
internal structure
singularity
m(r) → ∞
r → finite

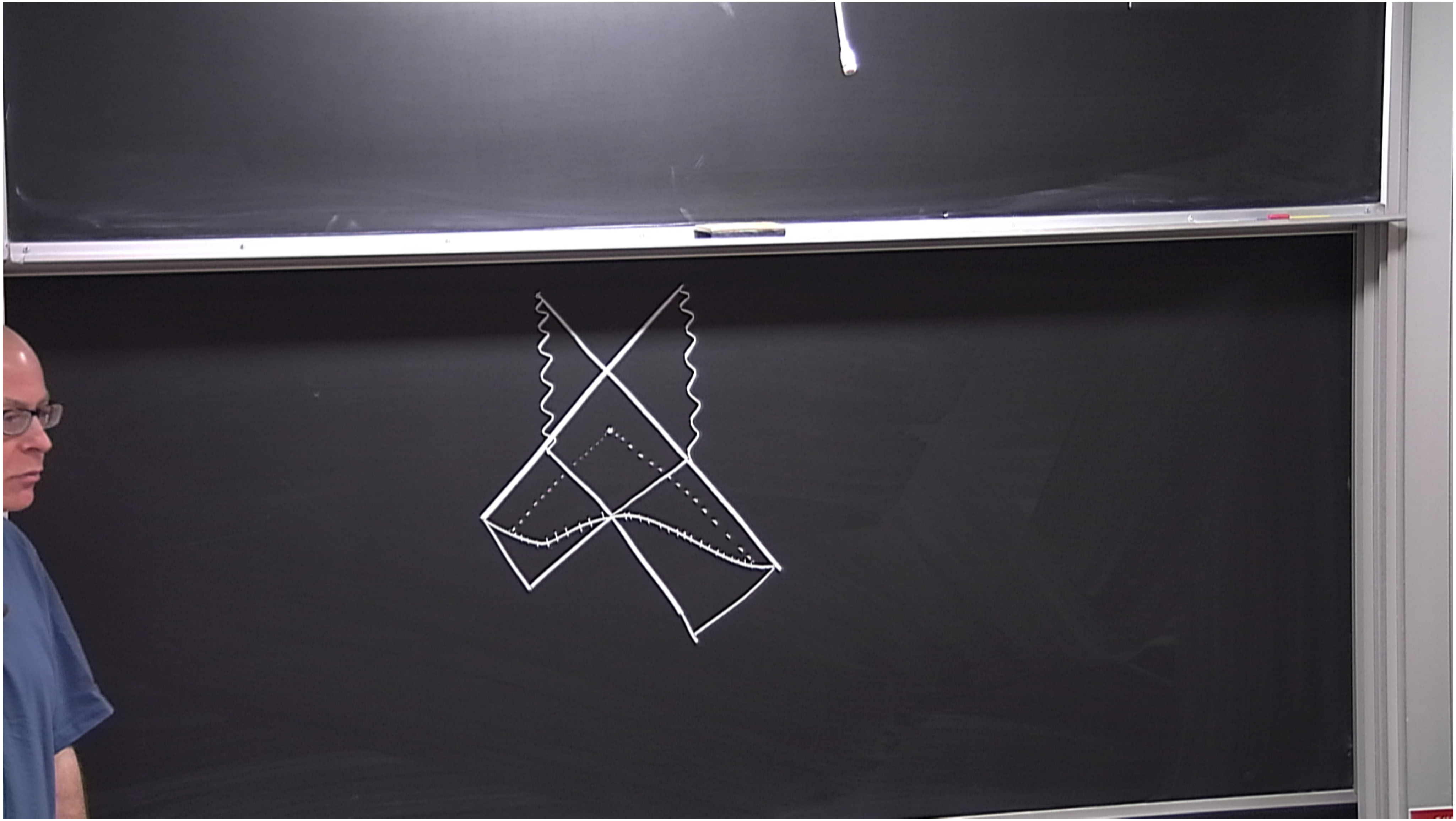


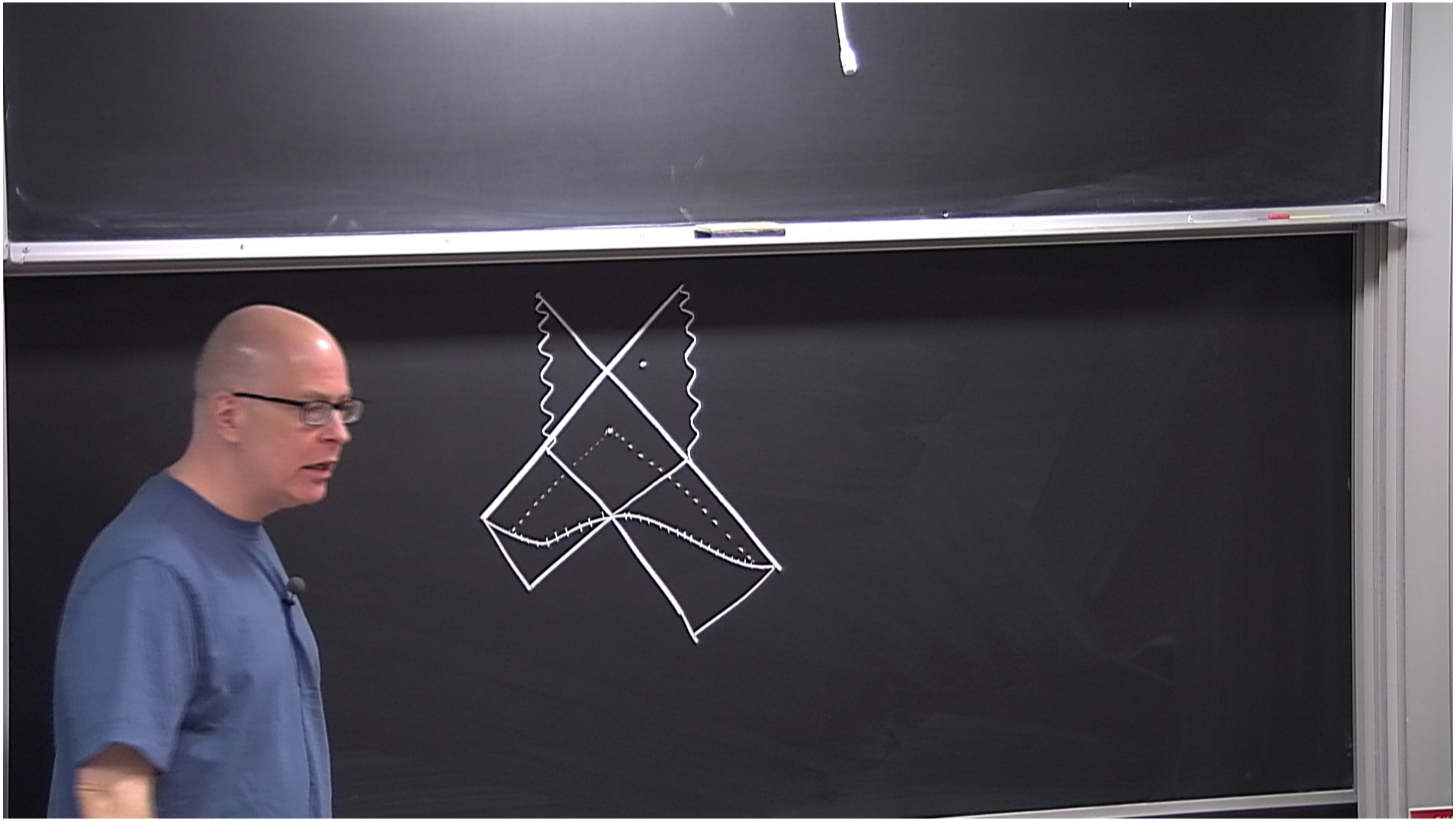


