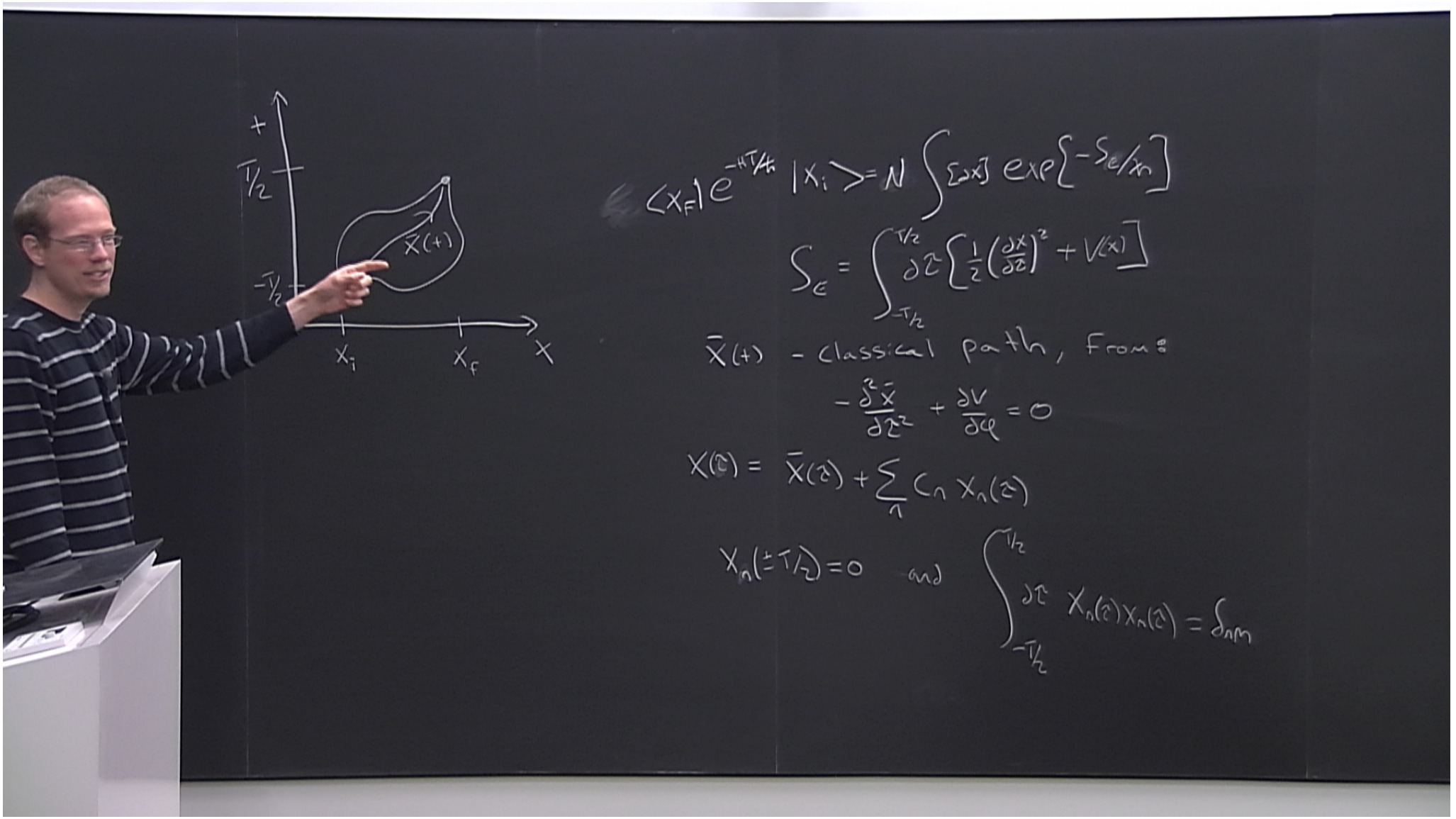
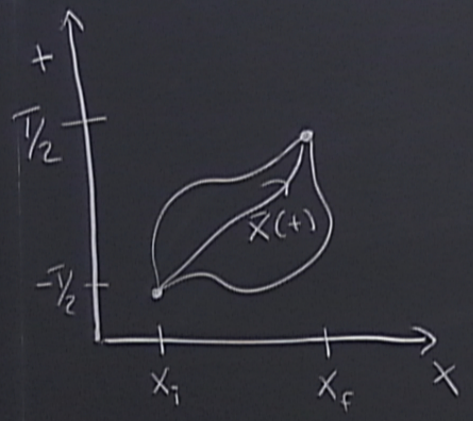
Title: Explorations in Cosmology - Lecture 11

Date: Mar 26, 2012 10:15 AM

URL: http://pirsa.org/12030037

Abstract:


+
T/2
-T/2
$\bar{x}(t)$
x_i
x_f
x
$\langle x_F | e^{-iHT/\hbar} | x_i \rangle = N \int \mathcal{D}x \exp\left[-S_E/\hbar\right]$
$S_E = \int_{-T/2}^{T/2} d\tau \left\{ \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + V(x) \right\}$
$\bar{x}(t)$ - classical path, From:
$-\frac{d^2\bar{x}}{d\tau^2} + \frac{dV}{dq} = 0$
$x(\tau) = \bar{x}(\tau) + \sum_n c_n x_n(\tau)$
$x_n(\pm T/2) = 0$ and $\int_{-T/2}^{T/2} d\tau \; x_n(\tau) x_n(\tau) = \delta_{nm}$



$$\langle x_F | e^{-iHT/\hbar} | x_i \rangle = N \int \mathcal{D}x \exp\left[-S_E/\hbar\right]$$

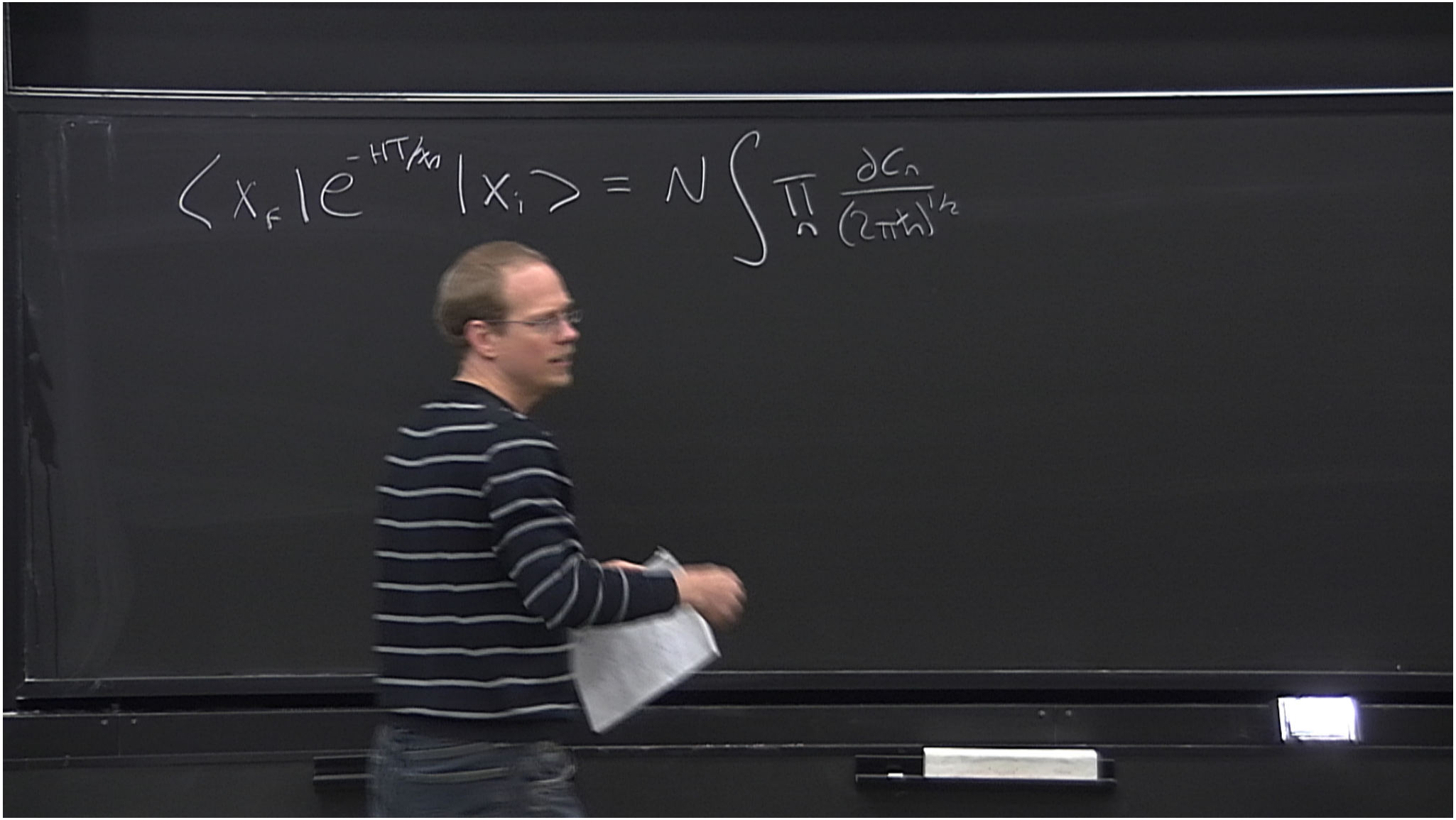
$$S_E = \int_{-T/2}^{T/2} d\tau \left[\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + V(x)\right]$$

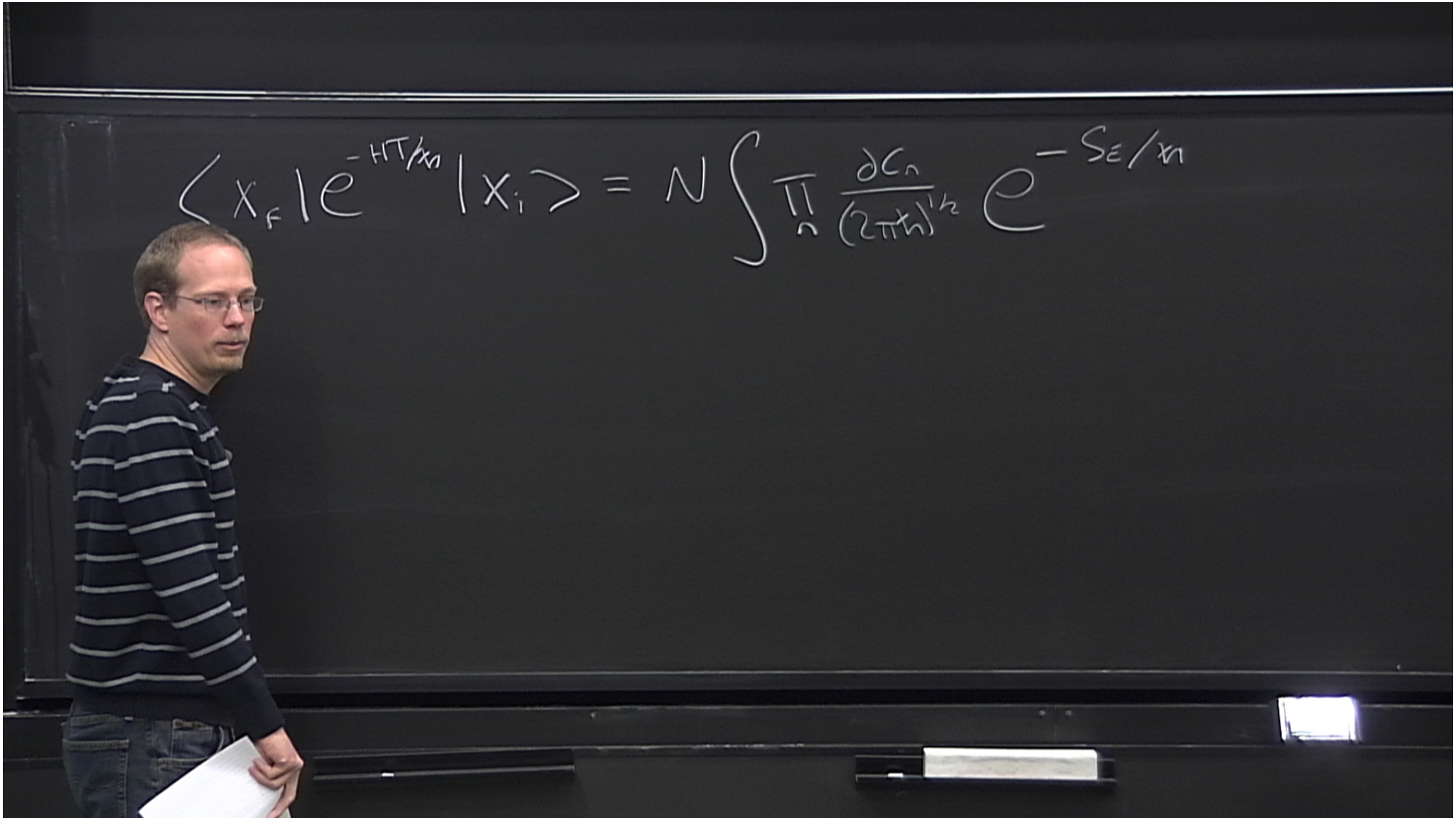
$\bar{x}(t)$ - classical path, From:

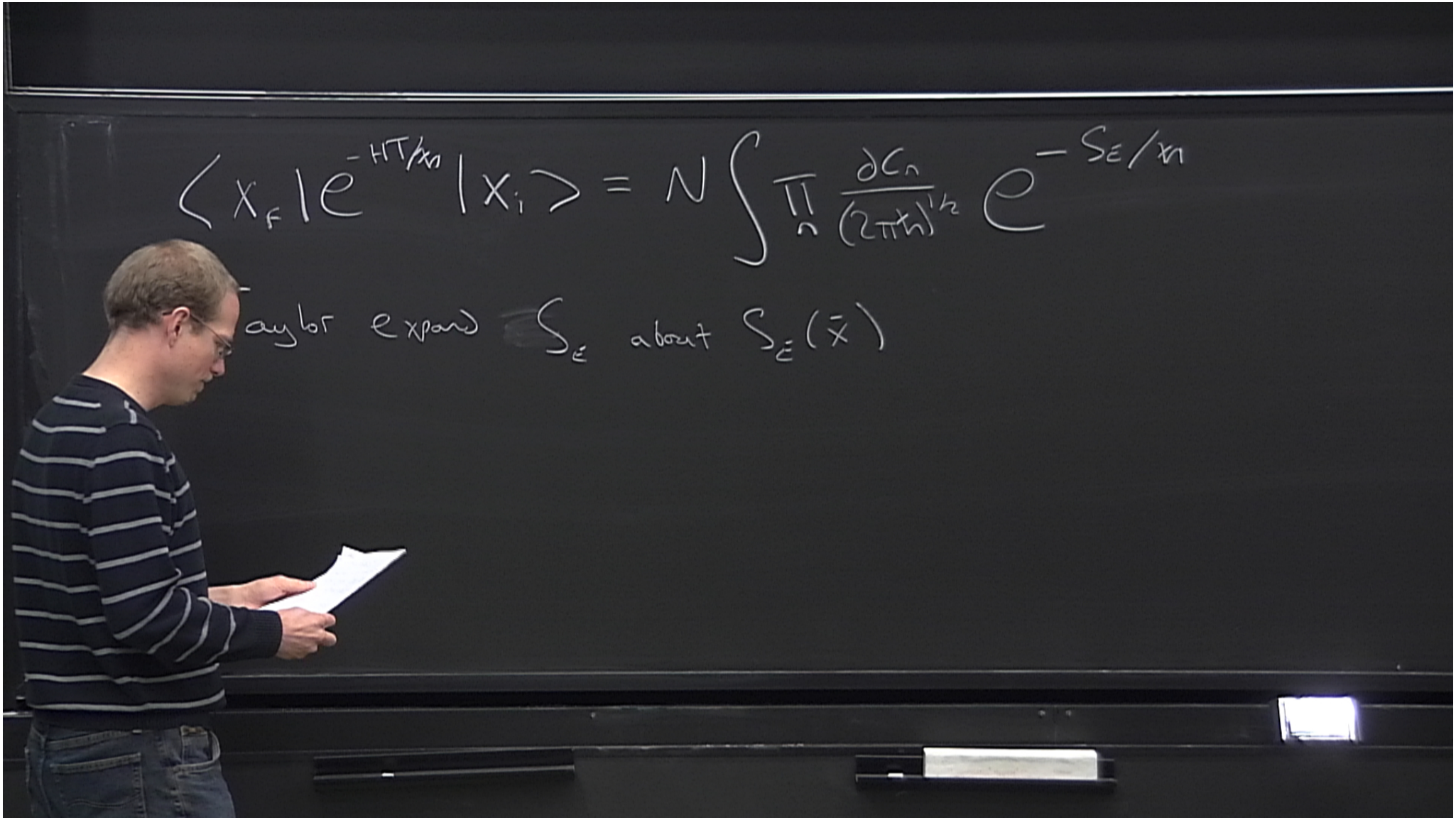
$$-\frac{d^2\bar{x}}{d\tau^2} + \frac{dV}{dq} = 0$$

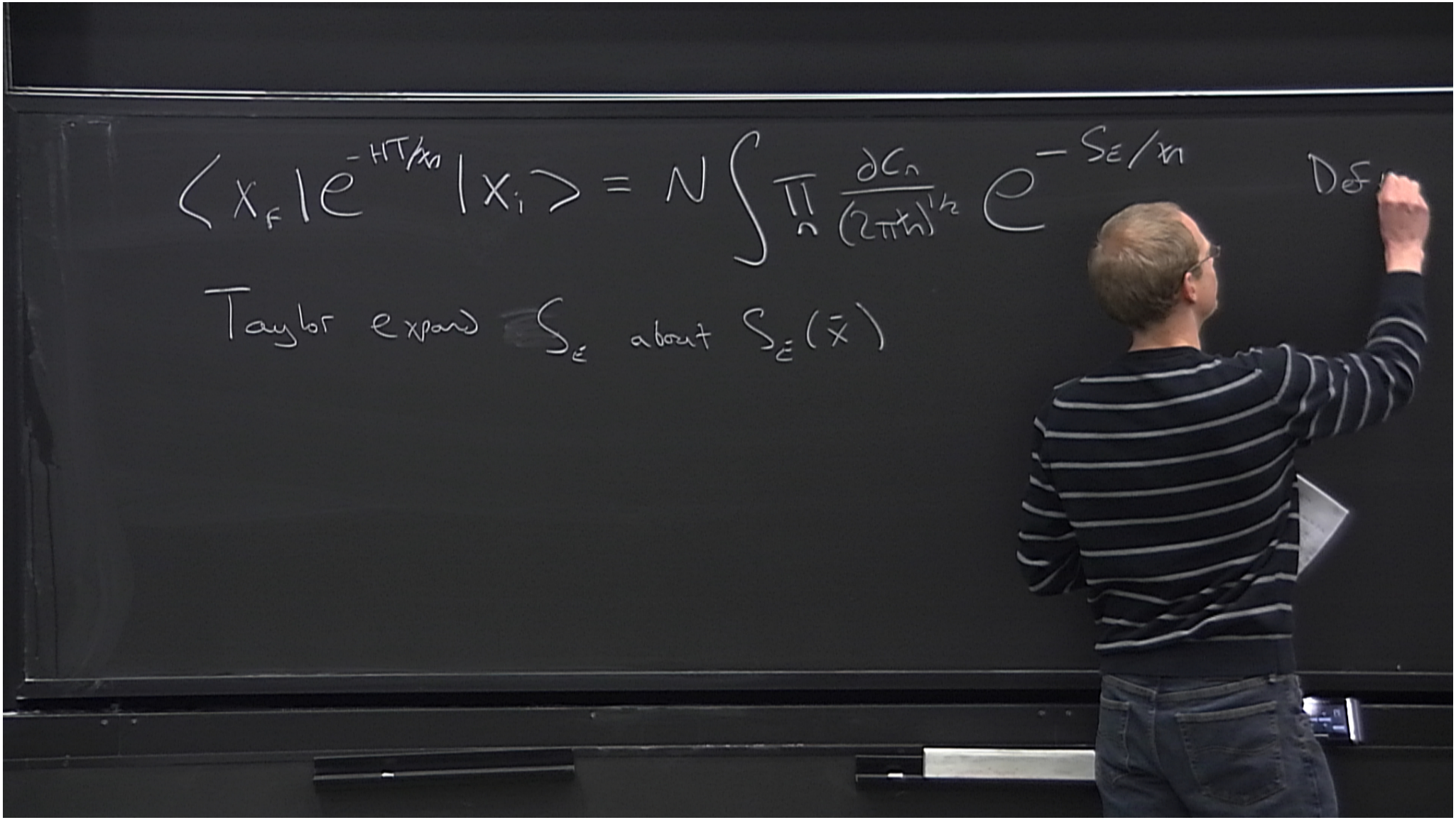
$$x(\tau) = \bar{x}(\tau) + \sum_n c_n x_n(\tau)$$

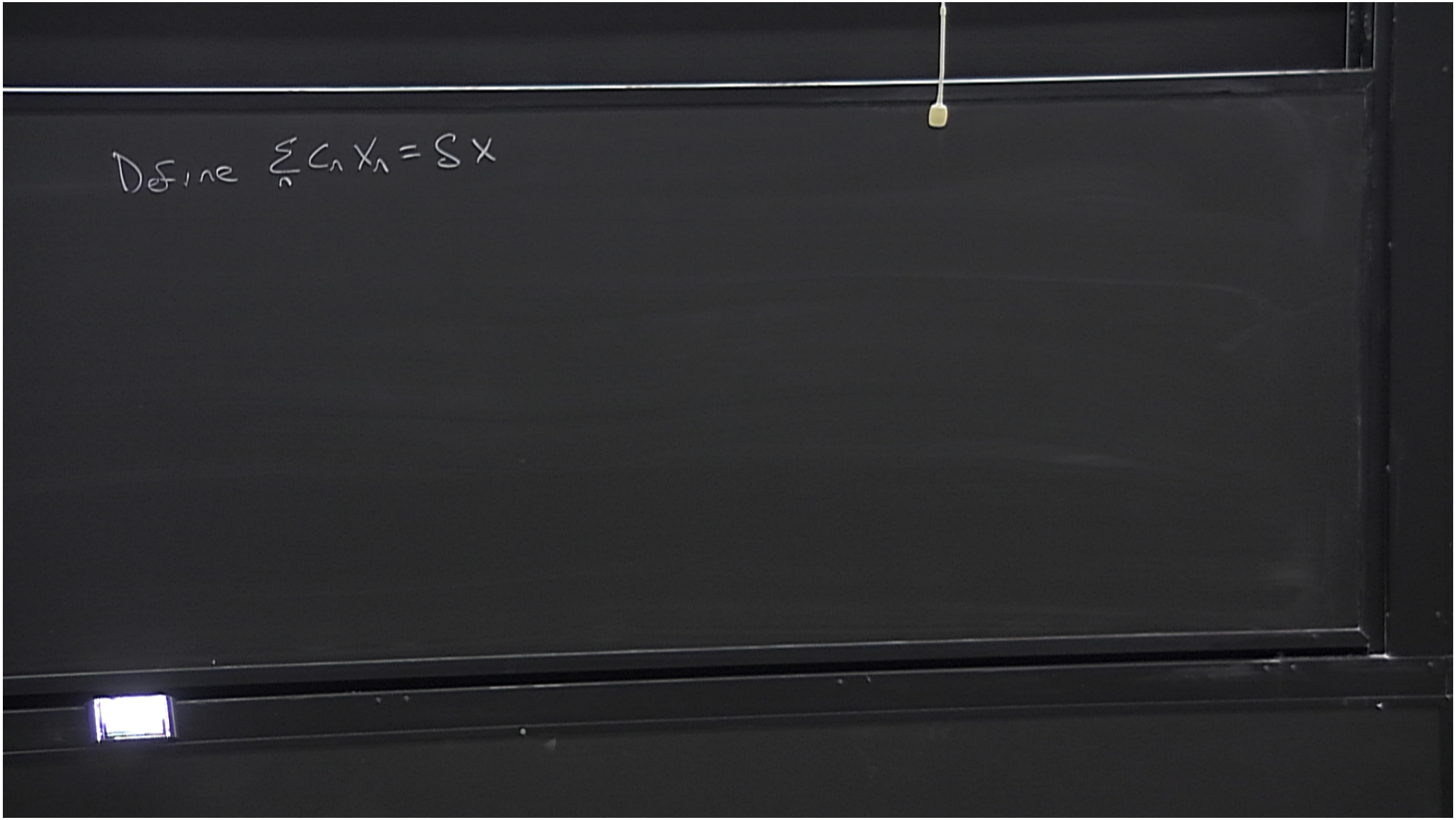
$$x_n(\pm T/2) = 0 \quad \text{and} \quad \int_{-T/2}^{T/2} d\tau \; x_n(\tau) x_n(\tau) = \delta_{nm}$$

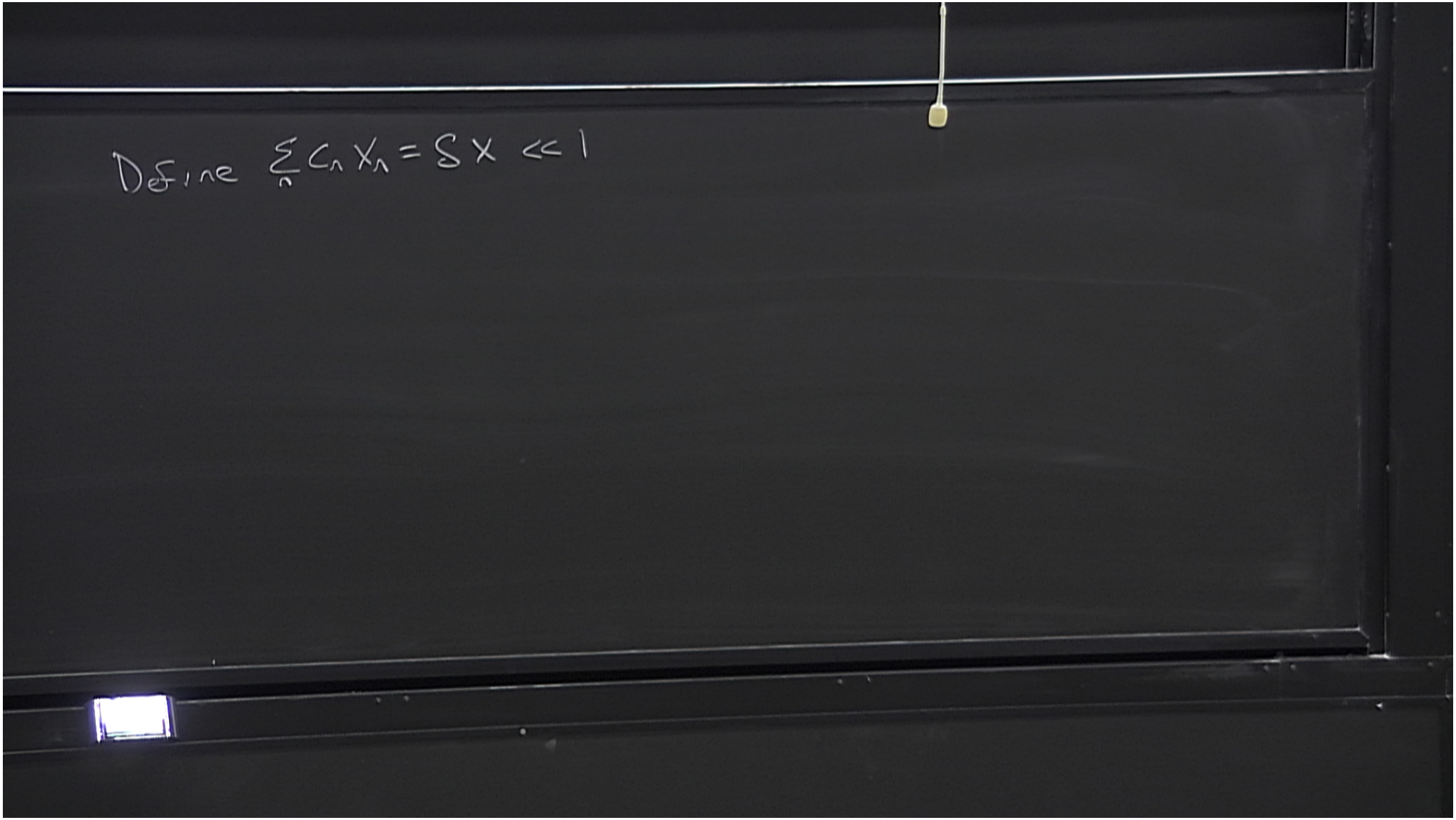

$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dC_n}{(2\pi\hbar)^{1/2}}$


$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dC_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$


