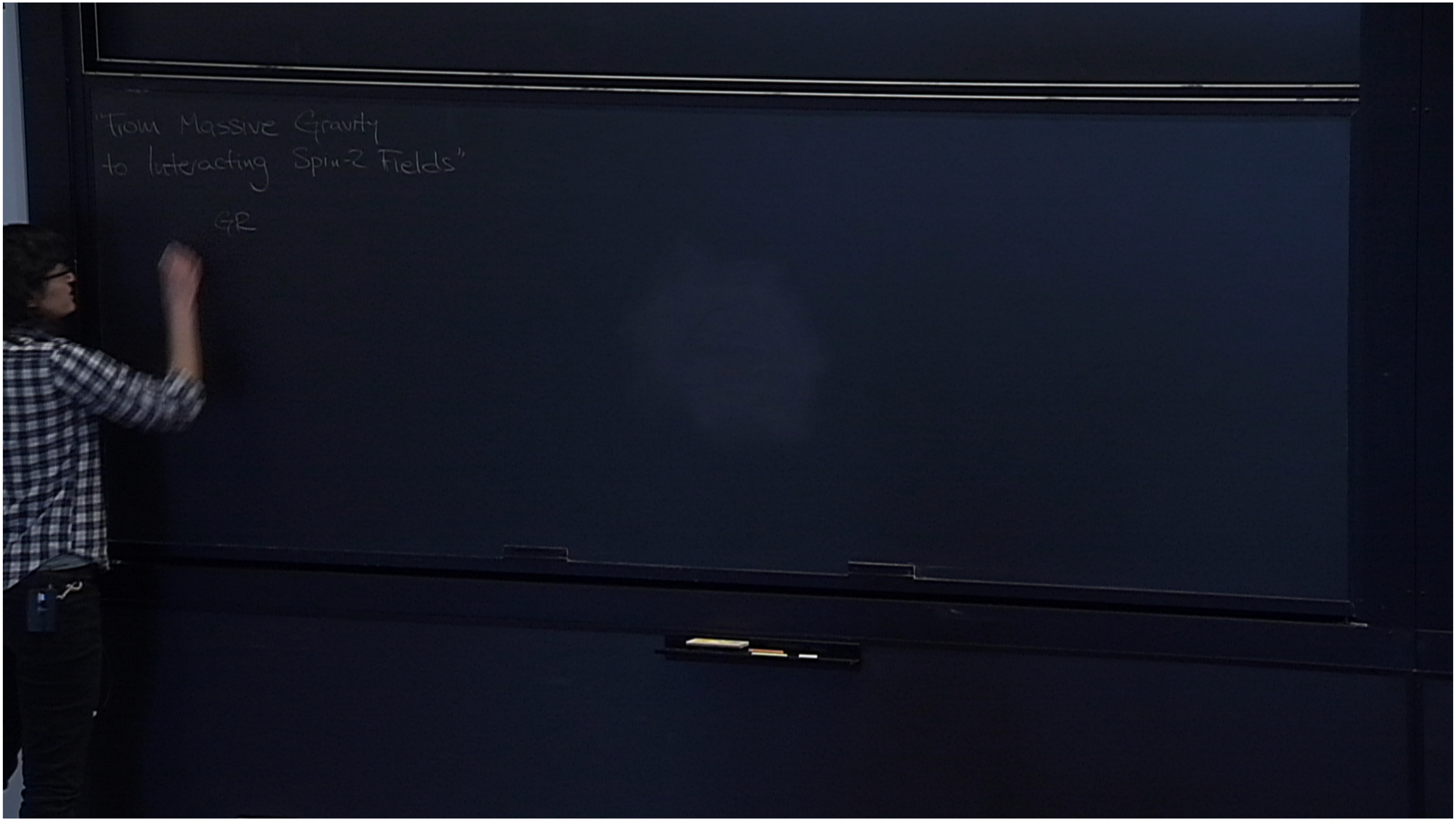
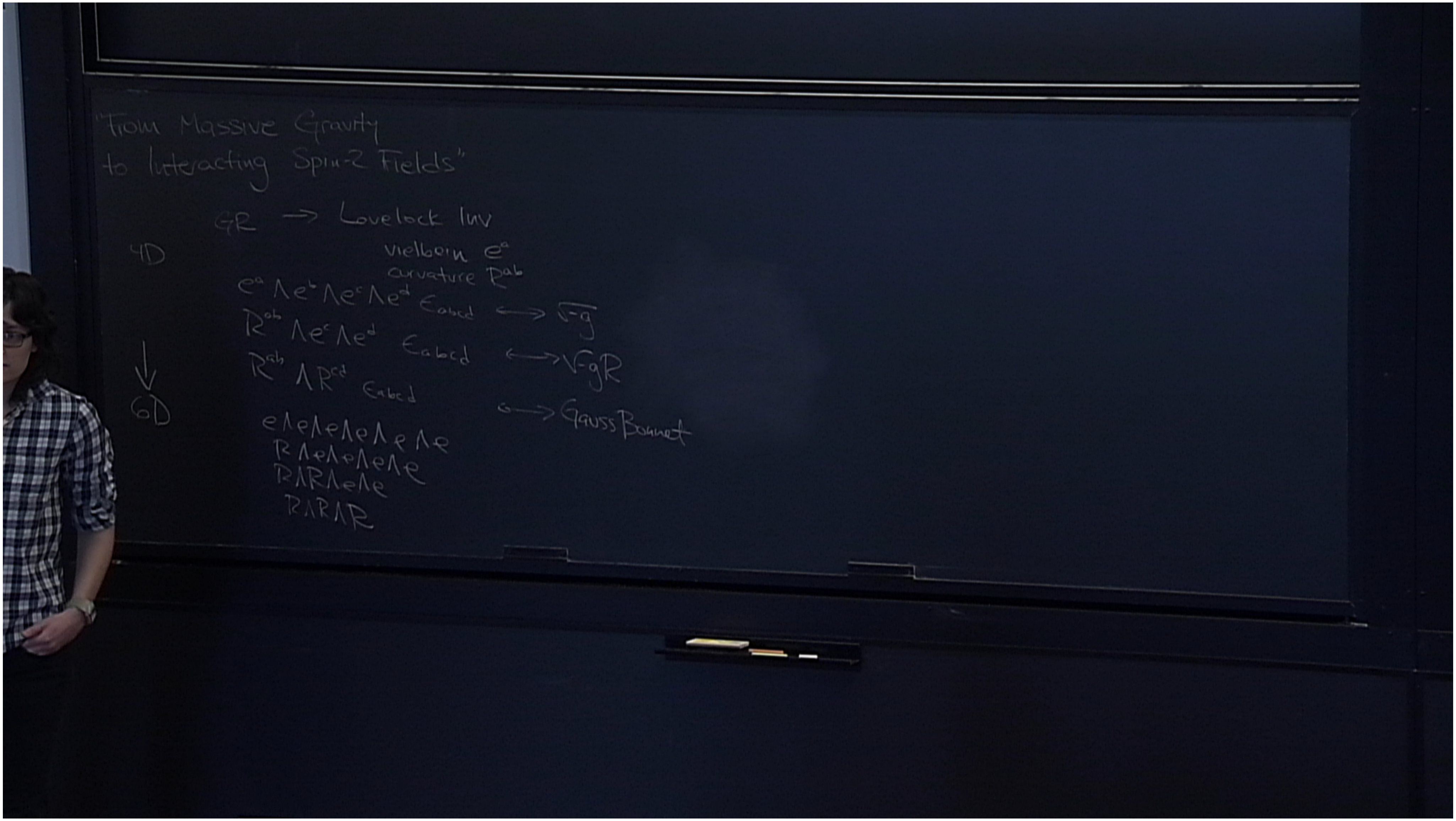
Title: From Massive Gravity to Interacting Spin-2 Fields

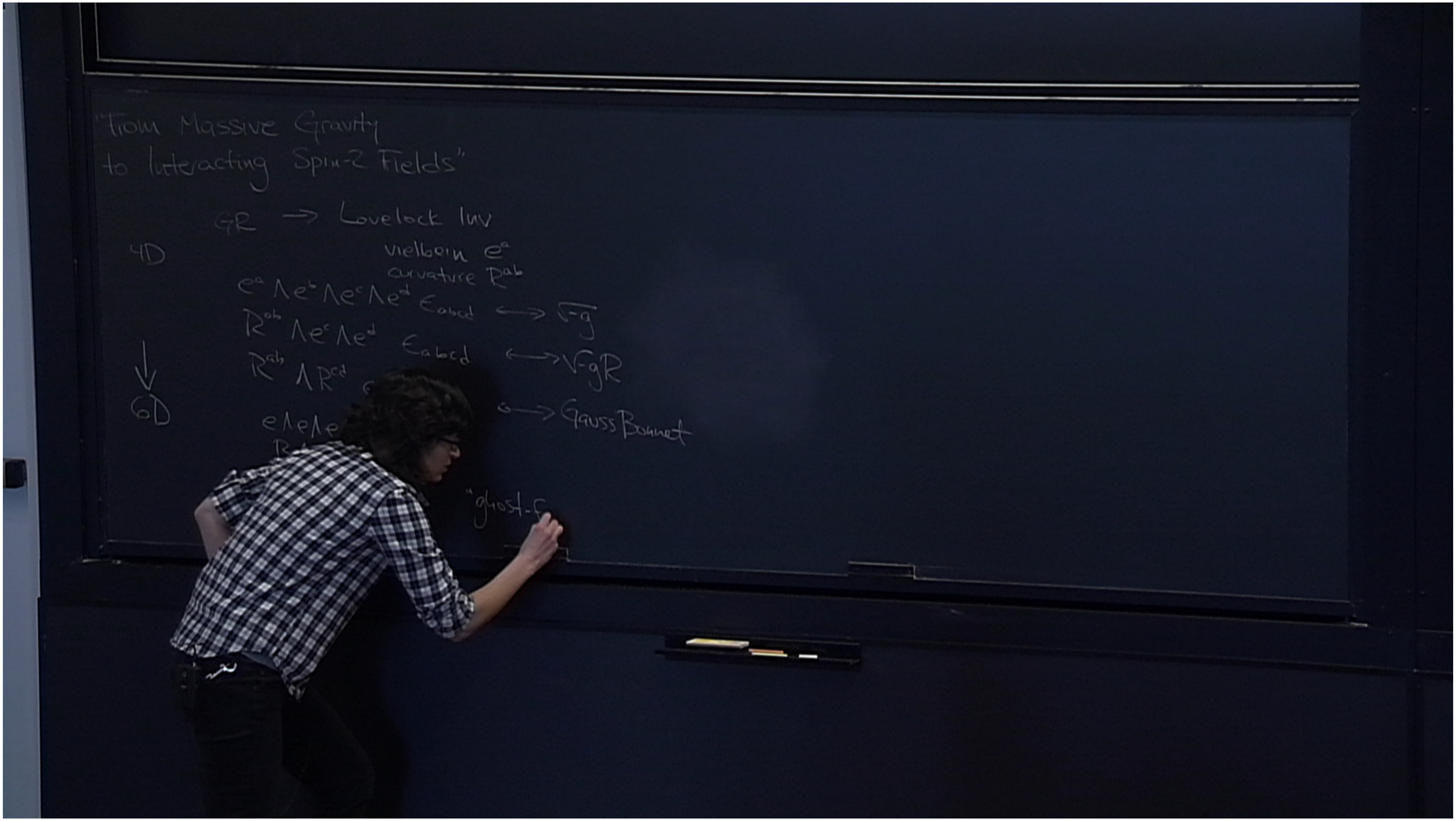
Date: Mar 06, 2012 11:00 AM

URL: http://pirsa.org/12030063

Abstract: Recent progress in massive gravity has made it possible to construct consistent theories of interacting spin-2 fields. In this talk I'll describe these developments, focusing on the resolution of the Boulware-Deser ghost problem and the promotion of massive gravity to a bimetric theory of gravity with two dynamical, interacting spin-2 fields. I'll then discuss the generalization of these bimetric theories to theories of multiple interacting spin-2 fields.


"From Massive Gravity
to Interacting Spin-2 Fields"
GR


"From Massive Gravity
to Interacting Spin-2 Fields"
GR → Lovelock Inv
4D
vielbein e^a
curvature R^ab
e^a ∧ e^b ∧ e^c ∧ e^d ε_abcd ⟷ √-g
R^ab ∧ e^c ∧ e^d ε_abcd ⟷ √-g R
R^ab ∧ R^cd ε_abcd ⟷ Gauss Bonnet
6D
e∧e∧e∧e∧e∧e
R∧e∧e∧e∧e
R∧R∧e∧e
R∧R∧R


"From Massive Gravity
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GR → Lovelock Inv
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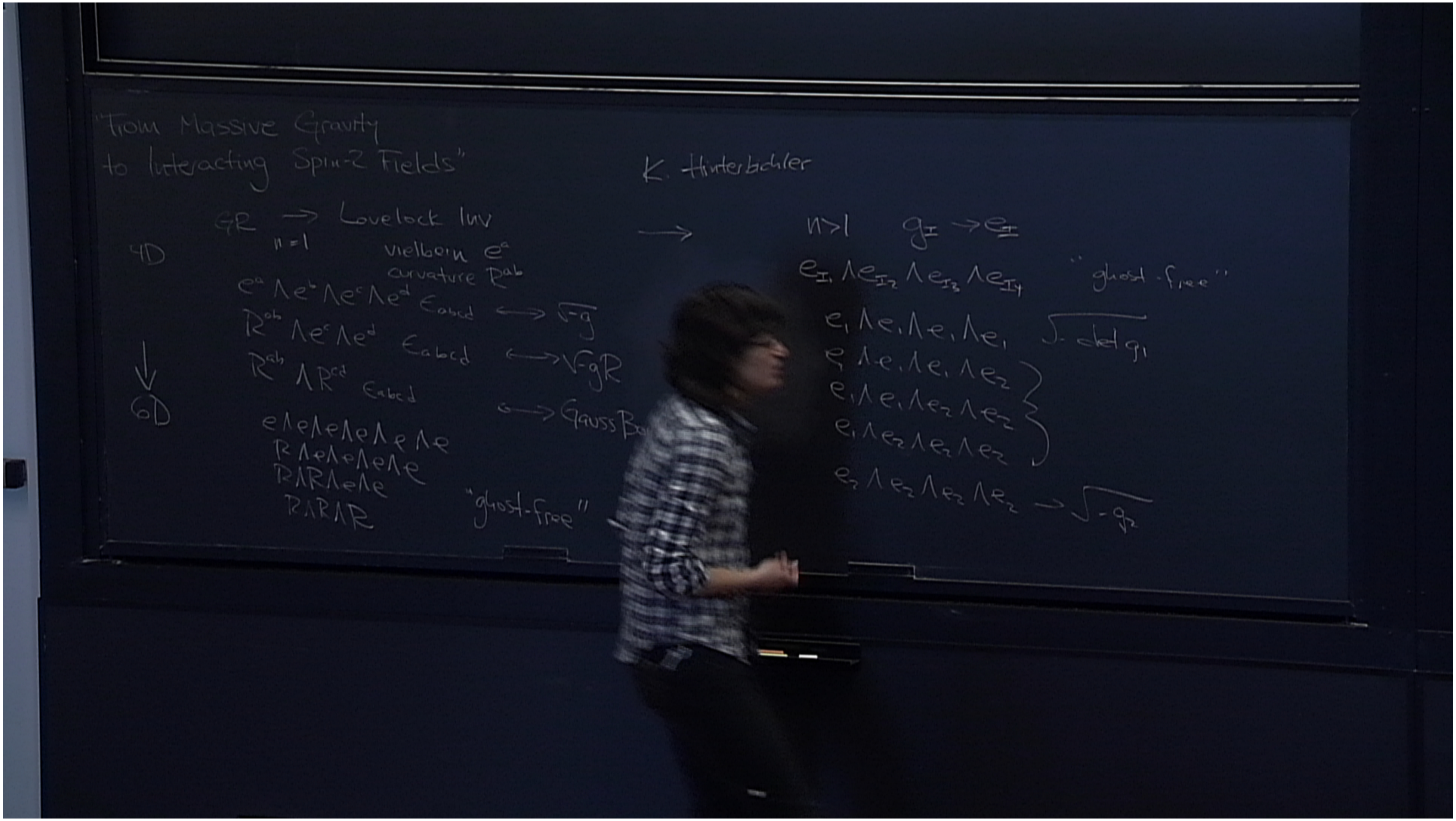
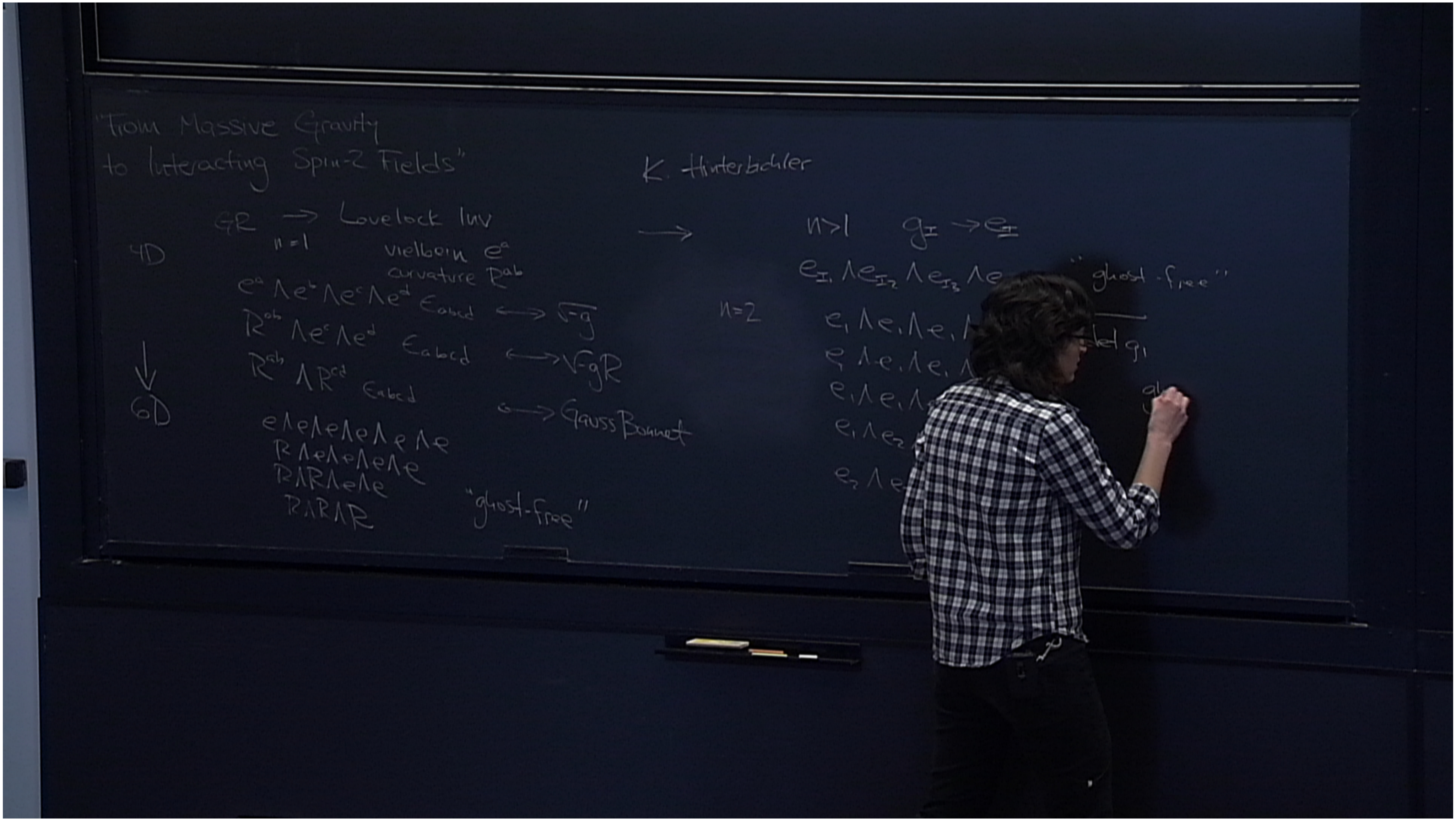
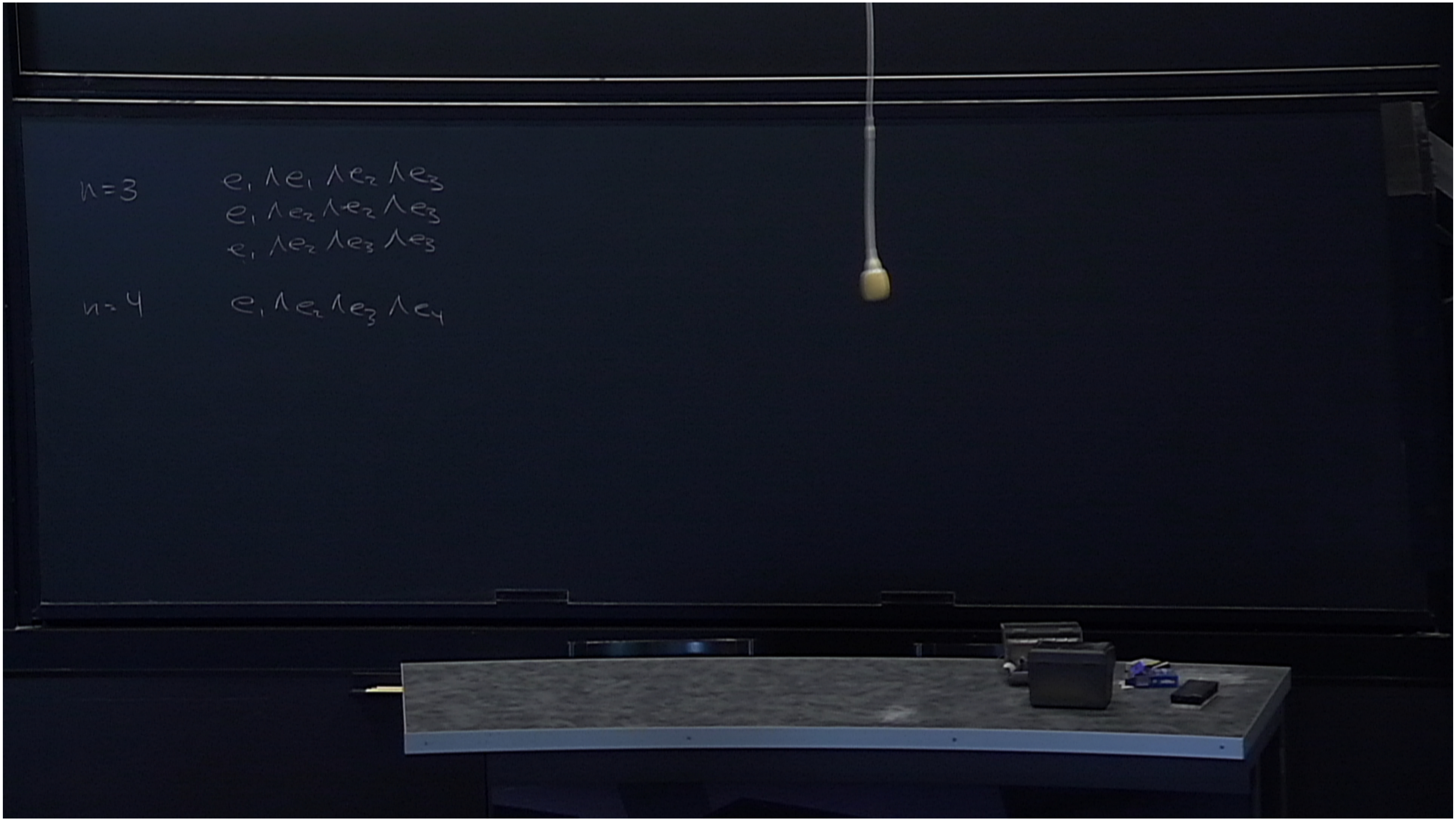
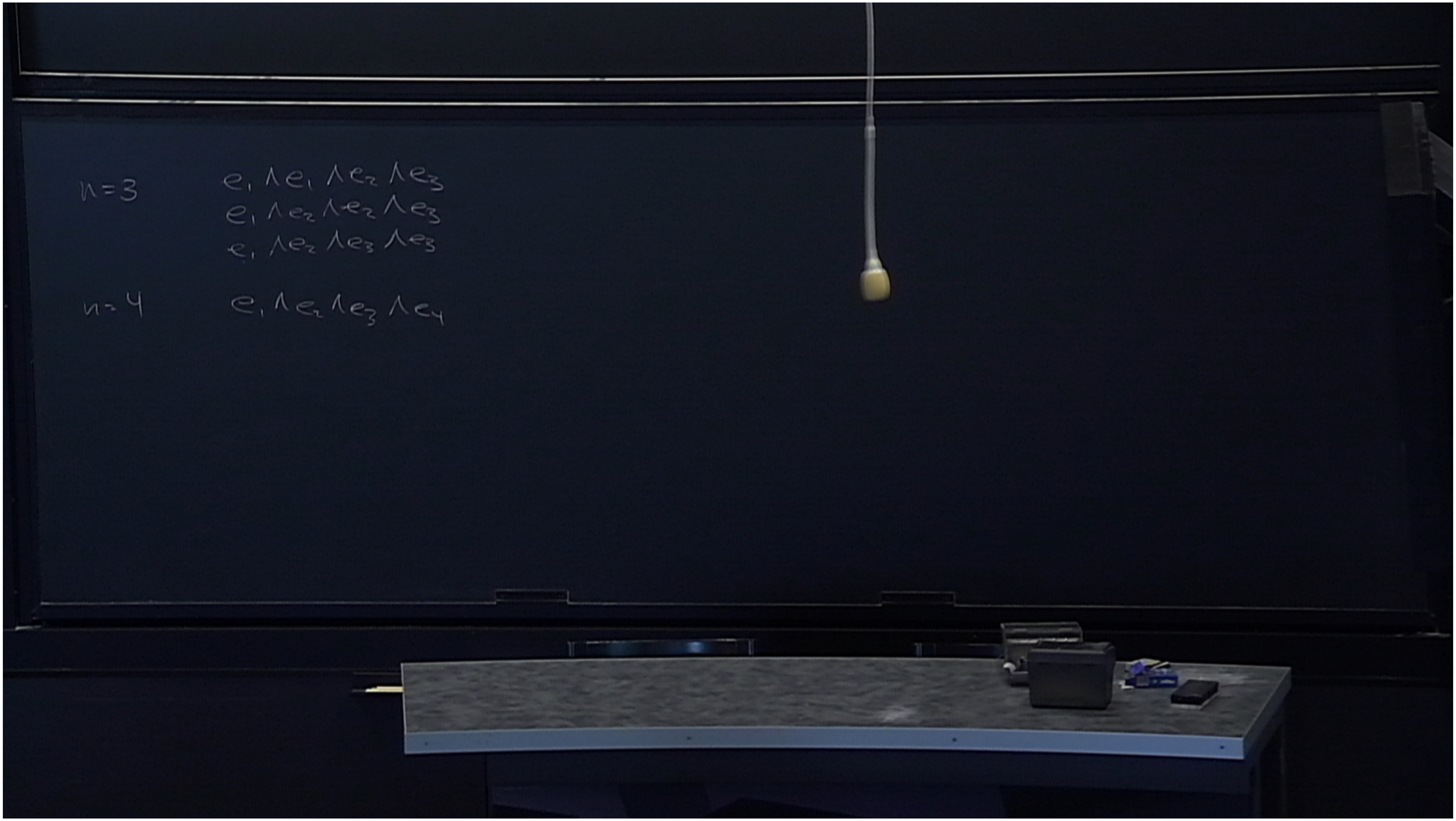
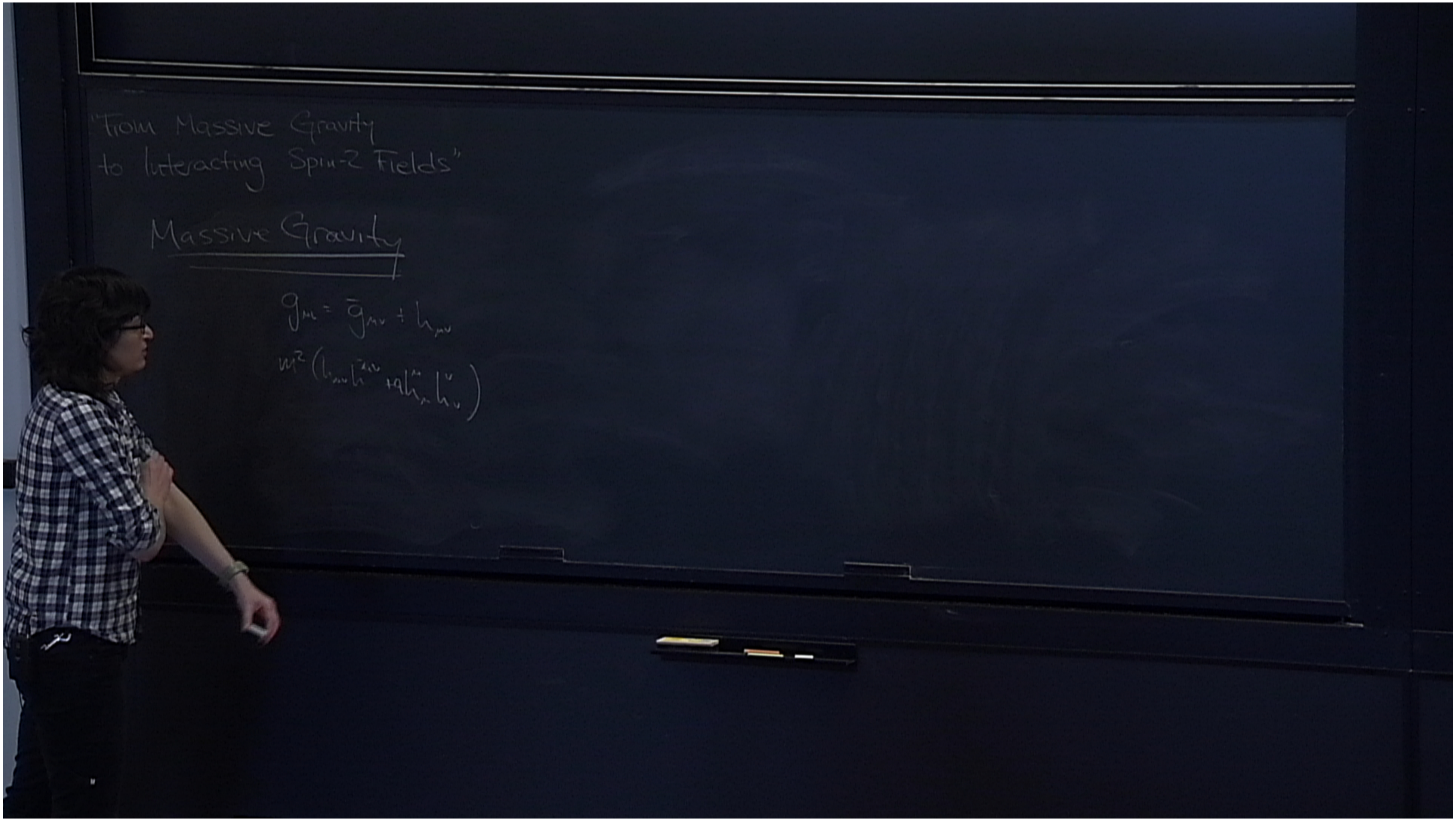
vielbein e^a
curvature R^ab
e^a ∧ e^b ∧ e^c ∧ e^d ε_abcd ⟷ √-g
R^ab ∧ e^c ∧ e^d ε_abcd ⟷ √-g R
R^ab ∧ R^cd
6D
e∧e∧e
⟷ Gauss Bonnet
"ghost-f

