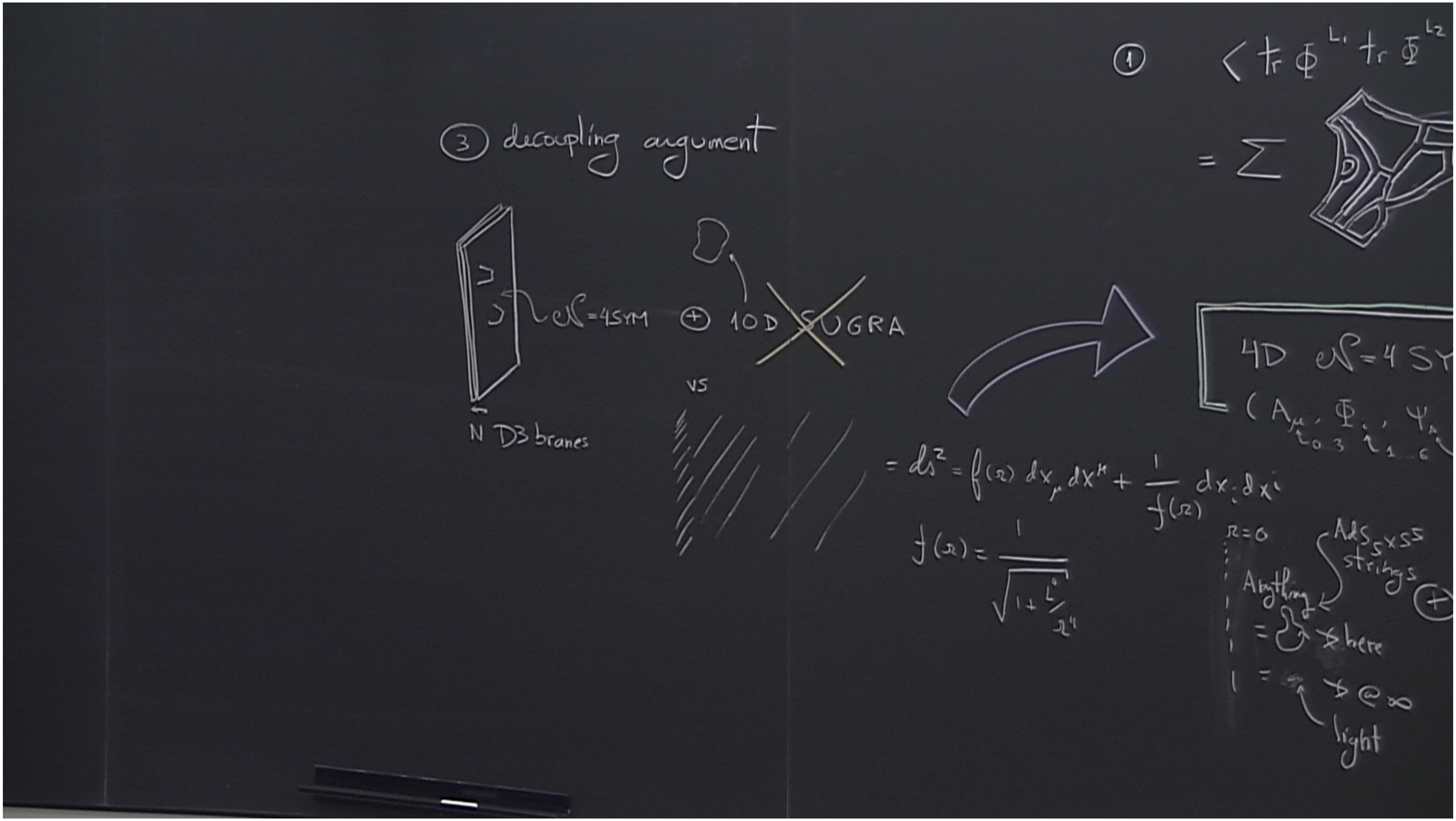
Title: Explorations in String Theory - Lecture 4

Date: Mar 15, 2012 11:30 AM

URL: http://pirsa.org/12030045

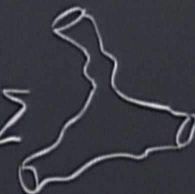
Abstract:


4D N=4 SYM = strings in AdS5 x S5
SYM
Gravity background
AdS5
S5
strings
10D SUGRA


③ decoupling argument
N D3 branes
N=4 SYM ⊕ 10D SUGRA
vs
① < tr Φ^L1 tr Φ^L2
= Σ
4D N=4 SYM
(A_μ , Φ_i , Ψ_A
ds² = f(r) dx_μ dx^μ + 1/f(r) dx_i dx^i
f(r) = 1/√(1 + L⁴/r⁴)
r = 0
AdS5×S5
strings
Anything
here
@ ∞
light

① $\langle \mathrm{tr}\,\Phi^{L_1}\,\mathrm{tr}\,\bar{\Phi}^{L_2}\,\mathrm{tr}\,\phi^{L_3} \rangle$

't Hooft expansion

$= \sum$  $= \sum$ 

②

~~SUGRA~~

$$\boxed{4D\ \mathcal{N}=4\ \text{SYM} = \text{Strings in } AdS_5 \times S^5}$$

$$(A_{\mu}, \Phi_{i}, \Psi_{A}) \quad {\scriptstyle \mu = 0..3,\ i = 1..6,\ A = 1..16}$$

$$ds^2 = \underbrace{\frac{r^2}{L^2} dx_\mu dx^\mu + \frac{L^2}{r^2} dr^2}_{AdS_5} + \underbrace{L^2 d\Omega^2_{S^5}}_{S^5}$$

