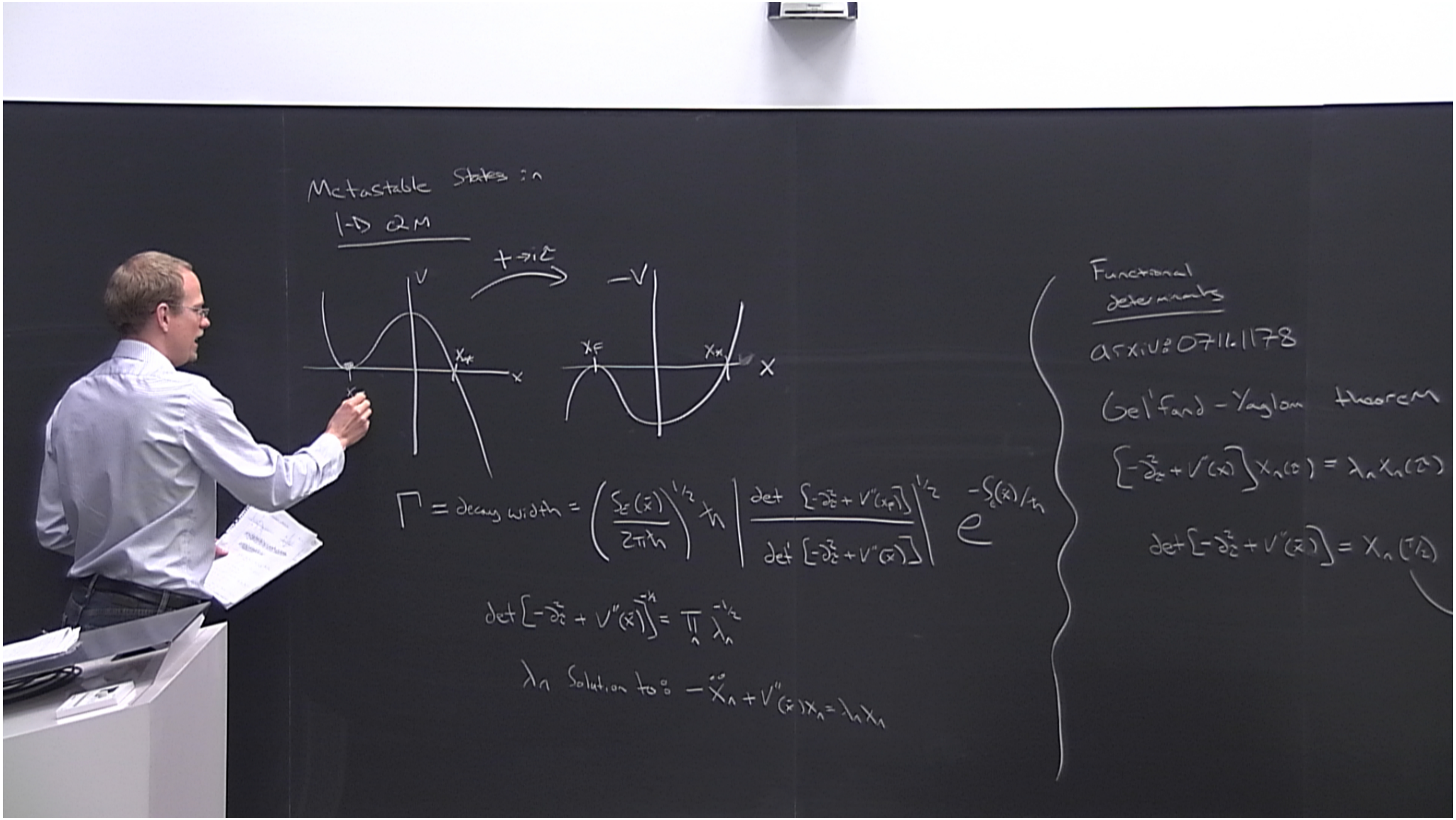
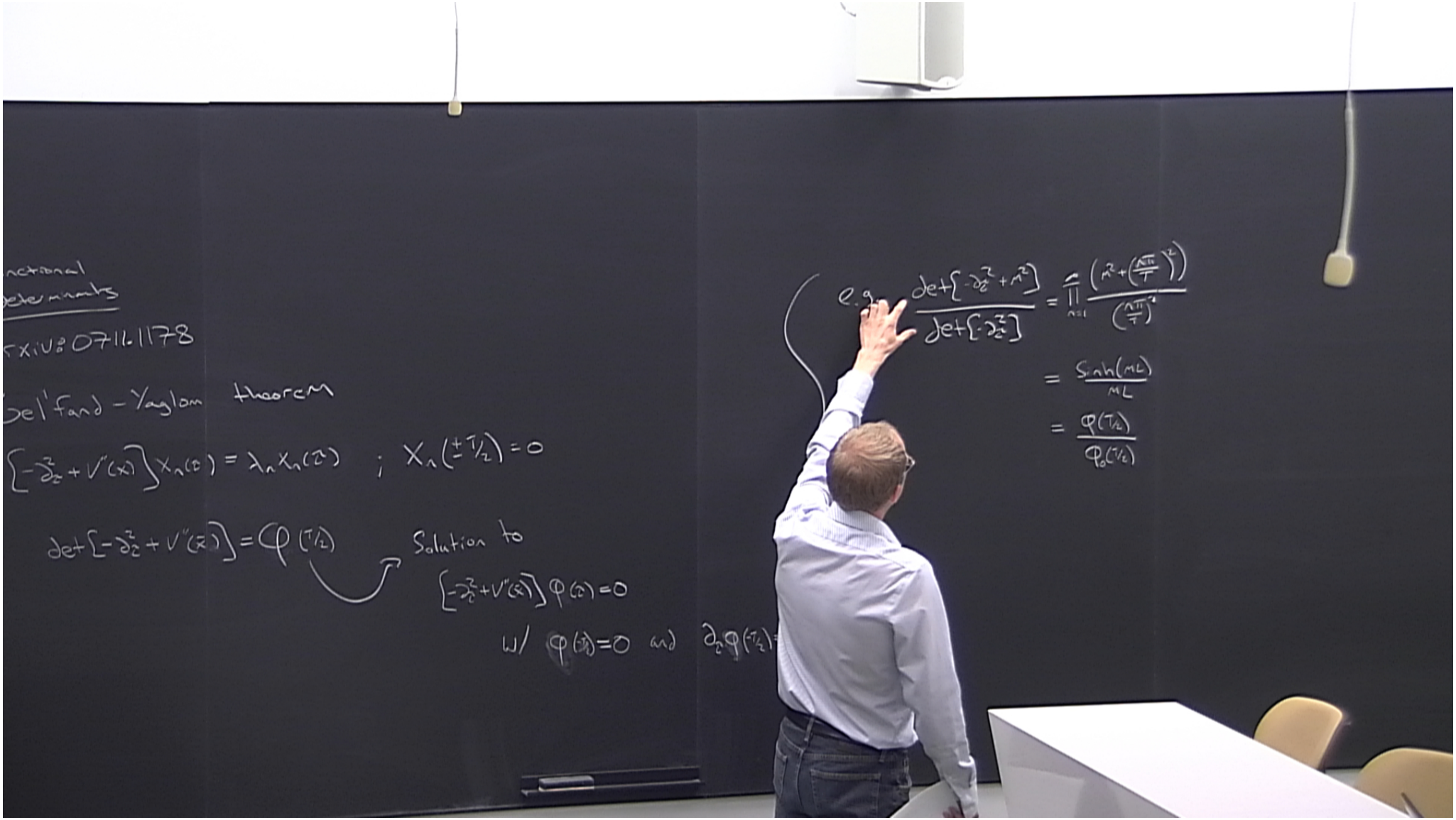
Title: Explorations in Cosmology - Lecture 12