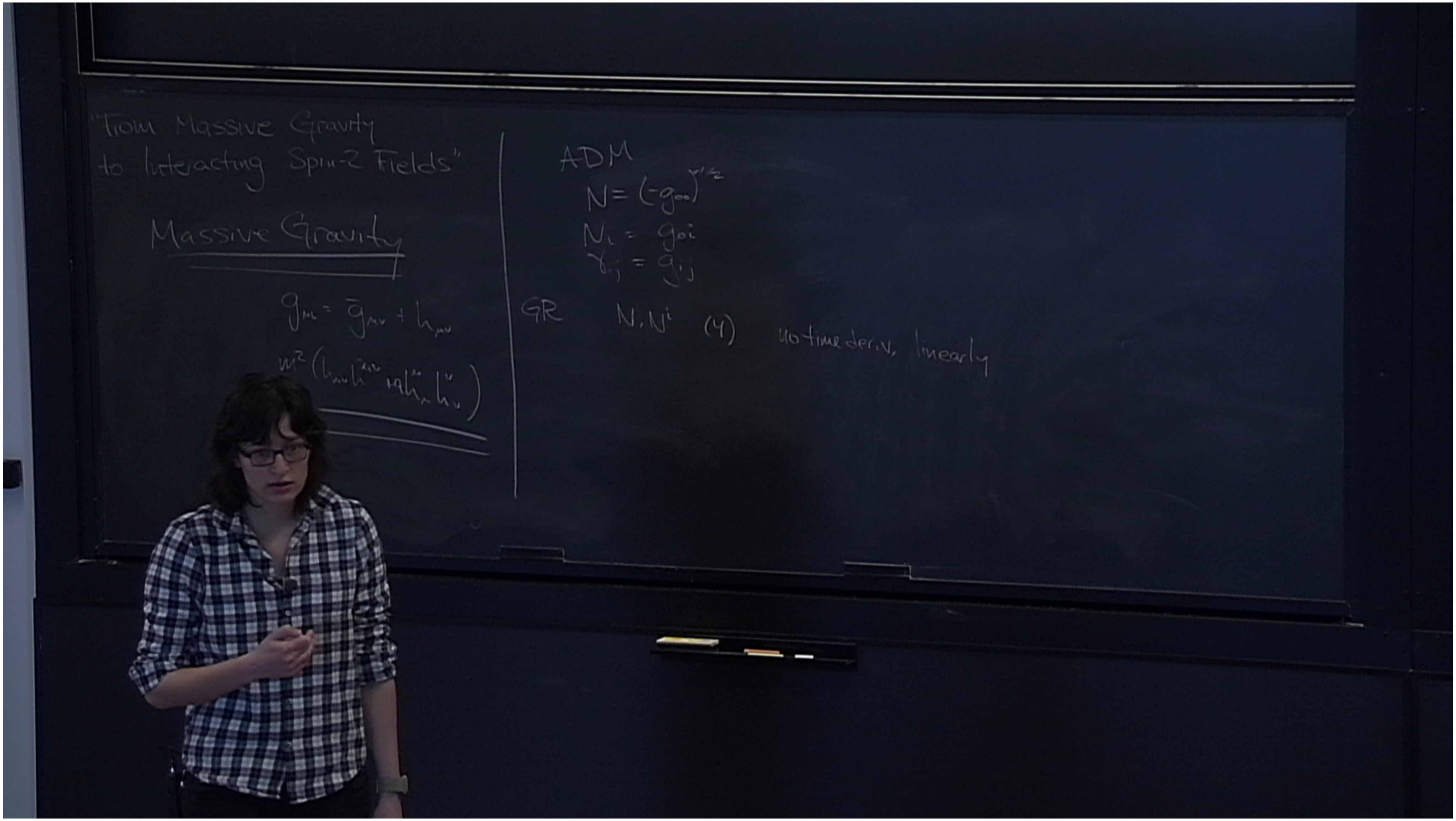

"From Massive Gravity
to Interacting Spin-2 Fields"
K. Hinterbichler
GR → Lovelock Inv
n=1
vielbein e^a
curvature R^ab
4D
6D
"ghost-free"
Gauss Bo
n>1
"ghost-free"

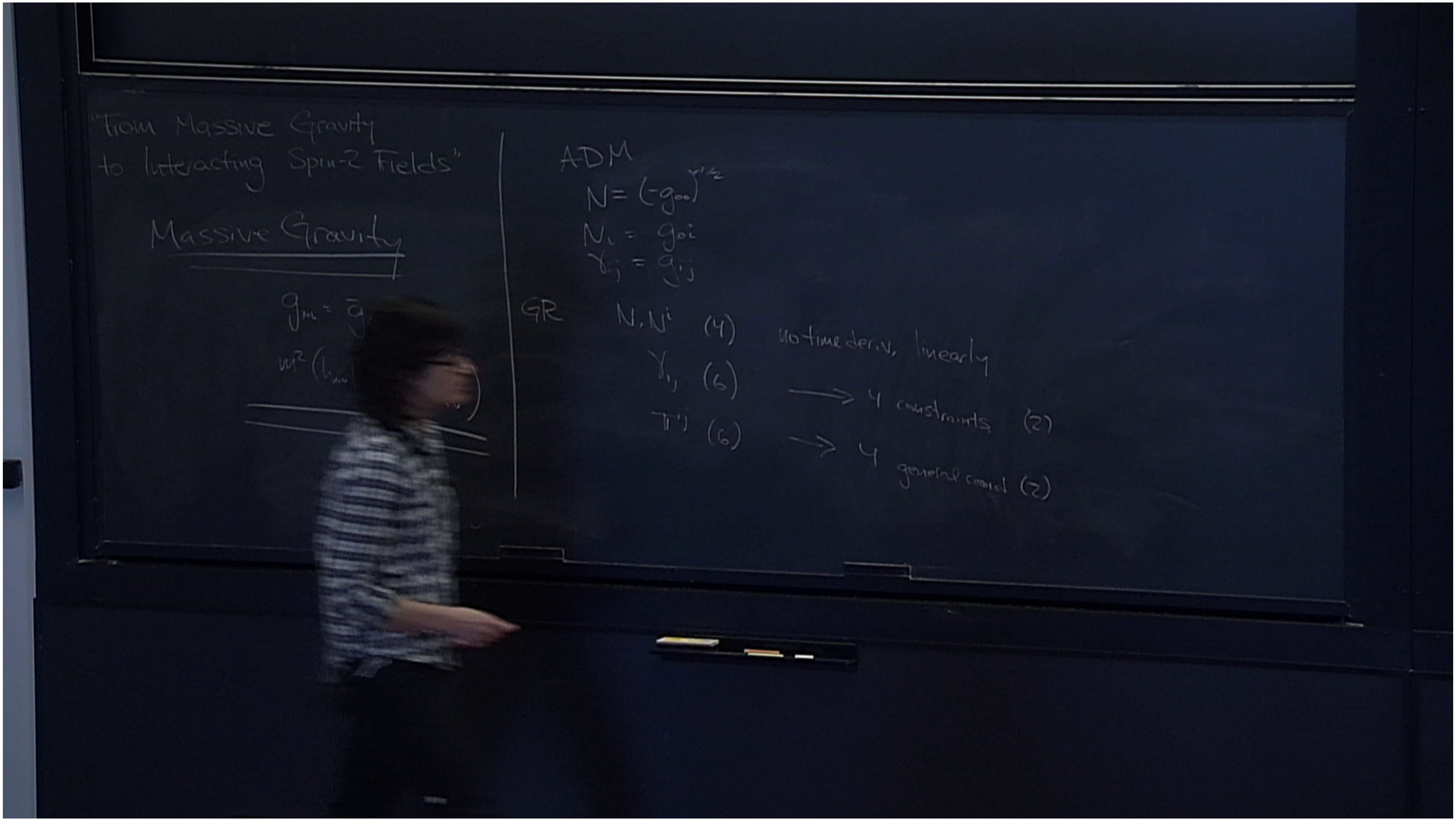

"From Massive Gravity
to Interacting Spin-2 Fields"
K. Hinterbichler
GR → Lovelock Inv
n=1
4D
vielbein e^a
curvature R^ab
e^a ∧ e^b ∧ e^c ∧ e^d ε_abcd ⟷ √-g
R^ab ∧ e^c ∧ e^d ε_abcd ⟷ √-g R
R^ab ∧ R^cd ε_abcd ⟷ Gauss Bonnet
6D
e∧e∧e∧e∧e∧e
R∧e∧e∧e∧e
R∧R∧e∧e
R∧R∧R
"ghost-free"
n>1
g_I → e_I
"ghost-free"
n=2
det g_1

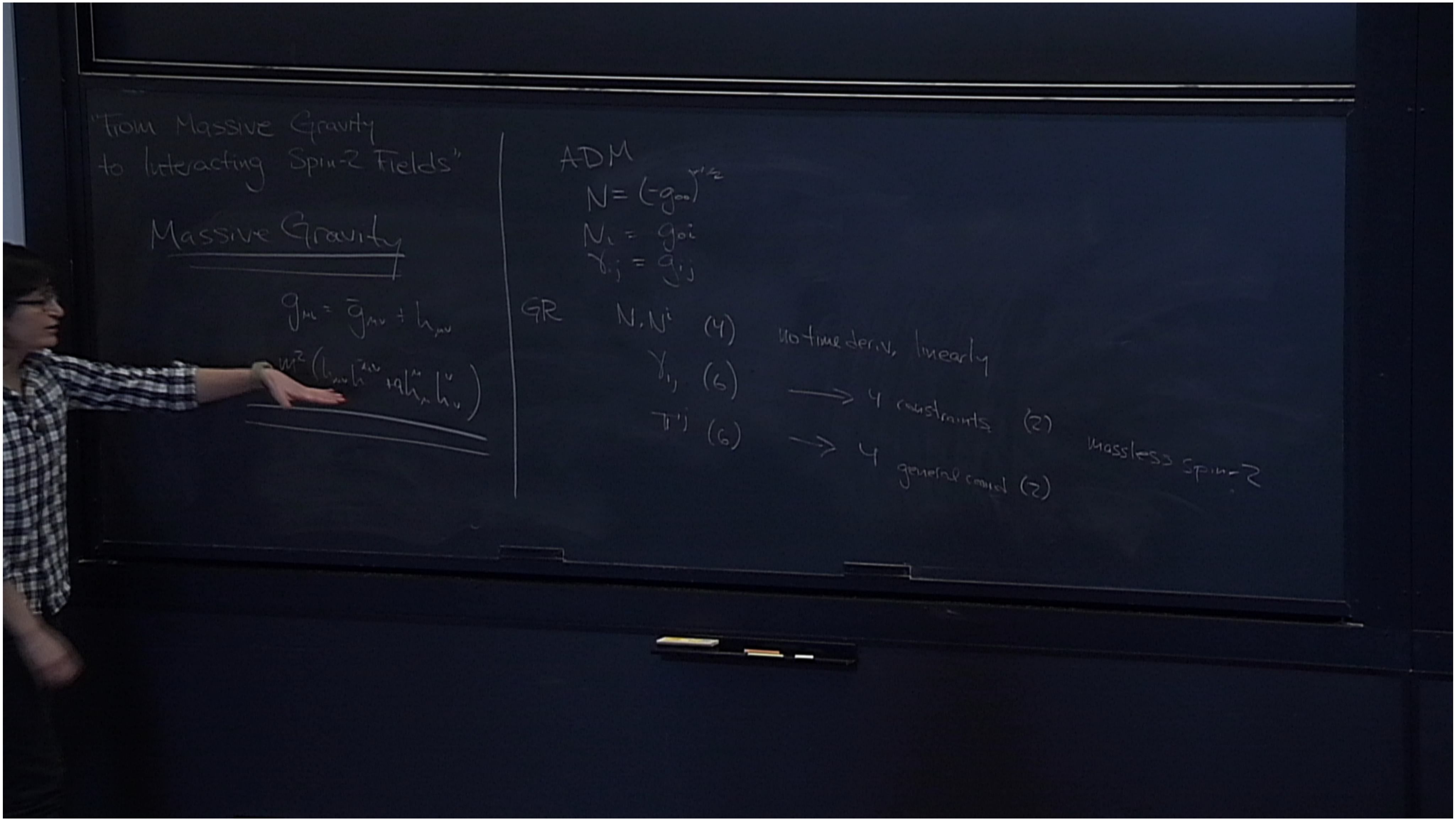

n=3
e1 ∧ e1 ∧ e2 ∧ e3
e1 ∧ e2 ∧ e2 ∧ e3
e1 ∧ e2 ∧ e3 ∧ e3
n=4
e1 ∧ e2 ∧ e3 ∧ e4

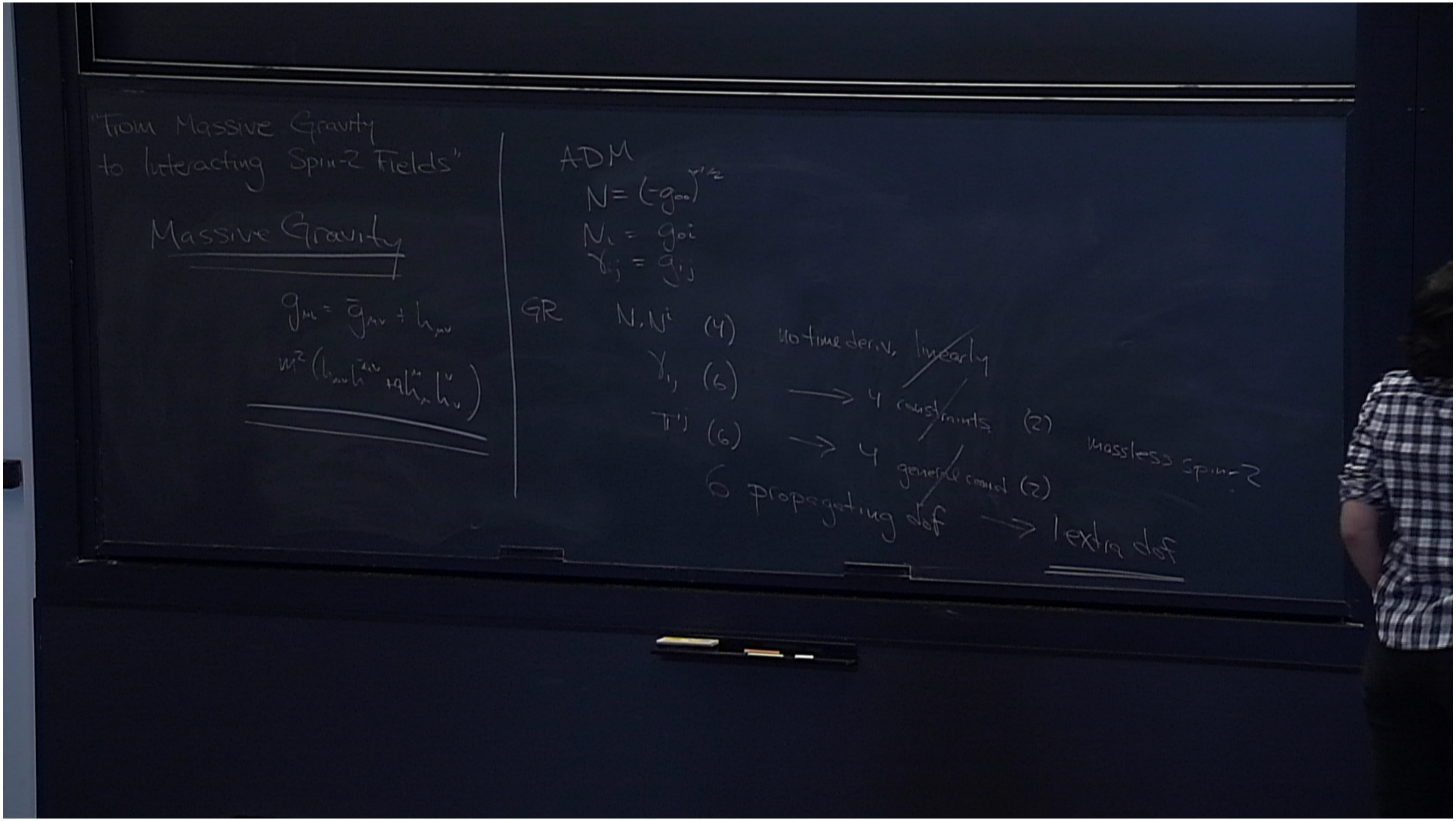

$n=3$
$e_1 \wedge e_1 \wedge e_2 \wedge e_3$
$e_1 \wedge e_2 \wedge e_2 \wedge e_3$
$e_1 \wedge e_2 \wedge e_3 \wedge e_3$
$n=4$
$e_1 \wedge e_2 \wedge e_3 \wedge e_4$

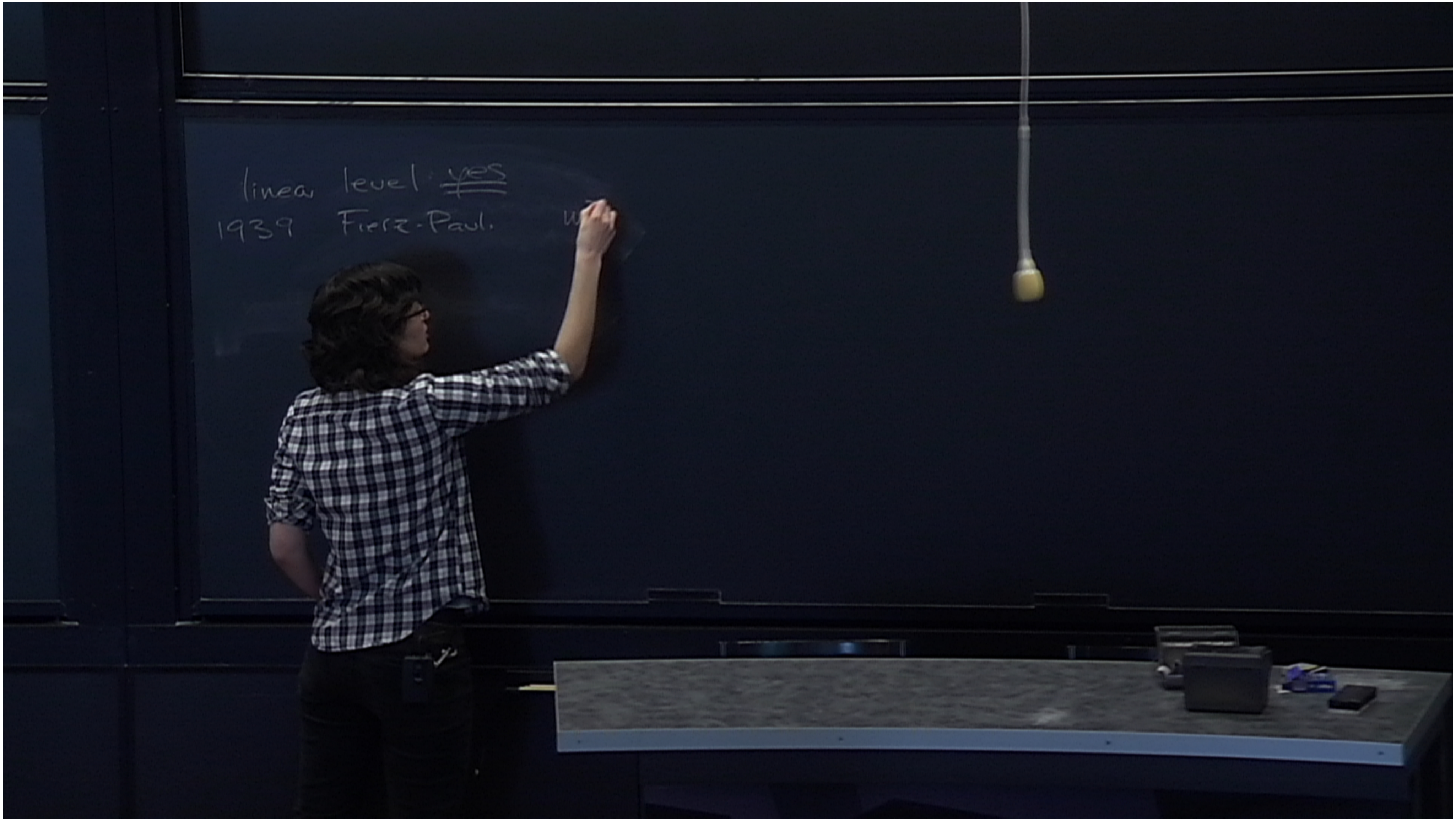

"From Massive Gravity
to Interacting Spin-2 Fields"
Massive Gravity
$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
$m^2 (h_{\mu\nu} h^{\mu\nu} + a h^{\mu}_{\mu} h^{\nu}_{\nu})$

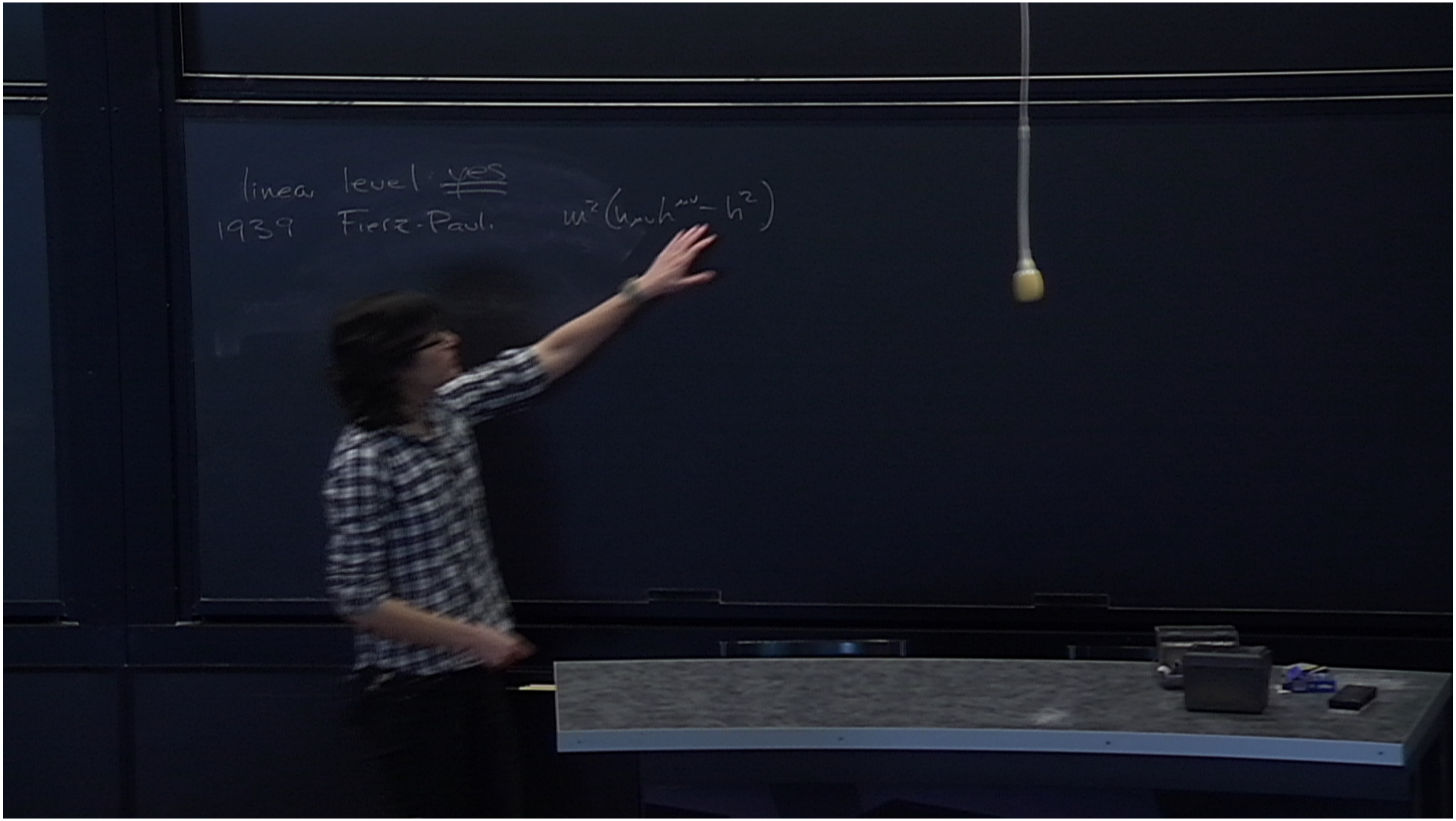

"From Massive Gravity to Interacting Spin-2 Fields"
Massive Gravity
ADM
GR
no time deriv, linearly

