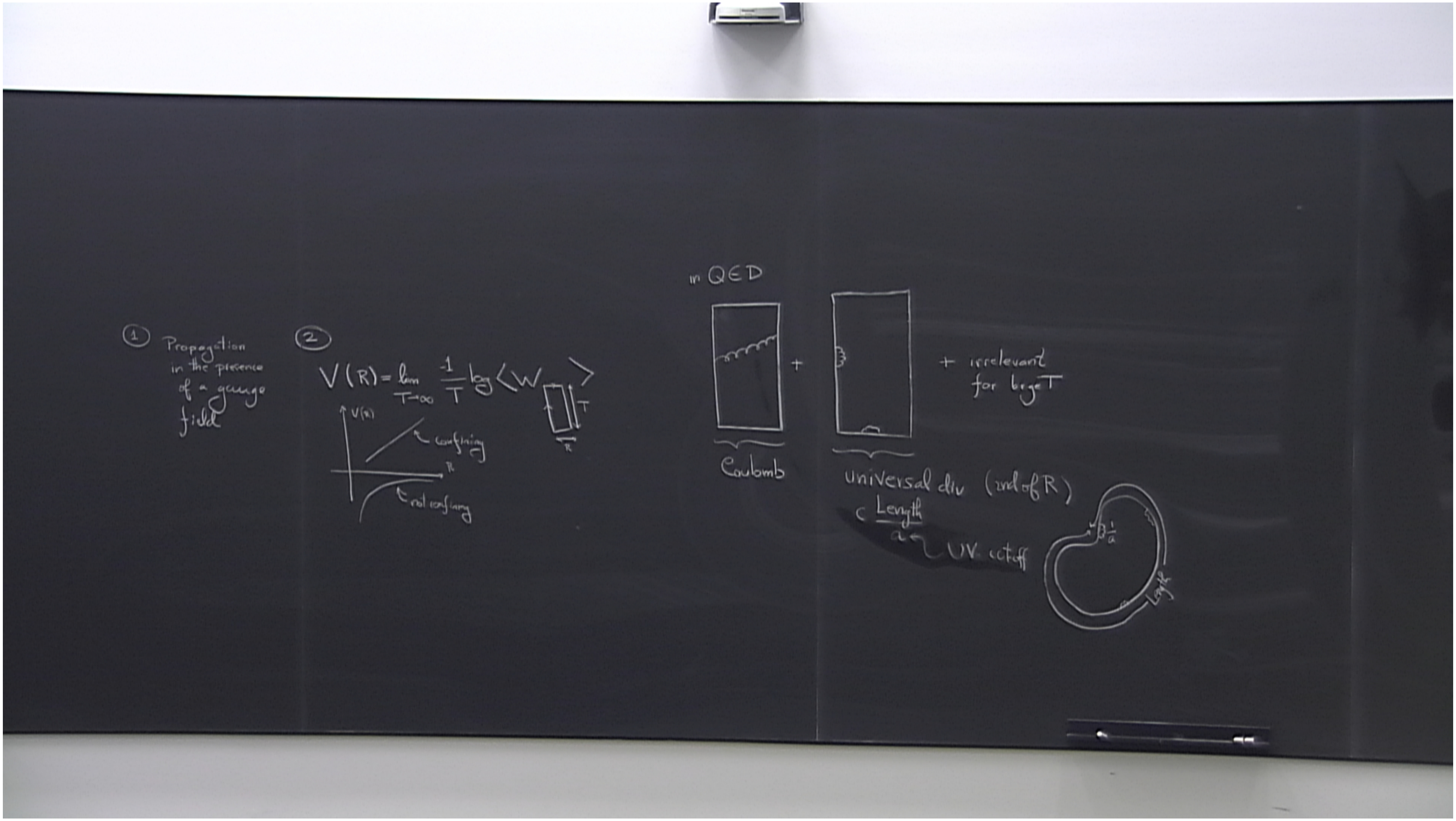
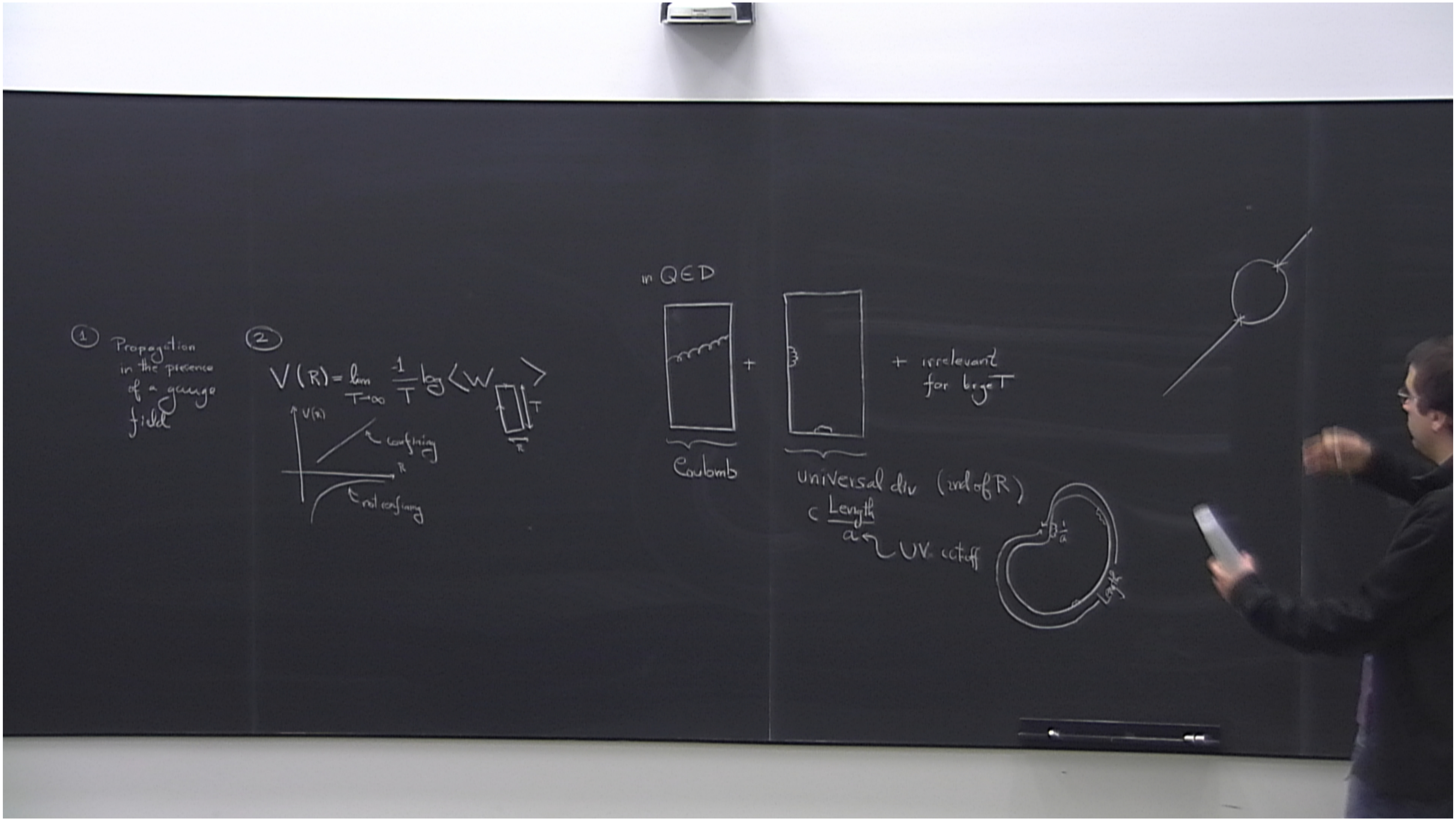
Title: Explorations in String Theory - Lecture 7

Date: Mar 20, 2012 11:30 AM

URL: http://pirsa.org/12030050

Abstract:


① Propagation in the presence of a gauge field
② V(R) = lim T→∞ -1/T log ⟨W⟩
V(R)
confining
R
not confining
T
in QED
+
+ irrelevant for large T
Coulomb
universal div (ind of R)
c Length/a
UV cutoff
1/a
Length



① Propagation in the presence of a gauge field
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V(R)
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UV cutoff
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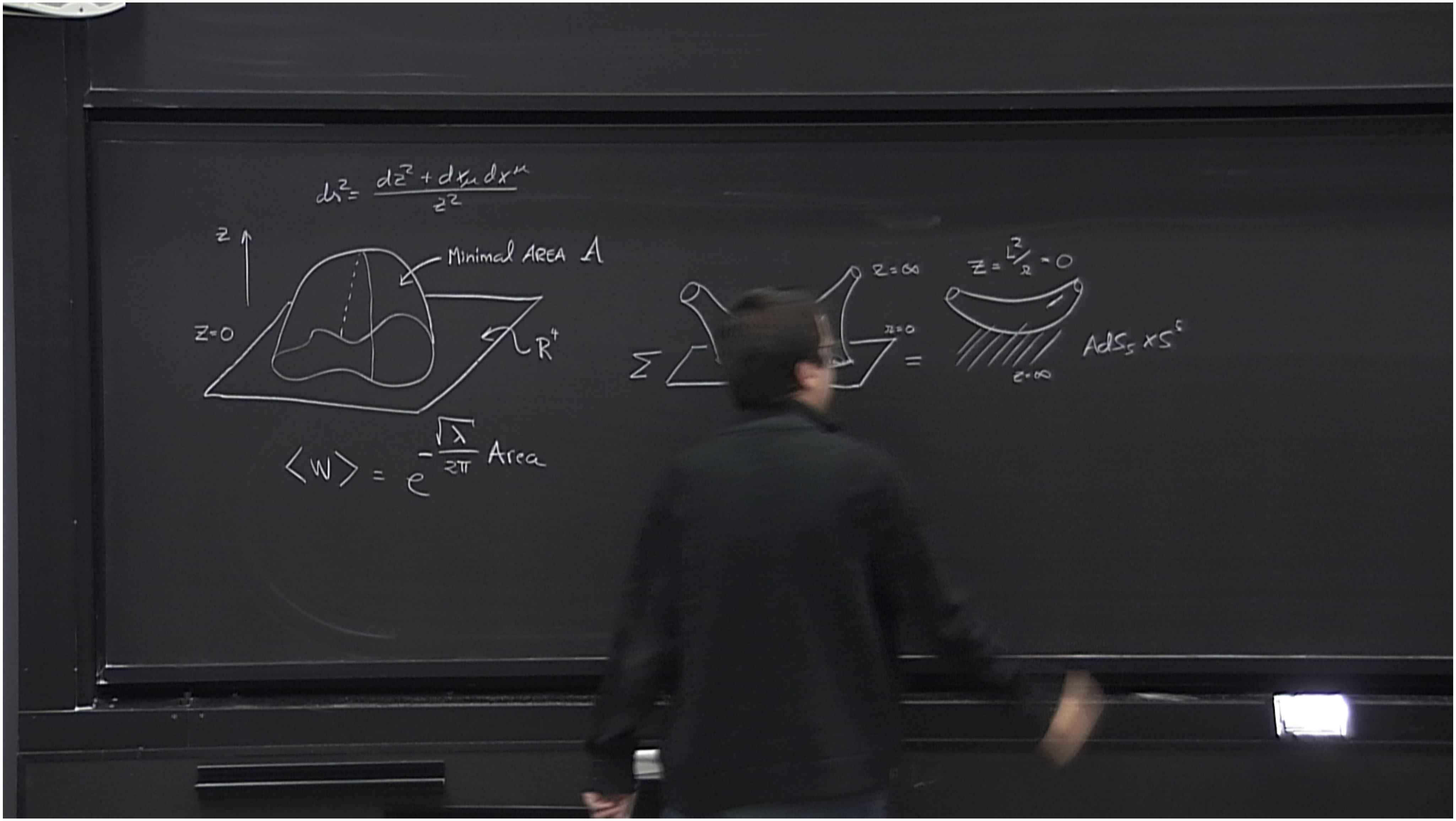
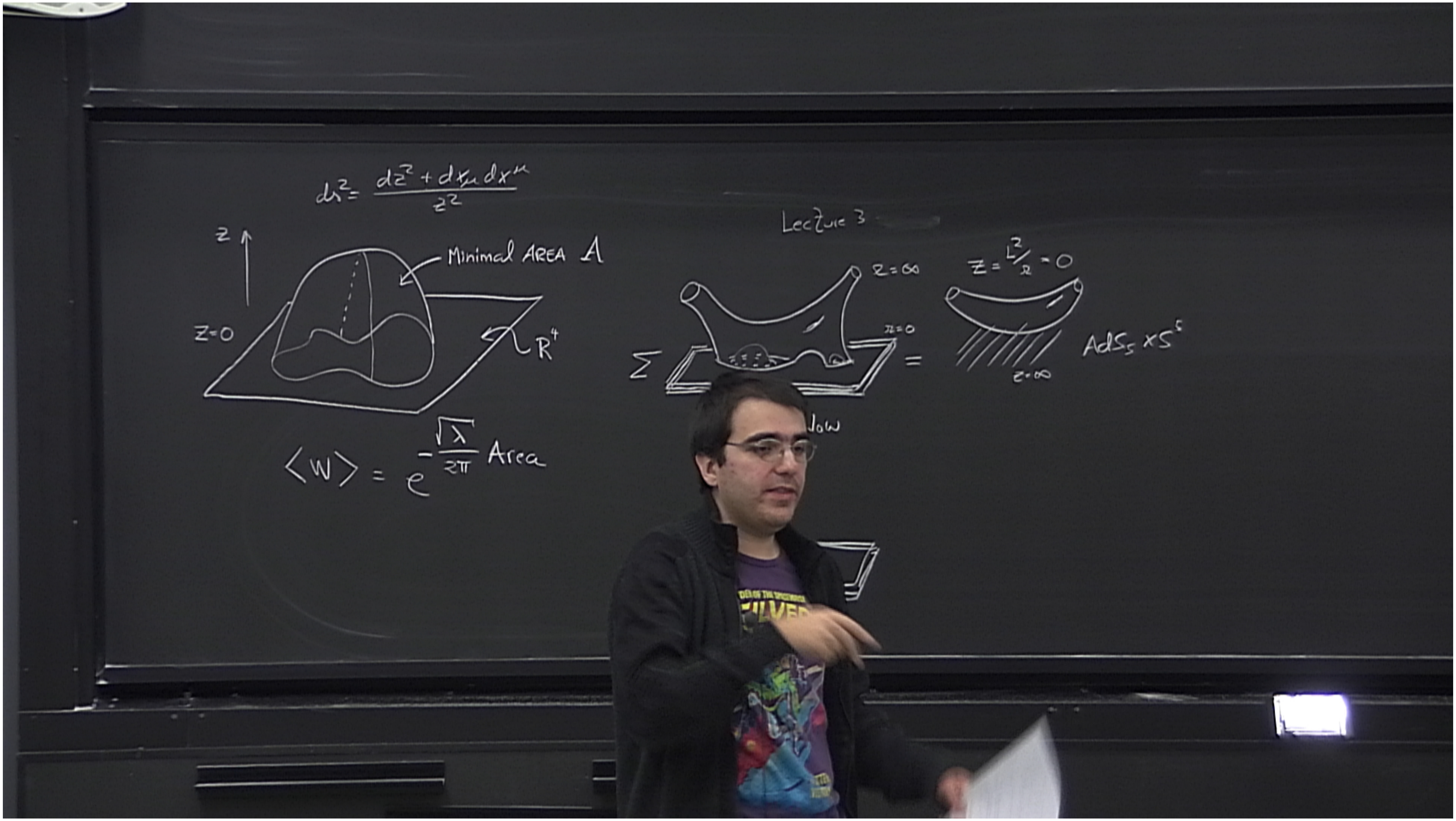
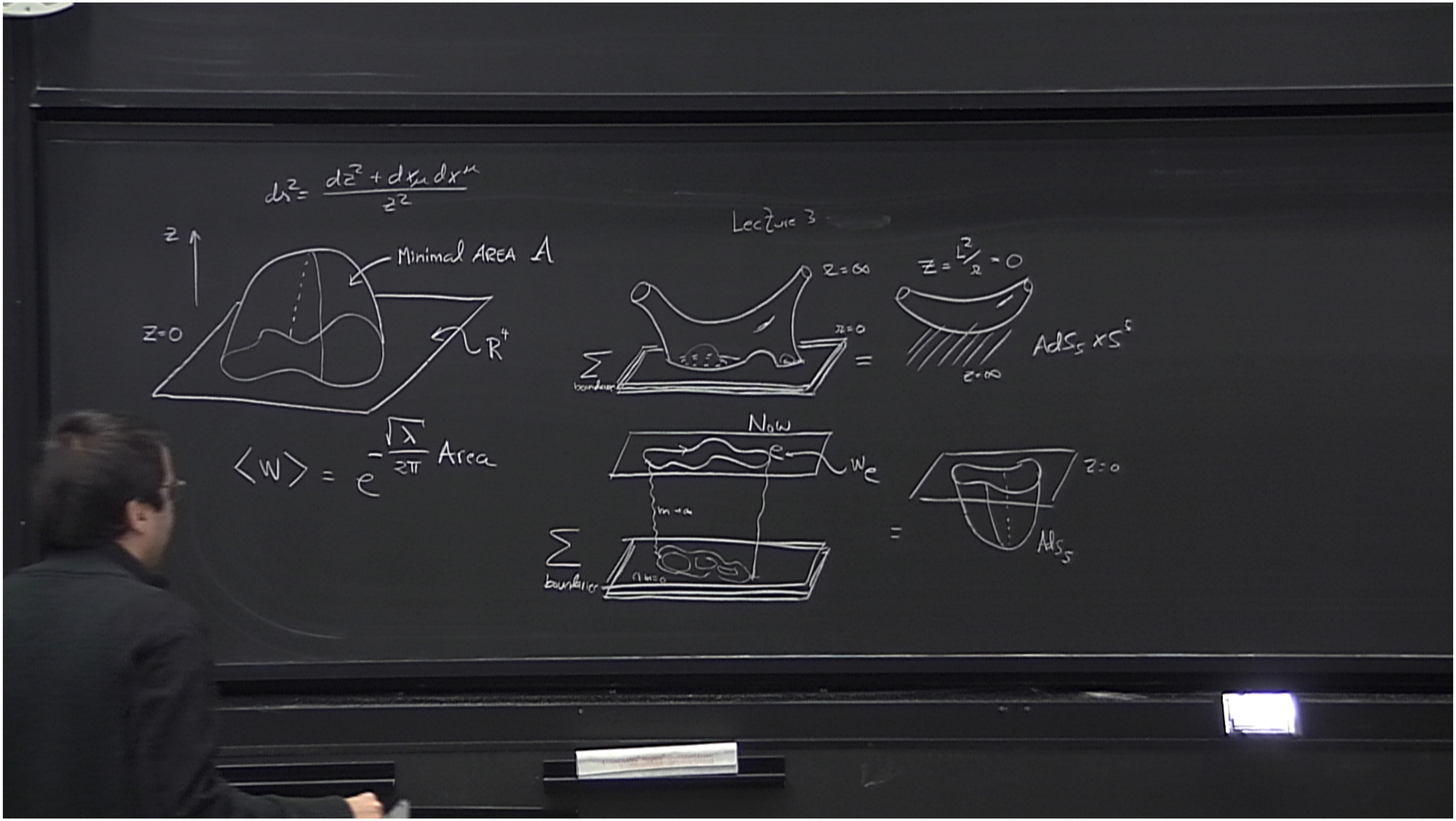
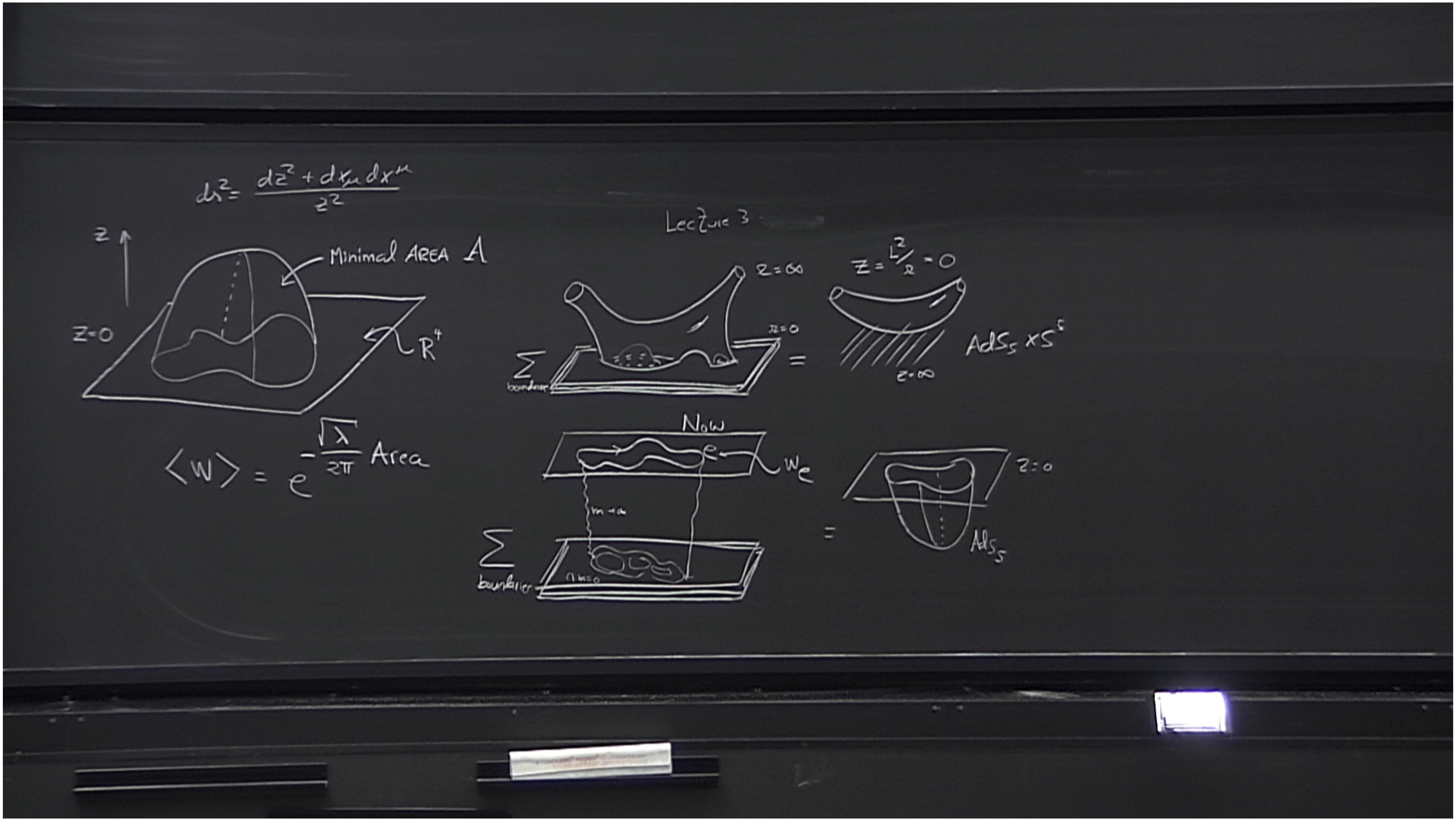

