

Title: From Massive Gravity to Interacting Spin-2 Fields

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Abstract: Recent progress in massive gravity has made it possible to construct consistent theories of interacting spin-2 fields. &nbsp;In this talk I'll describe these developments, focusing on the resolution of the Boulware-Deser ghost problem and the promotion of massive gravity to a bimetric theory of gravity with two dynamical, interacting spin-2 fields. &nbsp;I'll then discuss the generalization of these bimetric theories to theories of multiple interacting spin-2 fields.

"From Massive Gravity  
to Interacting Spin-2 Fields"

GR



# "From Massive Gravity to Interacting Spin-2 Fields"

GR  $\rightarrow$  Lovelock Inv

4D

vielbein  $e^a$   
curvature  $R^{ab}$

$$e^a \wedge e^b \wedge e^c \wedge e^d \wedge E_{abcd} \longleftrightarrow \sqrt{-g}$$

$$R^{ab} \wedge e^c \wedge e^d \wedge E_{abcd} \longleftrightarrow \sqrt{-g} R$$

$$R^{ab} \wedge R^{cd} \wedge E_{abcd} \longleftrightarrow \text{Gauss-Bonnet}$$

$\downarrow$   
6D

$$e^1 e^2 e^3 e^4 e^5 e^6$$

$$R^1 e^2 e^3 e^4 e^5 e^6$$

$$R^1 R^2 e^3 e^4 e^5 e^6$$

$$R^1 R^2 R^3 e^4 e^5 e^6$$



"From Massive Gravity  
to Interacting Spin-2 Fields"

GR  $\rightarrow$  Lovelock Inv  
 vielbein  $e^a$   
 curvature  $R^{ab}$

4D  
 $e^a \Lambda^b \Lambda^c \Lambda^d \epsilon^{abcd} \leftrightarrow \sqrt{-g}$   
 $R^{ab} \Lambda^c \Lambda^d \epsilon^{abcd} \leftrightarrow \sqrt{-g} R$   
 $R^{ab} \Lambda^c \Lambda^d \epsilon^{abcd} \leftrightarrow \sqrt{-g} R$   
 $\epsilon^a \epsilon^b \epsilon^c \epsilon^d \epsilon^{abcd} \leftrightarrow \text{Gauss-Bonnet}$

$\downarrow$   
 6D

"ghost-F"



"From Massive Gravity  
to Interacting Spin-2 Fields"

K. Hinterbichler

4D GR  $\rightarrow$  Lovelock Inv  $\rightarrow$  vielbein  $e^a$   
 $n=1$  curvature  $R^{ab}$   
 $e^a \wedge e^b \wedge e^c \wedge e^d \wedge E_{abcd} \leftrightarrow \sqrt{-g}$   
 $R^{ab} \wedge e^c \wedge e^d \wedge E_{abcd} \leftrightarrow \sqrt{-g} R$   
 $R^{ab} \wedge R^{cd} \wedge E_{abcd} \leftrightarrow \text{Gauss-Bonnet}$   
 $e_1 e_2 e_3 e_4$   
 $R_1 e_2 e_3 e_4$   
 $R_1 R_2 e_3 e_4$   
 $R_1 R_2 R_3$   
 "ghost-free"

$n > 1$   $g_{\mathbb{I}} \rightarrow e_{\mathbb{I}}$   
 $e_{\mathbb{I}_1} \wedge e_{\mathbb{I}_2} \wedge e_{\mathbb{I}_3} \wedge e_{\mathbb{I}_4}$  "ghost-free"  
 $e_1 \wedge e_1 \wedge e_1 \wedge e_1$   
 $e_1 \wedge e_1 \wedge e_1 \wedge e_2$   
 $e_1 \wedge e_1 \wedge e_2 \wedge e_2$   
 $e_1 \wedge e_2 \wedge e_2 \wedge e_2$   
 $e_2 \wedge e_2 \wedge e_2 \wedge e_2 \rightarrow \sqrt{-g_2}$

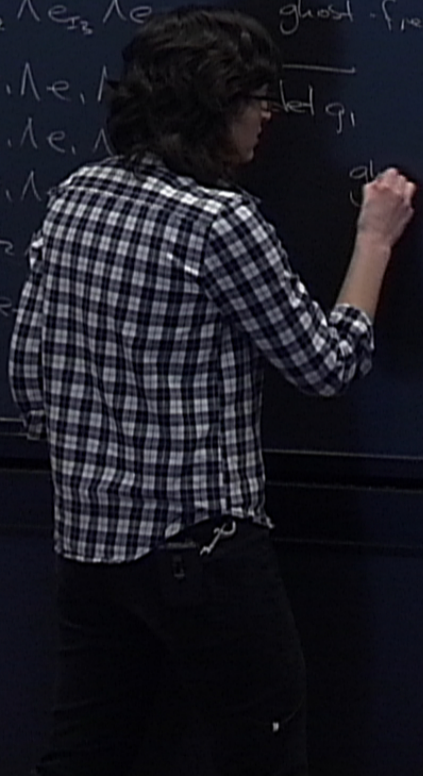


"From Massive Gravity  
to Interacting Spin-2 Fields"

K. Hinterbichler

4D GR  $\rightarrow$  Lovelock Inv  $\rightarrow$  vielbein  $e^a$  curvature  $R^{ab}$   
 $\parallel=1$   
 $e^a \wedge e^b \wedge e^c \wedge e^d \wedge E_{abcd} \leftrightarrow \sqrt{-g}$   
 $R^{ab} \wedge e^c \wedge e^d \wedge E_{abcd} \leftrightarrow \sqrt{-g} R$   
 $R^{ab} \wedge R^{cd} \wedge E_{abcd} \leftrightarrow \text{Gauss-Bonnet}$   
 $e_1 e_2 e_3 e_4 e_5 e_6$   
 $R_1 e_2 e_3 e_4 e_5 e_6$   
 $R_1 R_2 e_3 e_4 e_5 e_6$   
 $R_1 R_2 R_3 e_4 e_5 e_6$   
 $R_1 R_2 R_3 R_4$   
 "ghost-free"

$n > 1$   $g_{\mathbb{I}} \rightarrow e_{\mathbb{I}}$   
 $e_{\mathbb{I}_1} \wedge e_{\mathbb{I}_2} \wedge e_{\mathbb{I}_3} \wedge e_{\mathbb{I}_4}$  "ghost-free"  
 $n=2$   
 $e_1 \wedge e_2 \wedge e_3 \wedge e_4$   
 $e_1 \wedge e_2 \wedge e_3 \wedge e_4$   
 $e_1 \wedge e_2 \wedge e_3 \wedge e_4$   
 $e_1 \wedge e_2$   
 $e_3 \wedge e_4$   
 $g_{\mathbb{I}}$