$$= ds^2 = f(r)\, dx_\mu dx^\mu + \frac{1}{f(r)} dx_i dx^i$$

$$f(r) = \frac{1}{\sqrt{1 + L^4/r^4}}$$

$r = 0$

Anything

$AdS_5 \times S5$ strings ⊕

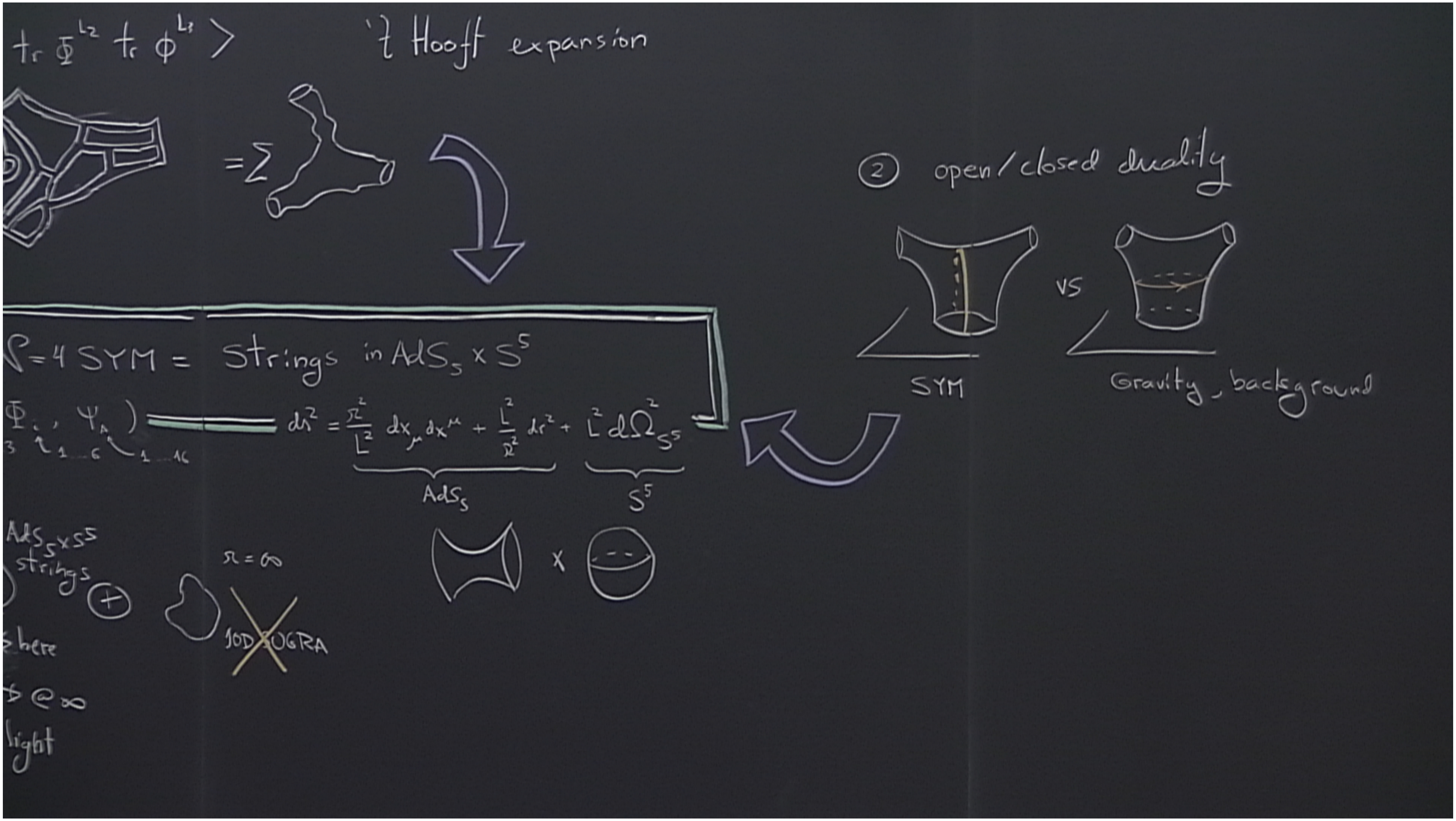
$r = \infty$

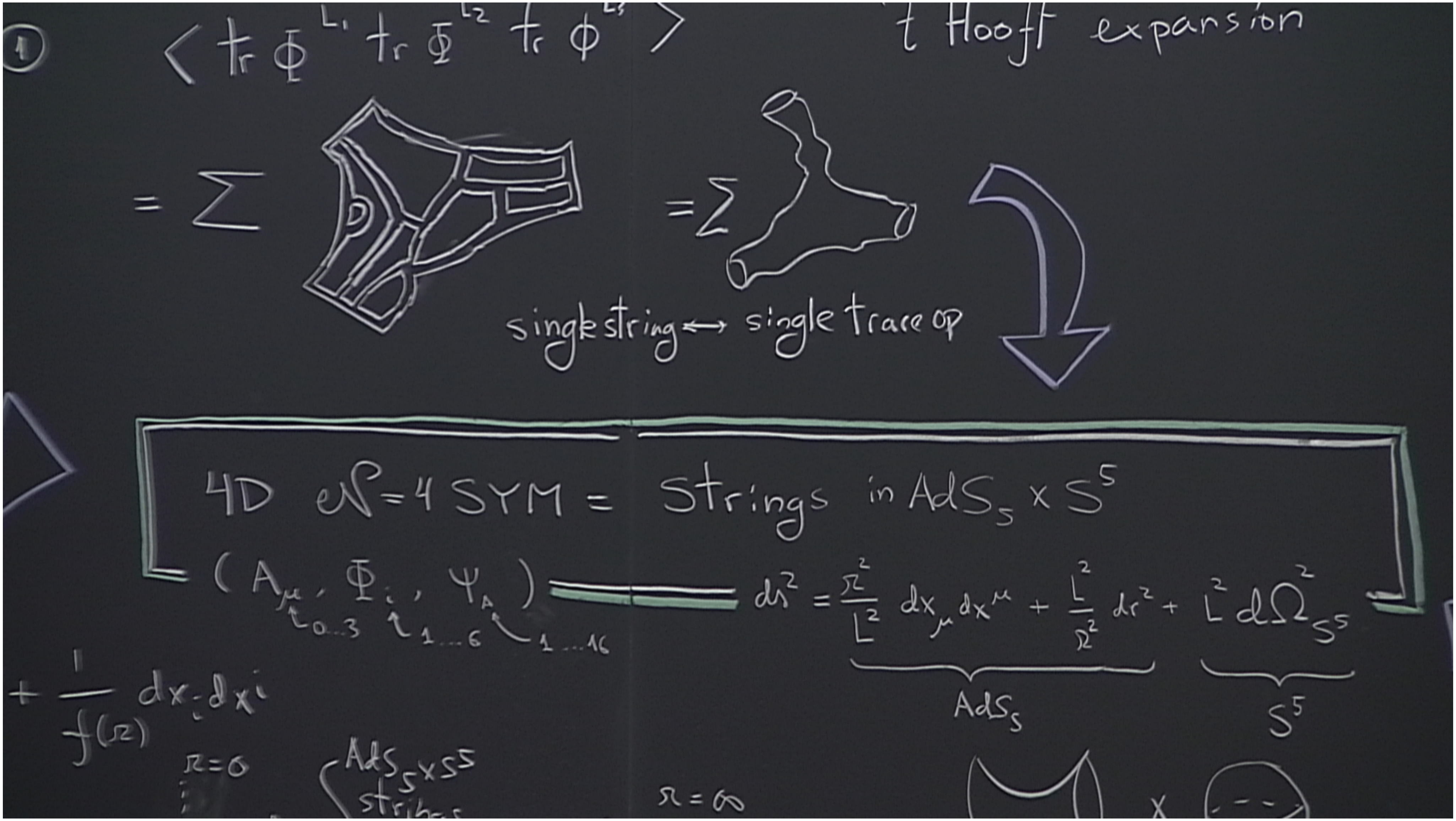
10D ~~SUGRA~~

= $\nexists$ here

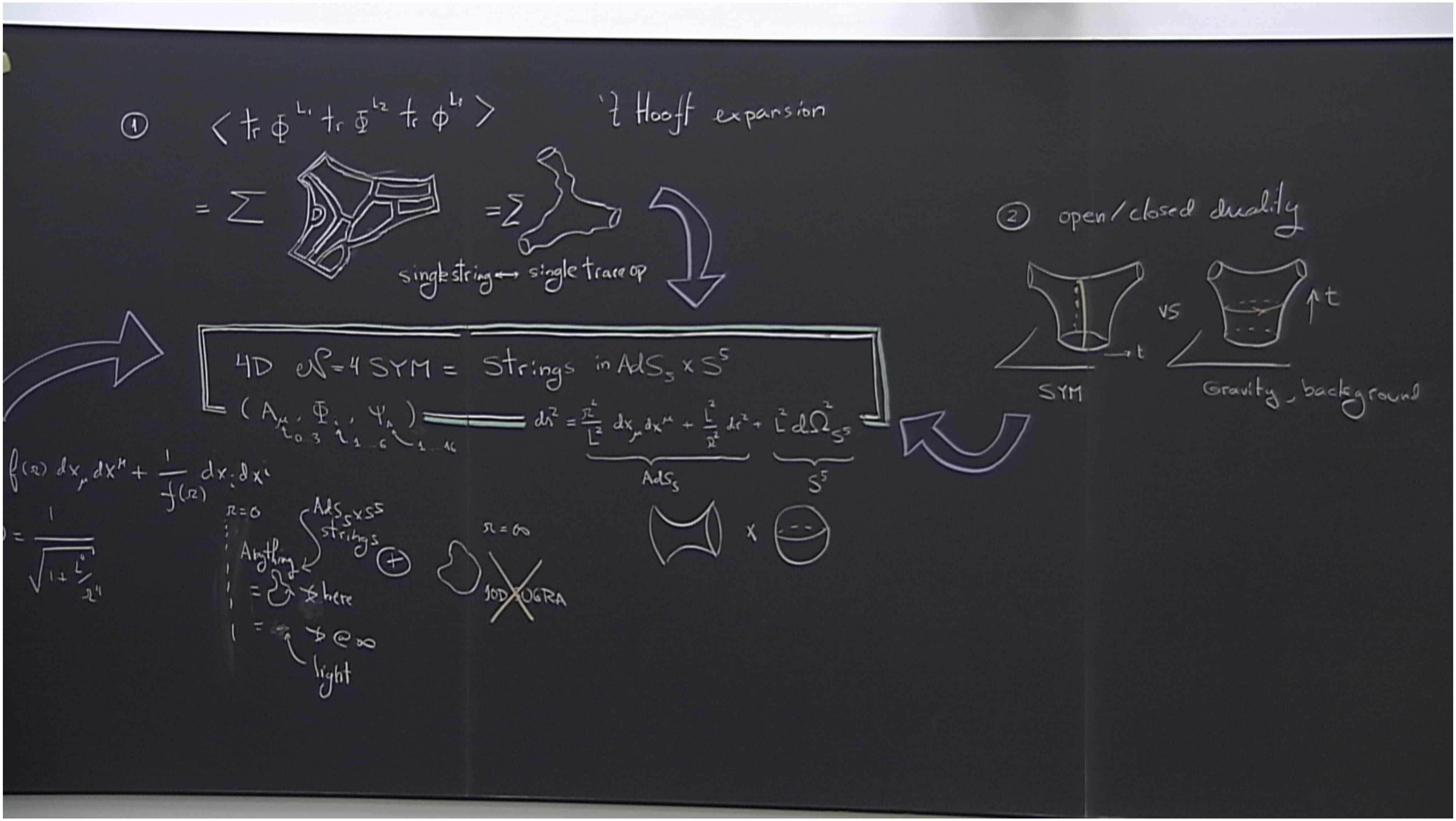
= $\nexists$ @ ∞

light

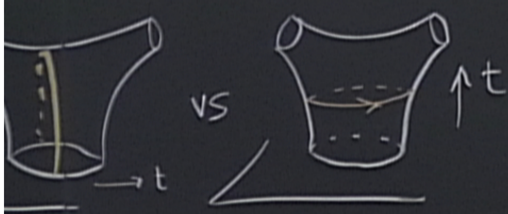

't Hooft expansion
② open/closed duality
vs
SYM
Gravity, background
N=4 SYM = Strings in AdS5 x S5
ds² = r²/L² dx_μ dx^μ + L²/r² dr² + L² dΩ²_S5
AdS5
S5
AdS5xS5 strings
r = ∞
SUGRA
here
@∞
light



1
$\langle \mathrm{tr}\,\Phi^{L_1}\,\mathrm{tr}\,\Phi^{L_2}\,\mathrm{tr}\,\phi^{L_3} \rangle$
't Hooft expansion
$= \Sigma$
$= \Sigma$
single string ⟷ single trace op
4D $\mathcal{N}=4$ SYM = Strings in $AdS_5 \times S^5$
(A_μ, Φ_i, Ψ_A)
0...3
1...6
1...16
$ds^2 = \frac{r^2}{L^2} dx_\mu dx^\mu + \frac{L^2}{r^2} dr^2 + L^2 d\Omega^2_{S^5}$
AdS_5
S^5
$+ \frac{1}{f(r)} dx_i dx^i$
$r = 0$
$AdS_5 \times S^5$
$r = \infty$


