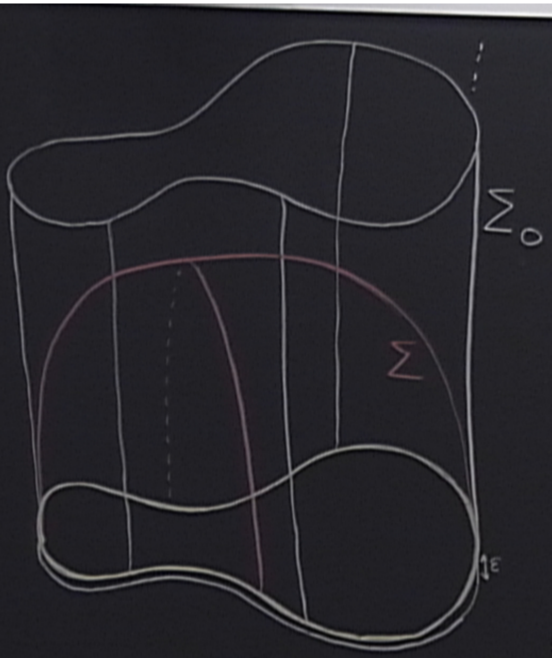
Title: Explorations in String Theory - Lecture 8

Date: Mar 21, 2012 11:30 AM

URL: http://pirsa.org/12030051

Abstract:



$$x^\mu(\sigma,z) = x^\mu(\sigma) + 0\,z + \frac{1}{2}\,x^\mu(\sigma)''\,z^2 - \frac{1}{3}\,\frac{\delta A}{\delta x_\mu(\sigma)}\,z^3 +$$

$$Z(\sigma,z) = z + 0\,z^2 - \frac{1}{3}(x'')^2\,z^3 + \dots\dots$$

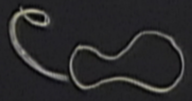
local

non-local

finite

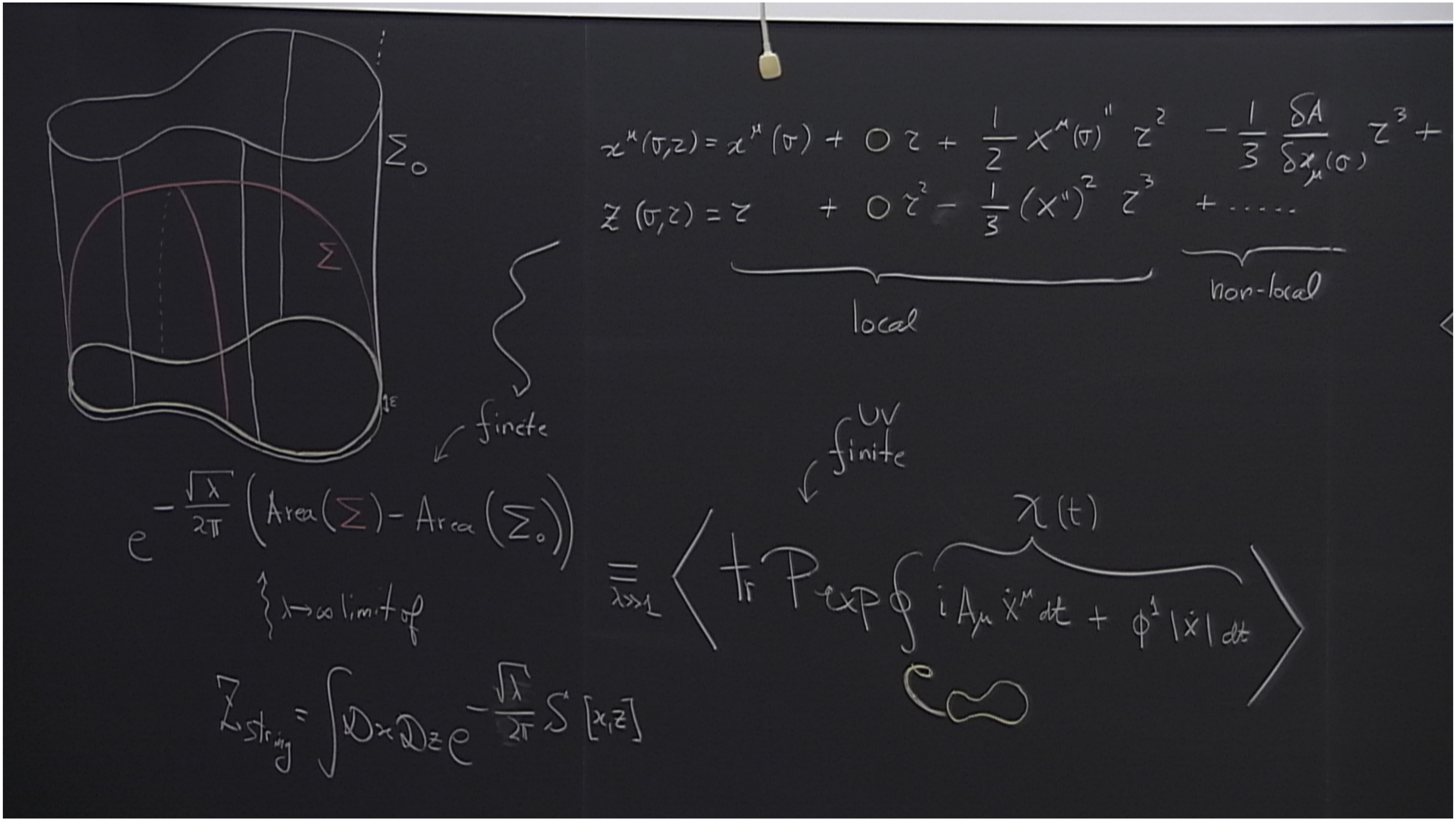
UV finite

$$e^{-\frac{\sqrt{\lambda}}{2\pi}\left(\mathrm{Area}(\Sigma)-\mathrm{Area}(\Sigma_0)\right)} \underset{\lambda\gg1}{\equiv} \left\langle \mathrm{tr}\, P \exp \oint_{C} \overbrace{i A_\mu \dot{x}^\mu dt + \phi^1 |\dot{x}|\, dt}^{\chi(t)} \right\rangle$$



$\lambda\to\infty$ limit of

$$Z_{\text{string}} = \int \mathcal{D}x\,\mathcal{D}z\, e^{-\frac{\sqrt{\lambda}}{2\pi} S[x,z]}$$



$$x^\mu(\sigma,z) = x^\mu(\sigma) + 0\,z + \frac{1}{2} x^\mu(\sigma)'' \, z^2 - \frac{1}{3}\frac{\delta A}{\delta x_\mu(\sigma)} z^3 +$$

$$\underbrace{Z(\sigma,z) = z + 0\,z^2 - \frac{1}{3}(x'')^2 z^3}_{\text{local}} \underbrace{+ \cdots\cdots}_{\text{non-local}}$$

$$e^{-\frac{\sqrt{\lambda}}{2\pi}\left(\text{Area}(\Sigma) - \text{Area}(\Sigma_0)\right)}$$

finite

$\Big\updownarrow$ $\lambda \to \infty$ limit of

$$Z_{\text{string}} = \int \mathcal{D}x\,\mathcal{D}z\, e^{-\frac{\sqrt{\lambda}}{2\pi} S[x,z]}$$

UV finite

$$\underset{\lambda \gg 1}{\equiv} \left\langle \text{tr}\, P \exp \oint_{C} \overbrace{i A_\mu \dot{x}^\mu dt + \phi^1 |\dot{x}|\, dt}^{x(t)} \right\rangle$$

$$- \frac{1}{3} \frac{\delta A}{\delta x_{\mu}(\sigma)} z^3 + g^{\mu}(\sigma) z^4 + \cdots$$

$$- - - - - -$$

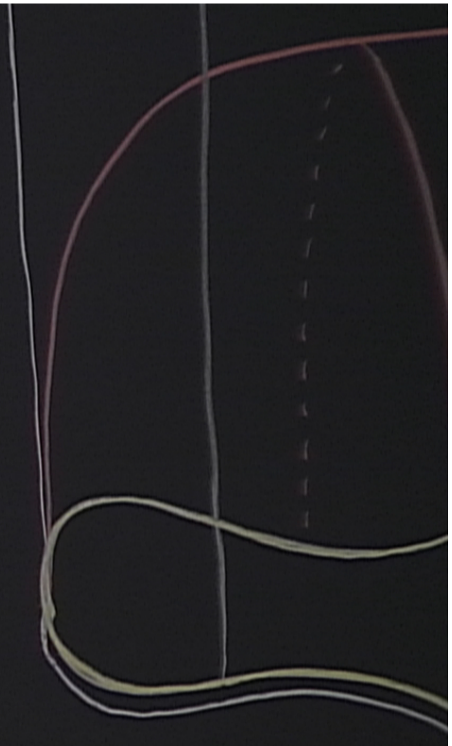
$$\underbrace{\qquad\qquad}_{\text{non-local}}$$

$$\langle X(t_1) X(t_2) \rangle = \frac{1}{4\pi^2} \frac{|\dot{X}(t_1)||\dot{X}(t_2)| - \dot{X}(t_1) \cdot \dot{X}(t_2)}{|X(t_1) - X(t_2)|^2}$$

$$| \, dt \rangle$$

$$z' \sim \frac{1}{z^2}$$

$$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$$



e^{-}

- - - - - -

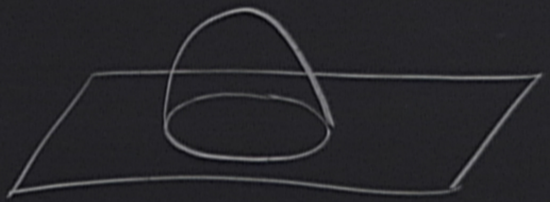
non-local

$$\langle X(t_1)X(t_2)\rangle = \frac{1}{4\pi^2}\,\frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$

finite as $t_1 -$

$| dt \rangle$

CIRCULAR WL



$x_1 = r\cos\theta$

$x_2 = r\sin\theta$

$Z = f(r)$

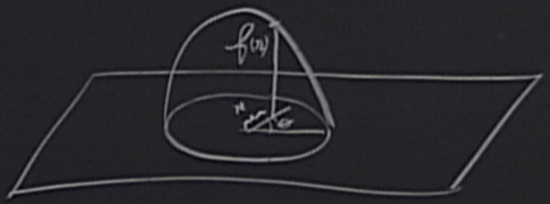
non-local

$|dt$

$$\langle X(t_1)\, X(t_2)\rangle = \frac{1}{4\pi^2}\,\frac{|\dot{x}(t_1)|\,|\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$

finite as $t_1 -$

CIRCULAR WL



$$x_1 = \overset{z}{r}\cos\overset{\sigma}{\theta}$$
$$x_2 = r\ \sin\theta$$
$$z = f(r)$$

$\partial x_\mu(\sigma)$

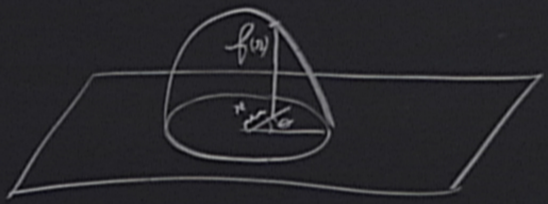
$- - - - - -$

$\underbrace{\qquad\qquad}_{\text{non-local}}$

$$\langle X(t_1) X(t_2) \rangle = \frac{1}{4\pi^2} \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$ finite as $t_1 -$

$| dt \rangle$

CIRCULAR WL



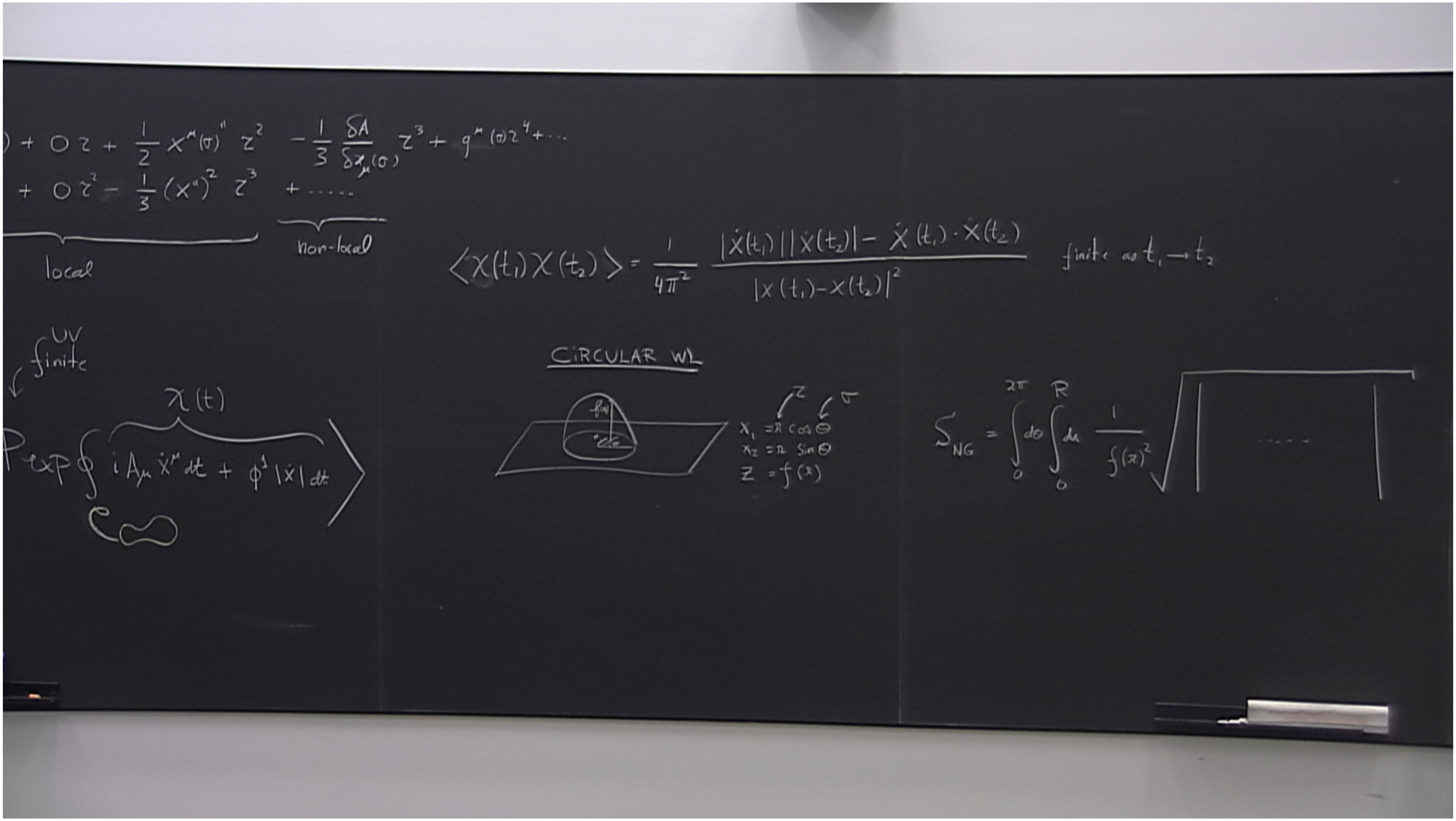
$$\overset{z}{\swarrow} \qquad \overset{\sigma}{\swarrow}$$

$x_1 = r \cos\theta$

$x_2 = r \sin\theta$

$Z = f(r)$

$$S_{NG} = \int_0^{2\pi} d\theta \int_0^R dr \, \frac{1}{f(r)^2} \sqrt{}$$



$$) + 0\,z + \frac{1}{2} x^{\mu}(\sigma)'' z^2 - \frac{1}{3} \frac{\delta A}{\delta x_{\mu}(\sigma)} z^3 + g^{\mu}(\sigma) z^4 + \cdots$$

$$\underbrace{+ 0\,z^2 - \frac{1}{3} (x'')^2 z^3}_{\text{local}} \underbrace{+ \cdots\cdots}_{\text{non-local}}$$

$$\langle X(t_1) X(t_2) \rangle = \frac{1}{4\pi^2} \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$

finite as $t_1 \to t_2$

UV finite

$$P\exp \oint_{C} \overbrace{i A_{\mu} \dot{x}^{\mu} dt + \phi^{1} |\dot{x}| dt}^{x(t)} \Big\rangle$$

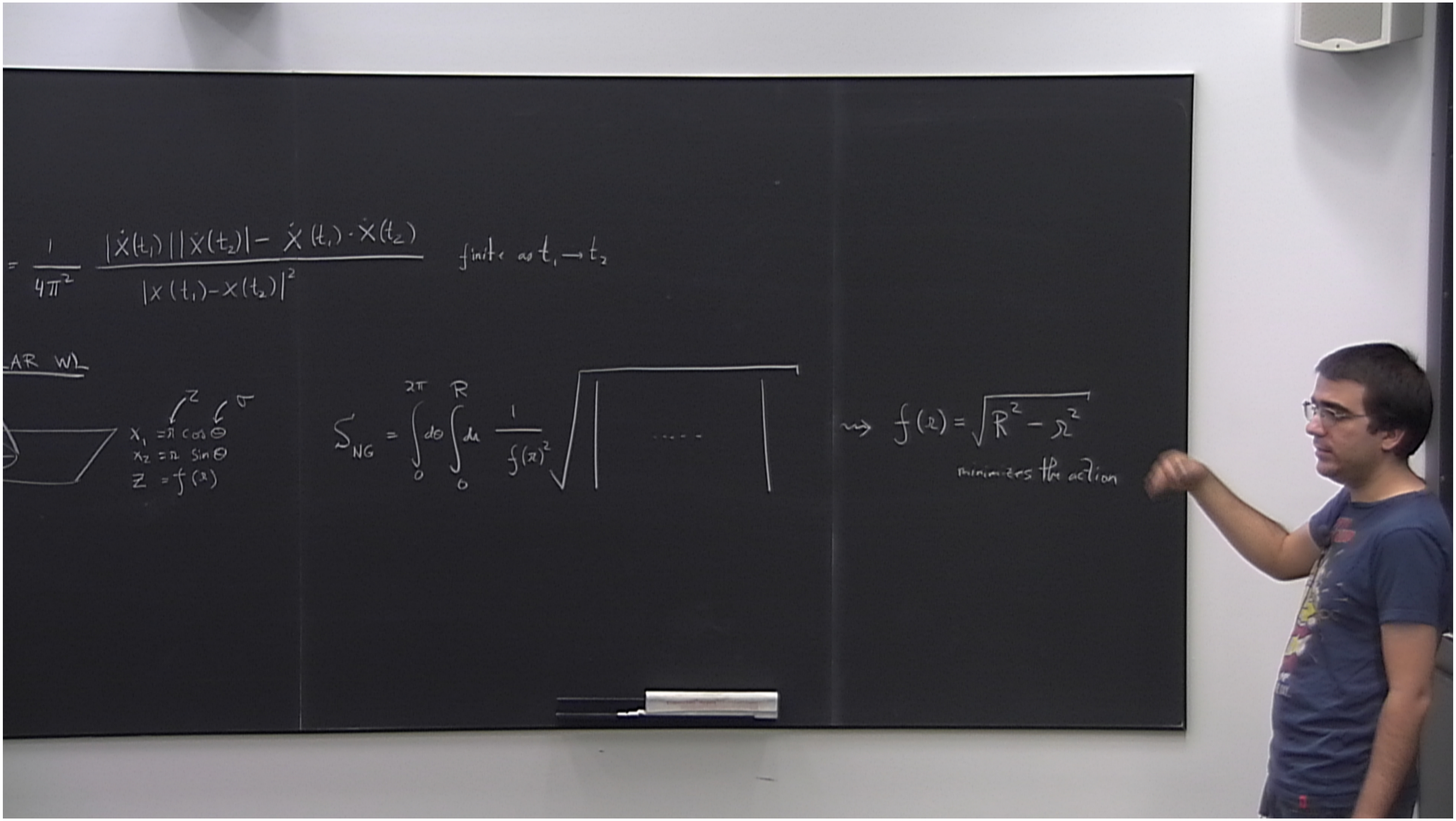
CIRCULAR WL

$$x_1 = r \cos\theta \quad \overset{z}{\swarrow} \quad \overset{\sigma}{\swarrow}$$

$$x_2 = r \sin\theta$$

$$z = f(r)$$

$$S_{NG} = \int_0^{2\pi} d\theta \int_0^{R} dr \, \frac{1}{f(r)^2} \sqrt{\Big|\quad \cdots\cdots \quad\Big|}$$



$$= \frac{1}{4\pi^2}\,\frac{|\dot{x}(t_1)|\,|\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$ finite as $t_1 \to t_2$