$n=3$   $e_1 \wedge e_1 \wedge e_2 \wedge e_3$   
 $e_1 \wedge e_2 \wedge e_2 \wedge e_3$   
 $e_1 \wedge e_2 \wedge e_3 \wedge e_3$

$n=4$   $e_1 \wedge e_2 \wedge e_3 \wedge e_4$



$n=3$   $e_1 \wedge e_1 \wedge e_2 \wedge e_3$   
 $e_1 \wedge e_2 \wedge e_2 \wedge e_3$   
 $e_1 \wedge e_2 \wedge e_3 \wedge e_3$

$n=4$   $e_1 \wedge e_2 \wedge e_3 \wedge e_4$



"From Massive Gravity  
to Interacting Spin-2 Fields"

Massive Gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$
$$m^2 (h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h_{\mu}^{\mu} h_{\nu}^{\nu})$$



"From Massive Gravity  
to Interacting Spin-2 Fields"

Massive Gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$
$$m^2 (h_{\mu\nu}^{\mu\nu} - \frac{1}{2} h_{\mu\nu}^{\mu\nu})$$

ADM

$$N = (-g_{00})^{1/2}$$

$$N_i = g_{0i}$$

$$\gamma_{ij} = g_{ij}$$

GR

$$N, N^i \quad (4)$$

no time deriv., linearly



"From Massive Gravity  
to Interacting Spin-2 Fields"

Massive Gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu}$$

$$m^2 (h_{\mu\nu})$$

ADM

$$N = (-g_{00})^{1/2}$$

$$N_i = g_{0i}$$

$$\gamma_{ij} = g_{ij}$$

GR

$$N, N^i \quad (4)$$

$$\gamma_{ij} \quad (6)$$

$$T^i_j \quad (6)$$

no time der. v, linearly

→ 4 constraints (2)

→ 4 general coord (2)



"From Massive Gravity  
to Interacting Spin-2 Fields"

Massive Gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$m^2 (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h^\alpha{}_\alpha)$$

ADM

$$N = (-g_{00})^{1/2}$$

$$N_i = g_{0i}$$

$$\gamma_{ij} = g_{ij}$$

GR

$$N, N^i \quad (4)$$

$$\gamma_{ij} \quad (6)$$

$$T^i_j \quad (6)$$

No time deriv. linearly

→ 4 constraints (2)

→ 4 general coord (2)

massless Spin-2



"From Massive Gravity to Interacting Spin-2 Fields"

Massive Gravity

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$m^2 (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h)$$

ADM

$$N = (-g_{00})^{1/2}$$

$$N_i = g_{0i}$$

$$\gamma_{ij} = g_{ij}$$

GR

$N, N^i$  (4)

$\gamma_{ij}$  (6)

$T^i_j$  (6)

no time der. v, linearly

→ 4 constraints (2)

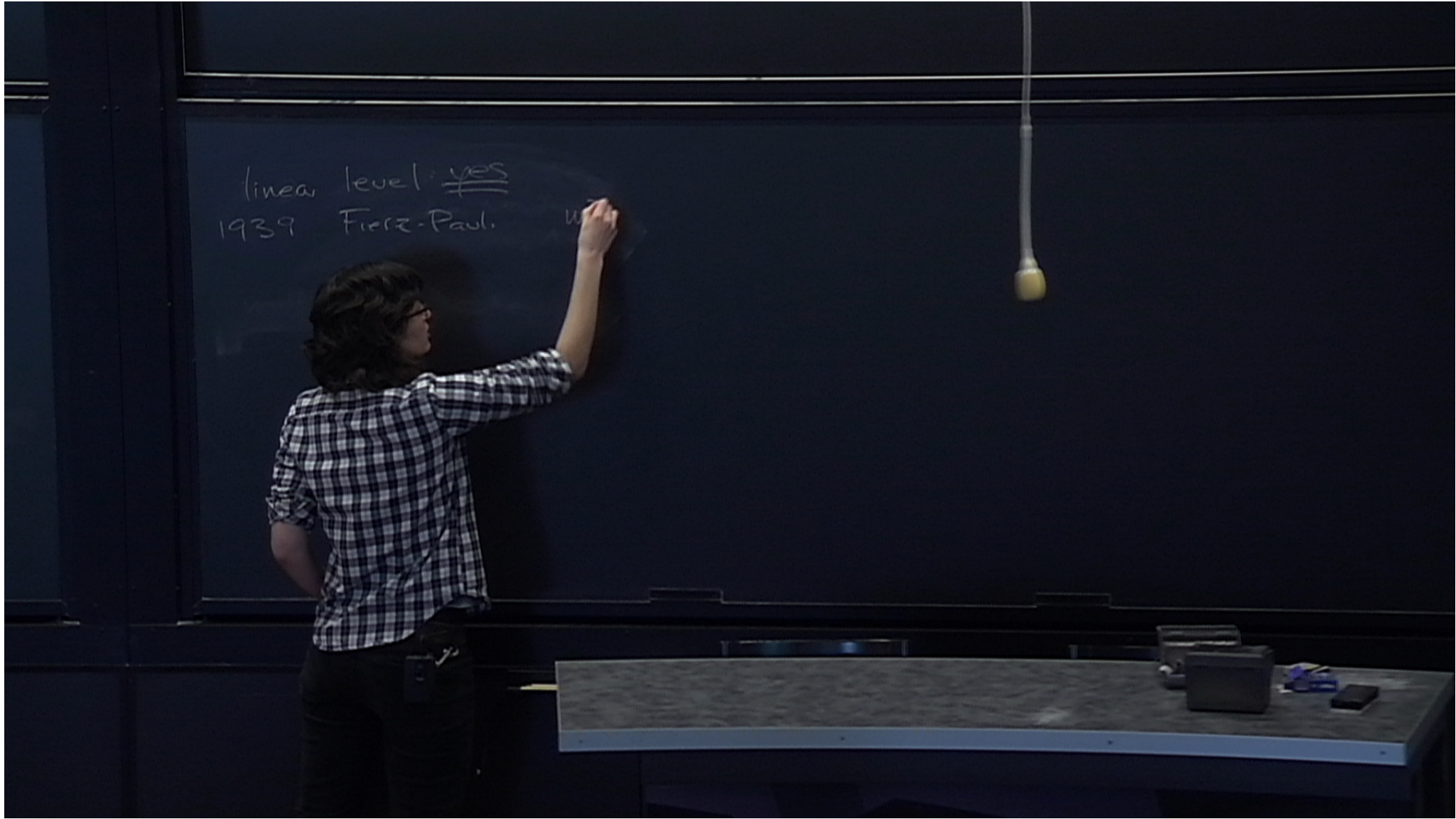
→ 4 general coord (2)