ds² = (dz² + dx_μ dx^μ) / z²
z
Minimal AREA A
z=0
R⁴
⟨W⟩ = e^(−√λ/2π Area)
Σ
z=∞
z=0
=
z = L²/r = 0
AdS5 × S5
z=∞

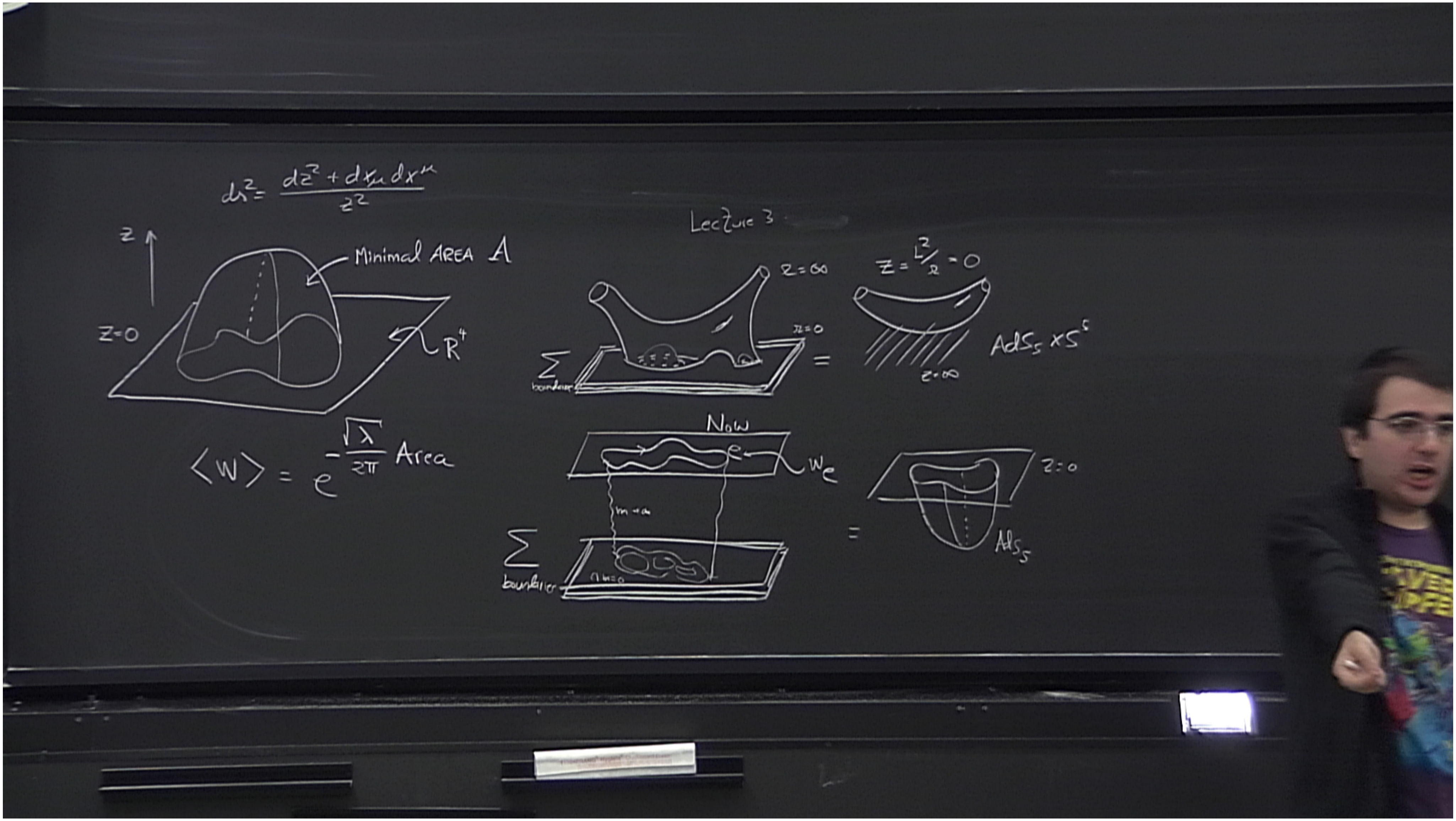


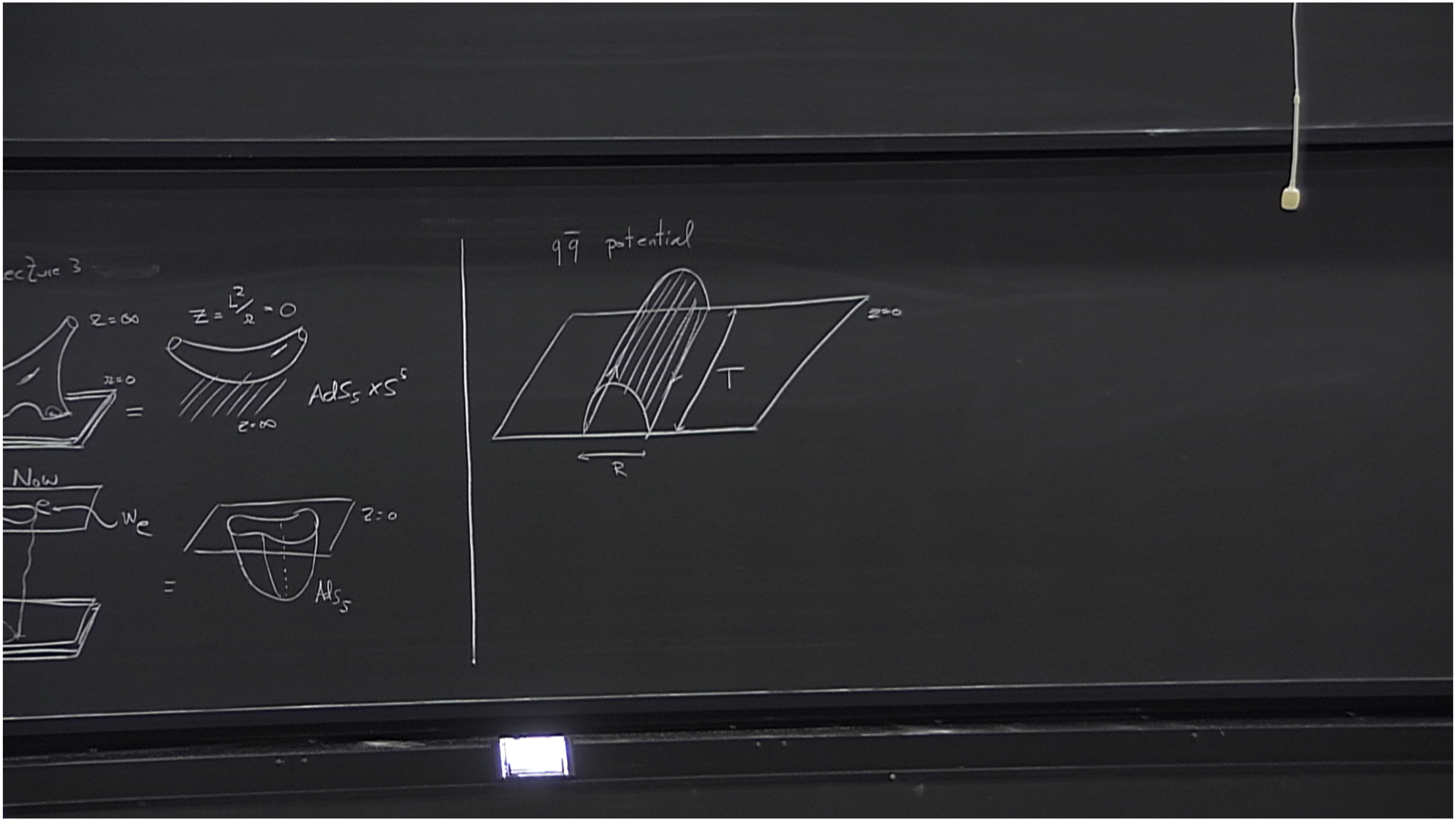
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
z
Minimal AREA A
z=0
R^4
$\langle W \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi} \text{Area}}$
Lecture 3
z=∞
z=0
$z = L^2/z = 0$
Σ
=
$AdS_5 \times S^5$
z=∞


ds² = (dz² + dx_μ dx^μ) / z²
Lecture 3
z
Minimal AREA A
Z=0
R⁴
⟨W⟩ = e^(−√λ/2π Area)
z=∞
z=0
AdS₅ x S⁵
Σ
Now
W_e
AdS₅

