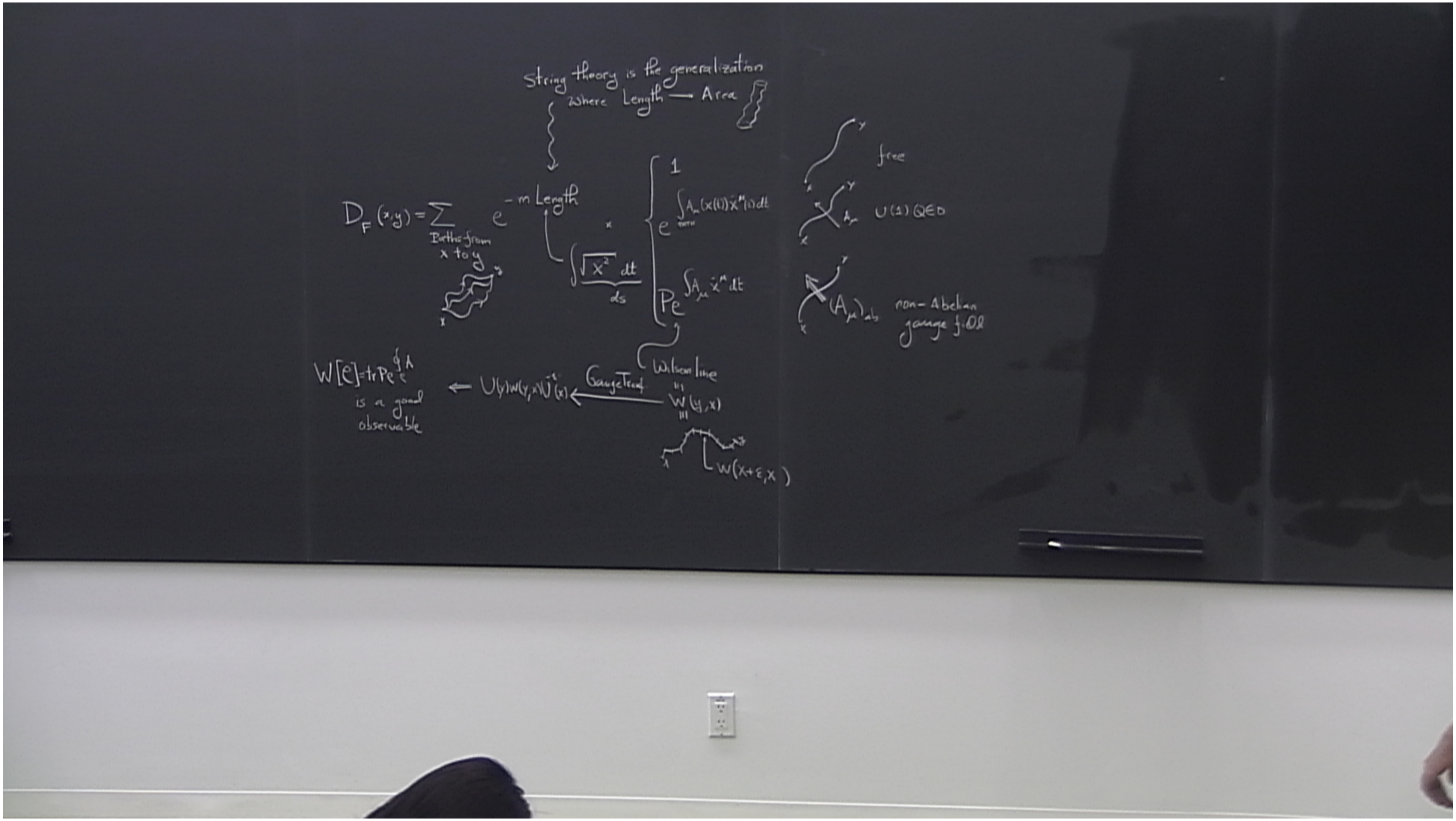
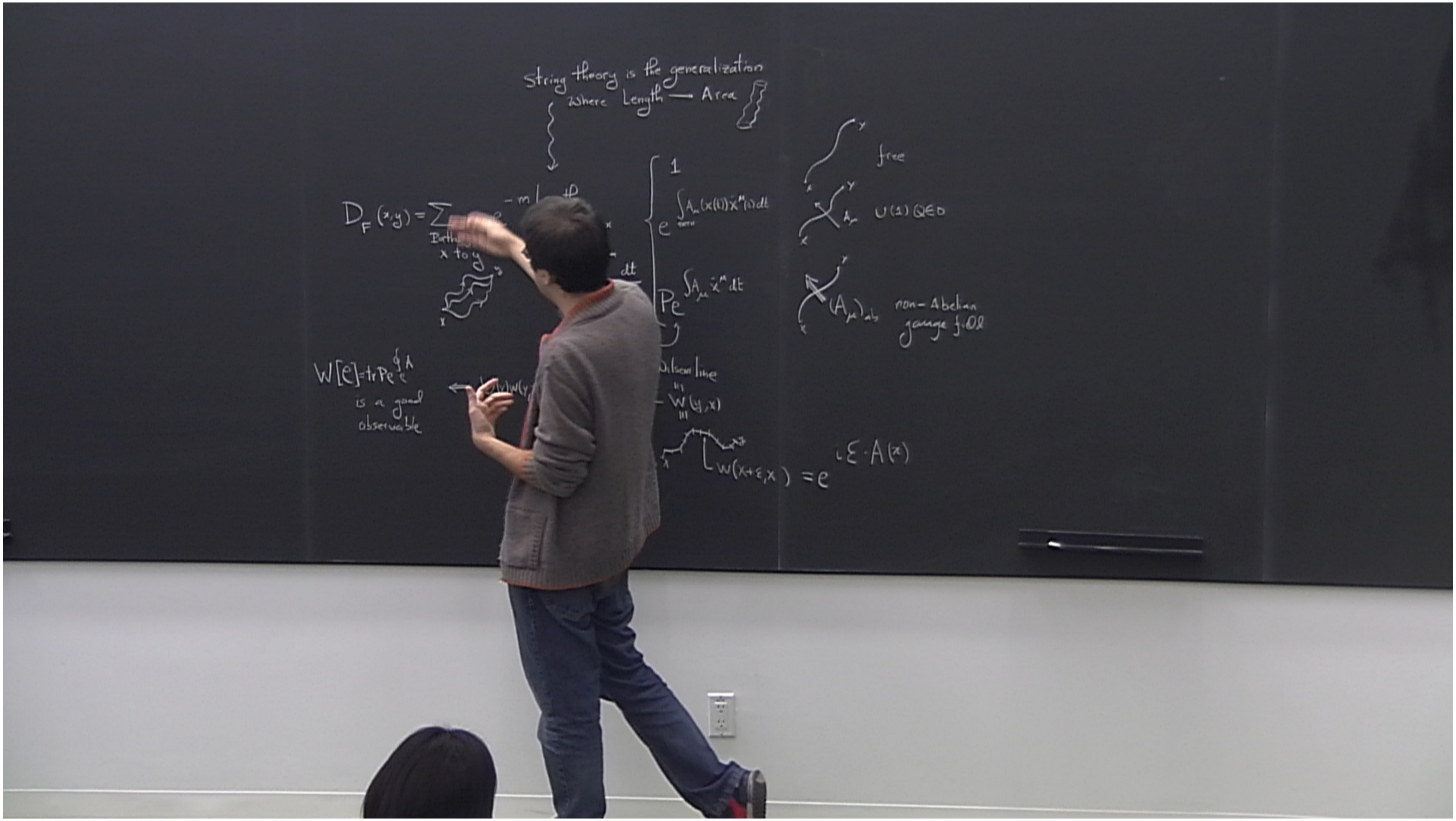
Title: Explorations in String Theory - Lecture 6

Date: Mar 19, 2012 11:30 AM

URL: http://pirsa.org/12030049

Abstract:


String theory is the generalization
where Length → Area
$D_F(x,y) = \sum_{\text{Paths from } x \text{ to } y} e^{-m\,\text{Length}}$
$\int \sqrt{\dot{x}^2}\,dt$
ds
free
U(1) QED
non-Abelian gauge field
Wilson line
W(y,x)
Gauge Trans
W[C] = tr P e^{∮ A}
is a good observable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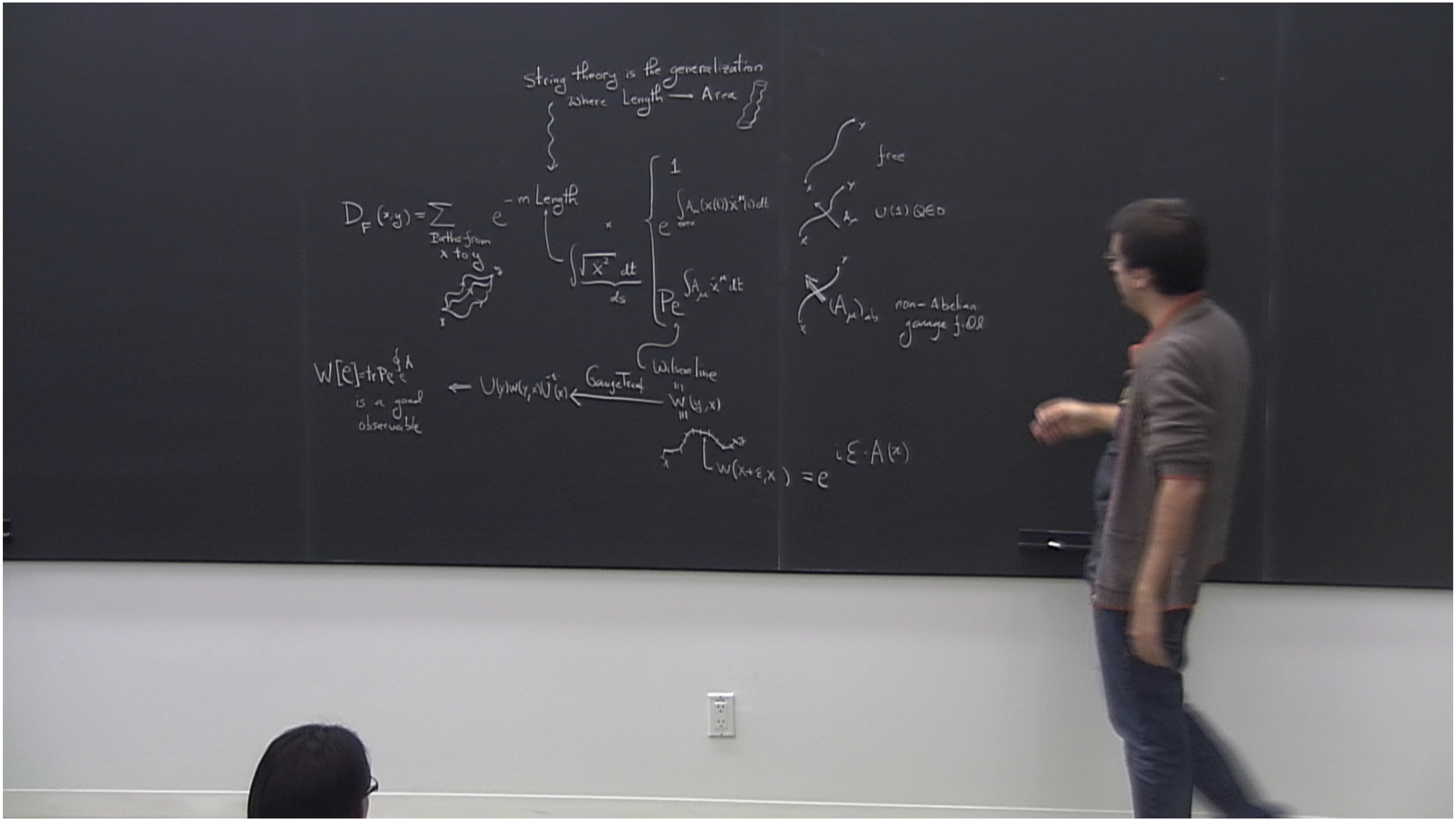
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$\int \sqrt{\dot{x}^2}\, dt$
ds
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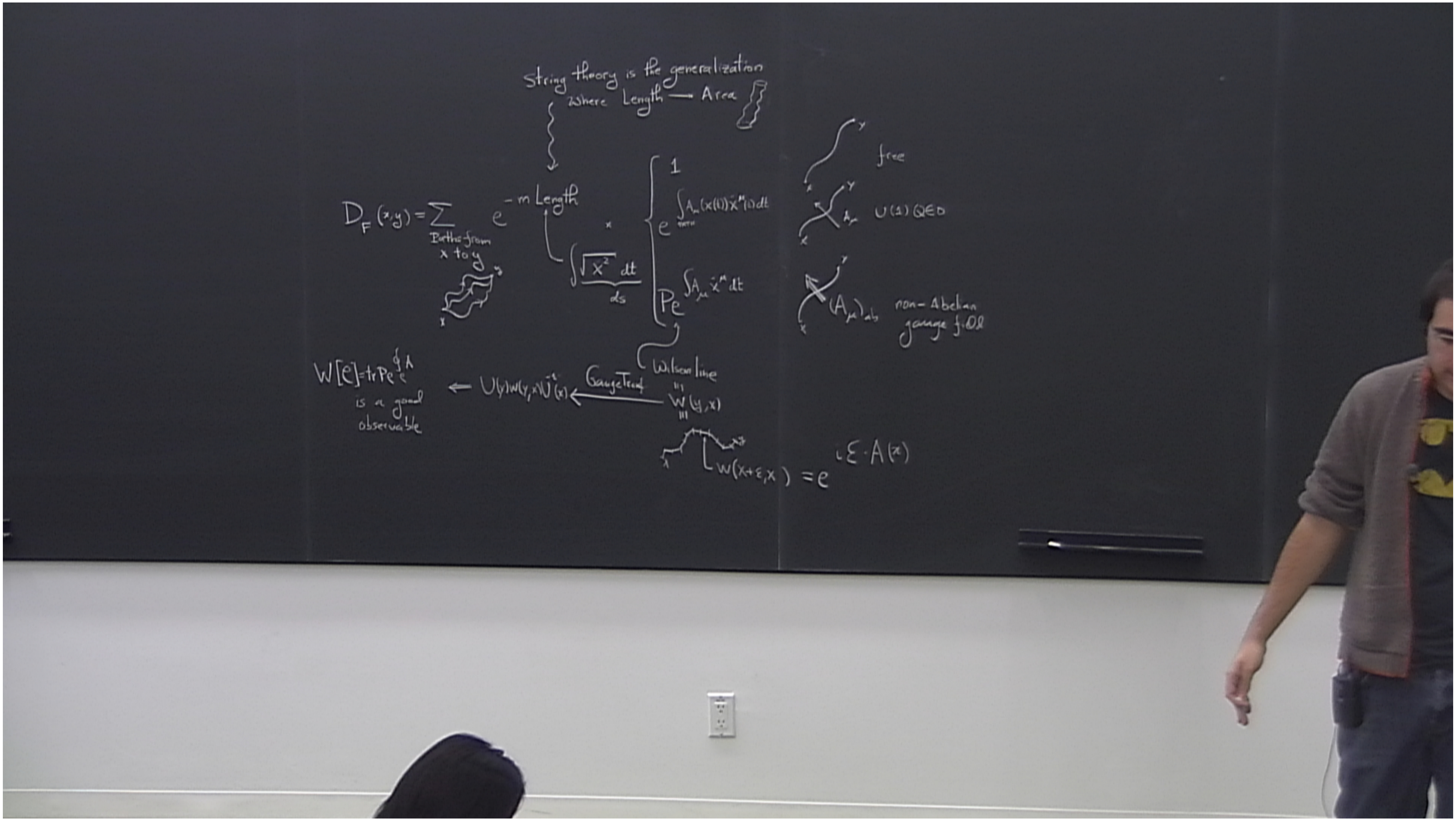
free
U(1) QED
non-Abelian gauge field
Wilson line
W(y,x)
Gauge Transf
$U(y)W(y,x)U^{-1}(x)$
W[C] = tr P e
is a good observable
$W(x+\varepsilon, x) = e^{i\varepsilon \cdot A(x)}$



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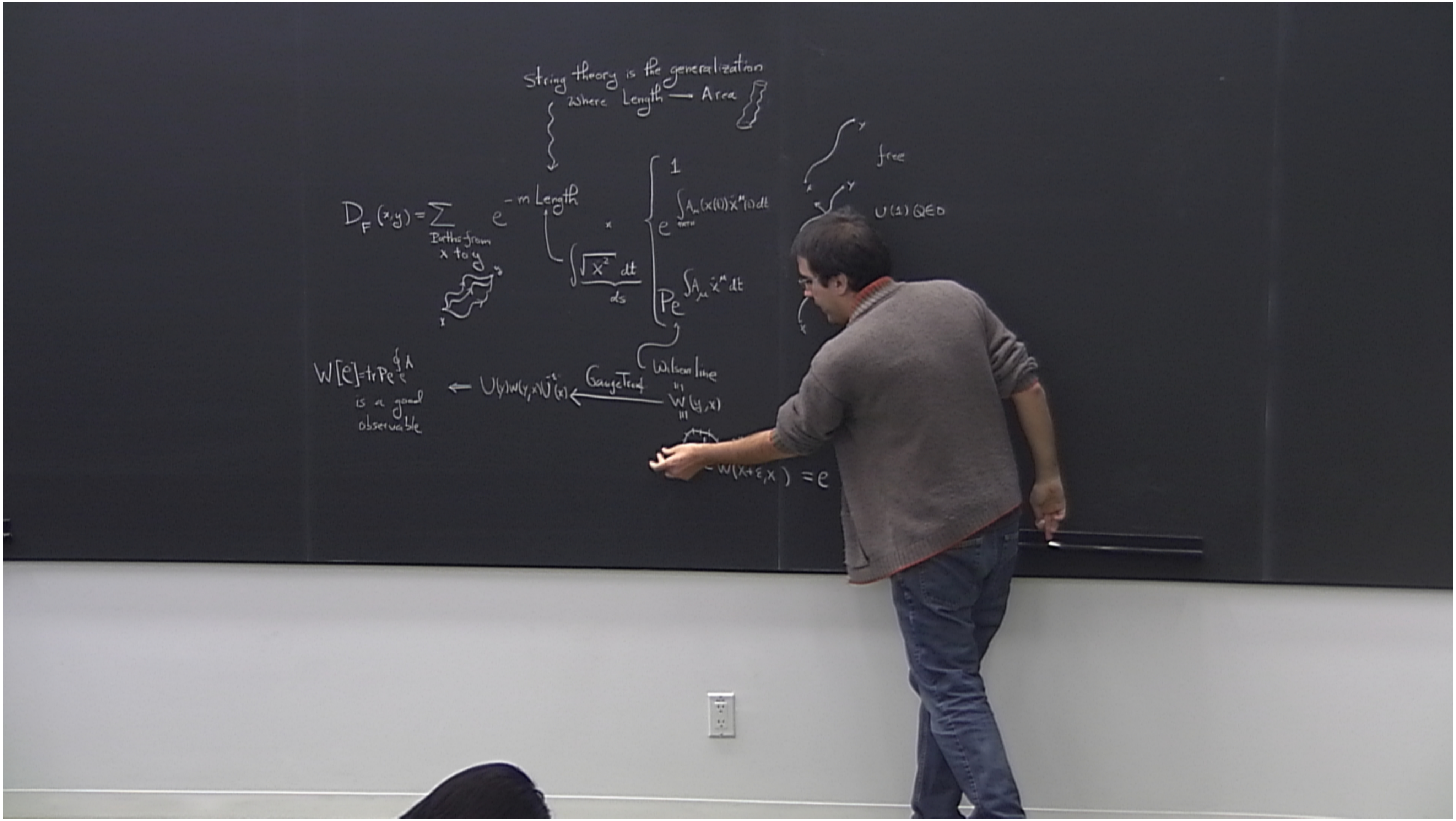
String theory is the generalization
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U(1) QED
Paths from
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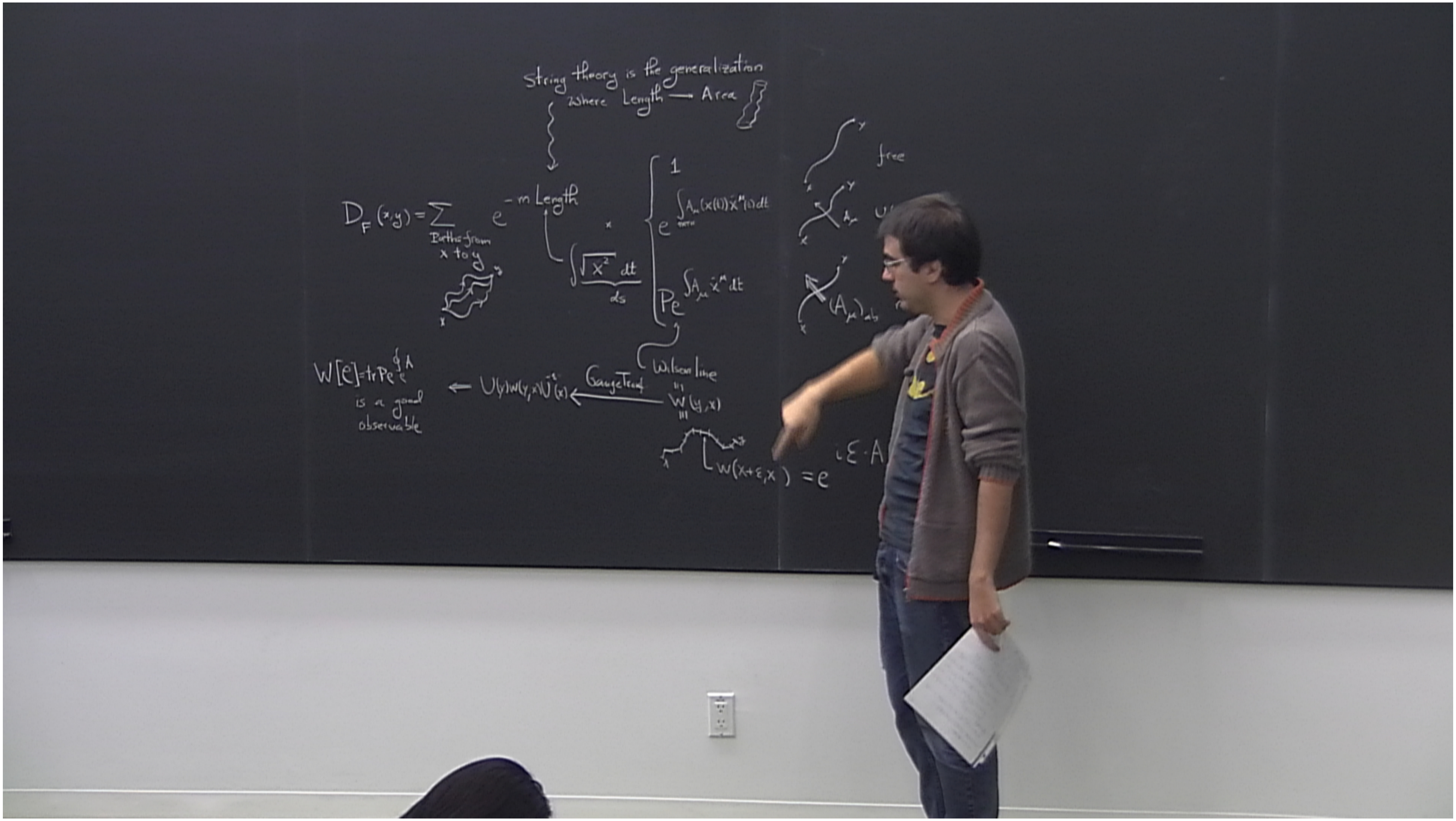
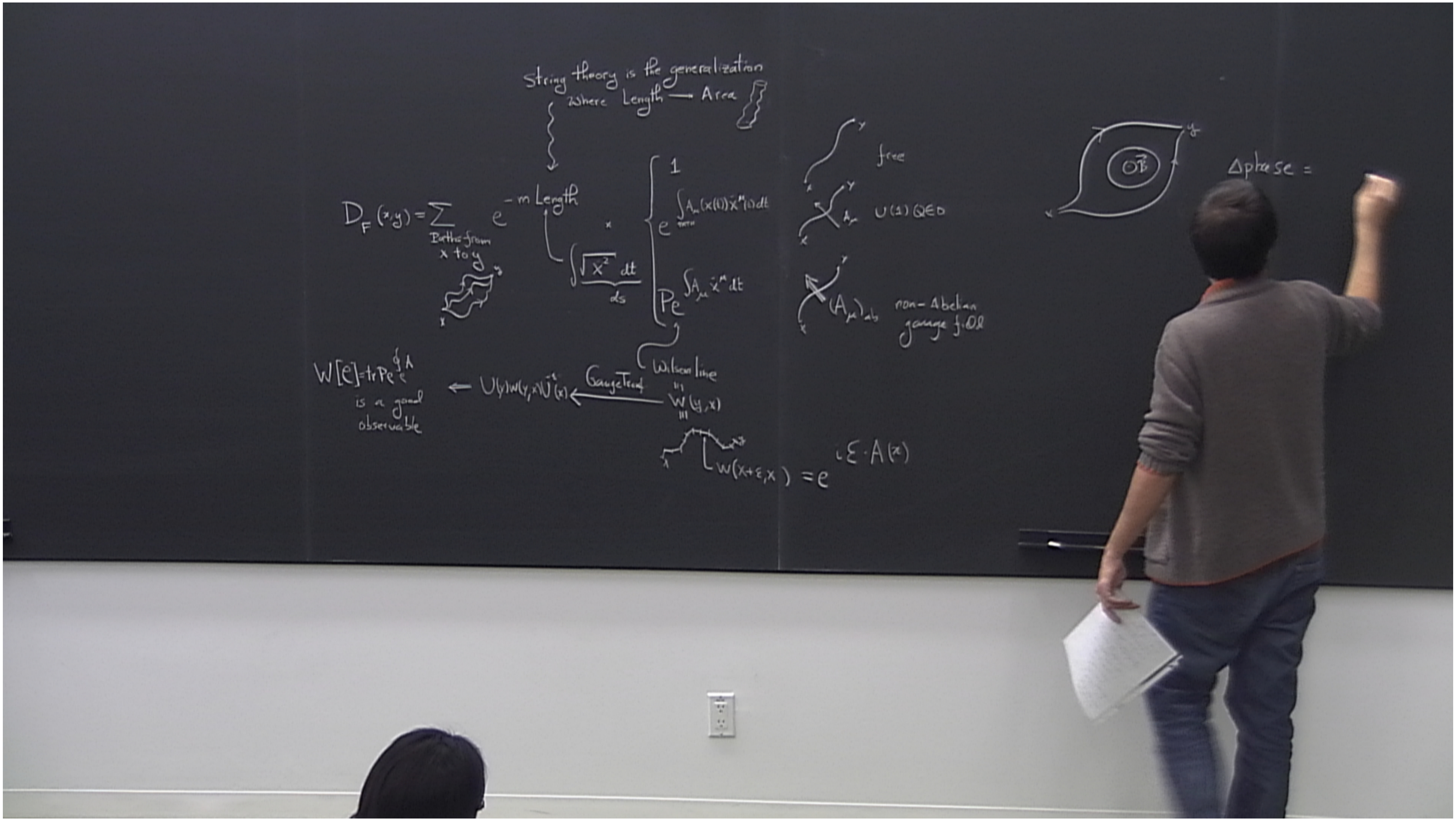
Wilson line
Gauge Transf
is a good
observable



String theory is the generalization
where Length → Area
$D_F(x,y) = \sum_{\text{Paths from } x \text{ to } y} e^{-m\,\text{Length}}$
$\int \sqrt{\dot{x}^2}\,dt$
ds
1
$e^{\int A_\mu(x(t))\dot{x}^\mu(t)\,dt}$
$Pe^{\int A_\mu \dot{x}^\mu dt}$
free
Wilson line
$W(y,x)$
Gauge Trans
$U(y)W(y,x)U^{-1}(x)$
$W[C] = \mathrm{tr}\,Pe^{\oint A}$
is a good observable
$W(x+\varepsilon,x) = e^{i\varepsilon\cdot A}$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$\int \sqrt{\dot{x}^2}\, dt$
ds
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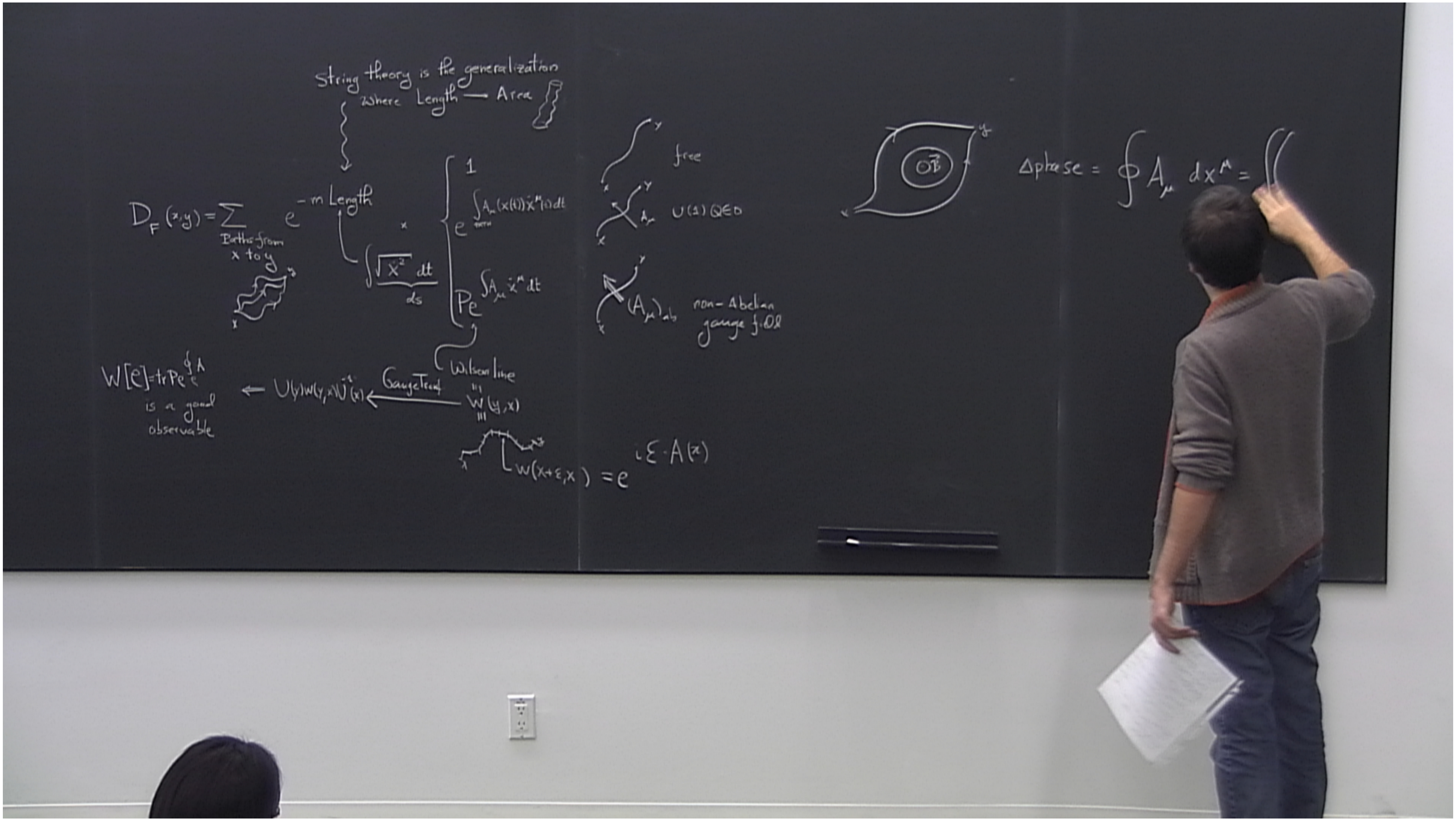
free
U(1) QED
non-Abelian gauge field
Wilson line
$W(y,x)$
GaugeTrans
$U(y)W(y,x)U^{-1}(x)$
$W[C] = \mathrm{tr}\, P e^{\oint_C A}$
is a good observable
$W(x+\epsilon, x) = e^{i\epsilon \cdot A(x)}$
Δphase =


String theory is the generalization
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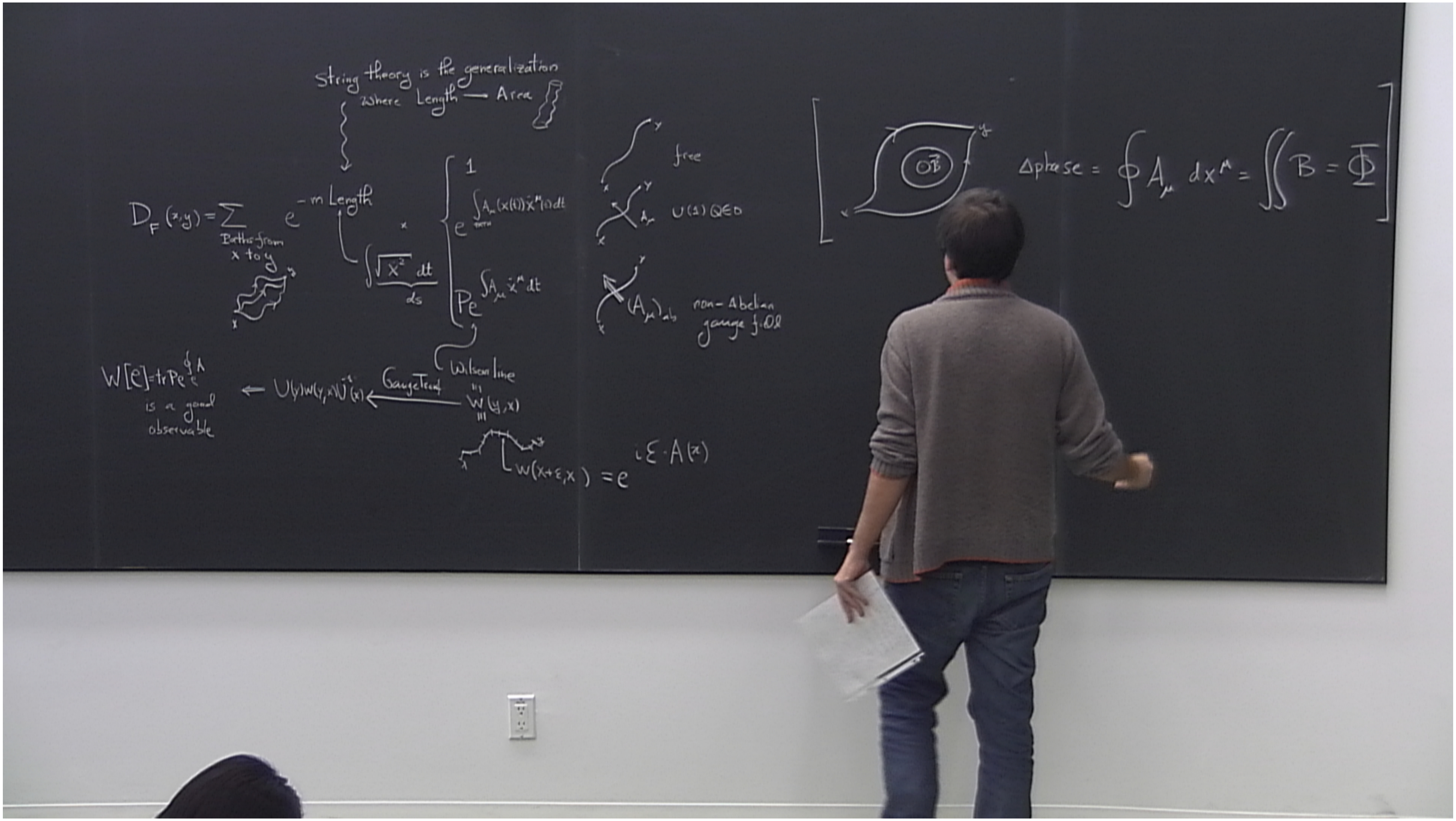
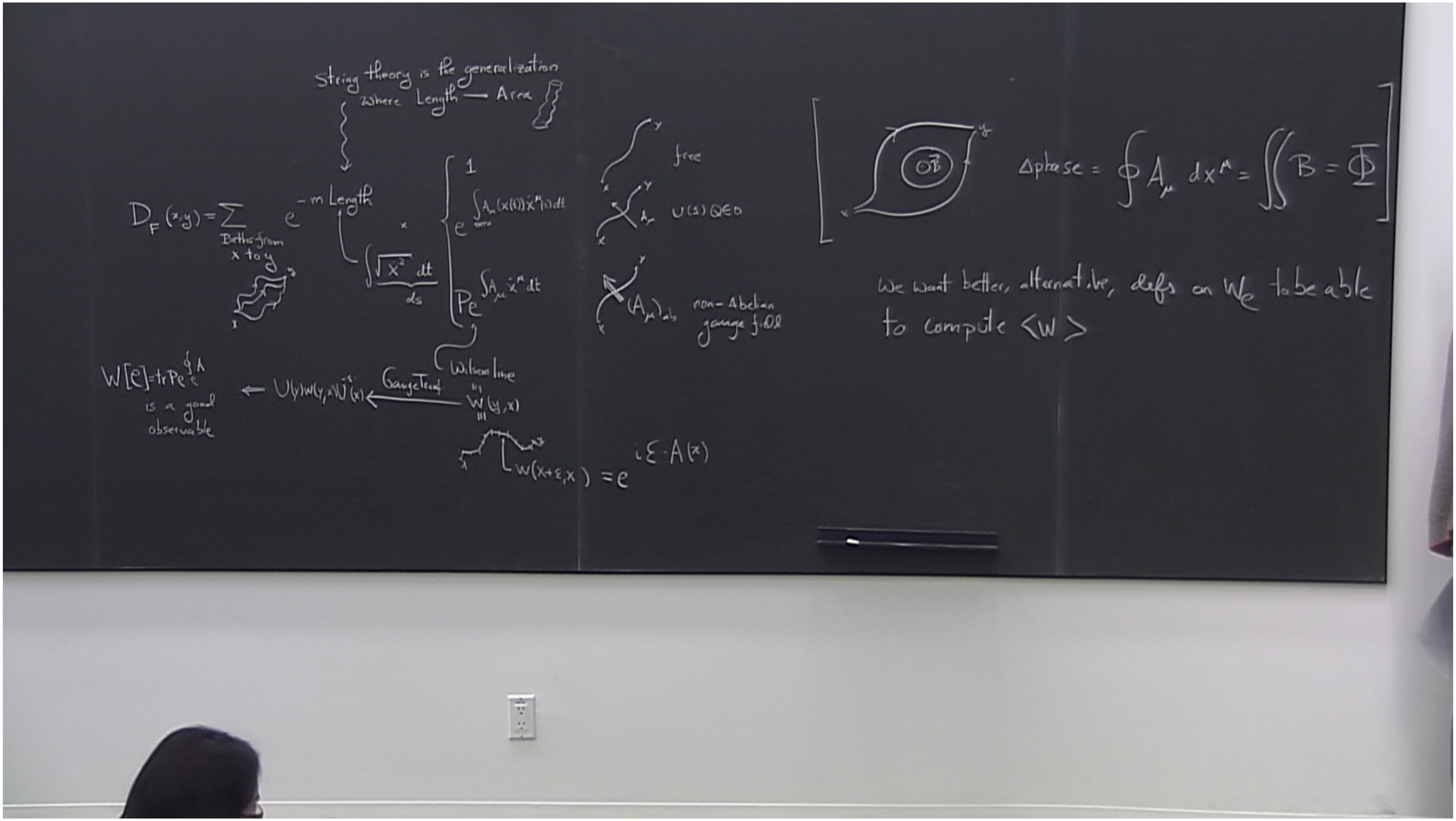
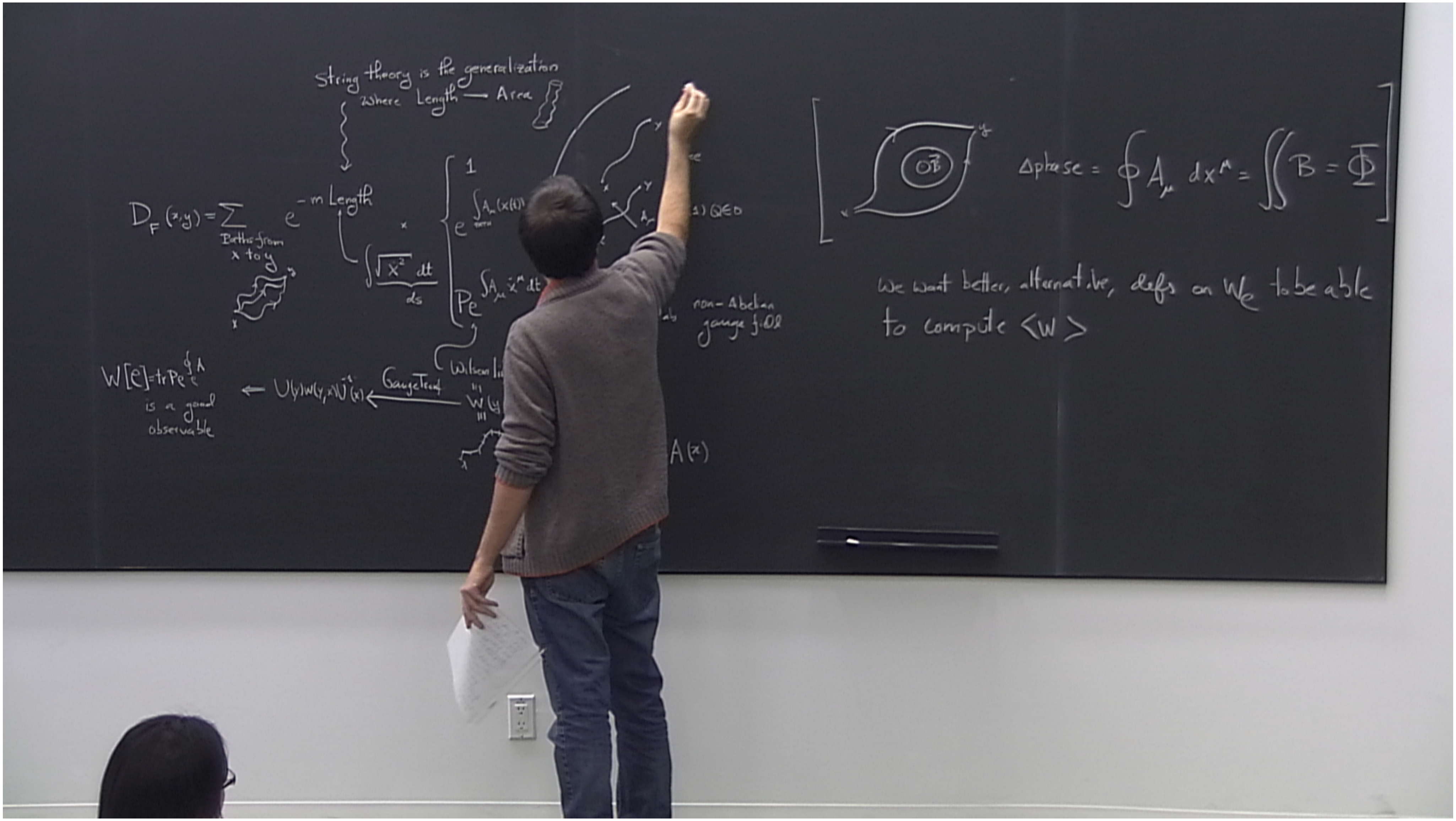
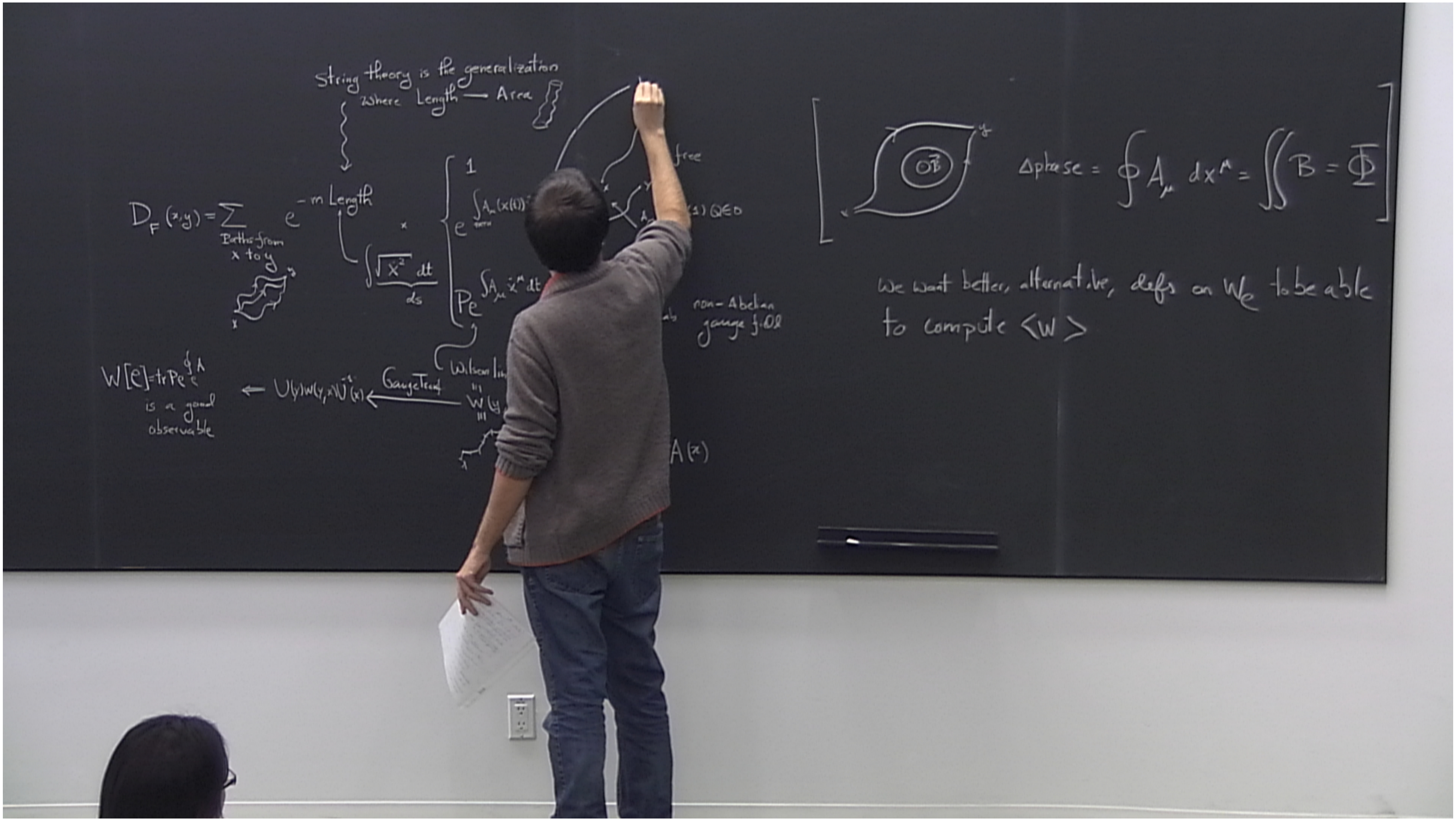
$D_F(x,y) = \sum_{\text{Paths from } x \text{ to } y} e^{-m\,\text{Length}}$
$\int \sqrt{\dot{x}^2}\, dt$
ds
Wilson line
$W(y,x)$
Gauge Trans
$\leftarrow U(y)W(y,x)U^{-1}(x) \leftarrow$
$W[C] = \mathrm{tr}\, P e^{\oint_C A}$
is a good observable
$W(x+\varepsilon, x) = e^{i\varepsilon\cdot A(x)}$
free
U(1) QED
$(A_\mu)_{ab}$ non-Abelian gauge field
$\Delta\text{phase} = \oint A_\mu\, dx^\mu = \iint$



String theory is the generalization
where Length → Area
free
U(1) QED
non-Abelian gauge field
Wilson line
Gauge Transf
is a good observable
Paths from x to y
Δphase =


String theory is the generalization
Where Length → Area
free
U(1) QED
non-Abelian gauge field
Δphase = ∮ A_μ dx^μ = ∬ B = Φ
We want better, alternative, defs on W_C to be able to compute ⟨W⟩
D_F (x,y) = Σ e^{-m Length}
Paths from x to y
Wilson line
Gauge Transf
W(y,x)
W[C] = tr P e^{∮ A}
is a good observable
W(x+ε,x) = e^{iε·A(x)}


String theory is the generalization
where Length → Area
D_F (x,y) = ∑ e^{-m Length}
Paths from x to y
∫√(ẋ²) dt
ds
1
Pe^{∫A_μ ẋ^μ dt}
QED
non-Abelian gauge field
W[C] = tr Pe^{∮_C A}
is a good observable
U(y)W(y,x)U⁻¹(x)
Gauge Trans
Wilson li
Δphase = ∮A_μ dx^μ = ∬B = Φ
We want better, alternative, defs on W_C to be able to compute ⟨W⟩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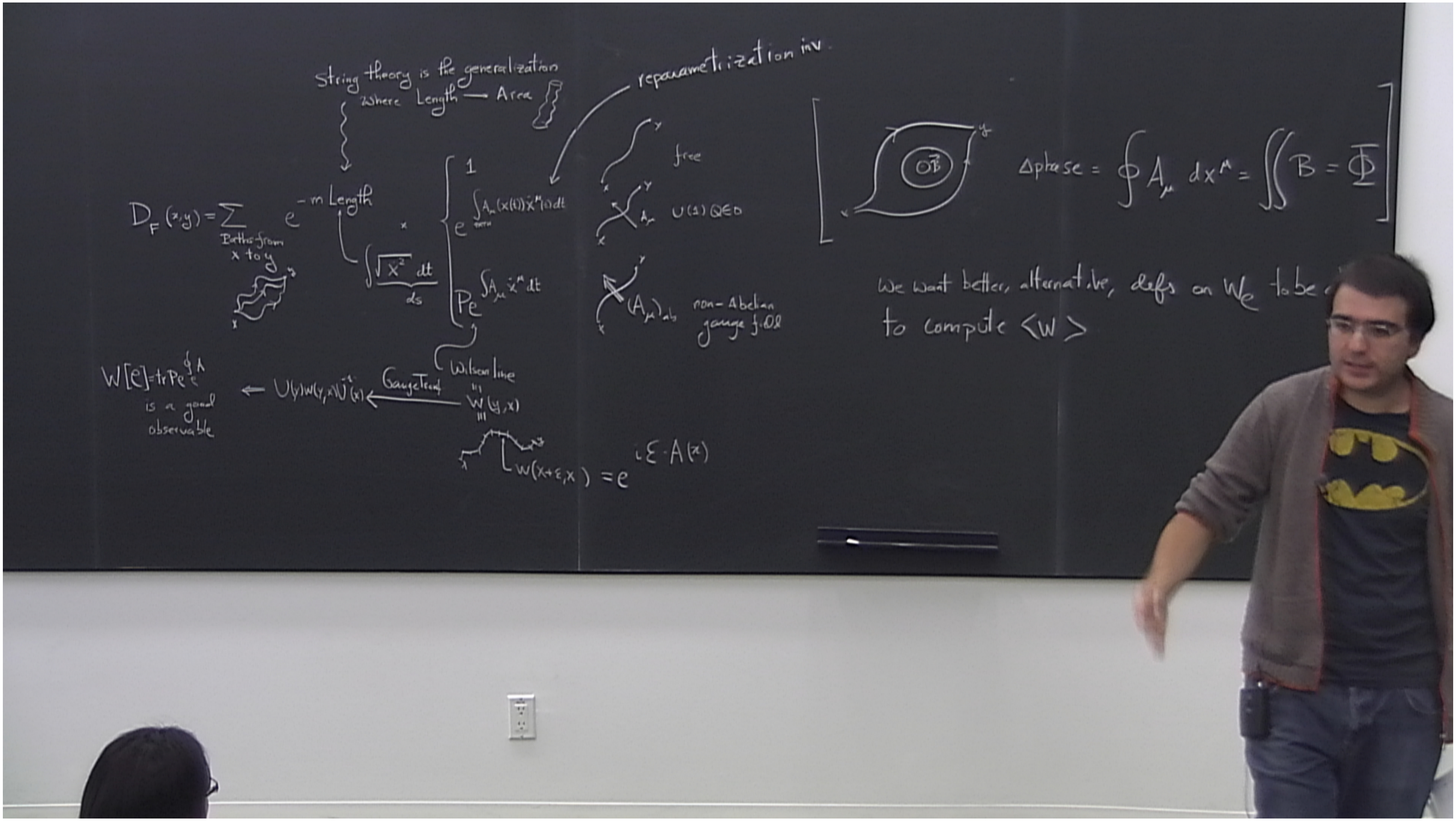
Gauge Transf
$W[\mathcal{C}] = \mathrm{tr}\, P e^{\oint_{\mathcal{C}} A}$
is a good observable
$\Delta\text{phase} = \oint A_\mu\, dx^\mu = \iint B = \Phi$
We want better, alternative, defs on $W_{\mathcal{C}}$ to be able to compute $\langle W \rangle$


String theory is the generalization
where Length → Area
reparametrization inv.
free
U(1) QED
non-Abelian gauge field
Paths from x to y
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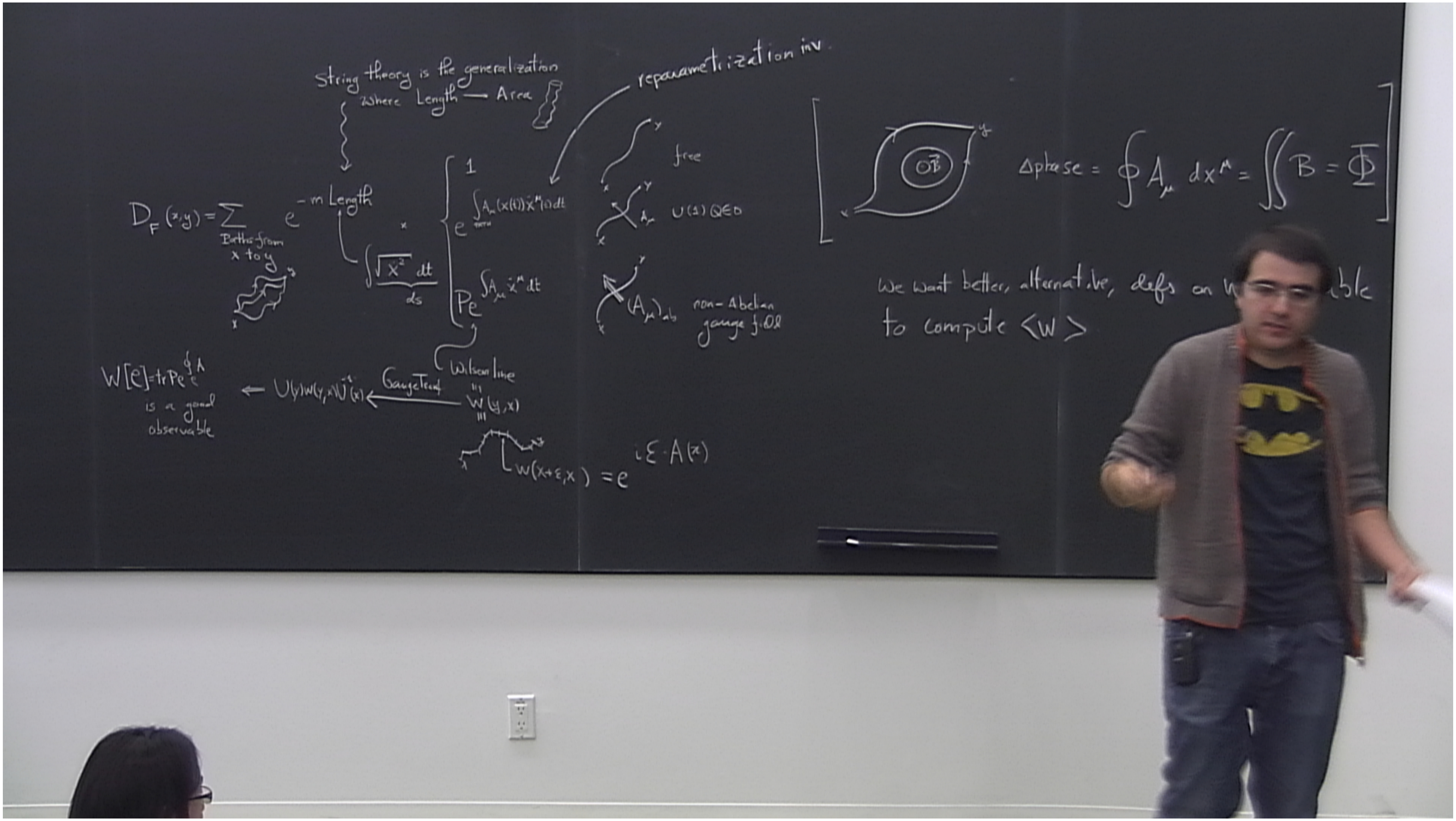
Δphase = ∮ A_μ dx^μ = ∬ B = Φ
We want better, alternative, defs on W_C to be
to compute <W>


