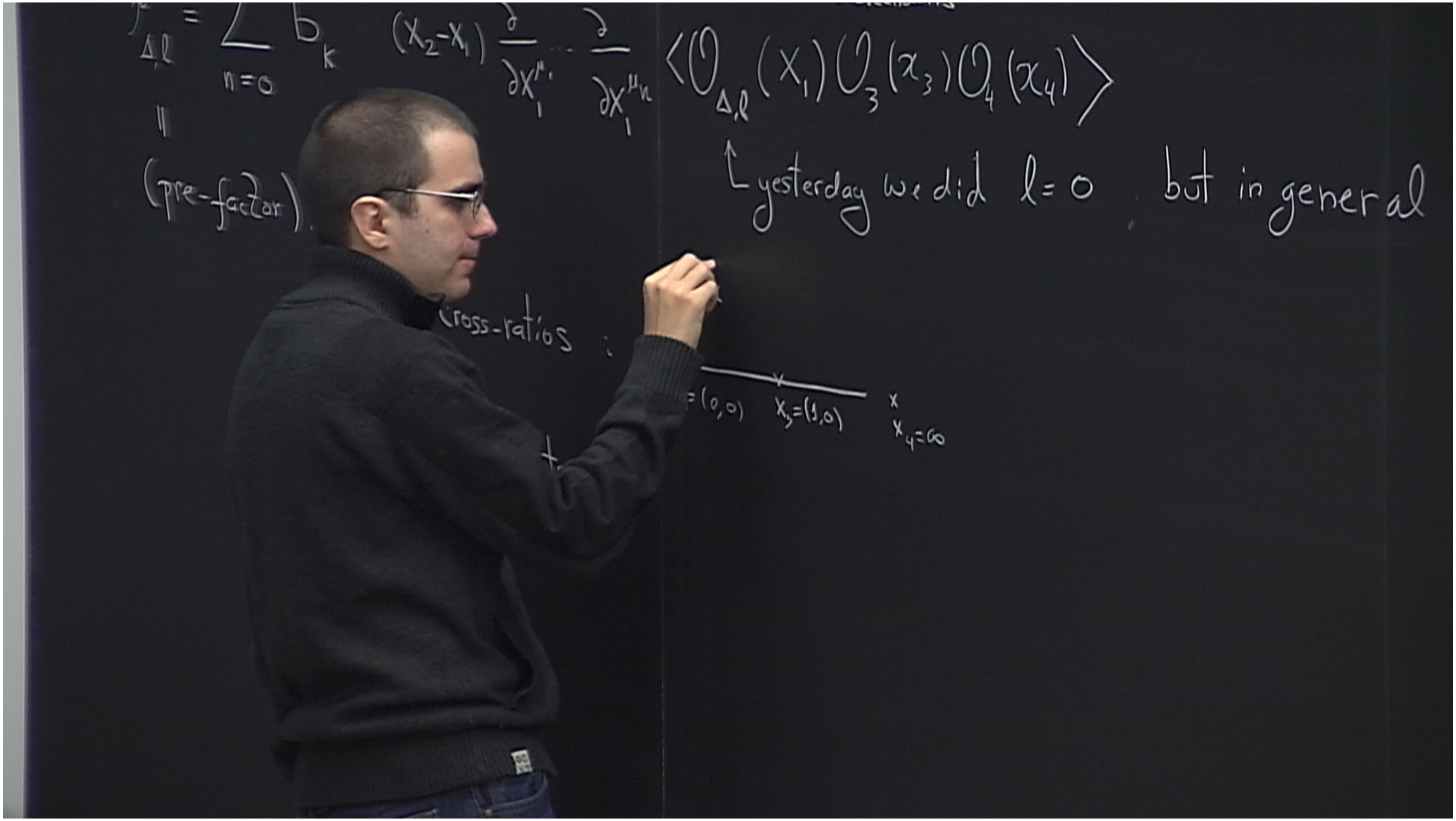
Title: Explorations in String Theory - Lecture 10

Date: Mar 23, 2012 11:30 AM

URL: http://pirsa.org/12030053

Abstract:



$\langle \mathcal{O}_{\Delta,\ell}(x_1)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$
(pre-factor)
yesterday we did $\ell = 0$ but in general
cross-ratios :
$x_3=(1,0)$
$x_4=\infty$

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_4(x_4) \rangle = \sum_{K} C_{12K}\, C_{K34}\, \mathcal{F}_K^{(12)(34)}$$

K: primaries

$\mathcal{F}_K$: conformal block containing all descendents

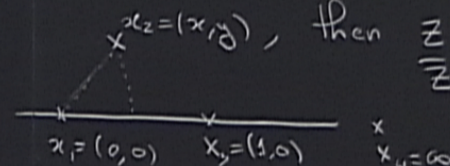
$$\mathcal{F}_{\Delta,\ell} = \sum_{n=0}^{\infty} b_K^{\mu_1 \cdots \mu_n} (x_2 - x_1) \frac{\partial}{\partial x_1^{\mu_1}} \cdots \frac{\partial}{\partial x_1^{\mu_n}} \langle \mathcal{O}_{\Delta,\ell}(x_1)\, \mathcal{O}_3(x_3)\, \mathcal{O}_4(x_4) \rangle$$

$\mathcal{O}_{\Delta,\ell}$: yesterday we did $\ell = 0$ but in general

$$\mathcal{F}_{\Delta,\ell} = (\text{pre-factor}) \times G_{\Delta,\ell}(\underbrace{z, \bar{z}}_{\text{cross-ratios}})$$

$G_{\Delta,\ell}$: Some (product of) Hypergeometrics

cross-ratios :



, then $z = x + iy$, $\bar{z} = x - iy$

so 1 complex CR or 2 scalar CR's

slight confusion yesterday?

general

CR

CR's

sion ?

day ?

$$0 \neq \langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_{\underbrace{\mu_1\cdots\mu_\ell}_{\text{sym traceless}}}(x_3) \rangle$$

fixed by Conf symmetry

only they can appear for $\mathcal{O}_1$ and $\mathcal{O}_2$ Scalars

sym traceless ← spin ℓ

(scalar is $\ell = 0$)

Casimir of the Conformal group

$$\parallel$$

$$\vec{J}^2 = -D^2 - \frac{1}{2}\left(K_\mu P_\mu + P_\mu K_\mu\right) + \frac{1}{2} M_{\mu\nu} M^{\mu\nu}$$

$$\vec{J}^2 |\mathcal{O}_{\Delta,l}\rangle = \left(\Delta(\Delta-d) + l(l+d-2)\right) |\mathcal{O}_{\Delta,l}\rangle$$

$J = (D, K, P, M)$

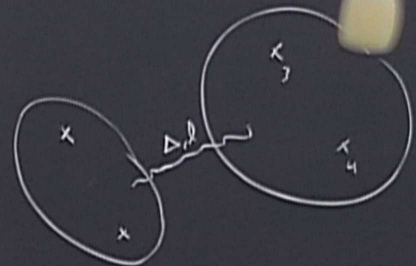
$M_{\mu\nu} M^{\mu\nu}$

$) \, |O_{\Delta,\ell}\rangle$

acts on x_4

$$\left[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + J_A^{(4)} \right] \langle O_1 \dots O_4 \rangle = 0 \quad \Longleftarrow \text{Conf invariance}$$

$$\mathcal{F}_K^{(12)(34)} = 0$$



$$\left[J^{(3)} + J^{(4)} \right]^2 \mathcal{F}_K^{(12)(34)} = \left(J^{(3)} + J^{(4)} \right)^2 \text{ acting \#}$$

$$\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_4(x_4)\rangle = \sum_{k} C_{12k}\, C_{k34}\, \mathcal{F}_k^{(12)(34)}$$

k: primaries Δ, ℓ

$\mathcal{F}_k$: conformal block containing all descendents

b_k: fixed by conf. sym

$$\mathcal{F}_{\Delta,\ell} = \sum_{n=0}^{\infty} b_k^{\mu_1\cdots\mu_n}(x_2-x_1)\frac{\partial}{\partial x_1^{\mu_1}}\cdots\frac{\partial}{\partial x_1^{\mu_n}}\,\#\,\langle \mathcal{O}_{\Delta,\ell}(x_1)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle$$

$\mathcal{O}_{\Delta,\ell}$: yesterday we did $\ell = 0$ but i

$$\mathcal{F}_{\Delta,\ell} = (\text{pre-factor}) \times G_{\Delta,\ell}\underbrace{(z,\bar{z})}_{\text{cross-ratios}}$$

$G_{\Delta,\ell}$: Some (product of) Hypergeometrics

cross-ratios:

$x_2 = (x,y)$, then $z = x+iy$, $\bar{z} = x-iy$

$x_1 = (0,0)$ $\quad x_3 = (1,0)$ $\quad x_4 = \infty$

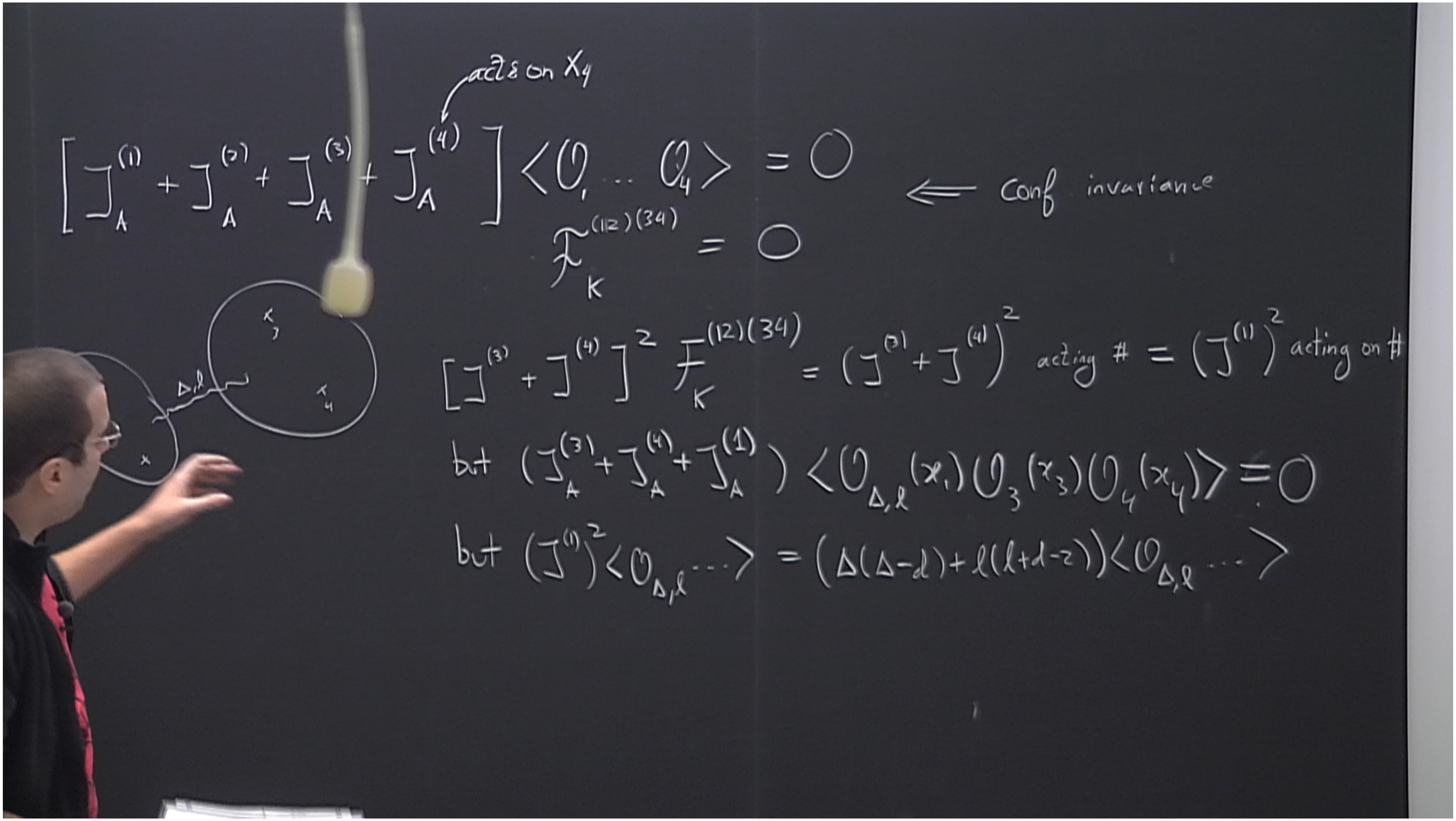
so 1 comple

or

2 real

slight con

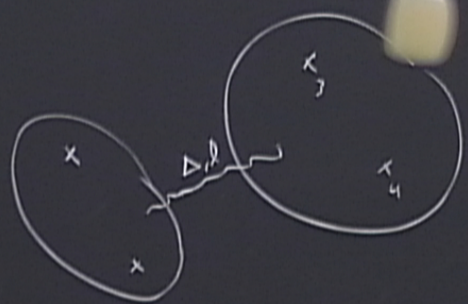
yest


acts on x_4
$\left[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + J_A^{(4)}\right] \langle O_1 \dots O_4 \rangle = 0 \Leftarrow$ Conf invariance
$\mathcal{F}_K^{(12)(34)} = 0$
x_3
x_4
Δ,ℓ
$\left[J^{(3)} + J^{(4)}\right]^2 \mathcal{F}_K^{(12)(34)} = \left(J^{(3)} + J^{(4)}\right)^2$ acting # $= \left(J^{(1)}\right)^2$ acting on #
but $\left(J_A^{(3)} + J_A^{(4)} + J_A^{(1)}\right) \langle O_{\Delta,\ell}(x_1) O_3(x_3) O_4(x_4) \rangle = 0$
but $\left(J^{(1)}\right)^2 \langle O_{\Delta,\ell} \dots \rangle = \left(\Delta(\Delta-d) + \ell(\ell+d-2)\right) \langle O_{\Delta,\ell} \dots \rangle$

acts on x_4

$$\left[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + J_A^{(4)}\right]\langle \mathcal{O}_1 \cdots \mathcal{O}_4 \rangle = 0 \quad \Longleftarrow \text{Conf. invariance}$$