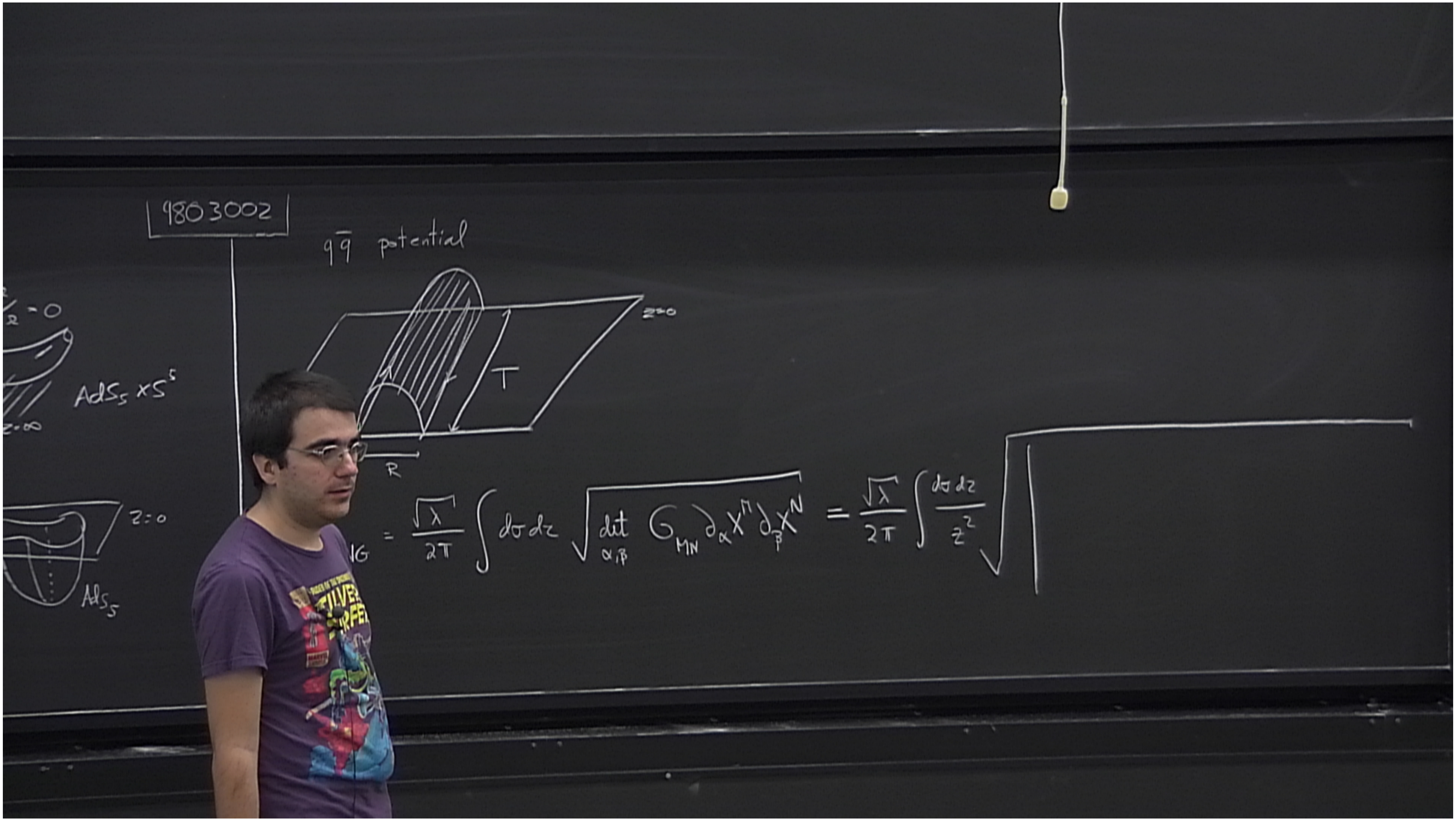


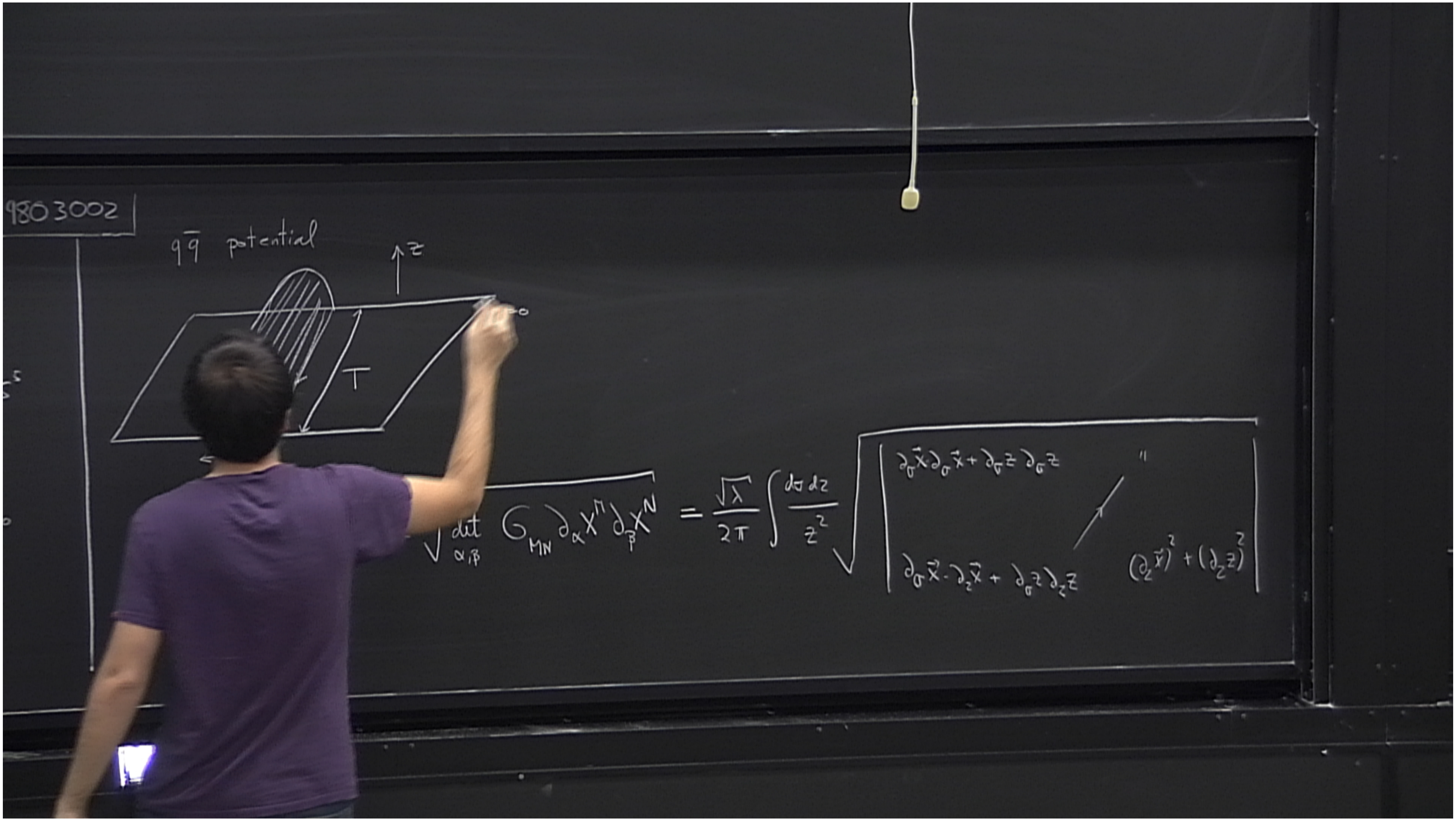
$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
Lecture 3
z
Minimal AREA A
z=0
R^4
$\langle W \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi} \text{Area}}$
z=∞
z=0
Σ
boundary
=
$z = \frac{L^2}{r} = 0$
z=∞
$AdS_5 \times S^5$
Now
W_e
Σ
boundary
z=0
=
AdS_5

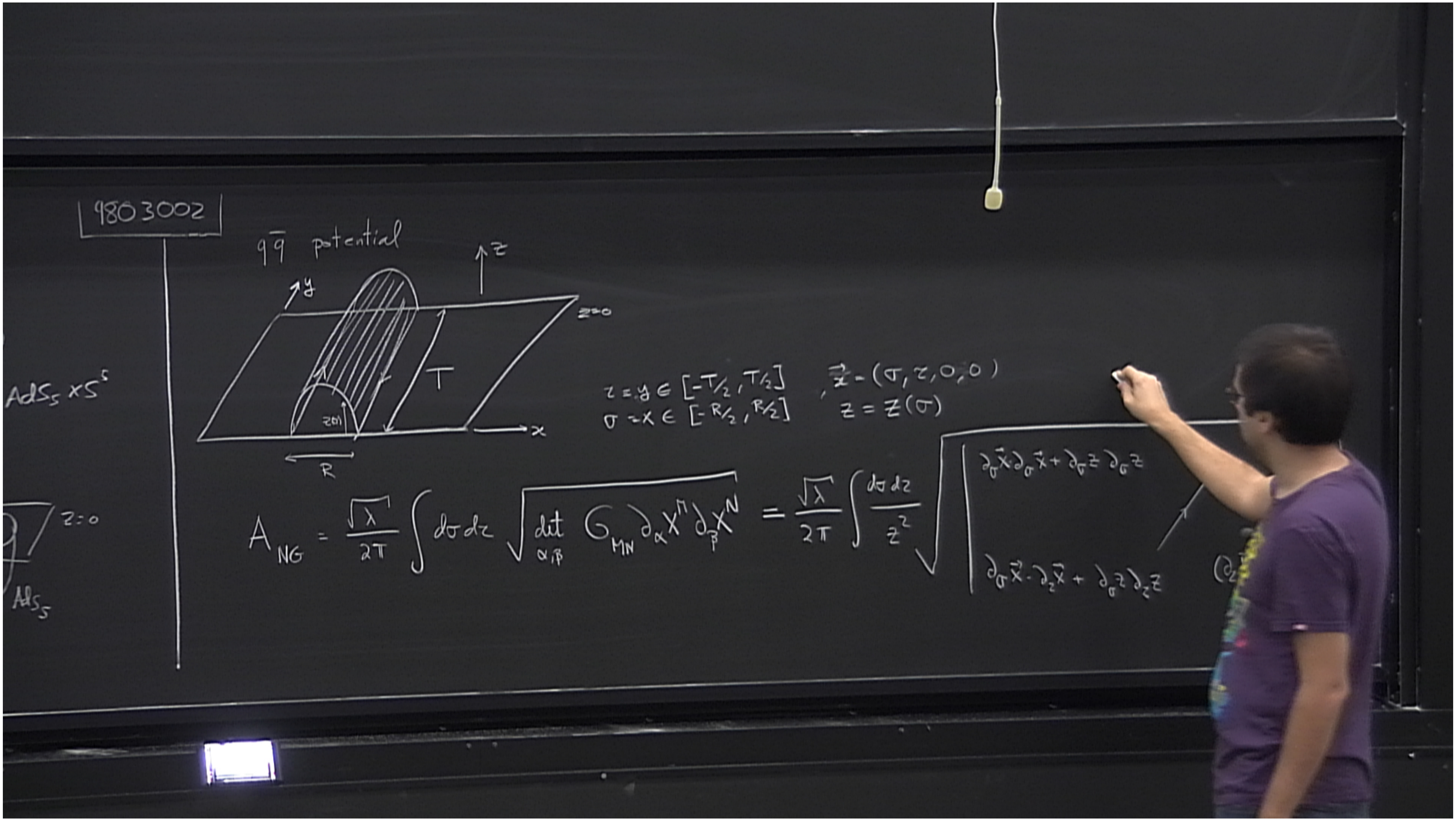

$ds^2 = \frac{dz^2 + dx_\mu dx^\mu}{z^2}$
z
Minimal AREA A
Z=0
R^4
$\langle W \rangle = e^{-\frac{\sqrt{\lambda}}{2\pi} \text{Area}}$
Lecture 3
z=∞
z=0
Σ
boundaries
=
$z = L^2/z = 0$
$AdS_5 \times S^5$
z=∞
Now
W_e
z=0
Σ
boundaries
=
AdS_5

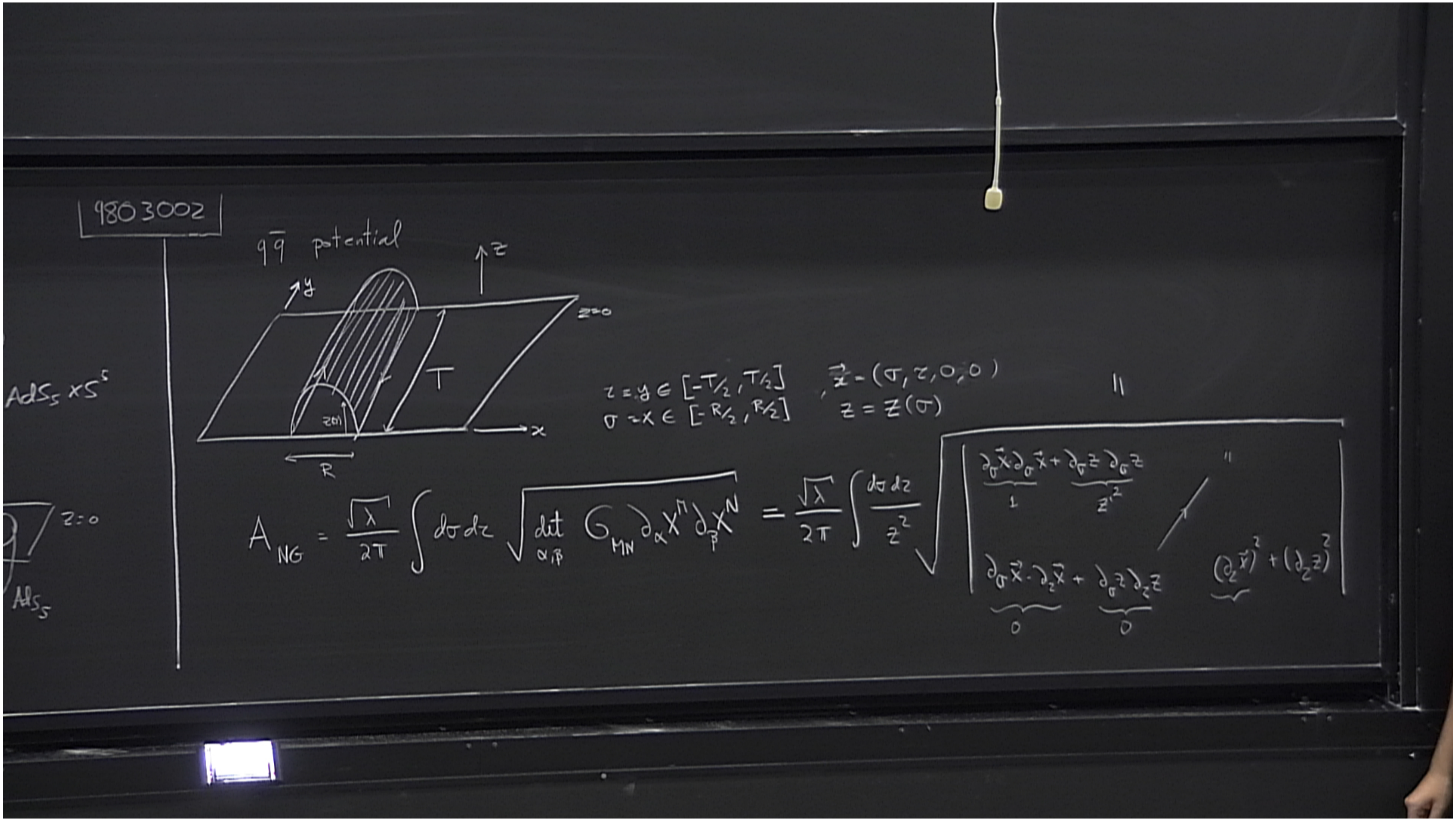


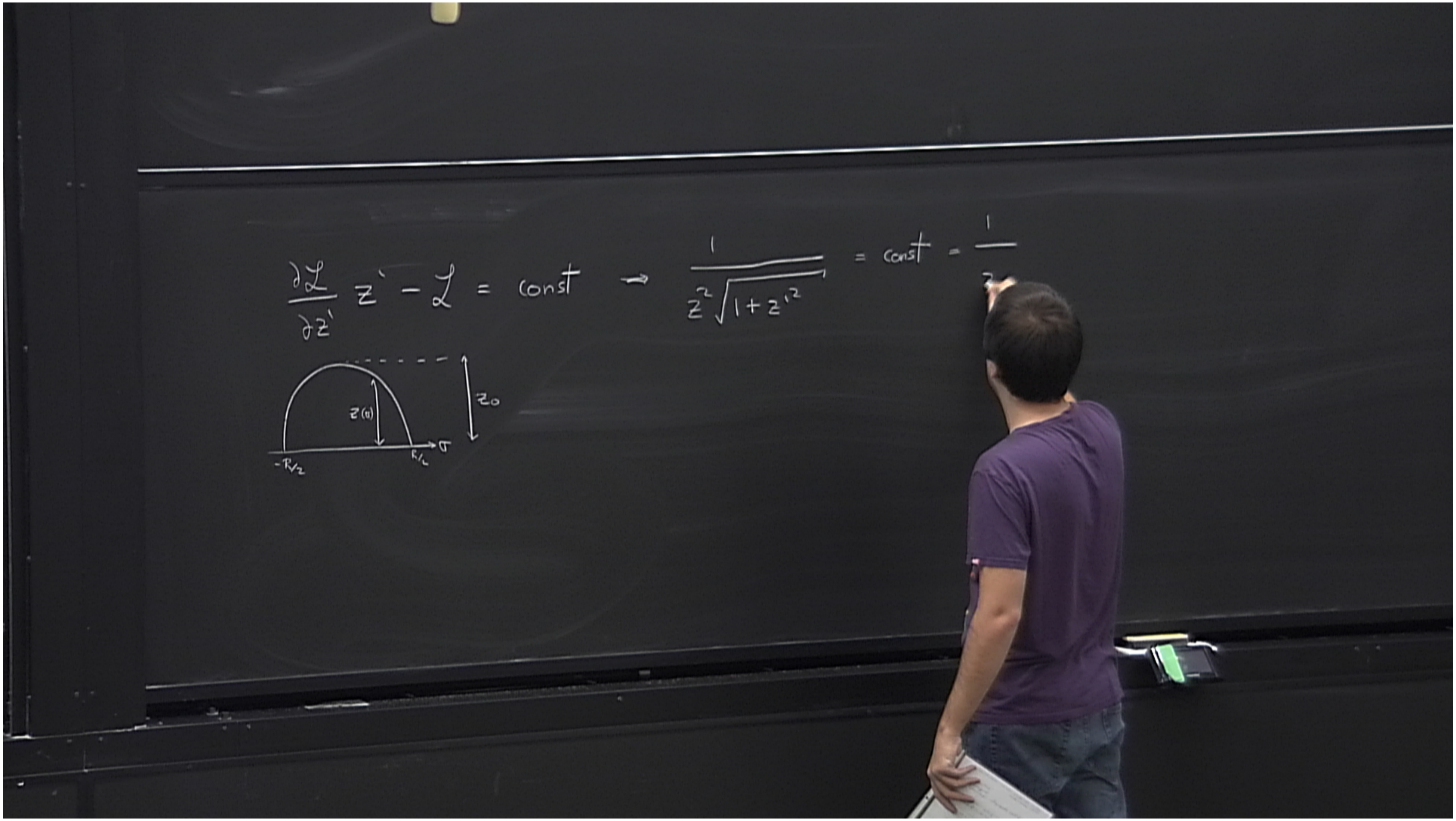
ecture 3
z = ∞
Z = L²/z = 0
z=0
AdS5 × S5
z=∞
Now
We
z=0
AdS5
qq̄ potential
z=0
T
R