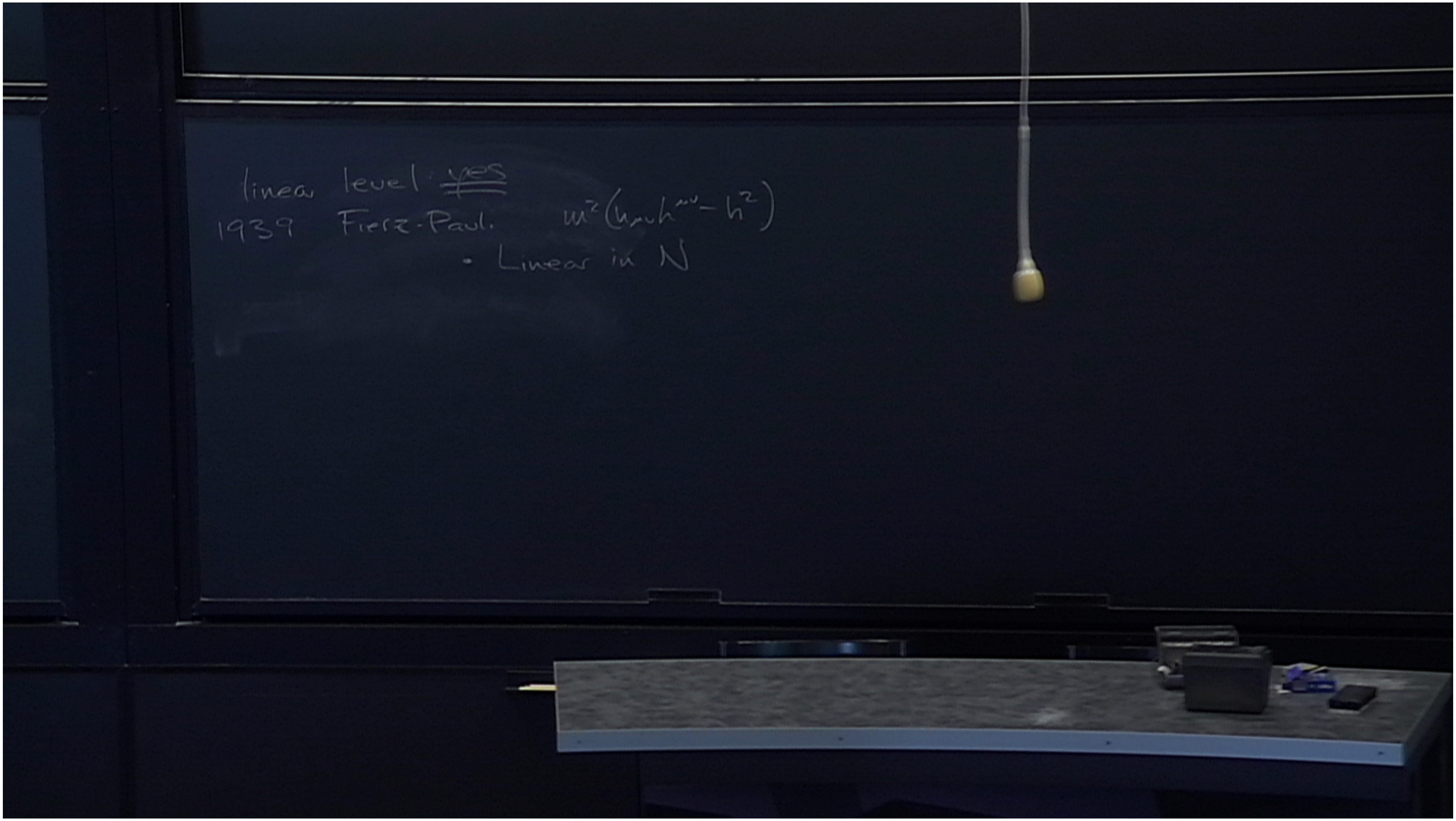

"From Massive Gravity
to Interacting Spin-2 Fields"
Massive Gravity
ADM
N = (-g00)^-1/2
Ni = g0i
γij = gij
GR
N, N^i (4) no time deriv, linearly
γij (6) → 4 constraints (2)
π^ij (6) → 4 general coord (2)

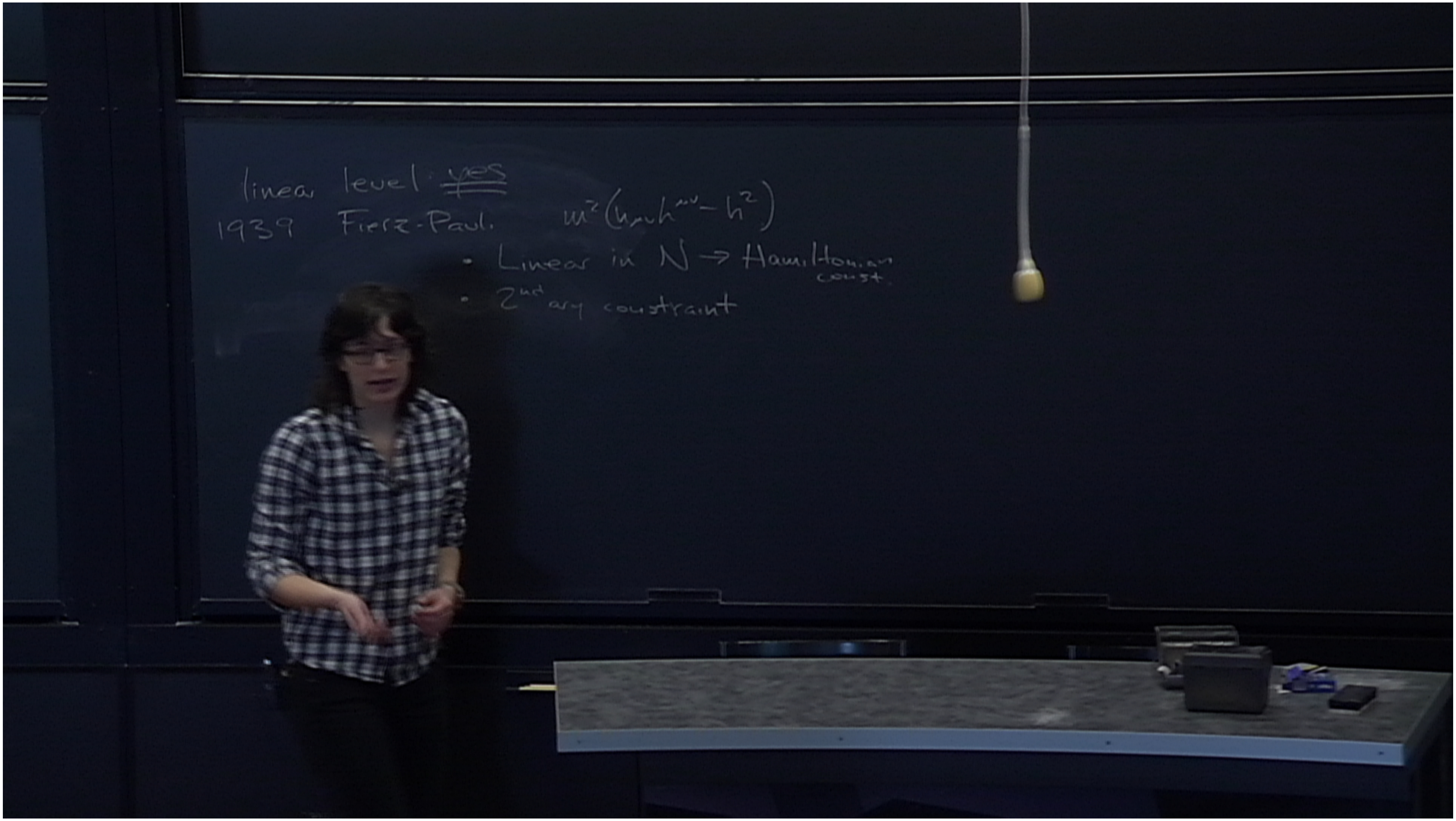

"From Massive Gravity
to Interacting Spin-2 Fields"
Massive Gravity
ADM
GR
no time deriv, linearly
4 constraints (2)
4 general coord (2)
massless spin-2

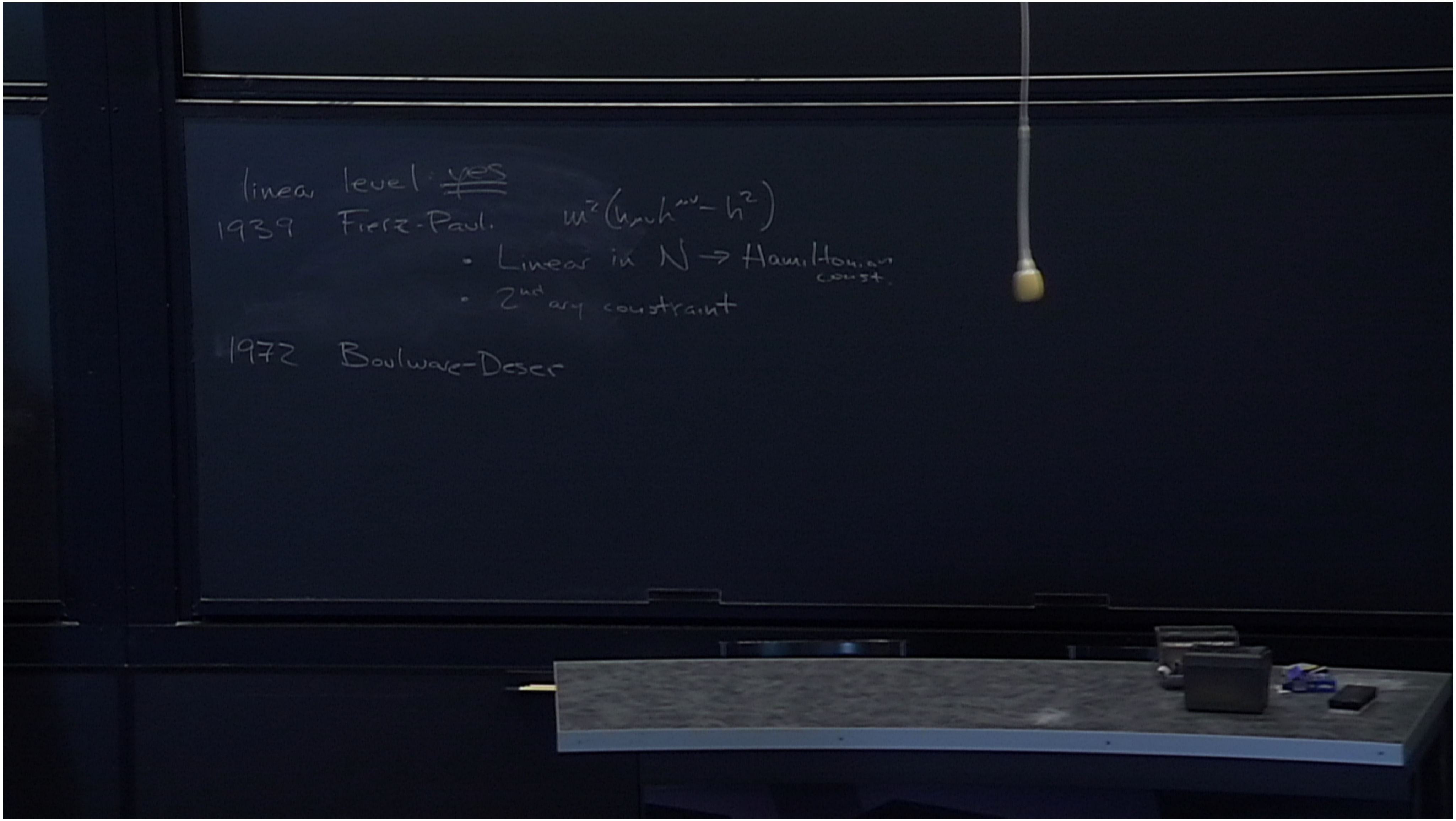

"From Massive Gravity
to Interacting Spin-2 Fields"
Massive Gravity
ADM
GR
no time deriv. linearly
4 constraints (2)
4 general coord (2)
massless spin-2
6 propagating dof
1 extra dof

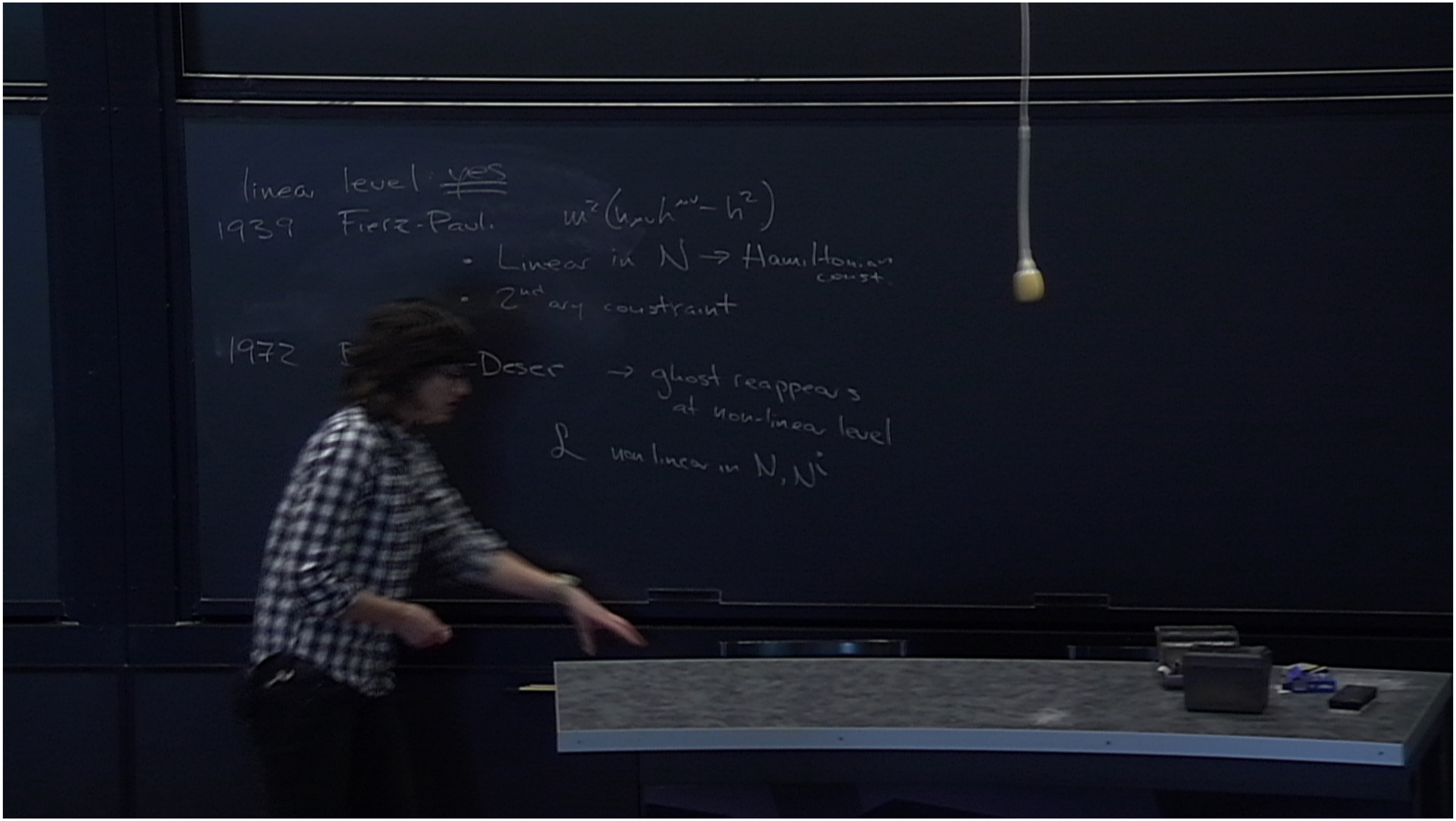

linear level: yes
1939 Fierz-Pauli

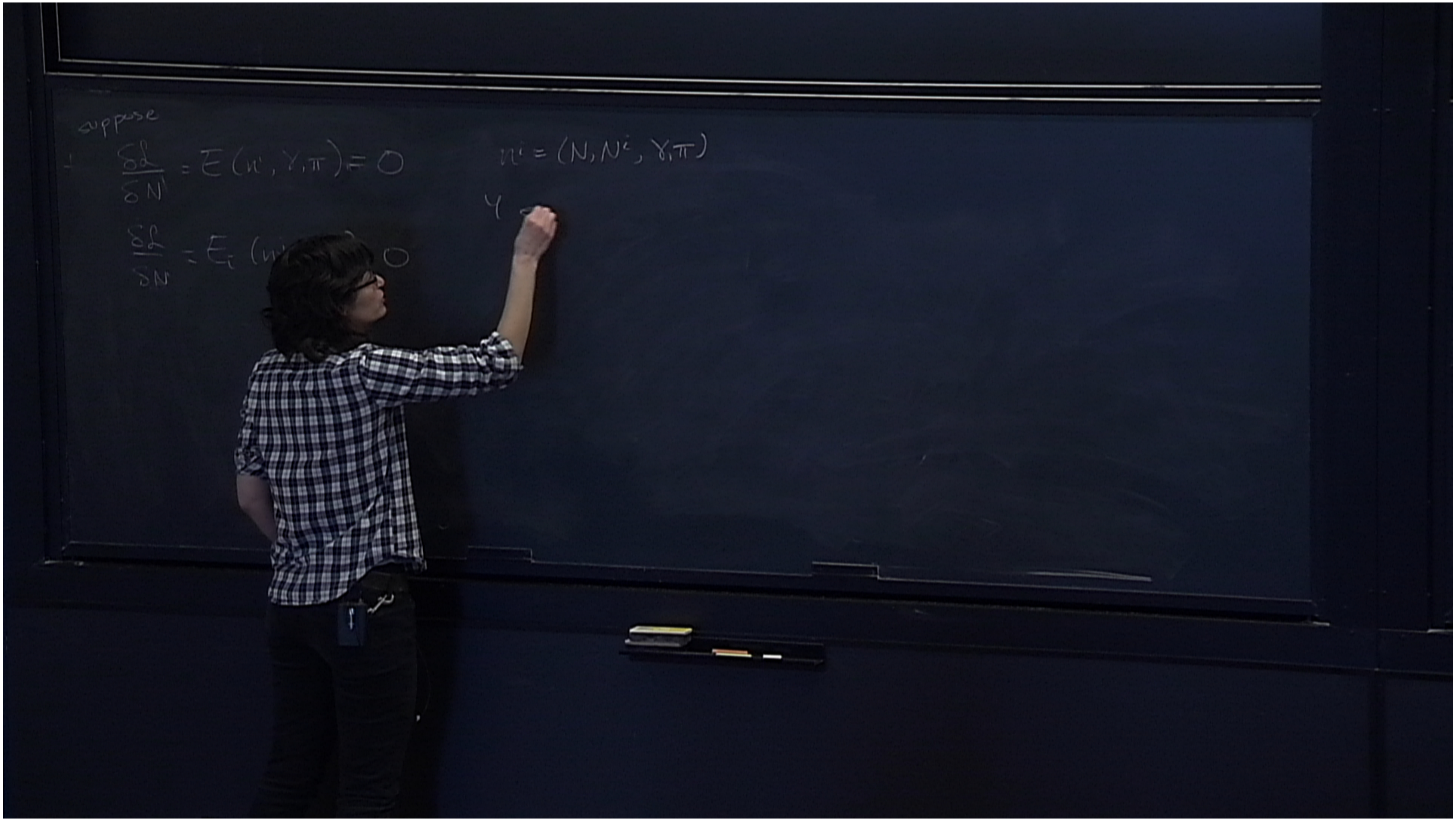

linear level: yes
1939 Fierz-Pauli
$m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$

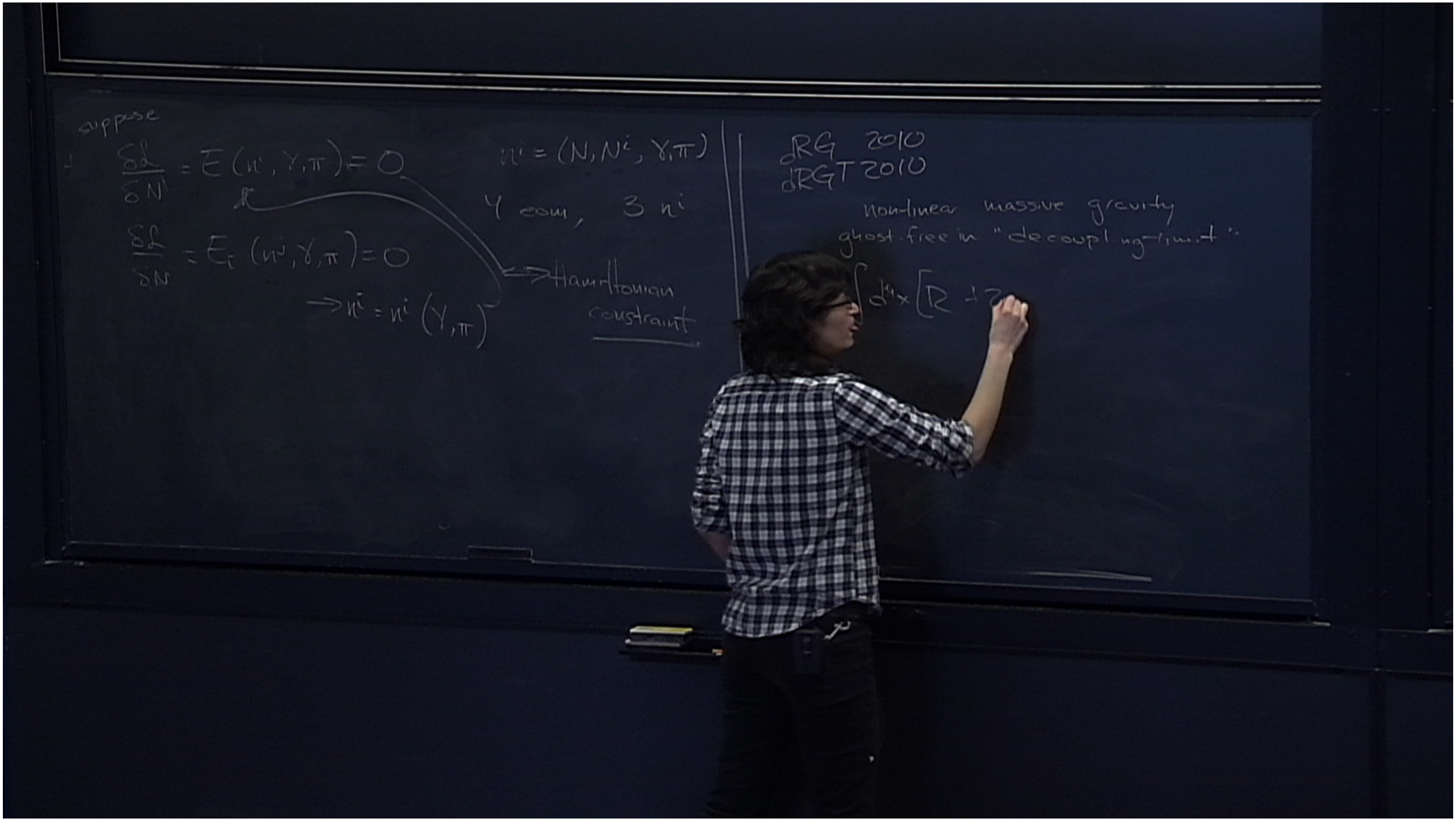

linear level: yes
1939 Fierz-Pauli. m²(h_μν h^μν − h²)
• Linear in N

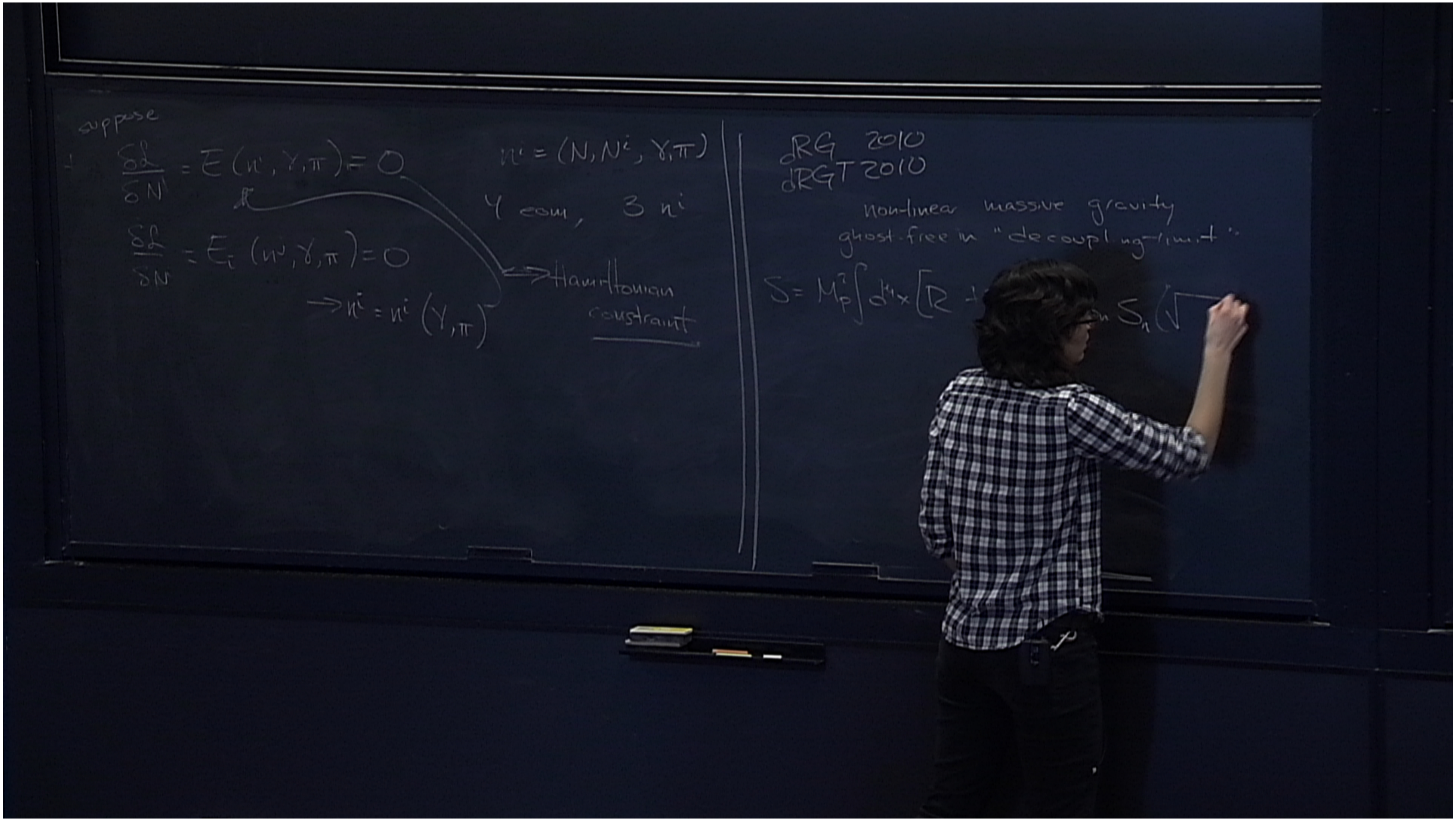

linear level: yes
1939 Fierz-Pauli
$m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$
• Linear in N → Hamiltonian const.
• 2nd ary constraint