6 propagating dof

massless spin-2

→ 1 extra dof







linear level: ~~yes~~  
1939 Fierz-Pauli

$$m^2 (\eta_{\mu\nu} h^{\mu\nu} - h^2)$$



linear level: ~~yes~~  
1939 Fierz-Pauli  $m^2(h^{\mu\nu} - h^2)$   
• Linear in  $N$



linear level: ~~yes~~

1939 Fierz-Pauli

$$m^2(\eta^{\mu\nu}h_{\mu\nu} - h^2)$$

- Linear in  $N \rightarrow$  Hamiltonian const.
- 2<sup>nd</sup> ary constraint



linear level: ~~yes~~

1939 Fierz-Pauli  $m^2(h^{\mu\nu} - h^2)$

- Linear in  $N \rightarrow$  Hamiltonian const.
- 2<sup>nd</sup> ary constraint

1972 Boulware-Deser



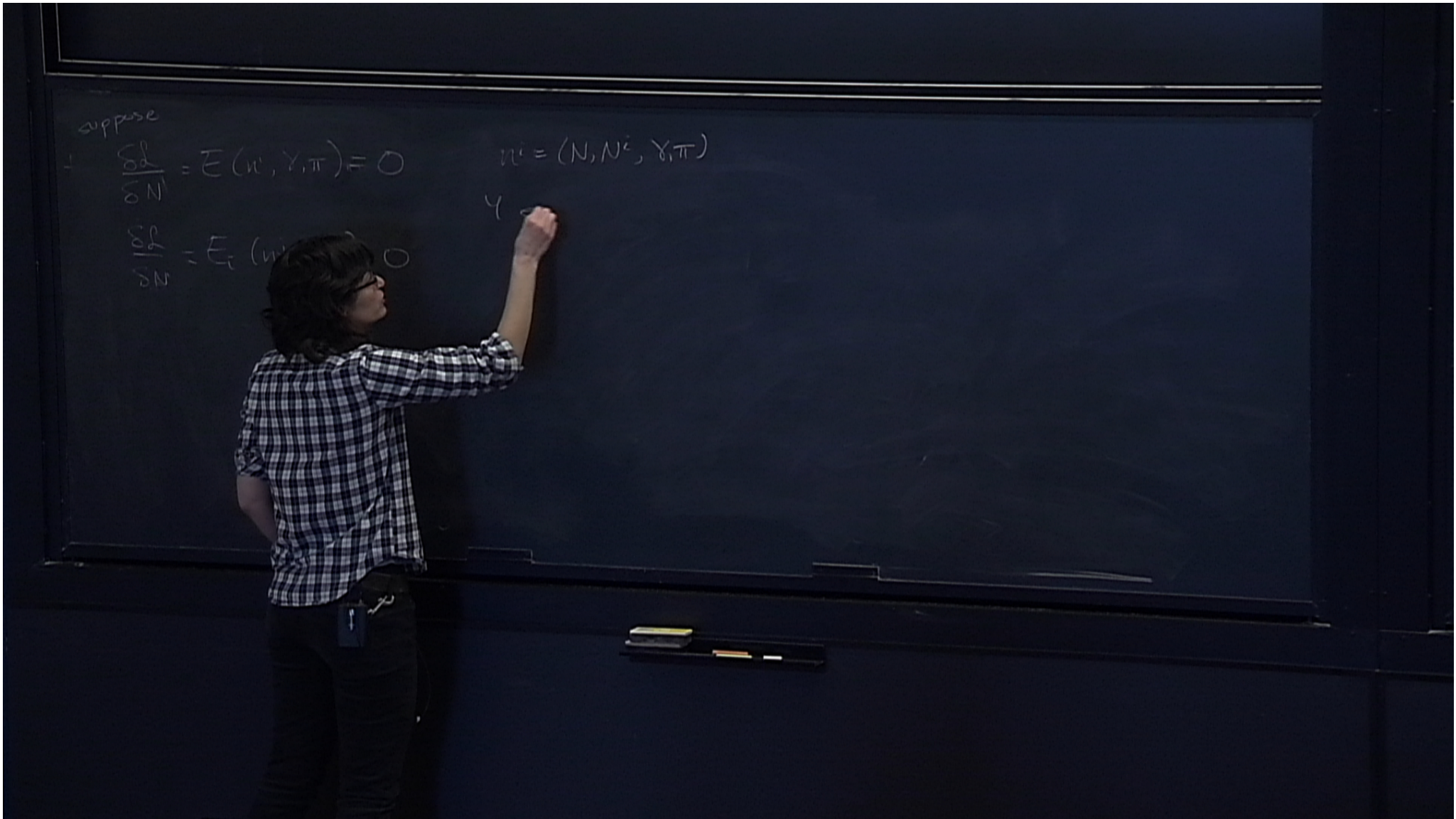
linear level: ~~yes~~

1939 Fierz-Pauli  $m^2(\eta_{\mu\nu} - h^2)$

- Linear in  $N \rightarrow$  Hamiltonian const.
- 2<sup>nd</sup> ary constraint

1972 Fierz-Deser  $\rightarrow$  ghost reappears  
at non-linear level  
 $\mathcal{L}$  non linear in  $N, N^i$







suppose

$$\frac{\delta \mathcal{L}}{\delta N} = E(N, \gamma, \pi) = 0$$

$$\frac{\delta \mathcal{L}}{\delta N^i} = E_i(N, \gamma, \pi) = 0$$

$$\rightarrow n^i = n^i(\gamma, \pi)$$

$$n^i = (N, N^i, \gamma, \pi)$$

4 eqn, 3  $n^i$

Hamiltonian  
constraint

RG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$\int d^4x [R - 2\Lambda]$$



suppose

$$\frac{\delta \mathcal{L}}{\delta N} = E(w, \gamma, \pi) = 0$$

$$\frac{\delta \mathcal{L}}{\delta N^i} = E_i(w, \gamma, \pi) = 0$$

$$\rightarrow n^i = n^i(\gamma, \pi)$$

$$n^i = (N, N^i, \gamma, \pi)$$

4 eqn, 3  $n^i$

Hamiltonian  
constraint

RG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_p^2 \int d^4x [R + \dots] S_n(\sqrt{\dots})$$





suppose

$$\frac{\delta \mathcal{L}}{\delta N} = \mathcal{E}(w, \gamma, \pi) = 0$$

$$\frac{\delta \mathcal{L}}{\delta N^i} = \mathcal{E}_i(w, \gamma, \pi) = 0$$

$$\rightarrow n^i = n^i(\gamma, \pi)$$

$$n^i = (N, N^i, \gamma, \pi)$$

4 eqn, 3  $n^i$

Hamiltonian  
constraint

DRG 2010  
dRGT 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_p^2 \int d^4x \left[ R + 2m^2 \sum_{n=0}^4 \beta_n S_n(\sqrt{g'f}) \right]$$

