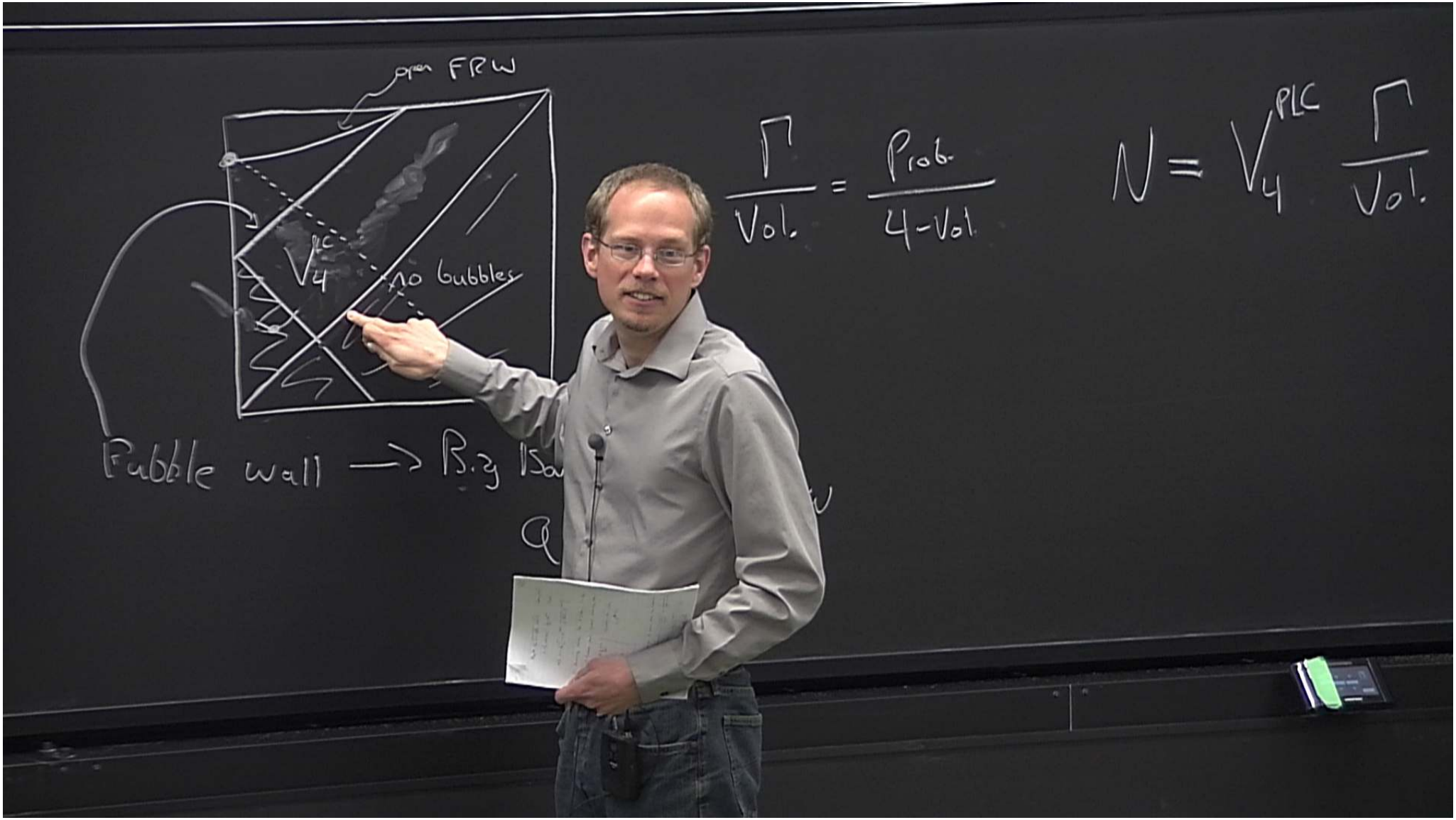
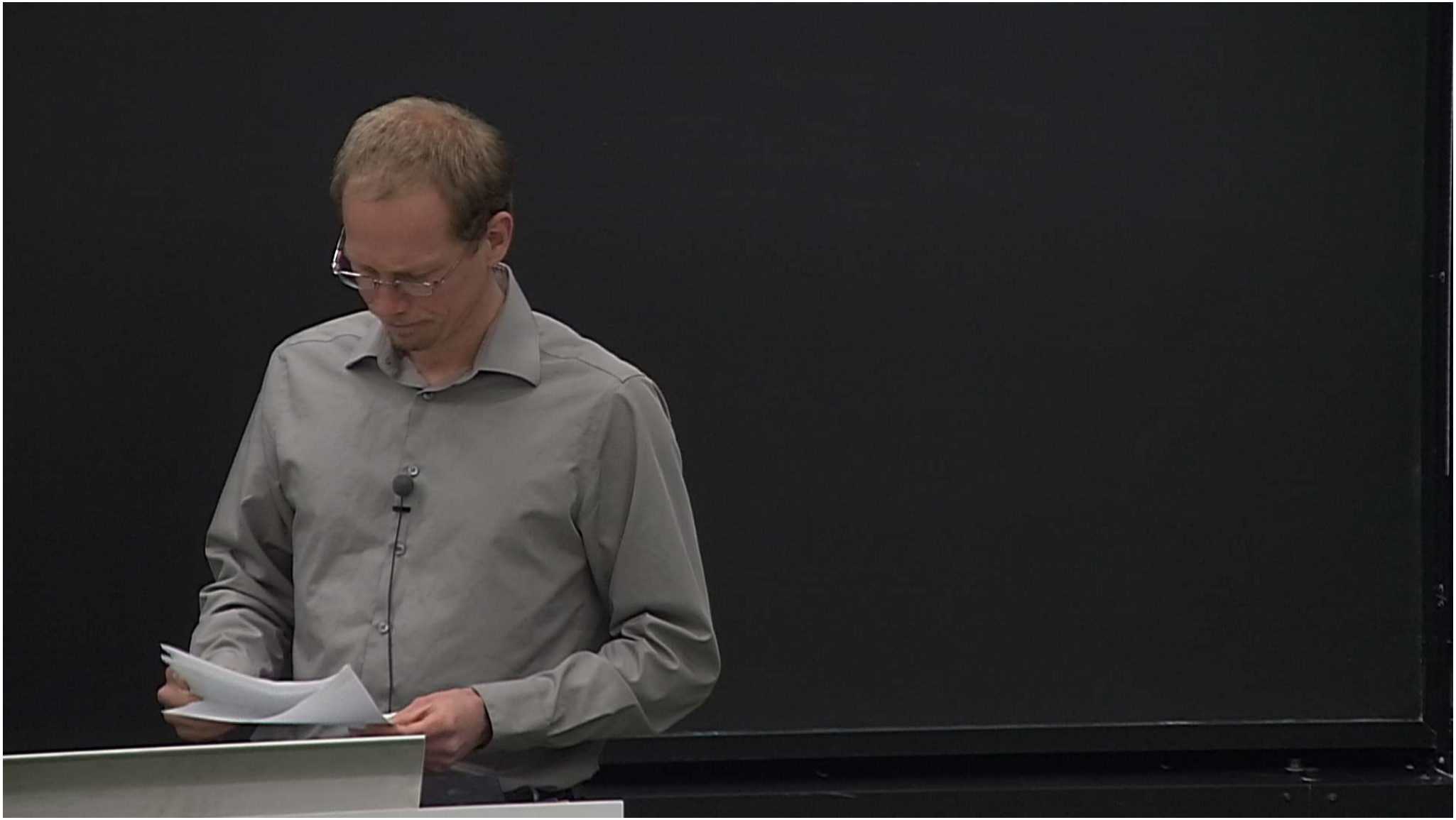
Title: Explorations in Cosmology - Lecture 14

Date: Mar 29, 2012 10:15 AM

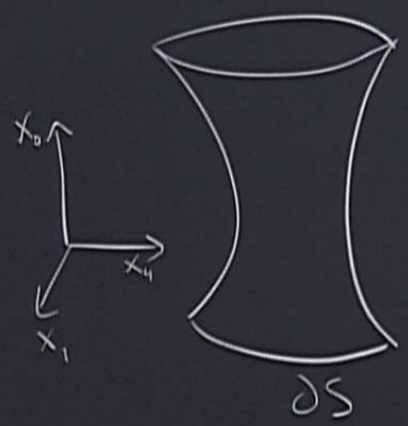
URL: http://pirsa.org/12030040

Abstract:


open FRW
V_4^{PLC}
no bubbles
bubble wall
$\frac{\Gamma}{Vol.} = \frac{Prob.}{4\text{-}Vol.}$
$N = V_4^{PLC} \frac{\Gamma}{Vol.}$



Yesterday



$dS - O(4,1)$

Bubble $- O(3,1)$ (Const. accel.)

$\Rightarrow$ Break 1-rotation

Wall is at $X_4 = \text{const.}$

$$X_4 = H_f^{-1}\sqrt{1 - H_f^2 R_0^2}\,\cosh(H_f t) \quad \text{(static)}$$

↳ Bubble nucleates @ $t=0$

$$= H_F \cosh(H_F t) - \frac{H_f}{2} r^2 e^{H_F t} \quad \text{(flat)}$$

$\Rightarrow$ for bubble formed @ $t=0$

$$r = H_F^{-1}\left[1 + e^{-2H_F t} - \sqrt{1 - H_F^2 R_0^2}\, e^{-H_F t}\right]$$