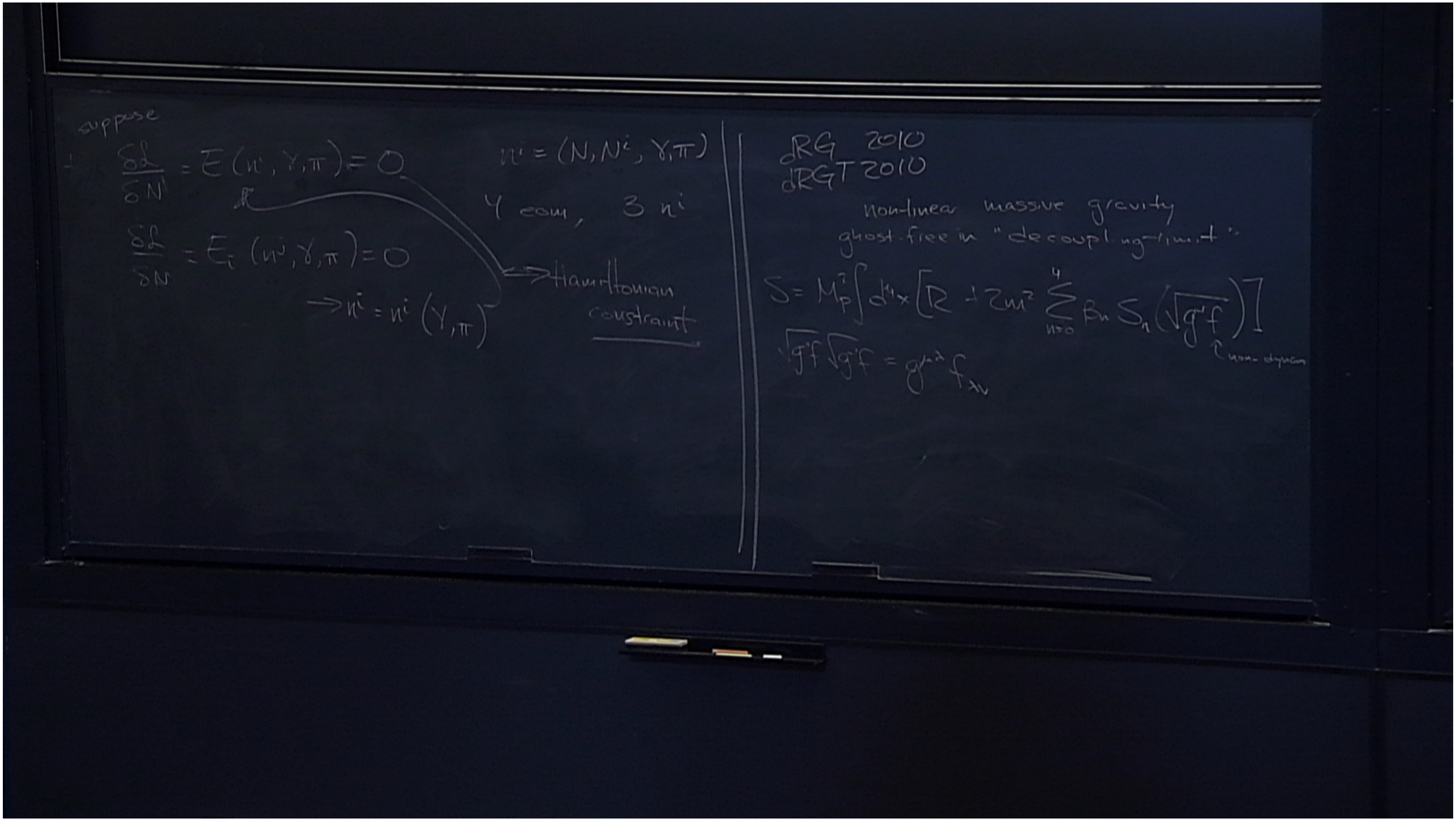

linear level: yes
1939 Fierz-Pauli $m^2(h_{\mu\nu}h^{\mu\nu}-h^2)$
• Linear in N → Hamiltonian const.
• 2ndary constraint
1972 Boulware-Deser

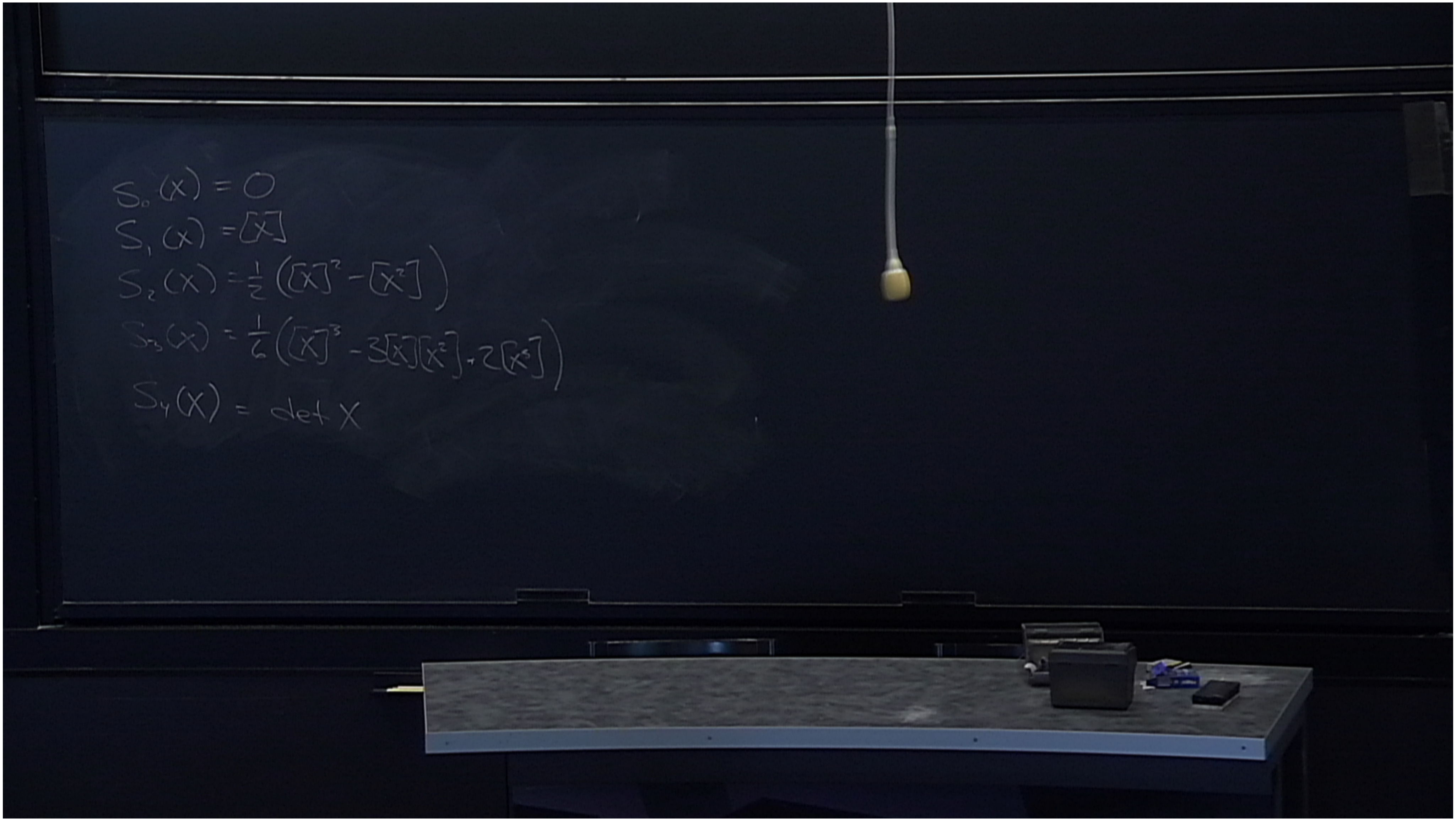

linear level: yes
1939 Fierz-Pauli
m²(h_μν h^μν − h²)
• Linear in N → Hamiltonian const.
• 2nd ary constraint
1972
Deser
→ ghost reappears at non-linear level
ℒ non linear in N, N^i


suppose
δL/δN = E(n^i, γ, π) = 0
n^i = (N, N^i, γ, π)

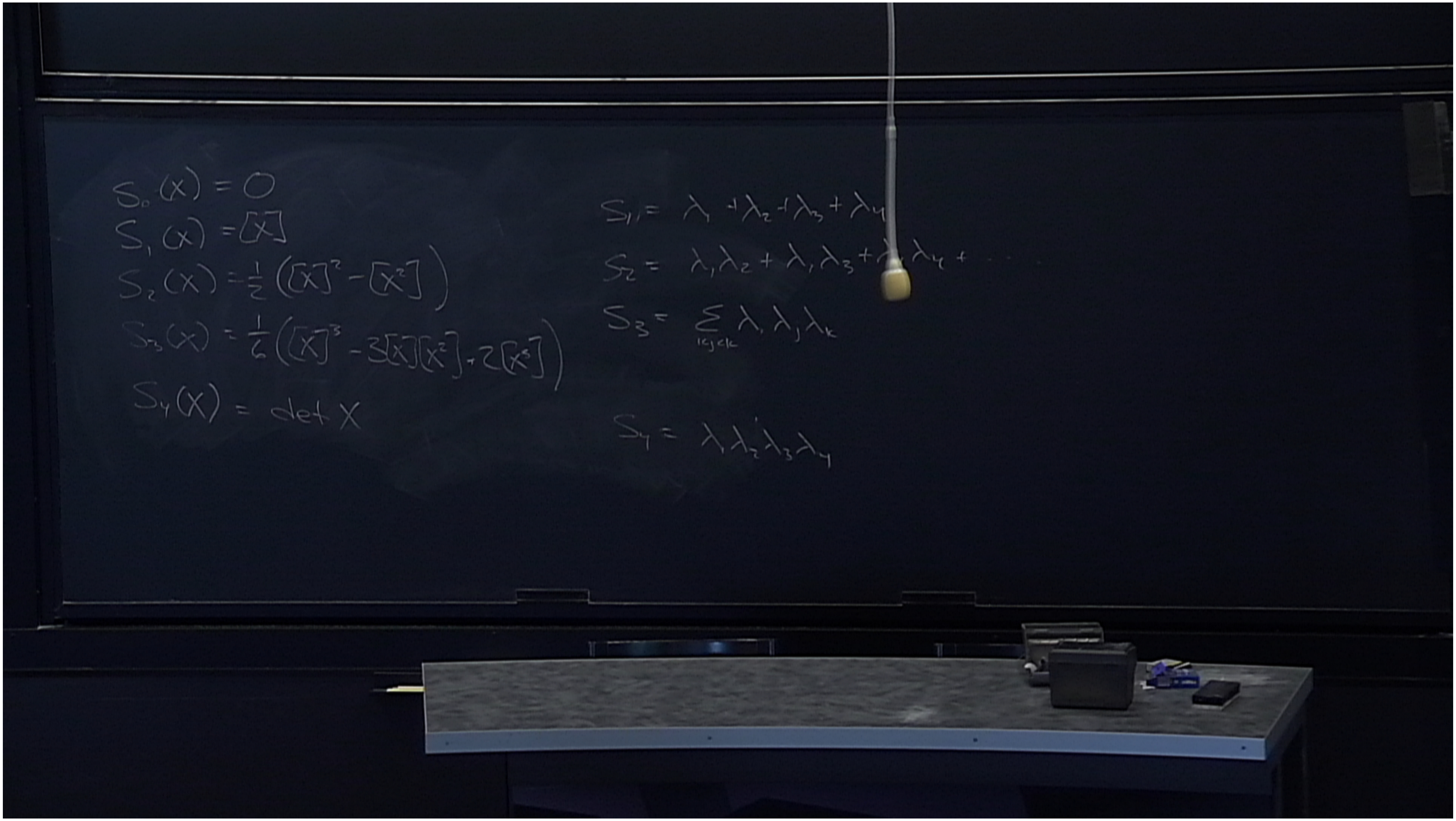

suppose
Y eom, 3 n^i
Hamiltonian constraint
dRG 2010
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"

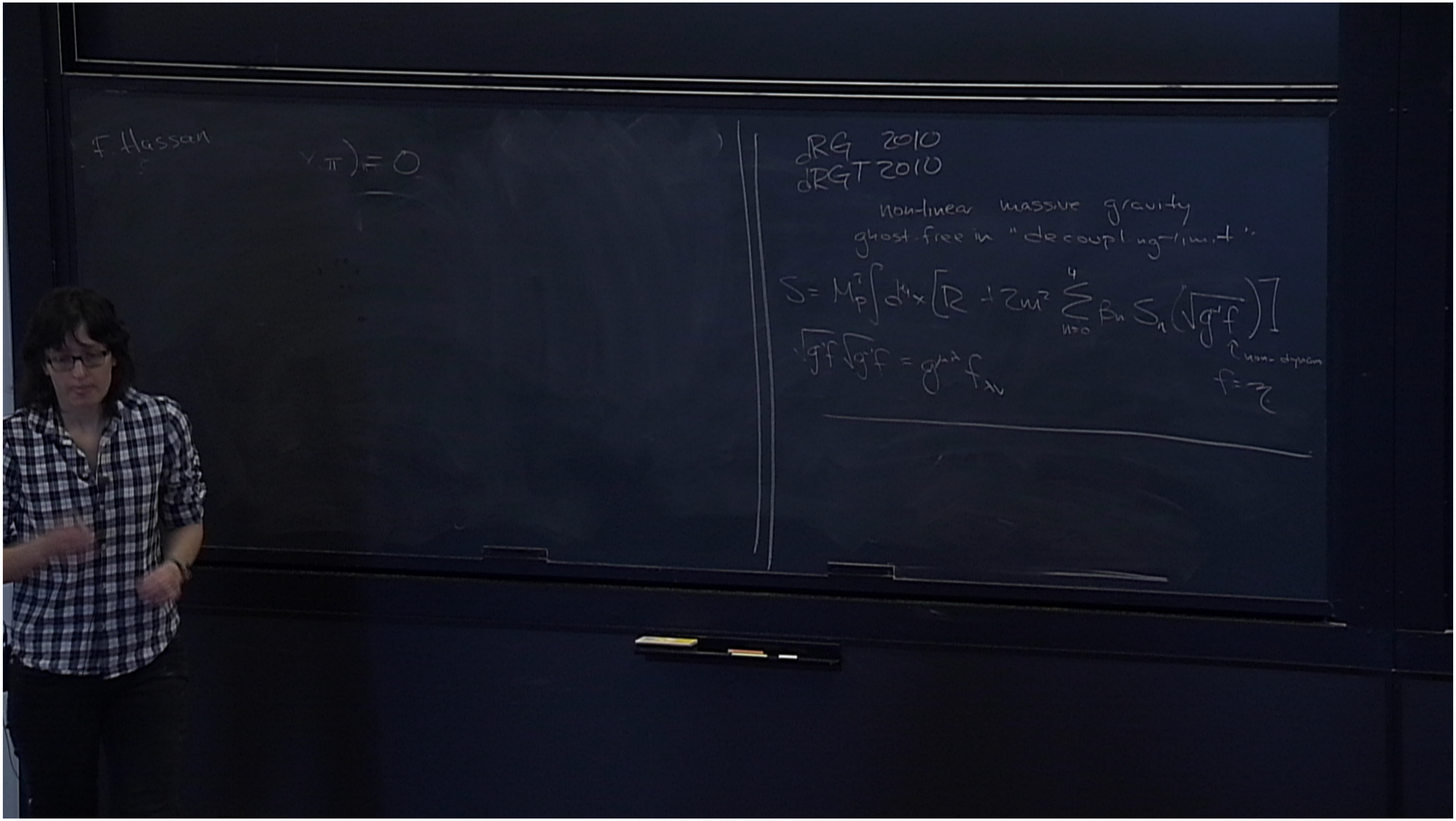

suppose
δℒ/δN = E(n^i, γ, π) = 0
n^i = (N, N^i, γ, π)
4 eom, 3 n^i
δℒ/δN^i = E_i(n^i, γ, π) = 0
⇒ n^i = n^i(γ, π)
⇔ Hamiltonian constraint
dRG 2010
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"
S = M_P^2 ∫ d^4x [R +

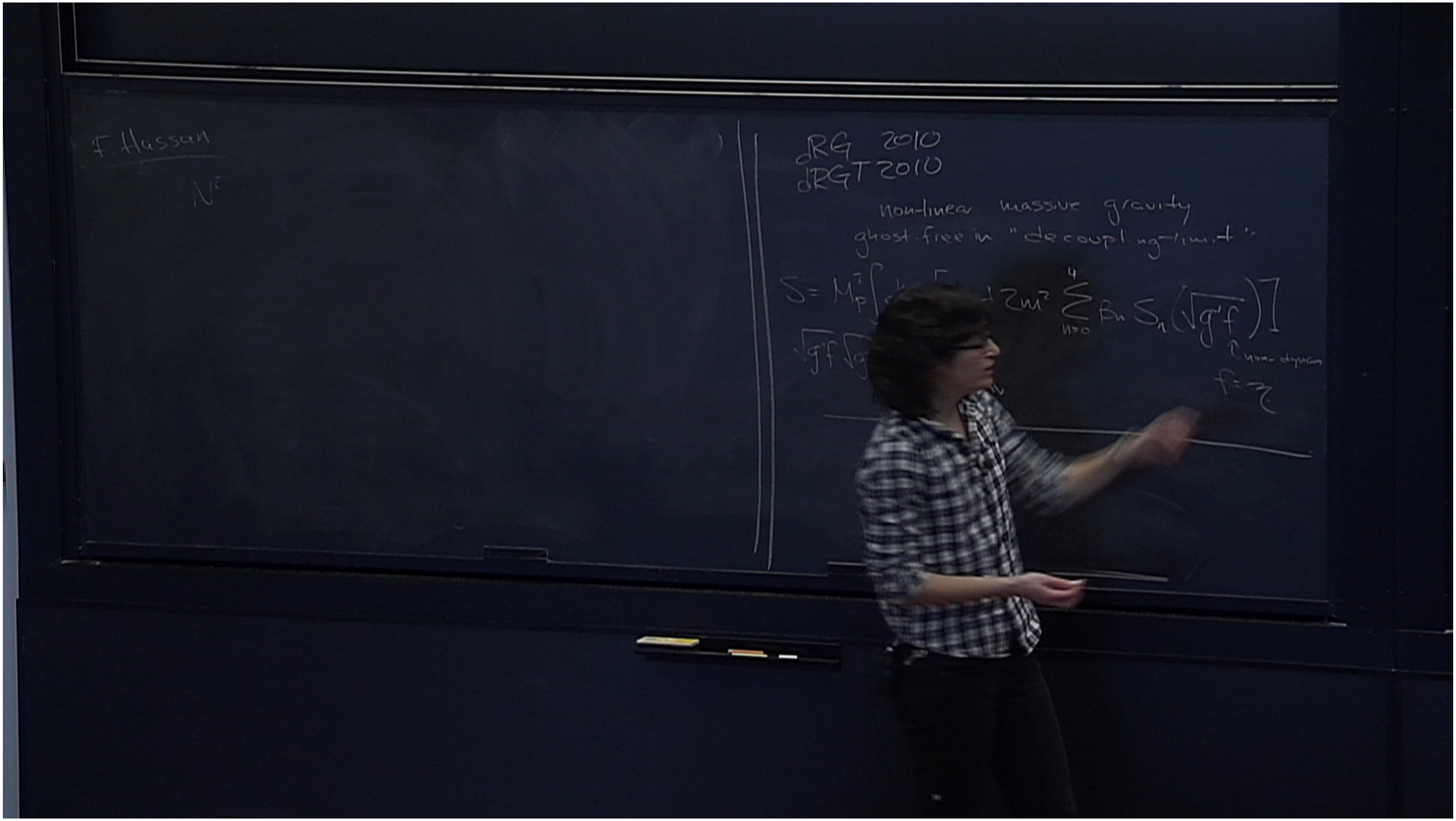

suppose
δL/δN = E(n, γ, π) = 0
n = (N, N^i, γ, π)
δL/δN^i = E_i(n^i, γ, π) = 0
4 eom, 3 n^i
Hamiltonian constraint
→ n^i = n^i(γ, π)
dRG 2010
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"
S = M_p^2 ∫ d^4x [R + 2m^2 Σ_{n=0}^{4} β_n S_n(√(g^{-1}f))]
non-dynam
√(g^{-1}f) √(g^{-1}f) = g^{μλ} f_{λν}


$S_0(X) = 0$
$S_1(X) = [X]$
$S_2(X) = \frac{1}{2}([X]^2 - [X^2])$
$S_3(X) = \frac{1}{6}([X]^3 - 3[X][X^2] + 2[X^3])$
$S_4(X) = \det X$


S0(x) = 0
S1(x) = [x]
S2(x) = 1/2 ([x]^2 − [x^2])
S4(x) = det


S0(X) = 0
S1(X) = [X]
S2(X) = 1/2 ([X]^2 − [X^2])
S3(X) = 1/6 ([X]^3 − 3[X][X^2] + 2[X^3])
S4(X) = det X
S1 = λ1 + λ2 + λ3 + λ4
S2 = λ1λ2 + λ1λ3 + λ1λ4 + . . .
S3 = Σ_{i<j<k} λi λj λk
S4 = λ1λ2λ3λ4