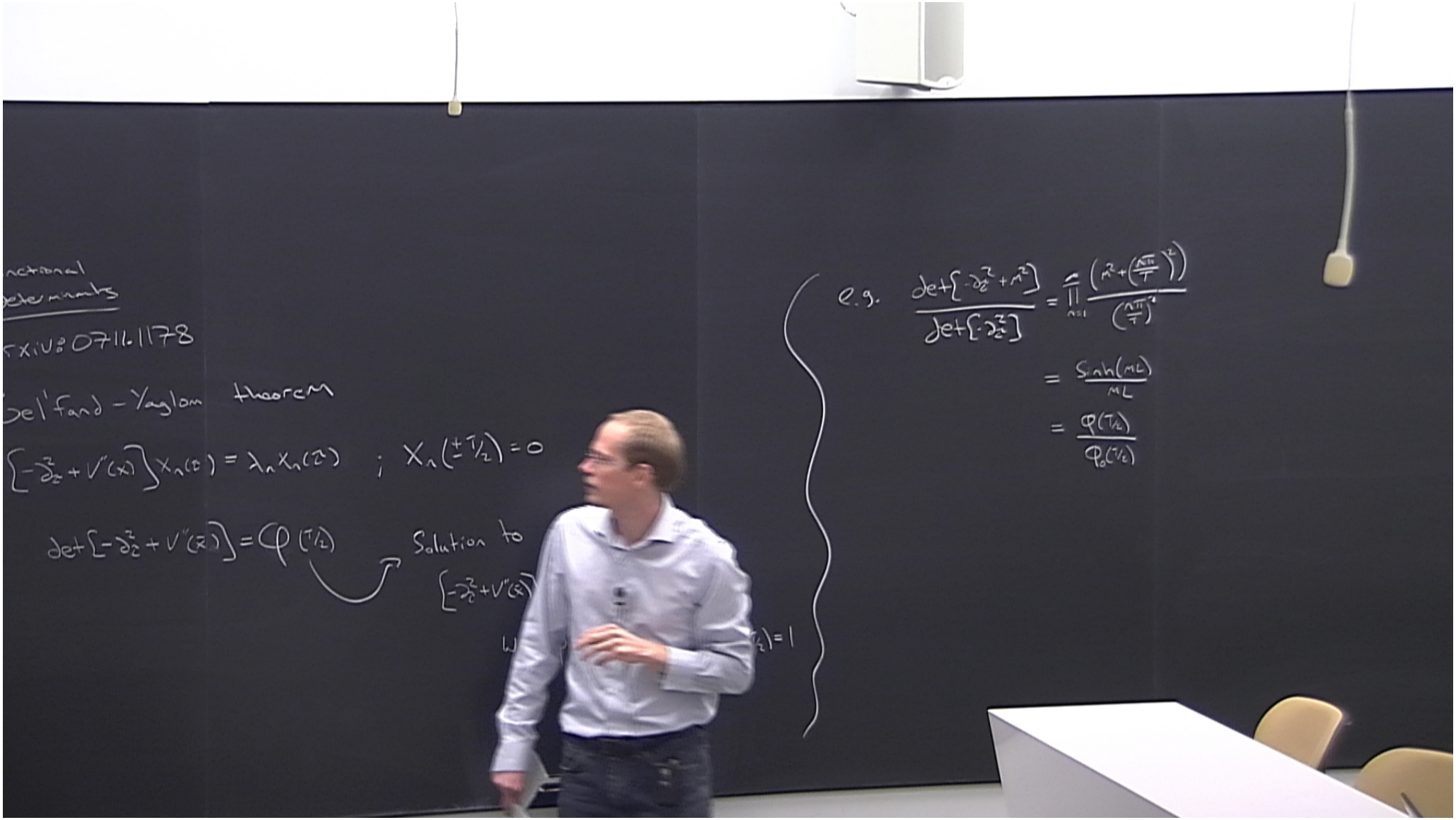
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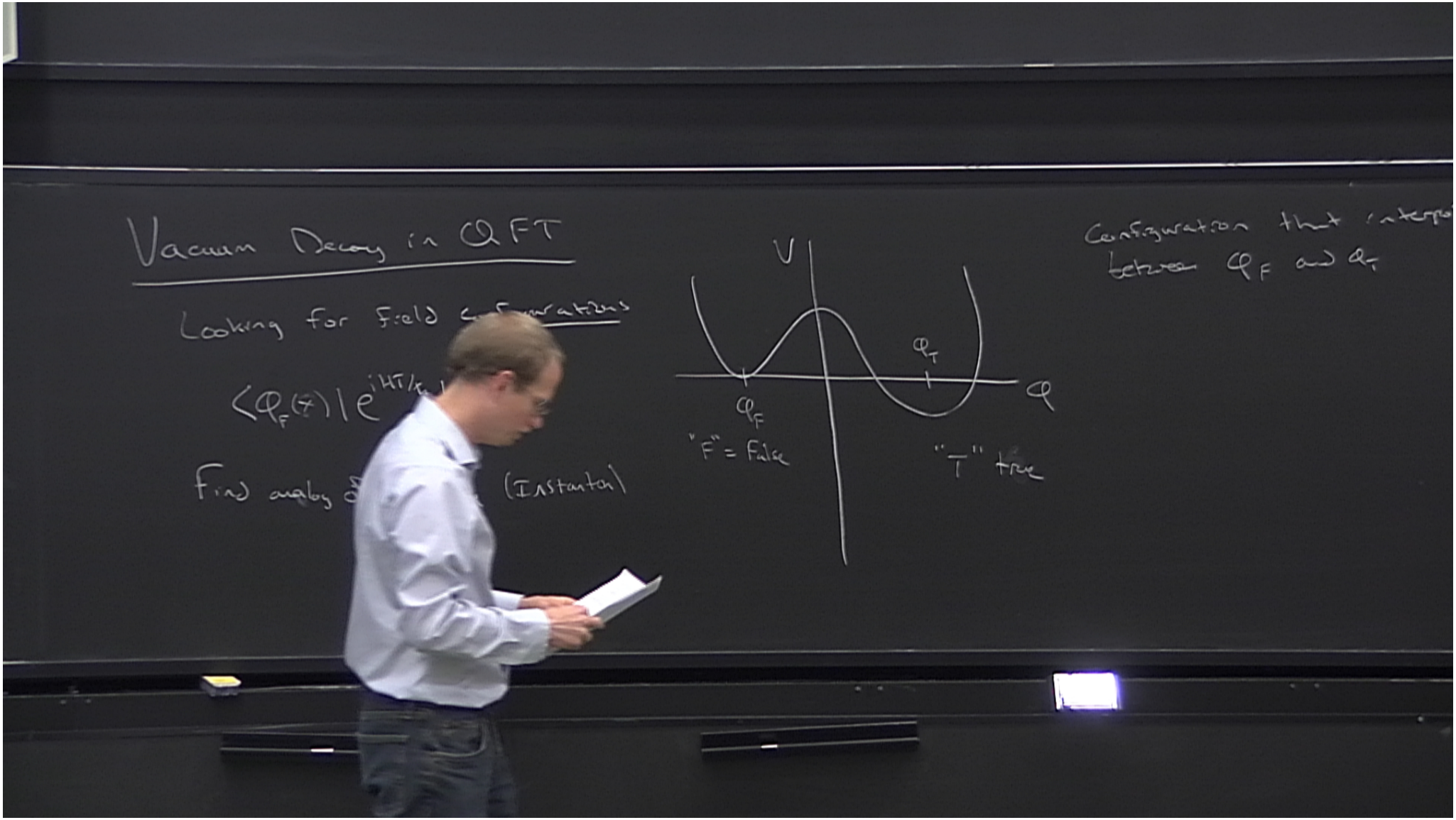
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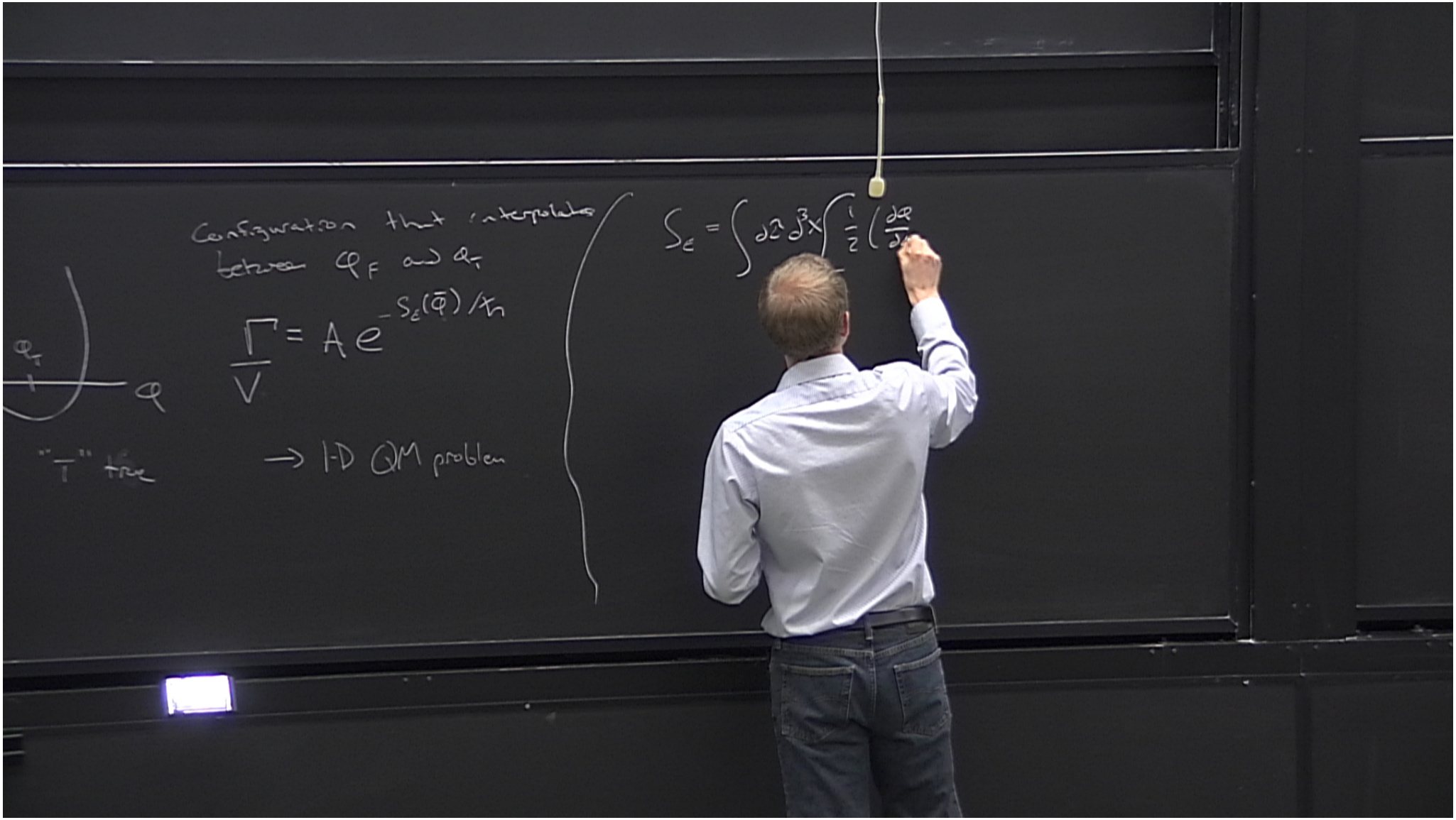
Abstract:

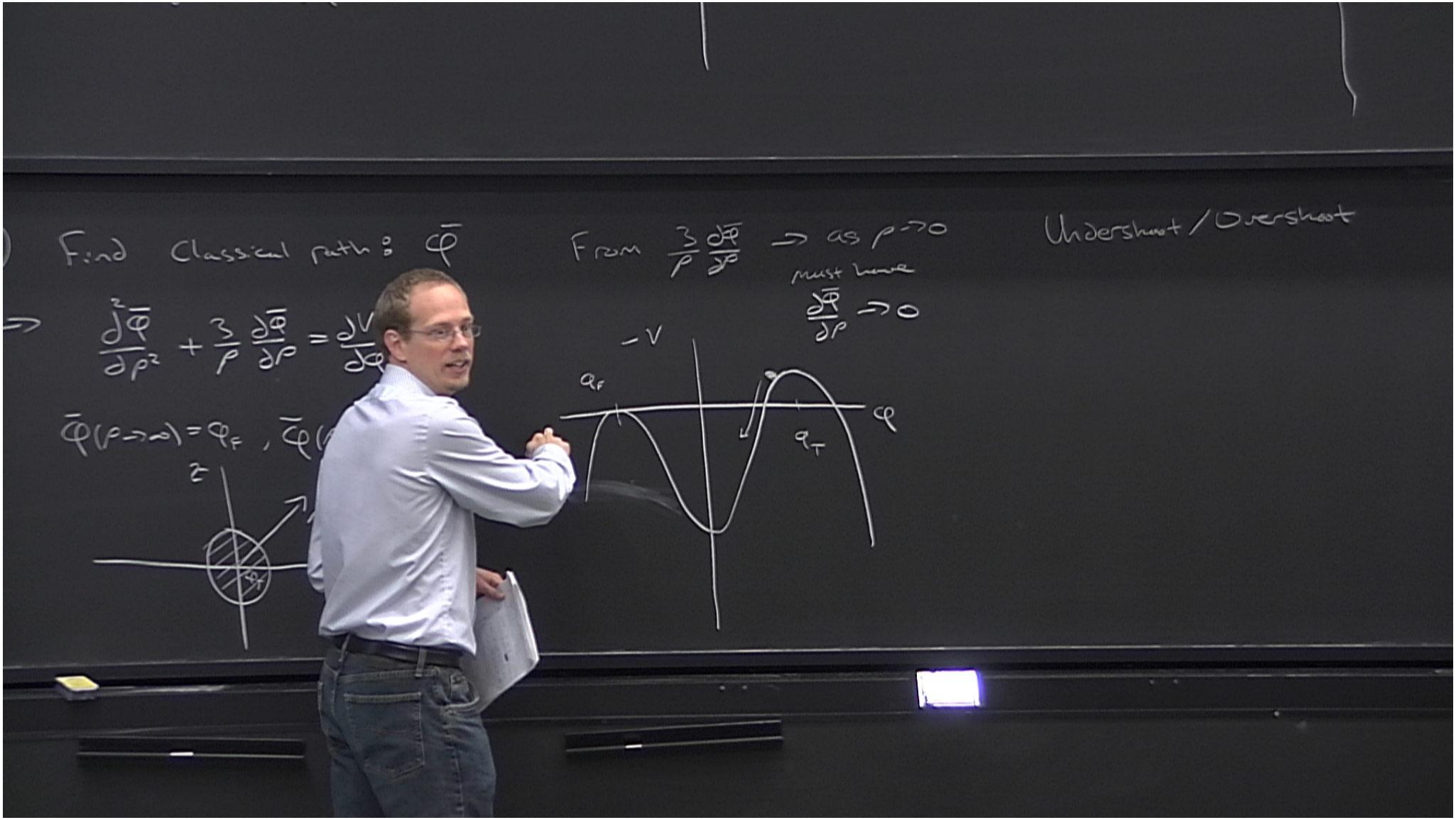

Metastable States in
1-D QM
V
x
$t \to -iE$
$-V$
x_F
x_T
x
Γ = decay width = $\left(\frac{S_E(\bar{x})}{2\pi\hbar}\right)^{1/2} \hbar \left|\frac{\det\left[-\partial_E^2 + V''(x_F)\right]}{\det\left[-\partial_E^2 + V''(\bar{x})\right]}\right|^{1/2} e^{-S_E(\bar{x})/\hbar}$
$\det\left[-\partial_E^2 + V''(\bar{x})\right]^{-\frac{1}{2}} = \prod_n \lambda_n^{-1/2}$
λ_n Solution to: $-\ddot{x}_n + V''(\bar{x})x_n = \lambda_n x_n$
Functional determinants
arXiv:0711.1178
Gel'fand - Yaglom theorem
$\left[-\partial_E^2 + V''(\bar{x})\right]x_n(E) = \lambda_n x_n(E)$
$\det\left[-\partial_E^2 + V''(\bar{x})\right] = x_n(T/2)$

