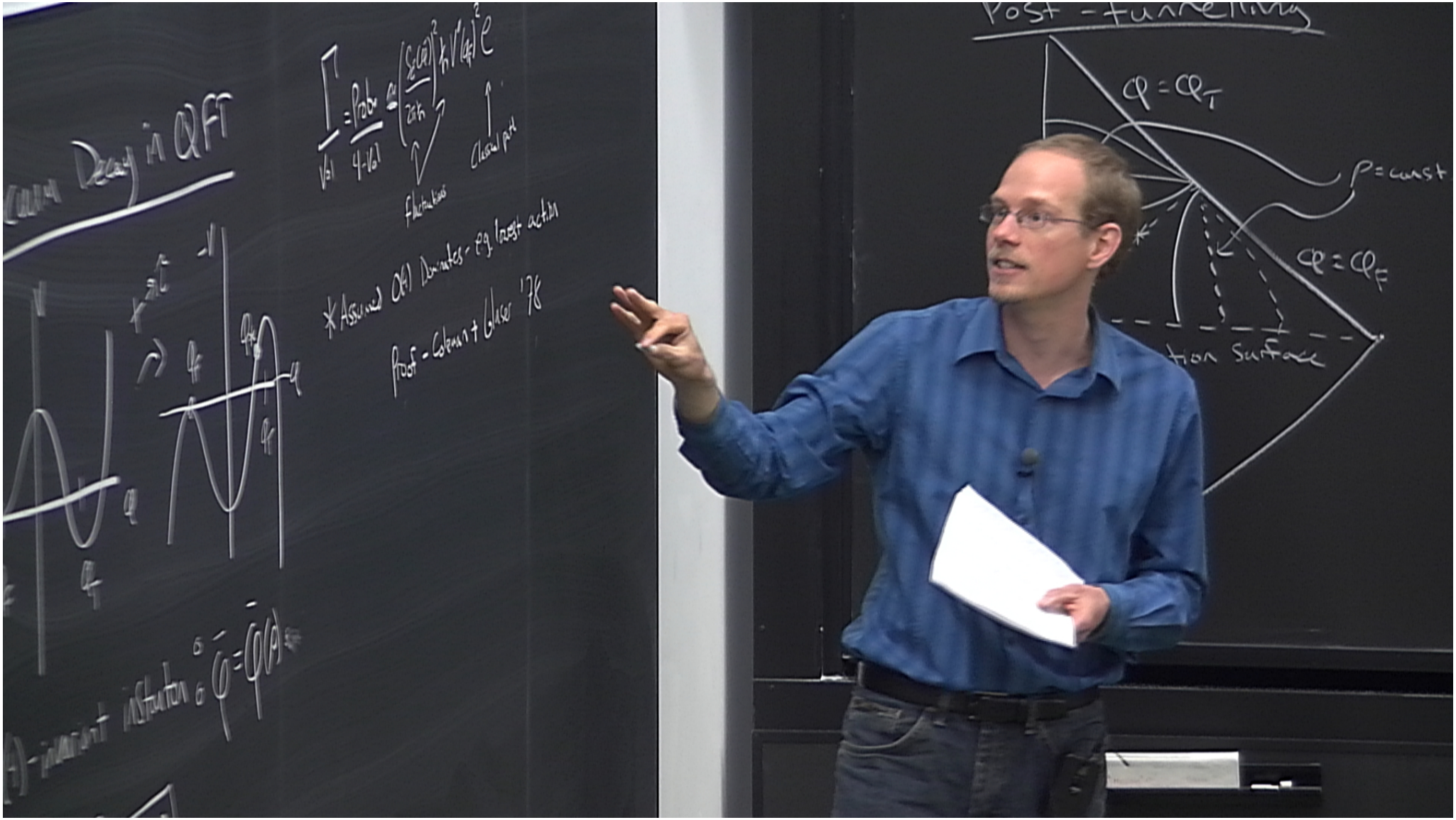
Title: Explorations in Cosmology - Lecture 13

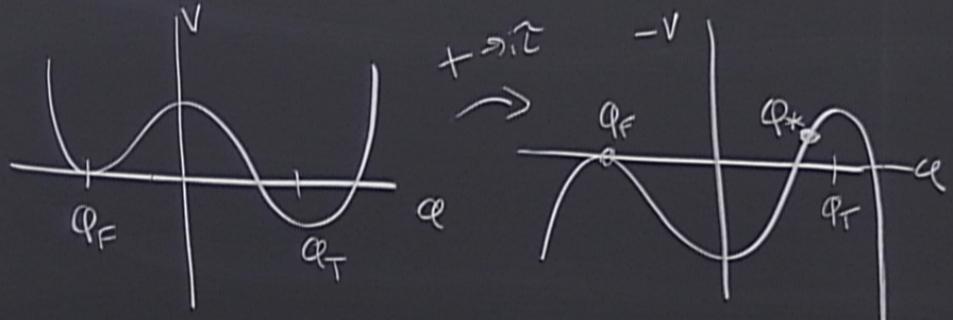
Date: Mar 28, 2012 10:15 AM

URL: http://pirsa.org/12030039

Abstract:


Decay in QFT
Prob
Vol
Vol
fluctuations
Classical part
Assumed O(4) Dominates - eg Largest action
Proof - Coleman + Glaser '78
Post - tunneling
φ = φ_T
ρ = const.
φ = φ_F
tion Surface

Vacuum Decay in QFT



$O(4)$ - invariant instanton : $\bar{\varphi} = \bar{\varphi}(\rho)$

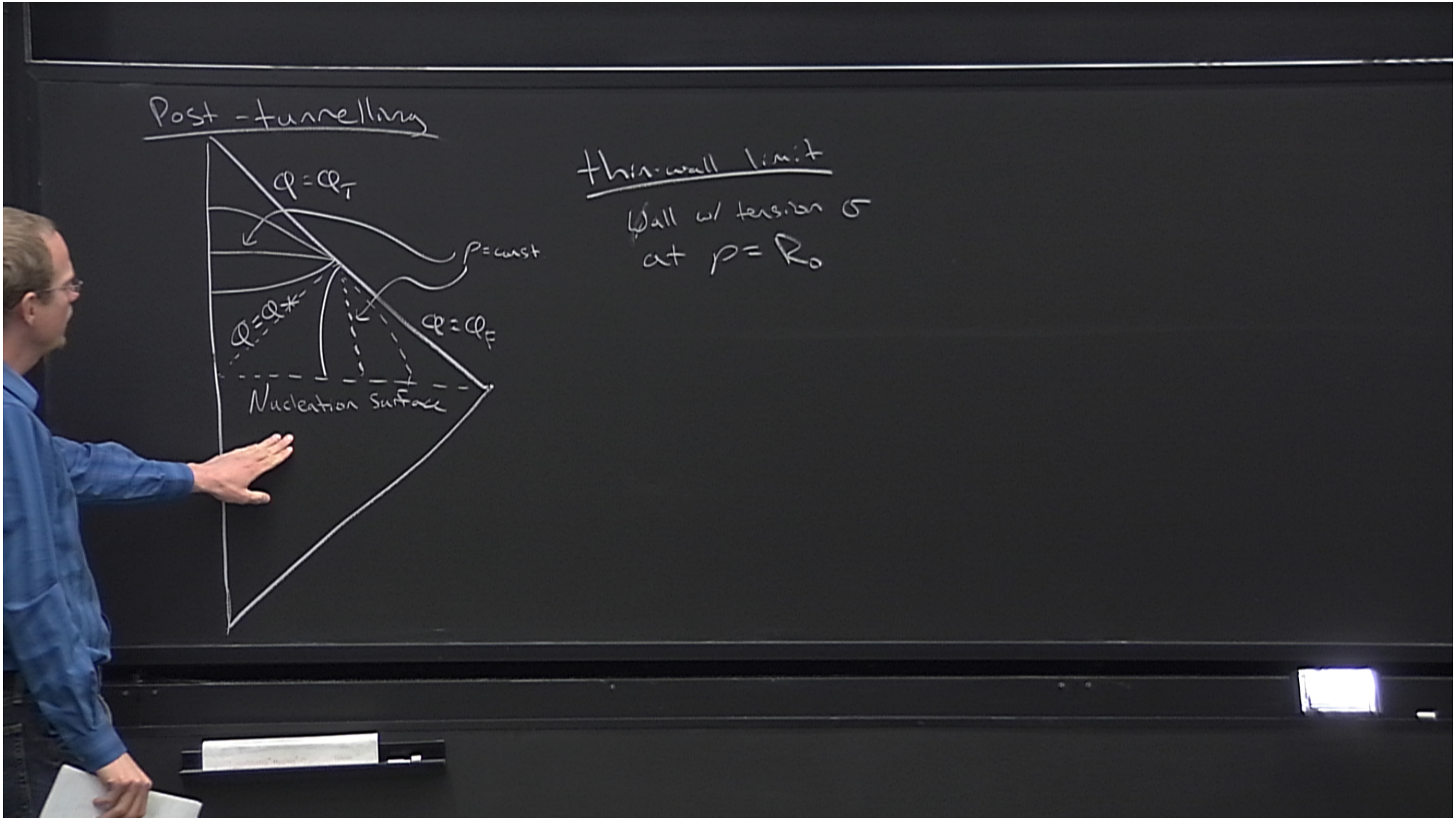
$\rho = \sqrt{\tau^2 + \vec{x}^2}$

$$\frac{\Gamma}{Vol} = \frac{\text{Prob}}{4\text{-Vol}} \simeq \left(\frac{S_E(\bar{\varphi})}{2\pi\hbar}\right)^2 \hbar\, V''(\varphi_F)^2\, e^{-S_E(\bar{\varphi})/\hbar}$$

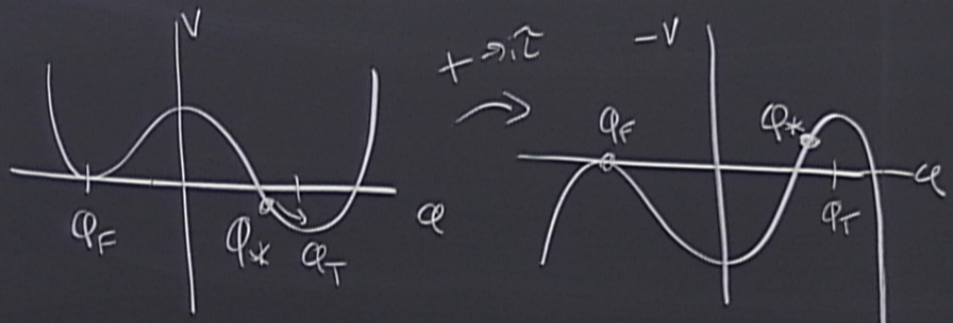
fluctuations — Classical path

* Assumed $O(4)$ Dominates - e.g. lowest action

Proof - Coleman + Glaser '78


Post-tunnelling
φ = φ_T
ρ = const
φ = φ*
φ = φ_F
Nucleation Surface
thin-wall limit
Wall w/ tension σ
at ρ = R_0