String theory is the generalization
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Paths from x to y
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Gauge Trans
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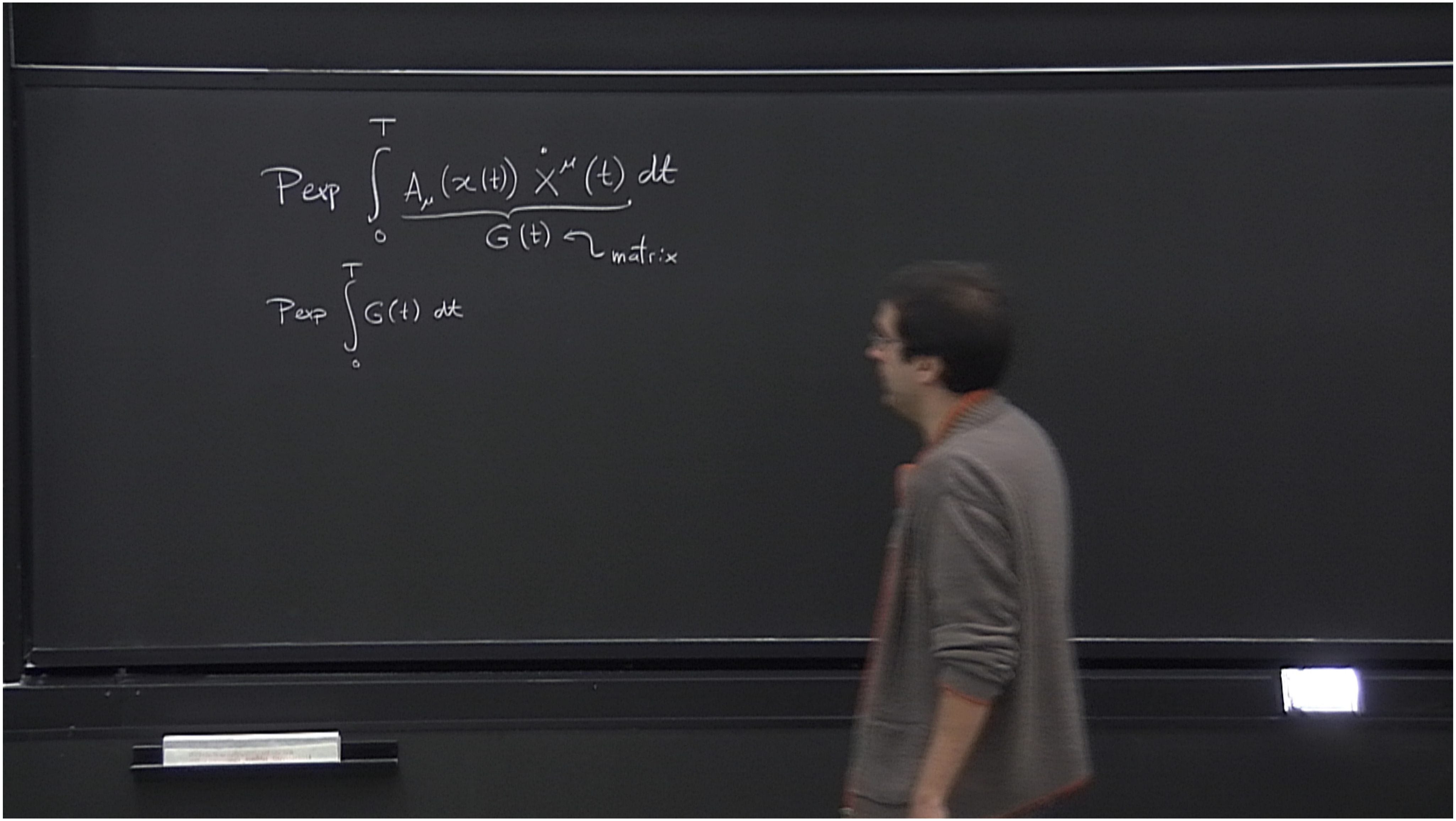
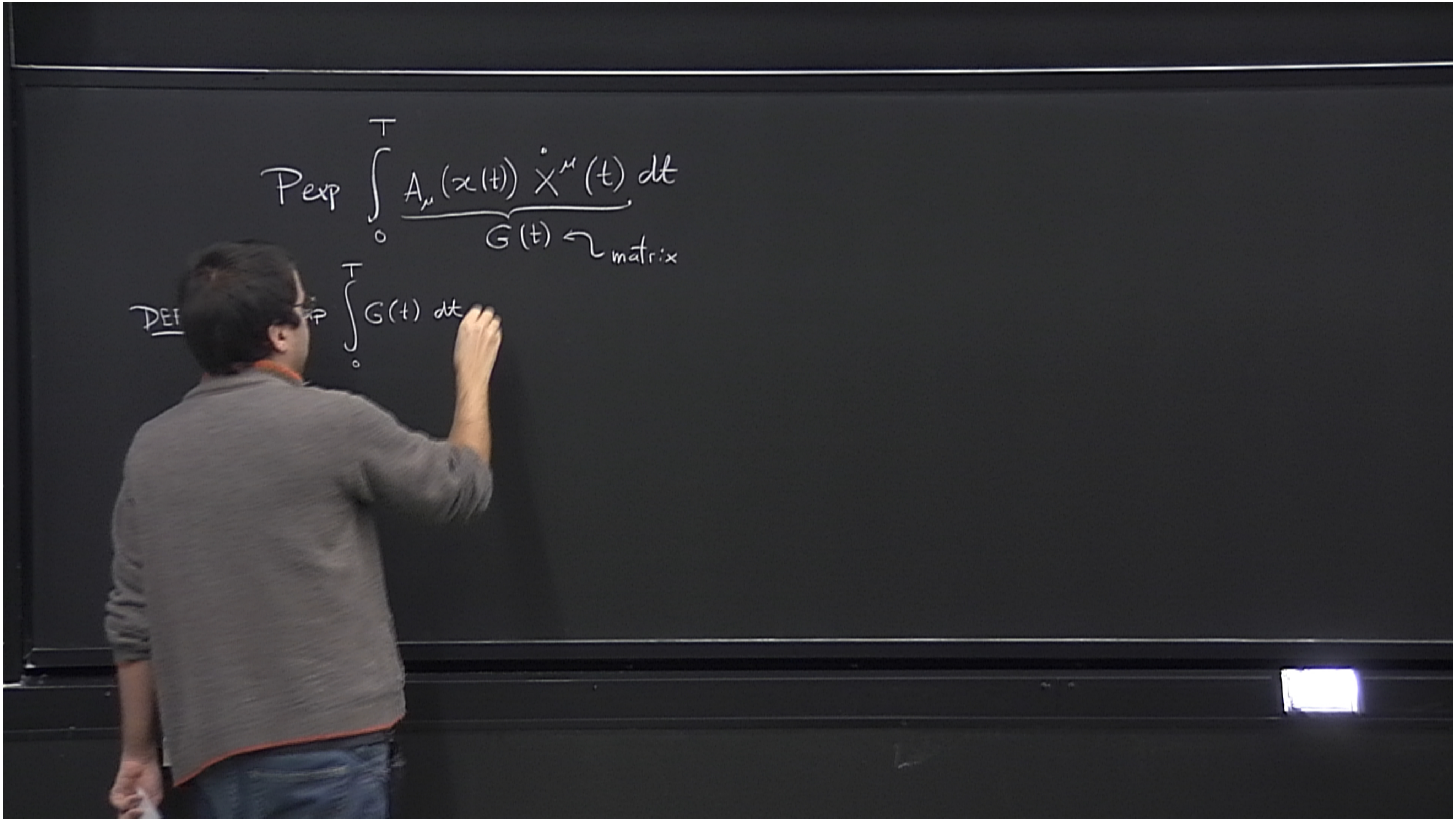
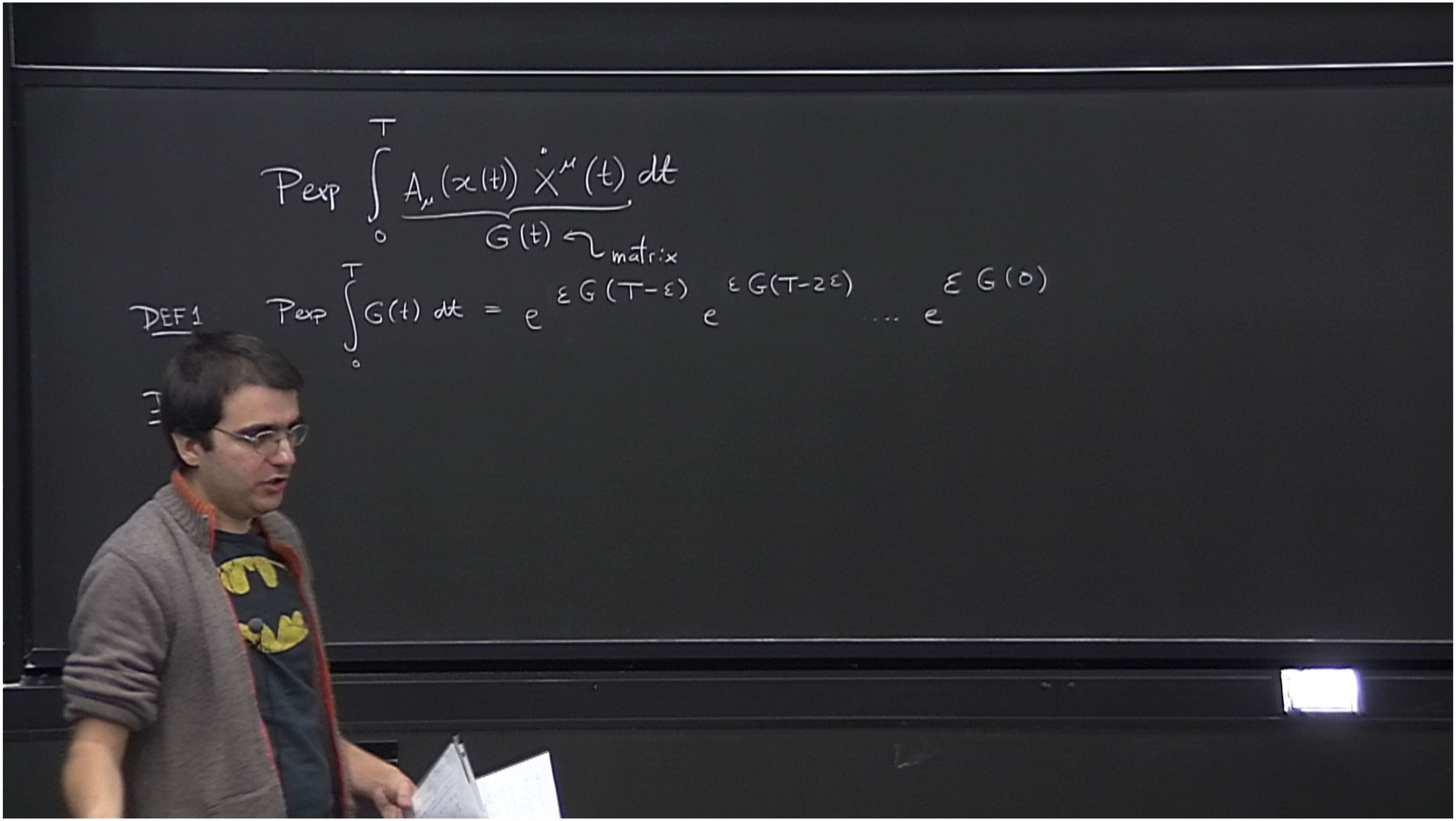
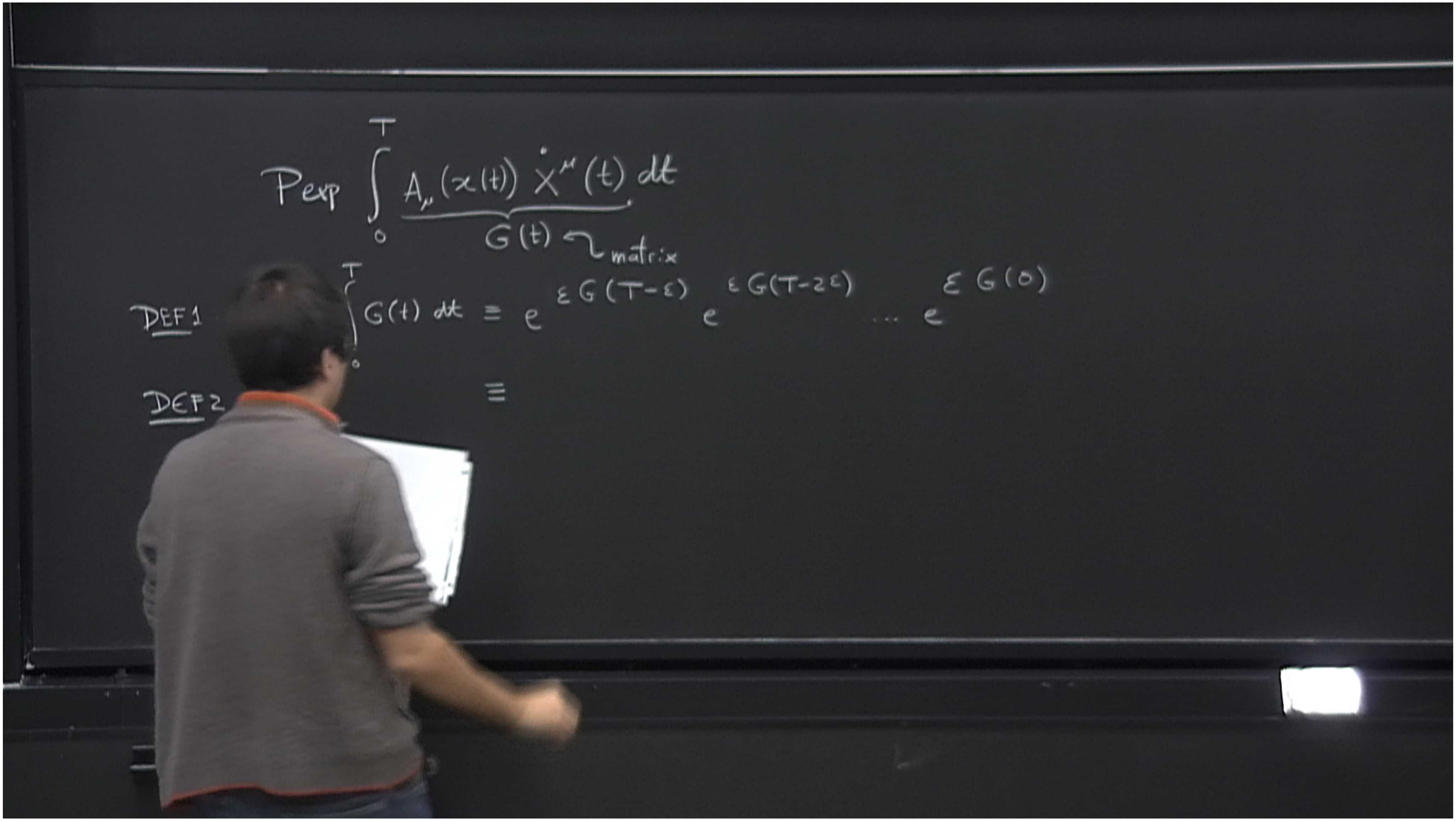
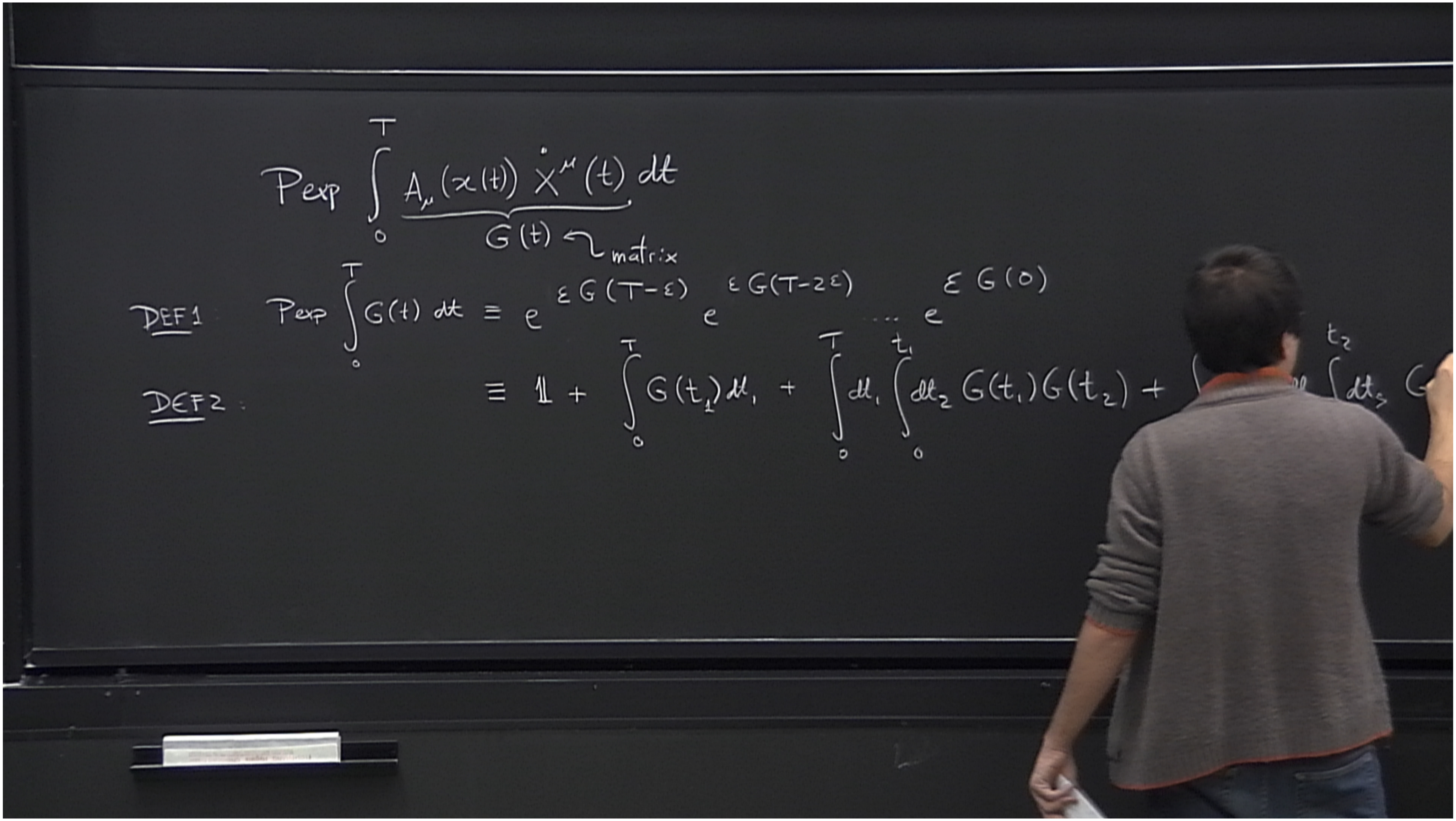
Δphase
We want better, alternative, defs on W
to compute ⟨W⟩

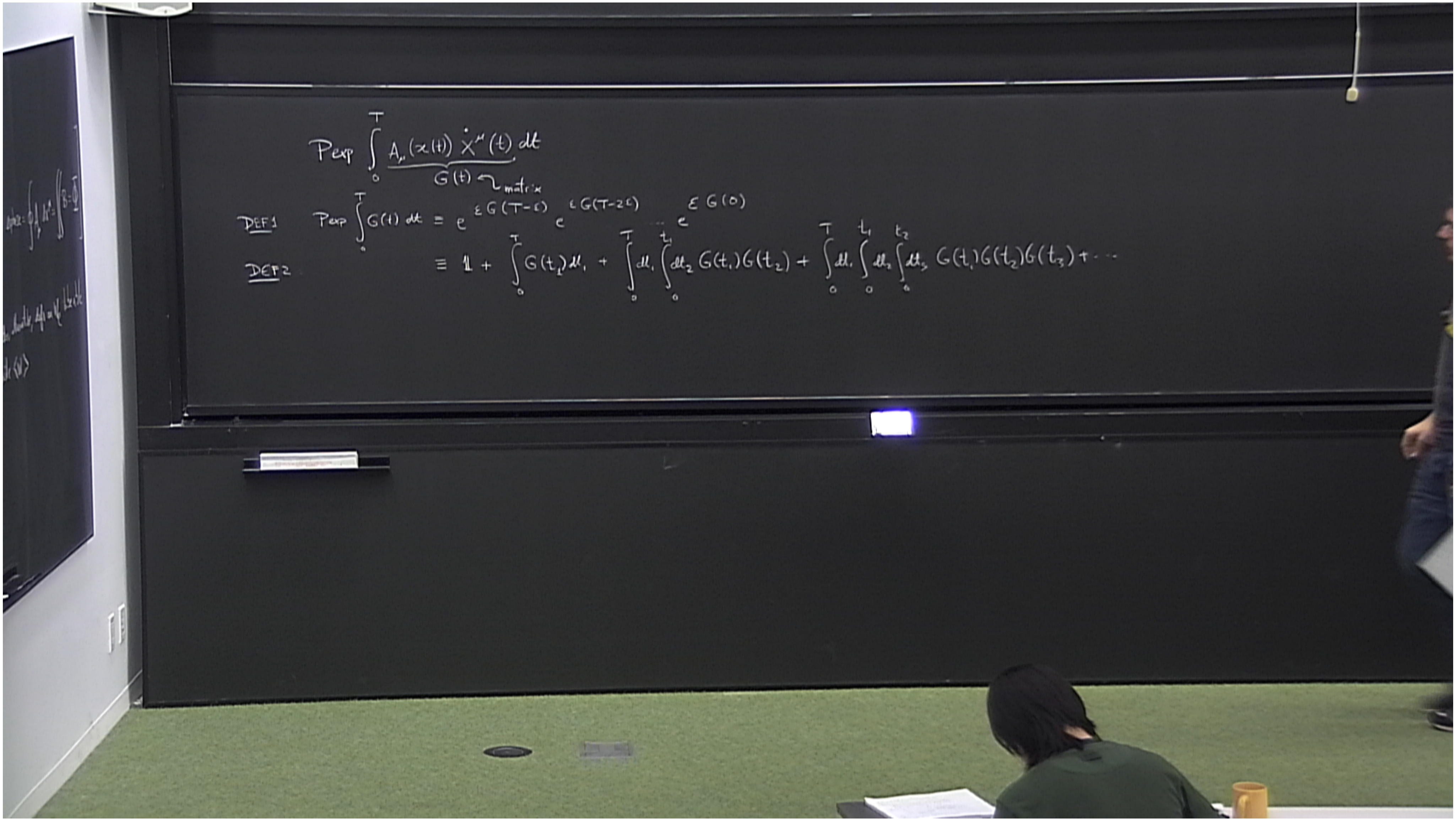

Pexp ∫₀ᵀ A_μ(x(t)) Ẋ^μ(t) dt
G(t) ← matrix
Pexp ∫₀ᵀ G(t) dt

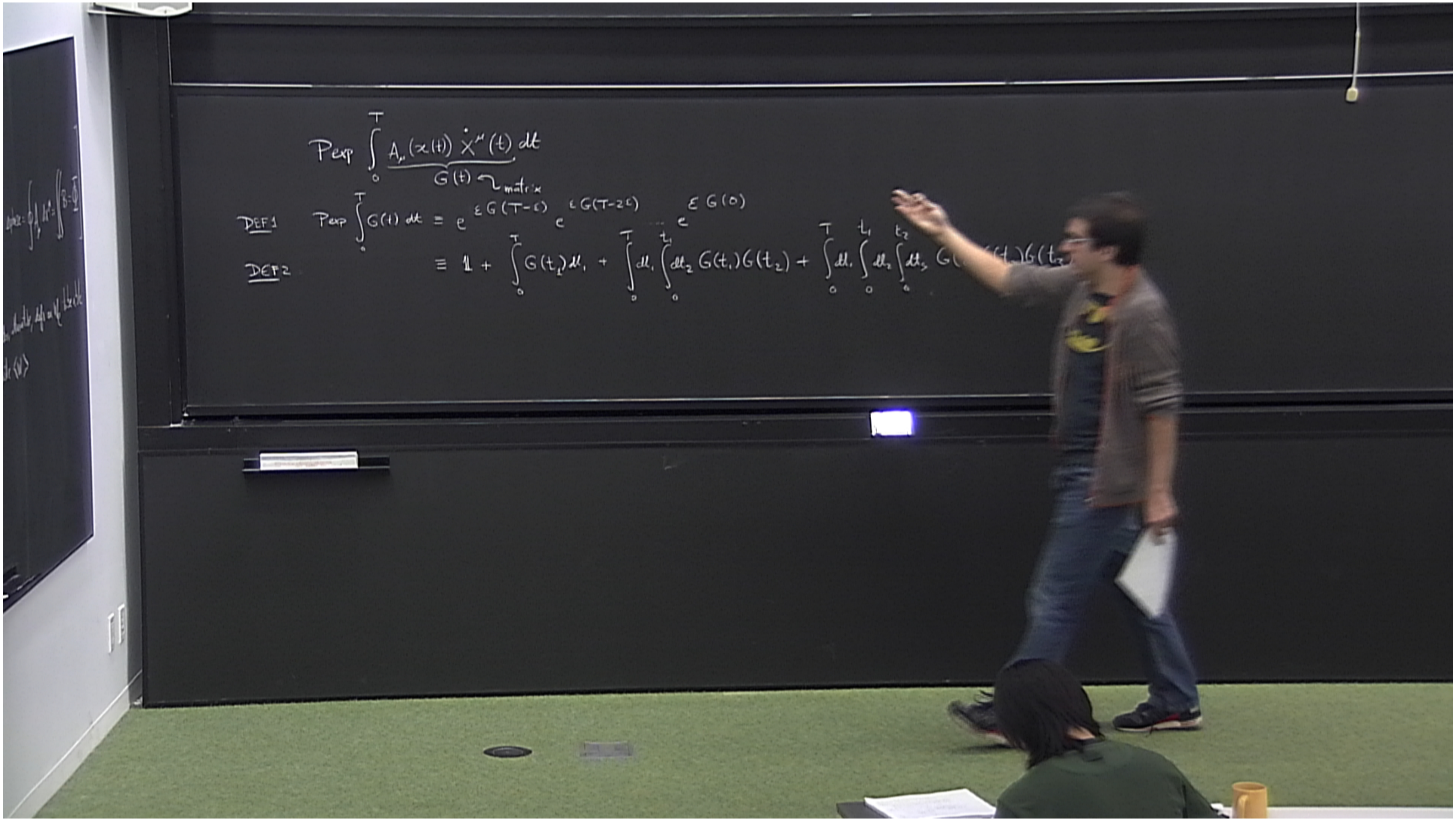

Pexp $\int_0^T A_\mu(x(t))\,\dot{x}^\mu(t)\,dt$
G(t) matrix
DEF
$\int_0^T G(t)\,dt$

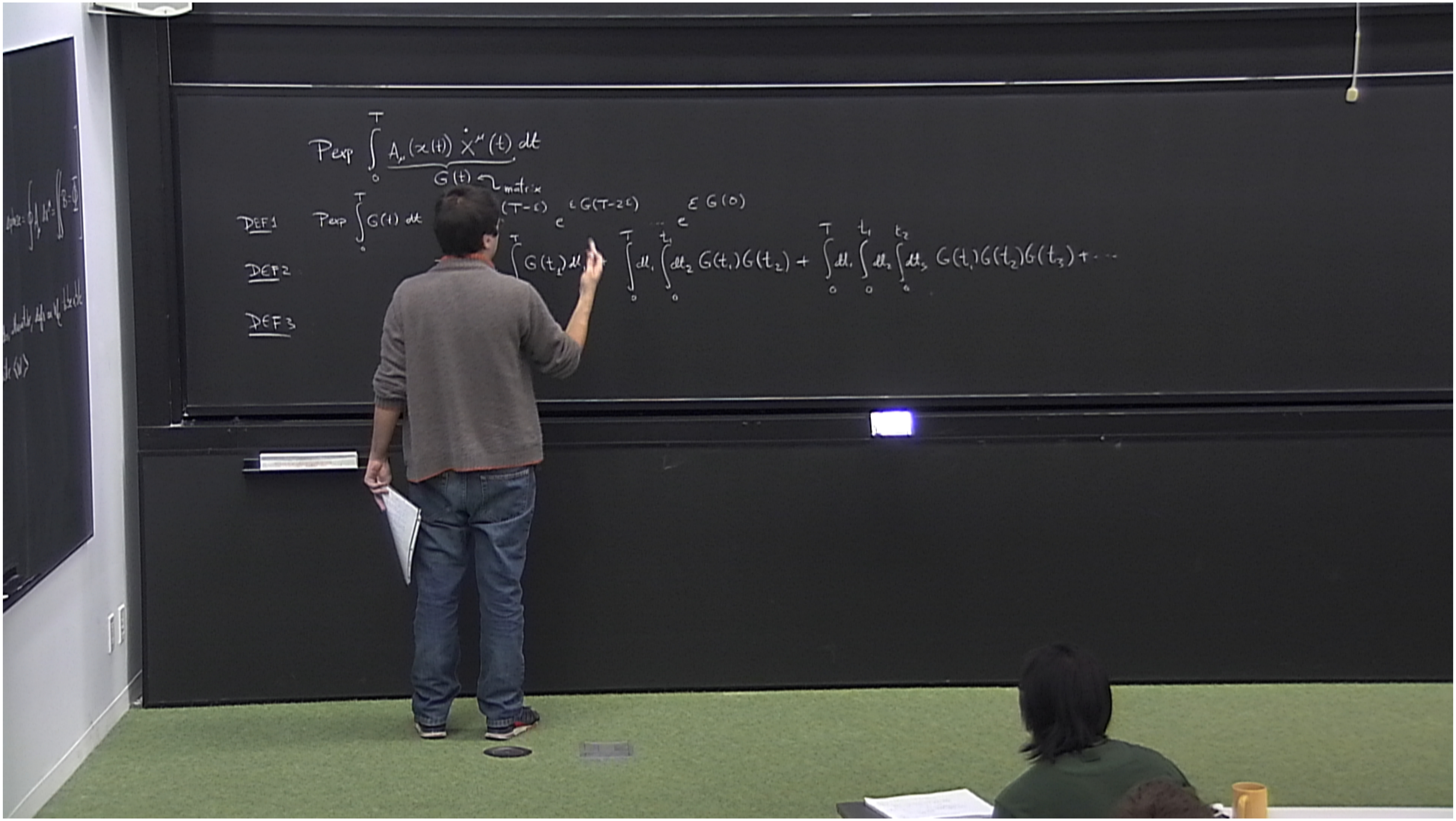

$\text{Pexp} \int_0^T \underbrace{A_\mu(x(t))\, \dot{x}^\mu(t)}_{G(t)} dt$
matrix
DEF 1
$\text{Pexp} \int_0^T G(t)\, dt = e^{\varepsilon G(T-\varepsilon)}\, e^{\varepsilon G(T-2\varepsilon)} \cdots e^{\varepsilon G(0)}$

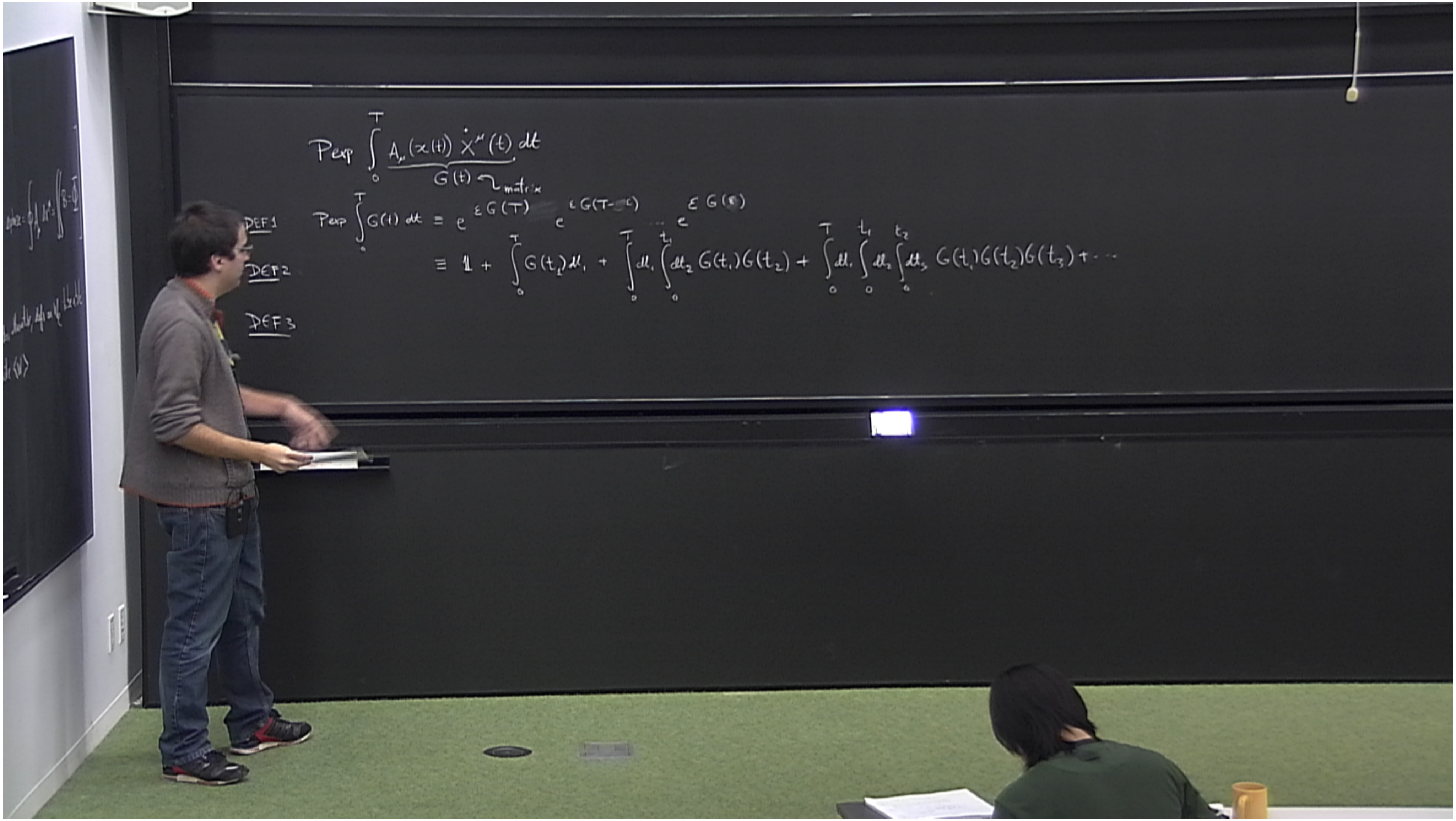

$P\exp \int_0^T A_\mu(x(t))\, \dot{x}^\mu(t)\, dt$
$G(t)$
matrix
DEF 1
$\int_0^T G(t)\, dt \equiv e^{\varepsilon G(T-\varepsilon)}\, e^{\varepsilon G(T-2\varepsilon)} \cdots e^{\varepsilon G(0)}$
DEF 2
$\equiv$

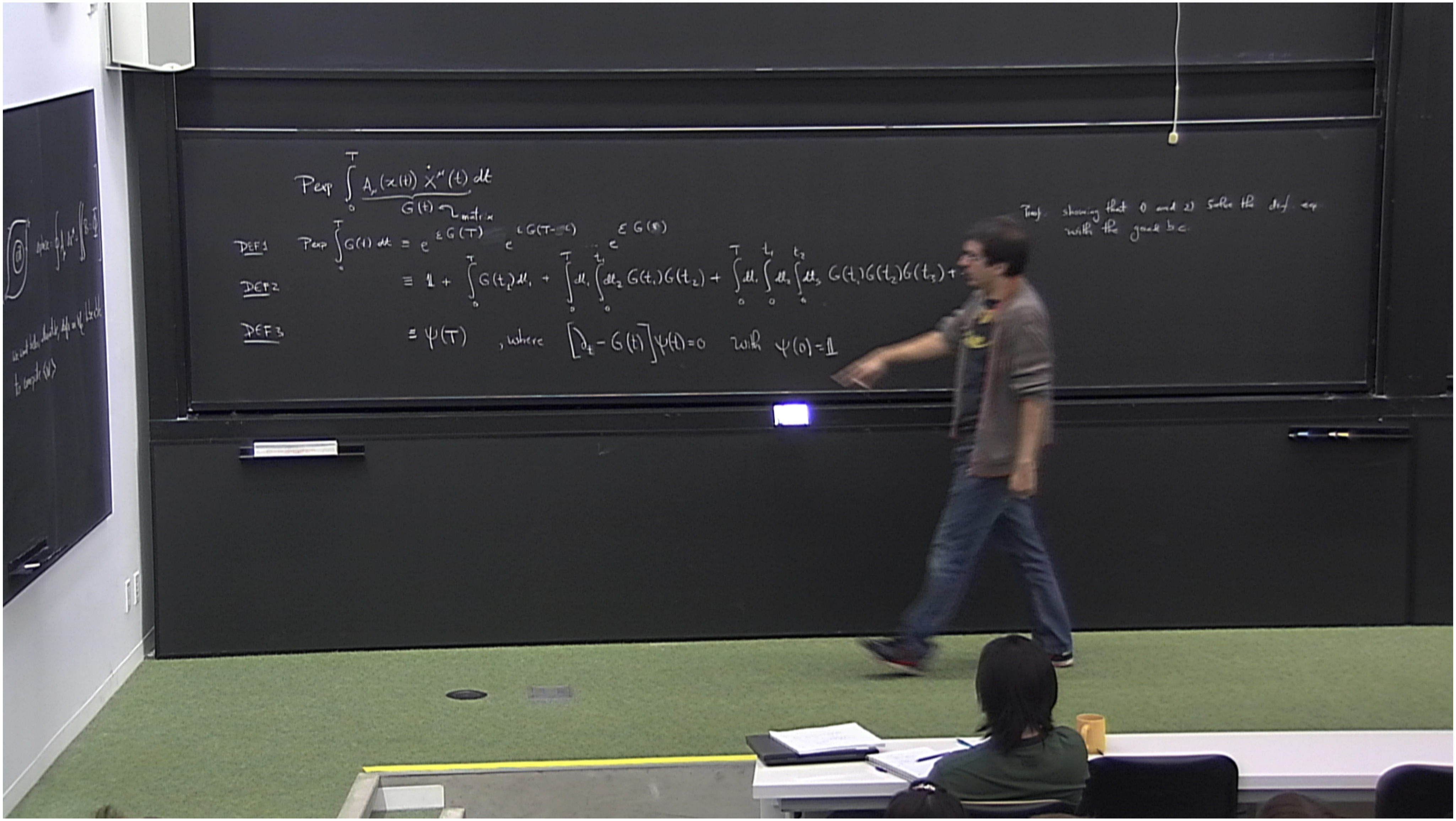

$P\exp \int_0^T A_\mu(x(t))\, \dot{x}^\mu(t)\, dt$
$G(t)$ matrix
DEF 1 $P\exp \int_0^T G(t)\, dt \equiv e^{\varepsilon G(T-\varepsilon)} e^{\varepsilon G(T-2\varepsilon)} \cdots e^{\varepsilon G(0)}$
DEF 2 : $\equiv \mathbb{1} + \int_0^T G(t_1)\, dt_1 + \int_0^T dt_1 \int_0^{t_1} dt_2\, G(t_1) G(t_2) +$

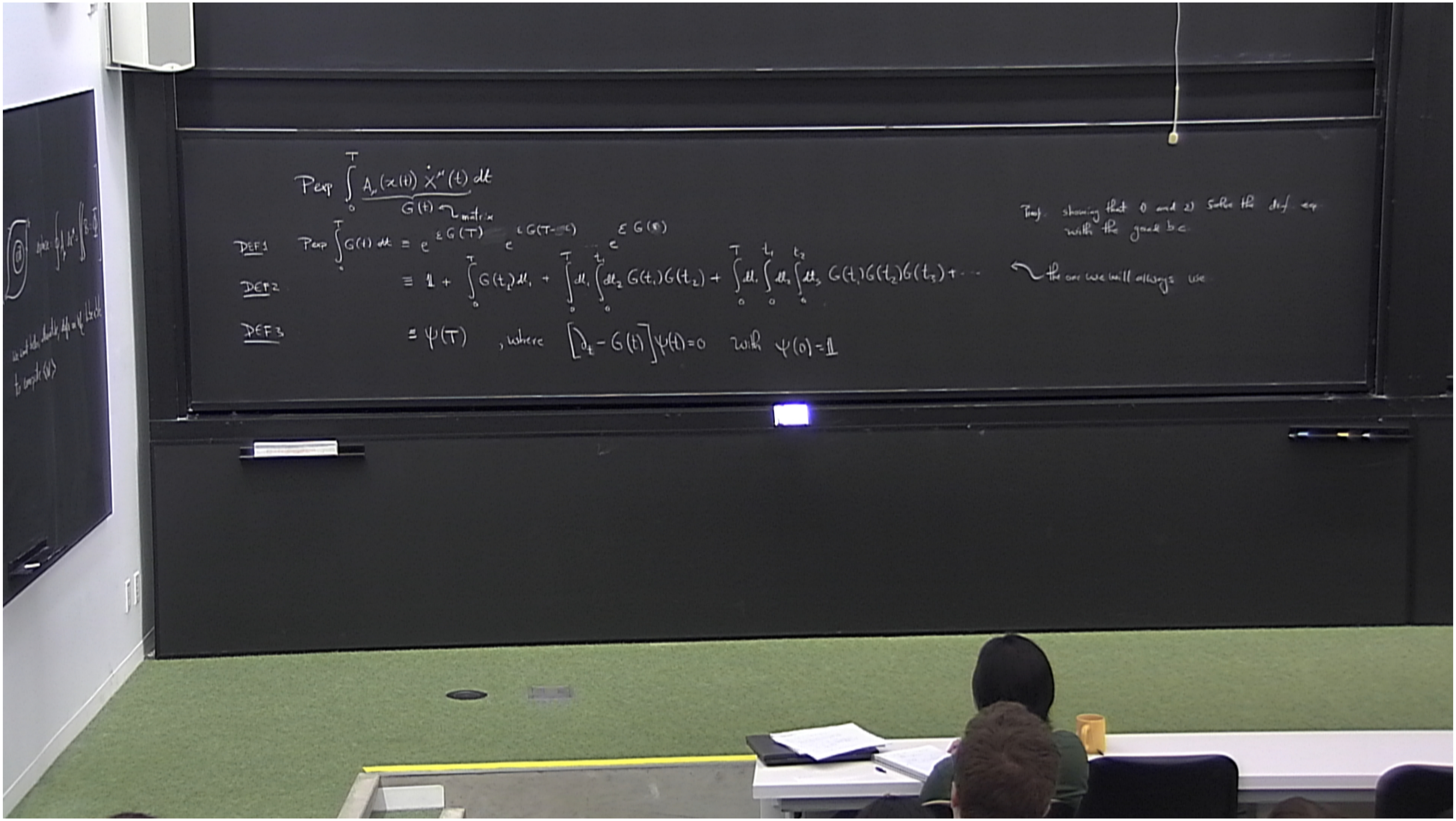

Pexp ∫_0^T A_μ(x(t)) Ẋ^μ(t) dt
G(t) matrix
DEF1 Pexp ∫_0^T G(t) dt = e^{εG(T−ε)} e^{εG(T−2ε)} ... e^{εG(0)}
DEF2 ≡ 1 + ∫_0^T G(t_1) dt_1 + ∫_0^T dt_1 ∫_0^{t_1} dt_2 G(t_1)G(t_2) + ∫_0^T dt_1 ∫_0^{t_1} dt_2 ∫_0^{t_2} dt_3 G(t_1)G(t_2)G(t_3) + ...

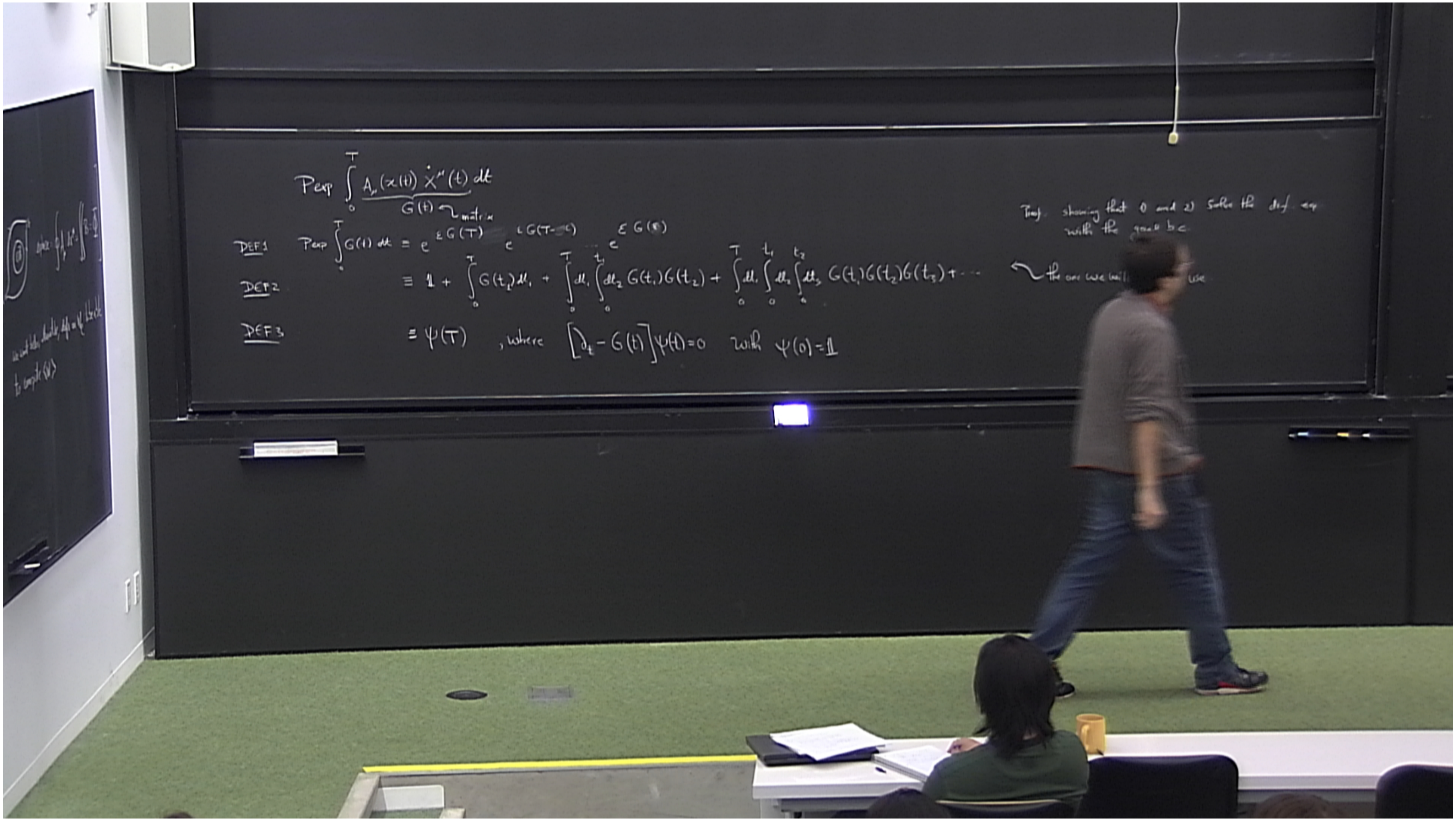

Pexp ∫ A_μ(x(t)) Ẋ^μ(t) dt
G(t) matrix
DEF 1
Pexp ∫ G(t) dt = e^{εG(T−ε)} e^{εG(T−2ε)} ... e^{εG(0)}
DEF 2
≡ 1 + ∫ G(t_1) dt_1 + ∫ dt_1 ∫ dt_2 G(t_1) G(t_2) + ∫ dt_1 ∫ dt_2 ∫ dt_3


Pexp ∫ A_μ(x(t)) ẋ^μ(t) dt
G(t) matrix
DEF 1 Pexp ∫ G(t) dt
DEF 2
DEF 3

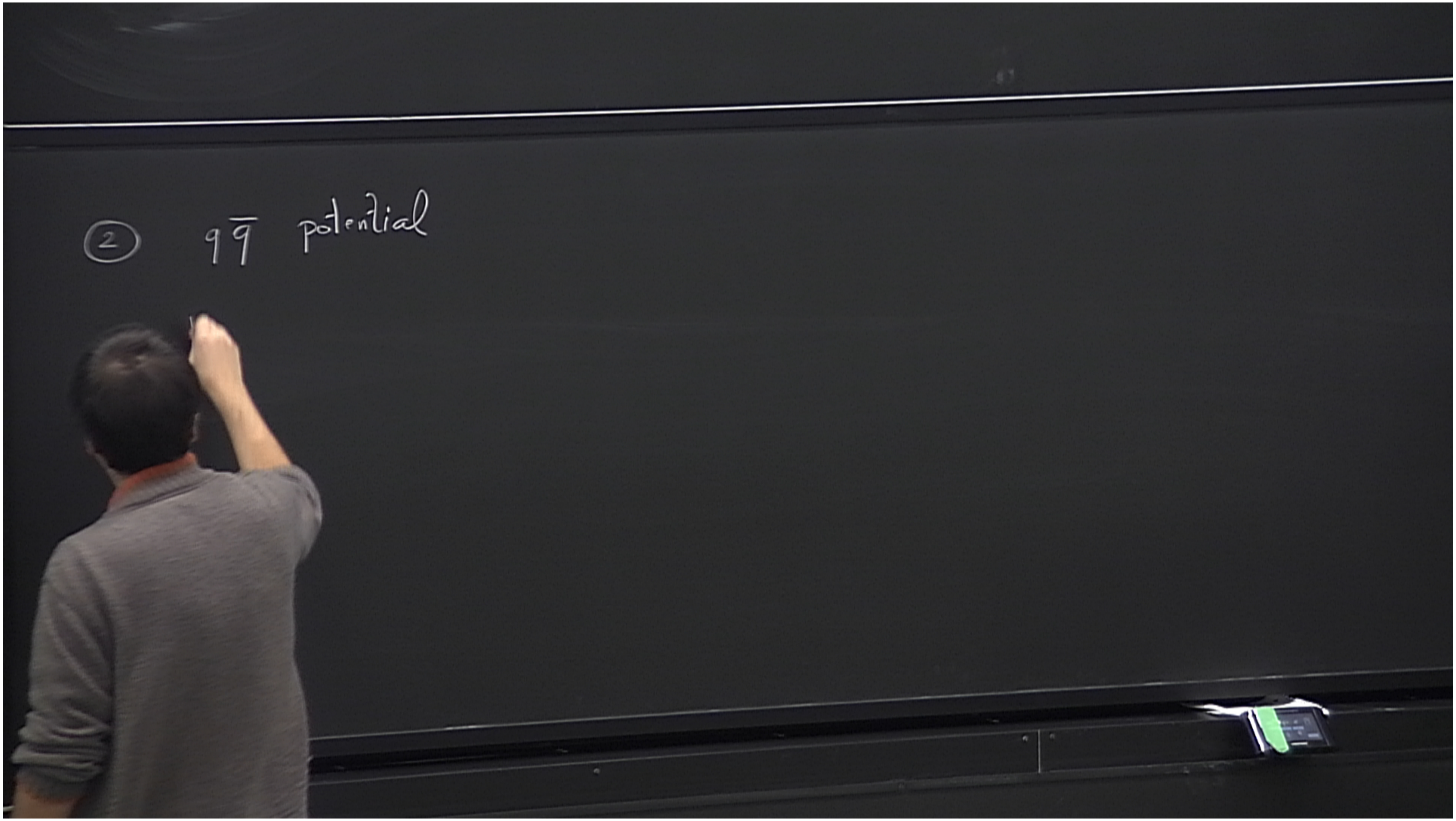

Pexp $\int_0^T A_\mu(x(t))\,\dot{x}^\mu(t)\,dt$
G(t) ~ matrix
DEF1
Pexp $\int_0^T G(t)\,dt = e^{\varepsilon G(T)}\, e^{\varepsilon G(T-}\cdots e^{\varepsilon G(}$
DEF2
$\equiv \mathbb{1} + \int_0^T G(t_1)\,dt_1 + \int_0^T dt_1 \int_0^{t_1} dt_2\, G(t_1)G(t_2) + \int_0^T dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3\, G(t_1)G(t_2)G(t_3) + \cdots$
DEF3

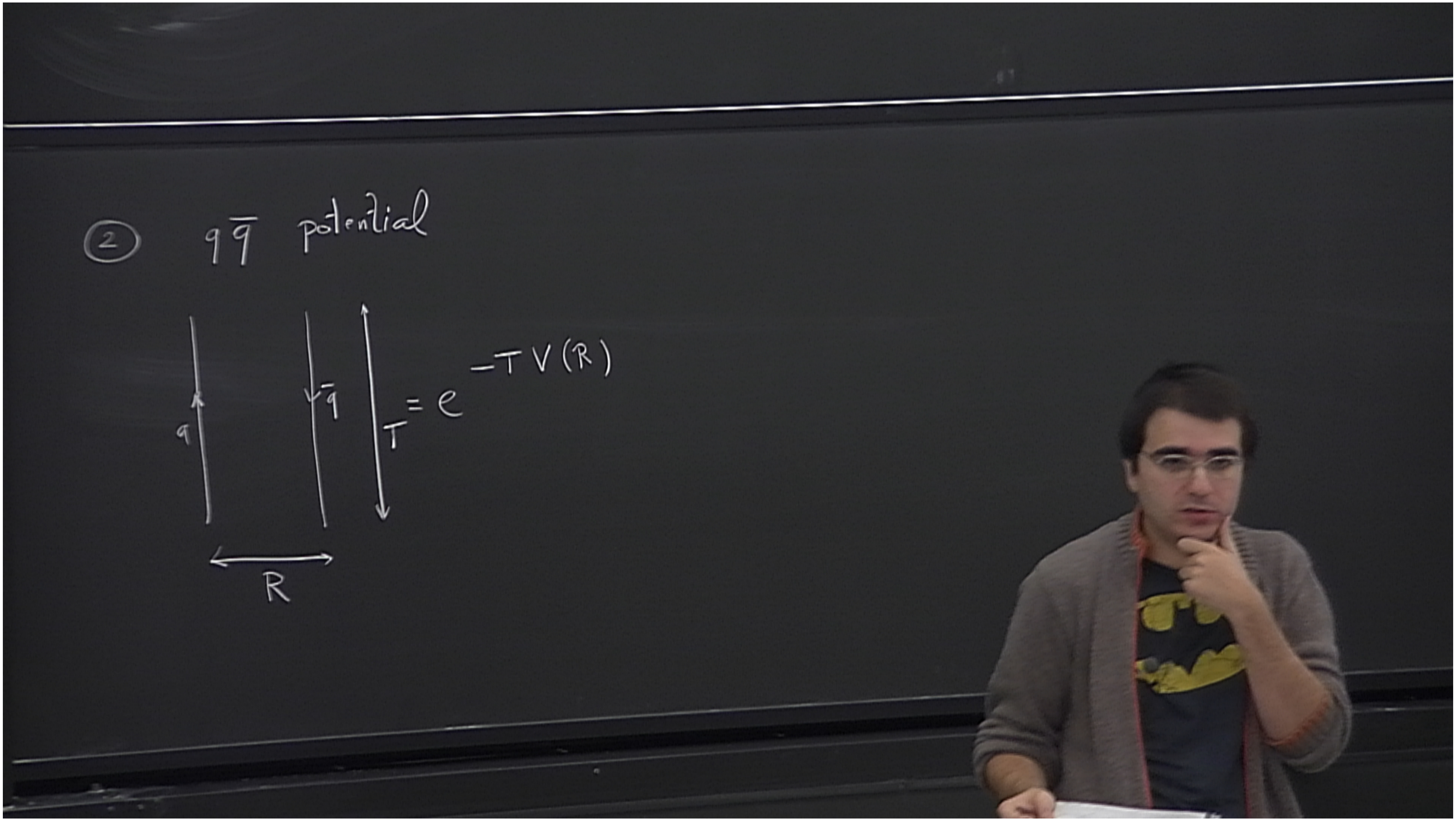

Pexp ∫ A_μ(x(t)) ẋ^μ(t) dt
G(t) ~ matrix
DEF 1 Pexp ∫ G(t) dt = e^{ε G(T)} e^{ε G(T−ε)} ... e^{ε G(ε)}
DEF 2 ≡ 𝟙 + ∫ G(t₁) dt₁ + ∫ dt₁ ∫ dt₂ G(t₁) G(t₂) + ∫ dt₁ ∫ dt₂ ∫ dt₃ G(t₁) G(t₂) G(t₃) + ...
DEF 3 ≡ ψ(T) , where [∂_t − G(t)] ψ(t) = 0 with ψ(0) = 𝟙
Proof: showing that 1) and 2) solve the dif. eq with the good b.c.

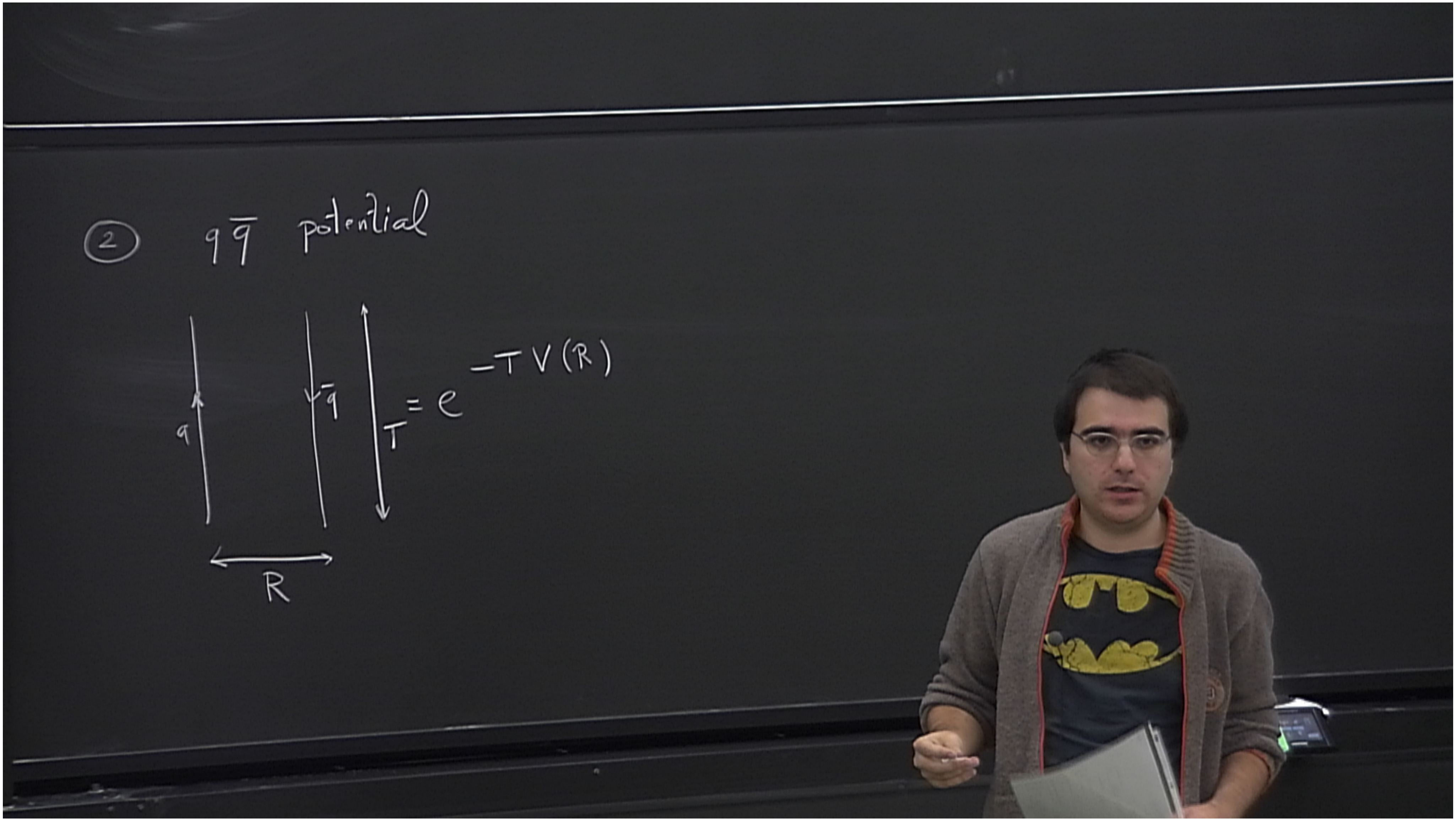

Pexp ∫ A_μ(x(t)) ẋ^μ(t) dt
G(t) ~ matrix
DEF 1 Pexp ∫ G(t) dt = e^{εG(T)} e^{εG(T−ε)} ... e^{εG(ε)}
DEF 2 ≡ 1 + ∫ G(t₁) dt₁ + ∫ dt₁ ∫ dt₂ G(t₁)G(t₂) + ∫ dt₁ ∫ dt₂ ∫ dt₃ G(t₁)G(t₂)G(t₃) + ...
the one we will always use
DEF 3 ≡ ψ(T) , where [∂_t − G(t)]ψ(t) = 0 with ψ(0) = 1
Proof: showing that 1) and 2) solve the dif. eq with the good b.c.