① ⟨ tr Φ^L1 tr Φ^L2 tr Φ^L3 ⟩
't Hooft expansion
= Σ
= Σ
single string ⟷ single trace op
② open/closed duality
vs
t
SYM
Gravity background
4D N=4 SYM = Strings in AdS5 × S5
(Aμ, Φi, Ψa)
ds² = r²/L² dxμ dx^μ + L²/r² dr² + L² dΩ²S5
AdS5
S5
f(r) dxμ dx^μ + 1/f(r) dxi dxi
r = 0
AdS5×S5 strings
Anything
here
@ ∞
light
r = ∞
10D SUGRA

open / closed duality



YM

Gravity background

Gauge Theory

$$g^2_{YM}$$

$$\lambda \ (= g^2_{YM} N)$$

String Theory

$$g_s = g^2_{YM}$$

$$L^4 / \alpha'^2 = \lambda$$

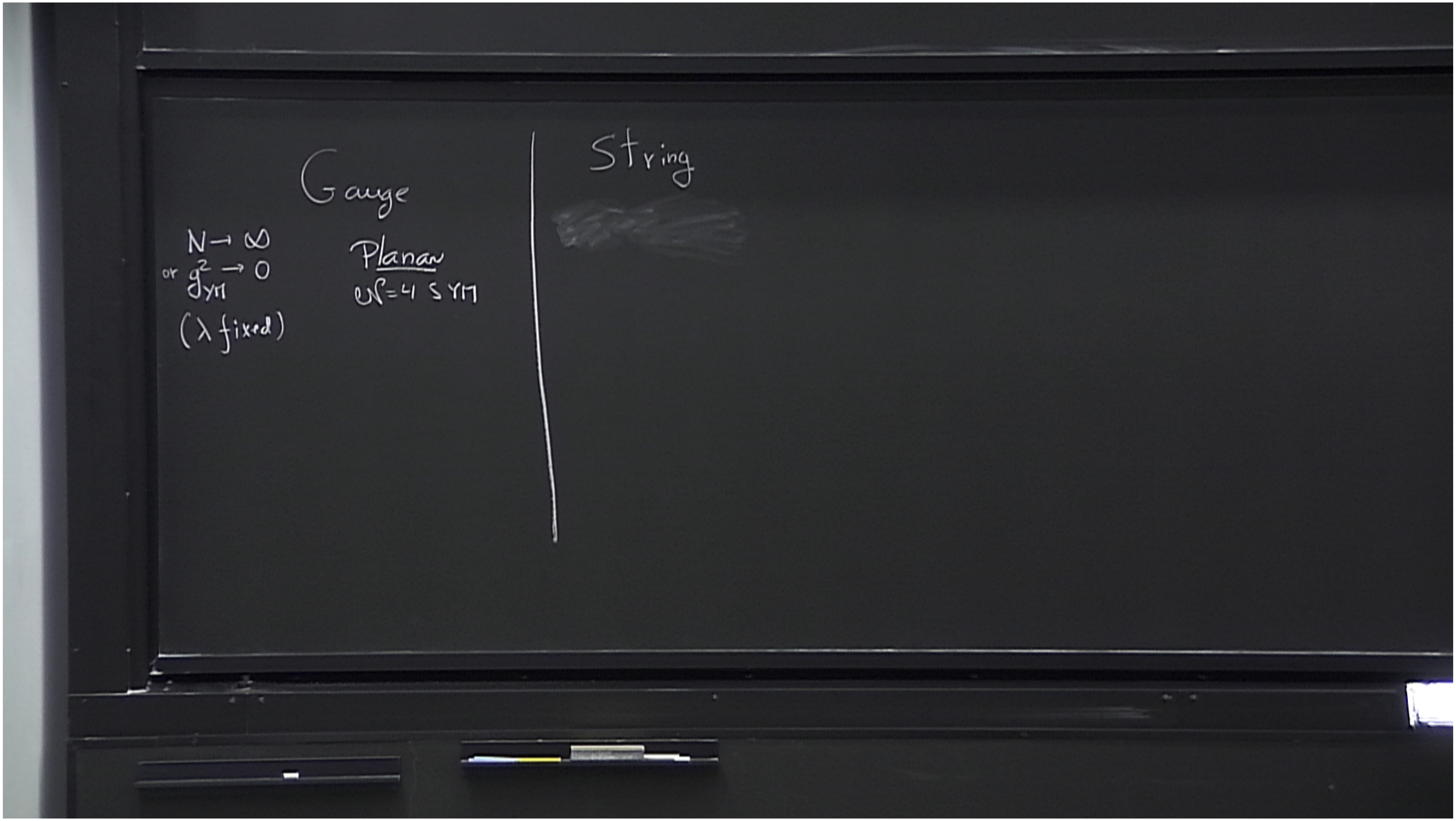
Action

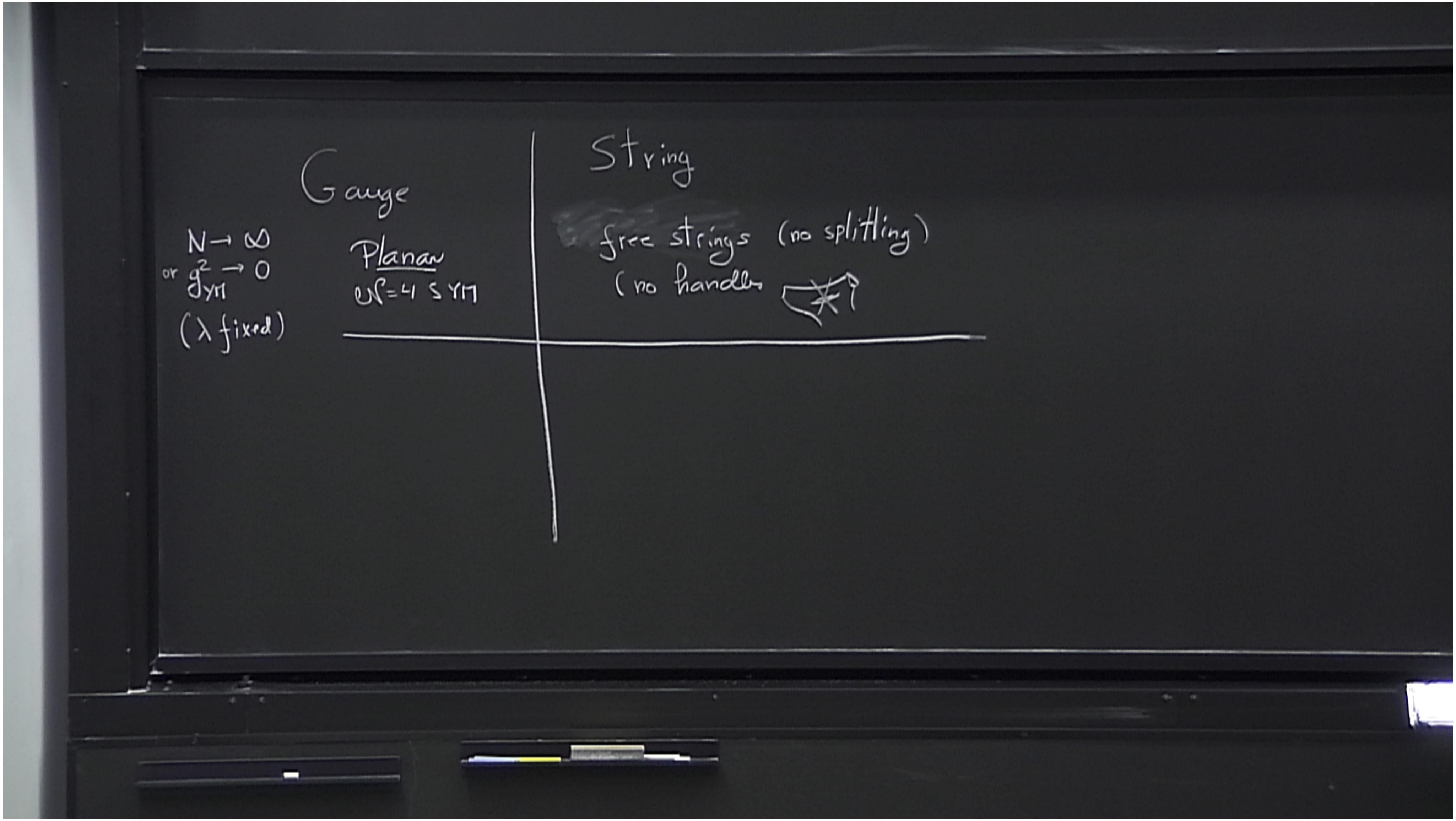
10D coordinates

$$\frac{1}{2\pi\alpha'} \int G_{\mu\nu}(X)\, \partial_\alpha X^\mu \partial_\beta X^\nu h^{\alpha\beta} \sqrt{h}$$

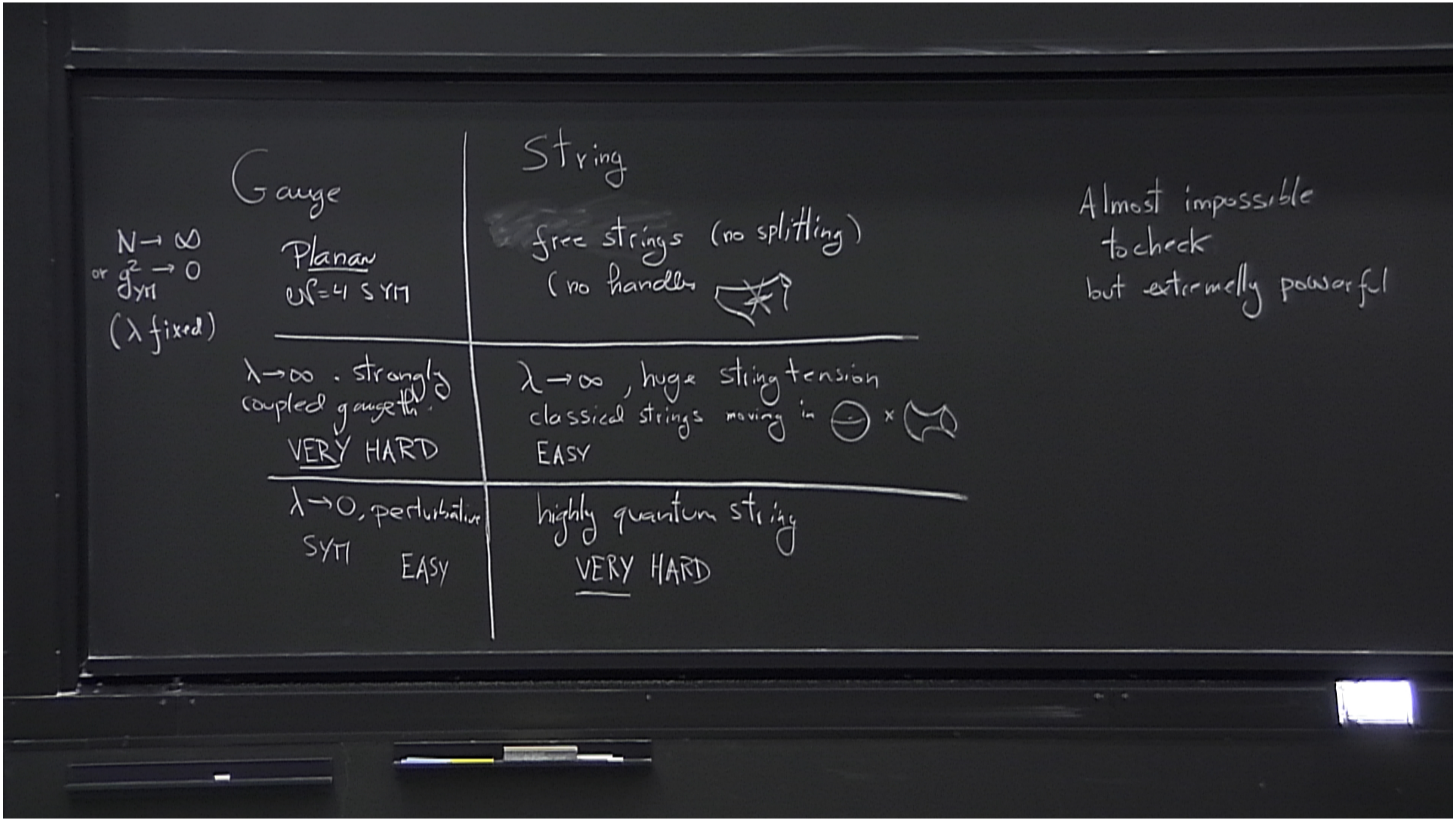
$$\Big\downarrow \ \begin{array}{l} \sigma \to L\sigma \\ x \to L x \end{array}$$

$$\frac{\sqrt{\lambda}}{2\pi} \int \left(\ldots \text{dimensionless} \ldots \right)$$