Vacuum Decay in QFT



$O(4)$ - invariant instanton : $\bar{\varphi} = \bar{\varphi}(\rho)$

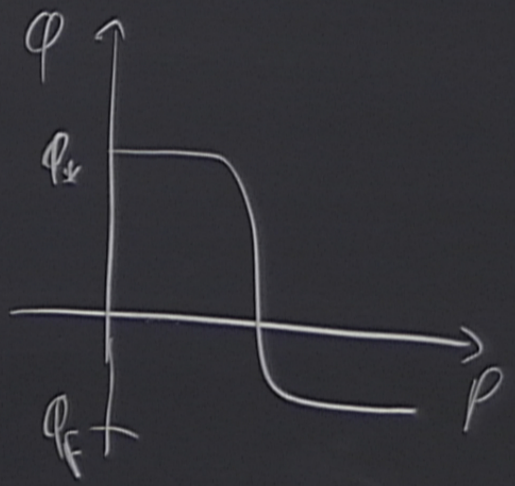
$\rho = \sqrt{\tilde{\tau}^2 + \vec{x}^2}$

$$\frac{\Gamma}{Vol} = \frac{\text{Prob}}{4\text{-Vol}} \simeq \left(\frac{S_E(\bar{\varphi})}{2\pi\hbar}\right)^2 \hbar\, V''(\varphi_F)^2\, e^{-S_E(\bar{\varphi})/\hbar}$$

fluctuations — Classical path

* Assumed $O(4)$ Dominates - e.g. lowest action

Proof - Coleman + Glaser '78



Back to cosmology – Vacuum decay w/ gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{M_P^2}{2} R \right]$$

Path integral

$$\langle \text{Final} | e^{iHT/\hbar} | \text{initial} \rangle = \int [Dg_{\mu\nu}][D\varphi] e^{iS/\hbar}$$

Hard to define

$\varphi(\vec{x})$

$g_{\mu\nu}(\vec{x})$

Back to cosmology – Vacuum decay w/ gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) - \frac{M_P^2}{2} R \right]$$

look for saddle points

$\bar{g}_{\mu\nu}, \bar{\varphi}$

then look at fluctuations

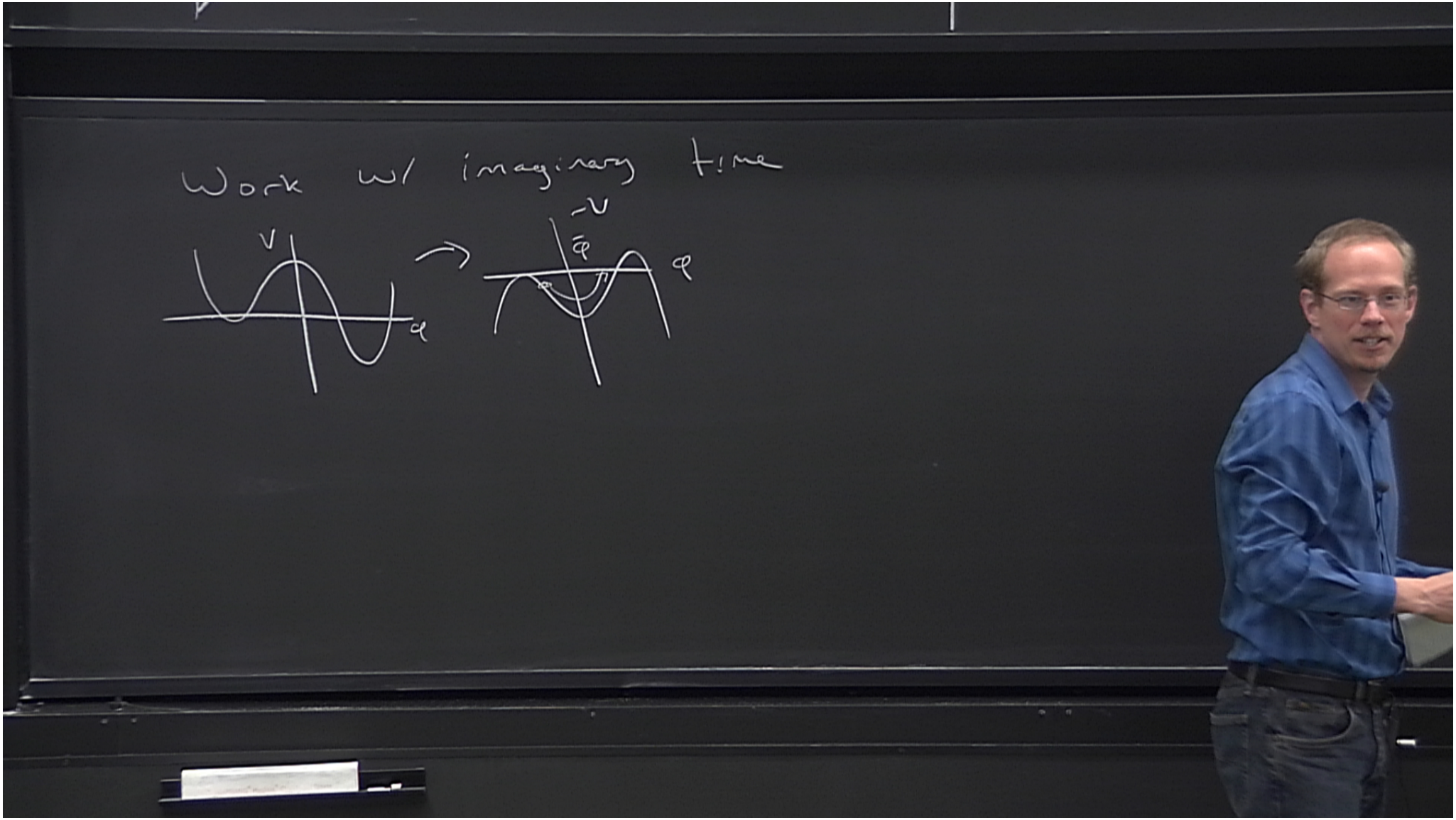
Path integral

Hard to define

$$\langle \text{final} | e^{-iHT/\hbar} | \text{initial} \rangle = \int [Dg_{\mu\nu}][D\varphi] e^{iS/\hbar}$$

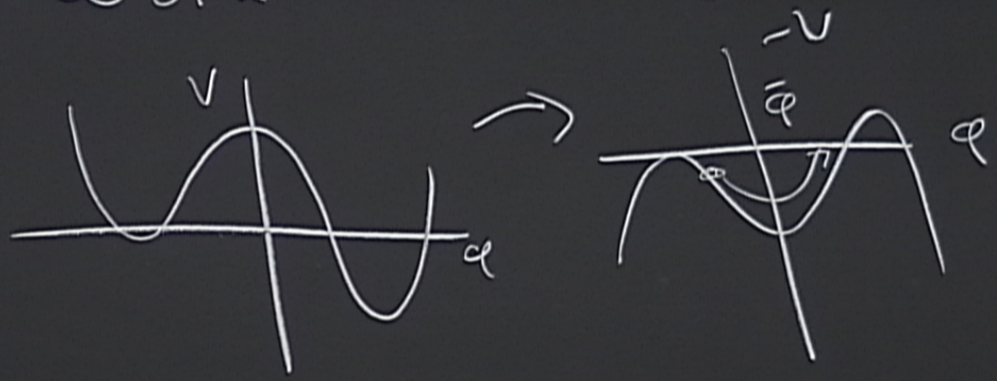
$\varphi(\vec{x})$

$g_{\mu\nu}(\vec{x})$


Work w/ imaginary time
V
q
-V
q
q

Work w/ imaginary time

$S_E =$



Assume $O(4)$ invariance

$$ds_E^2 = d\xi^2 + \rho^2(\xi)\, d\Omega_3^2$$

$$S_E = 2\pi^2 \int d\xi \left[\rho^3 \left(\frac{\dot{\varphi}^2}{2} + V(\varphi) \right) + 3M_P^2 \underbrace{\left(\rho^2 \ddot{\rho} + \rho \dot{\rho}^2 - \rho \right)}_{R} \right]$$

$2\pi^2 = \Omega_3$

$\dot{\varphi} \equiv \frac{d\varphi}{d\xi}$

Vary the action

$$\ddot{\varphi} + \frac{3\dot{\rho}}{\rho} \dot{\varphi} = V'$$

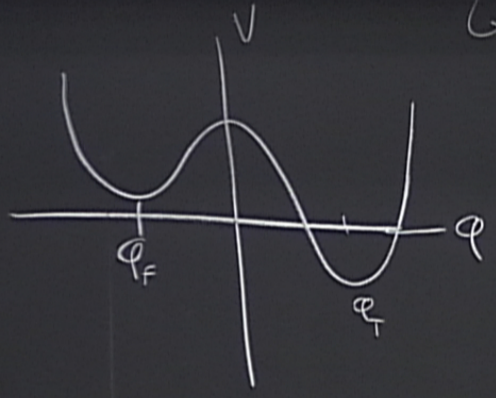
$$S_E = 2\pi^2 \int d\xi \left[\rho^3 \left(\frac{\dot{\varphi}^2}{2} + V(\varphi) \right) + 3M_p^2 \underbrace{\left(\rho^2 \ddot{\rho} + \rho \dot{\rho}^2 - \rho \right)}_{R} \right]$$

$2\pi^2 = \Omega_3$

$\dot{\varphi} \equiv \frac{d\varphi}{d\xi}$

Vary the action

$$\ddot{\varphi} + \frac{3\dot{\rho}}{\rho}\dot{\varphi} = V' \,, \qquad \dot{\rho}^2 = 1 + \frac{\rho^2}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 - V \right) \,, \qquad \ddot{\rho} = -\frac{\rho}{3M_p^2}\left(\dot{\varphi}^2 + V \right)$$