$$\mathcal{F}_K^{(12)(34)} = 0$$

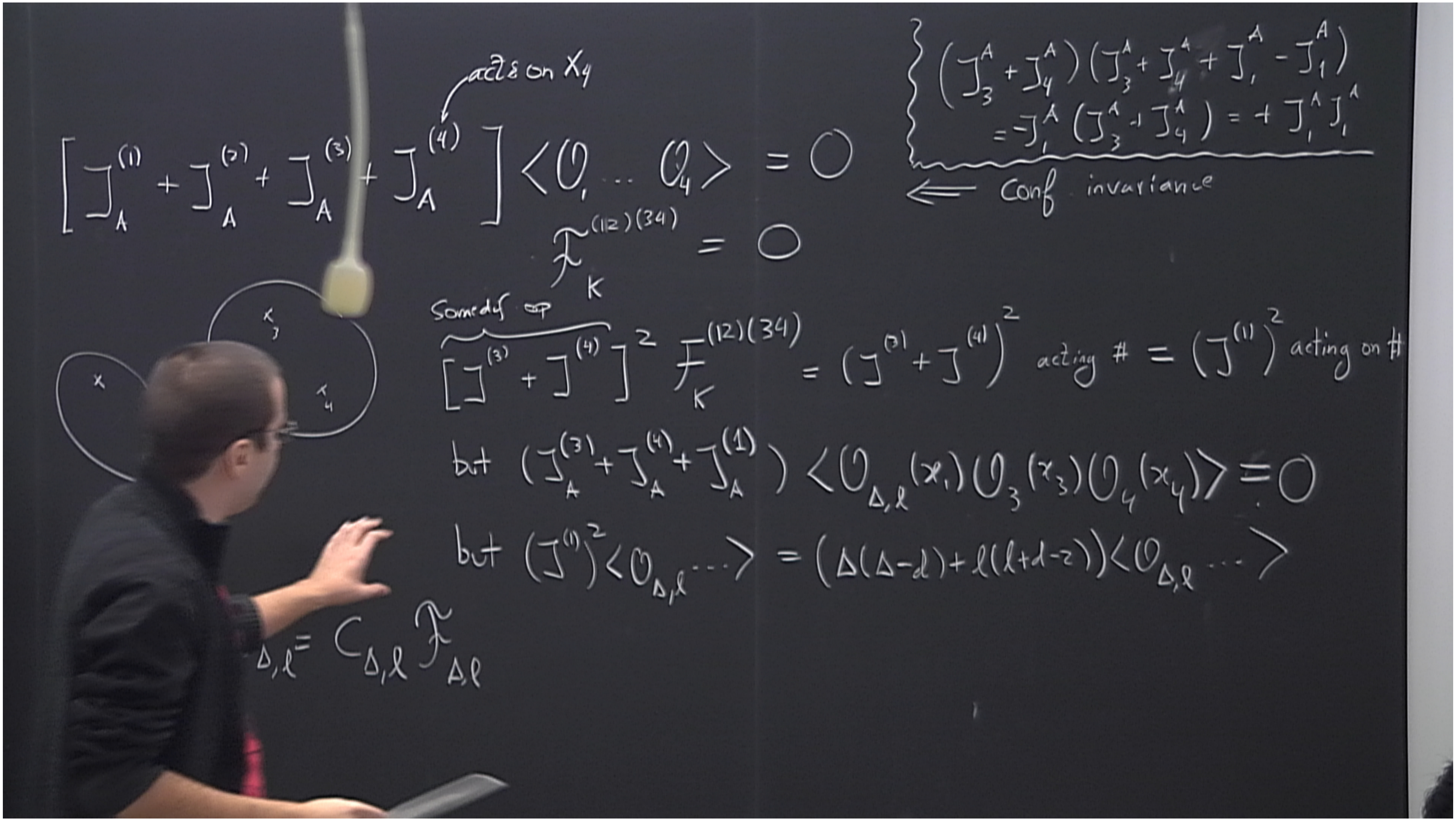


$$\underbrace{\left[J^{(3)} + J^{(4)}\right]^2}_{\text{Some dif op}} \mathcal{F}_K^{(12)(34)} = \left(J^{(3)} + J^{(4)}\right)^2 \text{ acting } \# = \left(J^{(1)}\right)^2 \text{ acting on } \#$$

but $\left(J_A^{(3)} + J_A^{(4)} + J_A^{(1)}\right)\langle \mathcal{O}_{\Delta,\ell}(x_1)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle = 0$

but $\left(J^{(1)}\right)^2 \langle \mathcal{O}_{\Delta,\ell} \cdots \rangle = \left(\Delta(\Delta - d) + \ell(\ell + d - 2)\right)\langle \mathcal{O}_{\Delta,\ell} \cdots \rangle$

$$[\text{Dif op}] \circ \mathcal{F}_{\Delta,\ell} = C_{\Delta,\ell}\, \mathcal{F}_{\Delta,\ell}$$

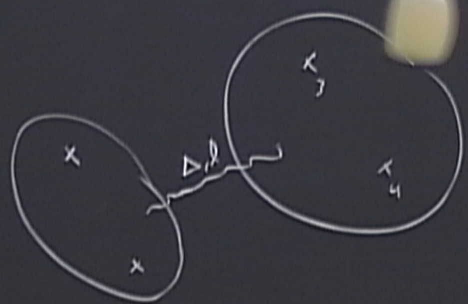

acts on x_4
$\left[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + J_A^{(4)}\right] \langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle = 0$
$\mathcal{F}_K^{(12)(34)} = 0$
$(J_3^A + J_4^A)(J_3^A + J_4^A + J_1^A - J_1^A)$
$= -J_1^A (J_3^A + J_4^A) = + J_1^A J_1^A$
$\Longleftarrow$ Conf. invariance
Some diff op
$\left[J^{(3)} + J^{(4)}\right]^2 \mathcal{F}_K^{(12)(34)} = (J^{(3)} + J^{(4)})^2$ acting # $= (J^{(1)})^2$ acting on #
but $(J_A^{(3)} + J_A^{(4)} + J_A^{(1)}) \langle \mathcal{O}_{\Delta,\ell}(x_1) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle = 0$
but $(J^{(1)})^2 \langle \mathcal{O}_{\Delta,\ell} \ldots \rangle = (\Delta(\Delta - d) + \ell(\ell + d - 2)) \langle \mathcal{O}_{\Delta,\ell} \ldots \rangle$
$_{\Delta,\ell} = C_{\Delta,\ell} \mathcal{F}_{\Delta,\ell}$

$$\Big\{ (J_3^A + J_4^A)(J_3^A + J_4^A + J_1^A - J_1^A)$$
$$= -J_1^A (J_3^A + J_4^A) = + J_1^A J_1^A$$

$\Longleftarrow$ Conf invariance

$$\Big[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + \overset{\text{acts on } x_4}{J_A^{(4)}}\Big] \langle O_1 \dots O_4 \rangle = 0$$

$$\mathcal{F}_K^{(12)(34)} = 0$$



$$\overbrace{\Big[J^{(3)} + J^{(4)}\Big]^2}^{\text{Some dif op}} \mathcal{F}_K^{(12)(34)} = (J^{(3)} + J^{(4)})^2 \text{ acting } \# = (J^{(1)})^2 \text{ acting on } \#$$

but $(J_A^{(3)} + J_A^{(4)} + J_A^{(1)}) \langle O_{\Delta,l}(x_1) O_3(x_3) O_4(x_4) \rangle = 0$

but $(J^{(1)})^2 \langle O_{\Delta,l} \dots \rangle = (\Delta(\Delta-d) + l(l+d-2)) \langle O_{\Delta,l} \dots \rangle$

$$[\text{Dif op}] \cdot \mathcal{F}_{\Delta,l} = C_{\Delta,l} \mathcal{F}_{\Delta,l}$$

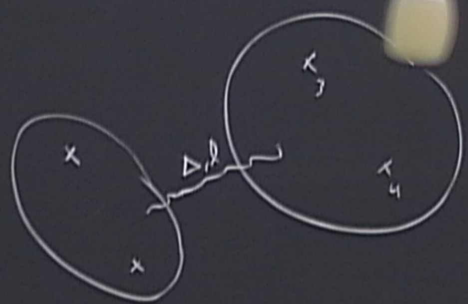
$$\left(J_3^A + J_4^A\right)\left(J_3^A + J_4^A + J_1^A - J_1^A\right)$$
$$= -J_1^A\left(J_3^A + J_4^A\right) = + J_1^A J_1^A$$

$\Longleftarrow$ Conf. invariance

acts on x_4

$$\left[J_A^{(1)} + J_A^{(2)} + J_A^{(3)} + J_A^{(4)}\right]\langle \mathcal{O}_1 \dots \mathcal{O}_4\rangle = 0$$

$$\mathcal{F}_K^{(12)(34)} = 0$$



$$\underbrace{\left[J^{(3)} + J^{(4)}\right]^2}_{\text{Some dif op}} \mathcal{F}_K^{(12)(34)} = \left(J^{(3)} + J^{(4)}\right)^2 \text{ acting } \# = \left(J^{(1)}\right)^2 \text{ acting on } \#$$

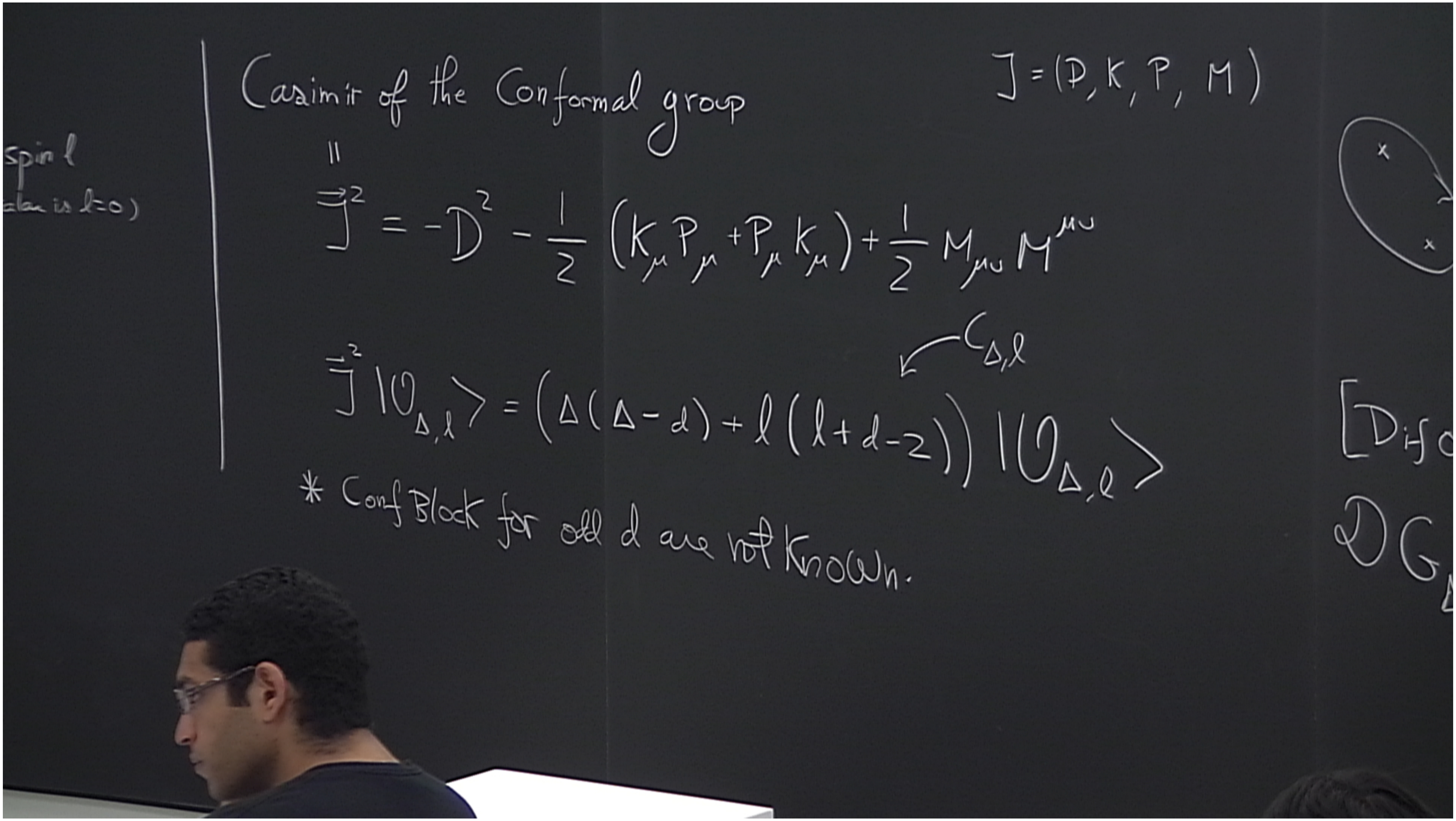
but $\left(J_A^{(3)} + J_A^{(4)} + J_A^{(1)}\right)\langle \mathcal{O}_{\Delta,l}(x_1)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle = 0$

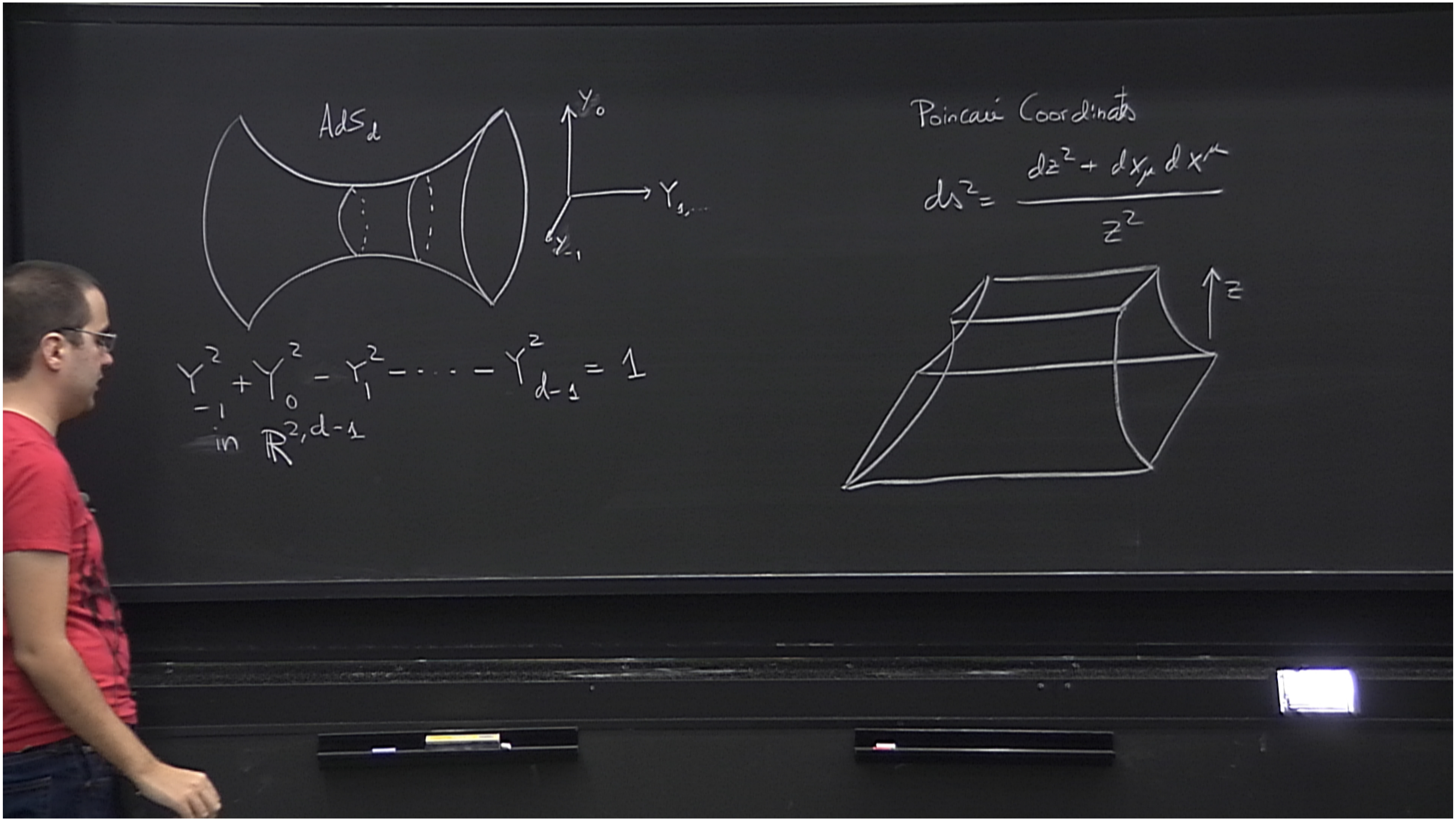
but $\left(J^{(1)}\right)^2\langle \mathcal{O}_{\Delta,l}\dots\rangle = \left(\Delta(\Delta-d) + l(l+d-2)\right)\langle \mathcal{O}_{\Delta,l}\dots\rangle$

$$[\text{Dif op}] \circ \mathcal{F}_{\Delta,l} = C_{\Delta,l}\,\mathcal{F}_{\Delta,l}$$