$\sqrt{g'f} \sqrt{g'f} = g^{\mu\nu} f_{\mu\nu}$  ↑ non-dynamical



$$S_0(x) = 0$$

$$S_1(x) = [x]$$

$$S_2(x) = \frac{1}{2} ([x]^2 - [x^2])$$

$$S_3(x) = \frac{1}{6} ([x]^3 - 3[x][x^2] + 2[x^3])$$

$$S_4(x) = \det X$$



$$S_0(x) = 0$$

$$S_1(x) = [x]$$

$$S_2(x) = \frac{1}{2}([x]^2 - [x^2])$$

$$S_3(x) = \frac{1}{6}([x]^3 - 3[x][x^2])$$

$$S_4(x) = \det x$$

$S_4$



$$S_0(x) = 0$$

$$S_1(x) = [x]$$

$$S_2(x) = \frac{1}{2}([x]^2 - [x^2])$$

$$S_3(x) = \frac{1}{6}([x]^3 - 3[x][x^2] + 2[x^3])$$

$$S_4(x) = \det X$$

$$S_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$$

$$S_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \dots$$

$$S_3 = \sum_{1 \leq j < k} \lambda_i \lambda_j \lambda_k$$

$$S_4 = \lambda_1 \lambda_2 \lambda_3 \lambda_4$$



F. Hassan

$$\chi(\pi) = 0$$

dRG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_P^2 \int d^4x \left[ R + 2m^2 \sum_{n=0}^4 \beta_n S_n(\sqrt{g^{\mu\nu}} f) \right]$$

$\sqrt{g^{\mu\nu}} \sqrt{g^{\rho\sigma}} f = g^{\mu\rho} f_{\nu\sigma}$

$f = \tau$  (non-dynamical)



F. Hassan  
 $N^2$

dRG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_P^2 \int d^4x \sqrt{g} \left[ \frac{1}{2} R + \sum_{n=2}^4 \beta_n S_n(\sqrt{g} f) \right]$$

$f = \tau$   
non-dynamical



F. Hassan

$N^E - M^E$

RG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_P^2 \int d^4x \left[ R + 2m^2 \sum_{n=0}^4 \beta_n S_n(\sqrt{g}f) \right]$$

$\sqrt{g}f \sqrt{g}f = g^{\mu\nu} f_{,\mu\nu}$

$f = \frac{z}{M, M^i}$

non-dynamical



F. Hassan

$$N^i - M^i = (MS_j + ND_j) n^j \rightarrow N^i \rightarrow n^j$$

$$\sqrt{1 - n^i n^i} D = \sqrt{(\gamma - D_n D_n) F}$$

$$\dot{\gamma}_i = \frac{1}{2} H_0(n, \gamma, \pi) + N C(n, \gamma, \pi)$$
$$= C_i(n, \gamma, \pi)$$

RG 2010  
dRG 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_P^2 \int d^4x \left[ R + 2m^2 \sum_{n=0}^4 \beta_n S_n(\sqrt{g^{\mu\nu}} f) \right]$$

$\uparrow$  non-dynamical  
 $f = \tilde{\gamma}$   
 $M, M^i$

$$\sqrt{g^{\mu\nu}} \sqrt{g^{\mu\nu}} f = g^{\mu\nu} f_{,\mu\nu}$$



F. Hassan

$$N^i - M^i = (MS_j + ND_j) n^j \rightarrow N^i \rightarrow n^j$$

$$\sqrt{1-n^i n^i} D = \sqrt{(\gamma - D_n D_n) F}$$

$$\frac{\delta L}{\delta t} = C(n, \gamma, \pi) = 0$$

$$L = \int d^3x \left[ H_0(n, \gamma, \pi) + N C(n, \gamma, \pi) \right] \rightarrow \text{Hamiltonian constraint}$$

$$C_i(n, \gamma, \pi) = 0 \rightarrow n = n(\gamma, \pi)$$

RG 2010  
DRGT 2010

non-linear massive gravity  
ghost-free in "decoupling limit"

$$S = M_P^2 \int d^4x \left[ R + 2m^2 \sum_{n=0}^4 \beta_n S_n(\sqrt{g} f) \right]$$

non-dynamical  
 $f = \tau$   
 $M, M^i$

$$\sqrt{g} f \sqrt{g} f = g^{\mu\nu} f_{\mu\nu}$$



F. Hassan

$$N^i - M^i = (MS_j + ND_j) \vec{n} \rightarrow N^i \rightarrow n^i$$

$$\sqrt{1 - n^i n^i} D = \sqrt{(\gamma - D_n D_n) F}$$

$$\frac{\delta \mathcal{L}}{\delta t} = C(n, \gamma, \pi) = 0$$

$$\mathcal{L} = \pi_j \dot{y}_j + \dots + N C(n, \gamma, \pi) \rightarrow \text{Hamiltonian constraint}$$

$$\frac{\delta \mathcal{L}}{\delta n^i} = 0$$

$$-(MS_j + ND_j) \vec{n} = 0$$

$$\rightarrow n = n(\gamma, \pi)$$



F. Hassan

$$N^i - M^i = (MS_j + ND_j) n^j \rightarrow N^i \rightarrow n^j$$

$$\sqrt{1 - \dot{u}^2} D = \sqrt{(\gamma - D_n D_n) F}$$

$$\mathcal{L} = \pi_j \dot{y}_j - H_0(u, \gamma)$$

$$\frac{\delta \mathcal{L}}{\delta u} = C_i(u, \gamma, \pi)$$

$$\rightarrow C_i(u)$$

$$\frac{\delta \mathcal{L}}{\delta t} = C(u, \gamma, \pi) = 0$$

$C(u, \gamma, \pi) \rightarrow$  Hamiltonian constraint

$$\left[ \begin{array}{l} \pi_j \dot{y}_j \\ H_0(u, \gamma) \end{array} \right] = 0$$

$$\frac{d}{dt} C = 0$$

$$\sum C(u, \gamma, \pi)_{PE} = 0 \quad H = \int d^3x (H_0 - NC)$$



F. Hassan

$$N^i - M^i = (MS_j + ND_j) \dot{w}^j \rightarrow N^i \rightarrow n^j$$

$$\sqrt{1 - n^i n_i} D = \sqrt{(\gamma - D_n D_n) F}$$

$$\frac{\delta L}{\delta N} = C(u, \gamma, \pi) = 0$$

$$L = \pi_j \dot{y}^j - H_0(u, \gamma, \pi) + N C(u, \gamma, \pi) \rightarrow \text{Hamiltonian constraint}$$

$$\frac{\delta L}{\delta u^i} = C_i(u, \gamma, \pi) \left[ \frac{\delta}{\delta u^k} (MS_j + ND_j) \dot{w}^j \right] = 0$$