Gravity - Zero of Potential

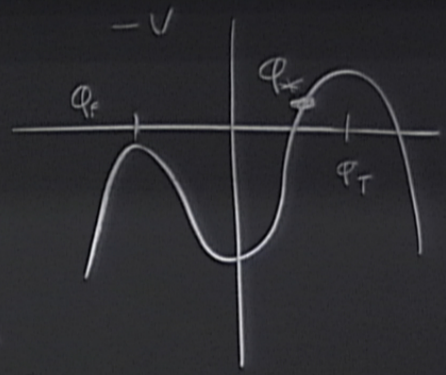
Matters

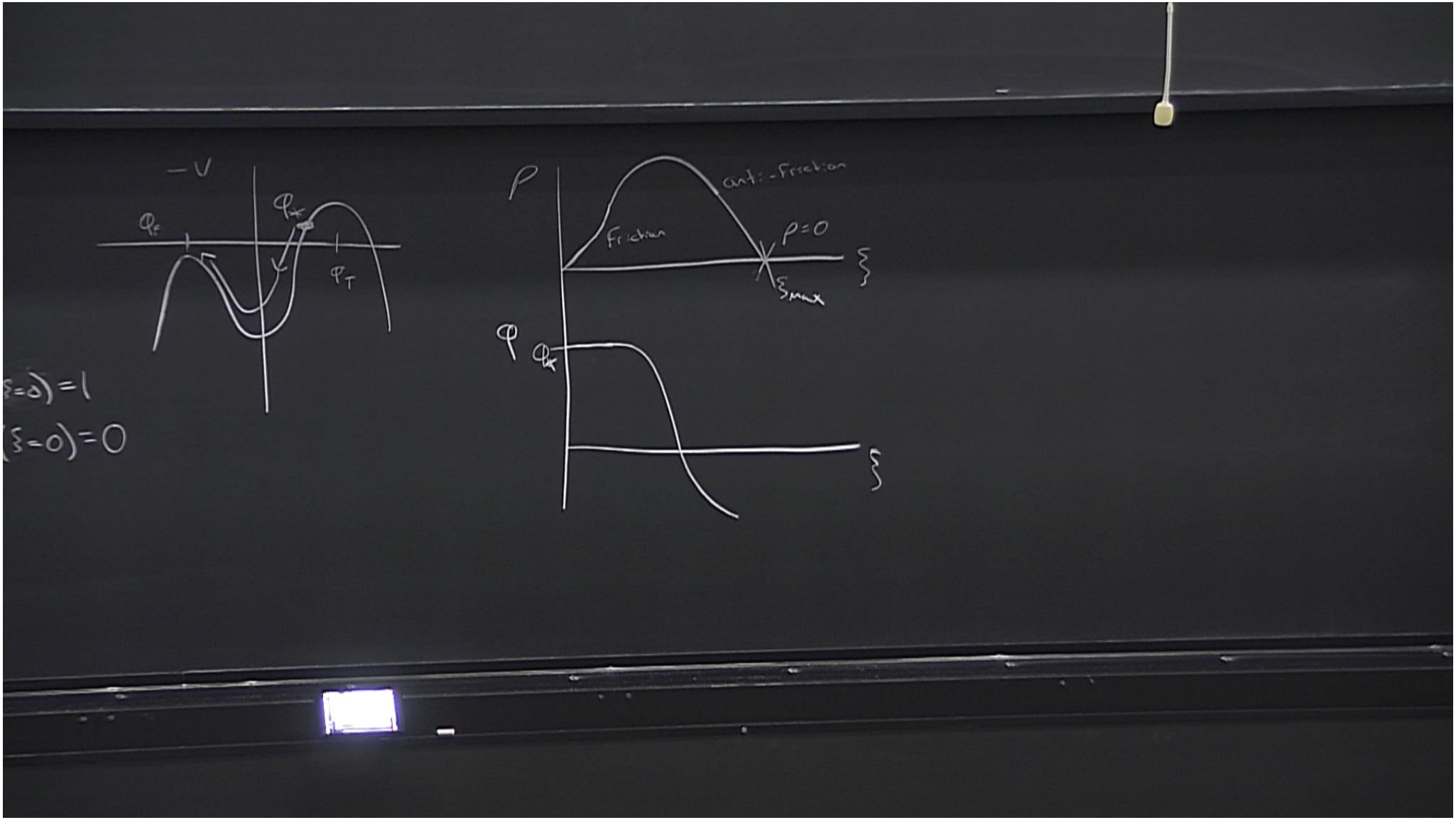
Find $\bar{\varphi}$ and $\bar{\rho}$

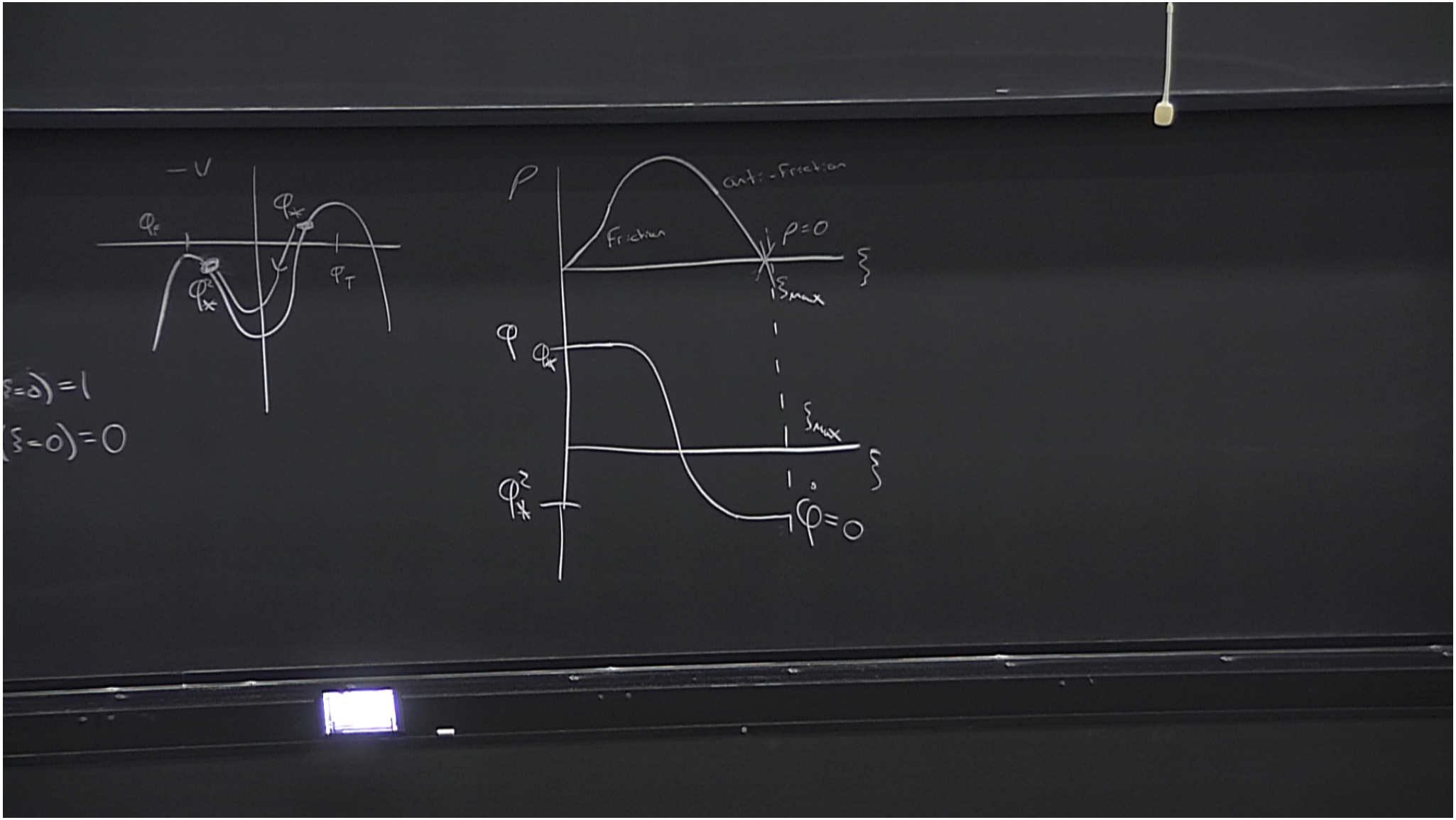
At $\xi = 0$, $\rho = 0$ $\Rightarrow \dot{\rho}(\xi=0) = 1$