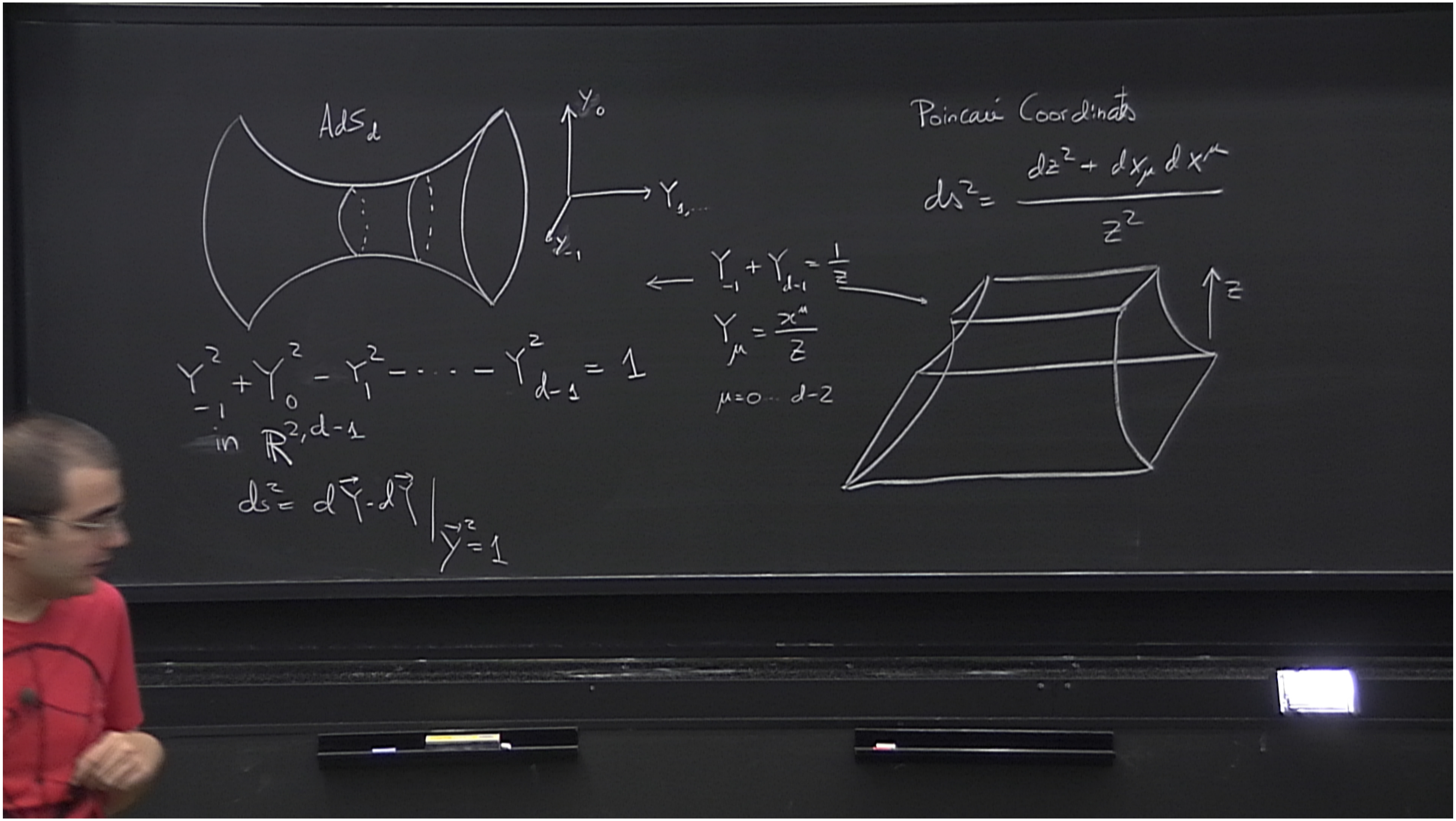
$$\mathcal{D}\,G_{\Delta,l} = C_{\Delta,l}\,G_{\Delta,l}$$

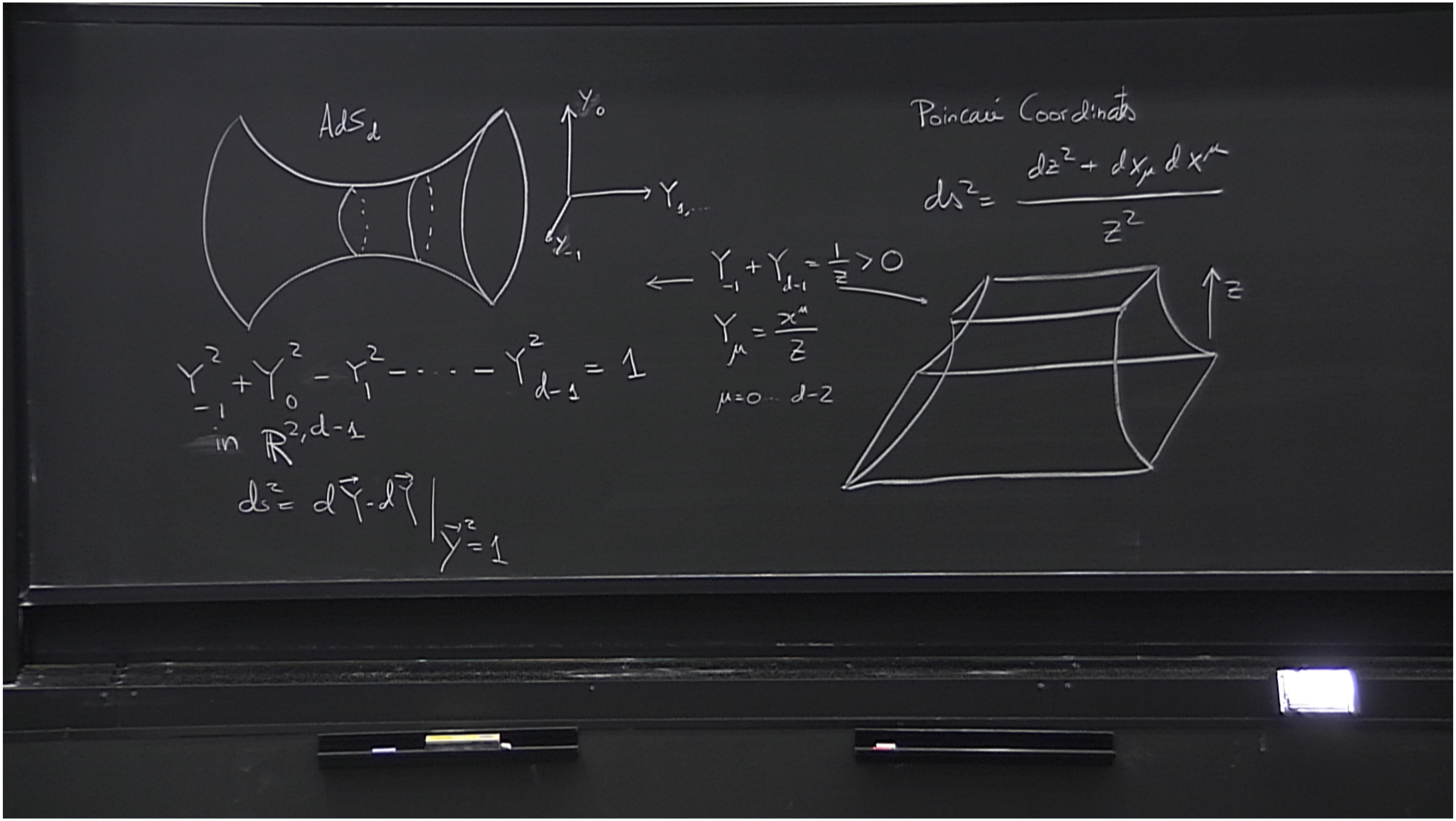
$$\mathcal{D} = z^2(1-z^2)\frac{\partial^2}{\partial z^2} + \frac{(\Delta_1-\Delta_2)(\Delta_3-\Delta_4)}{4}(z+\bar{z}) + \dots$$

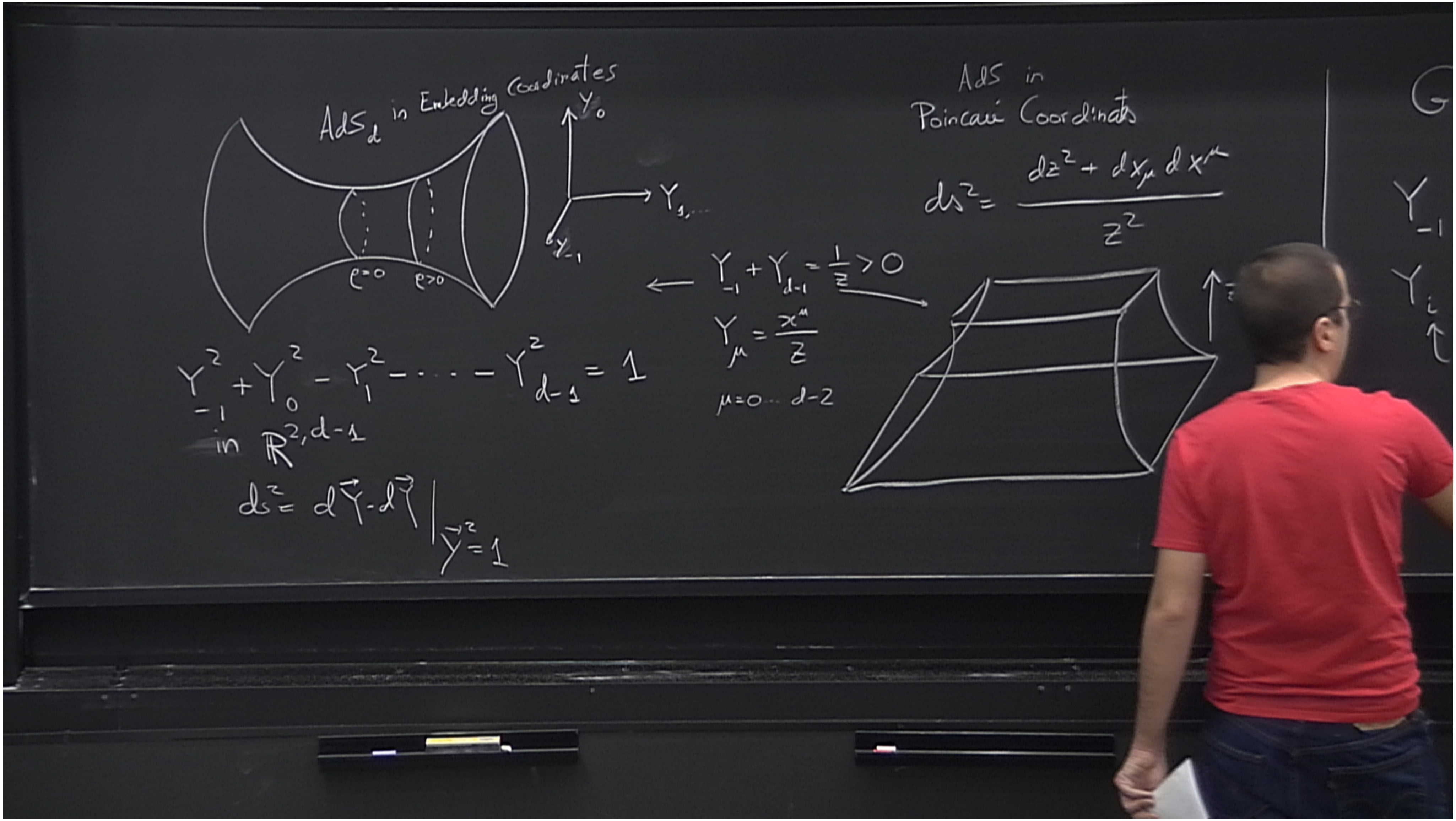

Casimir of the Conformal group
$J = (D, K, P, M)$
spin l
$\vec{J}^2 = -D^2 - \frac{1}{2}(K_\mu P_\mu + P_\mu K_\mu) + \frac{1}{2} M_{\mu\nu} M^{\mu\nu}$
$C_{\Delta,l}$
$\vec{J}^2 |\mathcal{O}_{\Delta,l}\rangle = (\Delta(\Delta - d) + l(l+d-2)) |\mathcal{O}_{\Delta,l}\rangle$
* Conf Block for odd d are not known.

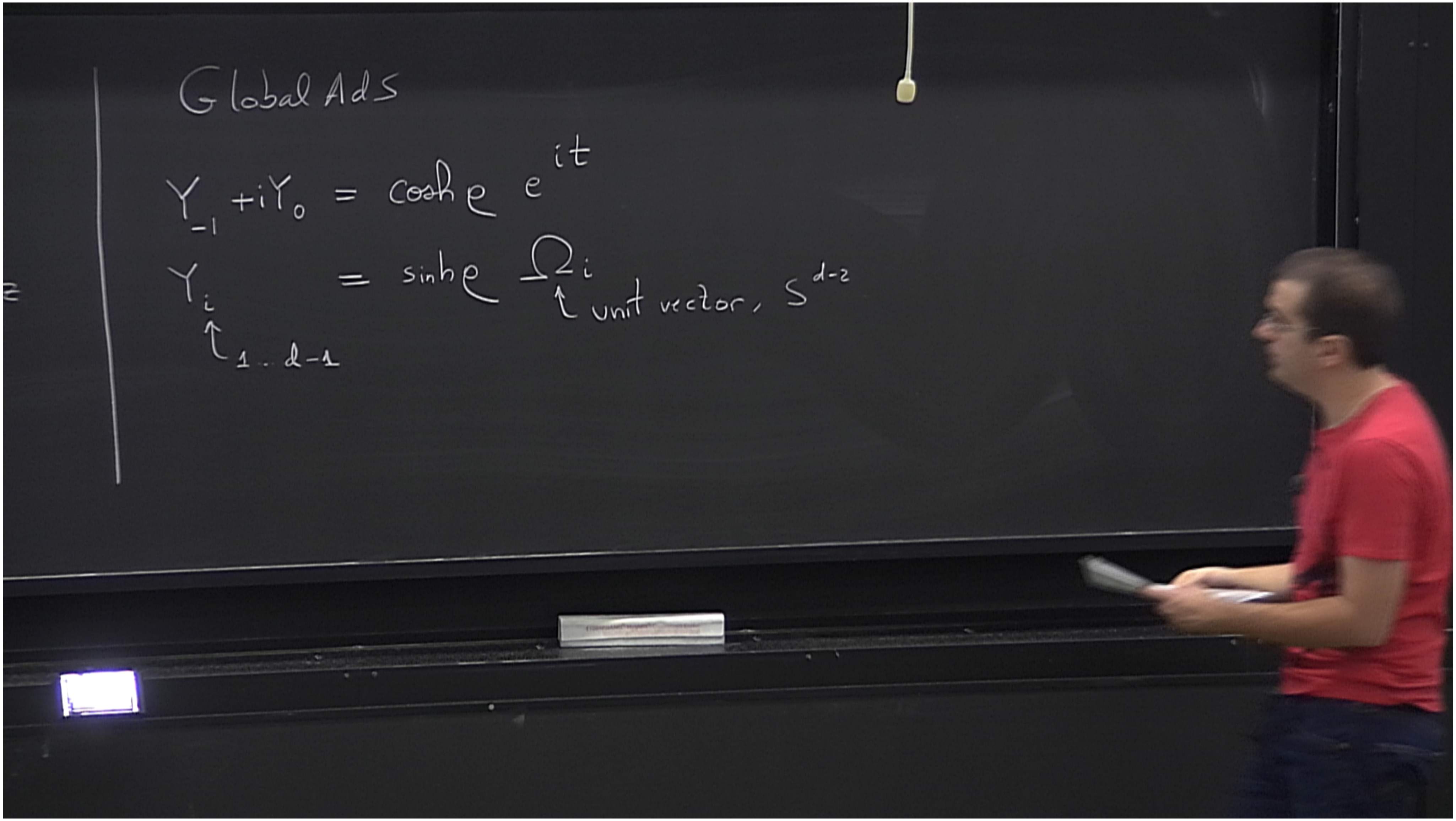


AdS_d
Y_0
$Y_1, \ldots$
Y_{-1}
$Y_{-1}^2 + Y_0^2 - Y_1^2 - \cdots - Y_{d-1}^2 = 1$
in $\mathbb{R}^{2,d-1}$
Poincaré Coordinates
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
z


