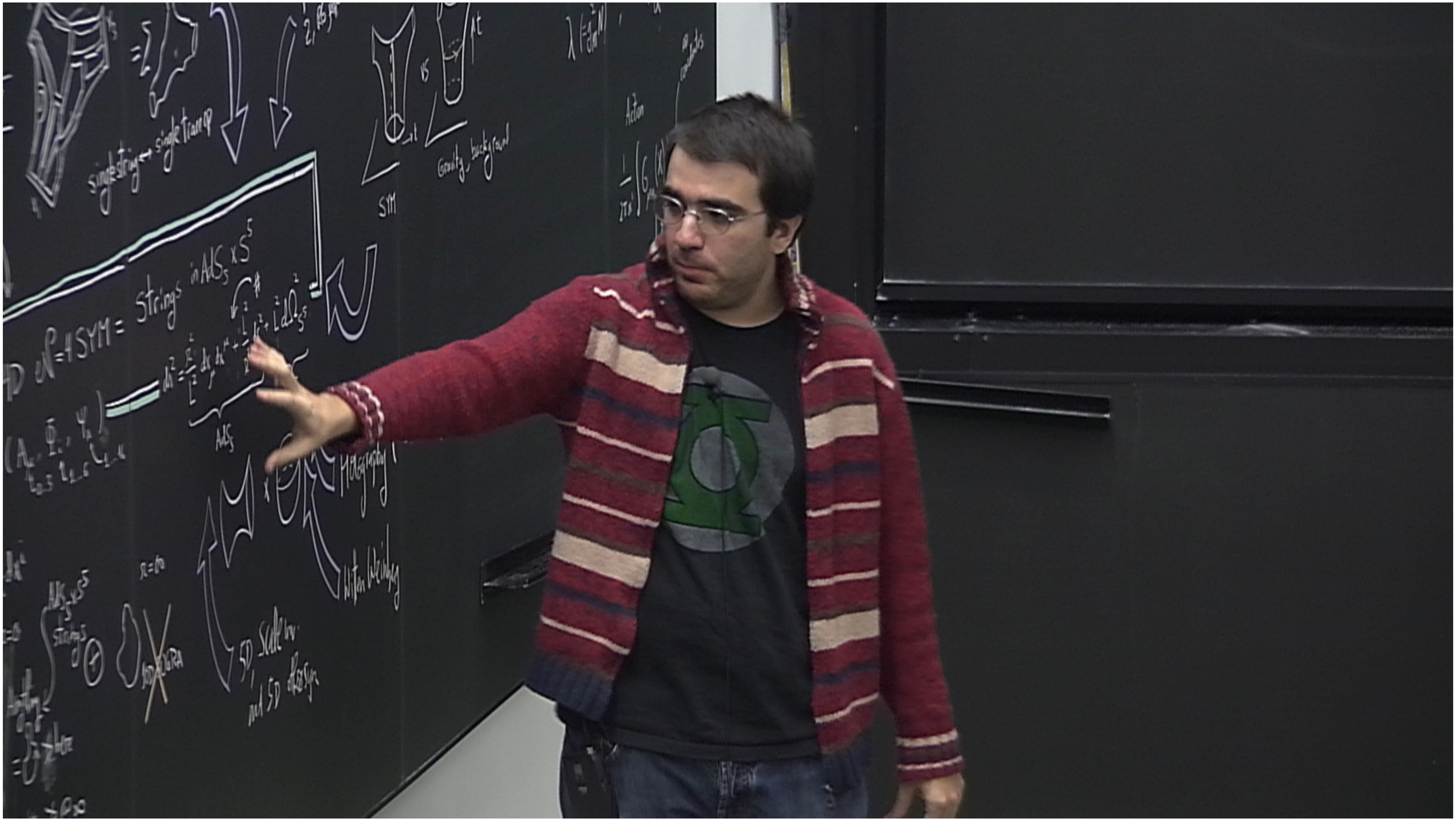
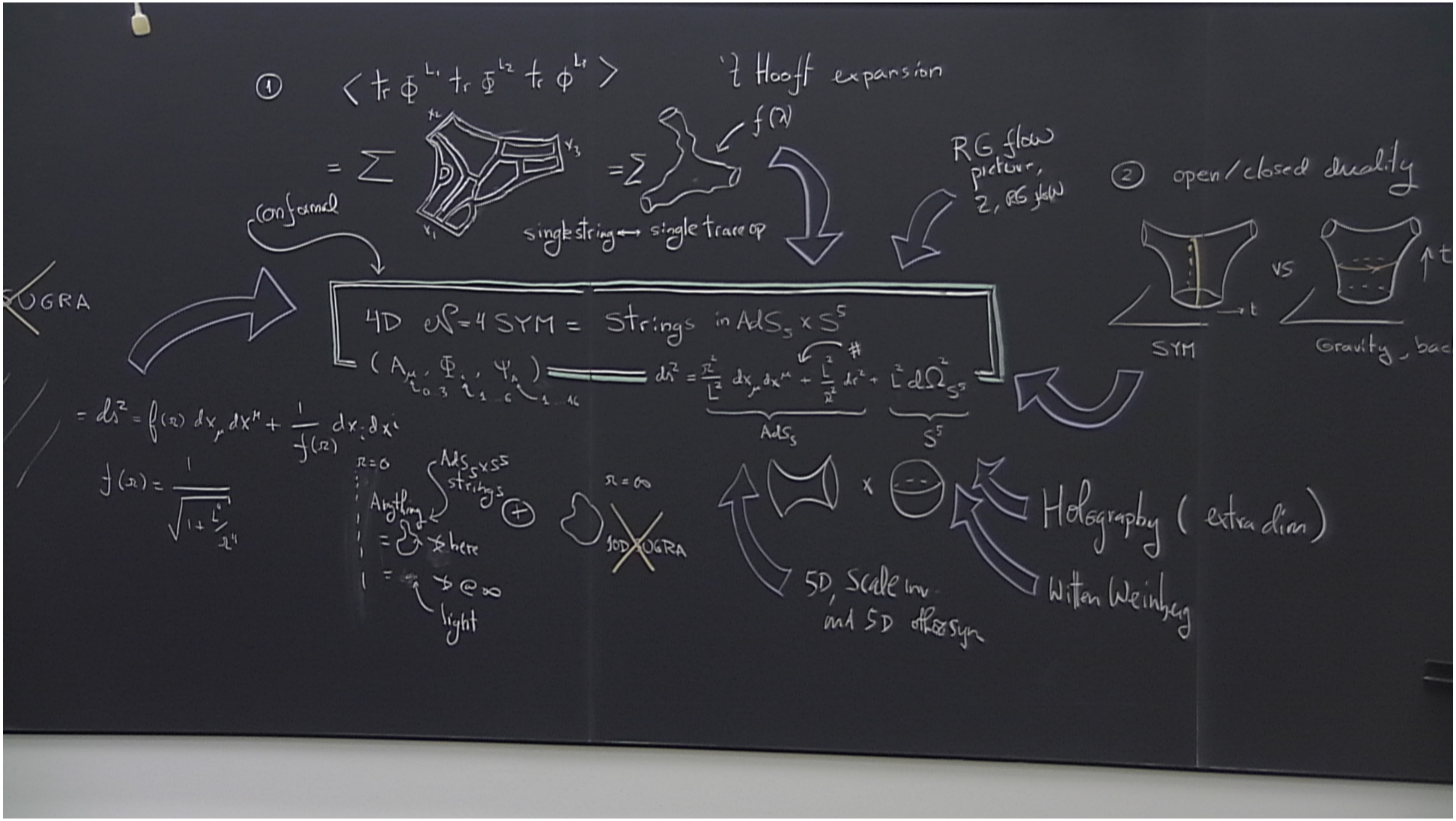
Title: Explorations in String Theory - Lecture 5

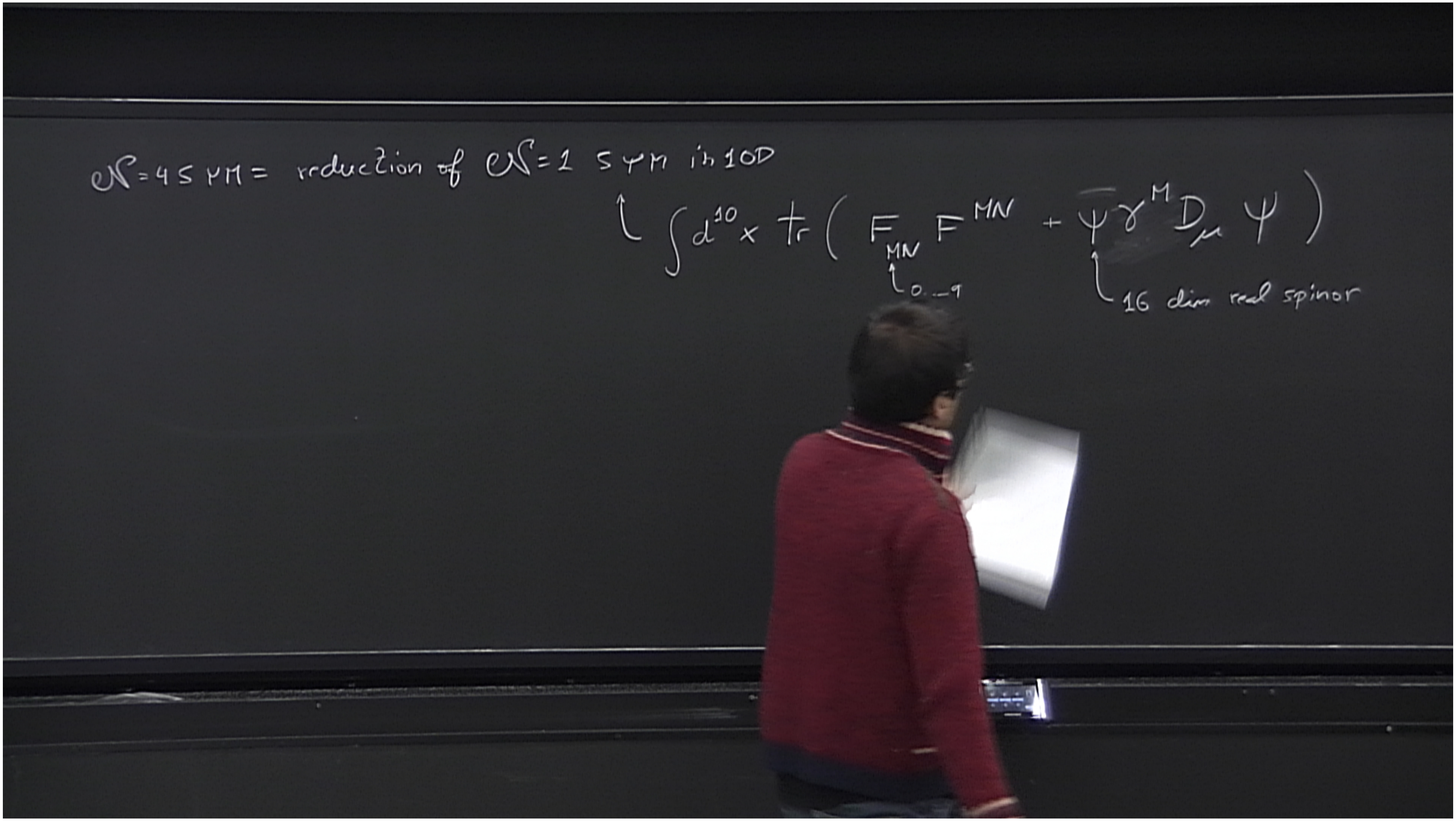
Date: Mar 16, 2012 11:30 AM

URL: http://pirsa.org/12030046

Abstract:


SYM
Gravity background
AdS5
Holography


① ⟨ tr Φ^L1 tr Φ^L2 tr Φ^L3 ⟩
't Hooft expansion
= Σ
= Σ
f(λ)
conformal
single string ⟷ single trace op
RG flow picture, Z, RG flow
② open/closed duality
vs
SYM
Gravity, bac
SUGRA
4D N=4 SYM = Strings in AdS5 × S5
(Aμ , Φi , Ψa)
ds² = r²/L² dxμ dx^μ + L²/r² dr² + L² dΩ²S5
AdS5
S5
= ds² = f(r) dxμ dx^μ + 1/f(r) dxi dxi
f(r) = 1/√(1 + L⁴/r⁴)
r = 0
r = ∞
AdS5×S5 strings
Anything
⊕
10D SUGRA
light
Holography (extra dim)
5D, Scale inv
Witten Weinberg


N = 4 SYM = reduction of N = 1 SYM in 10D
∫ d^10 x tr (F_MN F^MN + ψ̄ γ^M D_μ ψ)
0 ... 9
16 dim real spinor

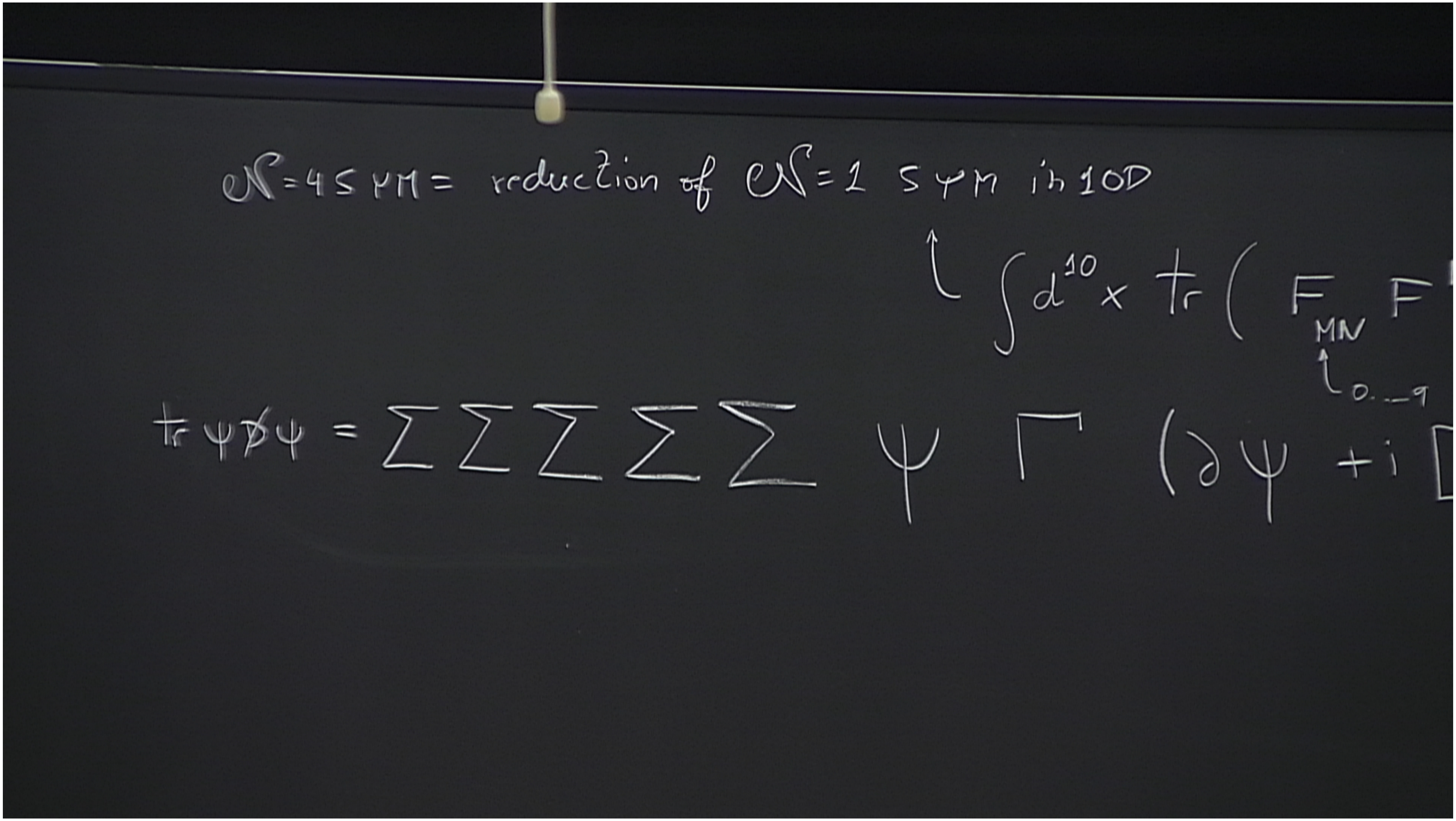
$\mathcal{N}=1$ SYM in 10D

$$\int d^{10}x \, \mathrm{tr}\left(F_{MN} F^{MN} + \bar{\psi} \Gamma^{M} D_{\mu} \psi \right)$$

$M, N = 0 \dots 9$

ψ: 16 dim real spinor

$$\Gamma^{M}\Gamma^{N} + \Gamma^{N}\Gamma^{M} = \eta^{MN} \mathbb{1}$$


N=4 SYM = reduction of N=1 SYM in 10D
∫d^10 x tr(F_MN F
MN
0...9
tr ψ D̸ ψ = Σ Σ Σ Σ Σ ψ Γ (∂ψ + i [

$\mathcal{N}=4$ SYM = reduction of $\mathcal{N}=1$ SYM in 10D

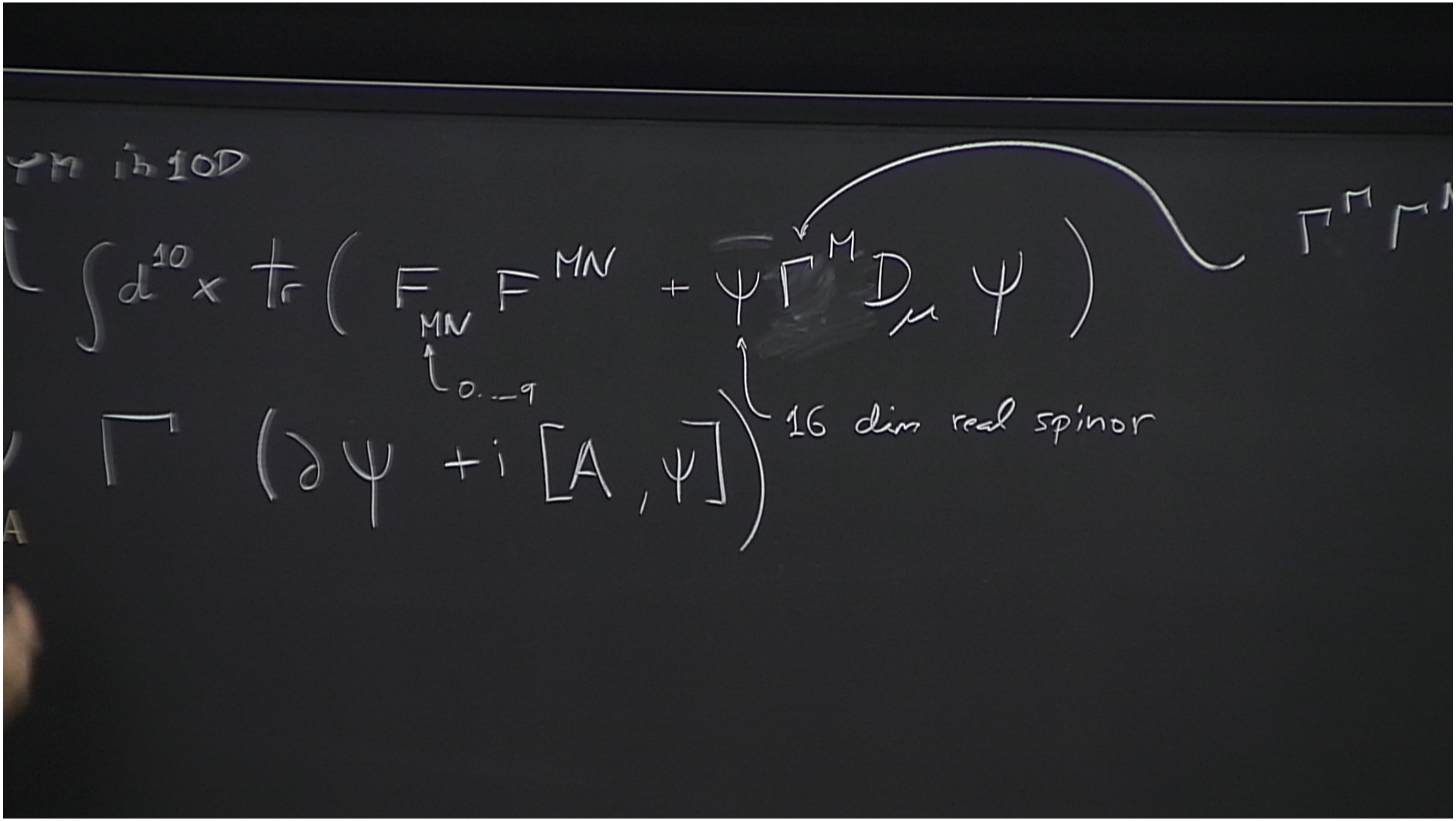
$$\int d^{10}x \, \mathrm{tr}\left(F_{MN} F^{MN} + \bar{\psi} \Gamma^M D_M \psi \right)$$