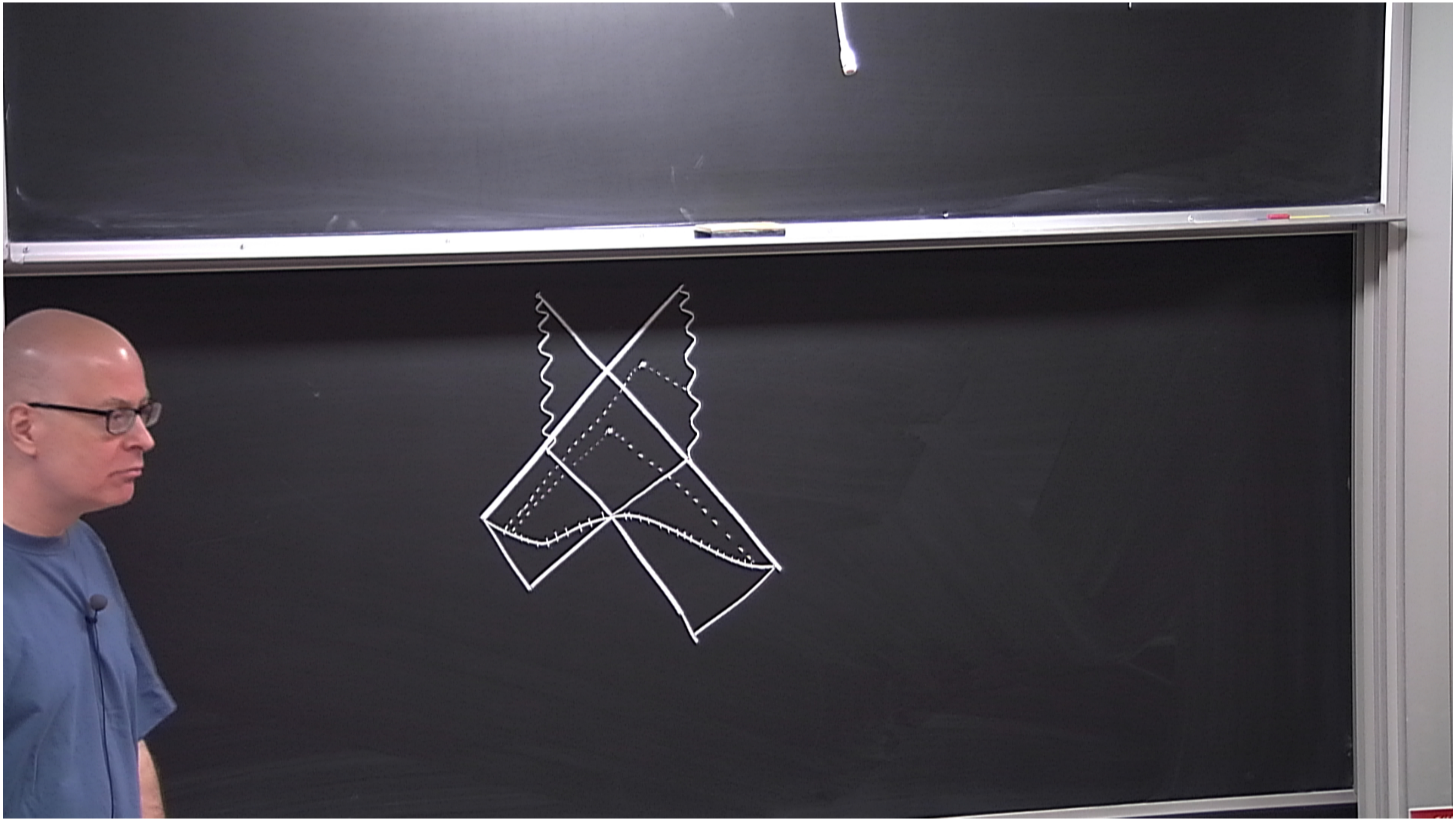


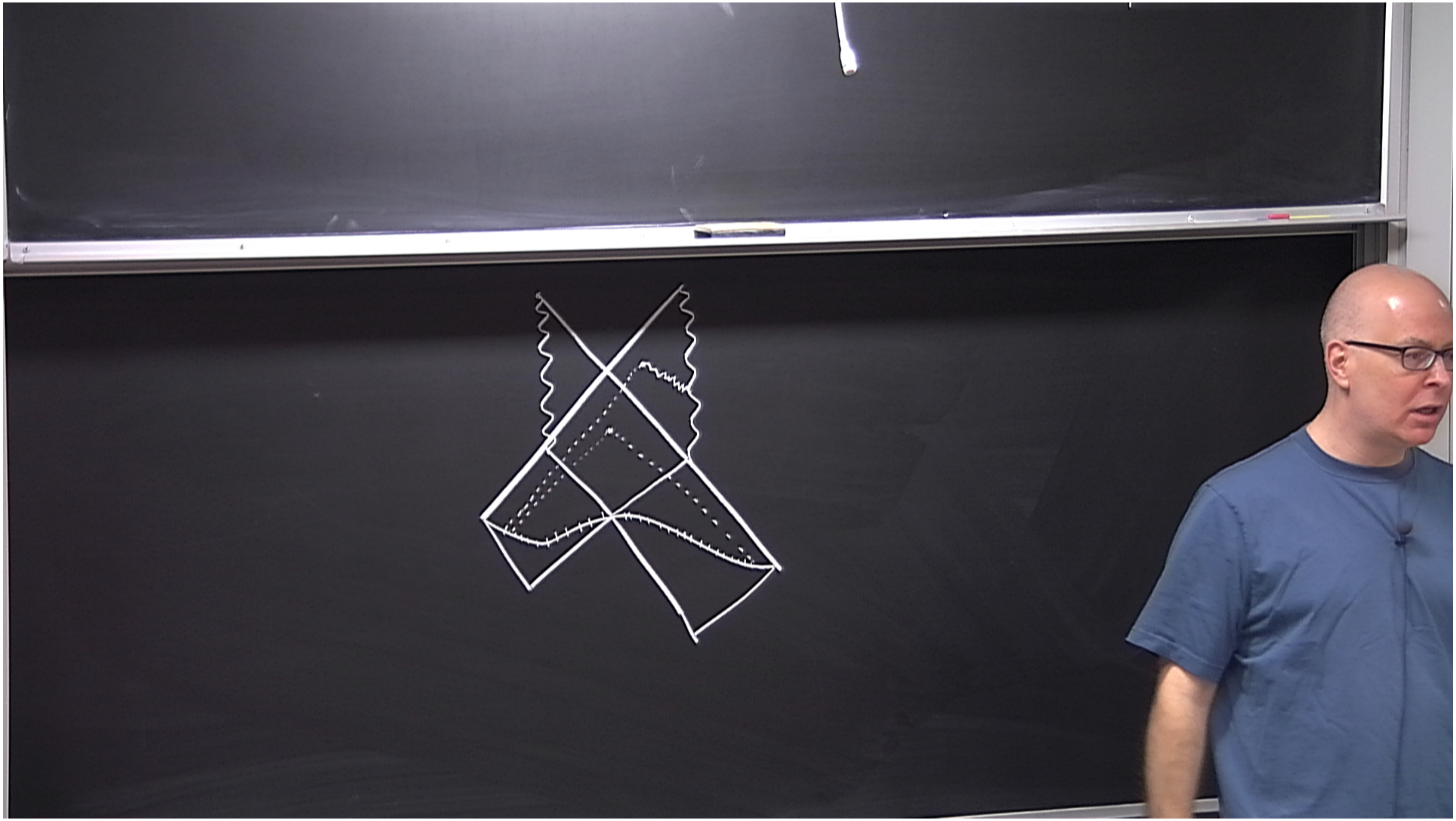


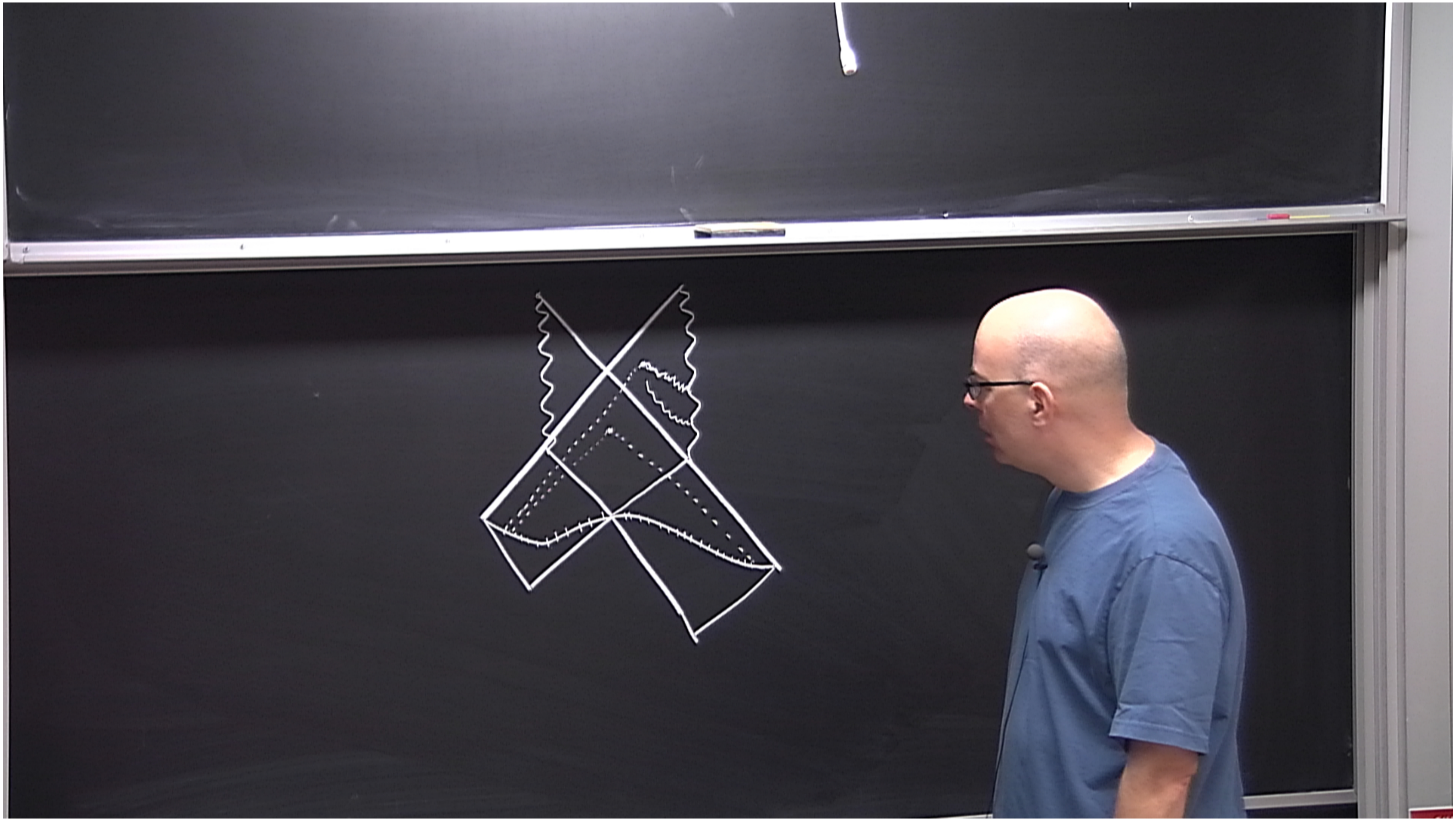