AR WL

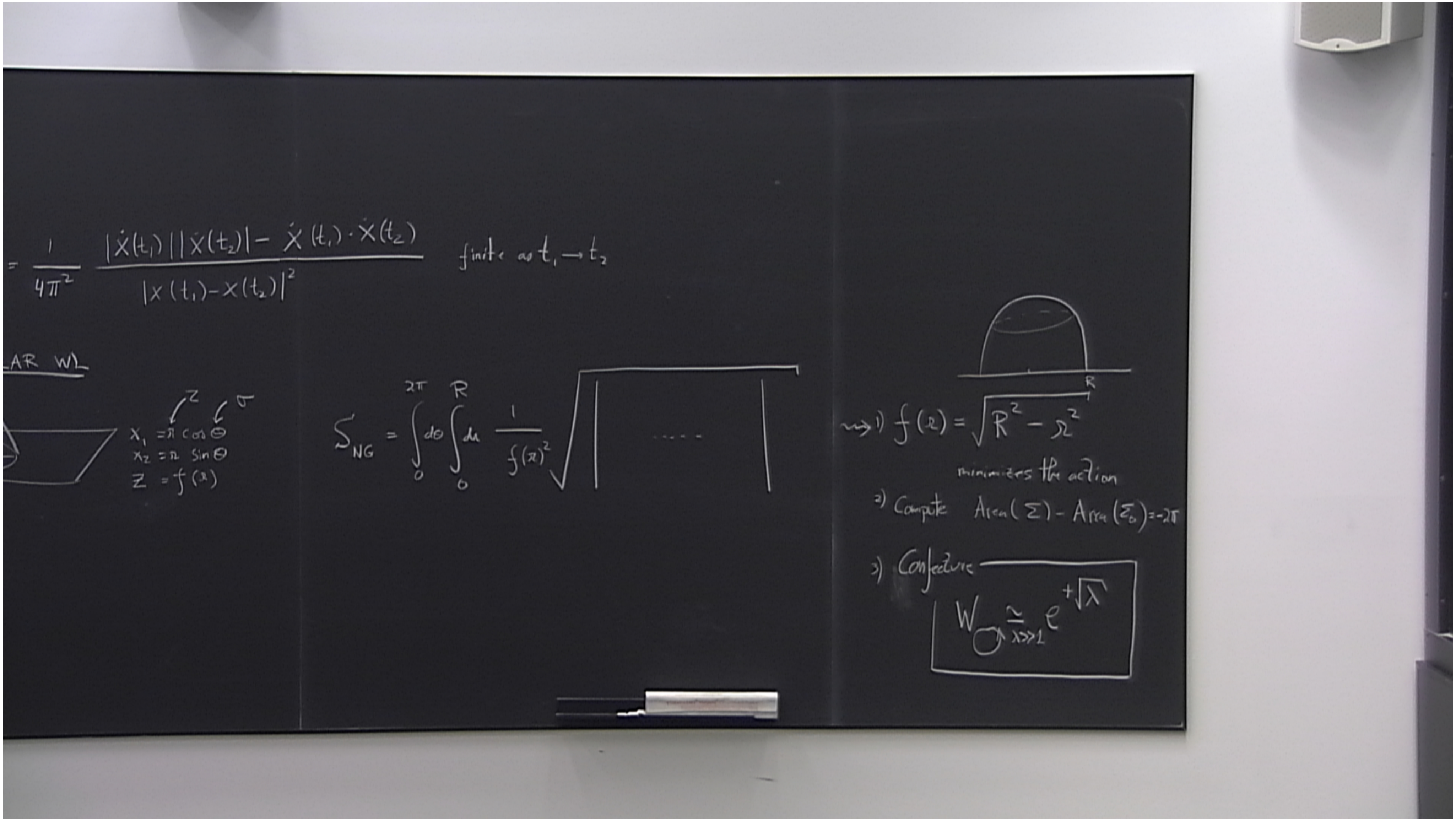
$x_1 = \overset{\tau}{r}\cos\overset{\sigma}{\theta}$

$x_2 = r\sin\theta$

$Z = f(r)$

$$S_{NG} = \int_0^{2\pi} d\theta \int_0^{R} dr\, \frac{1}{f(r)^2}\sqrt{\Big|\ \cdots\ \Big|} \rightsquigarrow f(r) = \sqrt{R^2 - r^2}$$

minimizes the action



$$= \frac{1}{4\pi^2} \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$ finite as $t_1 \to t_2$

AR WL

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$$S_{NG} = \int_0^{2\pi} d\theta \int_0^{R} dr \frac{1}{f(r)^2} \sqrt{\Big| \quad \cdots\cdots \quad \Big|}$$

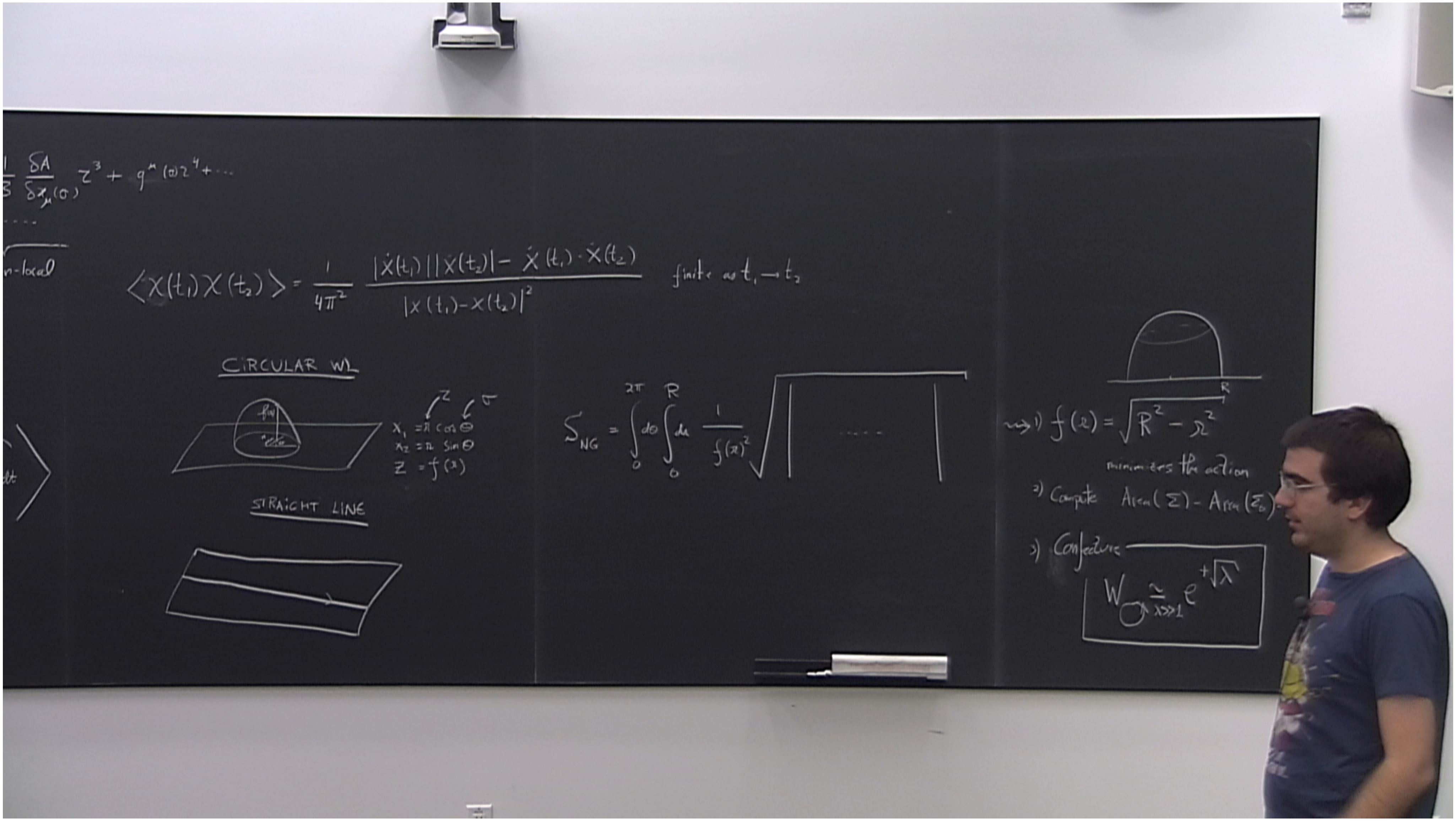
$\rightsquigarrow$ 1) $f(r) = \sqrt{R^2 - r^2}$

minimizes the action

2) Compute $\text{Area}(\Sigma) - \text{Area}(\Sigma_0) = -2\pi$

3) Conjecture

$$\boxed{W_{\bigcirc} \underset{\lambda \gg 1}{\simeq} e^{+\sqrt{\lambda}}}$$



$$\frac{1}{3}\frac{\delta A}{\delta x_\mu(\sigma)} z^3 + g^\mu(\sigma) z^4 + \cdots$$

n-local

$$\langle X(t_1) X(t_2) \rangle = \frac{1}{4\pi^2} \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$ finite as $t_1 \to t_2$

CIRCULAR WL

$$x_1 = r\cos\theta \qquad x_2 = r\sin\theta \qquad z = f(r)$$

z, σ

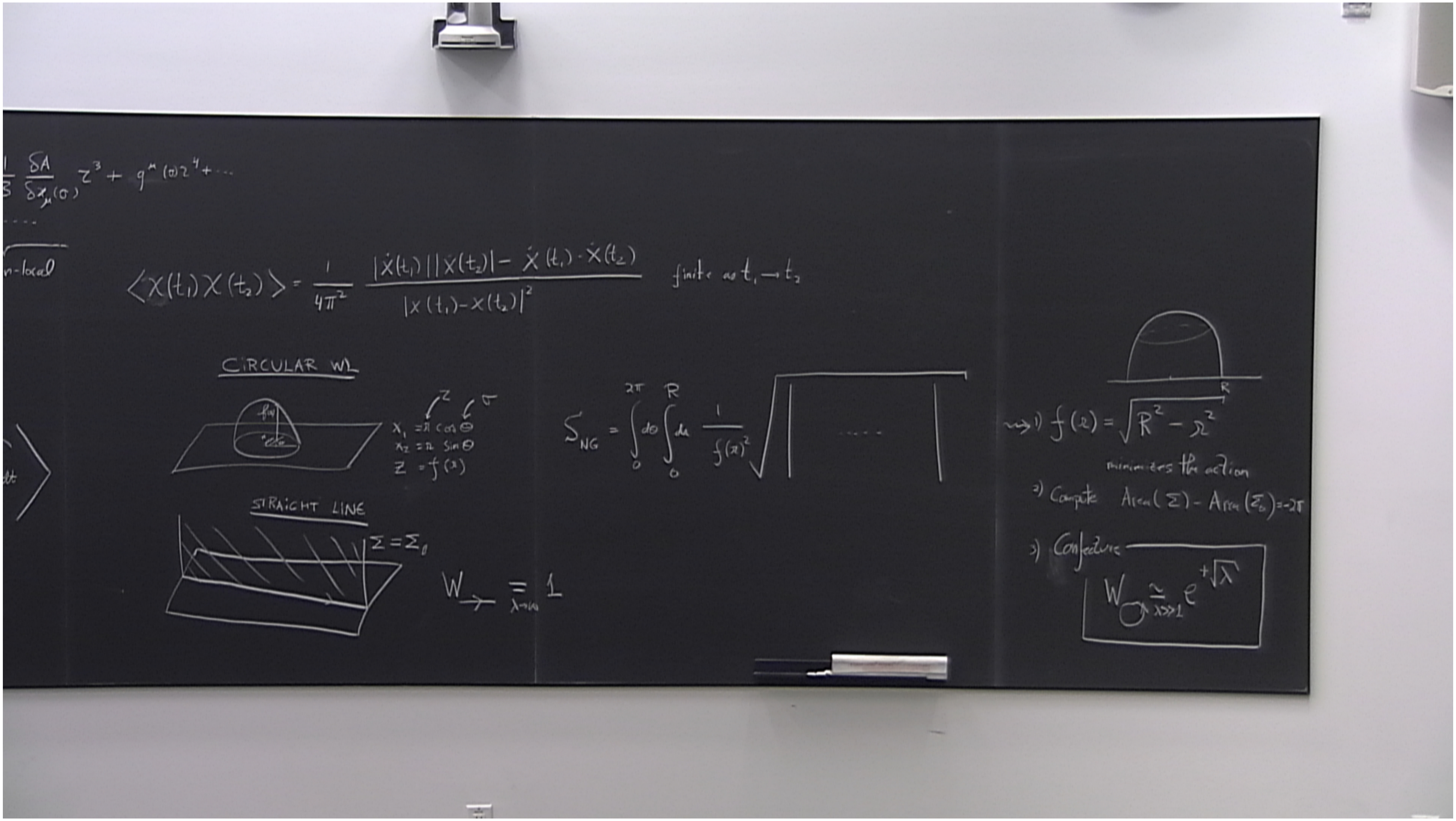
$$S_{NG} = \int_0^{2\pi} d\theta \int_0^R dr \frac{1}{f(r)^2} \sqrt{\left| \cdots \right|}$$

STRAIGHT LINE

$\rightsquigarrow$ 1) $f(r) = \sqrt{R^2 - r^2}$ minimizes the action

2) Compute $\text{Area}(\Sigma) - \text{Area}(\Sigma_0)$

3) Conjecture $\boxed{W_{\bigcirc} \underset{\lambda \gg 1}{\simeq} e^{+\sqrt{\lambda}}}$



$$\frac{1}{3}\frac{\delta A}{\delta x_\mu(\sigma)}z^3 + g^\mu(\sigma)z^4 + \cdots$$

n-local

$$\langle \chi(t_1)\chi(t_2)\rangle = \frac{1}{4\pi^2}\frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$

finite as $t_1 \to t_2$

CIRCULAR WL

$x_1 = r\cos\theta$

$x_2 = r\sin\theta$

$z = f(r)$

$$S_{NG} = \int_0^{2\pi} d\theta \int_0^R dr\, \frac{1}{f(r)^2}\sqrt{\;\cdots\cdots\;}$$

STRAIGHT LINE

$\Sigma = \Sigma_0$

$$W_{\to} \underset{\lambda\to\infty}{=} 1$$

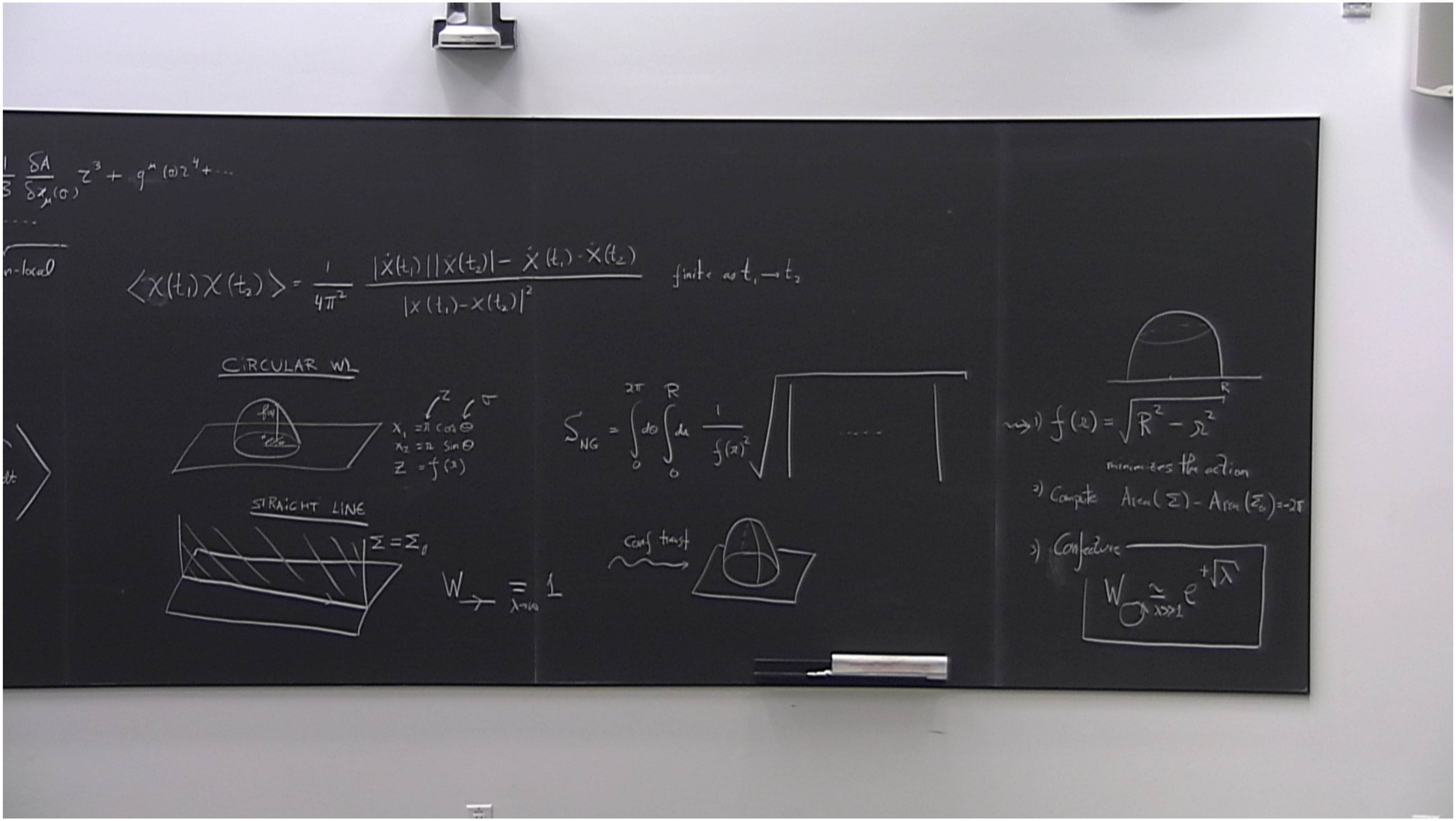
$\rightsquigarrow$ 1) $f(r) = \sqrt{R^2 - r^2}$

minimizes the action

2) Compute $\text{Area}(\Sigma) - \text{Area}(\Sigma_0) = -2\pi$

3) Conjecture

$$\boxed{W_{\bigcirc} \underset{\lambda\gg1}{\simeq} e^{+\sqrt{\lambda}}}$$



$\frac{1}{3}\frac{\delta A}{\delta x_\mu(\sigma)} z^3 + g^\mu(\sigma) z^4 + \cdots$

n-local

$$\langle X(t_1) X(t_2) \rangle = \frac{1}{4\pi^2} \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2}$$ finite as $t_1 \to t_2$

CIRCULAR WL

$x_1 = r\cos\theta$

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$$S_{NG} = \int_0^{2\pi} d\theta \int_0^R dr \, \frac{1}{f(r)^2} \sqrt{\cdots\cdots}$$