F. Hassan
dRG 2010
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"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dRG 2010
dRGT 2010
non-linear massive gravity
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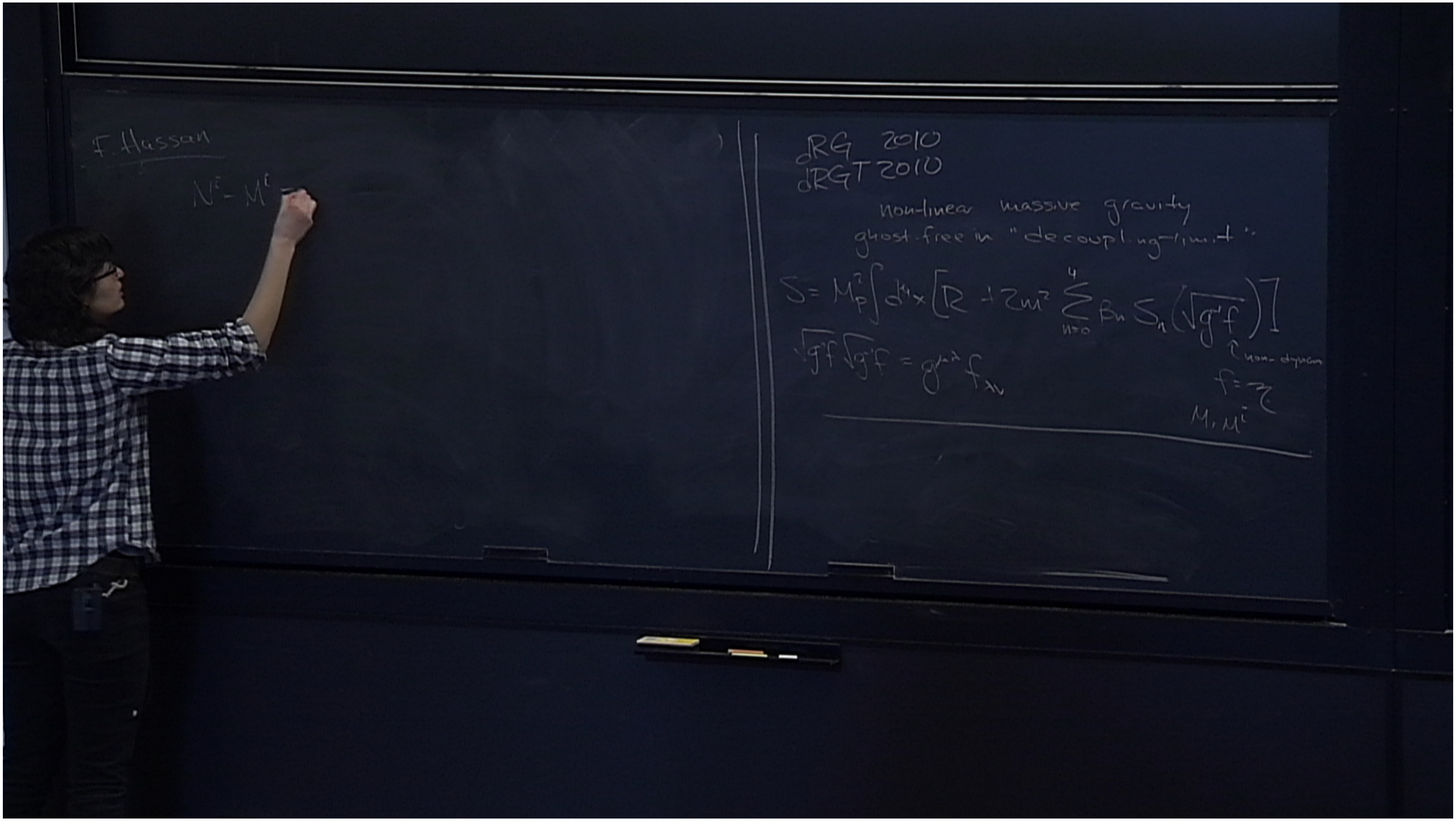
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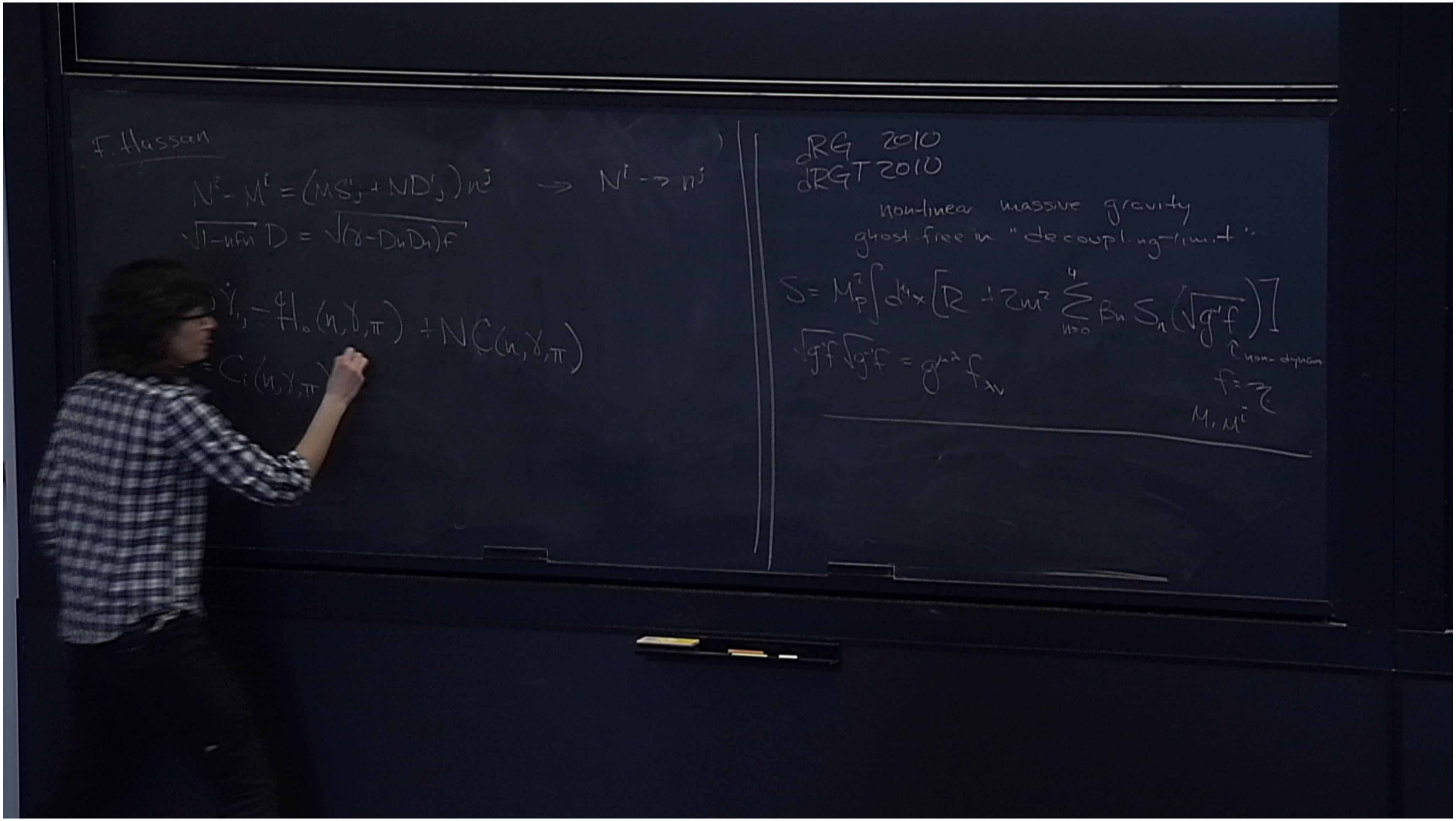
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"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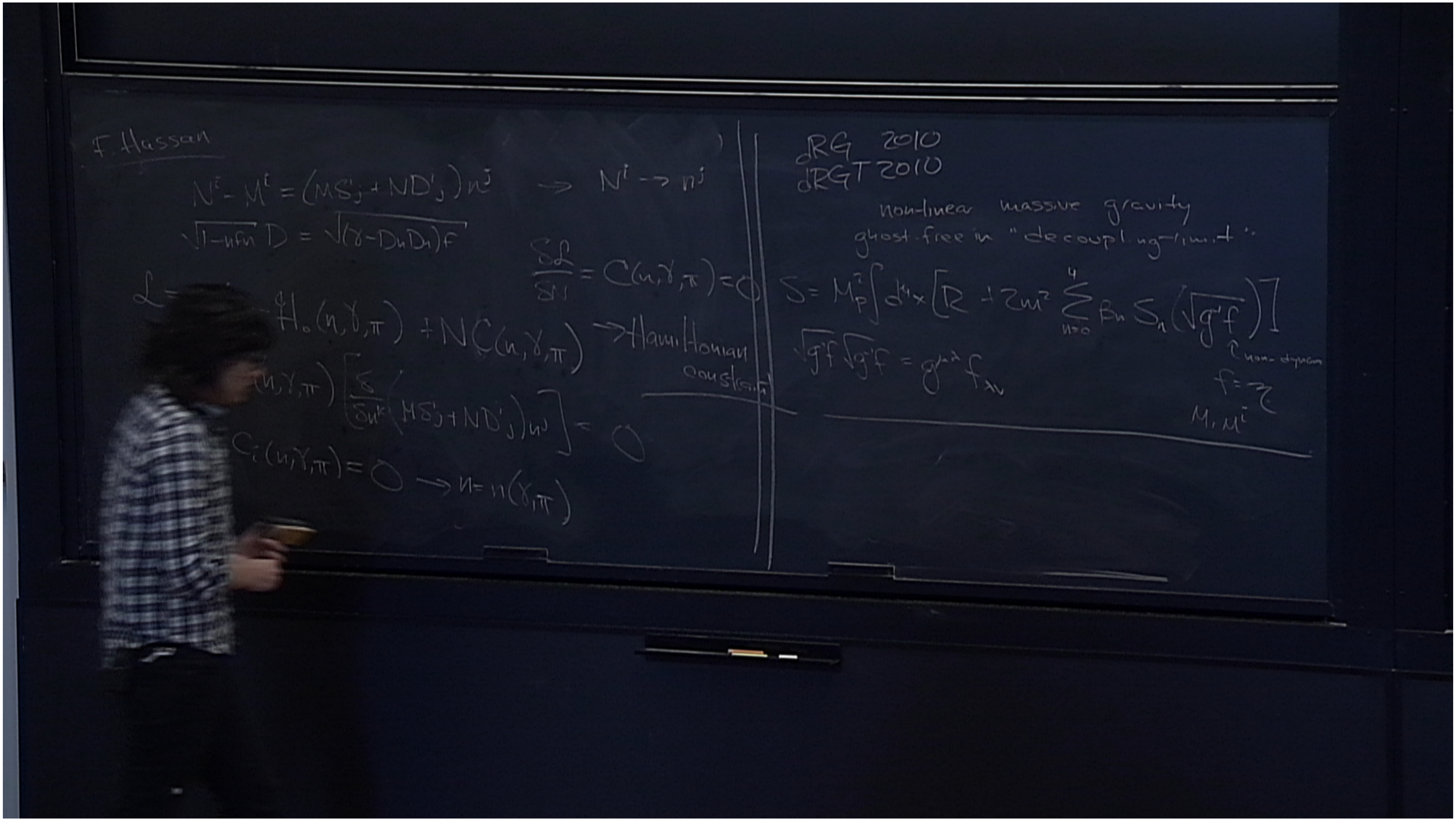
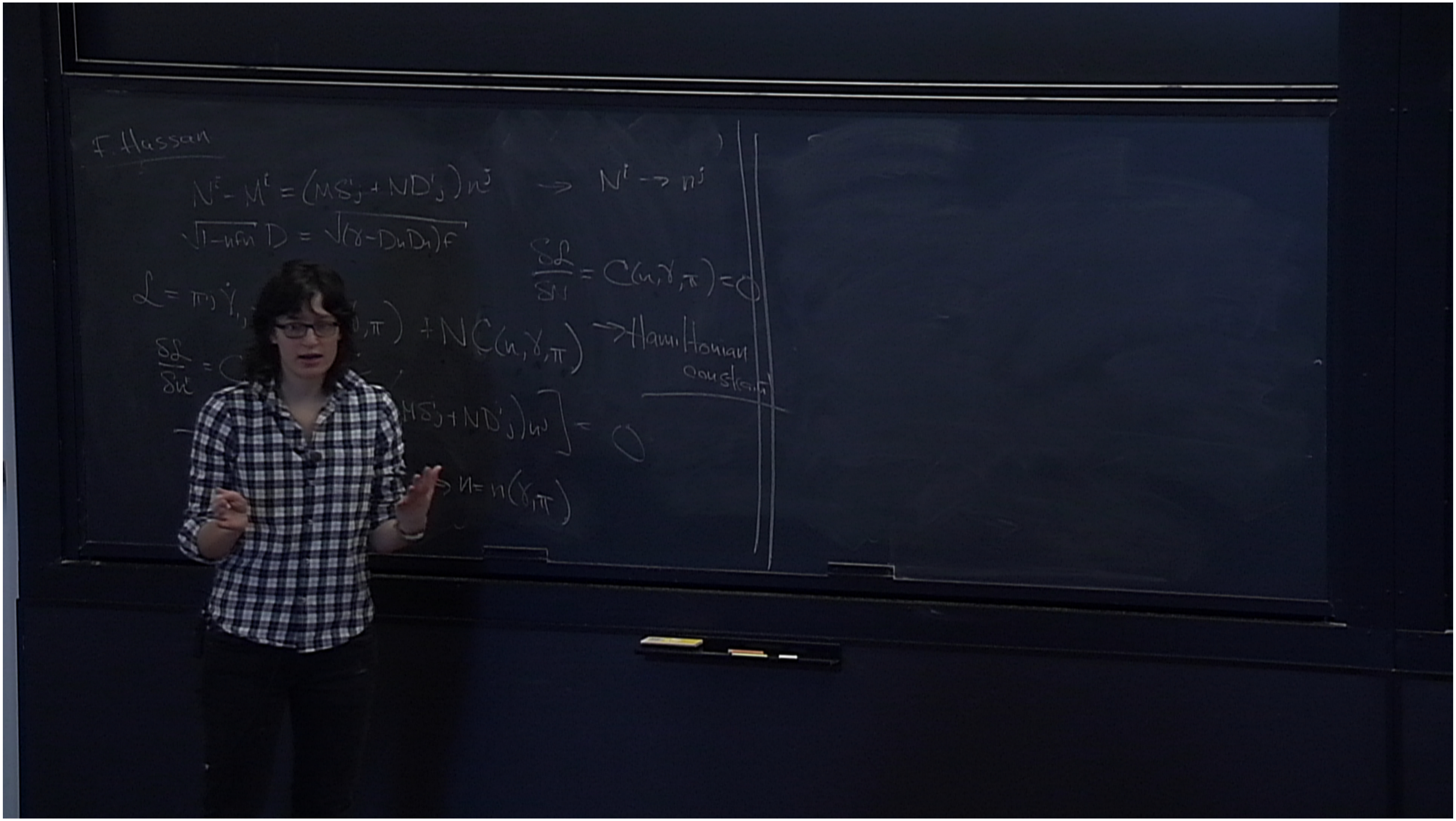
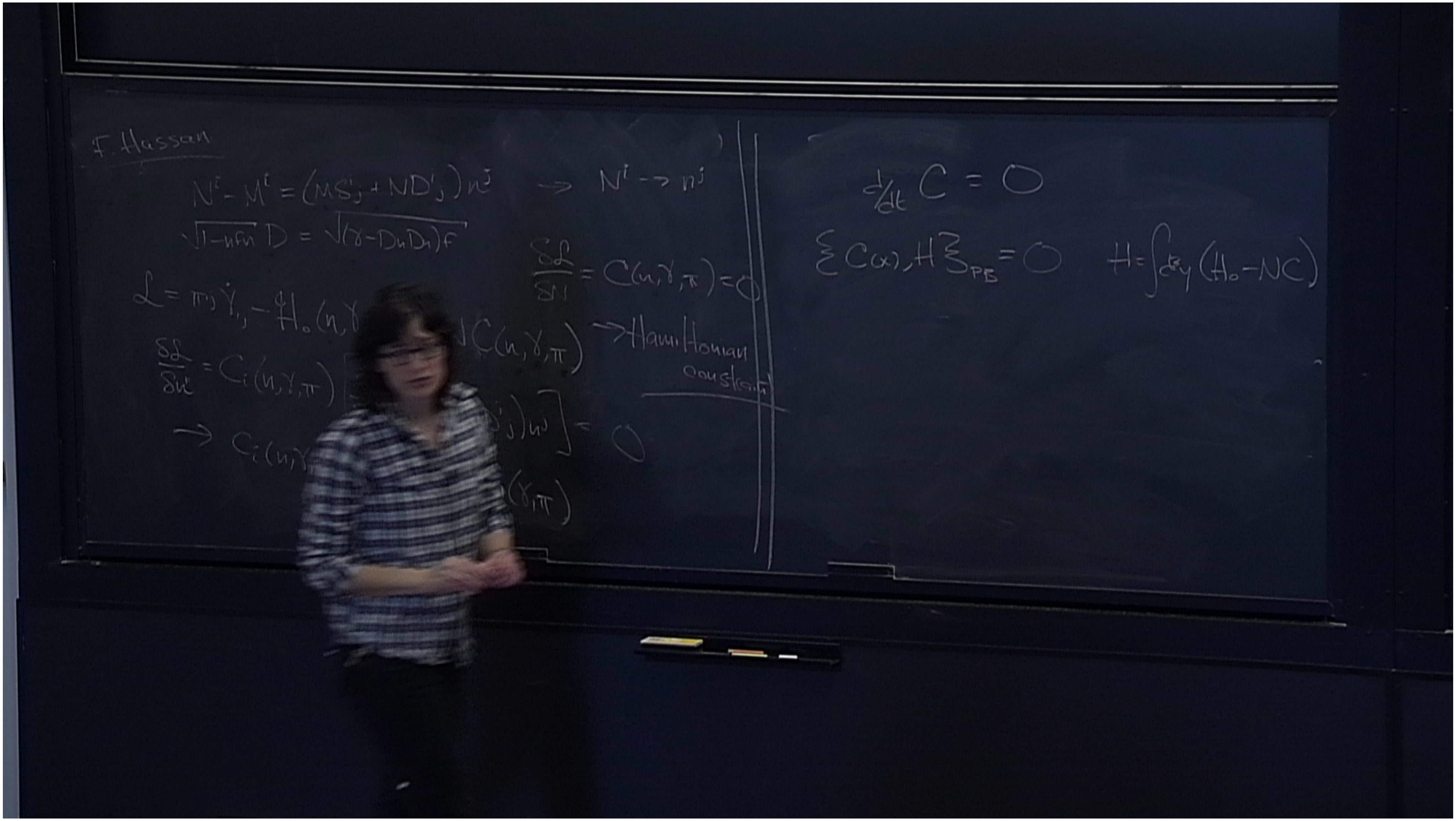
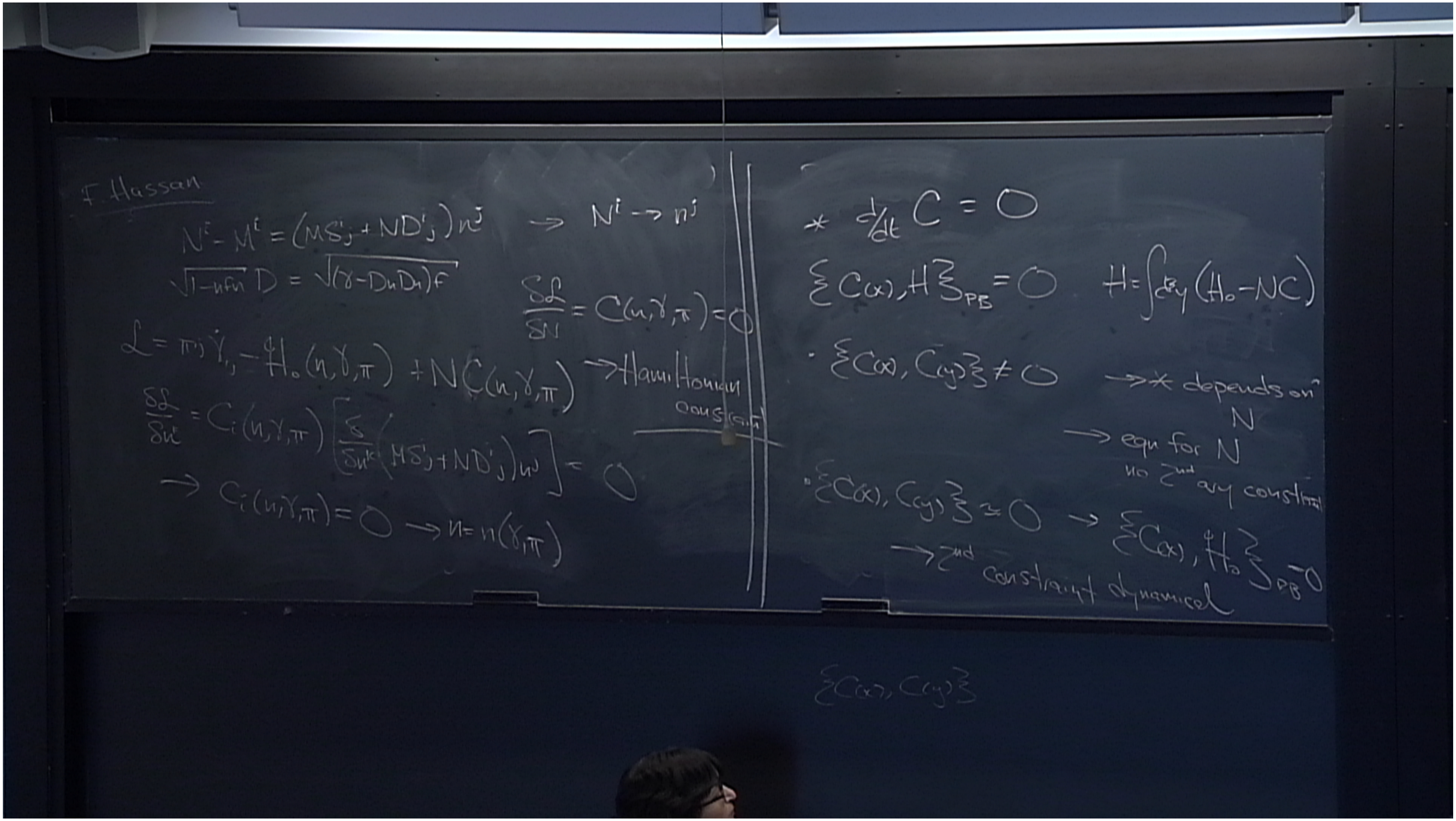
dRG 2010
dRGT 2010
non-linear massive gravity
ghost free in "decoupling limit"

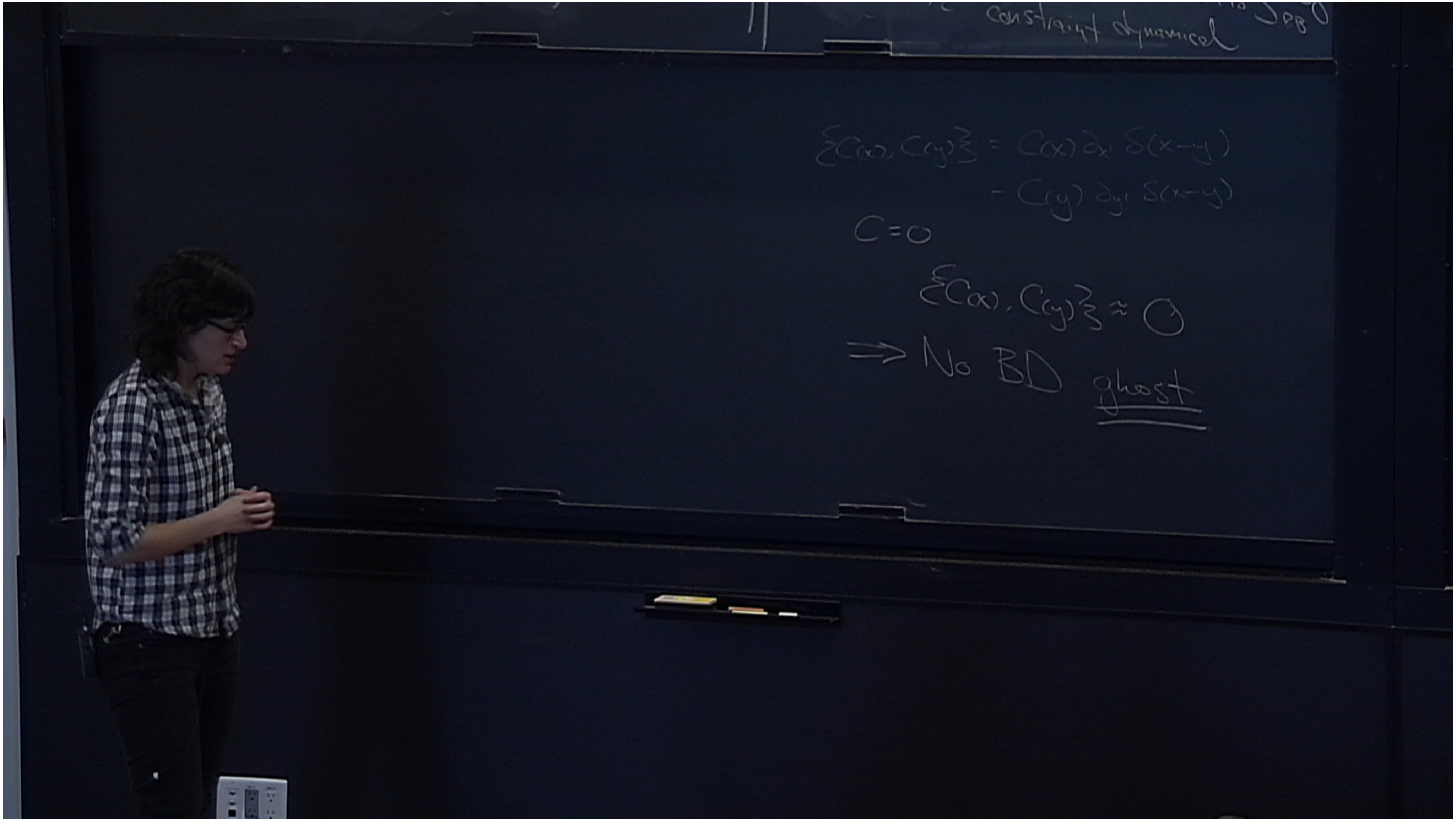


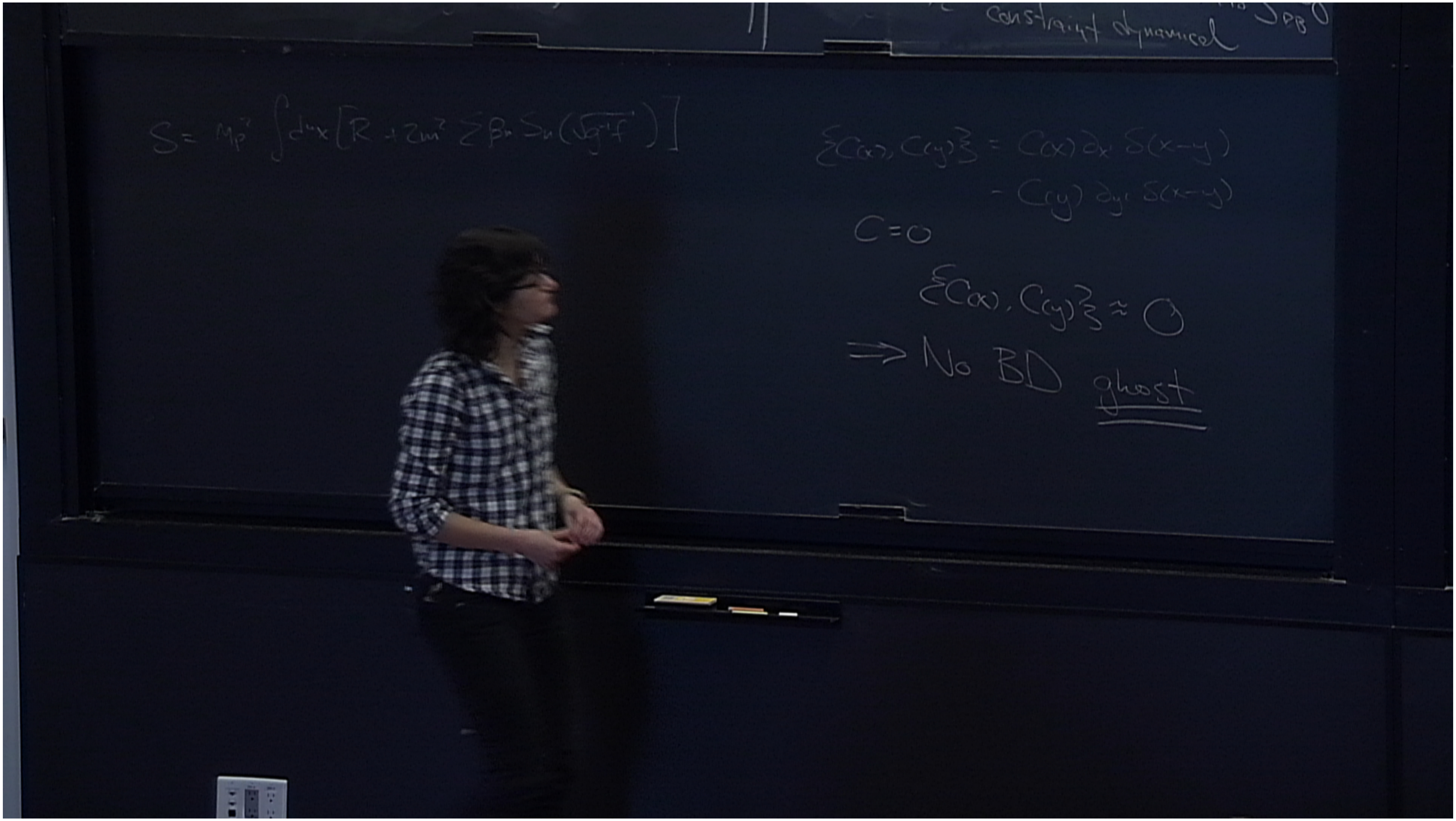
F. Hassan
dRG 2010
dRGT 2010
non-linear massive gravity
ghost-free in "decoupling-limit"
Hamiltonian
constraint