Gauge
String
$N \to \infty$
or $g^2_{YM} \to 0$
(λ fixed)
Planar
$\mathcal{N}=4$ SYM


Gauge
String
N → ∞
or $g^2_{YM} \to 0$
(λ fixed)
Planar
$\mathcal{N}=4$ SYM
free strings (no splitting)
(no handles

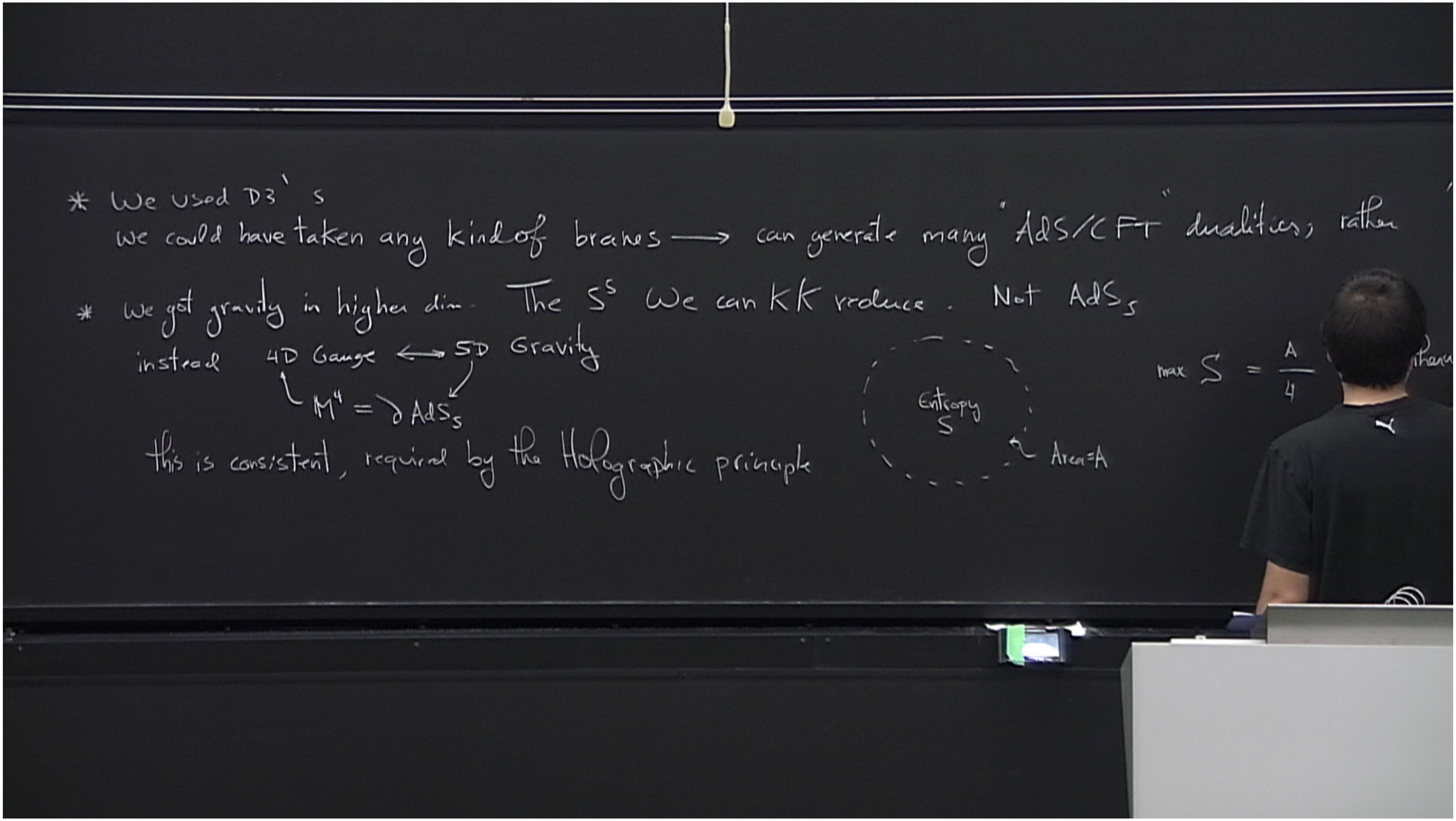
	Gauge	String
$N \to \infty$ or $g^2_{YM} \to 0$ (λ fixed)	Planar $\mathcal{N}=4$ SYM	free strings (no splitting) (no handles
	$\lambda \to \infty$, strongly coupled gauge th. VERY HARD	$\lambda \to \infty$, huge string tension classical strings moving in $\ominus \times$ EASY
	$\lambda \to 0$, perturbative SYM EASY	highly quantum string



Gauge
String
$N \to \infty$
or $g^2_{YM} \to 0$
(λ fixed)
Planar
$\mathcal{N}=4$ SYM
free strings (no splitting)
(no handles
$\lambda \to \infty$. strongly coupled gauge th.
VERY HARD
$\lambda \to \infty$, huge string tension
classical strings moving in $\times$
EASY
$\lambda \to 0$, perturbative SYM
EASY
highly quantum string
VERY HARD
Almost impossible to check
but extremelly powerful

* We used D3's
we could have taken any kind of branes $\longrightarrow$ can generate many "AdS/CFT" dualitie

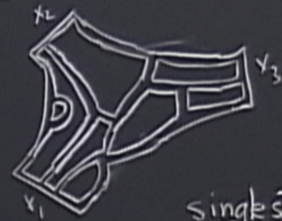
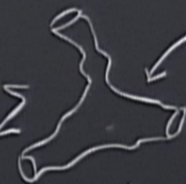
* We got gravity in higher dim. The S^5 We can KK reduce. Not AdS_5
instead 4D Gauge $\longleftrightarrow$ 5D Gravity
$M^4 = \partial AdS_5$


* We used D3's
we could have taken any kind of branes → can generate many "AdS/CFT" dualities; rather
* we got gravity in higher dim. The S^5 we can KK reduce. Not AdS_5
instead 4D Gauge ⟷ 5D Gravity
M^4 = ∂ AdS_5
this is consistent, required by the Holographic principle
Entropy S
Area=A
max S = A/4

* it was "good" that we did not find 4D gauge = 4D gravity

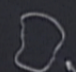
... argument

① $\langle \mathrm{tr}\,\Phi^{L_1}\,\mathrm{tr}\,\Phi^{L_2}\,\mathrm{tr}\,\Phi^{L_3} \rangle$ 't Hooft expansion

$= \sum$  $= \sum$  $f(\lambda)$

single string $\longleftrightarrow$ single trace op

conformal

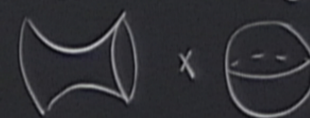


SYM $\oplus$ 10D $\cancel{\text{SUGRA}}$

vs

$$\boxed{\text{4D } \mathcal{N}=4 \text{ SYM} = \text{Strings in } AdS_5 \times S^5}$$

$$(\underbrace{A_\mu}_{0\ldots3},\ \underbrace{\Phi_i}_{1\ldots6},\ \underbrace{\Psi_A}_{1\ldots16}) \;=\!=\!=\; ds^2 = \underbrace{\frac{r^2}{L^2}dx_\mu dx^\mu + \frac{L^2}{r^2}dr^2}_{AdS_5} + \underbrace{L^2 d\Omega^2_{S^5}}_{S^5}$$



$$= ds^2 = f(r)\,dx_\mu dx^\mu + \frac{1}{f(r)}\,dx_i dx^i$$

$$f(r) = \frac{1}{\sqrt{1+L^4/r^4}}$$

$r=0$ Anything $\leftarrow$ $AdS_5 \times S^5$ strings $\oplus$  $r=\infty$ 10D $\cancel{\text{SUGRA}}$

= ... here

= ... @ ∞

light

$M^4 = \partial AdS_5$

this is consistent, required by the Holographic principle

Entropy S — Area=A

max $S = \frac{\;}{4}$, $S > \frac{A}{4}$

* it was "good" that we did not find 4D gauge = 4D gravity

Th: Weinberg & Witten: A QFT w/ poincaré inv. stress tensor $T_{\mu\nu}$, forbids

they show that $\langle k, \text{spin}2 \,|\, T_{\mu\nu} \,|\, k', \text{spin}2 \rangle$ behaves in a crazy way

* guessing $dx_\mu dx^\mu$ 0,1,2,3, scaling $x \to \lambda x$ should be a sym.