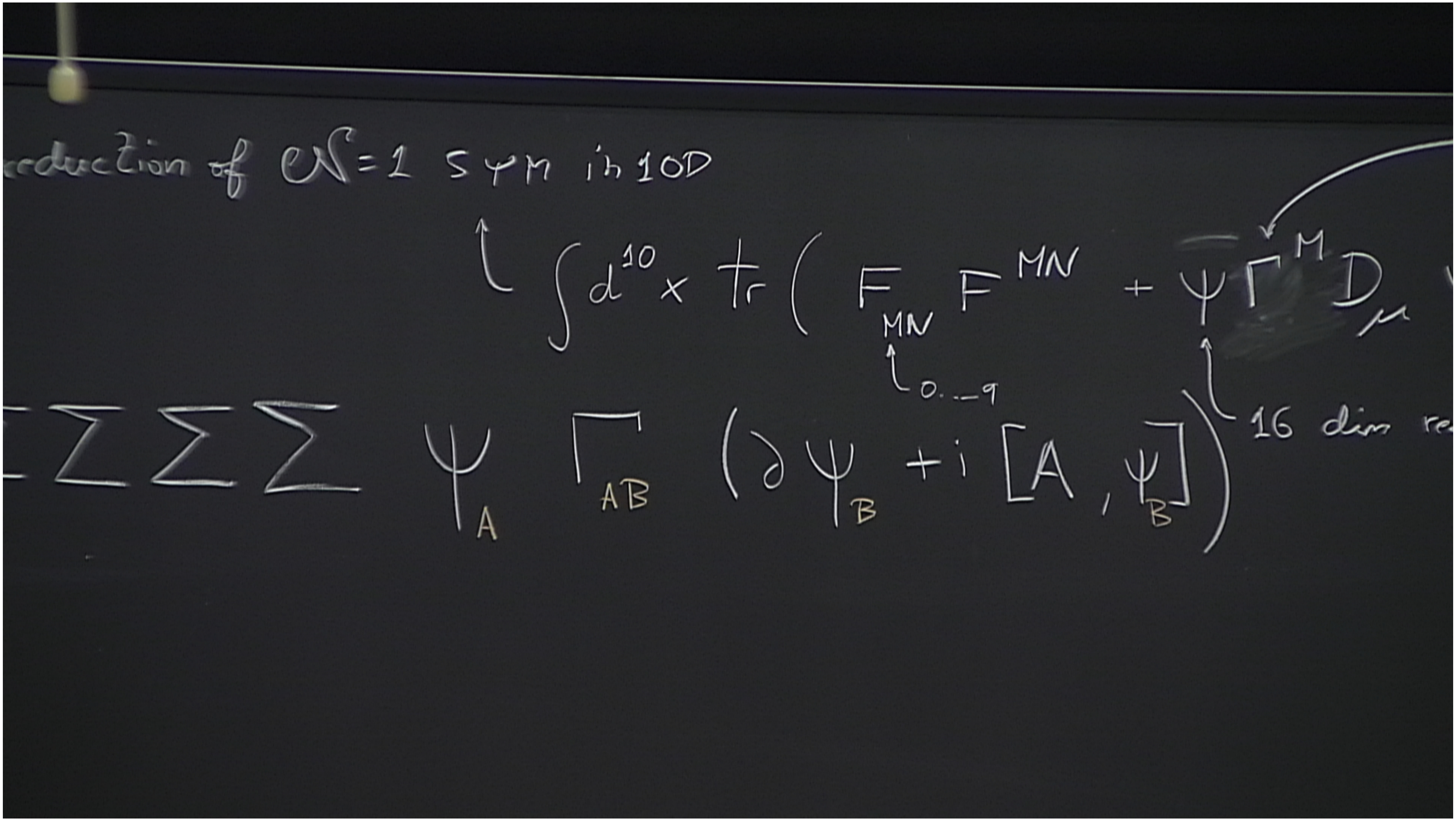
$M, N = 0 \ldots 9$

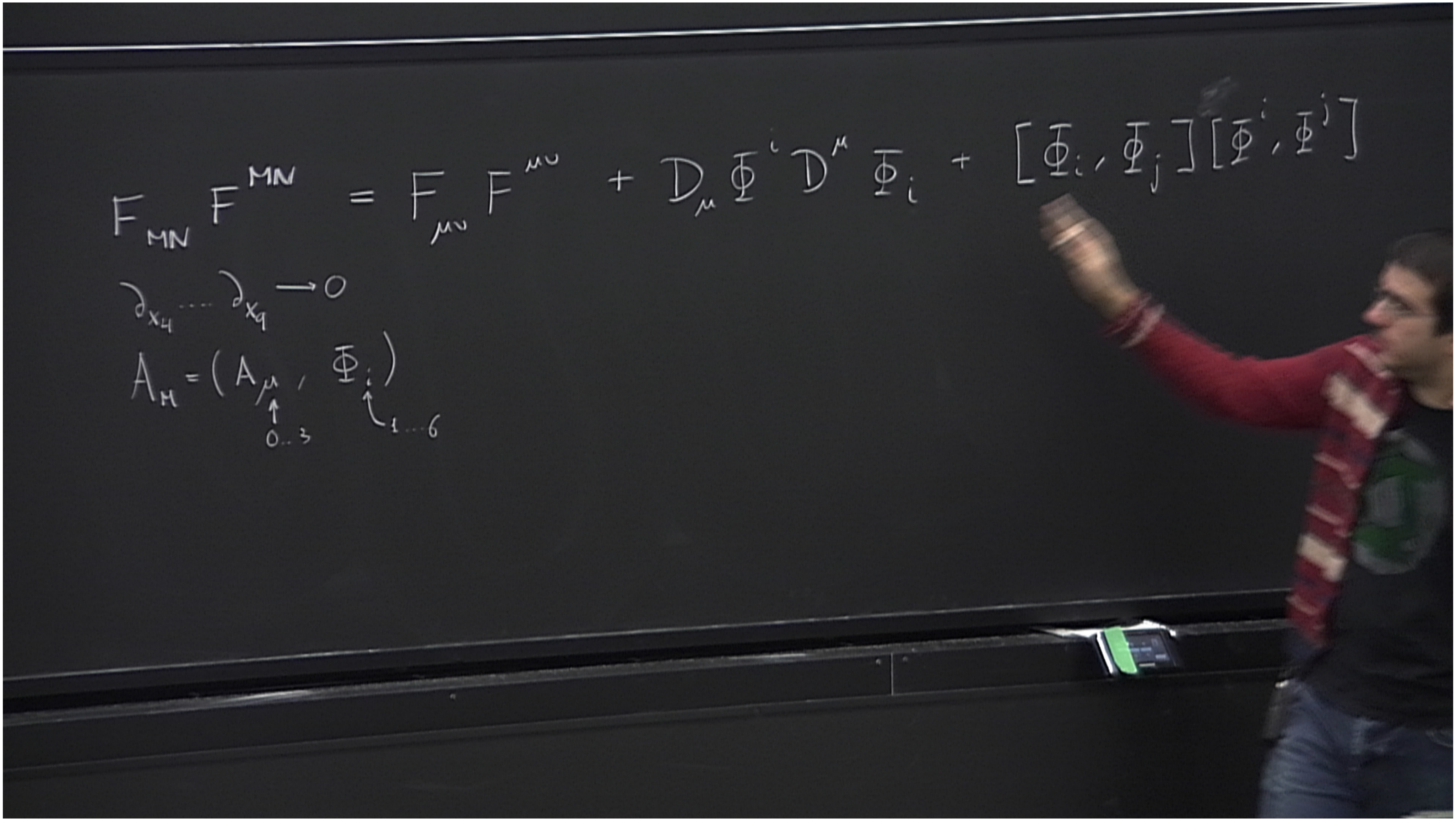
ψ: 16 dim real spinor

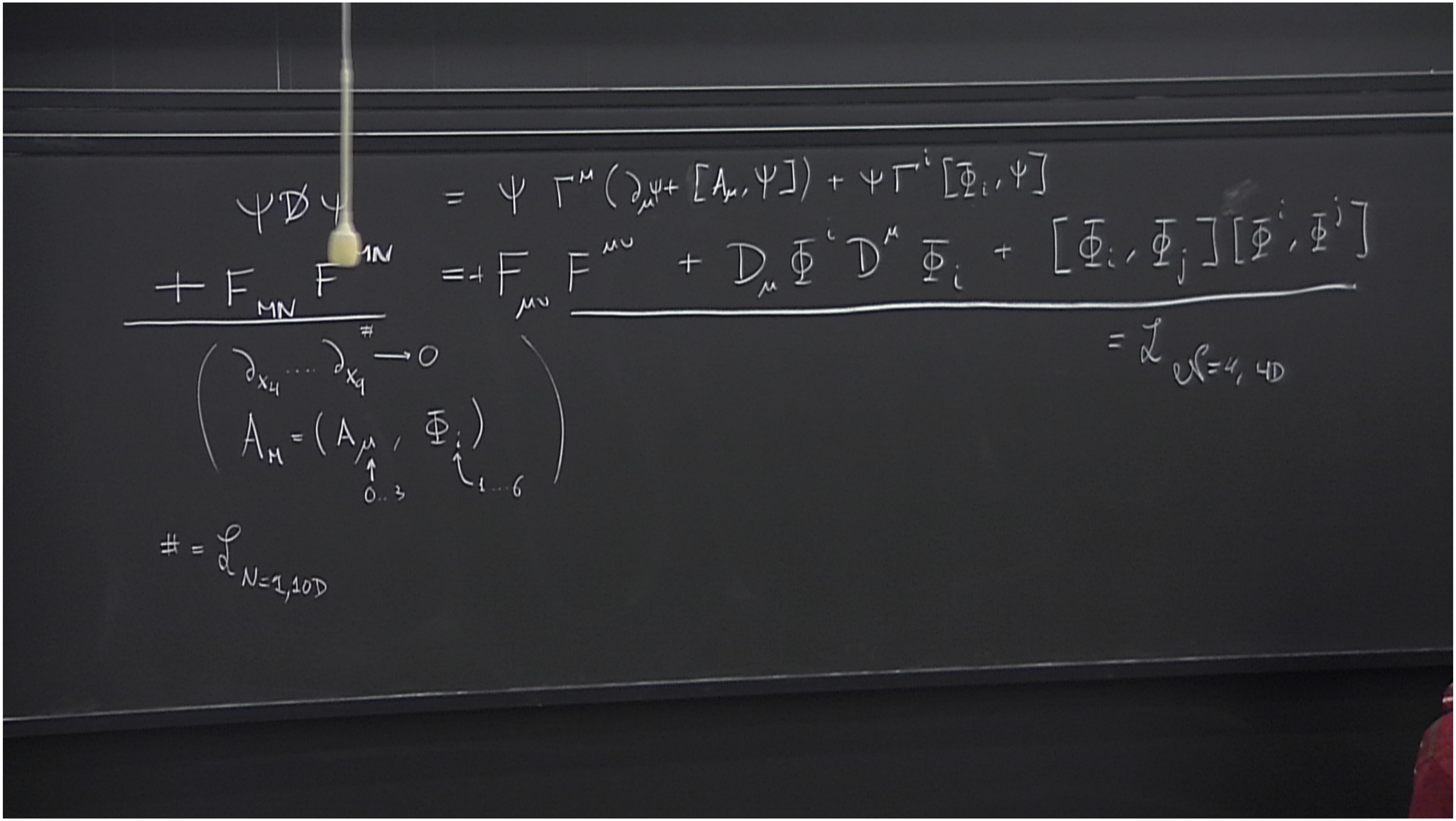
$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = \eta^{MN} \mathbb{1}$$

$$\mathrm{tr}\, \psi \not{D} \psi = \sum \sum \sum \sum \sum \psi \, \Gamma \left(\partial \psi + i [A, \psi] \right)$$


YM in 10D
∫d^10 x tr(F_MN F^MN + ψ Γ^M D_μ ψ)
0...9
16 dim real spinor
Γ (∂ψ + i [A, ψ])


eduction of N=1 SYM in 10D
∫d^10 x tr(F_MN F^MN + ψ̄ Γ^M D_μ
0...9
16 dim re
Σ Σ Σ ψ_A Γ_AB (∂ψ_B + i [A, ψ_B]))


$F_{MN} F^{MN} = F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi^i D^\mu \Phi_i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$
$\partial_{x_4} \ldots \partial_{x_9} \longrightarrow 0$
$A_M = (A_\mu , \Phi_i)$
$0..3$
$1...6$


$\Psi \not{D} \Psi = \Psi \Gamma^\mu (\partial_\mu \Psi + [A_\mu, \Psi]) + \Psi \Gamma^i [\Phi_i, \Psi]$
$+ F_{MN} F^{MN} = + F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi^i D^\mu \Phi_i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$
$= \mathcal{L}_{\mathcal{N}=4,\, 4D}$
$\partial_{x_4} \ldots \partial_{x_9} \longrightarrow 0$
$A_M = (A_\mu, \Phi_i)$
$0..3$
$1...6$
$\# = \mathcal{L}_{N=1,\, 10D}$

$\bar{\Phi}^j]$

All fields are in the adjoint

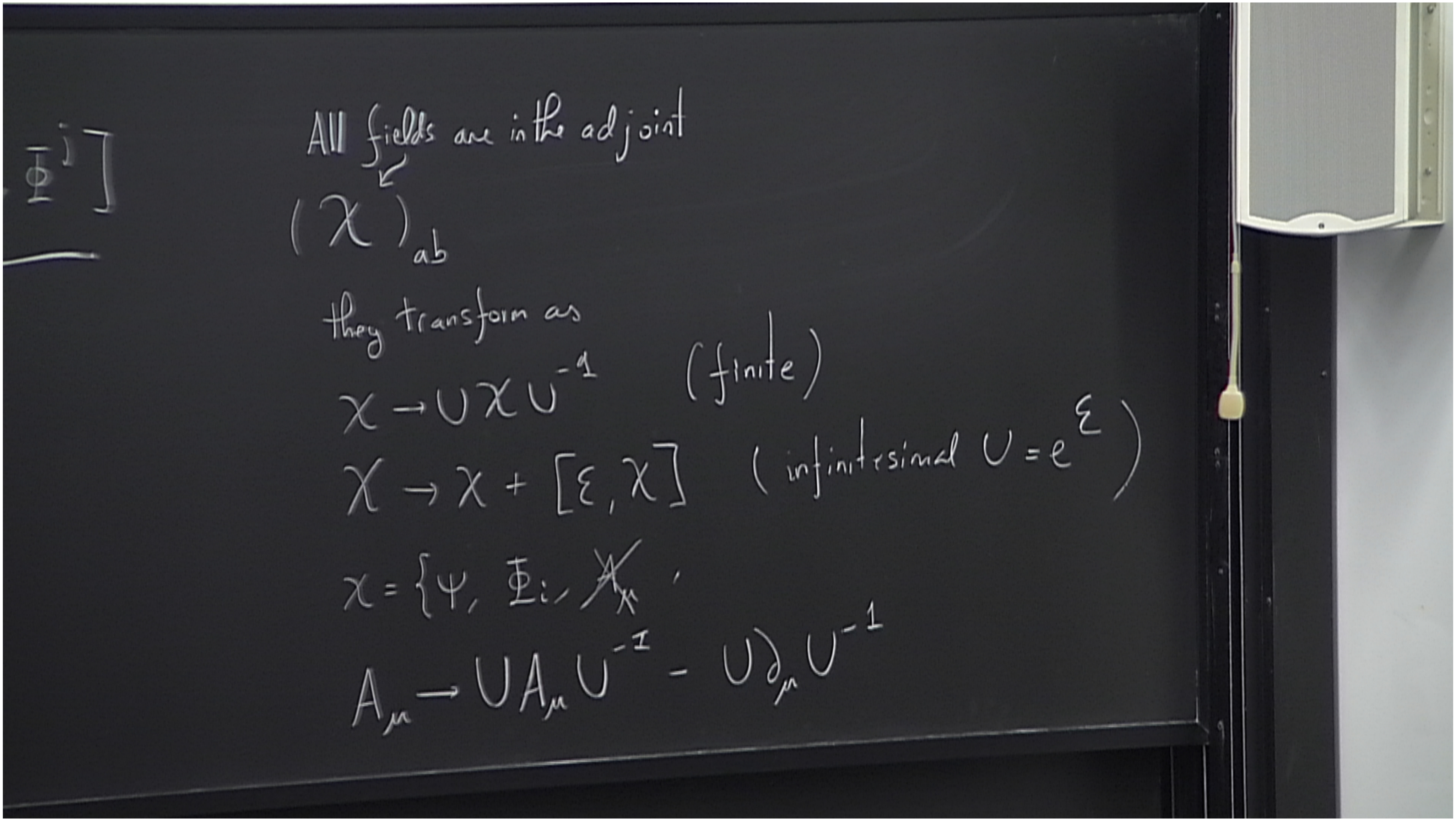
$(\chi)_{ab}$

they transform as

$\chi \to U \chi U^{-1}$ (finite)

$\chi \to \chi + [\varepsilon, \chi]$ (infinitesimal $U = e^{\varepsilon}$)

$\chi = \{\Psi, \Phi,$


Φ^j]
All fields are in the adjoint
(χ)_ab
they transform as
χ → U χ U^-1 (finite)
χ → χ + [ε, χ] (infintesimal U = e^ε)
χ = {Ψ, Φ_i, A_μ
A_μ → U A_μ U^-1 - U ∂_μ U^-1

$\bar{\Phi}^j]$

All fields are in the adjoint

$(\chi)_{ab}$

they transform as

$\chi \to U \chi U^{-1}$ (finite)

$\chi \to \chi + [\varepsilon, \chi]$ (infinitesimal $U = e^{\varepsilon}$)