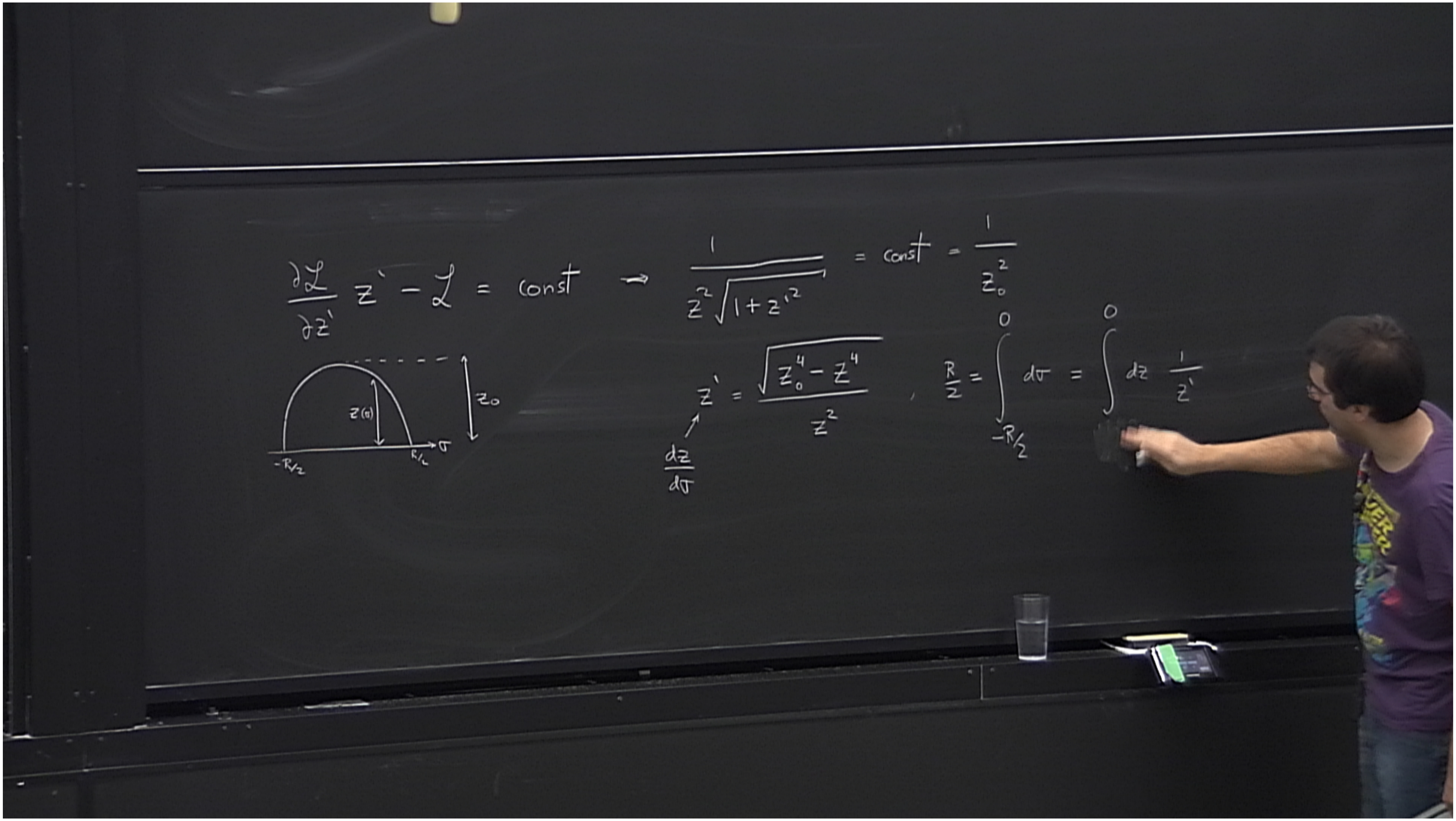

9803002
q q̄ potential
z=0
T
R
AdS5 × S5
z=∞
z=0
AdS5
= √λ/2π ∫ dσ dz √(det_{α,β} G_MN ∂_α X^M ∂_β X^N) = √λ/2π ∫ dσ dz/z² √

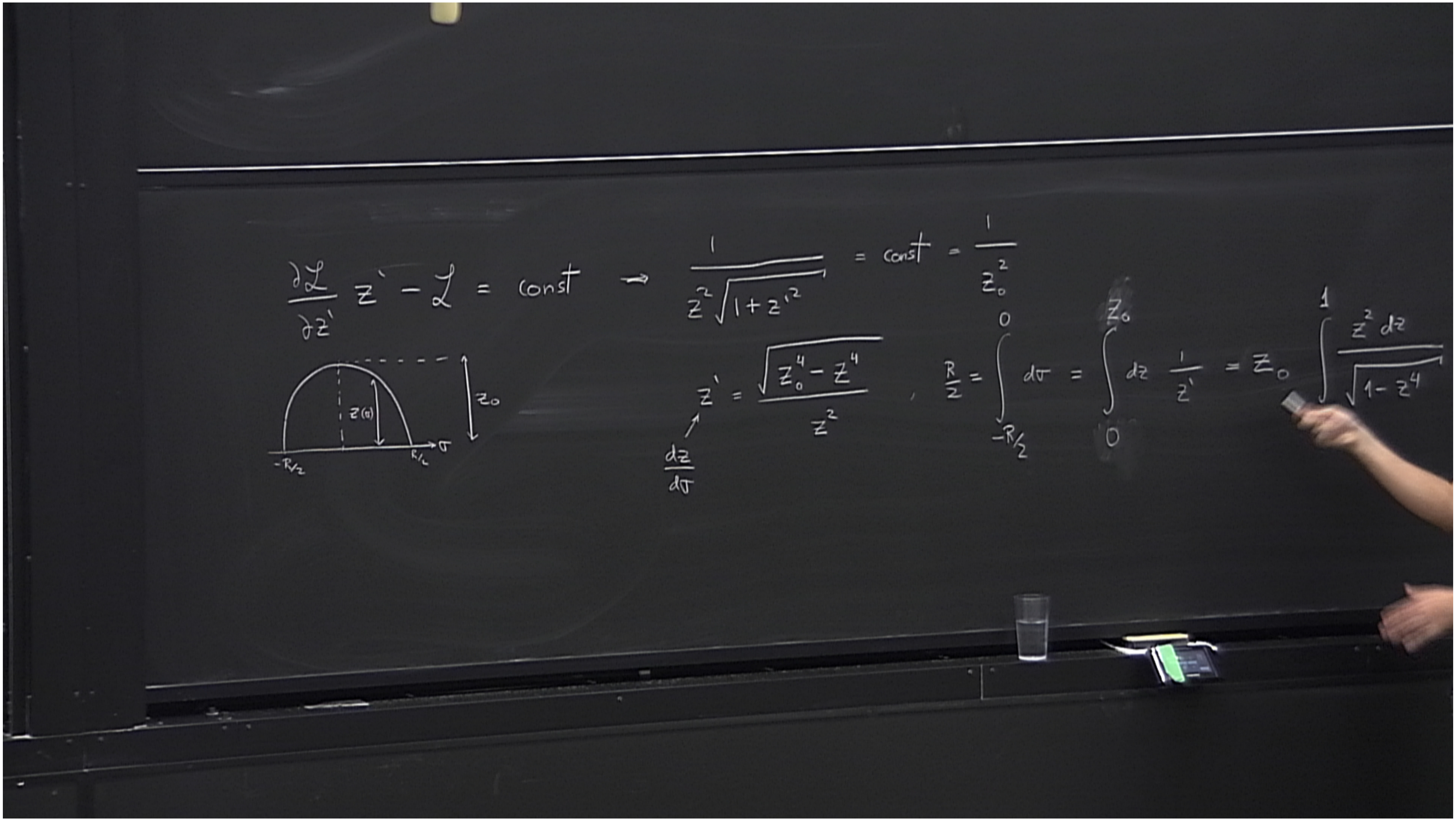

9803002
q q̄ potential
z
T
= √λ / 2π ∫ dσ dz / z²
∂σ x⃗ · ∂σ x⃗ + ∂σ z ∂σ z
∂σ x⃗ · ∂z x⃗ + ∂σ z ∂z z
(∂z x⃗)² + (∂z z)²

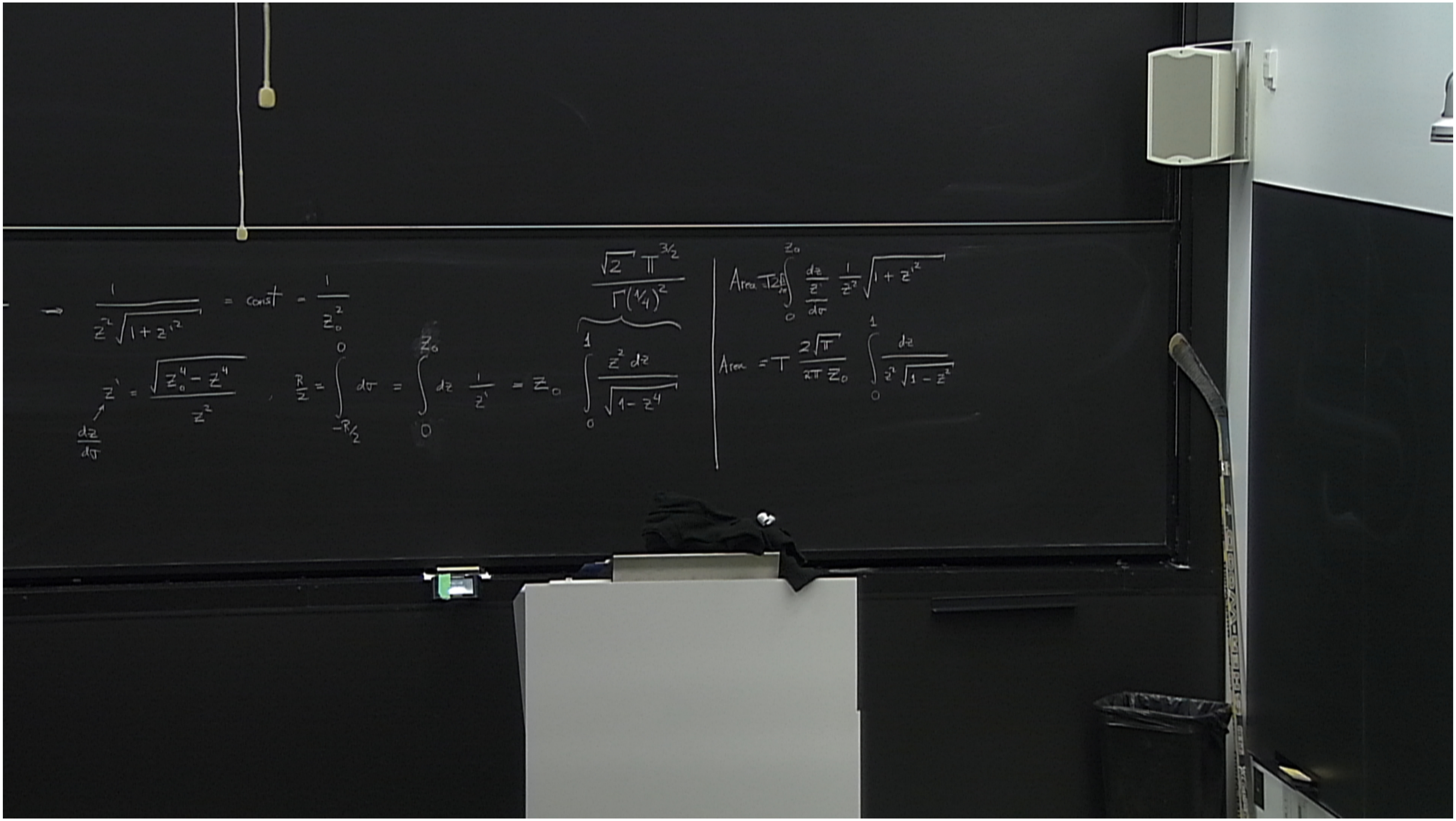

9803002
q q̄ potential
AdS5 × S5
z=0
AdS5
y
z
z=0
T
z(σ)
R
x
τ = y ∈ [-T/2, T/2]
σ = x ∈ [-R/2, R/2]
x⃗ = (σ, τ, 0, 0)
z = z(σ)
A_NG = √λ/2π ∫ dσ dτ √(det_{α,β} G_MN ∂_α X^M ∂_β X^N) = √λ/2π ∫ dσ dτ/z² √| ∂σx⃗·∂σx⃗ + ∂σz ∂σz   ∂σx⃗·∂τx⃗ + ∂σz ∂τz |

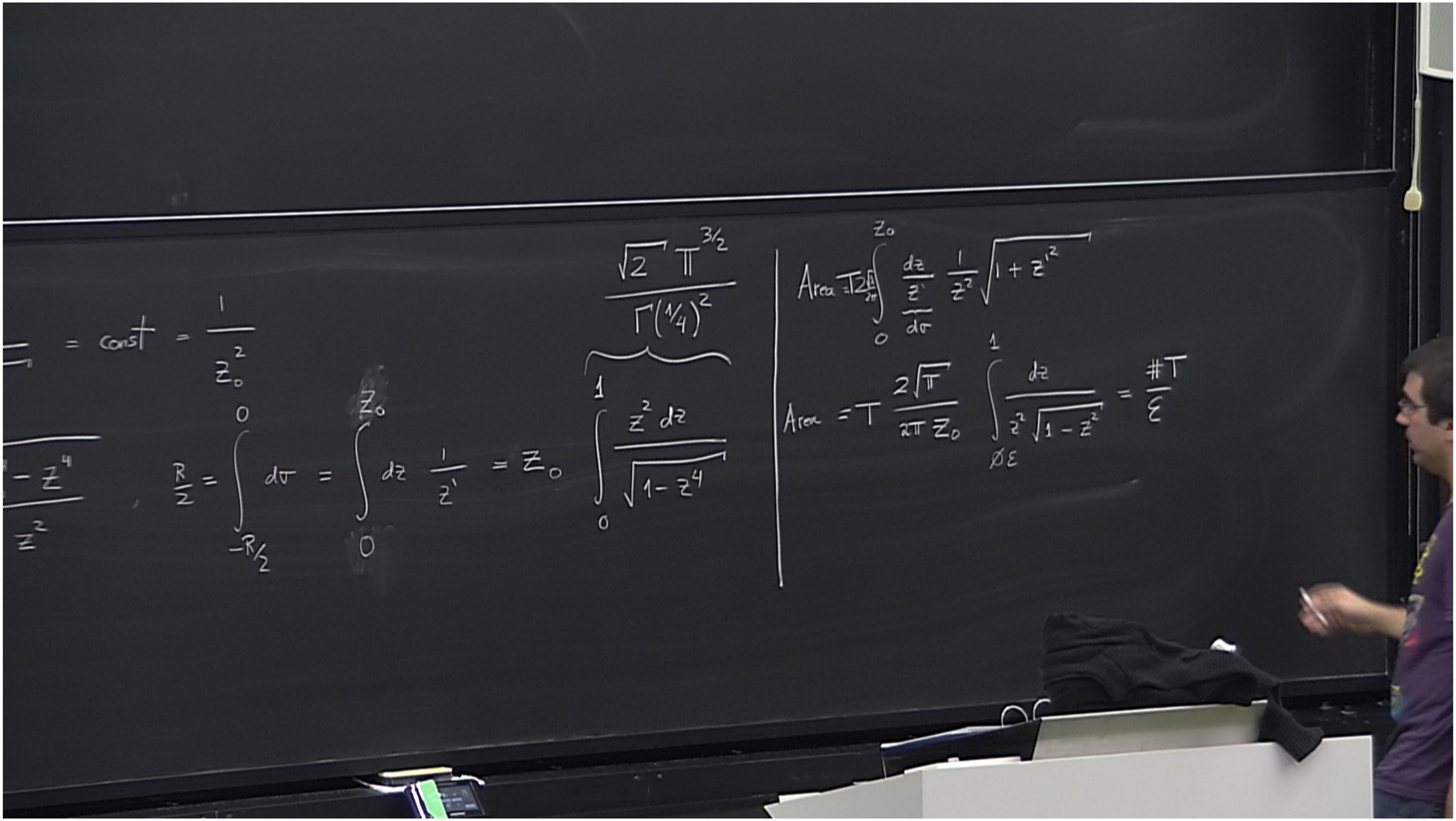

9803002
q q̄ potential
AdS5 × S5
AdS5
z=0
R
T
τ = y ∈ [-T/2, T/2]
σ = x ∈ [-R/2, R/2]
x⃗ = (σ, τ, 0, 0)
Z = Z(σ)
A_NG = √λ/2π ∫ dσ dτ √(det_{α,β} G_MN ∂_α X^M ∂_β X^N) = √λ/2π ∫ dσ dτ / z² √(...)