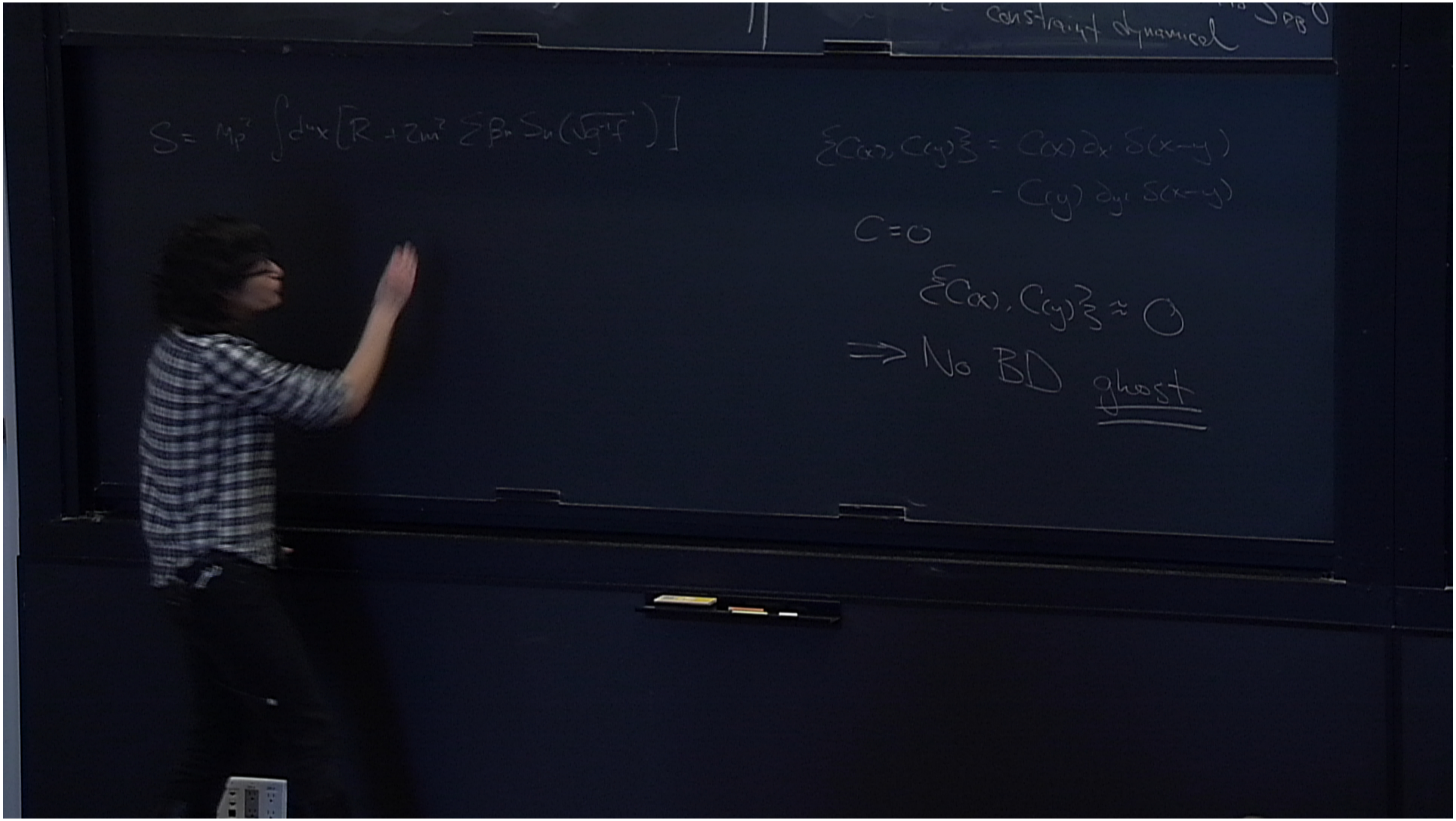


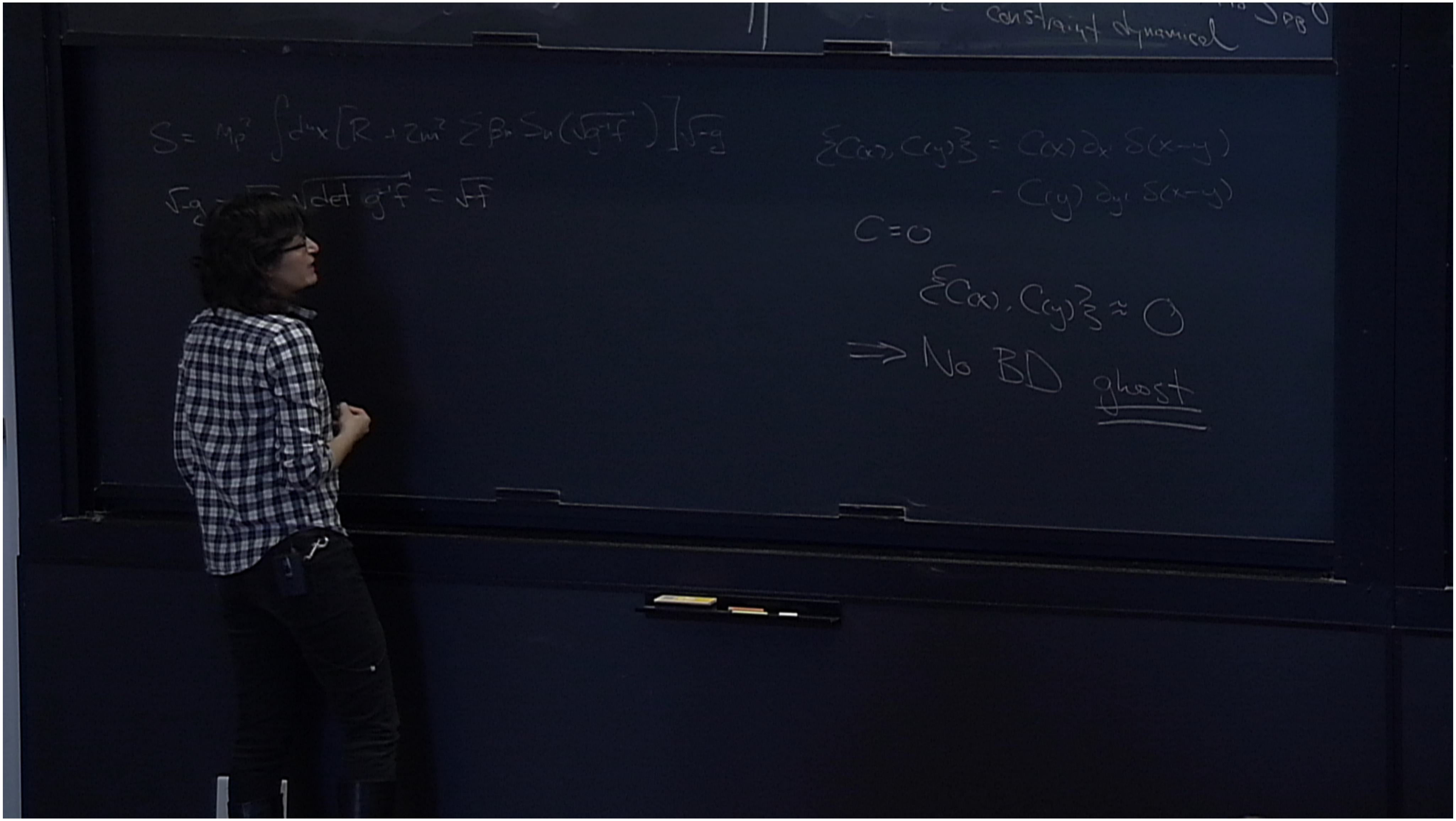

F. Hassan
→ Hamiltonian constraint
d/dt C = 0
{C(x), H}PB = 0
H = ∫d³y (H₀ − NC)

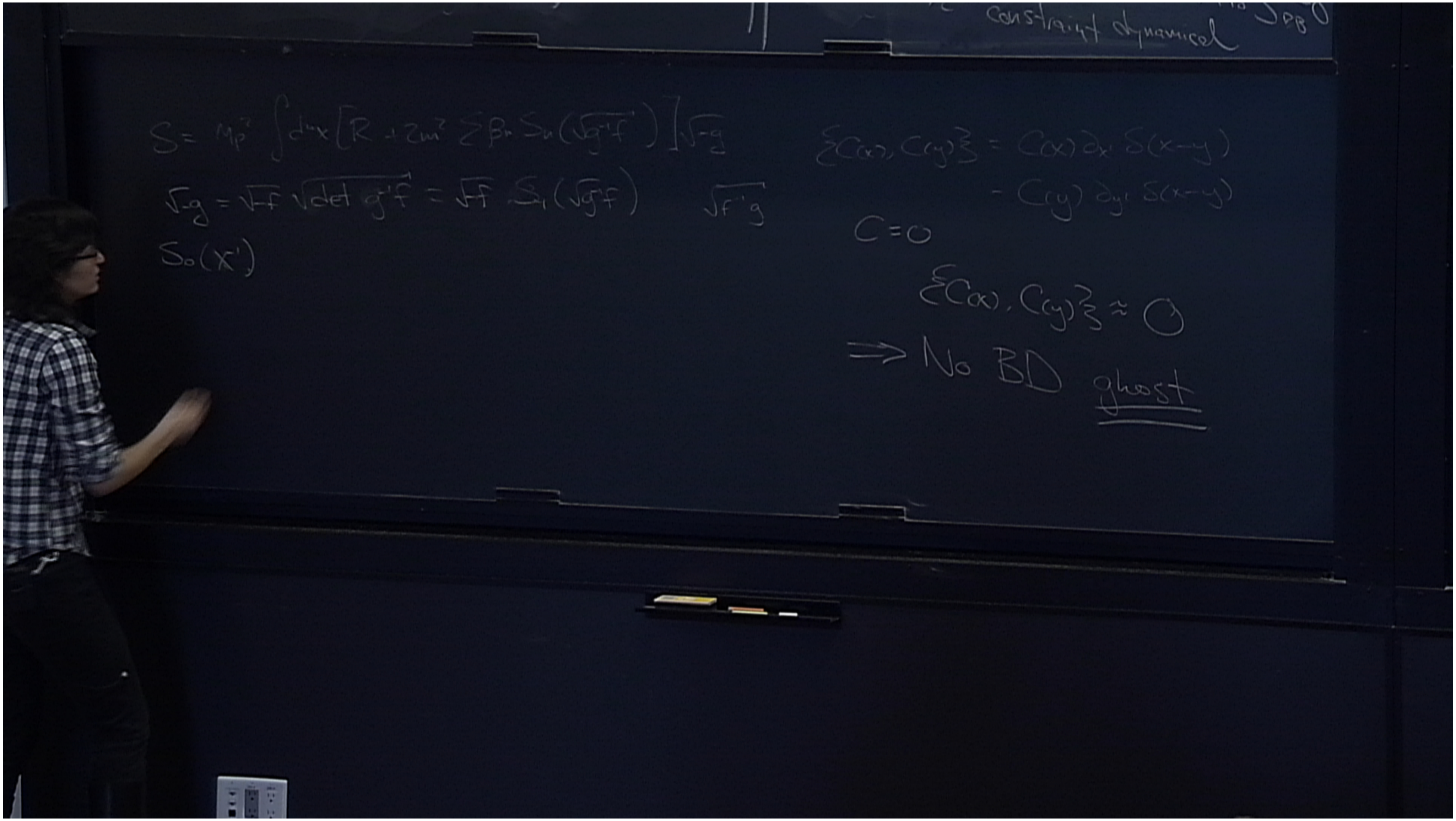

F. Hassan
N^i - M^i = (M δ^i_j + N D^i_j) n^j → N^i → n^j
√(1 - n f n) D = √((γ - D n D n) f)
δL/δN = C(n, γ, π) = 0
L = π^{ij} γ̇_{ij} - H_0(n, γ, π) + N C(n, γ, π) → Hamiltonian constraint
δL/δn^k = C_i(n, γ, π) [δ/δn^k (M δ^i_j + N D^i_j) n^j] = 0
→ C_i(n, γ, π) = 0 → n = n(γ, π)
* d/dt C = 0
{C(x), H}_PB = 0   H = ∫d³y (H_0 - NC)
· {C(x), C(y)} ≠ 0 → * depends on N
→ eqn for N
no 2nd any constraint
· {C(x), C(y)} ≈ 0 → {C(x), H_0}_PB ≈ 0
→ 2nd constraint dynamical

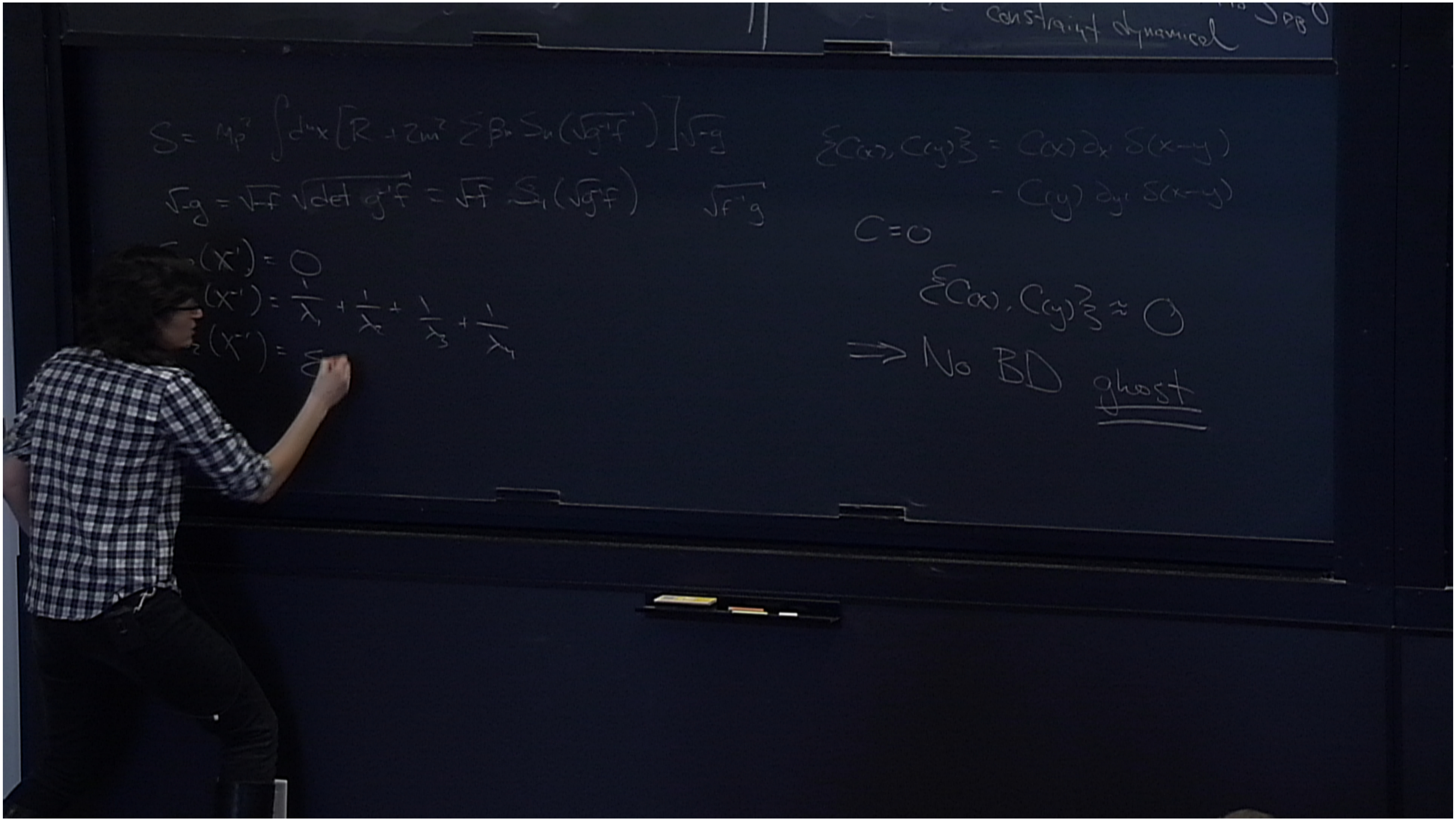

constraint dynamical
{C(x), C(y)} = C(x) ∂x δ(x−y)
− C(y) ∂y δ(x−y)
C=0
{C(x), C(y)} ≈ 0
⇒ No BD ghost

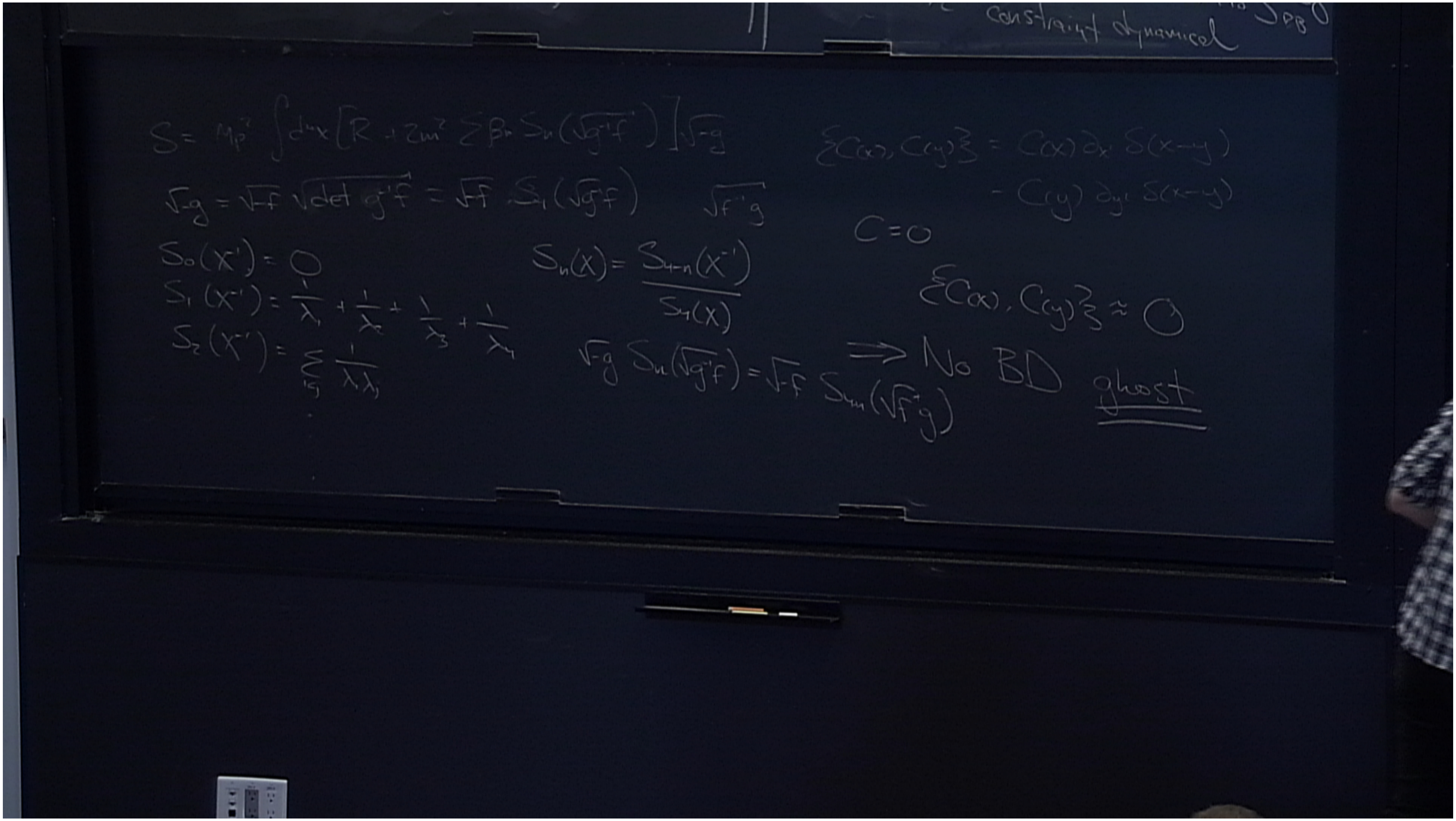

constraint dynamical
S = Mp² ∫d⁴x [R + 2m² Σ βn Sn(√(g⁻¹f))]
{C(x), C(y)} = C(x) ∂x δ(x−y)
− C(y) ∂y δ(x−y)
C = 0
{C(x), C(y)} ≈ 0
⇒ No BD ghost

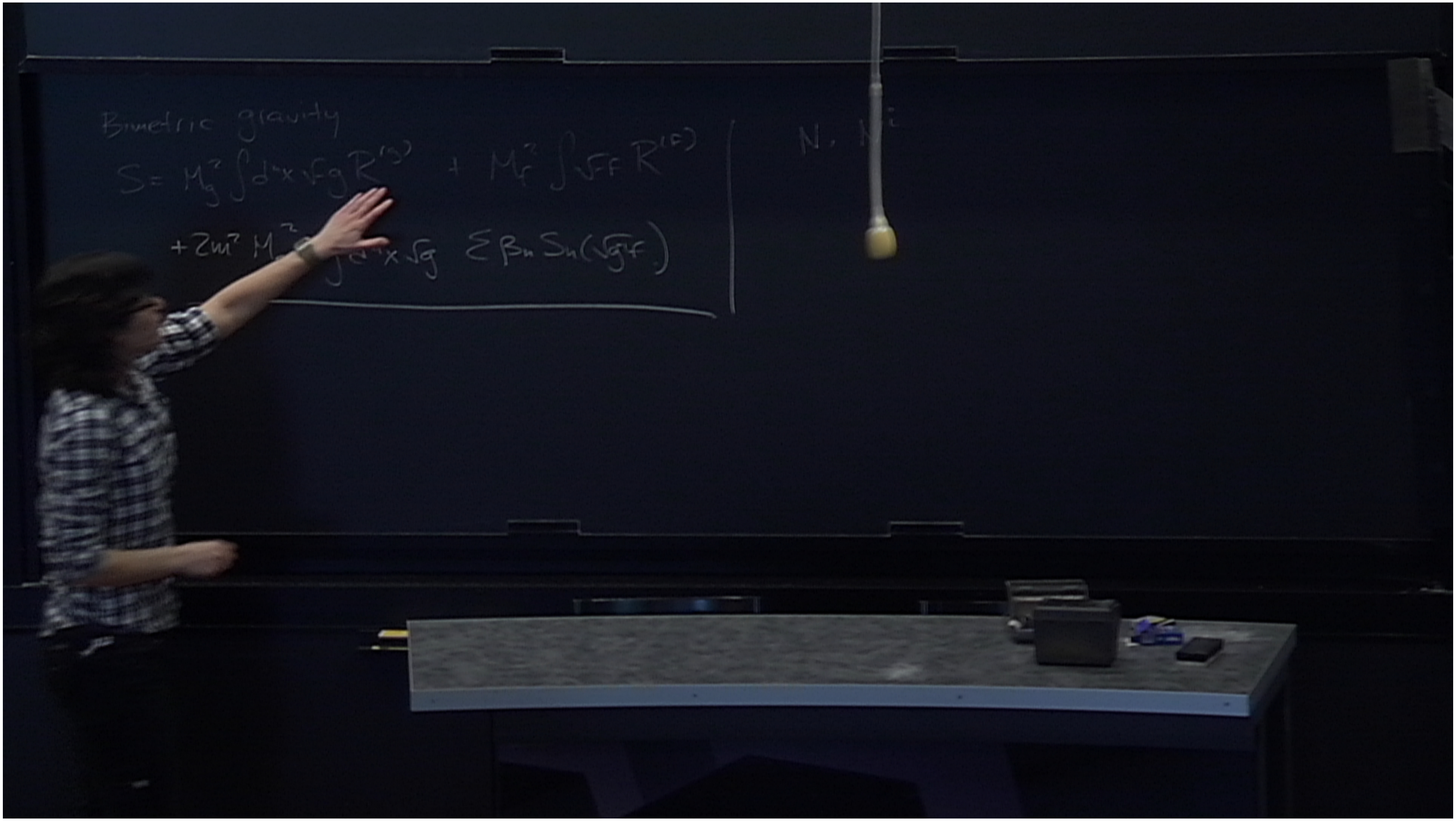

constraint dynamical
S = Mp² ∫d⁴x [R + 2m² Σ βn Sn(√g⁻¹f)]
{C(x), C(y)} = C(x) ∂x δ(x−y)
− C(y) ∂y δ(x−y)
C = 0
{C(x), C(y)} ≈ 0
⇒ No BD ghost

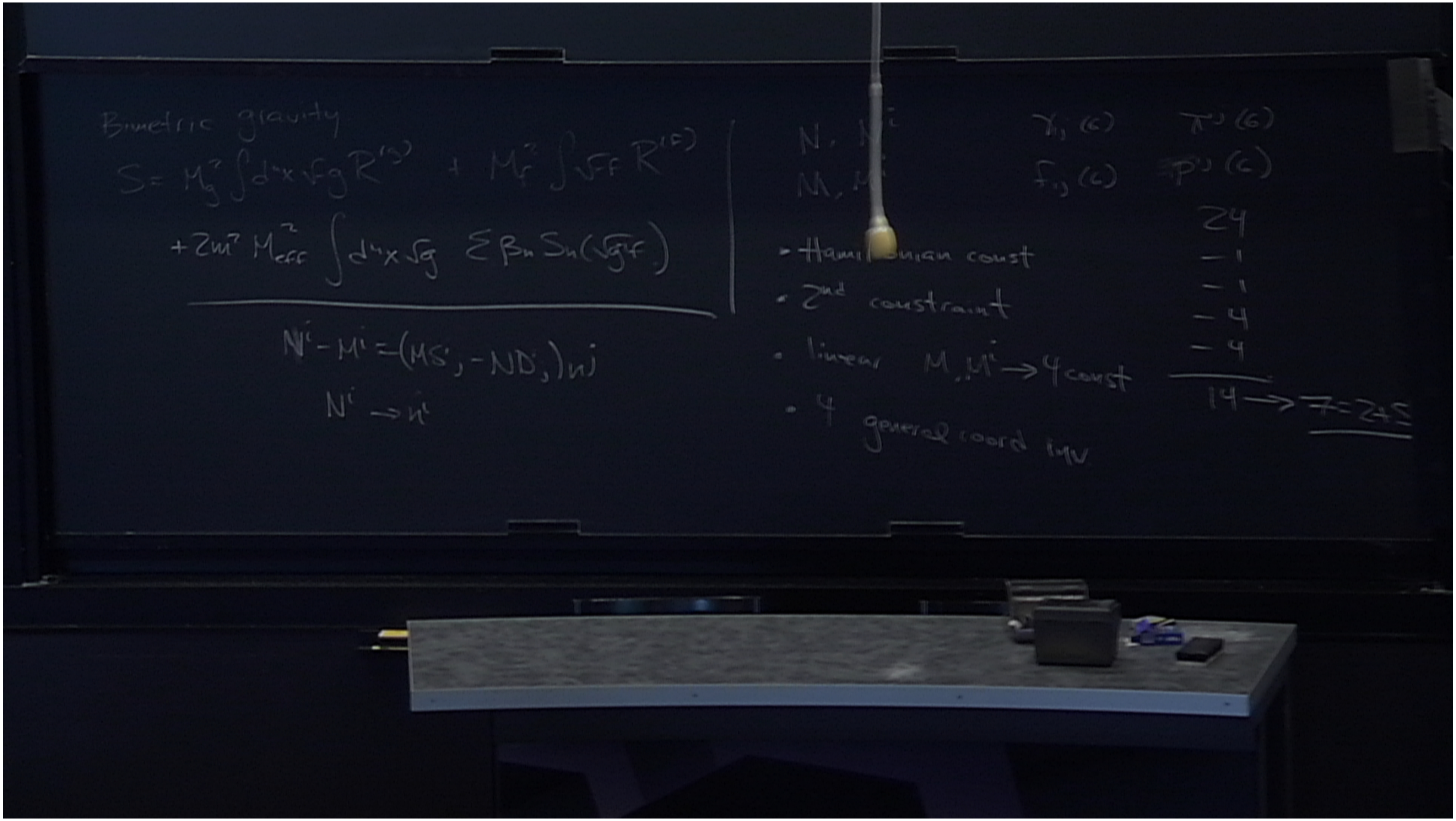

constraint dynamical
S = Mp² ∫d⁴x [R + 2m² Σ βn Sn(√(g⁻¹f))] √-g
{C(x), C(y)} = C(x) ∂x δ(x−y)
− C(y) ∂y δ(x−y)
C = 0
{C(x), C(y)} ≈ 0
⇒ No BD ghost

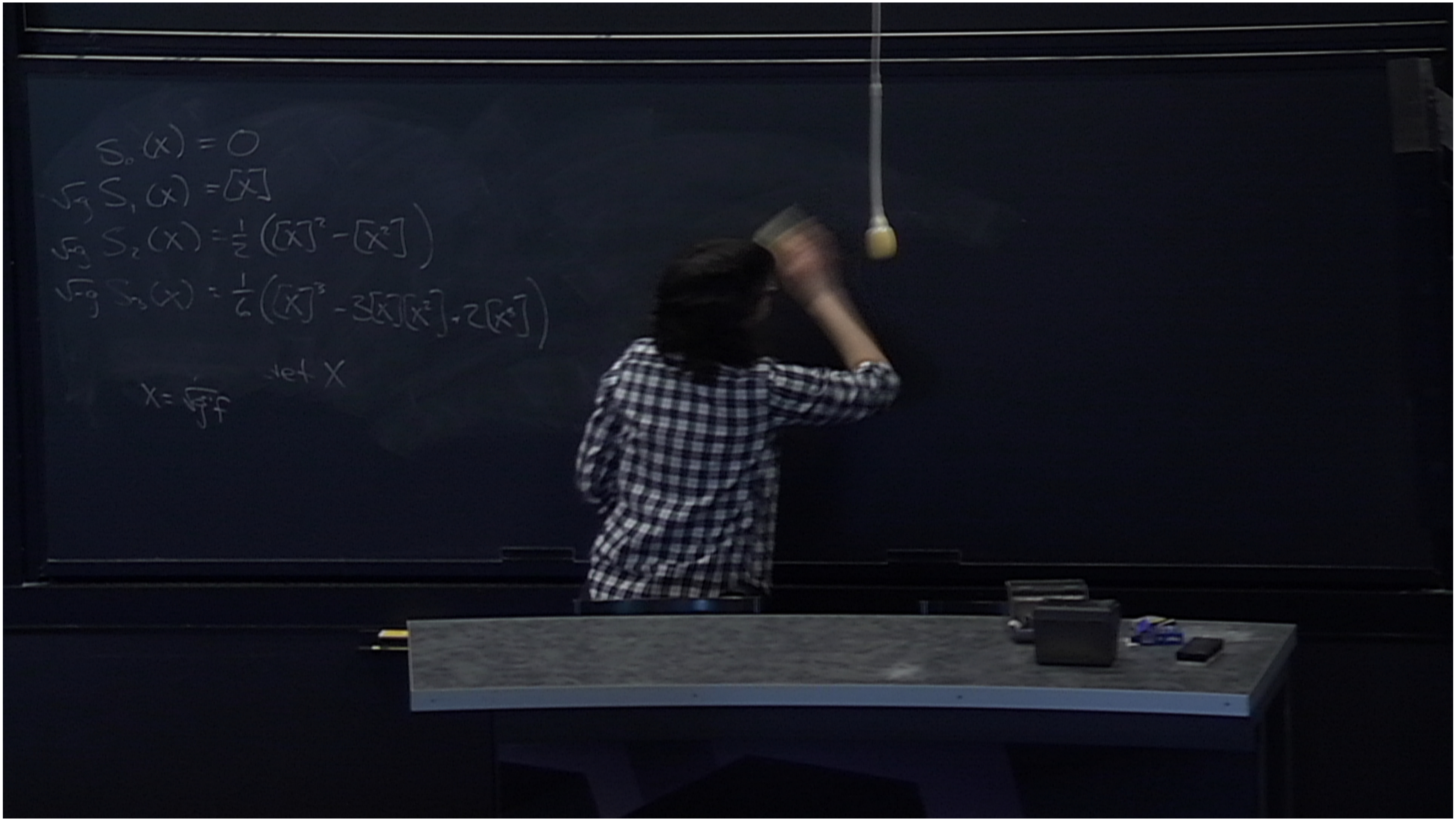

constraint dynamical
$S = M_P^2 \int d^4x \left[R + 2m^2 \sum \beta_n S_n(\sqrt{g^{-1}f}) \right] \sqrt{-g}$
$\sqrt{-g} = \sqrt{-f} \sqrt{\det g^{-1} f} = \sqrt{-f}\, S_4(\sqrt{g^{-1} f})$
$\sqrt{f^{-1} g}$
$S_0(X')$
$\{C(x), C(y)\} = C(x) \partial_x \delta(x-y)$
$- C(y) \partial_y \delta(x-y)$
$C = 0$
$\{C(x), C(y)\} \approx 0$
$\Rightarrow$ No BD ghost