Pexp
G(t)
matrix
DEF 1
DEF 2
DEF 3
, where
with
the one we will use

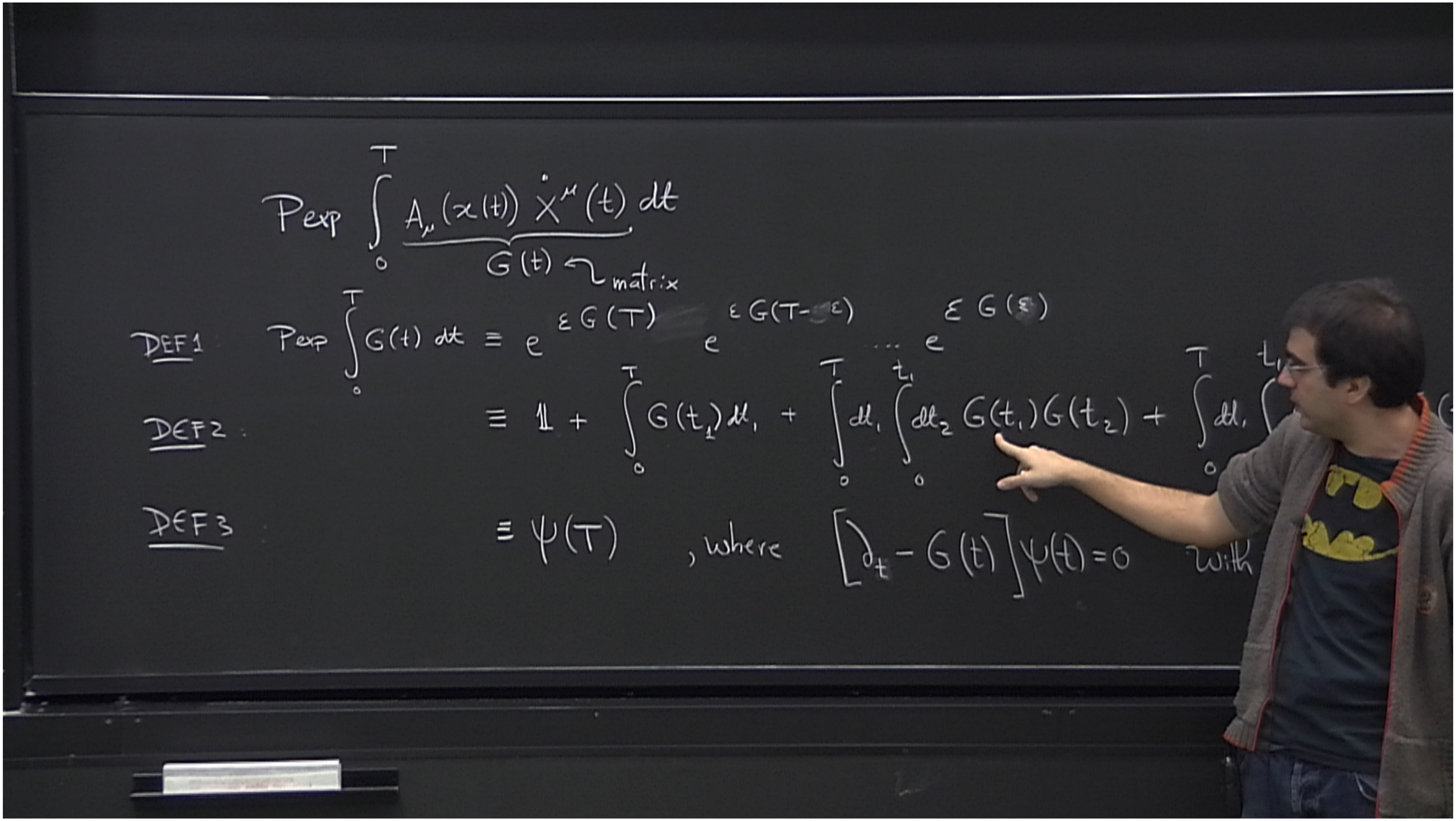
(2) $q\bar{q}$ potential


② $q\bar{q}$ potential

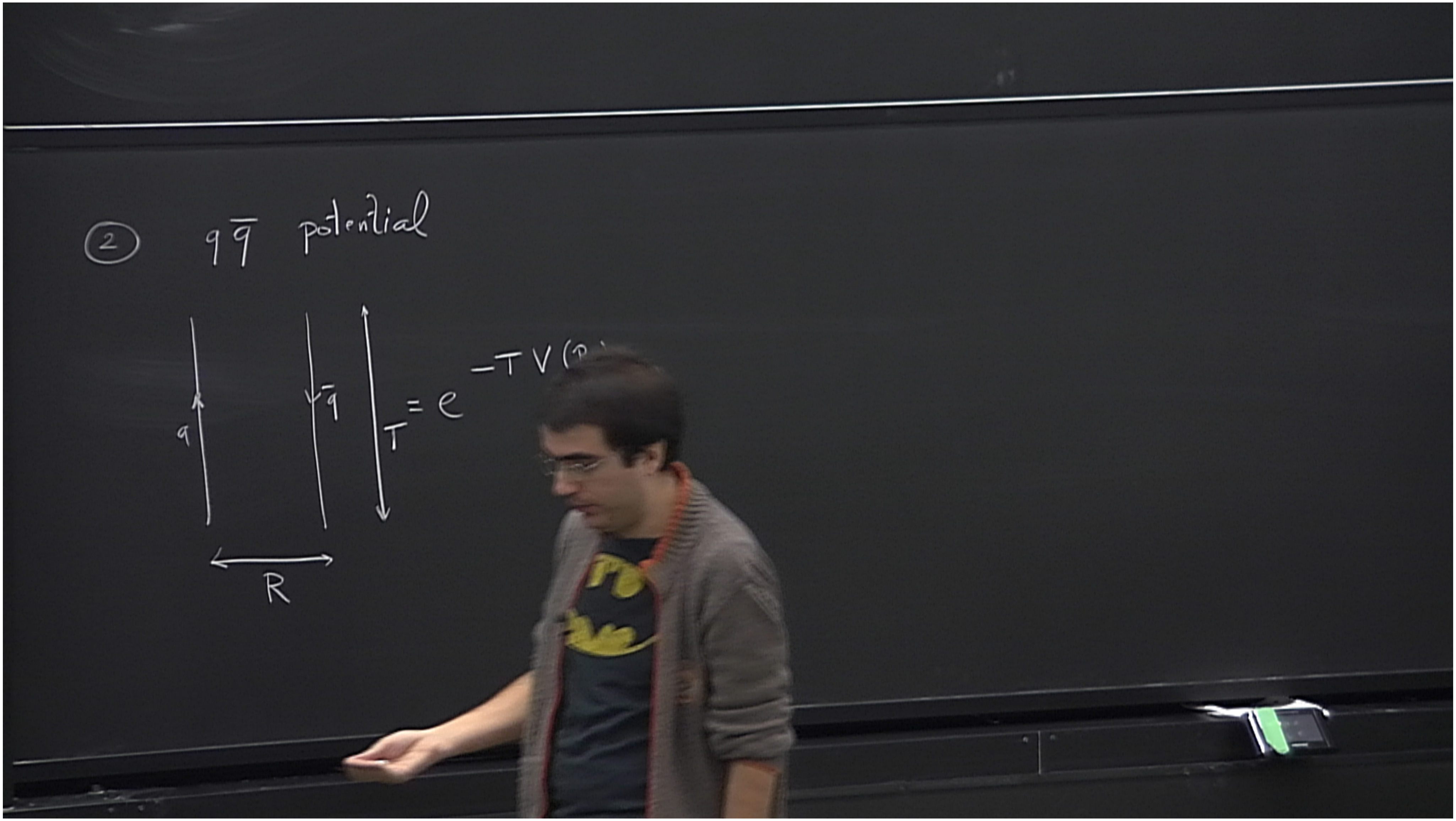

② q q̄ potential
q
q̄
T
= e^{−T V(R)}
R

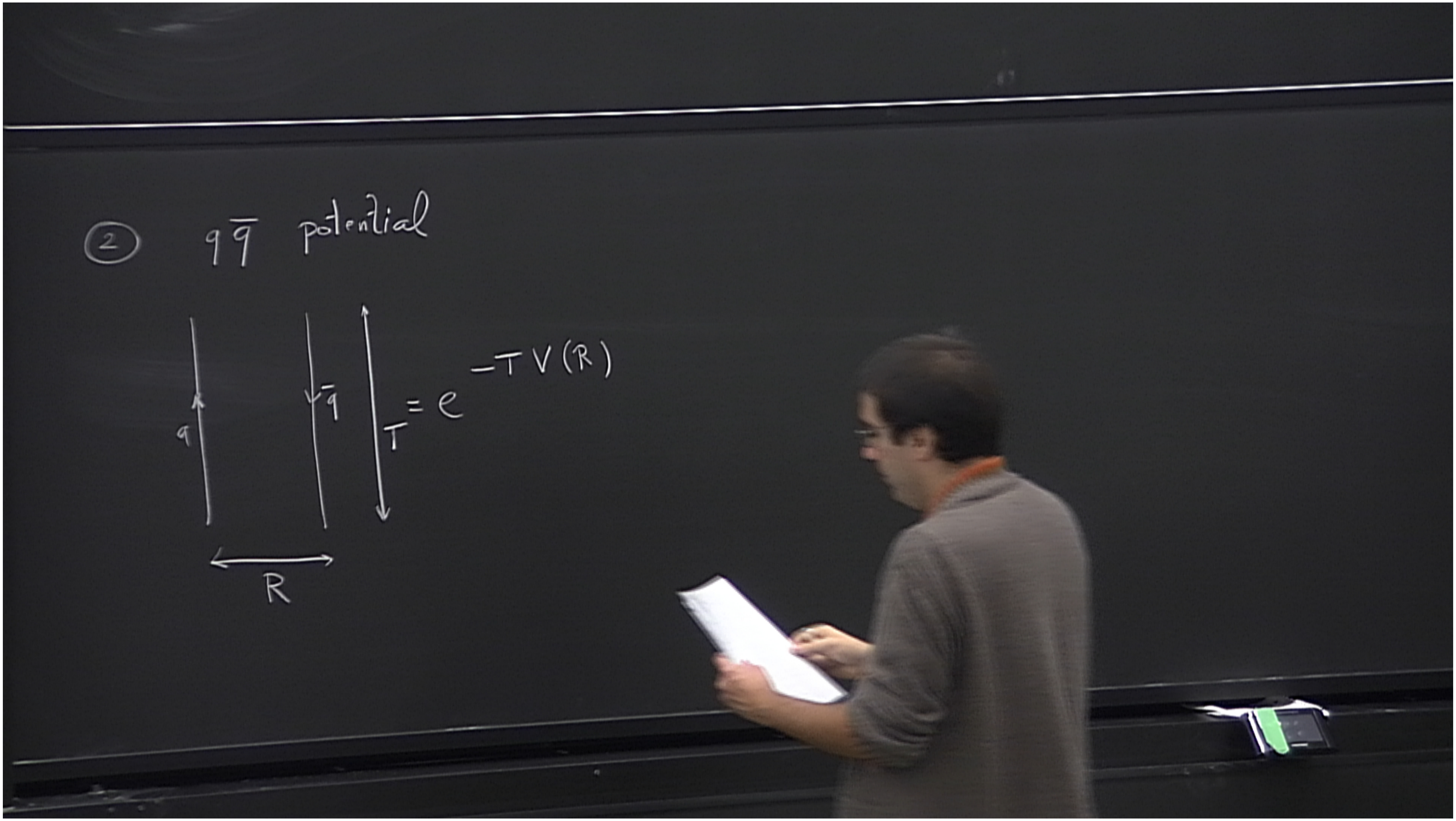


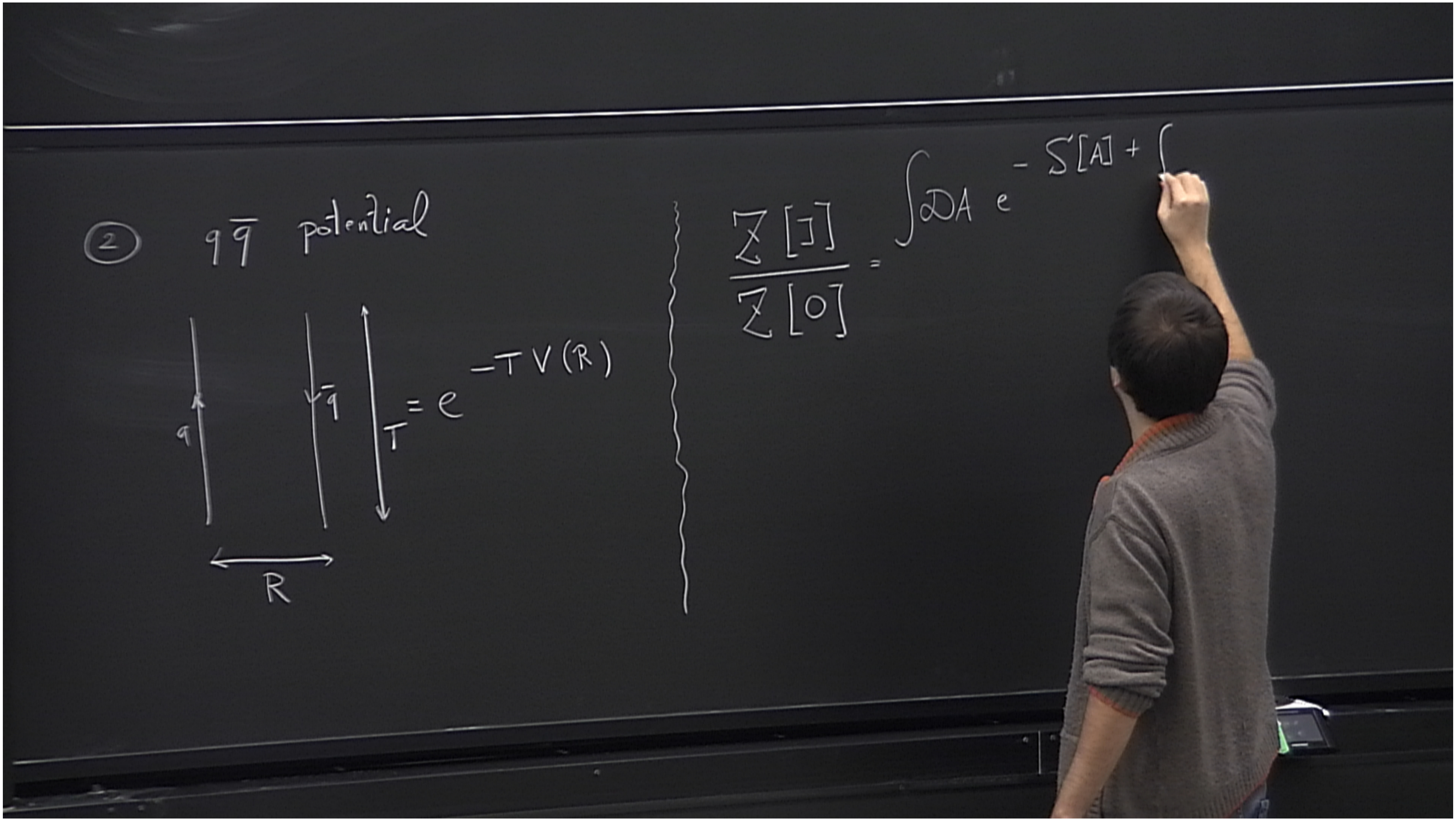
② q q̄ potential
q
q̄
T
= e^{-TV(R)}
R

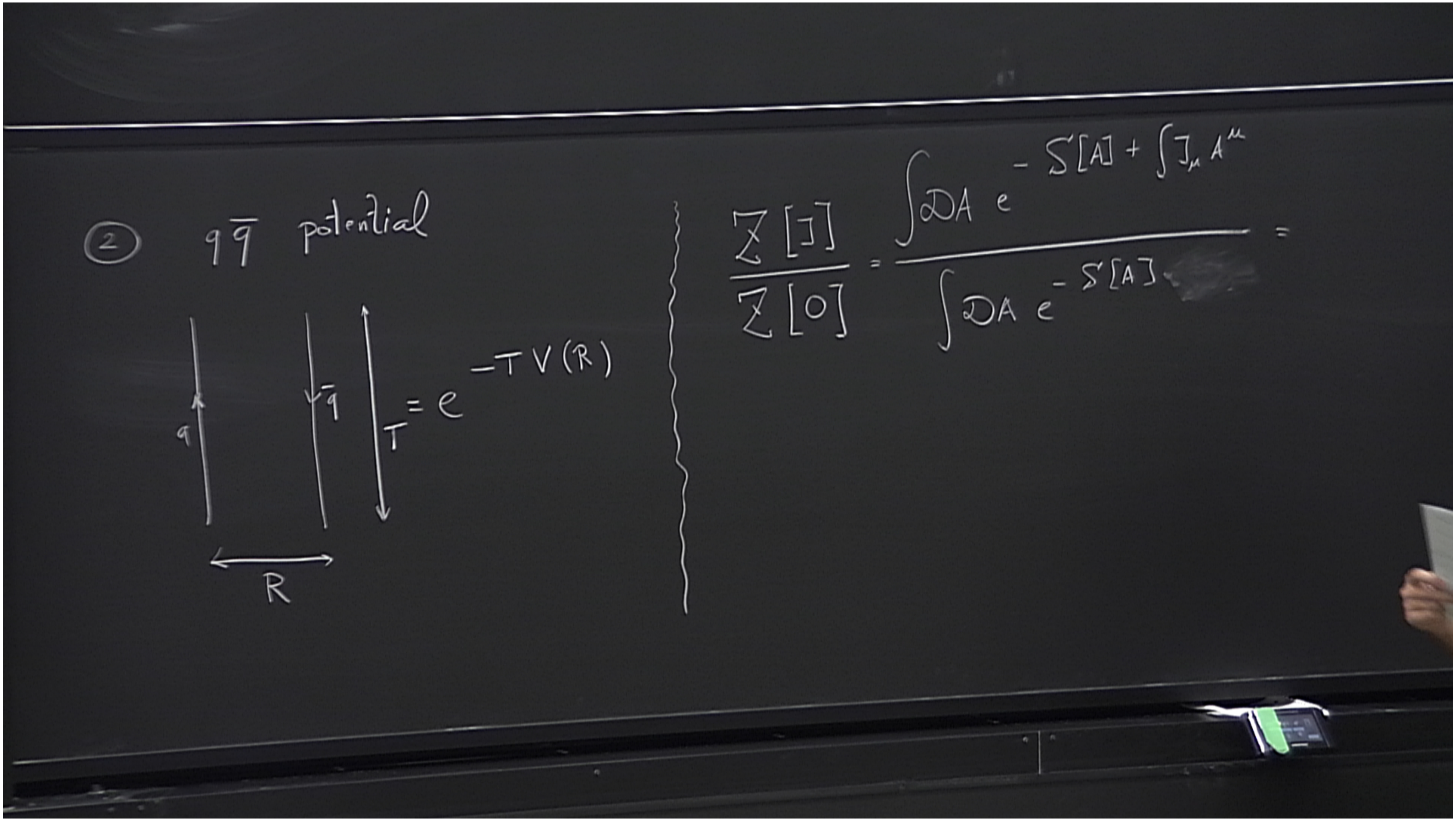


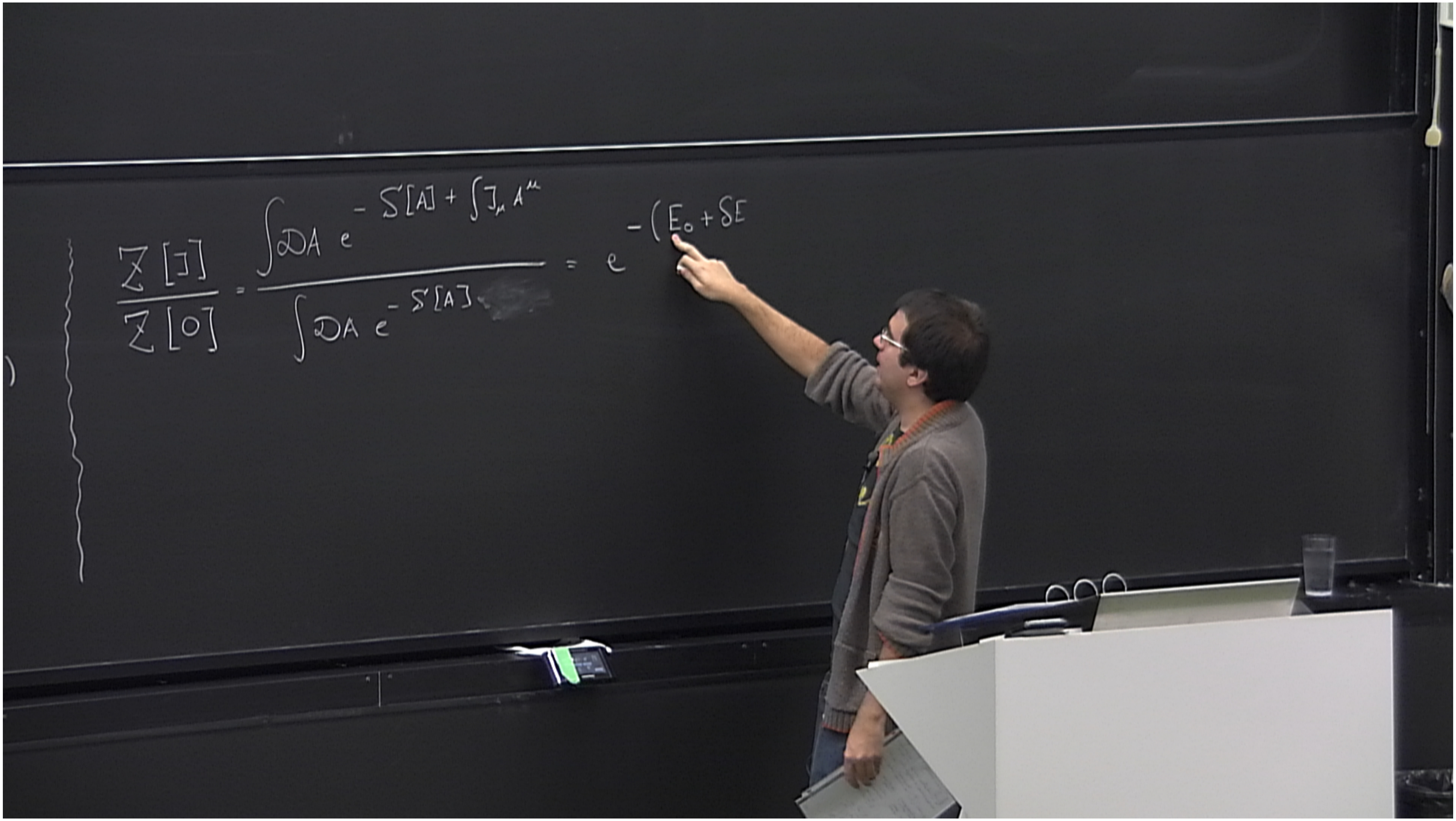
$\mathrm{Pexp} \int_0^T \underbrace{A_\mu(x(t))\, \dot{x}^\mu(t)}_{G(t)} dt$
$G(t) \leftarrow$ matrix
DEF 1 $\mathrm{Pexp} \int_0^T G(t)\, dt \equiv e^{\varepsilon G(T)}\, e^{\varepsilon G(T-\varepsilon)} \cdots e^{\varepsilon G(\varepsilon)}$
DEF 2 $\equiv \mathbb{1} + \int_0^T G(t_1)\, dt_1 + \int_0^T dt_1 \int_0^{t_1} dt_2\, G(t_1) G(t_2) + \int_0^T dt_1 \int^{t_1}$
DEF 3 $\equiv \psi(T)$, where $\left[\partial_t - G(t)\right] \psi(t) = 0$ with

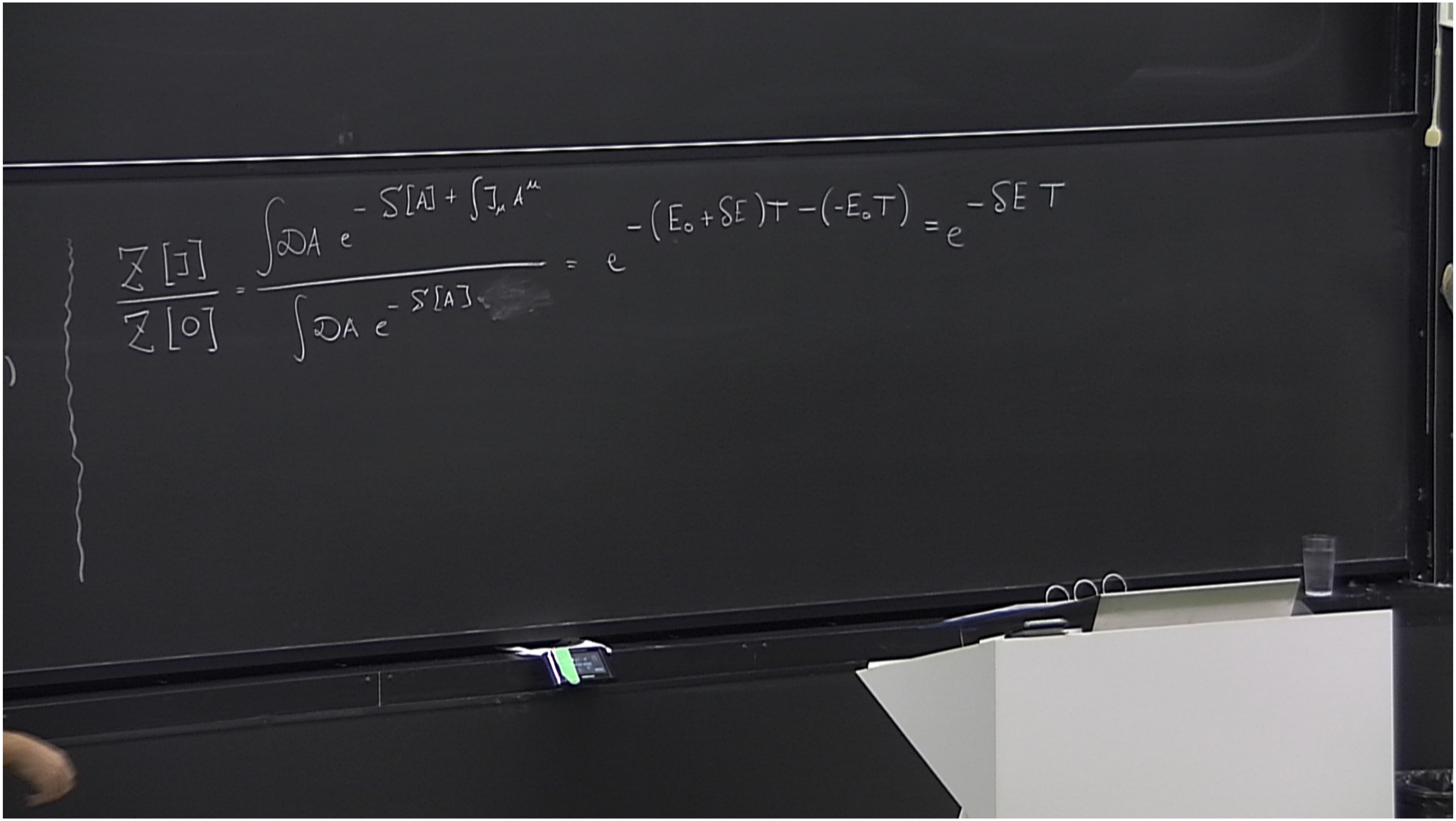

② $q\bar{q}$ potential
q
$\bar{q}$
T
$= e^{-TV(R)}$
R

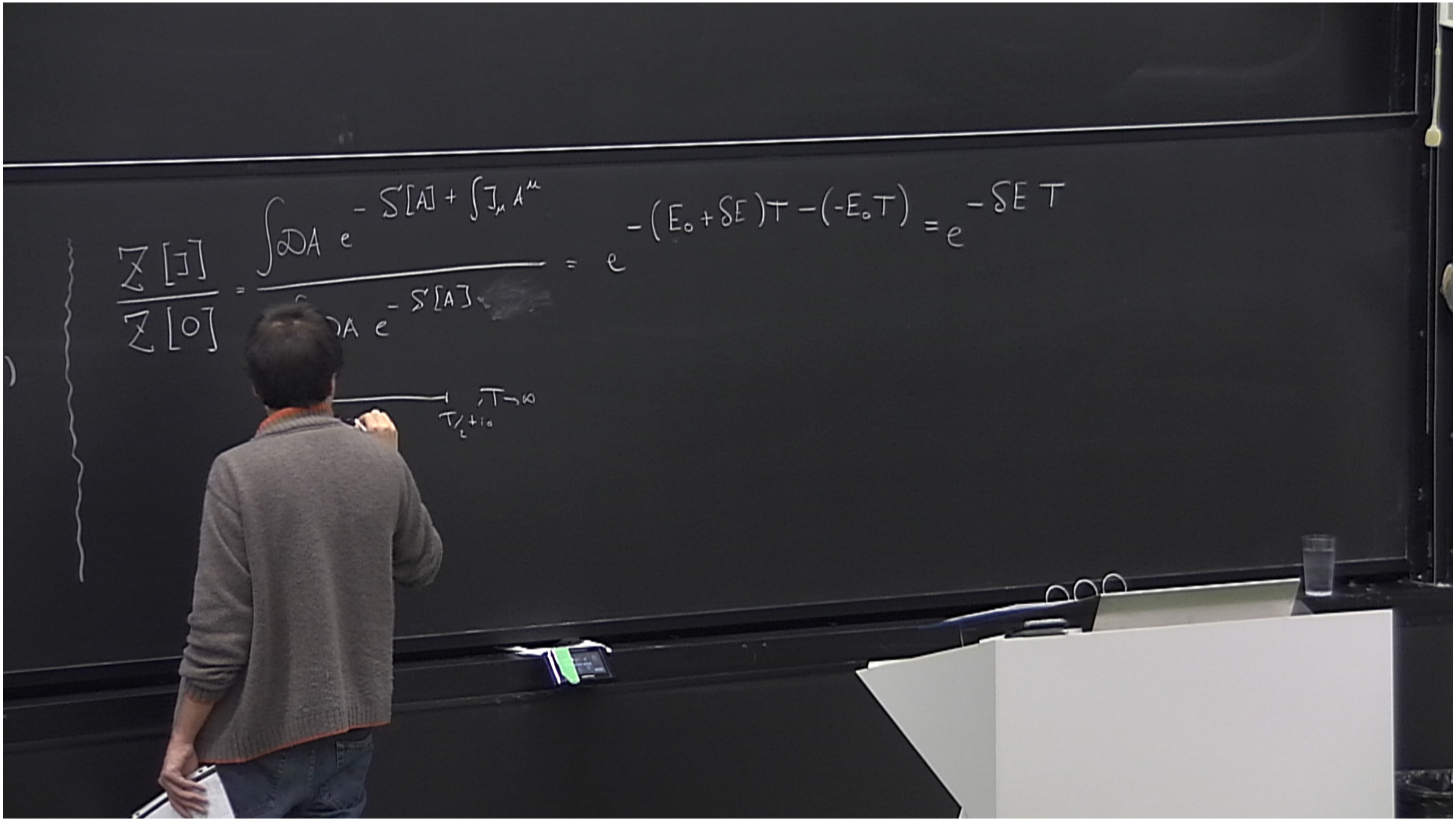

② q q̄ potential
q
q̄
T
= e^{−TV(R)}
R

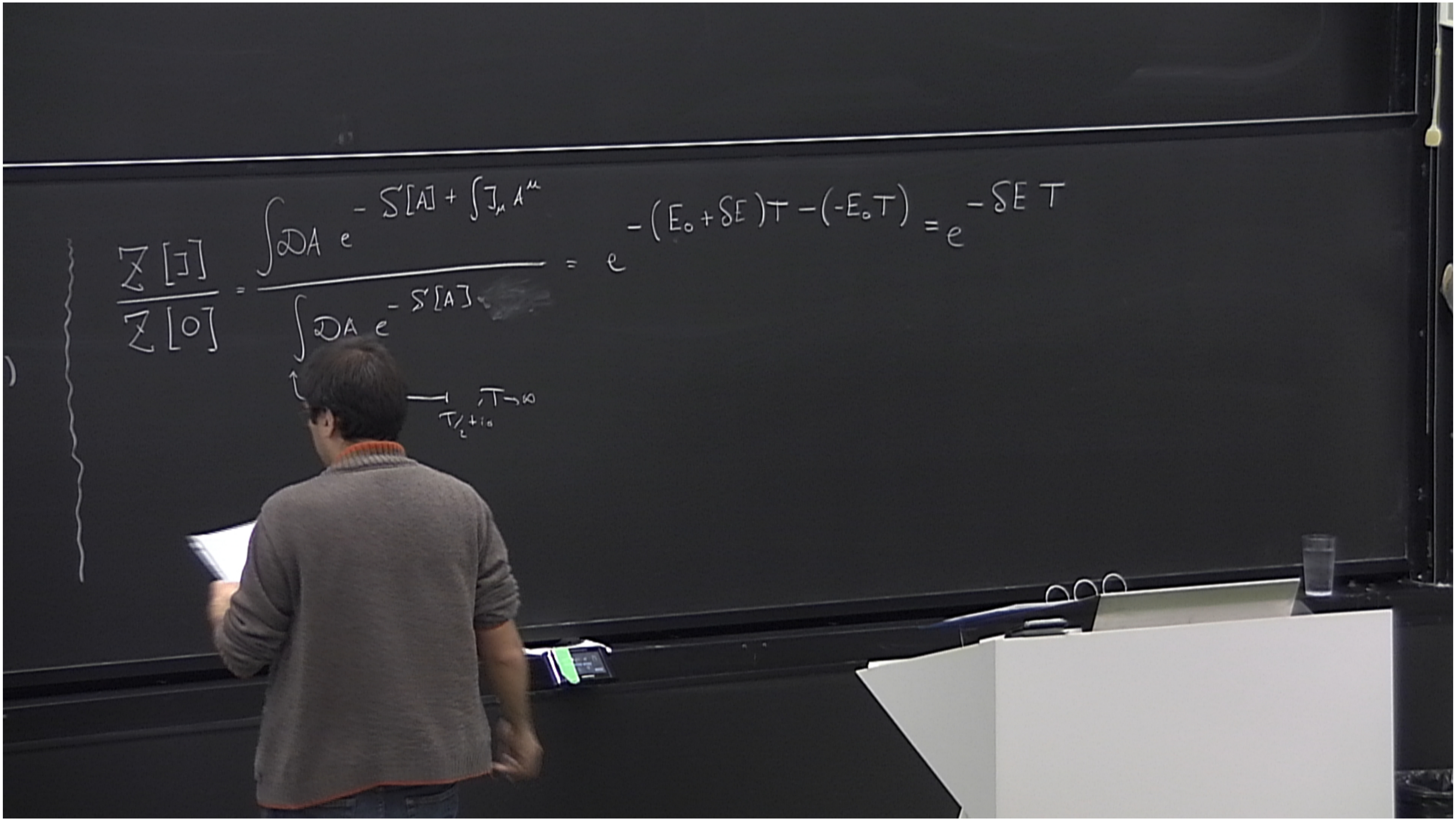

② q q̄ potential
q
q̄
T
R
= e^{-TV(R)}
Z[J] / Z[0] = ∫DA e^{- S[A] + ∫

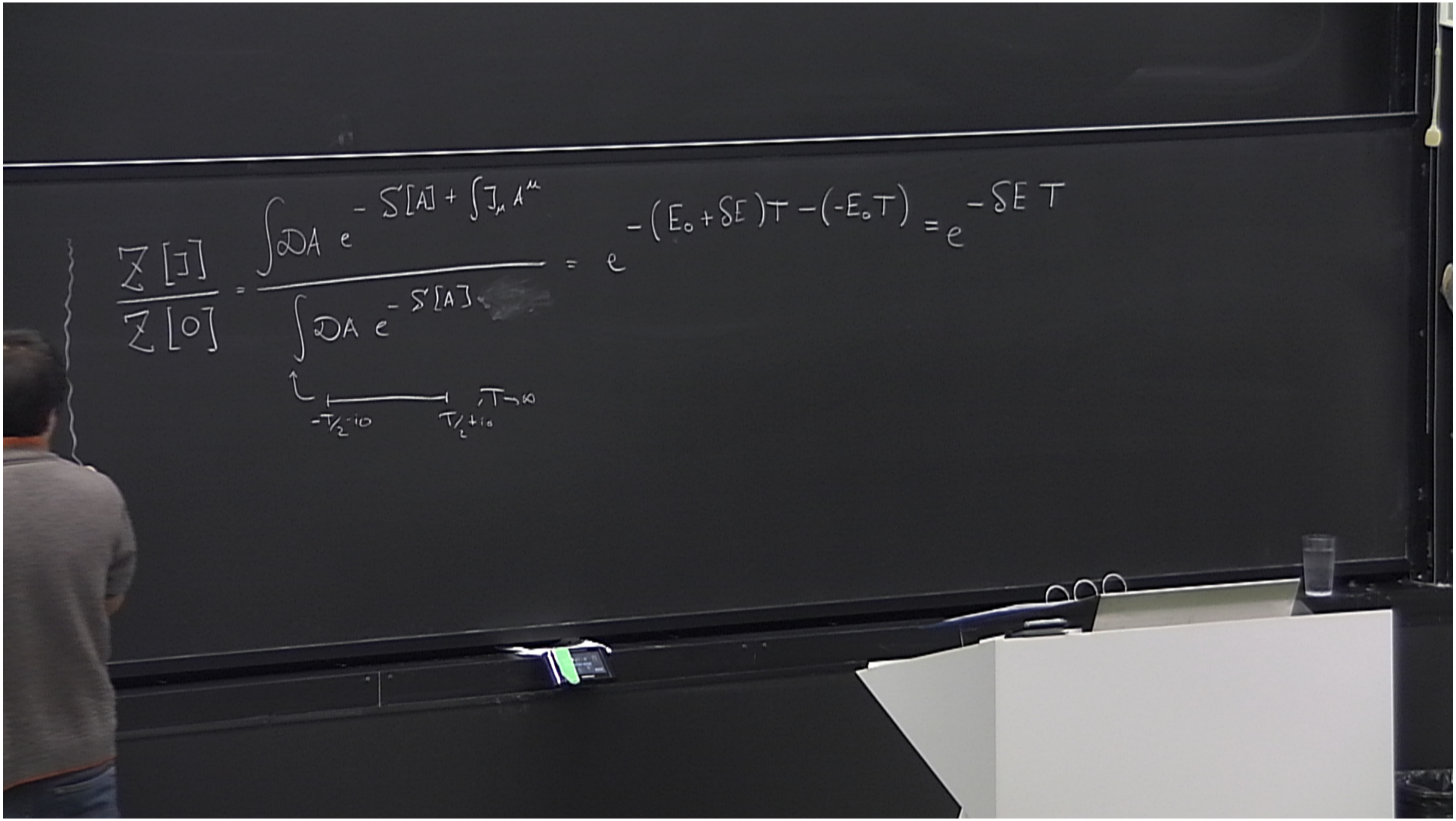

② $q\bar{q}$ potential
q
$\bar{q}$
T
R
$= e^{-TV(R)}$
$\frac{Z[J]}{Z[0]} = \frac{\int \mathcal{D}A \, e^{-S[A] + \int J_\mu A^\mu}}{\int \mathcal{D}A \, e^{-S[A]}} =$

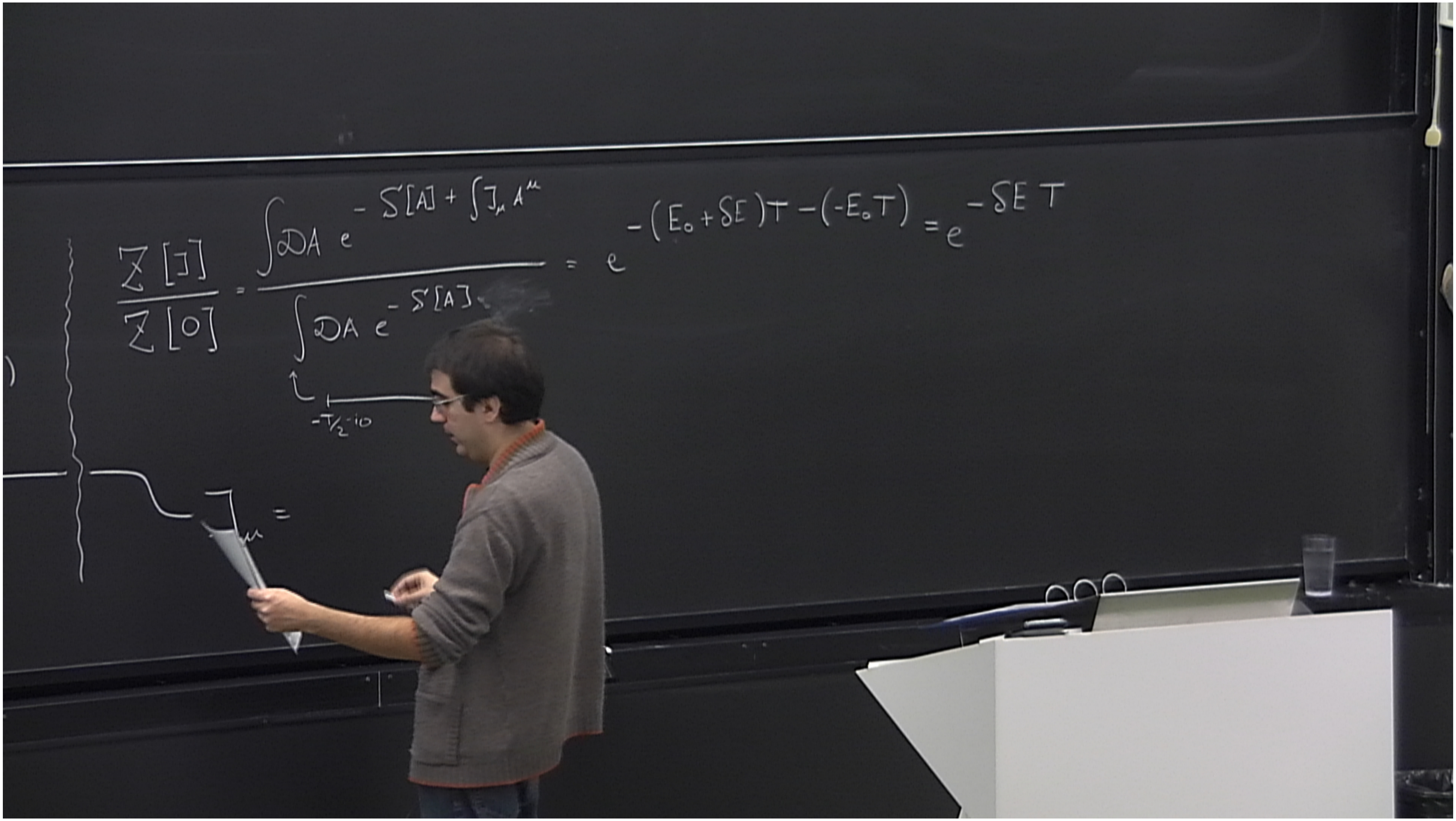

$\frac{Z[J]}{Z[0]} = \frac{\int \mathcal{D}A \, e^{-S[A] + \int J_\mu A^\mu}}{\int \mathcal{D}A \, e^{-S[A]}} = e^{-(E_0 + \delta E}$

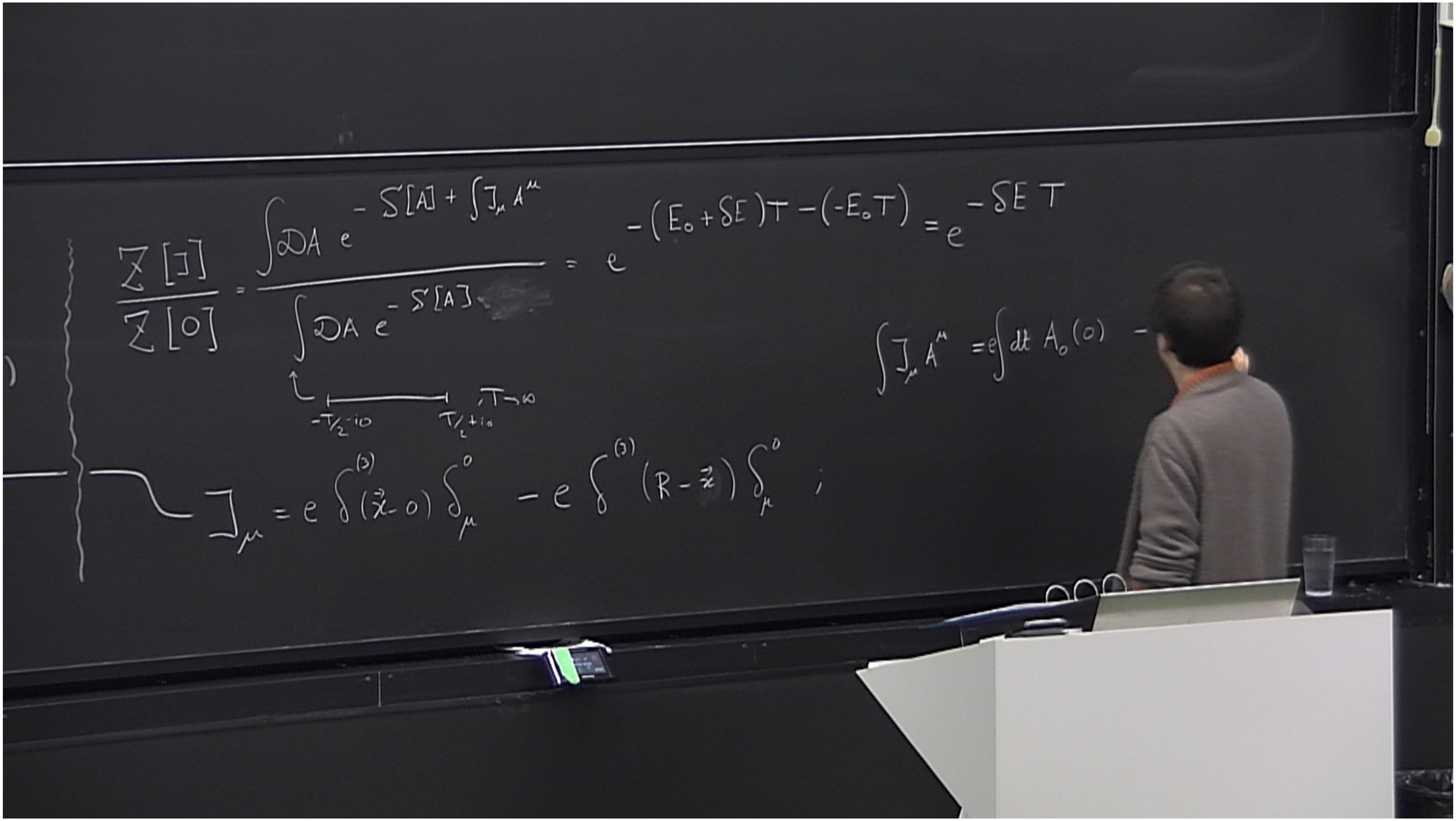

Z[J] / Z[0] = ∫DA e^{-S[A] + ∫J_μ A^μ} / ∫DA e^{-S[A]} = e^{-(E_0 + δE)T - (-E_0 T)} = e^{-δE T}

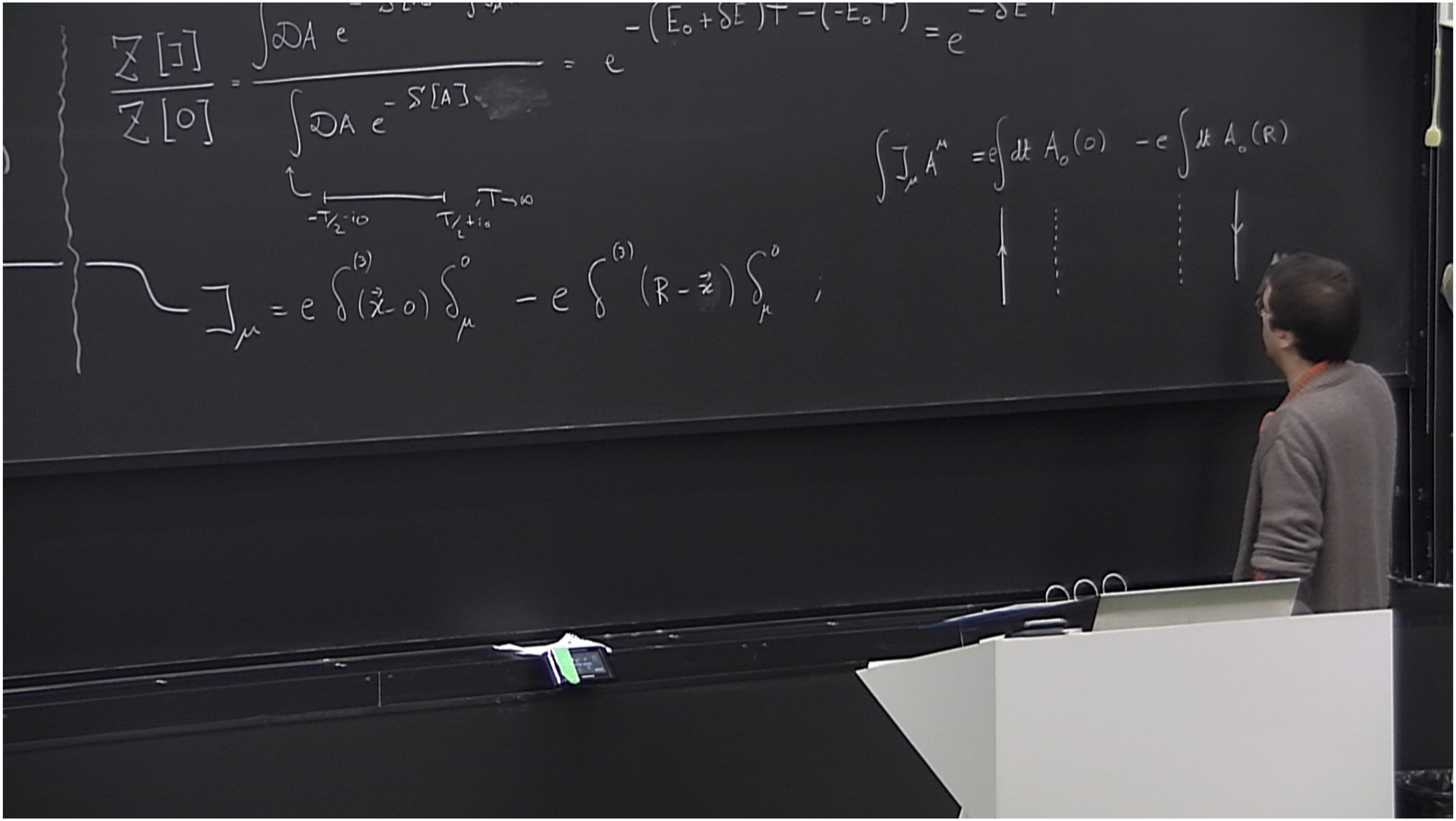

Z[J] / Z[0] = ∫DA e^(− S[A] + ∫J_μ A^μ) / ∫DA e^(− S[A]) = e^(−(E_0 + δE)T − (−E_0 T)) = e^(− δE T)
, T→∞
T/2 + i0

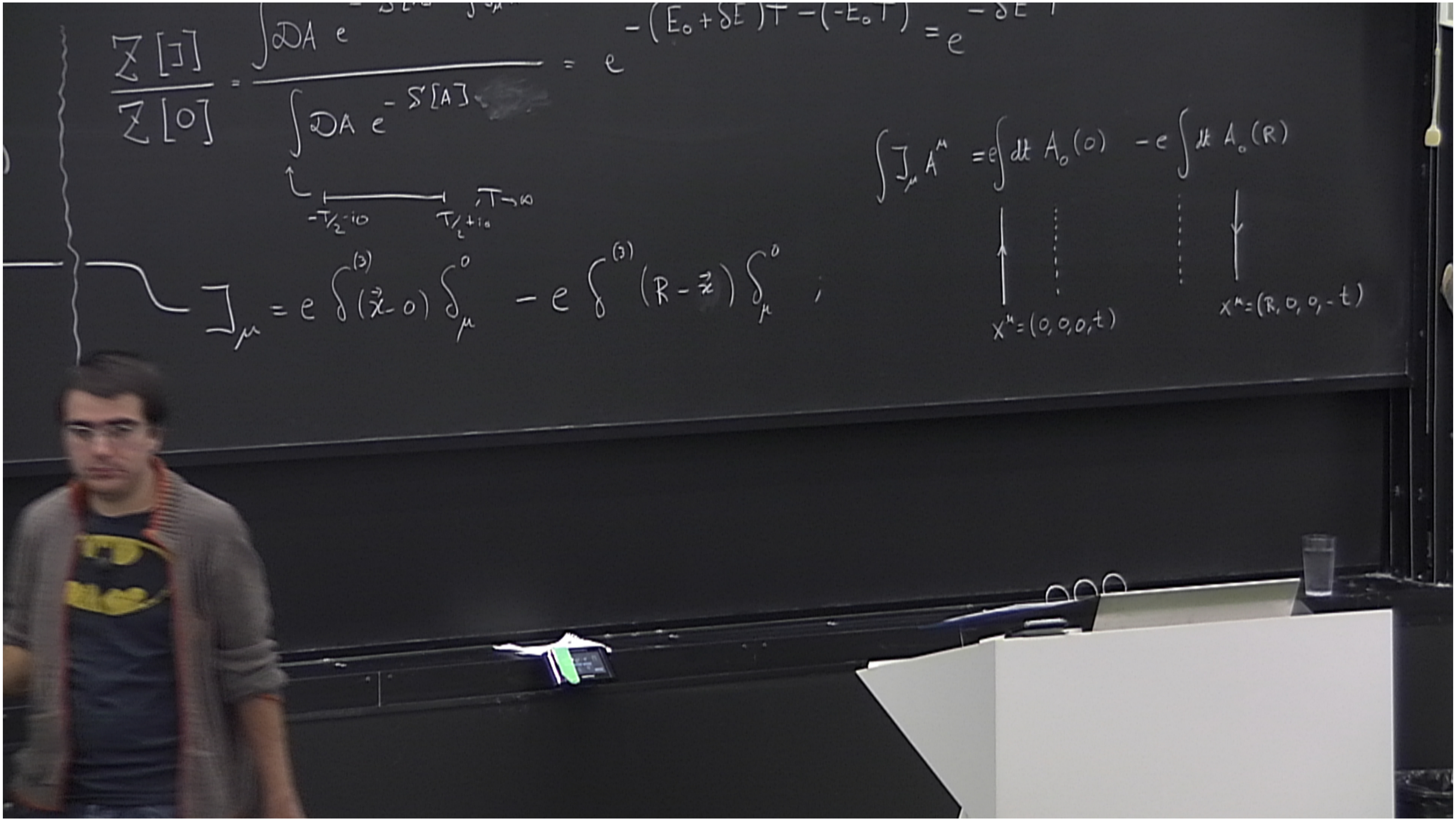

Z[J] / Z[0] = ∫DA e^{-S[A] + ∫J_μ A^μ} / ∫DA e^{-S[A]} = e^{-(E_0 + δE)T - (-E_0 T)} = e^{-δE T}
T/2 + iα
, T → ∞

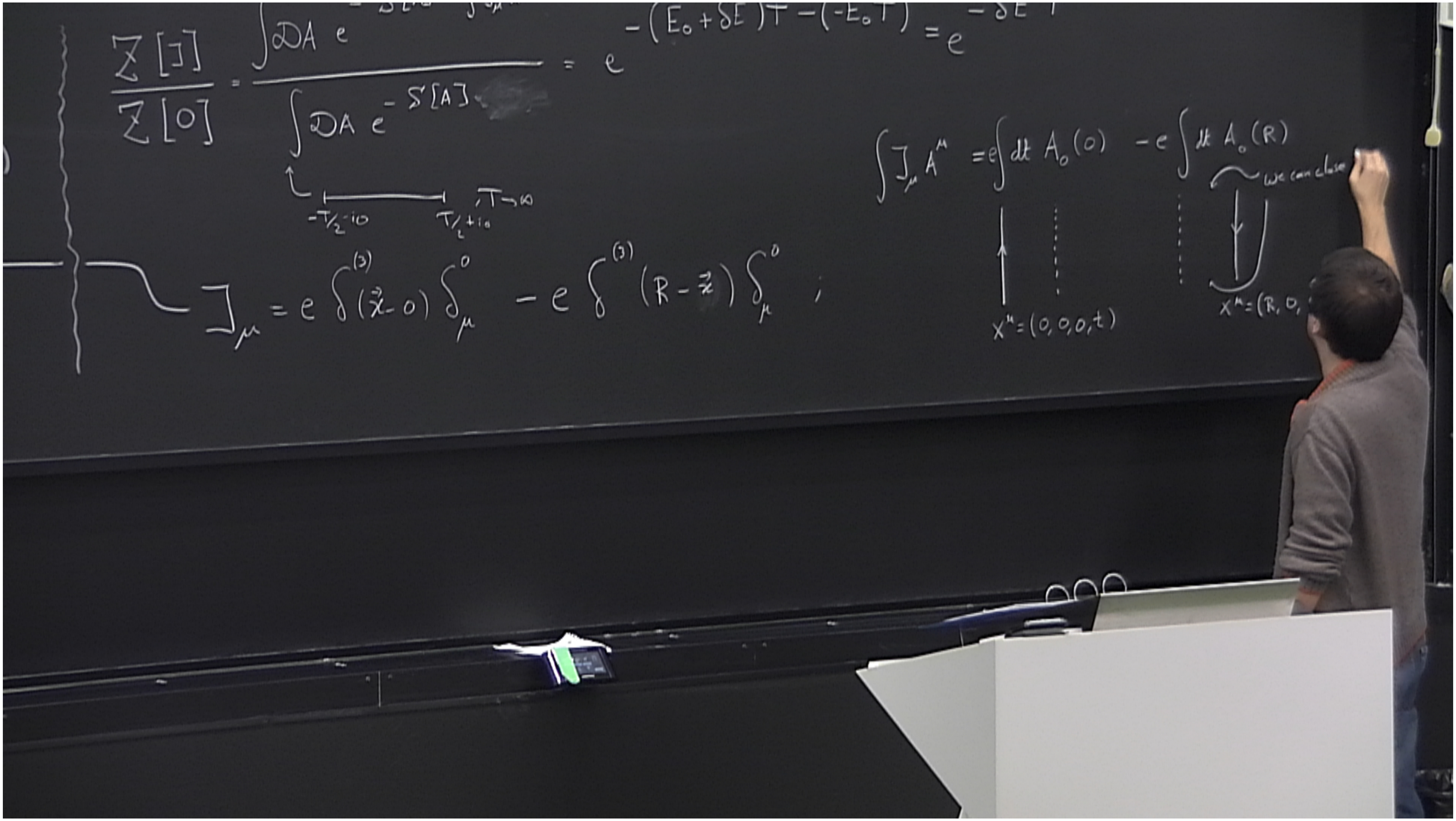

$\frac{Z[J]}{Z[0]} = \frac{\int \mathcal{D}A \, e^{-S'[A] + \int J_\mu A^\mu}}{\int \mathcal{D}A \, e^{-S'[A]}} = e^{-(E_0 + \delta E)T - (-E_0 T)} = e^{-\delta E \, T}$
$-T/2 - i0$
$T/2 + i0$
$, T \to \infty$