AdS$_d$
Y_0
$Y_1, \ldots$
Y_{-1}
$Y_{-1}^2 + Y_0^2 - Y_1^2 - \cdots - Y_{d-1}^2 = 1$
in $\mathbb{R}^{2,d-1}$
$ds^2 = d\vec{Y} \cdot d\vec{Y} \Big|_{\vec{Y}^2 = 1}$
$Y_{-1} + Y_{d-1} = \frac{1}{z}$
$Y_\mu = \frac{x^\mu}{z}$
$\mu = 0 \cdots d-2$
Poincaré Coordinates
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
z


AdS_d
Y_0
Y_1, ...
Y_-1
Y_-1^2 + Y_0^2 - Y_1^2 - . . . - Y_{d-1}^2 = 1
in R^{2,d-1}
ds^2 = dY · dY |_{Y^2 = 1}
Y_-1 + Y_{d-1} = 1/z > 0
Y_μ = x^μ / z
μ = 0 .. d-2
Poincaré Coordinates
ds^2 = (dz^2 + dx_μ dx^μ) / z^2
z


AdS$_d$ in Embedding Coordinates
Y_0
$Y_1, \ldots$
Y_{d-1}
$\rho=0$
$\rho>0$
$Y_{-1}^2 + Y_0^2 - Y_1^2 - \cdots - Y_{d-1}^2 = 1$
in $\mathbb{R}^{2,d-1}$
$ds^2 = d\vec{Y}\cdot d\vec{Y}\big|_{\vec{Y}^2=1}$
$Y_{-1} + Y_{d-1} = \frac{1}{z} > 0$
$Y_\mu = \frac{x^\mu}{z}$
$\mu = 0 \ldots d-2$
AdS in
Poincaré Coordinates
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
Y_{-1}
Y_i


Global AdS
$Y_{-1} + iY_0 = \cosh\rho \, e^{it}$
$Y_i = \sinh\rho \, \Omega_i$
unit vector, S^{d-2}
$1 .. d-1$

Global AdS

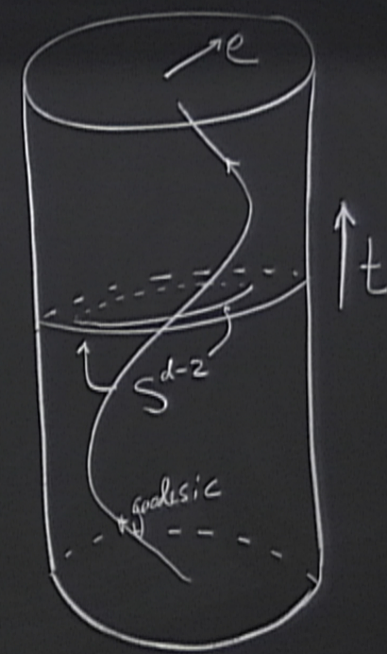
$$Y_{-1} + iY_0 = \cosh\rho \; e^{it}$$

$$Y_i = \sinh\rho \; \Omega_i$$

Ω_i: unit vector, S^{d-2}

i: $1 \ldots d-1$

$$ds^2 = -\cosh^2\rho \; dt^2 + d\rho^2 + \sinh^2\rho \; d\Omega^2_{S^{d-2}}$$



Global AdS

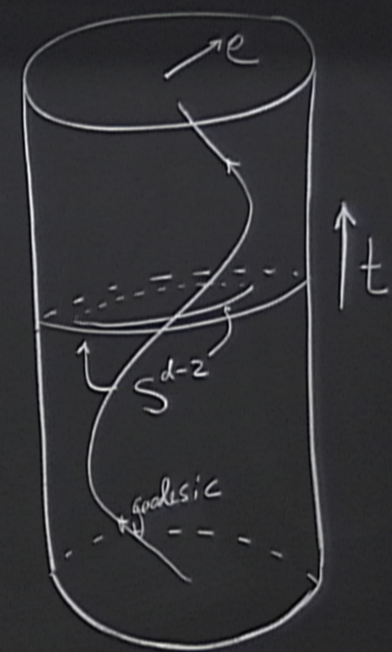
$$Y_{-1} + iY_0 = \cosh\rho \; e^{it} \qquad \text{naively} \quad t \sim t + 2\pi$$

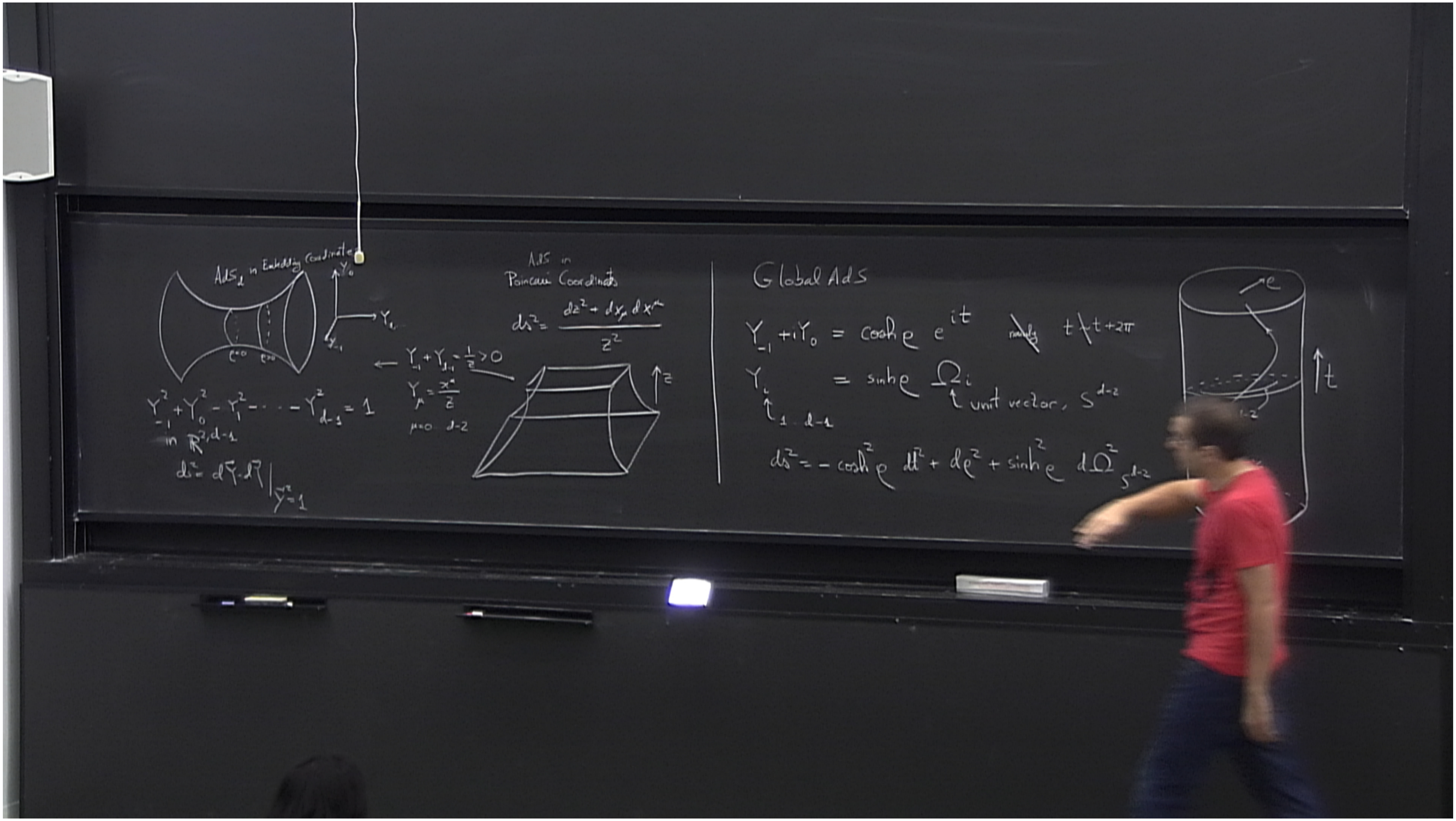
$$Y_i = \sinh\rho \; \Omega_i$$

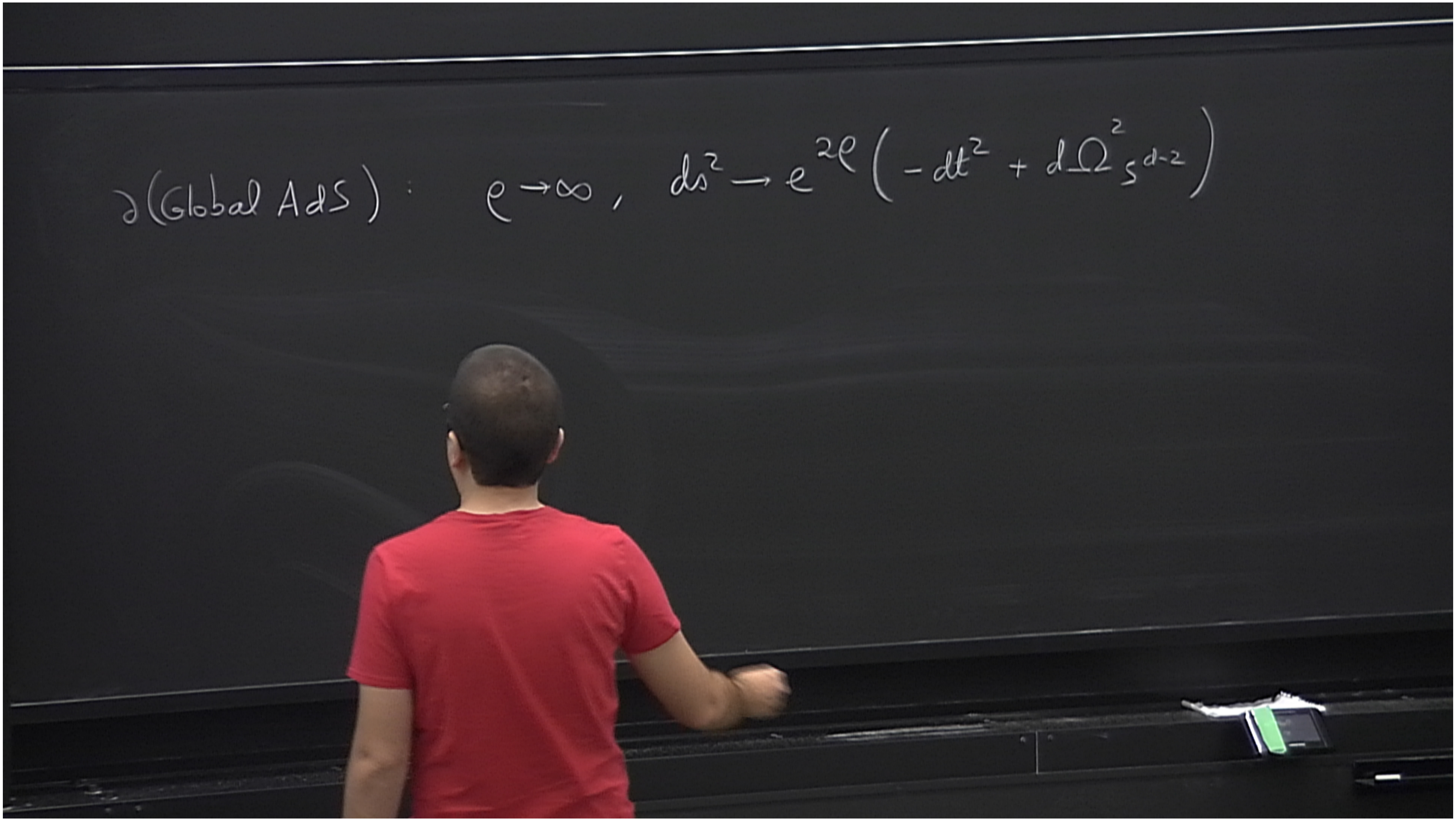
Ω_i: unit vector, S^{d-2}

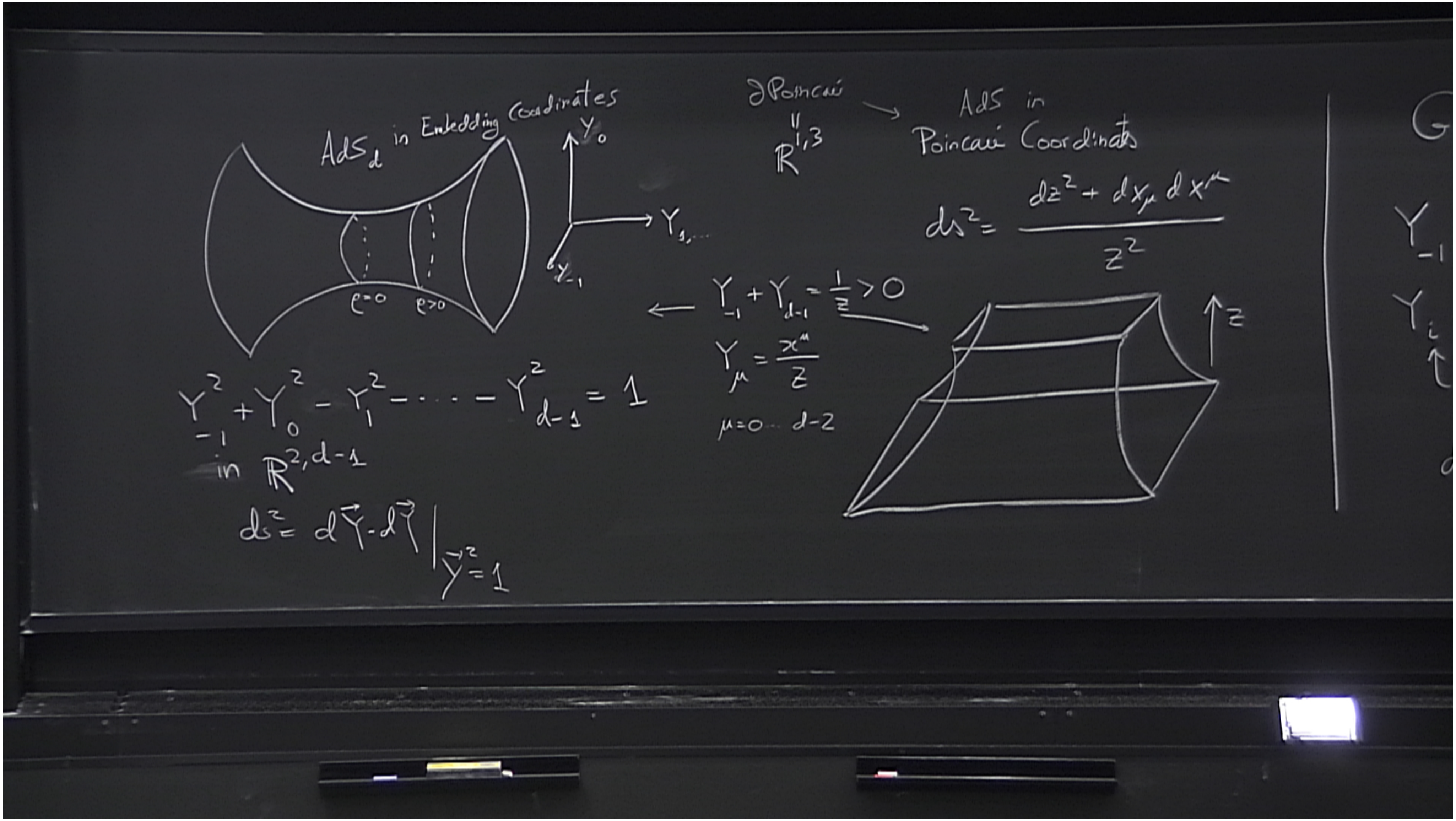
i: $1 \,..\, d-1$

$$ds^2 = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, d\Omega^2_{S^{d-2}}$$