STRAIGHT LINE

$\Sigma = \Sigma_0$

$$W_{\rightarrow} \underset{\lambda\to\infty}{=} 1$$

Conf transf

1) $f(r) = \sqrt{R^2 - r^2}$ minimizes the action

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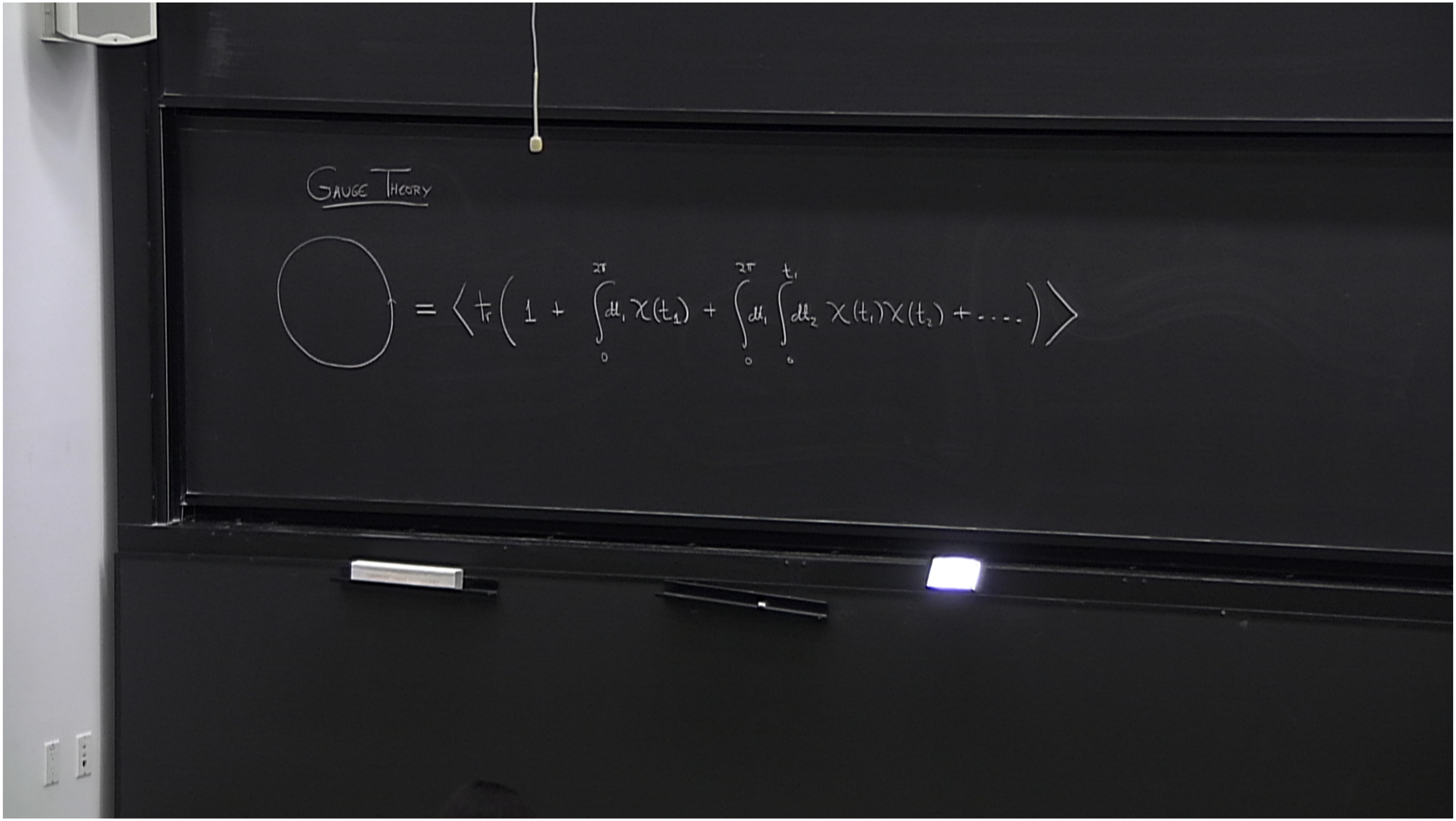
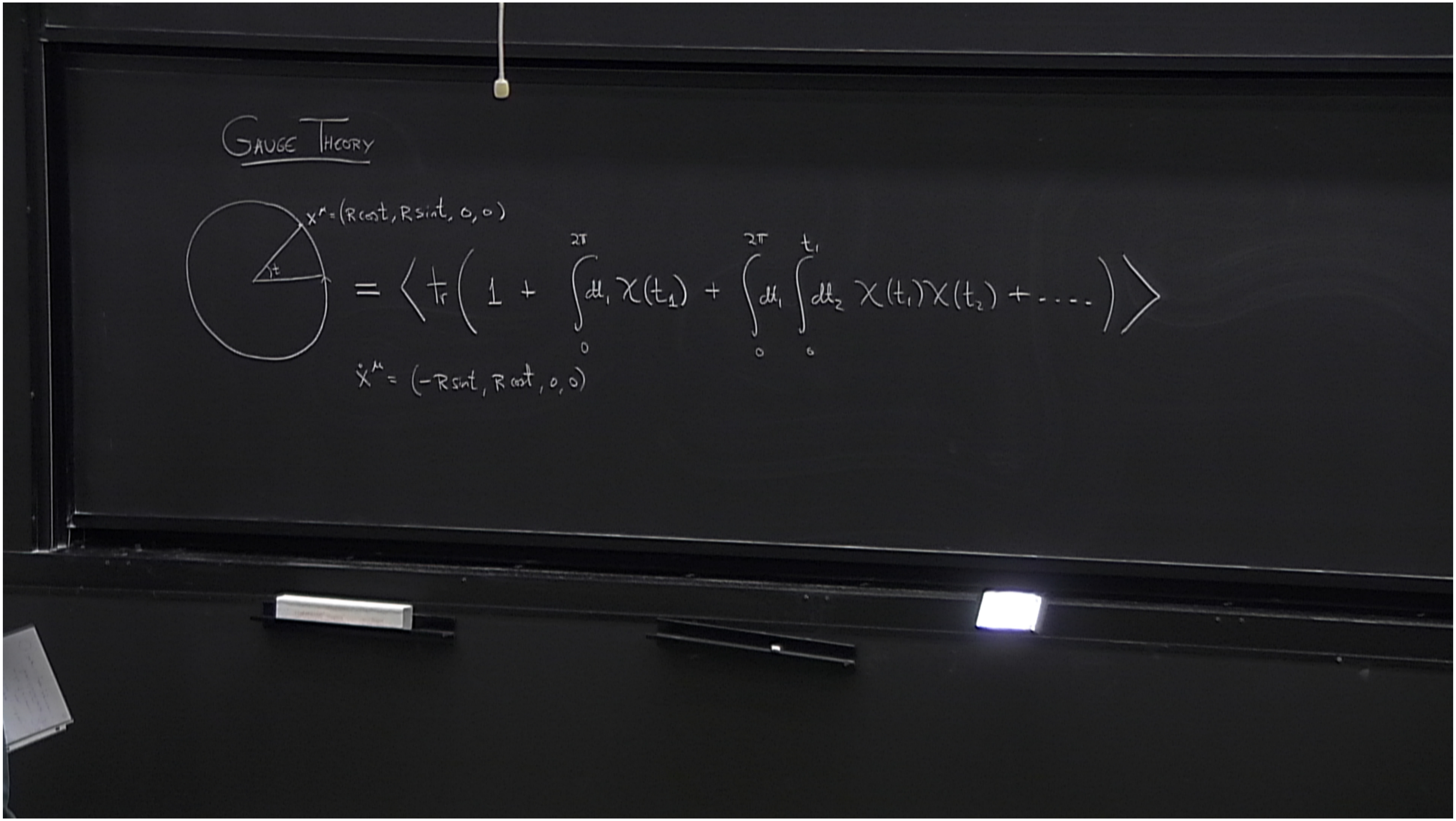


GAUGE THEORY

$$\bigcirc = \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, X(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, X(t_1) X(t_2) + \ldots \right) \right\rangle$$

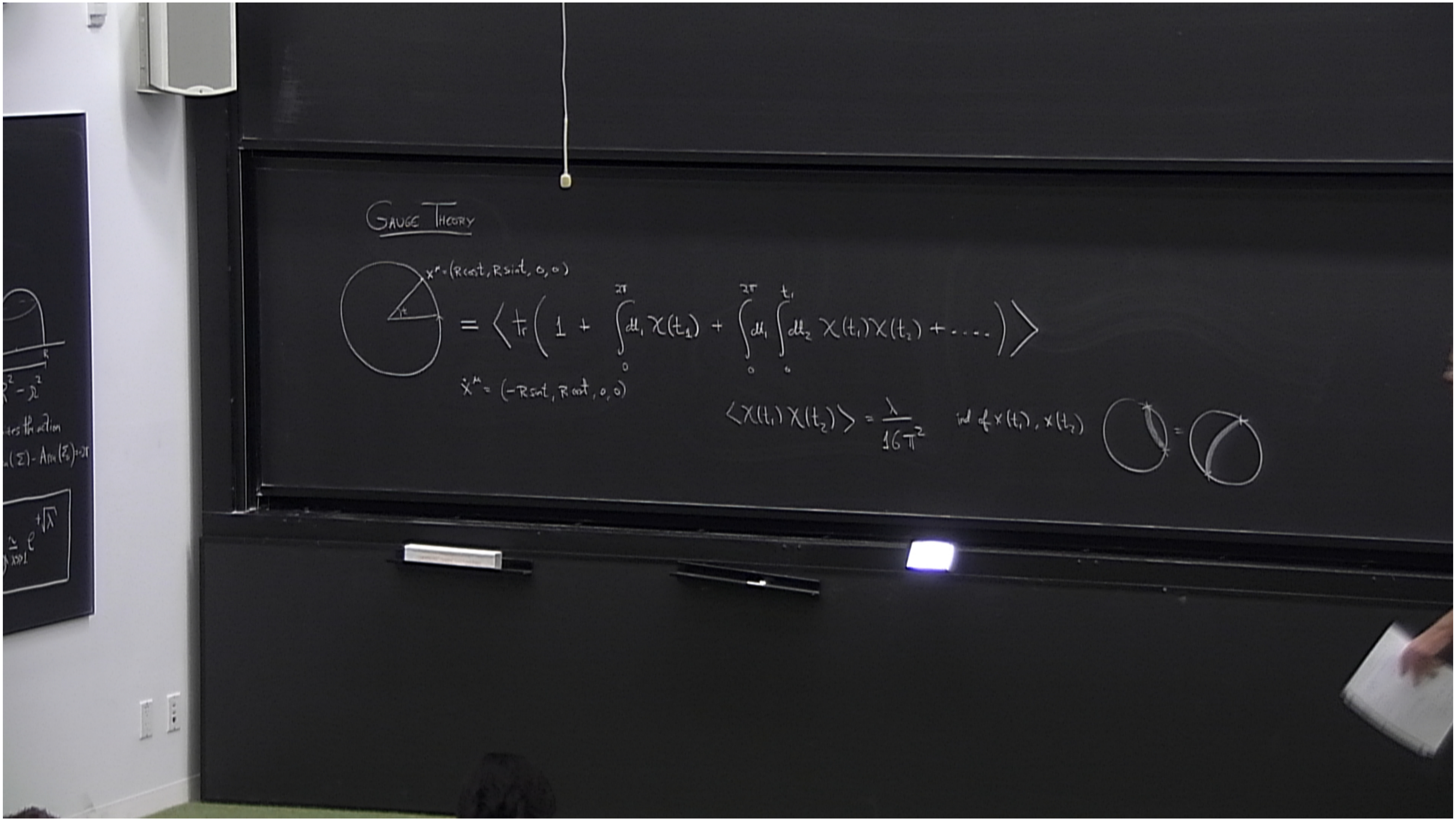


GAUGE THEORY

$x^\mu = (R\cos t, R\sin t, 0, 0)$

$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \ldots . \right) \right\rangle$$

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$



GAUGE THEORY

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$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \dots \right) \right\rangle$$

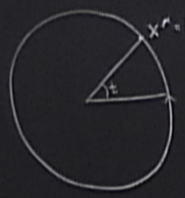
$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$$\langle \chi(t_1)\chi(t_2) \rangle = \frac{\lambda}{16\pi^2}$$

ind of $x(t_1)$, $x(t_2)$

GAUGE THEORY

$x^\mu = (R\cos t, R\sin t, 0, 0)$



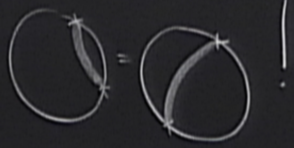
$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \dots \right) \right\rangle \overset{1\,\mathrm{loop}}{\equiv} 1 + \frac{\lambda}{16\pi^2}\, \frac{1}{2!} \cdot 2\pi \cdot 2\pi$$

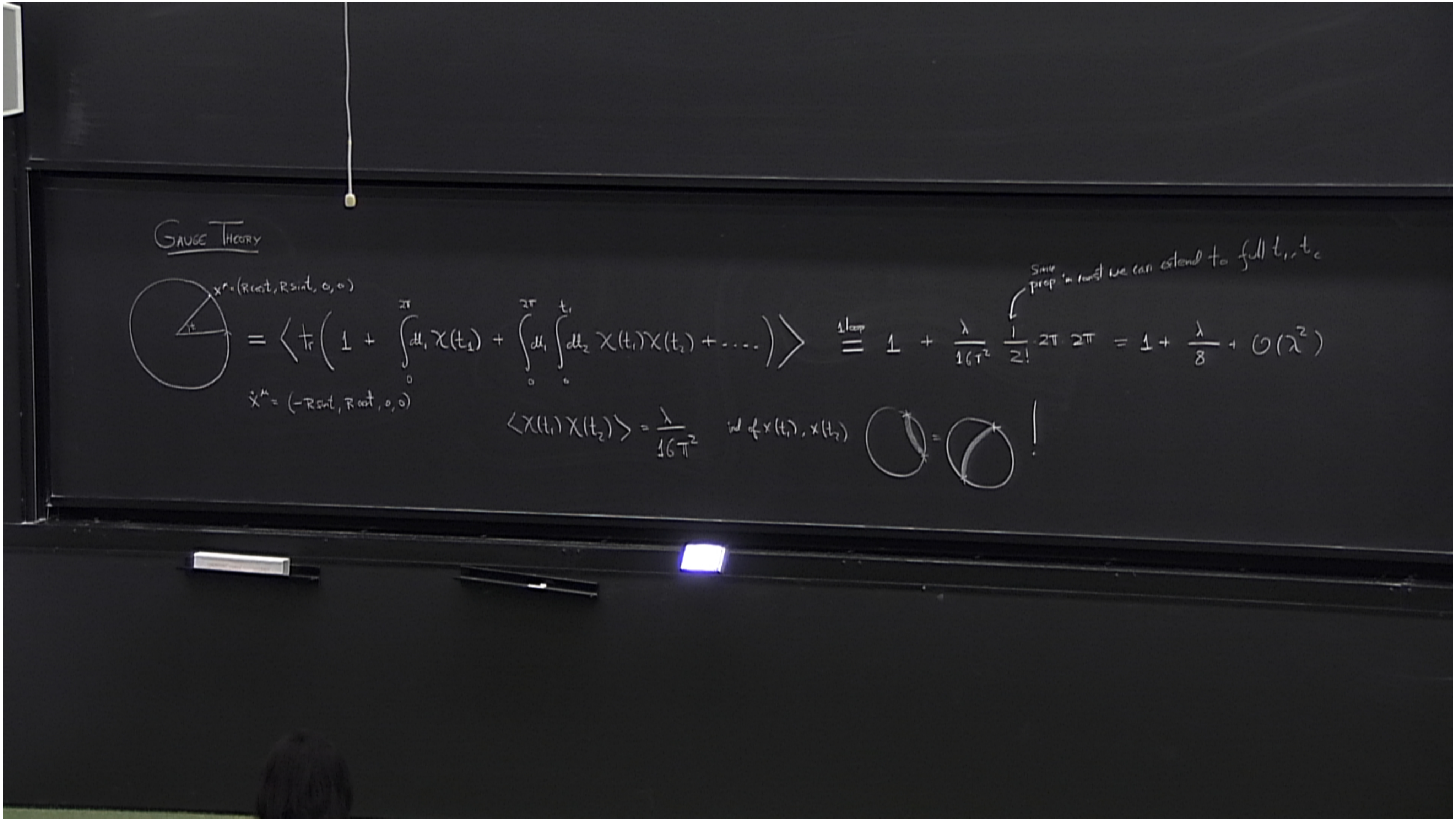
Since prop. is const we can extend to full t_1, t_2

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$$\langle \chi(t_1)\chi(t_2)\rangle = \frac{\lambda}{16\pi^2}$$

ind of $x(t_1), x(t_2)$





GAUGE THEORY

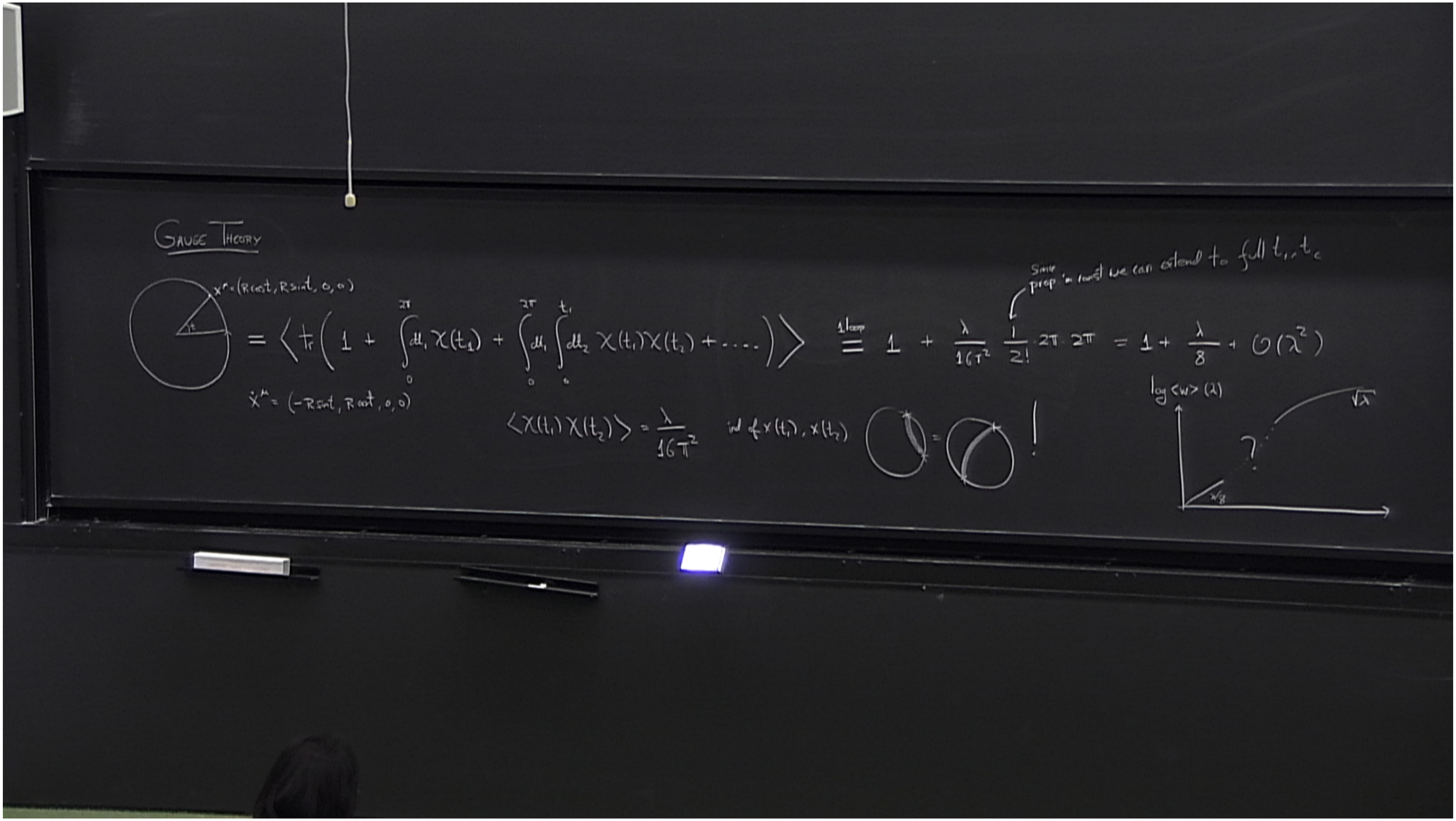
$x^\mu = (R\cos t, R\sin t, 0, 0)$

$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \ldots \right)\right\rangle \overset{1\,\mathrm{loop}}{=} 1 + \frac{\lambda}{16\pi^2}\frac{1}{2!}\, 2\pi \cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$$

Since prop is const we can extend to full t_1, t_c

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$\langle \chi(t_1)\chi(t_2)\rangle = \frac{\lambda}{16\pi^2}$ ind of $x(t_1), x(t_2)$ = !