$\rightarrow$ Reaches const. comoving size

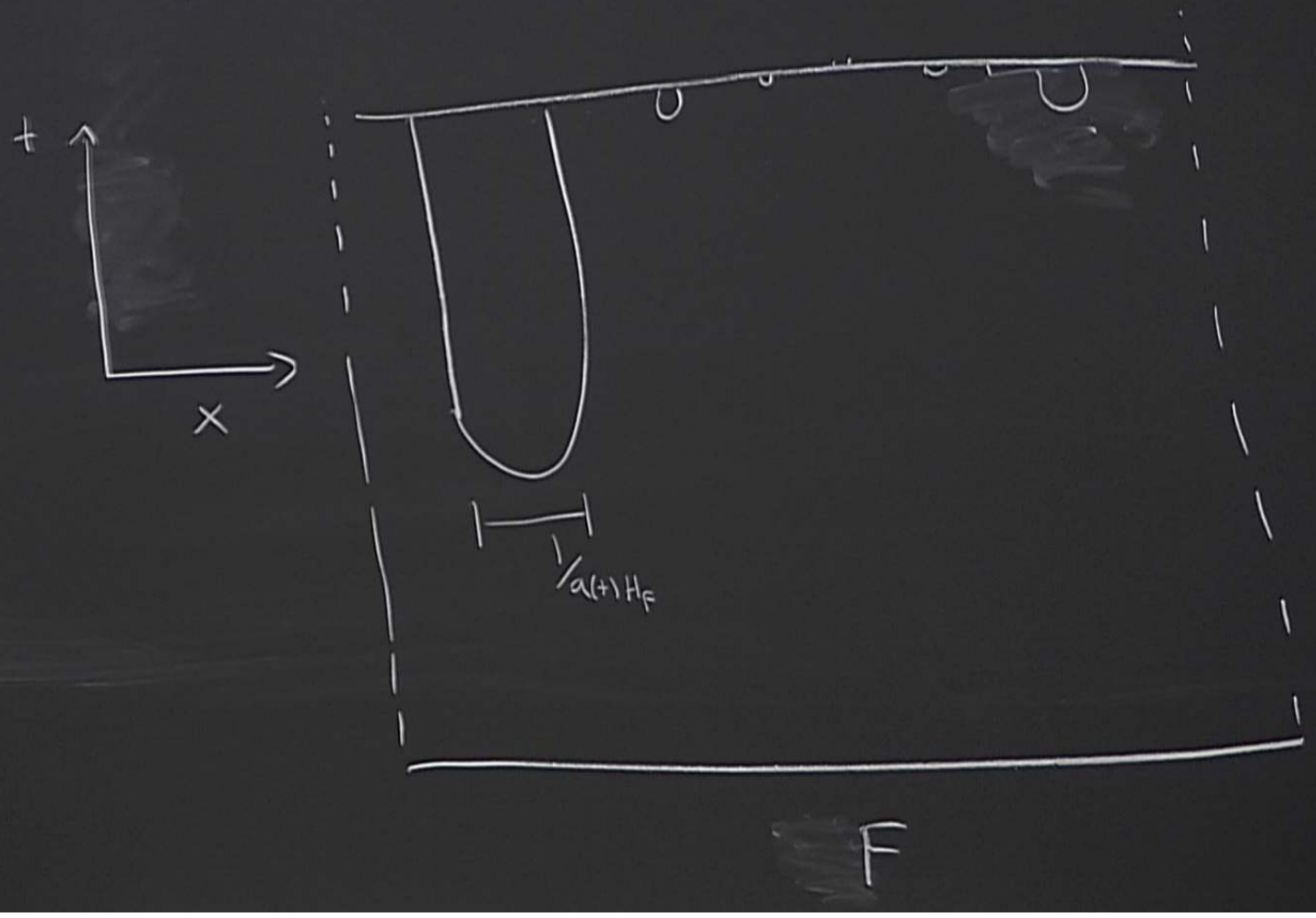
Bubble formed @ $t \neq 0$

$$r(t \to \infty) = \frac{1}{a(t) H_F}$$

static)

nucleates @ $t=0$

$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2$$



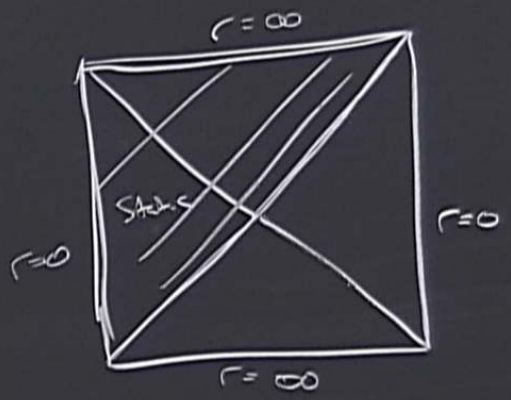
ubble formed @ $t=0$

$$= H_F^{-1}\left[1 + e^{-2H_F t} - \sqrt{1 - H_F^2 R_0^2}\, e^{-H_F t}\right]$$

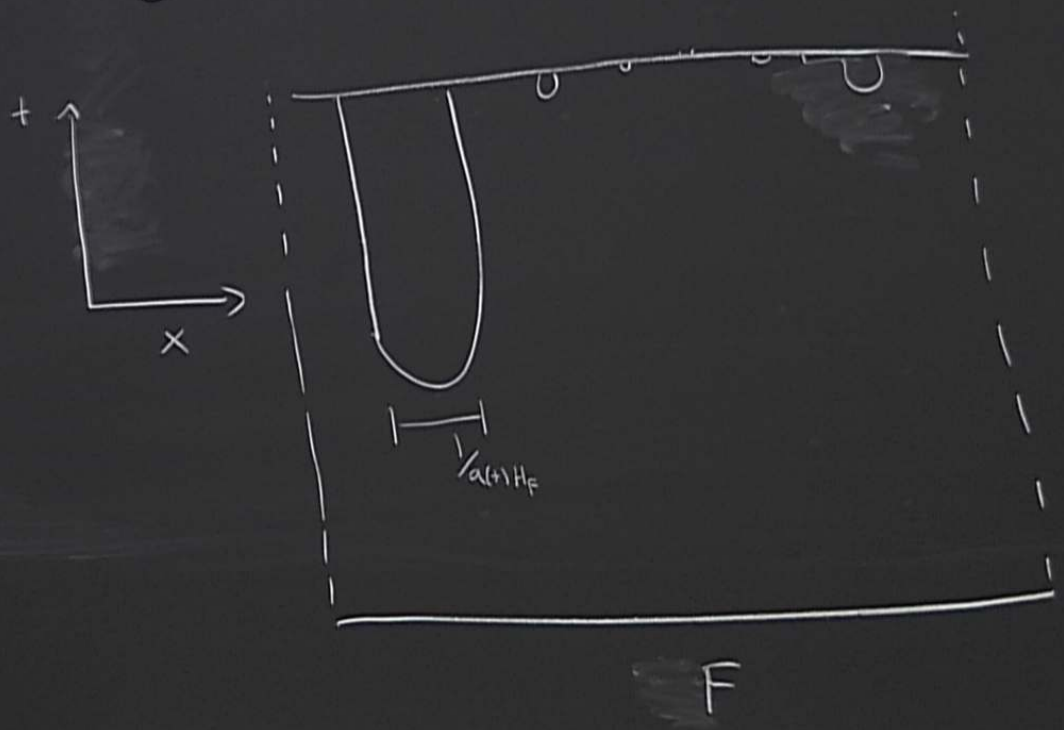
→ Reaches const. comoving size.

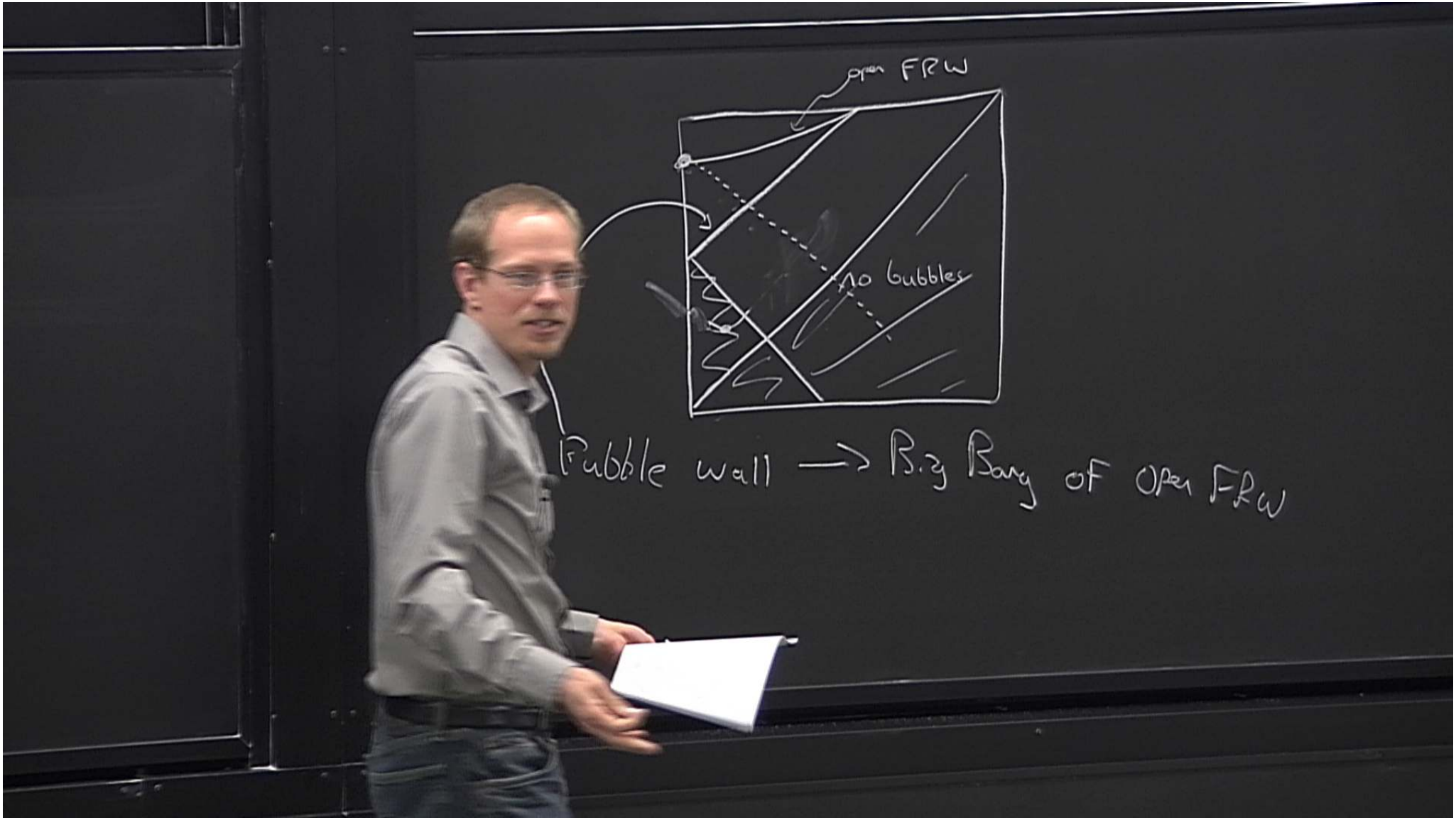
ble formed @ $t \neq 0$:

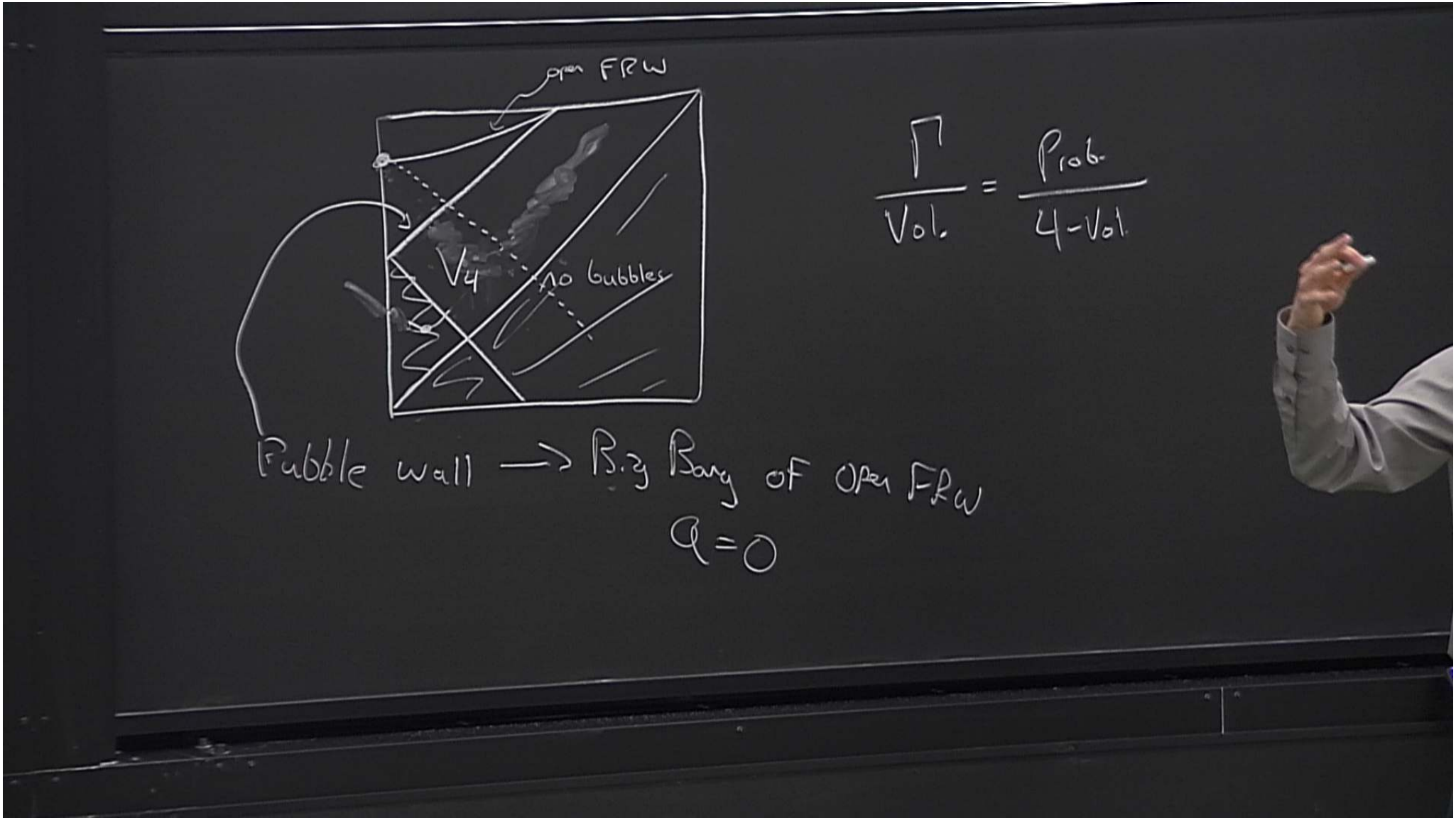
$$r(t \to \infty) = \frac{1}{a(t) H_F}$$

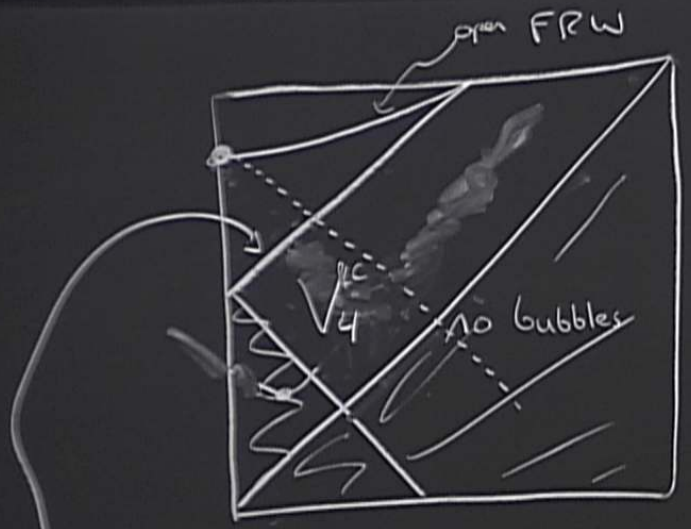


$$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$$




open FRW
no bubbles
Bubble wall → Big Bang of open FRW


open FRW
V_4
no bubbles
$\frac{\Gamma}{\text{Vol.}} = \frac{\text{Prob.}}{4\text{-Vol.}}$
Bubble wall → B.g Bang of open FRW
$q = 0$

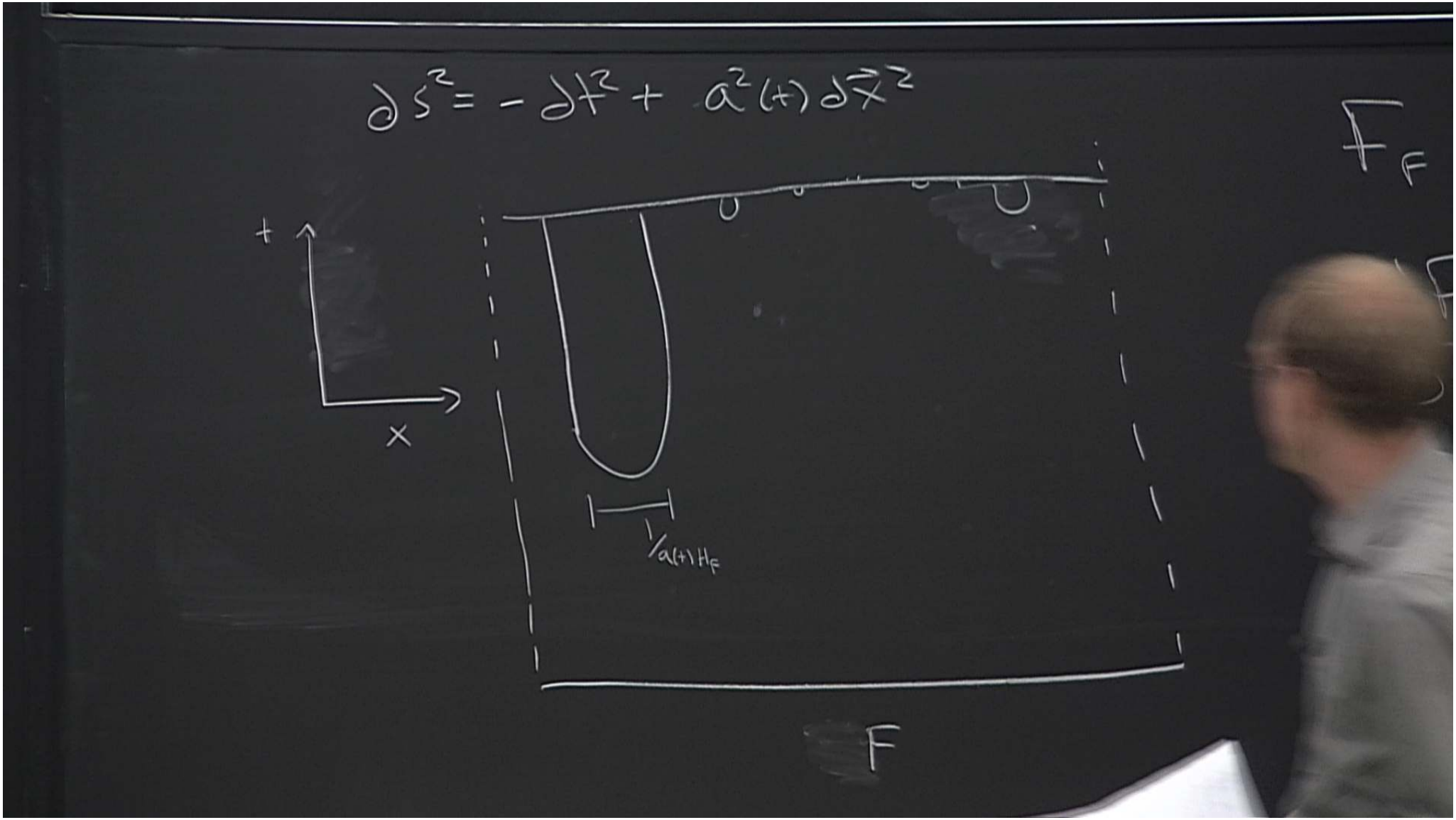


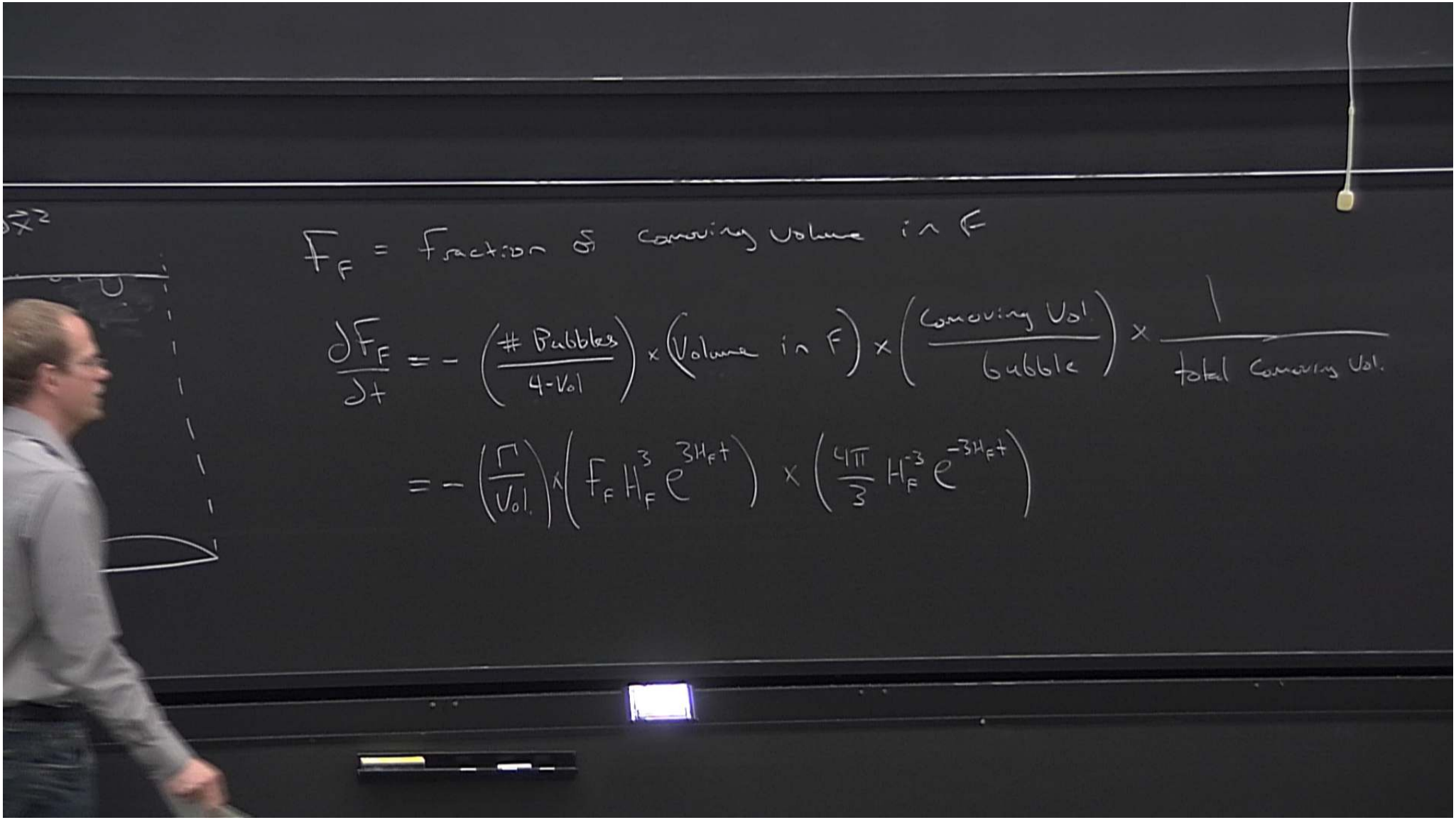
$$\frac{\Gamma}{\text{Vol.}} = \frac{\text{Prob.}}{4\text{-Vol.}}$$