$\chi = \{\Psi, \Phi_i, \cancel{A_\mu}, D_\mu \Phi_i$

$A_\mu \to U A_\mu U^{-1} - U \partial_\mu U^{-1}$

$\Phi^j]$

All fields are in the adjoint

$$(\chi)_{ab}$$

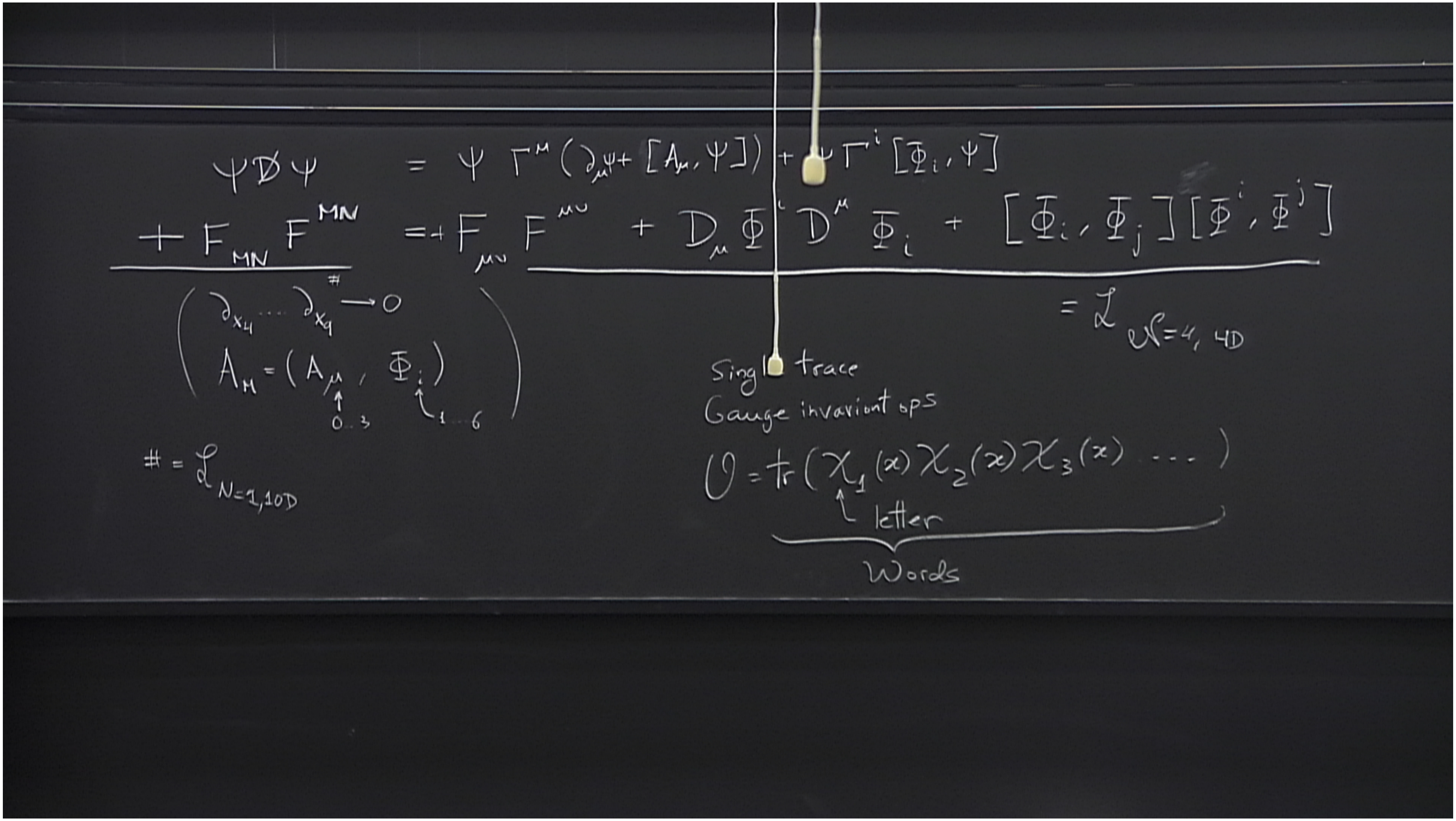
they transform as

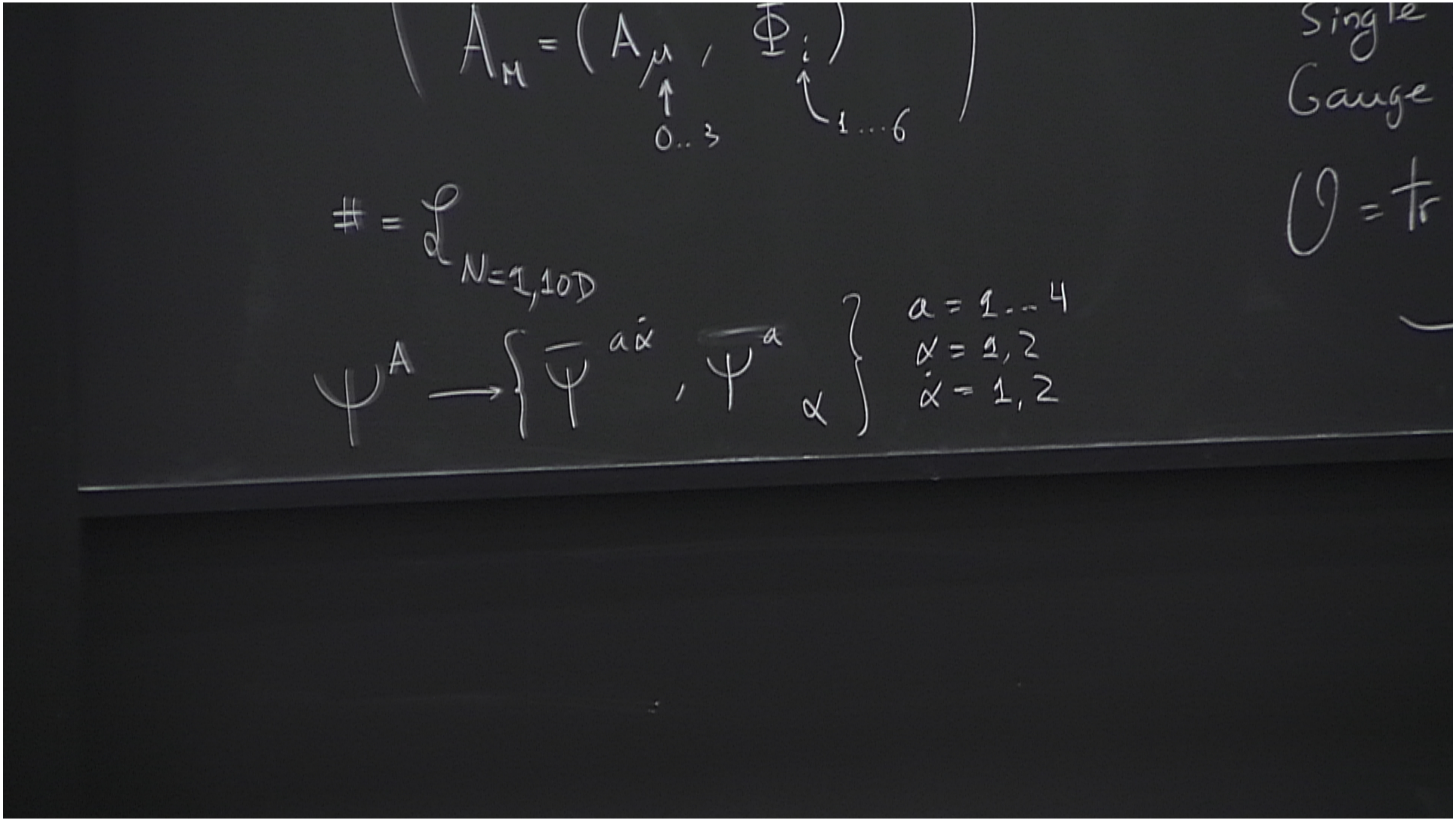
$$\chi \to U \chi U^{-1} \quad \text{(finite)}$$

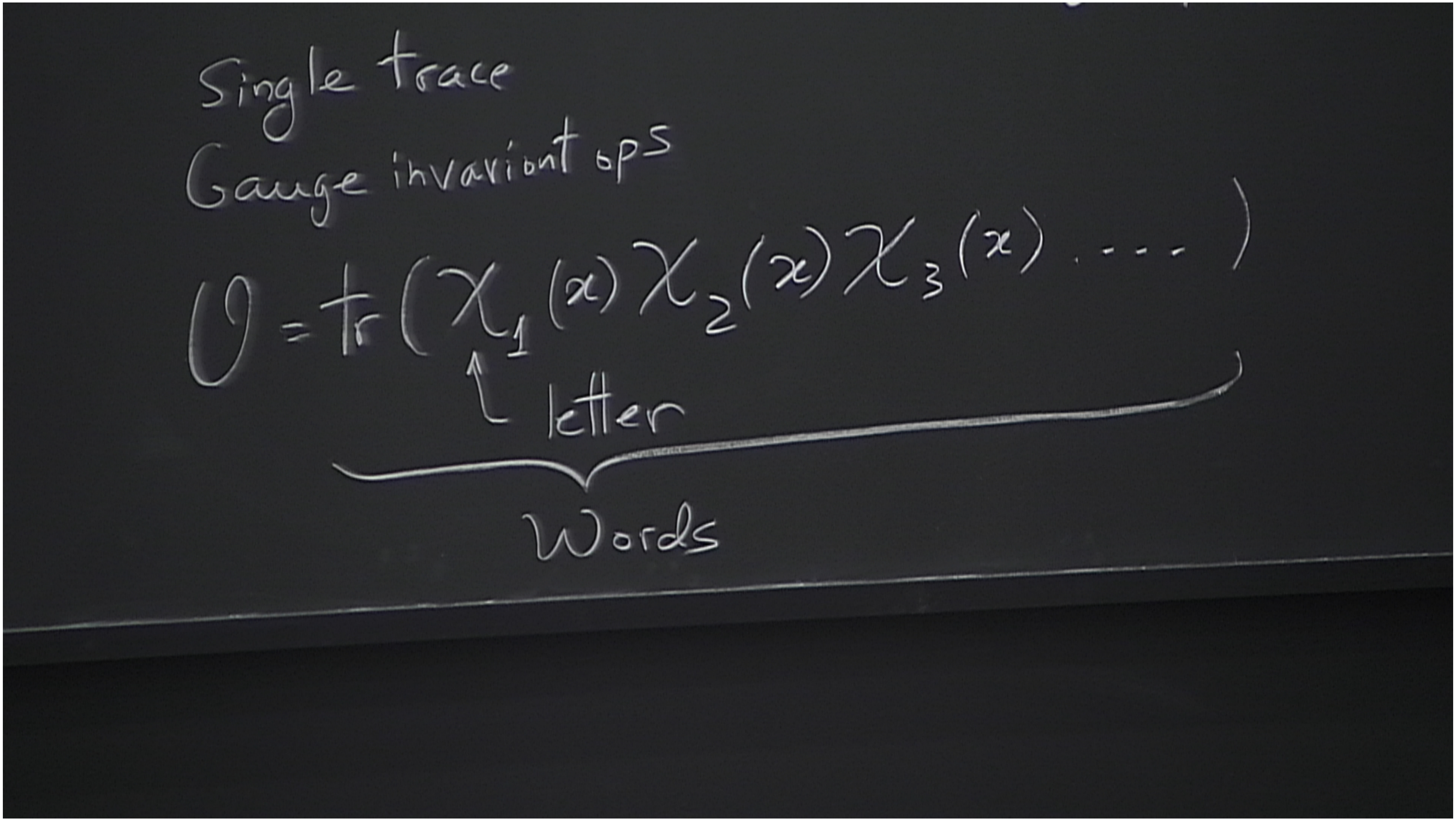
$$\chi \to \chi + [\varepsilon, \chi] \quad (\text{infinitesimal } U = e^{\varepsilon})$$

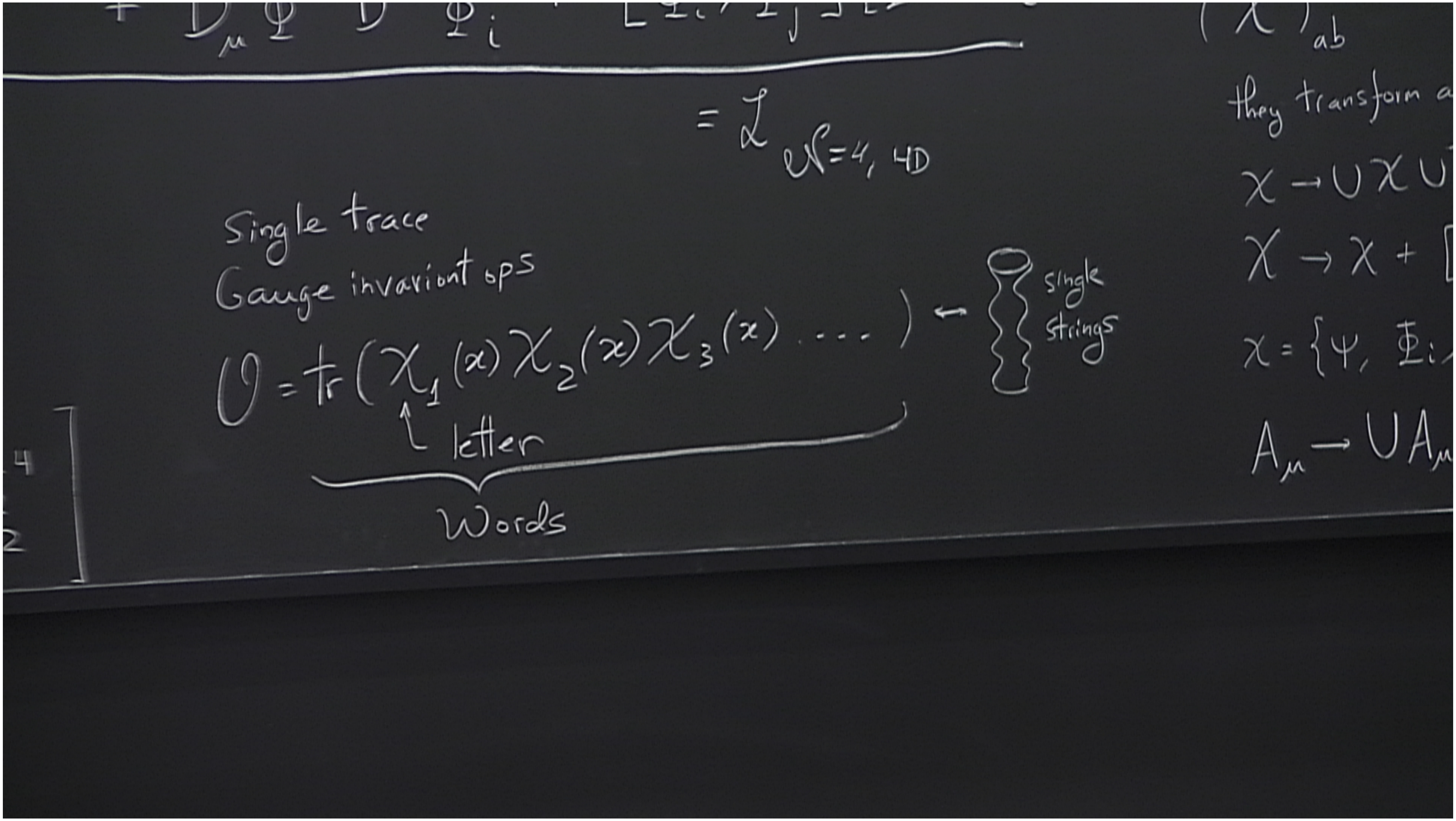
$$\chi = \{\Psi, \Phi_i, \cancel{A_\mu}, D_\mu \Phi_i, F_{\mu\nu}, D_\mu D_\nu \Psi_A\}$$

$$A_\mu \to U A_\mu U^{-1} - U \partial_\mu U^{-1}$$


$\psi \not{D} \psi = \psi \Gamma^{\mu} (\partial_\mu \psi + [A_\mu, \psi]) + \psi \Gamma^{i} [\Phi_i, \psi]$
$+ F_{MN} F^{MN} = + F_{\mu\nu} F^{\mu\nu} + D_\mu \Phi^i D^\mu \Phi_i + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$
$= \mathcal{L}_{\mathcal{N}=4,\ 4D}$
$\partial_{x_4} \dots \partial_{x_9} \to 0$
$A_M = (A_\mu, \Phi_i)$
$0 \dots 3$
$1 \dots 6$
$\# = \mathcal{L}_{N=1,\ 10D}$
Single trace
Gauge invariant ops
$\mathcal{O} = \mathrm{tr}(\chi_1(x) \chi_2(x) \chi_3(x) \dots)$
letter
Words


$A_M = (A_\mu, \Phi_i)$
0..3
1...6
$\# = \mathcal{L}_{N=1, 10D}$
$\psi^A \longrightarrow \{\bar{\psi}^{a\dot{\alpha}}, \psi^a{}_\alpha\}$
$a = 1 \ldots 4$
$\alpha = 1, 2$
$\dot{\alpha} = 1, 2$
Single
Gauge
$\mathcal{O} = \mathrm{tr}$


Single trace
Gauge invariant ops
$\mathcal{O} = \mathrm{tr}(\mathcal{X}_1(x)\mathcal{X}_2(x)\mathcal{X}_3(x)\ldots)$
letter
Words


they transform a
Single trace
Gauge invariant ops
single
strings
letter
Words

$\mathcal{N}=4$ SYM = reduction of $\mathcal{N}=1$ SYM in 10D

$$\int d^{10}x \, \mathrm{tr}\left(F_{MN} F^{MN} + \bar{\psi} \Gamma^M D_M \psi \right)$$

MN: $0 \ldots 9$

ψ: 16 dim real spinor

$$\mathrm{tr}\, \bar{\psi} \not{D} \psi = \sum_{M=0}^{9} \sum_{b=1}^{N} \sum_{a=1}^{N} \sum_{B=1}^{16} \sum_{A=1}^{16} \left(\psi_A\right)_{ab} \Gamma^M_{AB} \left(\partial_M \psi_B + i \left[A_M, \psi_B\right]\right)_{ba}$$

$\mathcal{N}=4$ SYM is a CFT

$$\mu \frac{\partial g_{YM}}{\partial \mu} = \beta = g_{YM}^3 \left(\frac{11}{3} N - \frac{1}{6} \#_{\text{scalars}} N - \frac{1}{6} \times \#\, \text{real fermions}\, N \right)$$

$\beta = 0$ @ all loops

$N=1, 10D$

$$\left[\Psi^A \longrightarrow \left\{ \bar\Psi^{a\dot\alpha},\ \bar\Psi^{a}{}_{\alpha} \right\}\right] \quad a = 1..4,\ \alpha = 1,2,\ \dot\alpha = 1,2$$