GAUGE THEORY

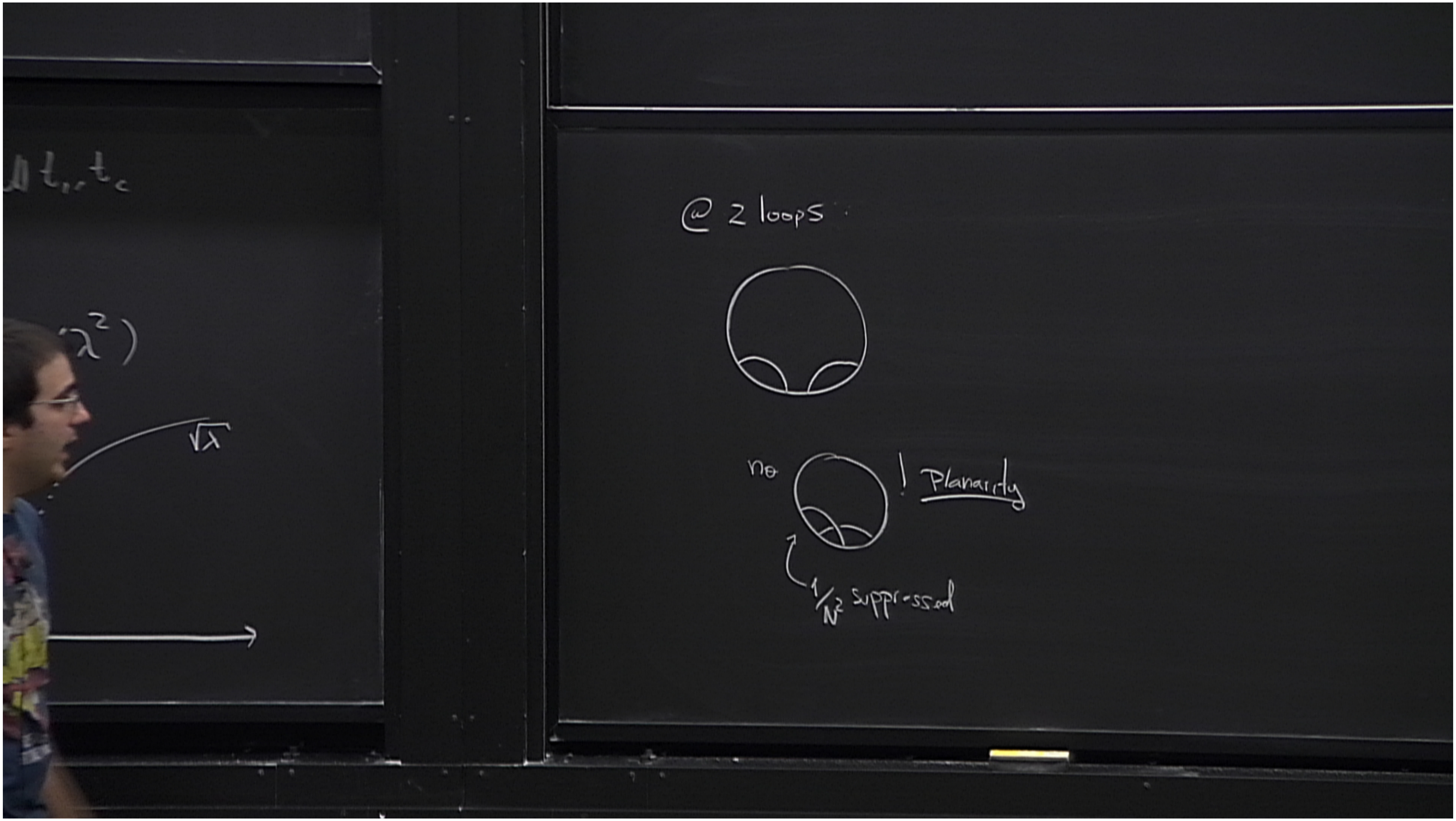
$x^\mu = (R\cos t, R\sin t, 0, 0)$

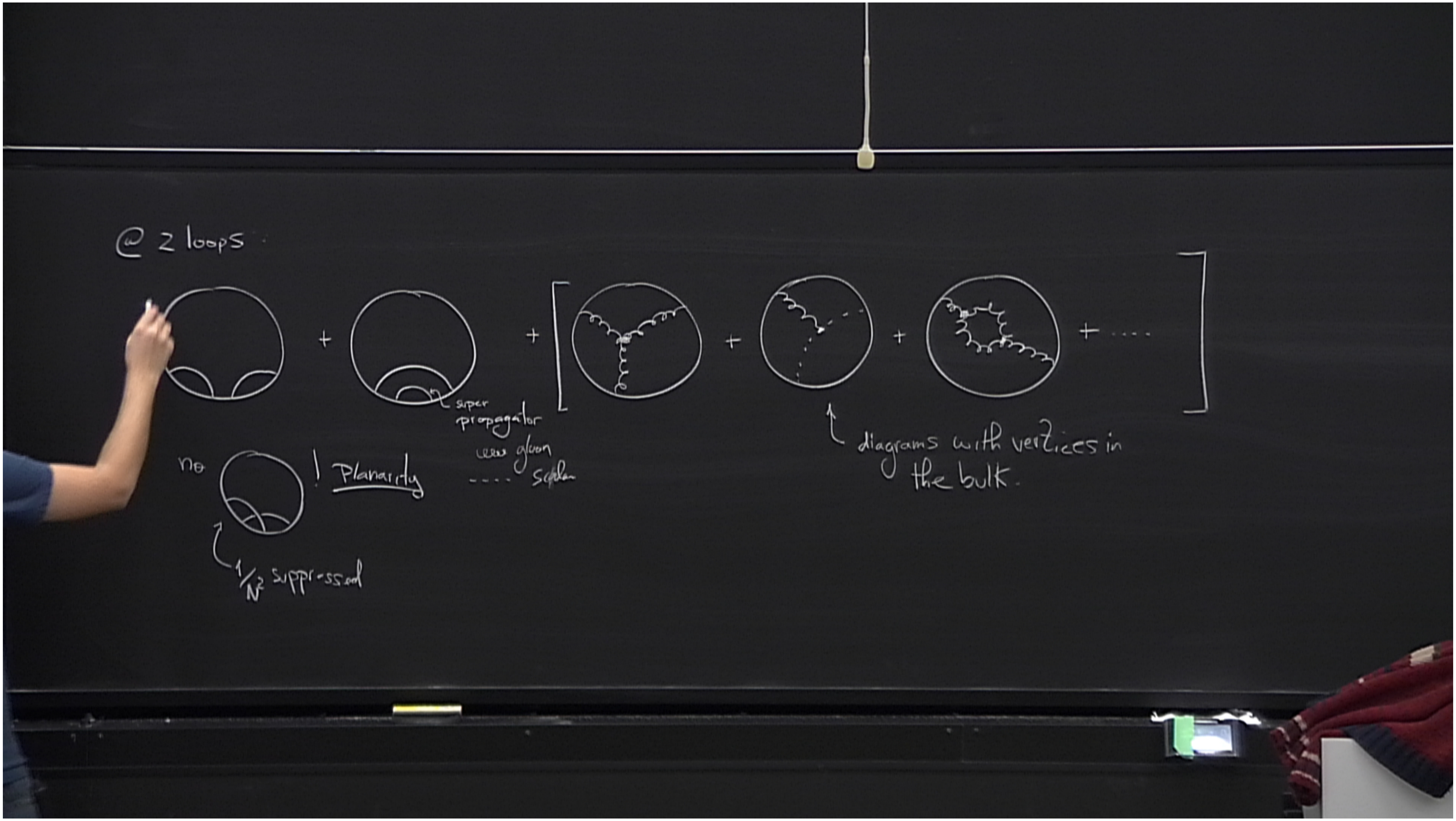
$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, X(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, X(t_1)X(t_2) + \dots\right)\right\rangle \overset{1\,\mathrm{loop}}{=} 1 + \frac{\lambda}{16\pi^2}\frac{1}{2!}\,2\pi\cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$$

Since prop is const we can extend to full t_1, t_2

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$\langle X(t_1)X(t_2)\rangle = \frac{\lambda}{16\pi^2}$ ind of $x(t_1), x(t_2)$


@ 2 loops
no
! Planarity
1/N^2 suppressed
√λ
λ^2)

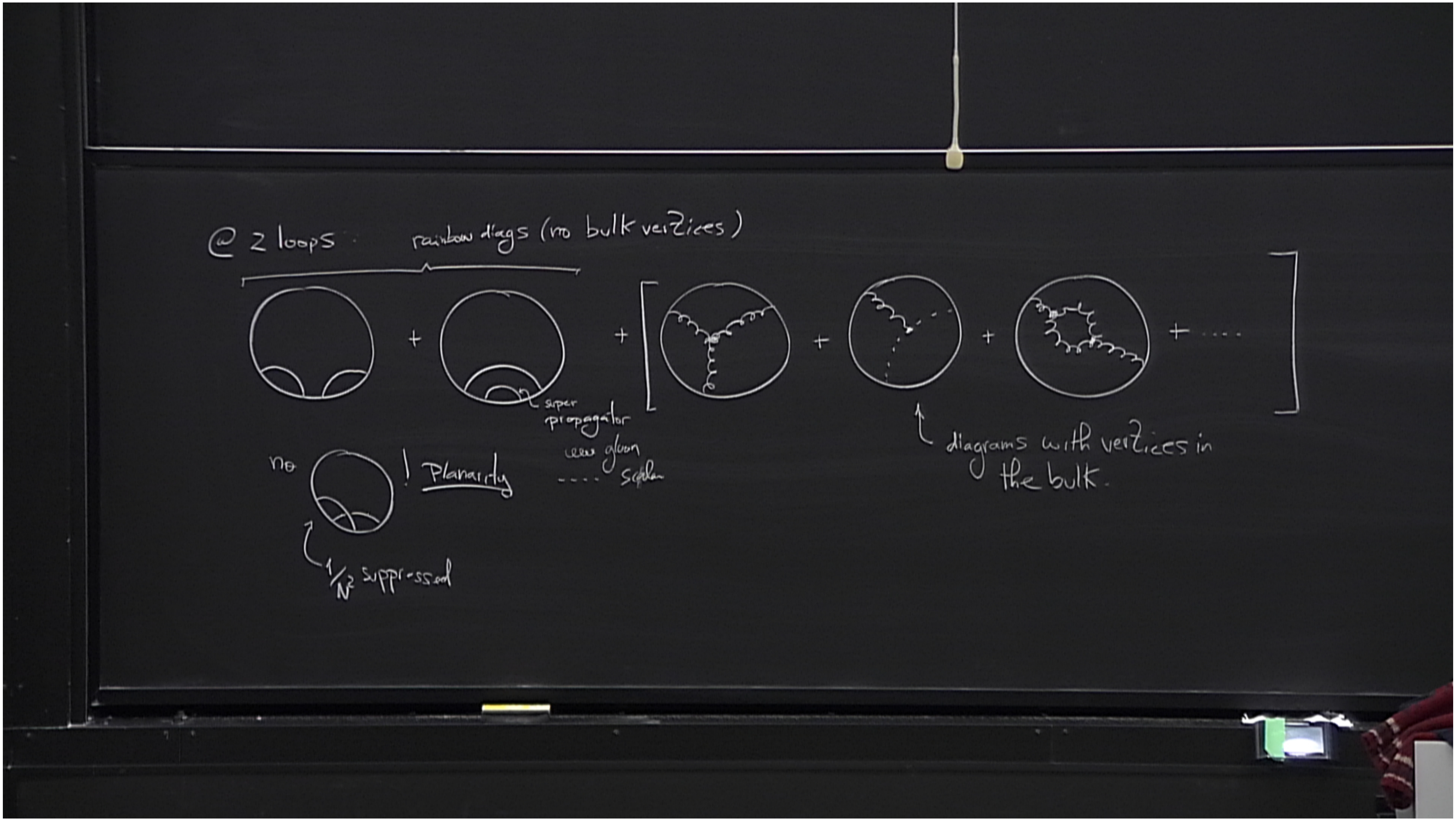


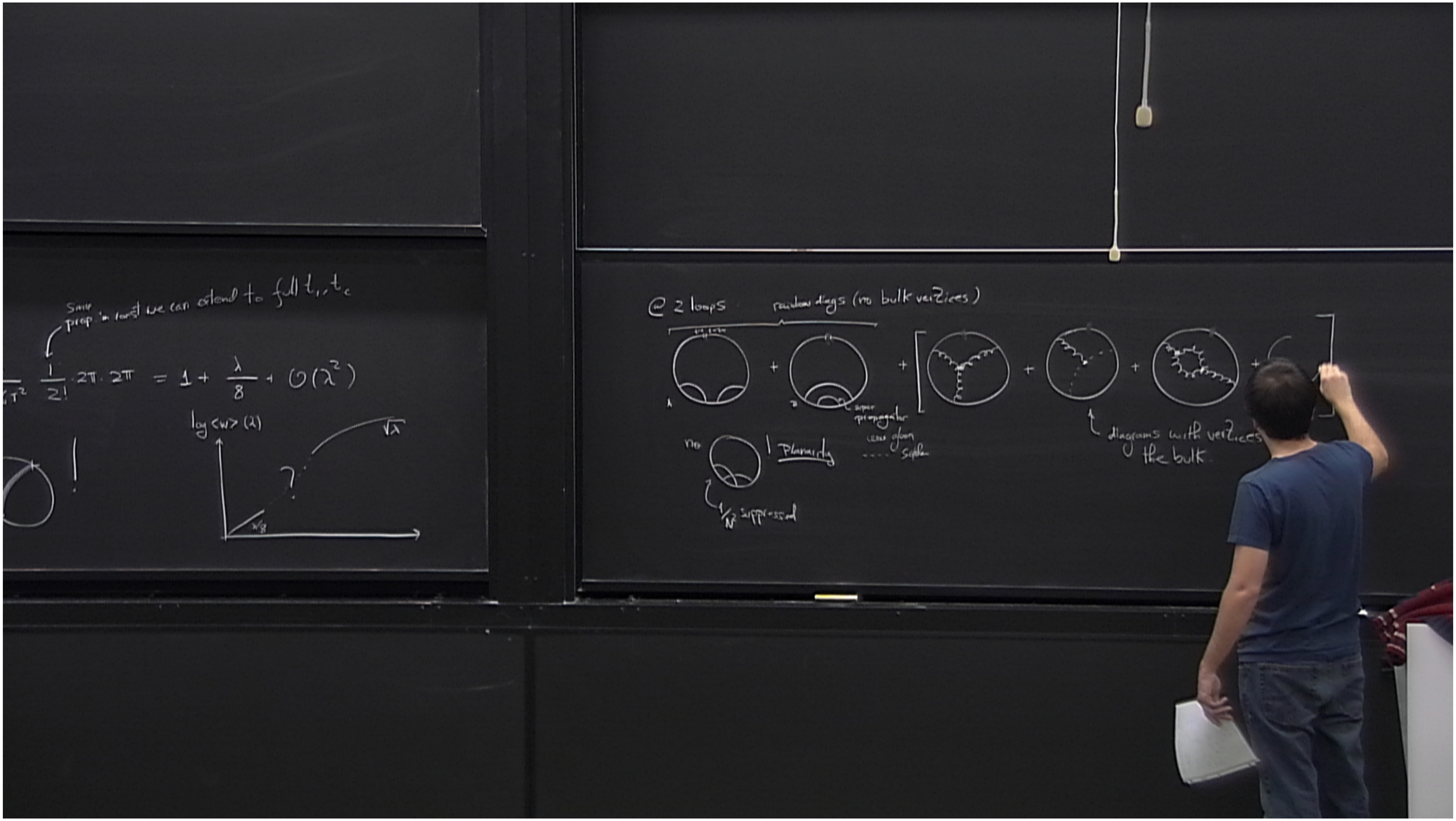
@ 2 loops

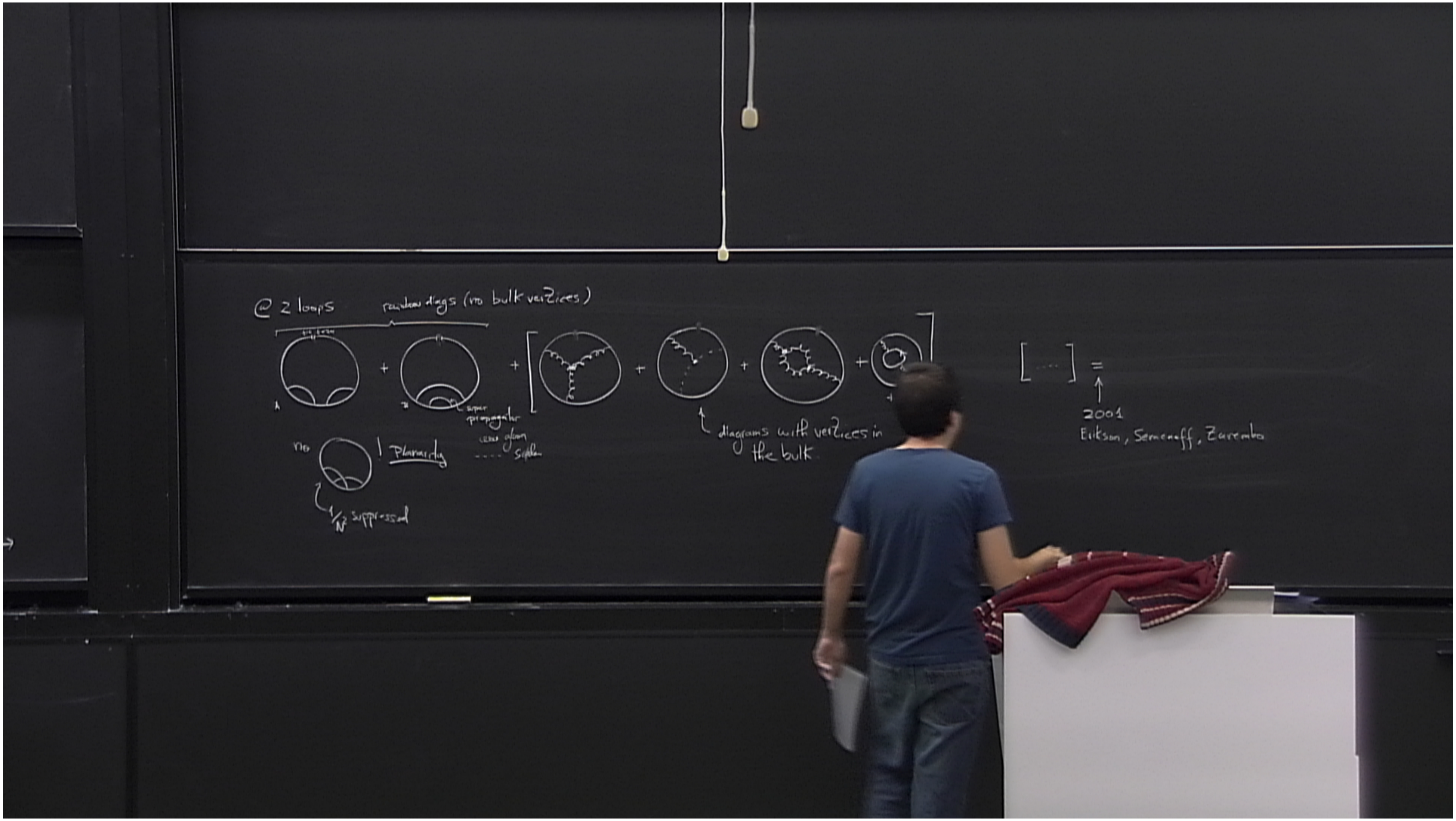
no ! Planarity

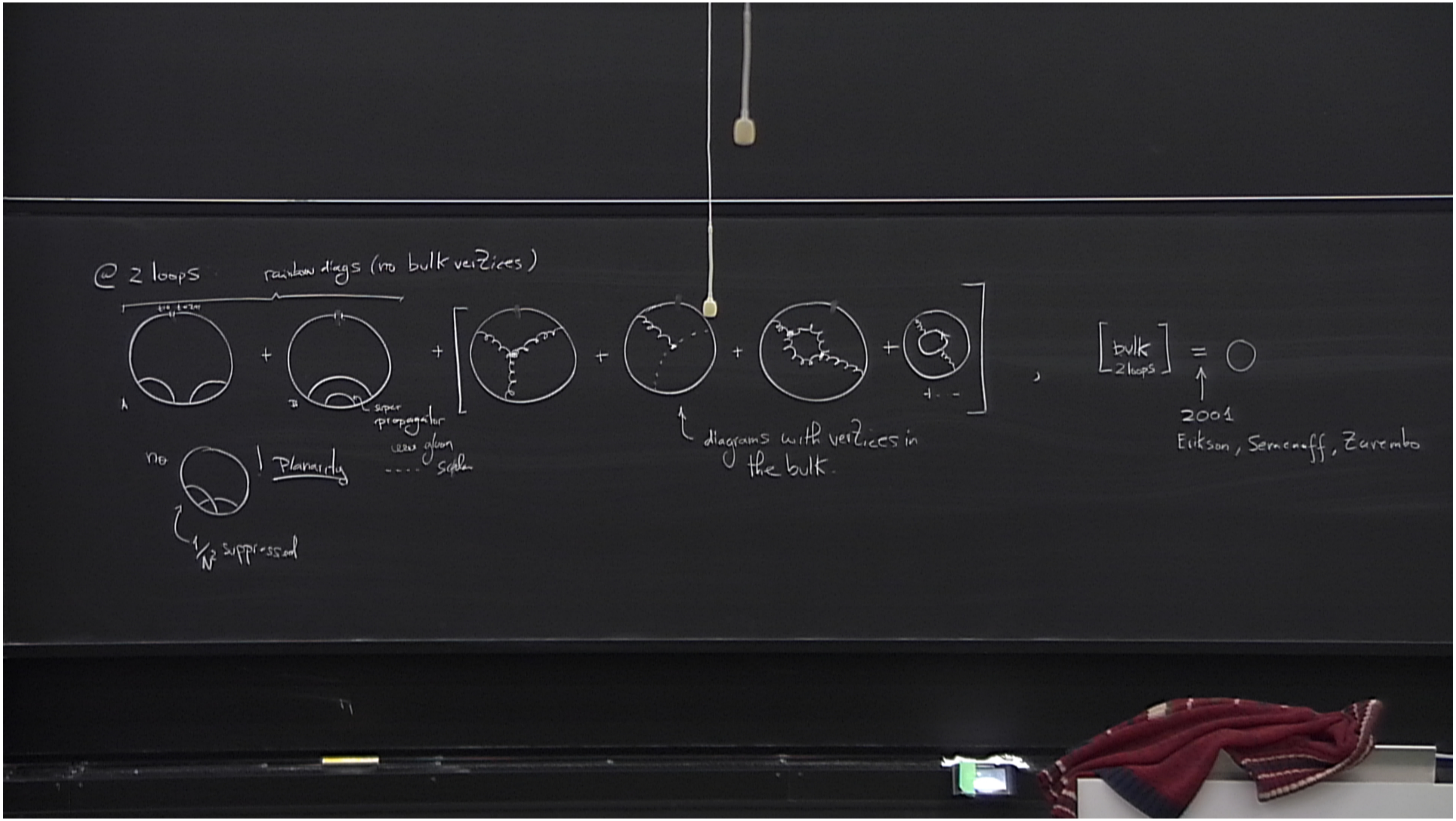
$1/N^2$ suppressed

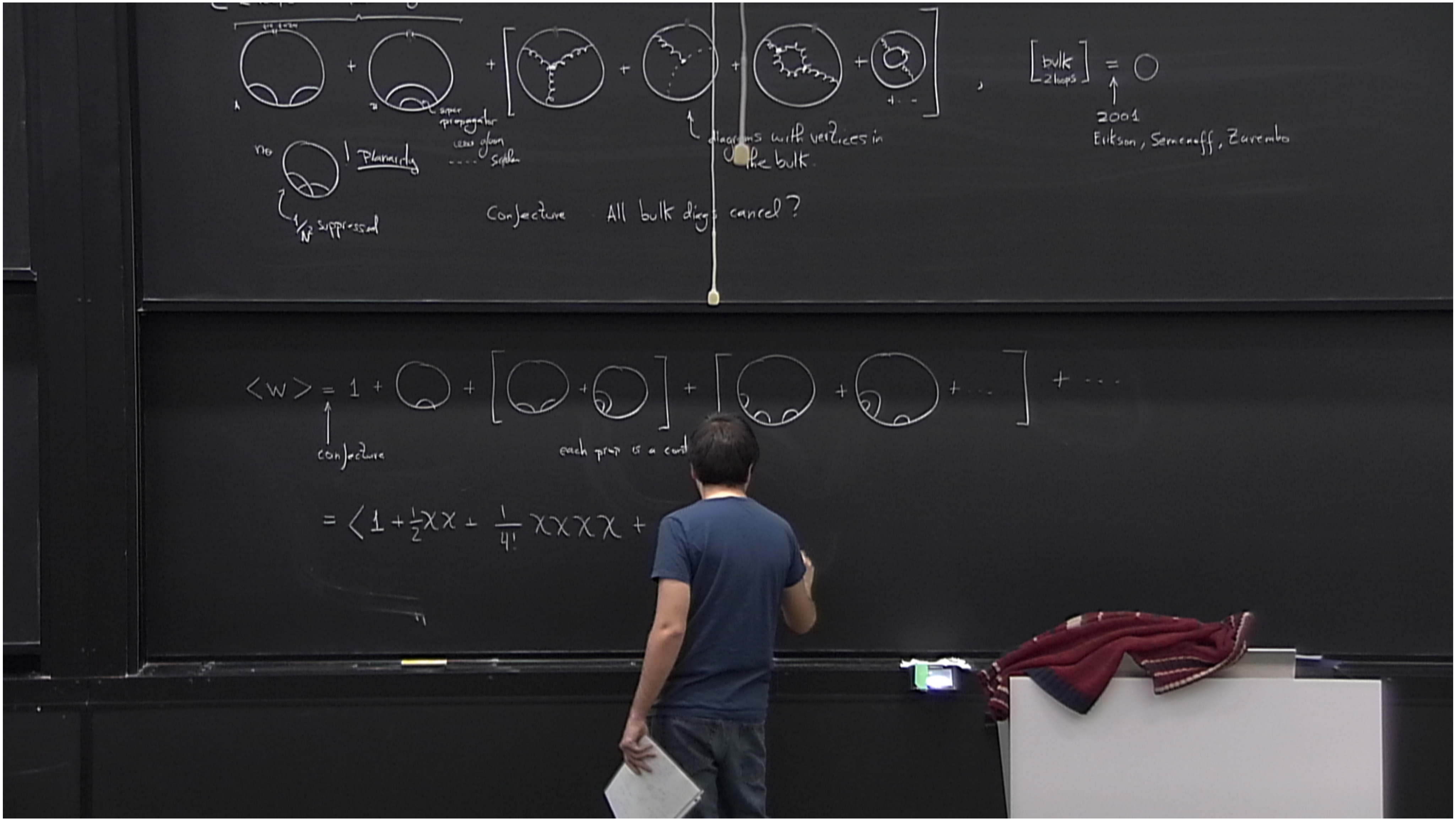
diagrams with vertices in the bulk.