$$N = V_4^{PLC} \frac{\Gamma}{\text{Vol.}}$$

Bubble wall → B.g Bang of open FRW

$$Q = 0$$


$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$
f_F
t
x
$1/a(t)H_F$
F


F_F = fraction of comoving volume in F
$\frac{dF_F}{dt} = -\left(\frac{\#\text{ Bubbles}}{4\text{-Vol}}\right) \times (\text{Volume in F}) \times \left(\frac{\text{Comoving Vol}}{\text{bubble}}\right) \times \frac{1}{\text{total Comoving Vol.}}$
$= -\left(\frac{\Gamma}{\text{Vol.}}\right) \times \left(f_F H_F^3 e^{3H_F t}\right) \times \left(\frac{4\pi}{3} H_F^{-3} e^{-3H_F t}\right)$

f_F = fraction of comoving volume in F

$$\frac{dF_F}{dt} = -\left(\frac{\#\ \text{Bubbles}}{4\text{-Vol}}\right) \times \left(\text{Volume in F}\right) \times \left(\frac{\text{Comoving Vol}}{\text{bubble}}\right) \times \frac{1}{\text{total Comoving Vol.}}$$

$$= -\left(\frac{\Gamma}{\text{Vol.}}\right) \times \left(f_F H_F^{-3} e^{3H_F t}\right) \times \left(\frac{4\pi}{3} H_F^{-3} e^{-3H_F t}\right) \times \frac{1}{H_F^{-3}}$$

$$= -\frac{4\pi}{3}\left(\frac{\Gamma}{\text{Vol.}}\right) H_F^{-3} f_F$$

f_F = fraction of comoving volume in F

$$\frac{df_F}{dt} = -\left(\frac{\#\ \text{Bubbles}}{4\text{-Vol}}\right) \times \left(\text{Volume in F}\right) \times \left(\frac{\text{Comoving Vol.}}{\text{bubble}}\right) \times \frac{1}{\text{total comoving Vol.}}$$

$$= -\left(\frac{\Gamma}{\text{Vol.}}\right) \times \left(f_F H_F^{-3} e^{3H_F t}\right) \times \left(\frac{4\pi}{3} H_F^{-3} e^{-3H_F t}\right) \times \frac{1}{H_F^{-3}}$$

$$= -\boxed{\frac{4\pi}{3}\left(\frac{\Gamma}{\text{Vol.}}\right) H_F^{-3}} f_F \rightarrow K_{TF}$$

$$\Rightarrow f_F = e^{-\kappa_{TF} t}$$ → All comoving vol. removed when κ

Define V_F, V_T total volume (physical) in T and F

$$\frac{dV_F}{dt} = -\kappa_{TF} V_F + 3H_F V_F$$

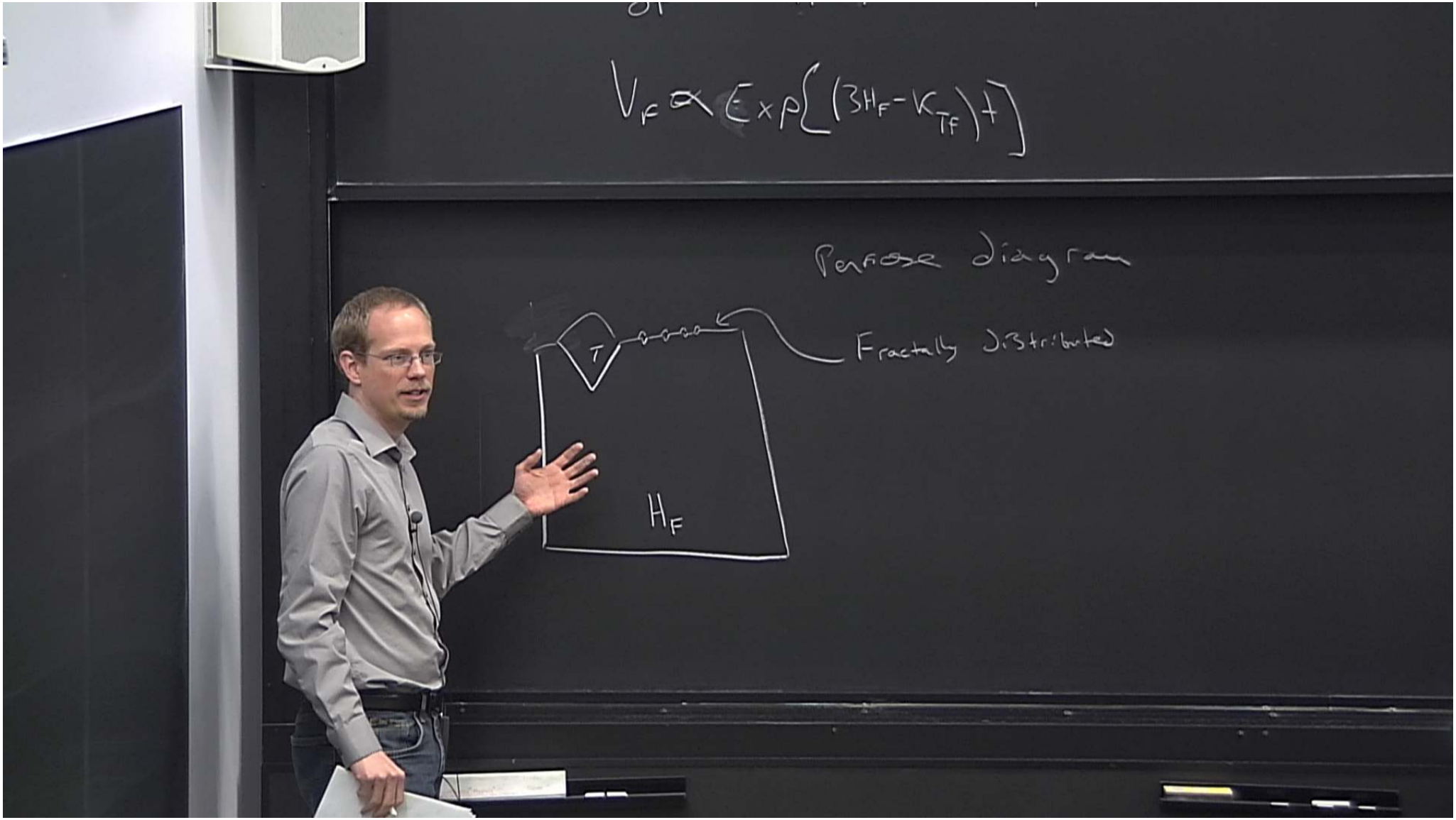
$$V_F \propto Exp\left[(3H_F - \kappa_{TF})t\right]$$