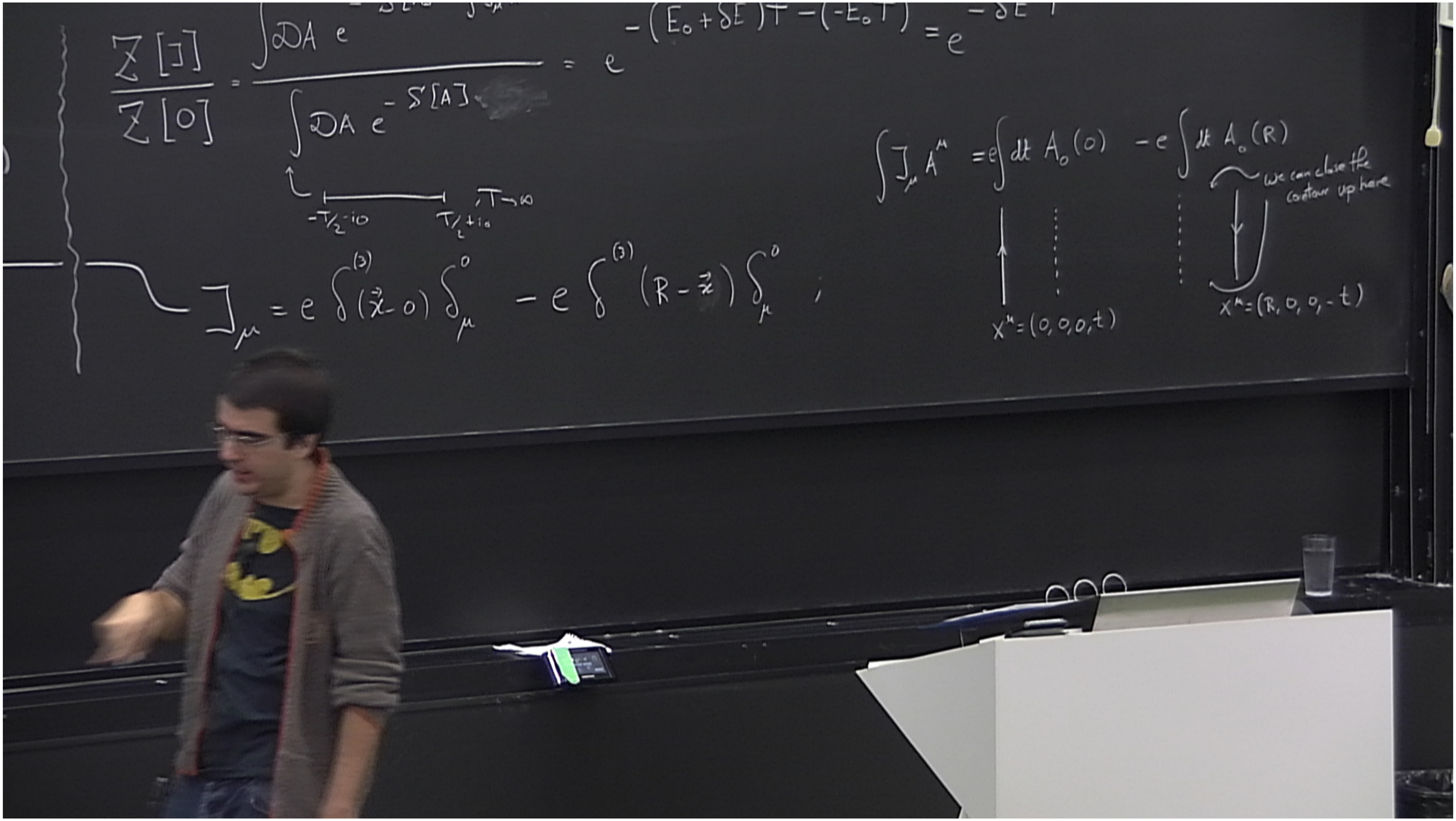

$\frac{Z[J]}{Z[0]} = \frac{\int \mathcal{D}A\, e^{-S[A] + \int J_\mu A^\mu}}{\int \mathcal{D}A\, e^{-S[A]}} = e^{-(E_0 + \delta E)T - (-E_0 T)} = e^{-\delta E T}$
$-T/2 - i\epsilon$
$J_\mu =$

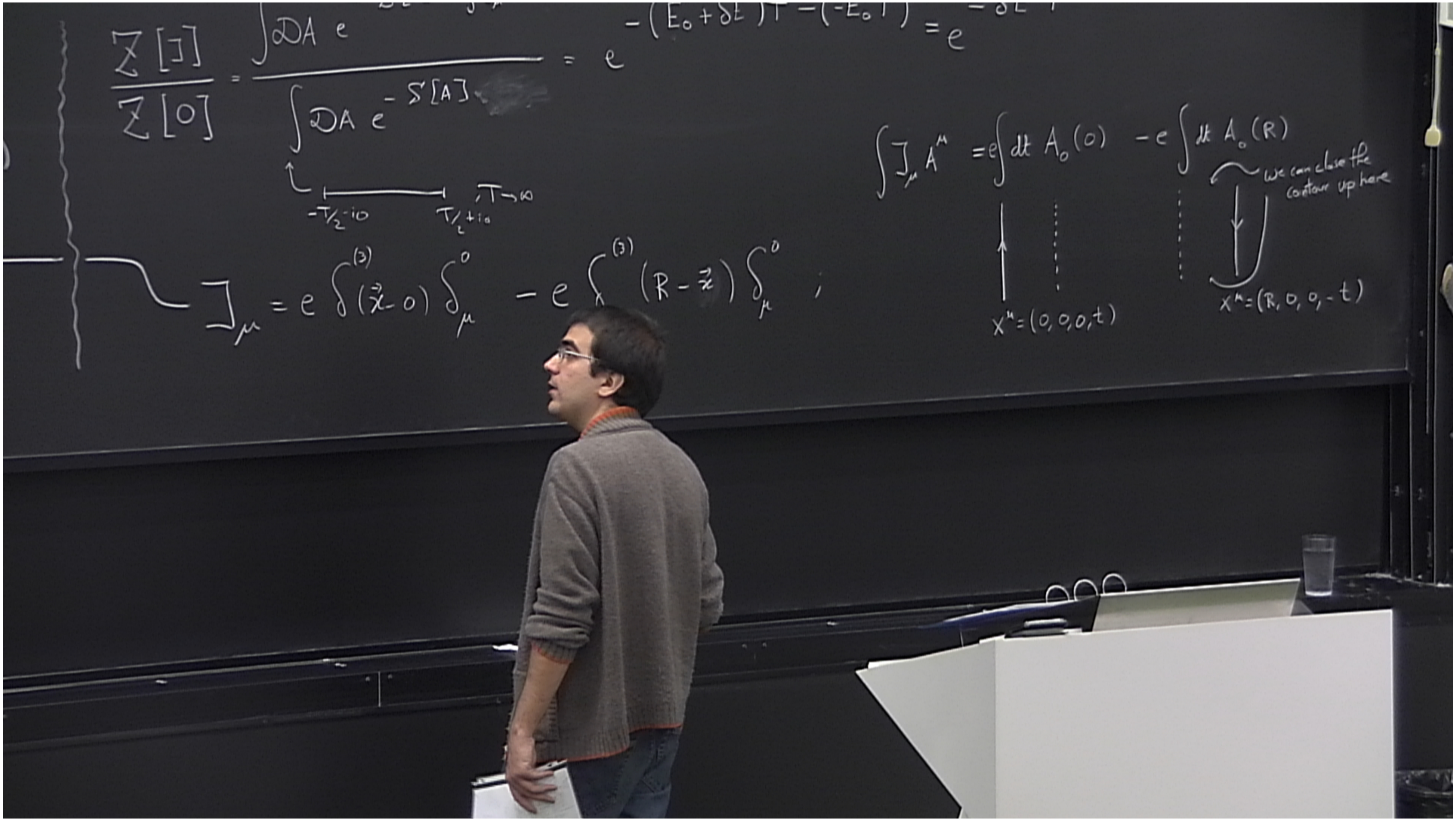

$\frac{Z[J]}{Z[0]} = \frac{\int \mathcal{D}A \, e^{-S[A] + \int J_\mu A^\mu}}{\int \mathcal{D}A \, e^{-S[A]}} = e^{-(E_0 + \delta E)T - (-E_0 T)} = e^{-\delta E \, T}$
$-T/2 - i0 \qquad T/2 + i0 \quad , T \to \infty$
$\int J_\mu A^\mu = e \int dt \, A_0(0) \; -$
$J_\mu = e \, \delta^{(3)}(\vec{x} - 0) \, \delta^0_\mu - e \, \delta^{(3)}(R - \vec{x}) \, \delta^0_\mu \; ;$

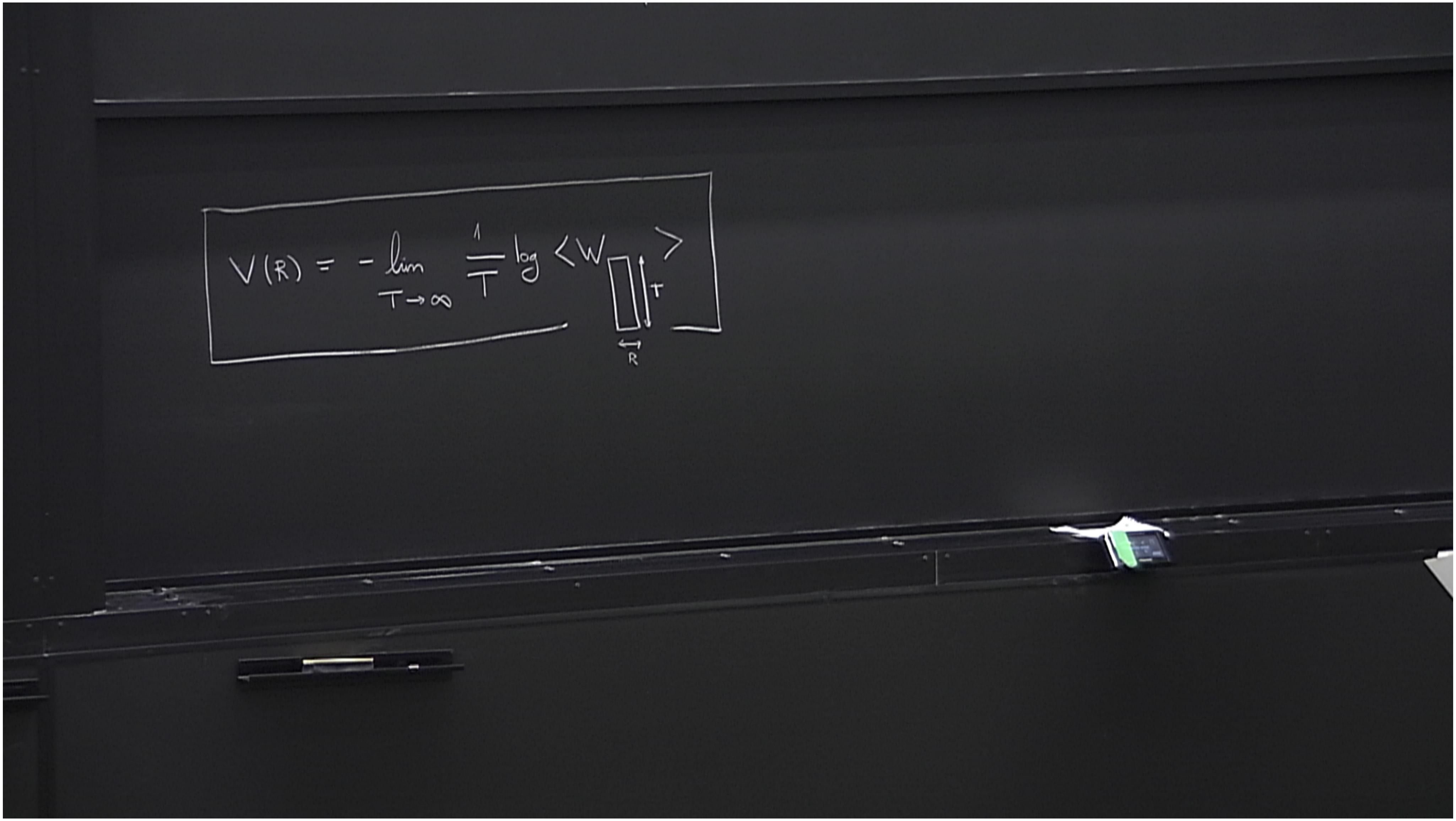


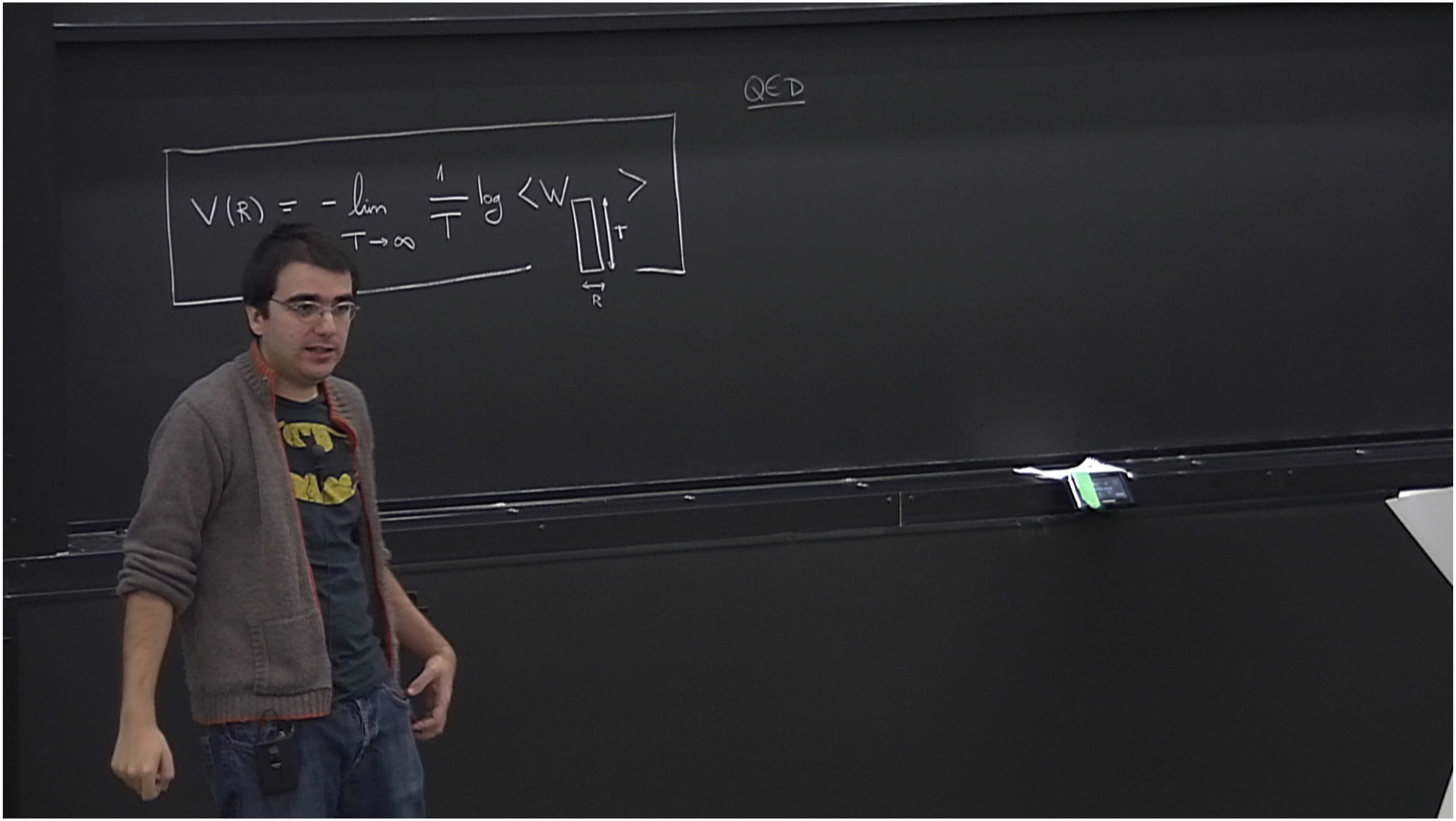

Z[J] / Z[0] = ∫DA e / ∫DA e^{- S[A]} = e^{-(E₀ + δE)T - (-E₀T)} = e^{-δE T}
-T/2 - iε   T/2 + iε , T → ∞
J_μ = e δ^{(3)}(x⃗ - 0) δ_μ^0 - e δ^{(3)}(R - x⃗) δ_μ^0 ;
∫ J_μ A^μ = e ∫ dt A₀(0) - e ∫ dt A₀(R)
x^μ = (0, 0, 0, t)
x^μ = (R, 0, 0, - t)

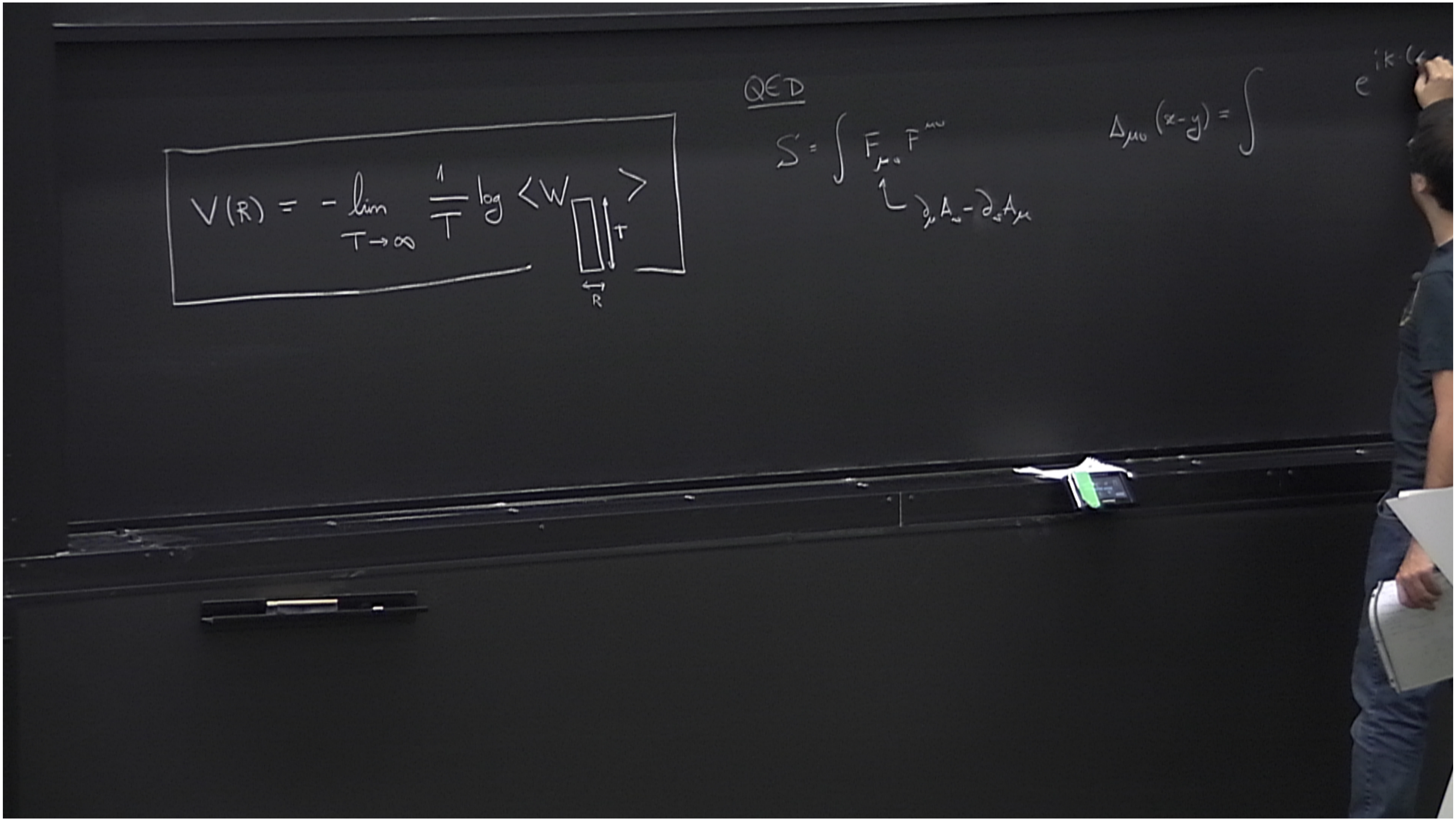

Z[J] / Z[0] = ∫DA e / ∫DA e^{-S[A]}
T→∞
J_μ = e δ^(3)(x - 0) δ_μ^0 - e δ^(3)(R - x) δ_μ^0 ;
∫J_μ A^μ = e∫dt A_0(0) - e∫dt A_0(R)
we can close
x^μ=(0,0,0,t)
x^μ=(R,0,

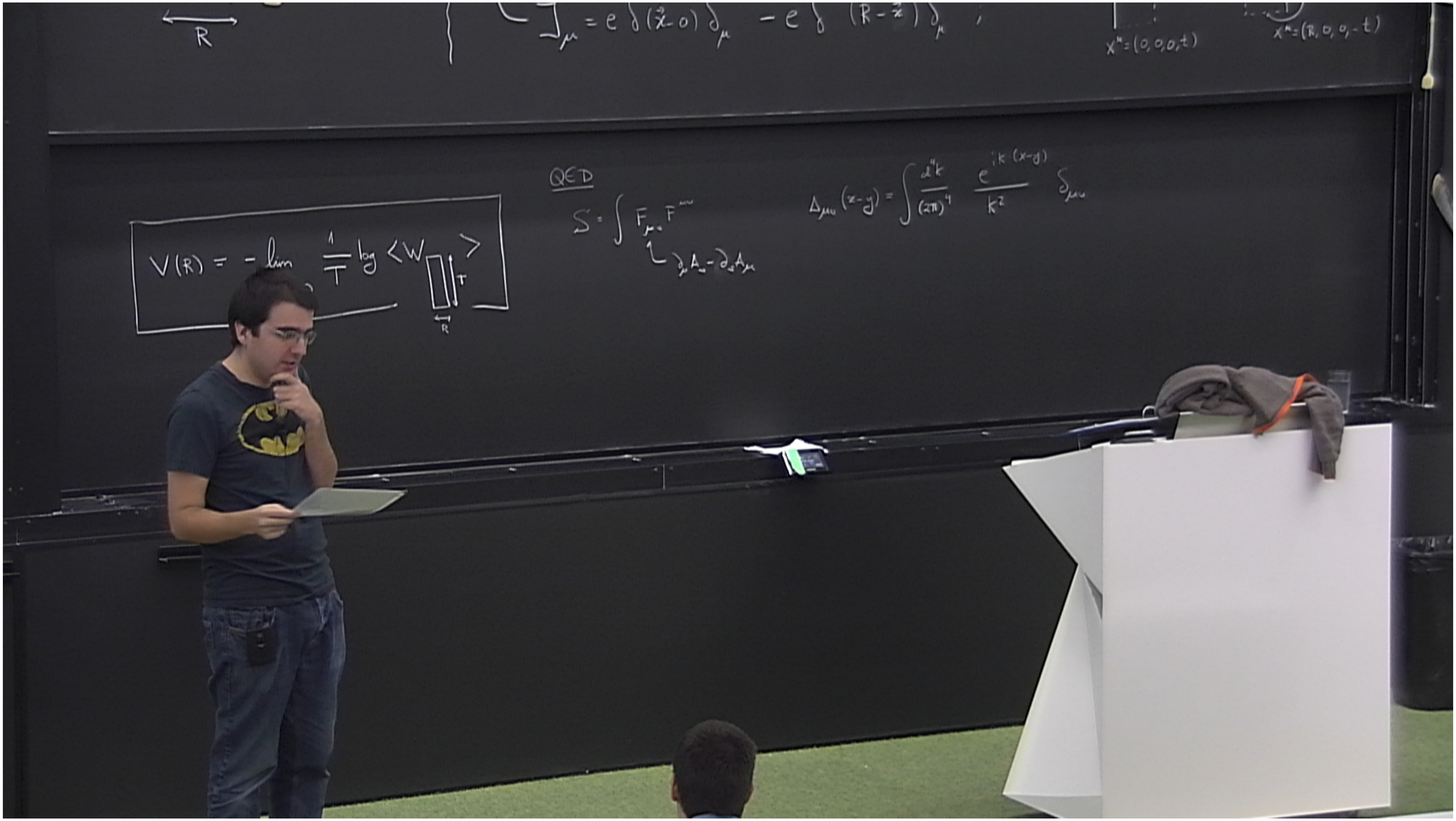

Z[J] / Z[0] = ∫DA e^{-S[A] ...} / ∫DA e^{-S[A]} = e^{-(E_0+δE)T - (-E_0 T)} = e^{-δE T}
-T/2 - i0   T/2 + i0   , T→∞
J_μ = e δ^{(3)}(x⃗ - 0) δ_μ^0 - e δ^{(3)}(R - x⃗) δ_μ^0 ;
∫ J_μ A^μ = e ∫dt A_0(0) - e ∫dt A_0(R)
we can close the contour up here
x^μ = (0, 0, 0, t)
x^μ = (R, 0, 0, -t)

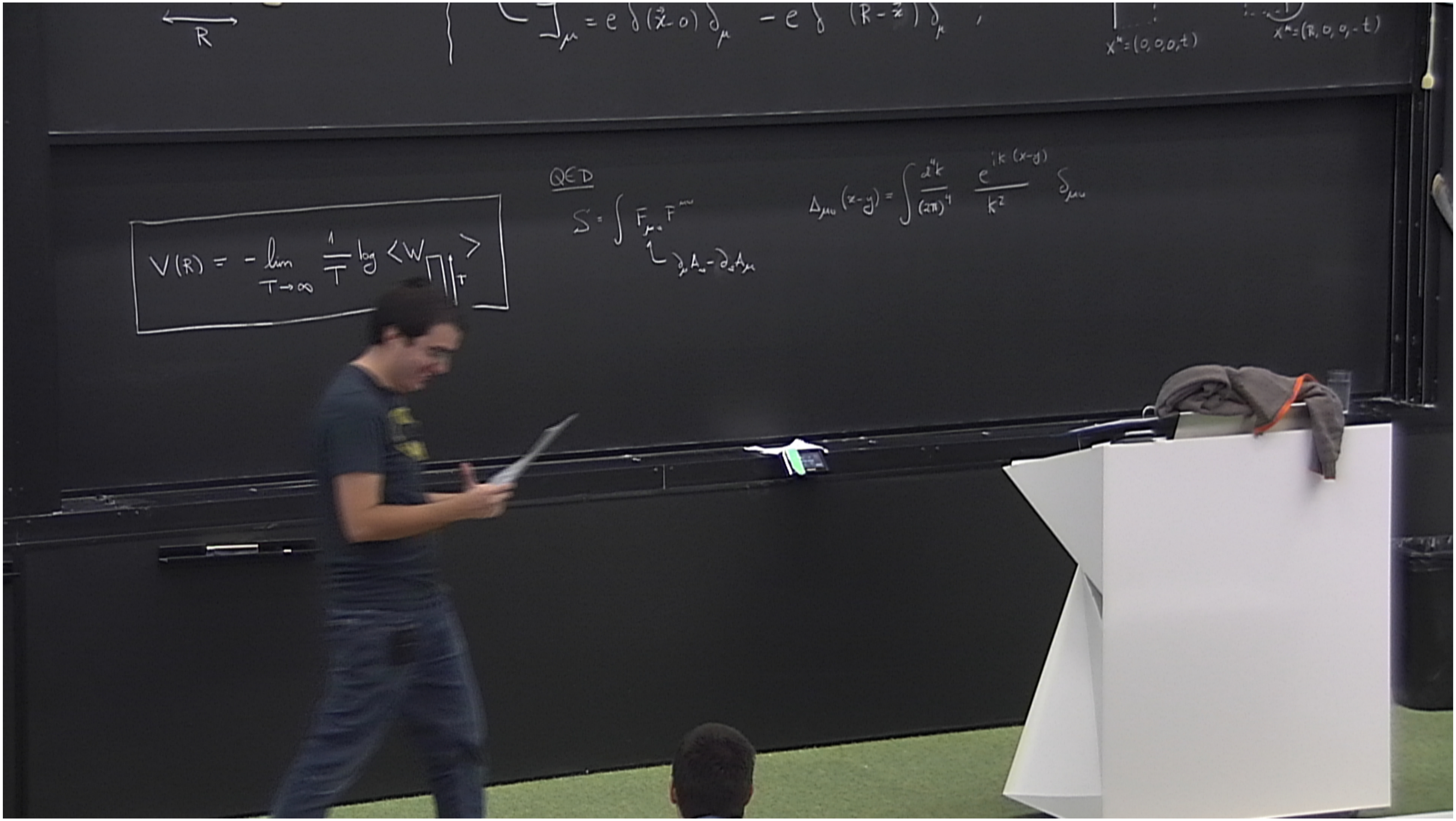

Z[J] / Z[0] = ∫DA e^{...} / ∫DA e^{-S[A]} = e^{-(E_0 + δE)T - (-E_0 T)} = e^{-δE T}
-T/2 - i0   T/2 + i0   , T → ∞
J_μ = e δ^(3)(x - 0) δ^0_μ - e δ^(3)(R - x) δ^0_μ ;
∫ J_μ A^μ = e ∫ dt A_0(0) - e ∫ dt A_0(R)
we can close the contour up here
x^μ = (0, 0, 0, t)
x^μ = (R, 0, 0, -t)


$V(R) = - \lim_{T \to \infty} \frac{1}{T} \log \langle W \rangle$
T
R

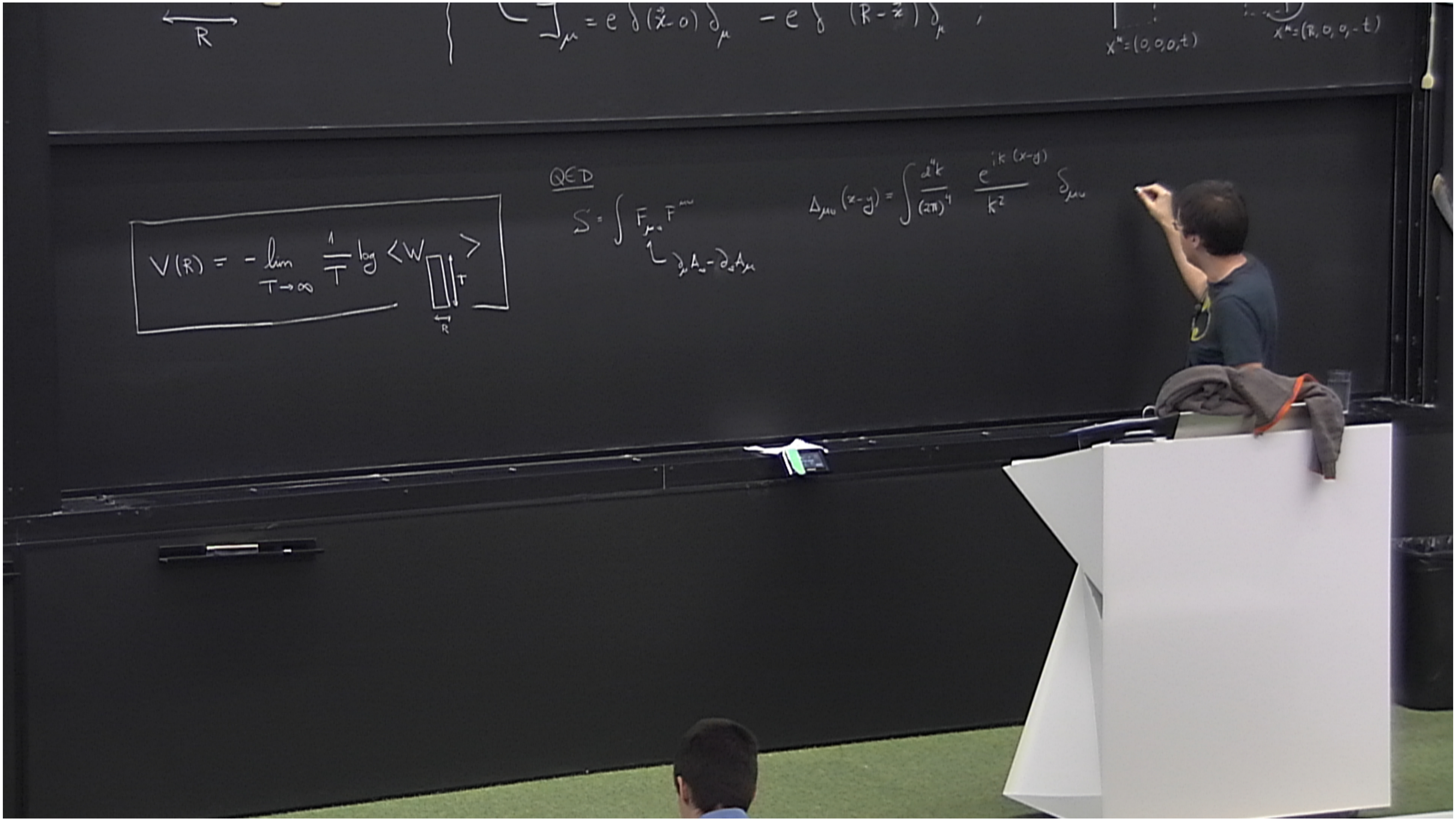

QED
V(R) = - lim T→∞ 1/T log ⟨W⟩
T
R


QED
$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log \langle W \rangle$
T
R
$S = \int F_{\mu\nu} F^{\mu\nu}$
$\partial_\mu A_\nu - \partial_\nu A_\mu$
$\Delta_{\mu\nu}(x-y) = \int$

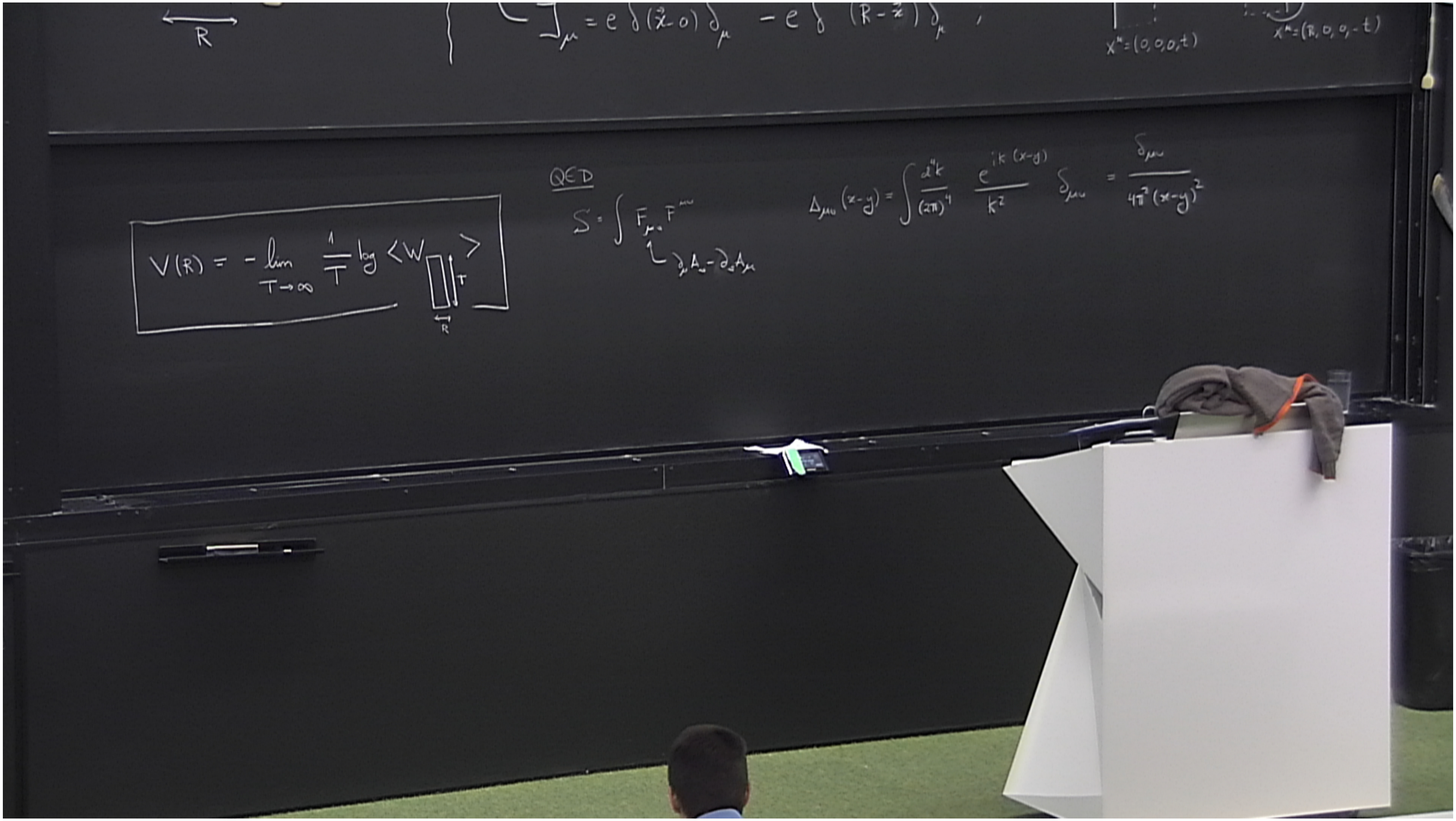

QED
$S = \int F_{\mu\nu} F^{\mu\nu}$
$\partial_\mu A_\nu - \partial_\nu A_\mu$
$\Delta_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2} \delta_{\mu\nu}$
$V(R) = -\lim \frac{1}{T} \log \langle W \rangle$

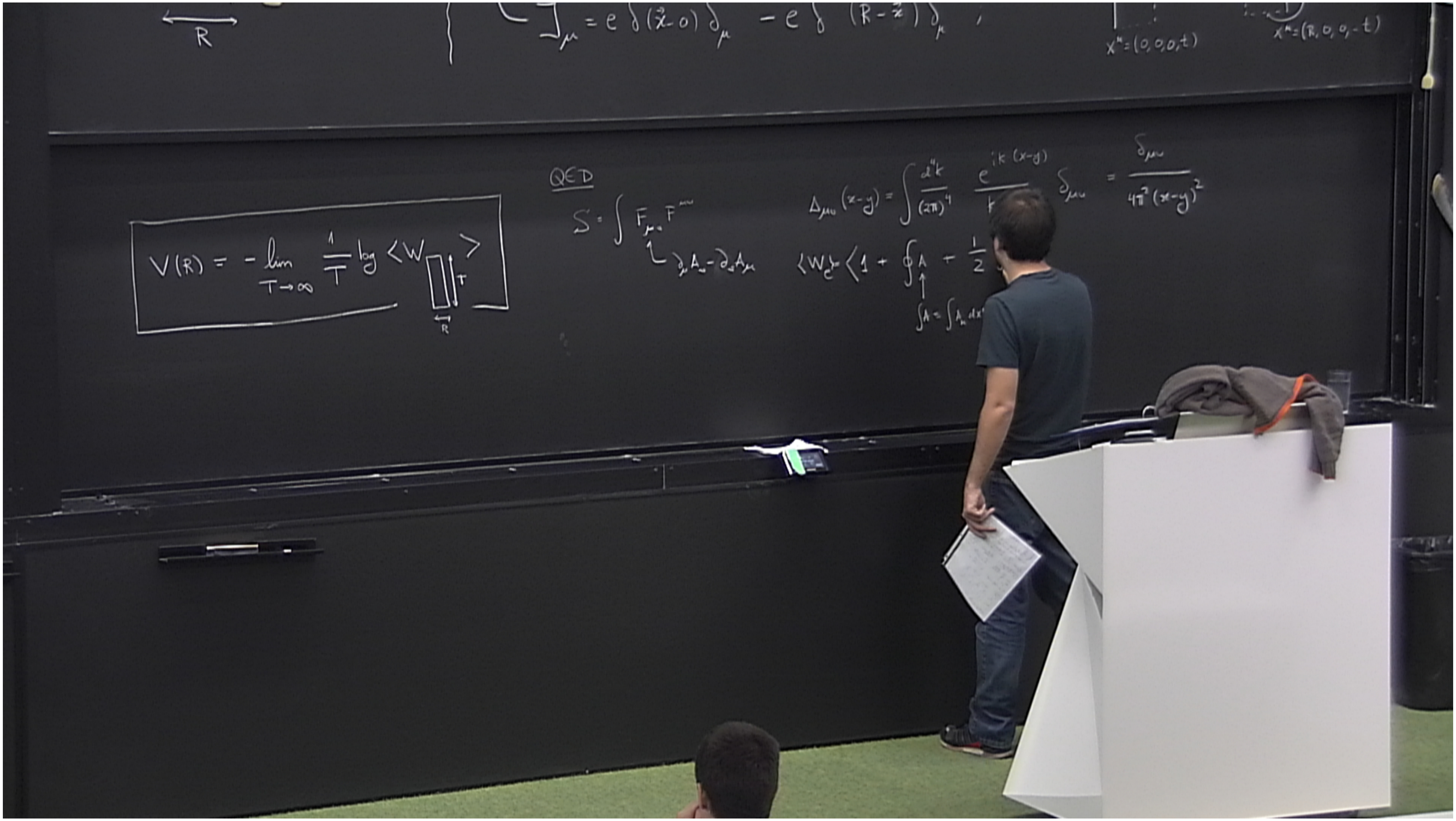


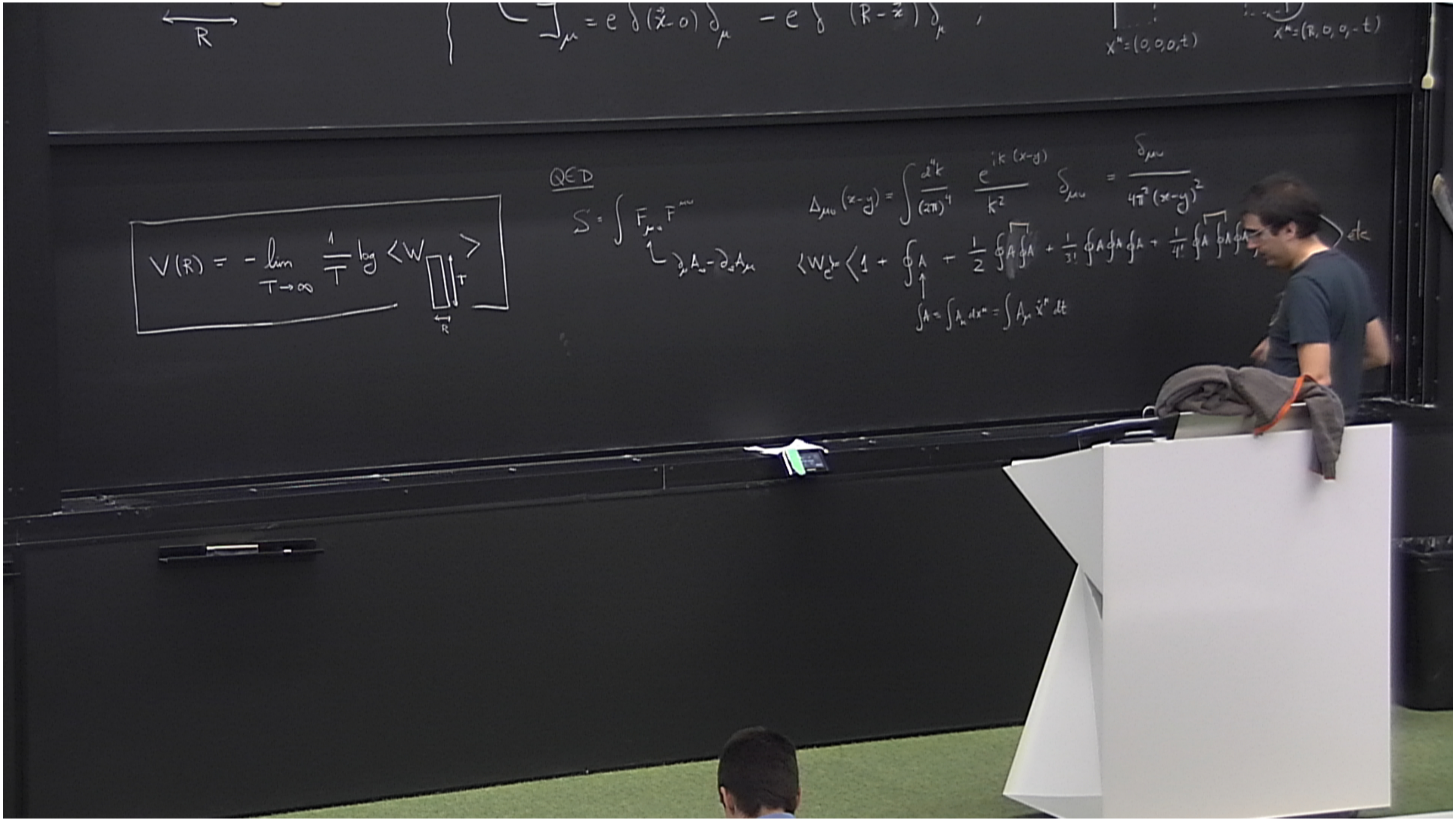
QED
$V(R) = -\lim_{T\to\infty} \frac{1}{T} \log \langle W \rangle$
$S = \int F_{\mu\nu} F^{\mu\nu}$
$\partial_\mu A_\nu - \partial_\nu A_\mu$
$\Delta_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2} \delta_{\mu\nu}$
$x^\mu = (0,0,0,t)$
$x^\mu = (R,0,0,-t)$

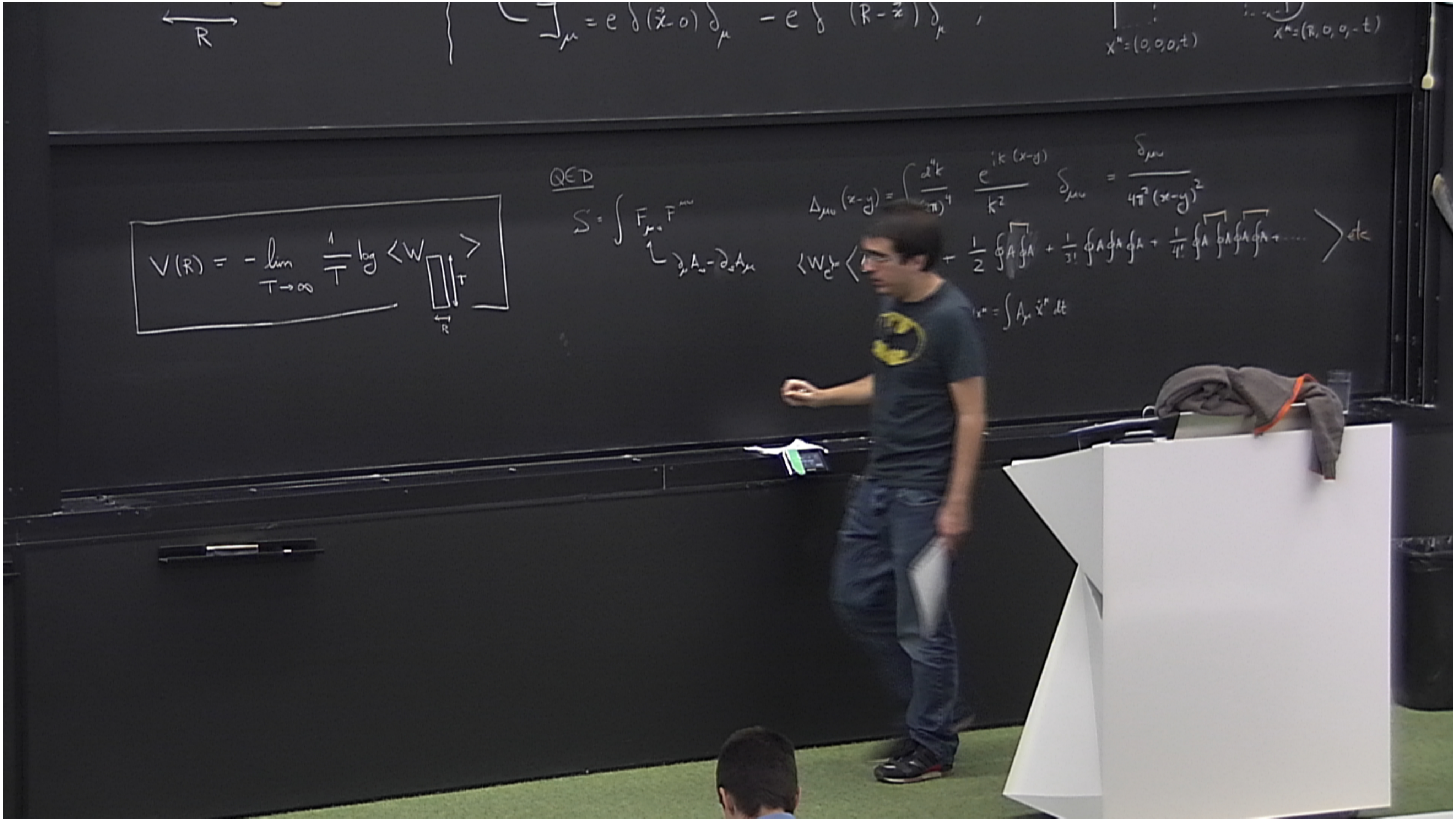


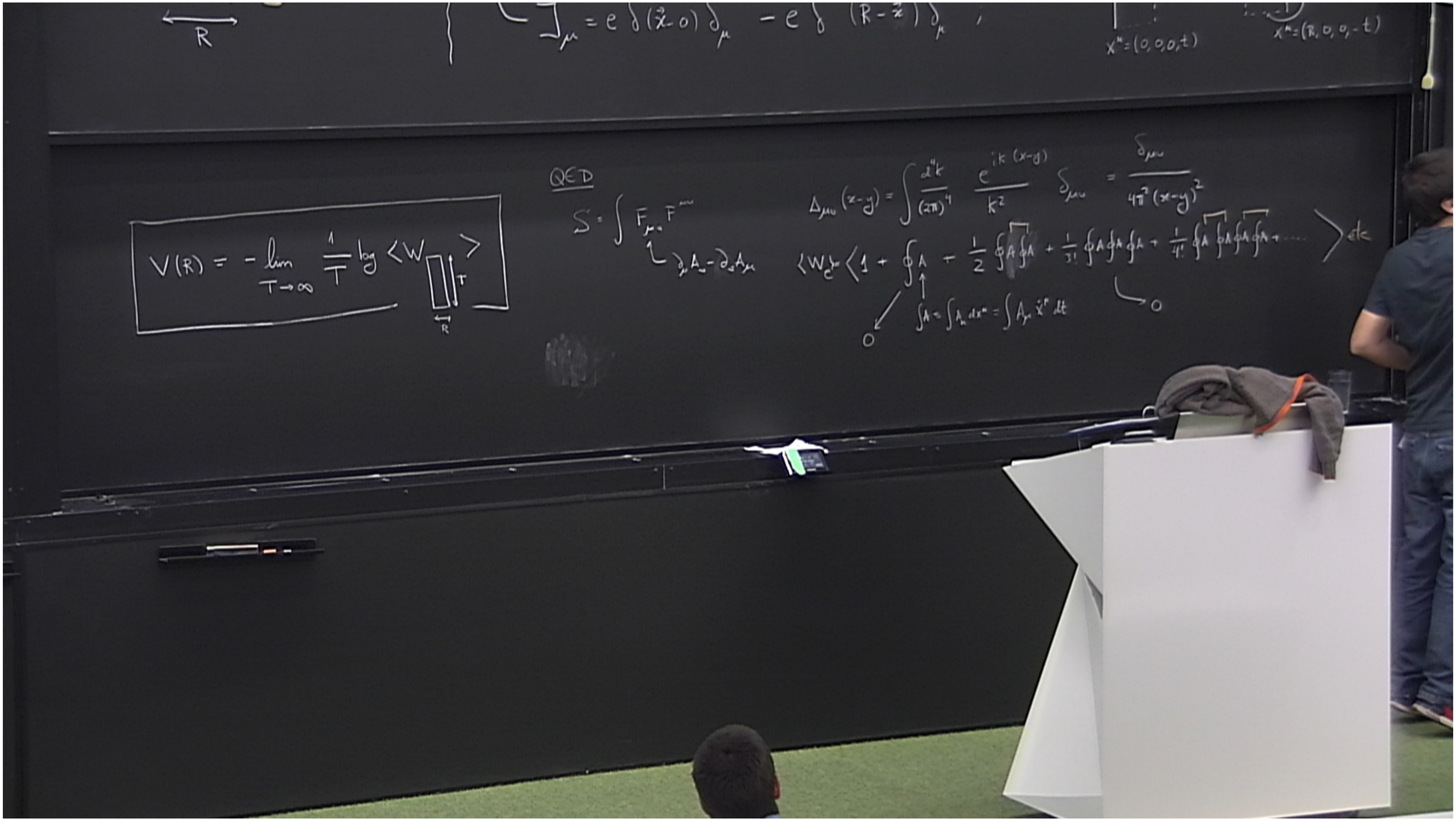
QED
V(R) = - lim T→∞ 1/T log <W>
S = ∫ F F
Δμν (x-y) = ∫ d⁴k/(2π)⁴ e^{ik(x-y)}/k² δμν
x^μ=(0,0,0,t)

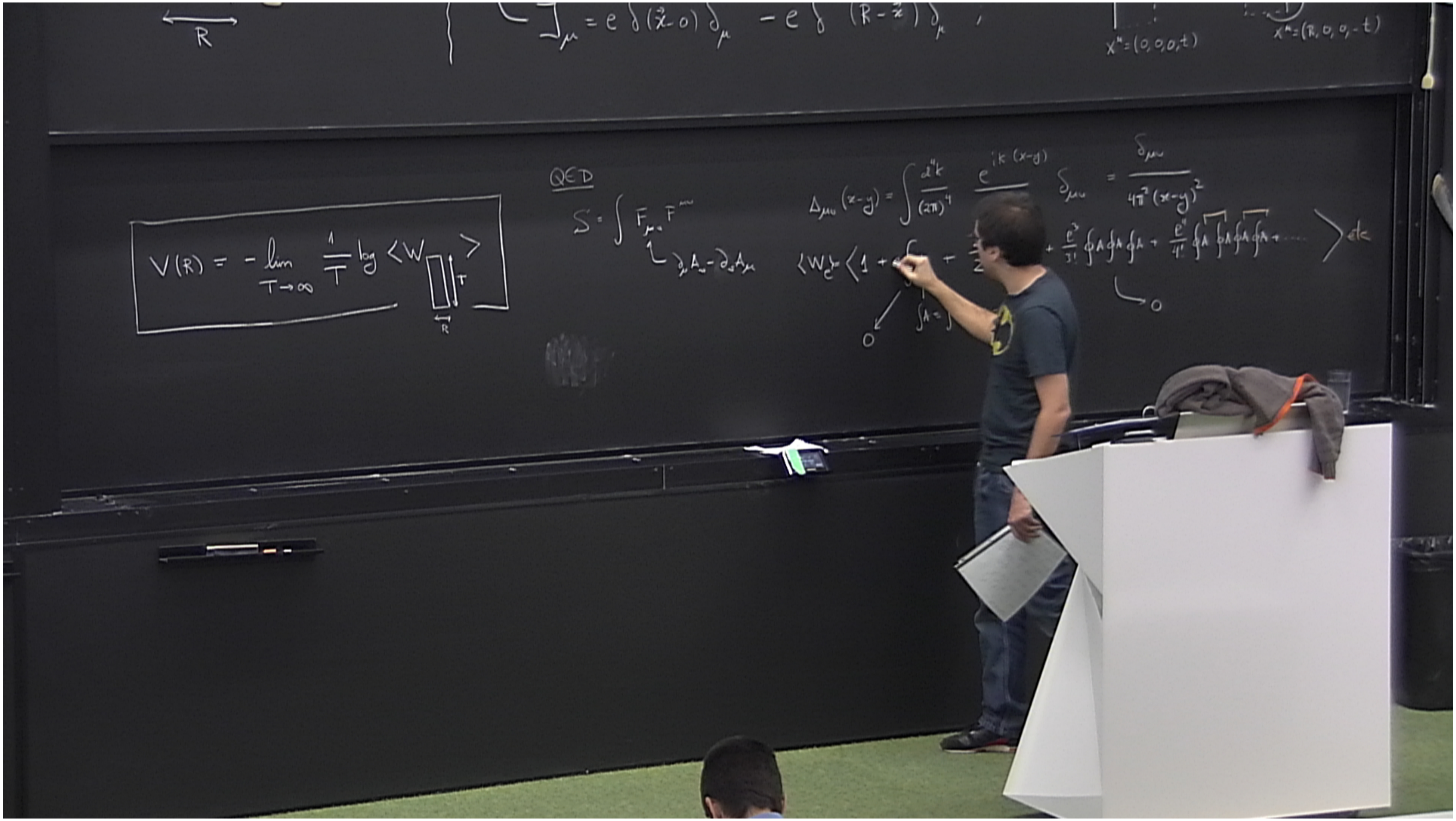

QED
S = ∫ F_μν F^μν
∂_μ A_ν − ∂_ν A_μ
V(R) = − lim_{T→∞} 1/T log ⟨W⟩
Δ_μν (x−y) = ∫ d⁴k/(2π)⁴ e^{ik(x−y)}/k² δ_μν = δ_μν / 4π² (x−y)²
x^μ = (0,0,0,t)
x^μ = (R,0,0,−t)