$M^4 = \partial AdS_5$

this is consistent, required by the Holographic principle

Entropy S

Area $= A$

max $S = \frac{\ }{4}$

$S > \frac{A}{4}$

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* guessing $dx_\mu dx^{\mu}$ (μ = 0,1,2,3), scaling $x \to \lambda x$ should be a sym.

add a coordinate z : $z \to z\lambda$, $ds^2 = \dfrac{dx_\mu dx^\mu + dz^2}{z^2}$ is the simplest promotion of $4D \to 5D$, z

argument

SYM ⊕ 10D $\cancel{\text{SUGRA}}$

vs

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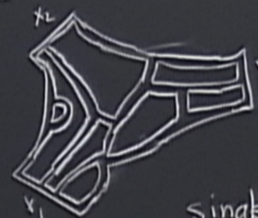
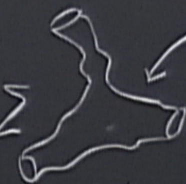
$= \Sigma$  $= \Sigma$  $f(\lambda)$

single string ⟷ single trace op

conformal

$$\boxed{4D\ \mathcal{N}=4\ \text{SYM} = \text{Strings in } AdS_5 \times S^5}$$

$$\big(\underset{\mu=0\ldots3}{A_\mu},\ \underset{i=1\ldots6}{\Phi_i},\ \underset{A=1\ldots16}{\Psi_A}\big) \;=\; ds^2 = \underbrace{\frac{r^2}{L^2}\, dx_\mu dx^\mu + \frac{L^2}{r^2}\, dr^2}_{AdS_5} + \underbrace{L^2 d\Omega^2_{S^5}}_{S^5}$$

#

 $\times$ 

$$= ds^2 = f(r)\, dx_\mu dx^\mu + \frac{1}{f(r)}\, dx_i dx^i$$

$$f(r) = \frac{1}{\sqrt{1 + L^4/r^4}}$$

$r=0$

$AdS_5 \times S^5$ strings ⊕ 10D $\cancel{\text{SUGRA}}$

$r=\infty$

Anything

= here

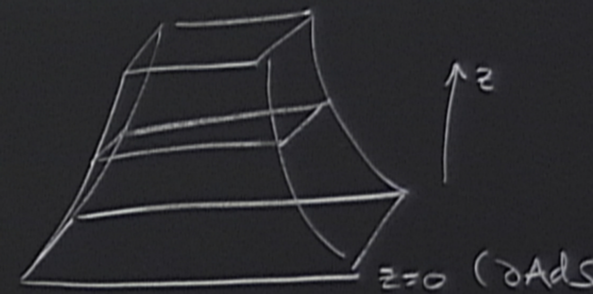
= @ ∞

light

$\mathbb{M}^4$. AdS^5

$ds^2 = dx_\mu dx^\mu$, $ds^2 = L^2 \left(\frac{dx_\mu dx^\mu + dz^2}{z^2} \right)$

$\partial AdS_5 : z = \varepsilon \to 0$, $ds^2 \propto dx_\mu dx^\mu = ds^2_{\mathbb{M}^4}$



M^4

$ds^2 = dx_\mu dx^\mu$,

conf sym

$X \to \lambda X$

$X \to X + C$

$X^\mu \longrightarrow X^\mu +$

AdS^5

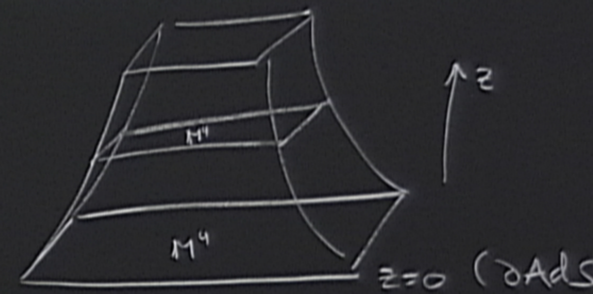
$$ds^2 = L^2 \left(\frac{dx_\mu dx^\mu + dz^2}{z^2} \right)$$

$\partial AdS_5 : z = \varepsilon \to 0$, $ds^2 \propto dx_\mu dx^\mu = ds^2_{M^4}$

isometries

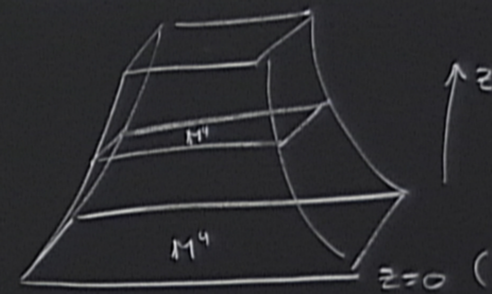
$X \to \lambda X$, $z \to \lambda z$

$X \to X + C$, $z \to z$



M^4 , AdS^5

$$ds^2 = dx_\mu dx^\mu , \qquad ds^2 = L^2 \left(\frac{dx_\mu dx^\mu + dz^2}{z^2} \right)$$



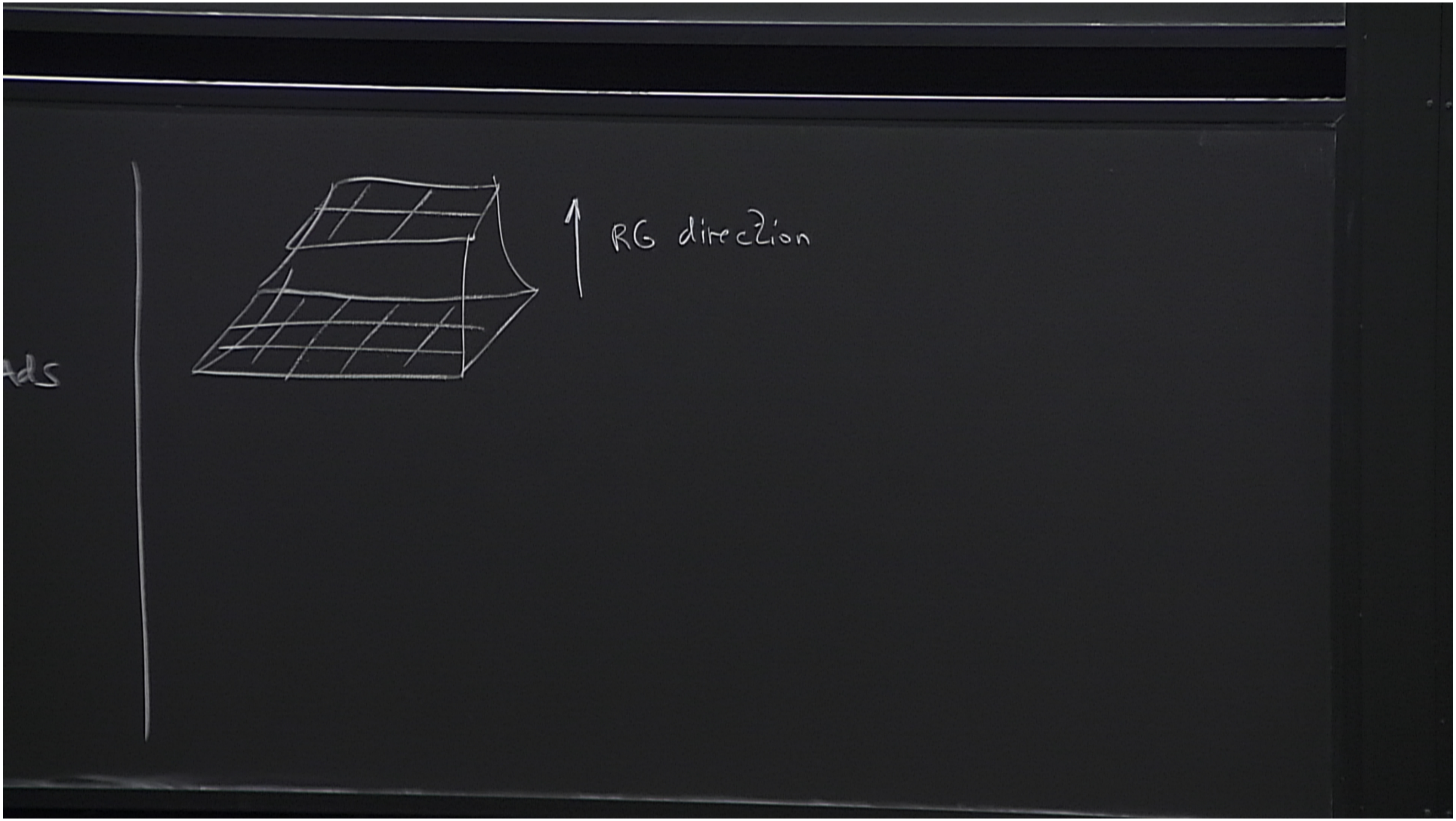
$$\partial AdS_5 : z = \varepsilon \to 0 , \quad ds^2 \propto dx_\mu dx^\mu = ds^2_{M^4}$$

conf sym ; isometries

$x \to \lambda x$; $x \to \lambda x , \ z \to \lambda z$

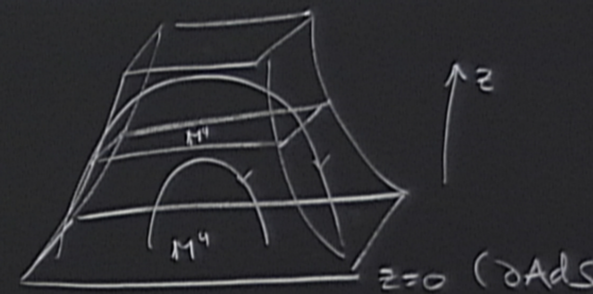
$x \to x + c$; $x \to x + c , \ z \to z$

$$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2 b\cdot x + b^2 x^2} , \quad x^\mu \to \frac{x^\mu + b^\mu (x^2 + z^2)}{1 + 2 b \cdot x + b^2 (x^2 + z^2)} , \quad z \to \frac{z}{1 + 2 b \cdot x + b^2 (x^2 + z^2)}$$


RG direction

M^4 , AdS^5

$ds^2 = dx_\mu dx^\mu$, $\quad ds^2 = L^2 \left(\dfrac{dx_\mu dx^\mu + dz^2}{z^2} \right)$