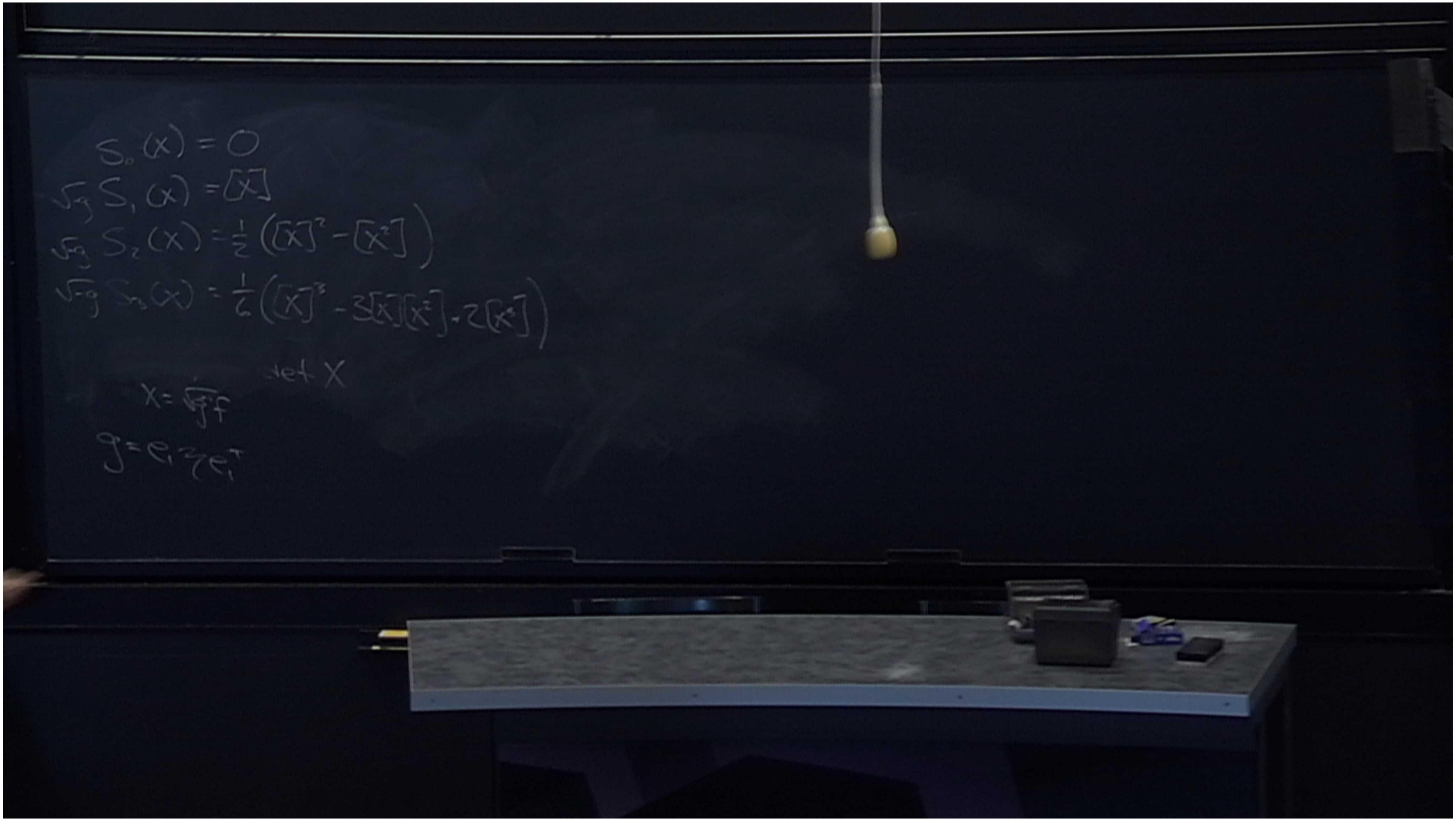

constraint dynamical
C=0
⇒ No BD ghost

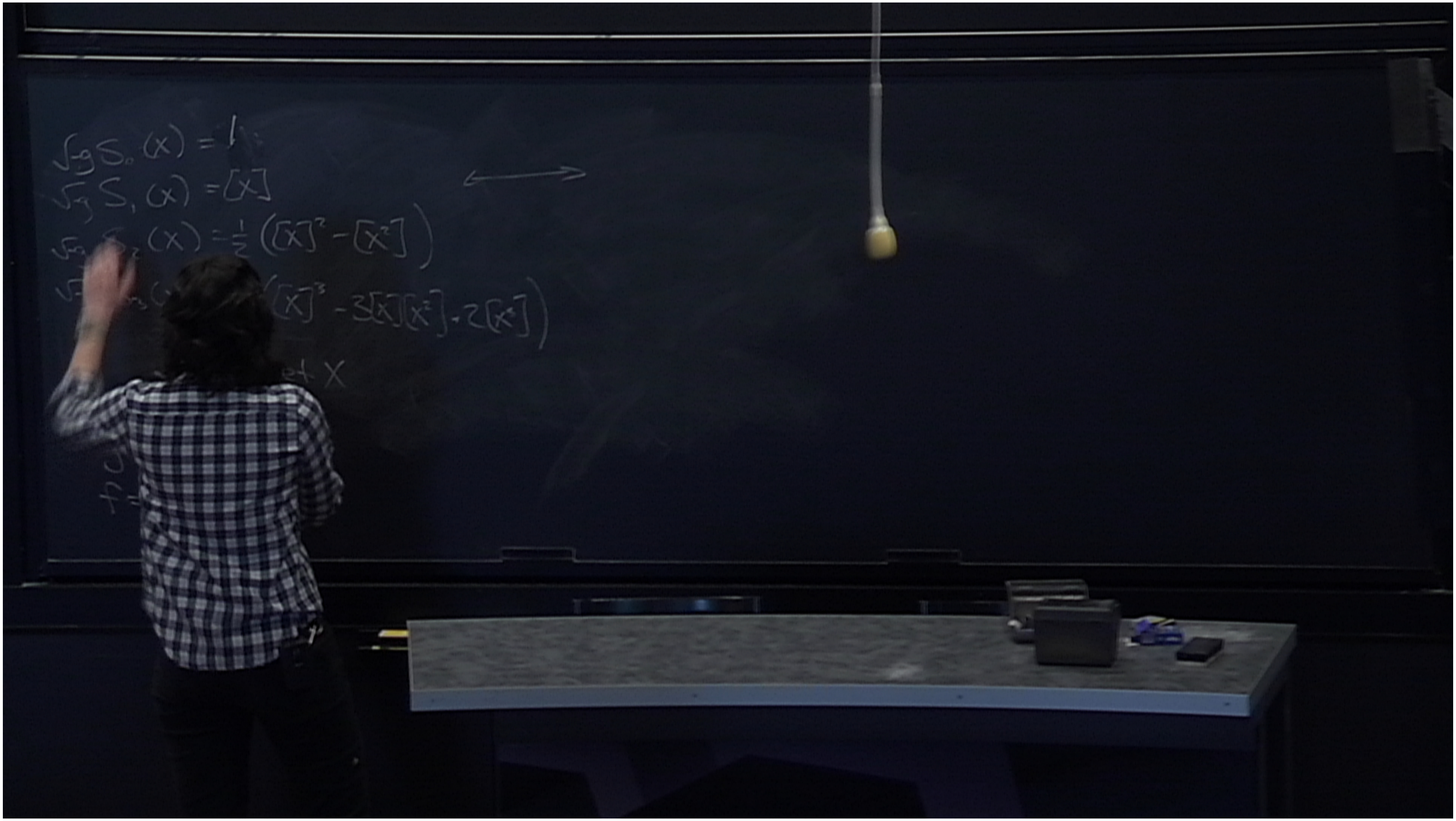

constraint dynamical
S = Mp² ∫d⁴x [R + 2m² Σ βn Sn(√(g⁻¹f))] √-g
{C(x), C(y)} = C(x) ∂x δ(x−y)
− C(y) ∂y δ(x−y)
√-g = √-f √det g⁻¹f = √-f S4(√g⁻¹f)
√f⁻¹g
C = 0
S0(X⁻¹) = 0
Sn(X) = S4-n(X⁻¹) / S4(X)
S1(X⁻¹) = 1/λ1 + 1/λ2 + 1/λ3 + 1/λ4
{C(x), C(y)} ≈ 0
S2(X⁻¹) = Σi<j 1/(λi λj)
√-g Sn(√g⁻¹f) = √-f S4-n(√f⁻¹g)
⟹ No BD ghost

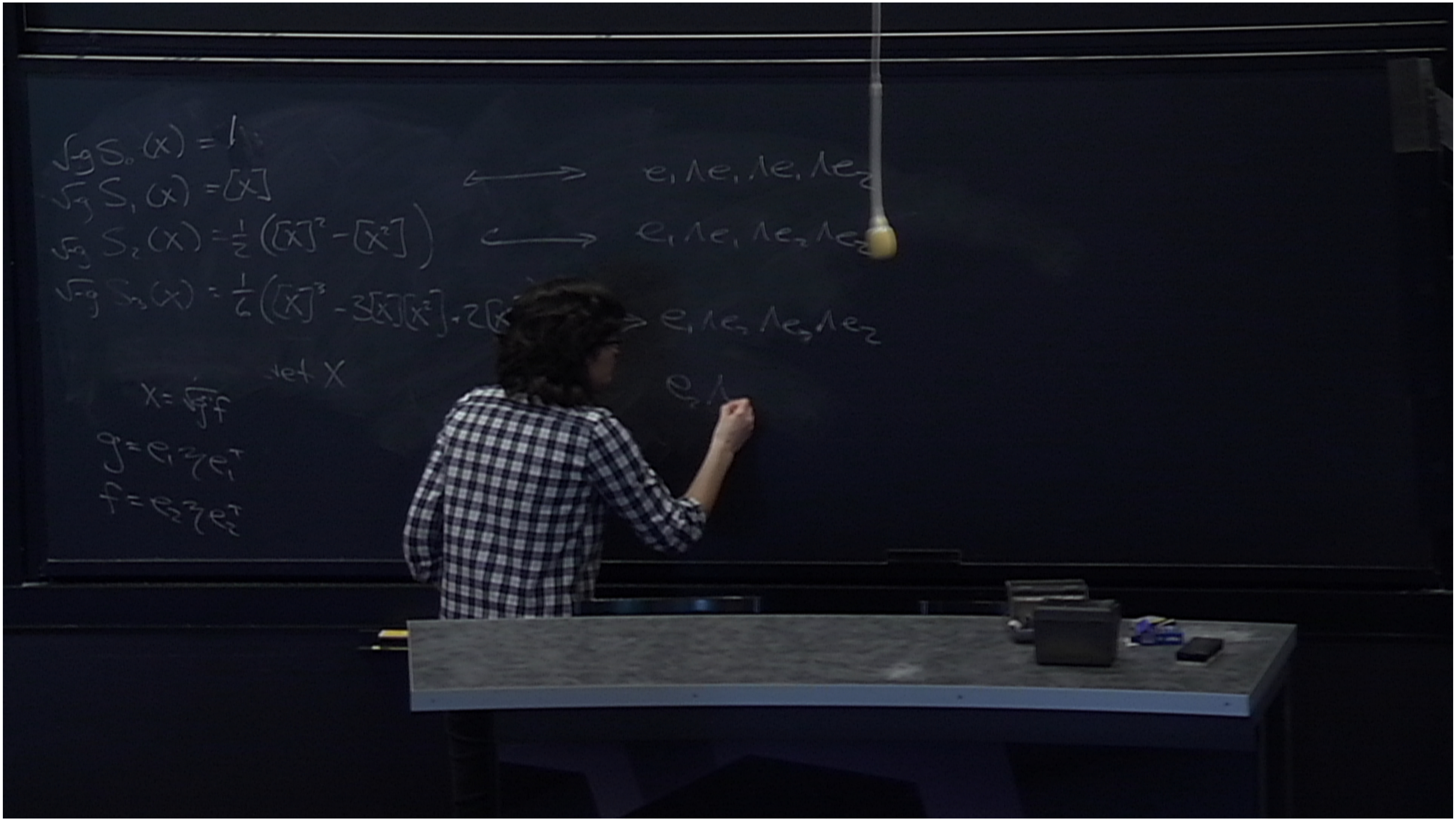

Bimetric gravity

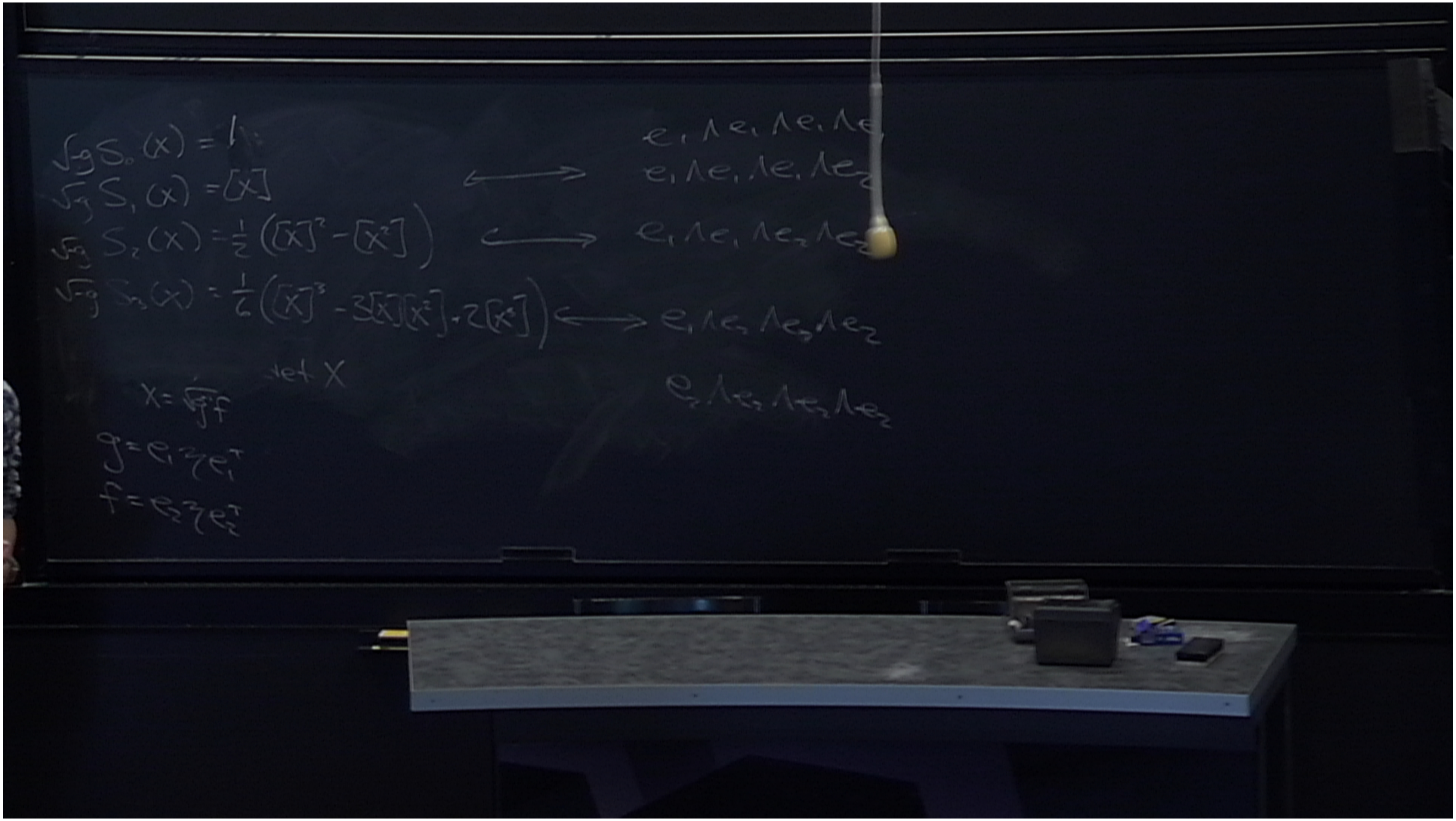

Bimetric gravity
Hamiltonian const
2nd constraint
linear
4 const
general coord inv
24
14


S0(x) = 0
√-g S1(x) = [X]
√-g S2(x) = 1/2 ([X]^2 - [X^2])
√-g S3(x) = 1/6 ([X]^3 - 3[X][X^2] + 2[X^3])
det X
X = √(g^-1 f)

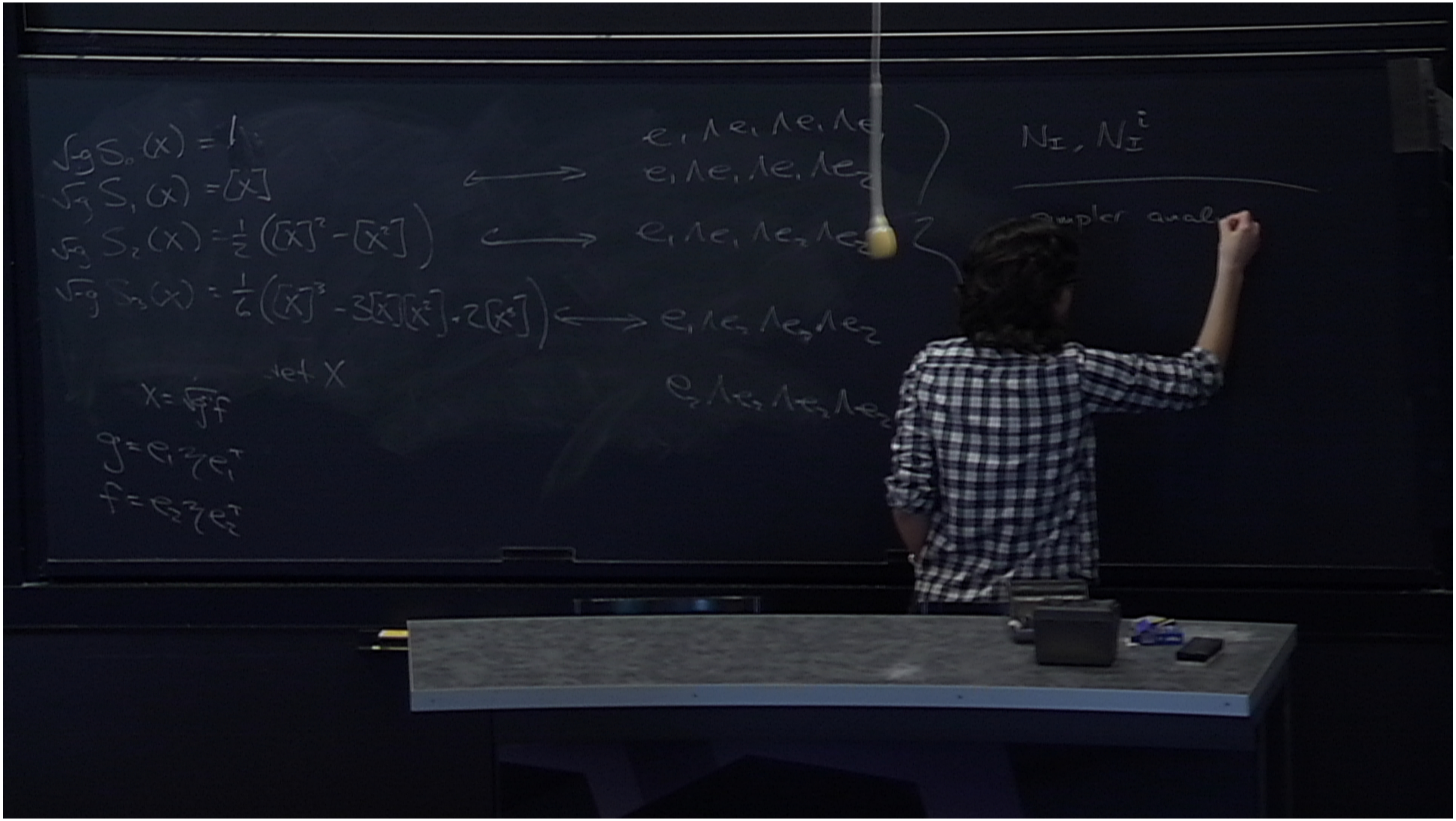

$S_0(X) = 0$
$\sqrt{-g}\, S_1(X) = [X]$
$\sqrt{-g}\, S_2(X) = \frac{1}{2}\left([X]^2 - [X^2]\right)$
$\sqrt{-g}\, S_3(X) = \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right)$
det X
$X = \sqrt{g^{-1}f}$
$g = e_1 \eta e_1^T$

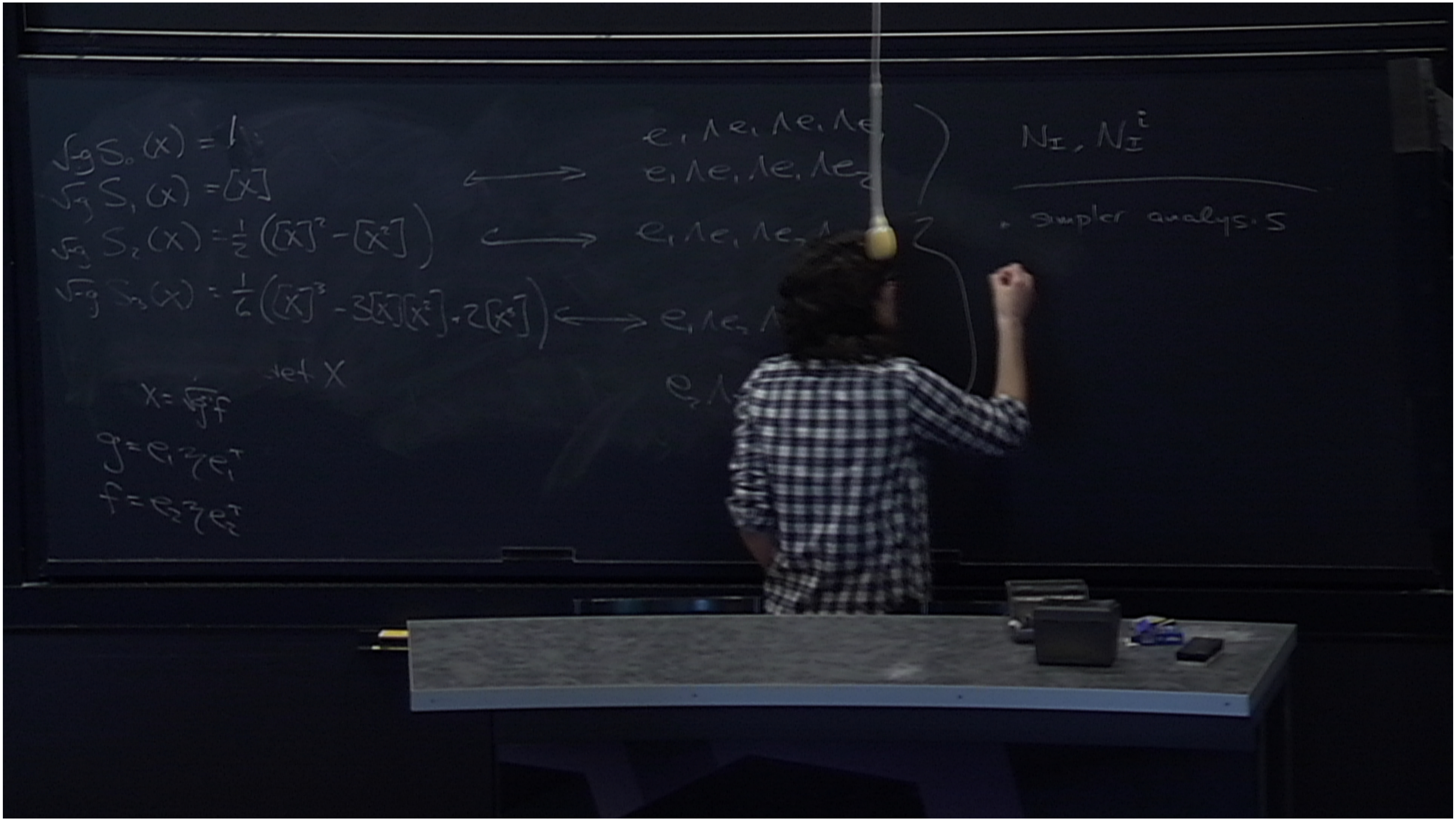

√g S₀ (x) = 1
√g S₁ (x) = [x]
√g S₂ (x) = ½ ([x]² − [x²])
([x]³ − 3[x][x²] + 2[x³])



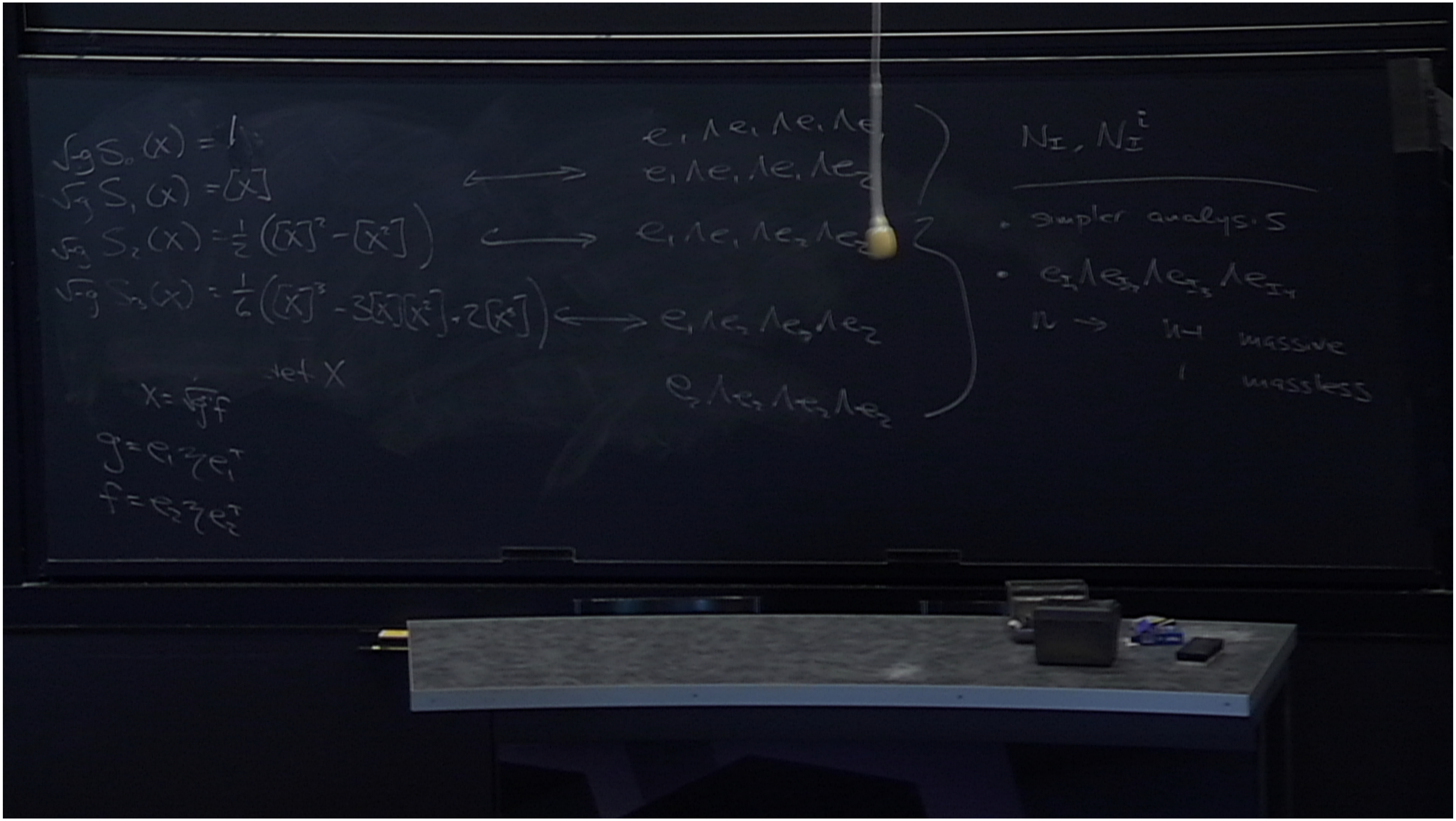


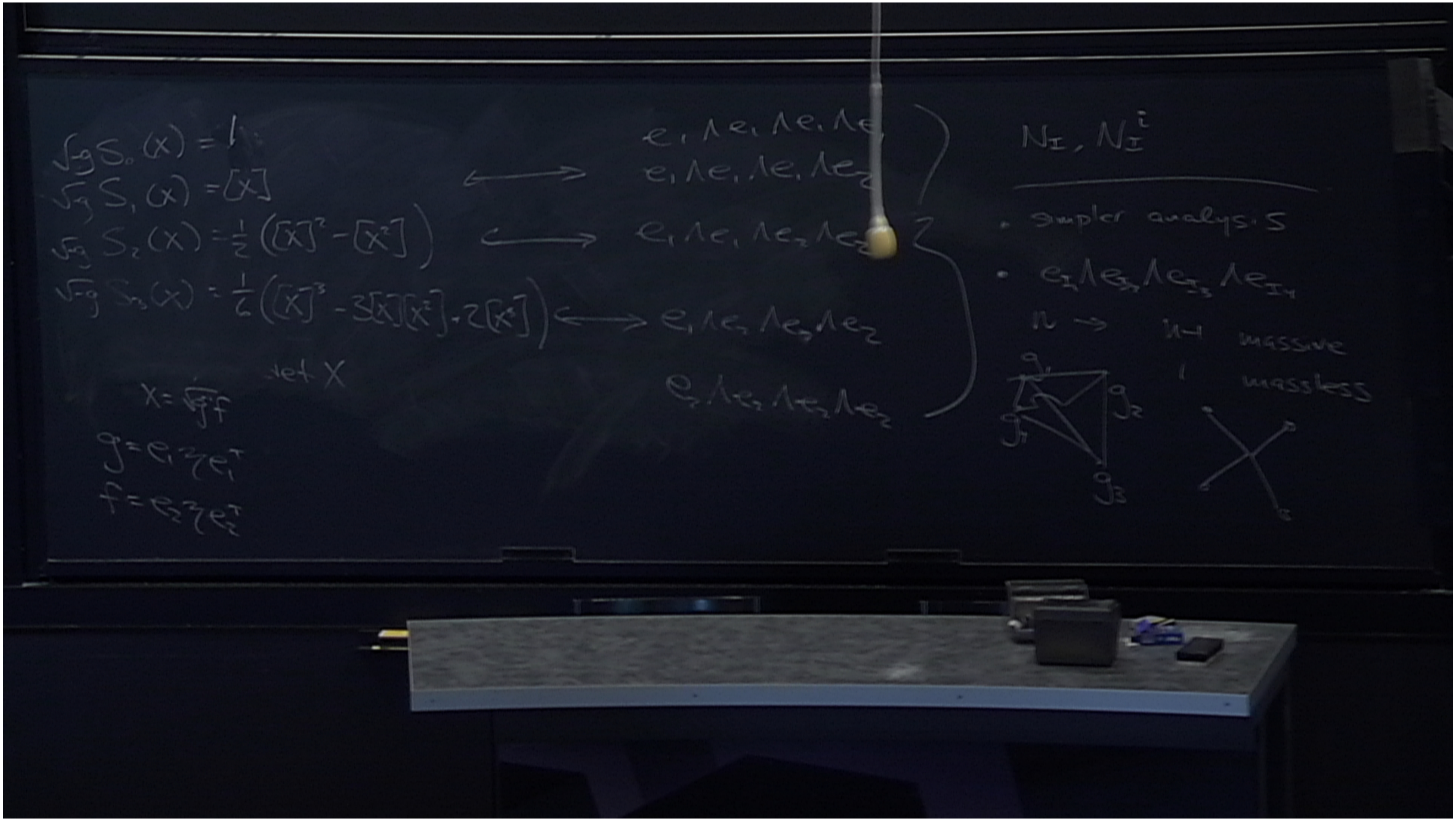
$\sqrt{-g}\, S_0(x) = 1$
$\sqrt{-g}\, S_1(x) = [X]$
$\sqrt{-g}\, S_2(x) = \frac{1}{2}\left([X]^2 - [X^2]\right)$
$\sqrt{-g}\, S_3(x) = \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right)$
det X
$X = \sqrt{g^{-1} f}$
$g = e_1 \eta e_1^T$
$f = e_2 \eta e_2^T$
$e_1 \wedge e_1 \wedge e_1 \wedge e_1$
$e_1 \wedge e_1 \wedge e_1 \wedge e_2$
$e_1 \wedge e_1 \wedge e_2 \wedge e_2$
$e_1 \wedge e_2 \wedge e_2 \wedge e_2$
$e_2 \wedge e_2 \wedge e_2 \wedge e_2$