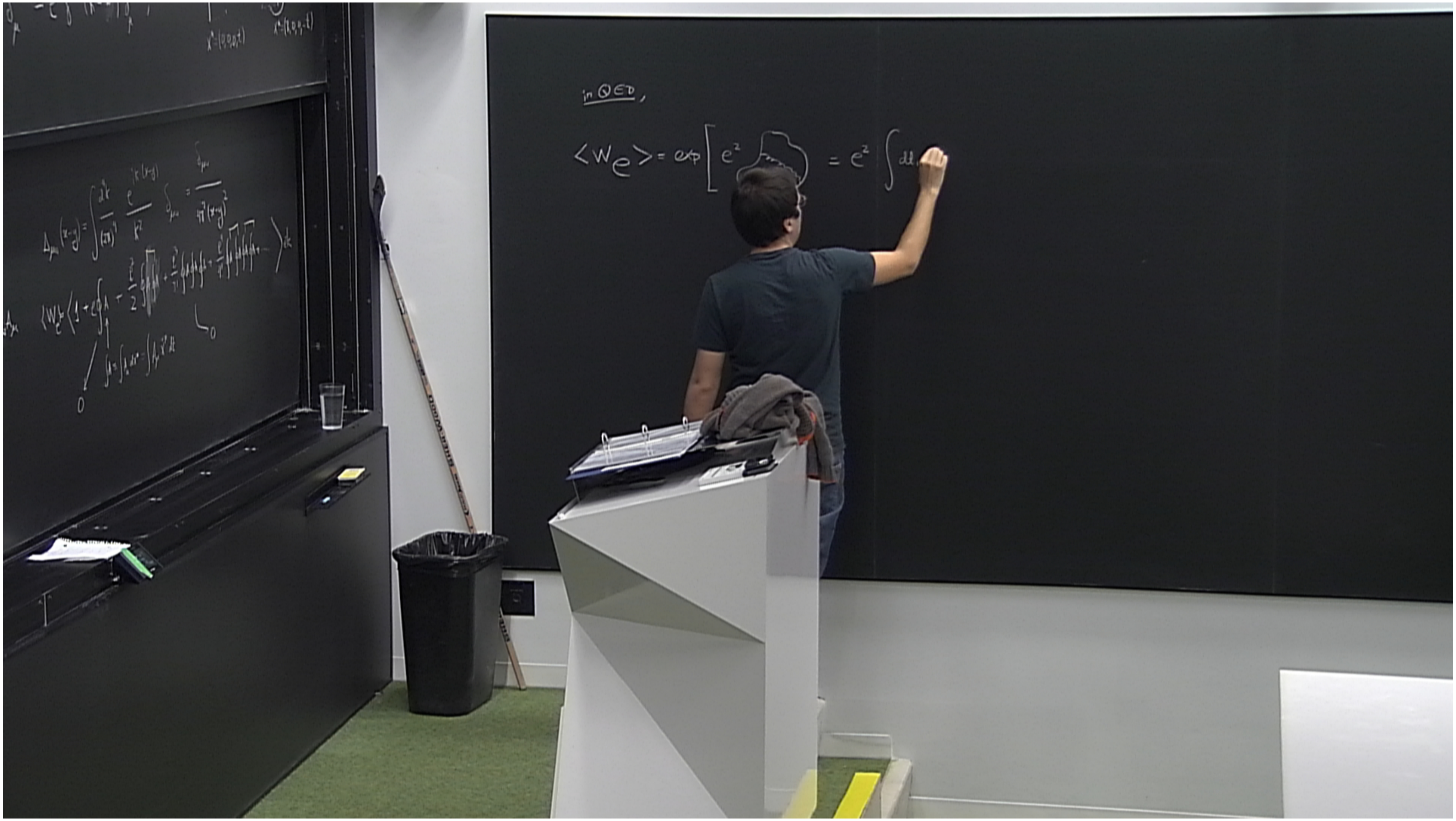


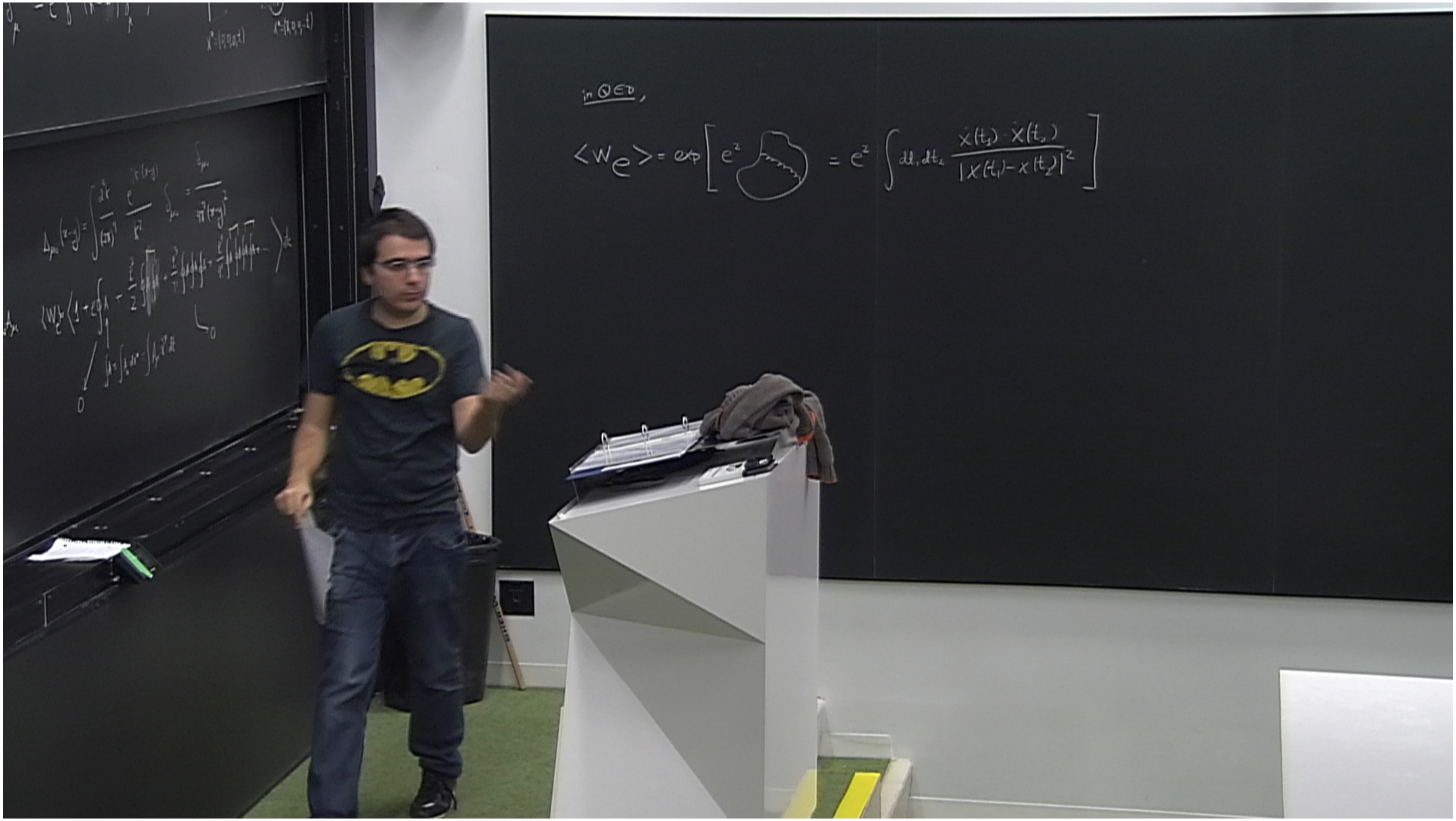

QED
V(R) = - lim T→∞ 1/T log ⟨W⟩
S = ∫ F_μν F^μν
∂_μ A_ν - ∂_ν A_μ
Δ_μν (x-y) = ∫ d^4k/(2π)^4 e^{ik(x-y)}/k^2 δ_μν = δ_μν / 4π^2 (x-y)^2
⟨W_C⟩ = ⟨1 + ∮A + 1/2 ∮A∮A + 1/3! ∮A∮A∮A + 1/4! ∮A∮A∮A∮A ...⟩ etc
∮A = ∫A_μ dx^μ = ∫A_μ ẋ^μ dt
x^μ = (0,0,0,t)
x^μ = (R,0,0,-t)

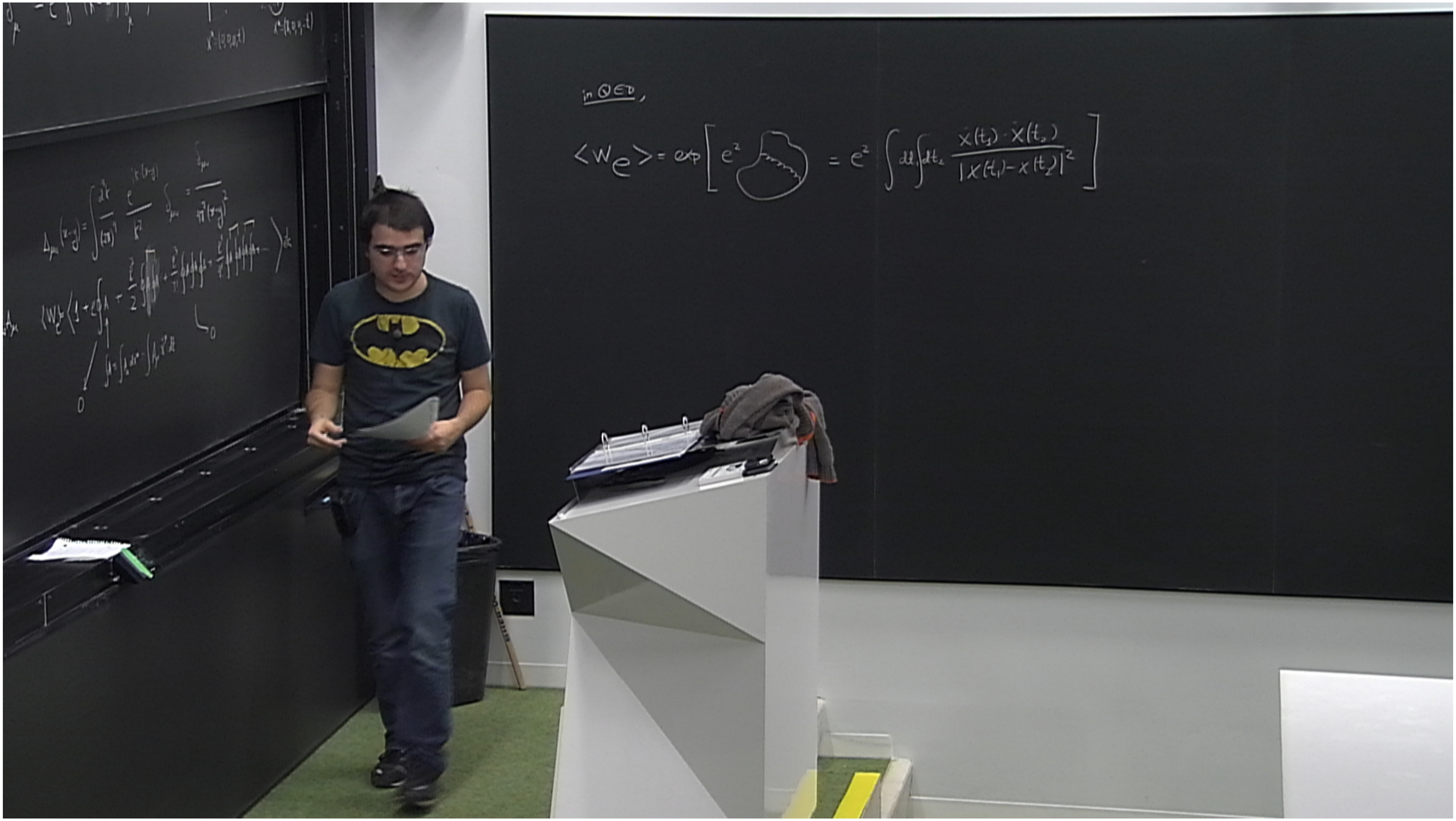


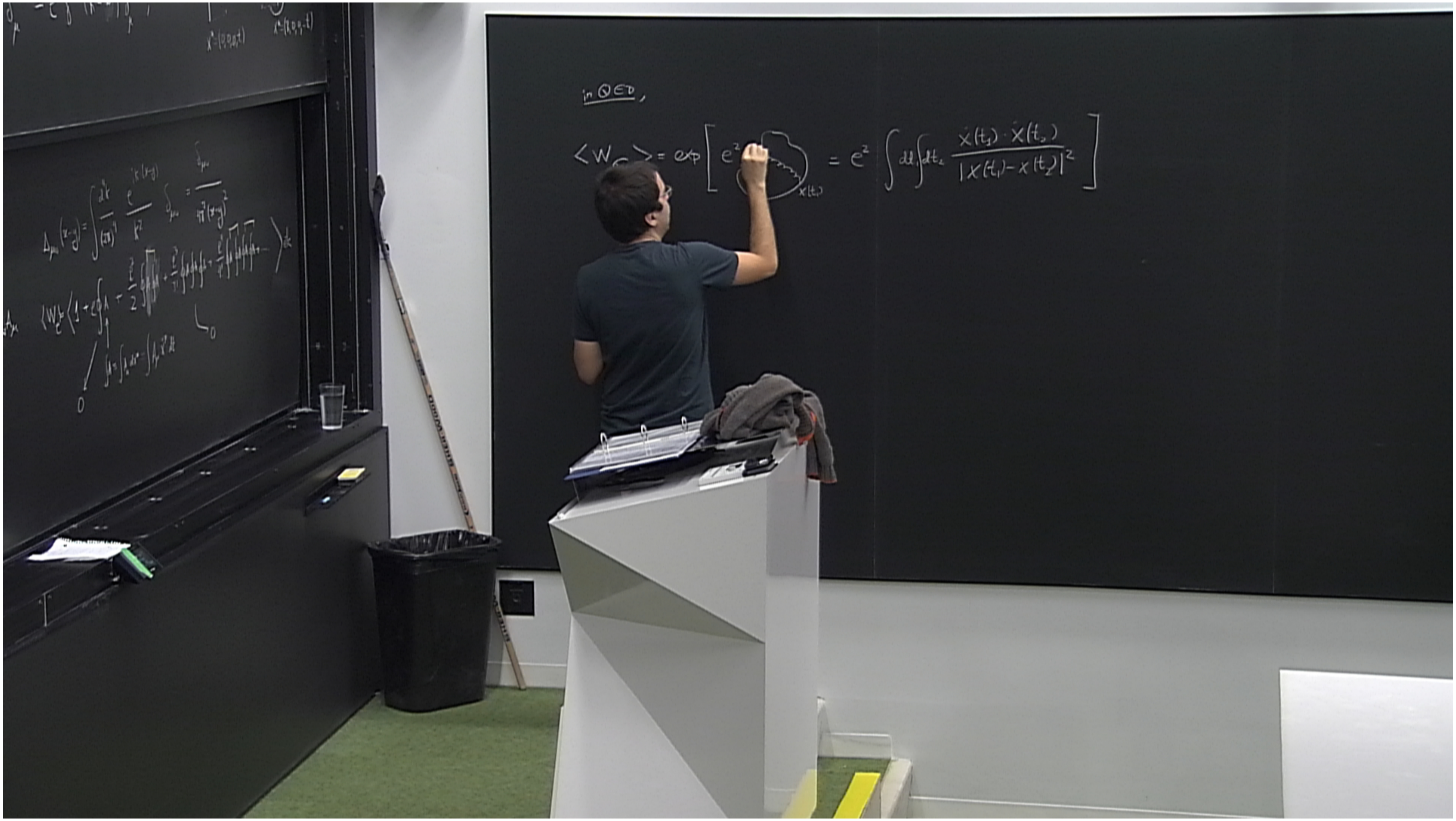

QED
V(R) = - lim T→∞ 1/T log ⟨W⟩
S = ∫ F_μν F^μν
∂_μ A_ν - ∂_ν A_μ
Δ_μν (x-y) = ∫ d^4k/(2π)^4 e^{ik(x-y)}/k^2 δ_μν = δ_μν/4π^2 (x-y)^2
⟨W_C⟩ = ⟨1 + ∮A - 1/2 ∮A∮A + 1/3! ∮A∮A∮A + 1/4! ∮A∮A∮A∮A + ...⟩ etc
∮A = ∫A_μ dx^μ = ∫A_μ ẋ^μ dt
x^μ = (0,0,0,t)
x^μ = (R,0,0,-t)

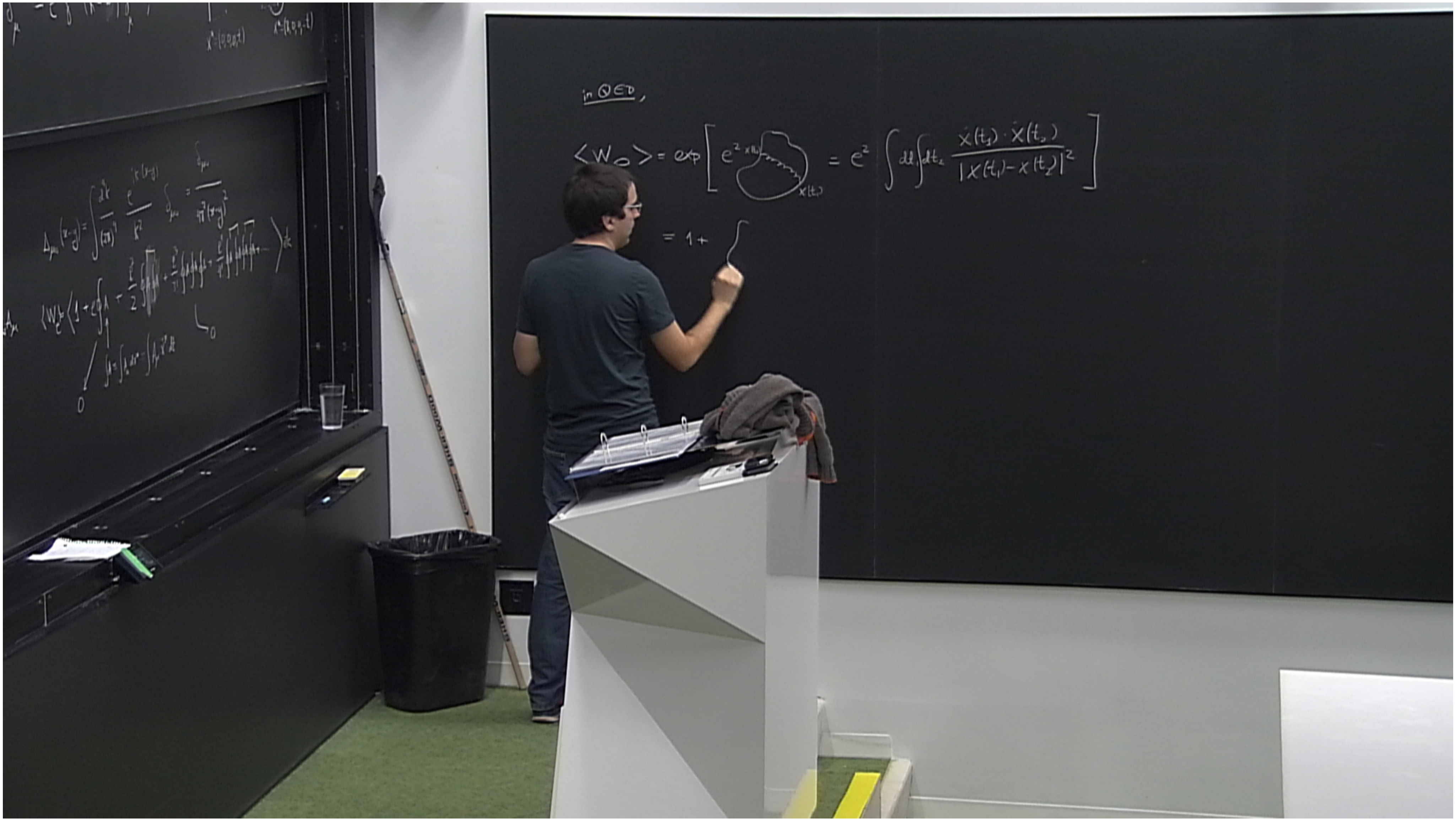


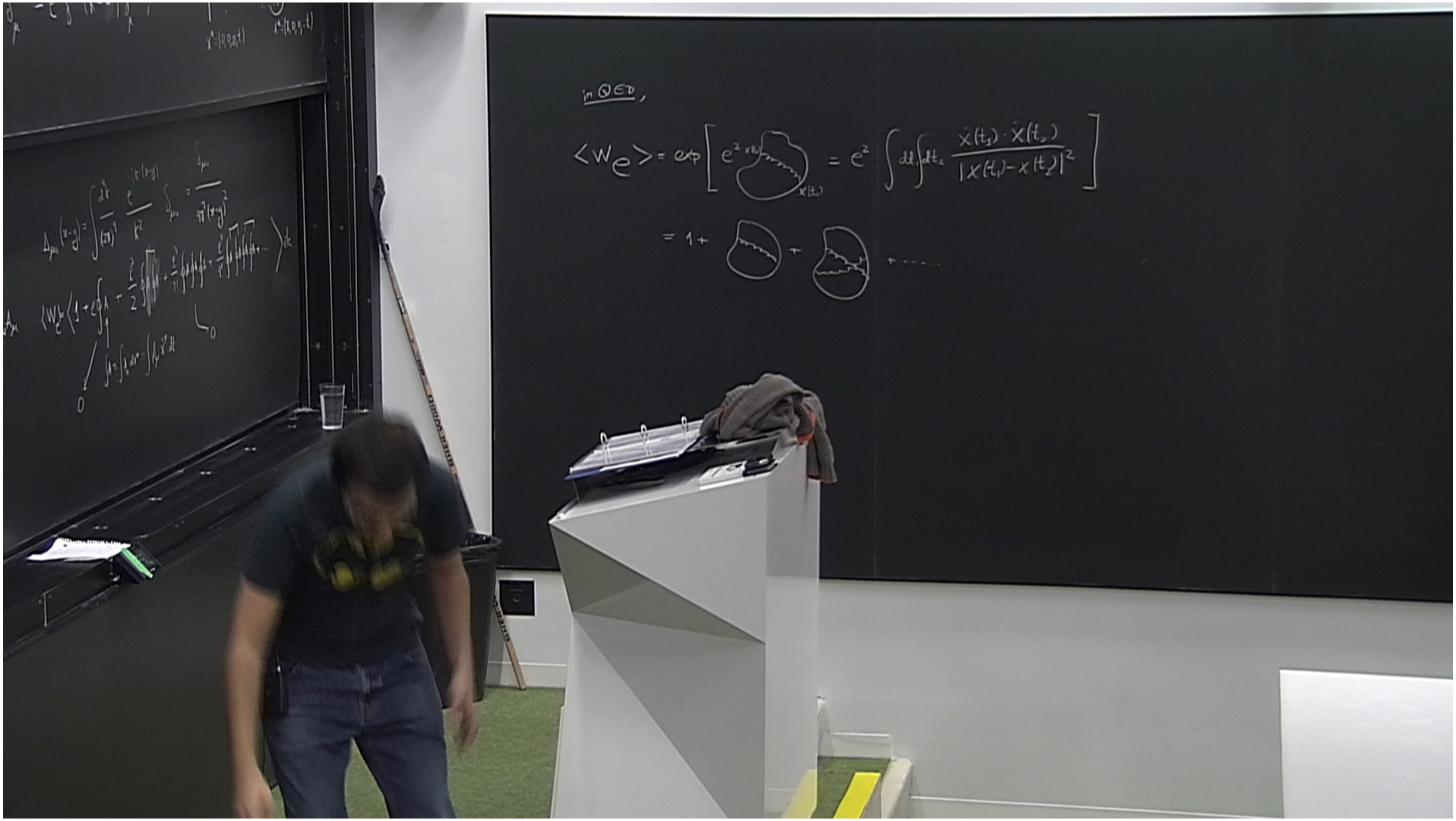


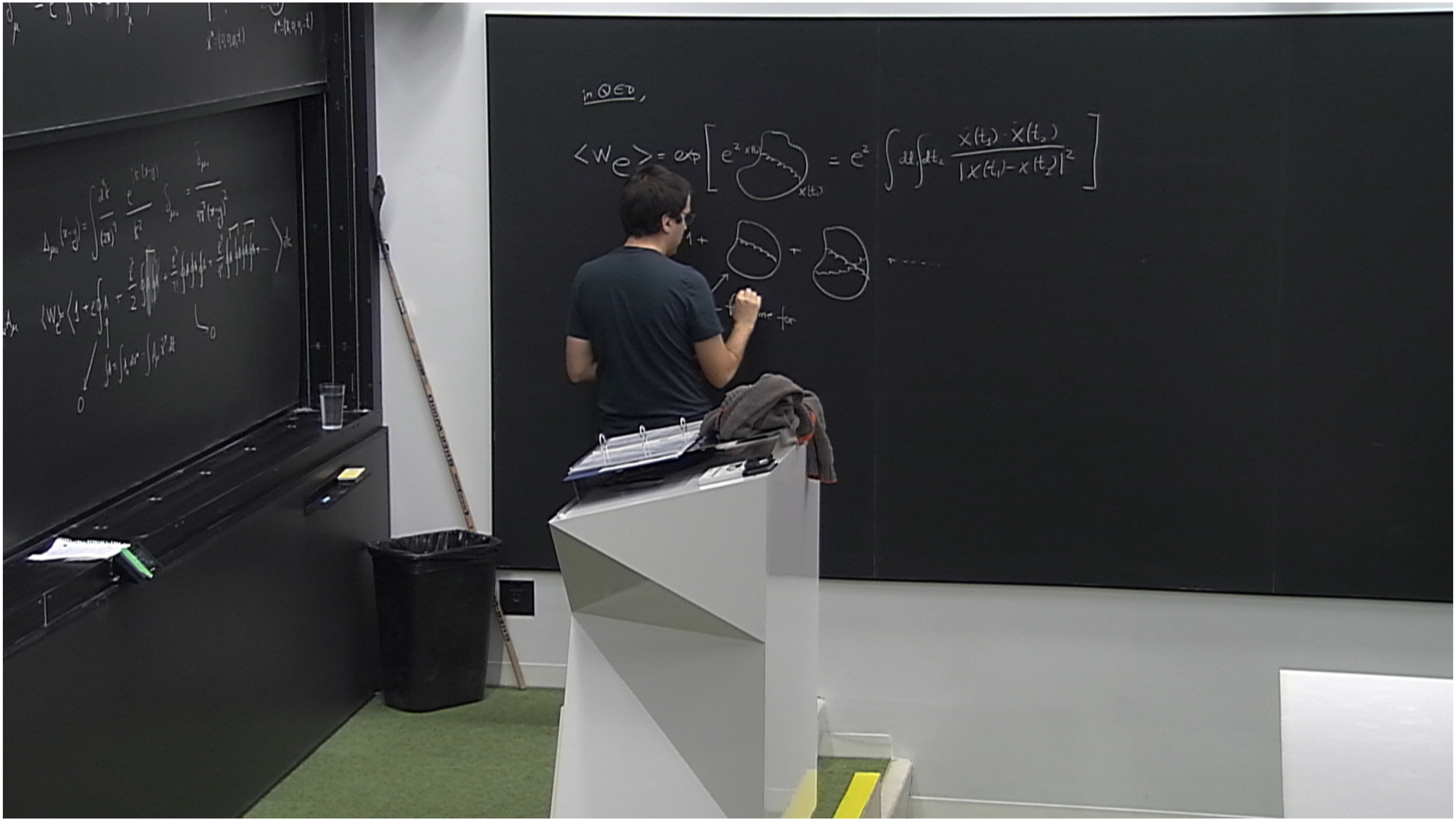

in QED,
$\langle W_e \rangle = \exp\left[e^2 \oint\oint = e^2 \int dt_1\, dt_2 \frac{\dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2} \right]$

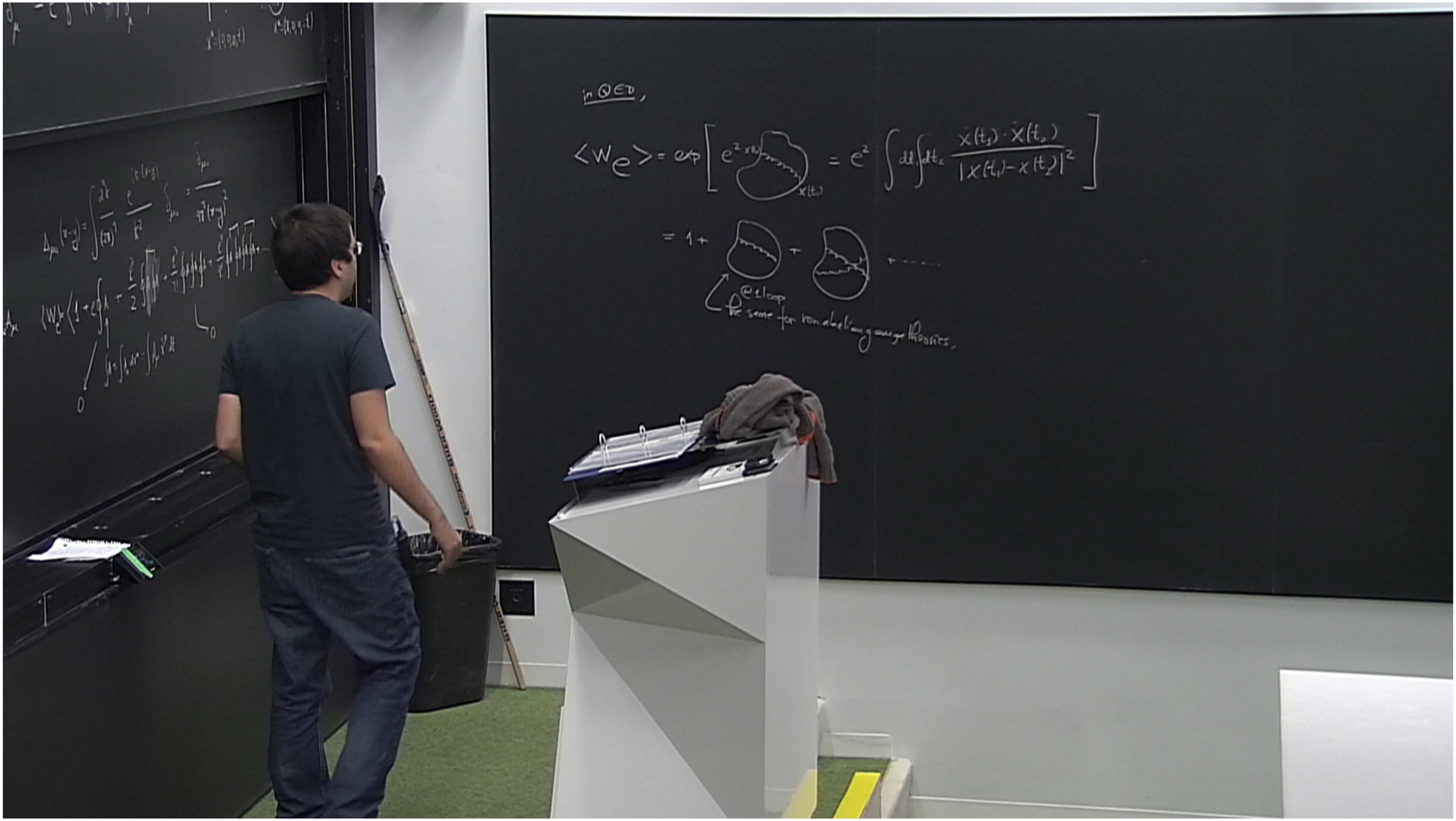

in QED,

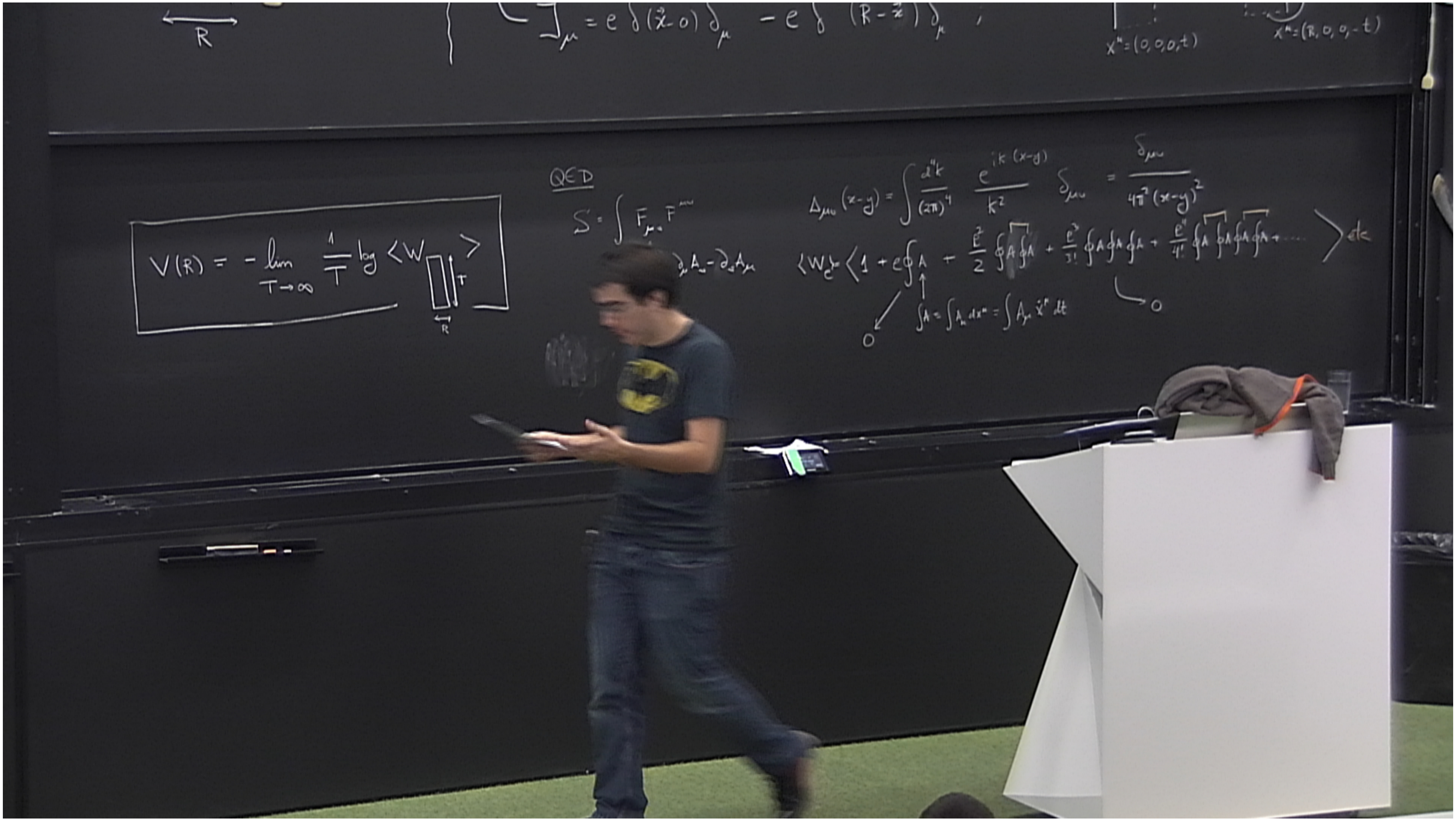

in QED,

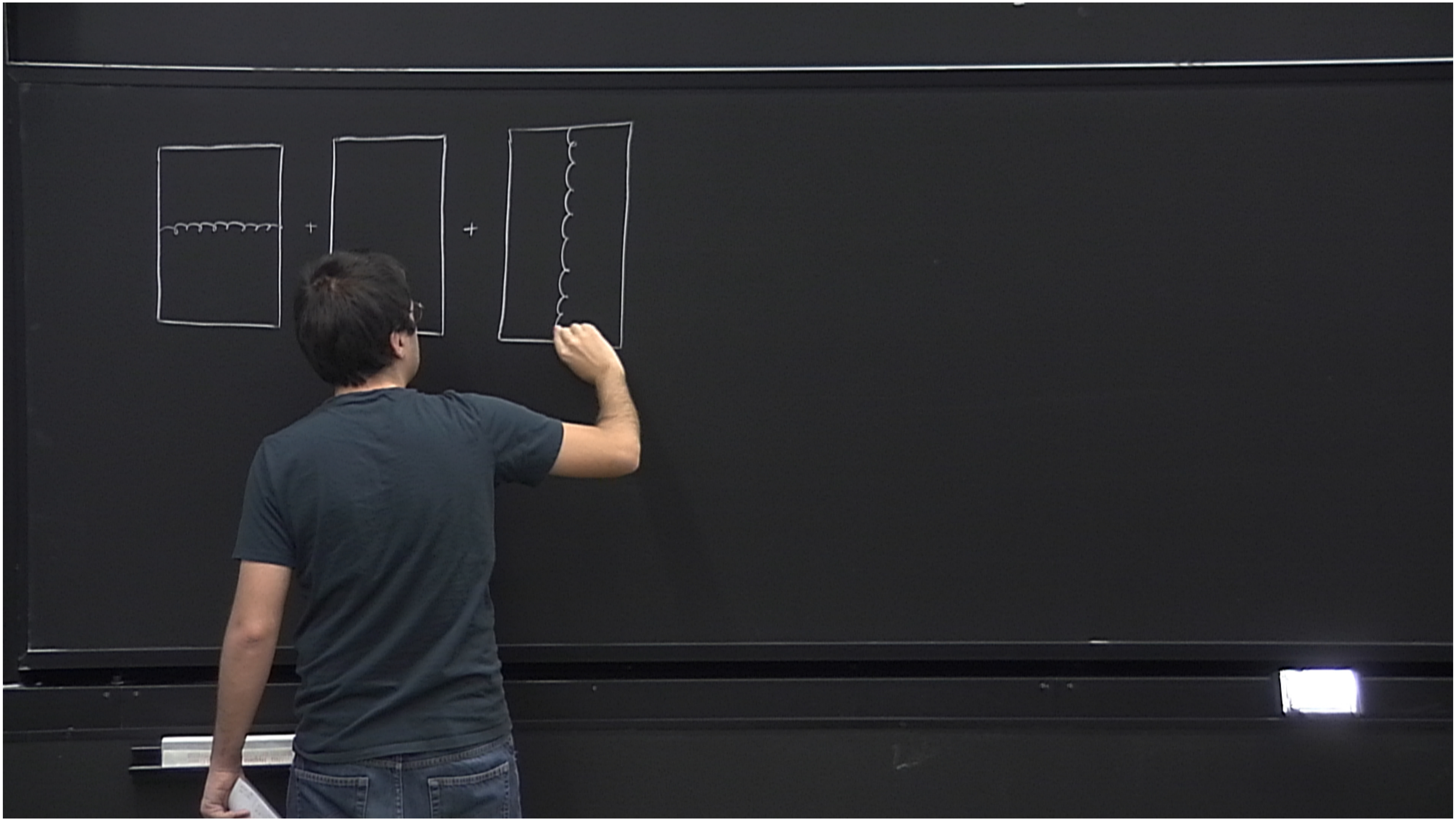

in QED,
= 1 +

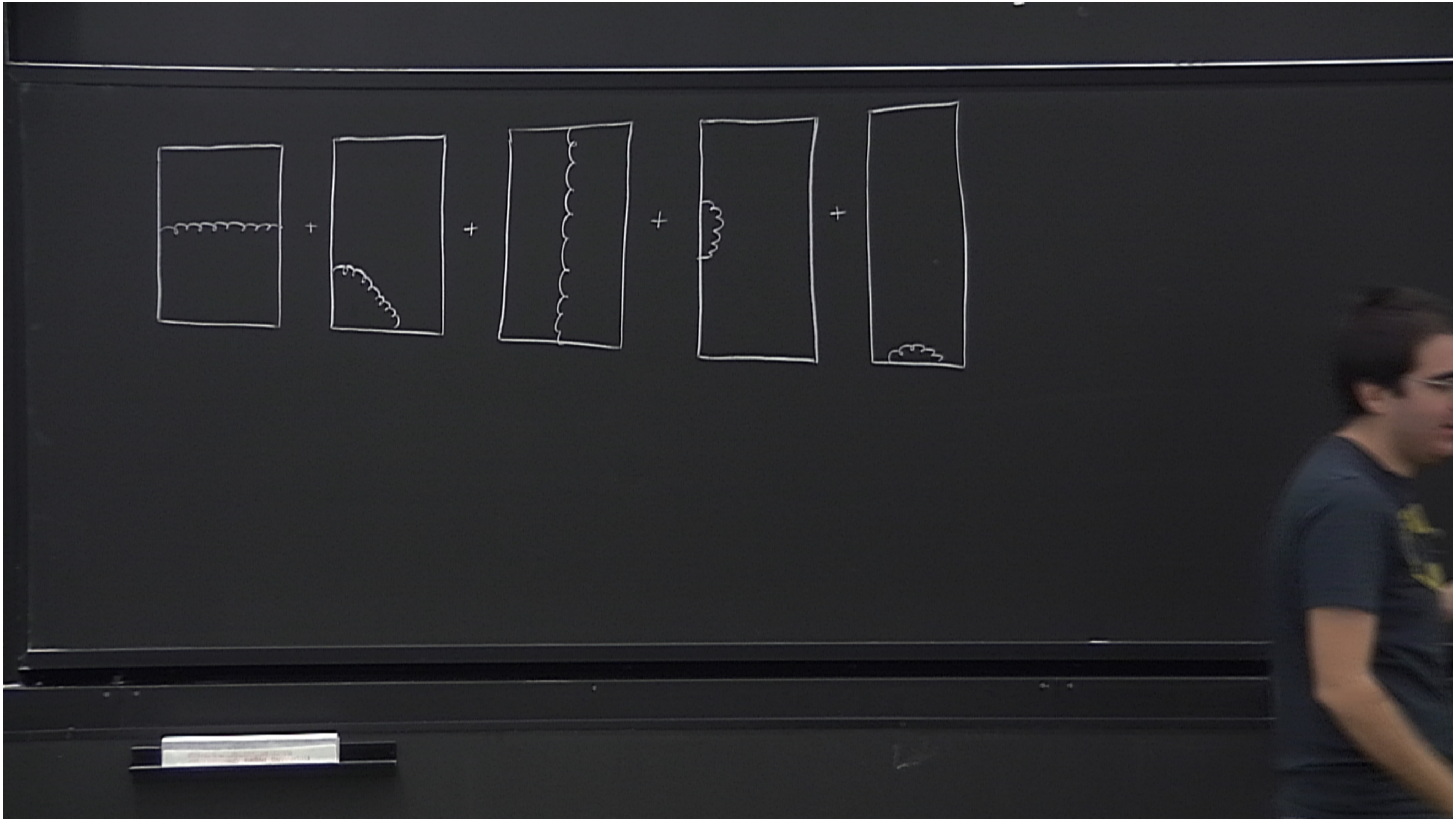

in QED,
= 1 +

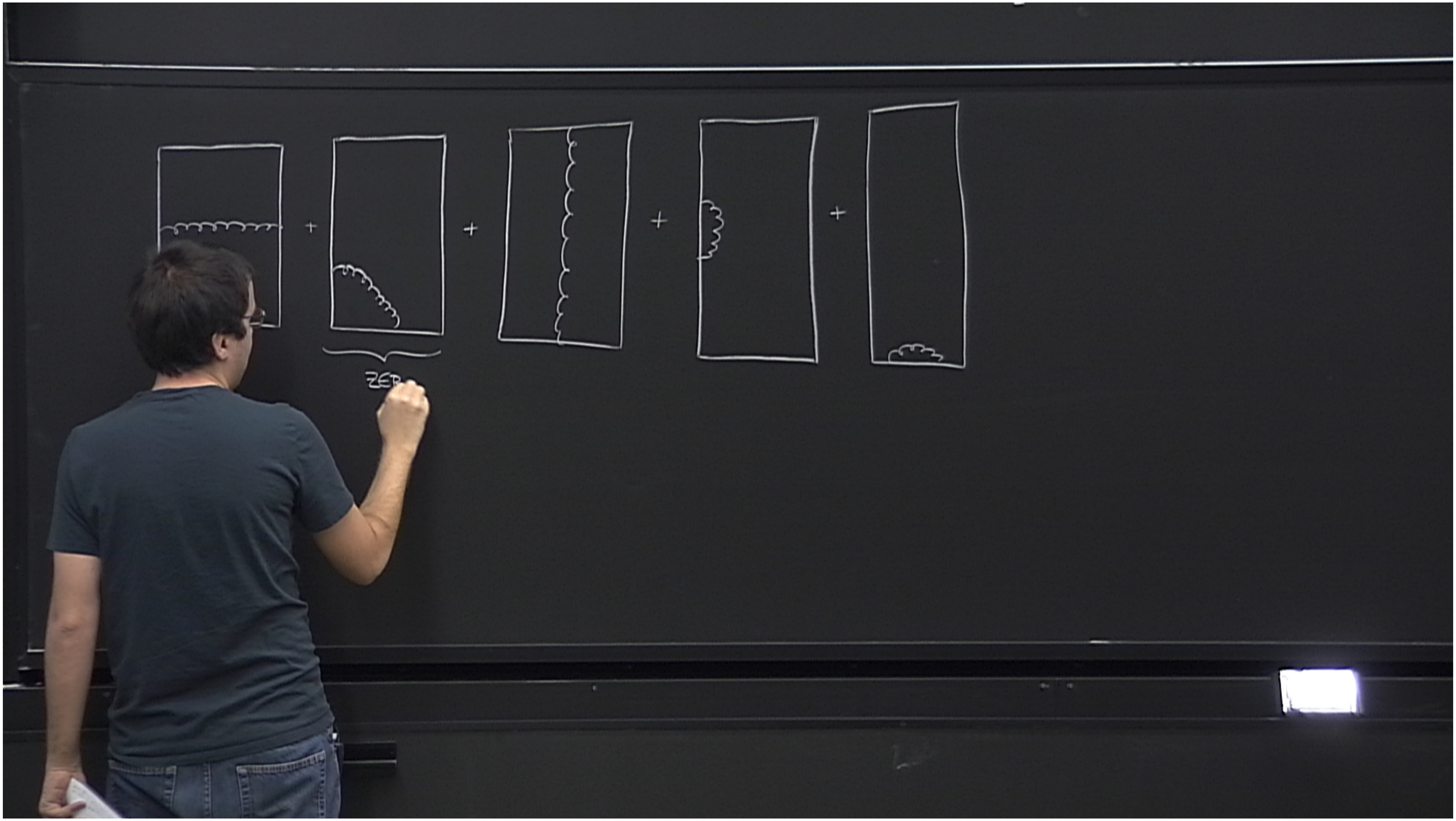

in QED,

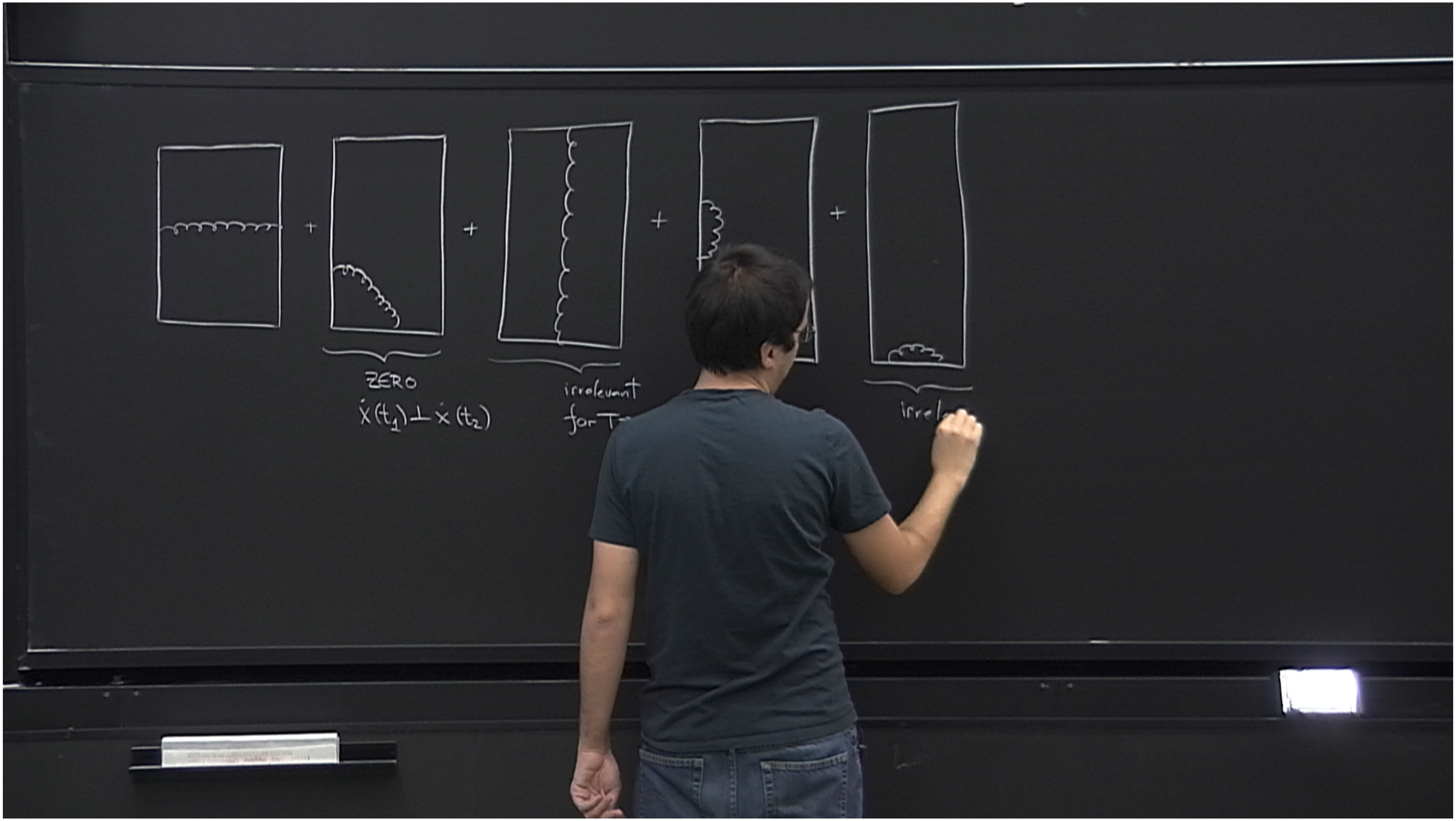

in QED,
$\langle W_e \rangle = \exp\left[e^2 \ldots = e^2 \int dt_1 \int dt_2 \frac{\dot{x}(t_1)\cdot\dot{x}(t_2)}{|x(t_1)-x(t_2)|^2} \right]$
$= 1 + \ldots + \ldots + \ldots$
@ 1 loop
the same for non abelian gauge theories.



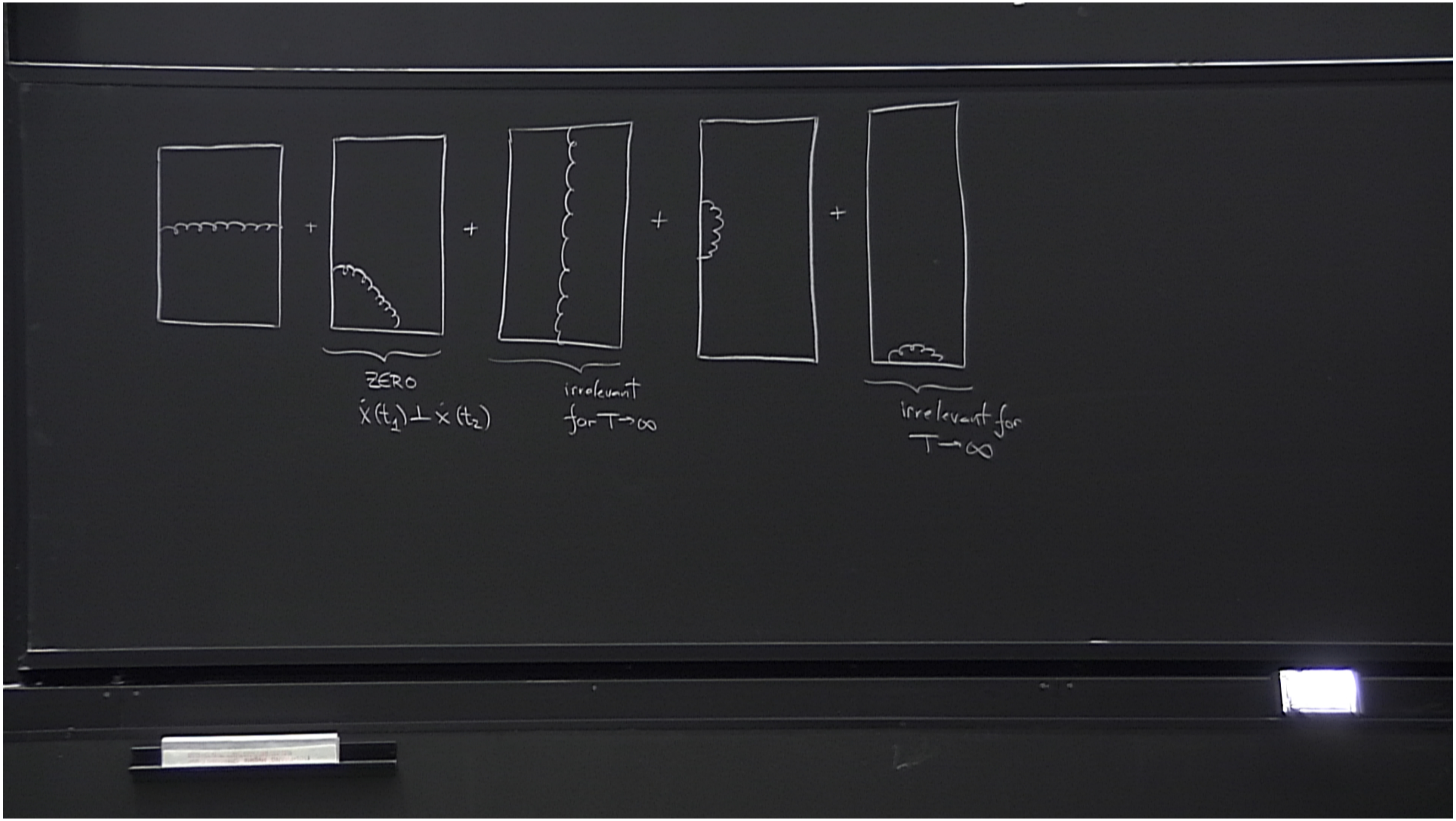


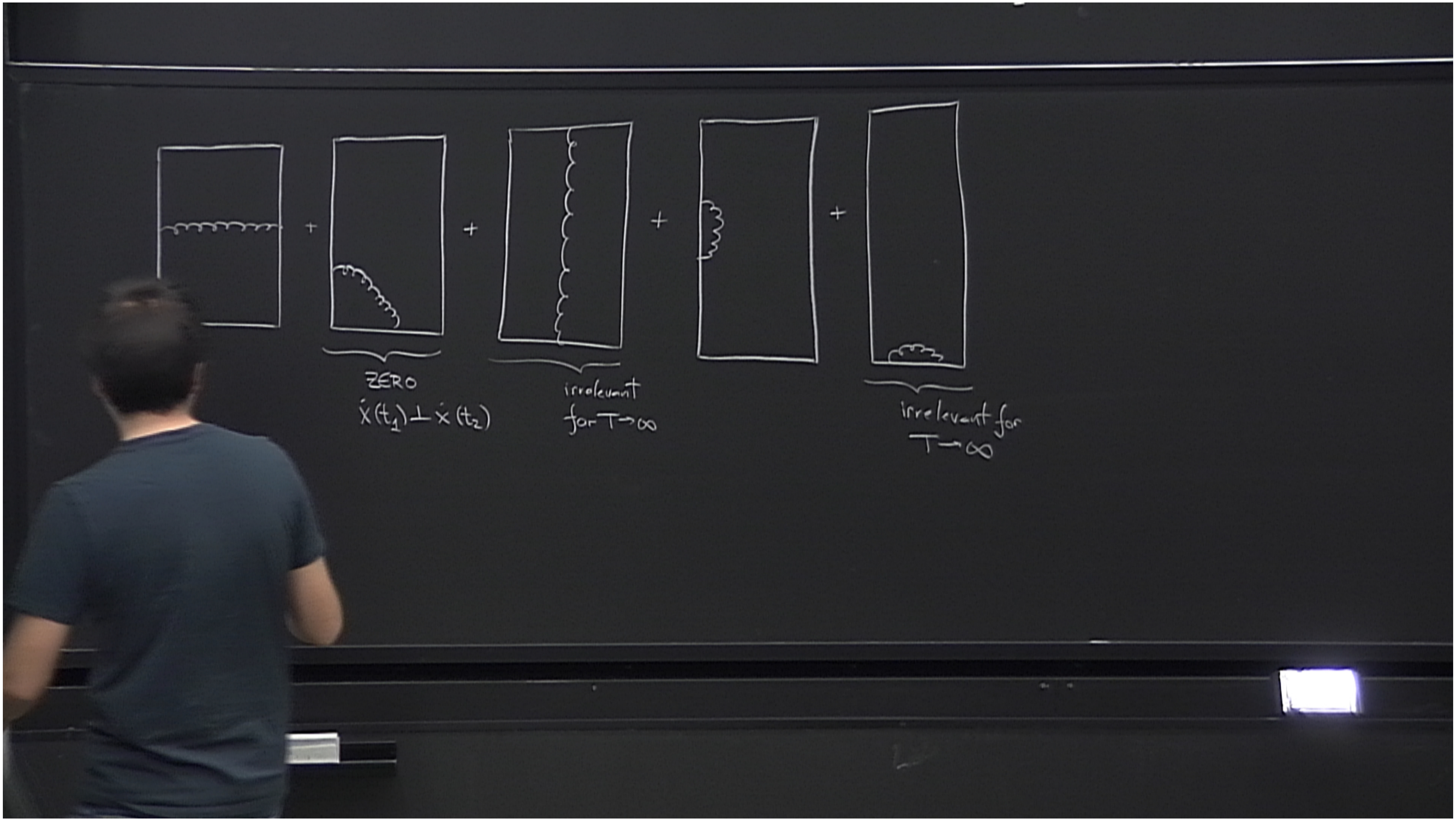




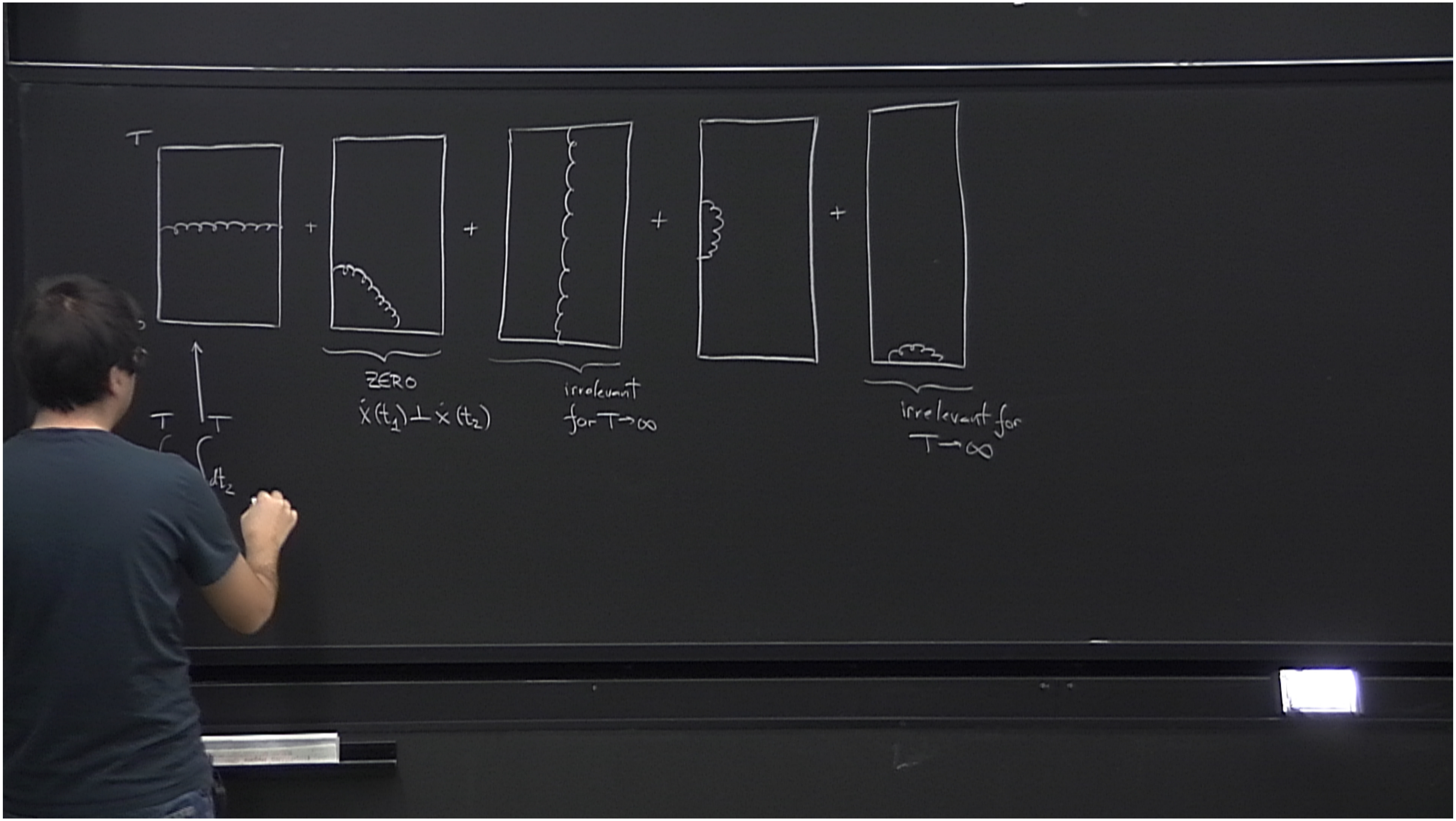


ZERO
$\dot{x}(t_1) \perp \dot{x}(t_2)$
irrelevant
for T
irrel

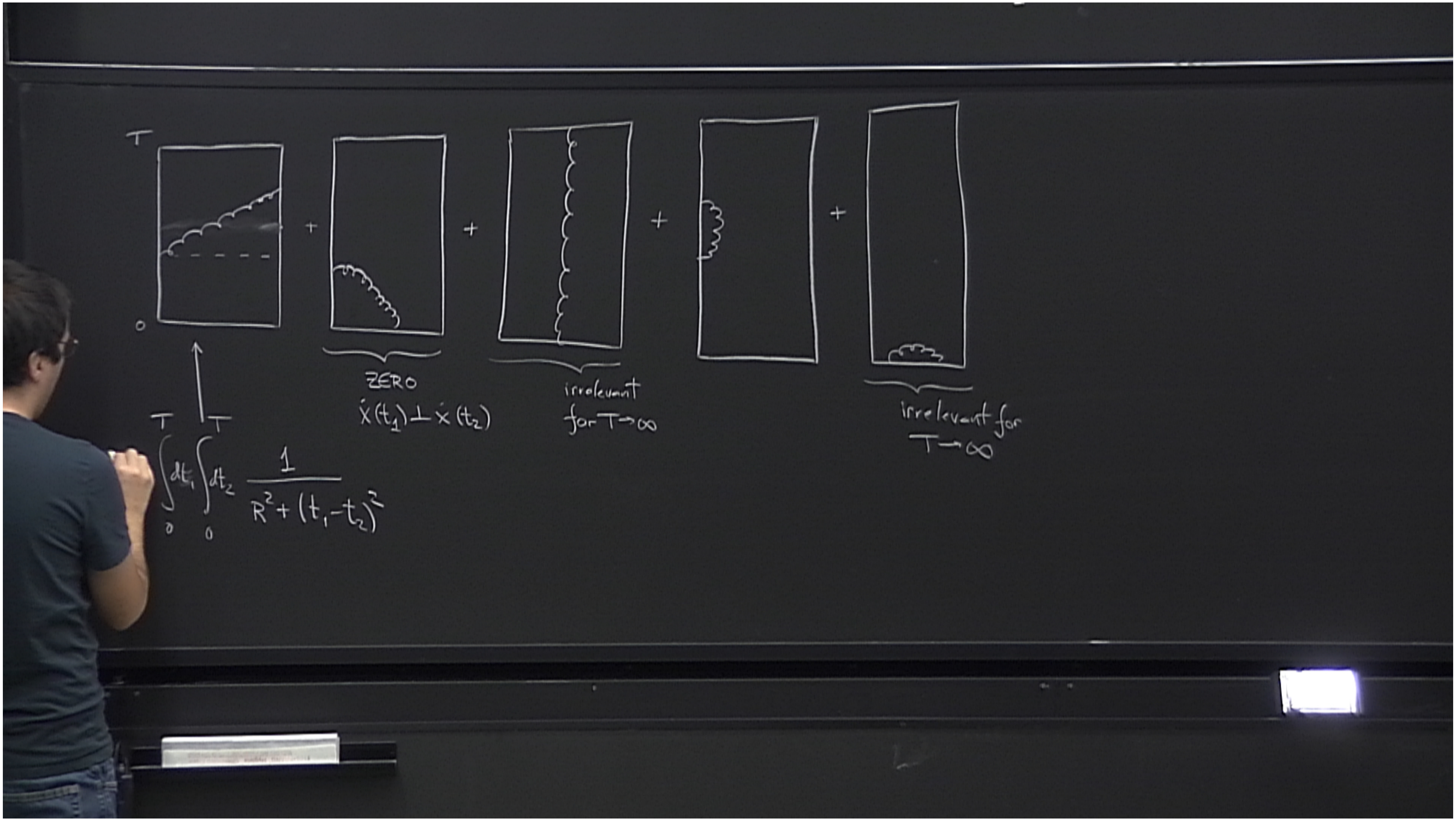

+
+
+
+
ZERO
$\dot{x}(t_1) \perp \dot{x}(t_2)$
irrelevant for $T \to \infty$
irrelevant for $T \to \infty$