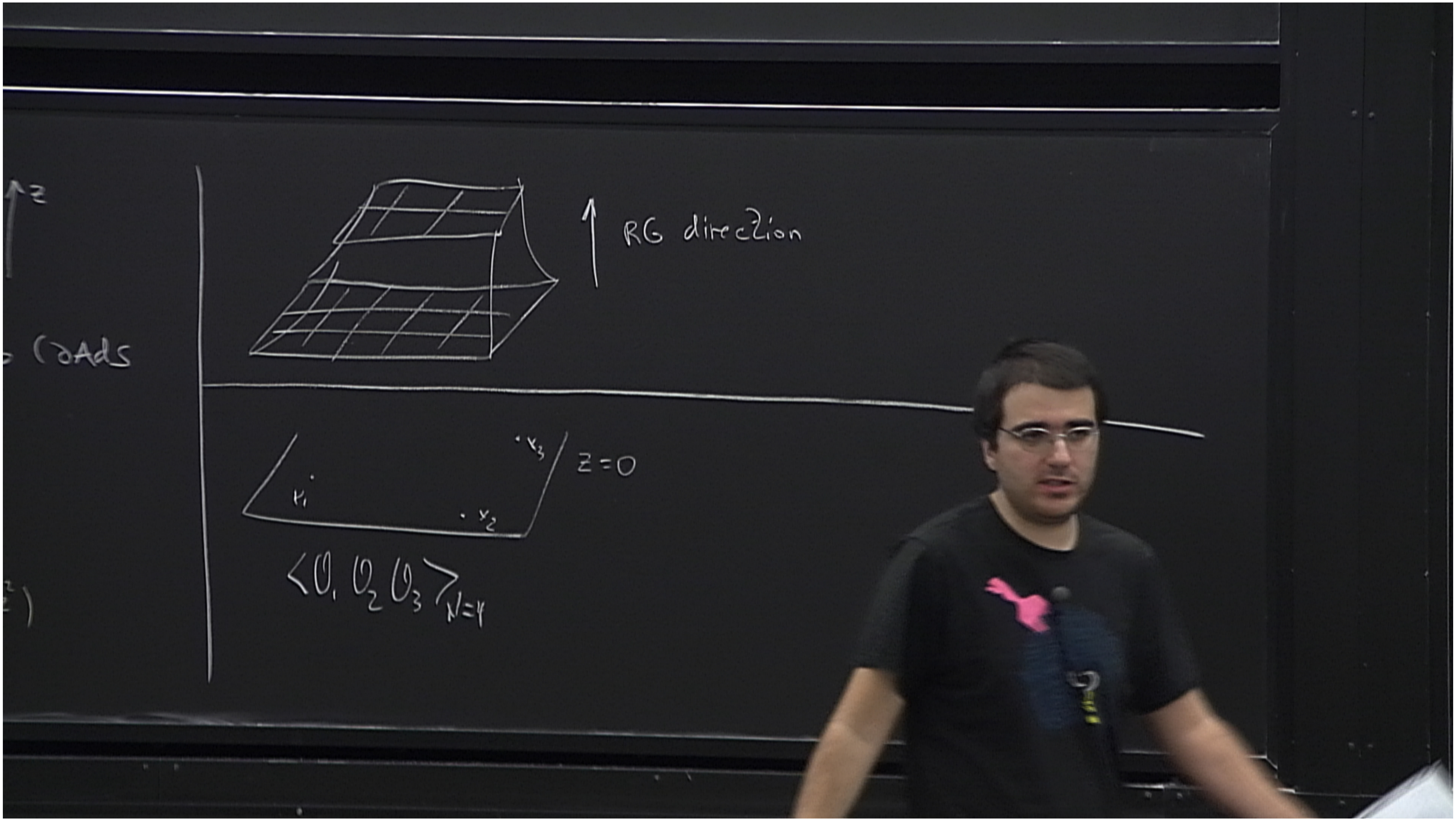
$\partial AdS_5 : z = \varepsilon \to 0 \,,\; ds^2 \propto dx_\mu dx^\mu = ds^2_{M^4}$

conf sym / isometries

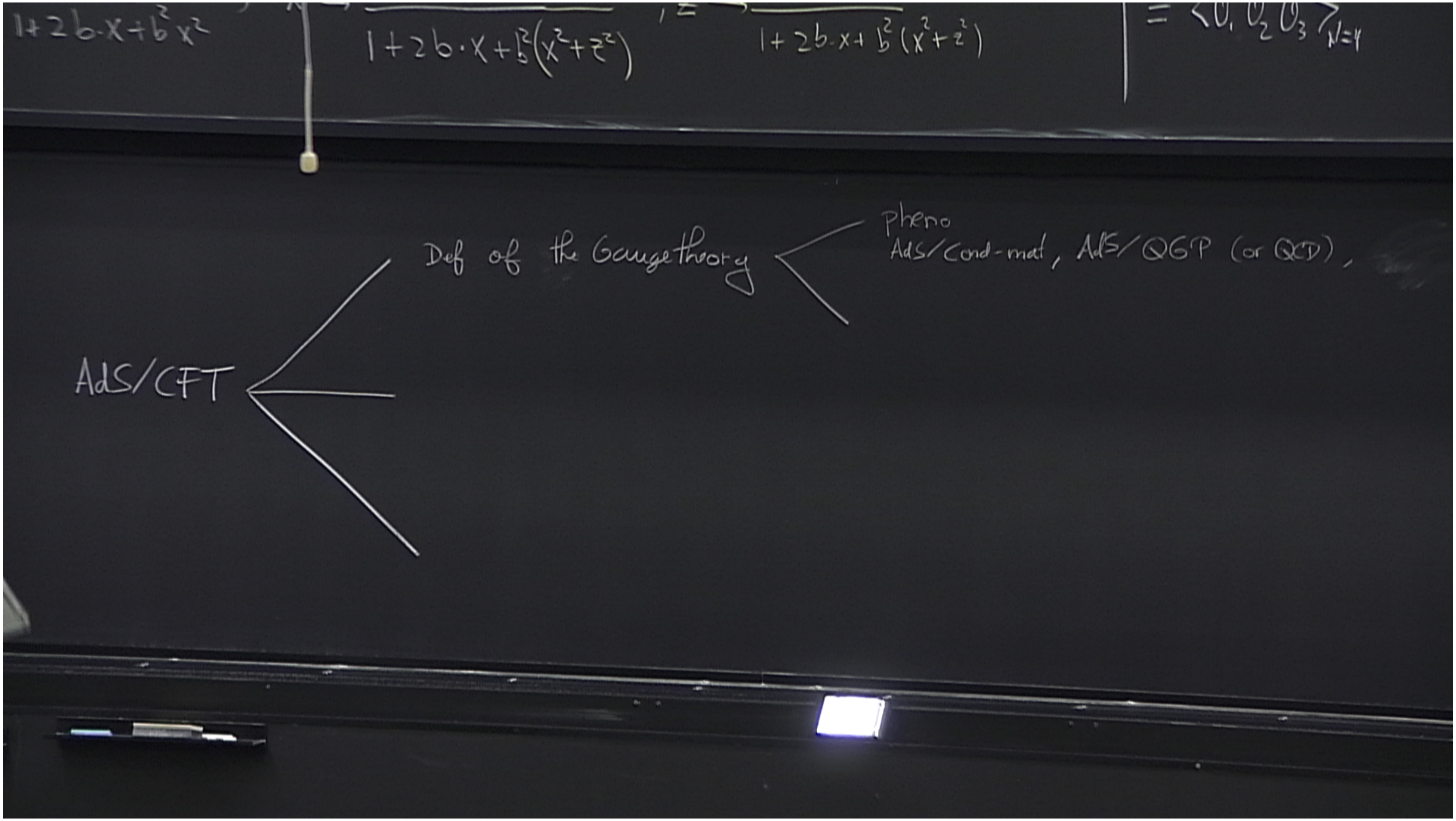
$x \to \lambda x \;,\quad x \to \lambda x,\; z \to \lambda z$

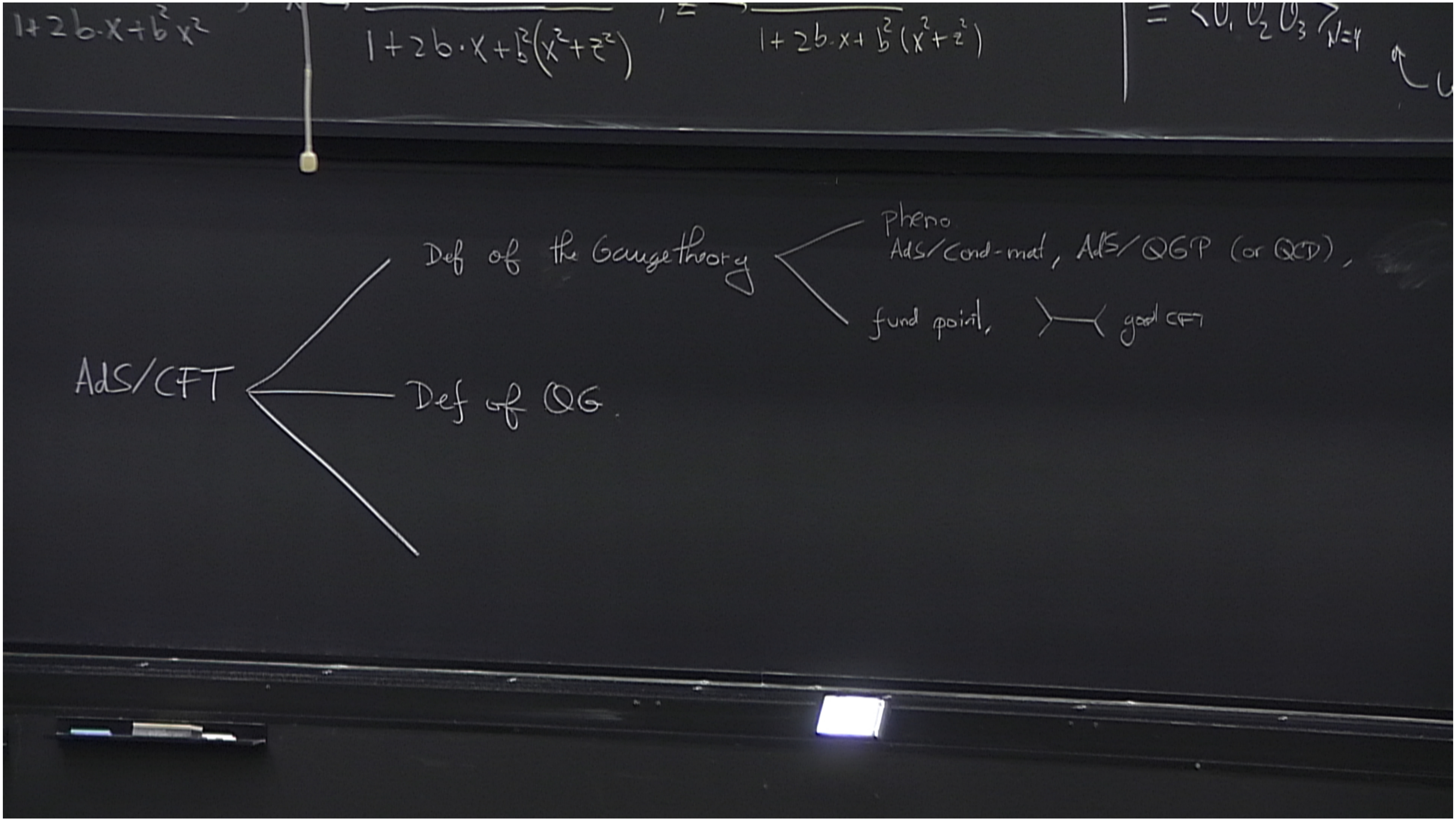
$x \to x + c \;,\quad x \to x + c,\; z \to z$