$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dC_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$
Taylor expand S_E about $S_E(\bar{x})$


$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dx_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$
Taylor expand S_E about $S_E(\bar{x})$


Define $\sum_n c_n X_n = S X$

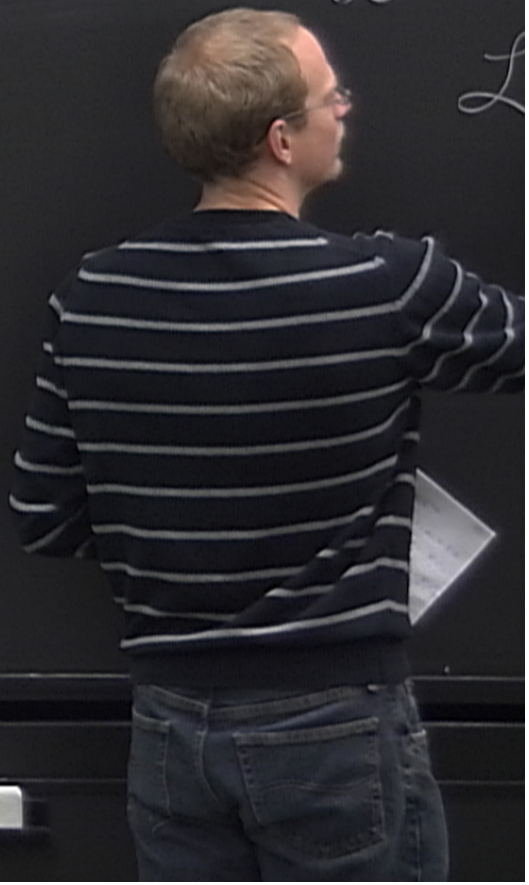

Define $\sum_n c_n \chi_n = S\chi \ll 1$

$$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dx_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$$

Define $\mathcal{L}$

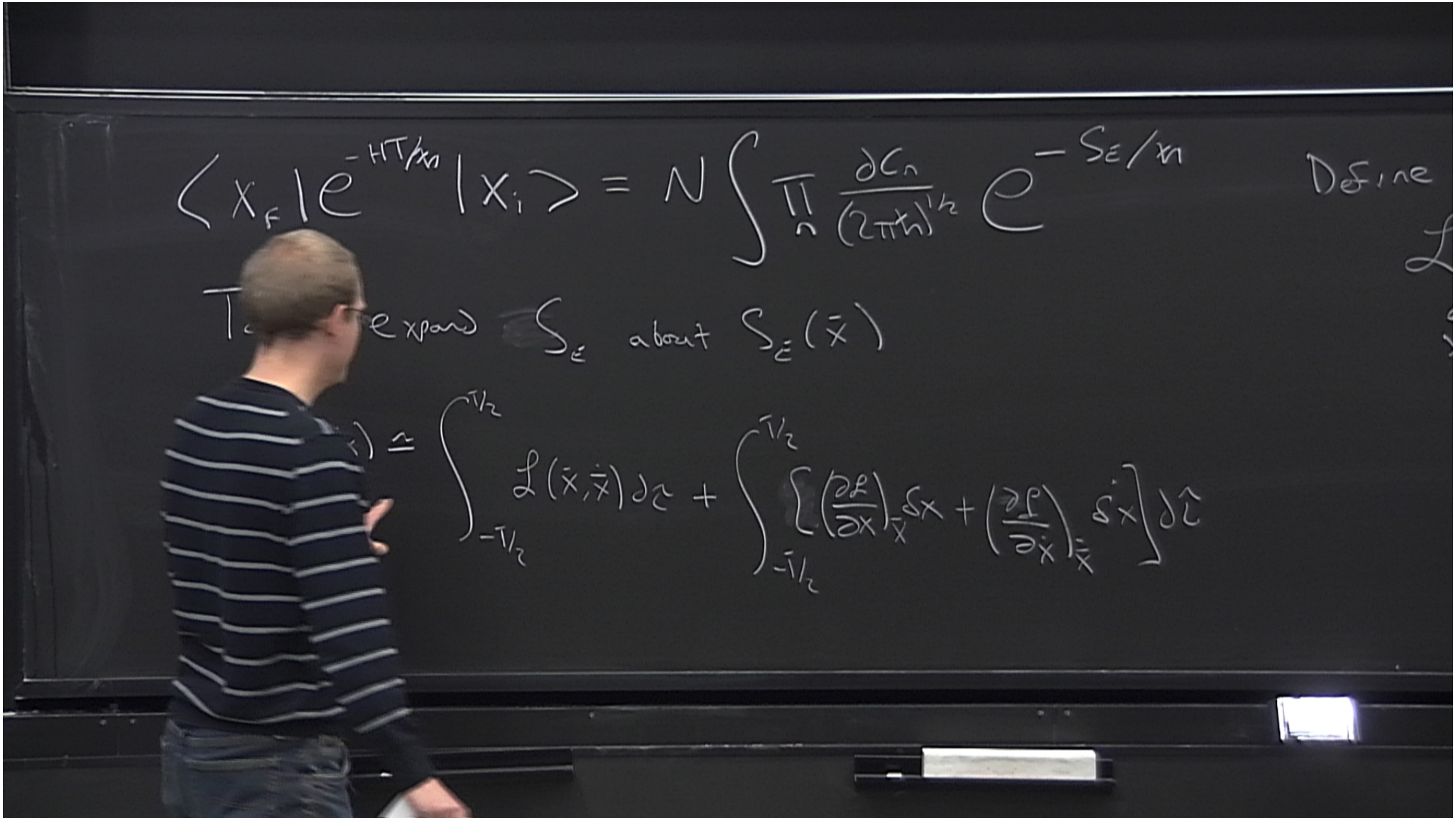
Taylor expand S_E about $S_E(\bar{x})$

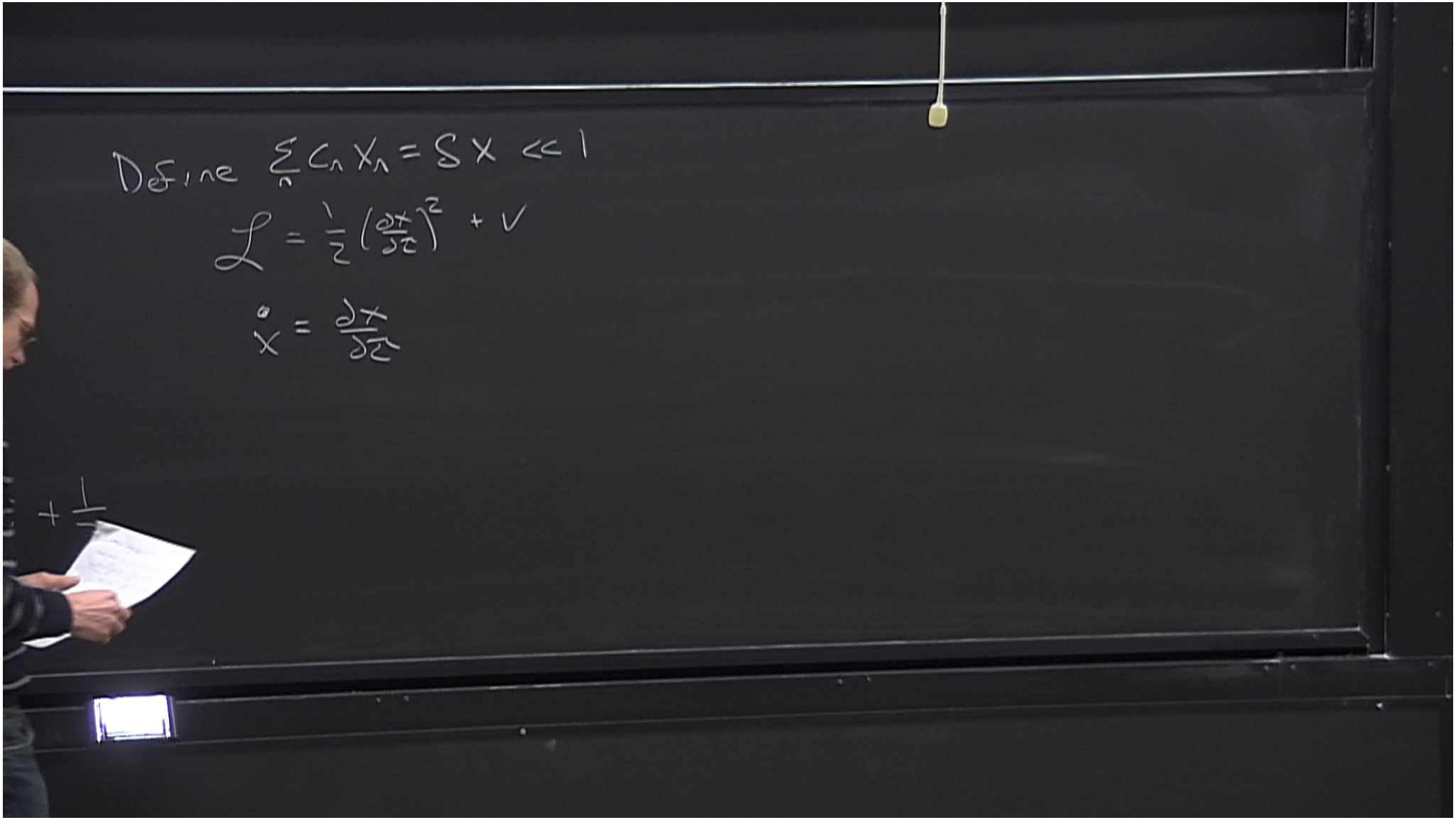
$$S_E(\bar{x} + \delta x) \simeq \int_{-T/2}^{T/2} \mathcal{L}(\bar{x}, \dot{\bar{x}})\, d\tau$$



Define $\sum_n c_n X_n = \delta X \ll 1$

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial x}{\partial t}\right)^2 + V$$


$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dC_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$
Define
expand S_E about $S_E(\bar{x})$
$\simeq \int_{-T/2}^{T/2} \mathcal{L}(\bar{x}, \dot{\bar{x}})\, d\tau + \int_{-T/2}^{T/2} \left[\left(\frac{\partial \mathcal{L}}{\partial x}\right)_{\bar{x}} \delta x + \left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right)_{\bar{x}} \delta \dot{x} \right] d\tau$