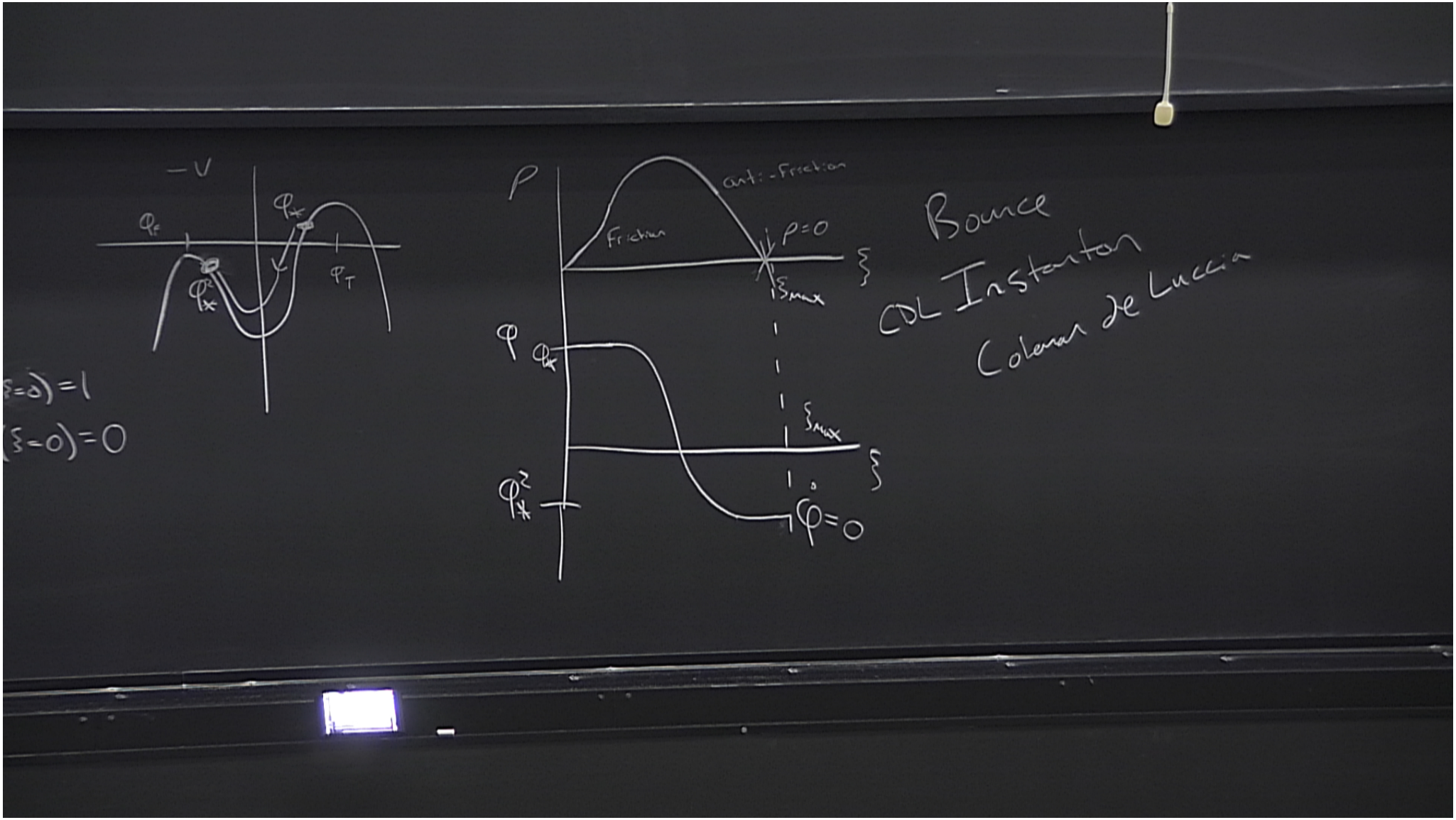
$\Rightarrow \dot{\varphi}(\xi=0) = 0$

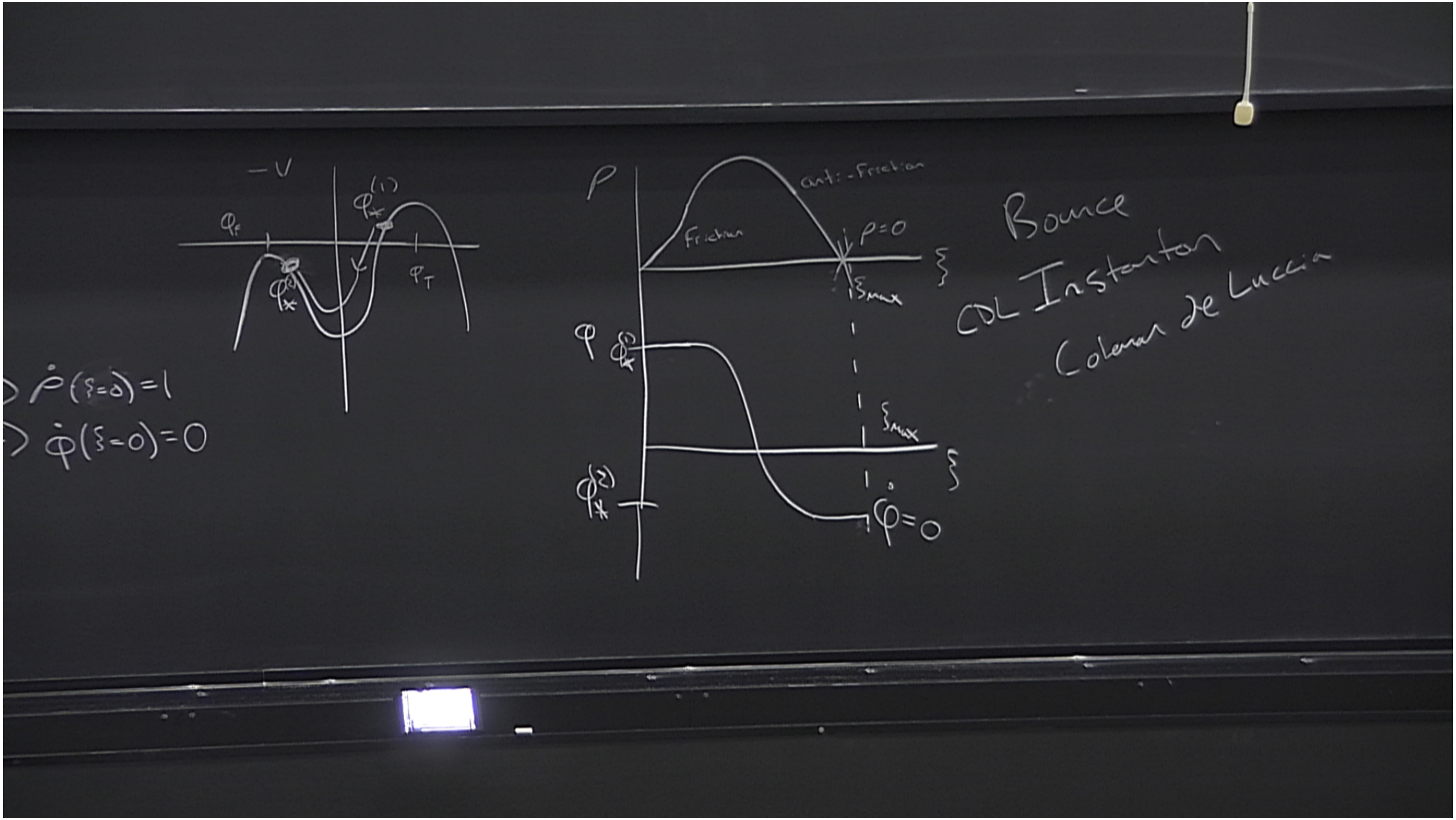
Need $\varphi_* = \varphi(\xi=0)$



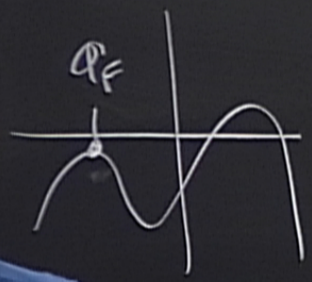

−V
φ_F
φ_{FT}
φ_T
$\xi=0)=1$
$\xi=0)=0$
P
anti-Friction
Friction
P=0
ξ_{max}
ξ
φ
φ_{FT}
ξ


−V
φ_F
φ_{1*}
φ^2_{1*}
φ_T
$\xi=0)=1$
$\xi=0)=0$
P
anti-Friction
Friction
P=0
ξ_{max}
φ
φ_{1*}
ξ_{max}
ξ
φ^2_{1*}
$\varphi=0$

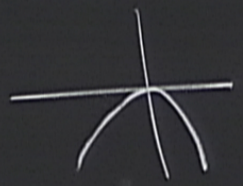

−V
φ_F
φ_{fx}
φ^2_{fx}
φ_T
ρ
Anti-Friction
Friction
ρ=0
ξ_{max}
φ
φ_{fx}
ξ_{max}
φ^2_{fx}
φ=0
ξ
Bounce
COL Instanton
Coleman de Luccia


−V
φ_F
$\varphi_*^{(1)}$
$\varphi_*^{(2)}$
φ_T
$\dot{\rho}(\xi=0)=1$
$\dot{\varphi}(\xi=0)=0$
ρ
Anti-Friction
Friction
ρ=0
ξ_{max}
φ
$\varphi_*^{(1)}$
ξ_{max}
$\varphi_*^{(2)}$
φ=0
ξ
Bounce
COL Instanton
Coleman de Luccia

Another Solution



No dynamics in field
but ρ will have dynamics

Analog of 

$$\ddot{\rho} = -\frac{\rho}{3M_p^2} V(q_F) \quad \Rightarrow \rho = \sqrt{\frac{3M_p^2}{V(q_F)}} \sin\left(\sqrt{\frac{V(q_F)}{3M_p^2}}\, \xi\right)$$

$$\rho(0) = 0, \quad \dot{\rho}(0) = 1$$

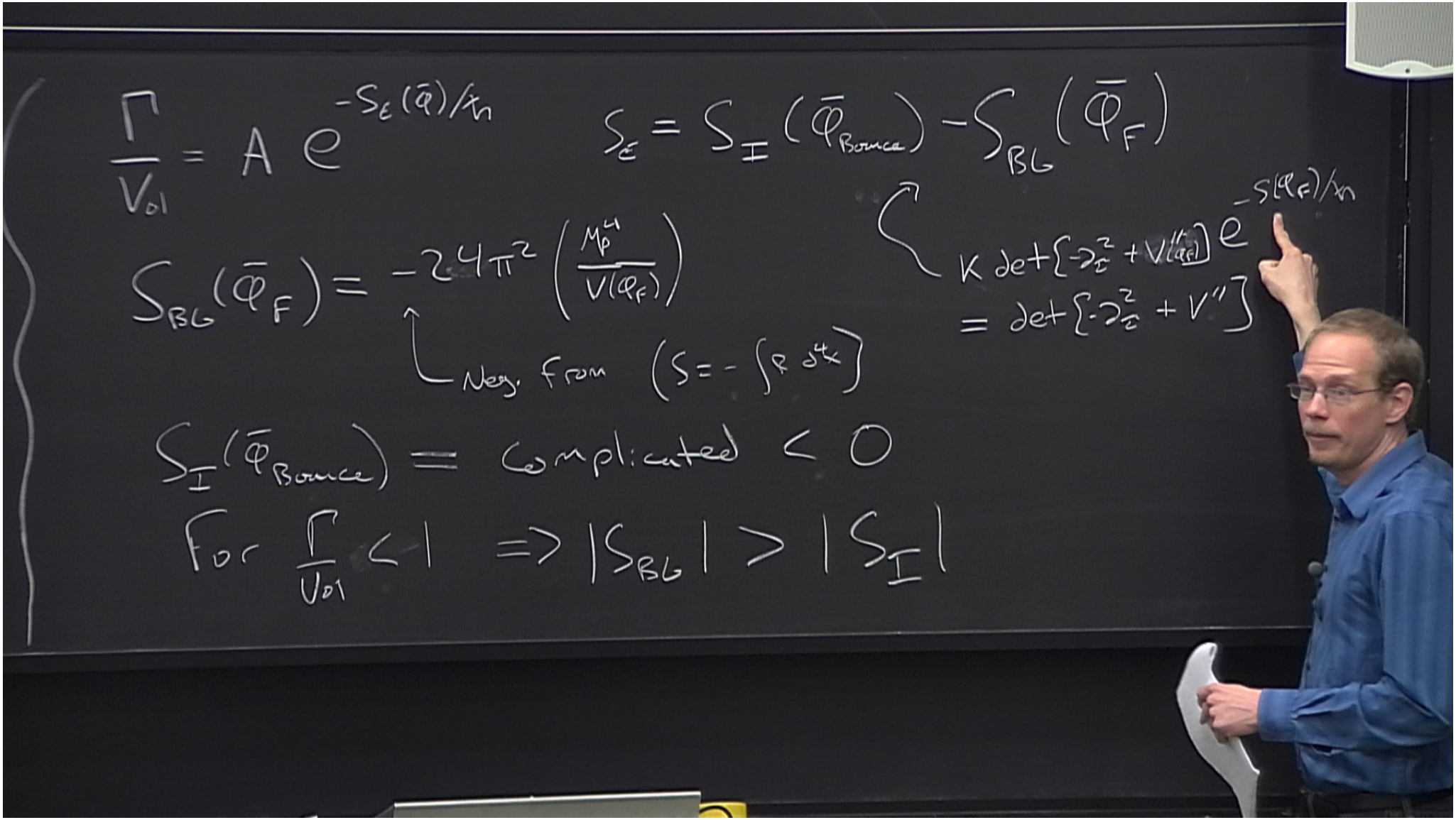
$$\frac{\Gamma}{V_{01}} = A\, e^{-S_E(\bar{\varphi})/\hbar} \qquad S_E = S_I(\bar{\varphi}_{Bounce}) - S_{BG}(\bar{\varphi}_F)$$