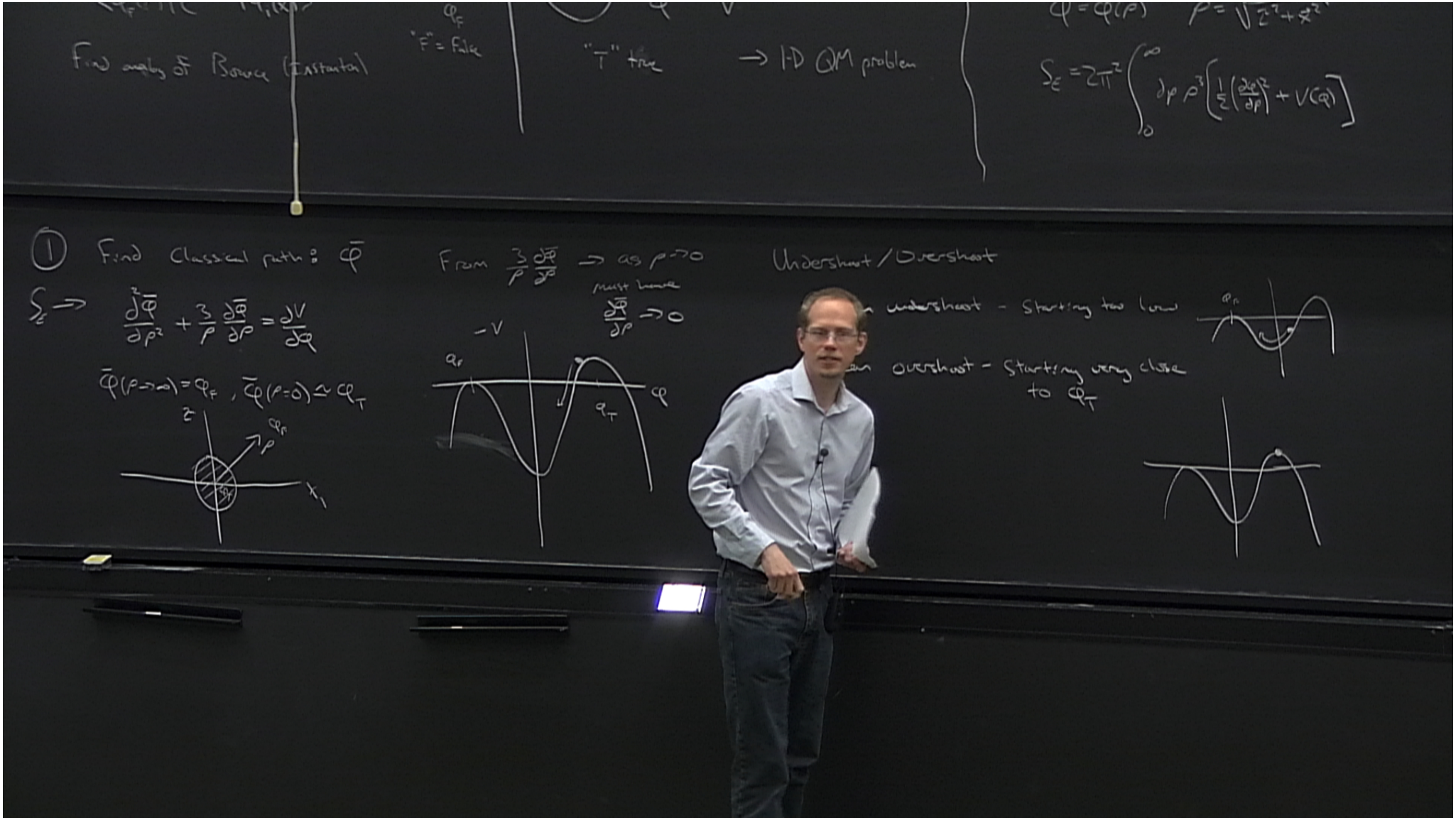

nctional
eterminants
rXiv: 0711.1178
Gel'fand - Yaglom theorem
$\left[-\partial_z^2 + V''(x)\right] X_n(z) = \lambda_n X_n(z)$; $X_n(\pm T/2) = 0$
$\det\left[-\partial_z^2 + V''(x)\right] = \varphi(T/2)$
Solution to
$\left[-\partial_z^2 + V''(x)\right] \varphi(z) = 0$
w/ $\varphi(\pm T/2) = 0$ and
e.g. $\frac{\det\left[-\partial_z^2 + m^2\right]}{\det\left[-\partial_z^2\right]} = \prod_{n=1} \frac{\left(m^2 + \left(\frac{2\pi n}{T}\right)^2\right)}{\left(\frac{2\pi n}{T}\right)^2}$
$= \frac{\sinh(mL)}{mL}$
$= \frac{\varphi(T/2)}{\varphi_0(T/2)}$

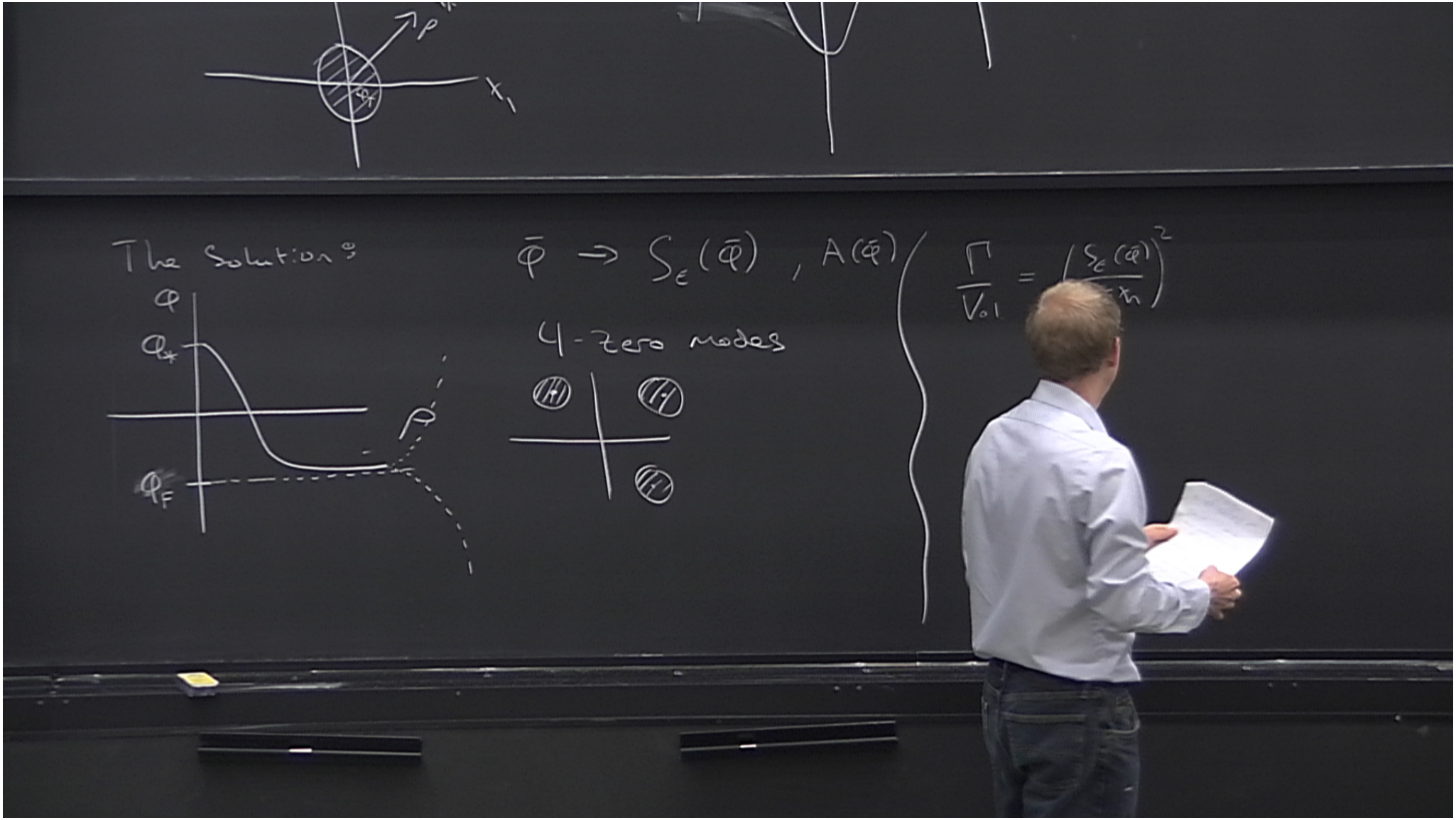

nctional
eterminants
rXiv:0711.1178
Gel'fand - Yaglom theorem
$\left[-\partial_x^2 + V''(x)\right] x_n(x) = \lambda_n x_n(x)$; $x_n(\pm T/2) = 0$
$\det\left[-\partial_x^2 + V''(x)\right] = \varphi(T/2)$
Solution to
$\left[-\partial_x^2 + V''(x)\right]$
$= 1$
e.g. $\frac{\det\left[-\partial_x^2 + m^2\right]}{\det\left[-\partial_x^2\right]} = \prod_{n=1} \frac{\left(m^2 + \left(\frac{n\pi}{T}\right)^2\right)}{\left(\frac{n\pi}{T}\right)^2}$
$= \frac{\sinh(mL)}{mL}$
$= \frac{\varphi(T/2)}{\varphi_0(T/2)}$