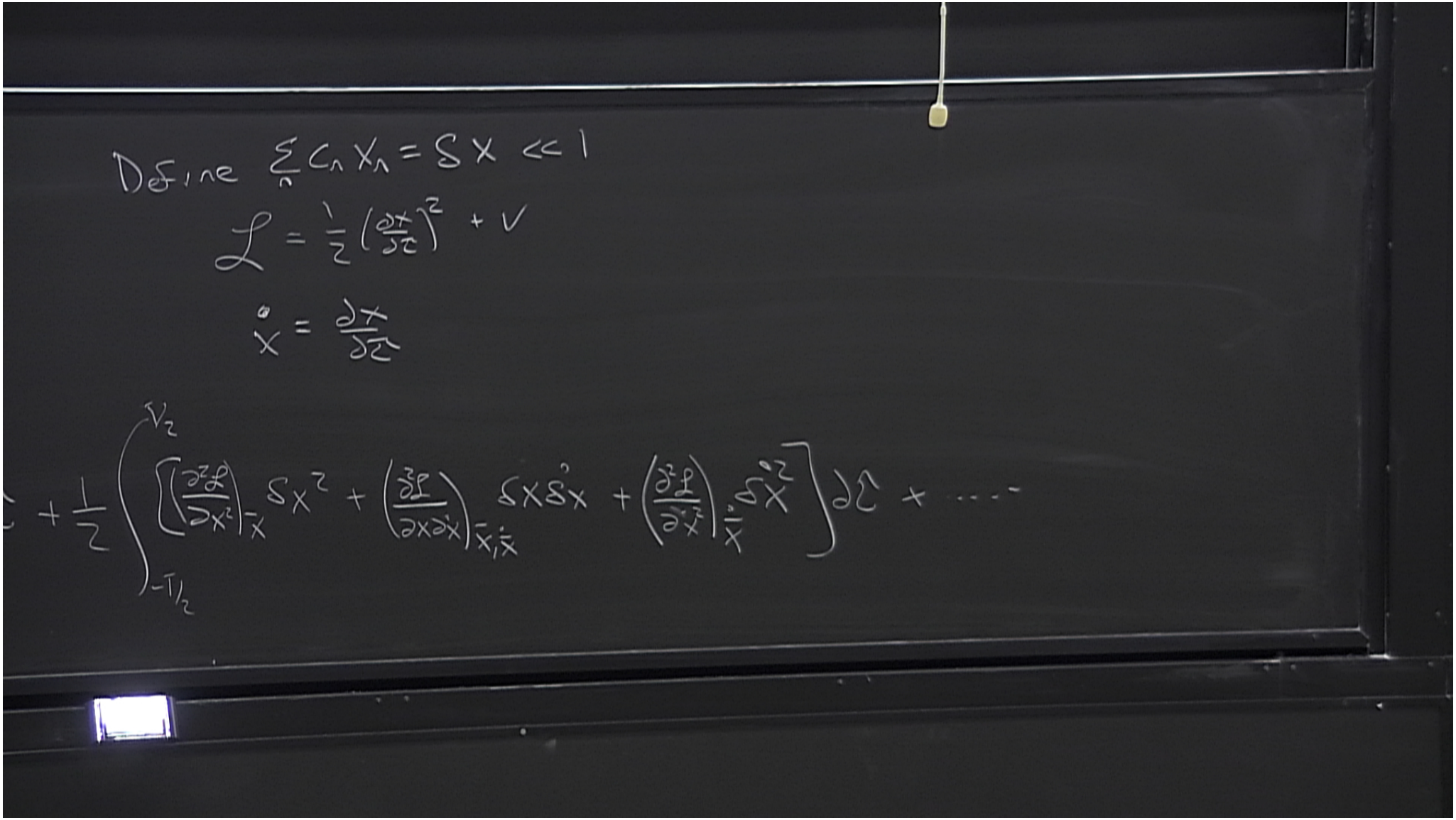

Define $\sum_n c_n X_n = S X \ll 1$
$\mathcal{L} = \frac{1}{2}\left(\frac{\partial x}{\partial z}\right)^2 + V$
$\dot{x} = \frac{\partial x}{\partial z}$
$+\frac{1}{2}$

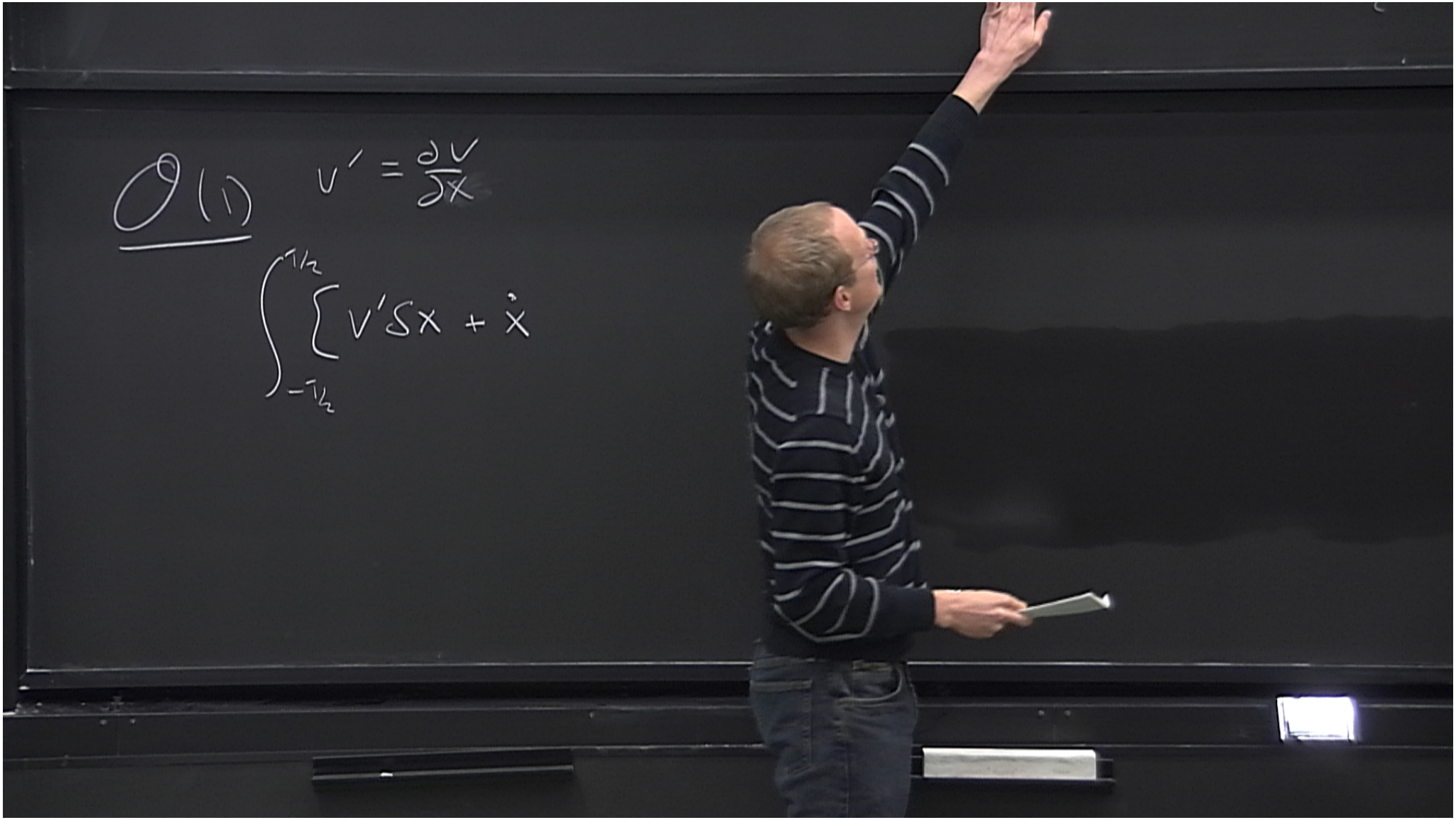
$$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dx_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$$