letter

Words

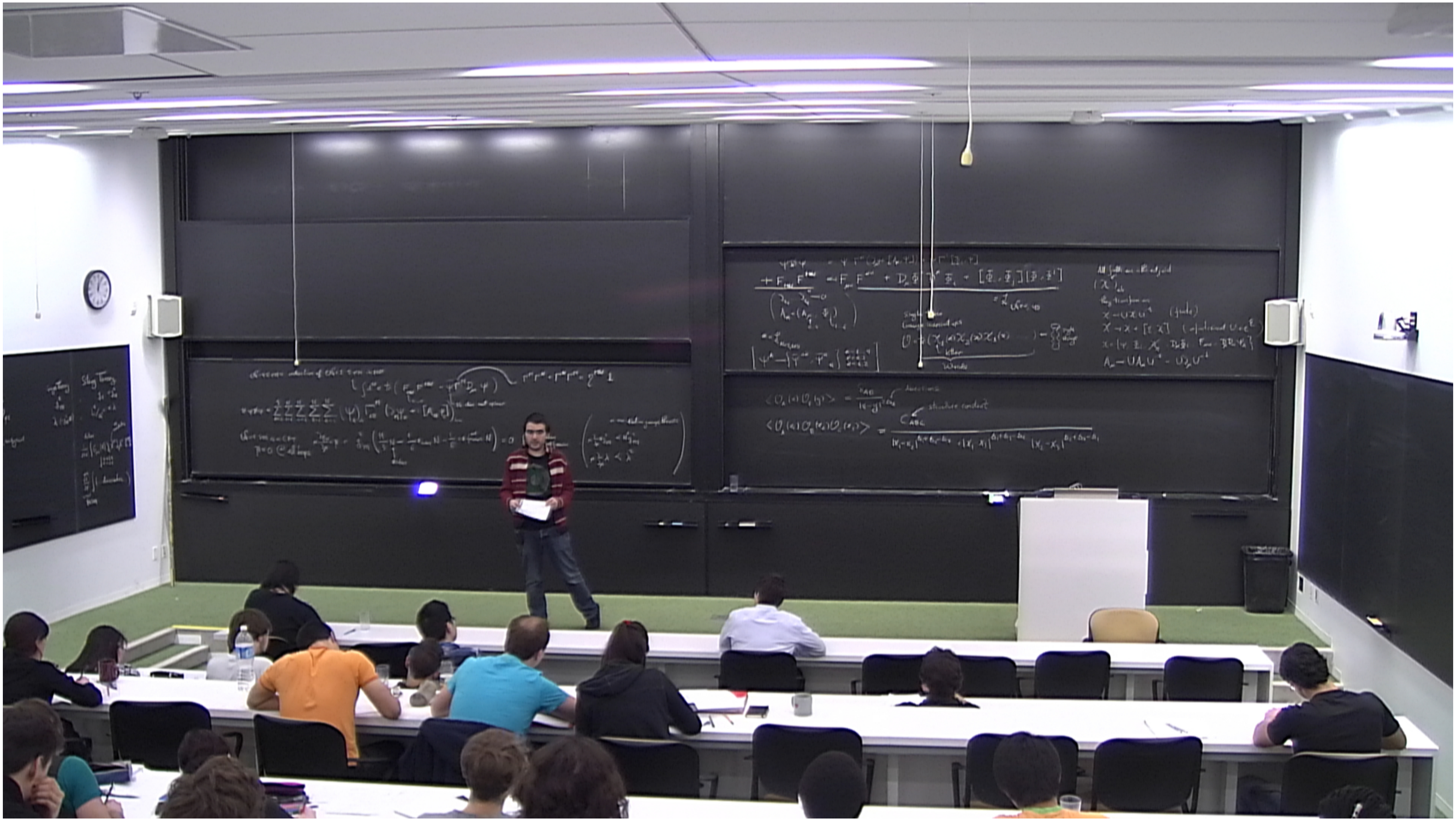
A_μ

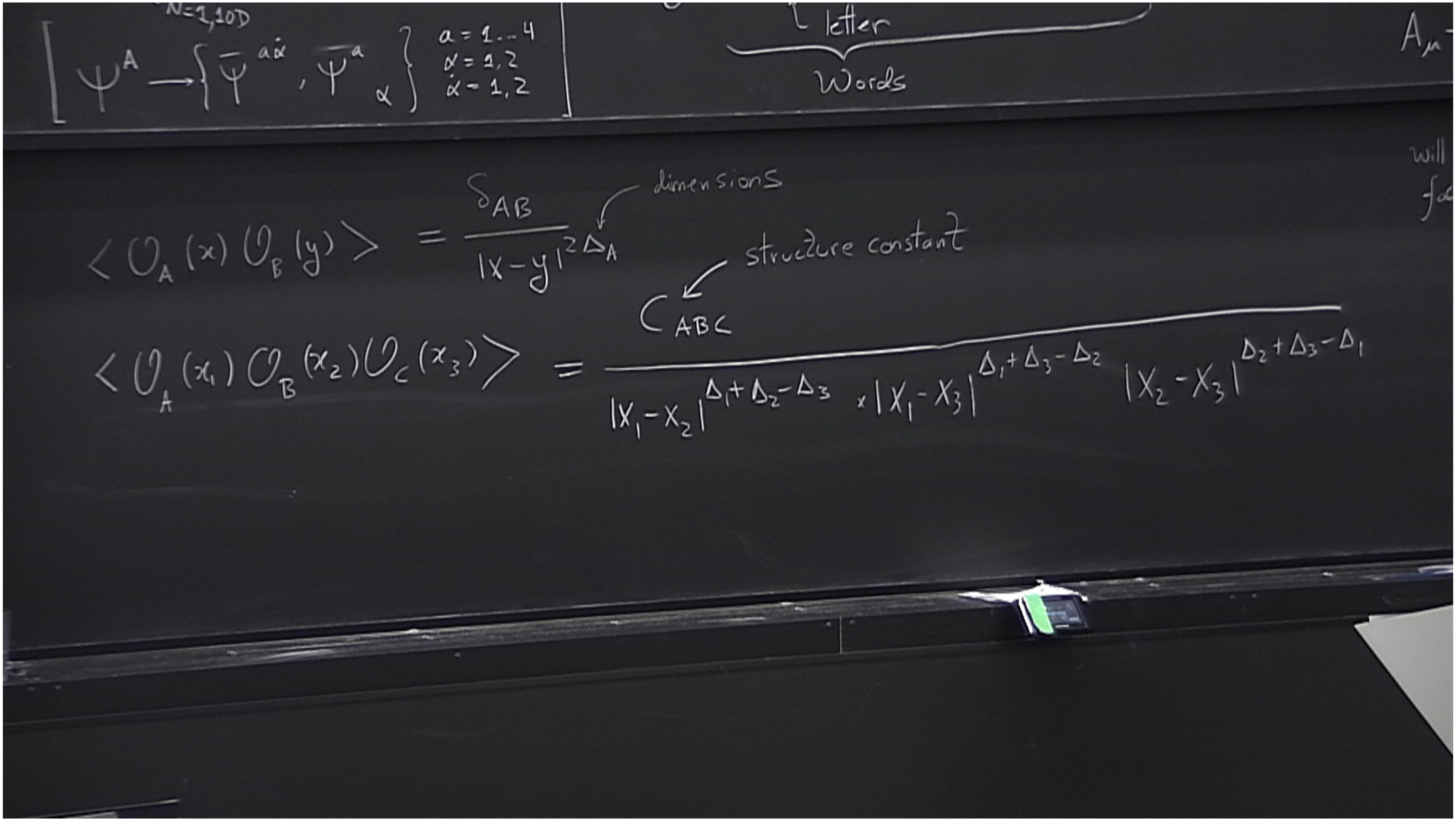
dimensions

$$\langle \mathcal{O}_A(x)\, \mathcal{O}_B(y) \rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta_A}}$$

structure constant

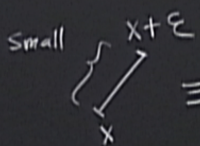
$$\langle \mathcal{O}_A(x_1)\, \mathcal{O}_B(x_2)\, \mathcal{O}_C(x_3) \rangle = \frac{C_{ABC}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3} \times |x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}\, |x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$




N=1, 10D
$\left[\Psi^A \longrightarrow \left\{ \bar{\Psi}^{a\dot{\alpha}}, \Psi^a{}_\alpha \right\}\right.$
a = 1...4
α = 1, 2
α̇ = 1, 2
letter
Words
A_μ
will
dimensions
$\langle \mathcal{O}_A(x)\,\mathcal{O}_B(y)\rangle = \frac{\delta_{AB}}{|x-y|^{2\Delta_A}}$
structure constant
$\langle \mathcal{O}_A(x_1)\,\mathcal{O}_B(x_2)\,\mathcal{O}_C(x_3)\rangle = \frac{C_{ABC}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}\times|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}\,|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$

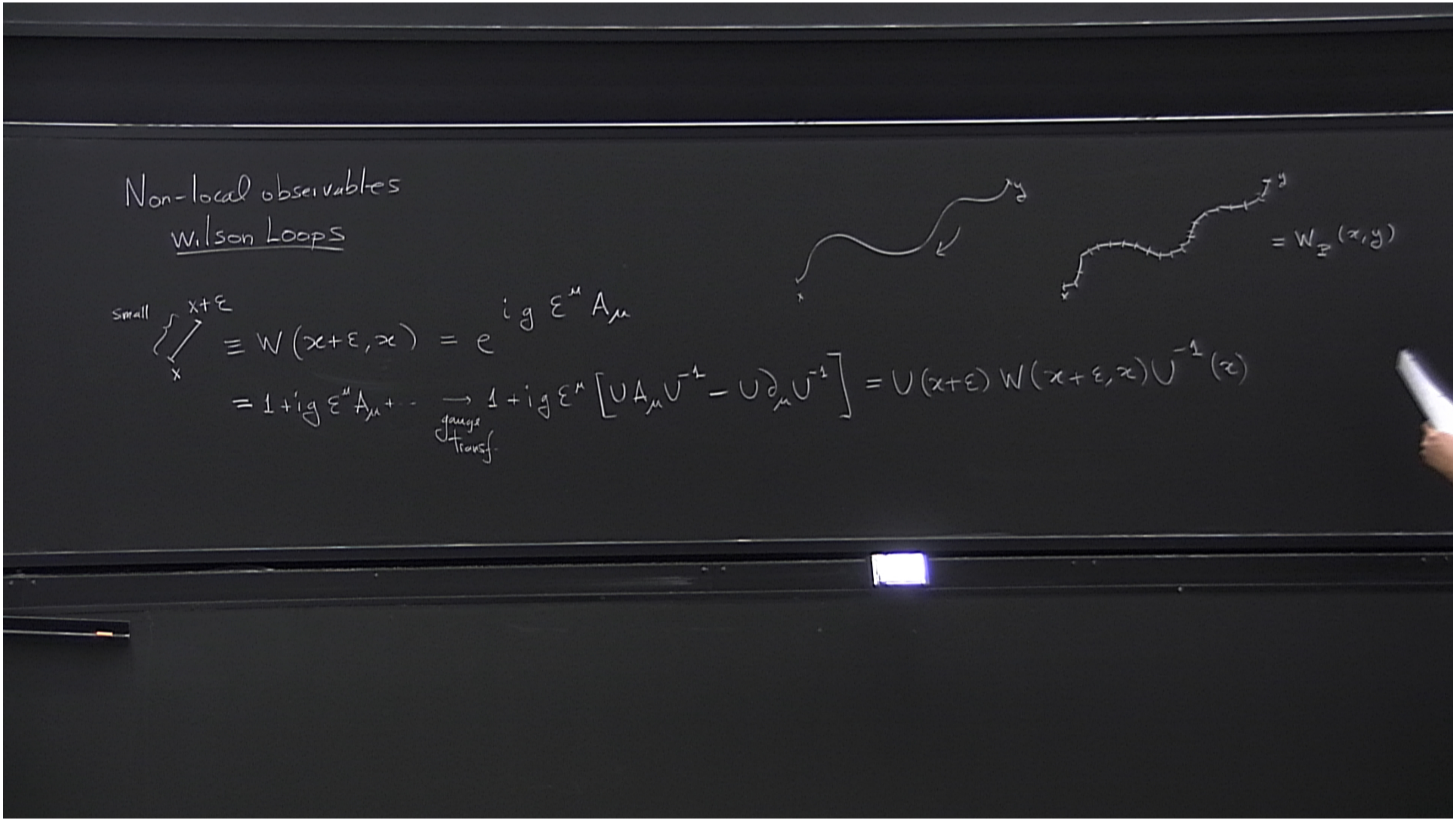
Non-local observables

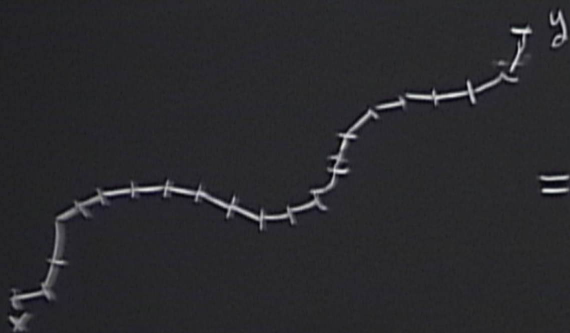
<u>Wilson Loops</u>



$$\equiv W(x+\varepsilon, x) = e^{ig\varepsilon^{\mu}A_{\mu}}$$

$$= 1 + ig\varepsilon^{\mu}A_{\mu} + \cdots \underset{\text{gauge transf.}}{\longrightarrow} 1 + ig\varepsilon^{\mu}\left[U A_{\mu} U^{-1} - U\partial_{\mu}U^{-1}\right] = U(x+\varepsilon)\, W(x+\varepsilon, x)\, U^{-1}(x)$$

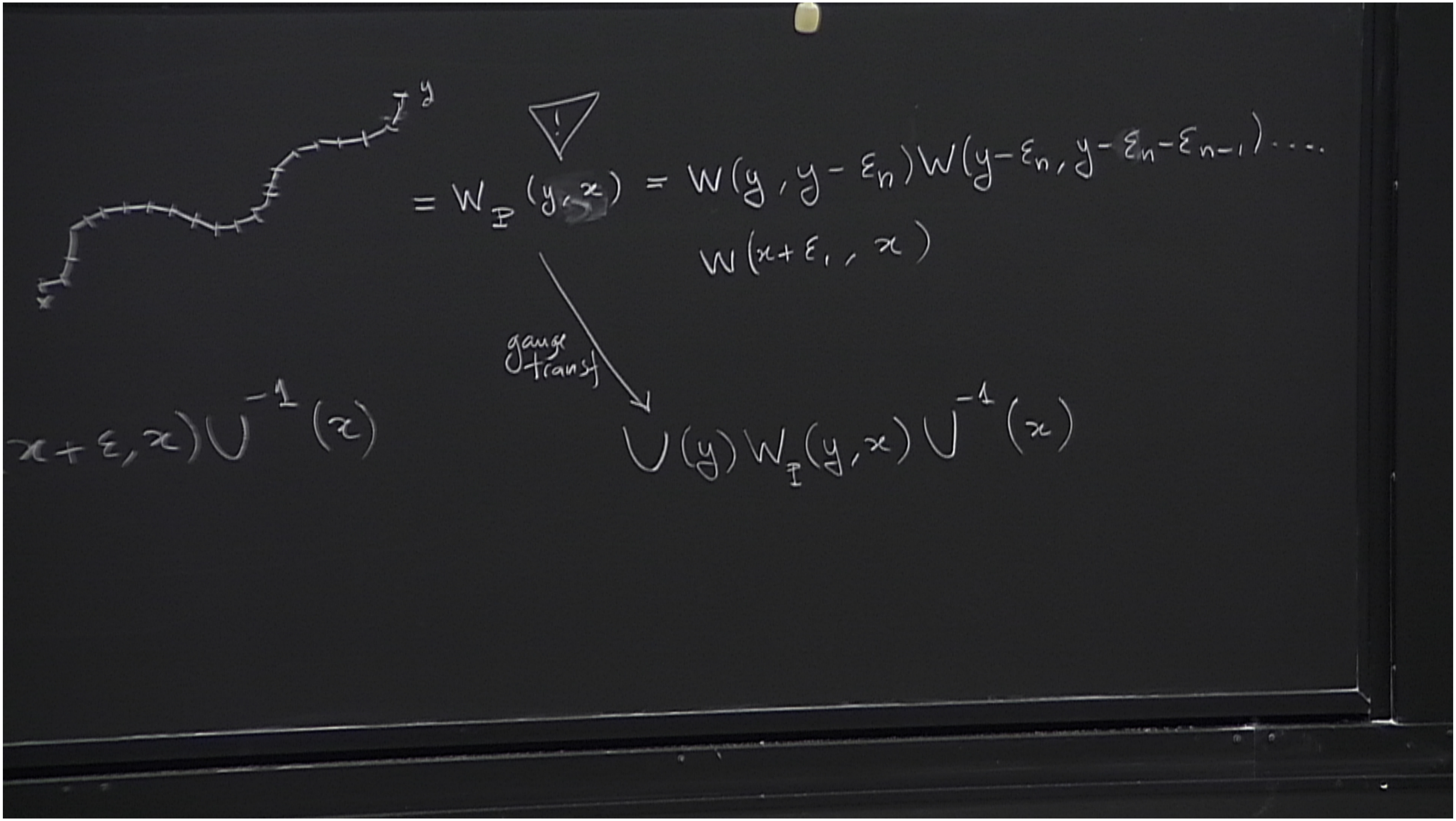

Non-local observables
Wilson Loops
small
x+ε
x
≡ W(x+ε, x) = e^{ig ε^μ A_μ}
= 1 + ig ε^μ A_μ + ... → 1 + ig ε^μ [U A_μ U^{-1} − U ∂_μ U^{-1}] = U(x+ε) W(x+ε, x) U^{-1}(x)
gauge transf.
x
y
= W_Σ(x, y)

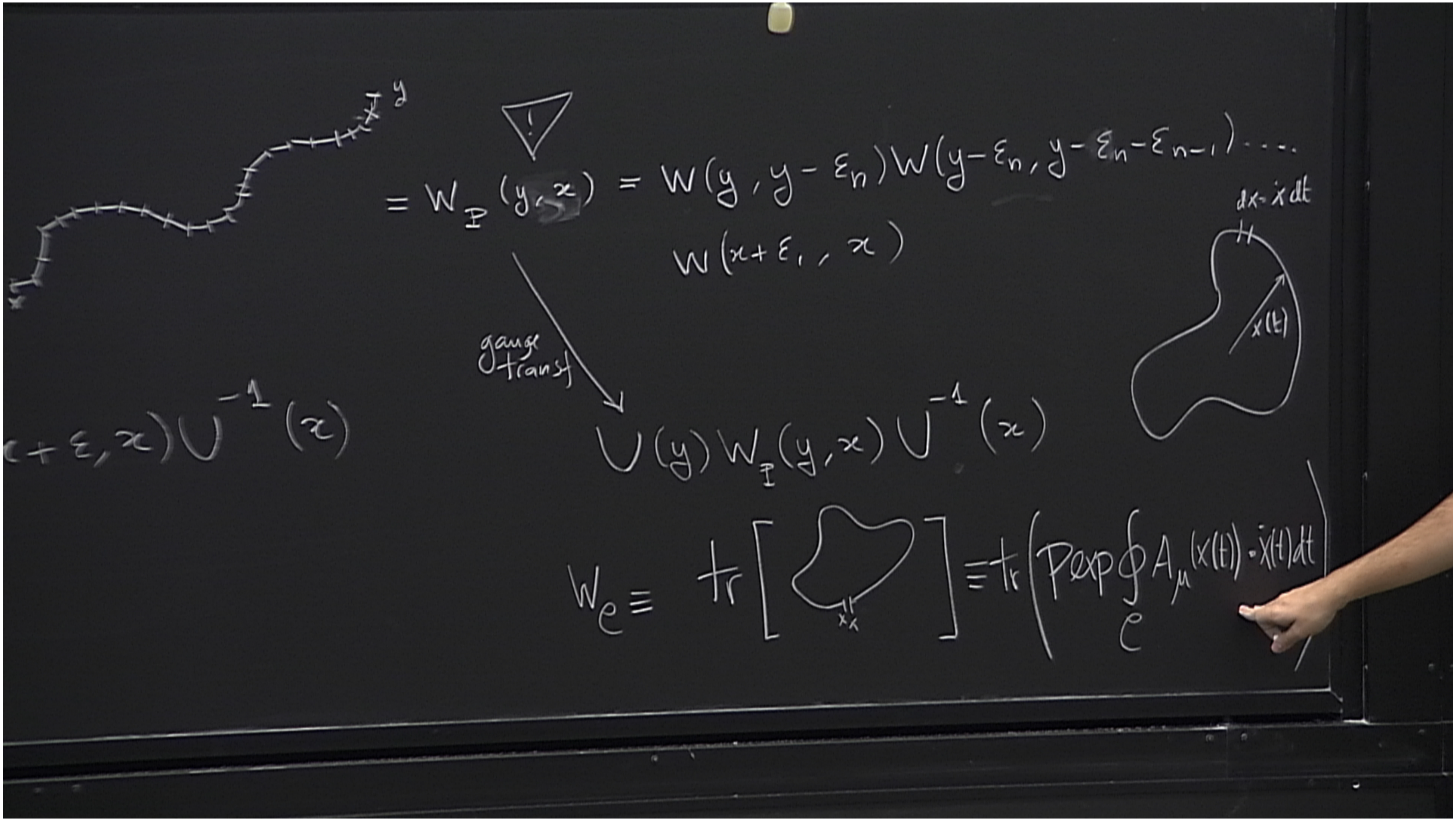


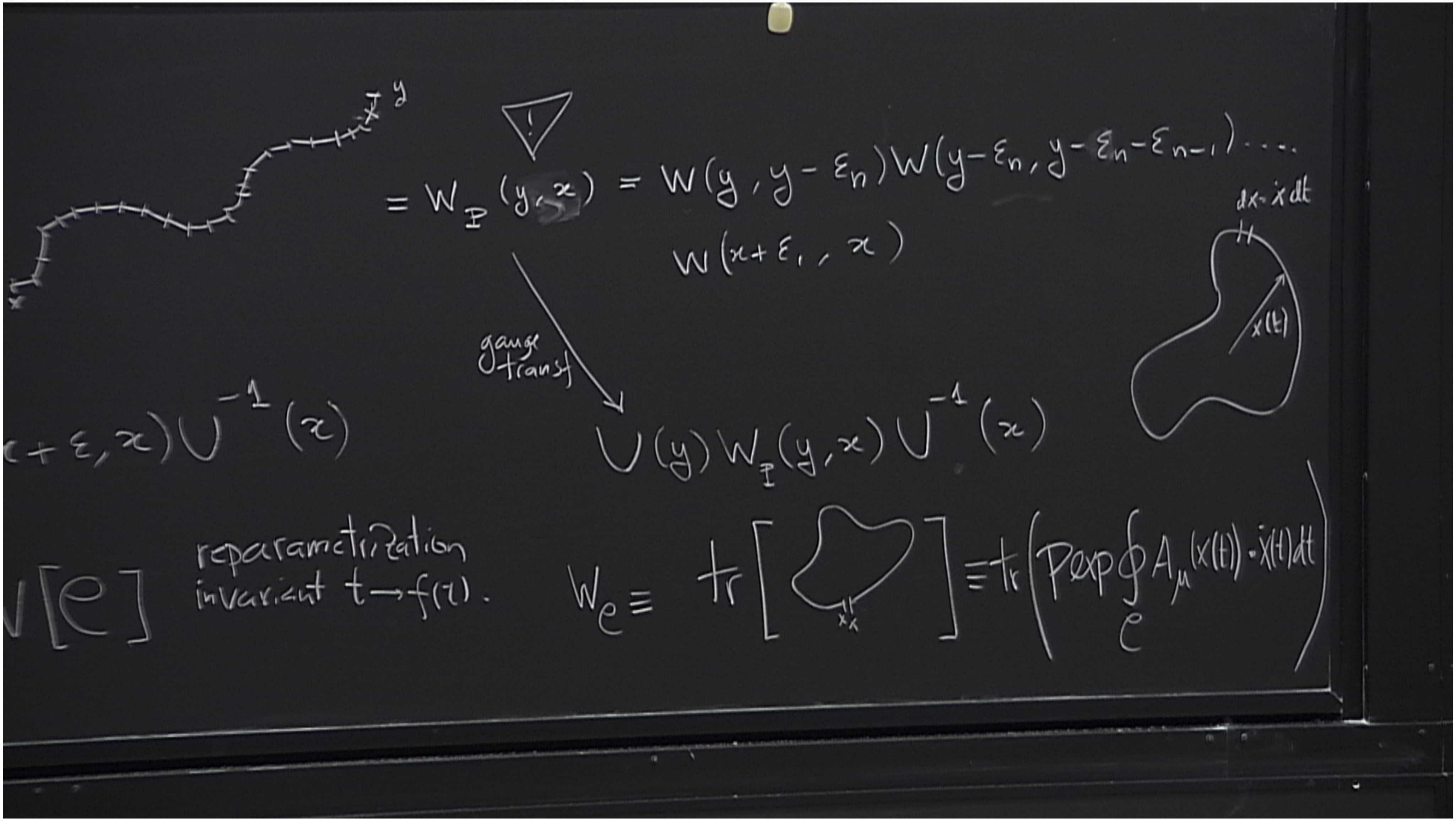
$$= W_P(x,y) = W(y, y-\varepsilon_n)\,W(y-\varepsilon_n, y-\varepsilon_n-\varepsilon_{n-1})\ldots$$
$$W(x+\varepsilon_1, x)$$

gauge transf

$$x+\varepsilon, x)\,U^{-1}(x)$$


y
x
!
$= W_P(y,x) = W(y, y-\varepsilon_n) W(y-\varepsilon_n, y-\varepsilon_n-\varepsilon_{n-1}) \ldots$
$W(x+\varepsilon_1, x)$
gauge transf
$x+\varepsilon, x) U^{-1}(x)$
$U(y) W_P(y,x) U^{-1}(x)$