$$\rightarrow C_i(u, \gamma, \pi) = 0 \rightarrow n = n(\gamma, \pi)$$

$$* \frac{d}{dt} C = 0$$

$$\sum \{C(x), H\}_{PB} = 0 \quad H = \int dx^y (H_0 - NC)$$

$$\cdot \sum \{C(x), C(y)\}_{PB} \neq 0 \rightarrow * \text{ depends on } N$$

$\rightarrow$  eqn for  $N$   
no 2<sup>nd</sup> ary constraint

$$\cdot \sum \{C(x), C(y)\}_{PB} = 0 \rightarrow \sum \{C(x), H_0\}_{PB} = 0$$

$\rightarrow$  2<sup>nd</sup> constraint dynamical

$$\sum \{C(x), C(y)\}_{PB}$$



$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost



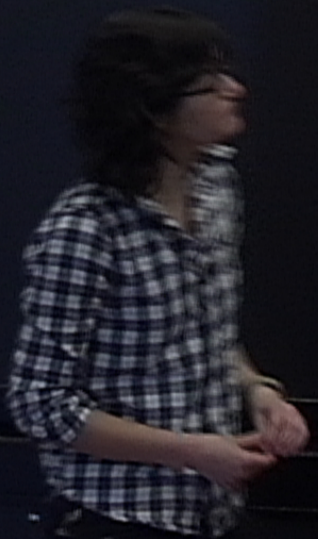
$$S = M_p^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g}^{-1/2} \dot{f})]$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost





$$S = M_p^2 \int d^4x [R + 2m^2 \sum p_a S_a(\sqrt{g}^{-1/2} \dot{f}^a)]$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost



$$S = M_p^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g}^{-1} F)] \sqrt{-g}$$

$$\sqrt{-g} = \sqrt{|\det g^{\mu\nu} F|} = \sqrt{F}$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost



$$S = M_p^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g}^{-1/2} F)] \sqrt{-g}$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g' F} = \sqrt{-F} S_0(\sqrt{g' F})$$

$S_0(x')$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost



$$S = M_p^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g}^{-1} F)] \sqrt{-g}$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^{\mu\nu} F} = \sqrt{-F} S_1(\sqrt{g}^{-1} F) \sqrt{F}^{-1} g$$

$$C(x) = 0$$

$$C(x) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$C(x) = \sum$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$\{C(x), C(y)\} \approx 0$$

⇒ No BD ghost



$$S = M_p^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g}^{-1}F)] \sqrt{-g}$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^{\mu\nu} F} = \sqrt{-F} S_1(\sqrt{g}^{-1}F) \quad \sqrt{-F} g$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y) - C(y) \partial_y S(x-y)$$

$$C=0$$

$$S_0(x') = 0$$

$$S_n(x) = \frac{S_{4-n}(x')}{S_4(x)}$$

$$\{C(x), C(y)\} \approx 0$$

$$S_1(x') = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$S_2(x') = \sum_{i,j} \frac{1}{\lambda_i \lambda_j}$$

$$\sqrt{-g} S_n(\sqrt{g}^{-1}F) = \sqrt{-F} S_{4-n}(\sqrt{F}g) \Rightarrow$$

No BD ghost



Bimetric gravity

$$S = M_g^2 \int d^4x \sqrt{g} R^{(g)} + M_f^2 \int \sqrt{F} R^{(f)}$$

$$+ 2m^2 M_g^2 \int d^4x \sqrt{g} \sum \beta_n S_n(\sqrt{g/f})$$

N. M.



Bimetric gravity

$$S = M_g^2 \int d^4x \sqrt{g} R^{(g)} + M_f^2 \int \sqrt{F} R^{(F)}$$

$$+ 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{g} \Sigma B_n S_n(\sqrt{g} F)$$