∂L/∂z' z' − L = const → 1/(z² √(1+z'²)) = const = 1/z
z(σ)
z0
σ
−R/2
R/2


$\frac{\partial \mathcal{L}}{\partial z'} z' - \mathcal{L} = \text{const} \rightarrow \frac{1}{z^2\sqrt{1+z'^2}} = \text{const} = \frac{1}{z_0^2}$
$z(\sigma)$
z_0
$-R/2$
$R/2$
σ
$z' = \frac{\sqrt{z_0^4 - z^4}}{z^2}$
$\frac{dz}{d\sigma}$
$\frac{R}{2} = \int_{-R/2}^{0} d\sigma = \int^{0} dz \, \frac{1}{z'}$


$\frac{\partial \mathcal{L}}{\partial z'} z' - \mathcal{L} = \text{const} \rightarrow \frac{1}{z^2\sqrt{1+z'^2}} = \text{const} = \frac{1}{z_0^2}$
$z(\sigma)$
z_0
σ
$-R/2$
$R/2$
$z' = \frac{\sqrt{z_0^4 - z^4}}{z^2}$
$\frac{dz}{d\sigma}$
$\frac{R}{2} = \int_{-R/2}^{0} d\sigma = \int_{0}^{z_0} dz \frac{1}{z'} = z_0 \int^{1} \frac{z^2 dz}{\sqrt{1-z^4}}$


1 / z^2 √(1 + z'^2) = const = 1 / z_0^2
z' = √(z_0^4 − z^4) / z^2
dz/dσ
R/2 = ∫_{−R/2}^{0} dσ = ∫_0^{z_0} dz 1/z' = z_0 ∫_0^1 z^2 dz / √(1 − z^4)
√2 π^{3/2} / Γ(1/4)^2
Area = T ∫_0^{z_0} dz/(z'/dσ) 1/z^2 √(1 + z'^2)
Area = T 2√π / 2π z_0 ∫_0^1 dz / z^2 √(1 − z^4)


$= \text{const} = \frac{1}{Z_0^2}$
$\frac{R}{2} = \int_{-R/2}^{0} d\sigma = \int_0^{Z_0} dz \frac{1}{z'} = Z_0 \int_0^1 \frac{z^2 dz}{\sqrt{1-z^4}}$
$\frac{\sqrt{2}\,\pi^{3/2}}{\Gamma(1/4)^2}$
$\text{Area} = T 2\pi \int_0^{Z_0} \frac{dz}{\frac{z'}{d\sigma}} \frac{1}{z^2}\sqrt{1+z'^2}$
$\text{Area} = T \frac{2\sqrt{\pi}}{2\pi Z_0} \int_{\varepsilon}^{1} \frac{dz}{z^2\sqrt{1-z^2}} = \frac{\# T}{\varepsilon}$

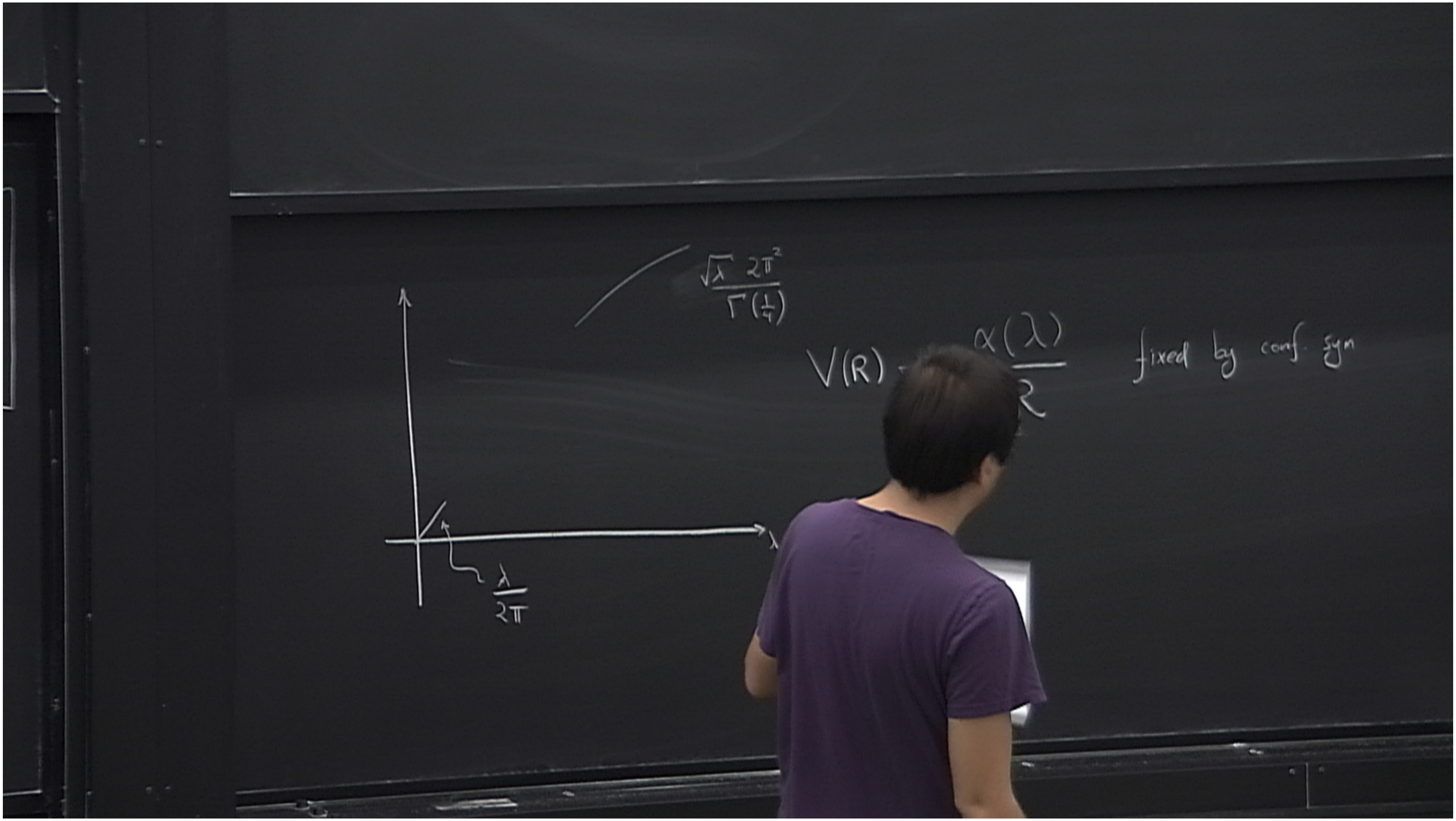
$$\ldots = \text{const} = \frac{1}{z_0^2}$$

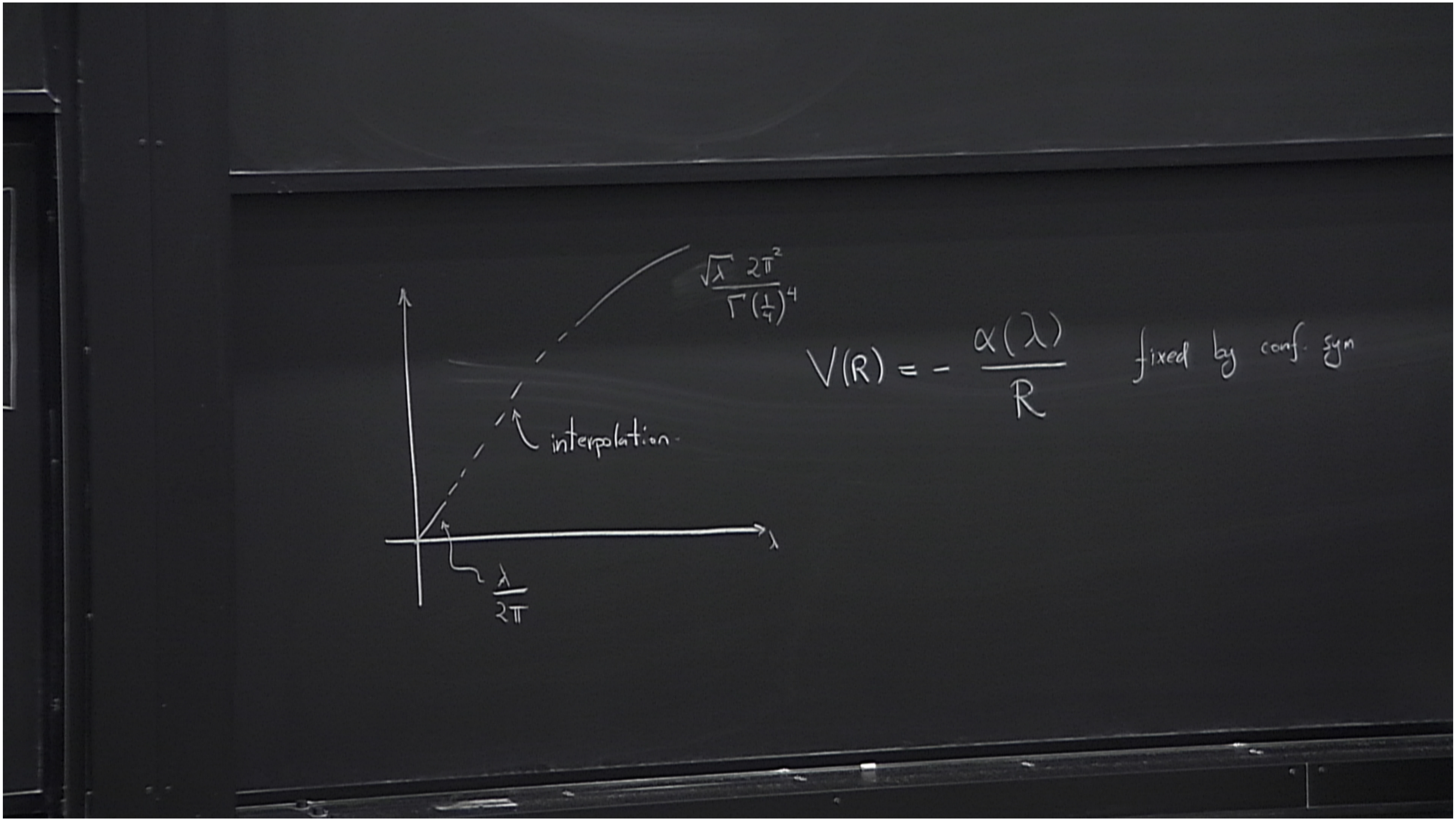
$$\frac{\ldots - z^4}{z^2}, \quad \frac{R}{2} = \int_{-R/2}^{0} d\sigma = \int_{0}^{z_0} dz \, \frac{1}{z'} = z_0 \underbrace{\int_0^1 \frac{z^2 \, dz}{\sqrt{1-z^4}}}_{\frac{\sqrt{2}\,\pi^{3/2}}{\Gamma(1/4)^2}}$$

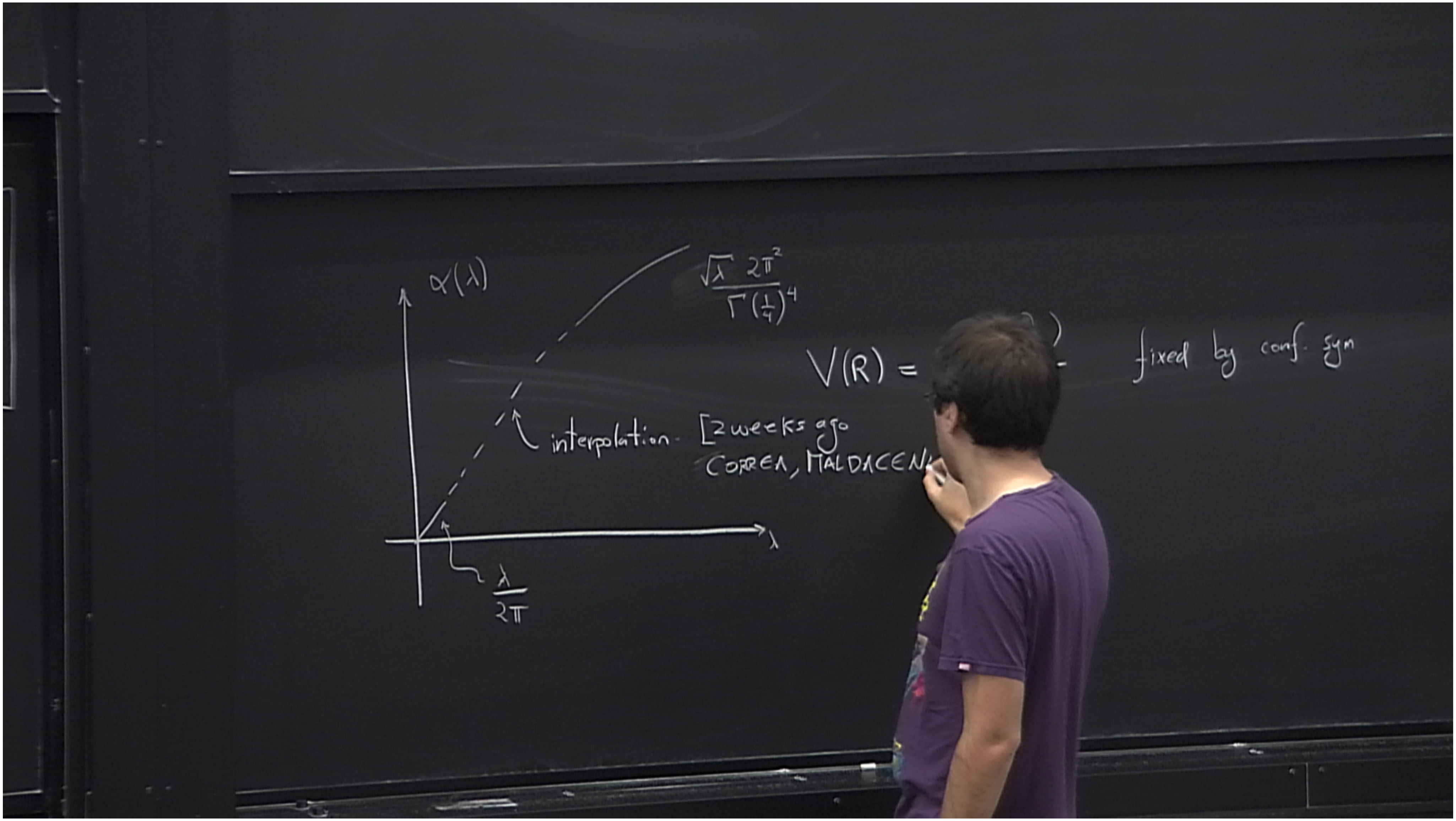
$$\text{Area} = T \frac{2\pi R}{2\pi} \int_0^{z_0} \frac{\frac{dz}{z'}}{d\sigma} \frac{1}{z^2} \sqrt{1+z'^2}$$

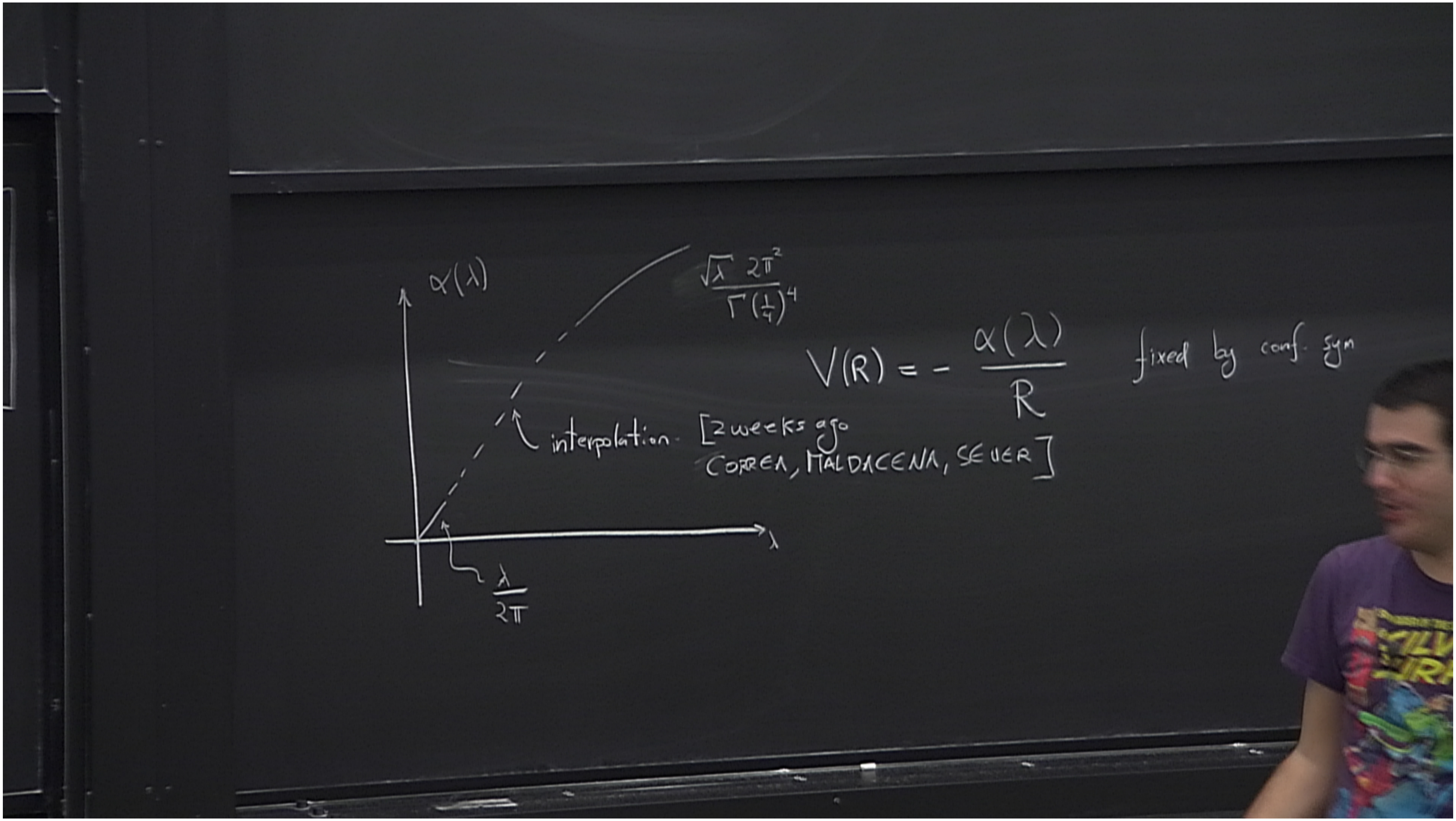
$$\text{Area} = T \frac{2\sqrt{\pi}}{2\pi z_0} \int_{\varepsilon R/z_0}^{1} \frac{dz}{z^2\sqrt{1-z^2}} = \frac{\# T}{\varepsilon} - \frac{2\pi^2 \sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)} \frac{1}{R}$$