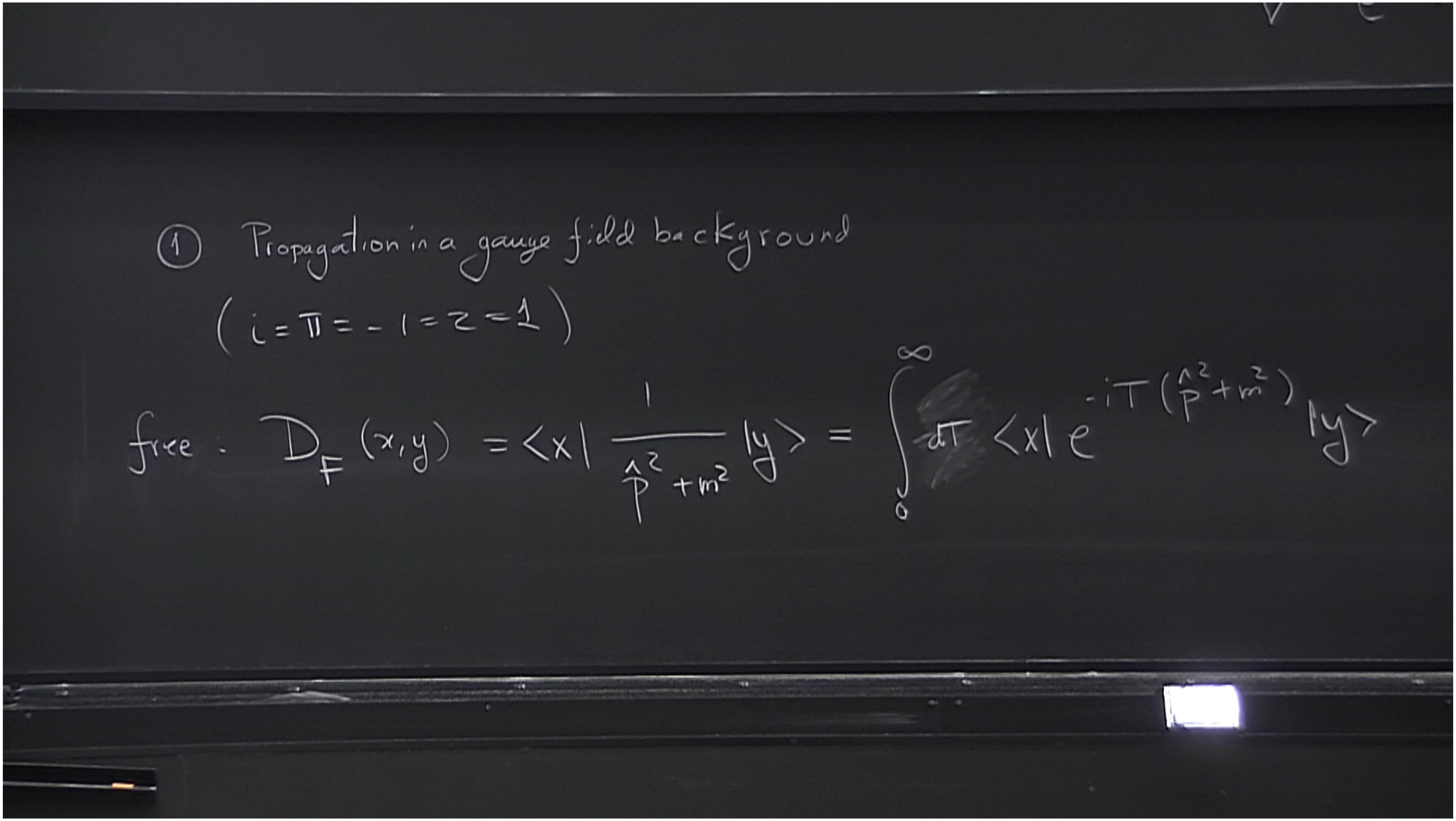

y
= W_P (y, x) = W(y, y − ε_n) W(y − ε_n, y − ε_n − ε_{n−1}) ....
W(x + ε_1, x)
gauge transf
dx = ẋ dt
x(t)
U(y) W_P(y, x) U^{-1}(x)
U^{-1}(x)
W_C ≡ tr[ ] ≡ tr(P exp ∮_C A_μ(x(t)) · ẋ(t) dt)


= W_P (y, x) = W(y, y - ε_n) W(y - ε_n, y - ε_n - ε_{n-1}) . . . .
W(x + ε_1, x)
gauge transf
x + ε, x) U^{-1}(x)
U(y) W_P(y, x) U^{-1}(x)
dx = ẋ dt
x(t)
W[C]
reparametrization invariant t → f(τ).
W_C ≡ tr[ ] ≡ tr(P exp ∮_C A_μ(x(t)) · ẋ(t) dt)

① Propagation in a gauge field background

$$(i = \pi = -1 = 2 = 1)$$

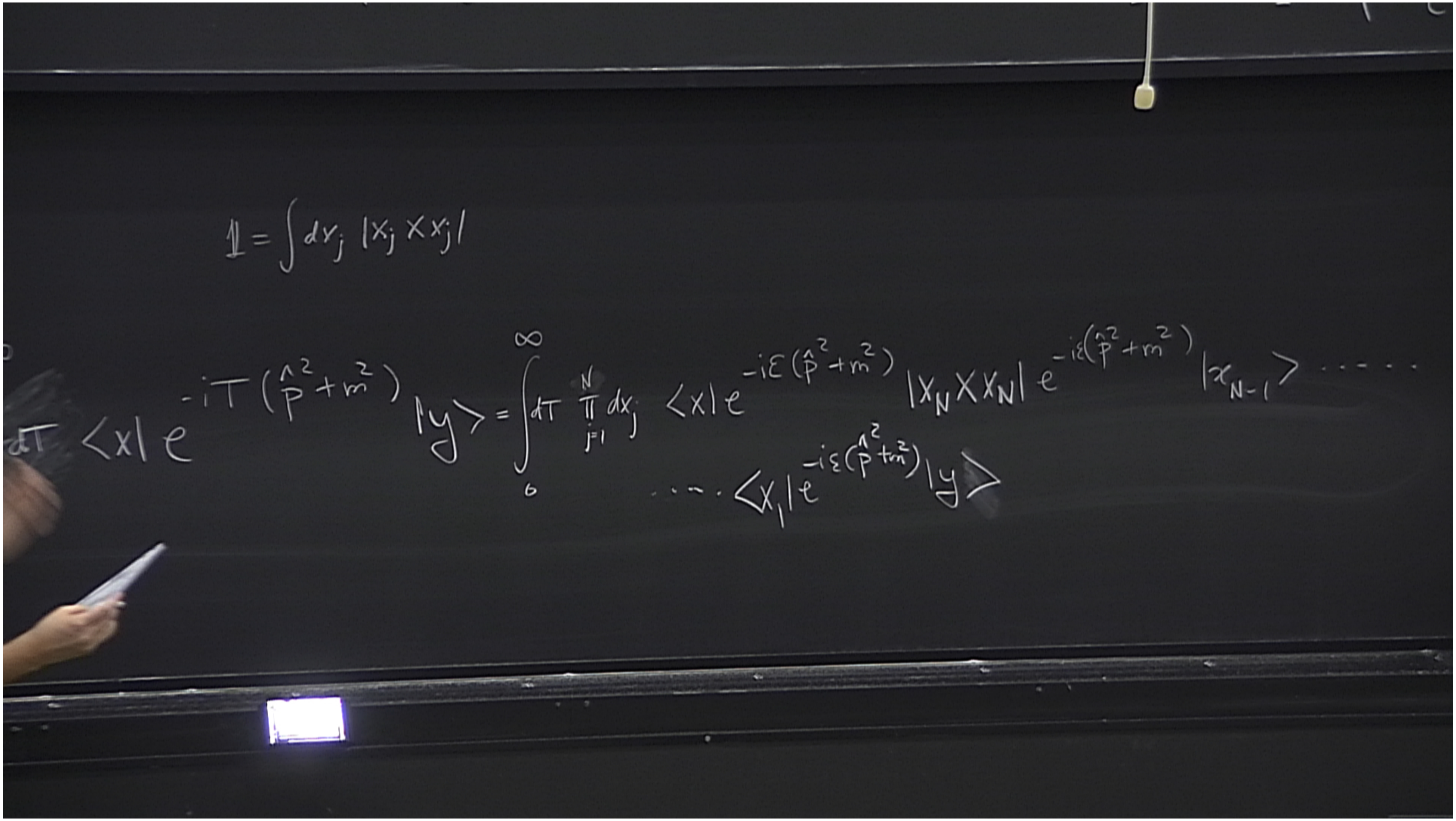
$$D_F(x,y)$$

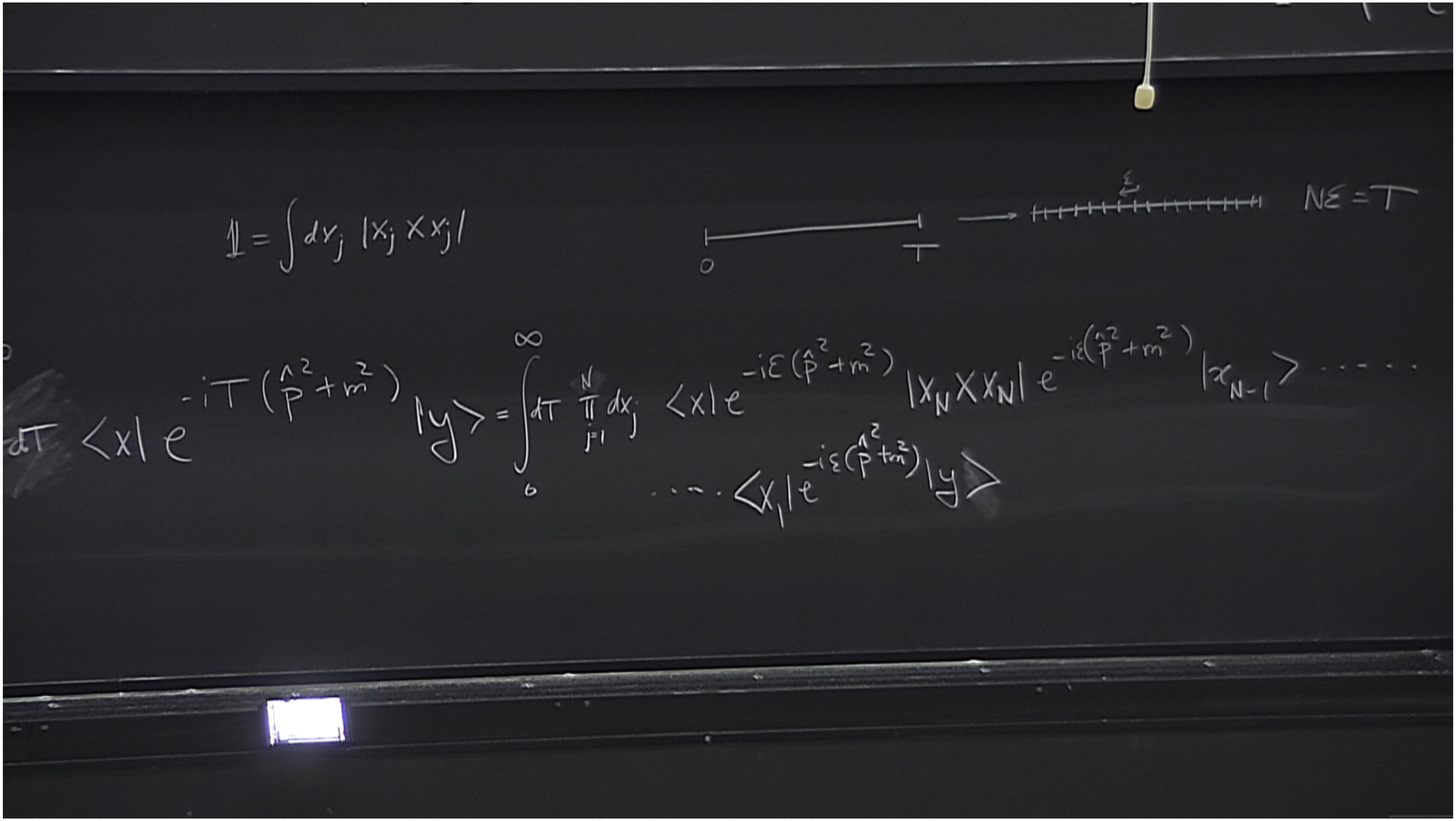

① Propagation in a gauge field background
(i = π = - 1 = 2 = 1)
free : D_F (x,y) = <x| 1/(p̂² + m²) |y> = ∫₀^∞ dT <x| e^{-iT(p̂² + m²)} |y>

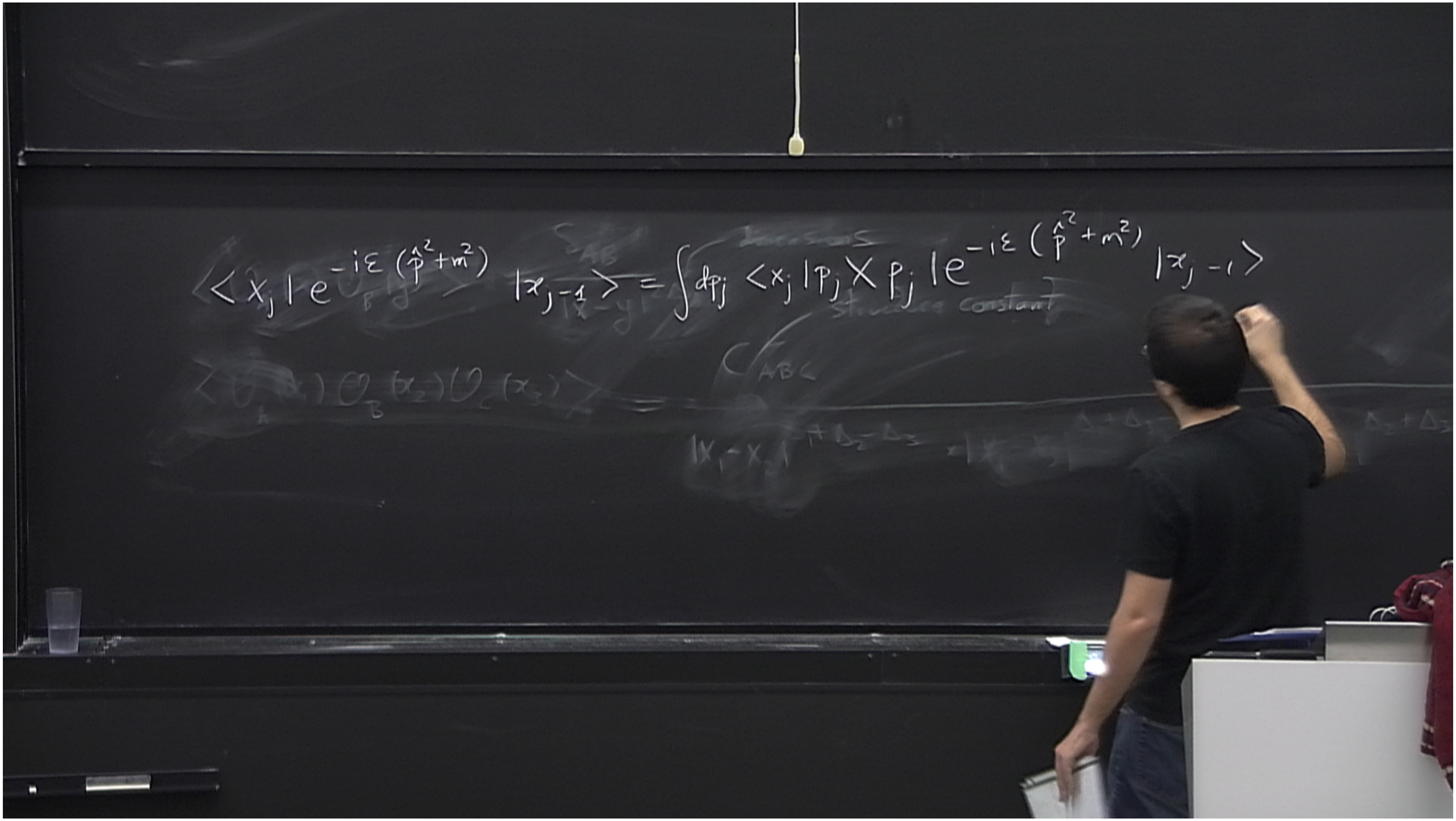
nd

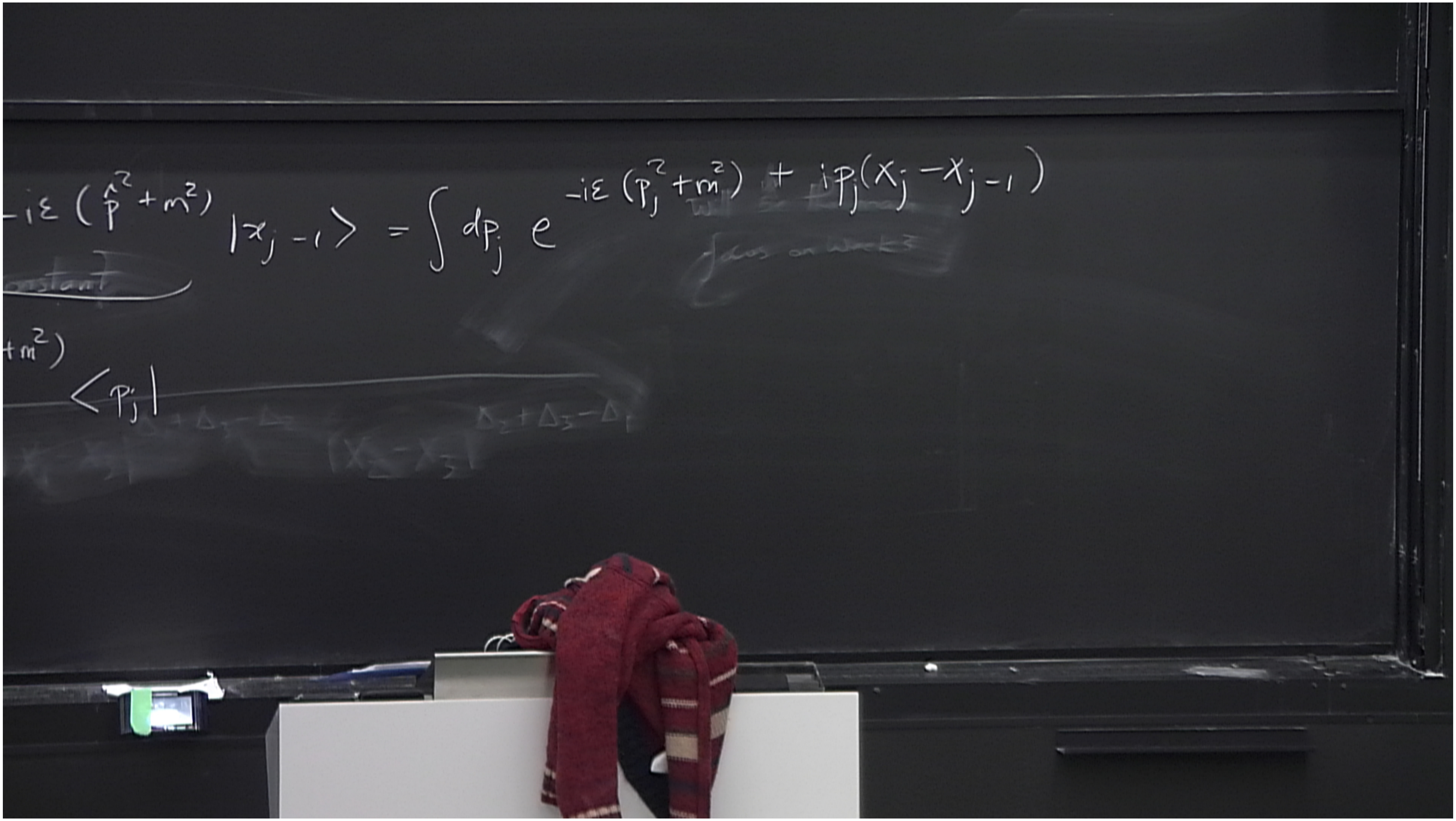
$$\mathbb{1} = \int dx_j \, |x_j\rangle\langle x_j|$$

$$= \int_0^\infty dT \, \langle x| e^{-iT(\hat{p}^2 + m^2)} |y\rangle = \int_0^\infty dT \prod_{j=1}^{N} dx_j$$