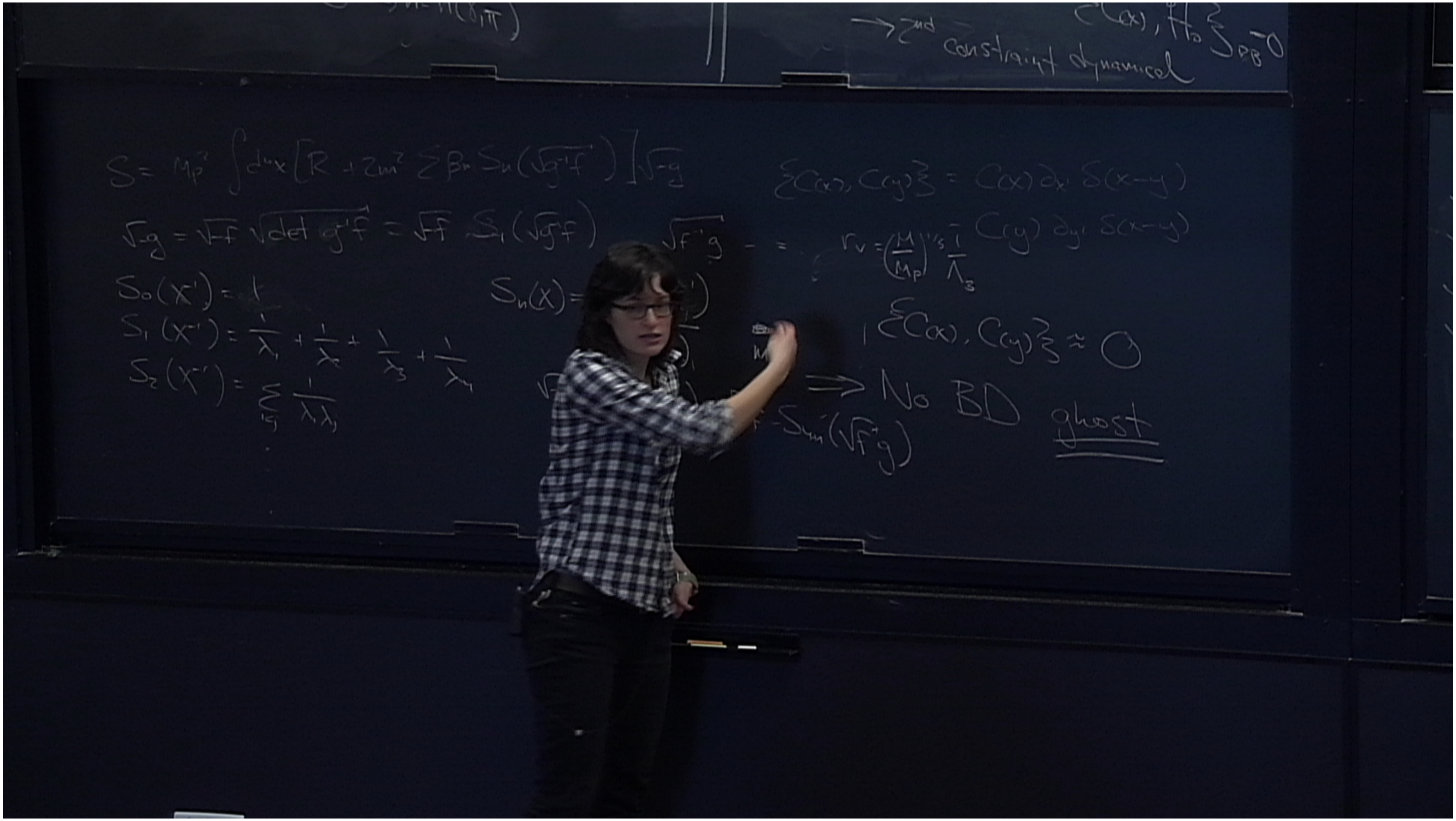

det X

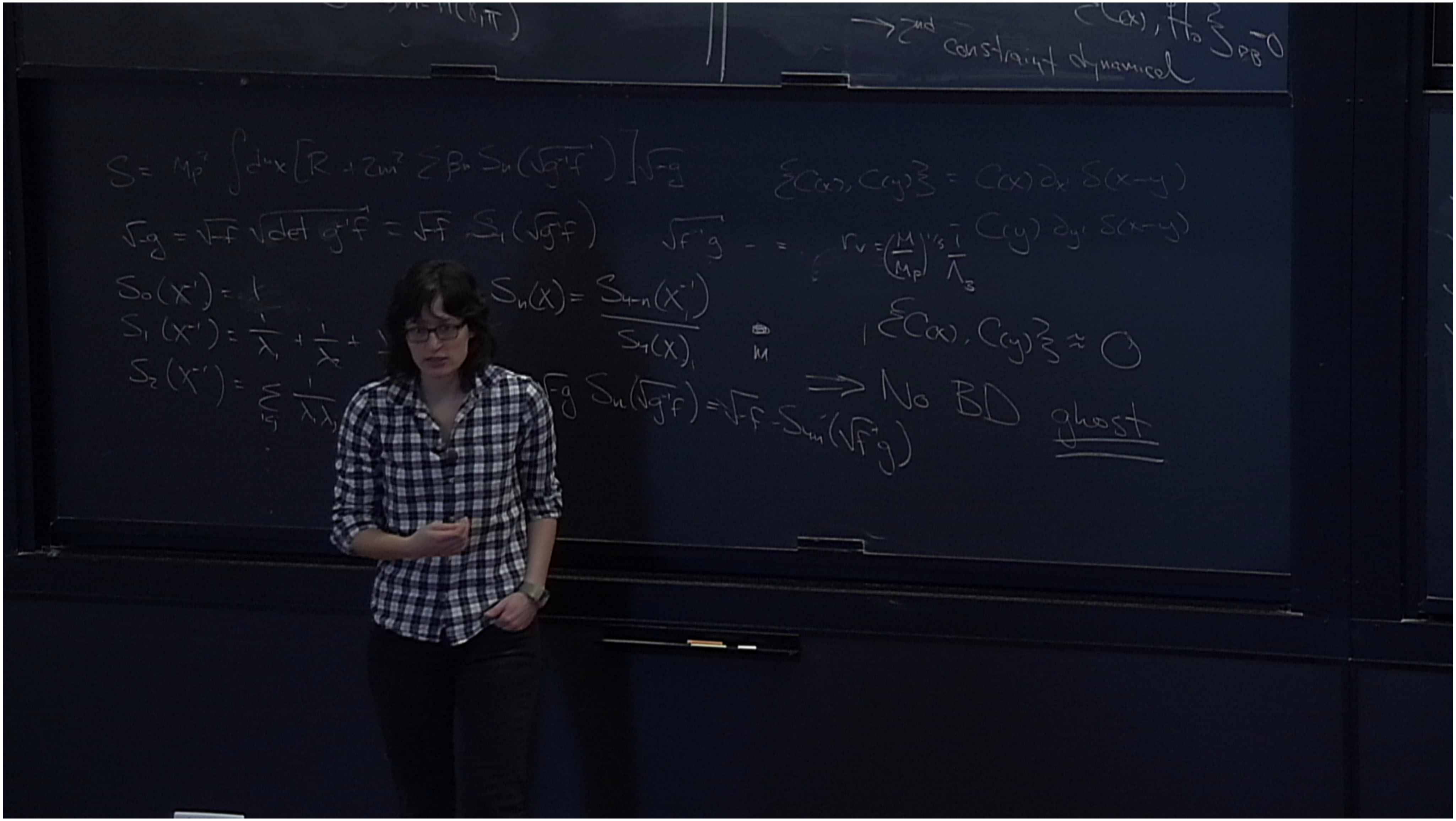


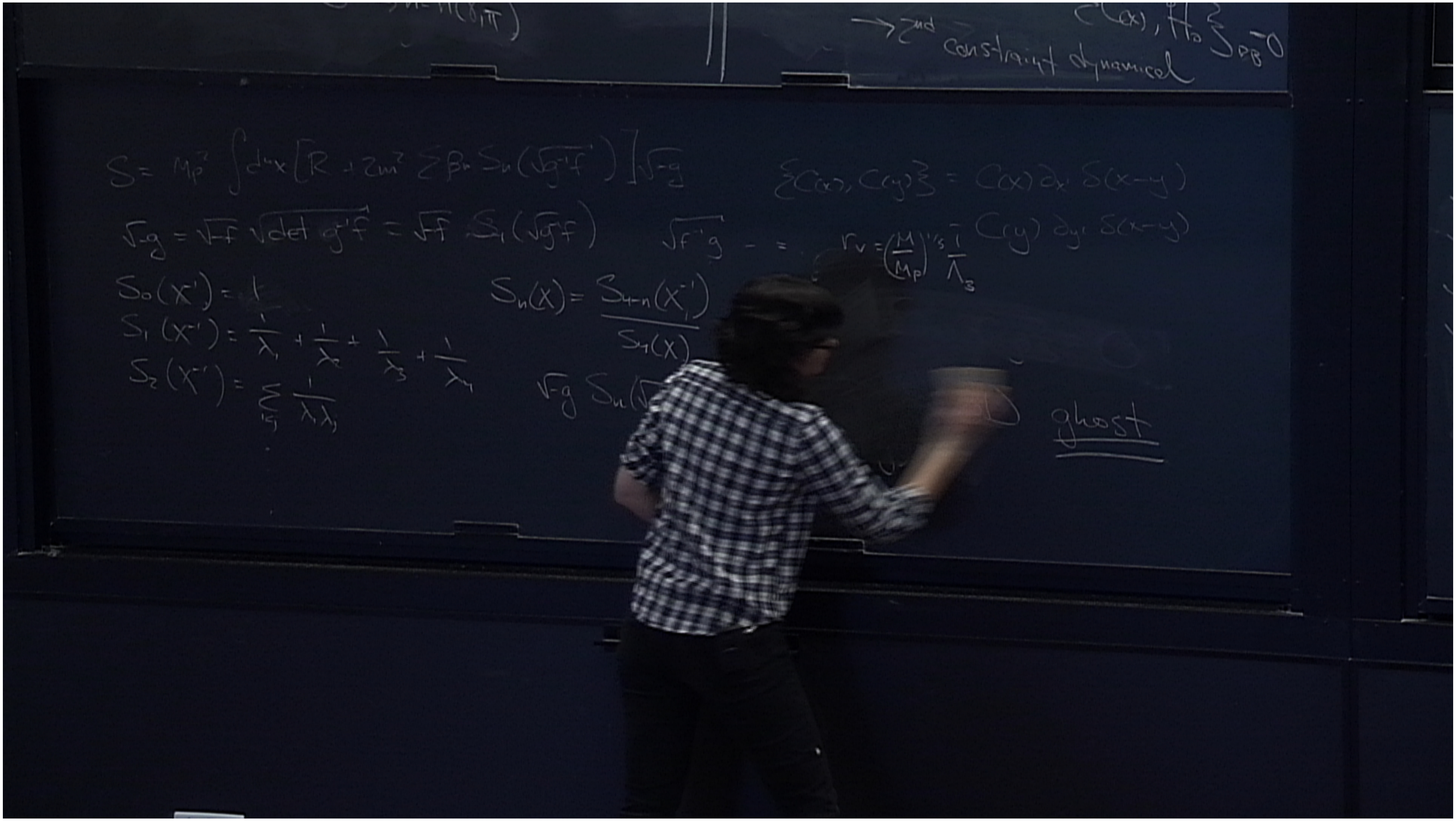
√-g S₀(x) = 1
√-g S₁(x) = [X]
√-g S₂(x) = ½([X]² − [X²])
√-g S₃(x) = ⅙([X]³ − 3[X][X²] + 2[X³])
det X
X = √g⁻¹f
g = e₁ η e₁ᵀ
f = e₂ η e₂ᵀ
N_I, N_I^i
simpler analysis

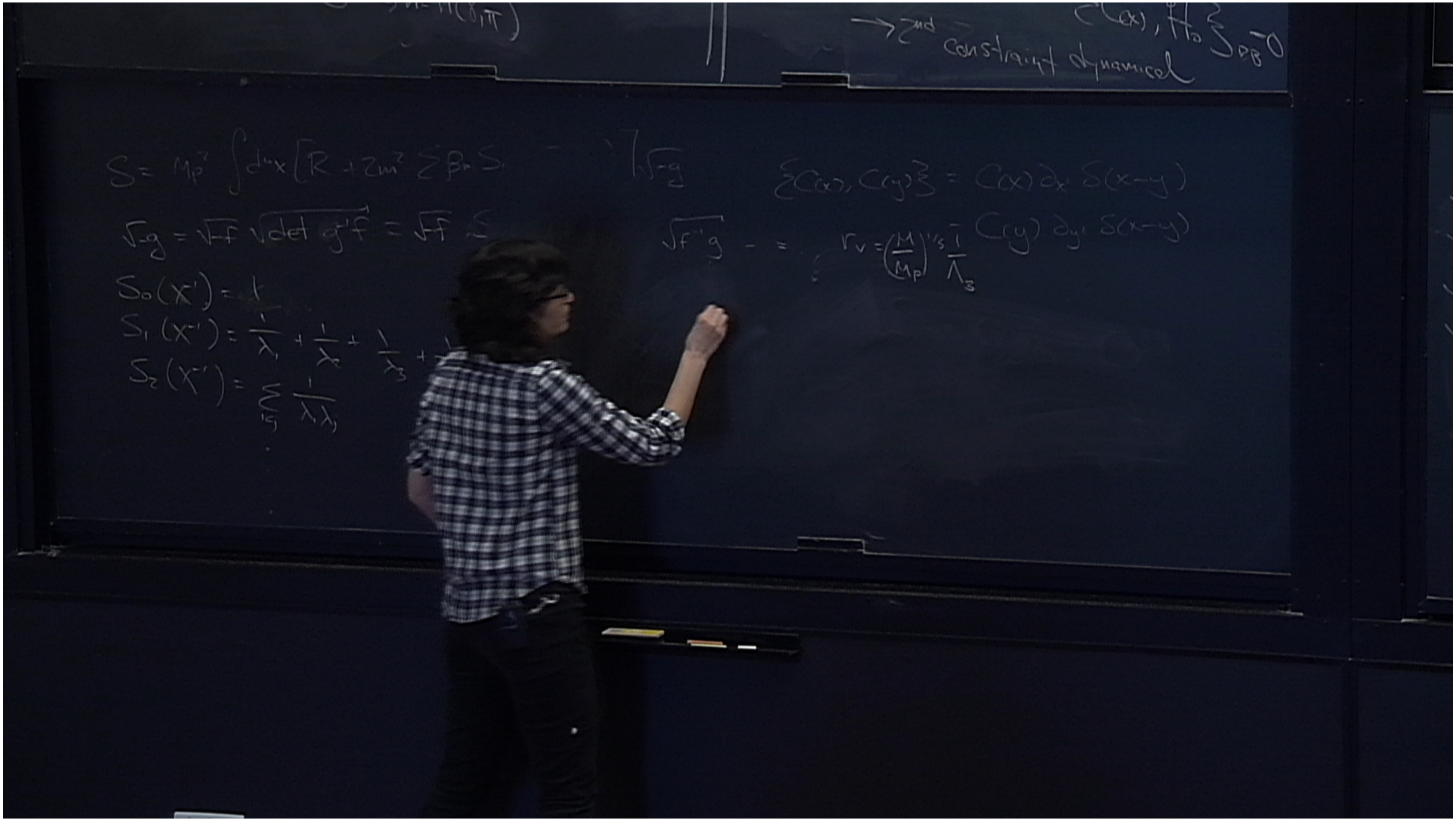

√-g S₀(x) = 1
√-g S₁(x) = [X]
√-g S₂(x) = ½([X]² − [X²])
√-g S₃(x) = ⅙([X]³ − 3[X][X²] + 2[X³])
det X
X = √g⁻¹f
g = e₁ η e₁ᵀ
f = e₂ η e₂ᵀ
N_I , N_I^i
• simpler analysis
massive
massless

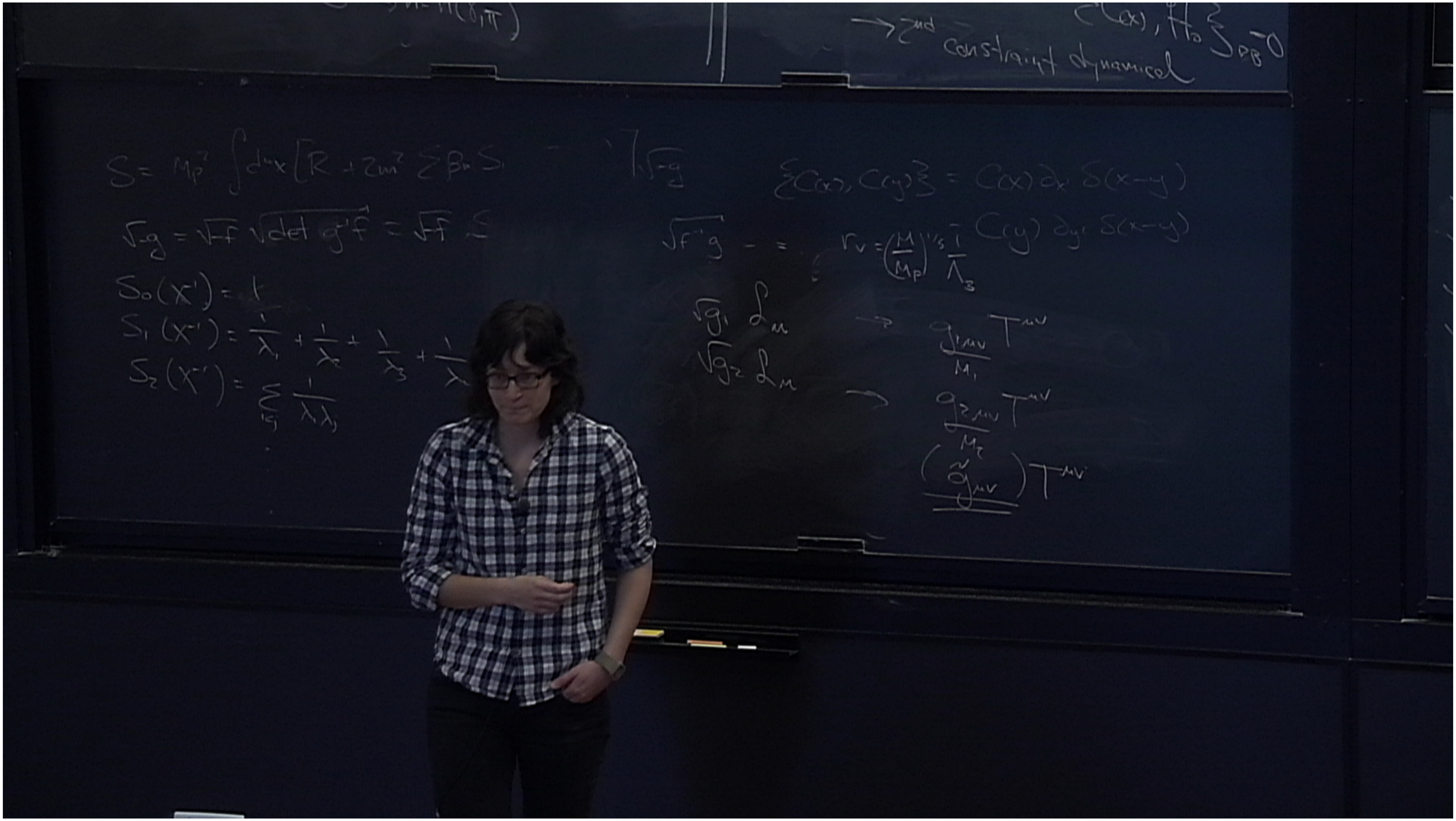

√-g S₀(x) = 1
√-g S₁(x) = [X]
√-g S₂(x) = 1/2 ([X]² − [X²])
√-g S₃(x) = 1/6 ([X]³ − 3[X][X²] + 2[X³])
det X
X = √(g⁻¹f)
g = e₁ η e₁ᵀ
f = e₂ η e₂ᵀ
e₁∧e₁∧e₁∧e₁
e₁∧e₁∧e₁∧e₂
e₁∧e₁∧e₂∧e₂
e₁∧e₂∧e₂∧e₂
e₂∧e₂∧e₂∧e₂
N_I, N_I^i
simpler analysis
e_I1∧e_I2∧e_I3∧e_I4
n → n-1 massive
1 massless
g₁
g₂
g₃

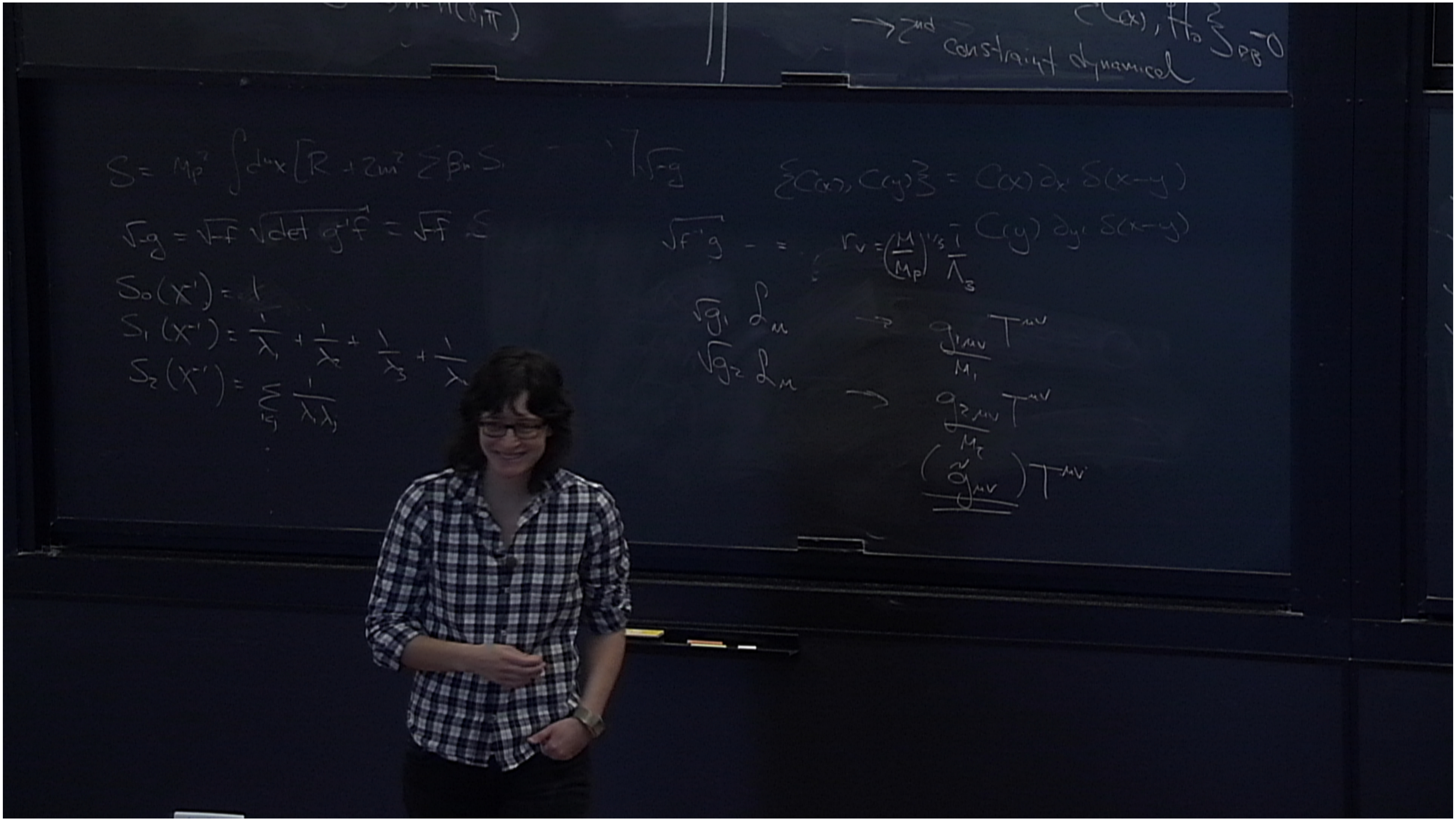

constraint dynamical
No BD ghost




constraint dynamical
ghost


constraint dynamical


constraint dynamical


2nd
constraint dynamical