When $K_{TF} < 3H_F$ → Volume grows forever

Eternal Inflation

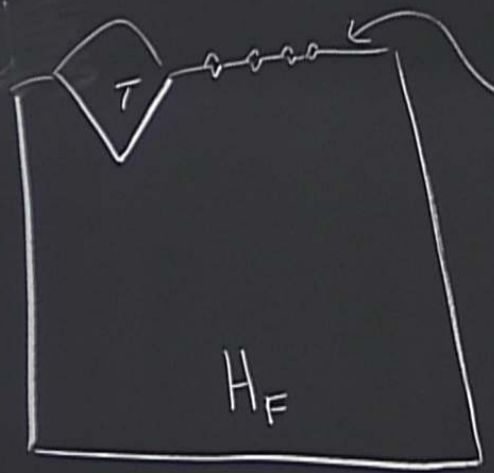
To avoid eternal inflation you need

1 Bubble per Hubble


$V_F \propto Exp[(3H_F - K_{TF})t]$
Penrose diagram
Fractally Distributed
T
H_F

$V_F \propto \exp\left[\left(3H_F^2 - K_{TF}\right)t\right]$

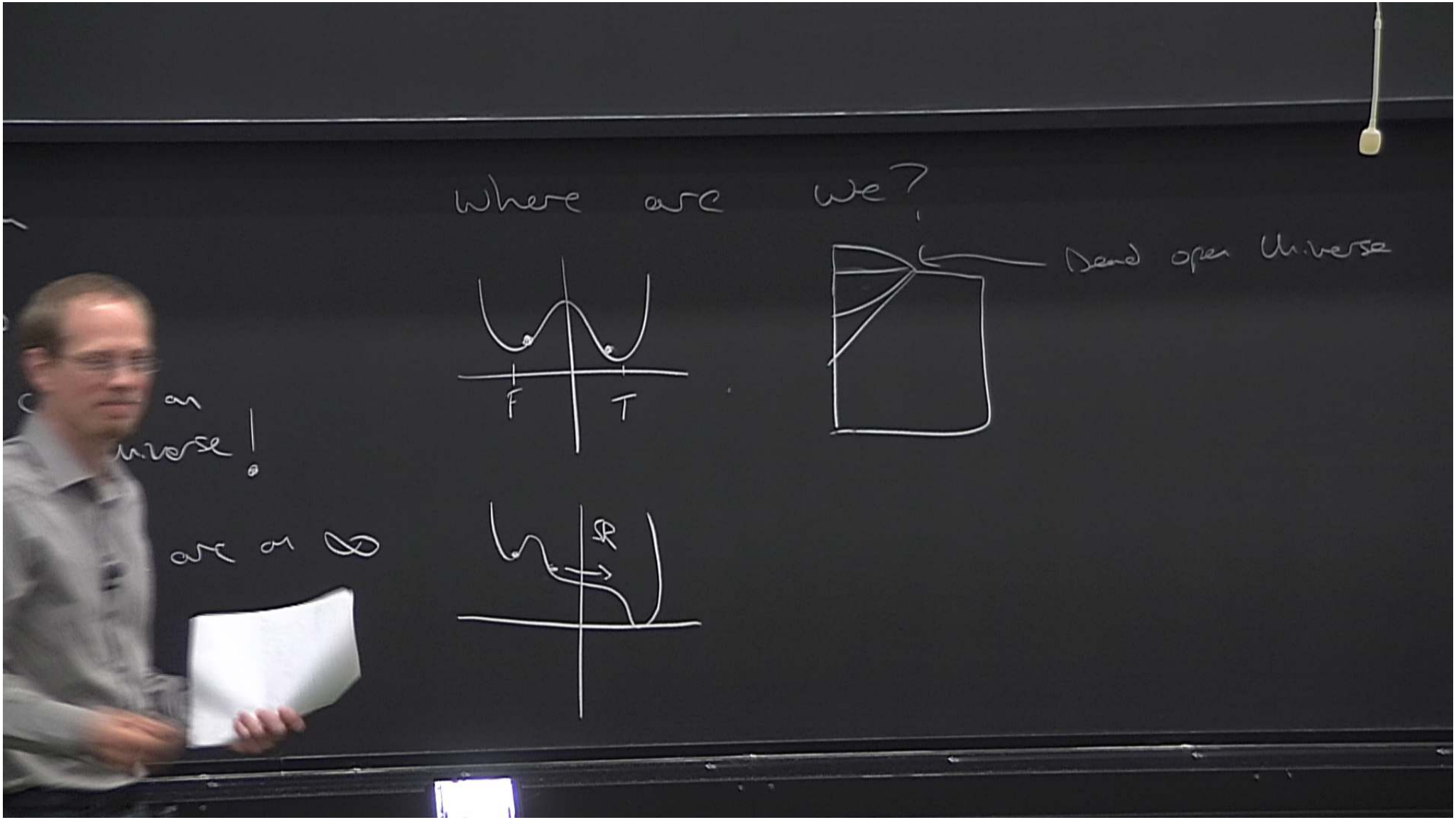
Penrose diagram

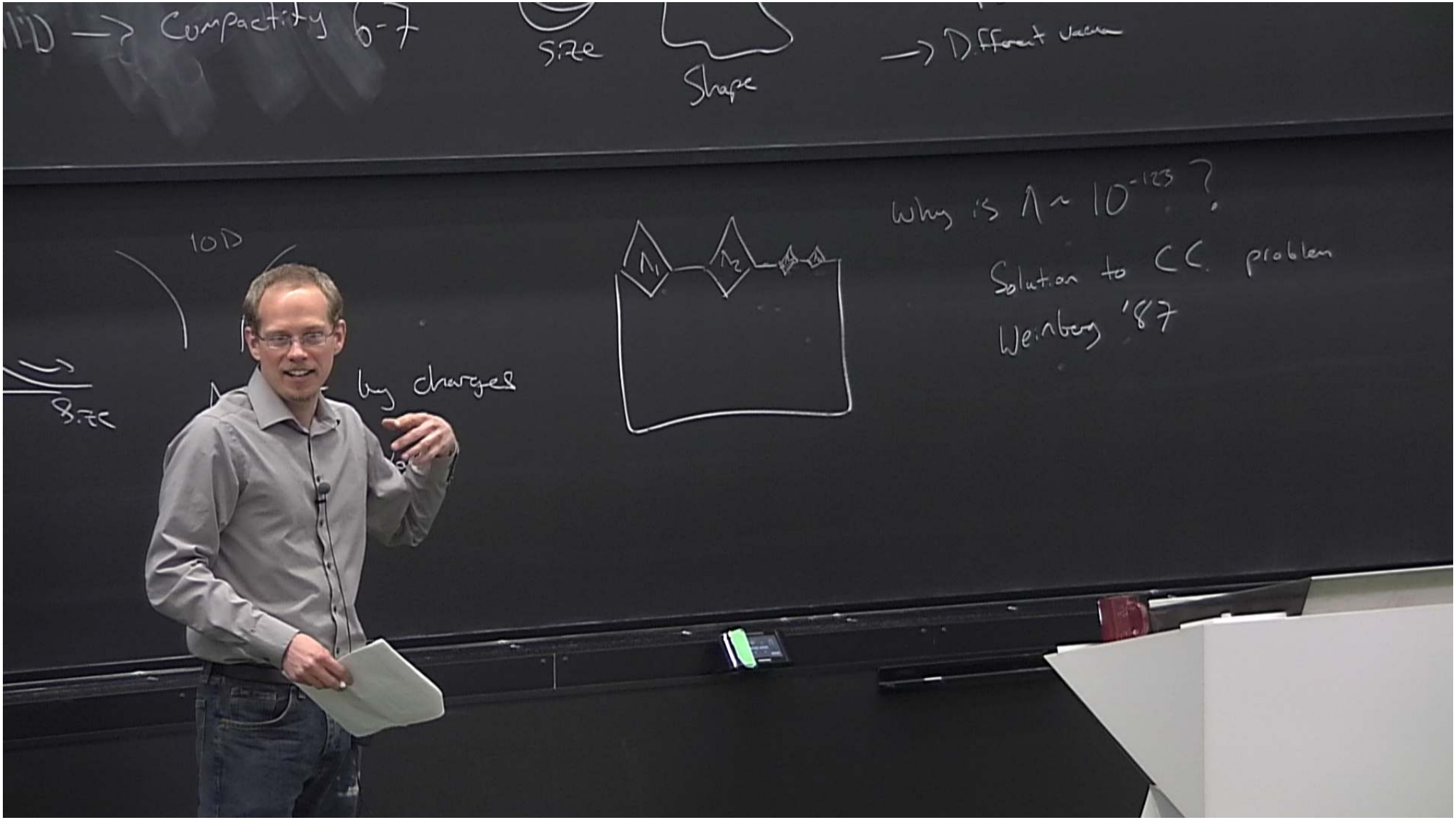


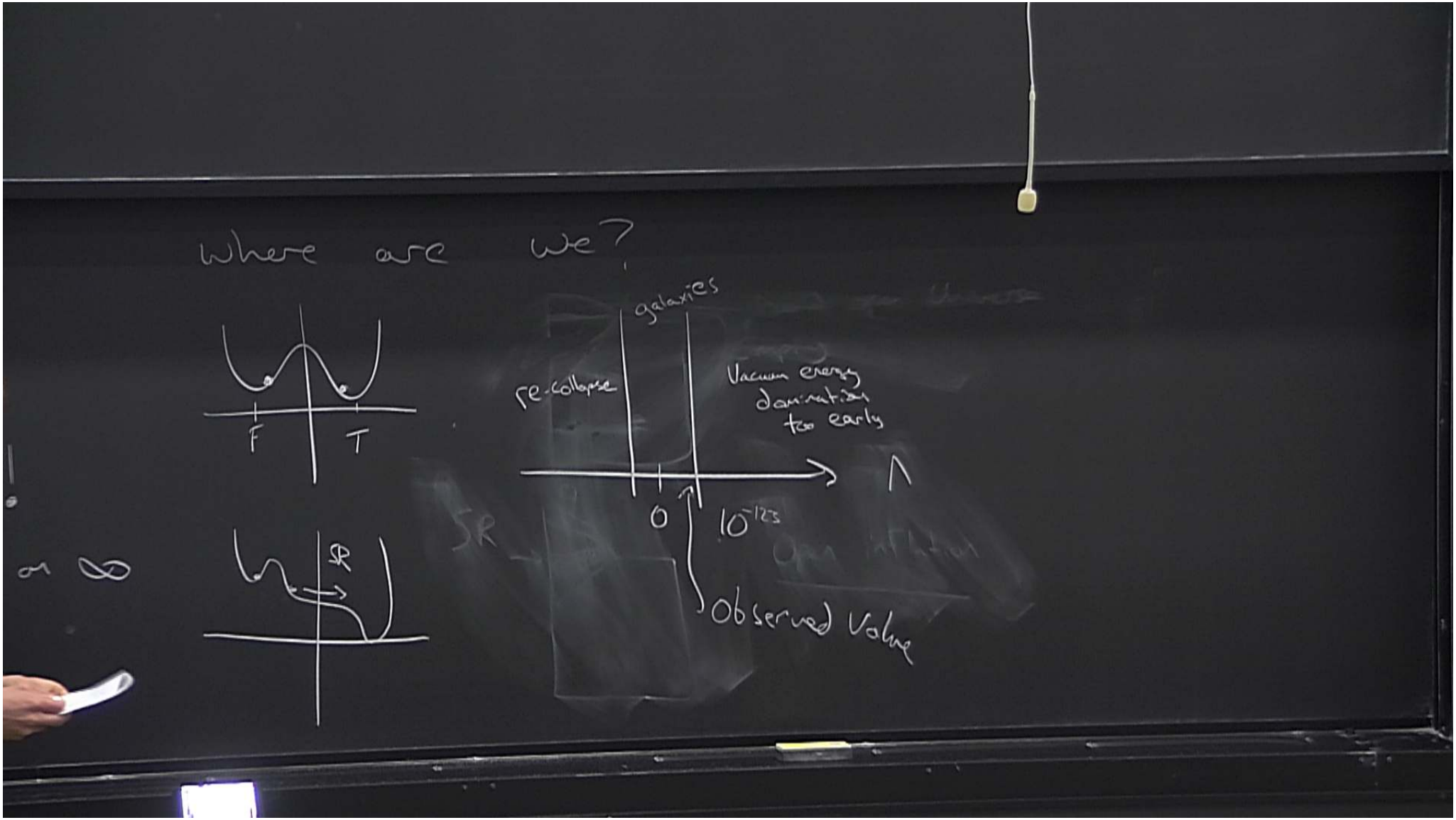
Fractally distributed

* Each bubble contains an ∞ open FRW universe!

* As $t \to \infty$, there are an ∞ # of bubbles!


Where are we?
Dead open Universe
F
T
Universe!
are or ∞


Compactify
Size
Shape
Why is Λ ~ 10^-123 ?
Solution to CC problem
Weinberg '87


Where are we?
F
T
R
galaxies
re-collapse
Vacuum energy dominates too early
Λ
0
10^{-123}
Observed Value

$V_c \propto E \times p \{ (3H_E - K) \}$

Quantum Cosmology

What is the quantum state of whole universe?

Restrict discussion to Universes described by

$$ds^2 = -N^2(t)dt + a^2(t) d\Omega^2$$

$V_c \propto \exp[(3H_E^{-2}\Lambda)]$

Quantum Cosmology

What is the quantum state of whole universe?