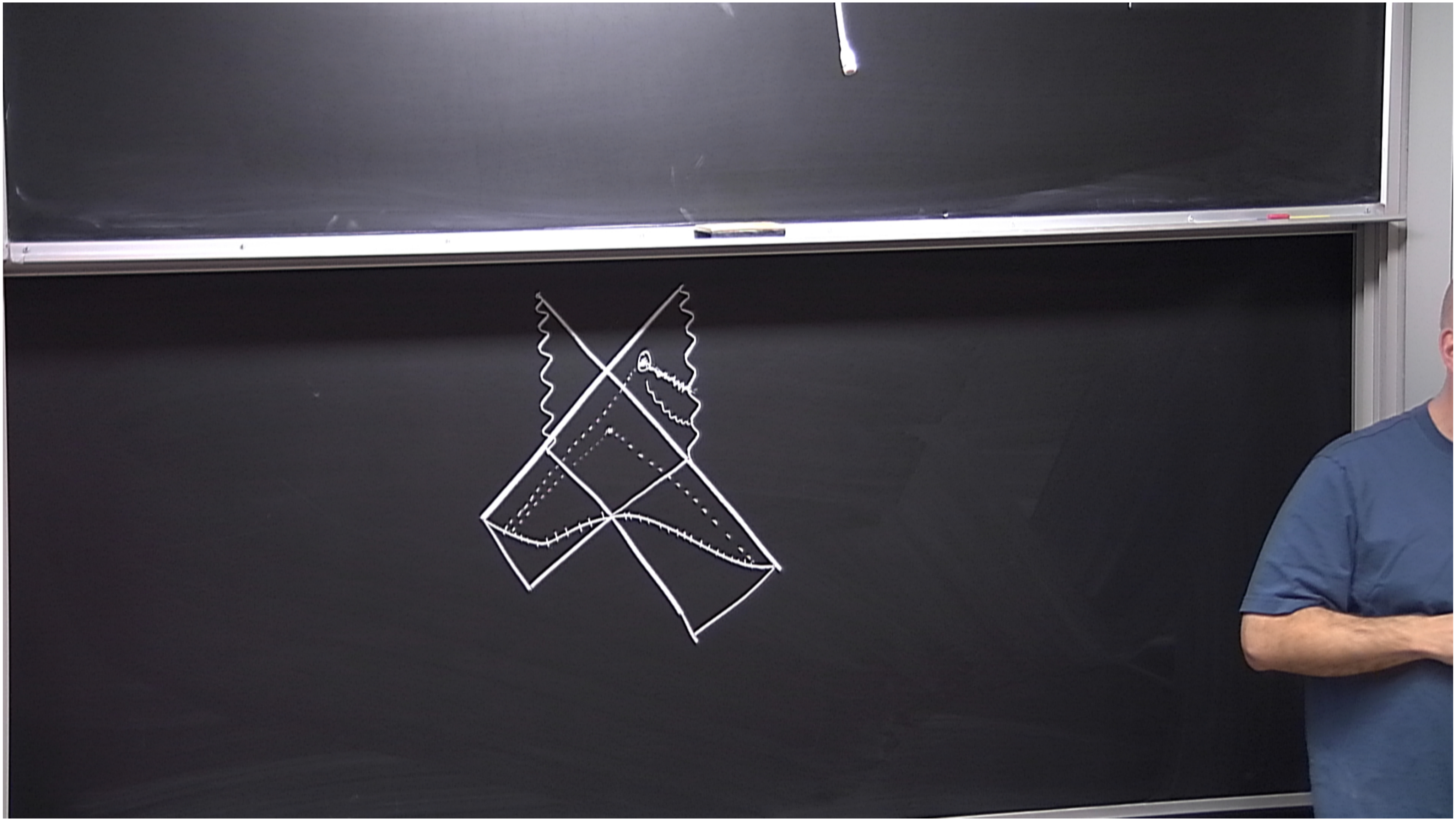


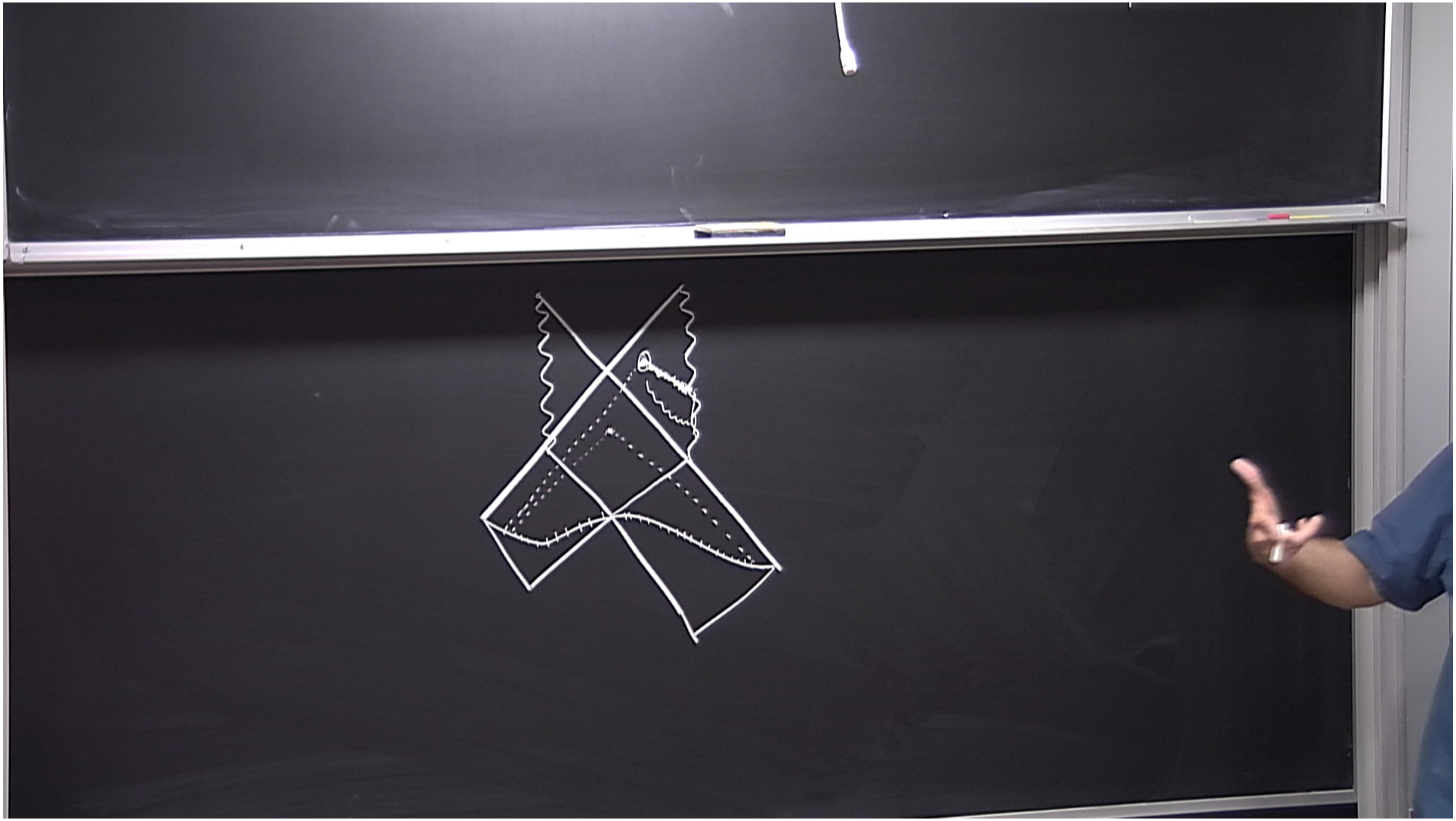


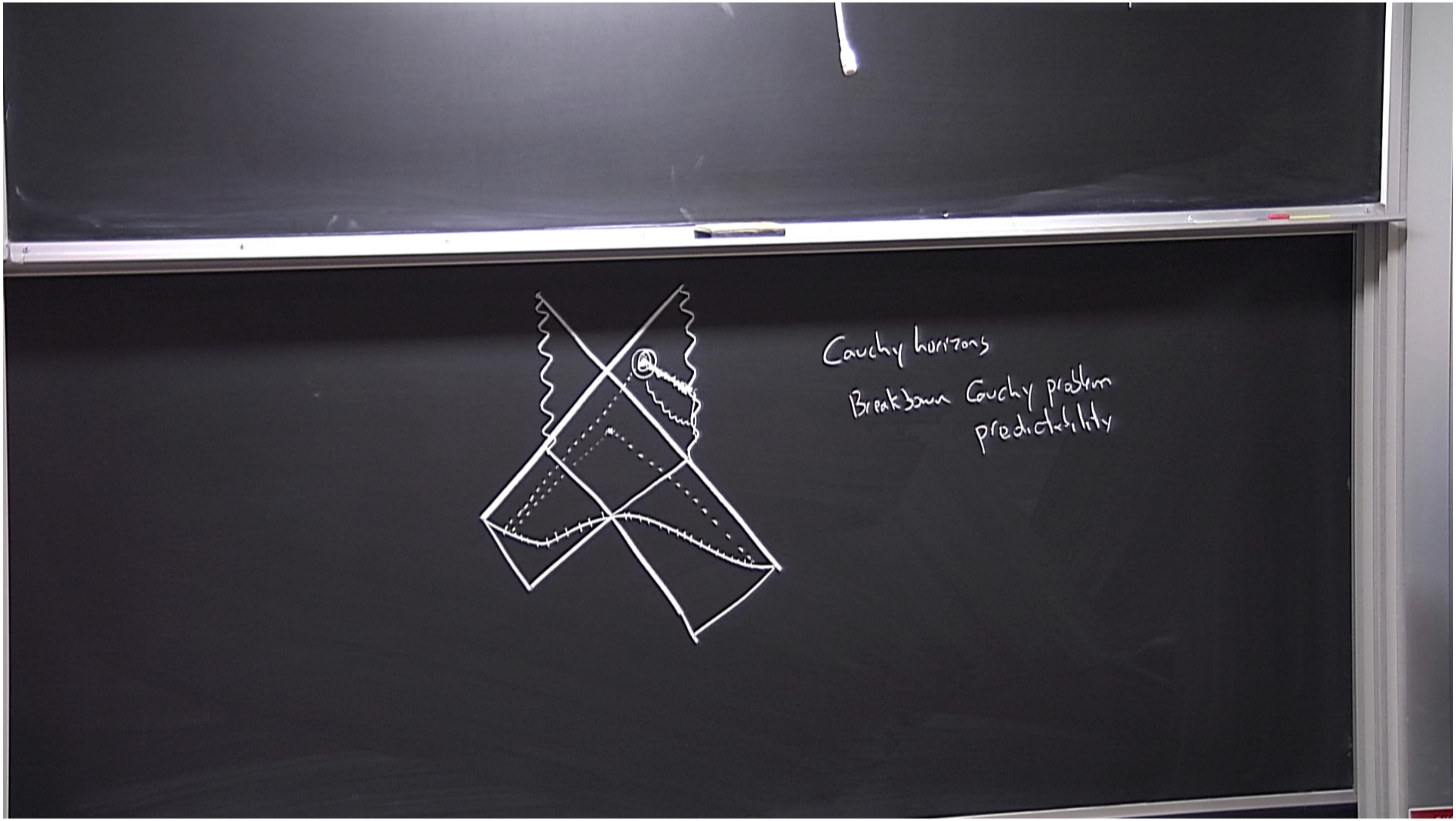