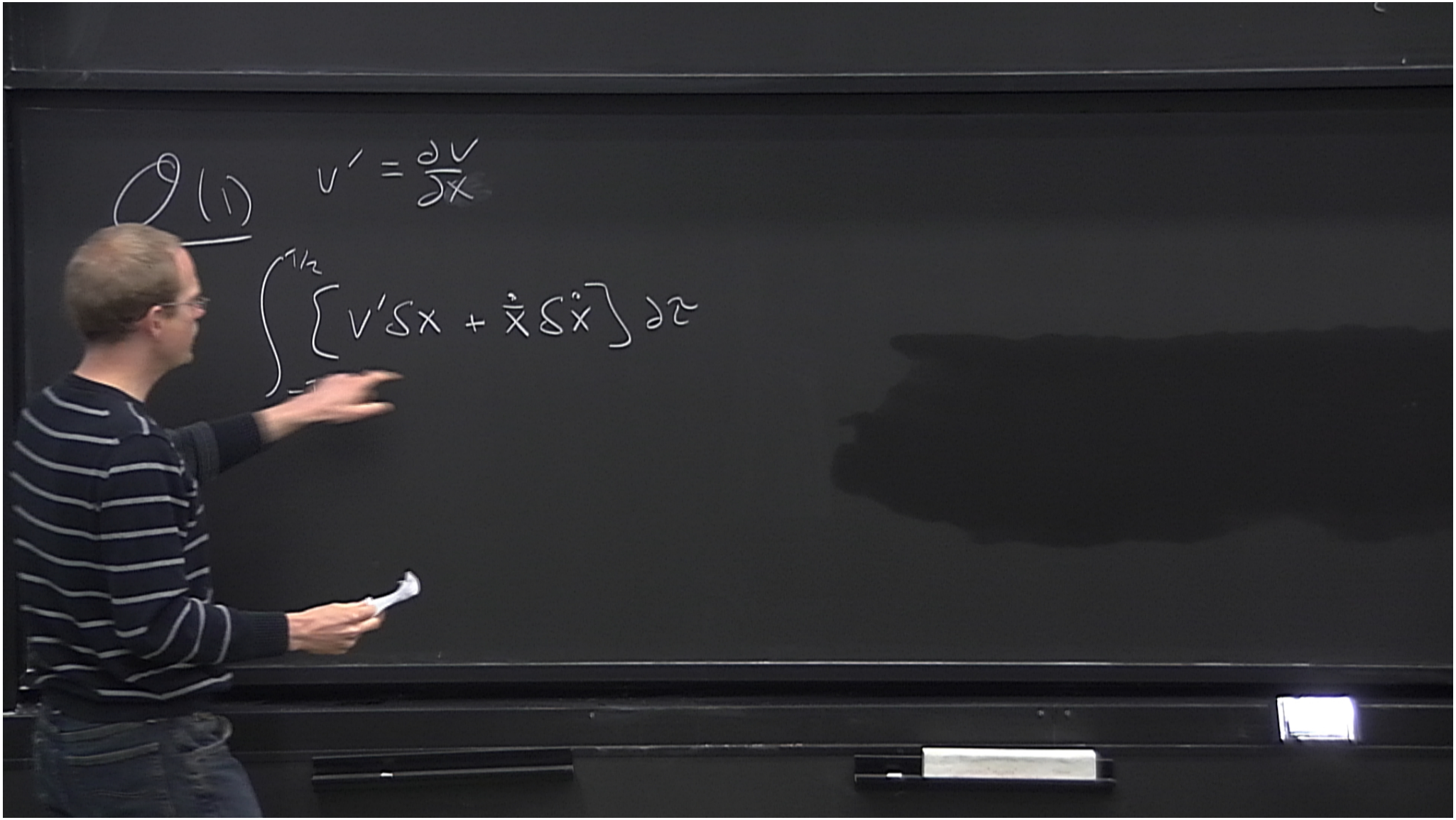
Define

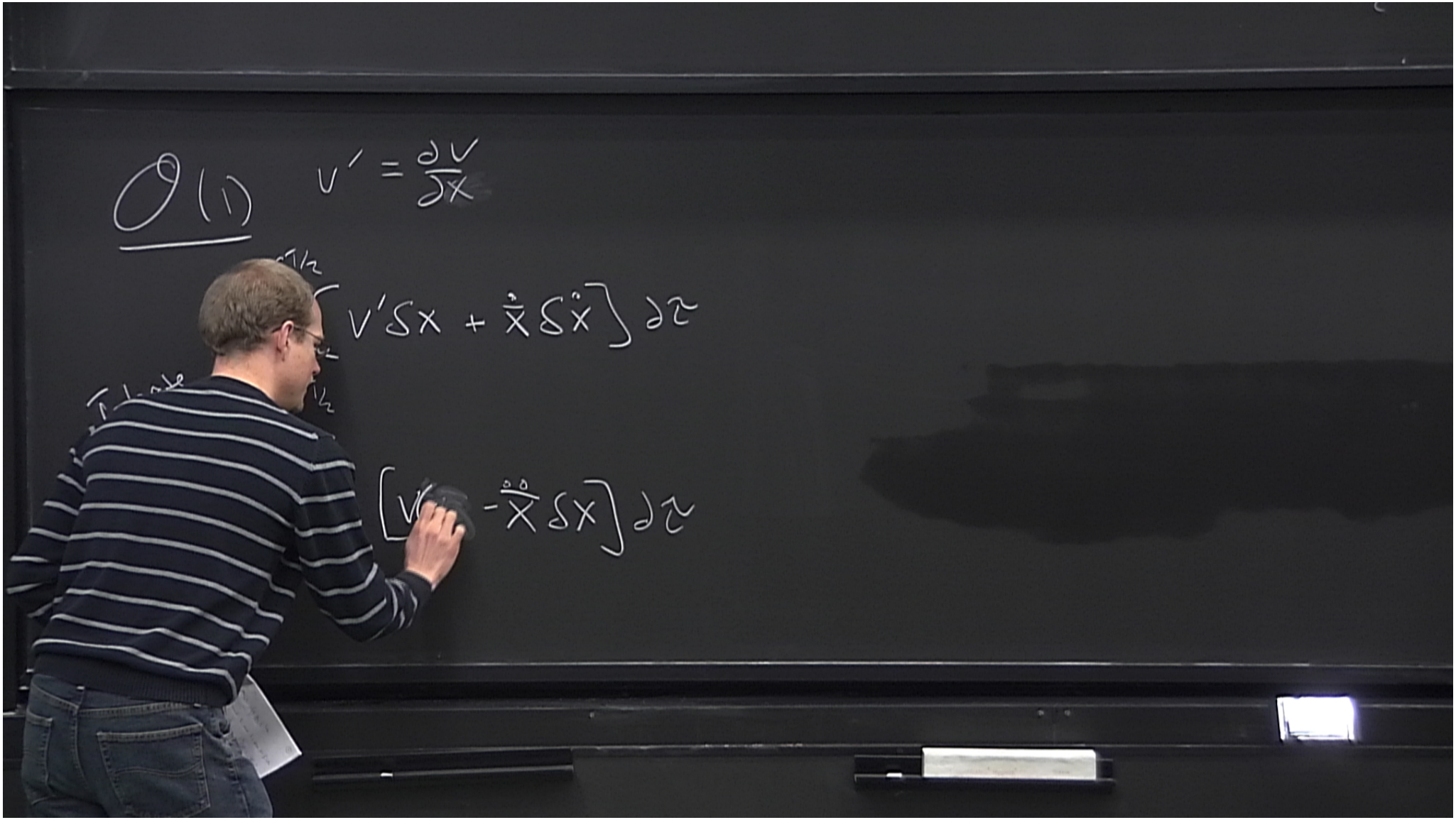
Taylor expand S_E about $S_E(\bar{x})$

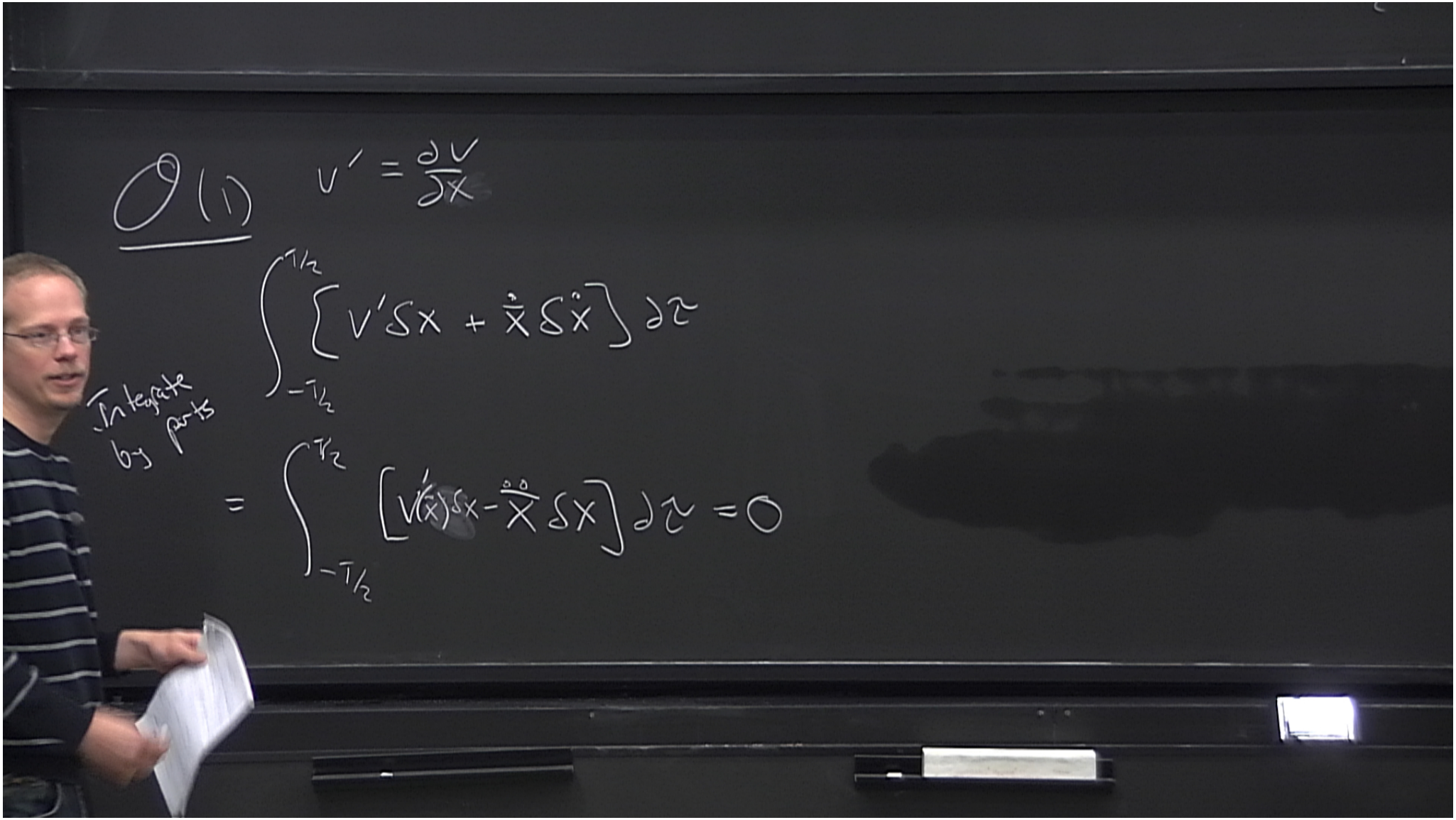
$$S_E(\bar{x}+\delta x) \simeq \int_{-T/2}^{T/2} \mathcal{L}(\bar{x},\dot{\bar{x}})\,d\tau + \int_{-T/2}^{T/2} \left\{ \left(\frac{\partial \mathcal{L}}{\partial x}\right)_{\bar{x}} \delta x + \left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right)_{\dot{\bar{x}}} \delta \dot{x} \right] d\tau +$$

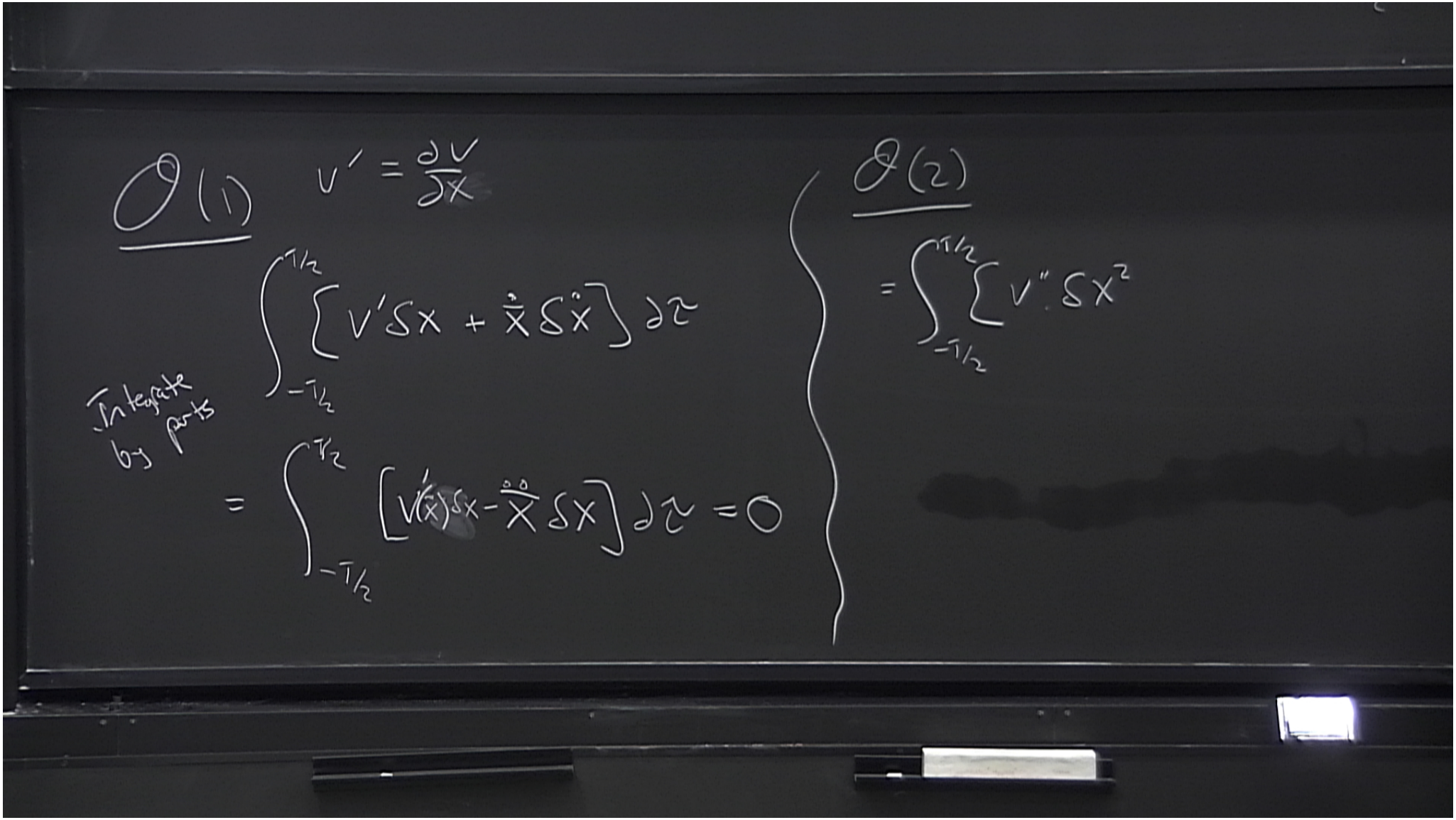

Define $\sum_n c_n X_n = \delta X \ll 1$
$\mathcal{L} = \frac{1}{2}\left(\frac{\partial x}{\partial \xi}\right)^2 + V$
$\dot{x} = \frac{\partial x}{\partial \xi}$
$+ \frac{1}{2}\int_{-T/2}^{T/2}\left[\left(\frac{\partial^2 \mathcal{L}}{\partial x^2}\right)_{\bar{x}} \delta x^2 + \left(\frac{\partial^2 \mathcal{L}}{\partial x \partial \dot{x}}\right)_{\bar{x},\dot{\bar{x}}} \delta x \delta \dot{x} + \left(\frac{\partial^2 \mathcal{L}}{\partial \dot{x}^2}\right)_{\dot{\bar{x}}} \delta \dot{x}^2\right] d\xi + \cdots$

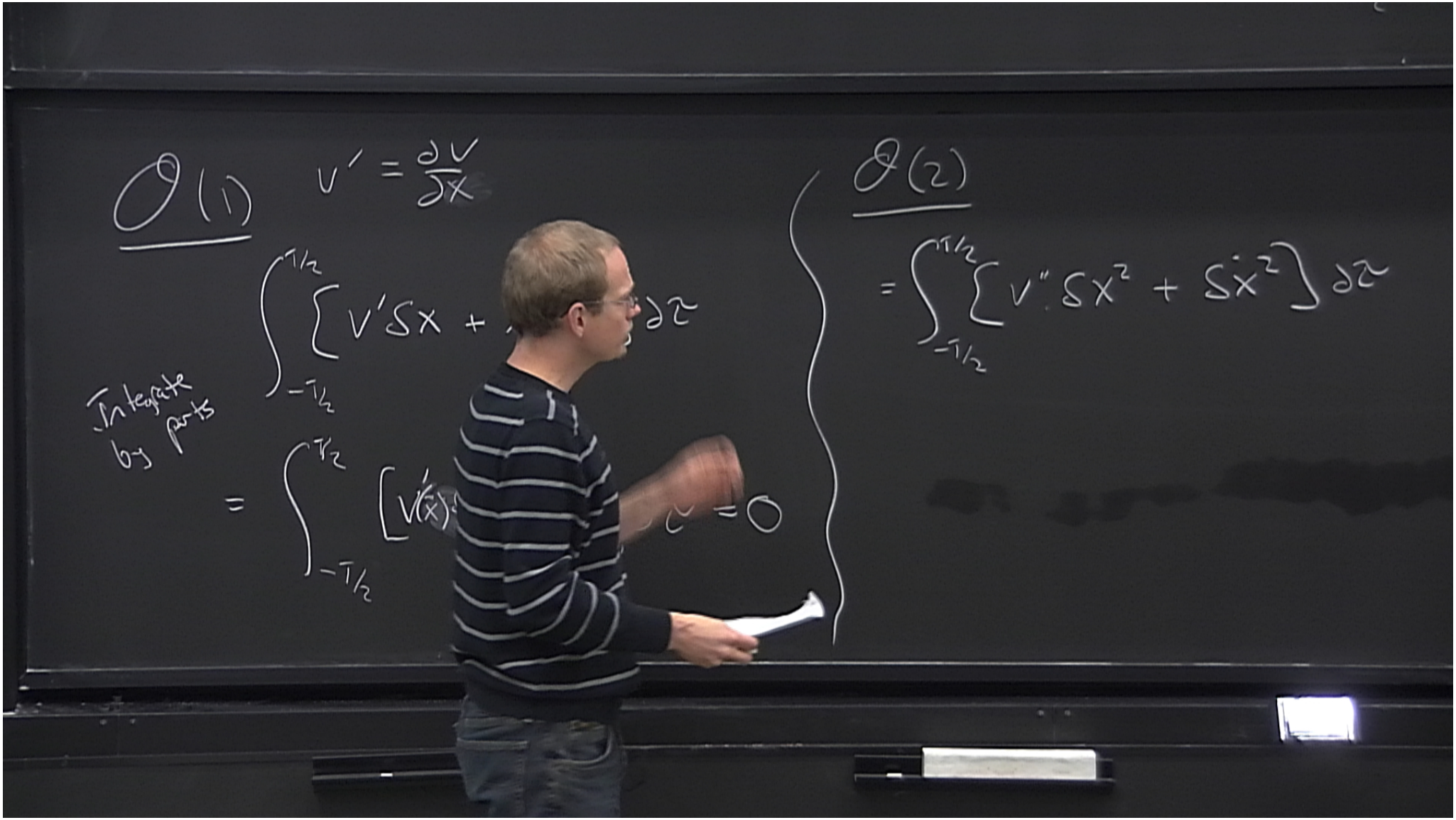

O(1)
$V' = \frac{\partial V}{\partial x}$
$\int_{-T/2}^{T/2} \{ V'\delta x + \dot{x}$