Restrict discussion to Universes described by

$$ds^2 = -N^2(t)dt + a^2(t)d\Omega_3^2$$

$V_c \propto Exp[3H_E^{-2}]$

Quantum Cosmology

What is the quantum state of whole universe?

Restrict discussion to Universes described by

$$ds^2 = -N^2(t)dt + a^2(t)d\Sigma_3^2 \sim \text{Minisuperspace}$$

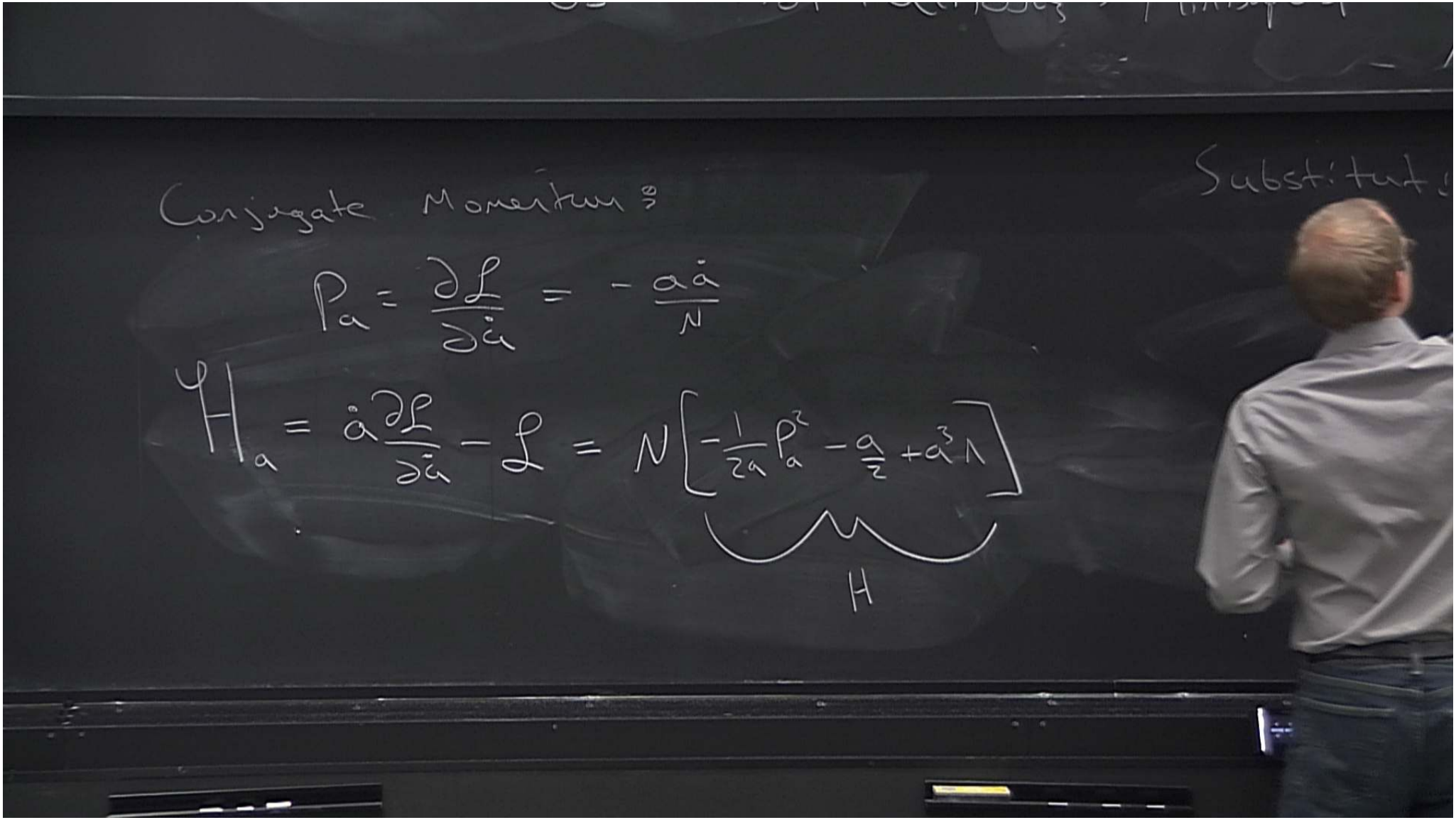
$$S = \frac{M_p^2}{2}\int d^4x\sqrt{-g}\left[R - 2\Lambda\right]$$

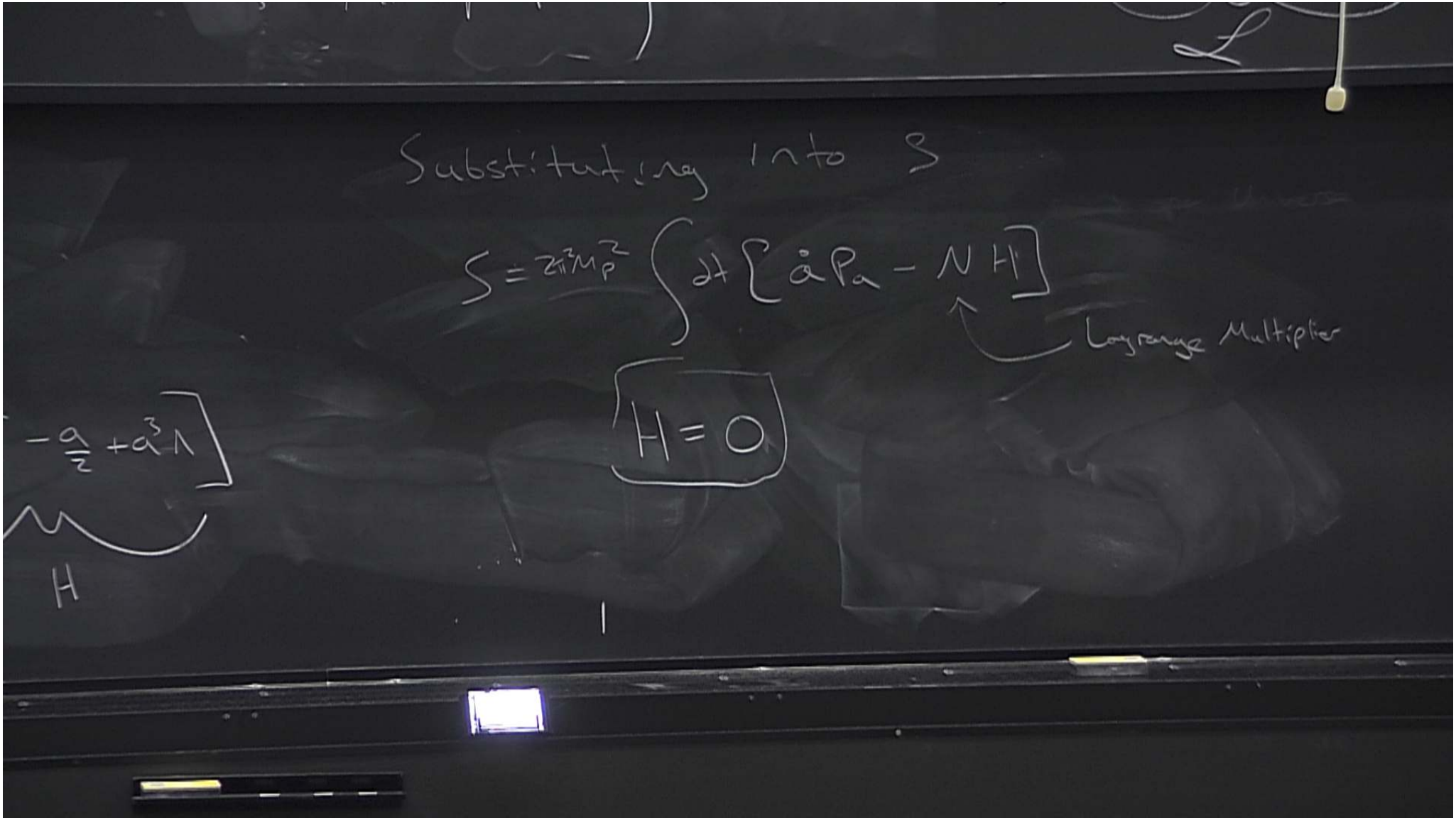
$$\sqrt{-g} = N a^3 \quad , \quad R = \frac{1}{a^2 N^3}\left[N^3 - a\dot{a}N + N(\dot{a}^2 + \ddot{a}a)\right]$$

Substitute + integrate by parts to remove $\ddot{a}$

$$S = 2\pi^2 M_p^2\int dt\left[\frac{Na}{2} - \frac{a\dot{a}^2}{2N} - Na^3\Lambda\right]$$

space

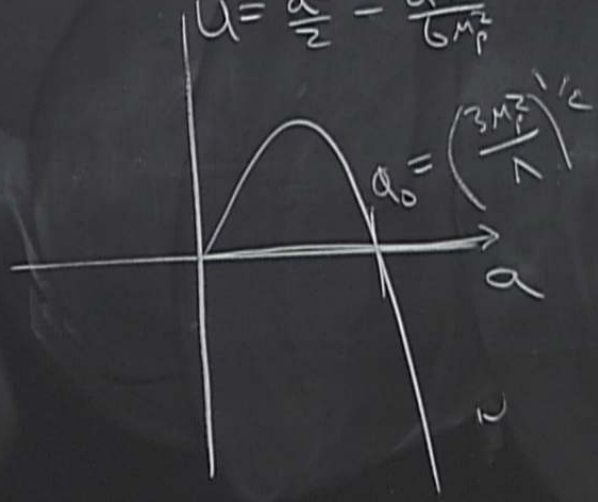

Conjugate Momentum:
$p_a = \frac{\partial \mathcal{L}}{\partial \dot{a}} = -\frac{a\dot{a}}{N}$
$\mathcal{H}_a = \dot{a}\frac{\partial \mathcal{L}}{\partial \dot{a}} - \mathcal{L} = N\left[-\frac{1}{2a}p_a^2 - \frac{a}{2} + a^3\Lambda\right]$
H