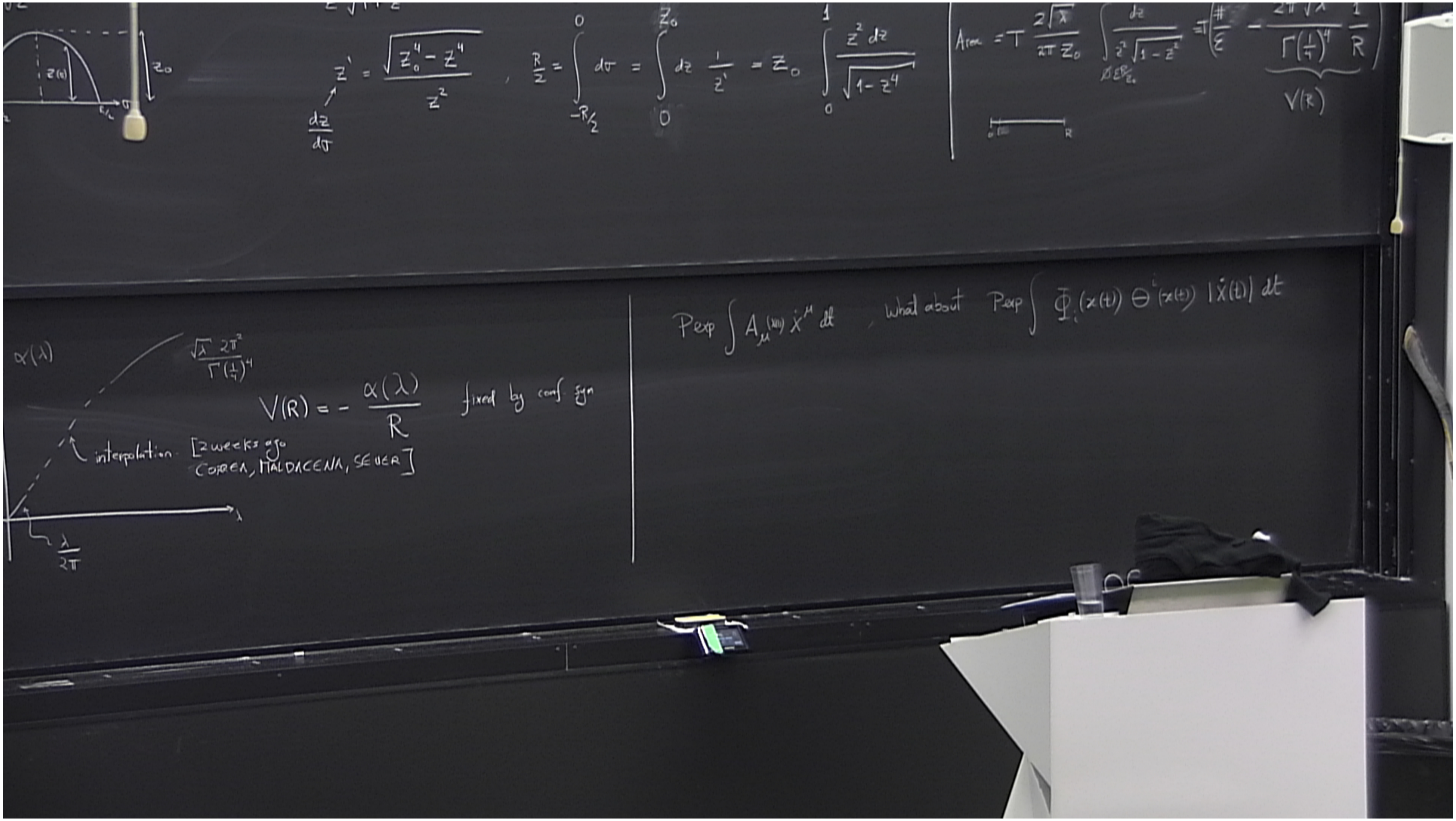
0 εR R


$\frac{\sqrt{\lambda}\ 2\pi^2}{\Gamma(\frac{1}{4})}$
$V(R)$
$\alpha(\lambda)$
fixed by conf. sym
$\frac{\lambda}{2\pi}$
λ

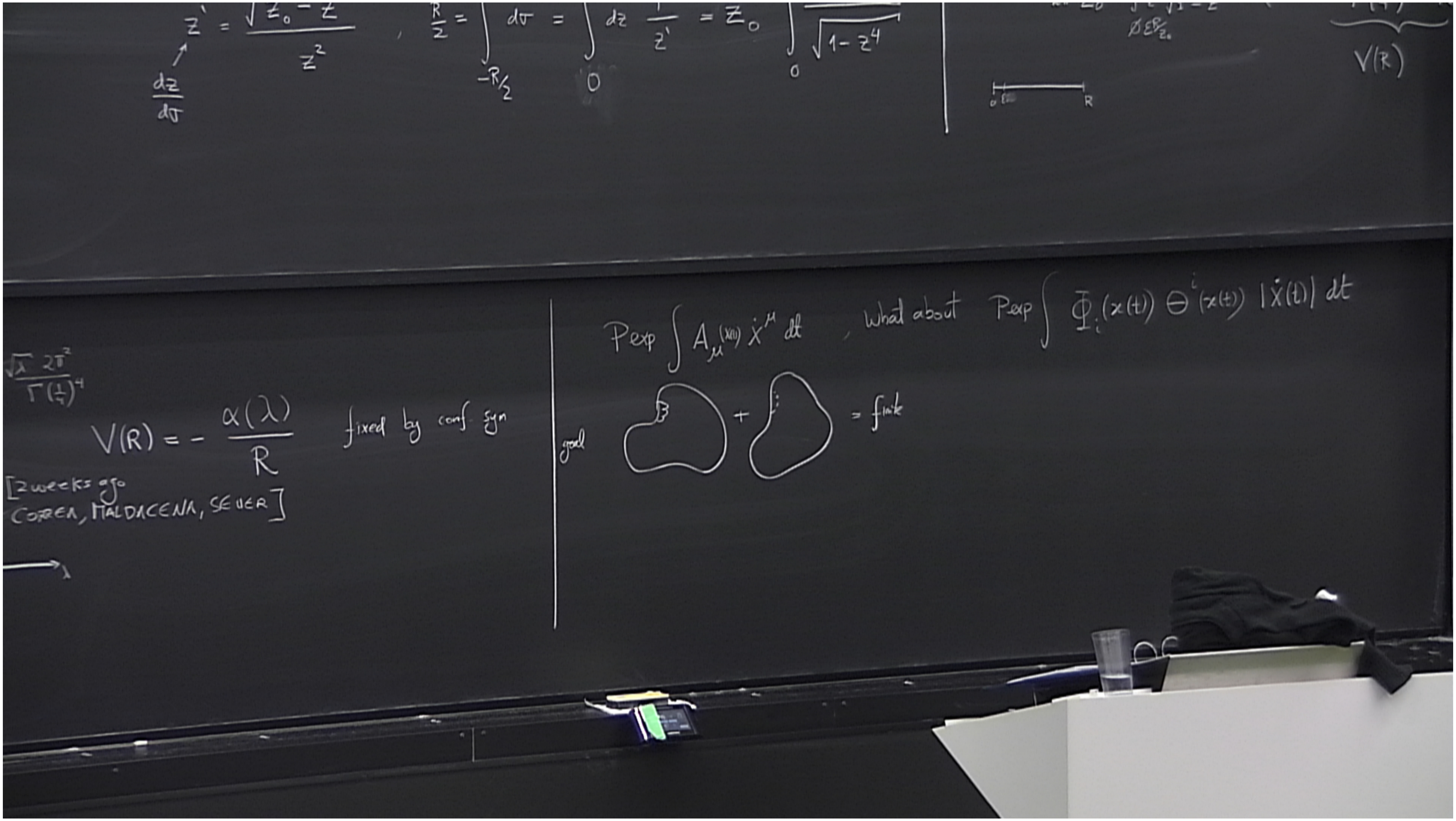

√λ 2π² / Γ(1/4)⁴
V(R) = − α(λ) / R
fixed by conf. sym
interpolation
λ / 2π
λ

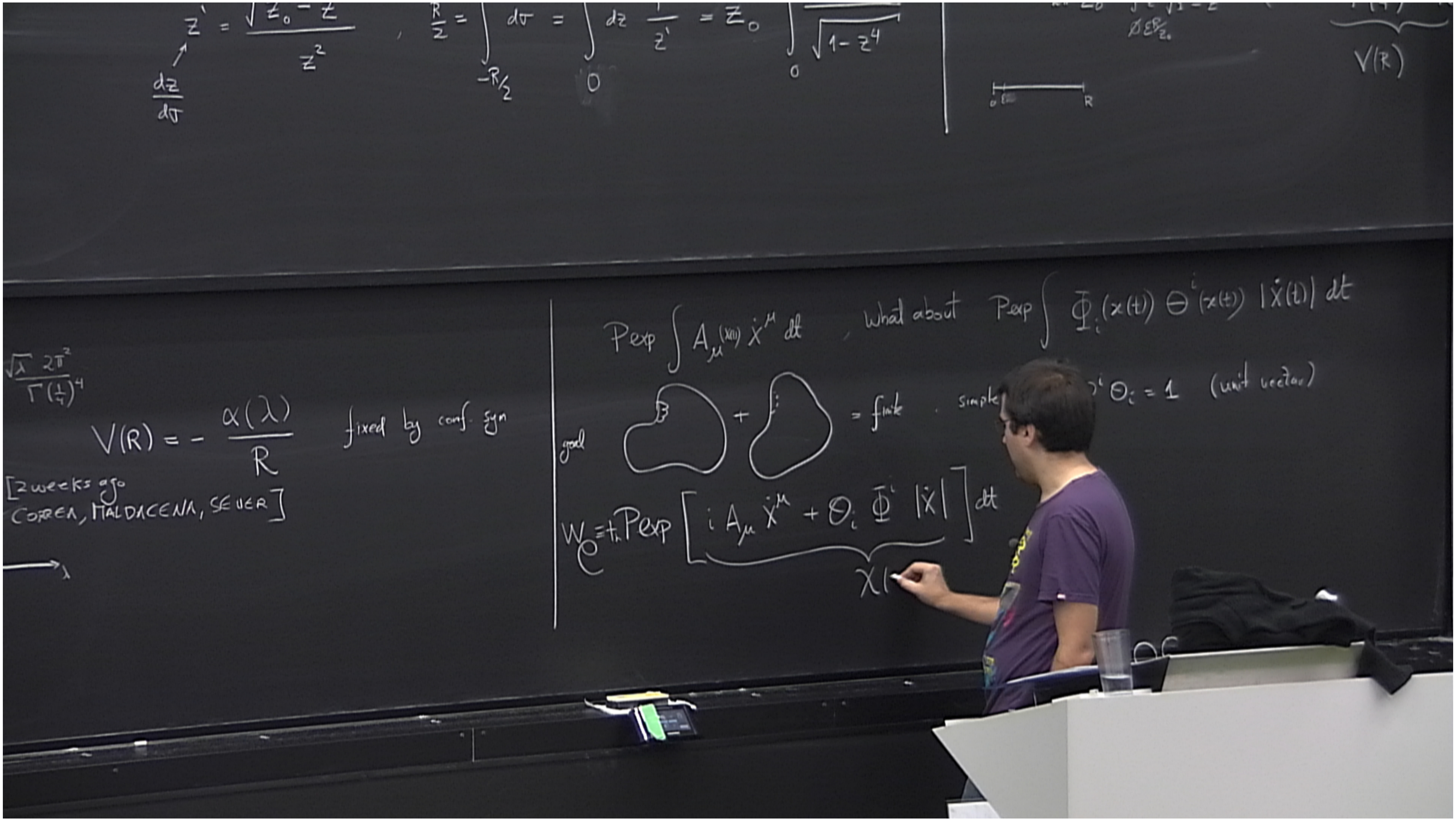

α(λ)
√λ 2π² / Γ(1/4)⁴
V(R) =
fixed by conf. sym
interpolation. [2 weeks ago
CORREA, MALDACEN
λ
λ/2π

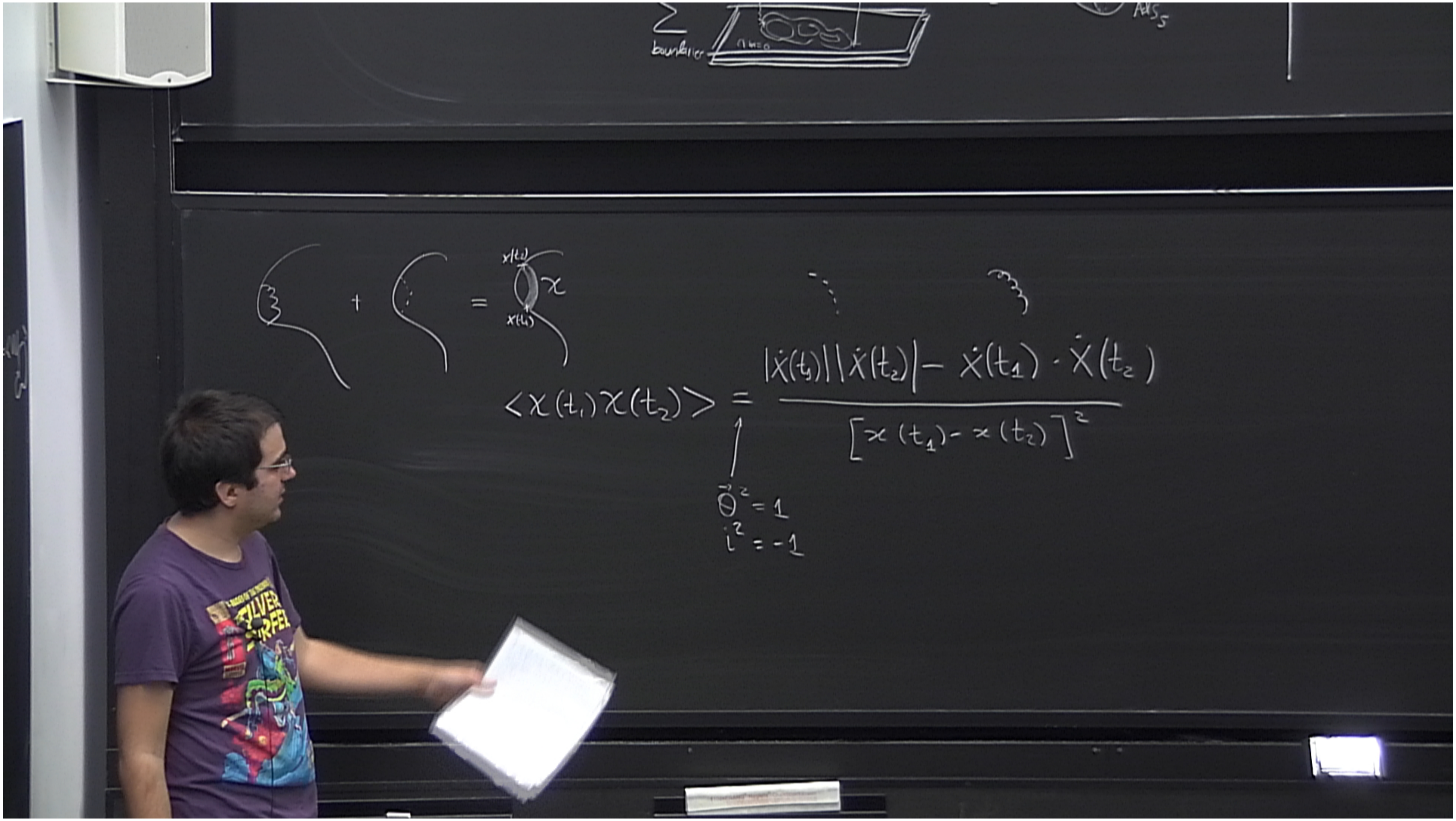

α(λ)
√λ 2π² / Γ(1/4)⁴
V(R) = - α(λ) / R
fixed by conf. sym
interpolation. [2 weeks ago
CORREA, MALDACENA, SEVER]
λ / 2π
λ