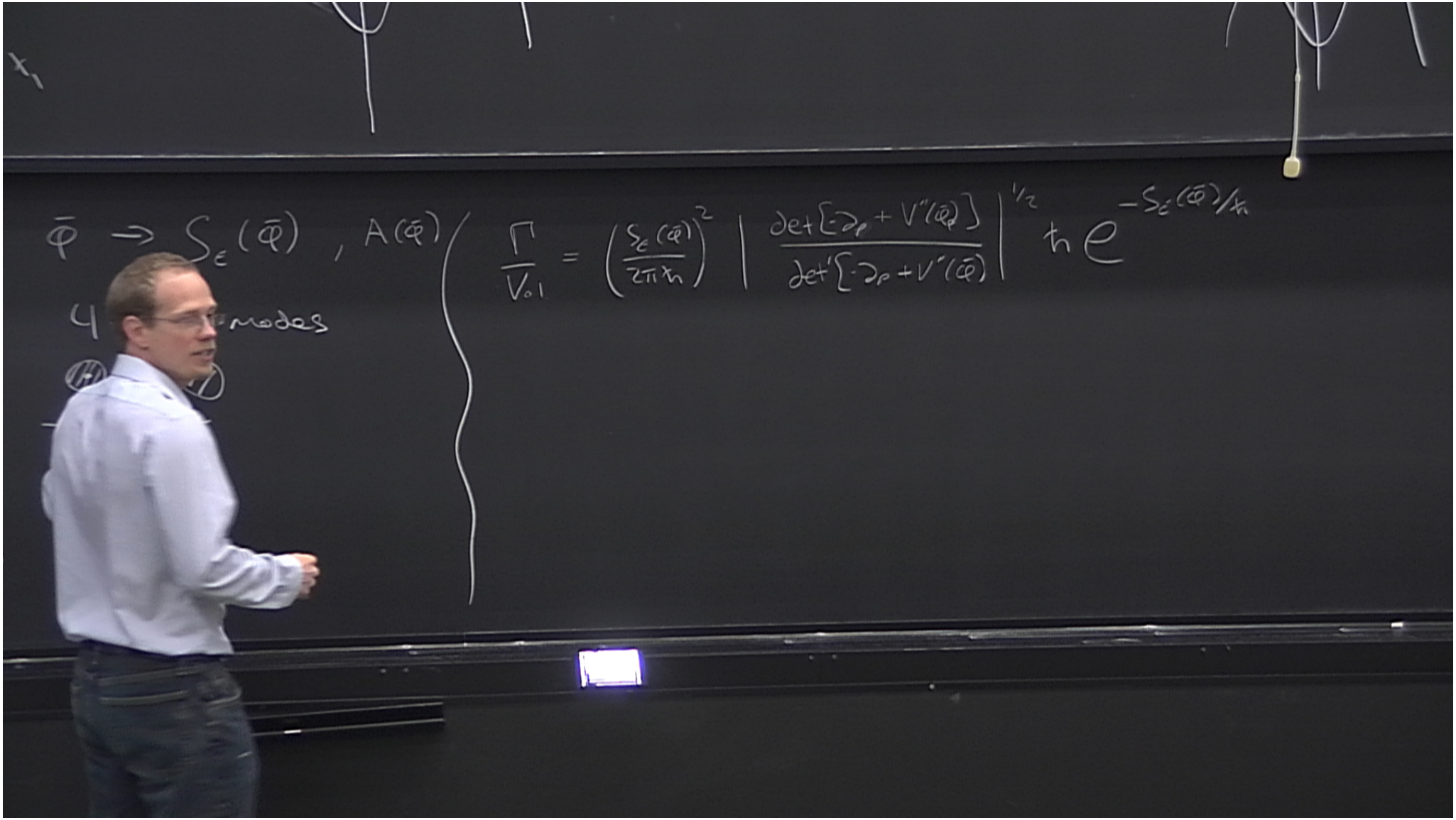

Vacuum Decay in QFT
Looking for field
Find analog
(Instanton)
V
φ_T
φ_F
φ
"F" = False
"T" true
Configuration that interpo
between φ_F and φ_T

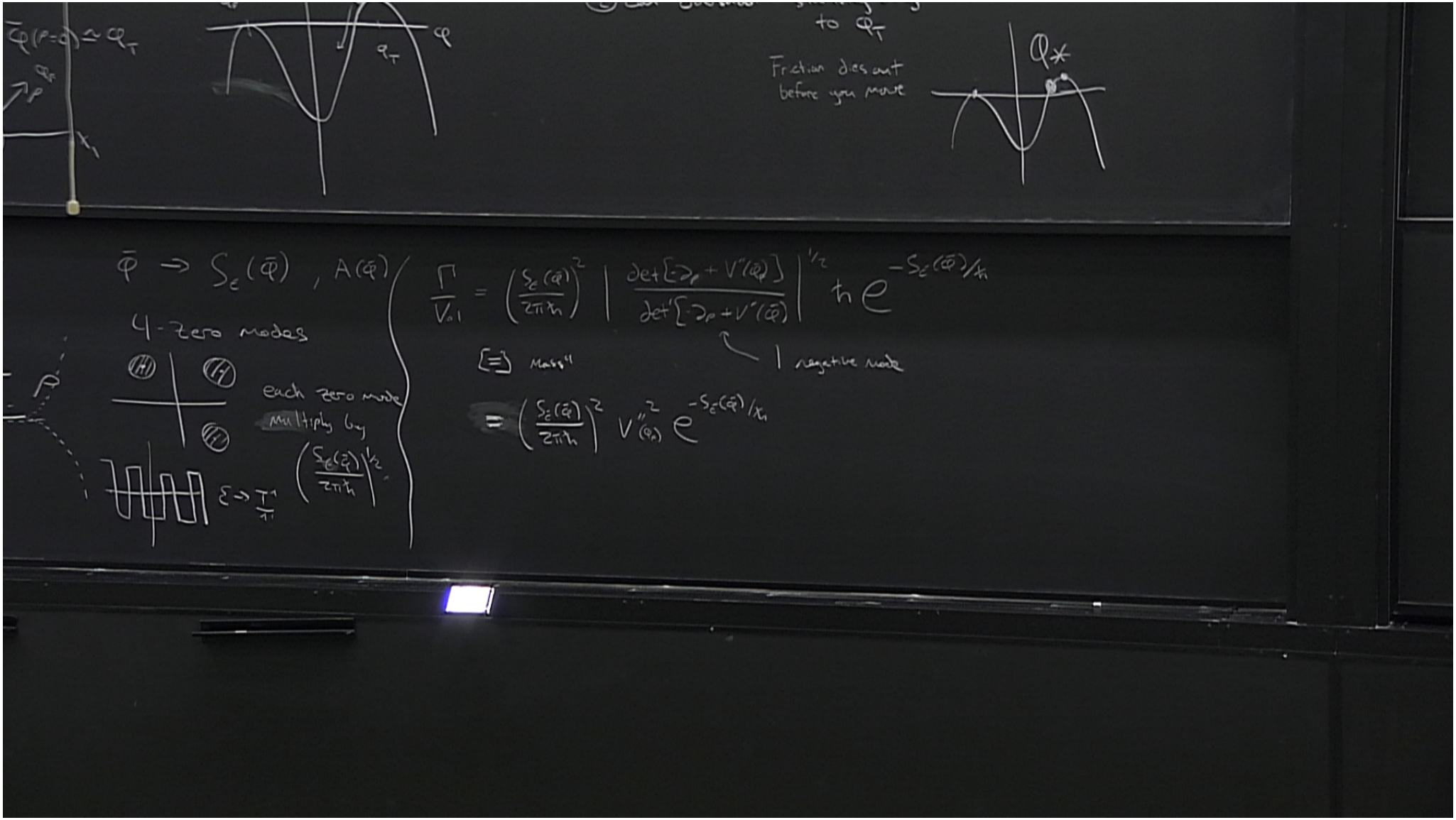

Configuration that interpolates between φ_F and φ_T
$\frac{\Gamma}{V} = A e^{-S_E(\bar{\varphi})/\hbar}$
φ_T
φ
"T" true
→ 1-D QM problem
$S_E = \int d\tau d^3x \left[\frac{1}{2} \left(\frac{d\varphi}{d\tau} \right.$

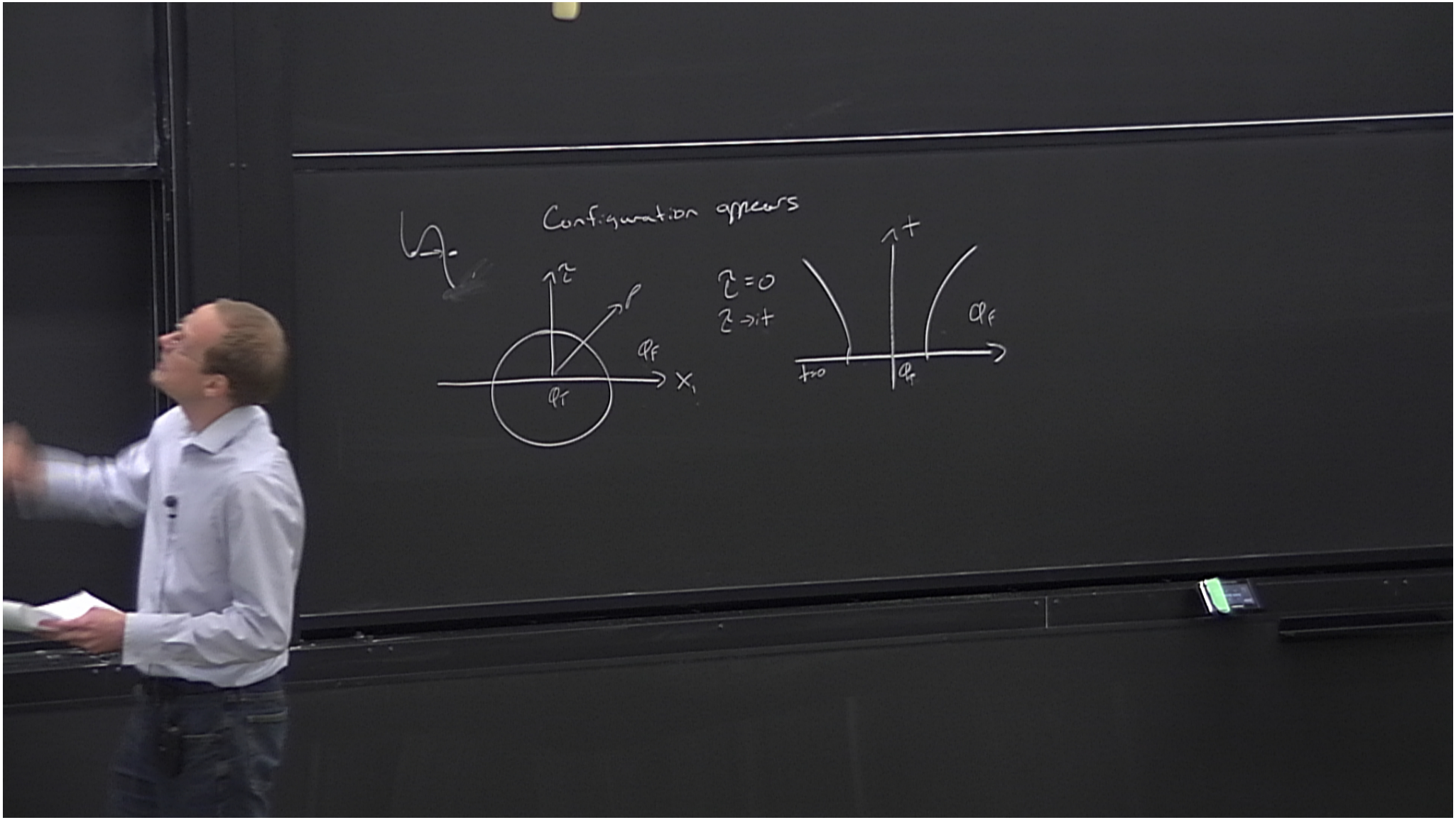

Find Classical path: $\bar{\varphi}$
From $\frac{3}{\rho}\frac{d\bar{\varphi}}{d\rho}$ → as $\rho \to 0$
must have
$\frac{d\bar{\varphi}}{d\rho} \to 0$
Undershoot / Overshoot
$\frac{d^2\bar{\varphi}}{d\rho^2} + \frac{3}{\rho}\frac{d\bar{\varphi}}{d\rho} = \frac{dV}{d\varphi}$
$\bar{\varphi}(\rho \to \infty) = \varphi_F$
$-V$
φ_F
φ_T
φ