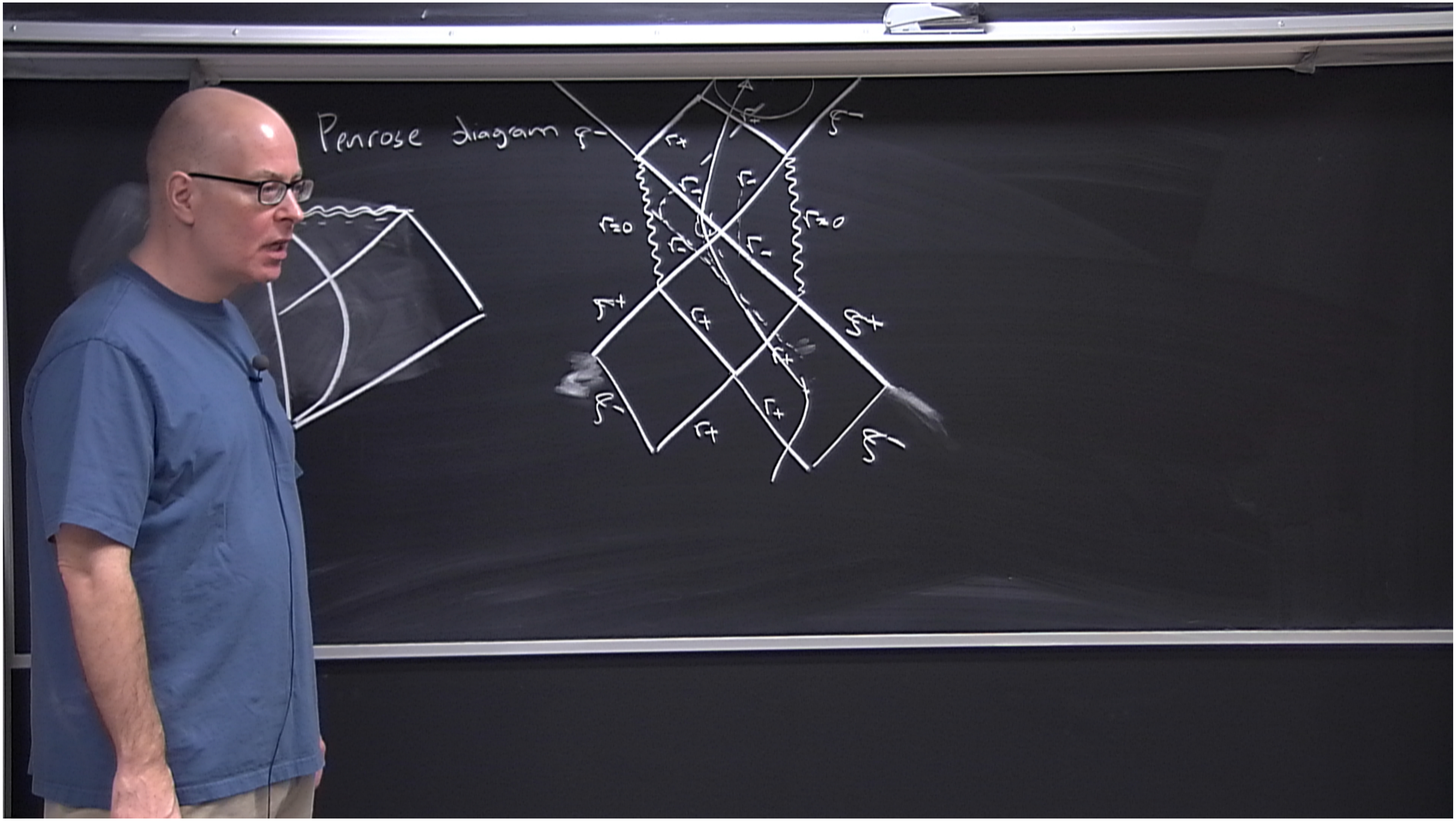


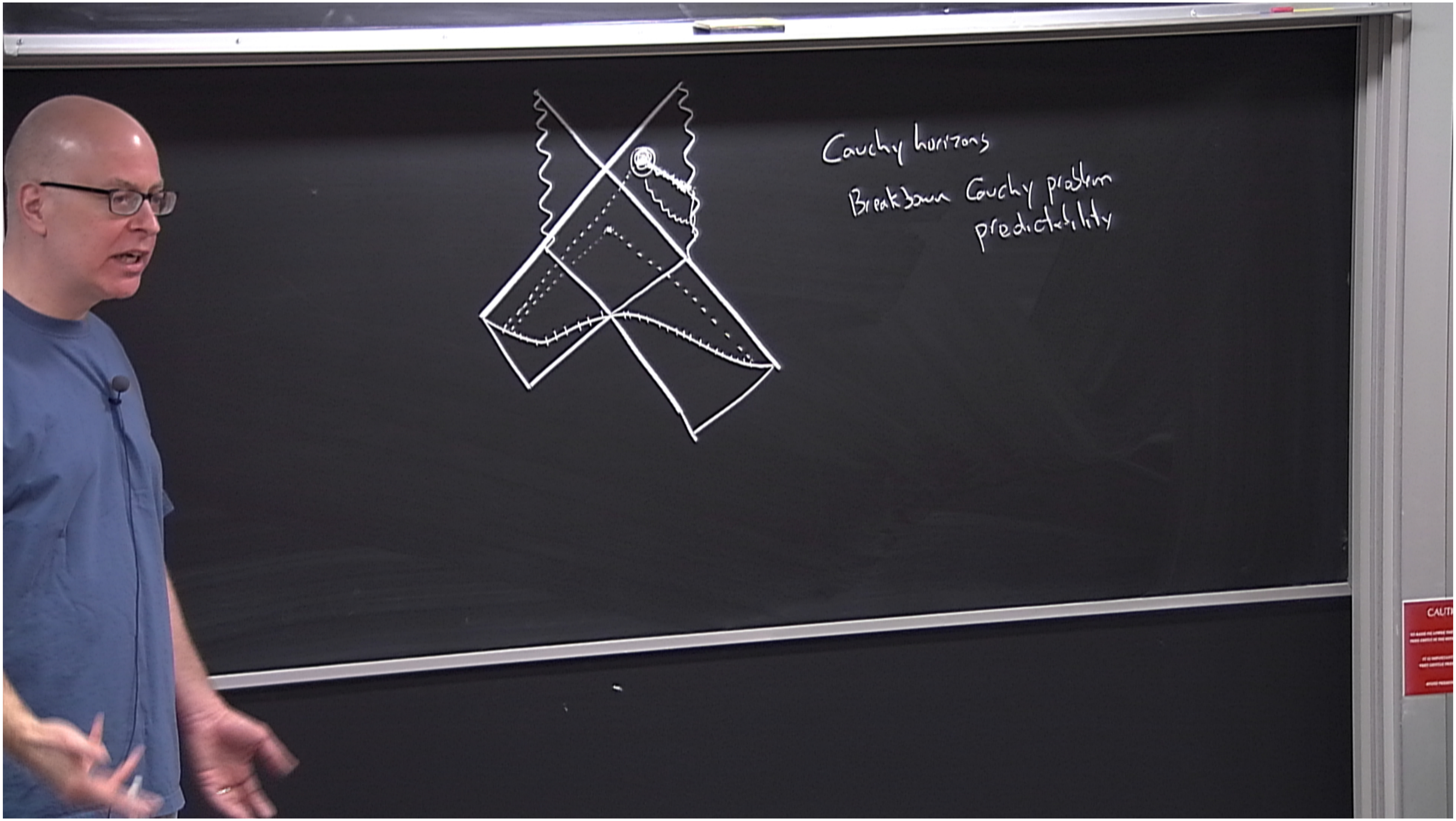


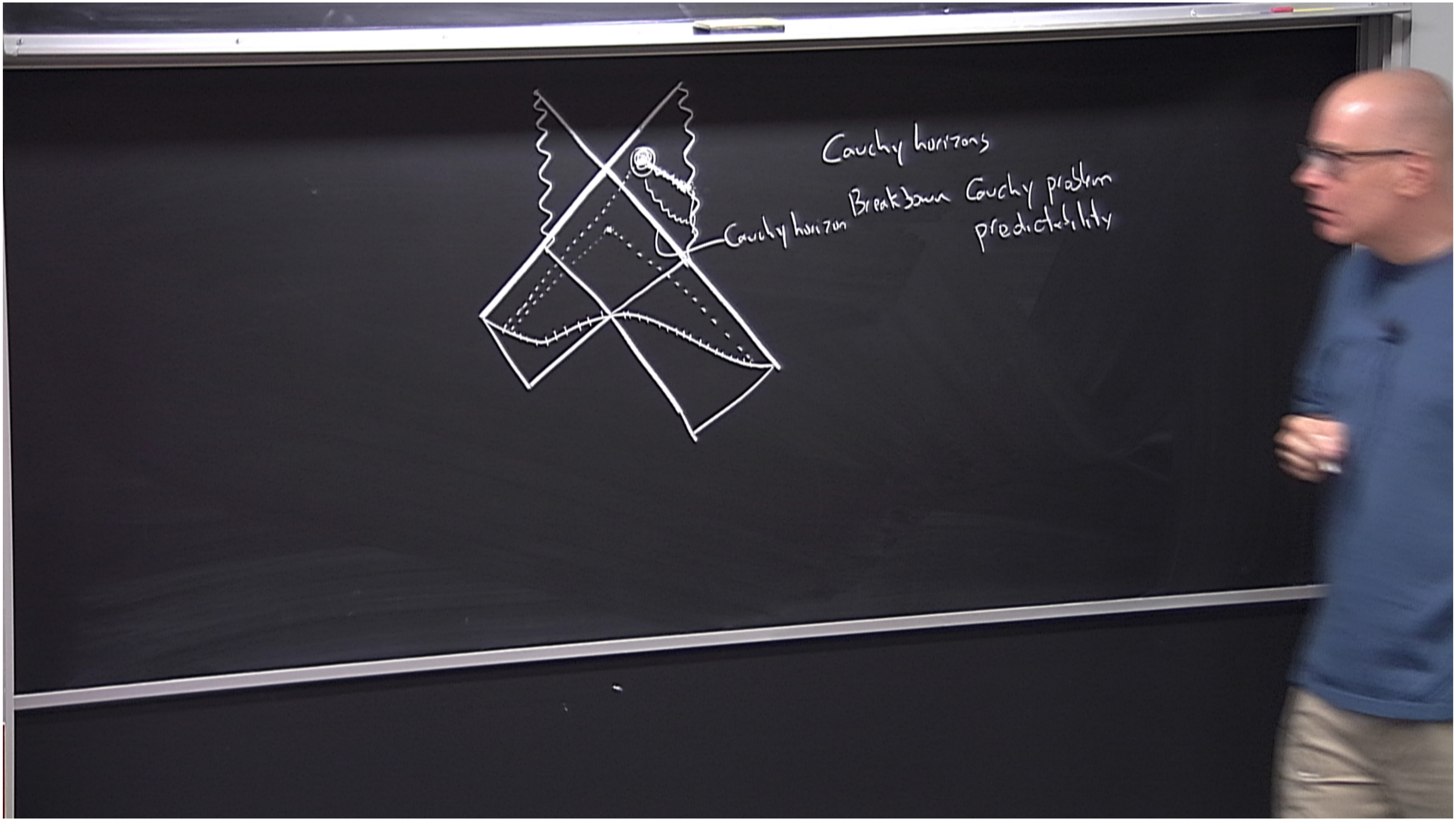