$$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2b\cdot x + b^2 x^2} \,,\quad x^\mu \to \frac{x^\mu + b^\mu (x^2 + z^2)}{1 + 2b\cdot x + b^2 (x^2 + z^2)} \,,\quad z \to \frac{z}{1 + 2b\cdot x + b^2 (x^2 + z^2)}$$

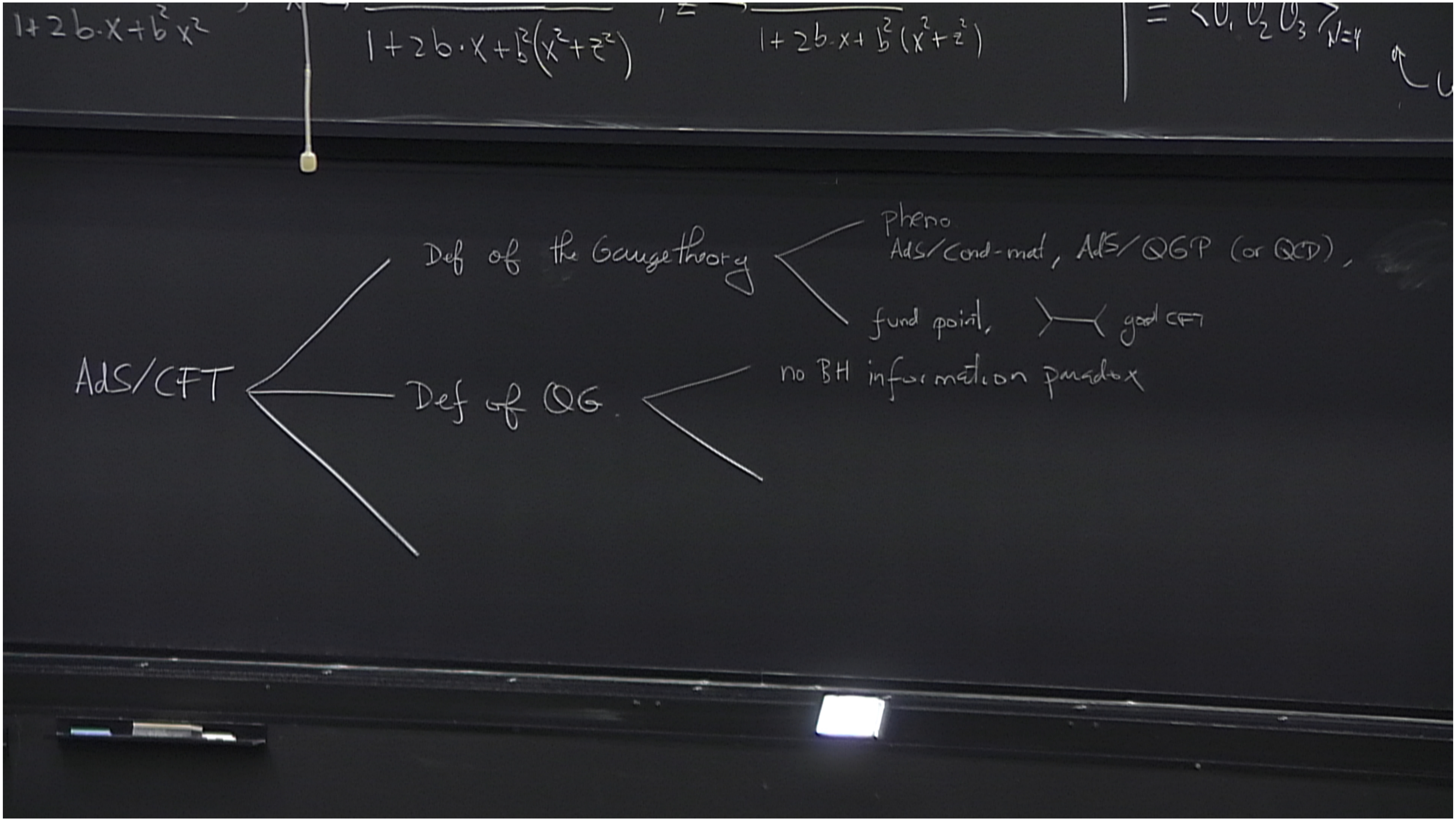


RG direction
z = 0
$\langle O_1 O_2 O_3 \rangle_{N=4}$


1+2b·x+b²x²
1+2b·x+b²(x²+z²)
1+2b·x+b²(x²+z²)
= ⟨O₁ O₂ O₃⟩ N=4
Def of the Gauge theory
pheno
AdS/Cond-mat, AdS/QGP (or QCD),
AdS/CFT



1+2b·x+b²x²
1+2b·x+b²(x²+z²)
1+2b·x+b²(x²+z²)
= ⟨O₁ O₂ O₃⟩ N=4
AdS/CFT
Def of the Gaugetheory
pheno
AdS/Cond-mat, AdS/QGP (or QCD),
fund point,
good CFT
Def of QG.


AdS/CFT
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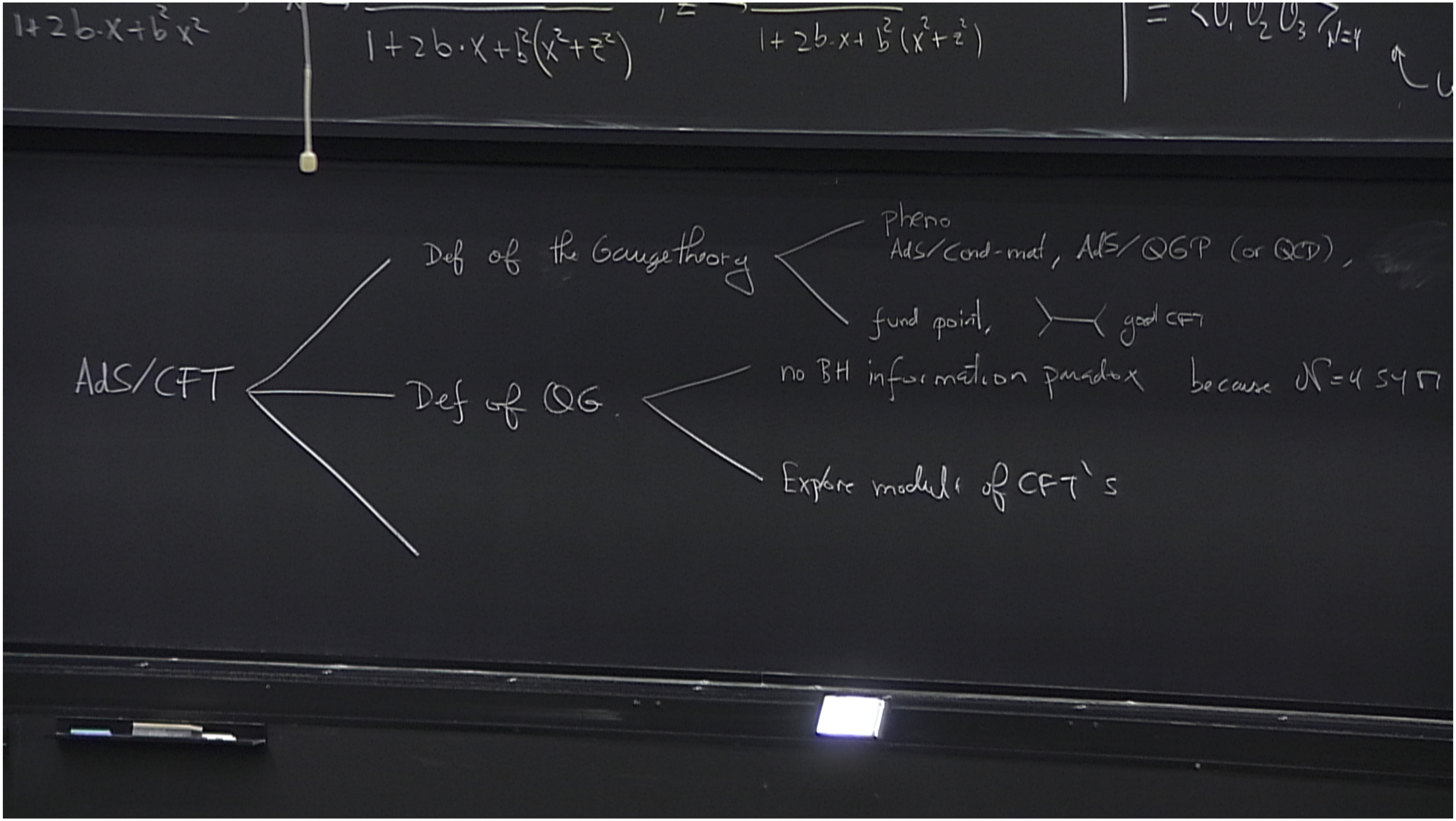
no BH information paradox