Substituting into S
$S = 2\pi^2 M_p^2 \int dt \left[\dot{a} P_a - N H \right]$
Lagrange Multiplier
$H = 0$
$-\frac{a}{2} + a^3 \Lambda$
H

$$\Rightarrow \frac{p_a^2}{2} + \frac{a^2}{2} - a^4 \frac{\Lambda}{6M_p^2} = 0$$

(Friedmann equation)

$$u = \frac{a^2}{2} - \frac{a^4 \Lambda}{6M_p^2}$$



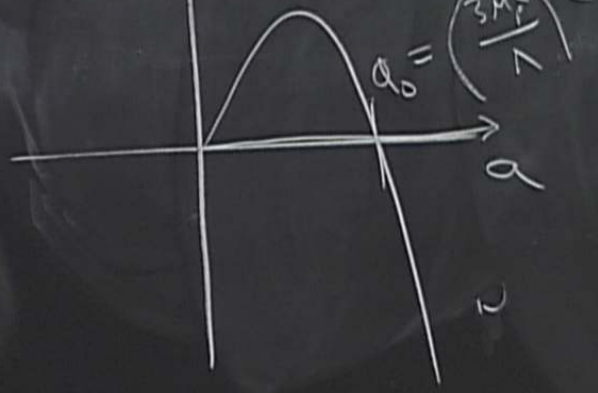
$$\hat{H}\,\psi(a) = 0$$

$$\hookrightarrow p_a \to i\frac{\partial}{\partial a}$$

$$\Rightarrow \frac{p_a^2}{2} + \frac{a^2}{2} - a^4 \frac{\Lambda}{6M_p^2} = 0$$

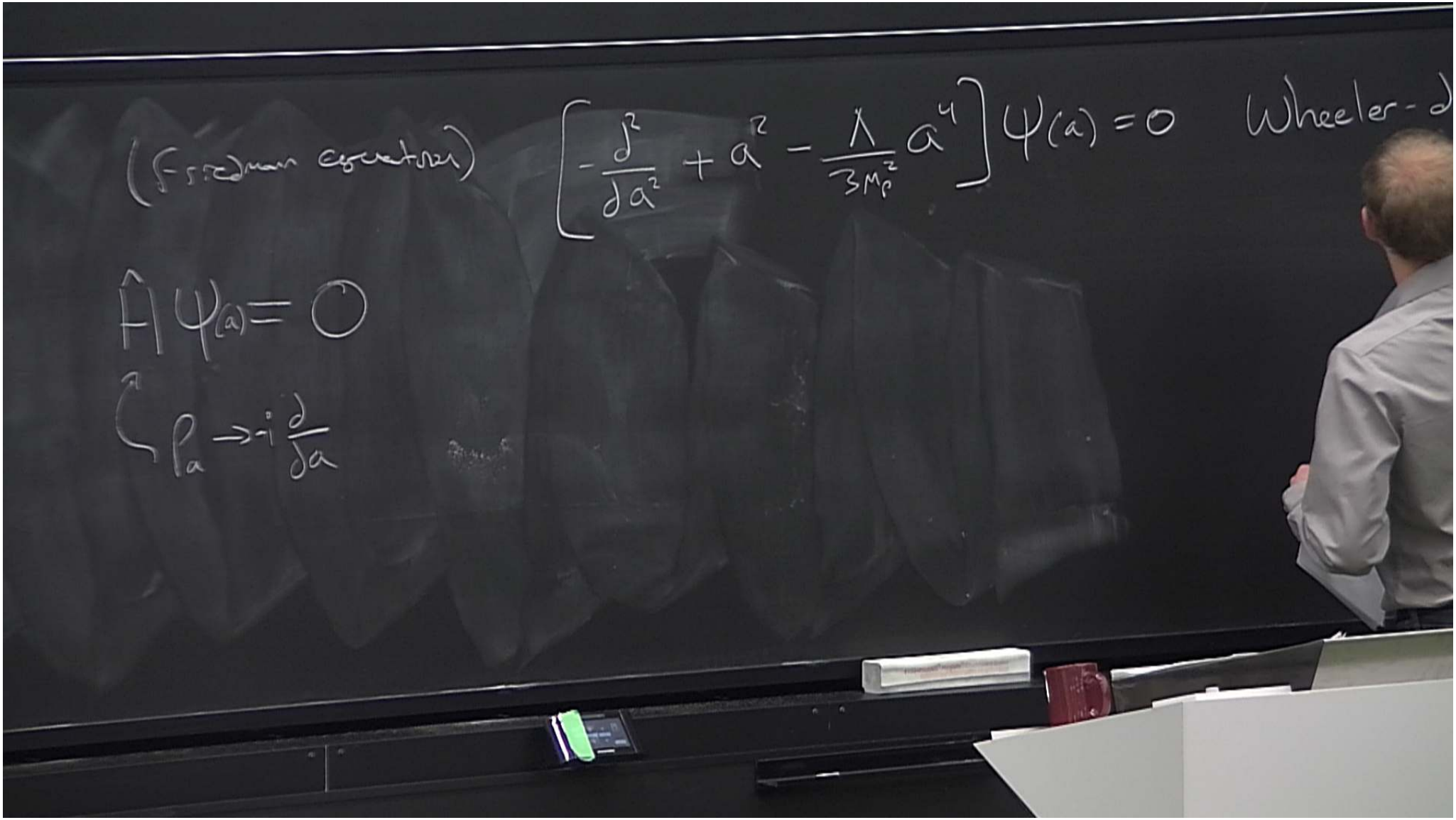
(Friedmann equation)

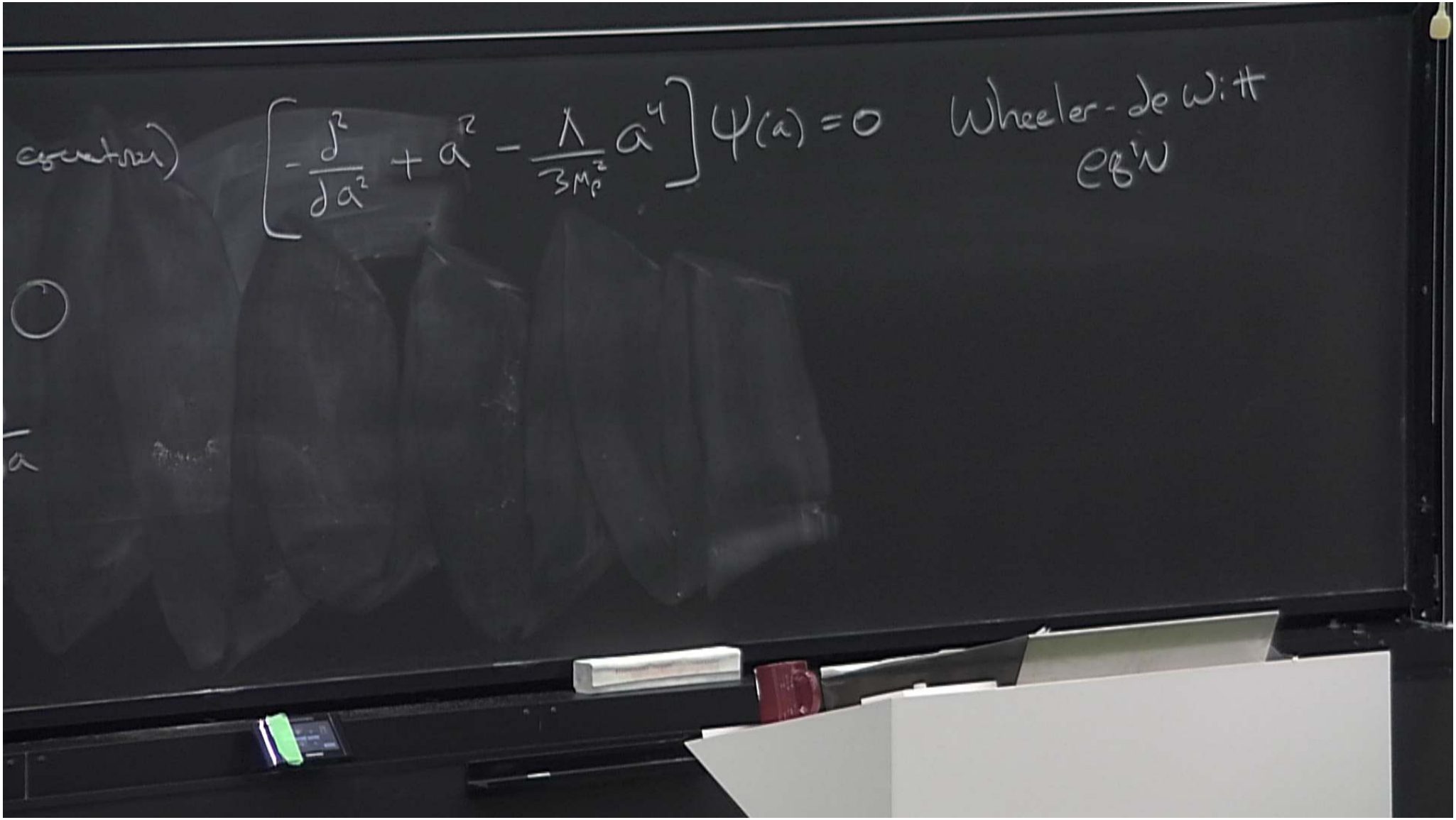
$$U = \frac{a^2}{2} - \frac{a^4 \Lambda}{6M_p^2}$$