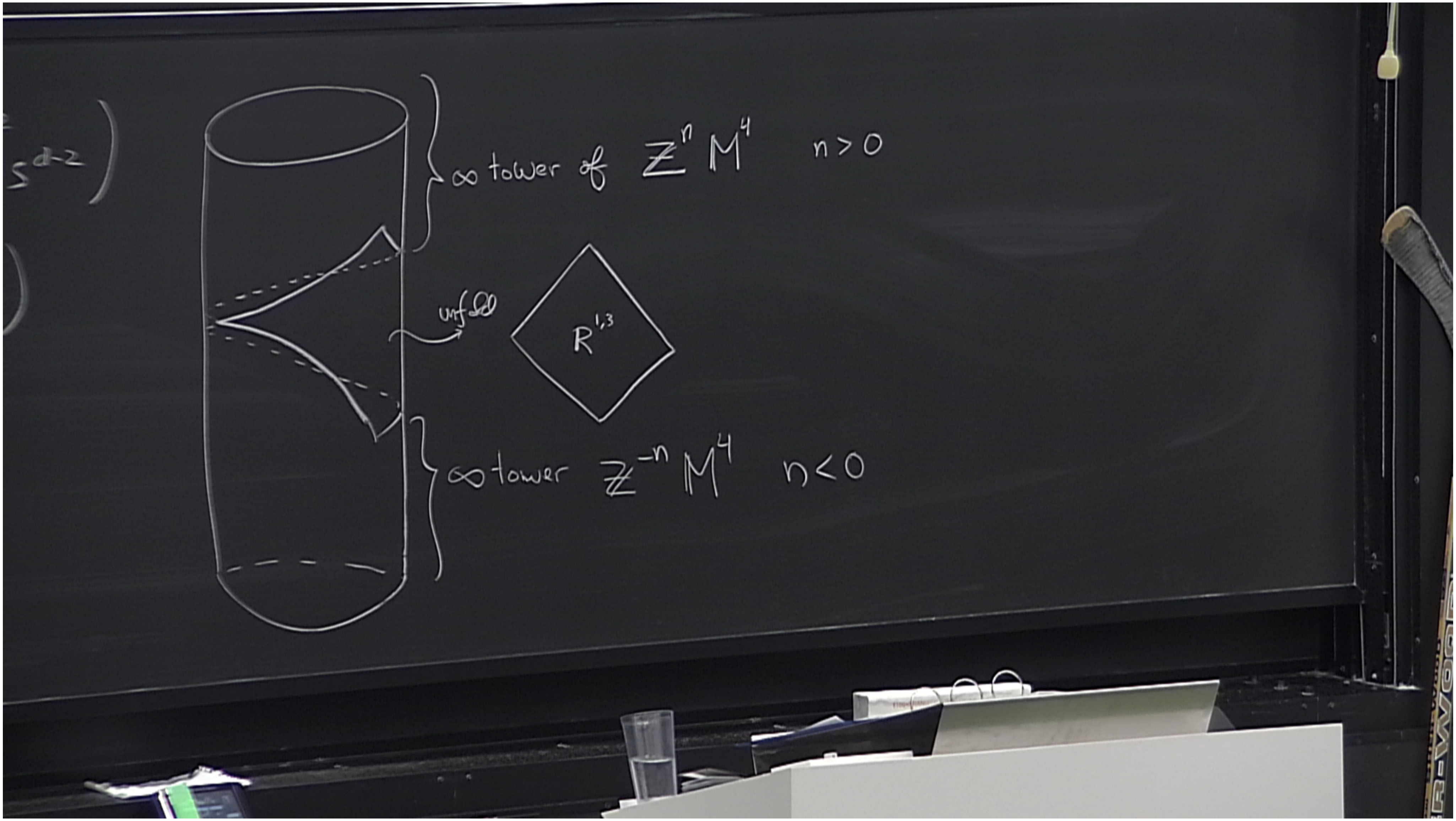


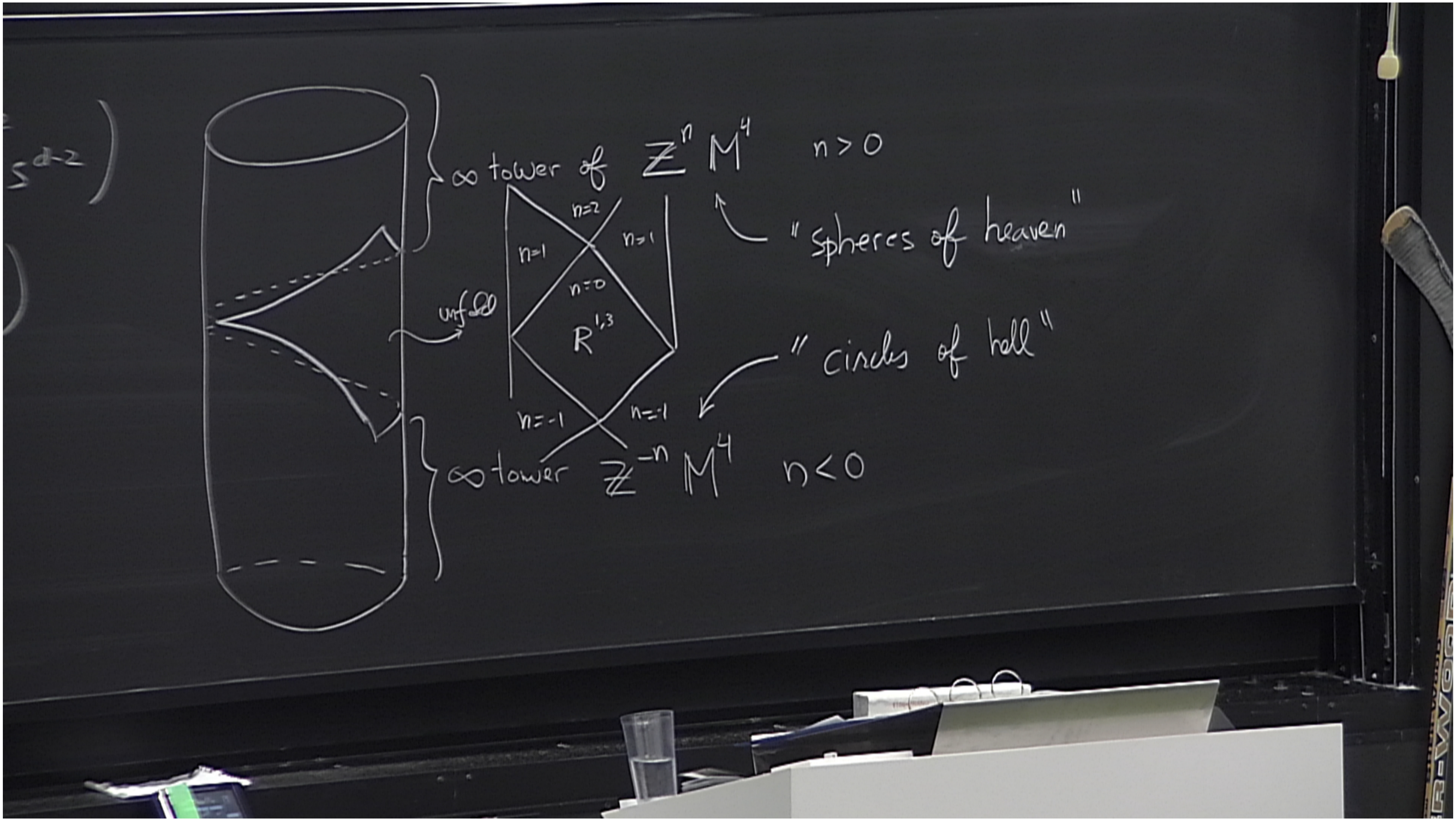

AdS_d in Embedding Coordinates
Y_0
Y_1
Y_{-1}
Y_{-1}^2 + Y_0^2 - Y_1^2 - ... - Y_{d-1}^2 = 1
in R^{2,d-1}
AdS in Poincaré Coordinates
ds^2 = (dz^2 + dx_μ dx^μ) / z^2
Y_{-1} + Y_{d-1} = 1/z > 0
Y_μ = x^μ / z
μ = 0 ... d-2
z
Global AdS
Y_{-1} + i Y_0 = cosh ρ e^{it}
Y_i = sinh ρ Ω_i
i = 1 ... d-1
unit vector, S^{d-2}
ds^2 = - cosh^2 ρ dt^2 + dρ^2 + sinh^2 ρ dΩ^2_{S^{d-2}}
t
ρ

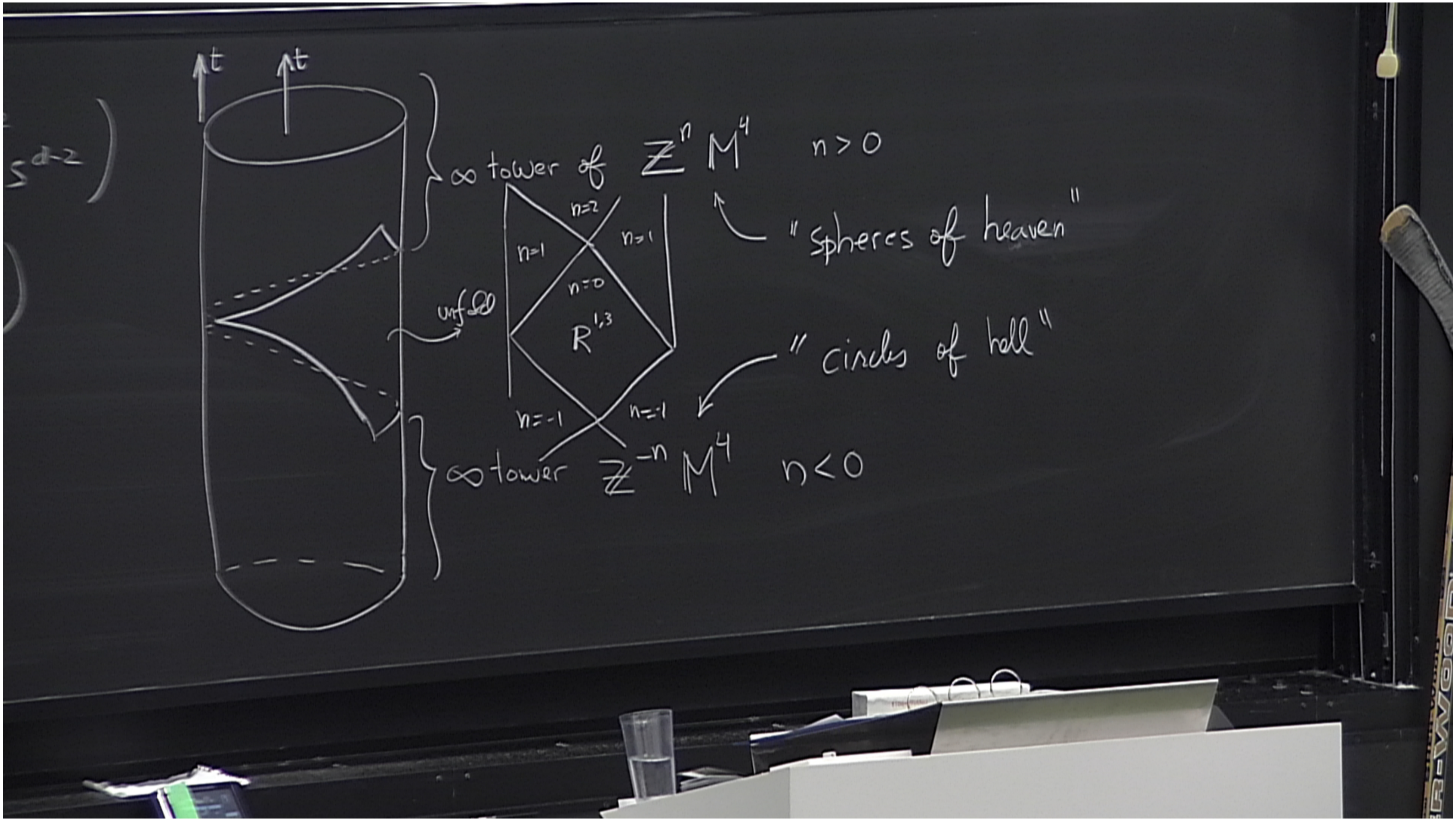

∂(Global AdS) : $\rho \to \infty$, $ds^2 \to e^{2\rho}\left(-dt^2 + d\Omega^2_{S^{d-2}}\right)$

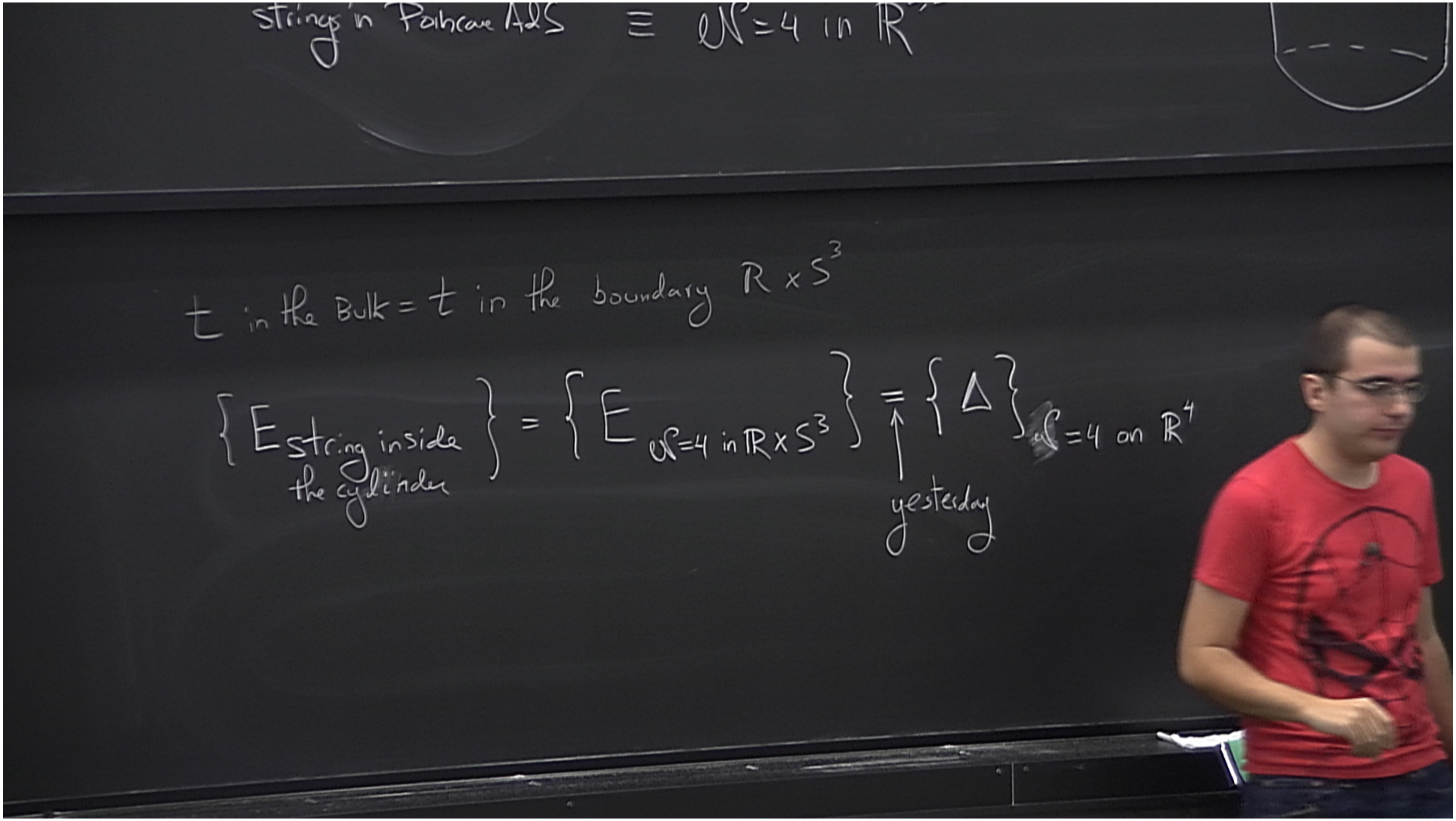


AdS_d in Embedding Coordinates
Y_0
$Y_1, \dots$
Y_{-1}
$\rho = 0$
$\rho > 0$
$Y_{-1}^2 + Y_0^2 - Y_1^2 - \dots - Y_{d-1}^2 = 1$
in $\mathbb{R}^{2,d-1}$
$ds^2 = d\vec{Y} \cdot d\vec{Y} \big|_{\vec{Y}^2 = 1}$
∂Poincaré $= \mathbb{R}^{1,3}$
AdS in Poincaré Coordinates
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
$Y_{-1} + Y_{d-1} = \frac{1}{z} > 0$
$Y_\mu = \frac{x^\mu}{z}$
$\mu = 0 \dots d-2$
z