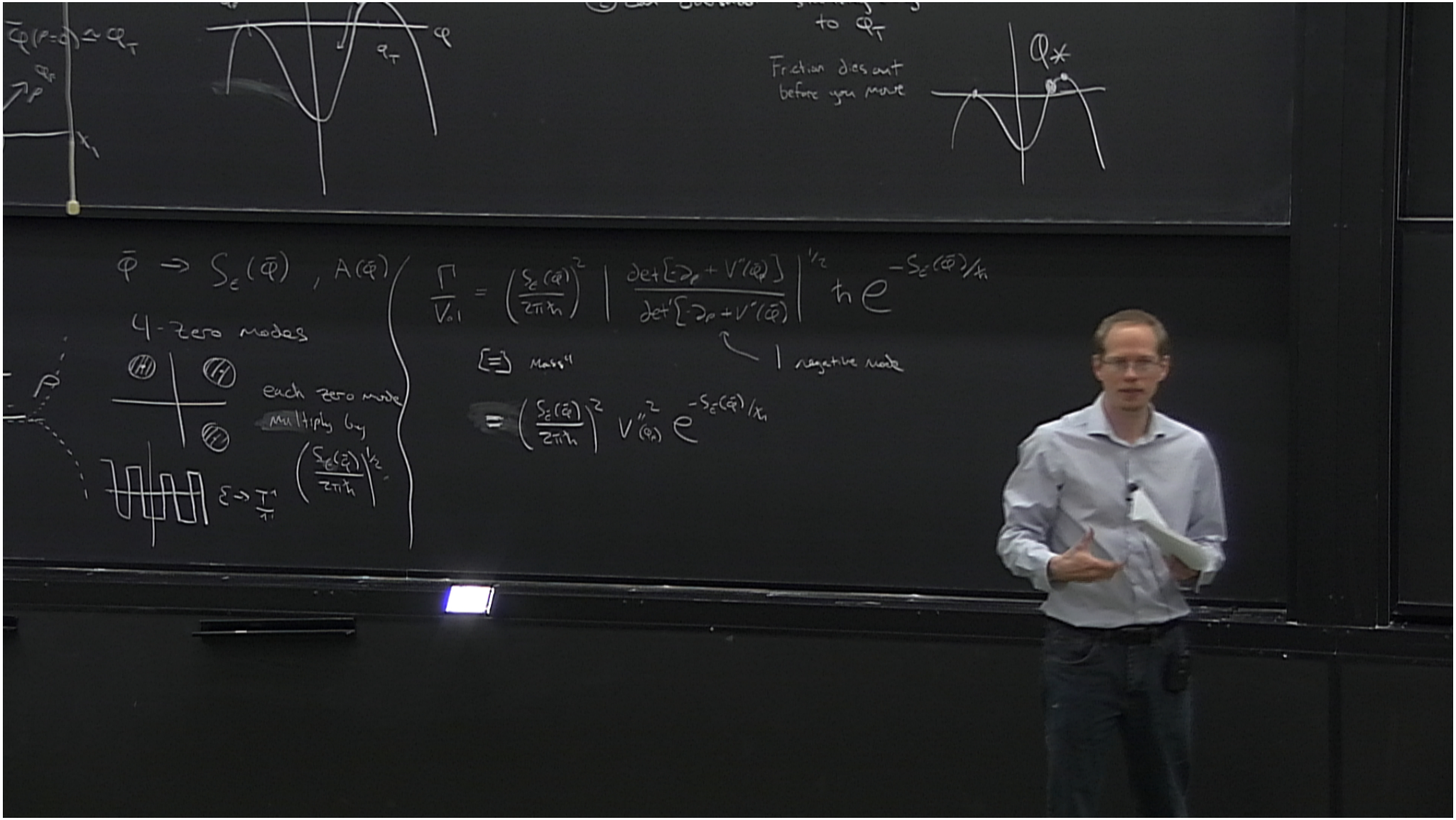

Find analog of Bounce (Instanton)
"F" = False
"T" true
→ 1-D QM problem
① Find Classical path: $\bar{\varphi}$
$\frac{d^2\bar{\varphi}}{d\rho^2} + \frac{3}{\rho}\frac{d\bar{\varphi}}{d\rho} = \frac{dV}{d\bar{\varphi}}$
$\bar{\varphi}(\rho\to\infty) = \varphi_F$, $\bar{\varphi}(\rho=0) \simeq \varphi_T$
From $\frac{3}{\rho}\frac{d\bar{\varphi}}{d\rho}$ → as $\rho\to 0$
must have
$\frac{d\bar{\varphi}}{d\rho} \to 0$
Undershoot / Overshoot
undershoot - Starting too low
overshoot - Starting very close to φ_T

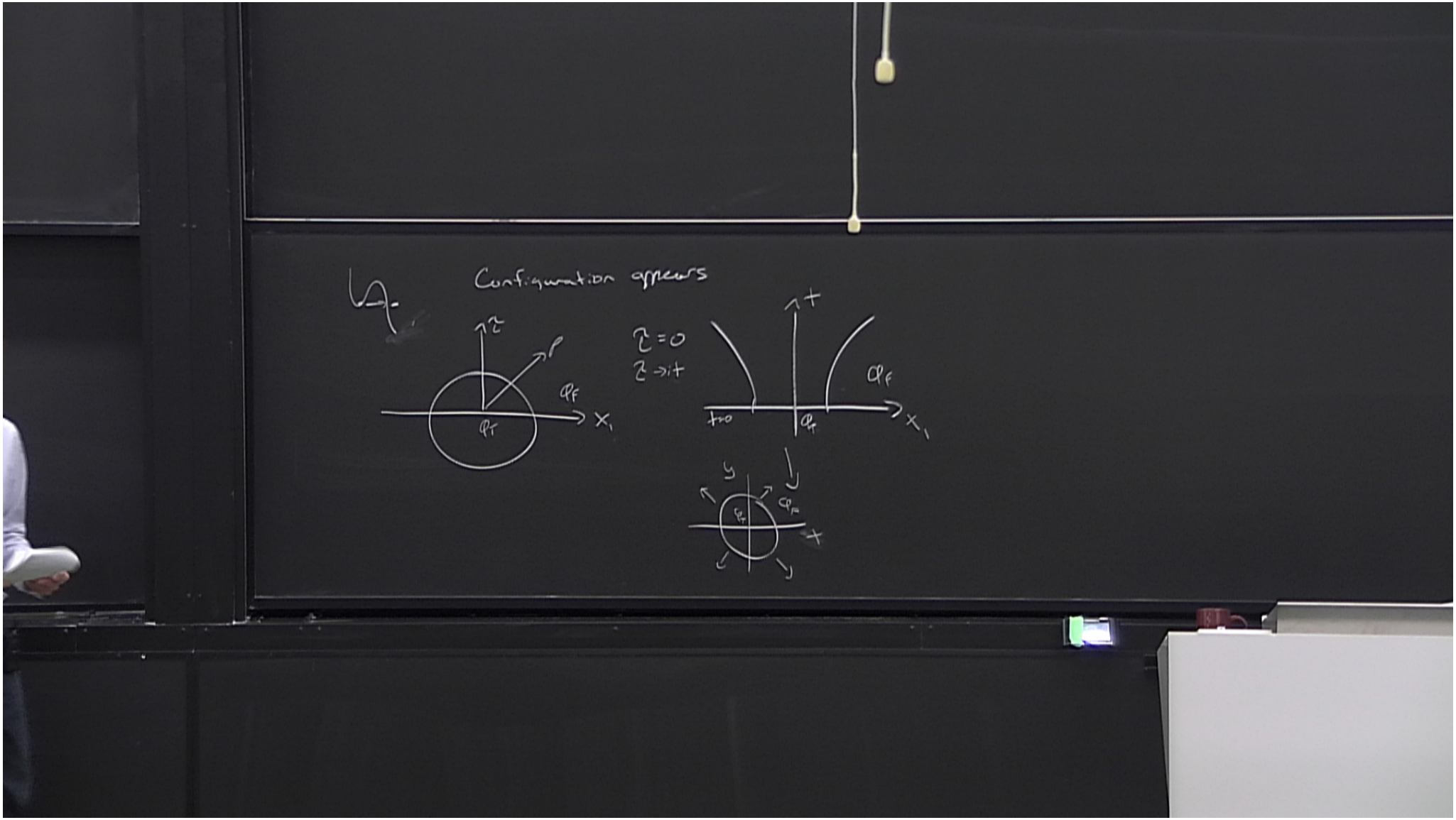

The Solution:
$\bar{\varphi} \rightarrow S_E(\bar{\varphi})$, $A(\bar{\varphi})$
$\frac{\Gamma}{V_{01}} = \left(\frac{S_E(\bar{\varphi})}{\pi}\right)^2$
4 - zero modes
φ
φ_*
φ_F
ρ

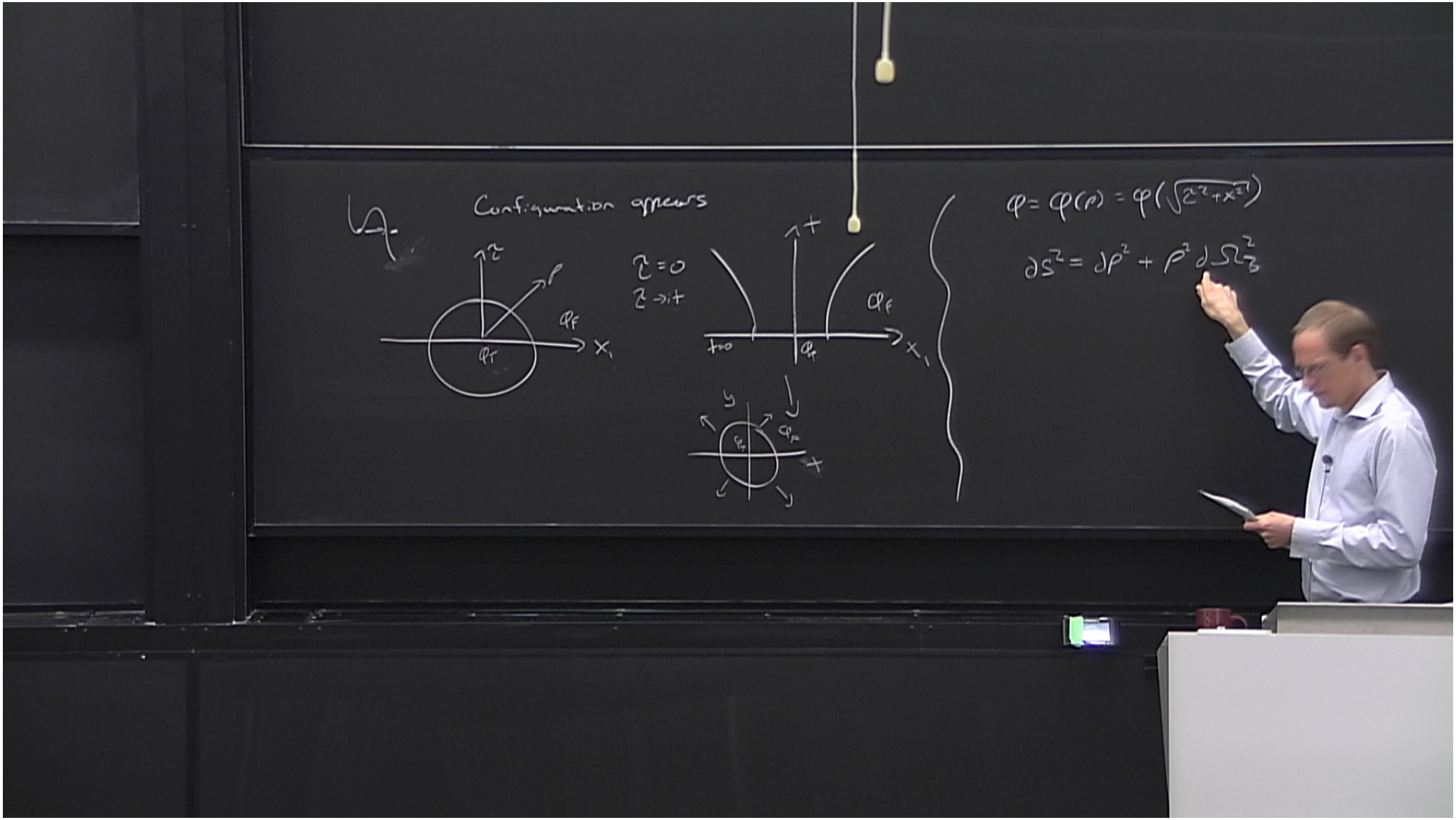

$\bar{\varphi} \to S_\varepsilon(\bar{\varphi})$, $A(\bar{\varphi})$
4 modes
$\frac{\Gamma}{V_{01}} = \left(\frac{S_\varepsilon(\bar{\varphi})}{2\pi\hbar}\right)^2 \left| \frac{\det[-\partial_\rho + V''(\bar{\varphi}_\rho)]}{\det'[-\partial_\rho + V''(\bar{\varphi})]} \right|^{1/2} \hbar\, e^{-S_\varepsilon(\bar{\varphi})/\hbar}$