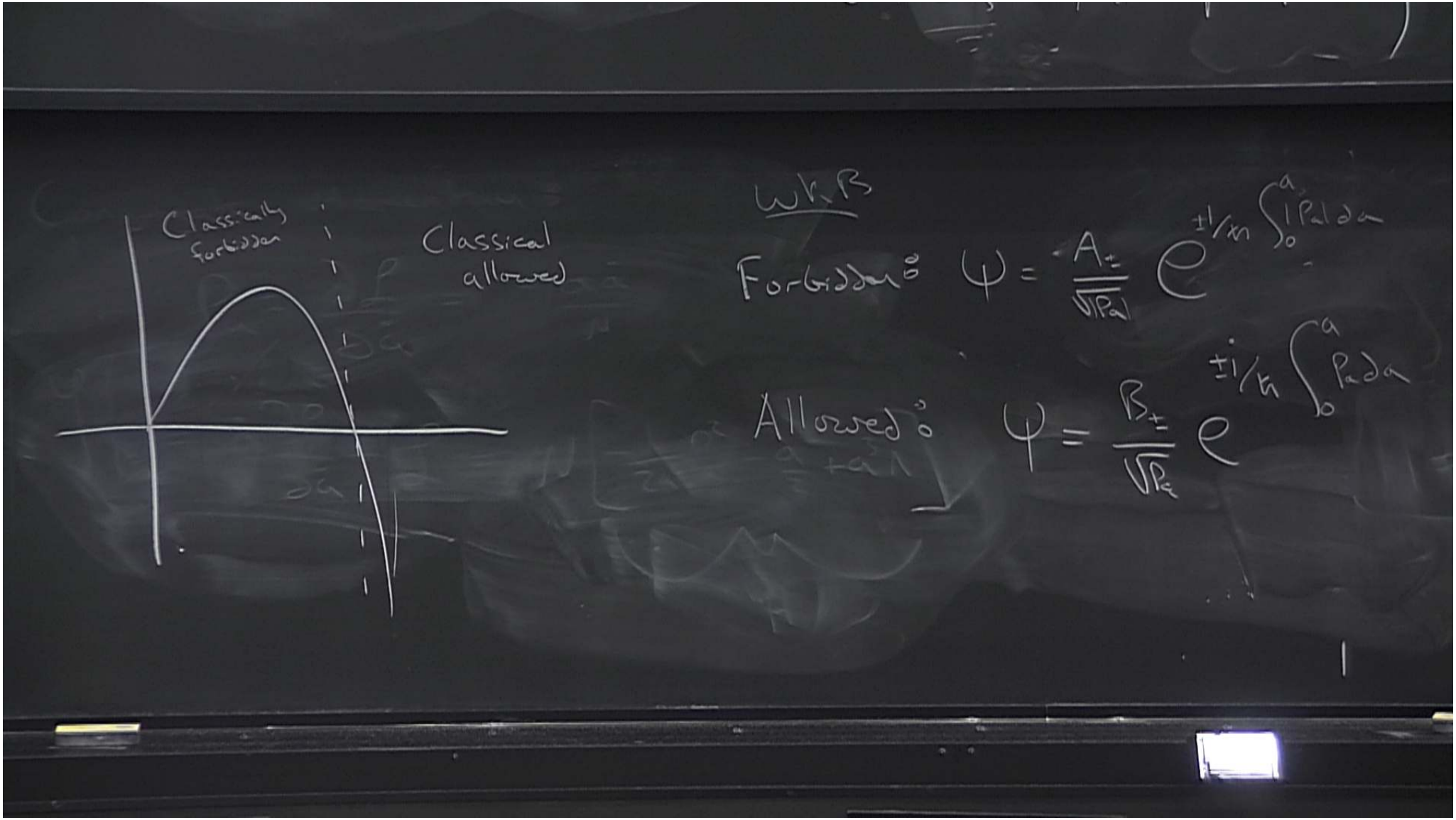


$$\hat{H}\,\psi(a) = 0$$

$$\hookrightarrow p_a \to -i\frac{\partial}{\partial a}$$


(Friedmann equation)
$\left[-\frac{\partial^2}{\partial a^2} + a^2 - \frac{\Lambda}{3M_p^2} a^4\right] \psi(a) = 0$
Wheeler-D
$\hat{H}\psi(a) = 0$
$\hookrightarrow p_a \to -i\frac{\partial}{\partial a}$

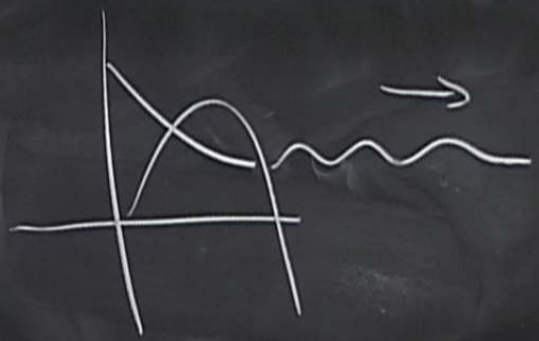

$\left[-\frac{d^2}{da^2} + a^2 - \frac{\Lambda}{3M_p^2} a^4\right] \psi(a) = 0$
Wheeler-deWitt eq'n


Classically forbidden
Classical allowed
WKB
Forbidden: $\psi = \frac{A_\pm}{\sqrt{|p|}} e^{\pm \frac{1}{\hbar}\int_a^x |p|\,dx}$
Allowed: $\psi = \frac{B_\pm}{\sqrt{p}} e^{\pm \frac{i}{\hbar}\int_a^x p\,dx}$

Wave fn of Universe

choosing $A_\pm$, $B_\pm$

Tunneling WF

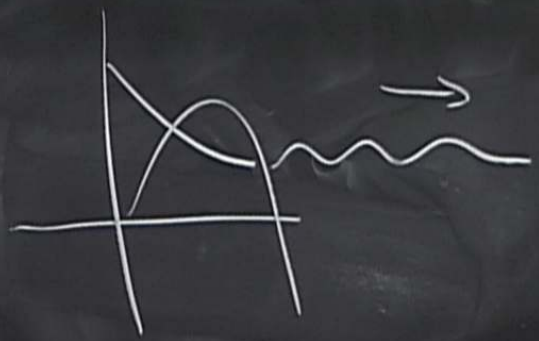


$$\frac{|\psi(a_0)|}{|\psi(0)|} = e^{-M_P^2/\Lambda}$$

Wave fn of Universe

choosing $A_\pm$, $B_\pm$

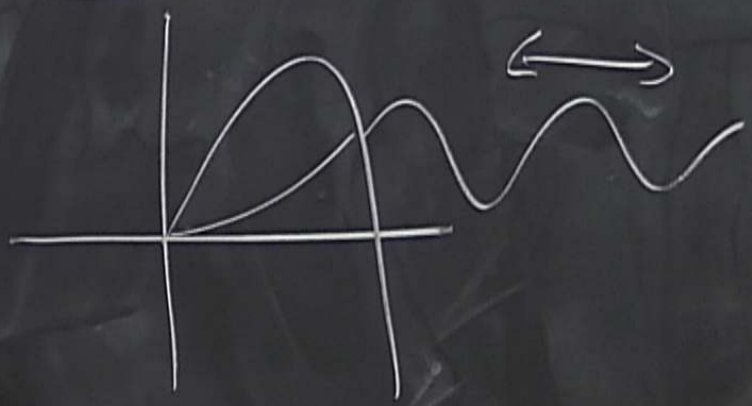
Tunneling WF



$$\frac{|\psi(a_0)|}{|\psi(0)|} = e^{-M_P^2/\Lambda}$$

Prefers large Λ

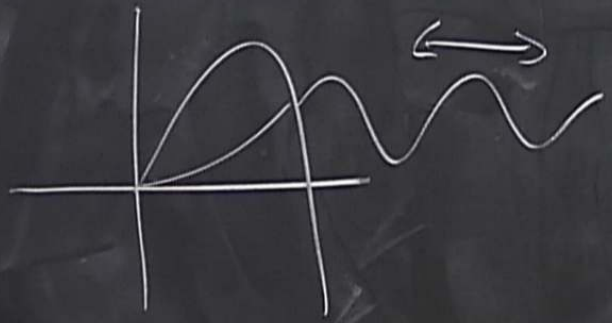
Hartle - Hawking WF



$$\frac{|\psi(a)|}{|\psi(0)|} =$$

$$\left[-\frac{d^2}{da^2} \right.$$

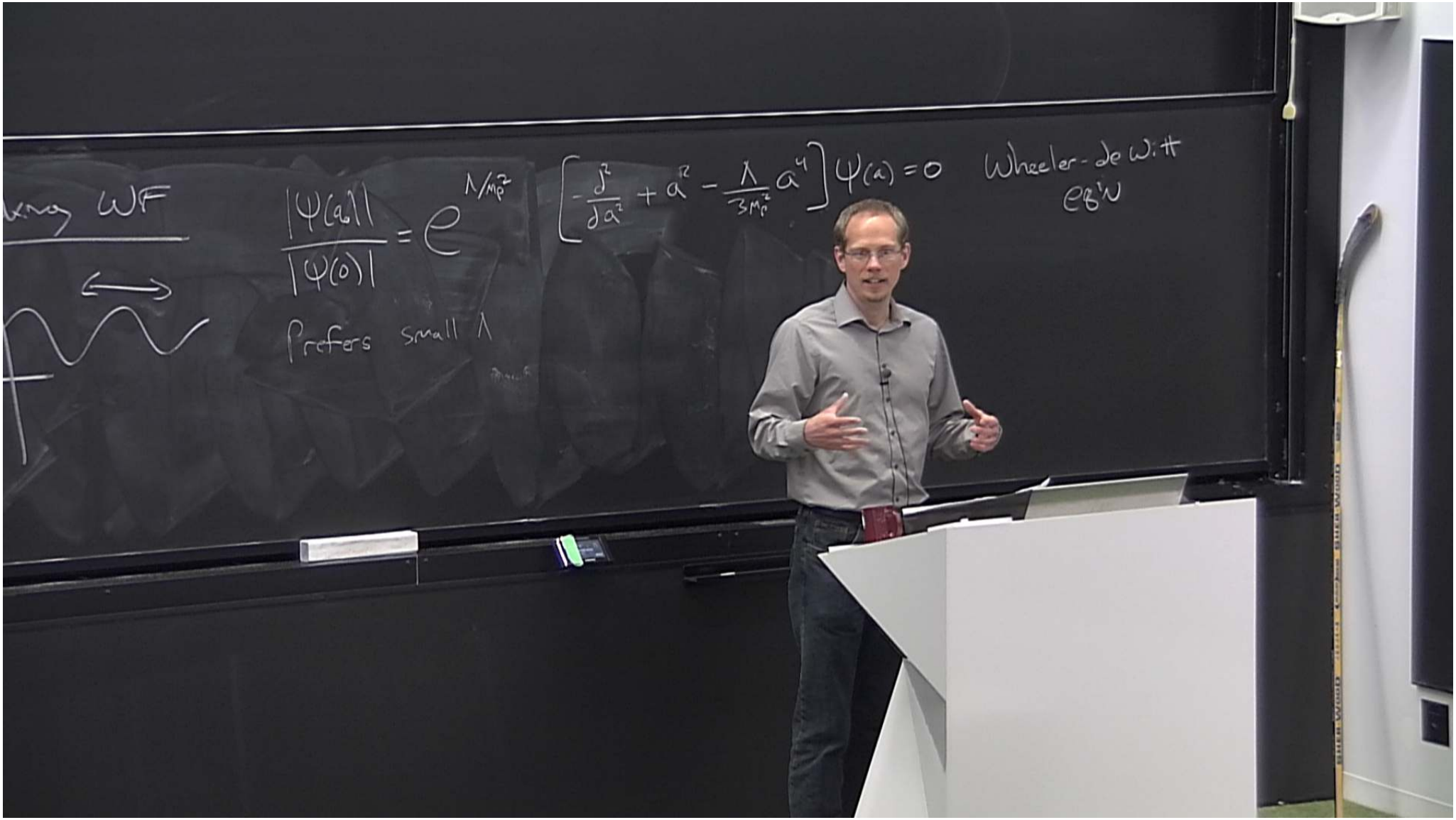
Hartle - Hawking WF



$$\frac{|\psi(a)|}{|\psi(0)|} = e^{\Lambda/M_P^2}$$

Prefers small Λ

$$\left[-\frac{\partial^2}{\partial a^2} + a^2 - \frac{\Lambda}{3M_P^2}a^4\right]\psi(a) = 0$$


WF
$\frac{|\Psi(a)|}{|\Psi(0)|} = e^{\Lambda/M_P^2}$
Prefers small Λ
$\left[-\frac{\partial^2}{\partial a^2} + a^2 - \frac{\Lambda}{3M_P^2} a^4\right] \Psi(a) = 0$
Wheeler-DeWitt
eqn