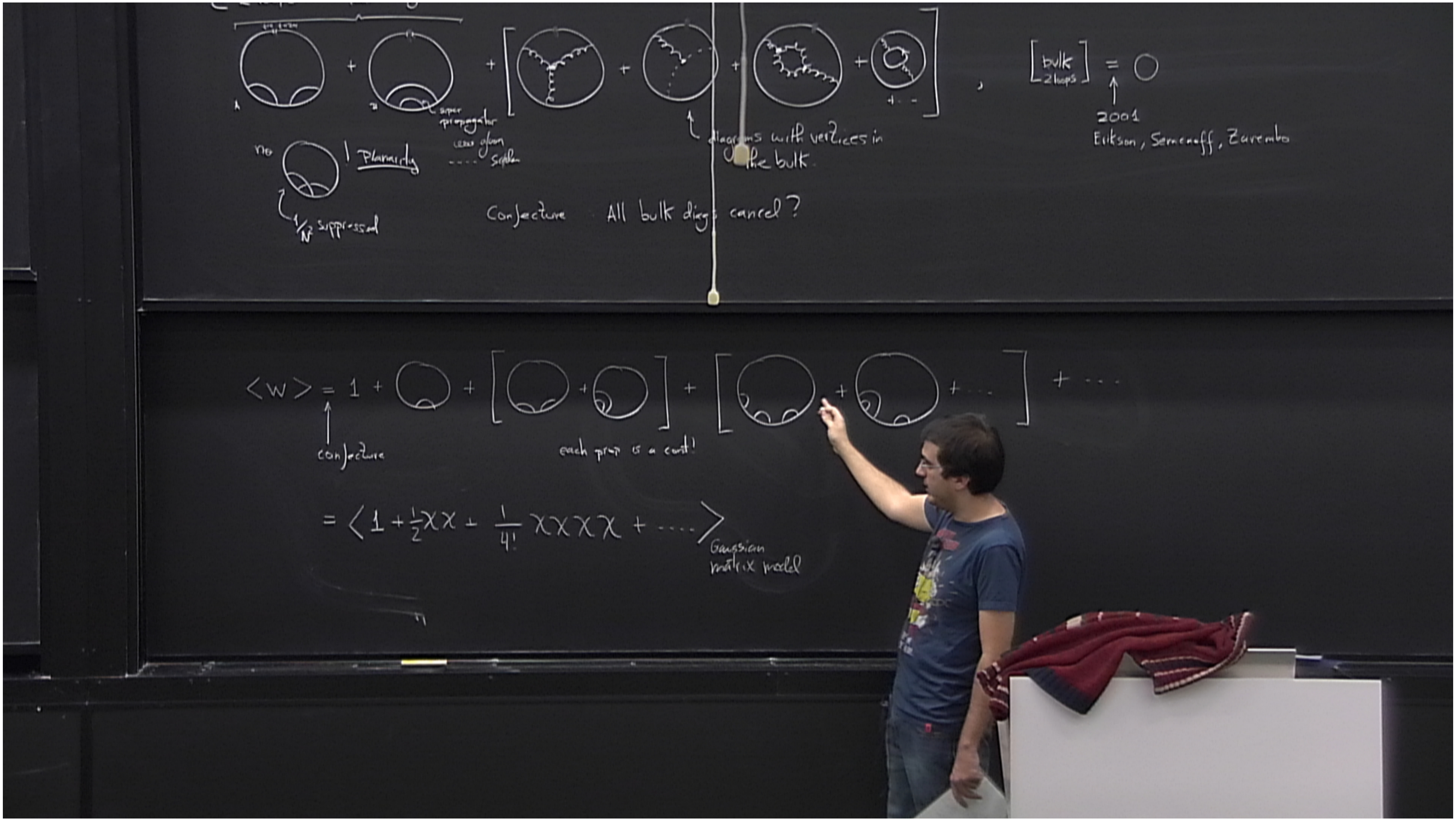

@ 2 loops
rainbow diags (no bulk vertices)
+
super propagator
gluon
scalar
no
! Planarity
1/N² suppressed
diagrams with vertices in the bulk.

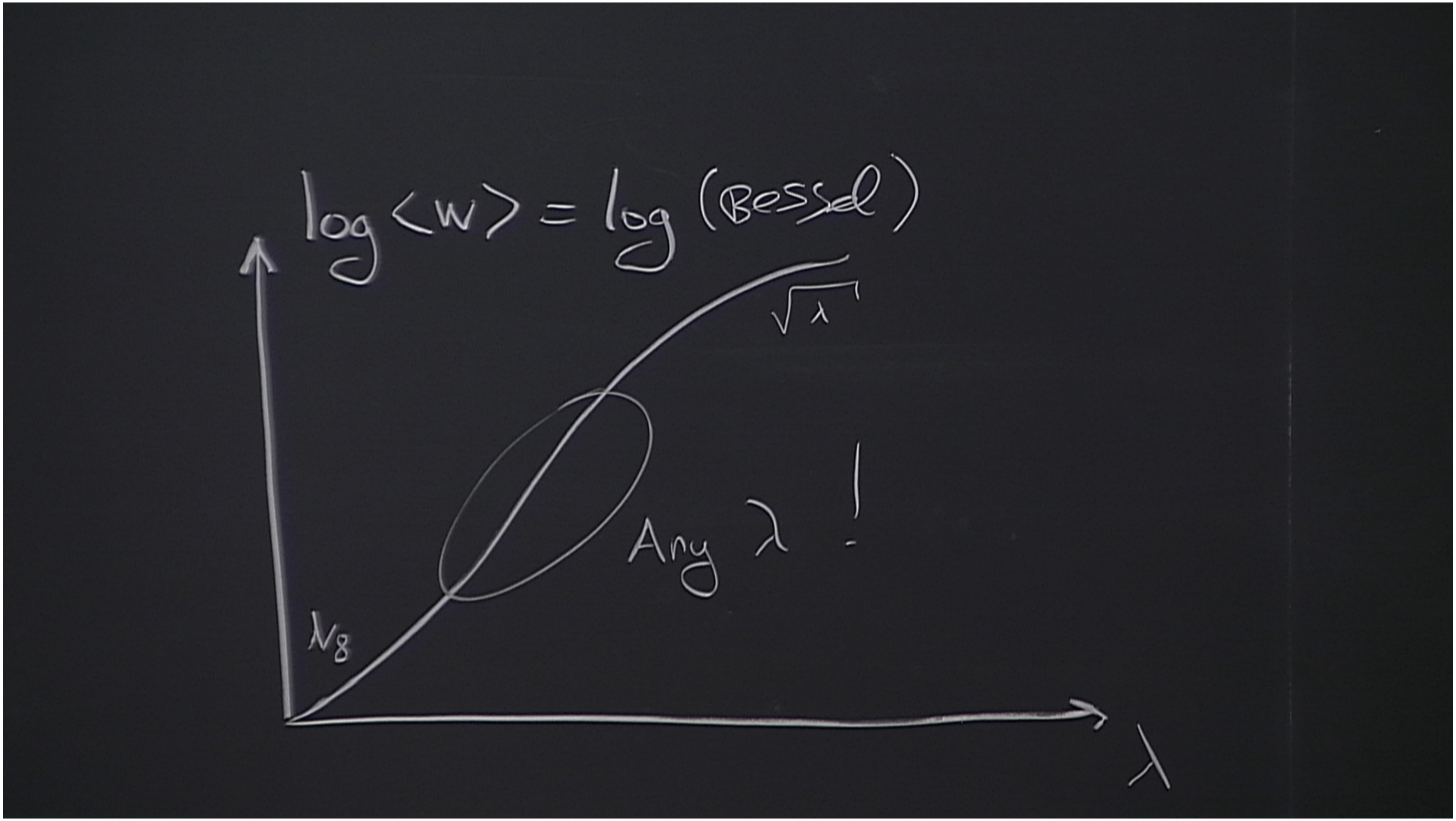

same prop. in const we can extend to full t_1, t_c
$\frac{1}{2!} \cdot 2\pi \cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$
!
$\log \langle W \rangle (\lambda)$
?
$\sqrt{\lambda}$
$\lambda/8$
@ 2 loops
rainbow diags (no bulk vertices)
+
+ [
+
+
+]
super propagator
gluon
scalar
no
! Planarity
$1/N^2$ suppressed
diagrams with vertices the bulk

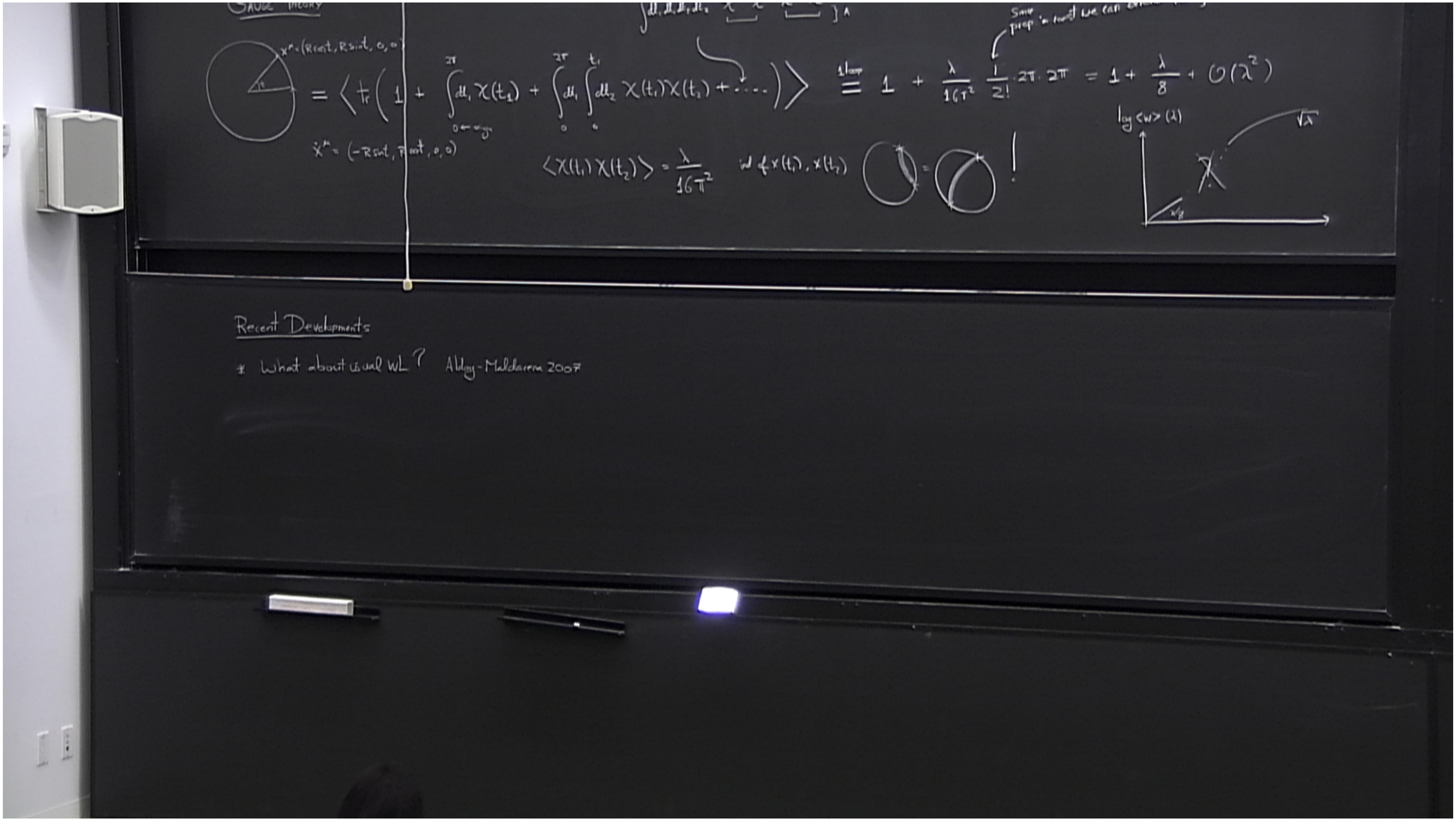

@ 2 loops
rainbow diags (no bulk vertices)
+
+
+
+
+
super propagator
scalar gluon
- - - - scalar
no
! Planarity
1/N² suppressed
diagrams with vertices in the bulk
[...] =
2001
Erikson, Semenoff, Zarembo


@ 2 loops
rainbow diags (no bulk vertices)
+
+
+
+
+
super propagator
gluon
scalar
no
! Planarity
1/N^2 suppressed
diagrams with vertices in the bulk.
bulk 2 loops = 0
2001
Erikson, Semenoff, Zarembo


super propagator
gluon
scalar
no
Planarity
1/N² suppressed
+ . . .
diagrams with vertices in the bulk
[bulk 2 loops] = 0
2001
Erikson, Semenoff, Zarembo
Conjecture   All bulk diag cancel ?
<w> = 1 + + [+] + [+ + . . .] + . . .
conjecture
each prop is a const
= < 1 + ½ XX + 1/4! XXXX +


super propagator
gluon
scalar
no
Planarity
1/N² suppressed
diagrams with vertices in the bulk
[bulk 2 loops] = 0
2001
Erikson, Semenoff, Zarembo
Conjecture All bulk diag's cancel?
<W> = 1 +
+ . . .
conjecture
each prop is a const!
= < 1 + ½ XX + 1/4! XXXX + …. >
Gaussian matrix model


log ⟨W⟩ = log (Bessel)
√λ
Any λ !
λ/8
λ



GAUGE THEORY

$x^\mu = (R\cos t, R\sin t, 0, 0)$

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$$= \left\langle \mathrm{tr}\left(1 + \int_{0}^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \cdots \right)\right\rangle \overset{1\,\mathrm{loop}}{=} 1 + \frac{\lambda}{16\pi^2}\frac{1}{2!}\cdot 2\pi\cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$$

0 ← origin

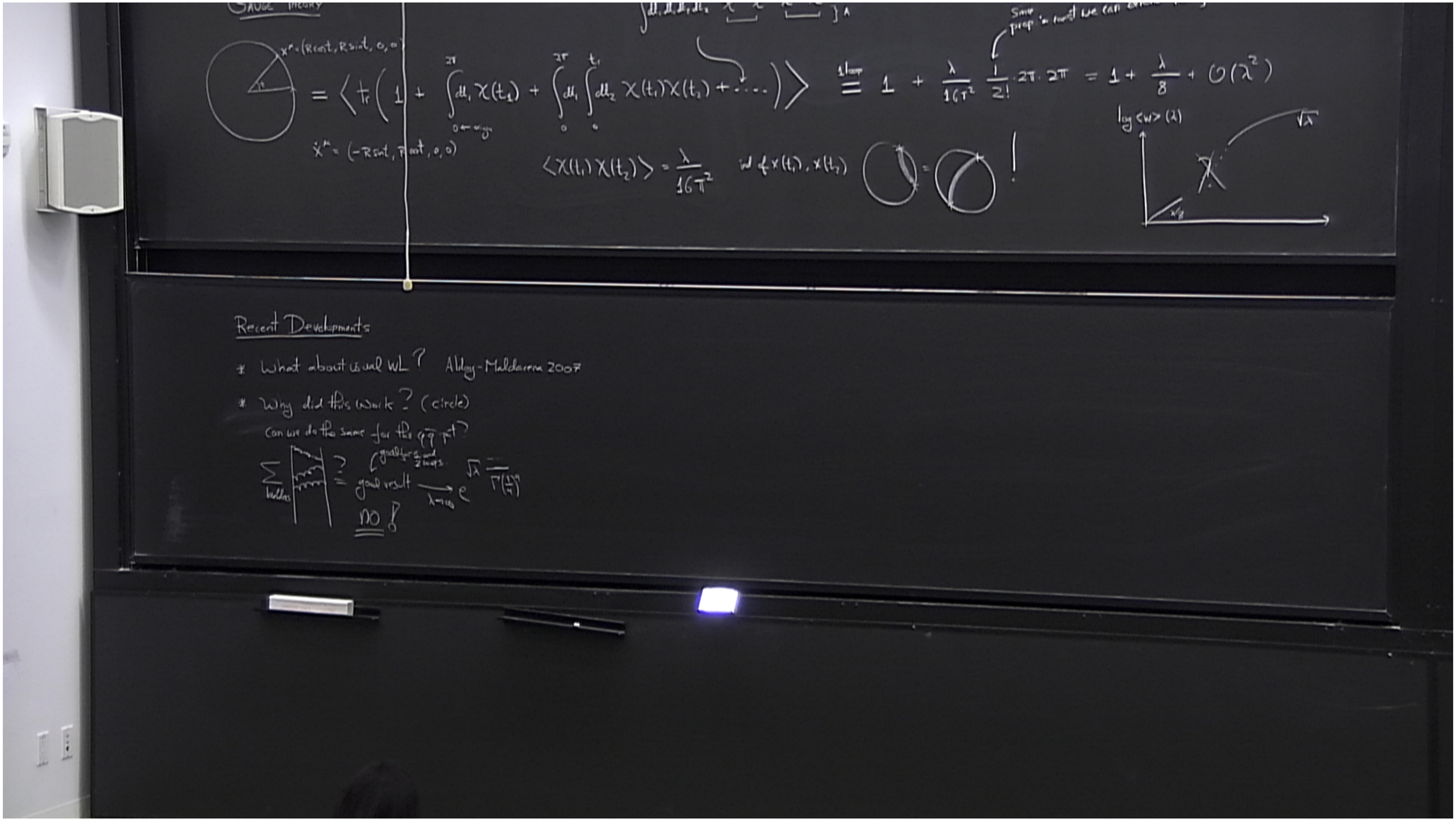
Since prop 'in const' we can

$$\langle \chi(t_1)\chi(t_2)\rangle = \frac{\lambda}{16\pi^2} \quad \text{ind of } x(t_1), x(t_2)$$

= !

Recent Developments

* What about usual WL? Alday-Maldacena 2007



GAUGE THEORY

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Since prop. in const we can ...

$$\langle \chi(t_1)\chi(t_2)\rangle = \frac{\lambda}{16\pi^2}$$ ind of $x(t_1), x(t_2)$

!

$\log \langle W\rangle(\lambda)$

$\sqrt{\lambda}$

$\lambda/8$

Recent Developments

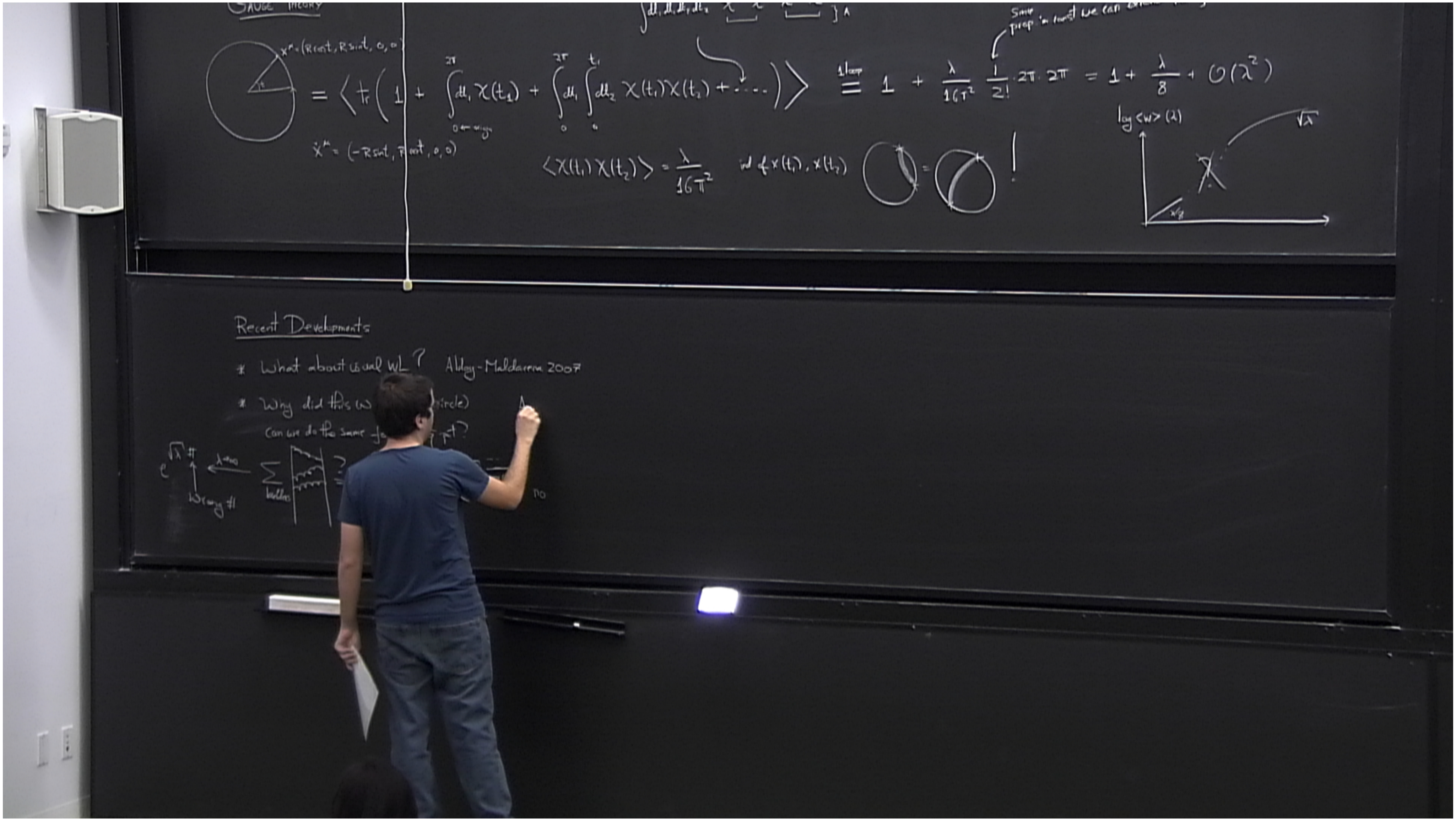
- * What about usual WL ? Alday - Maldacena 2007
- * Why did this work ? (circle)

Can we do the same for the $q\bar{q}$ pot ?