$$N^i - M^i = (M S^i, -N D^i)_{ij}$$

$$N^i \rightarrow \hat{n}^i$$

$N, M^i$

$M, M^i$

$\gamma_{ij}(g)$

$F_{ij}(f)$

$\gamma^{ij}(g)$

$F^{ij}(f)$

• Hamiltonian const

• 2nd constraint

• linear  $M, M^i \rightarrow 4$  const

• 4 general coord  $14 \rightarrow 7 = 2 \times 5$

24

- 1

- 1

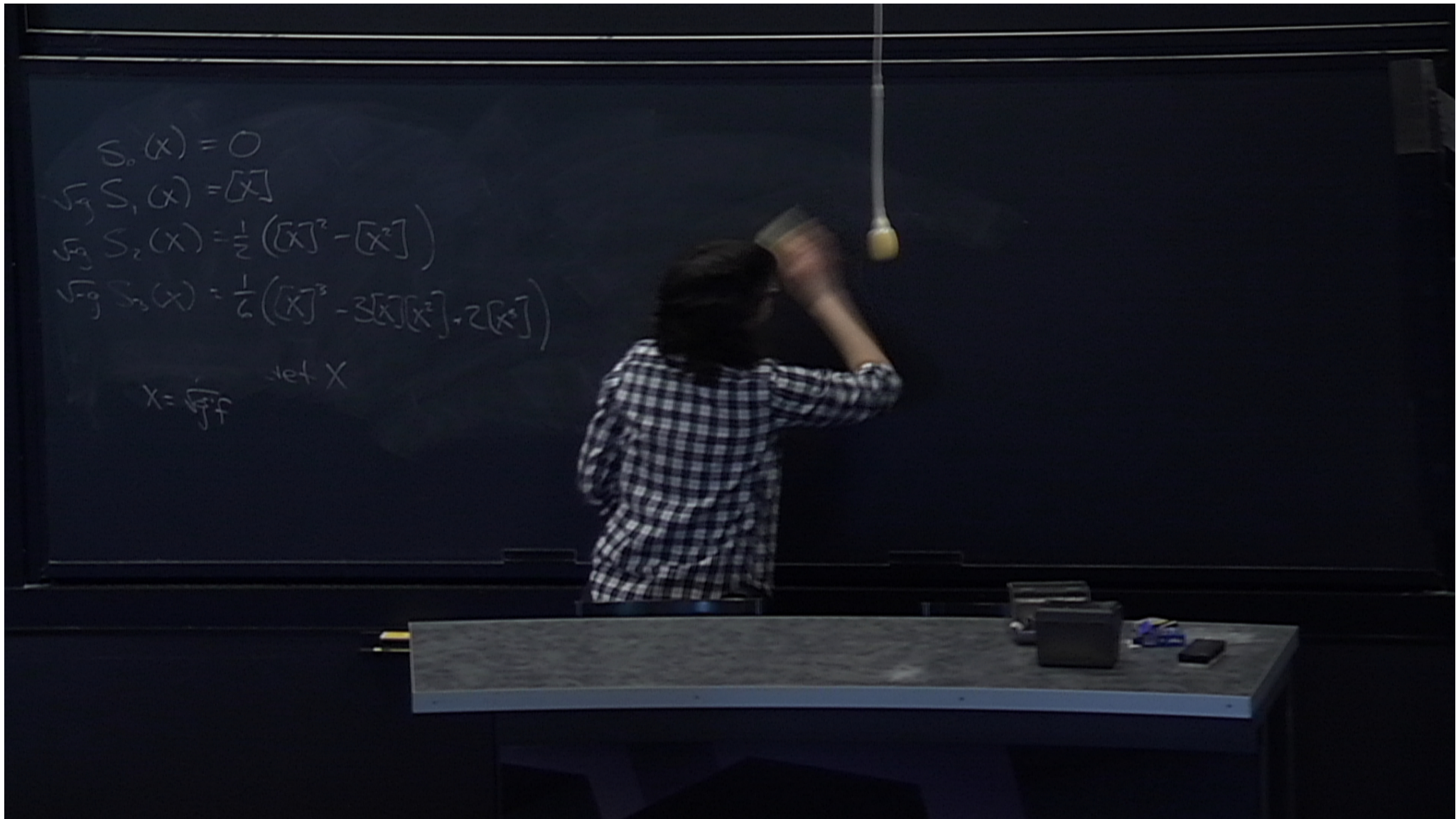
- 4

- 4

14

$\rightarrow 7 = 2 \times 5$







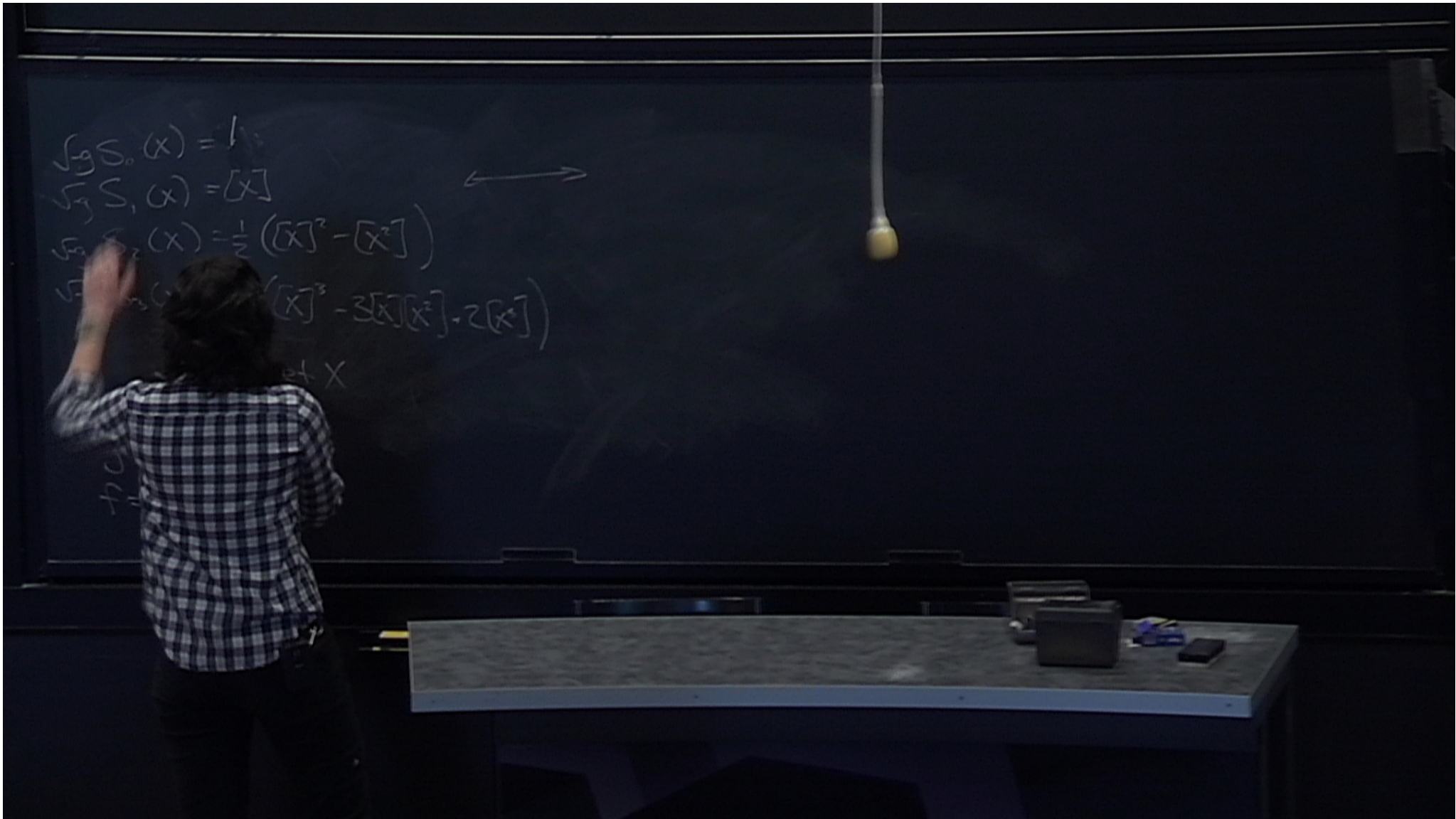
$$S_0(x) = 0$$
$$\sqrt{g} S_1(x) = [x]$$
$$\sqrt{g} S_2(x) = \frac{1}{2} ([x]^2 - [x^2])$$
$$\sqrt{g} S_3(x) = \frac{1}{6} ([x]^3 - 3[x][x^2] + 2[x^3])$$

def X

$$X = \sqrt{g} f$$

$$g = e_1 e_1^T$$





$$\begin{aligned}\sqrt[2]{S_0}(x) &= 1 \\ \sqrt[2]{S_1}(x) &= [x] \\ \sqrt[2]{S_2}(x) &= \frac{1}{2}([x]^2 - [x^2]) \\ \sqrt[2]{S_3}(x) &= \frac{1}{6}([x]^3 - 3[x][x^2] + 2[x^3])\end{aligned}$$