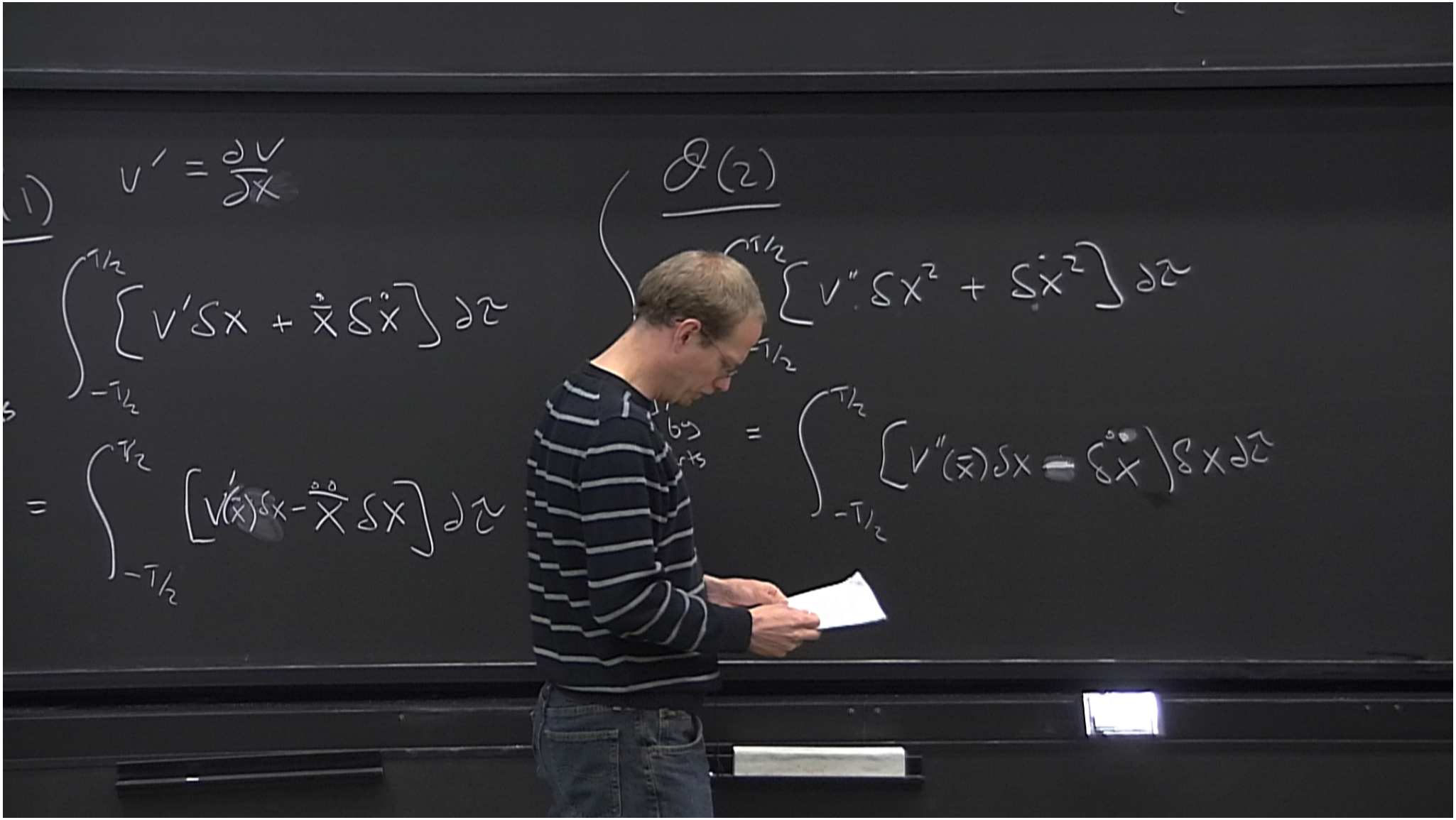

O(1)
$v' = \frac{\partial v}{\partial x}$
$\int_{-\pi/2}^{\pi/2} [v' \delta x + \dot{x} \delta \dot{x}] \, dz$

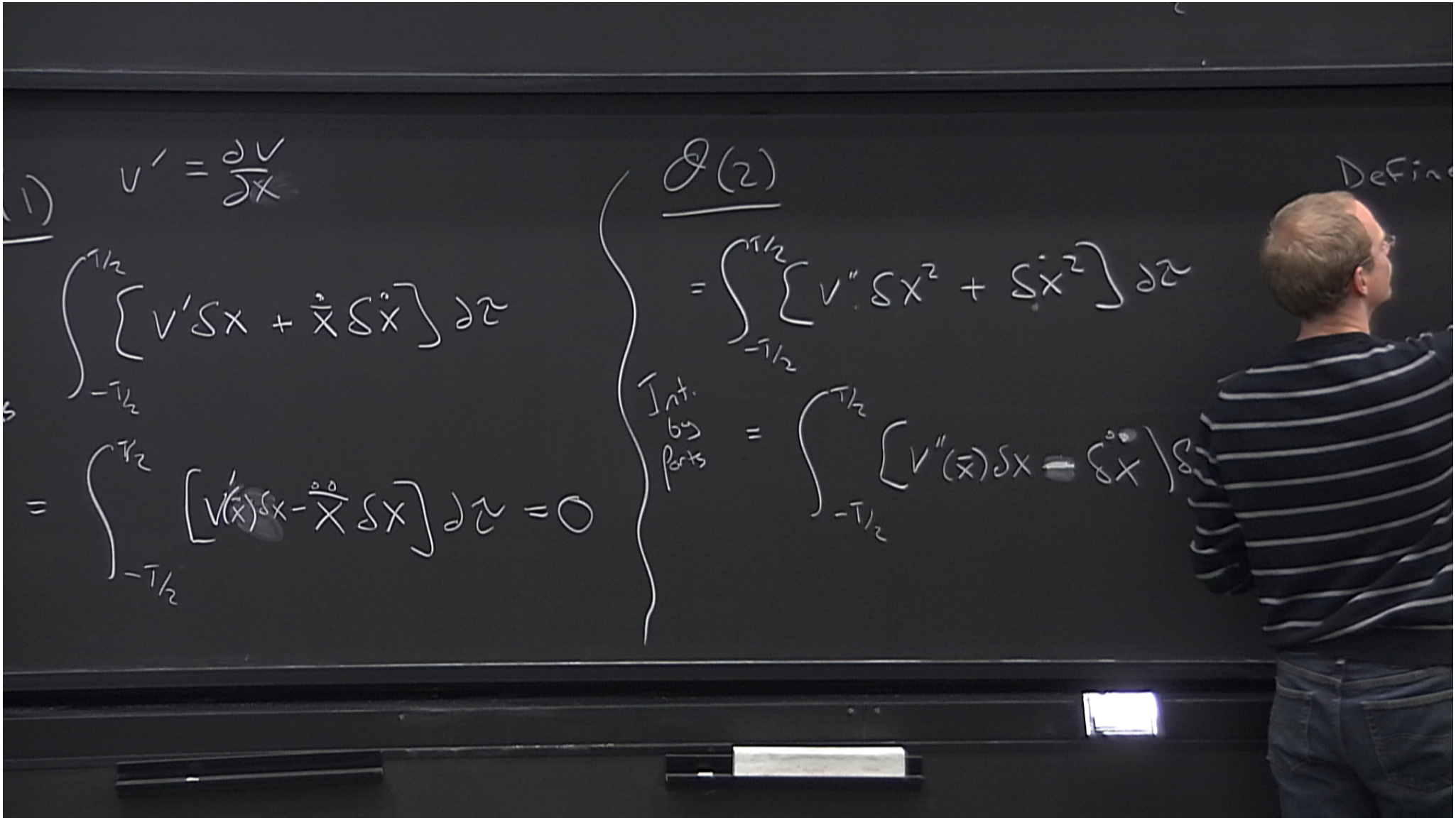

O(1)
v' = ∂v/∂x

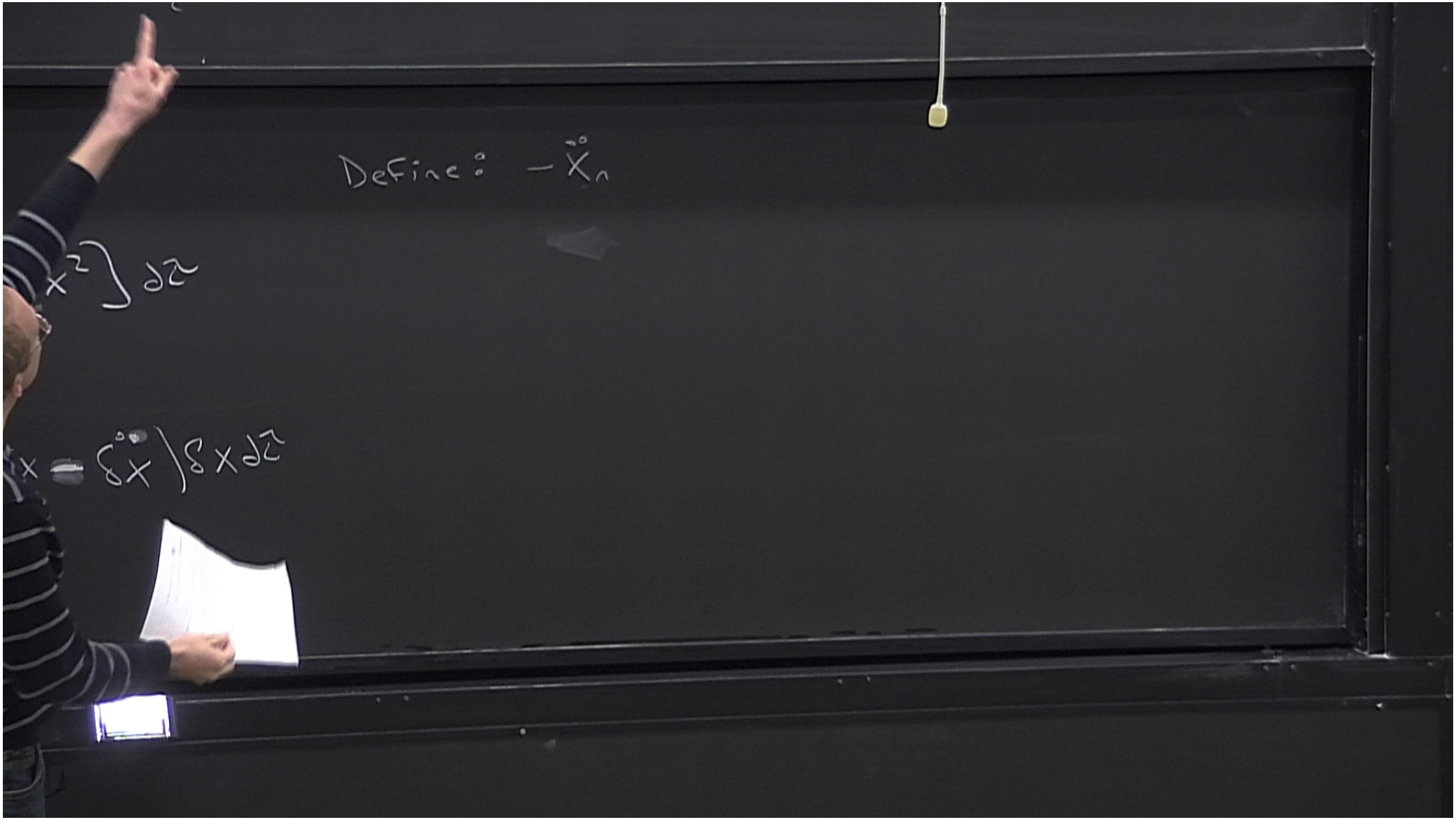

O(1)
$v' = \frac{\partial v}{\partial x}$
$\int_{-T/2}^{T/2} \left[v' \delta x + \dot{x} \delta \dot{x} \right] dt$
Integrate by parts
$= \int_{-T/2}^{T/2} \left[v(x) \delta x - \ddot{x} \delta x \right] dt = 0$