S^{d-2}
∞ tower of $Z^n M^4$ $n>0$
unfold
$R^{1,3}$
∞ tower $Z^{-n} M^4$ $n<0$


S^{d-2}
∞ tower of $Z^n M^4$
$n > 0$
"spheres of heaven"
$n=2$
$n=1$
$n=1$
$n=0$
unfold
$R^{1,3}$
"circles of hell"
$n=-1$
$n=-1$
∞ tower $Z^{-n} M^4$
$n < 0$


t
t
∞ tower of Z^n M^4 n > 0
n=2
n=1
n=1
n=0
R^{1,3}
unfold
" spheres of heaven "
" circles of hell "
n=-1
n=-1
∞ tower Z^{-n} M^4 n < 0
S^{d-2}

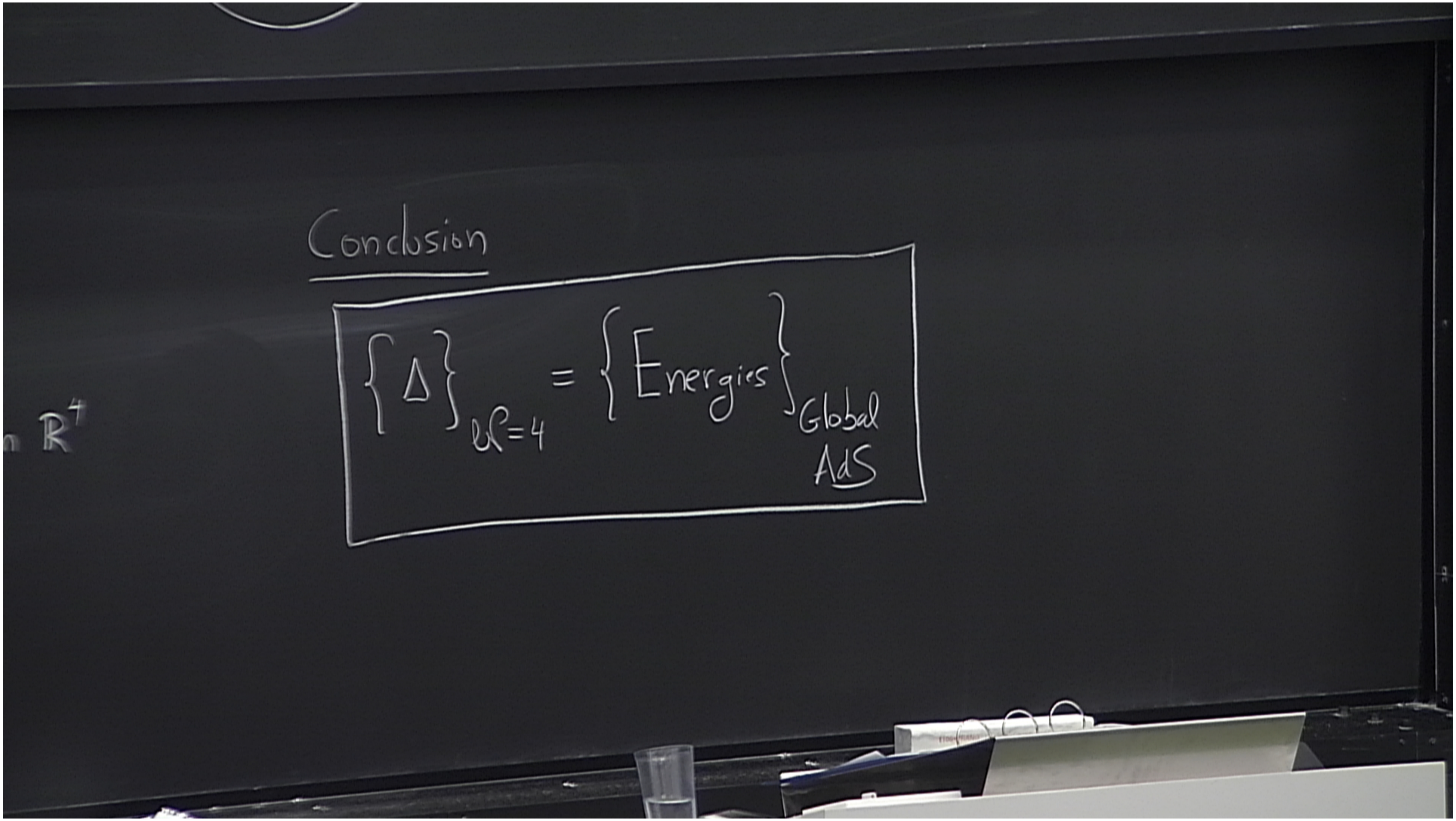

strings in Poincare AdS ≡ N=4 in R
t in the Bulk = t in the boundary R × S³
{E string inside the cylinder} = {E N=4 in R×S³} = {Δ} N=4 on R⁴
yesterday

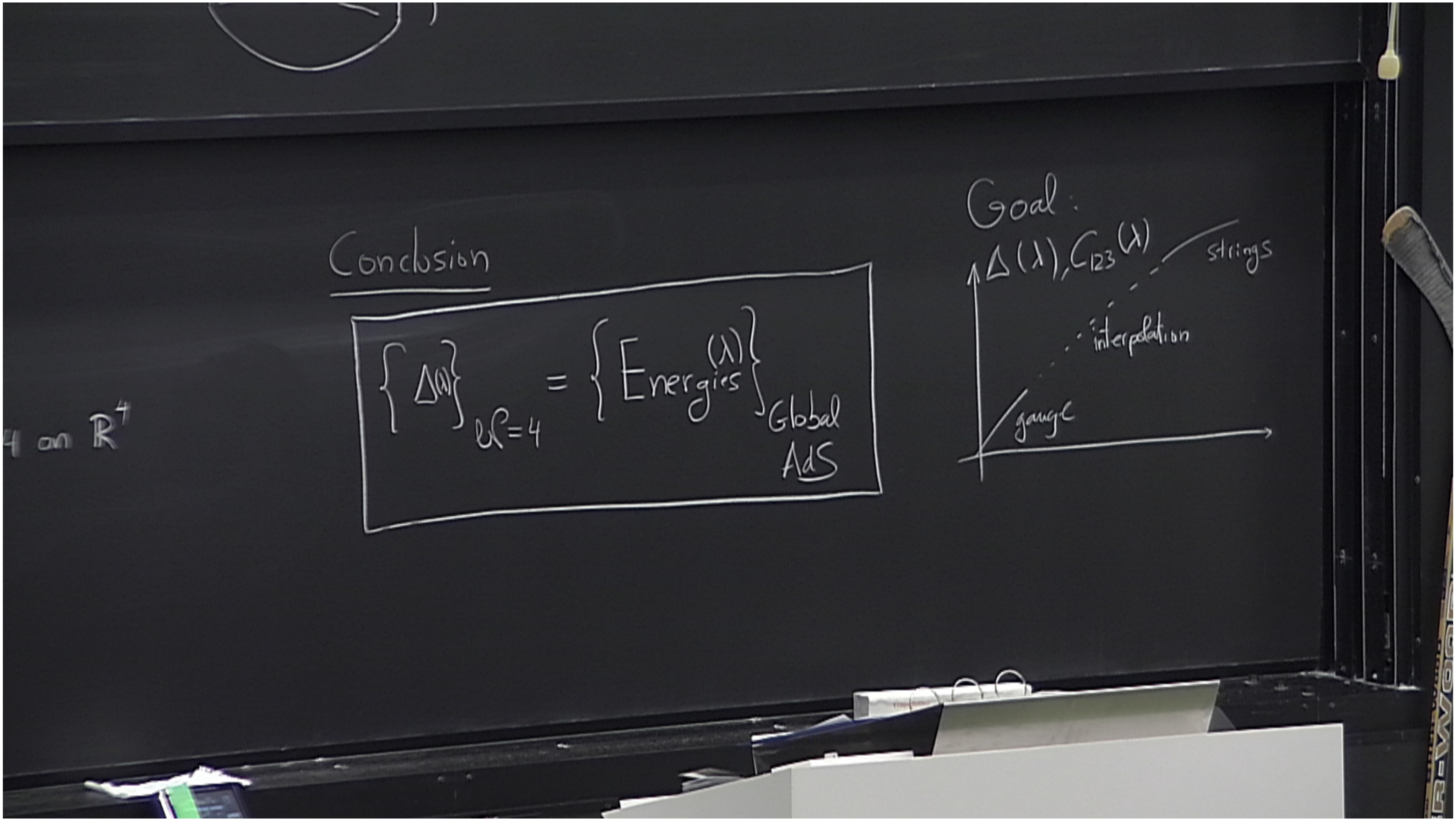
strings in Poincaré AdS $\equiv$ $\mathcal{N}=4$ in $\mathbb{R}$

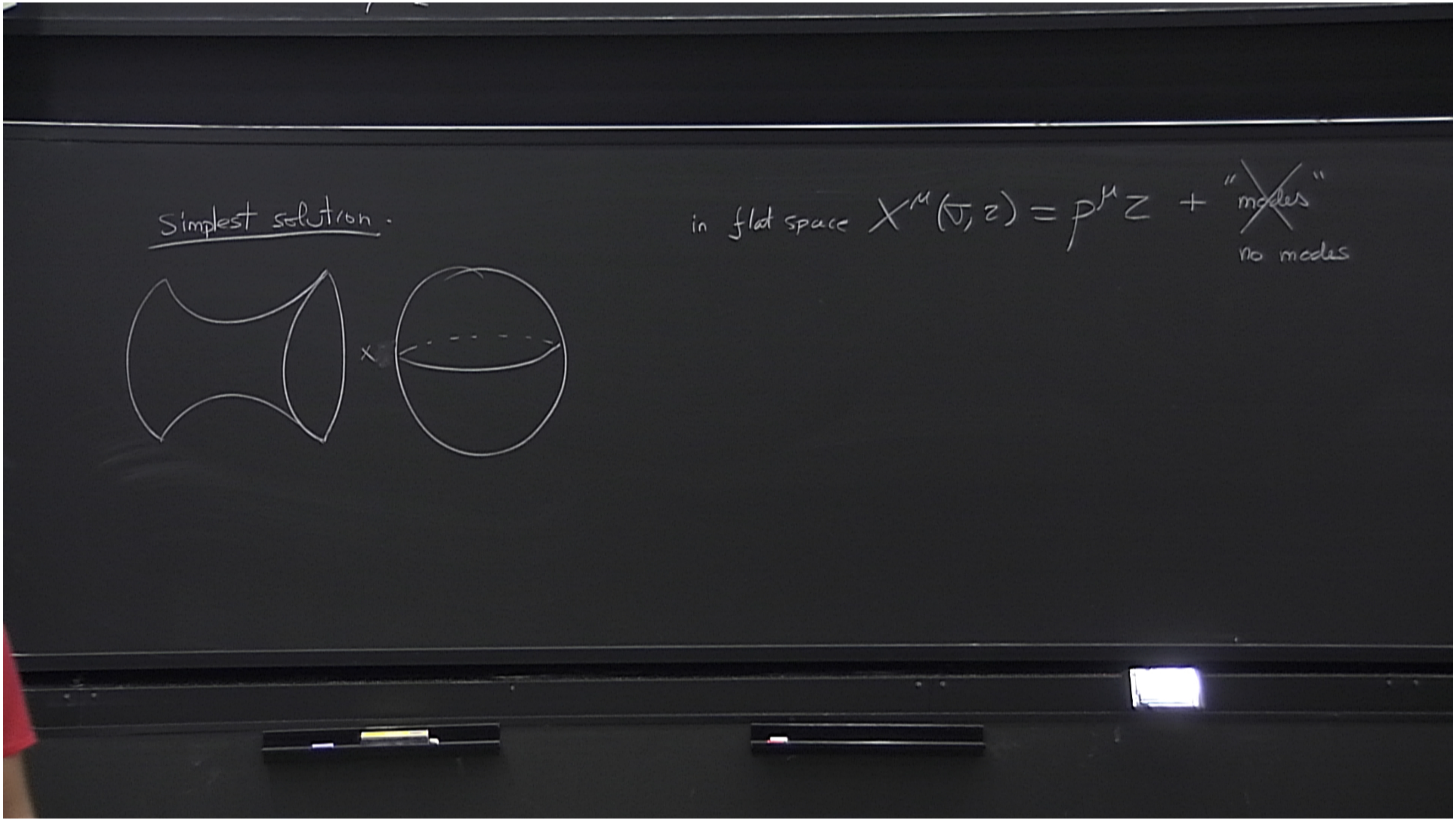
t in the Bulk $=$ t in the boundary $R \times S^3$

Conc

$$\left\{ E_{\substack{\text{string inside} \\ \text{the cylinder}}} \right\} = \left\{ E_{\mathcal{N}=4 \text{ in } \mathbb{R}\times S^3} \right\} \underset{\substack{\uparrow \\ \text{yesterday}}}{=} \left\{ \Delta \right\}_{\mathcal{N}=4 \text{ on } \mathbb{R}^4}$$


Conclusion
$\{\Delta\}_{\mathcal{N}=4} = \{\text{Energies}\}_{\text{Global AdS}}$
$\mathbb{R}^4$

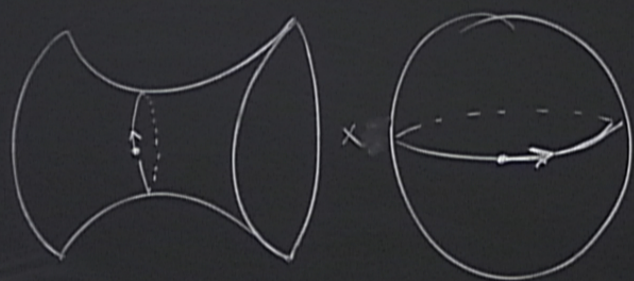

4 on $\mathbb{R}^4$
Conclusion
$\{\Delta(\lambda)\}_{N=4} = \{\text{Energies}(\lambda)\}_{\text{Global AdS}}$
Goal:
$\Delta(\lambda), C_{123}(\lambda)$
strings
interpolation
gauge