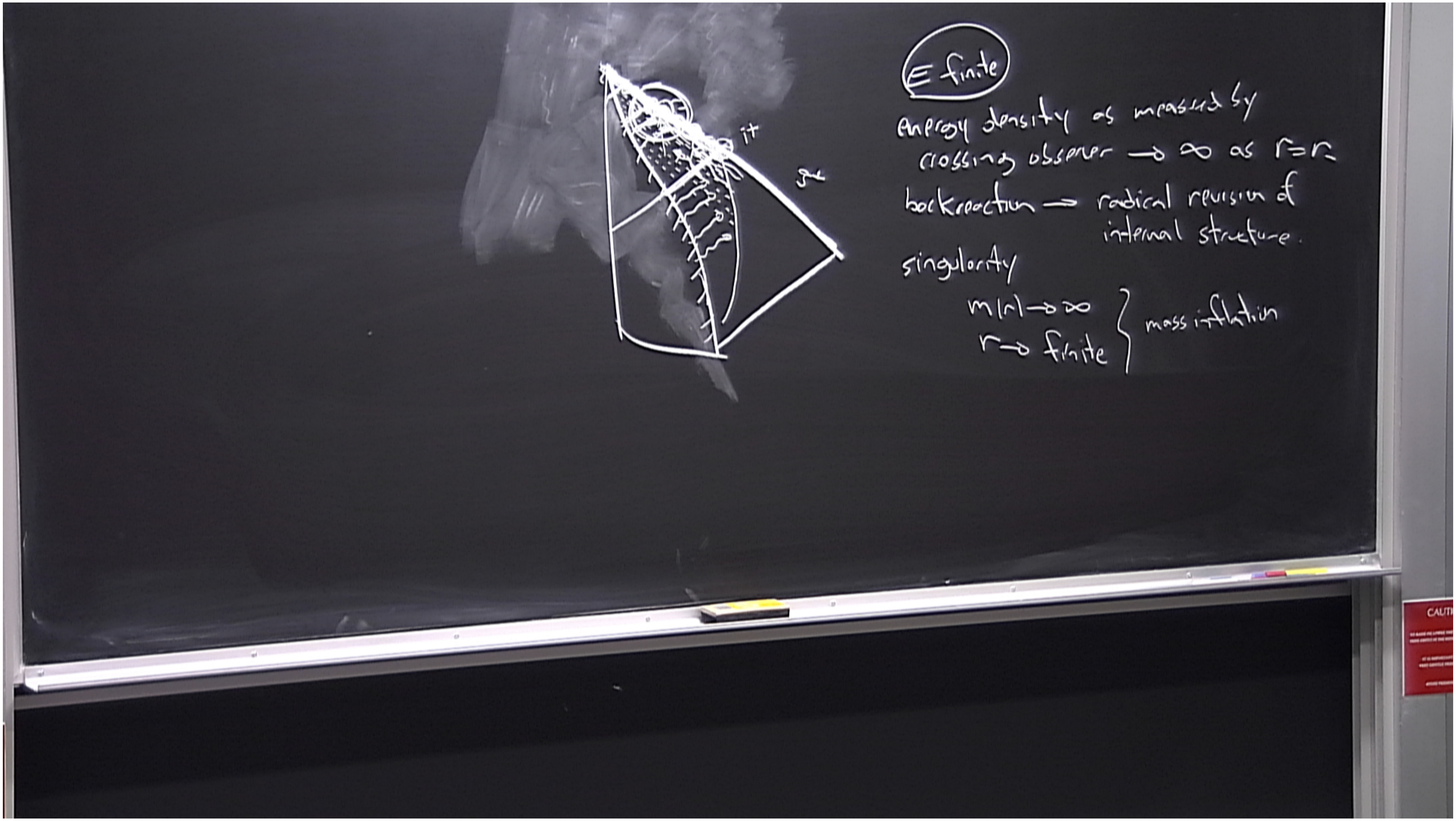


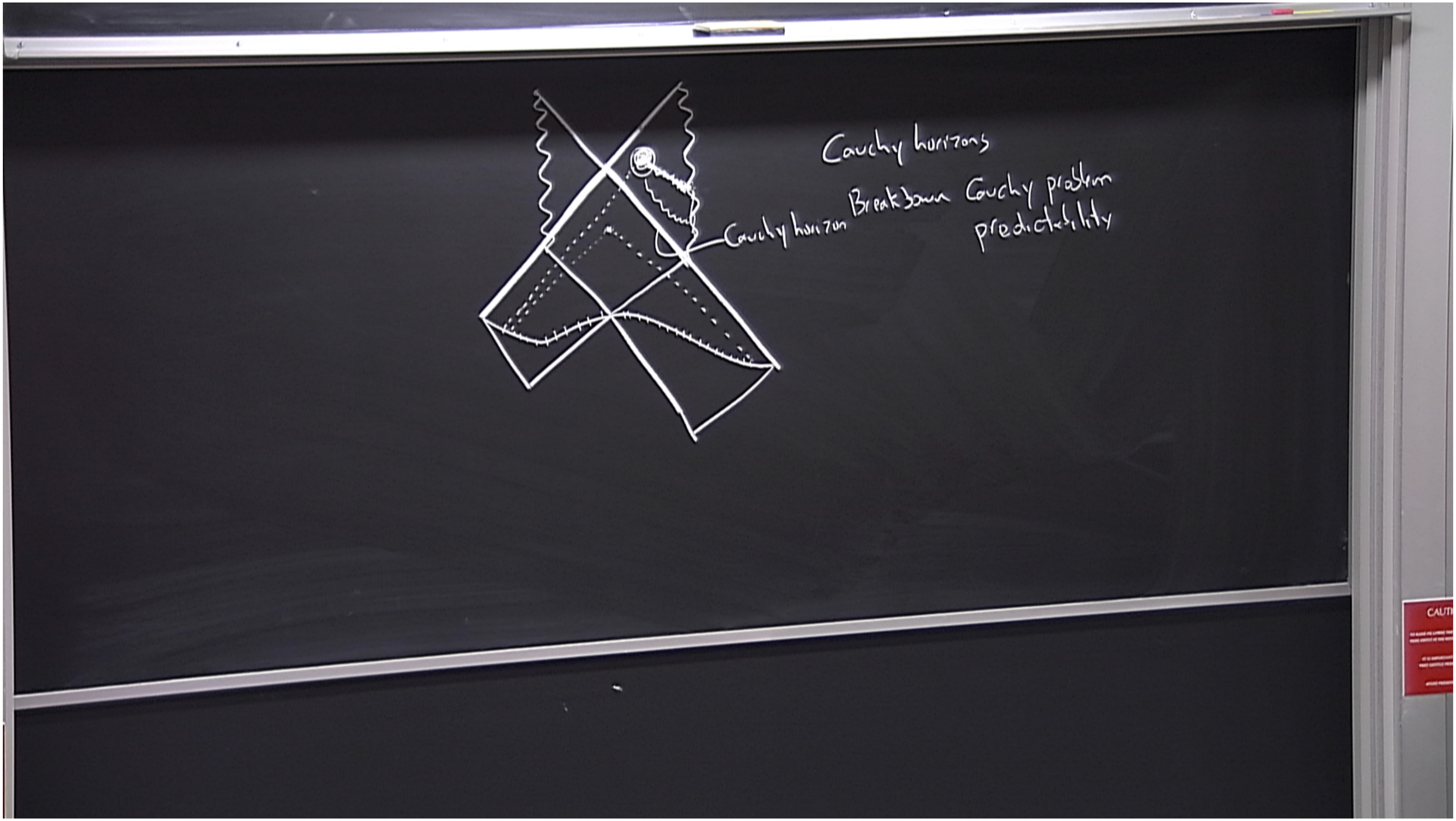

Cauchy horizons
Breakdown Cauchy problem
predictability


Penrose diagram
r=0
r=0


Cauchy horizons
Breakdown Cauchy problem
predictability


Cauchy horizons
Cauchy horizon
Breakdown Cauchy problem
predictability


E finite
energy density as measured by
crossing observer → ∞ as r=r₋
backreaction → radical revision of
internal structure.
singularity
m(r) → ∞
r → finite
mass inflation


Cauchy horizons
Cauchy horizon
Breakdown Cauchy problem
predictability