↔

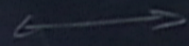


$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2} ([x]^2 - [x^2])$$

$$\sqrt{g} S_3(x) = \frac{1}{6} ([x]^3 - 3[x][x^2] + 2[x^3])$$



$$e_1, \lambda e_1, \lambda e_1, \lambda e_2$$



$$e_1, \lambda e_1, \lambda e_2, \lambda e_2$$

$$e_1, \lambda e_1, \lambda e_2, \lambda e_2$$

det X

$$X = \sqrt{g} f$$

$$g = e_1 \otimes e_1$$

$$f = e_2 \otimes e_2$$

$$e_1$$



$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2}([x]^2 - [x^2])$$

$$\sqrt{g} S_3(x) = \frac{1}{6}([x]^3 - 3[x][x^2] + 2[x^3])$$

$e_1, \lambda e_1, \lambda e_1, \lambda e_1$

$e_1, \lambda e_1, \lambda e_1, \lambda e_2$

$e_1, \lambda e_1, \lambda e_2, \lambda e_2$

$e_1, \lambda e_1, \lambda e_2, \lambda e_2$

$e_2, \lambda e_2, \lambda e_2, \lambda e_2$

det X

$$X = \sqrt{g} f$$

$$g = e_1 e_1^T$$

$$f = e_2 e_2^T$$



$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2}([x]^2 - [x^2])$$

$$\sqrt{g} S_3(x) = \frac{1}{6}([x]^3 - 3[x][x^2] + 2[x^3])$$

det X

$$X = \sqrt{g} f$$

$$g = e_1^T \gamma e_1^T$$

$$f = e_2^T \gamma e_2^T$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$N_{\mathbb{I}}, N_{\mathbb{I}}^i$$

complex anal



$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2}([x]^2 - [x^2])$$

$$\sqrt{g} S_3(x) = \frac{1}{6}([x]^3 - 3[x][x^2] + 2[x^3])$$

def X

$$x = \sqrt{f} f$$

$$g = e_1 e_1^T$$

$$f = e_2 e_2^T$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$N_I, N_I^i$$

• simpler analysis



$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2} ([x]^2 - [x^2])$$

$$\sqrt{g} S_3(x) = \frac{1}{6} ([x]^3 - 3[x][x^2] + 2[x^3])$$

det X

$$X = \sqrt{f} f$$

$$g = e_1 e_1^T$$

$$f = e_2 e_2^T$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_1$$

$$e_1, \lambda e_1, \lambda e_1, \lambda e_2$$

$$e_1, \lambda e_1, \lambda e_2, \lambda e_2$$

$$e_1, \lambda e_2, \lambda e_2, \lambda e_2$$

$$e_2, \lambda e_2, \lambda e_2, \lambda e_2$$

$$N_I, N_I^i$$

• simpler analysis

•  $e_2, \lambda e_2, \lambda e_2, \lambda e_{I+1}$

$n \rightarrow$   $n-1$  massive  
1 massless



$$\sqrt{g} S_0(x) = 1$$

$$\sqrt{g} S_1(x) = [x]$$

$$\sqrt{g} S_2(x) = \frac{1}{2} ([x]^2 - [x^2])$$

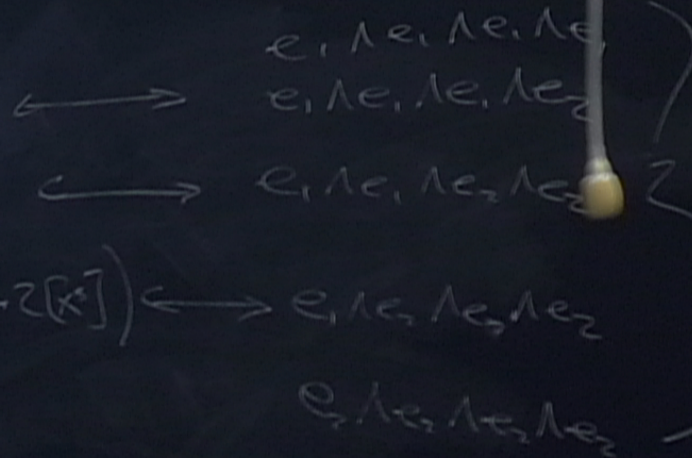
$$\sqrt{g} S_3(x) = \frac{1}{6} ([x]^3 - 3[x][x^2] + 2[x^3])$$

det X

$$X = \sqrt{g} f$$

$$g = e_1 e_1^T$$

$$f = e_2 e_2^T$$

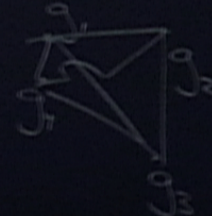


$N_I, N_I^i$

• super analysis

•  $e_2, \lambda e_3, \lambda e_{I3}, \lambda e_{I4}$

$N \rightarrow N-H$  massive  
massless





$\psi = \psi(\sigma, \pi)$

$\rightarrow$  2<sup>nd</sup> constraint dynamical  $\{C(x), H_0\} \approx 0$   
 $\{C(x), C(y)\} \approx 0$

$$S = M_P^2 \int d^4x [R + 2m^2 \sum \beta_n S_n(\sqrt{g} F)] \sqrt{-g}$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^i_j} = \sqrt{-F} S_1(\sqrt{g} F)$$

$$S_0(X^i) = 1$$

$$S_1(X^i) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$S_2(X^i) = \sum_{i,j} \frac{1}{\lambda_i \lambda_j}$$

$$S_n(X) = \dots$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y)$$

$$= \dots \quad v_n = \left(\frac{M}{M_P}\right)^{1/2} \frac{1}{\Lambda_s} C(y) \partial_{y^i} S(x-y)$$

$$\{C(x), C(y)\} \approx 0$$

$\Rightarrow$  No BD ghost



$$S = -11(0, \pi)$$

→ 2<sup>nd</sup> constraint dynamical  $\{C(x), H_0\} \approx 0$   
 $\{C(x), C(y)\} \approx 0$

$$S = M_P^2 \int d^4x [R + 2m^2 \sum p_n S_n(\sqrt{g}^{\mu\nu} F^{\mu\nu})] \sqrt{-g}$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y)$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^{\mu\nu} F} = \sqrt{-F} S_1(\sqrt{g}^{\mu\nu} F)$$

$$\sqrt{-g} = \dots \nu = \left(\frac{M}{M_P}\right)^{1/2} \frac{1}{\Lambda_3} C(y) \partial_y S(x-y)$$

$$S_0(X') = \frac{1}{\lambda}$$

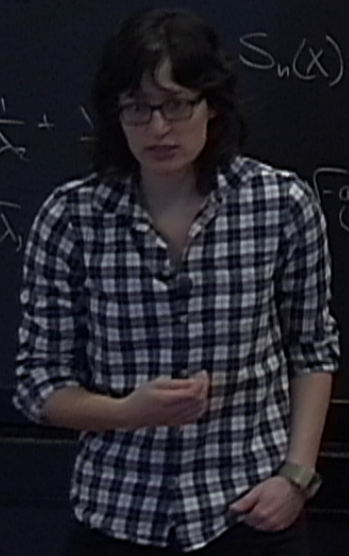
$$S_1(X') = \frac{1}{\lambda} + \frac{1}{\lambda} + \frac{1}{\lambda}$$

$$S_2(X') = \sum_{\lambda, \lambda} \frac{1}{\lambda \lambda}$$

$$S_n(X) = \frac{S_{4-n}(X')}{S_1(X)}$$

$$\{C(x), C(y)\} \approx 0$$

$$\sqrt{-g} S_n(\sqrt{g}^{\mu\nu} F) = \sqrt{-F} S_{4-n}(\sqrt{F}^{\mu\nu} g) \Rightarrow \text{No BD } \underline{\underline{\text{ghost}}}$$