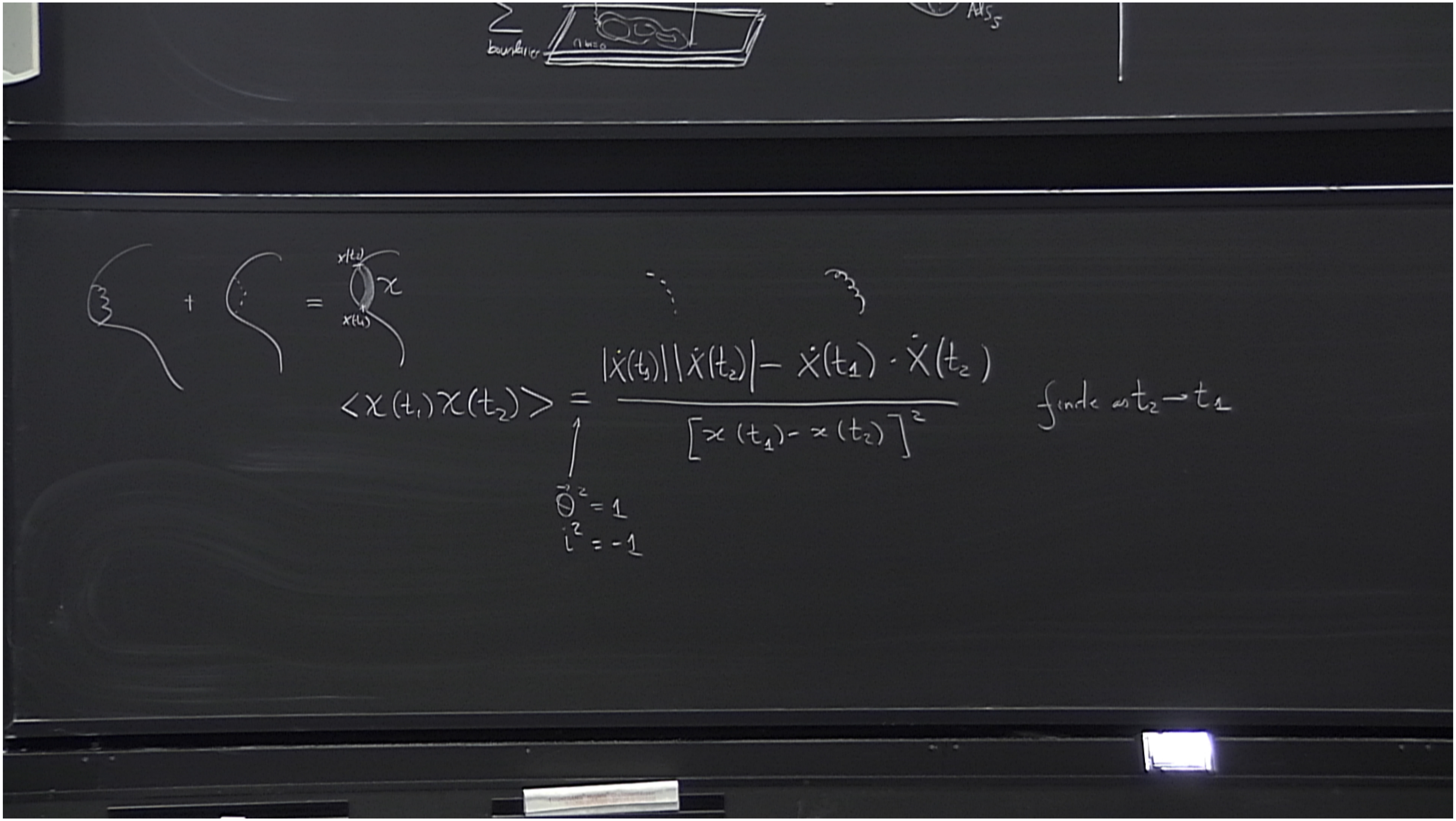
Area
V(R)
V(R) = - α(λ) / R
fixed by conf. sym
interpolation [2 weeks ago CORREA, MALDACENA, SEVER]
What about

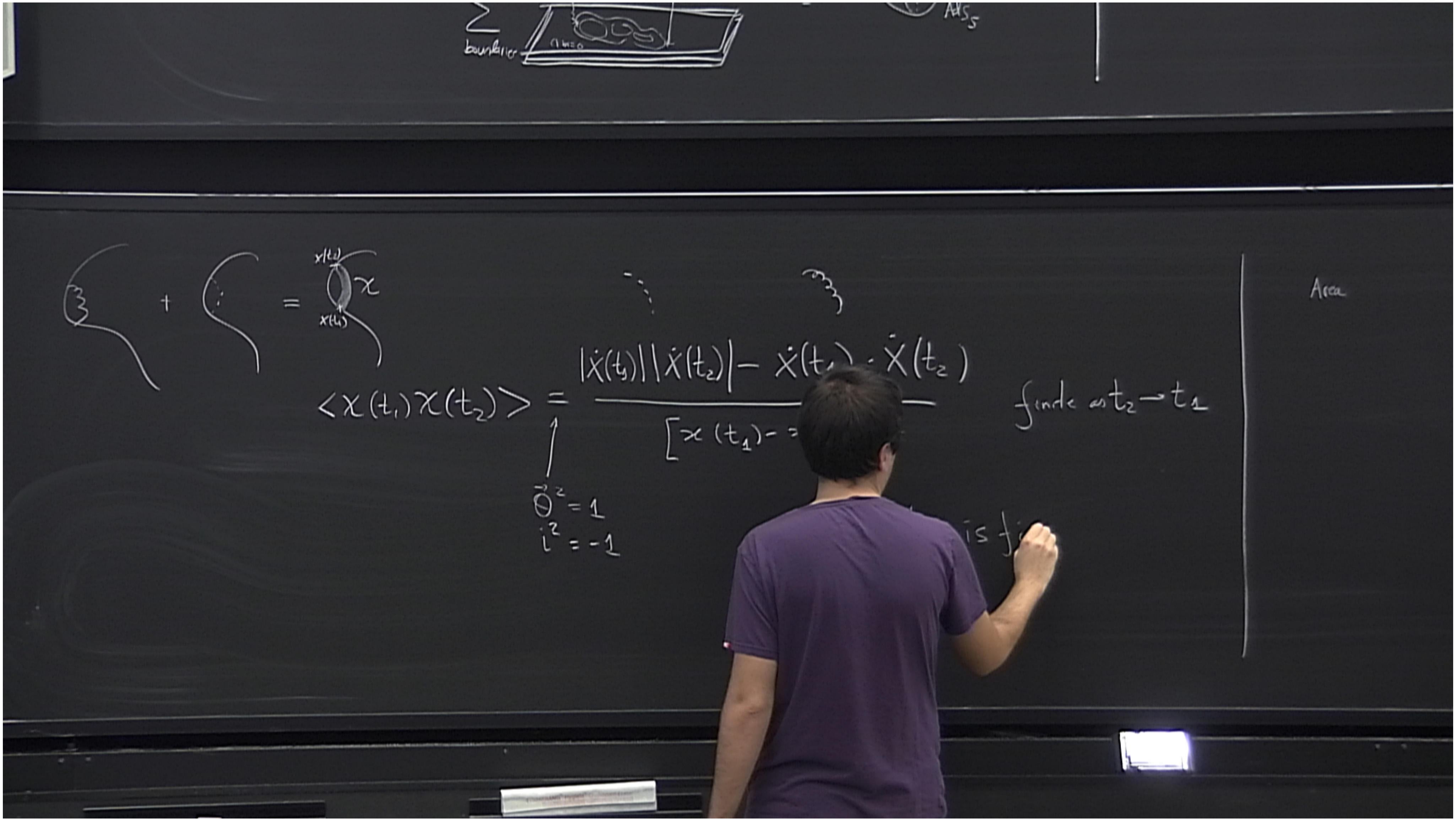

V(R) = - α(λ)/R
fixed by conf. sym
[2 weeks ago
Correa, Maldacena, Sever]
Pexp ∫ A_μ(x(t)) ẋ^μ dt
what about Pexp ∫ Φ_i(x(t)) Θ^i(x(t)) |ẋ(t)| dt
goal
= finite

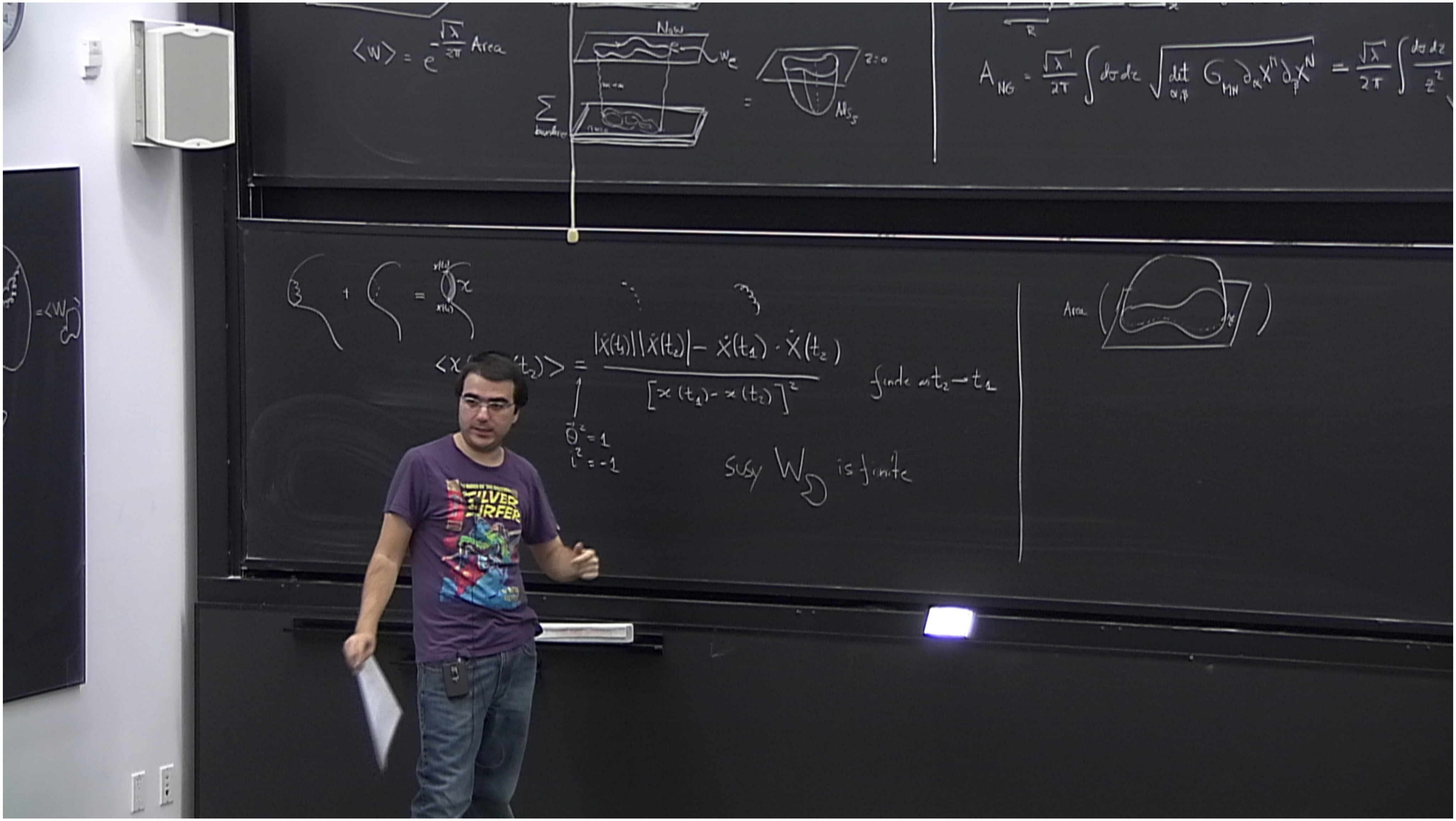

V(R) = − α(λ) / R
fixed by conf. sym
[2 weeks ago
CORREA, MALDACENA, SEVER]
Pexp ∫ A_μ(x(t)) ẋ^μ dt
what about Pexp ∫ Φ_i(x(t)) Θ^i(x(t)) |ẋ(t)| dt
= finite
(unit vector)
W_C = tr Pexp [i A_μ ẋ^μ + Θ_i Φ^i |ẋ|] dt

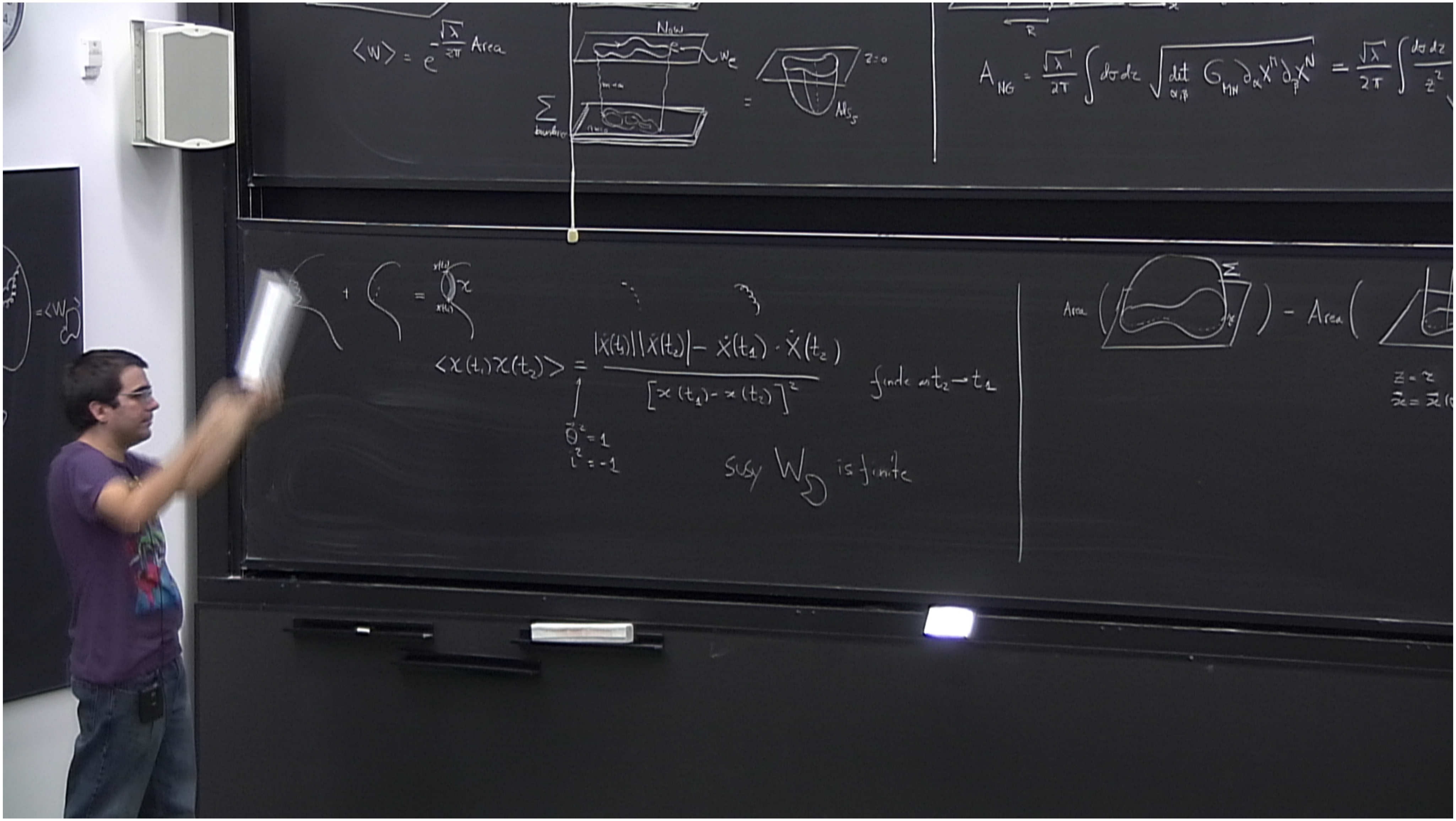


boundary
AdS5
x(t2)
x
x(t1)
<x(t1) x(t2)> = (|ẋ(t1)| |ẋ(t2)| − ẋ(t1) · ẋ(t2)) / [x(t1) − x(t2)]²
θ² = 1
i² = −1