$\mathbb{1} = \int dx_j \, |x_j\rangle\langle x_j|$
$dT \, \langle x| e^{-iT(\hat{p}^2 + m^2)} |y\rangle = \int_0^\infty dT \prod_{j=1}^{N} dx_j \, \langle x| e^{-i\varepsilon(\hat{p}^2+m^2)} |x_N\rangle\langle x_N| e^{-i\varepsilon(\hat{p}^2+m^2)} |x_{N-1}\rangle \cdots$
$\cdots \langle x_1| e^{-i\varepsilon(\hat{p}^2+m^2)} |y\rangle$


$\mathbb{1} = \int dx_j \, |x_j \rangle\langle x_j|$
0
T
ε
$N\varepsilon = T$
$d\tau \, \langle x| e^{-iT(\hat{p}^2 + m^2)} |y\rangle = \int_0^\infty d\tau \prod_{j=1}^{N} dx_j \, \langle x| e^{-i\varepsilon(\hat{p}^2+m^2)} |x_N\rangle\langle x_N| e^{-i\varepsilon(\hat{p}^2+m^2)} |x_{N-1}\rangle \cdots$
$\cdots \langle x_1| e^{-i\varepsilon(\hat{p}^2+m^2)} |y\rangle$

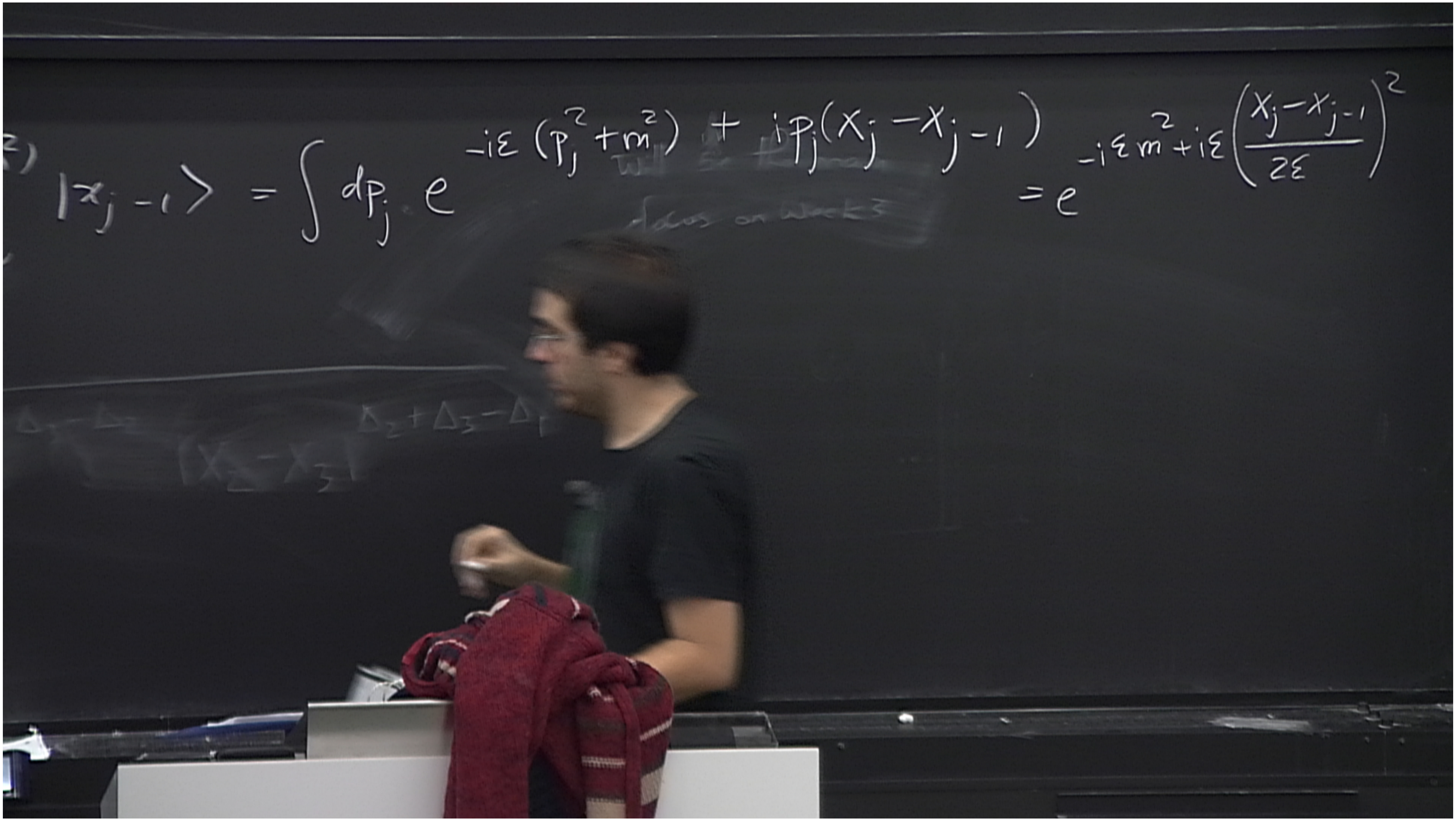

$\langle x_j | e^{-i\varepsilon(\hat{p}^2+m^2)} | x_{j-1} \rangle = \int dp_j \langle x_j | p_j \rangle \langle p_j | e^{-i\varepsilon(\hat{p}^2+m^2)} | x_{j-1} \rangle$


$-i\varepsilon(\hat{p}^2+m^2)\ |x_{j-1}\rangle = \int dp_j\ e^{-i\varepsilon(p_j^2+m^2)+ip_j(x_j-x_{j-1})}$
$+m^2)$
$\langle p_j|$

$$\langle x_j | e^{-i\varepsilon(\hat{p}^2+m^2)} | x_{j-1} \rangle = \int dp_j \, \langle x_j | p_j \rangle \langle p_j | e^{-i \cdots}$$

$$e^{-i\varepsilon(p_j^2+m^2 \cdots}$$

$$D_F(x,y) =$$


|x_{j-1}> = ∫ dp_j e^{-iε(p_j^2 + m^2) + ip_j(x_j - x_{j-1})}
= e^{-iεm^2 + iε((x_j - x_{j-1})/2ε)^2}
Δ_2 + Δ_3 - Δ_1

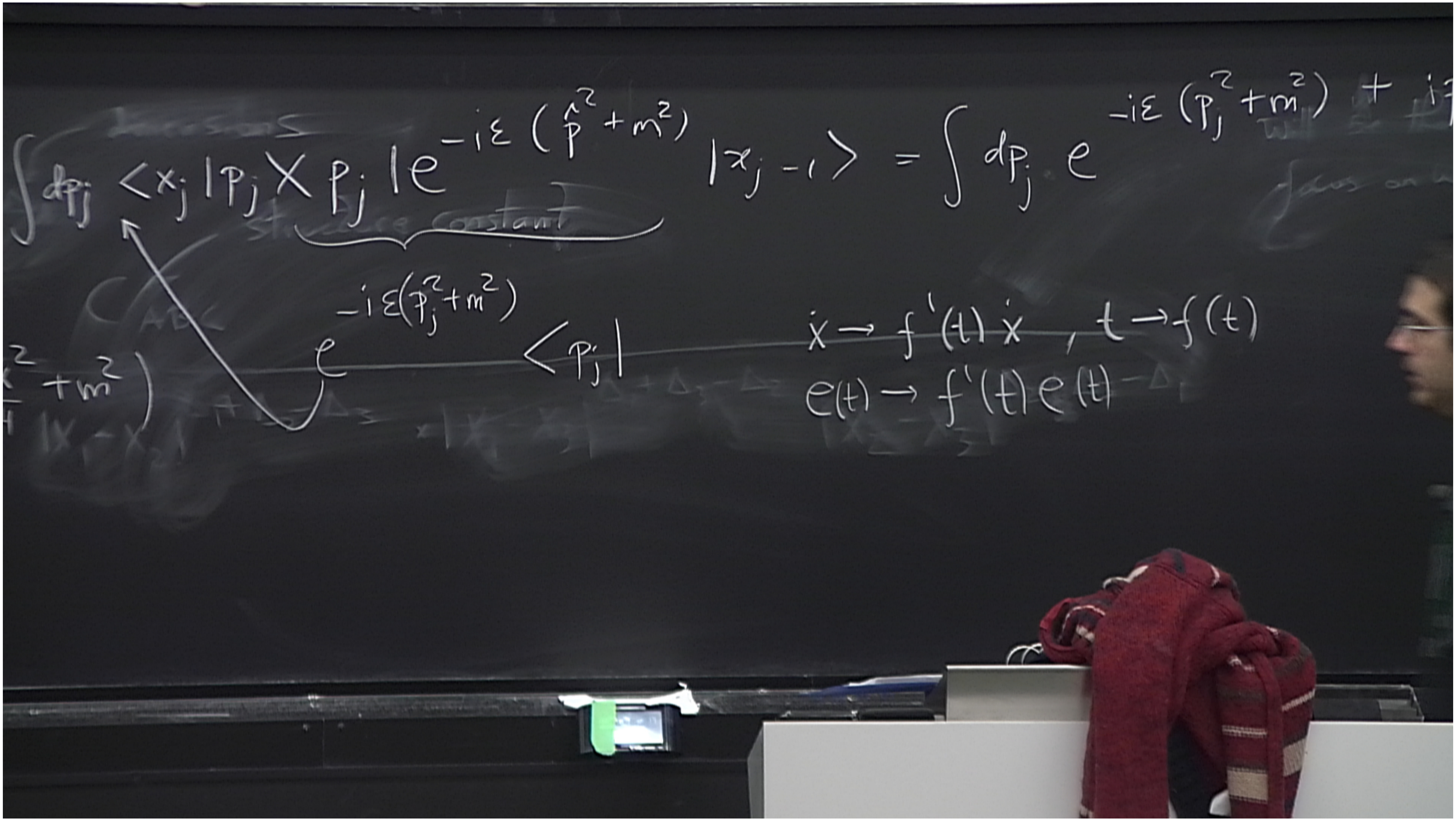
$$\langle x_j | e^{-i\varepsilon(\hat{p}^2+m^2)} | x_{j-1}\rangle = \int dp_j \langle x_j | p_j\rangle\langle p_j | e^{-i\varepsilon(\hat{p}^2+m^2)}$$

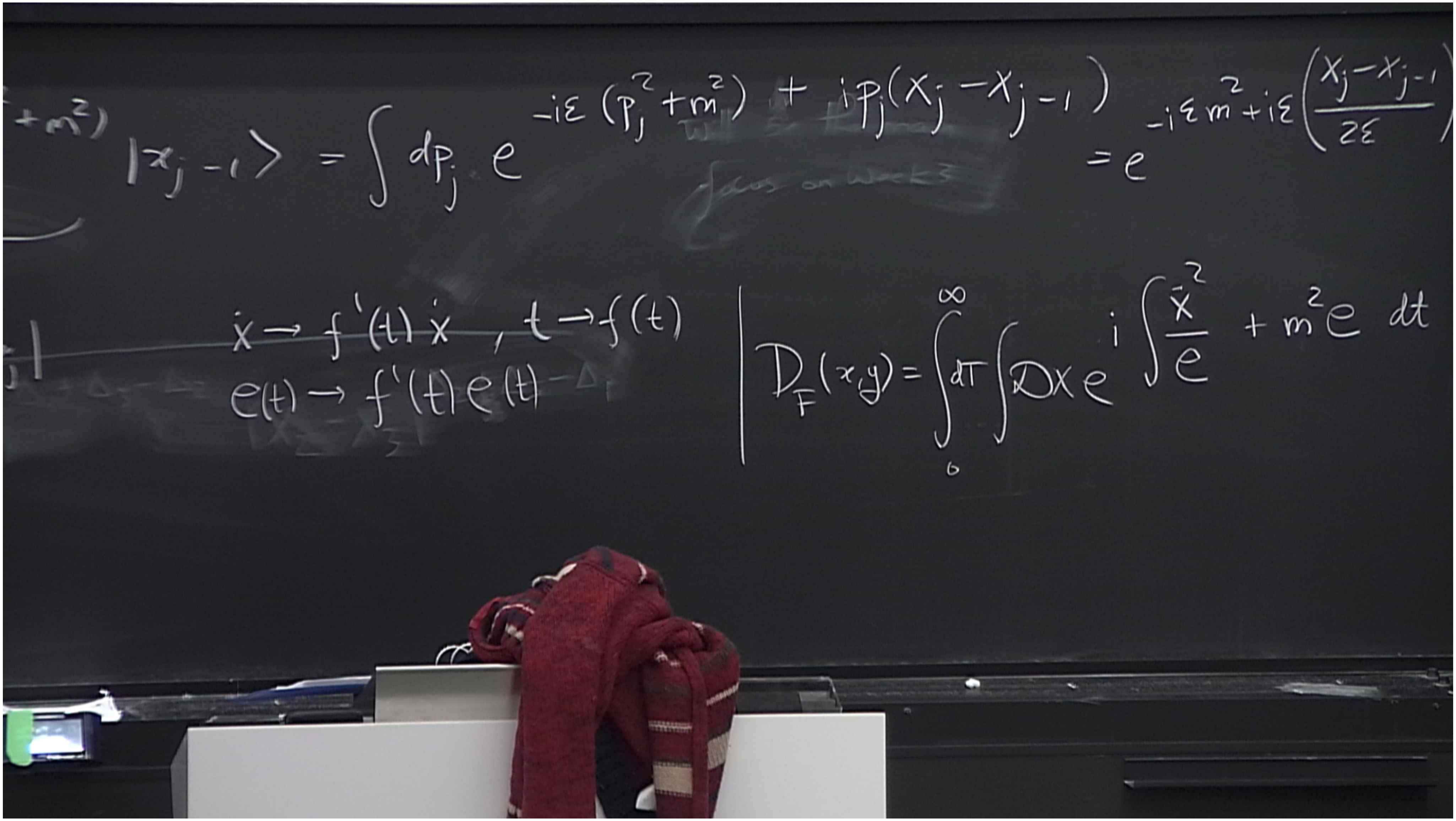
$$e^{-i\varepsilon(p_j^2+m^2)} \langle p_j |$$

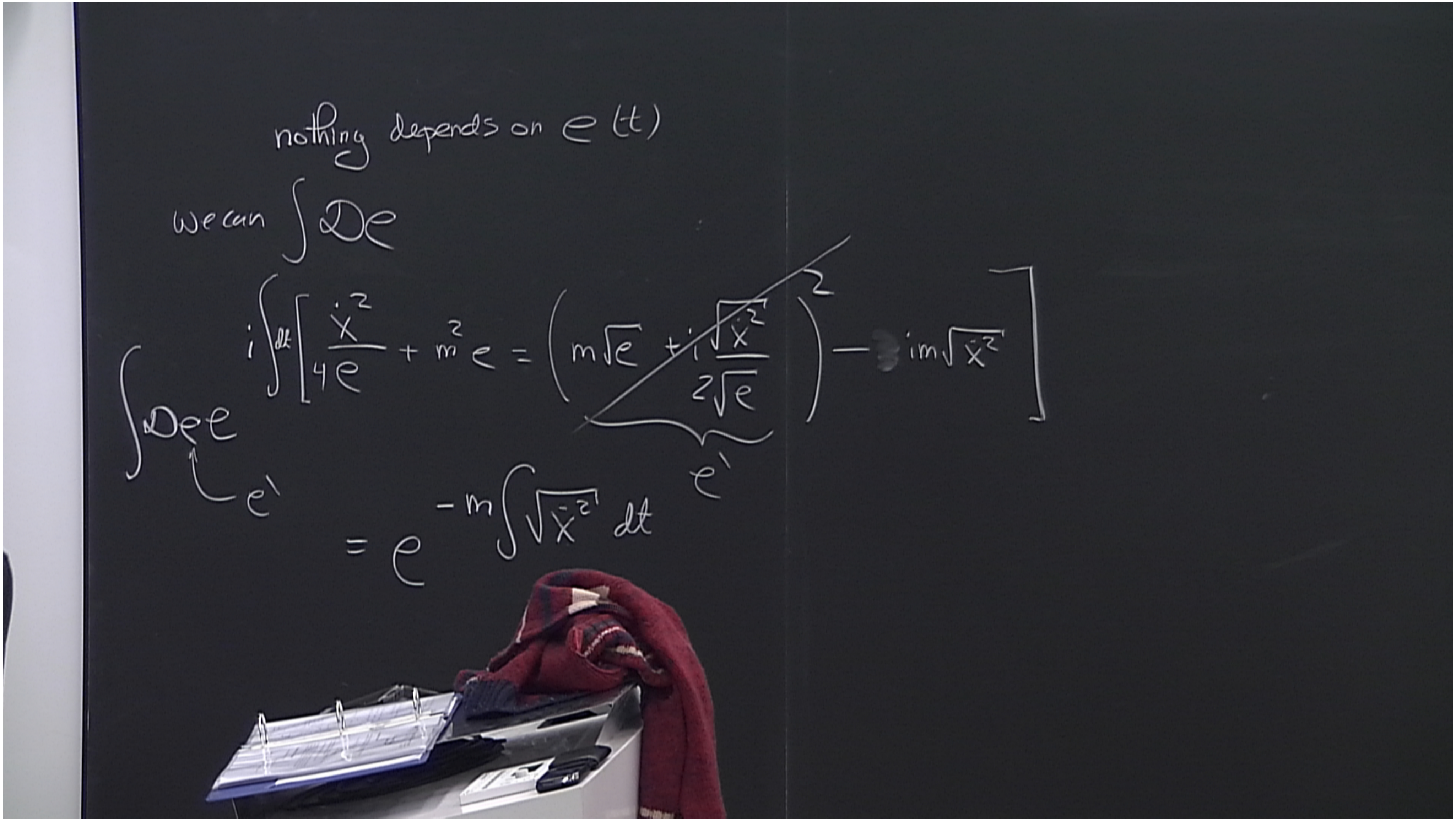
$$D_F(x,y) = \int_0^\infty dT \int_{\substack{X(0)=x \\ X(T)=y}} \mathcal{D}X\, e^{i\int dt\left(\ddot{X}^2+m^2\right)}$$