$\chi = \pi(0, \pi)$

$\rightarrow$  2nd constraint dynamical  $\{C(x), H_0\} \approx 0$

$$S = M_P^2 \int d^4x [R + 2m^2 \sum p_n S_n(\sqrt{g}^{-1/2})] \sqrt{-g}$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y)$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^{\mu\nu}} = \sqrt{-F} S_1(\sqrt{g}^{-1/2})$$

$$\sqrt{-g} = \dots r_v = \left(\frac{M}{M_P}\right)^{1/2} \frac{1}{\Lambda_s} C(y) \partial_y S(x-y)$$

$$S_0(X') = 1$$

$$S_n(X) = \frac{S_{4-n}(X')}{S_1(X)}$$

$$S_1(X') = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$S_2(X') = \sum_{i,j} \frac{1}{\lambda_i \lambda_j}$$

$$\sqrt{-g} S_n(\sqrt{g}^{-1/2})$$

ghost



$$\psi = \psi(0, \pi)$$

→ 2nd constraint dynamical  $\{C(x), H_0\} = 0$

$$S = M_P^2 \int d^4x \sqrt{-g} (R + 2m^2 \sum \phi_n S)$$

$$\sqrt{-g}$$

$$\{C(x), C(y)\} = C(x) \delta(x-y)$$

$$\sqrt{-g} = \sqrt{-F} \sqrt{\det g'^i_j} = \sqrt{-F} S$$

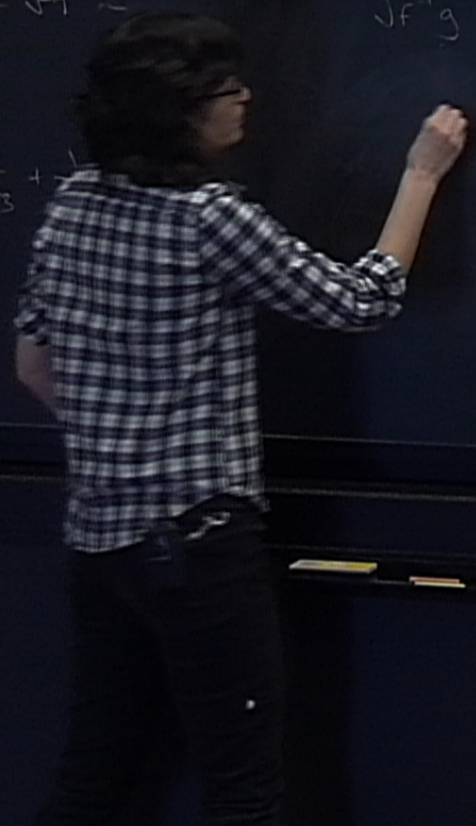
$$\sqrt{-g}$$

$$r_v = \begin{pmatrix} M \\ M_P \end{pmatrix} \frac{1}{\Lambda_s} C(y) \delta(x-y)$$

$$S_0(x') = \frac{1}{\lambda}$$

$$S_1(x') = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \dots$$

$$S_2(x') = \sum_{j=1}^M \frac{1}{\lambda_j}$$





$$\psi = \psi(\theta, \pi)$$

→ 2nd constraint dynamical  $\{C(x), H_0\} \approx 0$

$$S = M_P^2 \int d^4x \sqrt{-g} (R + 2m^2 \sum \beta_n S_n)$$

$$\sqrt{-g}$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y)$$

$$\sqrt{g} = \sqrt{F} \sqrt{\det g'^{\mu\nu}} = \sqrt{F} \sqrt{g}$$

$$\sqrt{g}$$

$$r_\nu = \left( \frac{M}{M_P} \right)^{1/2} \frac{1}{\Lambda_3} C(y) \partial_{y^\nu} S(x-y)$$

$$S_0(x') = \frac{1}{\lambda}$$

$$S_1(x') = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$S_2(x') = \sum_{\mu, \nu} \frac{1}{\lambda_{\mu\nu}}$$

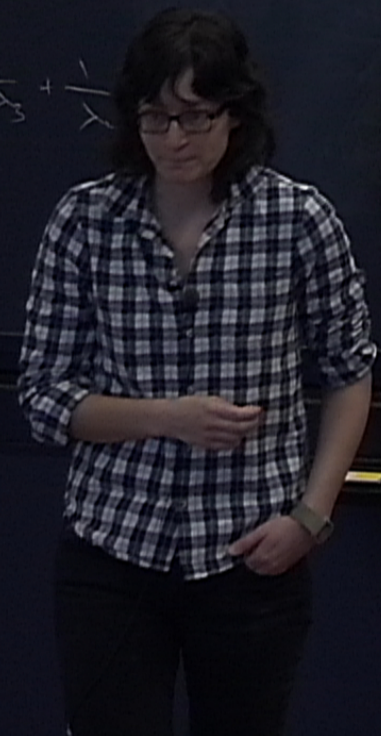
$$\sqrt{g_1} \mathcal{L}_m$$

$$\sqrt{g_2} \mathcal{L}_m$$

$$\rightarrow \frac{g_{1\mu\nu}}{M_1} T^{\mu\nu}$$

$$\rightarrow \frac{g_{2\mu\nu}}{M_2} T^{\mu\nu}$$

$$\left( \frac{g_{\mu\nu}}{M_P} \right) T^{\mu\nu}$$





$$\chi = \pi(0, \pi)$$

→ 2nd constraint dynamical  $\{C(x), H_0\} = 0$   
 $\int_{PB} = 0$

$$S = M_P^2 \int d^4x [R + 2m^2 \sum \beta_n S_n]$$

$$\sqrt{-g}$$

$$\{C(x), C(y)\} = C(x) \partial_x S(x-y)$$

$$\sqrt{g} = \sqrt{-F} \sqrt{\det g'^i{}_j} = \sqrt{-F} \sqrt{\det g''^i{}_j}$$

$$\sqrt{F} \sqrt{g} = \dots r_\nu = \left( \frac{M}{M_P} \right)^{1/2} \frac{1}{\Lambda_3} C(y) \partial_{y^\nu} S(x-y)$$

$$S_0(x') = \frac{1}{\lambda}$$

$$S_1(x') = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4}$$

$$S_2(x') = \sum_{i,j} \frac{1}{\lambda_{ij}}$$

$$\sqrt{g_1} L_m$$

$$\sqrt{g_2} L_m$$

$$\rightarrow \frac{g_{1\mu\nu}}{M_1} T^{\mu\nu}$$

$$\rightarrow \frac{g_{2\mu\nu}}{M_2} T^{\mu\nu}$$

$$\left( \frac{g_{\mu\nu}}{M_P} \right) T^{\mu\nu}$$