$$S_{BG}(\bar{\varphi}_F) = -24\pi^2 \left(\frac{M_p^4}{V(\varphi_F)}\right)$$

↑ Neg. from $\left(S = -\int R\, d^4x\right)$

$S_I(\bar{\varphi}_{Bounce}) =$ Complicated < 0

For $\frac{\Gamma}{V_{01}} < 1 \Rightarrow |S_{BG}| > |S_I|$

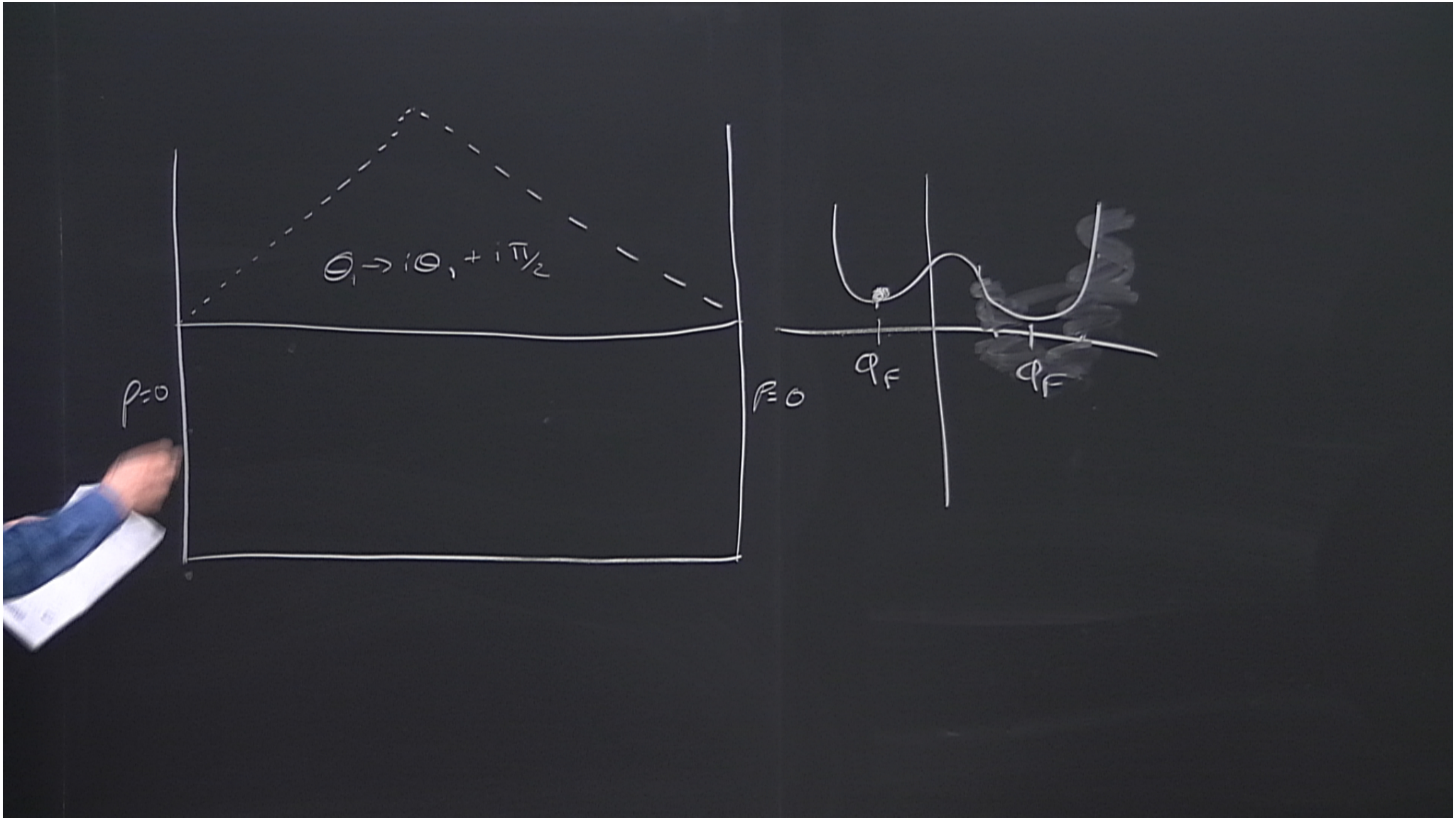

$\frac{\Gamma}{V_{01}} = A e^{-S_E(\bar{\varphi})/\hbar}$
$S_E = S_I(\bar{\varphi}_{Bounce}) - S_{BG}(\bar{\varphi}_F)$
$S_{BG}(\bar{\varphi}_F) = -24\pi^2 \left(\frac{M_p^4}{V(\varphi_F)}\right)$
Neg. from $(S = -\int R\, d^4x)$
$K \det\{-\partial_E^2 + V''(\varphi_F)\} e^{-S(\varphi_F)/\hbar}$
$= \det\{-\partial_E^2 + V''\}$
$S_I(\bar{\varphi}_{Bounce}) =$ Complicated < 0
For $\frac{\Gamma}{V_{01}} < 1 \Rightarrow |S_{BG}| > |S_I|$