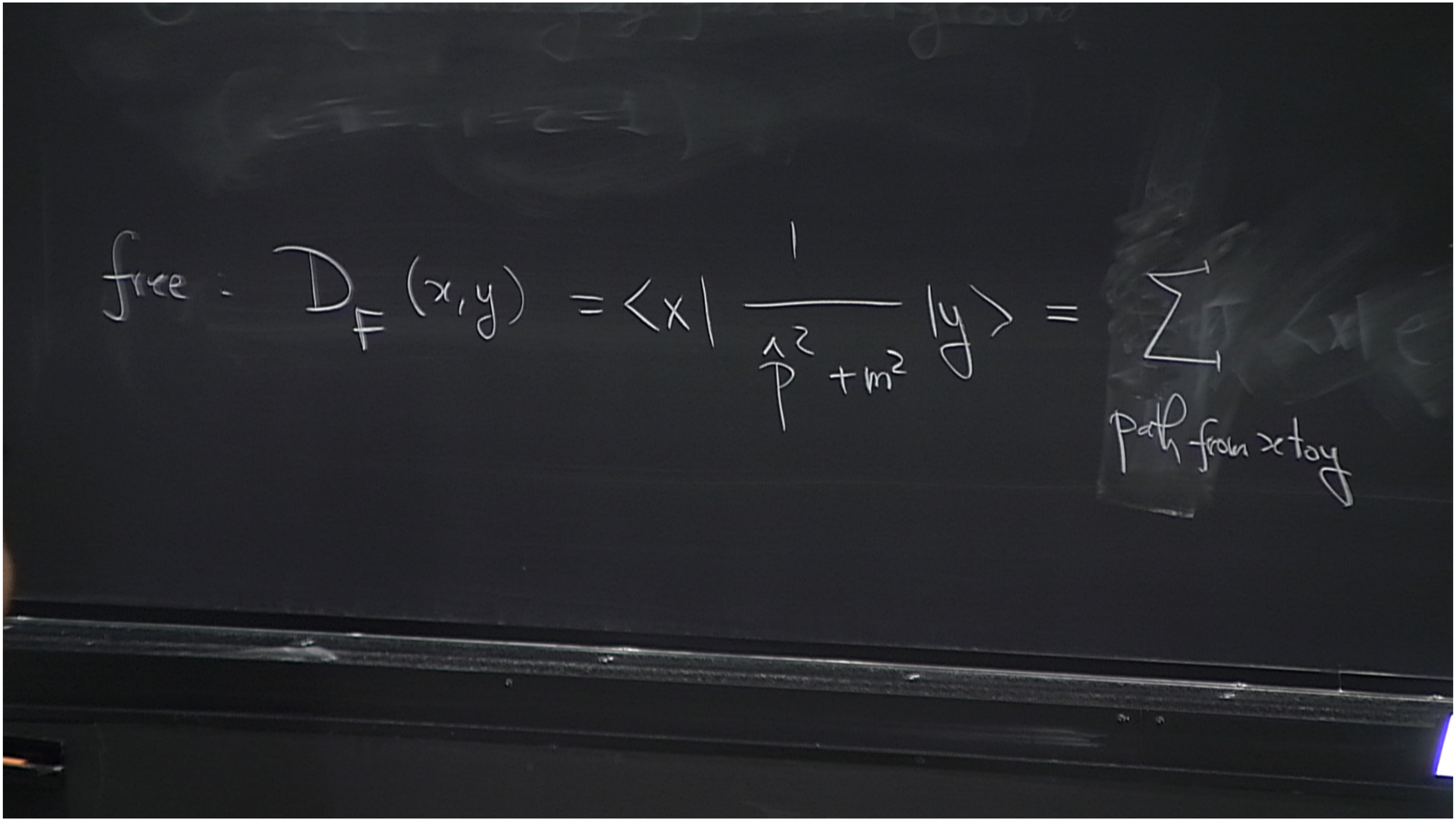
$$\langle x_j | e^{-i\varepsilon(\hat{p}^2+m^2)} | x_{j-1} \rangle = \int dp_j \, \langle x_j | p_j \rangle \underbrace{\langle p_j | e^{-i\varepsilon(\hat{p}^2+m^2)}}_{e^{-i\varepsilon(p_j^2+m^2)} \langle p_j |}$$

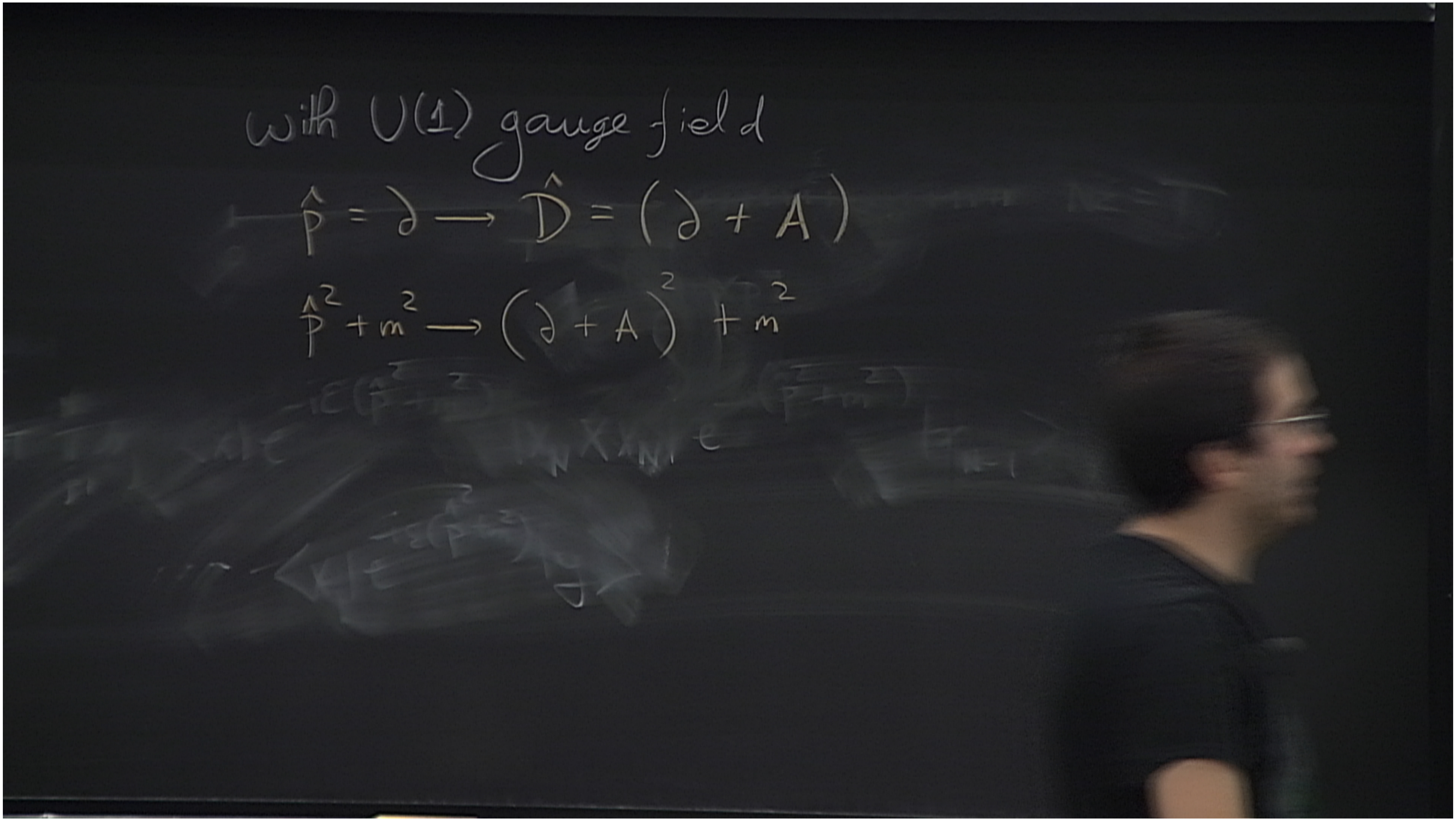
$$D_F(x,y) = \int_0^\infty dT \int_{\substack{x(0)=x \\ x(T)=y}} \mathcal{D}x \, e^{i\int dt \left(\frac{\dot{x}^2}{4} + m^2\right)}$$

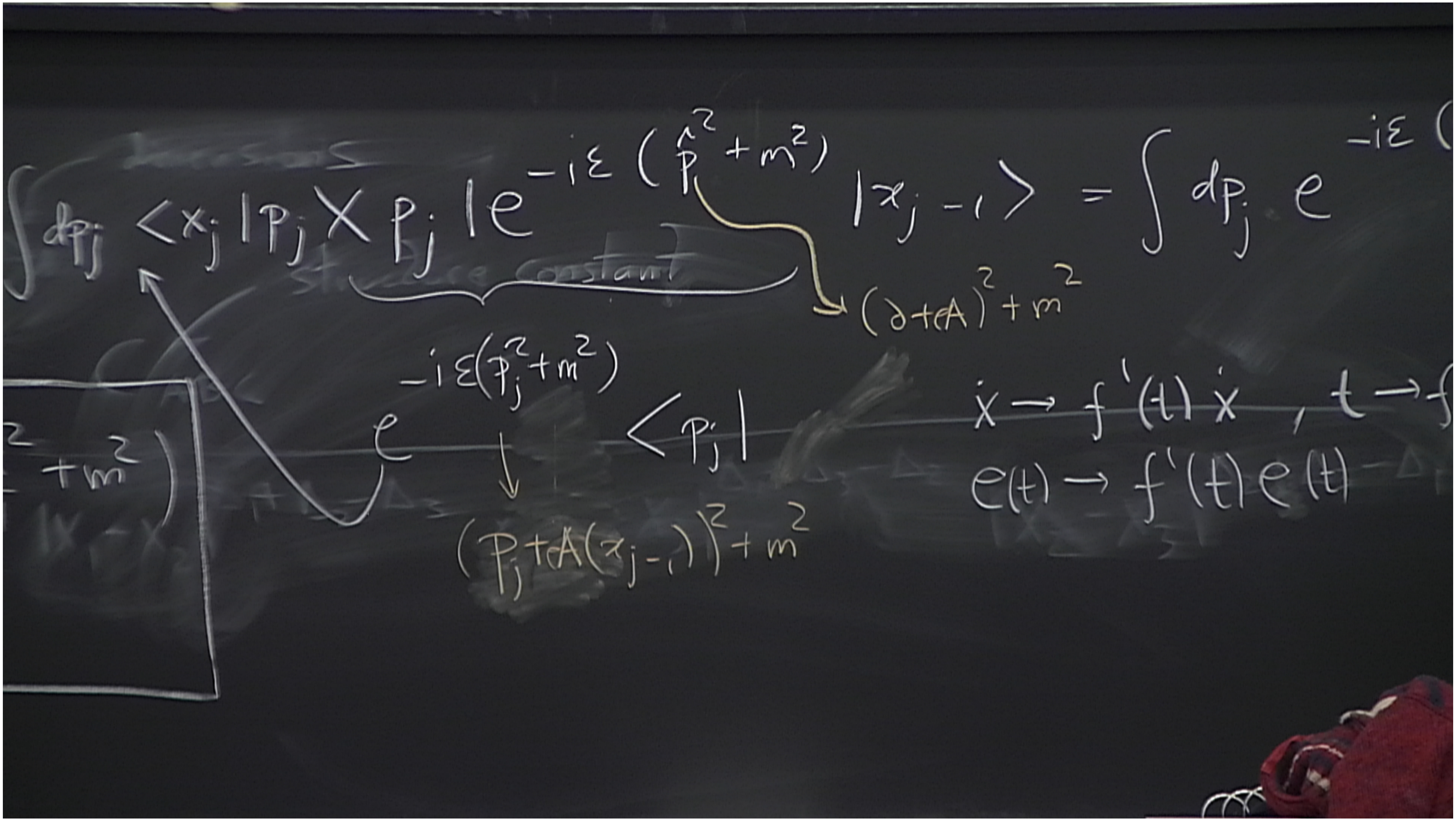

∫ dp_j ⟨x_j | p_j⟩⟨p_j | e^{-iε(p̂² + m²)} |x_{j-1}⟩ = ∫ dp_j e^{-iε(p_j² + m²) + i
constant
e^{-iε(p_j² + m²)} ⟨p_j|
ẋ → f'(t) ẋ , t → f(t)
e(t) → f'(t) e(t)

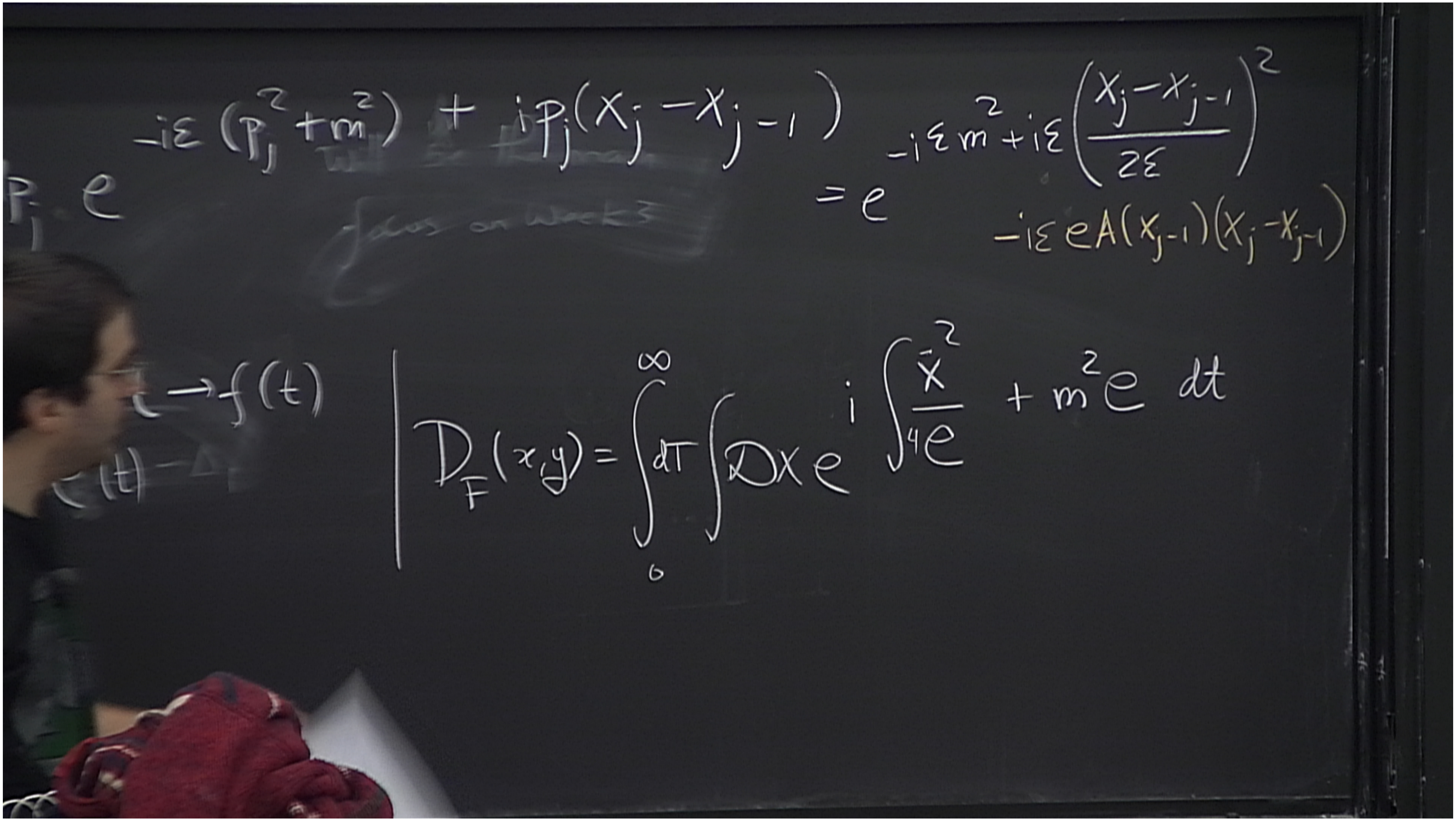

$|x_{j-1}\rangle = \int dp_j\, e^{-i\varepsilon(p_j^2+m^2) + ip_j(x_j - x_{j-1})}$
$= e^{-i\varepsilon m^2 + i\varepsilon\left(\frac{x_j - x_{j-1}}{2\varepsilon}\right)}$
$\dot{x} \to f'(t)\dot{x}, \quad t \to f(t)$
$e(t) \to f'(t)e(t)$
$D_F(x,y) = \int_0^\infty d\tau \int \mathcal{D}x\, e^{i\int \frac{\dot{x}^2}{e} + m^2 e\, dt}$


nothing depends on e(t)
we can $\int \mathcal{D}e$
$\int \mathcal{D}e\, e^{i\int dt\left[\frac{\dot{x}^2}{4e} + m^2 e = \left(m\sqrt{e} + i\frac{\sqrt{\dot{x}^2}}{2\sqrt{e}}\right)^2 - im\sqrt{\dot{x}^2}\right]}$
e'
e'
$= e^{-m\int\sqrt{\dot{x}^2}\, dt}$


free : $D_F(x,y) = \langle x | \frac{1}{\hat{p}^2 + m^2} | y \rangle = \sum$
path from x to y


with U(1) gauge field
$\hat{p} = \partial \longrightarrow \hat{D} = (\partial + A)$
$\hat{p}^2 + m^2 \longrightarrow (\partial + A)^2 + m^2$


∫dp_j <x_j|p_j><p_j| e^{-iε(p̂²+m²)} |x_{j-1}> = ∫dp_j e^{-iε(
constant
(∂+eA)²+m²
e^{-iε(p_j²+m²)} <p_j|
(p_j+eA(x_{j-1}))²+m²
ẋ → f'(t) ẋ , t → f
e(t) → f'(t) e(t)


$p_j\, e^{-i\varepsilon(p_j^2+m^2) + ip_j(x_j - x_{j-1})}$
$= e^{-i\varepsilon m^2 + i\varepsilon\left(\frac{x_j - x_{j-1}}{2\varepsilon}\right)^2 - i\varepsilon e A(x_{j-1})(x_j - x_{j-1})}$
$\to f(t)$
$D_F(x,y) = \int_0^\infty d\tau \int \mathcal{D}x\, e^{i\int \frac{\dot{x}^2}{4e} + m^2 e\, dt}$

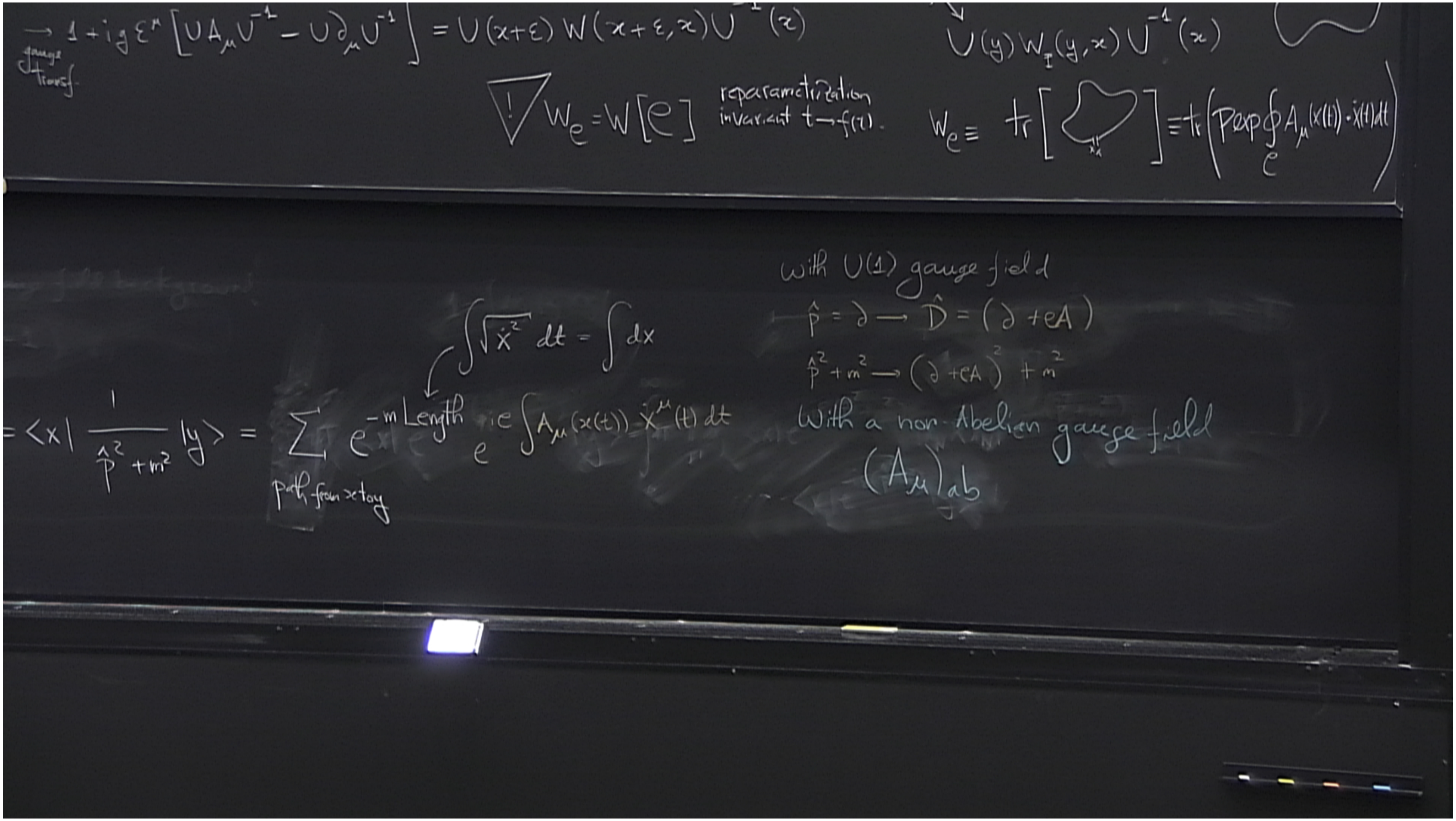
$$\int \sqrt{\dot{x}^2}\, dt = \int dx$$