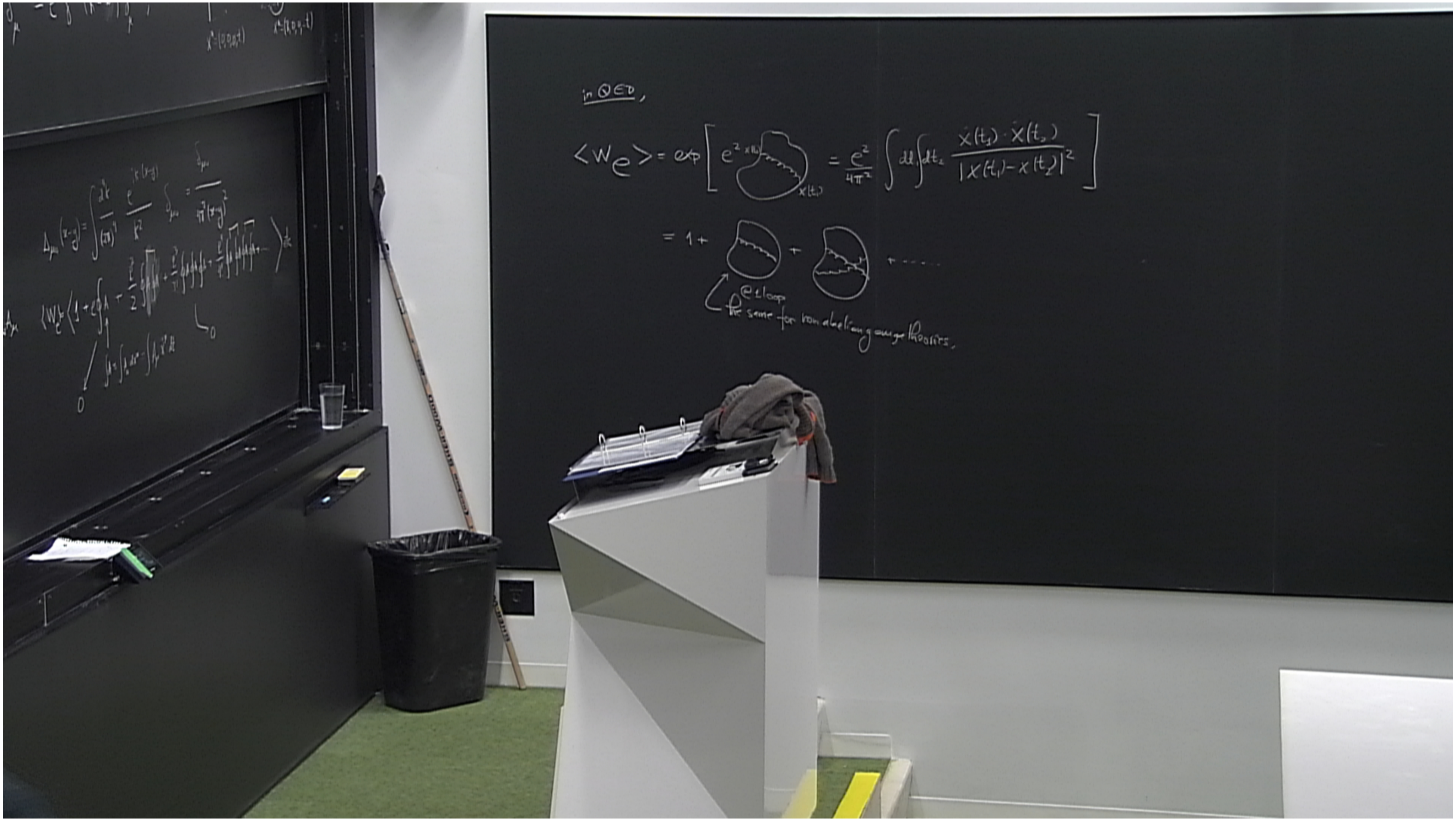


ZERO
$\dot{x}(t_1) \perp \dot{x}(t_2)$
irrelevant for $T \to \infty$
irrelevant for $T \to \infty$

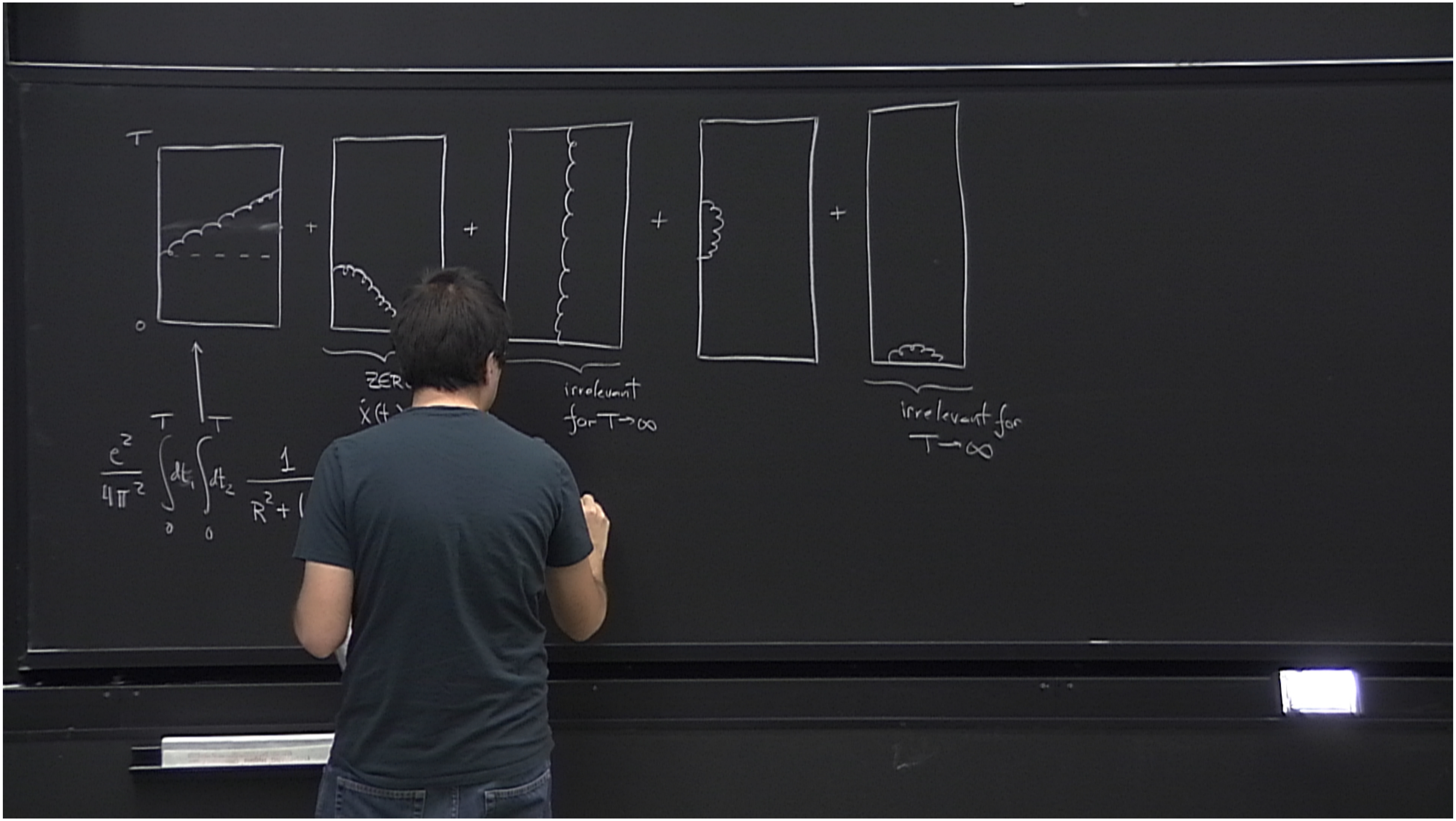


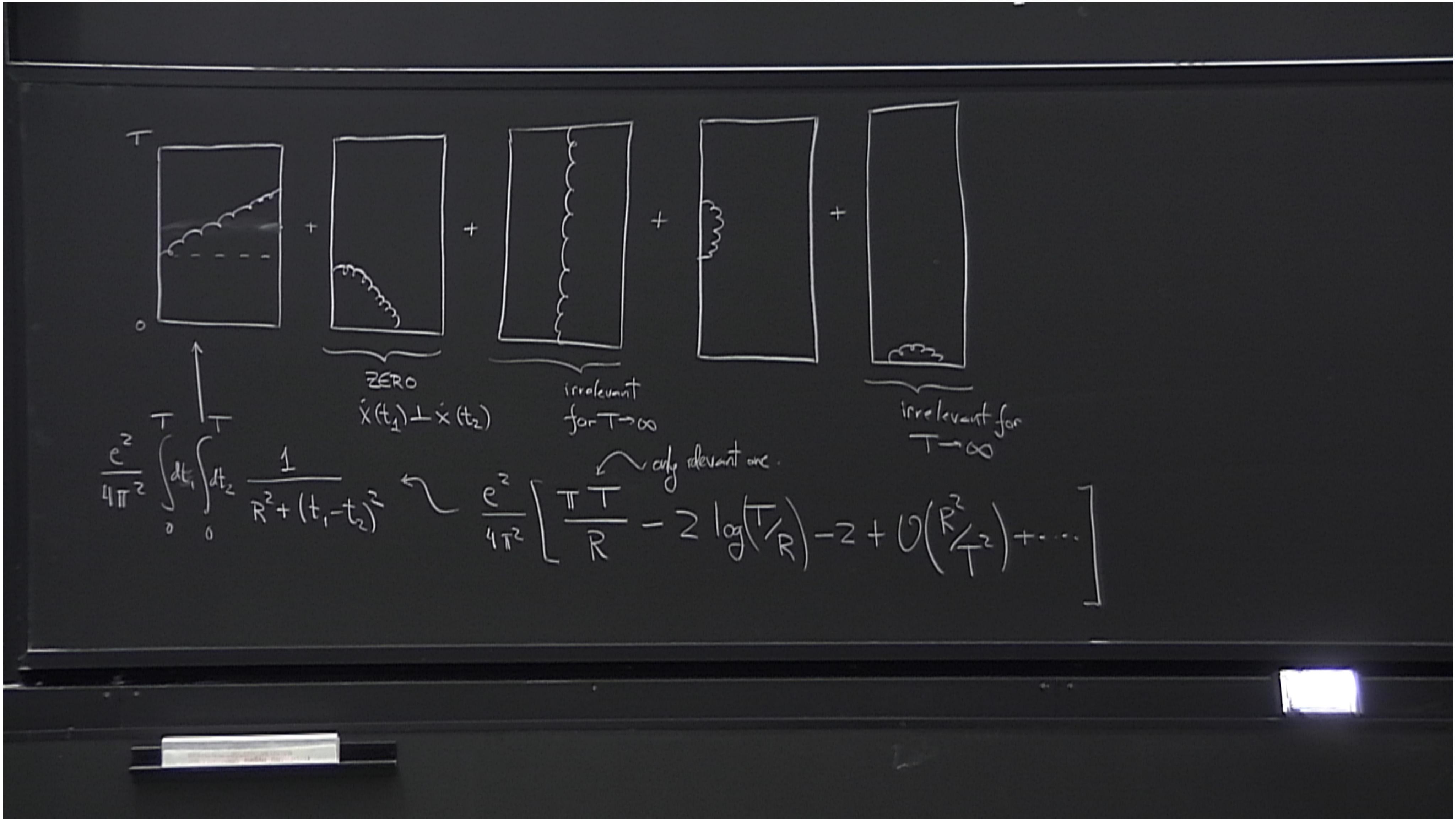
T
+
+
+
+
ZERO
ẋ(t1) ⊥ ẋ(t2)
irrelevant
for T→∞
irrelevant for
T→∞
T
T
dt2

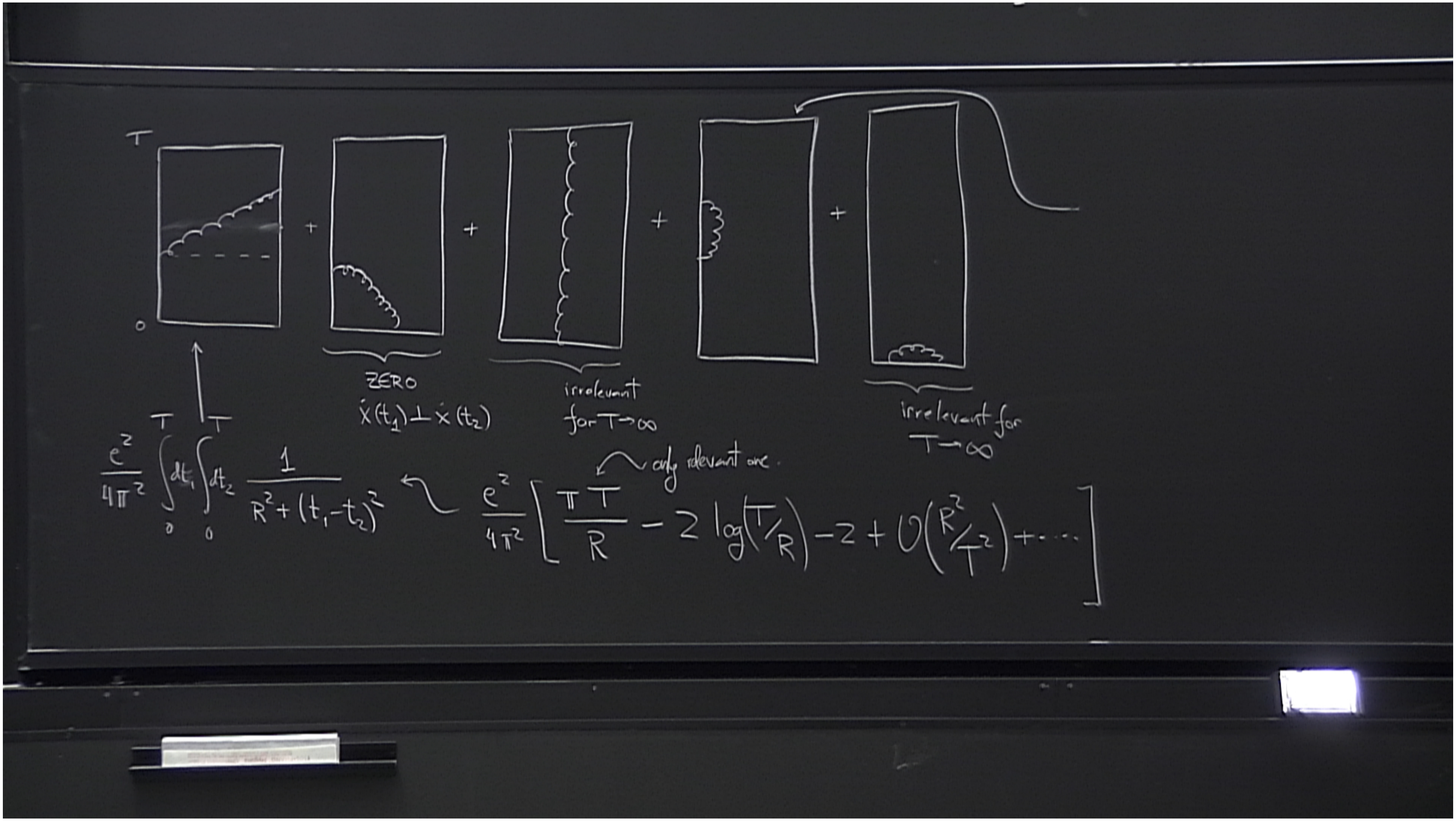

T
0
+
+
+
+
ZERO
$\dot{x}(t_1) \perp \dot{x}(t_2)$
irrelevant
for T→∞
irrelevant for
T→∞
$\int_0^T dt_1 \int_0^T dt_2 \frac{1}{R^2+(t_1-t_2)^2}$



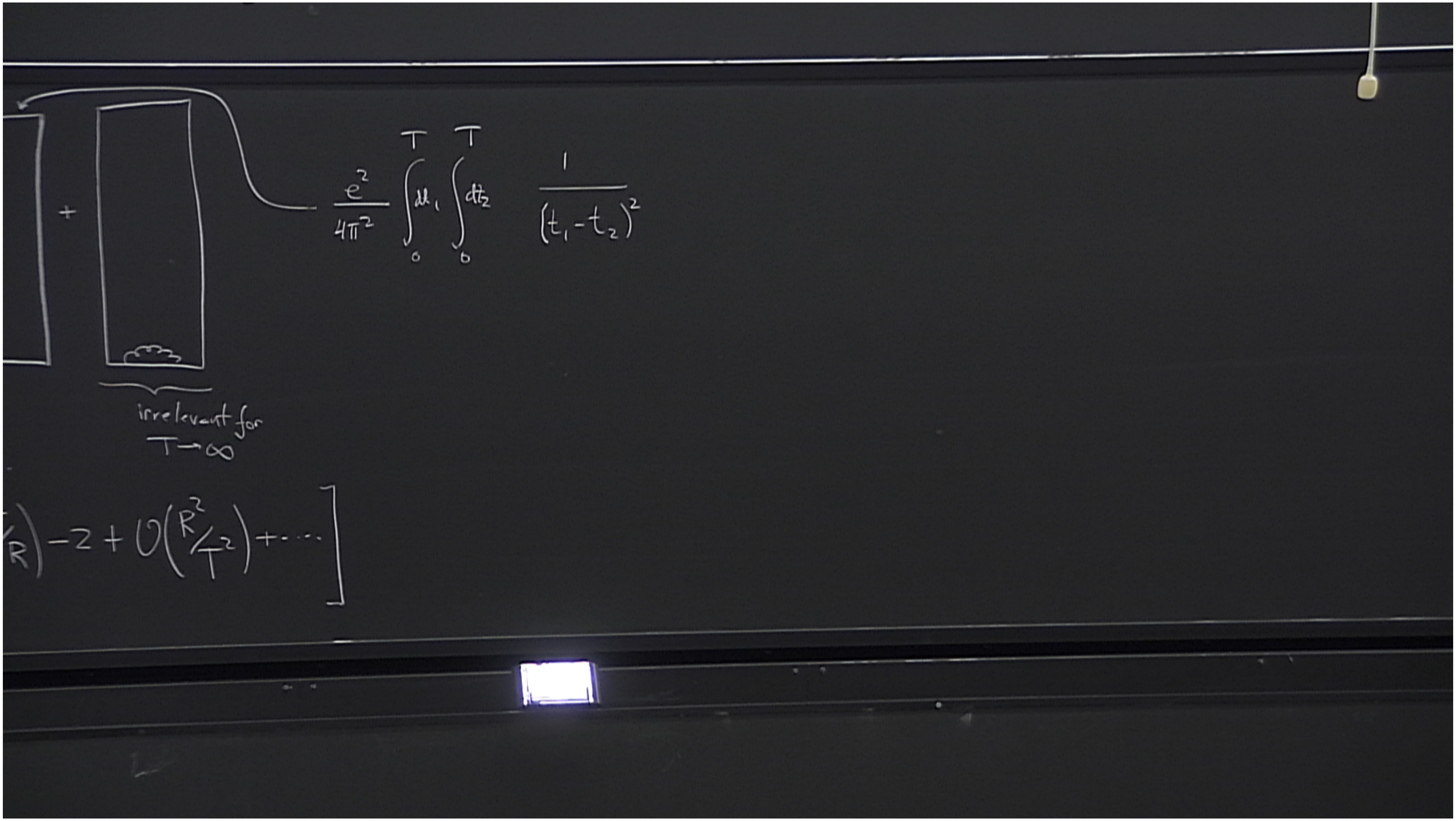
in QED,
= 1 +
the same for non abelian gauge theories,


T
0
+
+
+
+
ZER
irrelevant for T→∞
irrelevant for T→∞

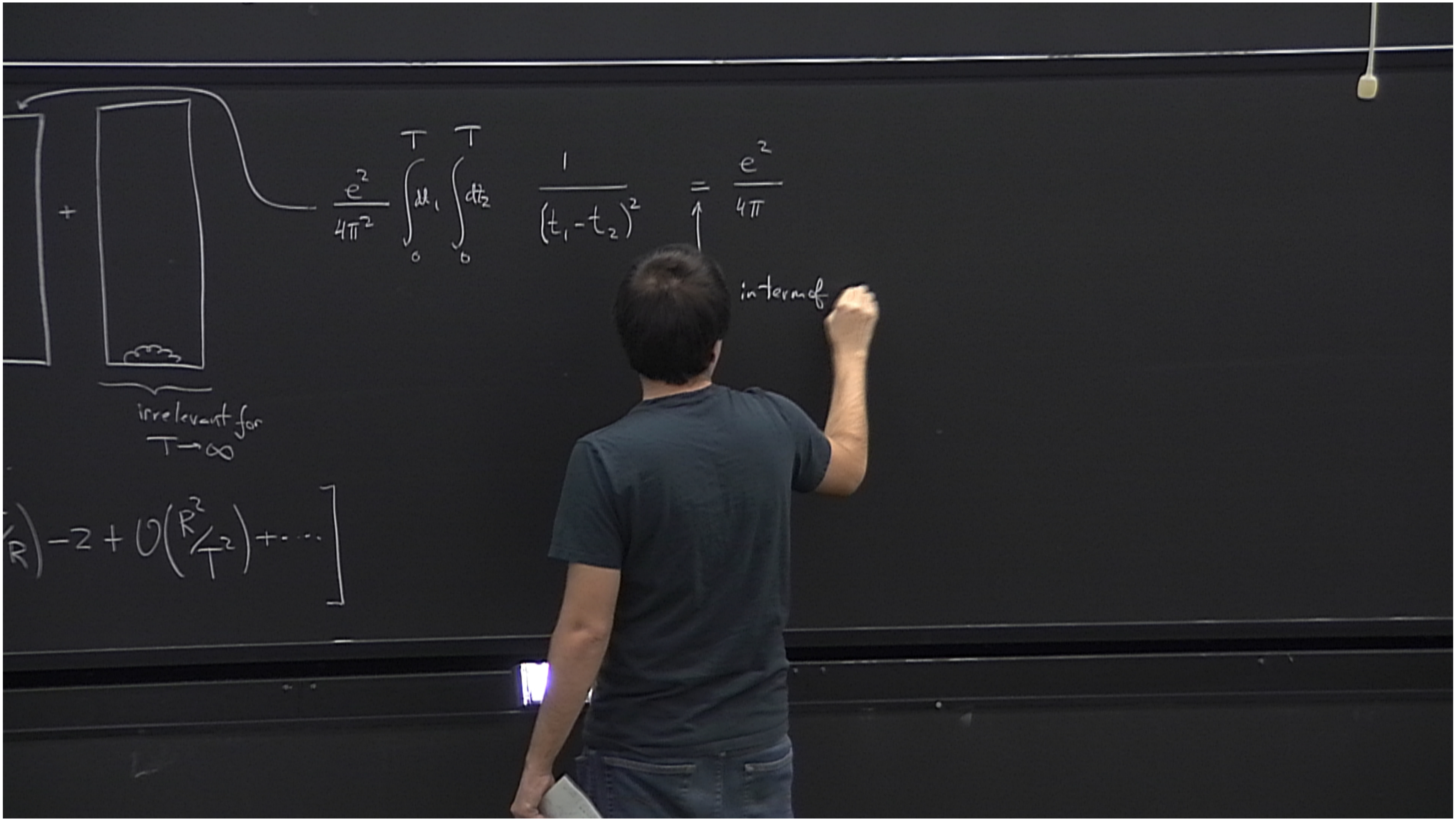

T
0
+
ZERO
$\dot{x}(t_1) \perp \dot{x}(t_2)$
irrelevant for $T \to \infty$
irrelevant for $T \to \infty$
only relevant one.
$\frac{e^2}{4\pi^2} \int_0^T dt_1 \int_0^T dt_2 \frac{1}{R^2 + (t_1 - t_2)^2} \sim \frac{e^2}{4\pi^2} \left[\pi \frac{T}{R} - 2 \log(T/R) - 2 + O(R^2/T^2) + \cdots \right]$



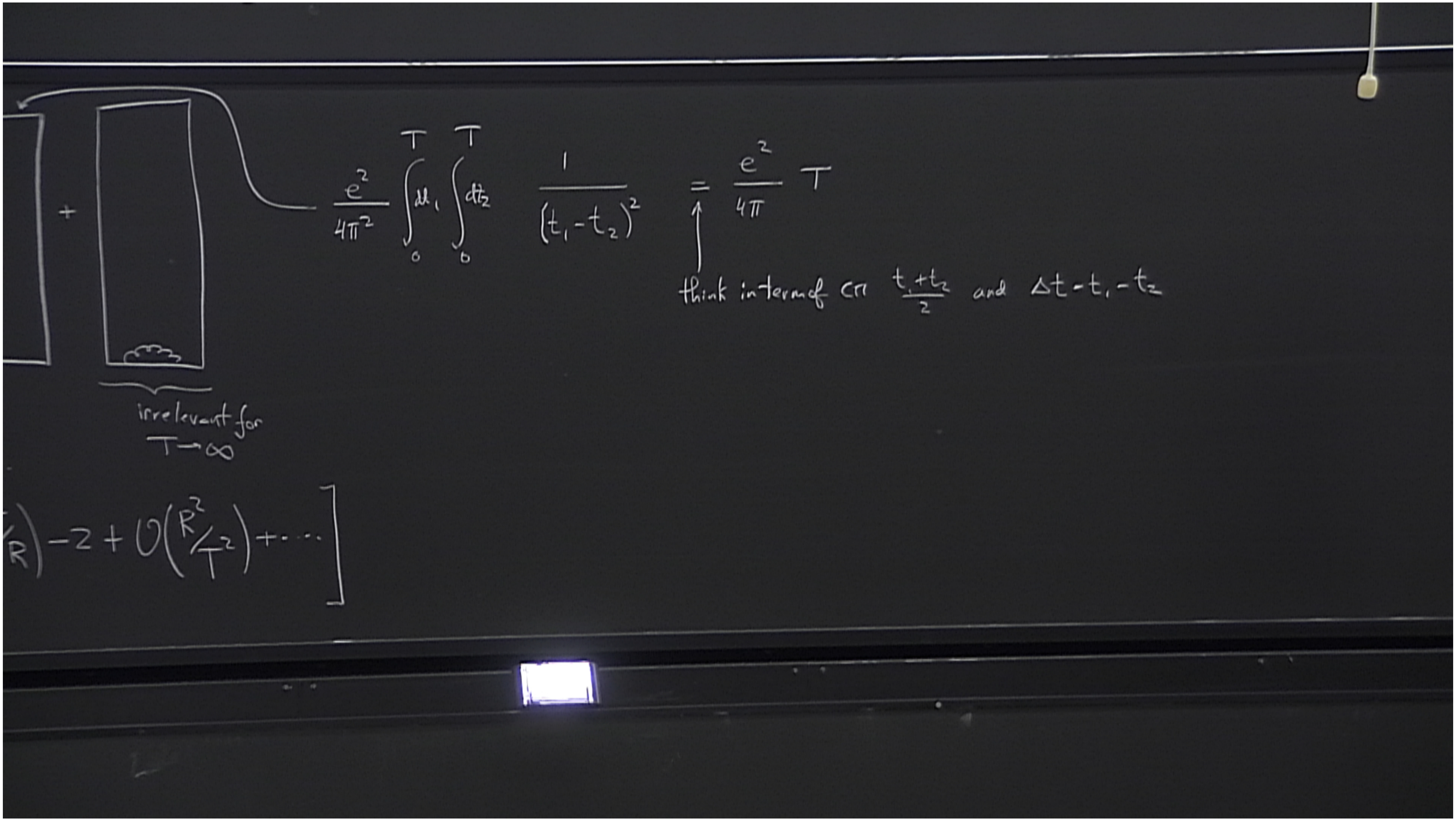
T
0
+
ZERO
ẋ(t1) ⊥ ẋ(t2)
irrelevant for T→∞
irrelevant for T→∞
only relevant one.
$\frac{e^2}{4\pi^2}\int_0^T dt_1\int_0^T dt_2 \frac{1}{R^2+(t_1-t_2)^2} \sim \frac{e^2}{4\pi^2}\left[\pi\frac{T}{R} - 2\log(T/R) - 2 + O(R^2/T^2) + \cdots\right]$

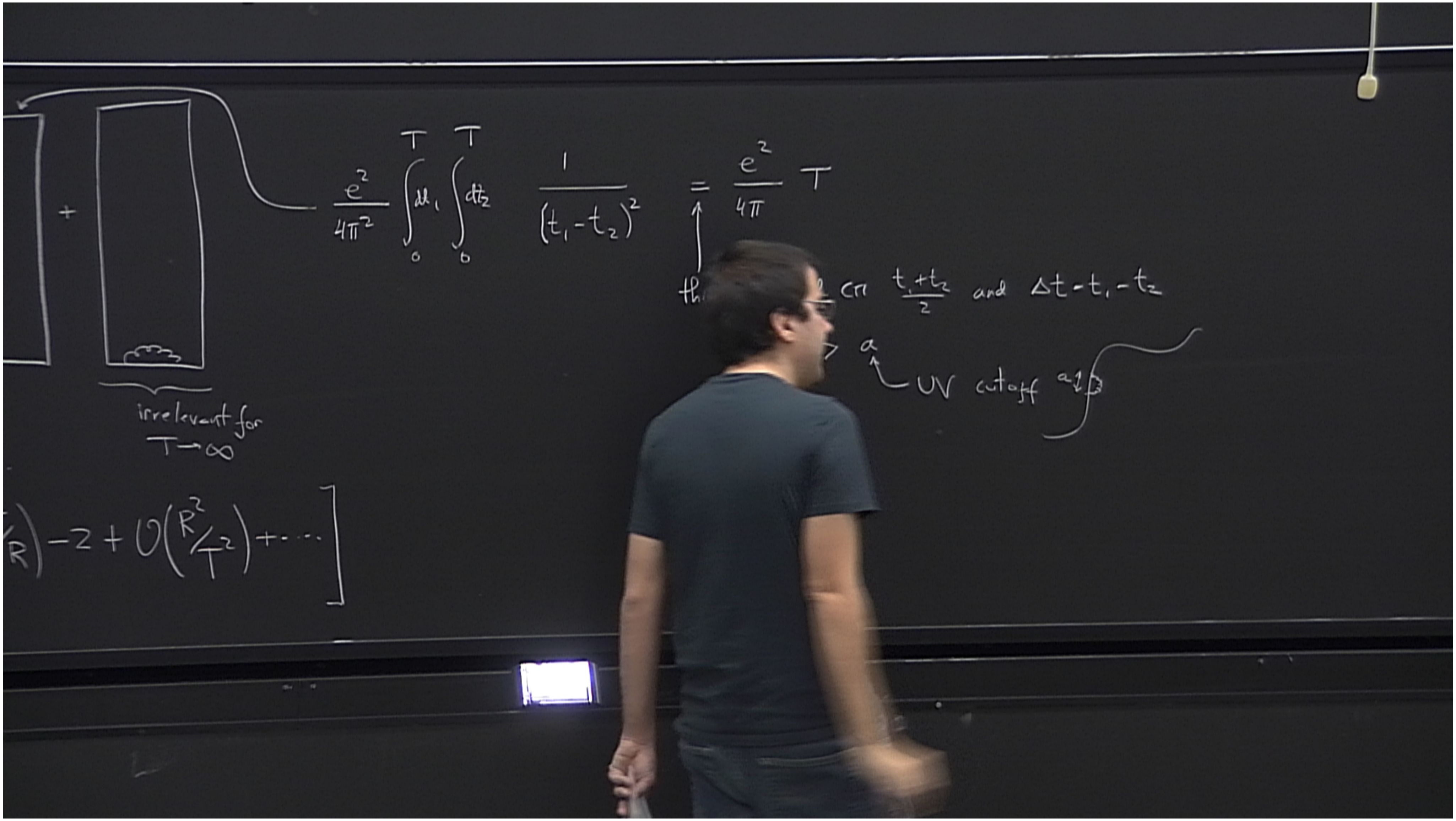


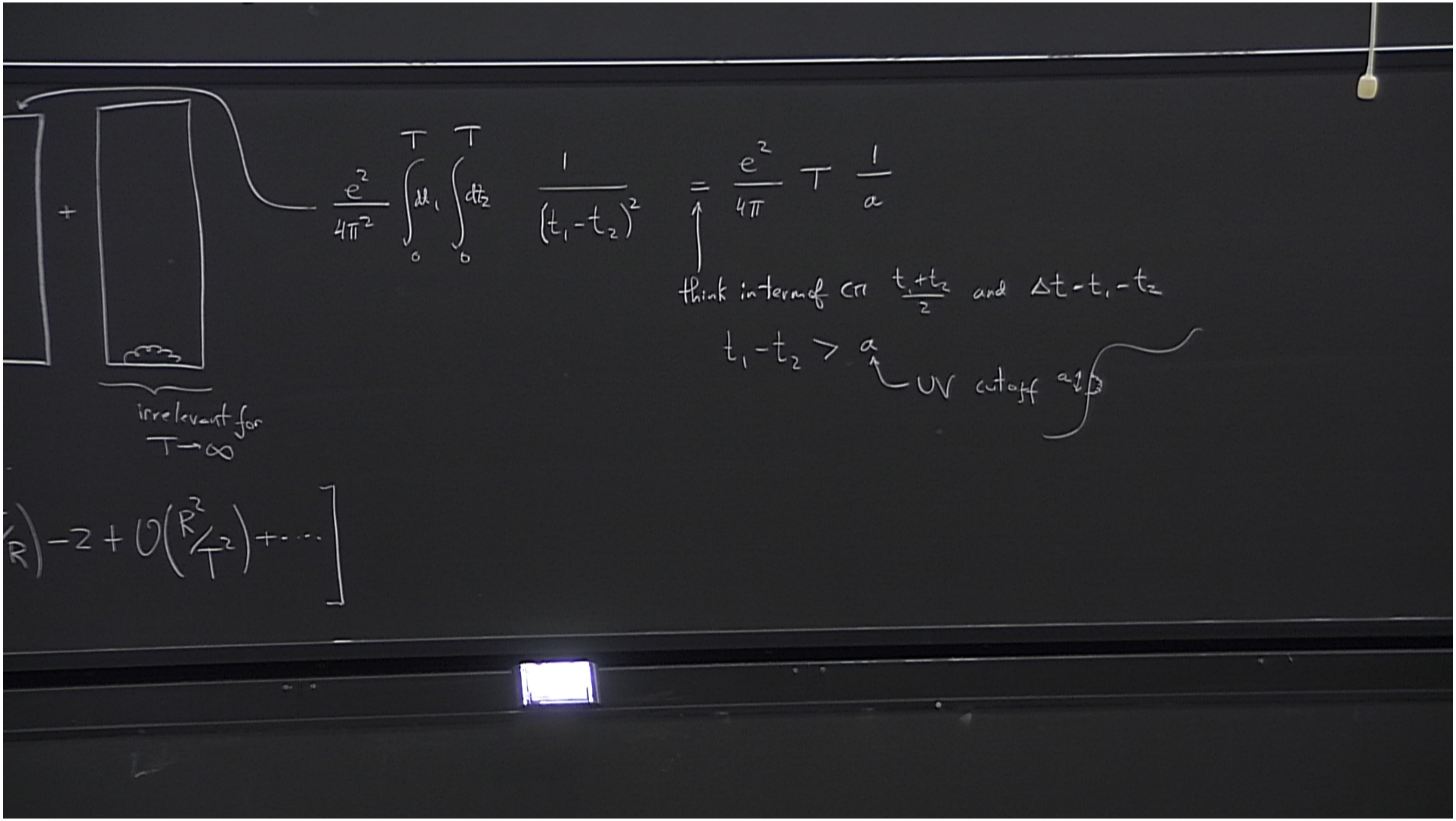
+
$\frac{e^2}{4\pi^2}\int_0^T dt_1 \int_0^T dt_2 \frac{1}{(t_1-t_2)^2}$
irrelevant for
$T\to\infty$
$R) - 2 + O(R^2/T^2) + \cdots]$

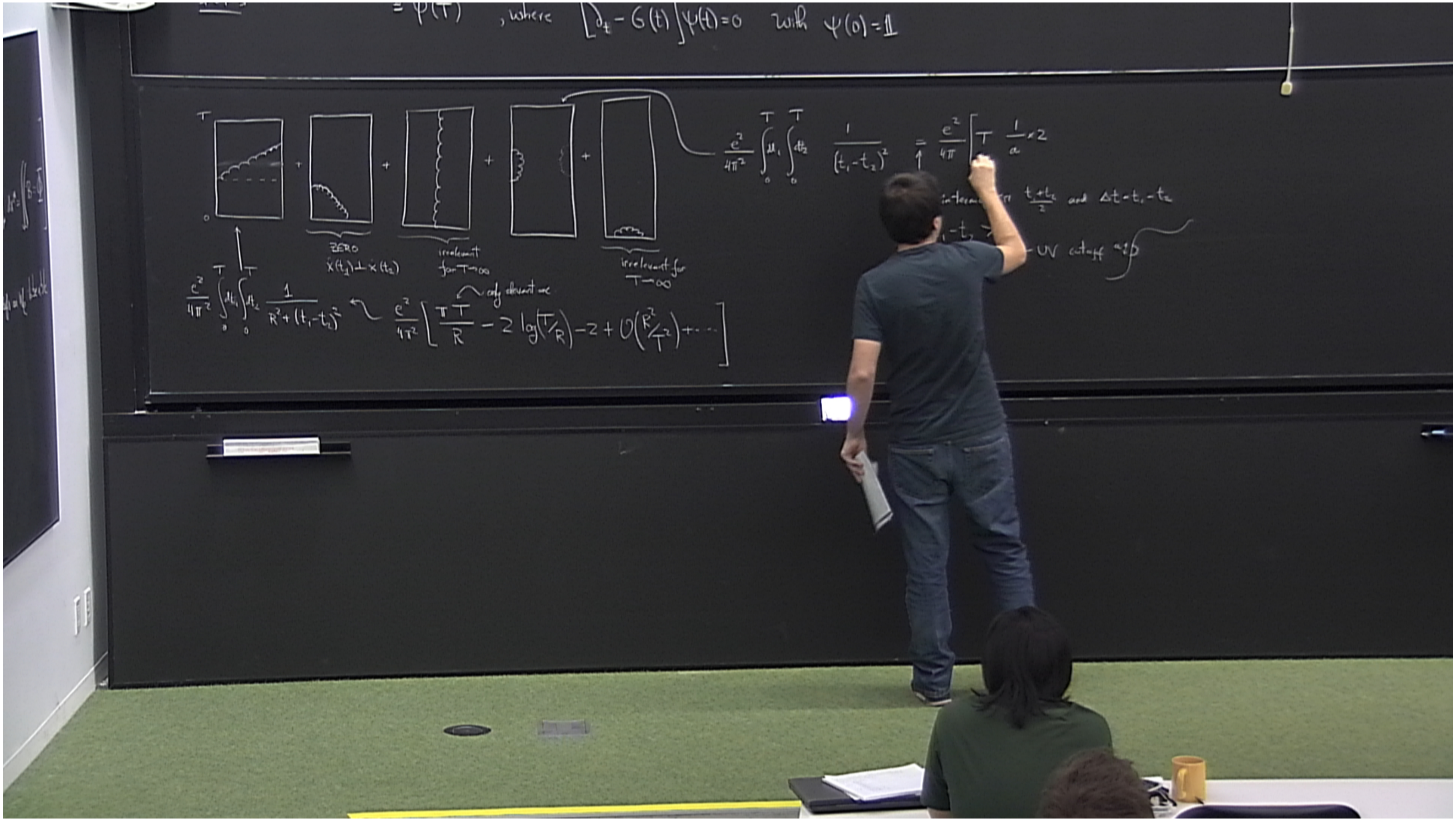


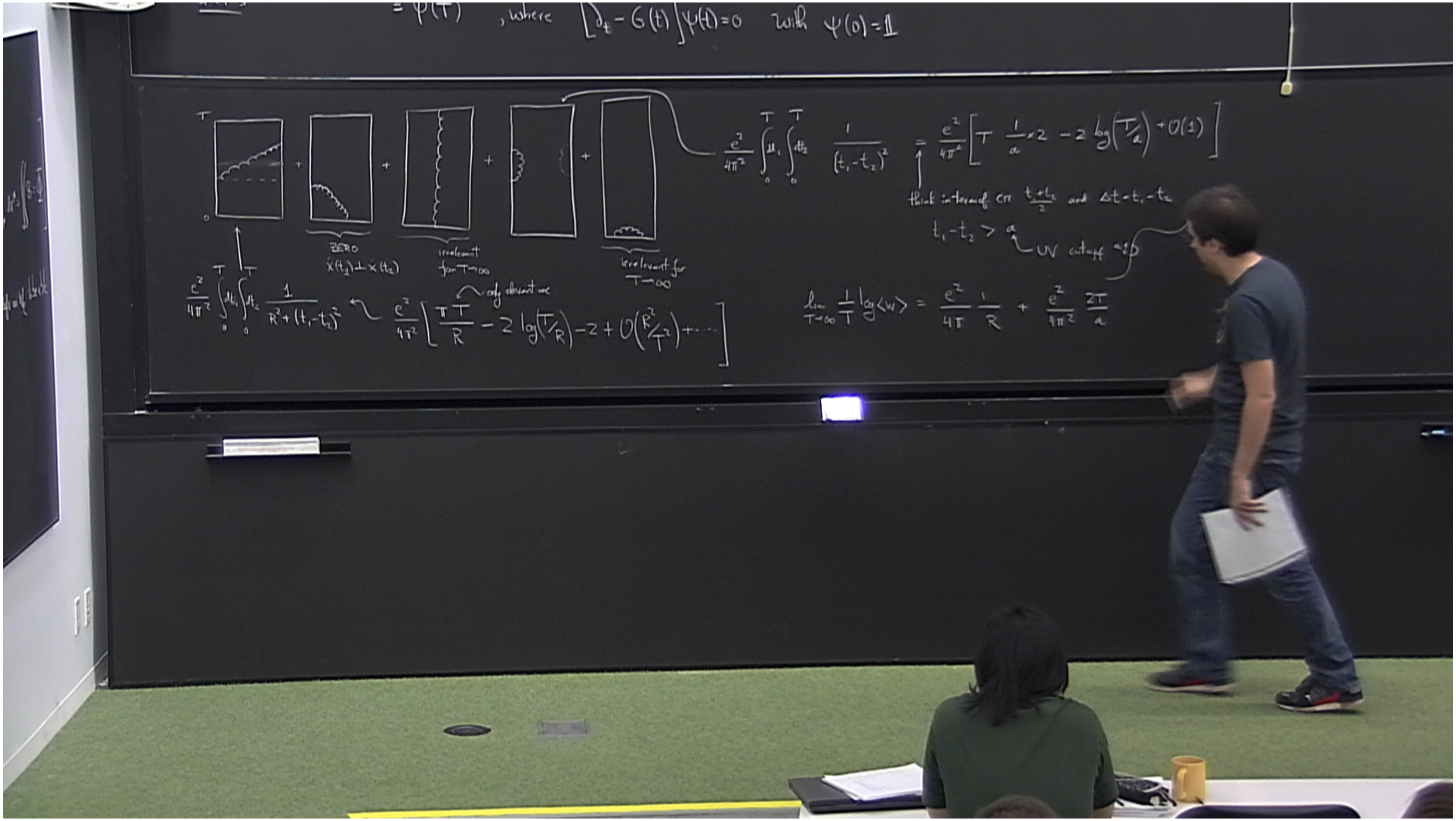
+
e^2 / 4π^2 ∫_0^T dt_1 ∫_0^T dt_2 1 / (t_1 - t_2)^2 = e^2 / 4π
irrelevant for
T → ∞
in term of
R) − 2 + O(R^2/T^2) + . . .]


+
$\frac{e^2}{4\pi^2}\int_0^T dt_1 \int_0^T dt_2 \frac{1}{(t_1-t_2)^2} = \frac{e^2}{4\pi} T$
think in terms of cm $\frac{t_1+t_2}{2}$ and $\Delta t = t_1 - t_2$
irrelevant for $T \to \infty$
$R) - 2 + O(R^2/T^2) + \cdots]$


$\frac{e^2}{4\pi^2}\int_0^T dt_1 \int_0^T dt_2 \frac{1}{(t_1-t_2)^2} = \frac{e^2}{4\pi} T$
irrelevant for
T → ∞
$-2 + O(R^2/T^2) + \cdots$
and $\Delta t = t_1 - t_2$
UV cutoff


e^2 / 4π^2 ∫_0^T dt_1 ∫_0^T dt_2 1 / (t_1 - t_2)^2 = e^2 / 4π T 1/a
think in terms of cm (t_1+t_2)/2 and Δt = t_1 - t_2
t_1 - t_2 > a
UV cutoff
irrelevant for T→∞
) -2 + O(R^2/T^2) + . . .]


where
with
ZERO
irrelevant for T→∞
irrelevant for T→∞
UV cutoff


, where
with
ZERO
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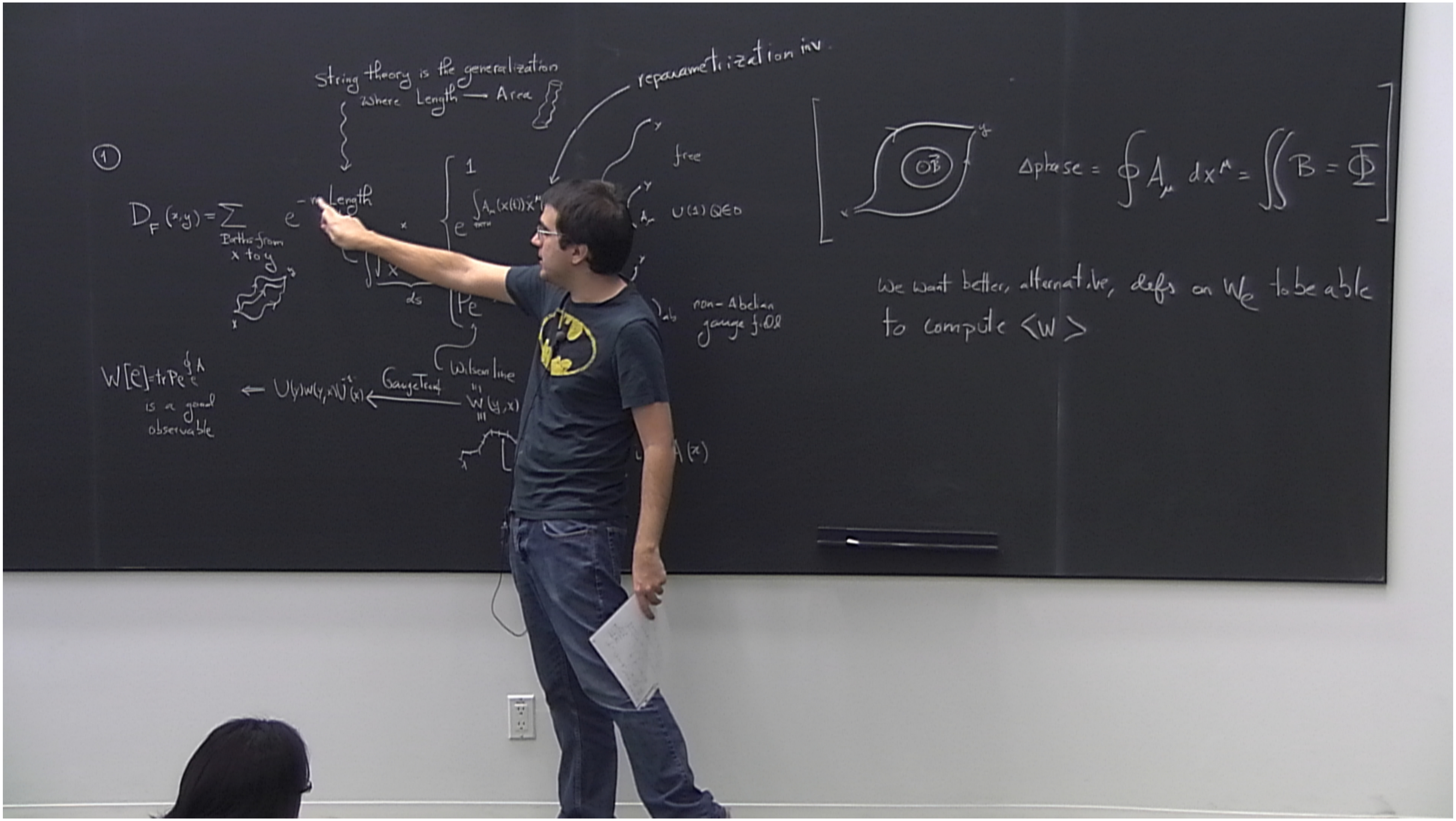
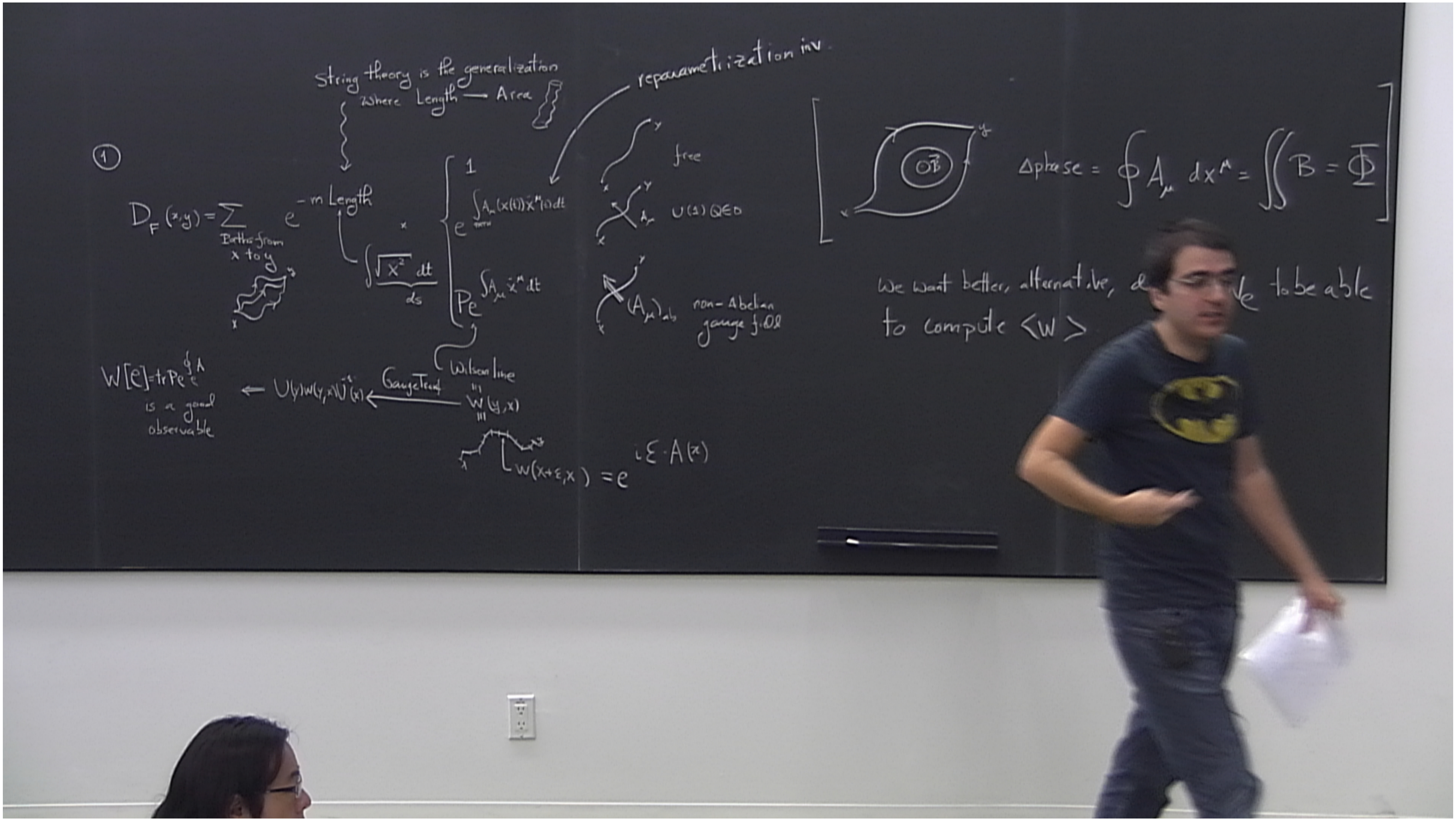
irrelevant for T→∞
only relevant one
think integral on
and
UV cutoff


String theory is the generalization
where Length → Area
reparametrization inv.
Paths from x to y
free
U(1) QED
non-Abelian gauge field
Wilson line
Gauge Transf
is a good observable
Δphase = ∮ A_μ dx^μ = ∬ B = Φ
We want better, alternative, defs on W_e to be able to compute ⟨W⟩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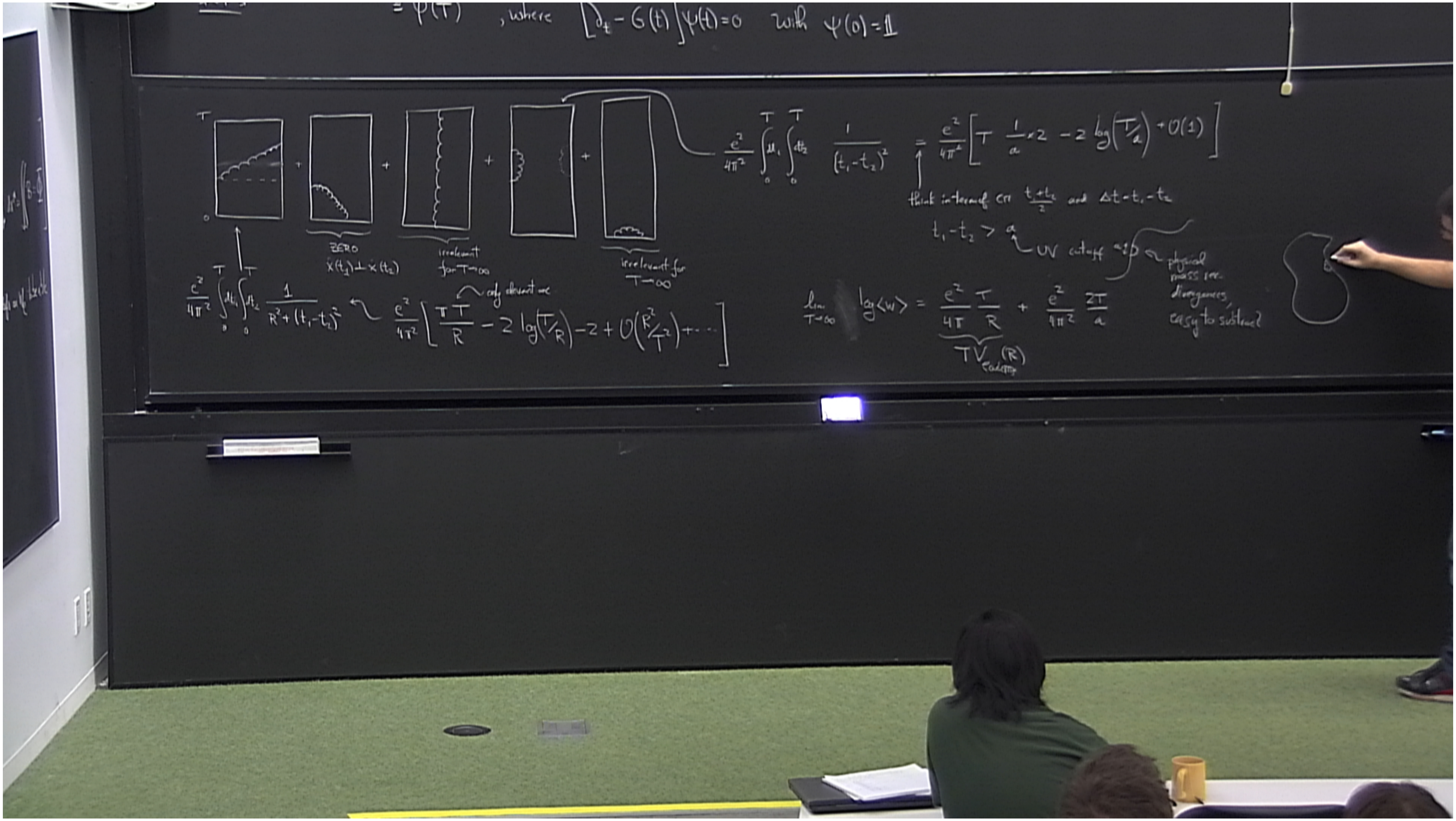

, where
ZERO
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irrelevant for T→∞
think in terms of
UV cutoff
physical mass ren. divergences, easy to subtract?

$$\int_0^T dt_2 \, \frac{1}{(t_1-t_2)^2} = \frac{e^2}{4\pi^2}\left[T \, \frac{1}{a} \times 2 - 2 \log\left(T/a\right) + O(1) \right]$$

think in term of cm $\frac{t_1+t_2}{2}$ and $\Delta t = t_1 - t_2$

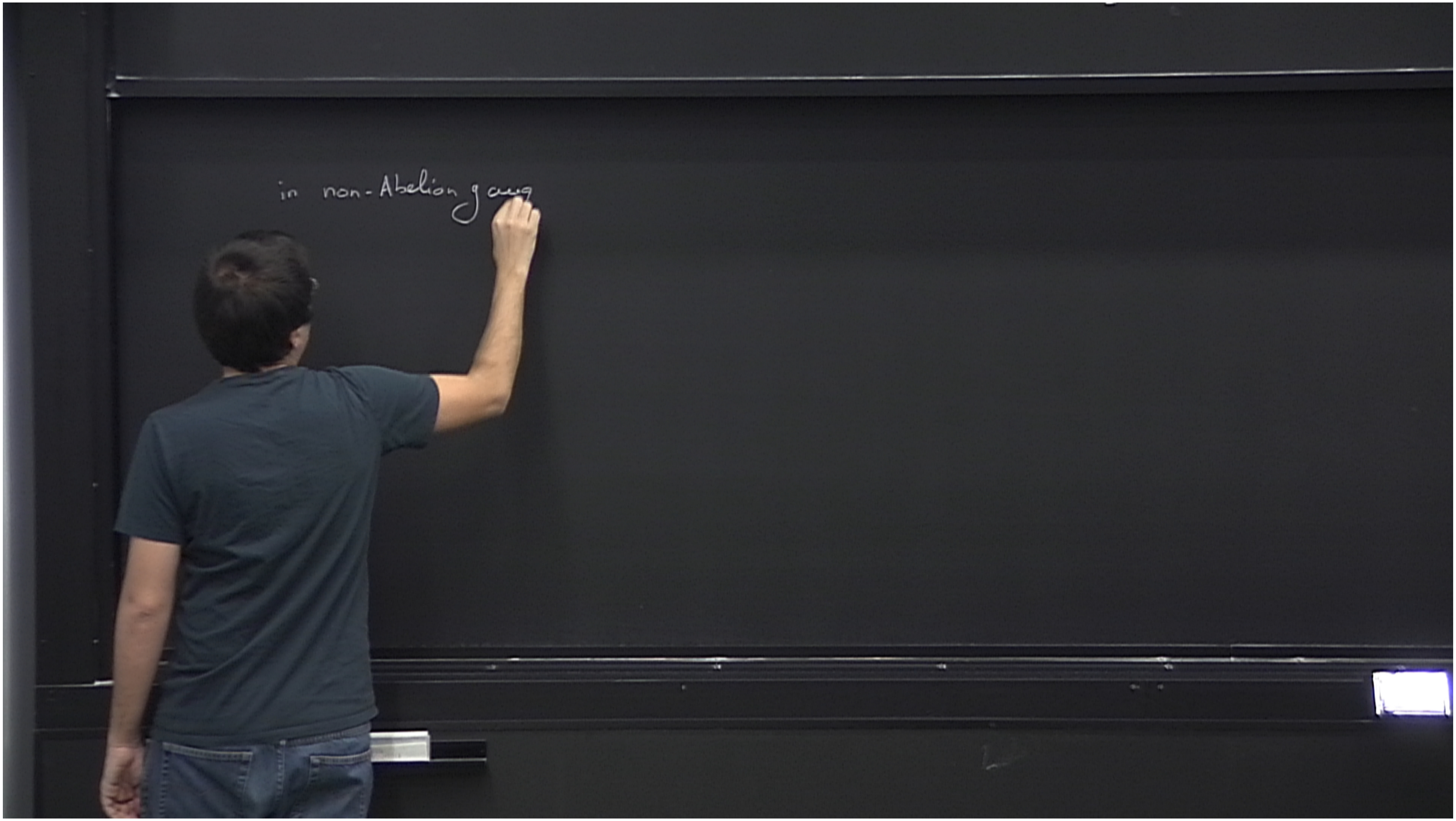
$t_1 - t_2 > a$ ← UV cutoff $a \to 0$ ~ physical mass ren. divergences, easy to subtract?