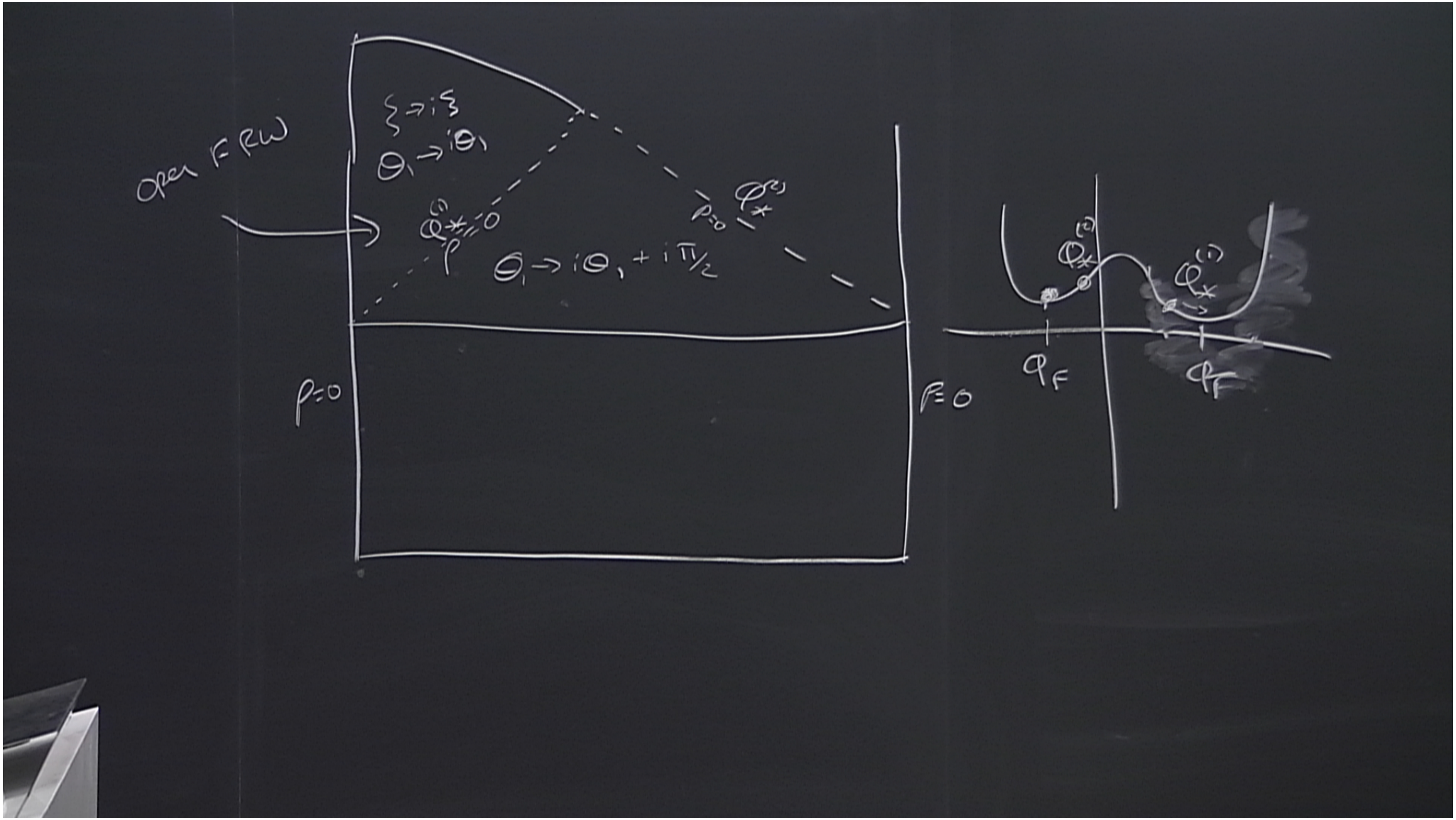
Pre-factor - A => hard to evaluate

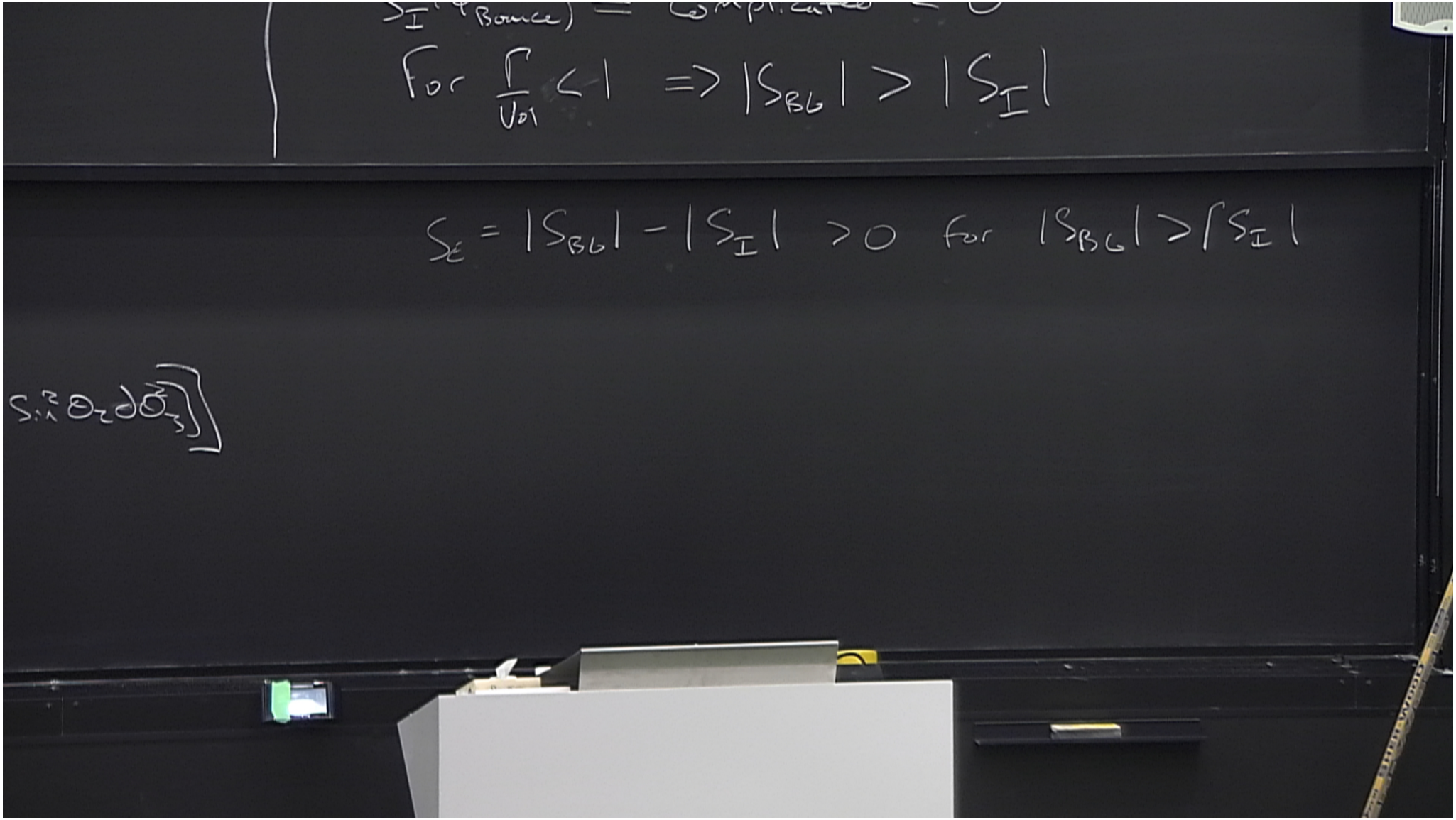
Post-tunelling Configuration

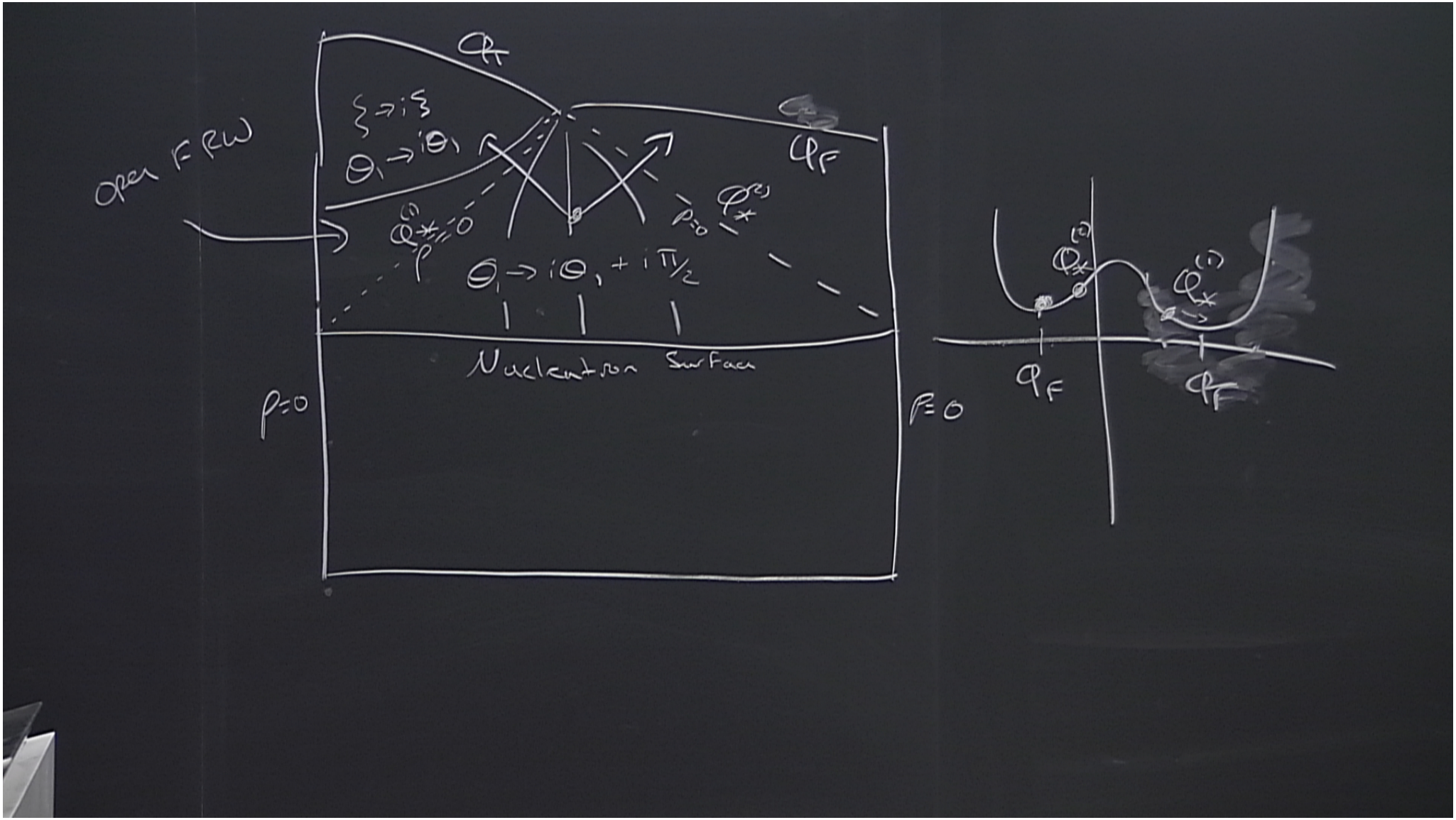
$$ds_E^2 = d\xi^2 + \rho^2(\xi)\left[d\theta_1^2 + \sin^2\theta_1\left[d\theta_2^2 + \sin^2\theta_2 d\theta_3^2\right]\right]$$

Again $\theta_1 \to i\theta_1 \to$ Wall


$\theta_1 \to i\theta_1 + i\pi/2$
$\rho = 0$
$r = 0$
q_F
q_F


open FRW
ξ → iξ
θ1 → iθ1
ρ=0
θ1 → iθ1 + iπ/2
ρ=0
ρ=0
qF
qF

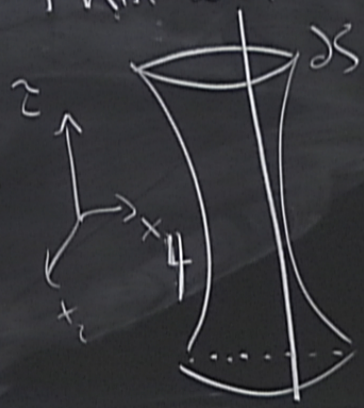

S_I (Bounce) = Simplified = 0
For $\frac{\Gamma}{v_{01}} < 1 \Rightarrow |S_{BG}| > |S_I|$
$S_E = |S_{BG}| - |S_I| > 0$ for $|S_{BG}| > |S_I|$


open FRW
φ_t
$\xi \to i\xi$
$\theta_1 \to i\theta_1$
φ_F
$\varphi^{(0)}_{*}$
$\rho=0$
$\varphi^{(1)}_{*}$
$\theta_1 \to i\theta_1 + i\pi/2$
Nucleation Surface
$\rho=0$
$\rho=0$
$\varphi^{(0)}_{*}$
$\varphi^{(1)}_{*}$
φ_F
φ_F

False Vacuum Eternal Inflation

What about more than 1 bubble?

Thin-wall limit



$\partial S \to O(4,1)$

∂S w/ bubble $\to O(3,1)$ $\rightsquigarrow$ Spherical Symmetry + Const. acceleration.

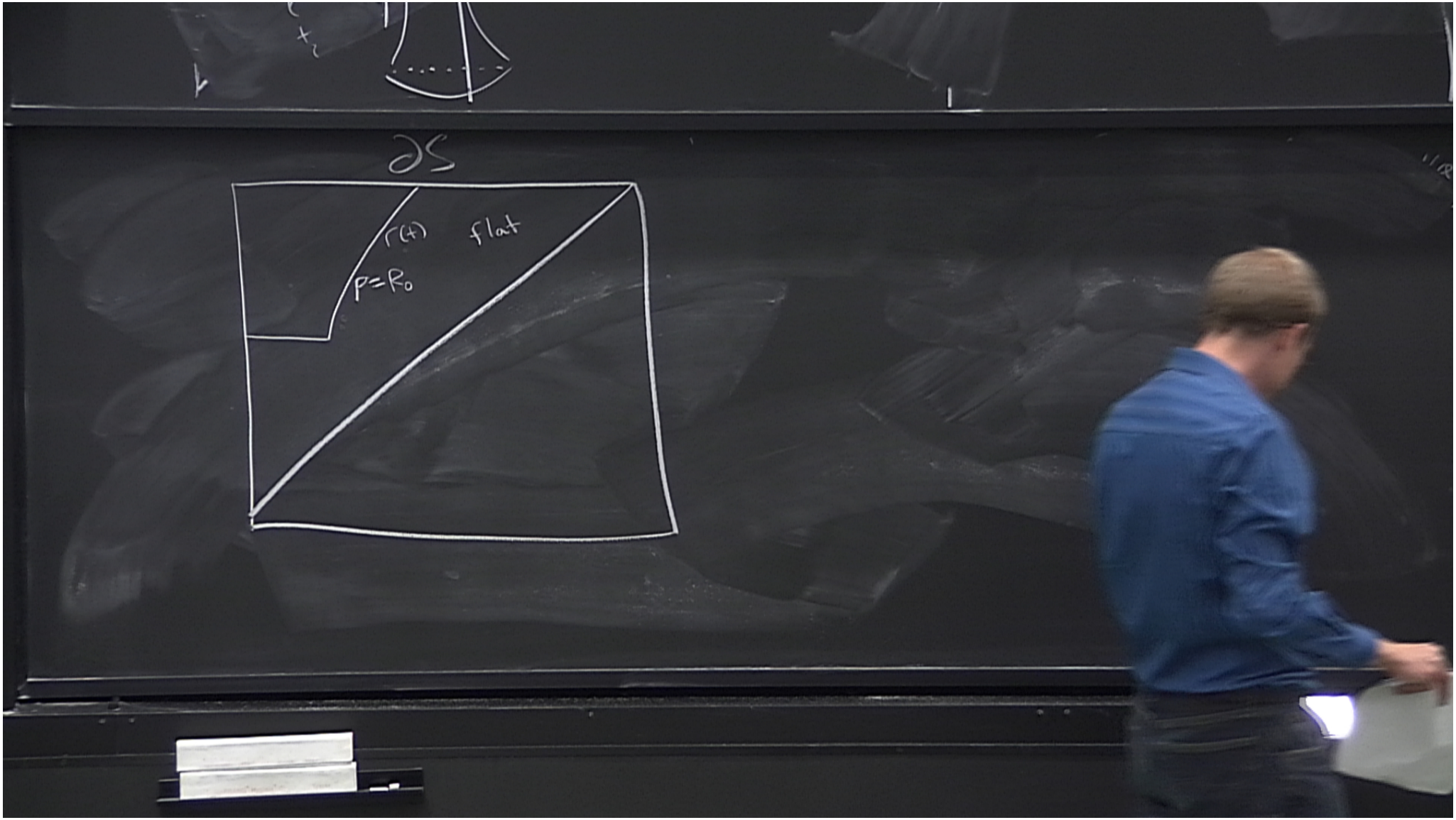
$$X_4 = H_F^{-1}\sqrt{1 - H_F^2 R_0^2}\,\cosh^{-1}(H_0) \qquad \text{(static)}$$