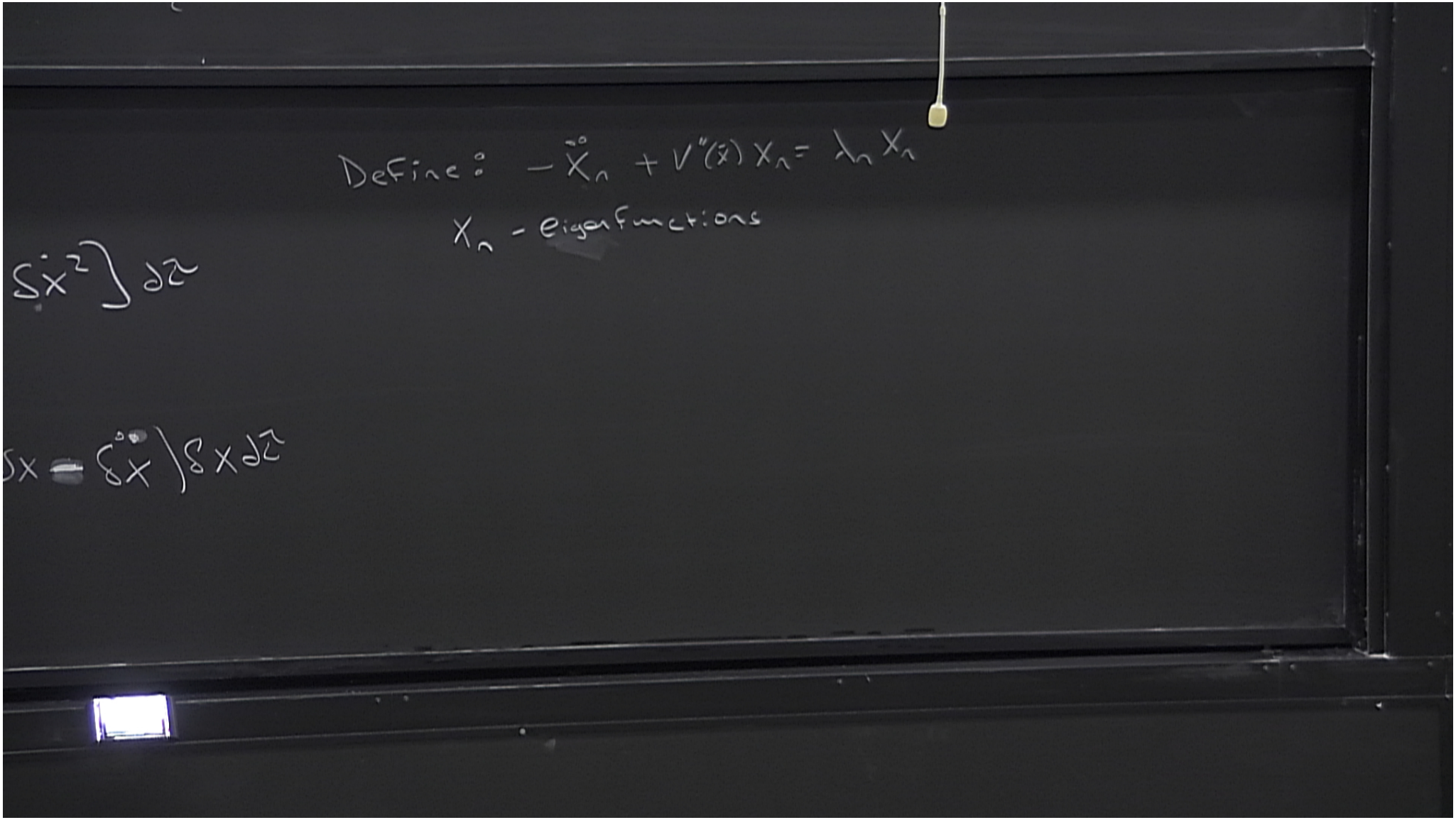

Q(1)
$V' = \frac{\partial V}{\partial x}$
$\int_{-\pi/2}^{\pi/2} [V' \delta x + \dot{x} \delta \dot{x}] \, dt$
Integrate by parts
$= \int_{-\pi/2}^{\pi/2} [V' \delta x - \ddot{x} \delta x] \, dt = 0$
Q(2)
$= \int_{-\pi/2}^{\pi/2} [V'' \delta x^2$

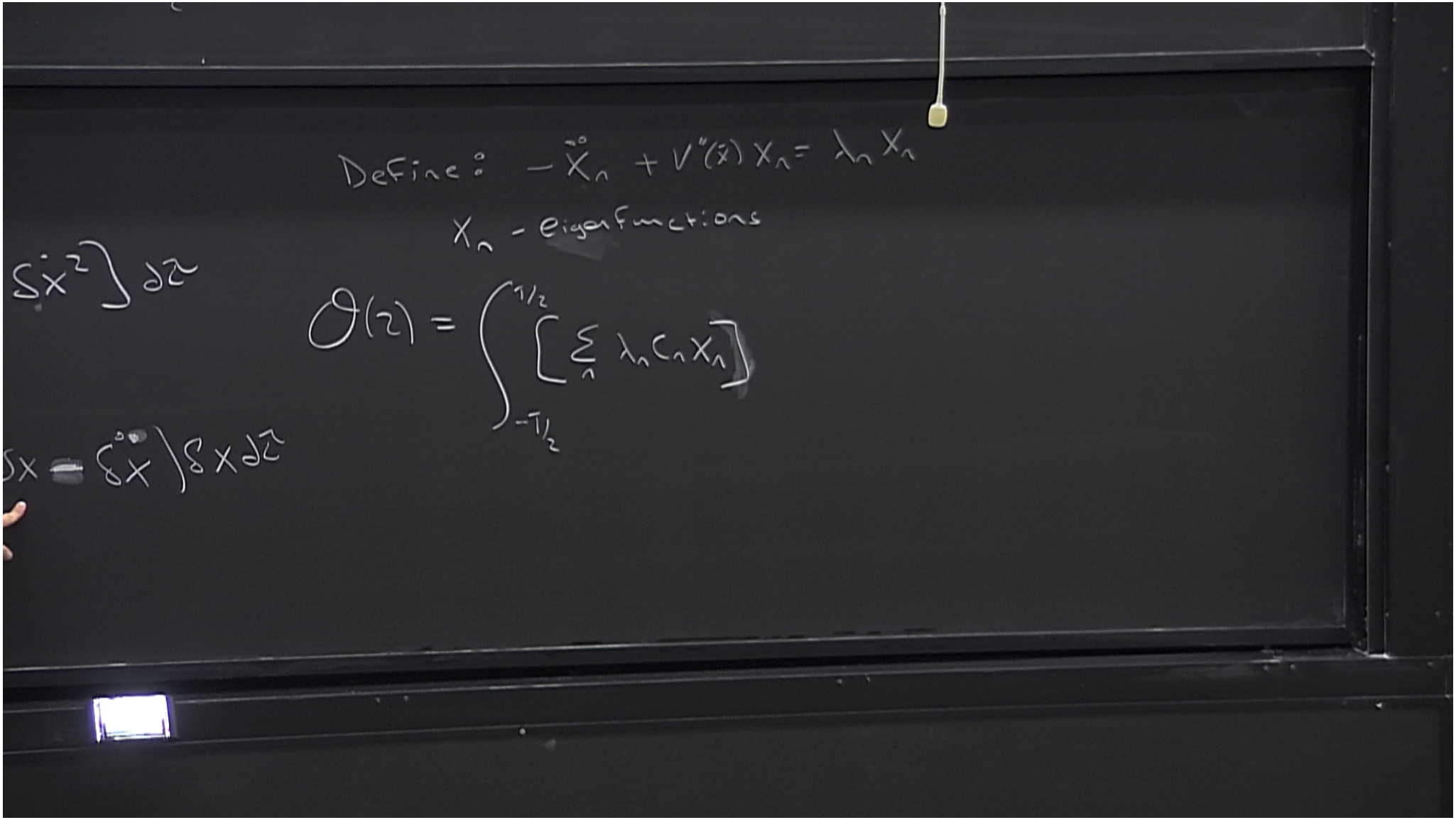

O(1)
V' = ∂V/∂x
Integrate by parts
O(2)

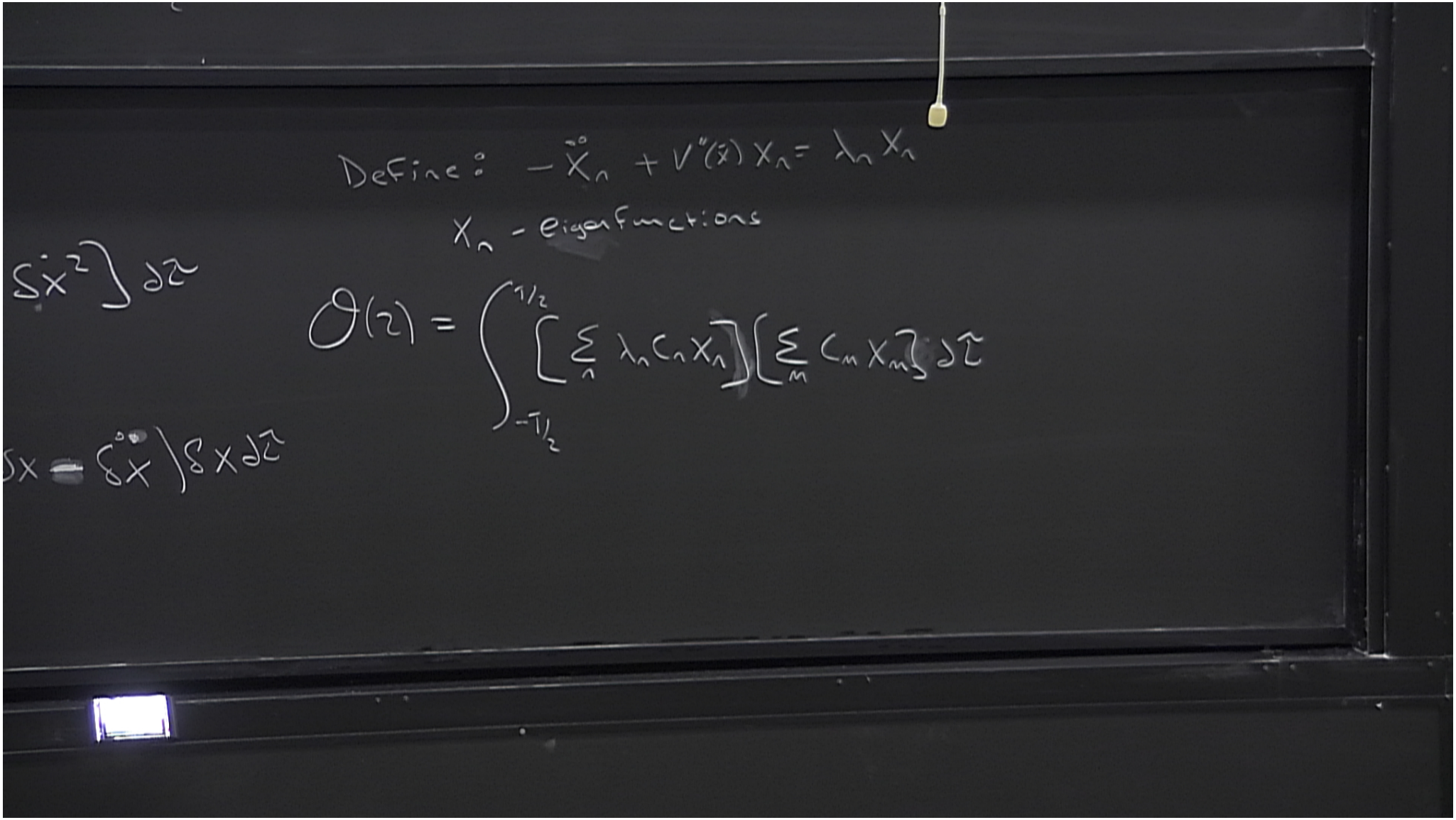

(1)
$V' = \frac{\partial V}{\partial x}$
$\int_{-T/2}^{T/2} \left[V' \delta x + \dot{x} \delta \dot{x} \right] d\tau$
$= \int_{-T/2}^{T/2} \left[V'(\bar{x}) \delta x - \ddot{x} \delta x \right] d\tau$
(2)
$\int_{-T/2}^{T/2} \left[V'' \delta x^2 + \delta \dot{x}^2 \right] d\tau$
$= \int_{-T/2}^{T/2} \left[V''(\bar{x}) \delta x - \ddot{\xi} \right] \delta x \, d\tau$


(1)
$V' = \frac{dV}{dx}$
$\int_{-T/2}^{T/2} [V'\delta x + \dot{x}\delta\dot{x}]\,dt$
$= \int_{-T/2}^{T/2} [V'(x)\delta x - \ddot{x}\delta x]\,dt = 0$
(2)
$= \int_{-T/2}^{T/2} [V''\delta x^2 + \delta\dot{x}^2]\,dt$
Int. by parts
$= \int_{-T/2}^{T/2} [V''(x)\delta x - \ddot{x}\delta x]\,\delta$
Defin


Define: $-\ddot{\vec{x}}_n$


Define: $-\ddot{X}_n + V''(\bar{x}) X_n = \lambda_n X_n$
X_n - eigenfunctions


Define: $-\ddot{X}_n + V''(\bar{x}) X_n = \lambda_n X_n$
X_n - eigenfunctions
$\theta(z) = \int_{-l/2}^{l/2} \left[\sum_n \lambda_n C_n X_n \right]$

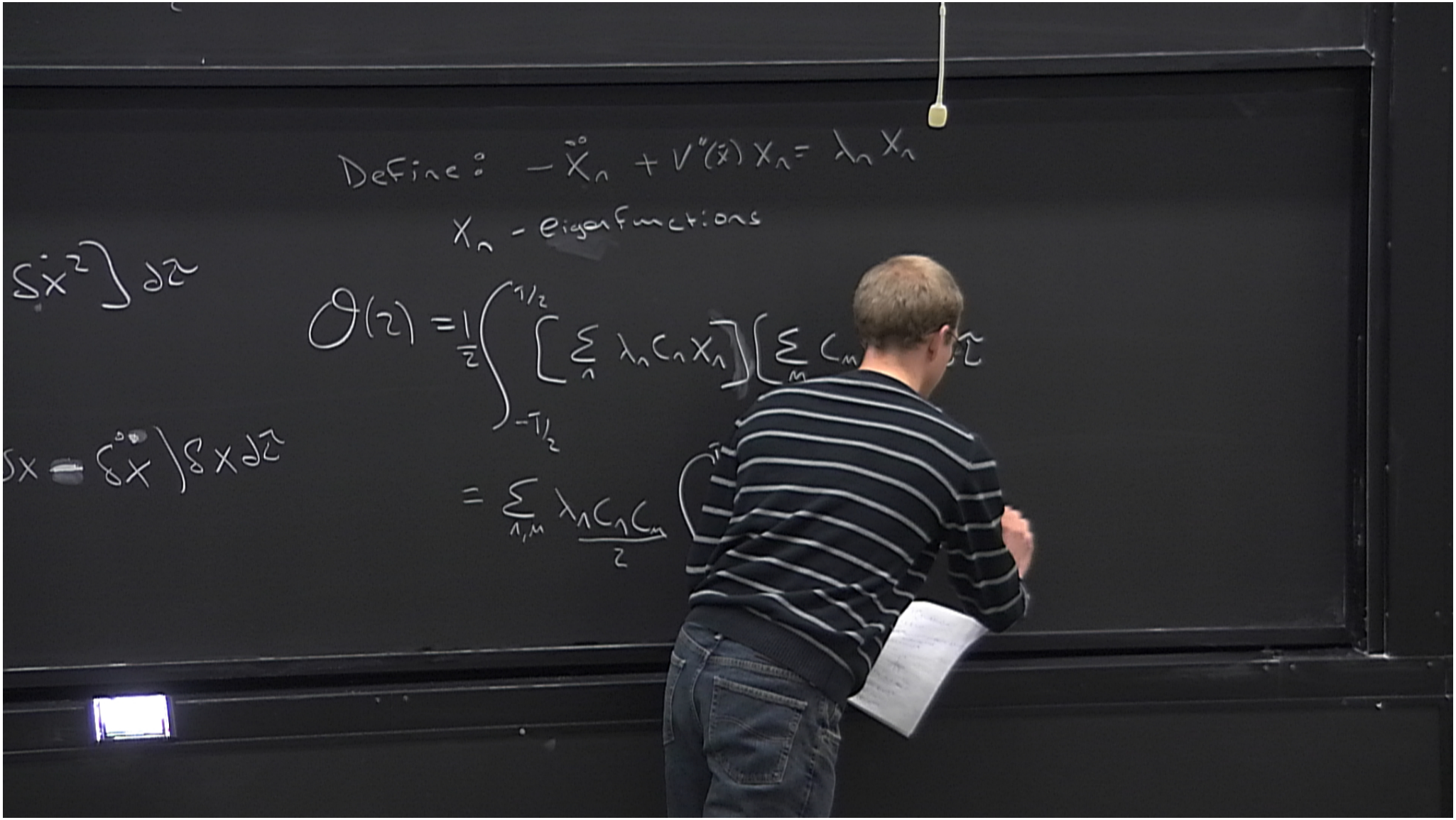

Define: $-\ddot{X}_n + V''(\bar{x}) X_n = \lambda_n X_n$
X_n - eigenfunctions
$\theta(\tau) = \int_{-T/2}^{T/2} \left[\sum_n \lambda_n C_n X_n\right]\left[\sum_m C_m X_m\right] d\tau$