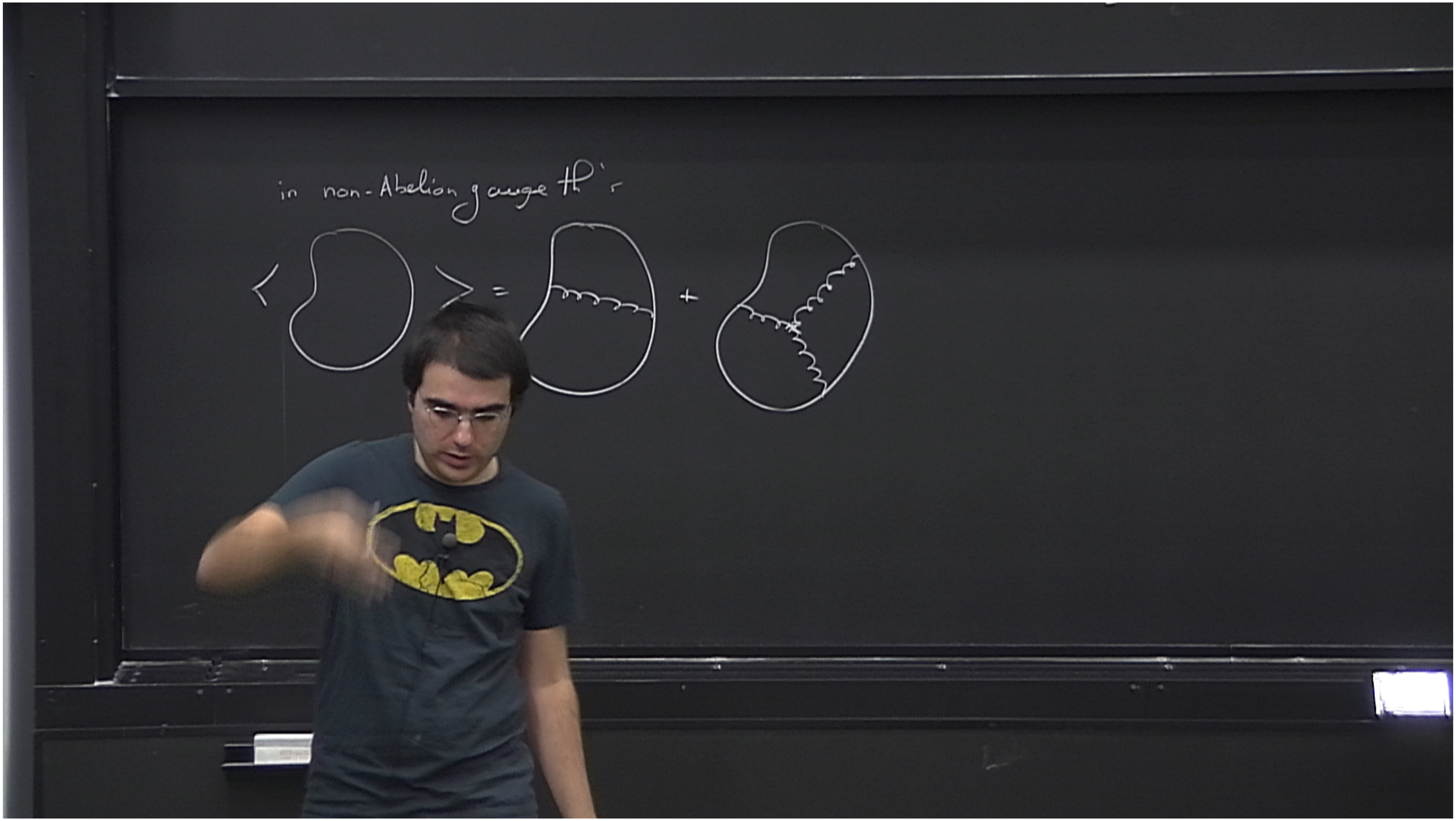
$$\lim_{T\to\infty} \log\langle W \rangle = \underbrace{\frac{e^2}{4\pi}\frac{T}{R}}_{T V_{\text{Coulomb}}(R)} + \frac{e^2}{4\pi^2}\frac{2T}{a}$$

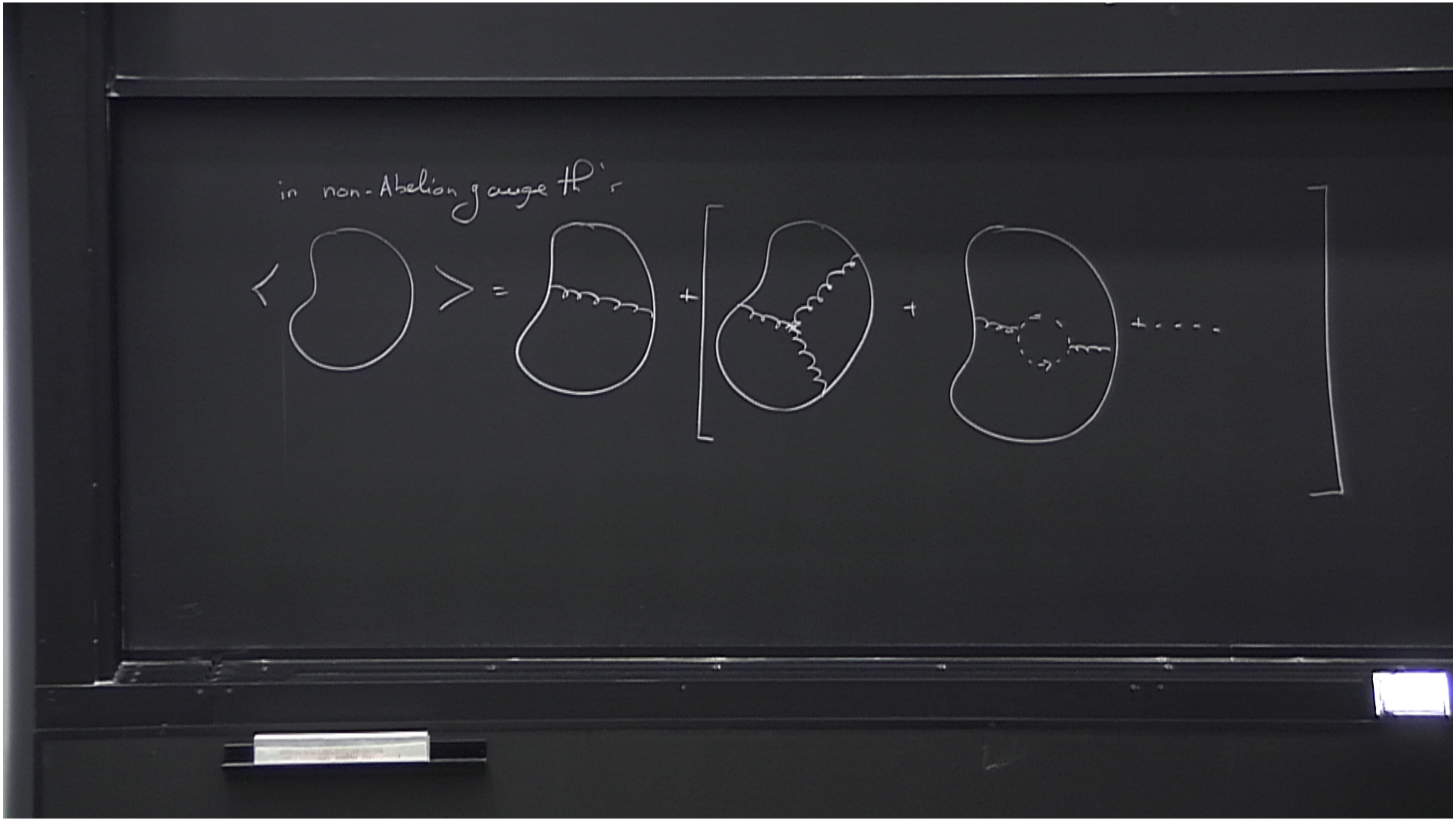
$\sim \frac{1}{a} \times \text{Length} \times \text{Const}$

Δx, difference

C.M.


in non-Abelian


in non-Abelian gauge th's: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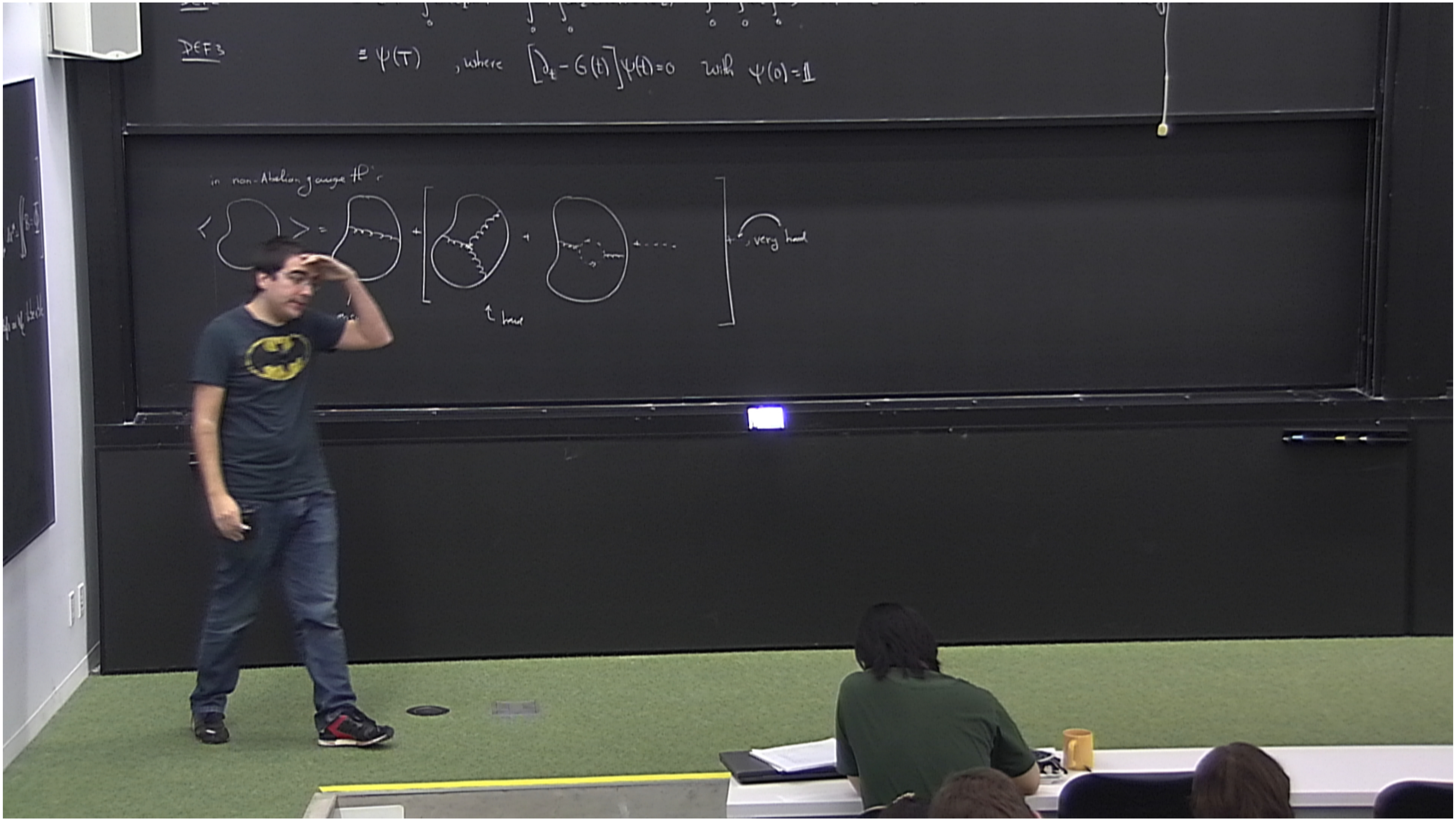
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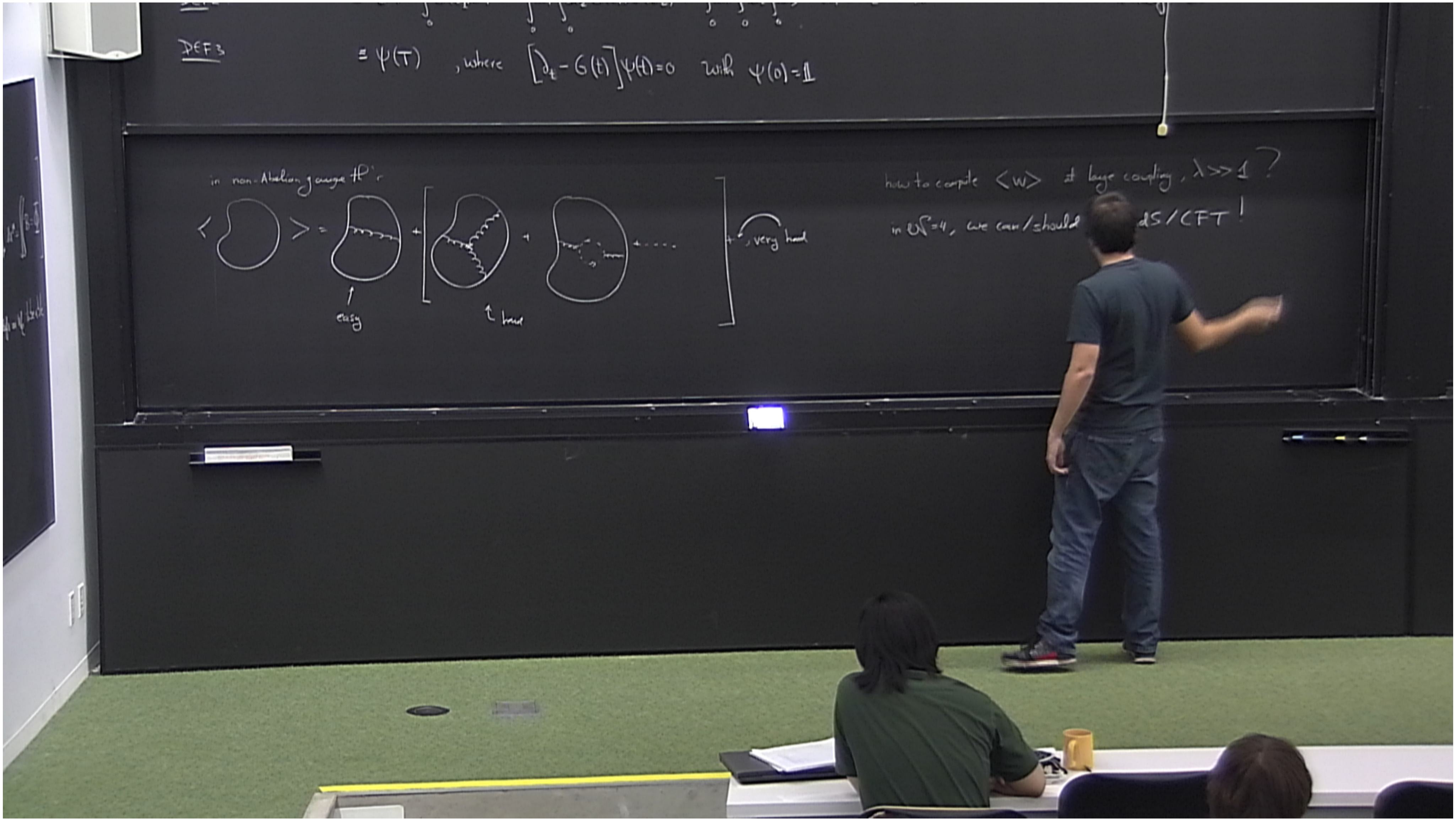
in non-Abelian gauge th'r
very hard
hard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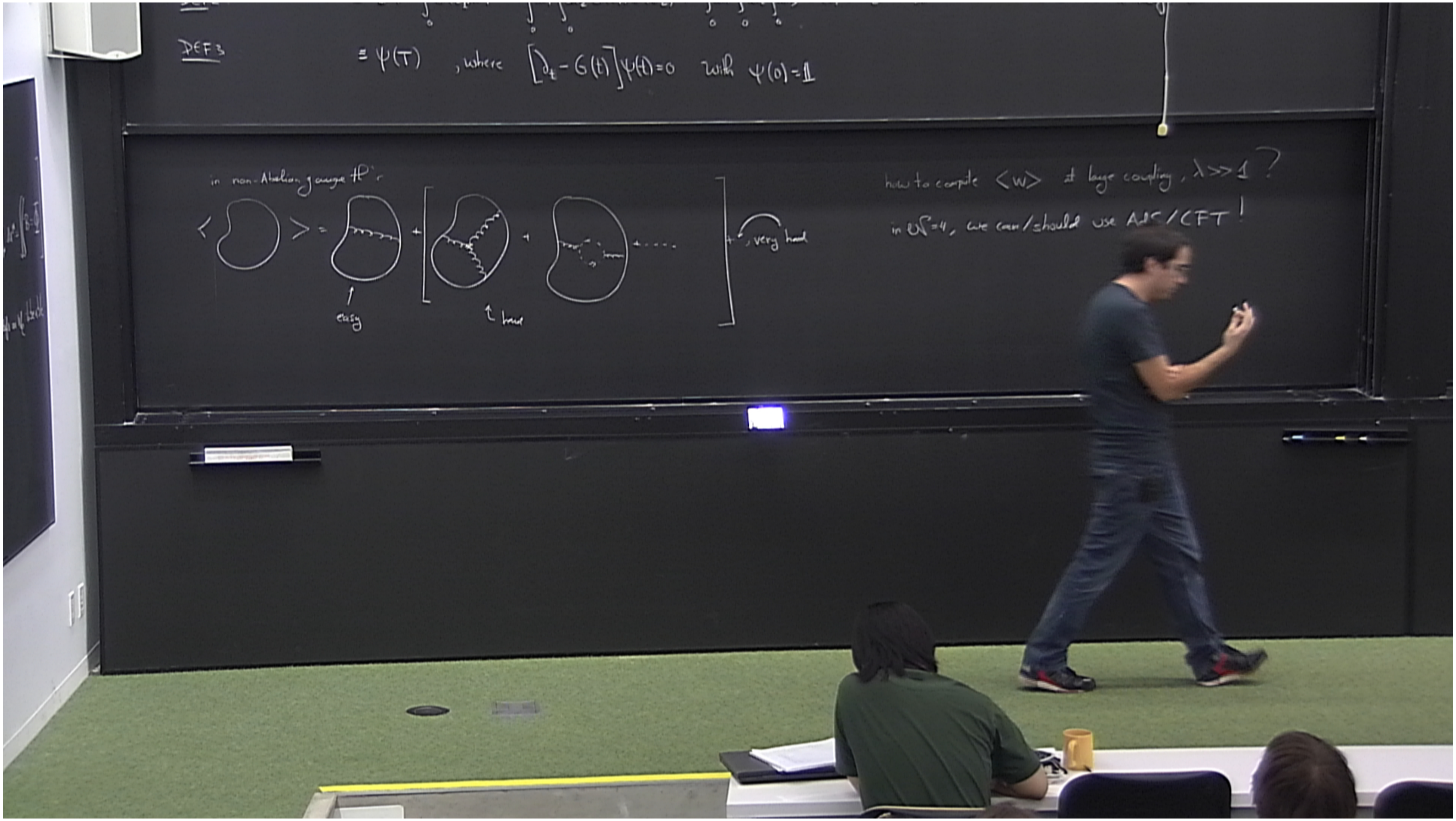
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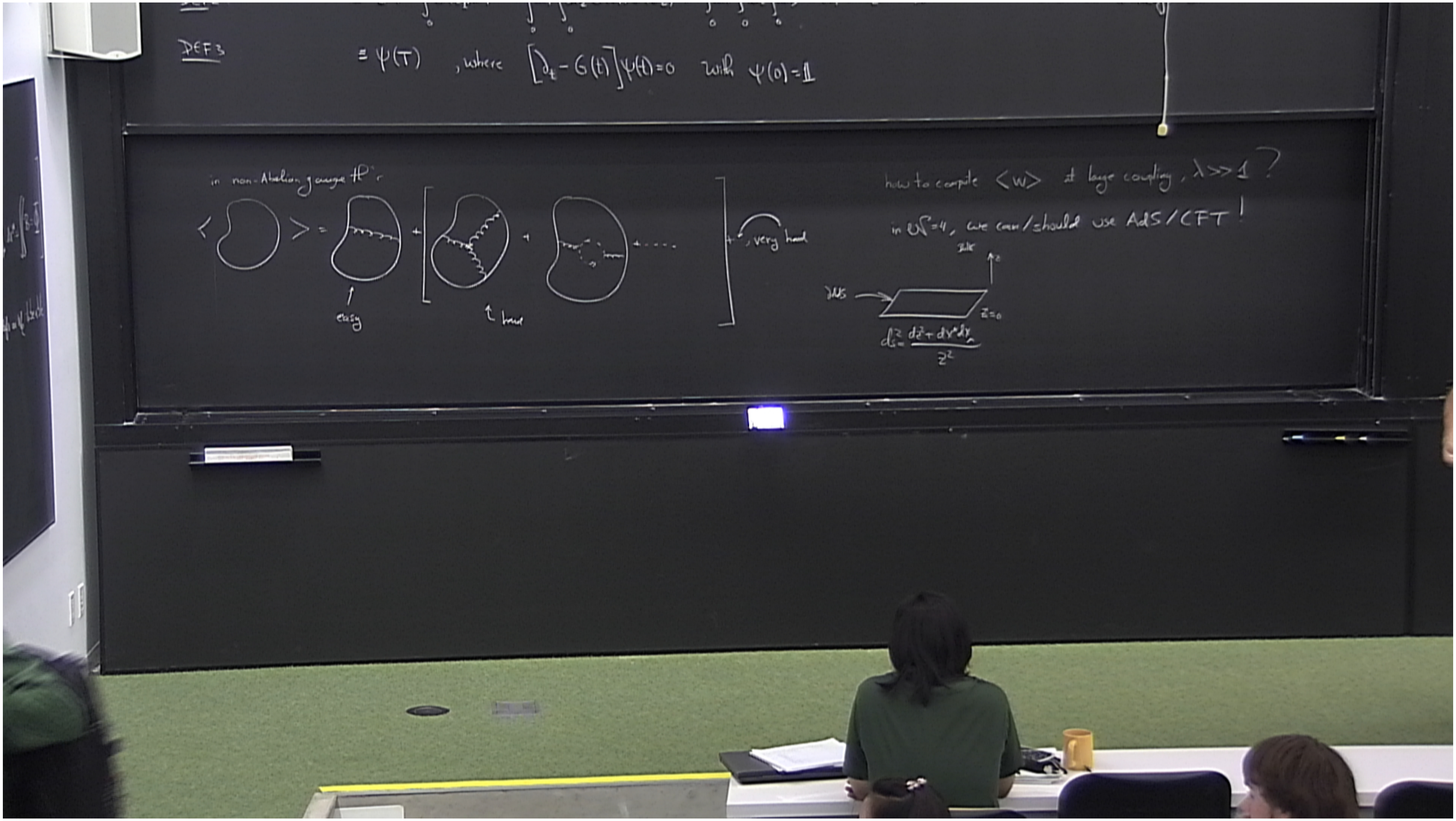
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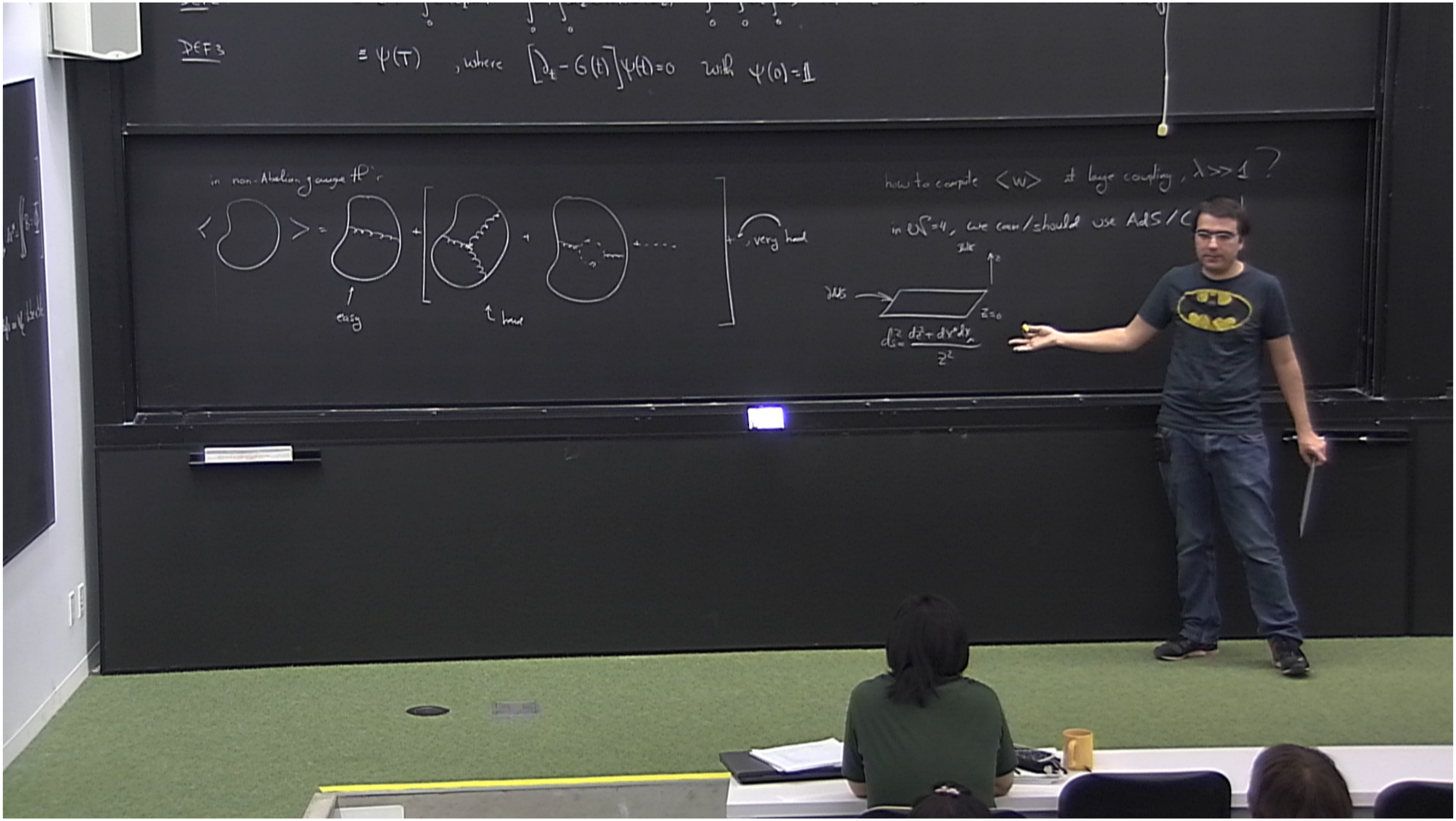
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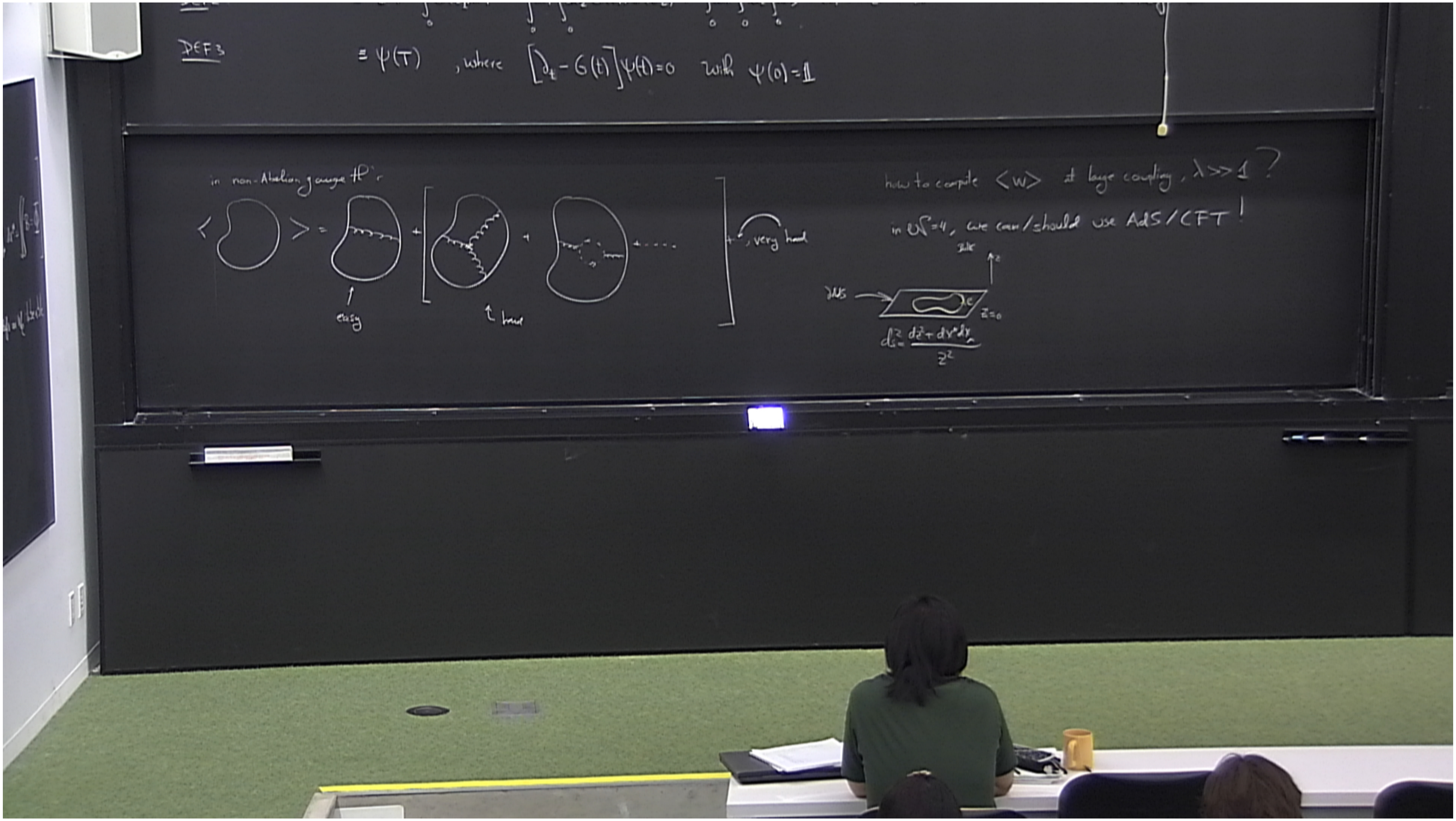
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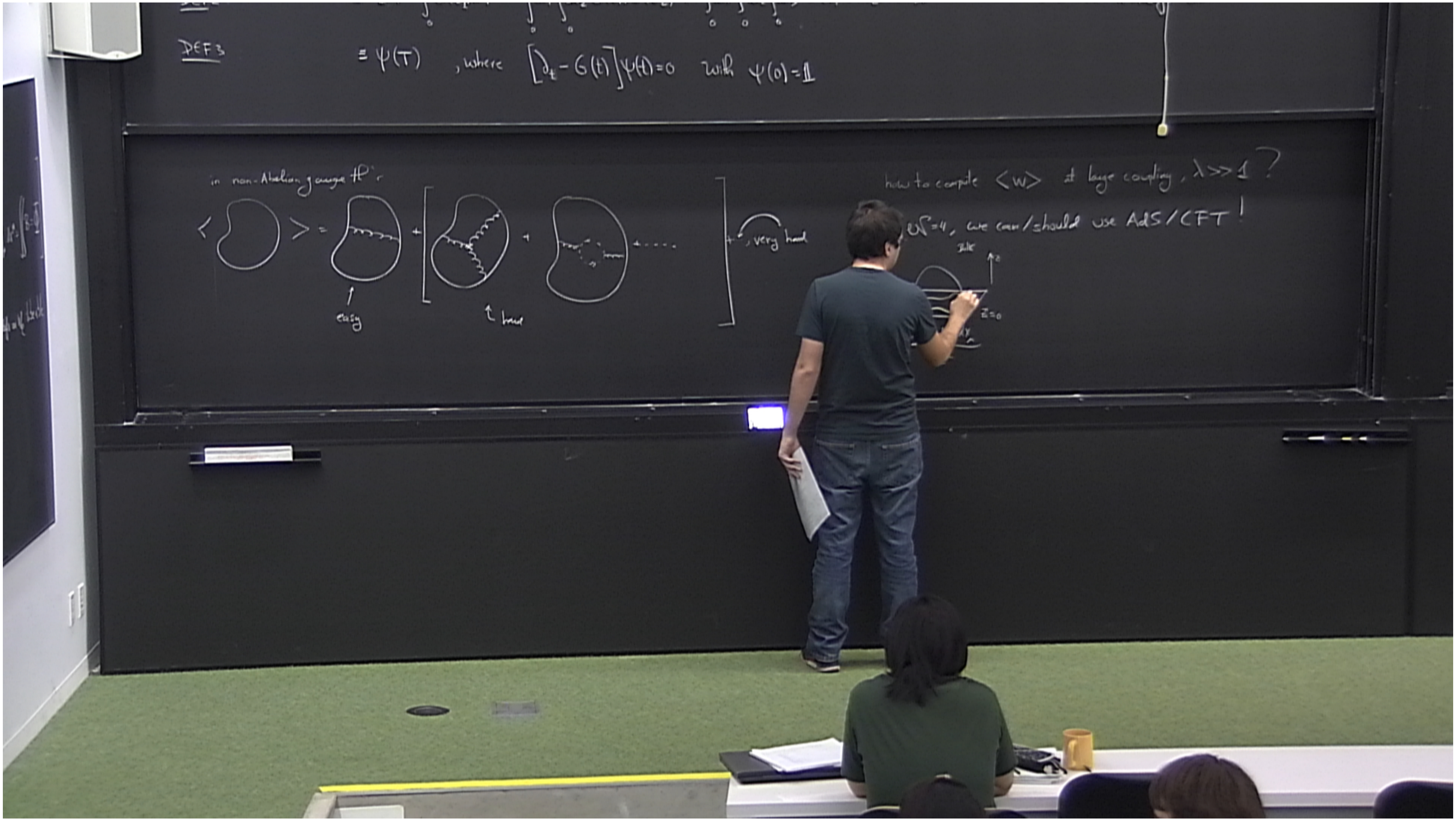
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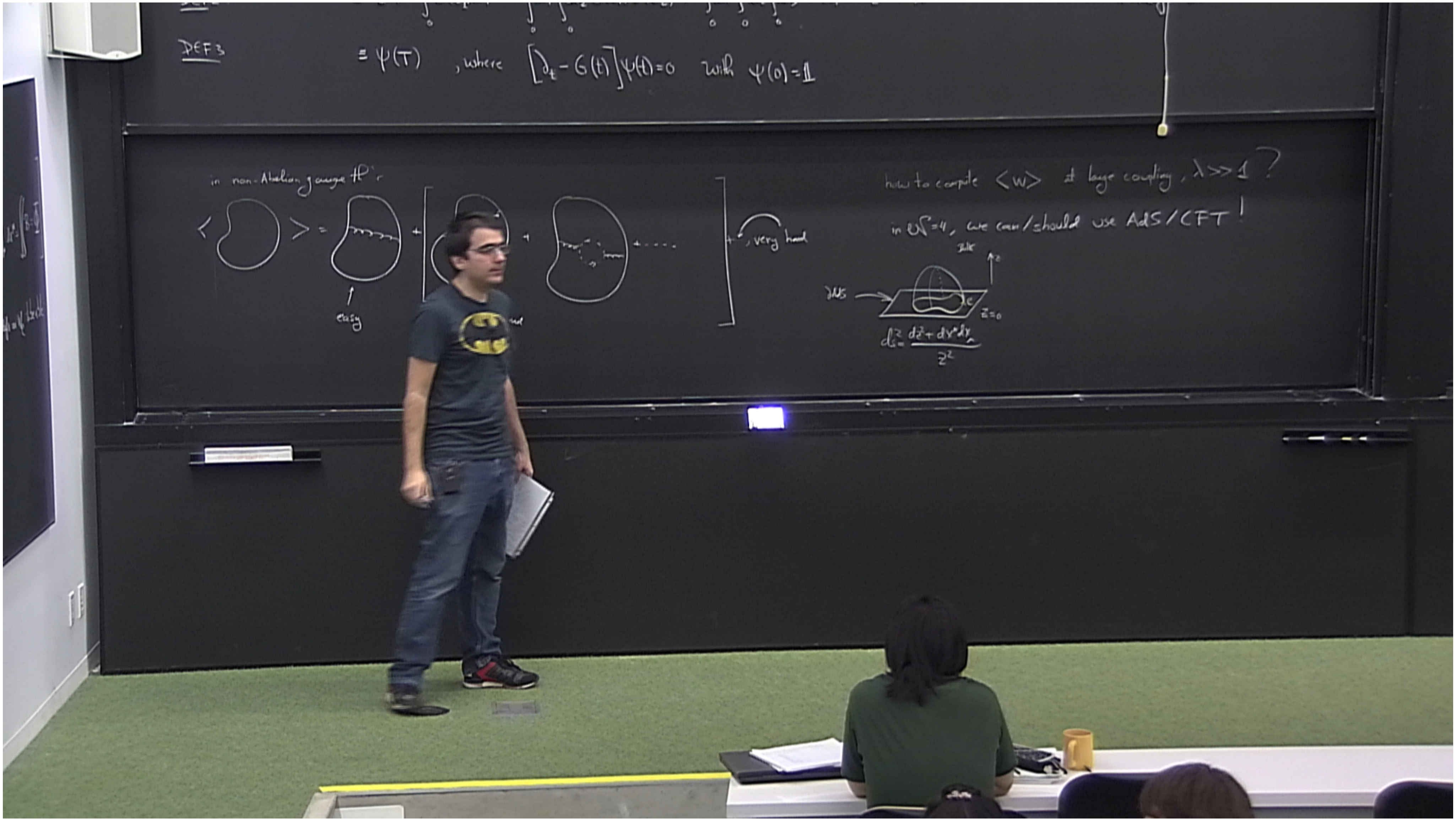
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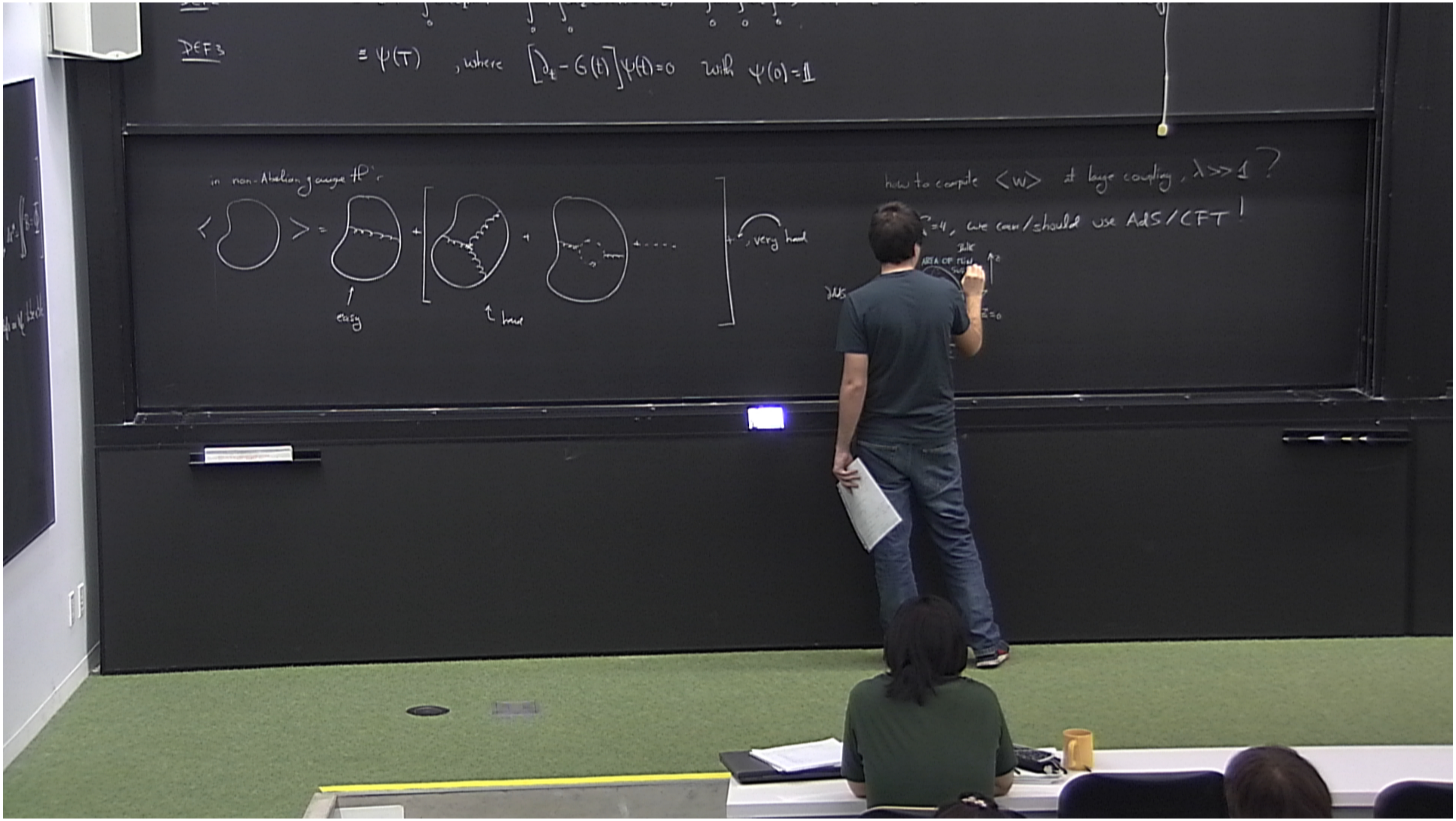
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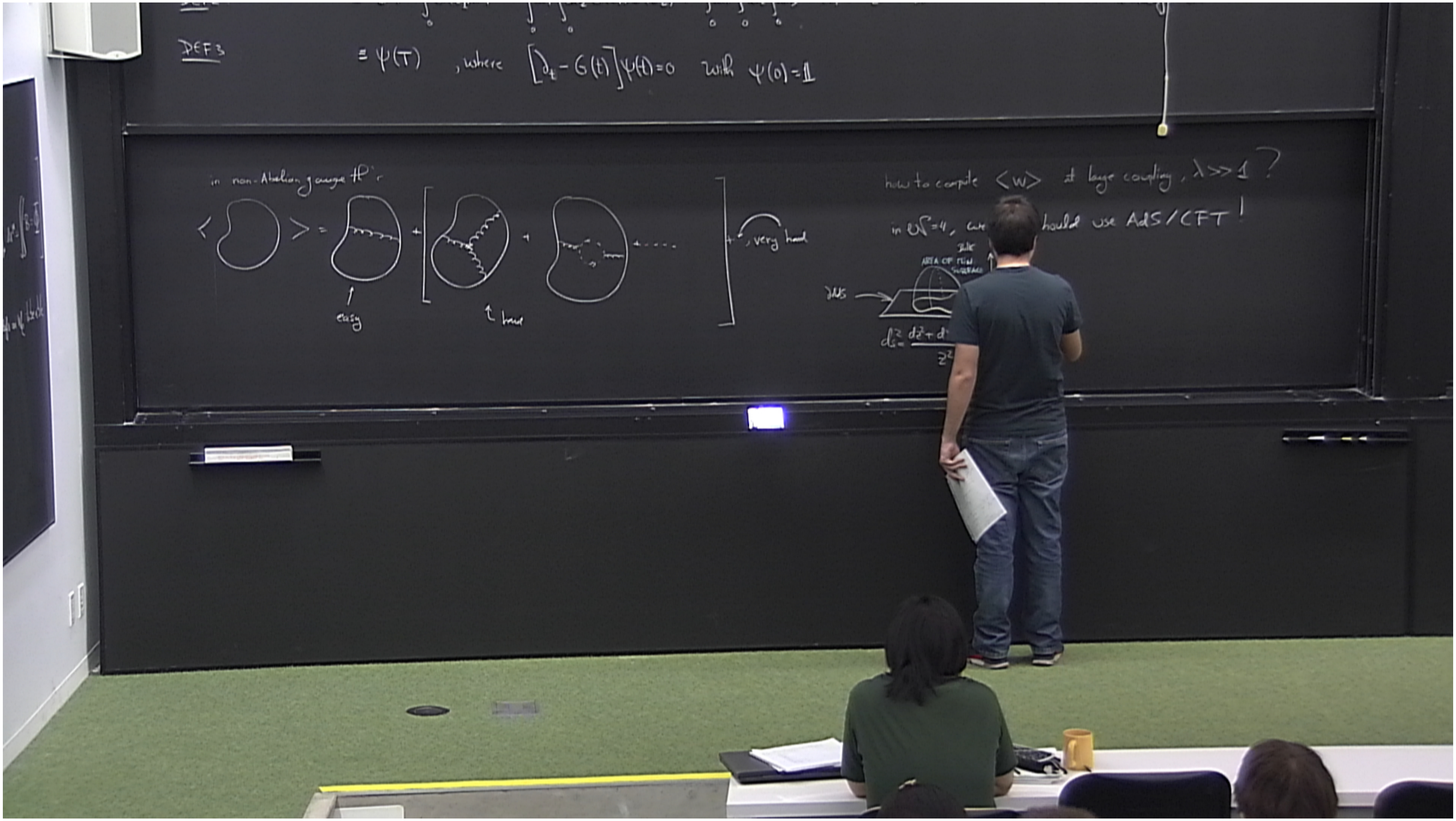
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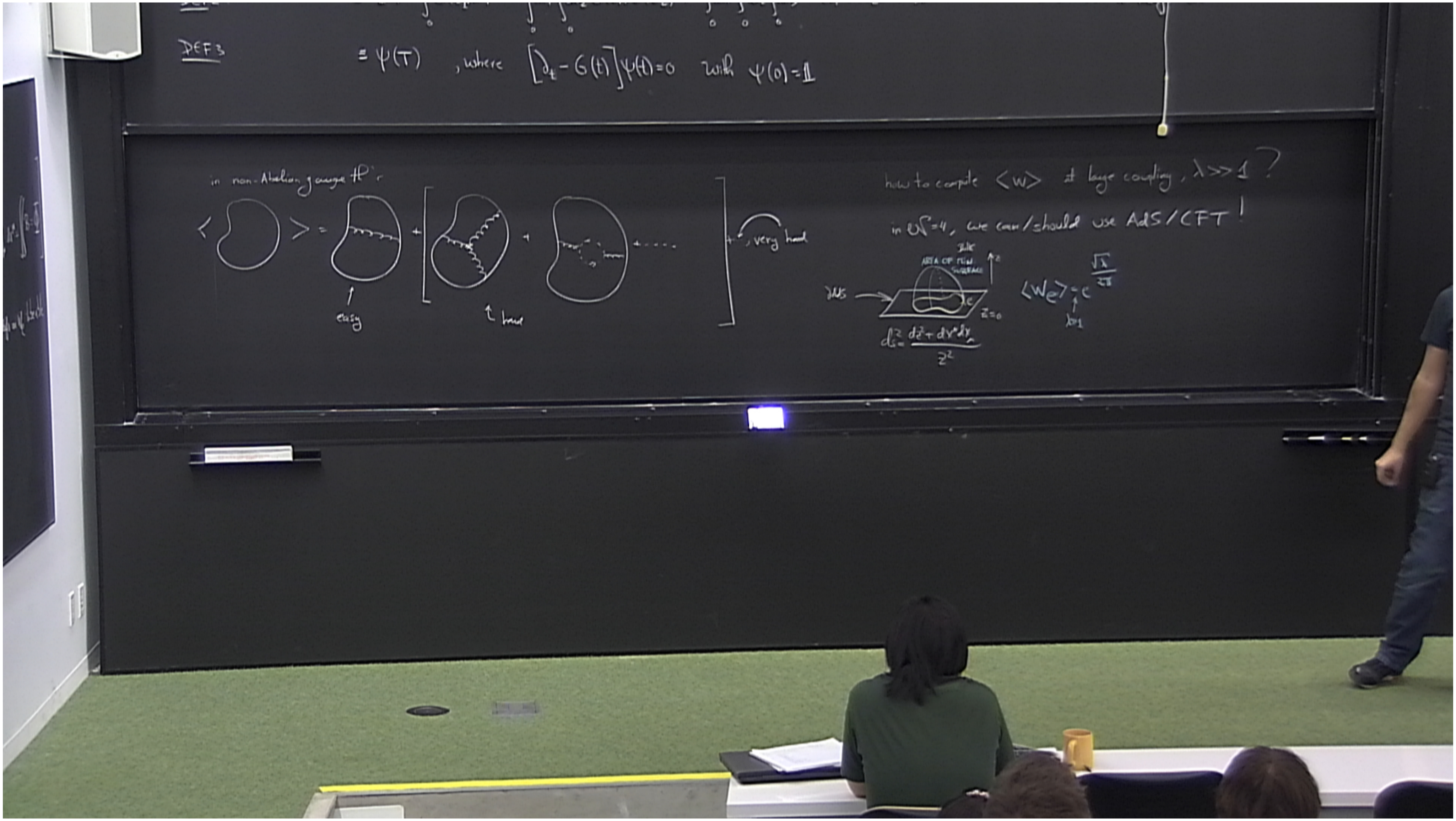
AdS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z=0
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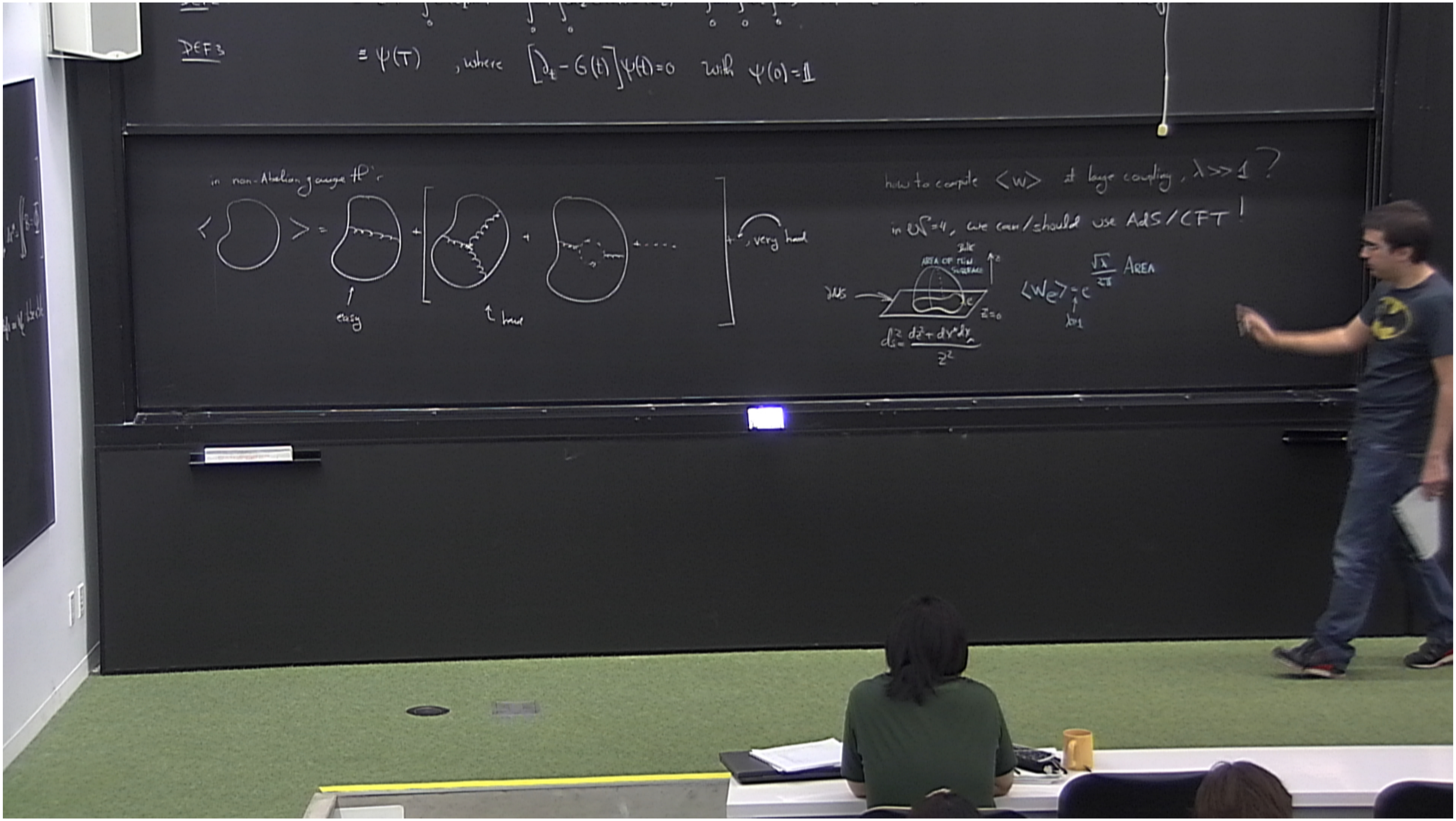
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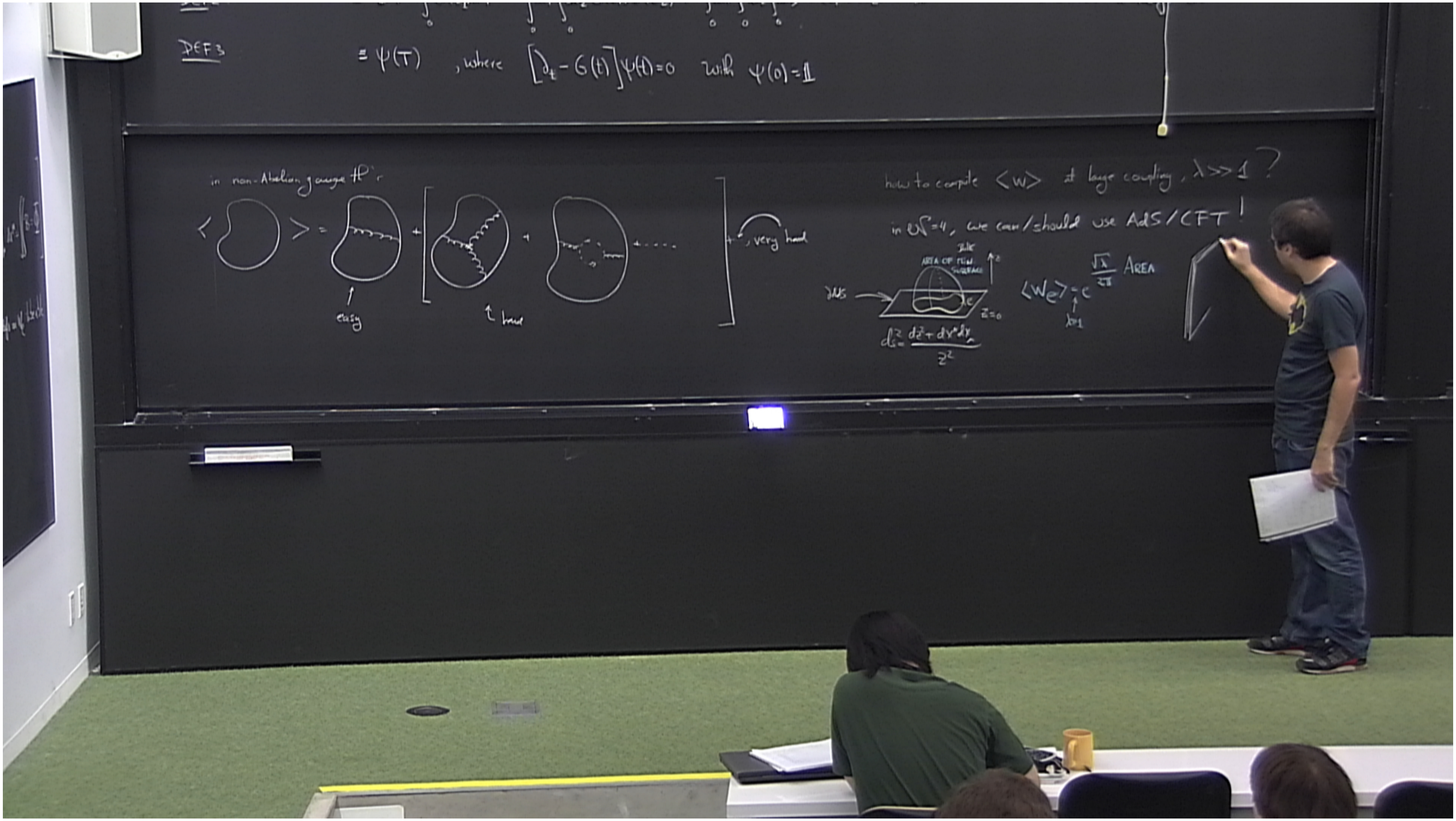
$\langle W_C \rangle = e^{\frac{\sqrt{\lambda}}{2\pi} \text{AREA}}$
$ds^2 = \frac{dz^2 + dx^\mu dx_\mu}{z^2}$