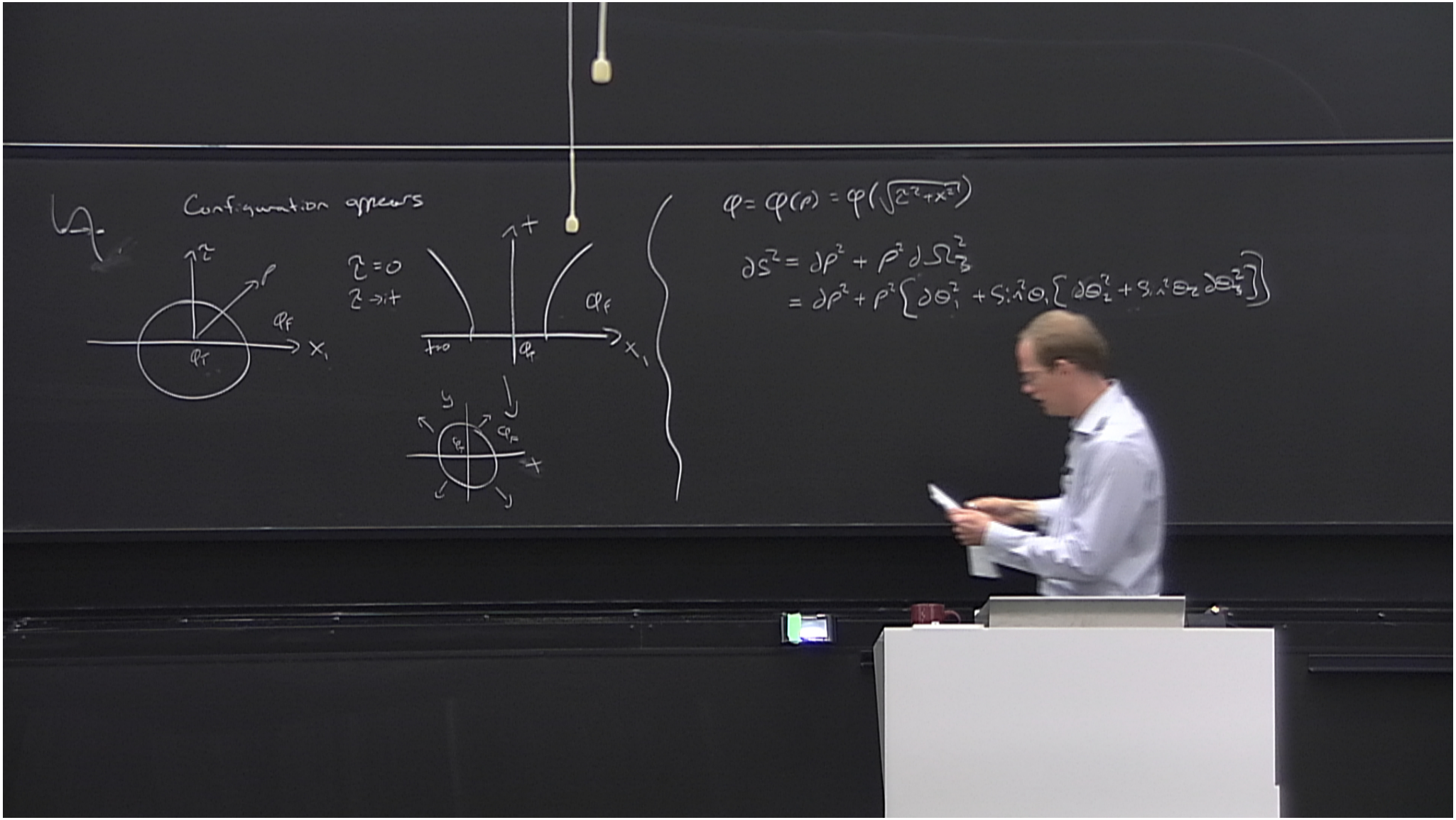

$\bar{\varphi}(\rho=0) \simeq \varphi_T$
φ_T
φ
to φ_T
Friction dies out before you move
φ_*
$\bar{\varphi} \to S_E(\bar{\varphi})$, $A(\bar{\varphi})$
$\frac{\Gamma}{V_{01}} = \left(\frac{S_E(\bar{\varphi})}{2\pi\hbar}\right)^2 \left|\frac{\det[-\partial_\rho^2 + V''(\varphi_F)]}{\det'[-\partial_\rho^2 + V''(\bar{\varphi})]}\right|^{1/2} \hbar\, e^{-S_E(\bar{\varphi})/\hbar}$
4 zero modes
each zero mode
Multiply by
$\left(\frac{S_E(\bar{\varphi})}{2\pi\hbar}\right)^{1/2}$
$\xi \to \frac{T}{\pi}$
Massless
1 negative mode
$= \left(\frac{S_E(\bar{\varphi})}{2\pi\hbar}\right)^2 V''^2_{(\varphi_F)}\, e^{-S_E(\bar{\varphi})/\hbar}$