$\sum_{\text{ladders}}$? = good result $\xrightarrow[\lambda\to\infty]{} e^{\sqrt{\lambda}\,\frac{\ldots}{\Gamma(\frac{1}{4})^4}}$

good for 1 and 2 loops

no !



GAUGE THEORY

$x^\mu = (R\cos t, R\sin t, 0, 0)$

$$= \left\langle \mathrm{tr}\left(1 + \int_{0 \leftarrow \text{origin}}^{2\pi} dt_1\, \chi(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, \chi(t_1)\chi(t_2) + \cdots \right) \right\rangle \overset{1\,\text{loop}}{=} 1 + \frac{\lambda}{16\pi^2}\frac{1}{2!}\cdot 2\pi \cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$$

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$\langle \chi(t_1)\chi(t_2) \rangle = \frac{\lambda}{16\pi^2}$ ind of $x(t_1), x(t_2)$

Since prop. is const we can

$\log \langle W \rangle (\lambda)$

$\sqrt{\lambda}$

$\lambda/8$

Recent Developments

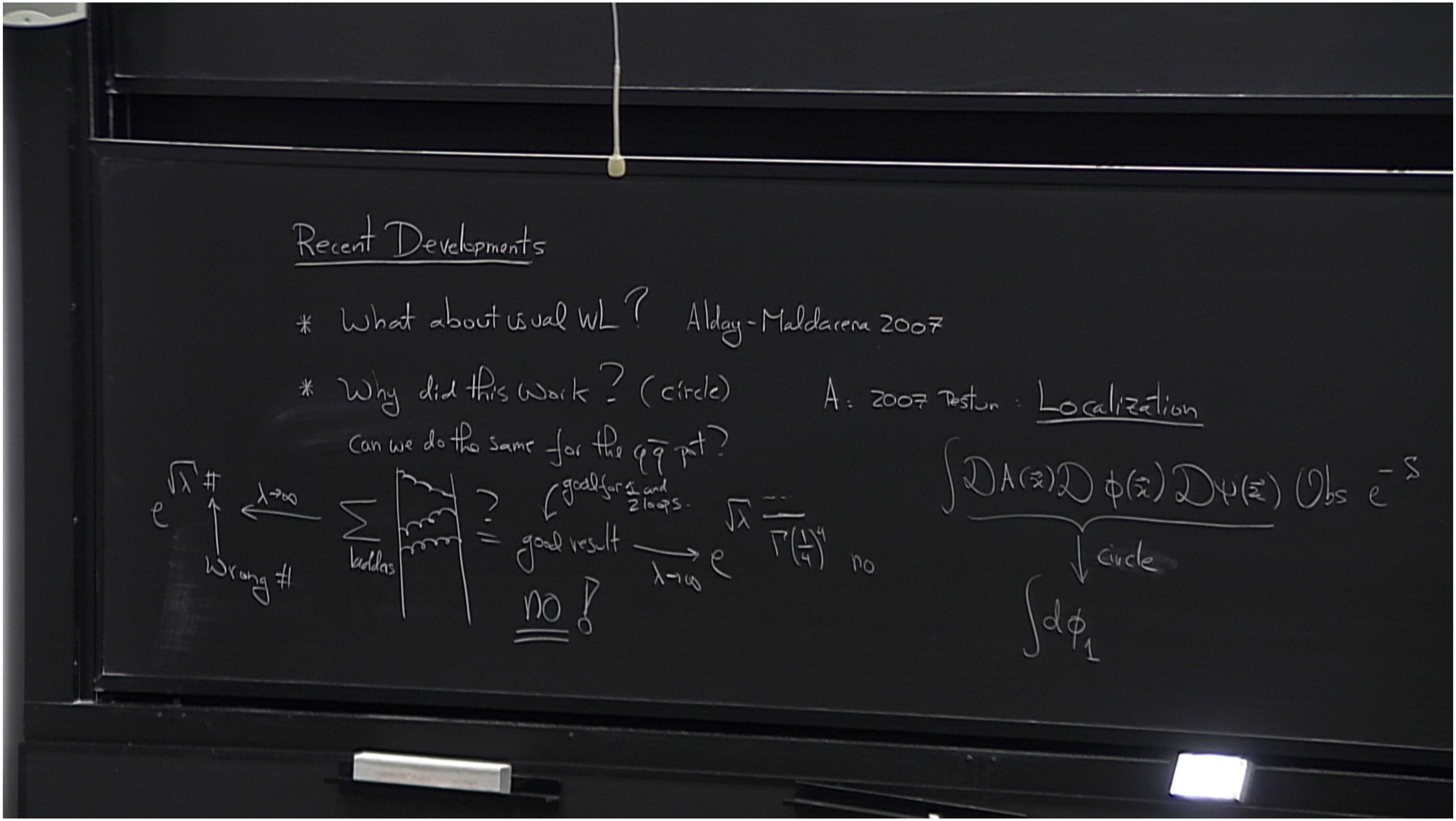
- What about usual WL? Alday-Maldacena 2007
- Why did this w… circle)

Can we do the same fo… pt?

$e^{\sqrt{\lambda}\,\#} \xleftarrow{\lambda\to\infty} \sum_{\text{ladders}}$?

Wrong #!

no



Recent Developments

- * What about usual WL? Alday - Maldacena 2007
- * Why did this work? (circle) A: 2007 Pestun: Localization

Can we do the same for the $q\bar{q}$ pot?

$$e^{\sqrt{\lambda}\,\#} \xleftarrow{\lambda\to\infty} \sum_{\text{ladders}} \overset{?}{=} \text{good result} \xrightarrow[\lambda\to\infty]{} e^{\sqrt{\lambda}\frac{--}{\Gamma\left(\frac{1}{4}\right)^4}} \quad \text{no}$$

Wrong #

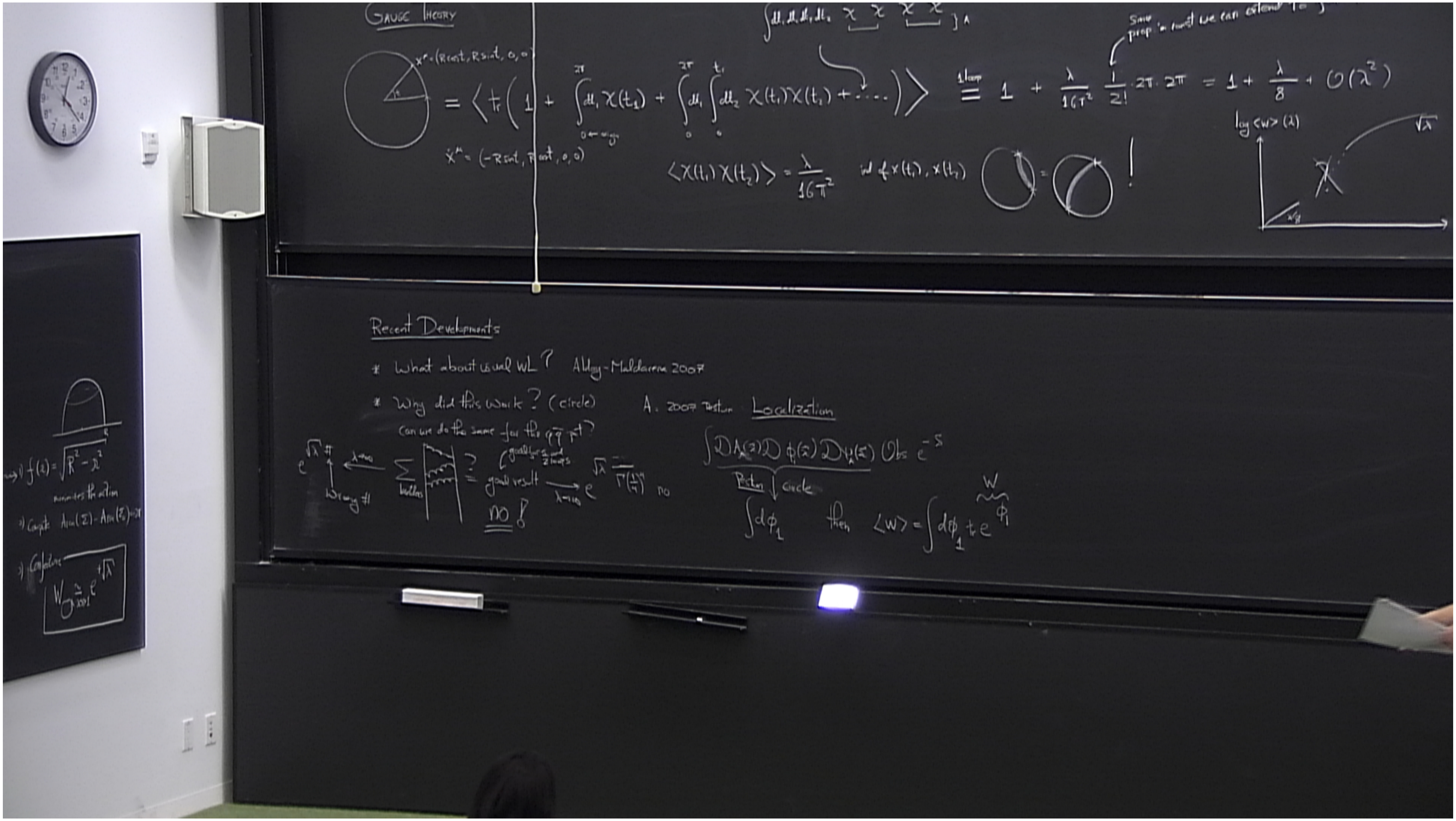
good for 1 and 2 loops.

no!

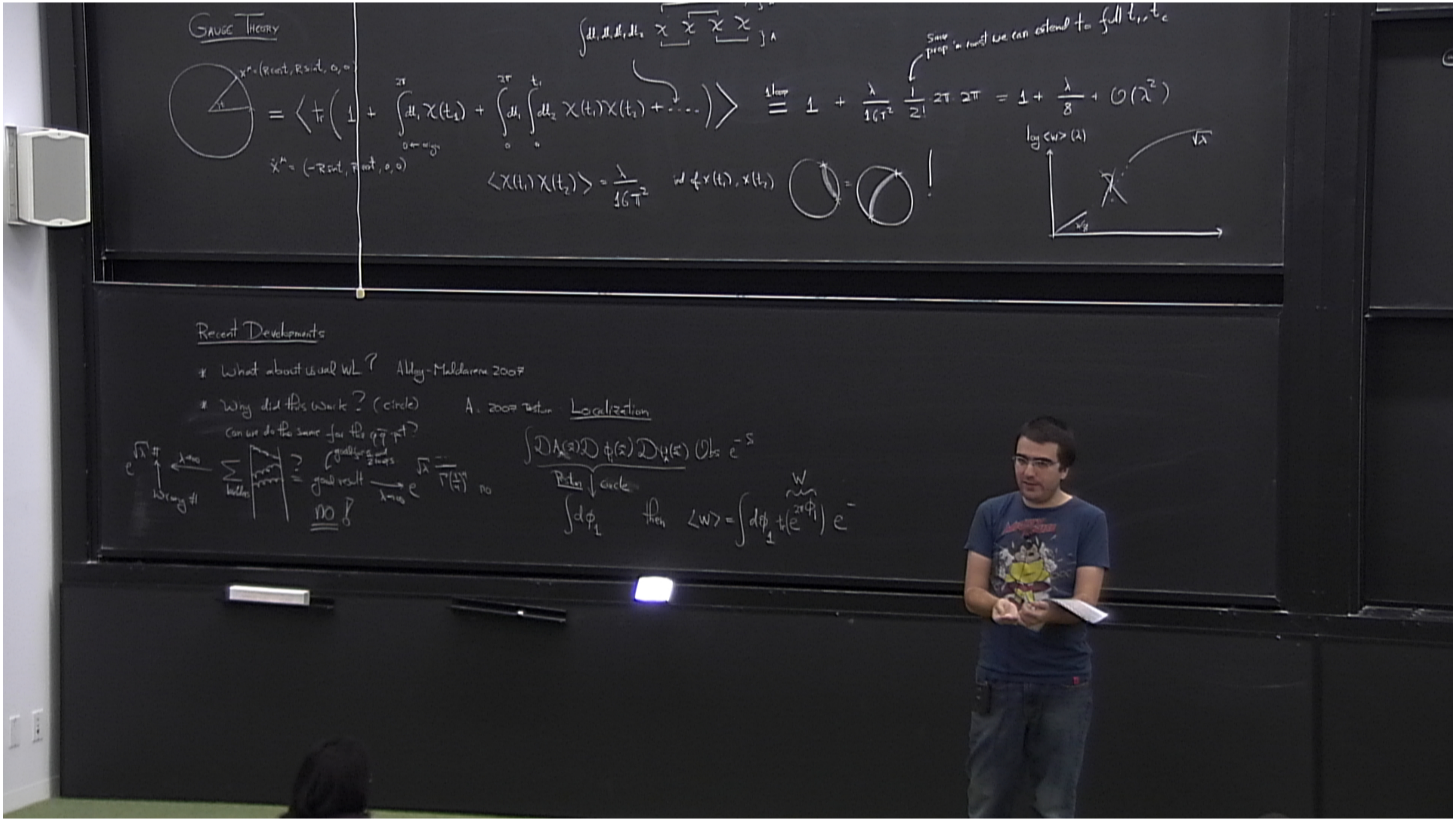
$$\underbrace{\int \mathcal{D}A(\vec{x})\,\mathcal{D}\phi(\vec{x})\,\mathcal{D}\psi(\vec{x})}\ \mathcal{O}bs\ e^{-S}$$

$$\Big\downarrow \text{circle}$$

$$\int d\phi_1$$



GAUGE THEORY
$\langle \chi(t_1)\chi(t_2)\rangle = \frac{\lambda}{16\pi^2}$
$= 1 + \frac{\lambda}{8} + O(\lambda^2)$
Recent Developments
What about usual WL? Alday-Maldacena 2007
Why did this work? (circle) A. 2007 Pestun Localization
Can we do the same for the $q\bar{q}$ pot?
no!
then $\langle W\rangle = \int d\phi_1 \, tr \, e^{\phi_1}$
Conjecture



GAUGE THEORY

$x^\mu = (R\cos t, R\sin t, 0, 0)$

$$= \left\langle \mathrm{tr}\left(1 + \int_0^{2\pi} dt_1\, X(t_1) + \int_0^{2\pi} dt_1 \int_0^{t_1} dt_2\, X(t_1)X(t_2) + \cdots\right)\right\rangle \overset{1\,\mathrm{loop}}{=} 1 + \frac{\lambda}{16\pi^2}\frac{1}{2!}\,2\pi\cdot 2\pi = 1 + \frac{\lambda}{8} + \mathcal{O}(\lambda^2)$$

0 ← vev

$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$\int dt_1 dt_1 dt_1 dt_2\ \underbrace{x\ x}\ \overbrace{x\ \underbrace{x\ x}}\ \} \lambda$

Since prop is const we can extend to full t_1, t_2

$\langle X(t_1)X(t_2)\rangle = \frac{\lambda}{16\pi^2}$ ind of $x(t_1), x(t_2)$ = !

$\log\langle W\rangle(\lambda)$

$\sqrt{\lambda}$

$\lambda/8$

Recent Developments

* What about usual WL ? Alday-Maldacena 2007

* Why did this work ? (circle) A. 2007 Pestun Localization

Can we do the same for the $q\bar{q}$ pot ?

$e^{\sqrt{\lambda}\,\#} \xleftarrow{\lambda\to\infty} \sum_{\text{ladders}}$? = good result $\xrightarrow{\lambda\to\infty} e^{\sqrt{\lambda}\frac{\cdots}{\Gamma(\frac{1}{4})^4}}$ no

wrong #!

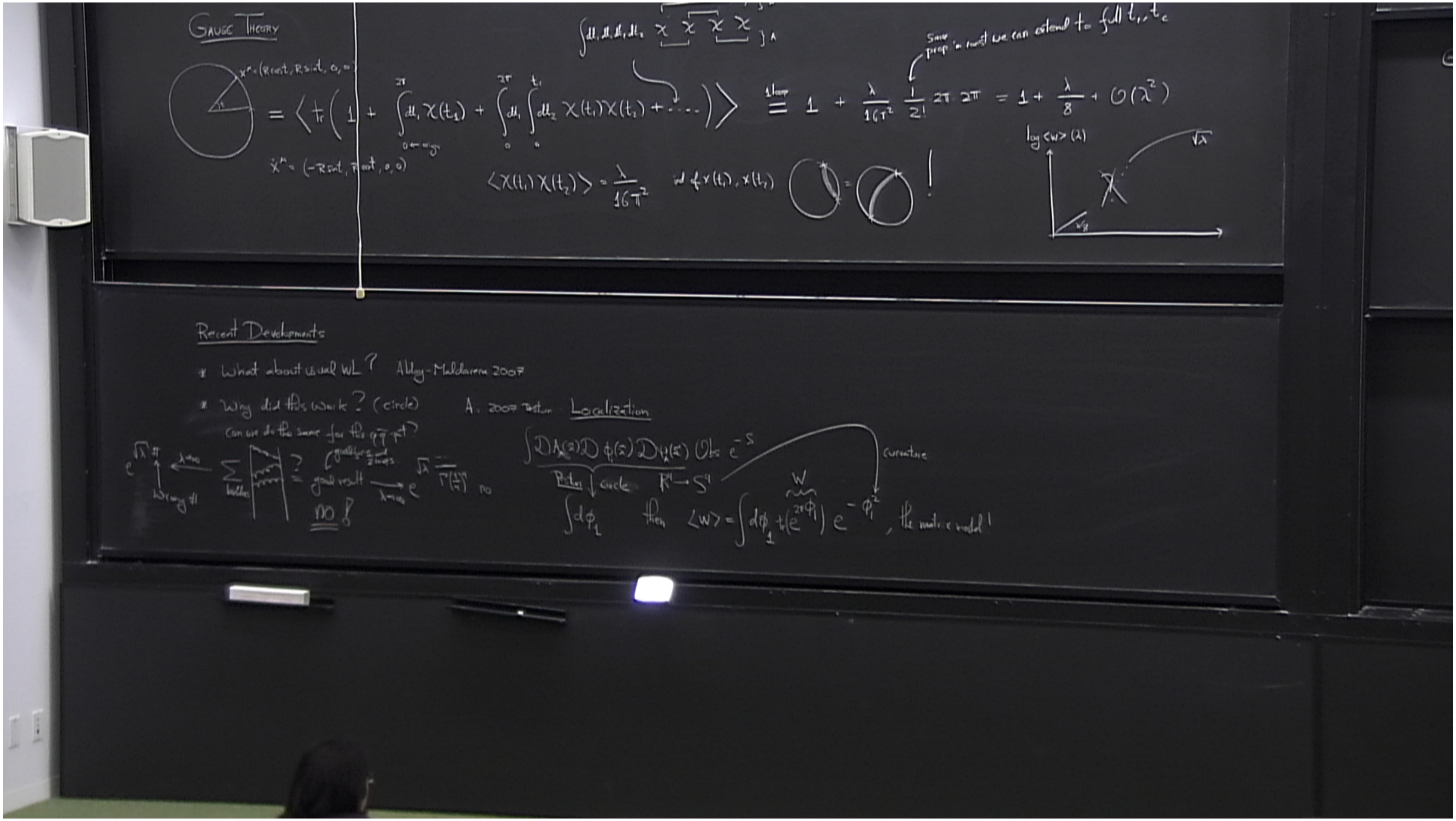
good for 8 and 2 loops

NO !

$$\underbrace{\int \mathcal{D}A_\mu(x)\,\mathcal{D}\phi_i(x)\,\mathcal{D}\psi_A(x)}\ \mathrm{Obs}\ e^{-S}$$

Pestun ↓ circle

$\int d\phi_1$ then $\langle W\rangle = \int d\phi_1\, \mathrm{tr}(\overbrace{e^{2\pi\phi_1}}^{W})\, e^{-}$



GAUGE THEORY

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$\int dt_1 dt_2 dt_3 dt_4\; \chi\; \chi\; \chi\; \chi \;\} \lambda$

Since prop is const we can extend to full t_1, t_2

$$\langle \chi(t_1)\chi(t_2) \rangle = \frac{\lambda}{16\pi^2}$$ ind of $x(t_1), x(t_2)$

= !