Simplest solution.
in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"
no modes
×

Simplest solution.

in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"

no modes



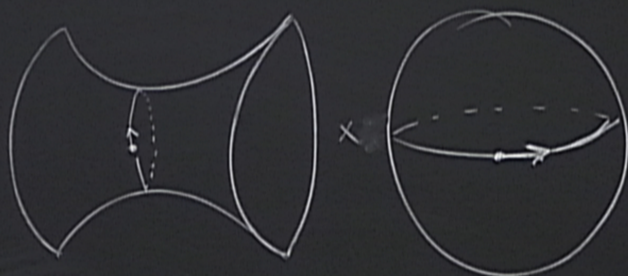
$$X_1^2 + \dots + X_6^2 = 1$$

$$X_1 + i X_2 = e^{iJ\tau}$$

Simplest solution.

in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"

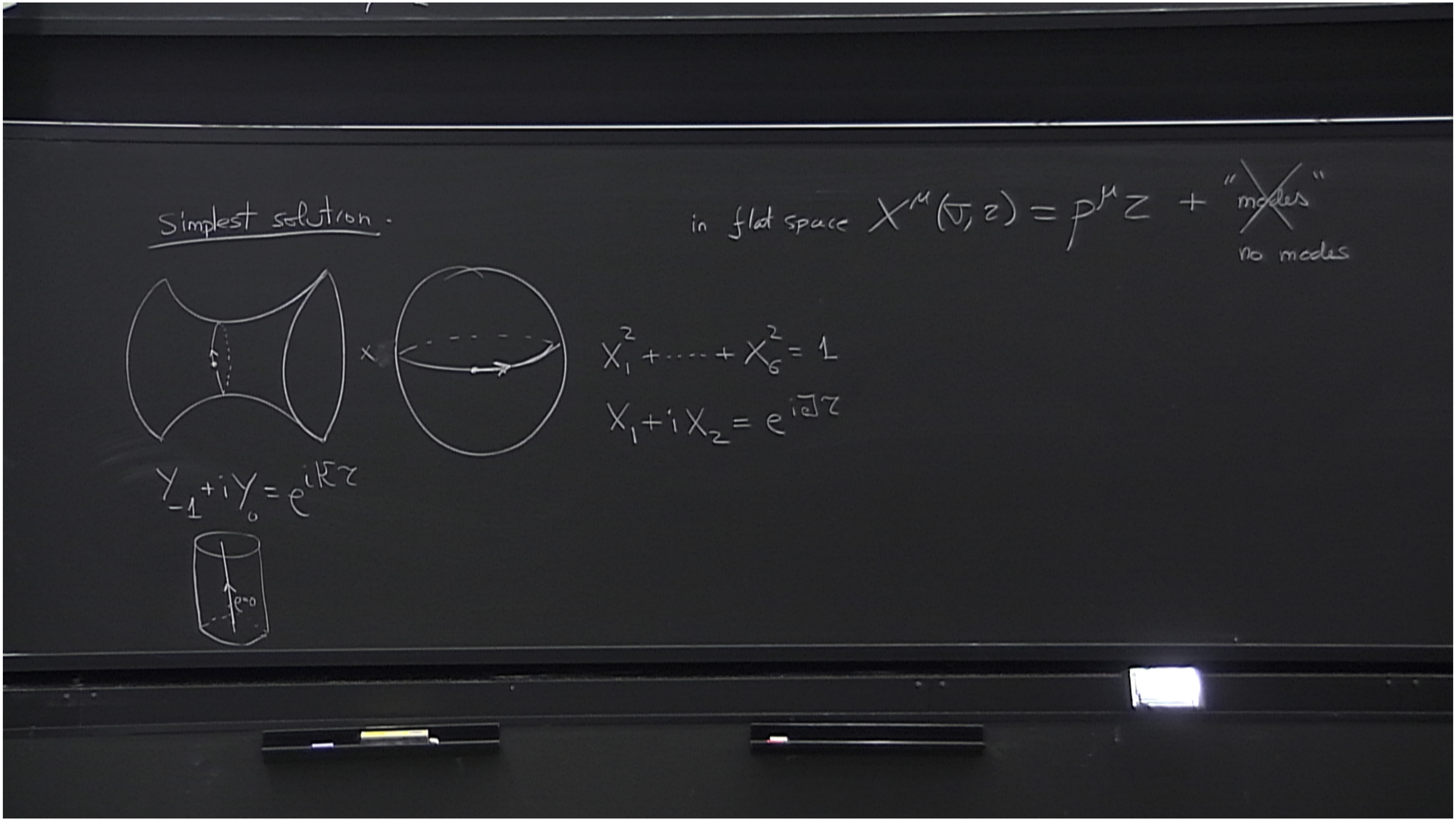
no modes

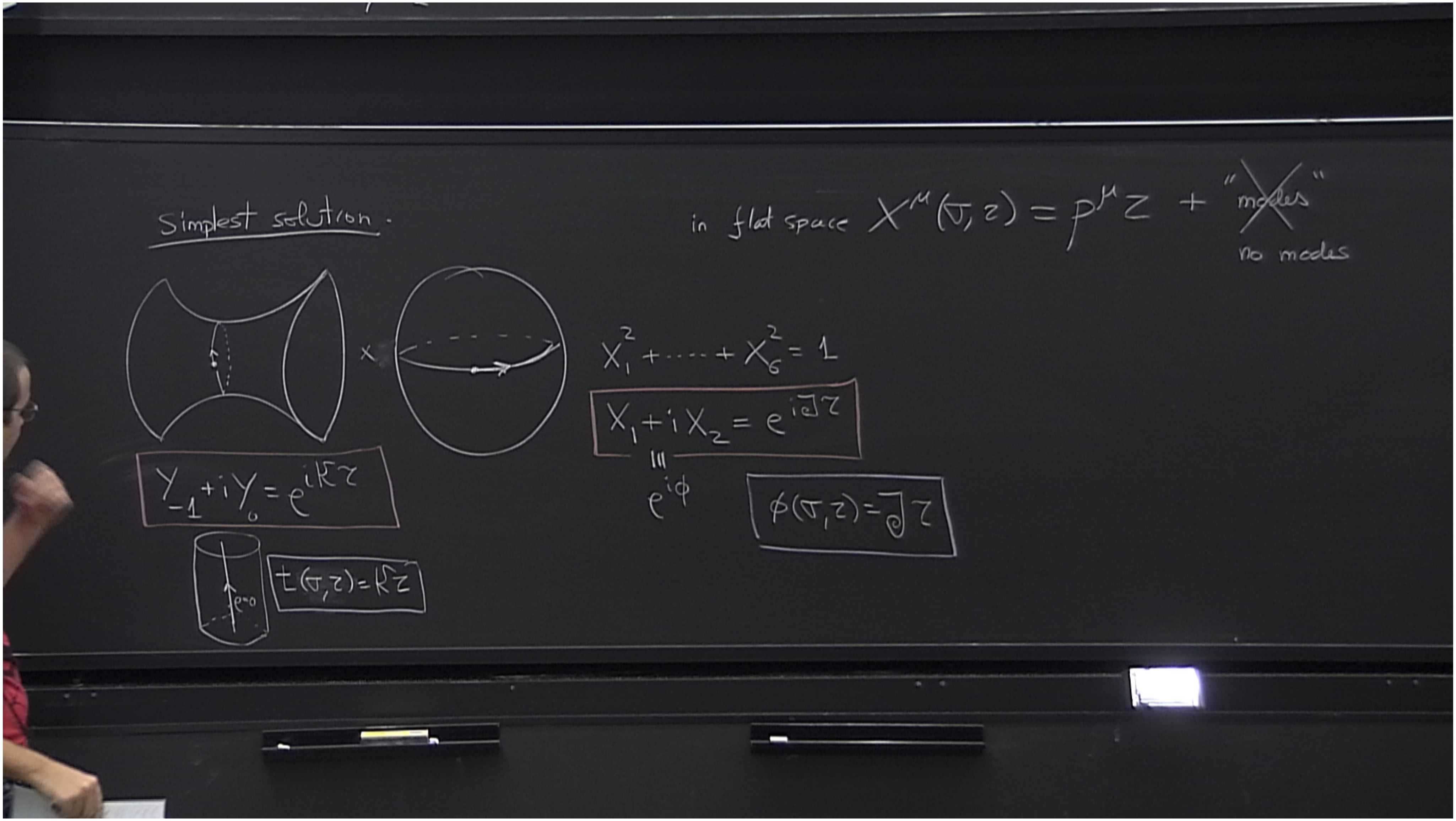


$$X_1^2 + \cdots + X_6^2 = 1$$

$$X_1 + i X_2 = e^{iJ\tau}$$

$$Y_{-1} + i Y_0 = e^{i\kappa\tau}$$

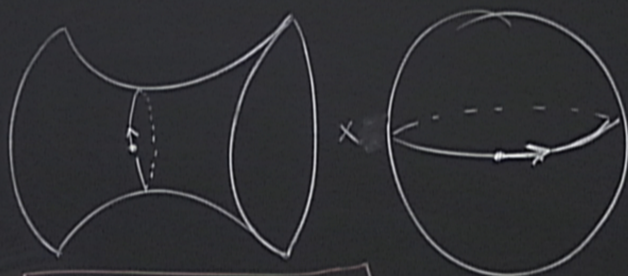

Simplest solution.
in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + \text{"}\cancel{\text{modes}}\text{"}$
no modes
×
$X_1^2 + \cdots + X_6^2 = 1$
$X_1 + i X_2 = e^{iJ\tau}$
$Y_{-1} + i Y_0 = e^{i\kappa\tau}$
$\rho = 0$


Simplest solution.
in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"
no modes
$X_1^2 + \cdots + X_6^2 = 1$
$X_1 + i X_2 = e^{iJ\tau}$
$e^{i\phi}$
$\phi(\sigma,\tau) = J\tau$
$Y_{-1} + i Y_0 = e^{i\kappa\tau}$
$t(\sigma,\tau) = \kappa\tau$
$\rho = 0$

Simplest solution.

in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"

no modes



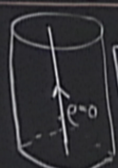
$$X_1^2 + \cdots + X_6^2 = 1$$

$$\boxed{X_1 + i X_2 = e^{iJ\tau}}$$

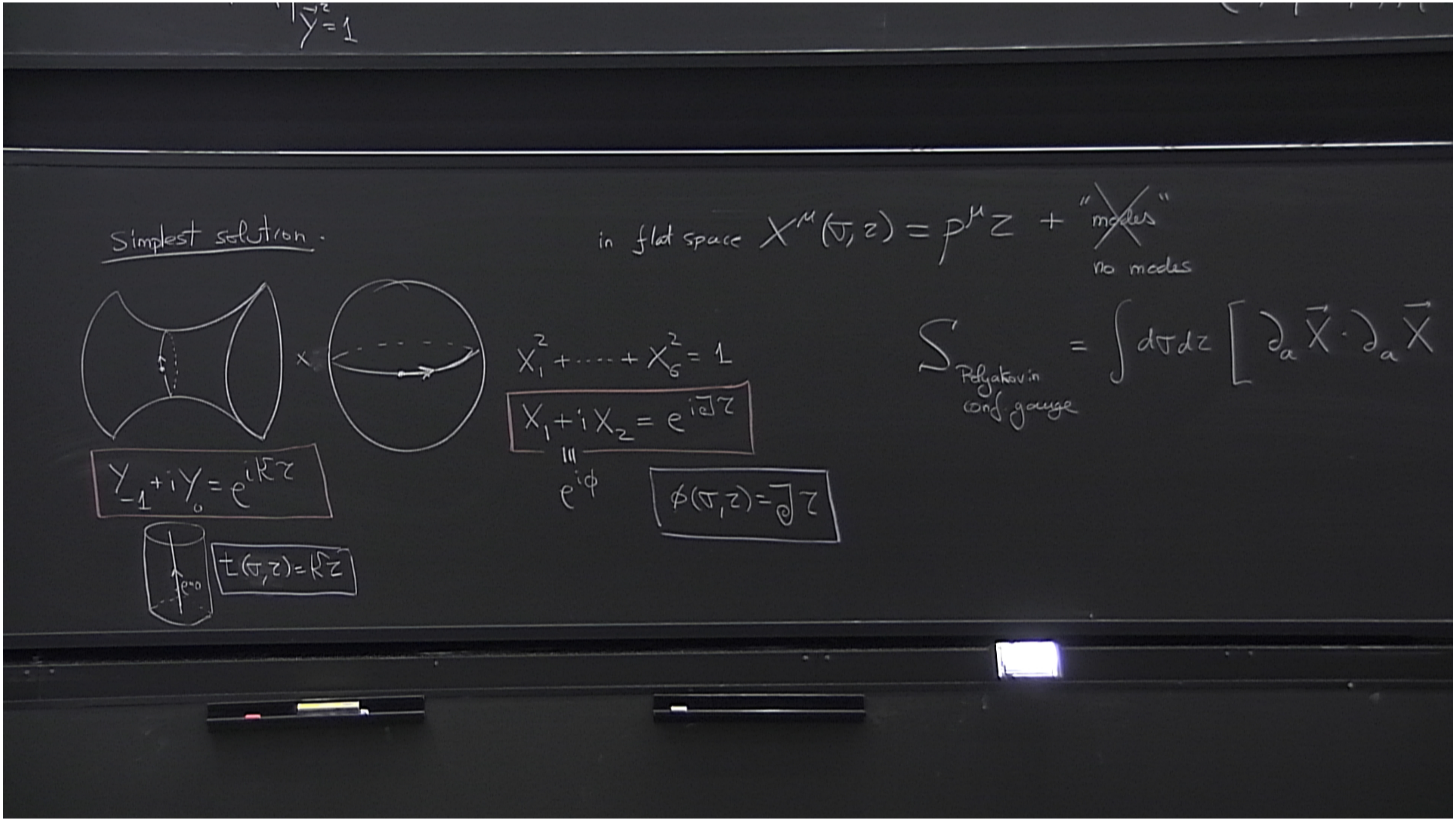
$$\equiv e^{i\phi}$$

$$\boxed{\phi(\sigma,\tau) = J\tau}$$

$$\boxed{Y_{-1} + i Y_0 = e^{i\kappa\tau}}$$



$$\boxed{t(\sigma,\tau) = \kappa\tau}$$



$y = 1$
Simplest solution.
in flat space $X^\mu(\sigma,\tau) = p^\mu \tau + $ "~~modes~~"
no modes
$S_{\text{Polyakov in conf. gauge}} = \int d\sigma d\tau \, [\, \partial_a \vec{X} \cdot \partial_a \vec{X}$
$X_1^2 + \cdots + X_6^2 = 1$
$X_1 + i X_2 = e^{iJ\tau}$
$\equiv e^{i\phi}$
$\phi(\sigma,\tau) = J\tau$
$Y_{-1} + i Y_0 = e^{i\kappa\tau}$
$t(\sigma,\tau) = \kappa\tau$
$\rho = 0$

$$X^\mu(\sigma,\tau) = p^\mu \tau + \text{"}\cancel{\text{modes}}\text{"}$$

no modes

$$S_{\substack{\text{Polyakov in}\\ \text{conf. gauge}}} = \int d\sigma\, d\tau \Big[\partial_a \vec{X} \cdot \partial_a \vec{X} + \lambda_{x} \big(\vec{X} \cdot \vec{X} - 1 \big)$$

$$\boxed{) = \partial \tau}$$

$$X^\mu(\sigma,\tau) = p^\mu \tau + \text{“}\cancel{\text{modes}}\text{”}$$

no modes

$$S_{\substack{\text{Polyakov in} \\ \text{conf. gauge}}} = \int d\sigma\, d\tau \Big[\partial_a \vec{X} \cdot \partial_a \vec{X} + \lambda_X (\vec{X}\cdot\vec{X} - 1) - \partial_a \vec{Y} \cdot \partial_a \vec{Y} - \lambda_Y (\vec{Y}\cdot\vec{Y} - 1) \Big]$$

$$) = \partial \tau$$

$$X^\mu(\sigma,\tau) = p^\mu \tau + \cancel{\text{"modes"}}$$

no modes

$$S_{\substack{\text{Polyakov in} \\ \text{conf. gauge}}} = \int d\sigma\, d\tau \Big[\partial_a \vec{X}\cdot\partial_a \vec{X} + \lambda_X (\vec{X}\cdot\vec{X} - 1) - \partial_a \vec{Y}\cdot\partial_a \vec{Y} - \lambda_Y (\vec{Y}\cdot\vec{Y} - 1) \Big] + \text{fermions}$$

$$\boxed{\;) = \partial \tau}$$

$$X^\mu(\tau, z) = p^\mu z + \text{"}\cancel{\text{modes}}\text{"}$$

no modes

$$S_{\substack{\text{Polyakov in}\\ \text{conf. gauge}}} = \int d\tau\, dz \left[\partial_a \vec{X} \cdot \partial_a \vec{X} + \lambda_X (\vec{X} \cdot \vec{X} - 1) - \partial_a \vec{Y} \cdot \partial_a \vec{Y} - \lambda_Y (\vec{Y} \cdot \vec{Y} - 1) \right] + \text{fermions}$$