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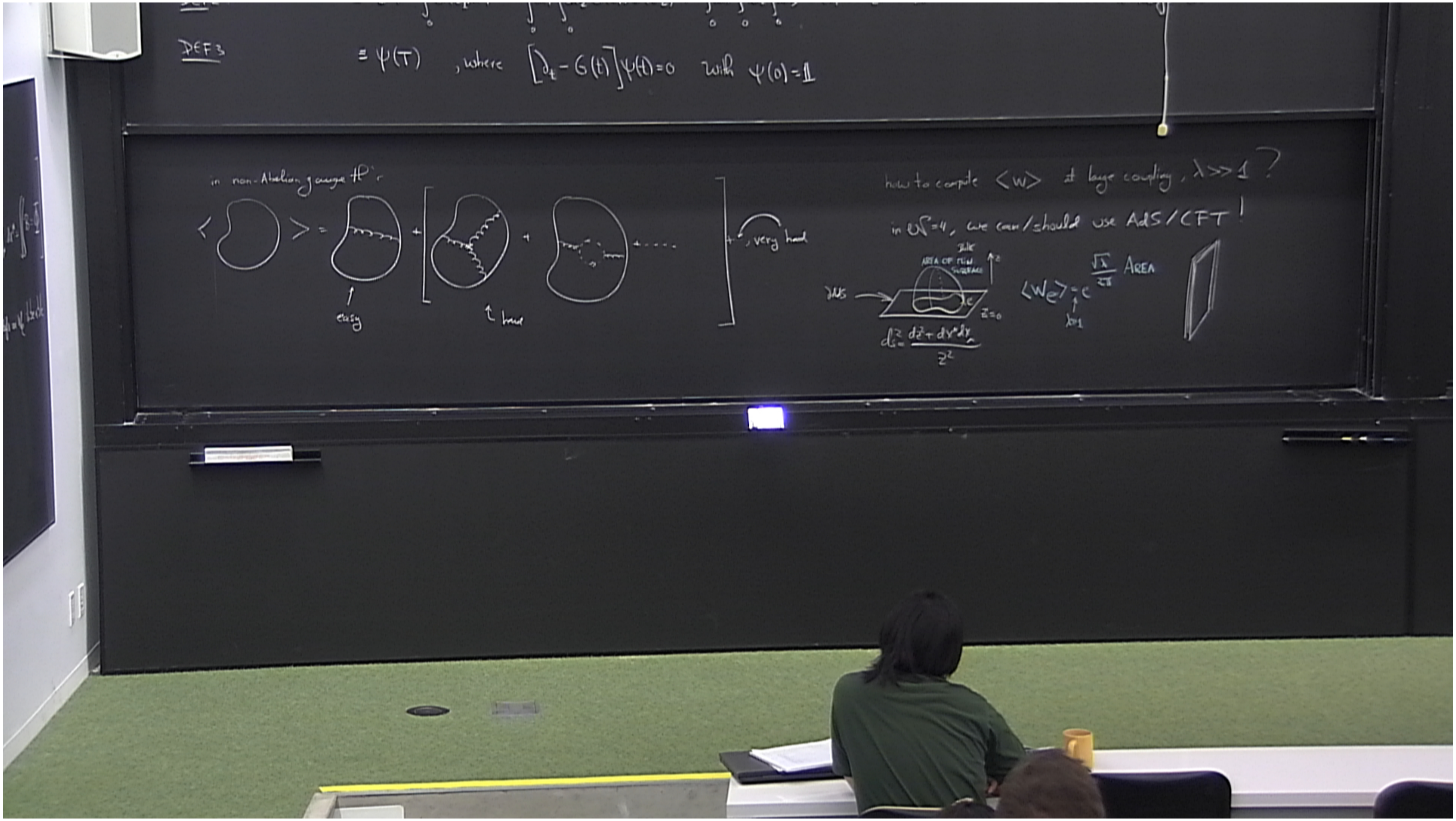
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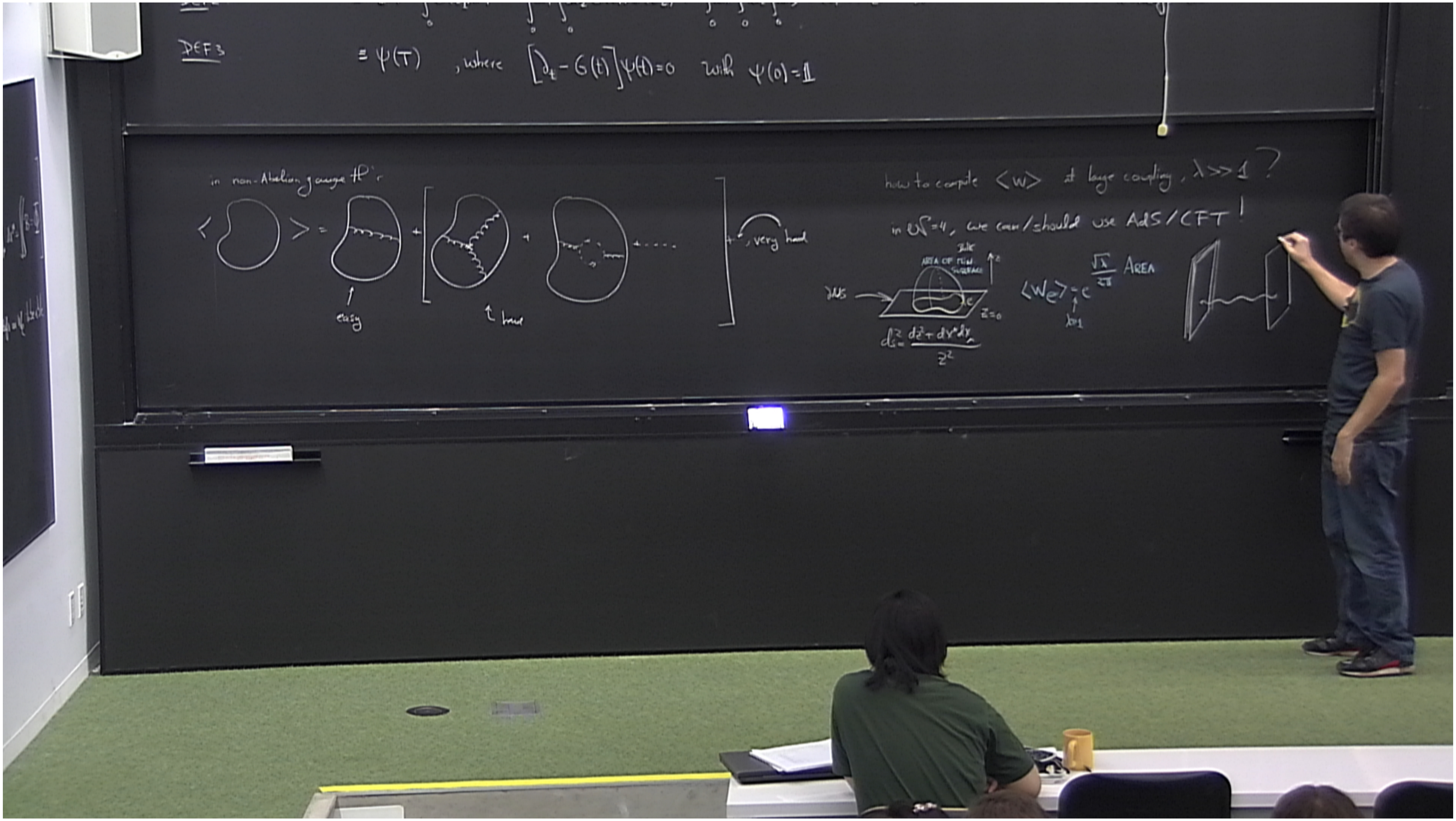
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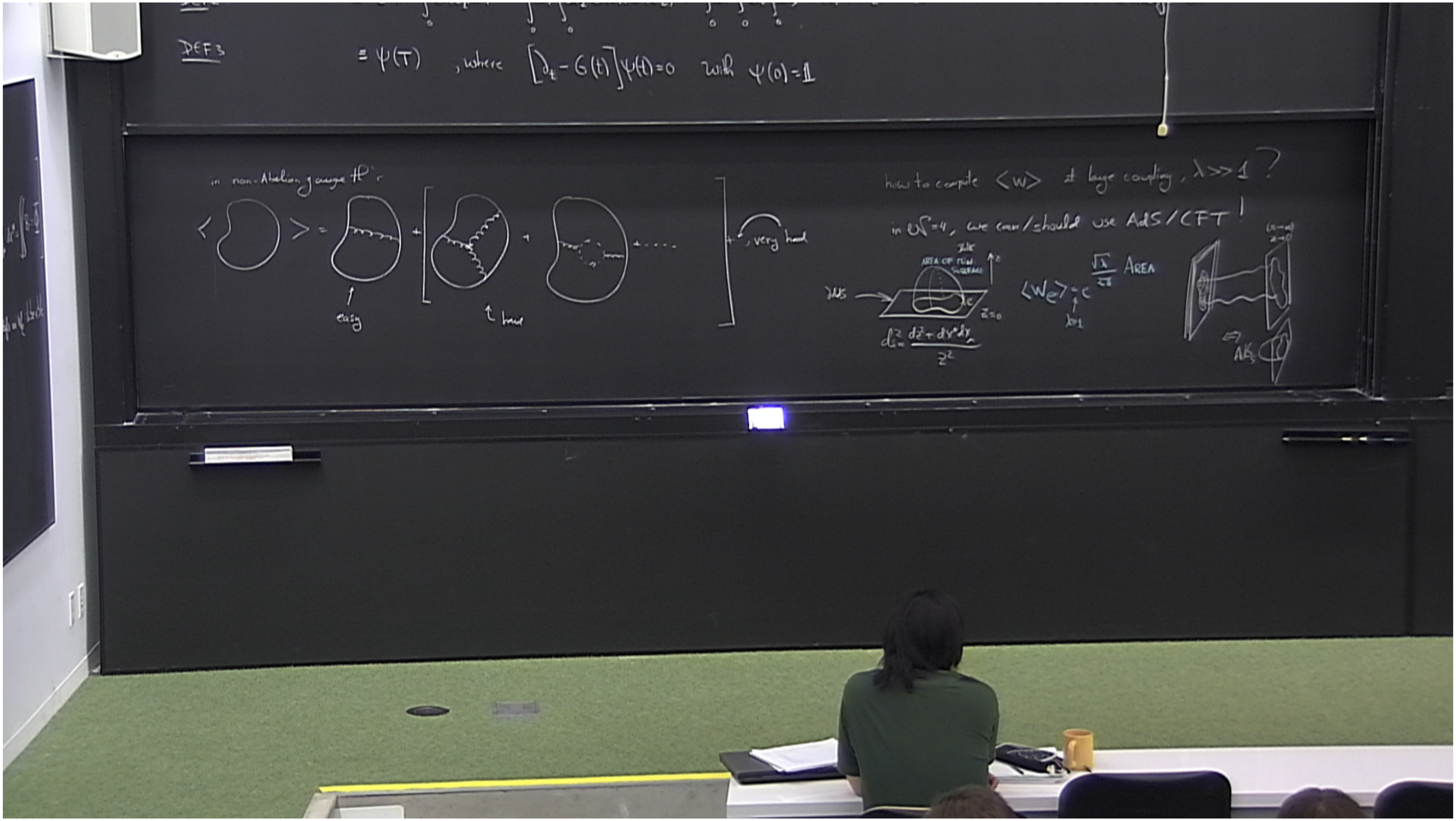

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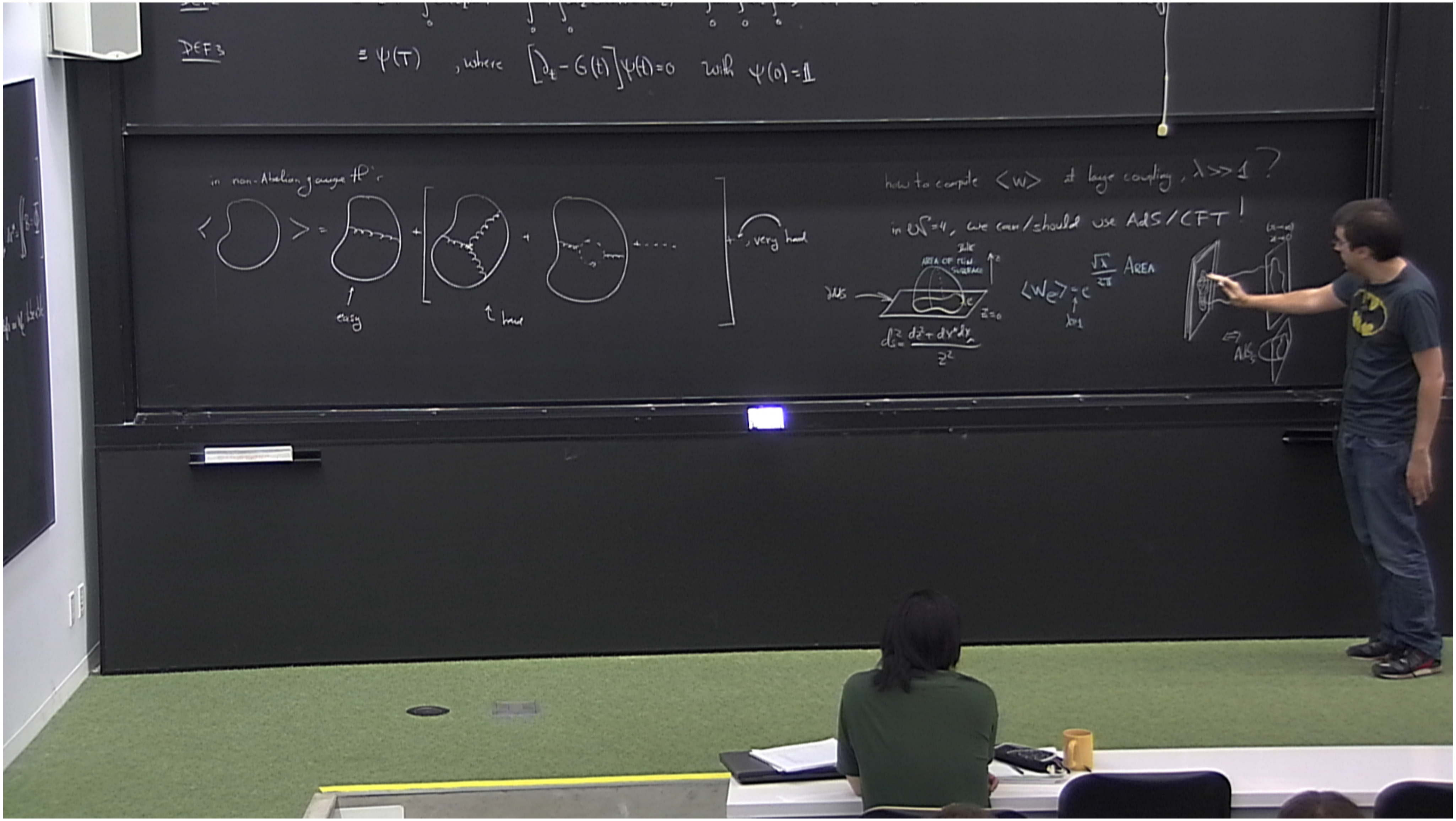
λ>>1
AdS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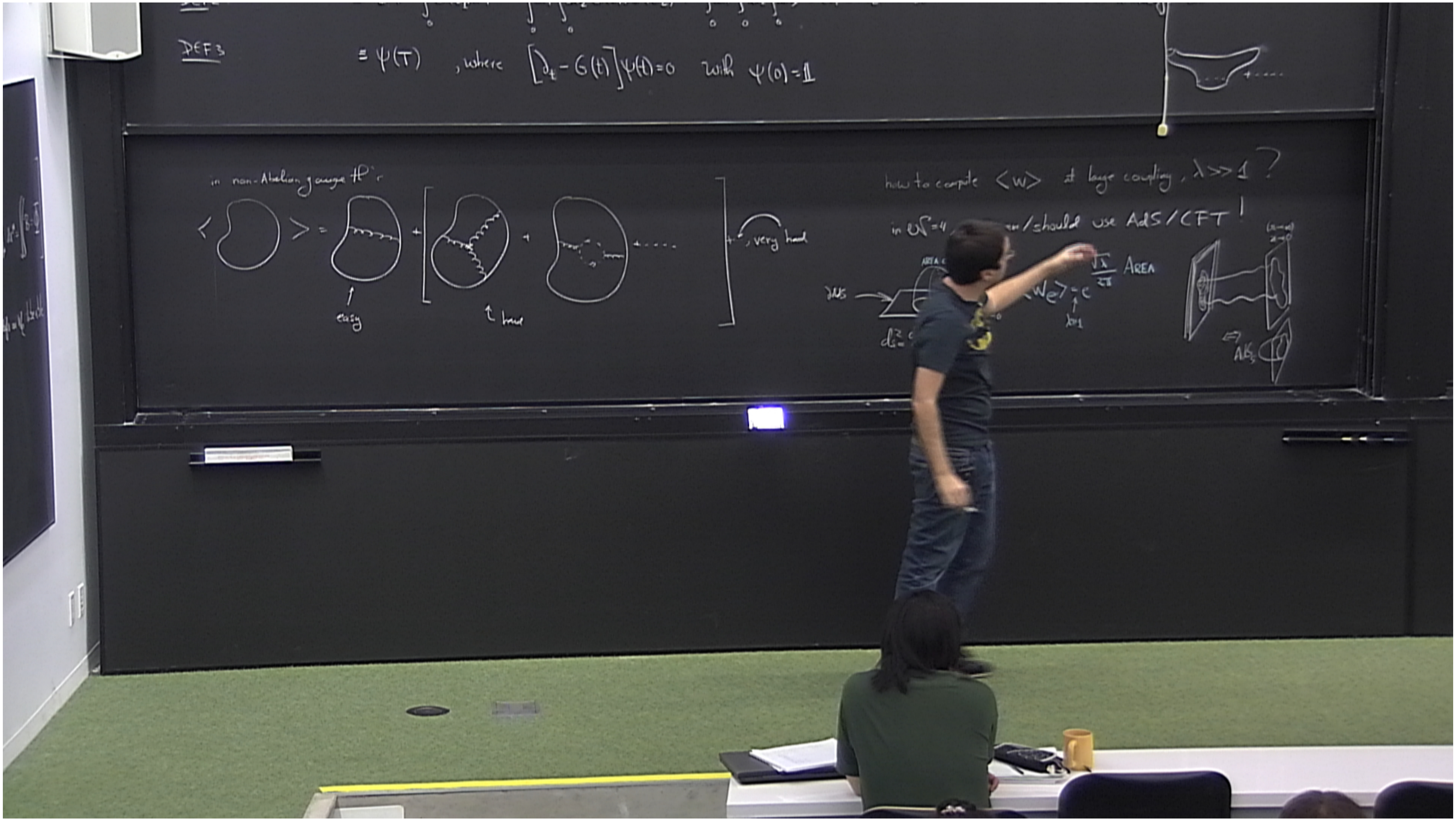
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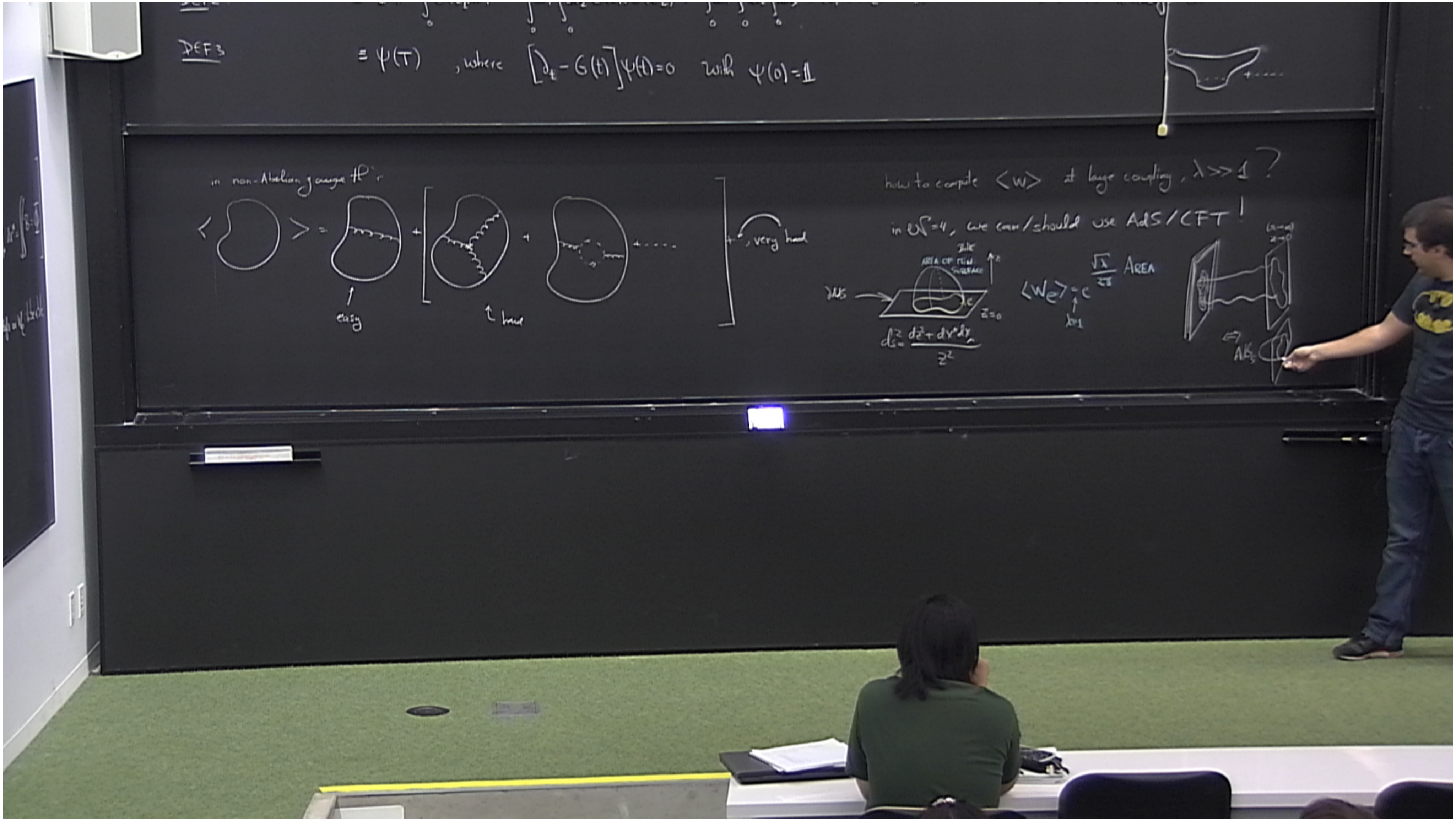
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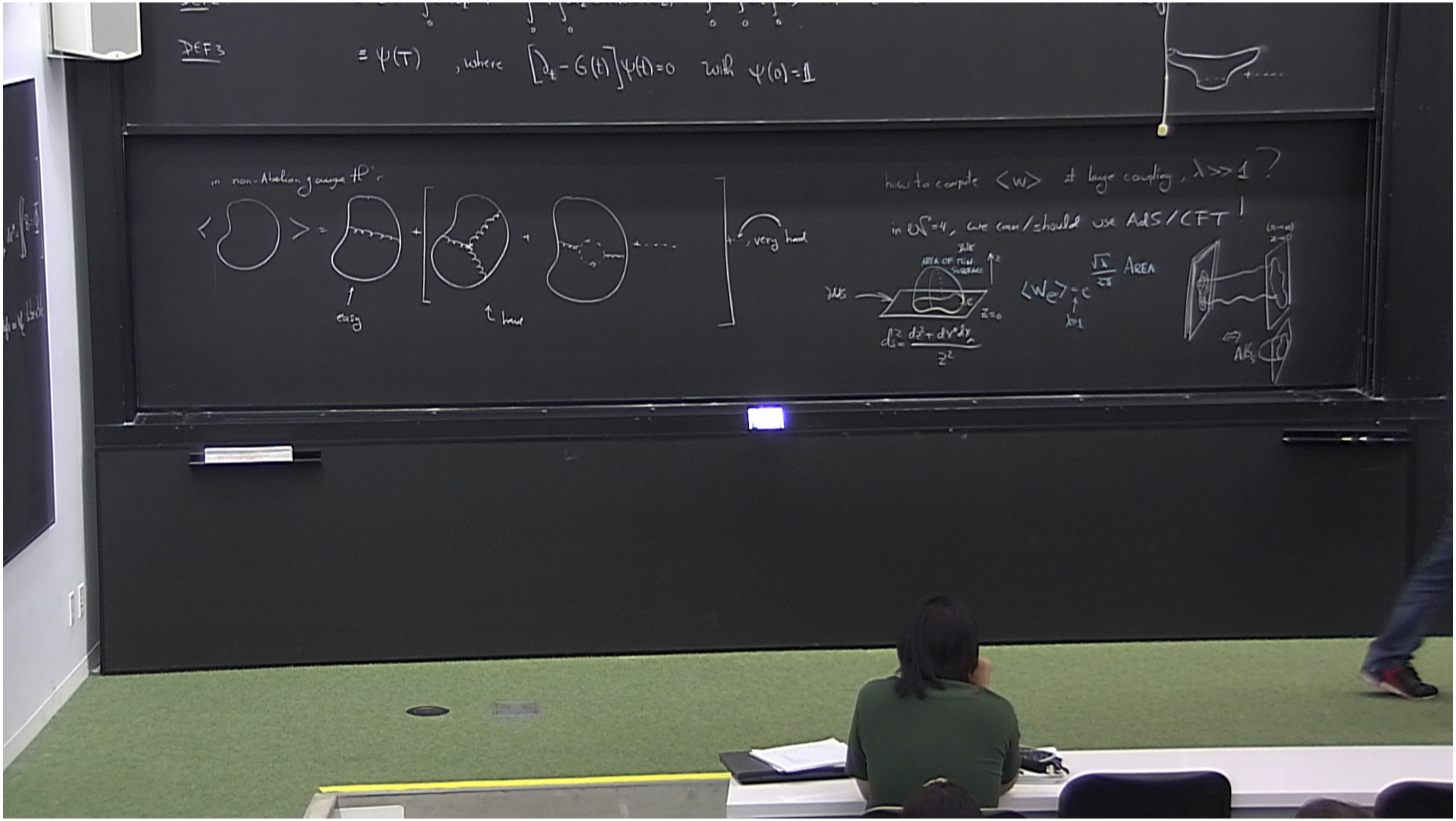
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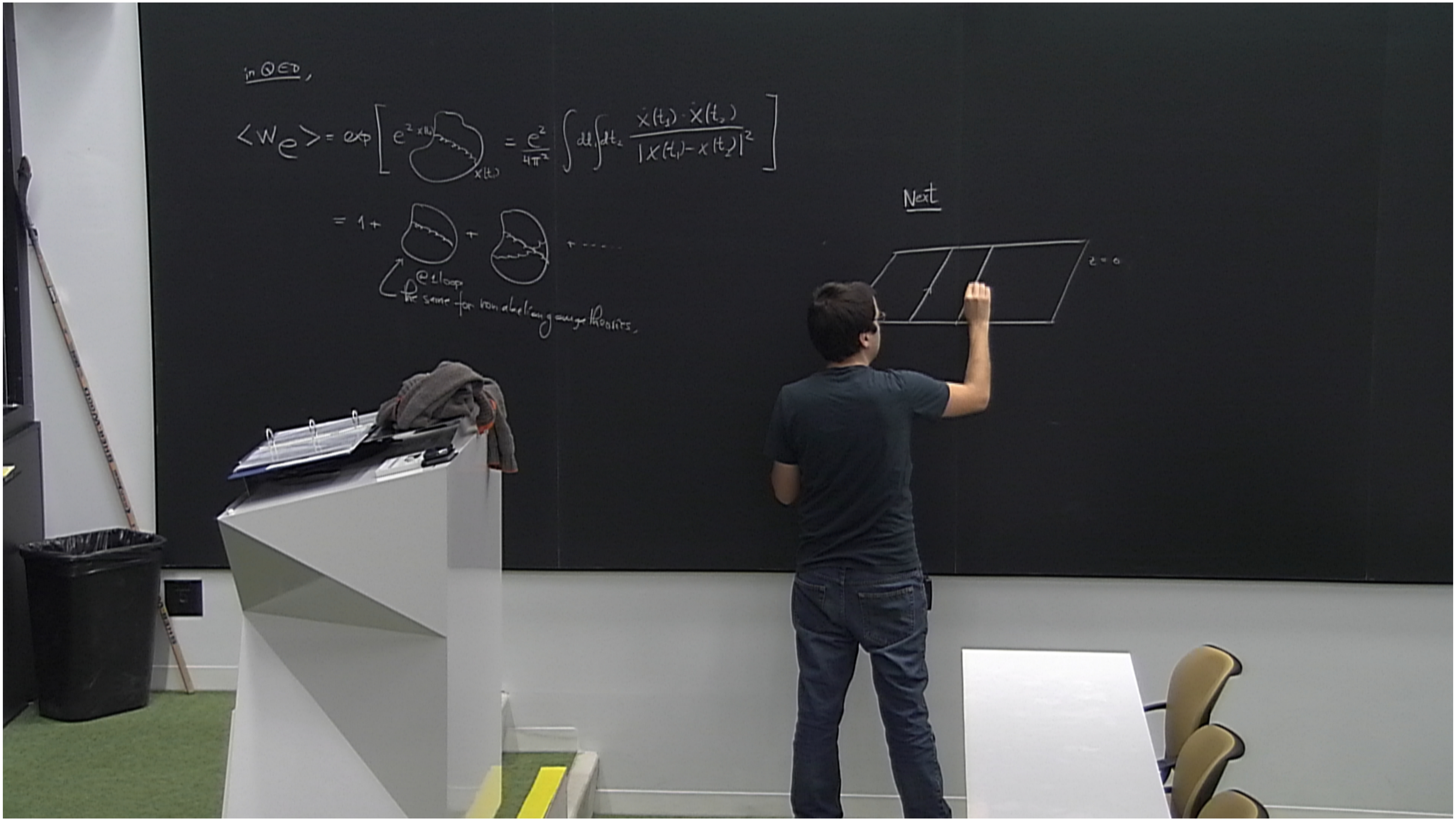
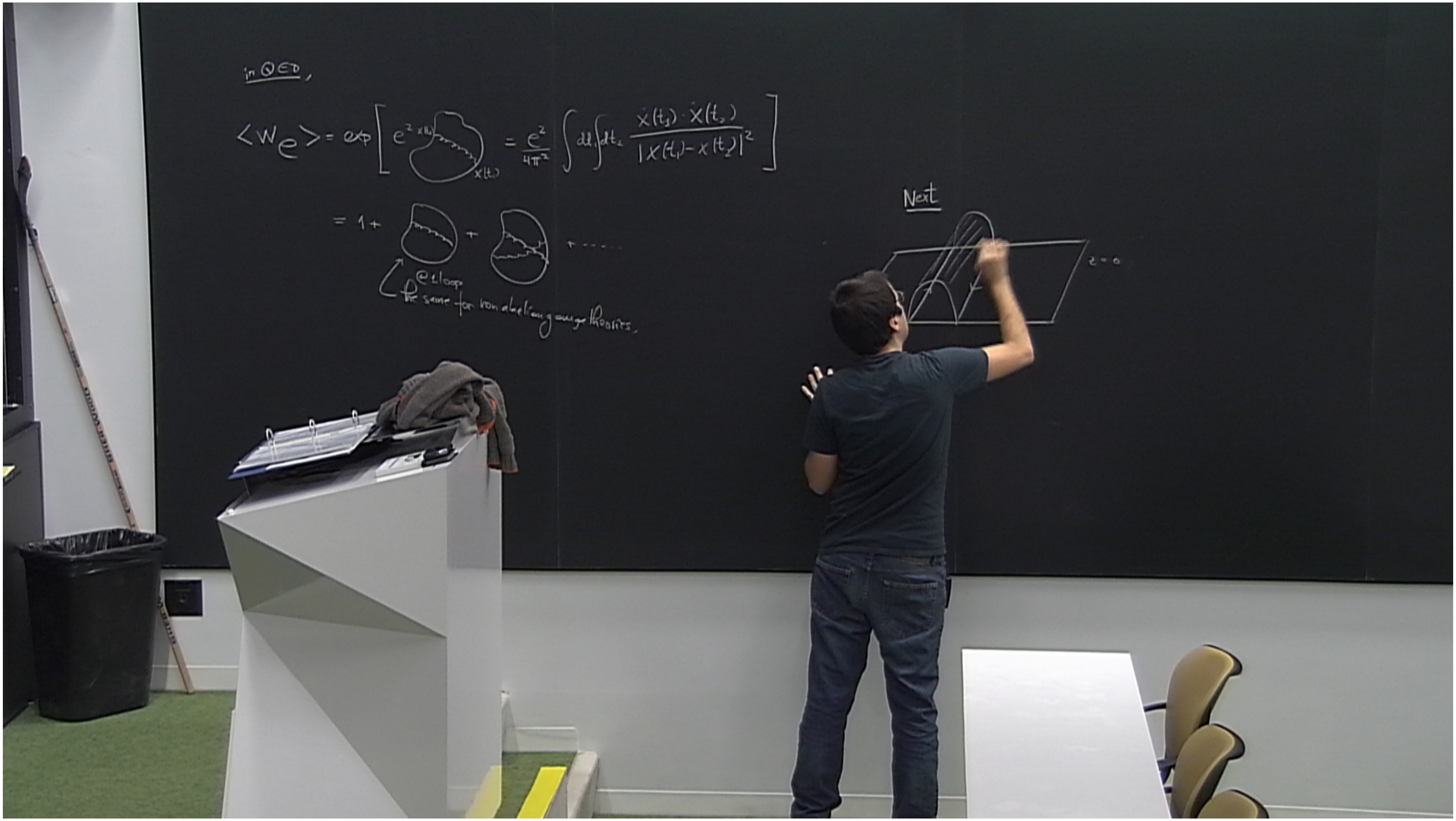

in QED,
Next
= 1 +
@ 1 loop
the same for non abelian gauge theories.
z = 0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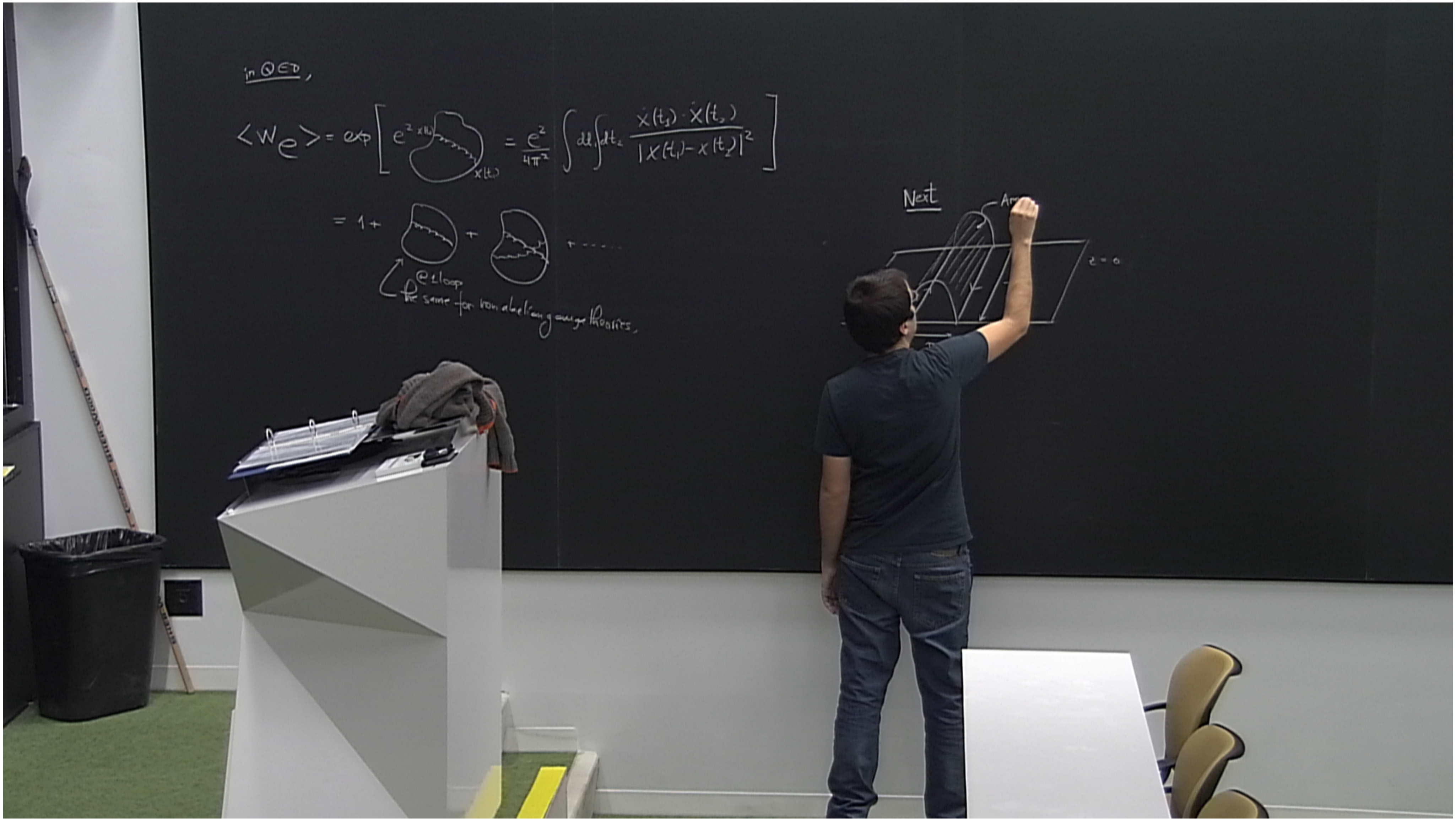

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Area?
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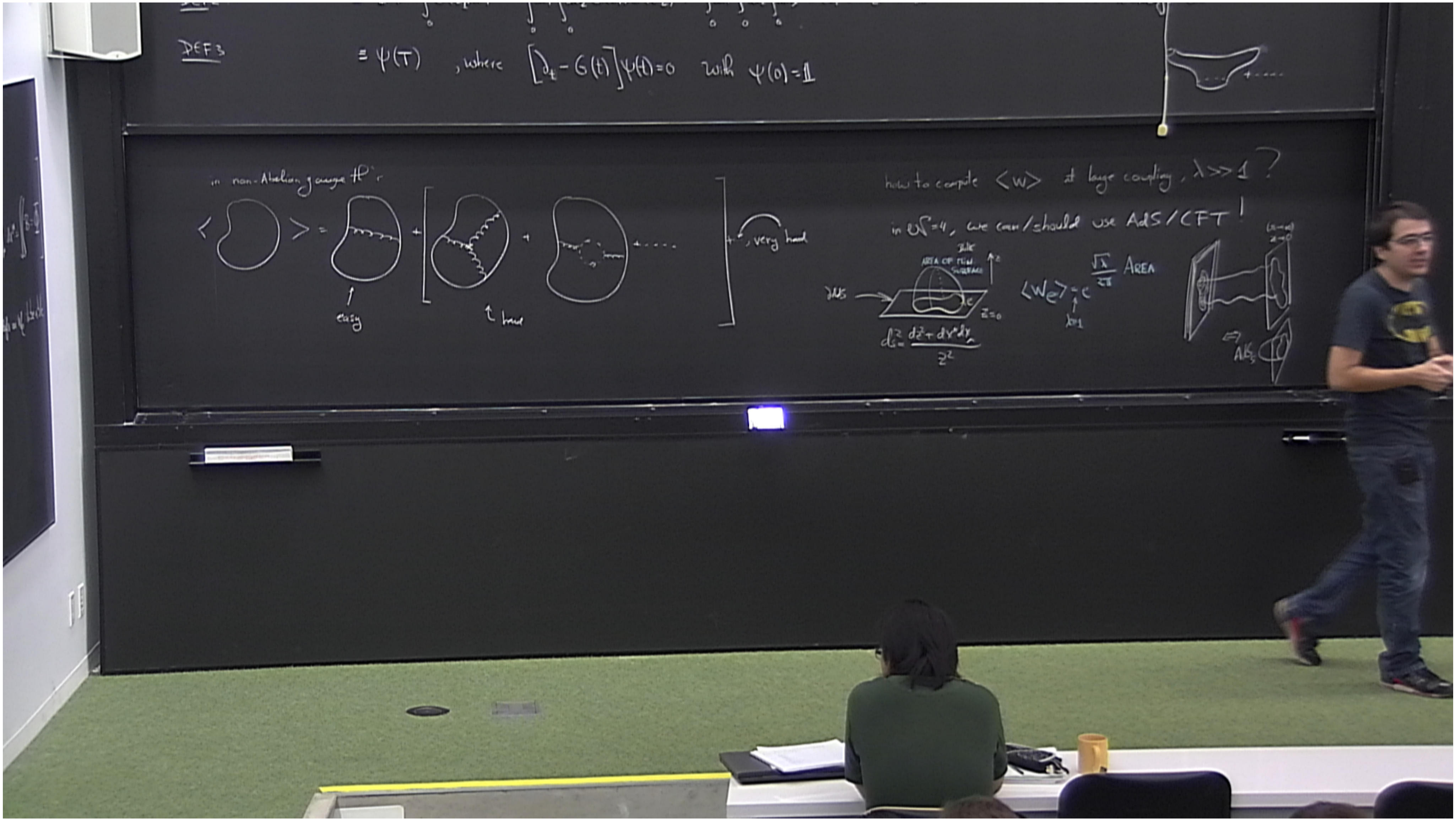

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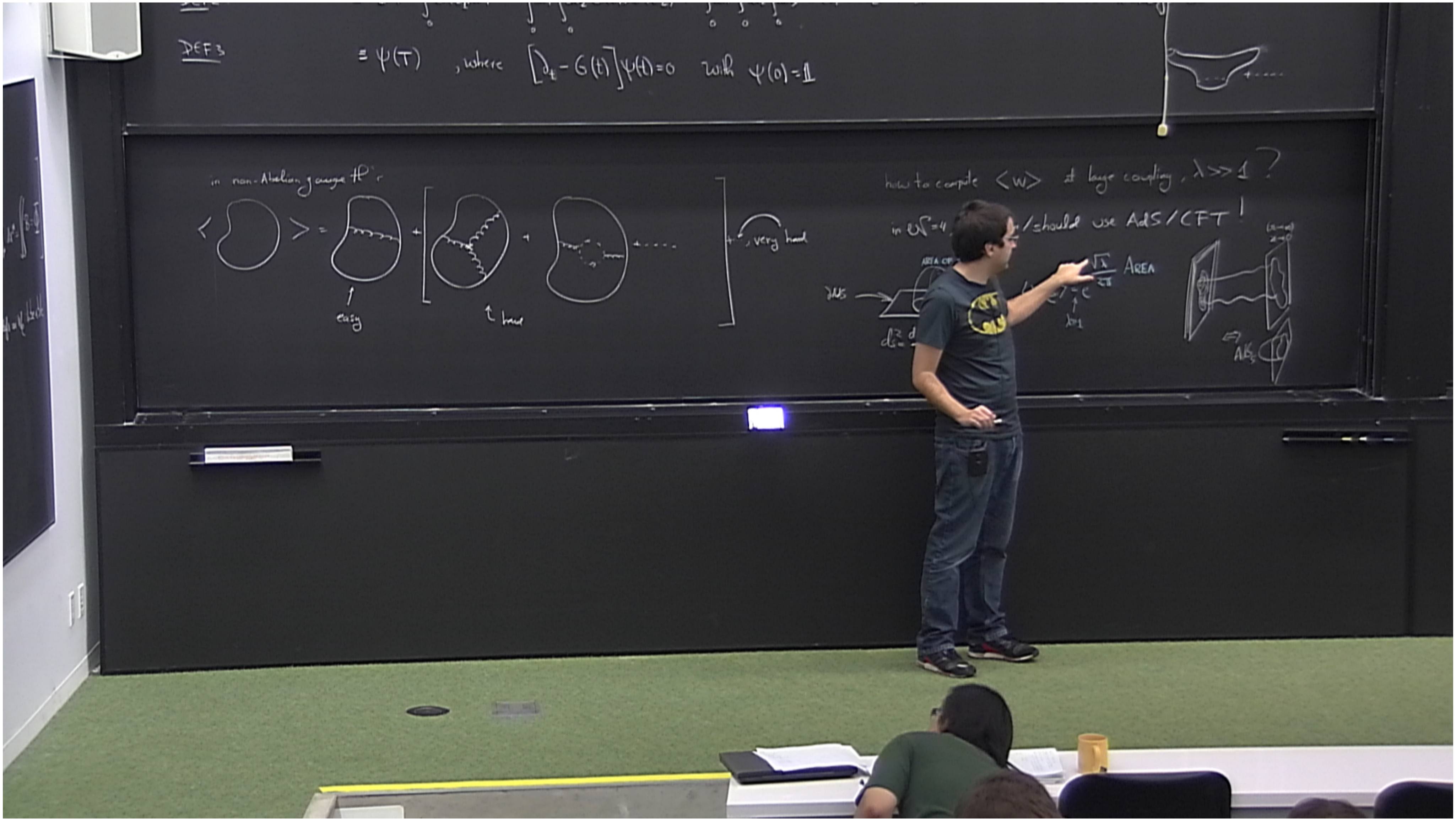
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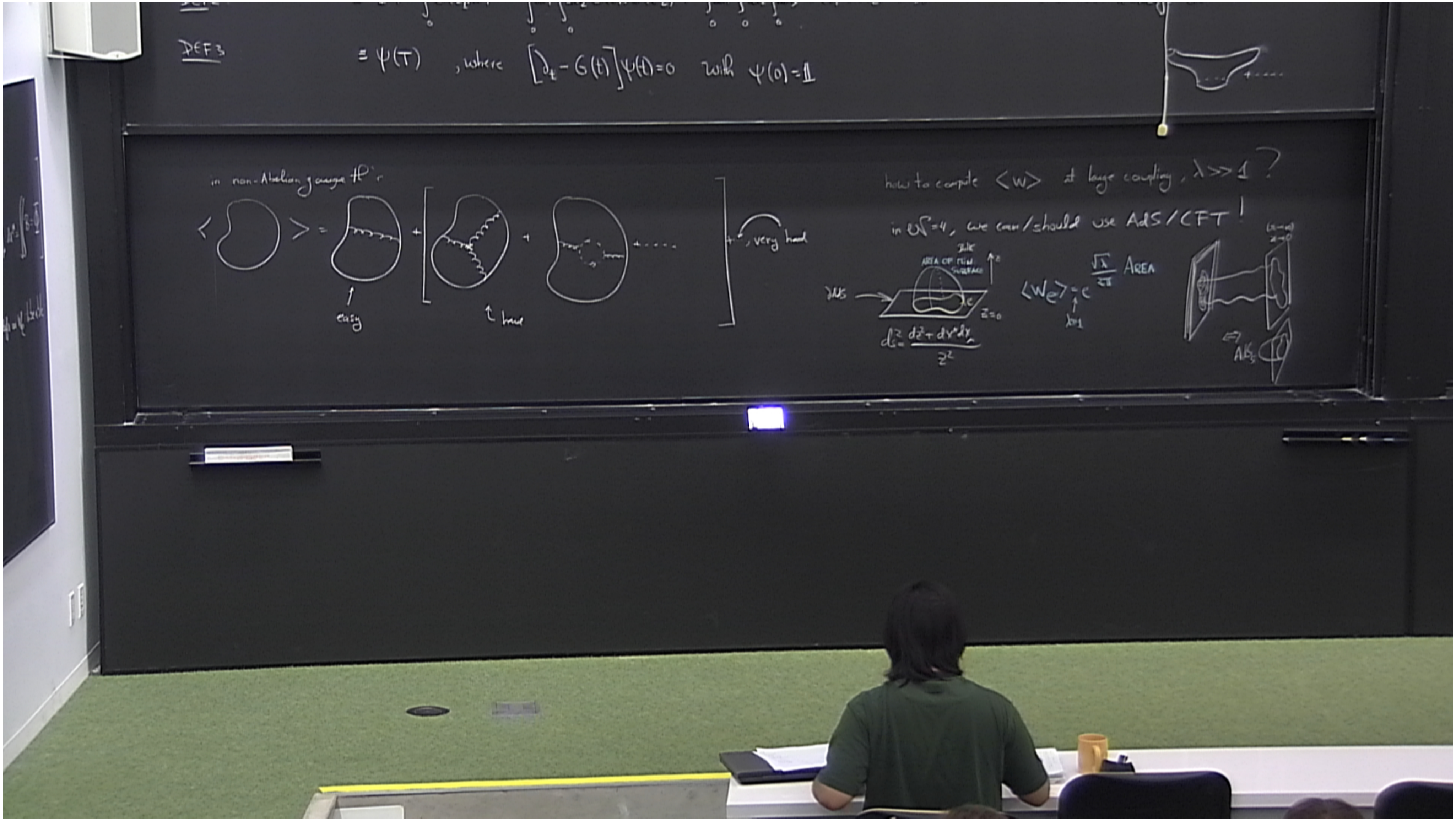
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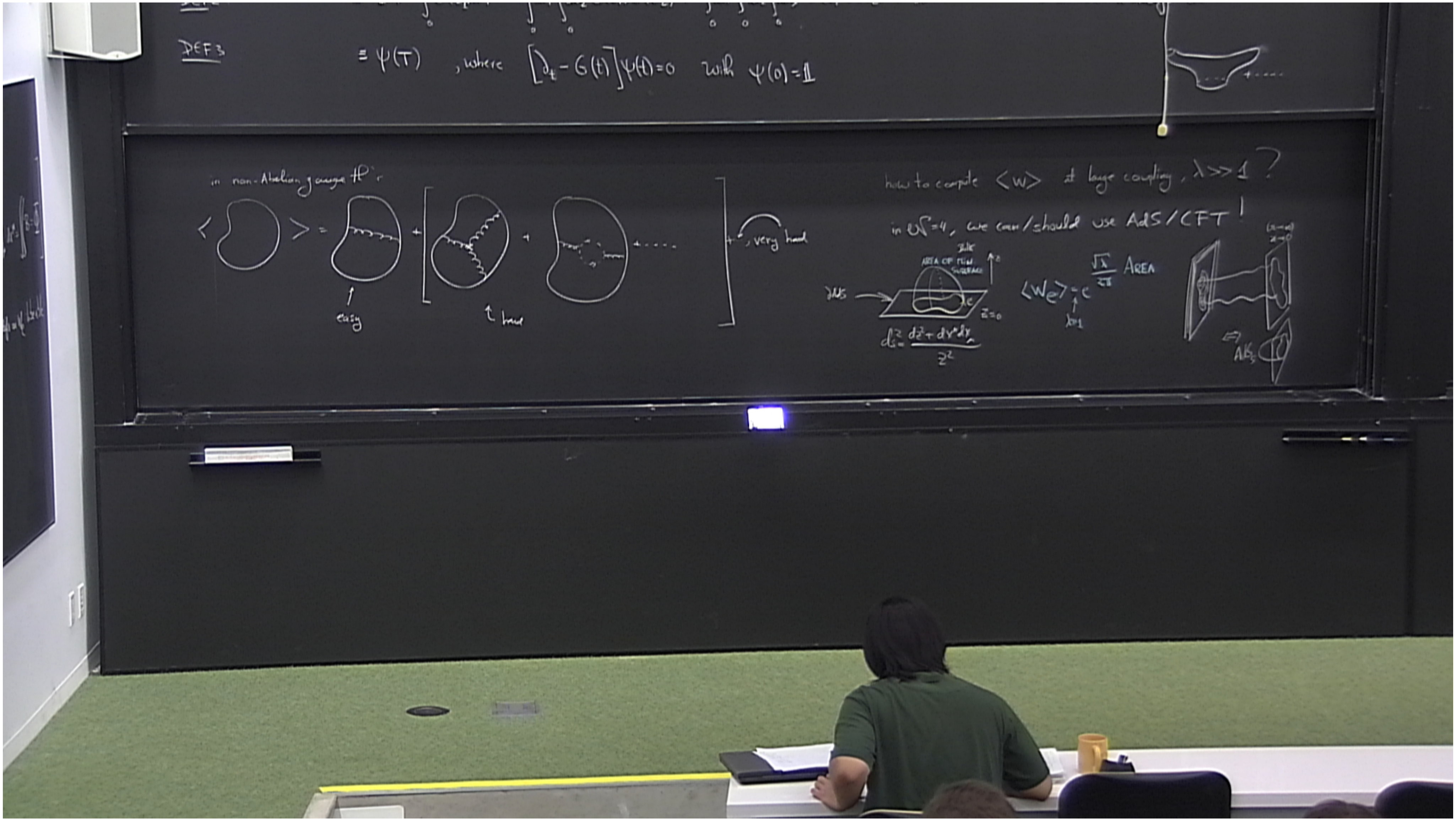
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