$\log \langle W \rangle (\lambda)$

$\sqrt{\lambda}$

$\lambda/8$

Recent Developments

- What about usual WL? Alday-Maldacena 2007
- Why did this work? (circle) A. 2007 Pestun Localization

Can we do the same for the $q\bar{q}$ pot?

$e^{\sqrt{\lambda}\,\#} \xleftarrow{\lambda\to\infty} \sum_{\text{ladders}}$? = good result $\xrightarrow{\lambda\to\infty} e^{\sqrt{\lambda}\,\frac{\cdots}{\Gamma(\frac{1}{4})^4}}$ no

wrong #!

good for small λ / 2 lines

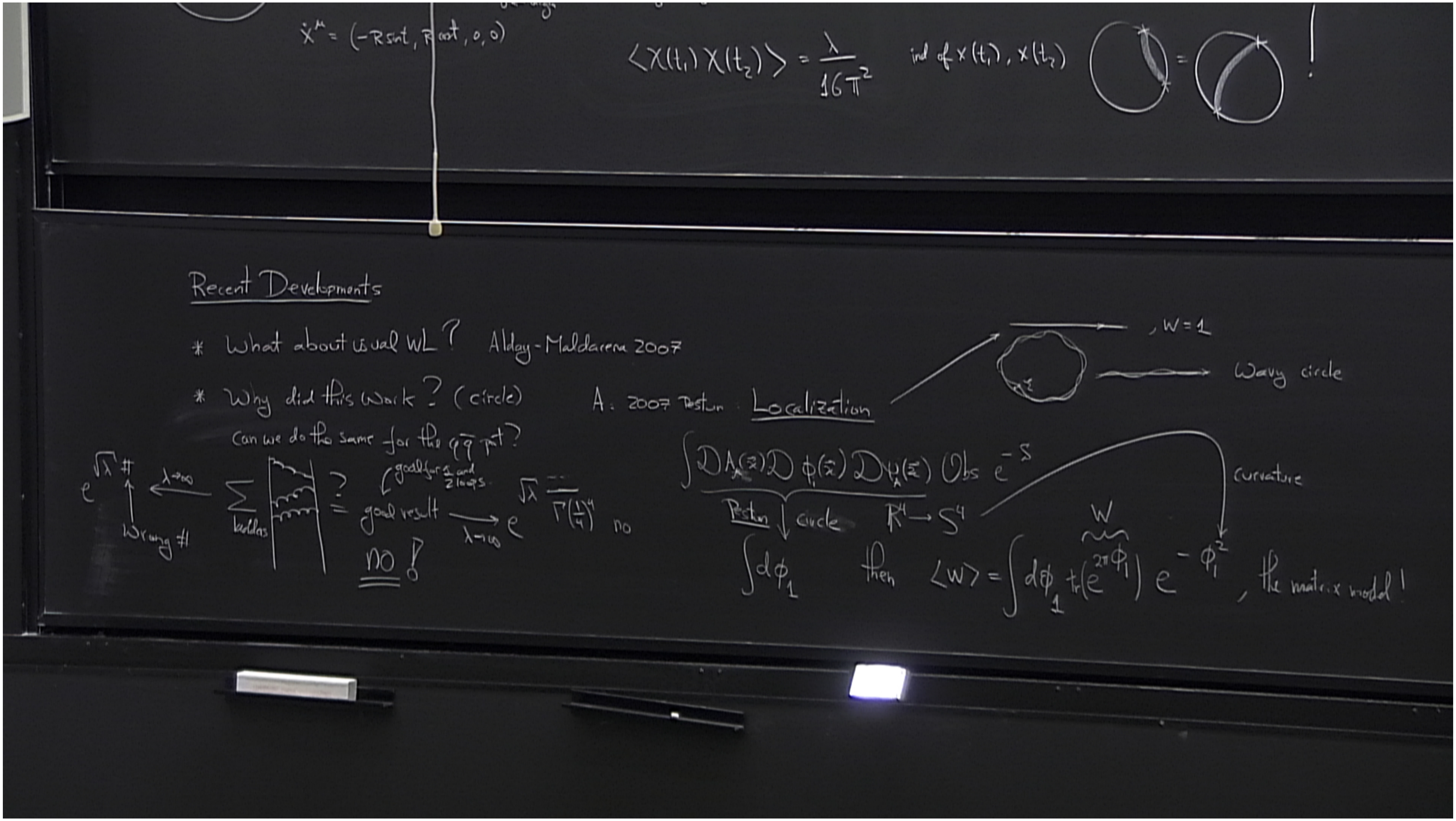
NO!

$$\int \mathcal{D}A_\mu(x)\, \mathcal{D}\phi_i(x)\, \mathcal{D}\psi_\lambda(x)\; \mathcal{O}bs\; e^{-S}$$

Pestun $\downarrow$ circle $\mathbb{R}^4 \to S^4$

curvature

$\int d\phi_1$ then $\langle W \rangle = \int d\phi_1\, \mathrm{tr}(\overset{W}{\widetilde{e^{2\pi\phi_1}}})\, e^{-\phi_1^2}$, the matrix model!



$\dot{x}^\mu = (-R\sin t, R\cos t, 0, 0)$

$\langle X(t_1) X(t_2) \rangle = \frac{\lambda}{16\pi^2}$ ind of $x(t_1), x(t_2)$ = !

Recent Developments

* What about usual WL? Aldey-Maldacena 2007

* Why did this work? (circle) A: 2007 Pestun Localization

Can we do the same for the $q\bar{q}$ pot?

$$e^{\sqrt{\lambda}\,\#} \xleftarrow{\lambda\to\infty} \sum_{\text{ladders}} \; ? = \text{good result} \xrightarrow[\lambda\to\infty]{} e^{\sqrt{\lambda}\frac{--}{\Gamma(\frac{1}{4})^4}} \text{ no}$$

Wrong #

good for 1 and 2 loops.

no!

, W = 1

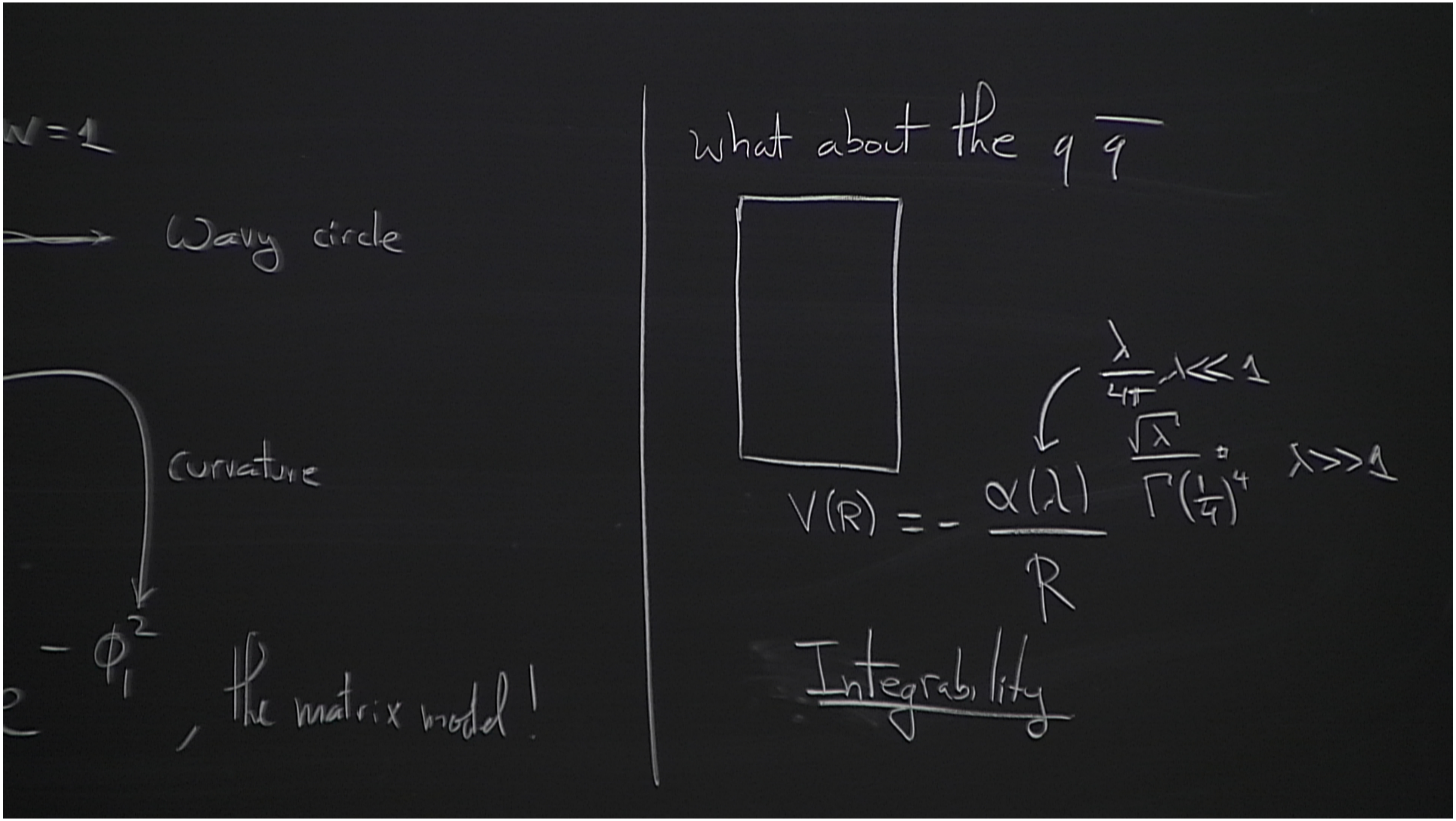
Wavy circle

$$\underbrace{\int \mathcal{D}A_\mu(\vec{x})\,\mathcal{D}\phi_i(\vec{x})\,\mathcal{D}\psi_A(\vec{x})}_{\text{Pestun}}\ \mathcal{O}bs\ e^{-S}$$

curvature

Pestun ↓ circle $R^4 \to S^4$

$\int d\phi_1$ then $\langle W \rangle = \int d\phi_1\, \mathrm{tr}(\overbrace{e^{2\pi\phi_1}}^{W})\, e^{-\phi_1^2}$, the matrix model!



$N = 1$

$\longrightarrow$ Wavy circle

curvature

$e^{-\phi_1^2}$, the matrix model!

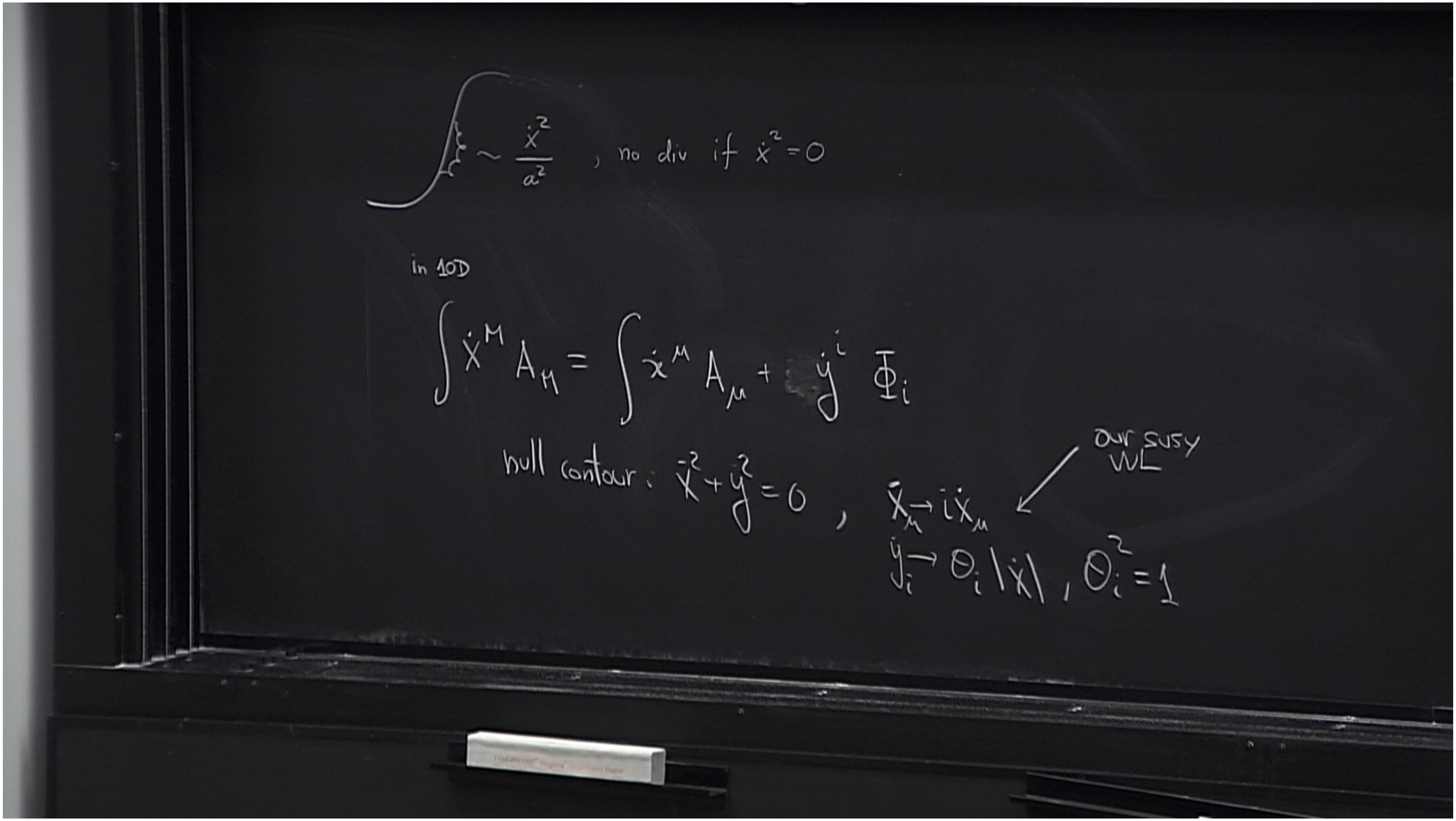
What about the $q\bar{q}$

$$V(R) = -\frac{\alpha(\lambda)}{R}$$

$\frac{\lambda}{4\pi}$ $\lambda \ll 1$

$\frac{\sqrt{\lambda}}{\Gamma(\frac{1}{4})^4}$ $\lambda \gg 1$

Integrability



$\sim \frac{\dot{x}^2}{a^2}$, no div if $\dot{x}^2=0$

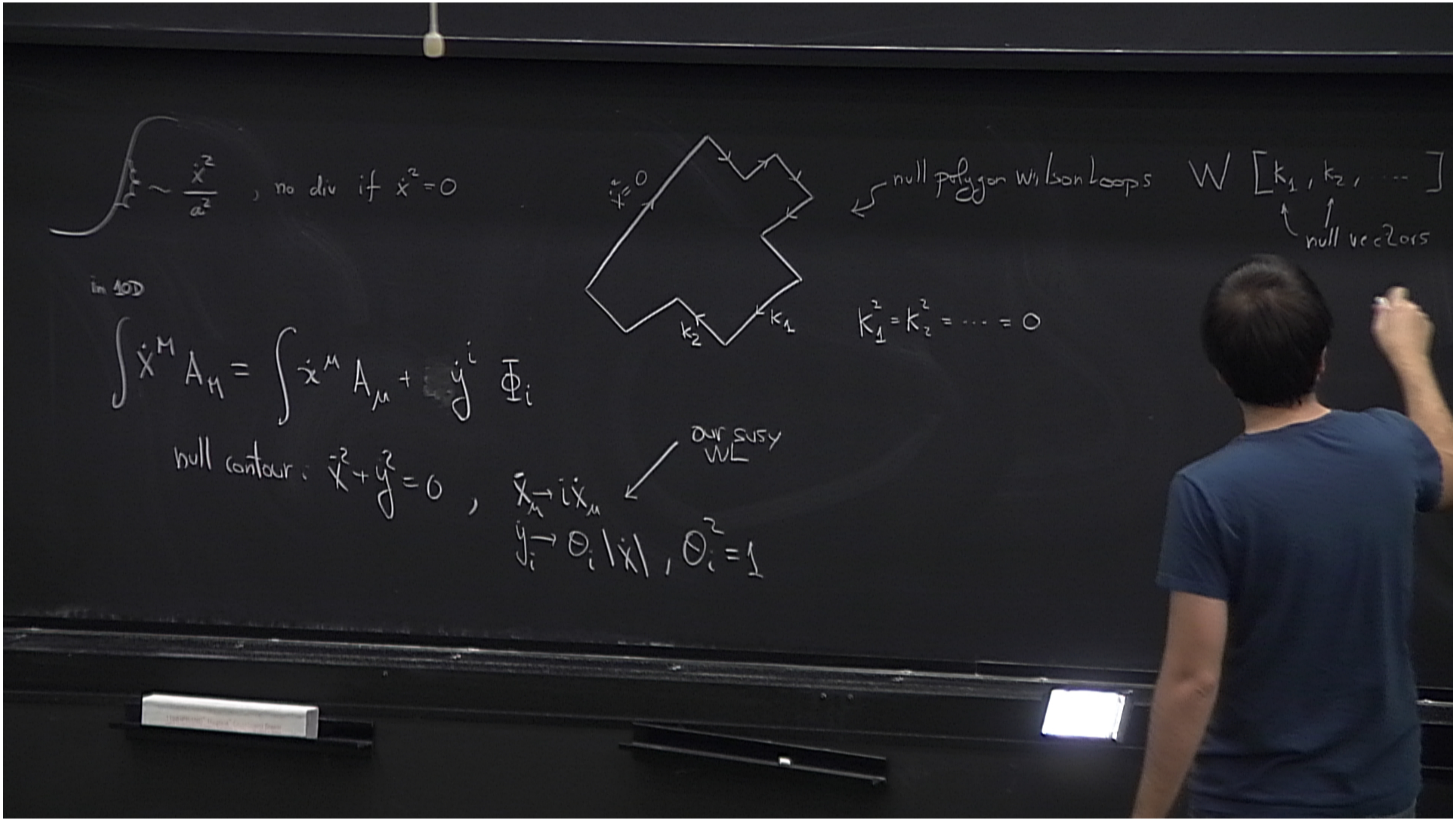
in 10D

$$\int \dot{X}^M A_M = \int \dot{x}^\mu A_\mu + \dot{y}^i \Phi_i$$

null contour: $\dot{x}^2+\dot{y}^2=0$, $\dot{x}_\mu \to i\dot{x}_\mu$

$\dot{y}_i \to \theta_i |\dot{x}|$, $\theta_i^2=1$

our susy WL



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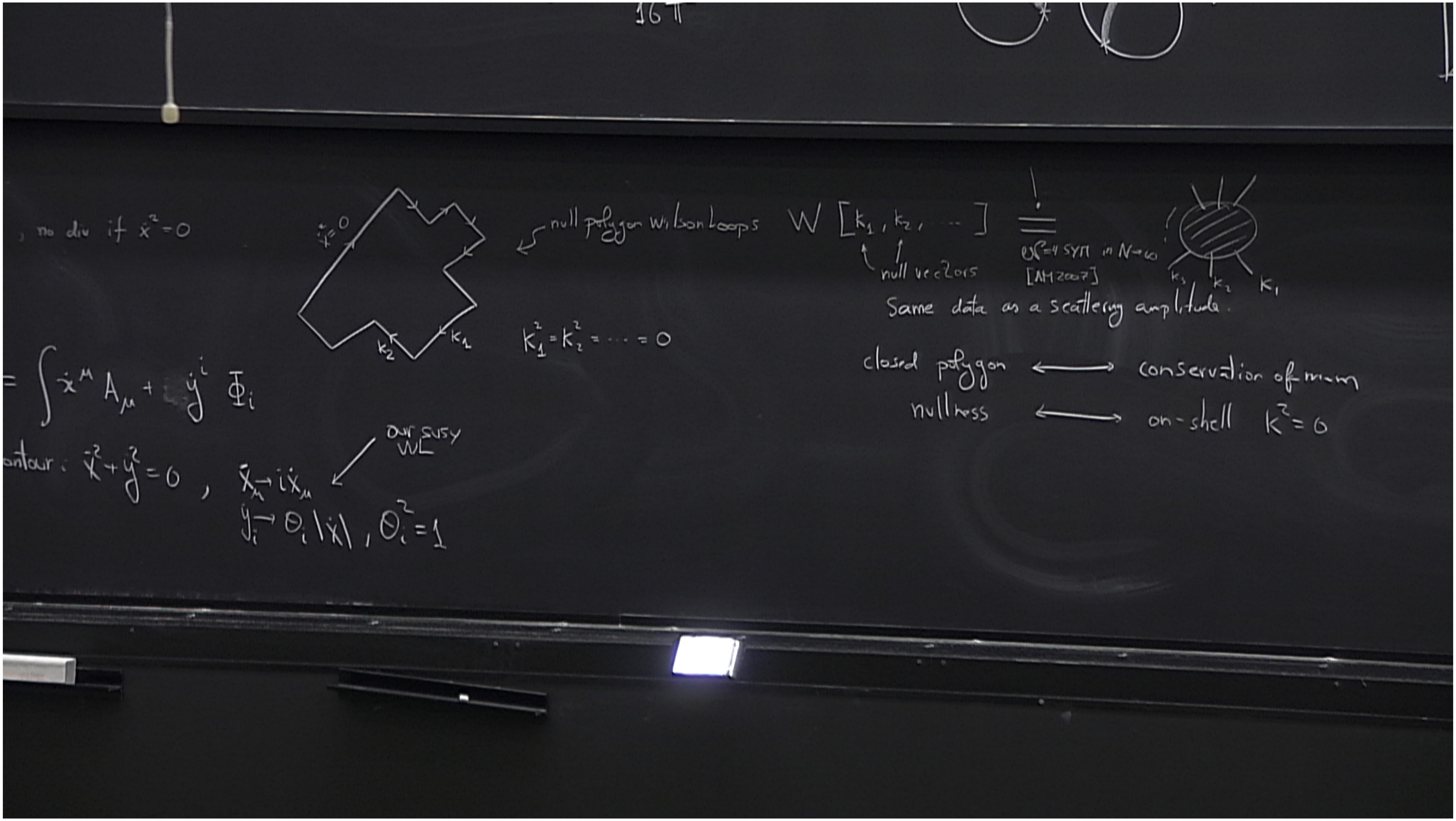
$\dot{y}_i \to \theta_i |\dot{x}|$, $\theta_i^2 = 1$

our susy WL

null polygon Wilson Loops $W[k_1, k_2, \cdots]$

null vectors

$k_1^2 = k_2^2 = \cdots = 0$



16π

, no div if $\dot{x}^2 = 0$

$\dot{x}^2 = 0$

$k_1 \quad k_2$

null polygon Wilson Loops

$k_1^2 = k_2^2 = \cdots = 0$

$W[k_1, k_2, \cdots] \overset{!}{\equiv}$ $k_3 \quad k_2 \quad k_1$

null vectors

$\mathcal{N}=4$ SYM in $N \to \infty$

[AM 2007]