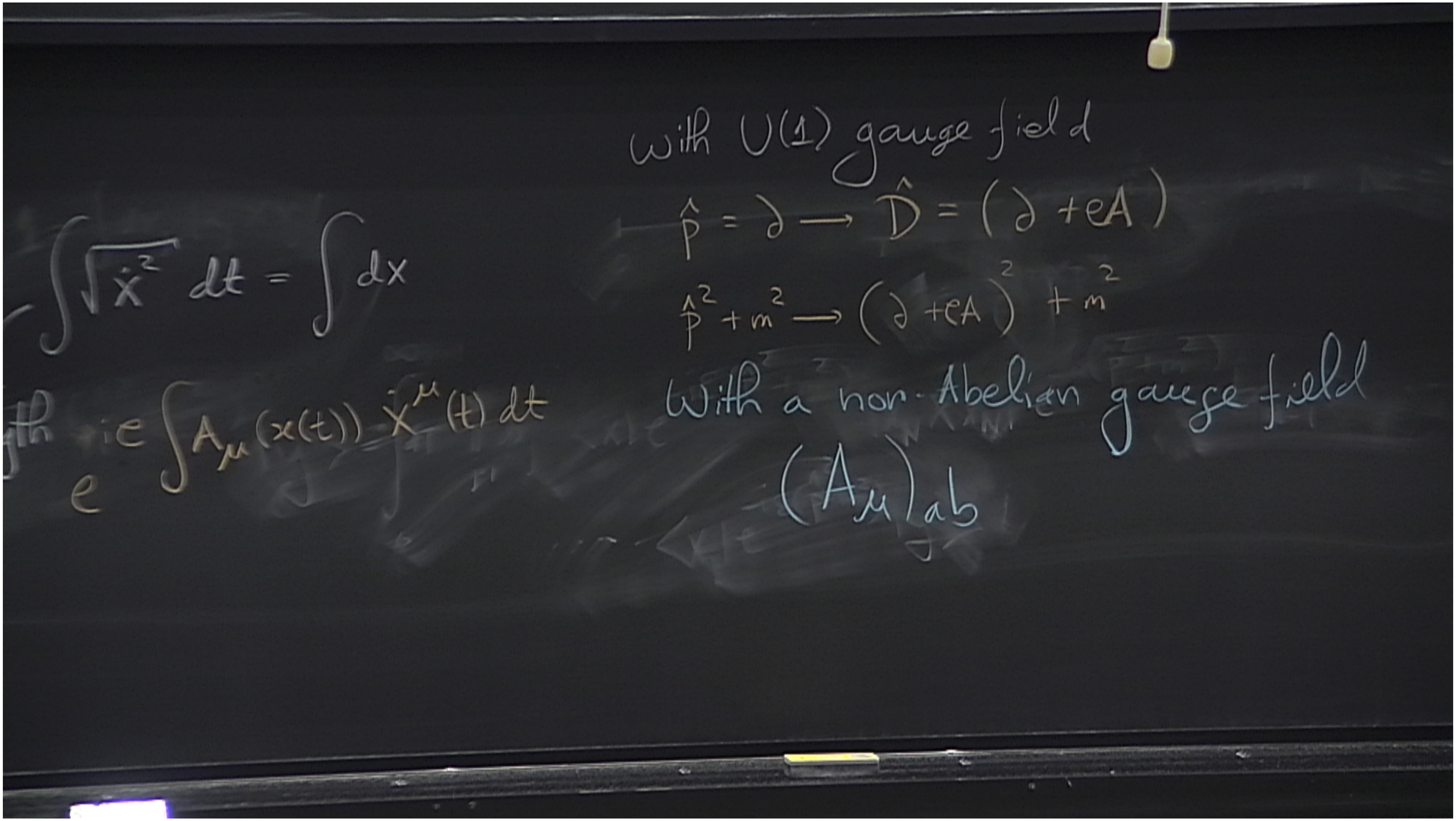
$$\sum_{\text{path from } x \text{ to } y} e^{-m\,\text{Length} + ie\int A_\mu(x(t))\,\dot{x}^\mu(t)\,dt}$$

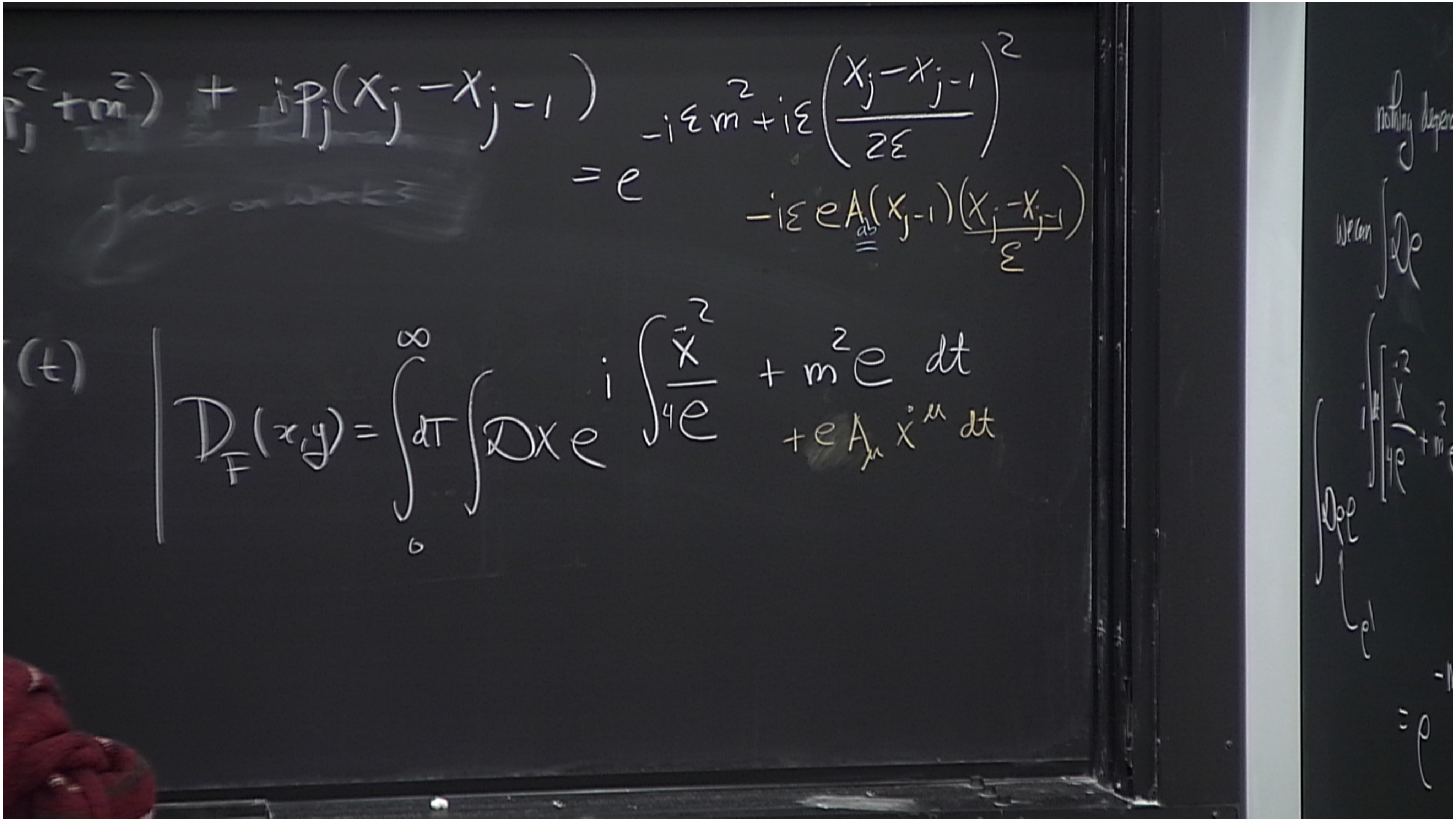
with U(1) gauge field

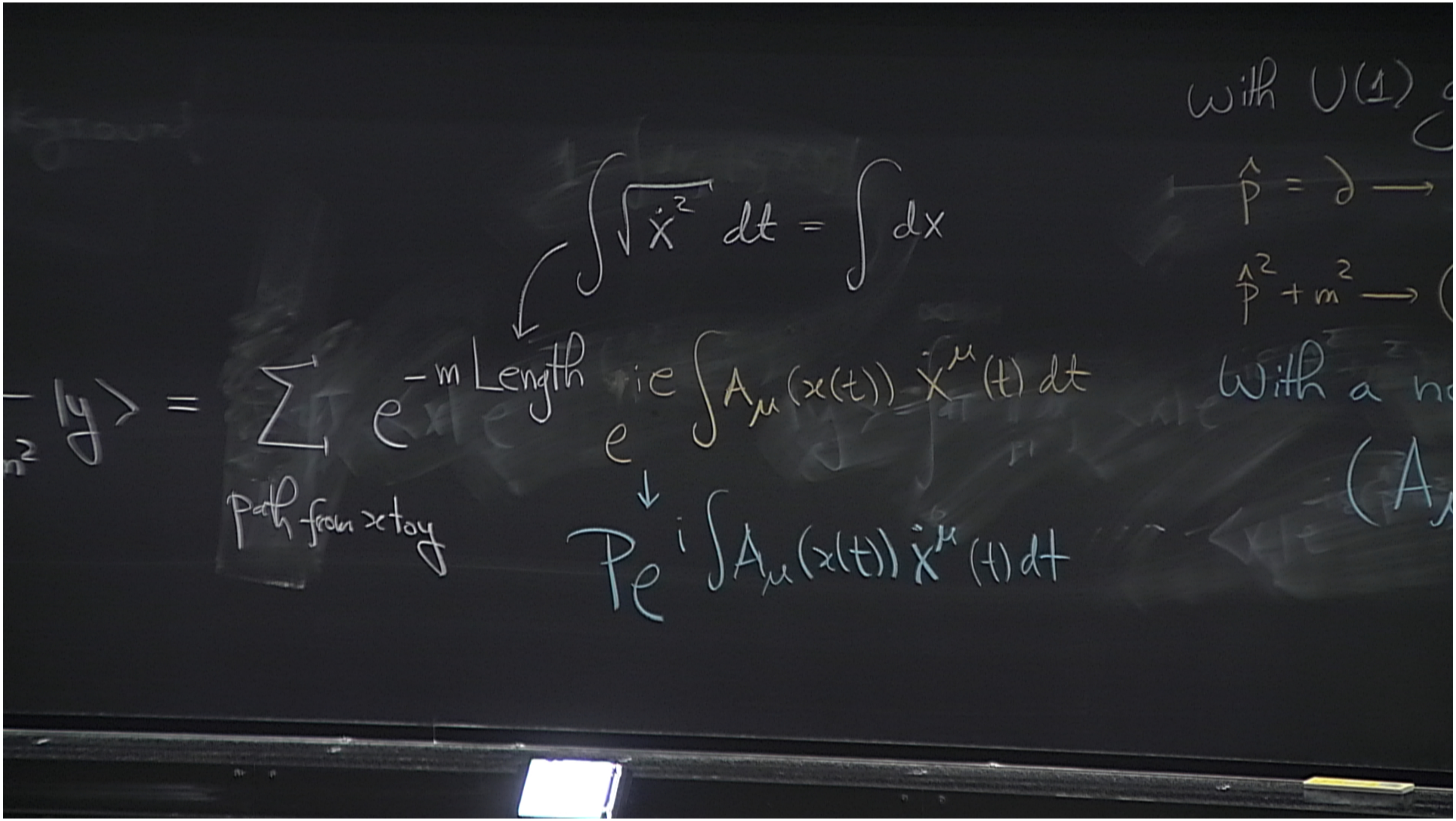
$$\hat{p} = \partial \longrightarrow \hat{D} = (\partial + eA)$$

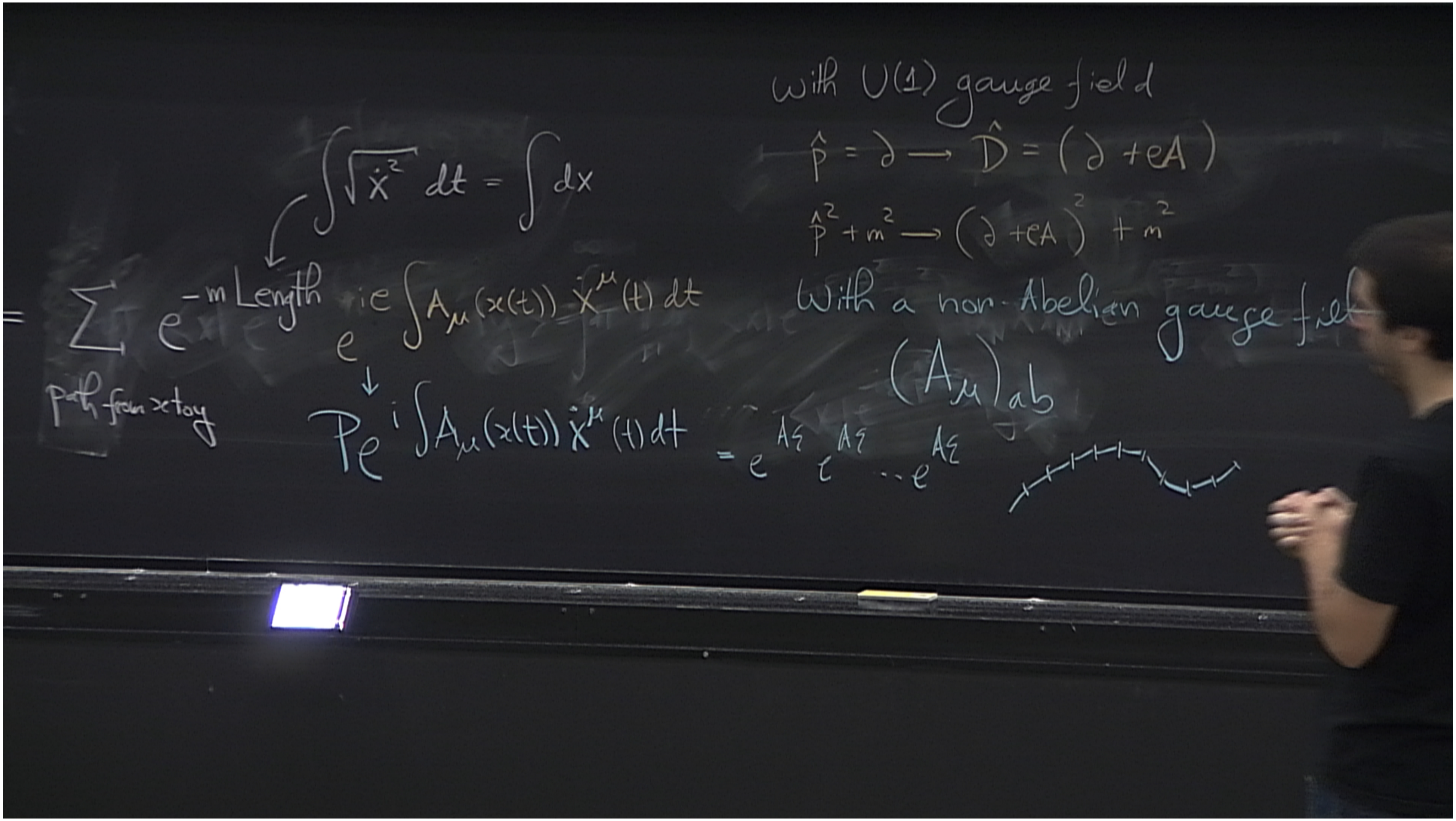
$$\hat{p}^2 + m^2 \longrightarrow (\partial + eA)^2 + m^2$$


→ 1 + ig ε^μ [U A_μ U^{-1} − U ∂_μ U^{-1}] = U(x+ε) W(x+ε, x) U^{-1}(x)
gauge transf.
U(y) W_I(y, x) U^{-1}(x)
W_C = W[C]
reparametrization invariant t → f(τ).
W_C ≡ tr[ ] ≡ tr(P exp ∮_C A_μ(x(t)) · ẋ(t) dt)
with U(1) gauge field
p̂ = ∂ → D̂ = (∂ + eA)
p̂² + m² → (∂ + eA)² + m²
With a non-Abelian gauge field
(A_μ)_ab
∫√(ẋ²) dt = ∫dx
= ⟨x| 1/(p̂² + m²) |y⟩ = Σ e^{−m Length} e^{ie ∫A_μ(x(t)) ẋ^μ(t) dt}
path from x to y


with U(1) gauge field
$\hat{p} = \partial \longrightarrow \hat{D} = (\partial + eA)$
$\hat{p}^2 + m^2 \longrightarrow (\partial + eA)^2 + m^2$
$\int \sqrt{\dot{x}^2}\, dt = \int dx$
$e^{ie \int A_\mu(x(t))\, \dot{x}^\mu(t)\, dt}$
With a non-Abelian gauge field
$(A_\mu)_{ab}$


$= e^{-i\varepsilon m^2 + i\varepsilon \left(\frac{x_j - x_{j-1}}{2\varepsilon}\right)^2 - i\varepsilon e A_{ab}(x_{j-1})\left(\frac{x_j - x_{j-1}}{\varepsilon}\right)}$
$D_F(x,y) = \int_0^\infty d\tau \int \mathcal{D}x \, e^{i\int \frac{\dot{x}^2}{4e} + m^2 e \, dt + e A_\mu \dot{x}^\mu dt}$


with U(1)
$\int \sqrt{\dot{x}^2}\, dt = \int dx$
$\hat{p} = \partial \longrightarrow$
$\hat{p}^2 + m^2 \longrightarrow$
$|y\rangle = \sum e^{-m \text{Length}}$
path from x to y
$e^{ie \int A_\mu (x(t)) \dot{x}^\mu (t)\, dt}$
$P e^{i \int A_\mu (x(t)) \dot{x}^\mu (t)\, dt}$
With a


with U(1) gauge field
$\hat{p} = \partial \longrightarrow \hat{D} = (\partial + eA)$
$\hat{p}^2 + m^2 \longrightarrow (\partial + eA)^2 + m^2$
$\int \sqrt{\dot{x}^2}\, dt = \int dx$
$= \sum e^{-m\,\mathrm{Length}}\, e^{ie\int A_\mu(x(t))\,\dot{x}^\mu(t)\,dt}$
path from x to y
$P e^{i\int A_\mu(x(t))\,\dot{x}^\mu(t)\,dt} = e^{A\epsilon} e^{A\epsilon} \cdots e^{A\epsilon}$
With a non-Abelian gauge field
$(A_\mu)_{ab}$