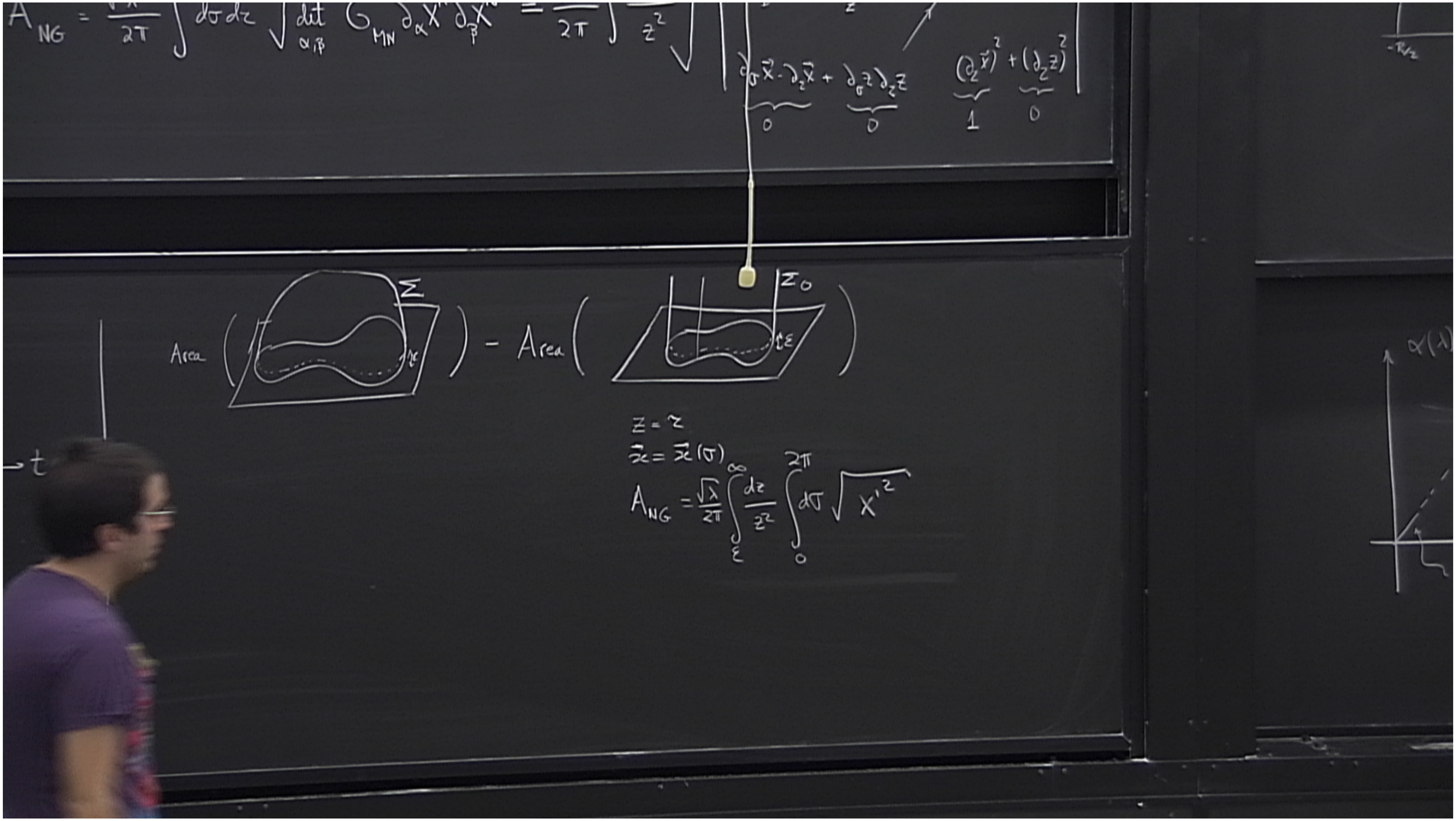

boundaries
AdS_5
$x(t_2)$
$x(t_1)$
x
$\langle x(t_1) x(t_2) \rangle = \frac{|\dot{x}(t_1)||\dot{x}(t_2)| - \dot{x}(t_1) \cdot \dot{x}(t_2)}{[x(t_1) - x(t_2)]^2}$
finite as $t_2 \to t_1$
$\vec{\theta}^2 = 1$
$i^2 = -1$

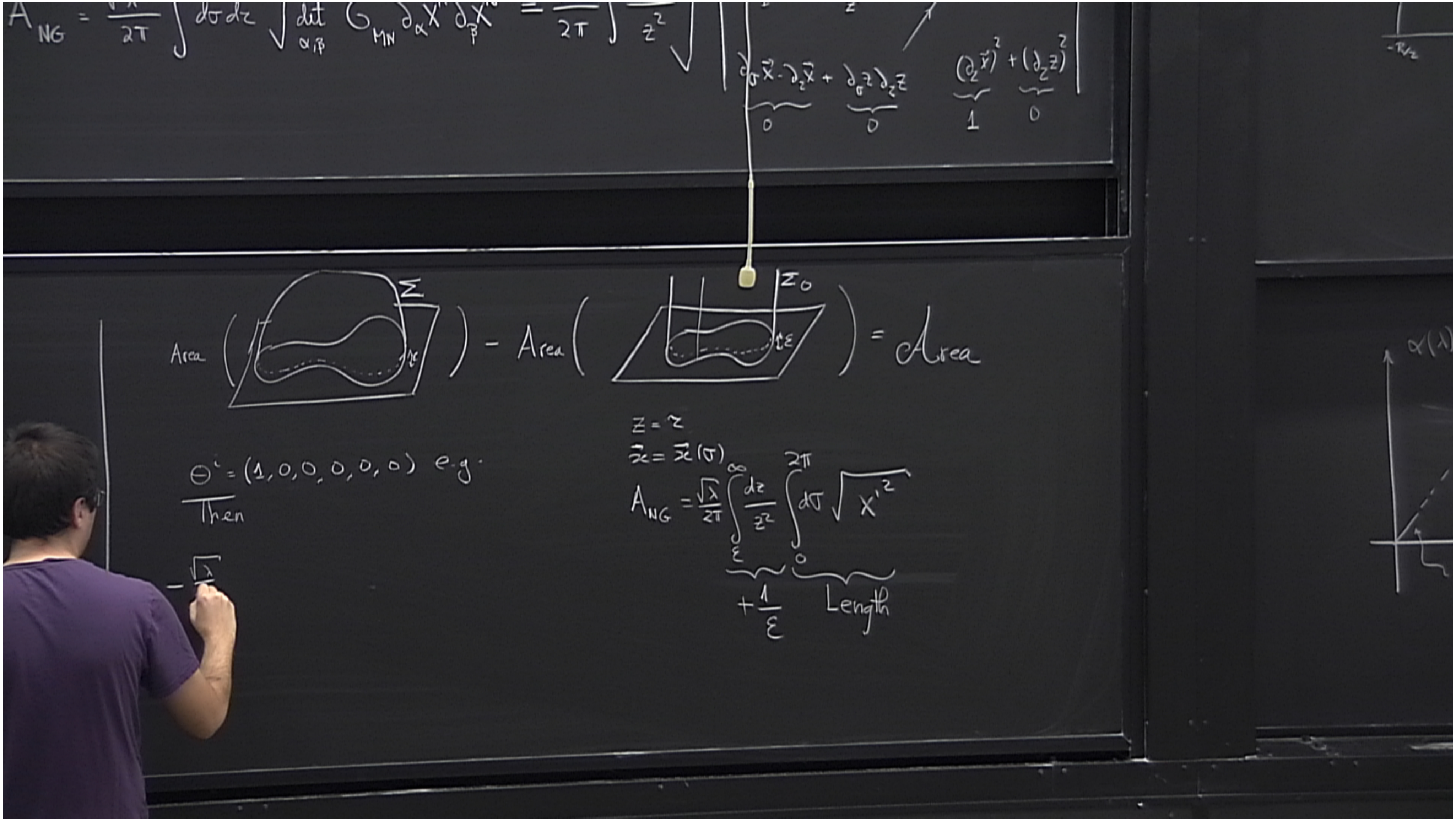

boundaries
AdS5
x(t2)
x(t1)
x
Area
<x(t1)x(t2)> = |ẋ(t1)||ẋ(t2)| − ẋ(t1) · ẋ(t2)
[x(t1)−
finite as t2 → t1
θ² = 1
i² = −1
is fi

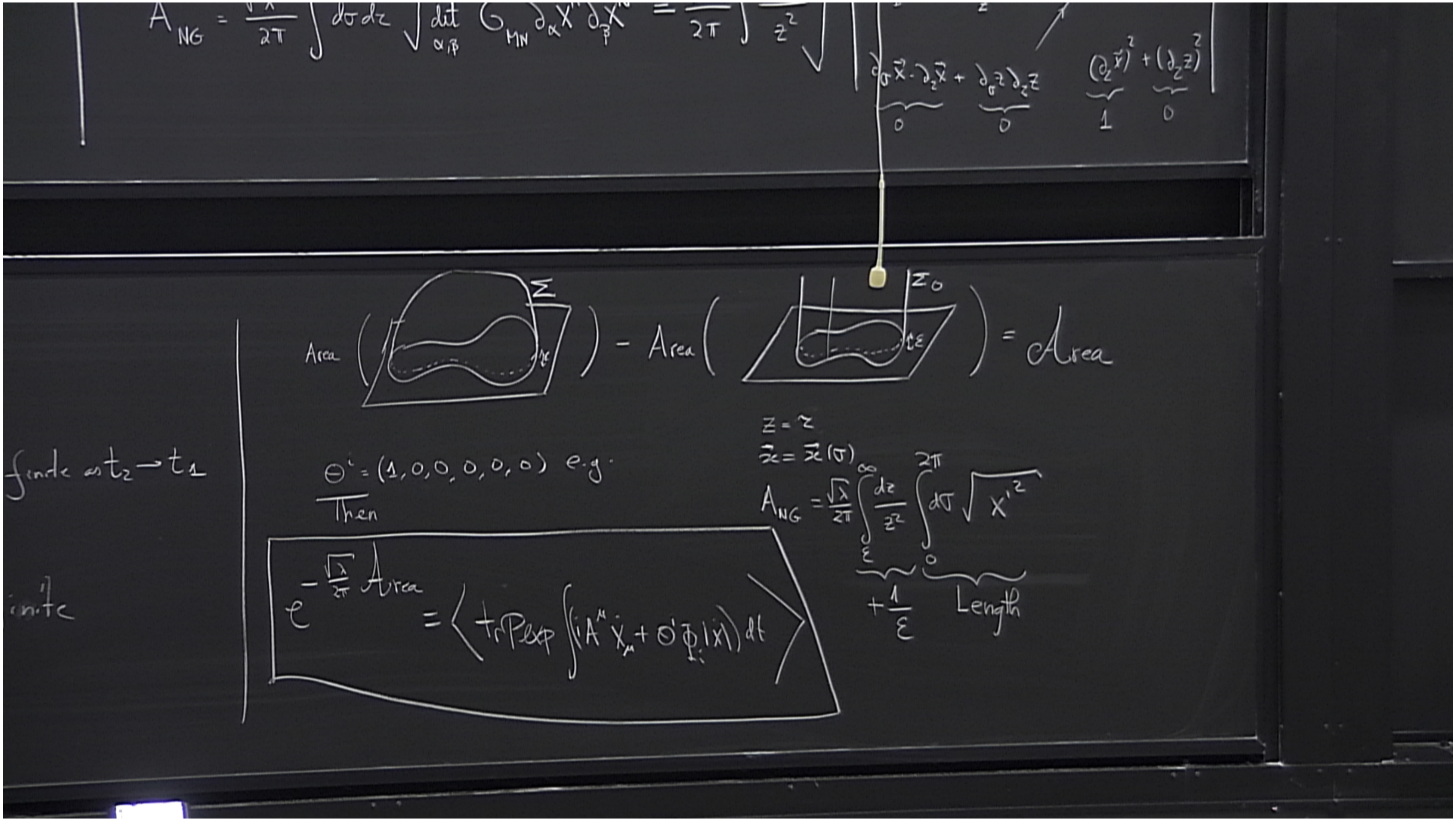


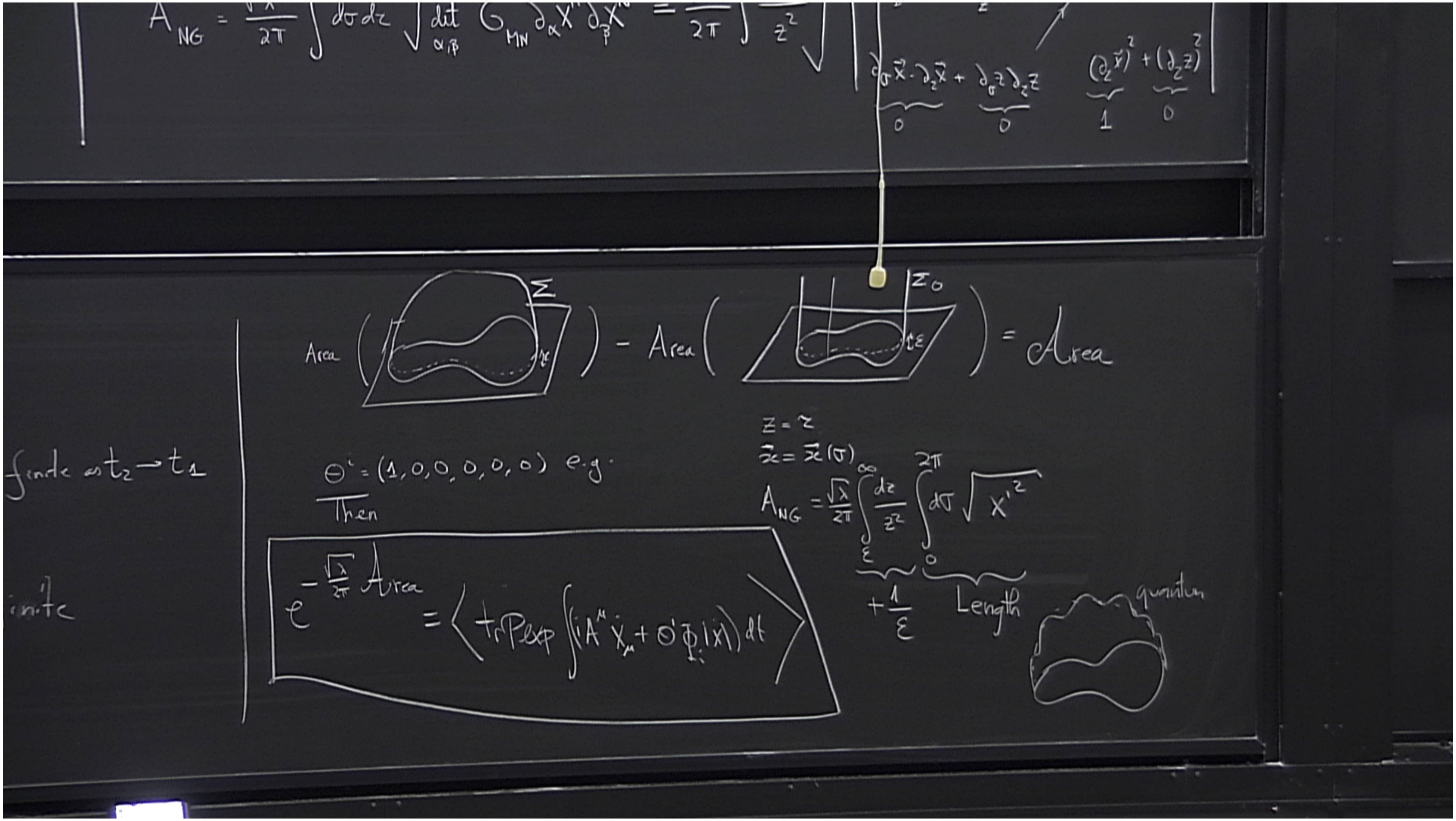


⟨W⟩ = e^{-√λ/2π Area}
N=4
W_C
z=0
AdS_5
A_NG = √λ/2π ∫ dσ dz √(det G_MN ∂_α X^M ∂_β X^N) = √λ/2π ∫ dσ dz / z^2
⟨x(t_1) x(t_2)⟩ = (|ẋ(t_1)| |ẋ(t_2)| − ẋ(t_1) · ẋ(t_2)) / [x(t_1) − x(t_2)]^2
finite as t_2 → t_1
θ^2 = 1
i^2 = −1
susy W is finite
Area( ) − Area(
Σ


A_NG = √λ/2π ∫ dσ dτ √ det_αβ G_MN ∂_α X^M ∂_β X^N
∂_σ x⃗ · ∂_τ x⃗ + ∂_σ z ∂_τ z
(∂_τ x⃗)² + (∂_τ z)²
0 0 1 0
Σ
Area ( ) − Area ( )
Σ_0
ε
z = τ
x⃗ = x⃗(σ)
A_NG = √λ/2π ∫_ε^∞ dz/z² ∫_0^2π dσ √ x'²


Area (Σ) − Area (Σ0) = Area
Z = z
x = x(σ)
Θ^i = (1, 0, 0, 0, 0, 0) e.g.
Then
A_NG = √λ/2π ∫_ε^∞ dz/z² ∫_0^2π dσ √(x'²)
+ 1/ε
Length


Area ( ) − Area ( ) = dArea
Σ
Σ₀
Θⁱ = (1, 0, 0, 0, 0, 0) e.g.
Then
Z = z
x = x(σ)
Length
finite as t₂ → t₁


Area
Σ
Area
Σ₀
ε
= dArea
finite as t₂ → t₁
finite
Θⁱ = (1, 0, 0, 0, 0, 0) e.g.
Then
e^{-√λ/2π Area} = ⟨ tr P exp ∮(iA^μ ẋ_μ + Θⁱ φ_i |ẋ|) dt ⟩
z = τ
x⃗ = x⃗(σ)
A_NG = √λ/2π ∫_ε^∞ dz/z² ∫_0^{2π} dσ √(x'²)
+ 1/ε
Length
quantum