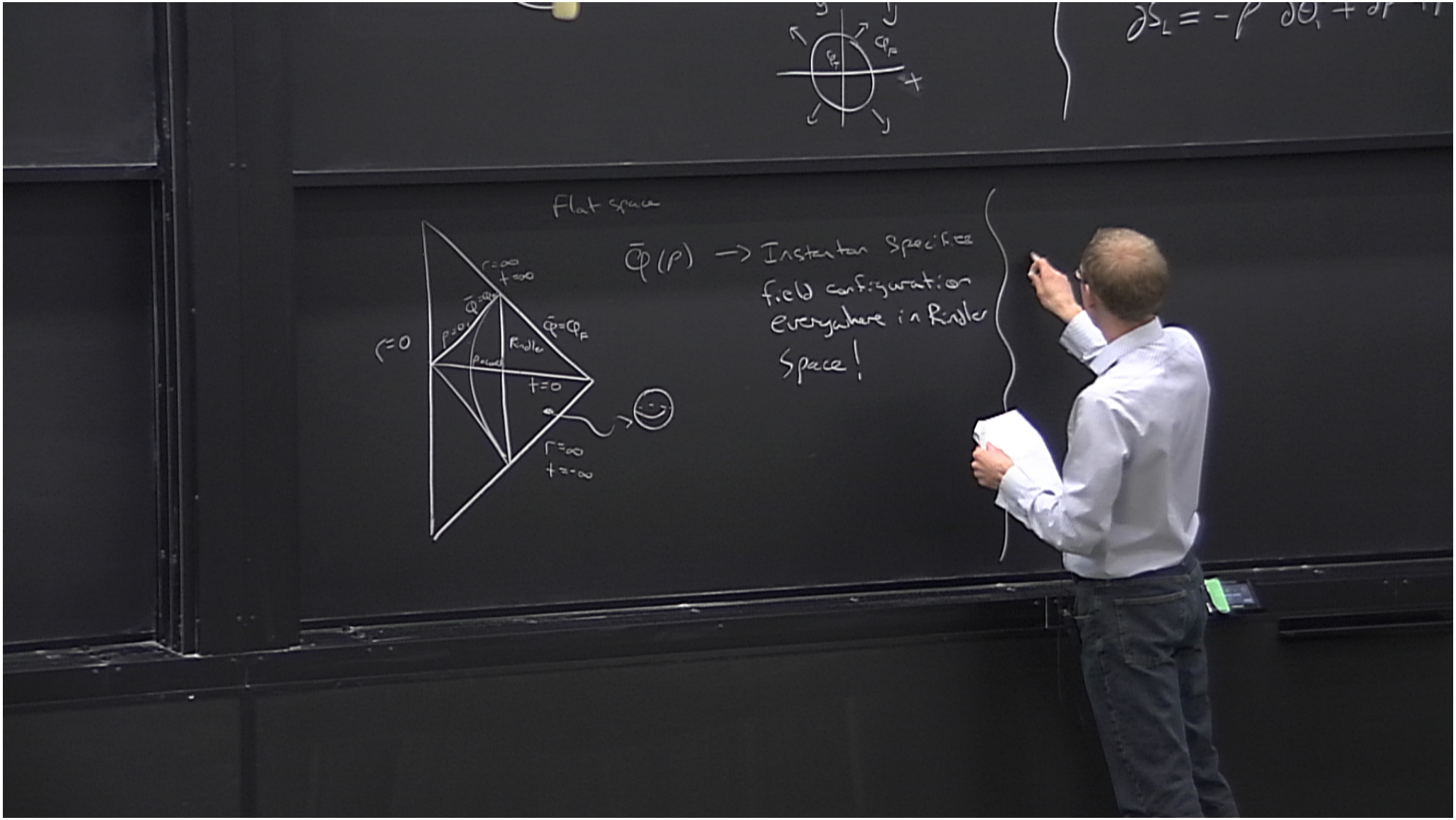

Configuration appears
τ
ρ
φ_F
φ_T
x_1
$\tau = 0$
$\tau \to it$
t
φ_F
t=0
φ_T

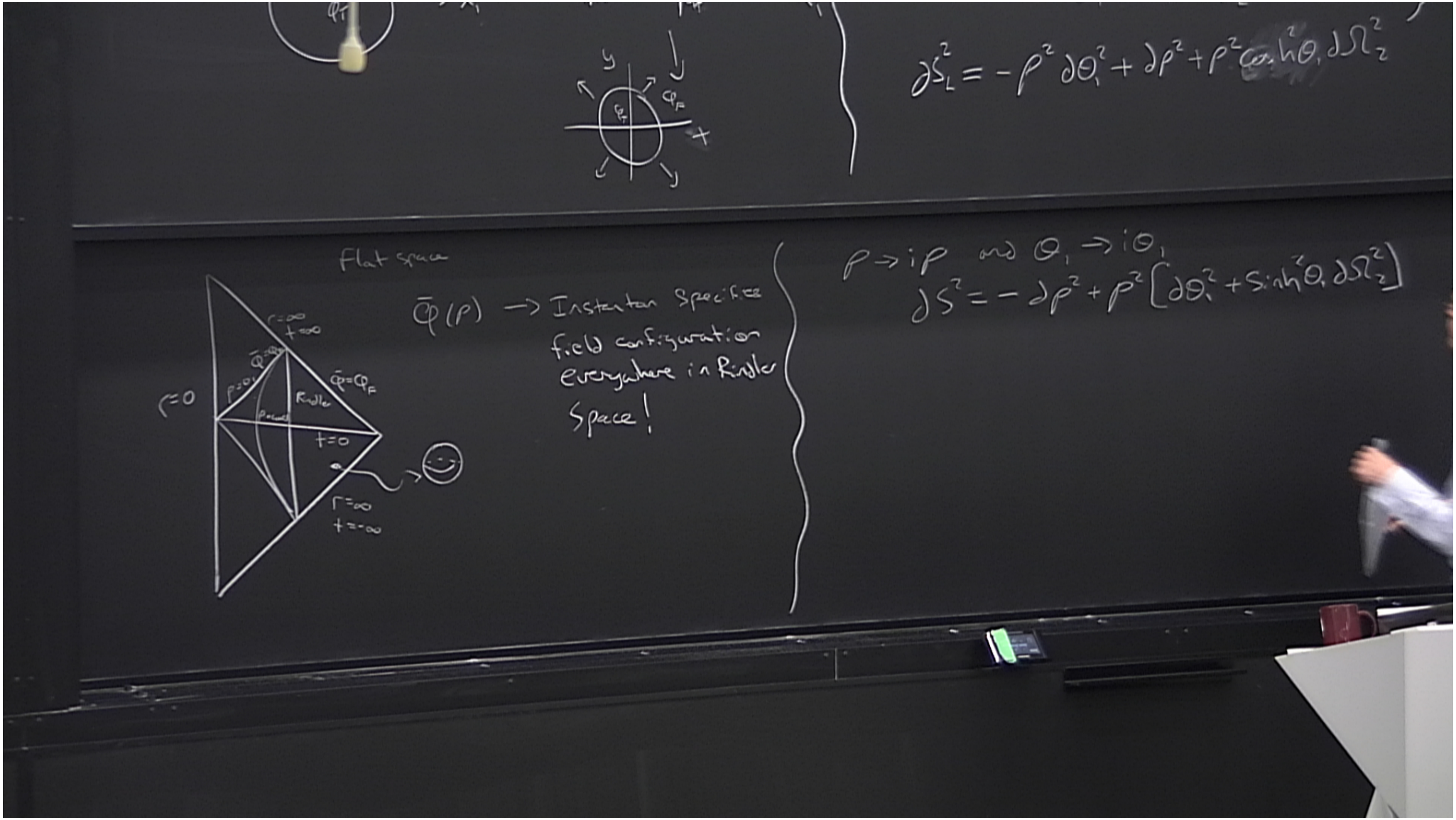

to q_T
Friction dies out before you move
q_*
$\bar{q} \to S_E(\bar{q})$, $A(\bar{q})$
4-zero modes
each zero mode
Multiply by
$\frac{\Gamma}{V_{ol}} = \left(\frac{S_E(\bar{q})}{2\pi\hbar}\right)^2 \left|\frac{\det[-\partial_\rho^2 + V''(q_0)]}{\det'[-\partial_\rho^2 + V''(\bar{q})]}\right|^{1/2} \hbar\, e^{-S_E(\bar{q})/\hbar}$
1 negative mode
Mass
$= \left(\frac{S_E(\bar{q})}{2\pi\hbar}\right)^2 V''^2_{(q_0)}\, e^{-S_E(\bar{q})/\hbar}$
$\xi \to \frac{T}{\pi}$
$\left(\frac{S_E(\bar{q})}{2\pi\hbar}\right)^{1/2}$

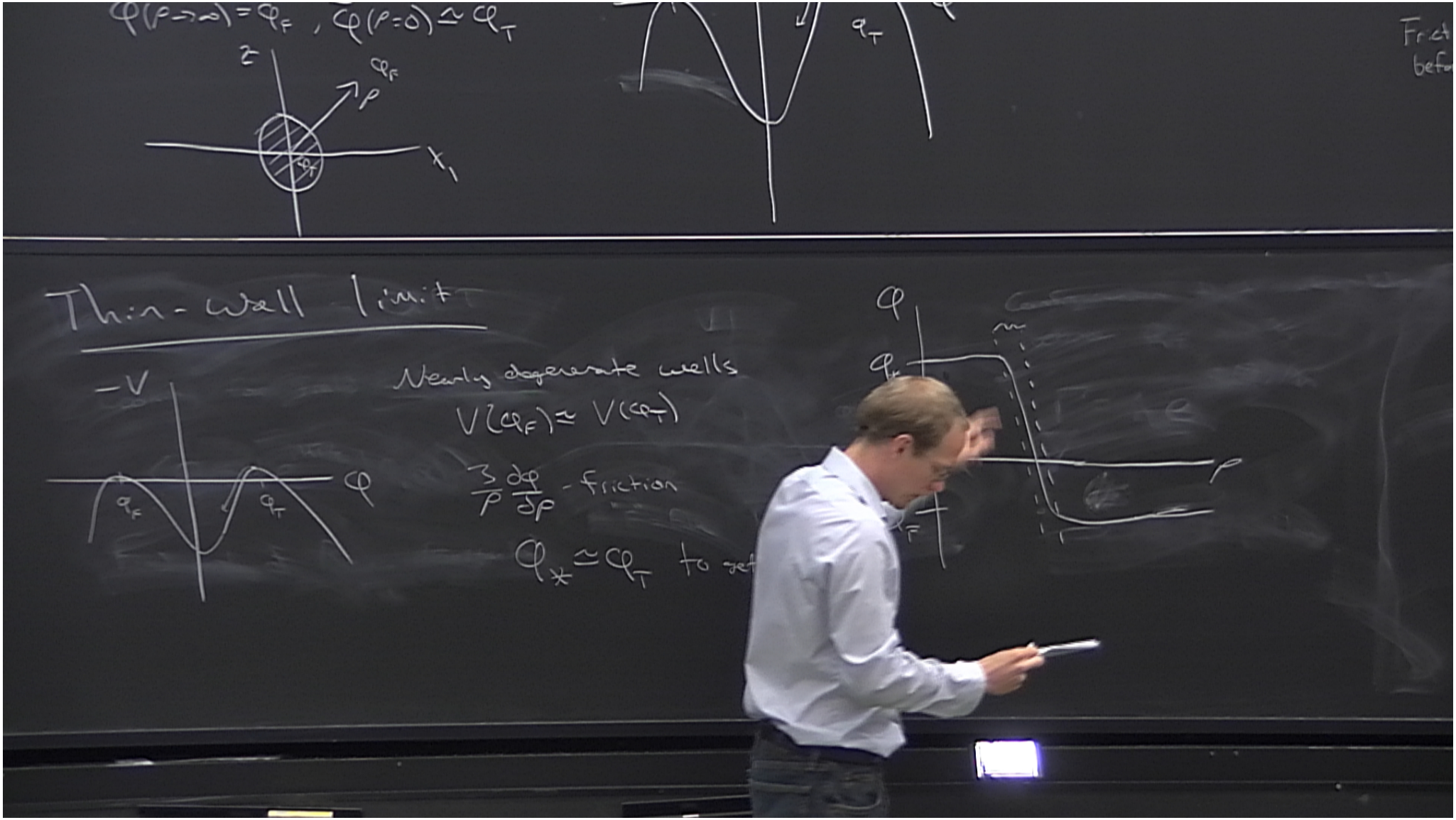

Configuration appears
τ
ρ
φF
φT
X1
τ = 0
τ → it
t
φF
t→0
φT
X1
y
φF
φT
t

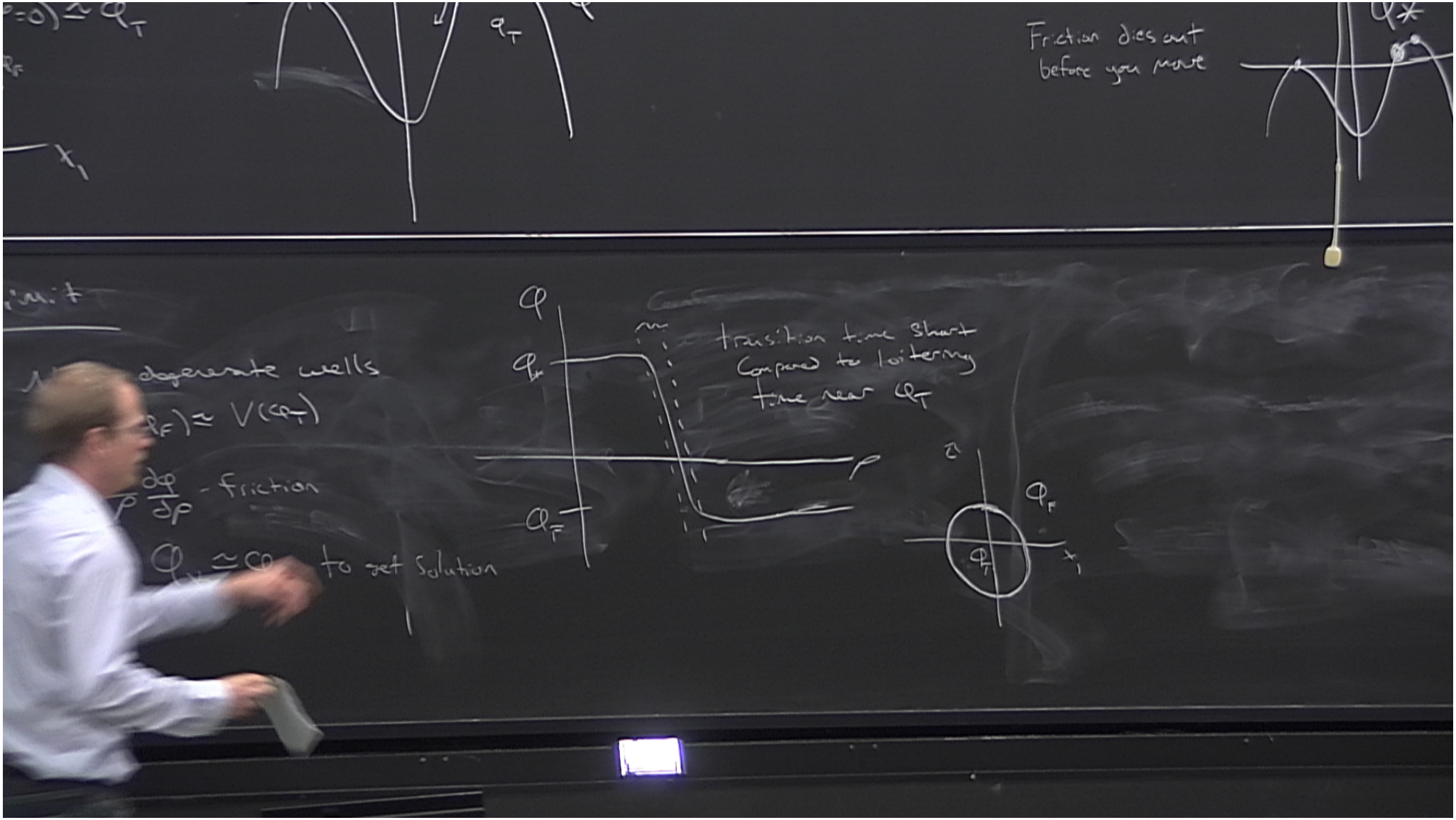

Configuration appears
τ
ρ
φF
φT
x1
τ = 0
τ → it
t
t→0
φT
φF
x1
y
φT
φF
x
φ = φ(ρ) = φ(√(τ² + x²))
ds² = dρ² + ρ² dΩ₃²