1+2b·x+b²x²
1+2b·x+b²(x²+z²)
1+2b·x+b²(x²+z²)
= ⟨O1 O2 O3⟩ N=4
AdS/CFT
Def of the Gaugetheory
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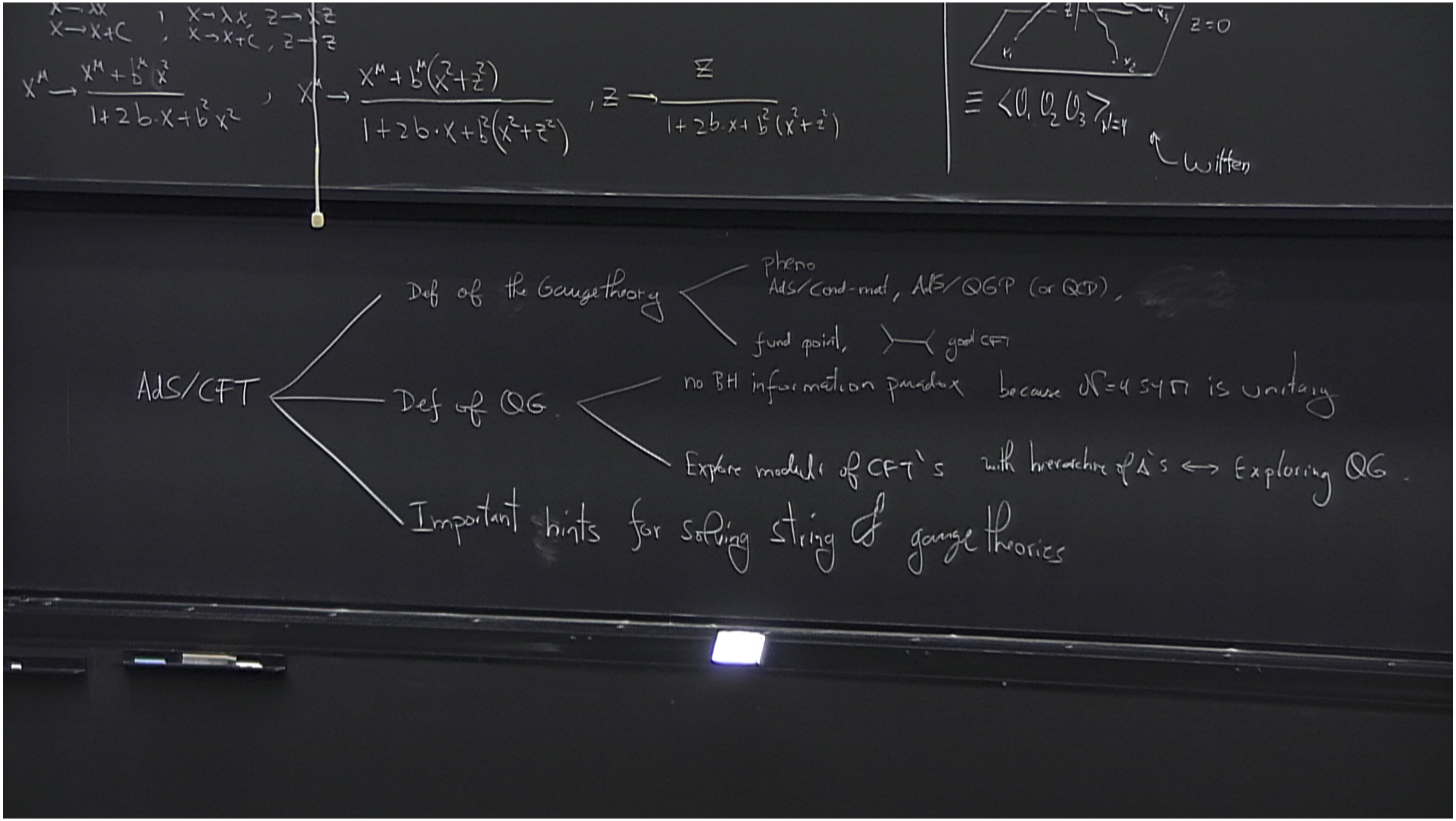
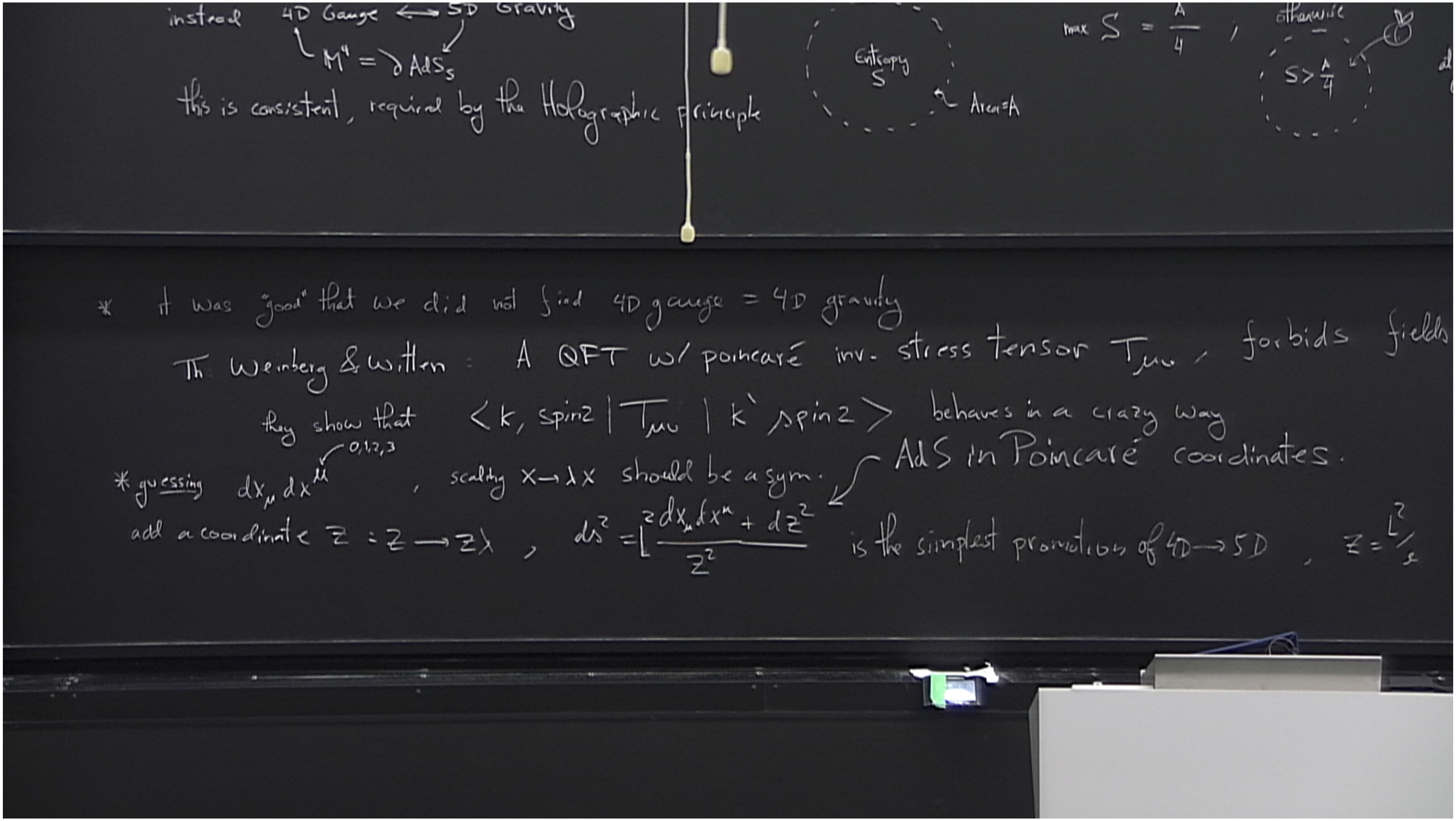
AdS/Cond-mat, AdS/QGP (or QCD),
fund point,
good CFT
Def of QG.
no BH information paradox because N=4 SYM
Explore moduli of CFT's



$x \to \lambda x$; $x \to \lambda x,\ z \to \lambda z$
$x \to x + c$; $x \to x + c,\ z \to z$
$x^\mu \to \frac{x^\mu + b^\mu x^2}{1 + 2b\cdot x + b^2 x^2}$, $x^\mu \to \frac{x^\mu + b^\mu (x^2 + z^2)}{1 + 2b\cdot x + b^2 (x^2 + z^2)}$, $z \to \frac{z}{1 + 2b\cdot x + b^2 (x^2 + z^2)}$
$z = 0$
$\equiv \langle O_1 O_2 O_3 \rangle_{\mathcal{N}=4}$
Witten
AdS/CFT
Def of the Gauge theory
pheno
AdS/Cond-mat, AdS/QGP (or QCD),
fund point, good CFT
Def of QG.
no BH information paradox because $\mathcal{N}=4$ SYM is unitary
Explore moduli of CFT's with hierarchy of Δ's $\leftrightarrow$ Exploring QG.
Important hints for solving string & gauge theories


instead 4D Gauge $\longleftrightarrow$ 5D Gravity
$M^4 = \partial AdS_5$
this is consistent, required by the Holographic principle
Entropy S
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Th Weinberg & Witten: A QFT w/ poincaré inv. stress tensor $T_{\mu\nu}$, forbids fields
they show that $\langle k, \text{spin}2 \,|\, T_{\mu\nu} \,|\, k', \text{spin}2 \rangle$ behaves in a crazy way
0,1,2,3
* guessing $dx_\mu dx^\mu$, scaling $x \to \lambda x$ should be a sym.
AdS in Poincaré coordinates.
add a coordinate z: $z \to z\lambda$, $ds^2 = L^2 \frac{dx_\mu dx^\mu + dz^2}{z^2}$ is the simplest promotion of 4D $\longrightarrow$ 5D, $z = L^2/r$