Same data as a scattering amplitude.

closed polygon ⟷ conservation of mom

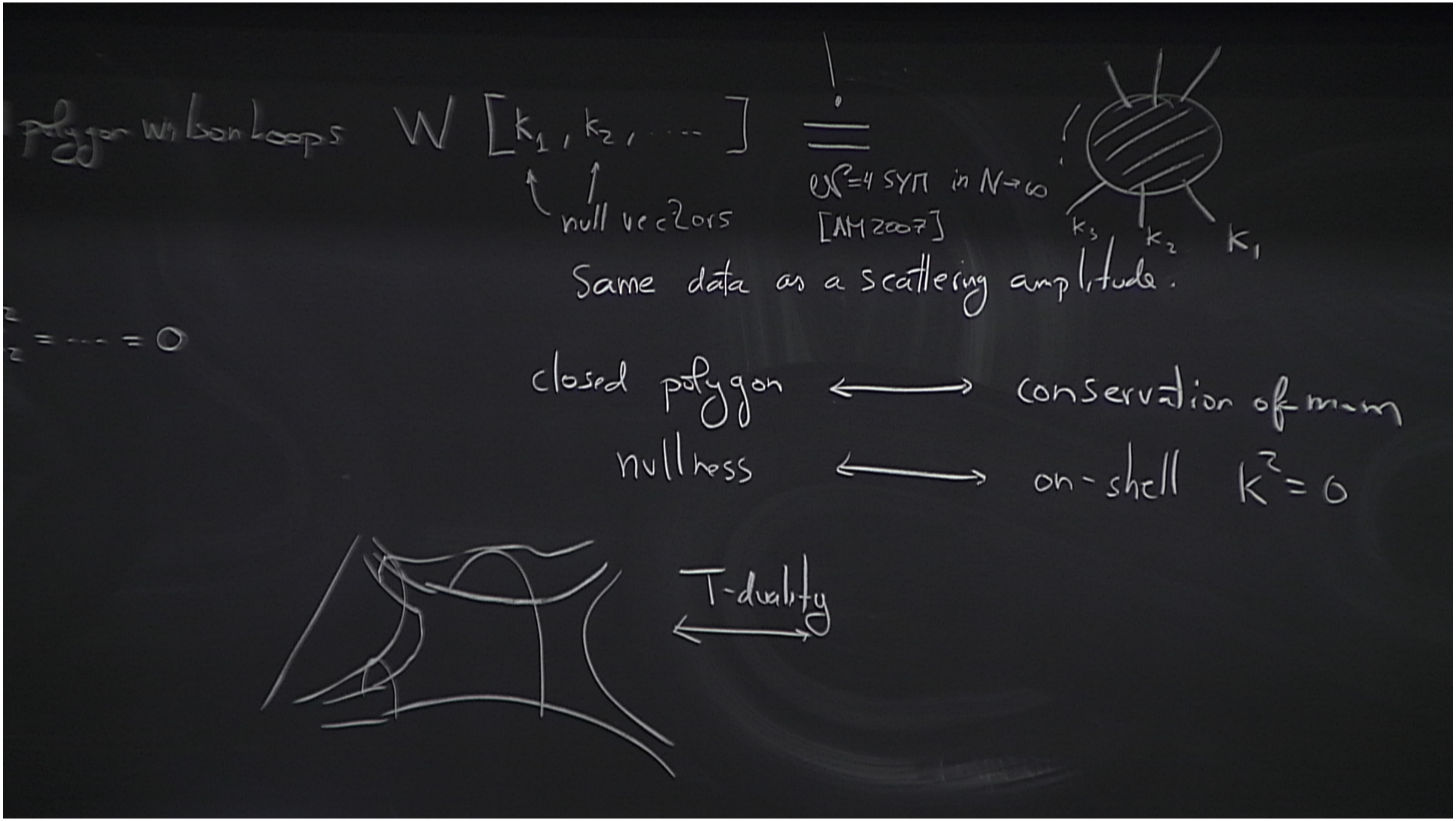
nullness ⟷ on-shell $k^2 = 0$

$= \int \dot{x}^\mu A_\mu + \dot{y}^i \Phi_i$

ontour: $\dot{x}^2 + \dot{y}^2 = 0$, $\dot{x}_\mu \to i\dot{x}_\mu$

$y_i \to \theta_i |\dot{x}|$, $\theta_i^2 = 1$

our susy WL



polygon Wilson Loops $W[k_1, k_2, \dots] \overset{!}{=}$

null vectors

$\mathcal{N}=4$ SYM in $N \to \infty$
[AM 2007]

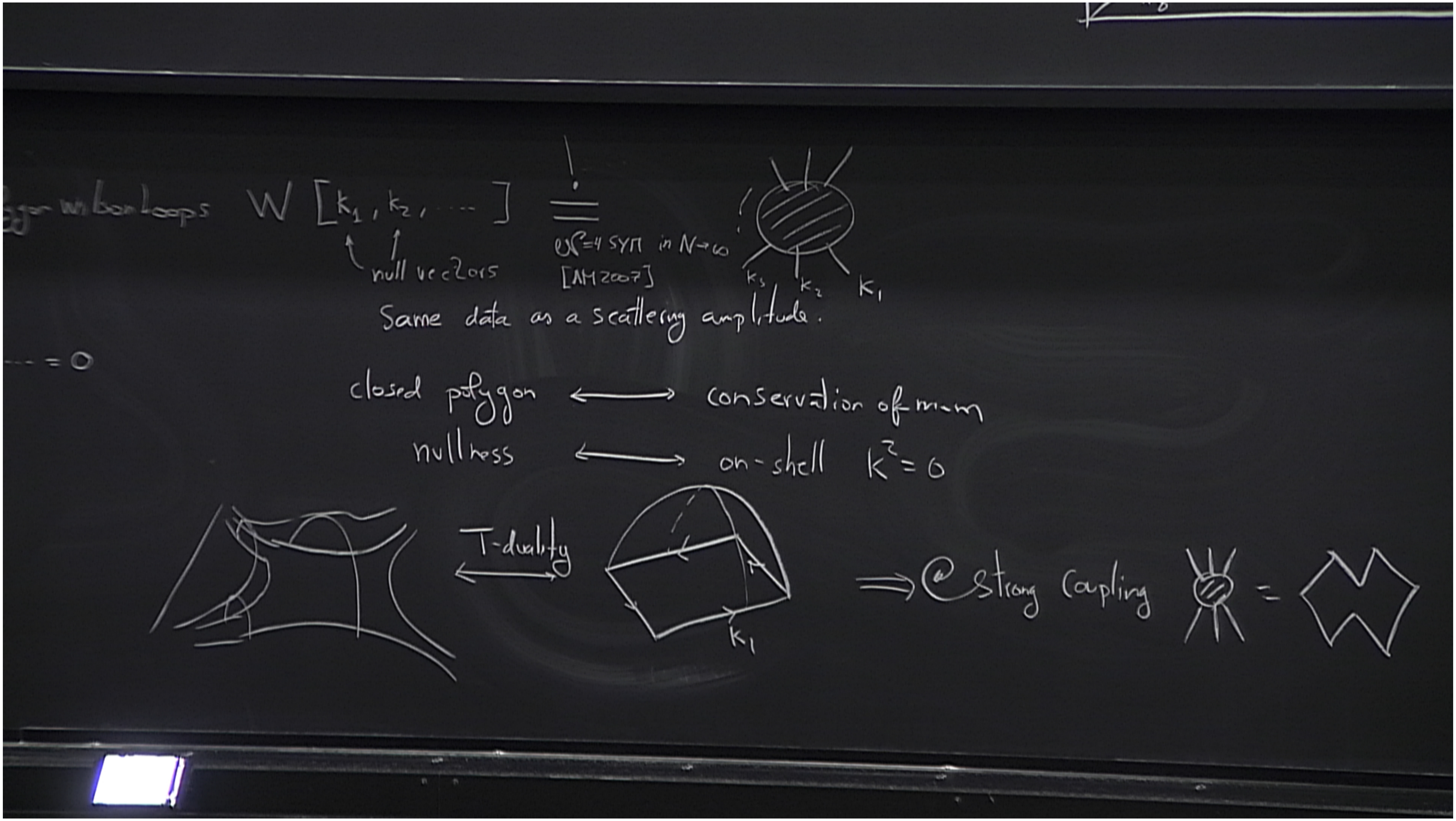
Same data as a scattering amplitude.

$= \cdots = 0$

closed polygon ⟷ conservation of mom

nullness ⟷ on-shell $k^2 = 0$

T-duality ⟷



gon Wilson Loops $W[k_1, k_2, \cdots] \overset{!}{=}$

$\mathcal{N}=4$ SYM in $N \to \infty$

[AM 2007]

null vectors

$\cdots = 0$

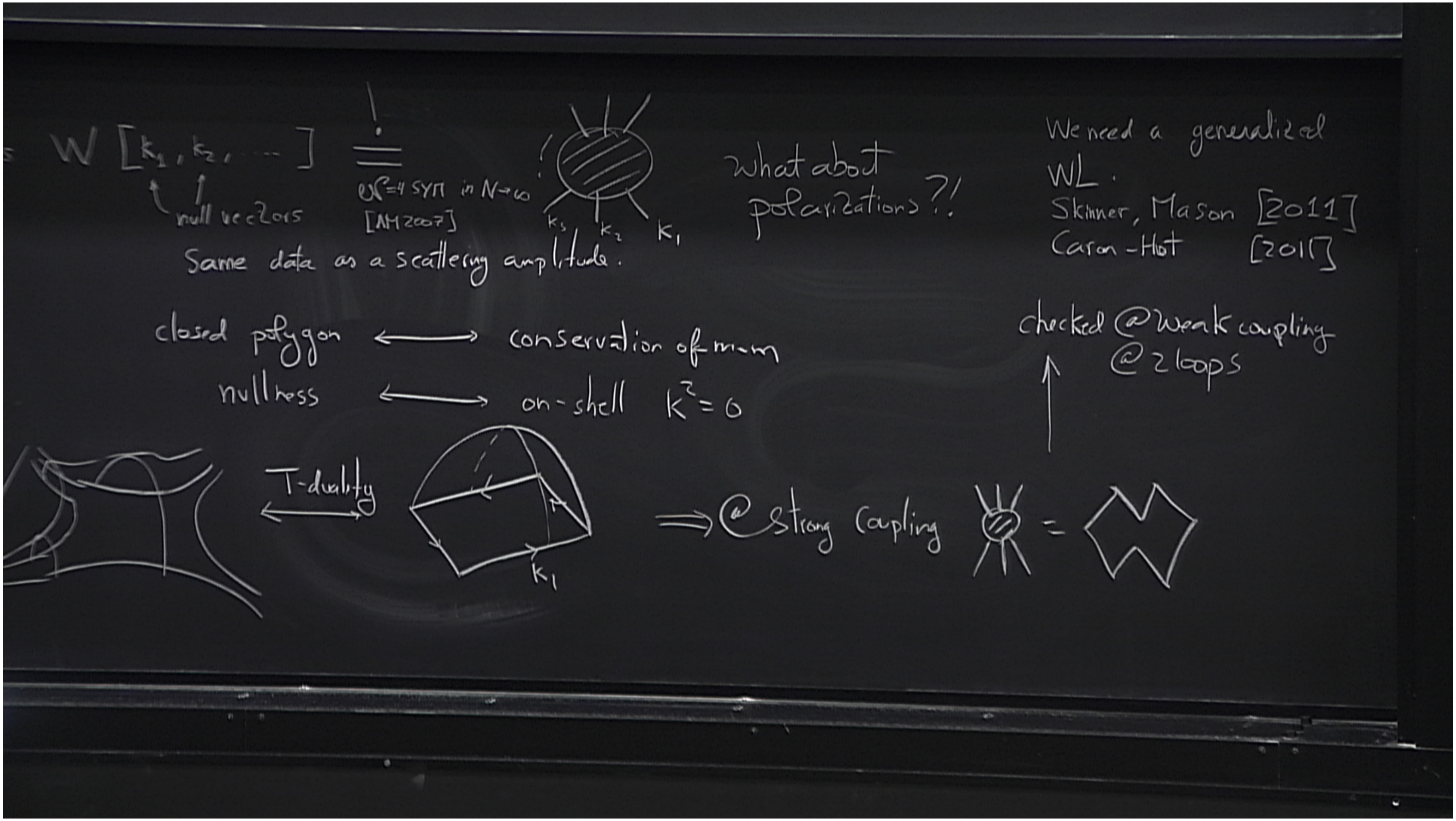
Same data as a scattering amplitude.

closed polygon $\longleftrightarrow$ conservation of mom

nullness $\longleftrightarrow$ on-shell $k^2 = 0$

T-duality $\longleftrightarrow$

$\Longrightarrow$ @ strong coupling $=$



$$W[k_1, k_2, \ldots] \overset{!}{\equiv}$$

null vectors

$\mathcal{N}=4$ SYM in $N \to \infty$

[AM 2007]

Same data as a scattering amplitude.

What about polarizations?!

We need a generalized WL.

Skinner, Mason [2011]

Caron-Hot [2011]

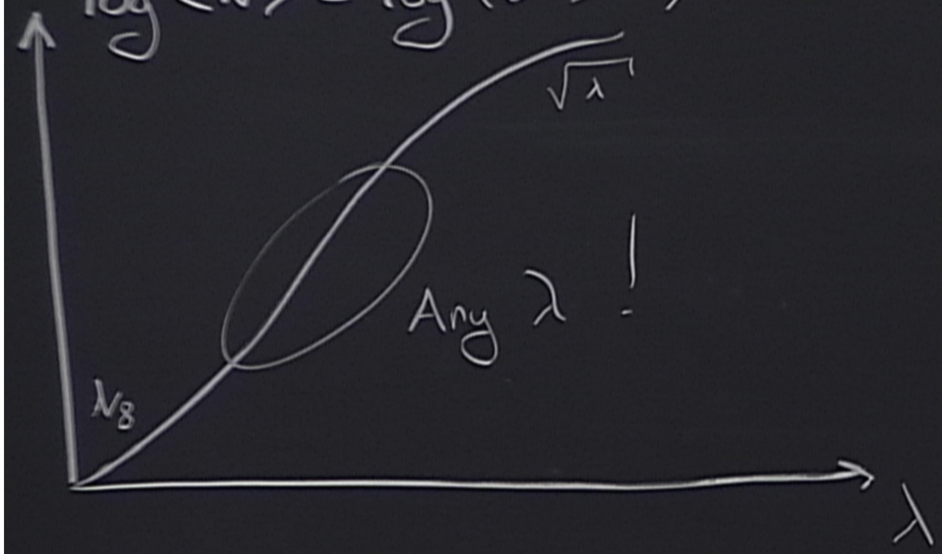
closed polygon ⟷ conservation of m-m

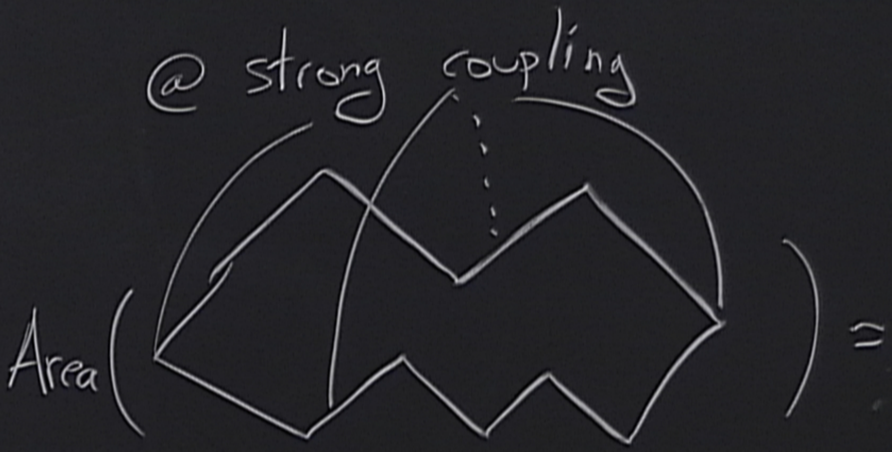
nullness ⟷ on-shell $k^2 = 0$

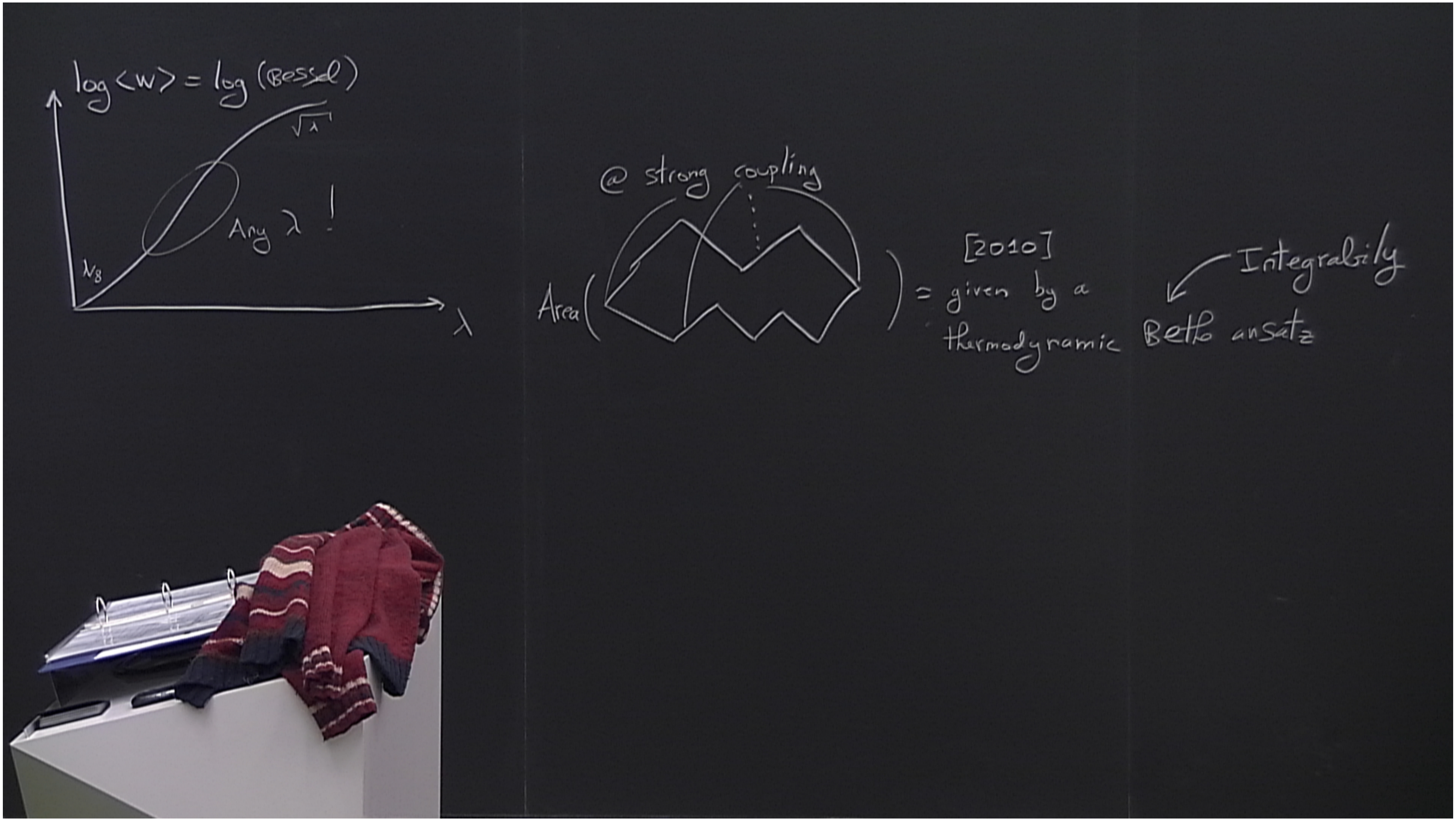
checked @ weak coupling @ 2 loops

⟹ @ strong coupling

$$\log \langle W \rangle = \log(\text{Bessel})$$







$\log\langle W\rangle = \log(\text{Bessel})$

@ strong coupling

Area( ) = given by a thermodynamic Bethe ansatz

[2010]

Integrability