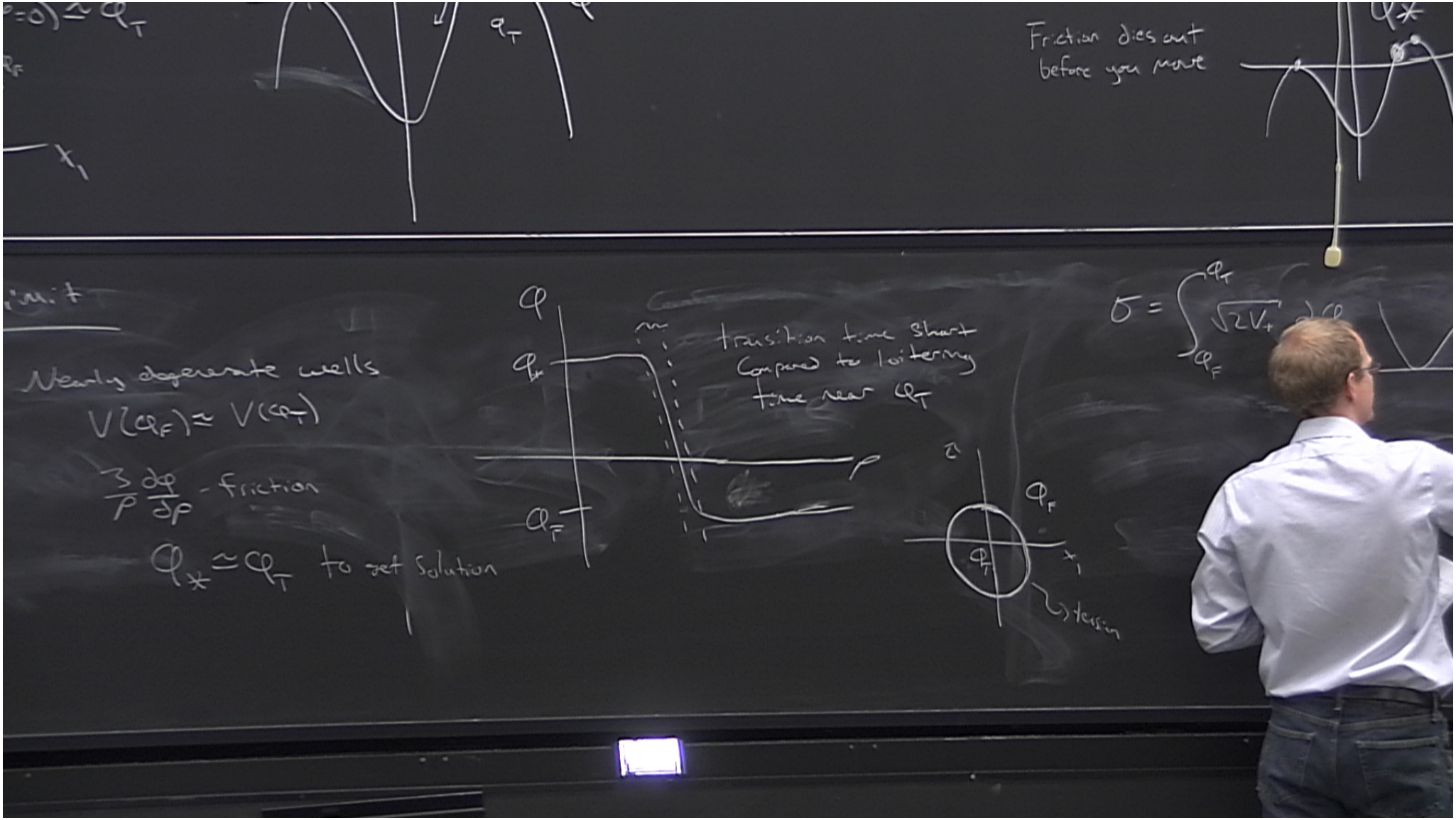

Configuration appears
z
ρ
φ_F
φ_T
x_1
z = 0
z → +t
+
φ_F
t→0
φ_T
x_1
y
φ_F
φ_T
+
$\varphi = \varphi(\rho) = \varphi(\sqrt{z^2+x^2})$
$ds^2 = d\rho^2 + \rho^2 d\Omega_3^2$
$= d\rho^2 + \rho^2 \{ d\theta_1^2 + \sin^2\theta_1 [d\theta_2^2 + \sin^2\theta_2 d\theta_3^2] \}$


Flat space
$\bar{\varphi}(\rho)$ → Instanton Specifies field configuration everywhere in Rindler Space!
r=0
r=∞
t=∞
$\bar{\varphi}=\varphi_F$
Rindler
t=0
r=∞
t=-∞


φ_F
y
φ_F
+
$ds_L^2 = -\rho^2 d\theta_1^2 + d\rho^2 + \rho^2 \cosh^2\theta_1 d\Omega_2^2$
Flat space
$\bar{\varphi}(\rho)$ → Instanton Specific field configuration everywhere in Rindler Space!
$\rho \to i\rho$ and $\theta_1 \to i\theta_1$
$ds^2 = -d\rho^2 + \rho^2 [d\theta_1^2 + \sinh^2\theta_1 d\Omega_2^2]$
r=0
r=∞
t=∞
$\bar{\varphi}=\varphi_F$
Rindler
t=0
r=∞
t=-∞

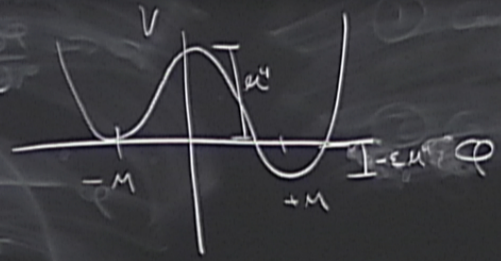

$\varphi(\rho\to\infty)=\varphi_F$, $\varphi(\rho=0)\simeq\varphi_T$
φ_F
ρ
x_1
φ_T
Thin-wall limit
Nearly degenerate wells
$V(\varphi_F)\simeq V(\varphi_T)$
$-V$
φ
φ_F
φ_T
$\frac{3}{\rho}\frac{d\varphi}{d\rho}$ - friction
$\varphi_*\simeq\varphi_T$ to


Friction dies out before you move
degenerate wells
friction
to get Solution
transition time Short
Compared to loitering
time near


Friction dies out before you move
Nearly degenerate wells
$V(q_F) \approx V(q_T)$
$\frac{3}{p}\frac{dp}{dp}$ - friction
$q_* \simeq q_T$ to get solution
transition time short compared to loitering time near q_T
$\sigma = \int_{q_F}^{q_T} \sqrt{2V_+}\, dq$
tension

Coleman '77

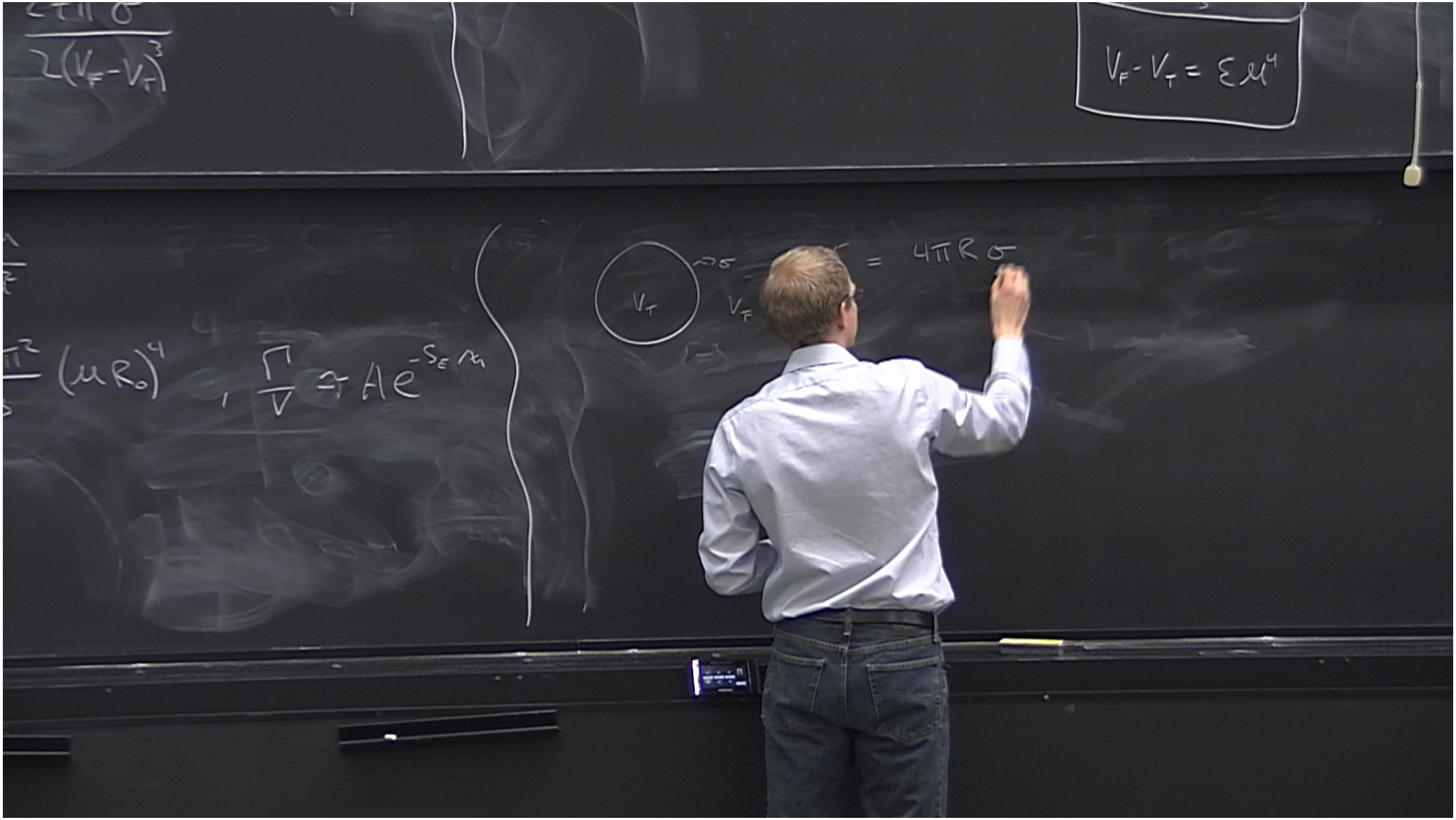
Example: $V(\varphi) = \mu^4 \left(\left(\frac{\varphi}{M} \right)^2 - 1 \right)^2 + \mu^3 \varepsilon \frac{\varphi}{M}$



① $\sigma = \int_{-M}^{M} \sqrt{2V_+} = \int_{-M}^{M} \sqrt{2}\,\mu^2 \left(\left(\frac{\varphi}{M} \right)^2 - 1 \right) d\varphi$

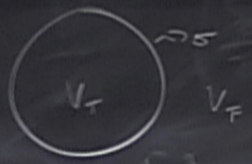
$\sigma = \frac{4\sqrt{2}}{3} M\mu^2$

$V_F - V_T = \varepsilon\mu^4$


$V_F - V_T = \varepsilon M^4$
V_T
V_F
$= 4\pi R\sigma$
$\frac{\Gamma}{V} \sim A e^{-S_E/\hbar}$


$V_F - V_T = \varepsilon \mu^4$
V_T
V_F
$\frac{4\pi R^2 \sigma}{\sqrt{1-\dot{R}^2}} - \frac{4\pi}{3} R^3 (V_F - V_T)$
$\frac{\Gamma}{V} \sim A e^{-S_E/\hbar}$
$R = 0$
$\Rightarrow R_0 = \frac{3\sigma}{V_F - V_T}$

$V_F - V_T = \varepsilon \mu^4$



$$\mathcal{E} = \frac{4\pi R^2 \sigma}{\sqrt{1-\dot{R}^2}} - \frac{4\pi}{3} R^3 (V_F - V_T)$$

Critical Bubble – $\mathcal{E} = 0$

at $t=0$ $\dot{R}=0$

$$\Rightarrow R_0 = \frac{3\sigma}{V_F - V_T}$$

$$R(t) = \sqrt{t^2 + R_0^2}$$

$t \sim R_0^{-1}$

Wall is moving at c!