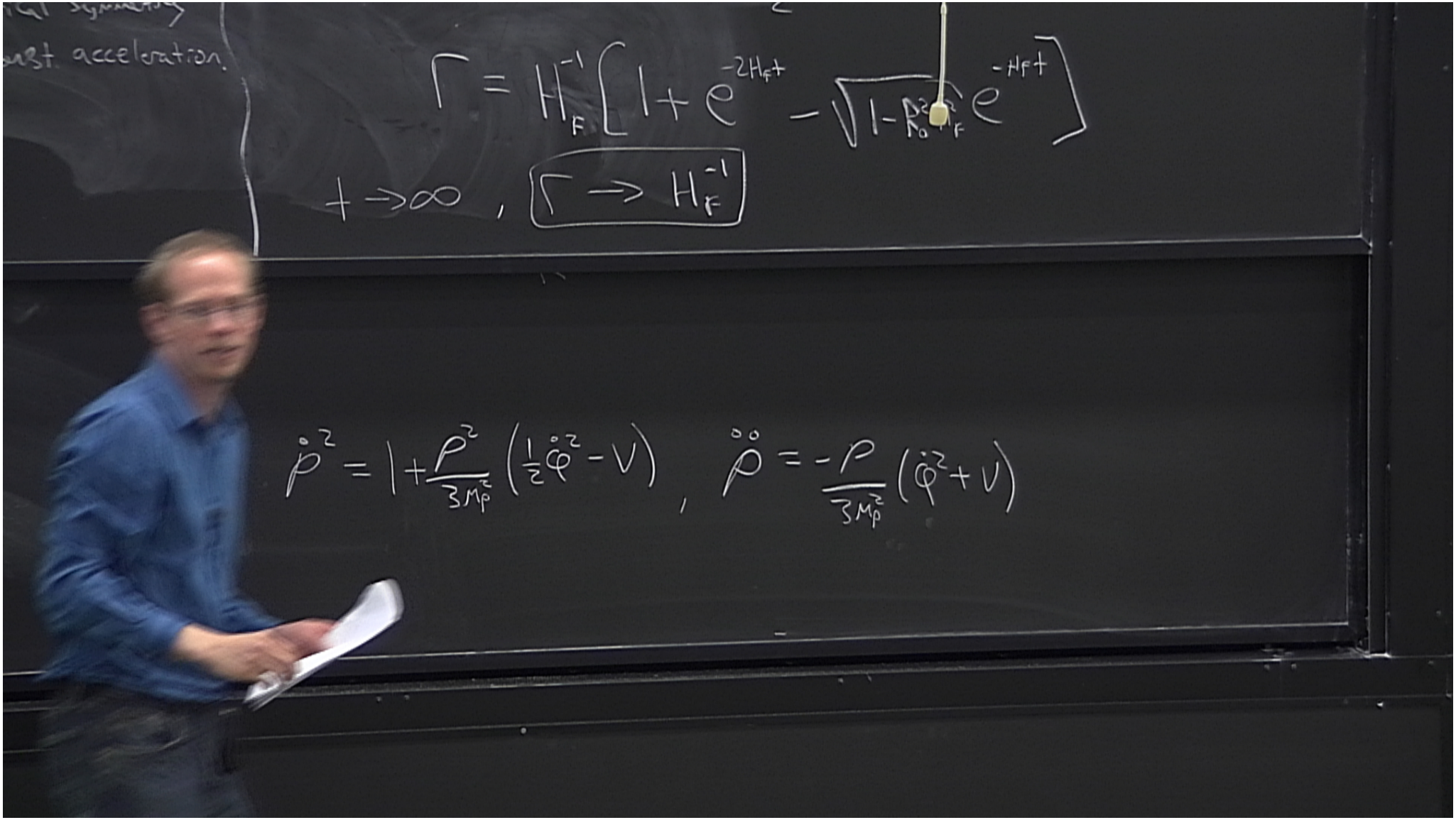
$$= \text{const.}$$

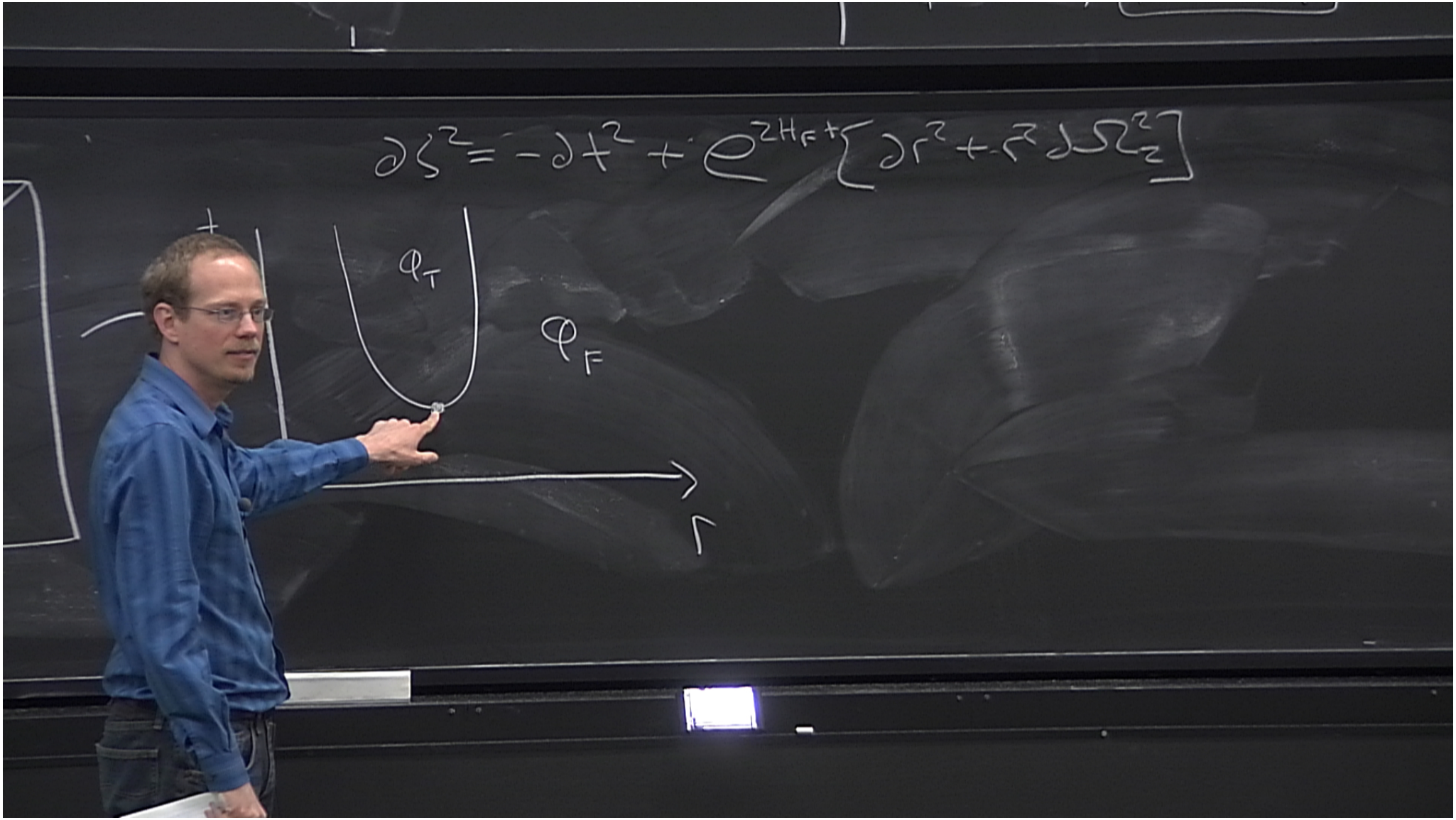
$$= H_F^{-1}\cosh(H_F t) - \frac{H_F}{2} r^2 e^{H_F t} \qquad \text{(flat)}$$

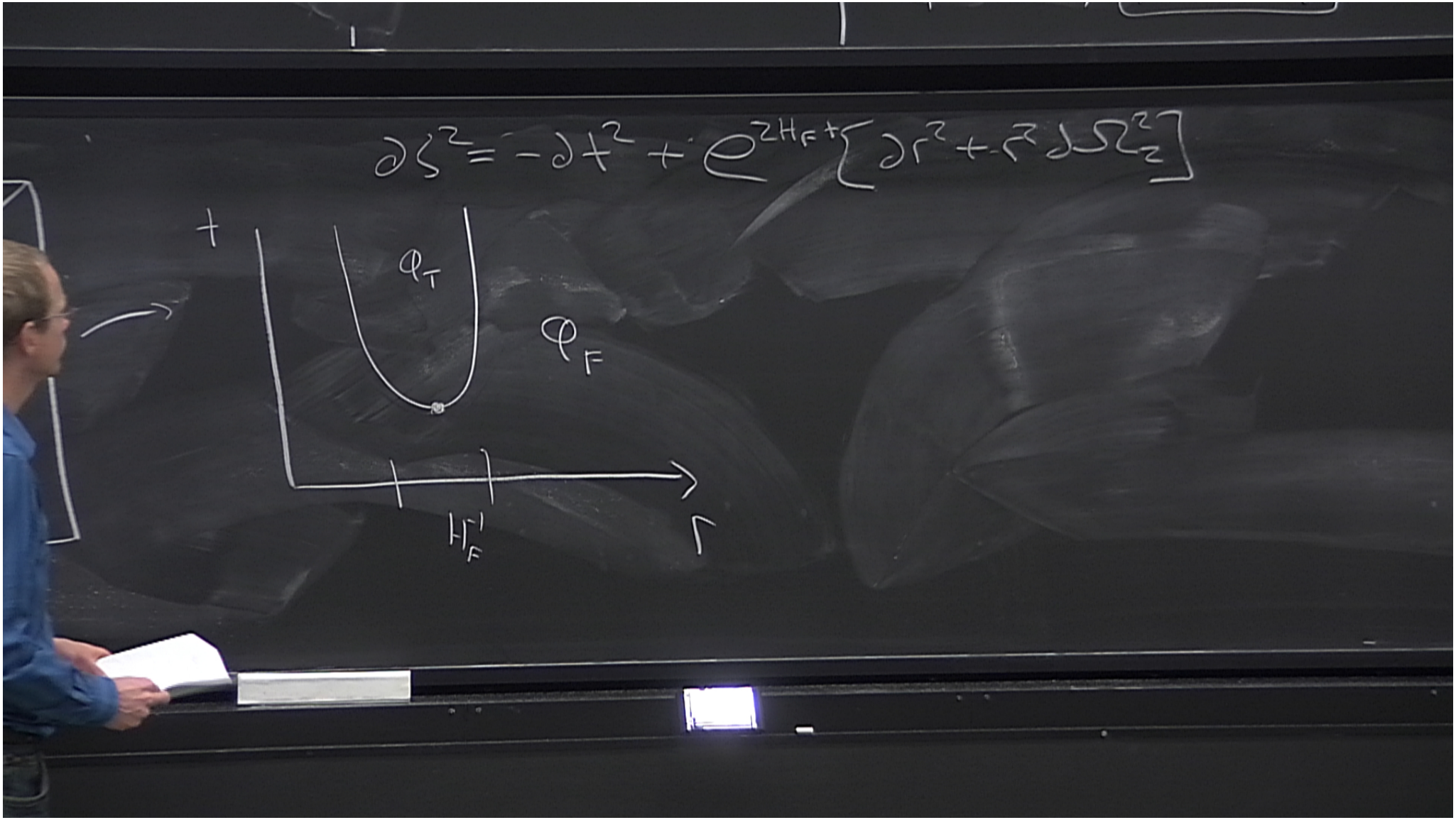
$$\Gamma = H_F^{-1}\left[1 + e^{-2H_F t} - \sqrt{1 - \frac{R_0^2}{H_F^2}}\, e^{-H_F t}\right]$$