$++++++$

$--++++$

Global AdS

$Y = 1$

$\vec{X}(\sigma + 2\pi, \tau) = \vec{X}(\sigma, \tau)$, same for $\vec{Y}$

Virasoro. $(\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y}) \cdot (\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y})$ = same thing with $\vec{Y} \to \vec{X}$

E.o.m for $\vec{X}$: $\partial^a \partial_a \vec{X} - \lambda_x \vec{X} = 0$

E.o.m for λ_x: $\vec{X}^2 = 1$

$\vec{Y} = 1$

$\vec{X}(\sigma + 2\pi, \tau) = \vec{X}(\sigma, \tau)$, same for $\vec{Y}$

Virasoro: $(\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y}) \cdot (\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y})$ = same thing with $\vec{Y} \to \vec{X}$

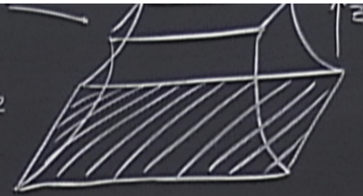
E.o.m for $\vec{X}$: $\partial^a \partial_a \vec{X} - \lambda_X \vec{X} = 0$ # $\vec{X} \cdot \# \Rightarrow \lambda = \vec{X} \cdot \partial^2 \vec{X}$

E.o.m for λ_X: $\vec{X}^2 = 1$

$Y_{-1}^2 + Y_0^2 - Y_1^2 - \dots - Y_{d-1}^2 = 1$ in $\mathbb{R}^{2,d-1}$

$ds^2 = d\vec{Y}\cdot d\vec{Y}\big|_{\vec{Y}^2=1}$

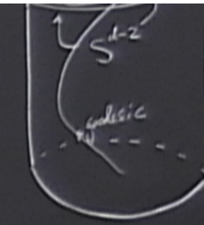
$Y_\mu = \frac{x^\mu}{z}$, $\mu = 0 \dots d-2$



Y_i, $i = 1 \dots d-1$, unit vector, S^{d-2}

$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega^2_{S^{d-2}}$

$\rho(\sigma,\tau),\ t(\sigma,\tau),\ \phi(\sigma,\tau)$



$\vec{X}(\sigma+2\pi,\tau) = \vec{X}(\sigma,\tau)$, same for $\vec{Y}$

Virasoro: $(\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y})\cdot(\partial_\tau \vec{Y} \pm \partial_\sigma \vec{Y}) =$ something with $\vec{Y}\to\vec{X}$

$\vec{X}\cdot\vec{X} = 1 \xRightarrow{\partial} \vec{X}\cdot\partial\vec{X} = 0 \xRightarrow{\partial} \partial\vec{X}\cdot\partial\vec{X} + \vec{X}\cdot\partial^2\vec{X} = 0$

E.o.m for $\vec{X}$: $\partial^a\partial_a \vec{X} - \lambda_X \vec{X} = 0$ # $\vec{X}\cdot$ # $\Rightarrow \lambda = \vec{X}\cdot\partial^2\vec{X} = -\partial\vec{X}\cdot\partial\vec{X}$

E.o.m for λ_X: $\vec{X}^2 = 1$

$$-Y_{-1}^2+Y_0^2-Y_1^2-\cdots-Y_{d-1}^2=1 \quad \text{in } \mathbb{R}^{2,d-1}$$

$$Y_\mu=\frac{x^\mu}{z} \qquad \mu=0\ldots d-2$$

$$ds^2=d\vec{Y}\cdot d\vec{Y}\Big|_{\vec{Y}^2=1}$$

$$\uparrow_{1\ldots d-1} \qquad \uparrow \text{unit vector}, \ S^{d-2}$$

$$ds^2=-\cosh^2\rho\, dt^2+d\rho^2+\sinh^2\rho\, d\Omega^2_{S^{d-2}}$$

$$\rho(\sigma,\tau),\ t(\sigma,\tau),\ \phi(\sigma,\tau)$$

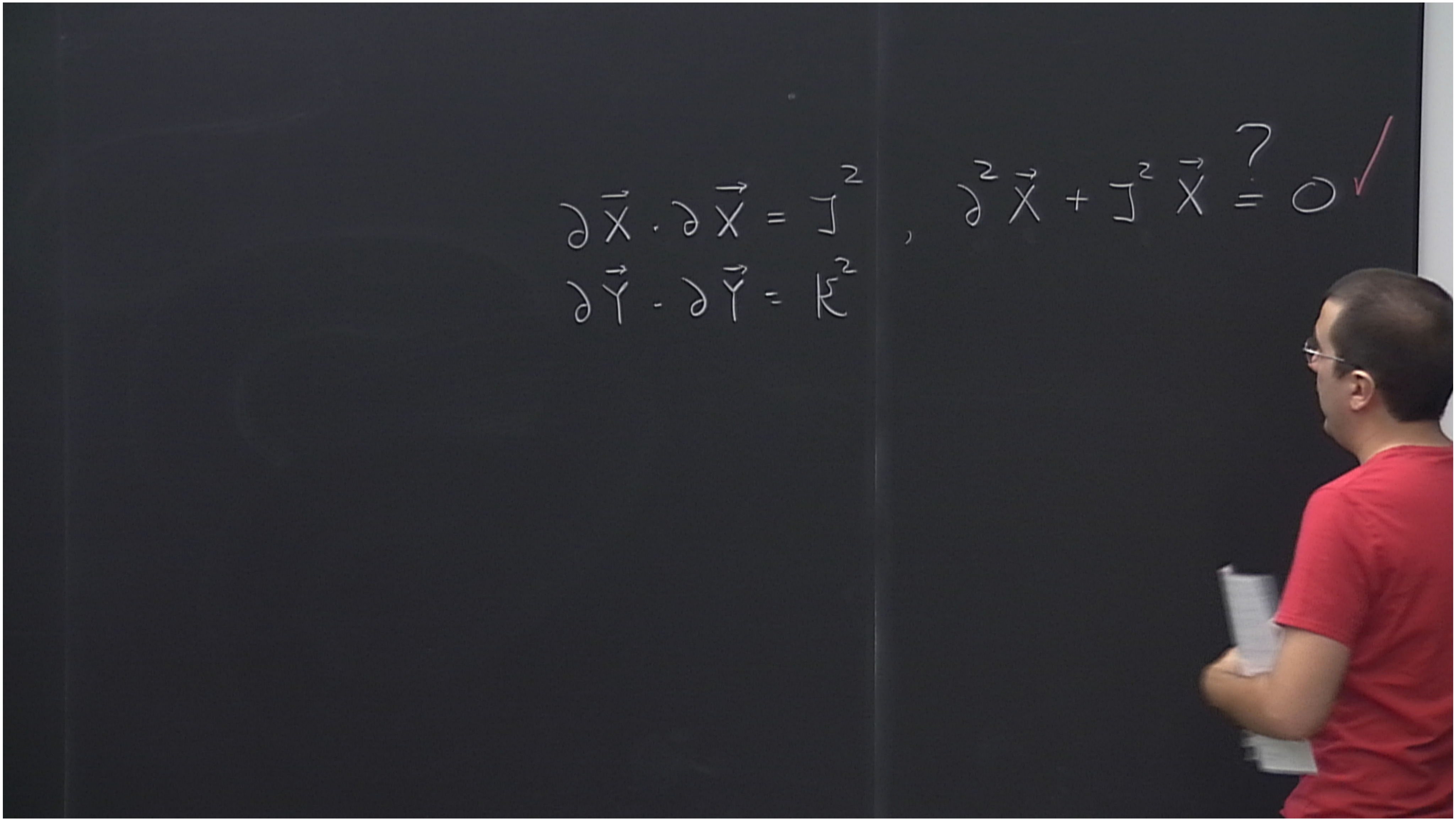
$$\vec{X}(\sigma+2\pi,\tau)=\vec{X}(\sigma,\tau)\ , \quad \text{same for } \vec{Y}$$

Virasoro: $(\partial_\tau\vec{Y}\pm\partial_\sigma\vec{Y})\cdot(\partial_\tau\vec{Y}\pm\partial_\sigma\vec{Y})=$ same thing with $\vec{Y}\to\vec{X}$

$$\vec{X}\cdot\vec{X}=1 \overset{\partial}{\Longrightarrow} \vec{X}\cdot\partial\vec{X}=0 \overset{\partial}{\Longrightarrow} \partial\vec{X}\,\partial\vec{X}+\vec{X}\cdot\partial^2\vec{X}=0$$

E.o.m for $\vec{X}$: $\partial^a\partial_a\vec{X}-\lambda_X\vec{X}=0$ # $\qquad \vec{X}\cdot\#\Rightarrow\lambda=\vec{X}\cdot\partial^2\vec{X}=-\partial\vec{X}\cdot\partial\vec{X} \to$ EOM: $\boxed{\partial^2\vec{X}+(\partial\vec{X}\cdot\partial\vec{X})\vec{X}=0}$

E.o.m for λ_X: $\vec{X}^2=1$


$\partial\vec{X}\cdot\partial\vec{X} = J^2$, $\partial^2\vec{X} + J^2\vec{X} \overset{?}{=} 0$ ✓
$\partial\vec{Y}\cdot\partial\vec{Y} = K^2$

$$\partial\vec{X}\cdot\partial\vec{X} = J^2 \quad , \quad \partial^2\vec{X} + J^2\vec{X} \stackrel{?}{=} 0 \ \checkmark$$

$$\partial\vec{Y}\cdot\partial\vec{Y} = K^2 \quad , \quad \partial^2\vec{Y} + K^2\vec{Y} = 0 \ \checkmark$$

Virasoro : $K = J$

What is the physical meaning of K, J?

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0 \right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma \; \dot{t}$$

$$J_1 = \text{Angular mom} =$$

∂

∂

Vi

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0 \right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma \, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(X_1 \dot{X}_2 - X_2 \dot{X}_1 \right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma \, \dot{\phi}$$

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0 \right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(X_1 \dot{X}_2 - X_2 \dot{X}_1 \right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{\phi}$$

$$\Delta = \text{Energy} = \sqrt{\lambda}\, \kappa$$

$$J_1 = \sqrt{\lambda}\, \mathcal{J}$$

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma\left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma\left(X_1 \dot{X}_2 - X_2 \dot{X}_1\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{\phi}$$

$$\Delta = \text{Energy} = \sqrt{\lambda}\,\kappa$$

$$J_1 = \sqrt{\lambda}\, J$$

$$\Delta = J_1$$

$$\partial\vec{X}\cdot\partial\vec{X} = J^2, \quad \partial^2\vec{X} + J^2\vec{X} \overset{?}{=} 0$$

$$\partial\vec{Y}\cdot\partial\vec{Y} = \kappa^2, \quad \partial^2\vec{Y} + \kappa^2\vec{Y} = 0$$

Virasoro: $\kappa = J$

What is the physical meaning of κ, J?

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(X_1 \dot{X}_2 - X_2 \dot{X}_1\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{\phi}$$

$$\Delta = \text{Energy} = \sqrt{\lambda}\,\kappa$$

$$J_1 = \sqrt{\lambda}\, J$$

$$\partial\vec{X}\cdot\partial\vec{X} = J^2 \;, \quad \partial^2\vec{X} + J^2\vec{X} \overset{?}{=} 0 \;\checkmark$$

$$\partial\vec{Y}\cdot\partial\vec{Y} = \kappa^2 \;, \quad \partial^2\vec{Y} + \kappa^2\vec{Y} = 0 \;\checkmark$$

Virasoro: $\kappa = J$

What is the physical meaning of κ, J?

$$\Delta = J_1 \longleftrightarrow \text{dual operator } \mathcal{O}$$

$$\Delta_{\mathcal{O}} \xrightarrow[\lambda \gg 1]{} J_1$$

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma\,\left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma\,\left(X_1\dot{X}_2 - X_2\dot{X}_1\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\,\dot{\phi}$$

$$\Delta = \text{Energy} = \sqrt{\lambda}\,\kappa$$
$$J_1 = \sqrt{\lambda}\,J$$

$$\partial\vec{X}\cdot\partial\vec{X} = J^2 \,, \quad \partial^2\vec{X} + J^2\vec{X} \overset{?}{=} 0 \;\checkmark$$
$$\partial\vec{Y}\cdot\partial\vec{Y} = \kappa^2 \,, \quad \partial^2\vec{Y} + \kappa^2\vec{Y} = 0 \;\checkmark$$

Virasoro: $\kappa = J$

What is the physical meaning of κ, J?

$$\Delta = J_1 \longleftrightarrow \text{dual operator } \mathcal{O} = \mathrm{Tr}(\phi_1 + i\phi_2)^{J_1}$$
$$\Delta_{\mathcal{O}} \xrightarrow[\lambda \gg 1]{} J_1$$

Noether Charges:

$$\text{Energy} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(Y_0 \dot{Y}_{-1} - Y_{-1}\dot{Y}_0\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{t}$$

$$J_1 = \text{Angular mom} = \frac{\sqrt{\lambda}}{4\pi}\int d\sigma \left(X_1 \dot{X}_2 - X_2 \dot{X}_1\right) = \frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma\, \dot{\phi}$$

$$\Delta = \text{Energy} = \sqrt{\lambda}\,\kappa$$
$$J_1 = \sqrt{\lambda}\,J$$

$$\partial\vec{X}\cdot\partial\vec{X} = J^2 \,, \quad \partial^2\vec{X} + J^2\vec{X} \overset{?}{=} 0$$
$$\partial\vec{Y}\cdot\partial\vec{Y} = \kappa^2 \,, \quad \partial^2\vec{Y} + \kappa^2\vec{Y} = 0$$

Virasoro: $\kappa = J$

What is the physical meaning of κ, J?

$$\Delta = J_1 \longleftrightarrow \text{dual operator } \mathcal{O} = \mathrm{Tr}(\phi_1 + i\phi_2)^{J_1}$$

$$\Delta_{\mathcal{O}} \xrightarrow{\lambda \gg 1} J_1$$
$$\xrightarrow{\lambda \to 0} J_1 = \#\text{ scalars!}$$