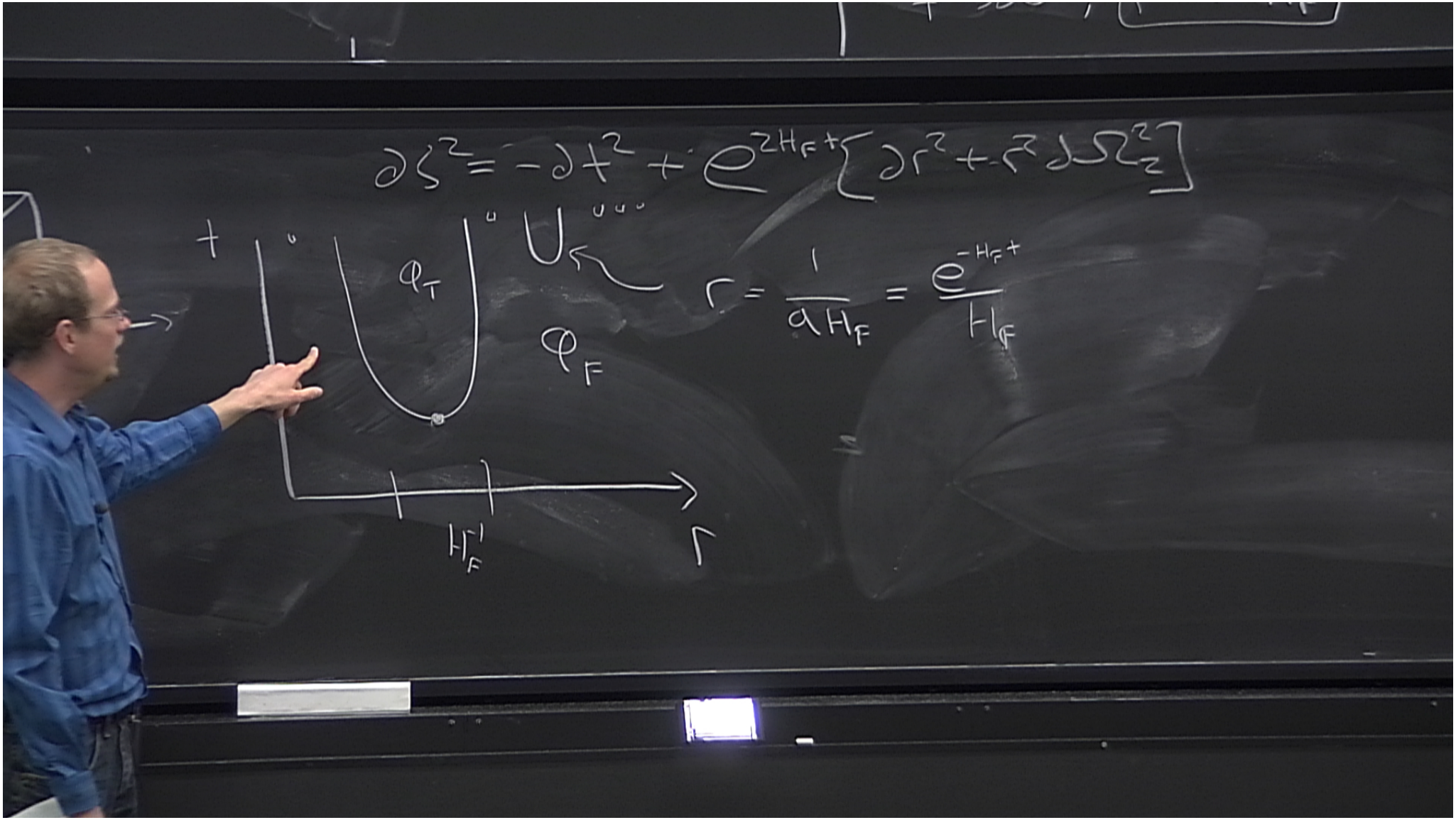
$$+ V(\phi)\Big) + 3M_P^2\left(\rho^2\ddot{\rho}^0 + \rho\dot{\rho}^2 - \rho\right)\Big]$$

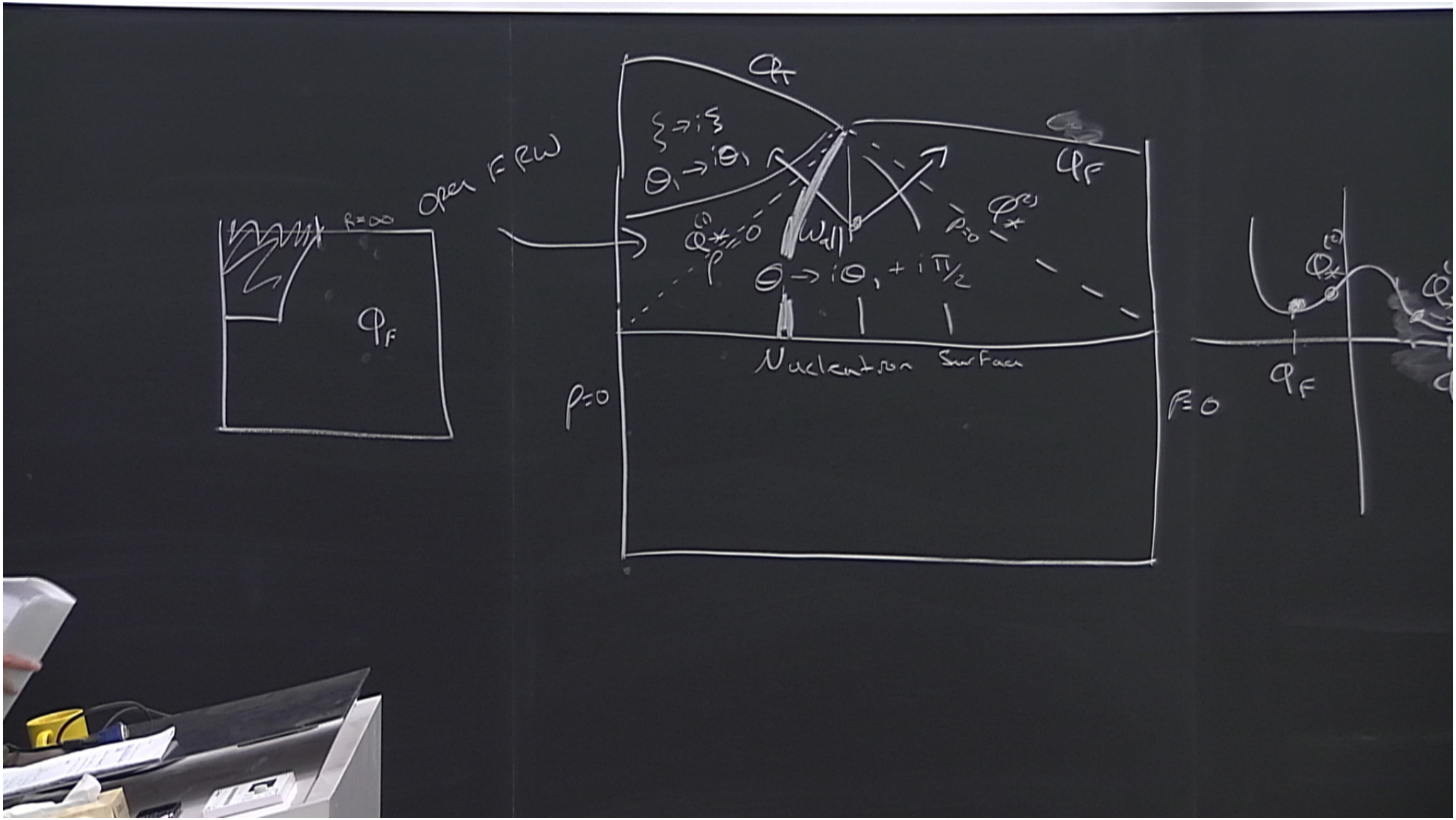

∂Σ
r(t)
flat
p=R₀


acceleration.
$\Gamma = H_F^{-1}\left[1 + e^{-2H_F t} - \sqrt{1 - \frac{\rho^2}{M_0^2}\frac{H_F^2}{}}\, e^{-H_F t}\right]$
$t \to \infty$, $\Gamma \to H_F^{-1}$
$\dot{\rho}^2 = 1 + \frac{\rho^2}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 - V\right)$, $\ddot{\rho} = -\frac{\rho}{3M_p^2}\left(\dot{\varphi}^2 + V\right)$


$ds^2 = -dt^2 + e^{2H_F t}[dr^2 + r^2 d\Omega_2^2]$
φ_T
φ_F
r


$ds^2 = -dt^2 + e^{2H_F t}[dr^2 + r^2 d\Omega_2^2]$
t
φ_T
φ_F
H_F^{-1}
r


$ds^2 = -dt^2 + e^{2H_F t}[dr^2 + r^2 d\Omega_2^2]$
q_T
q_F
U
$r = \frac{1}{aH_F} = \frac{e^{-H_F t}}{H_F}$
H_F^{-1}
r


open FRW
φ_F
$\rho=\infty$
φ_t
$\xi \to i\xi$
$\theta_1 \to i\theta_1$
φ_F
$\varphi^{(0)}_* = 0$
Wall
$\rho=0$
$\varphi^{(1)}_*$
$\theta \to i\theta_1 + i\pi/2$
Nucleation Surface
$\rho=0$
$\rho=0$
$\varphi^{(0)}_*$
φ_F