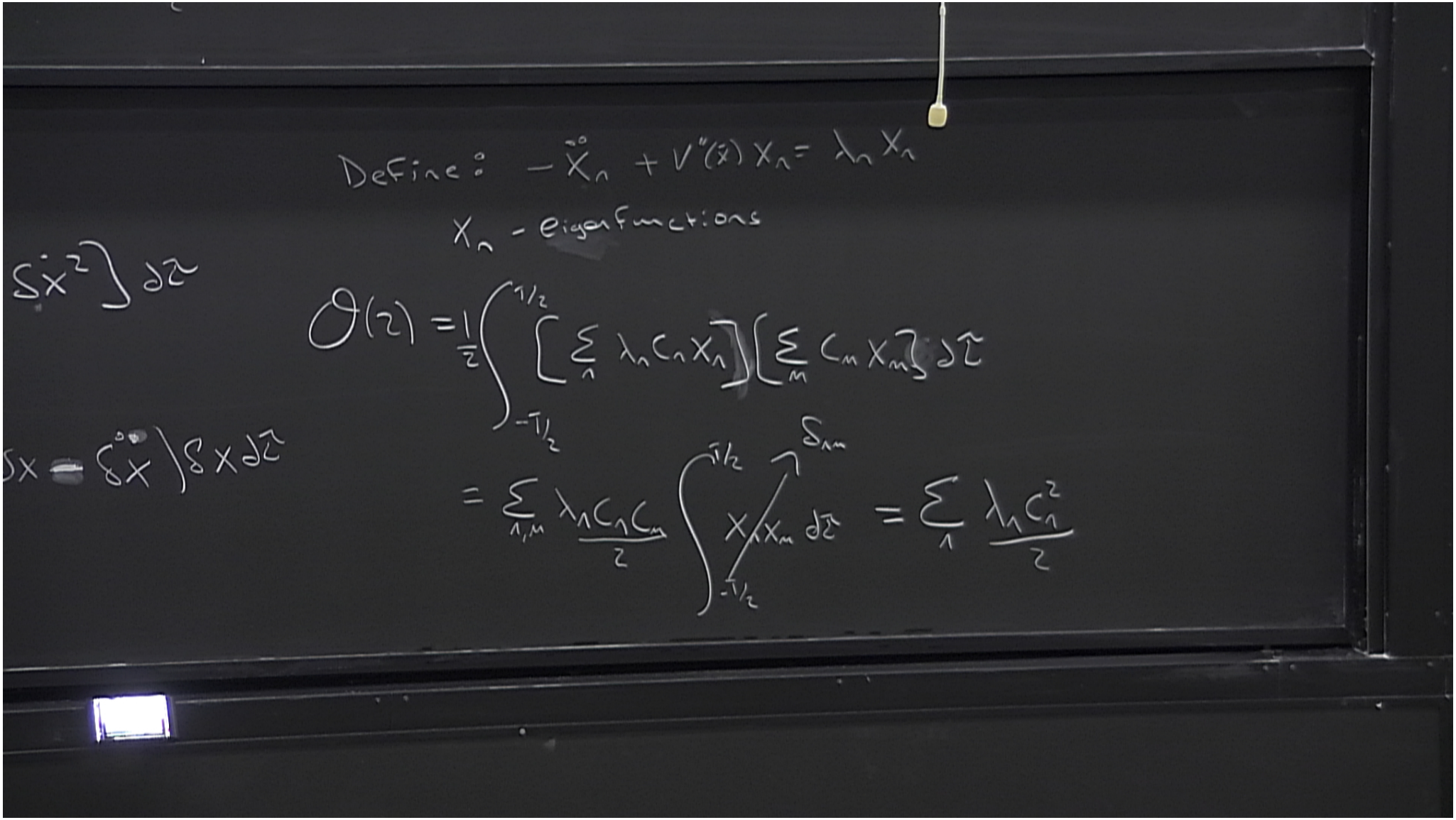
Define: $-\ddot{X}_n + V''(\bar{x})\, X_n = \lambda_n X_n$

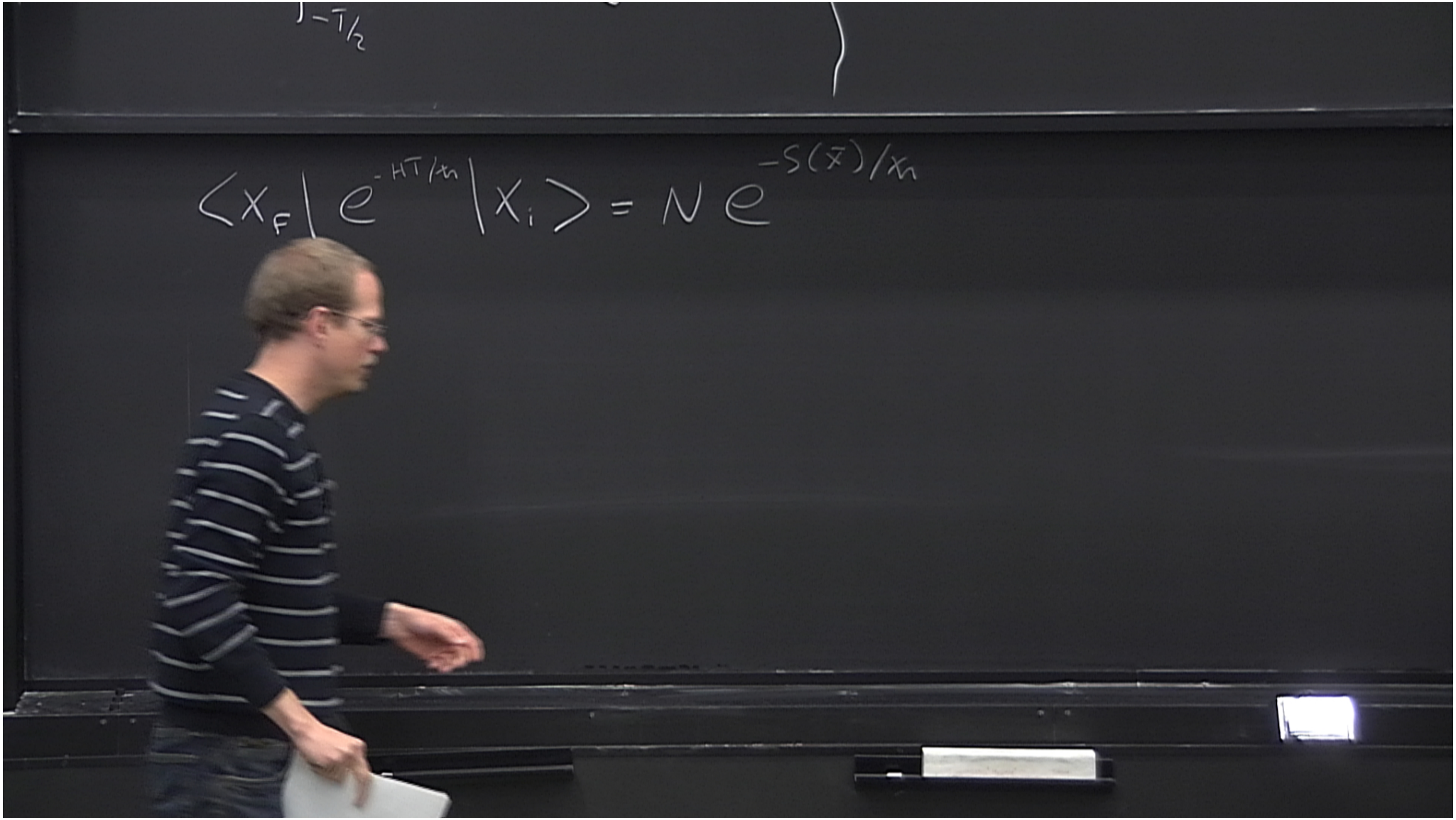
X_n - eigenfunctions

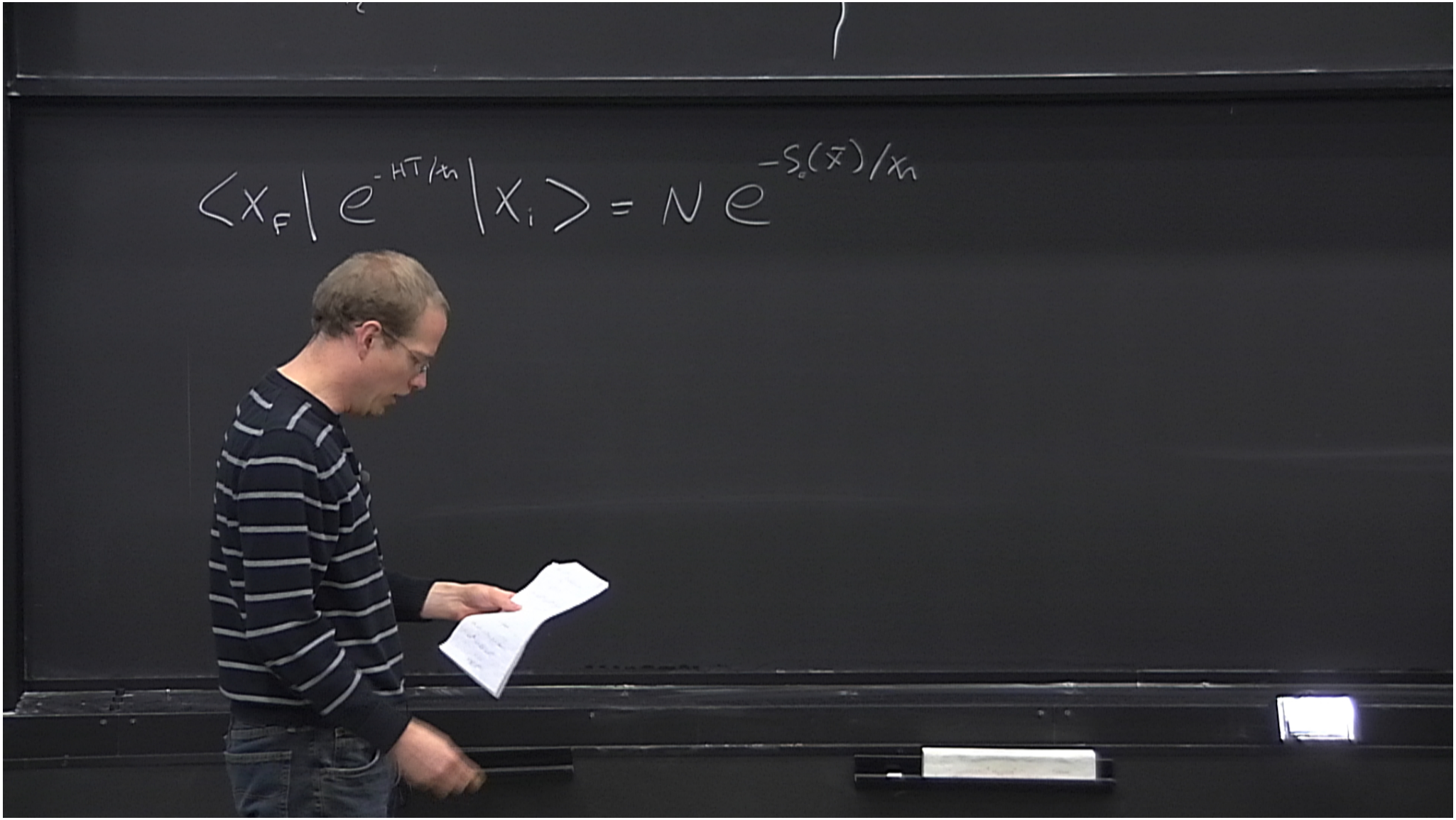
$$\theta(\tau) = \int_{-T/2}^{T/2} \left[\sum_n \lambda_n C_n X_n\right]\left[\sum_m C_m X_m\right] d\tau$$

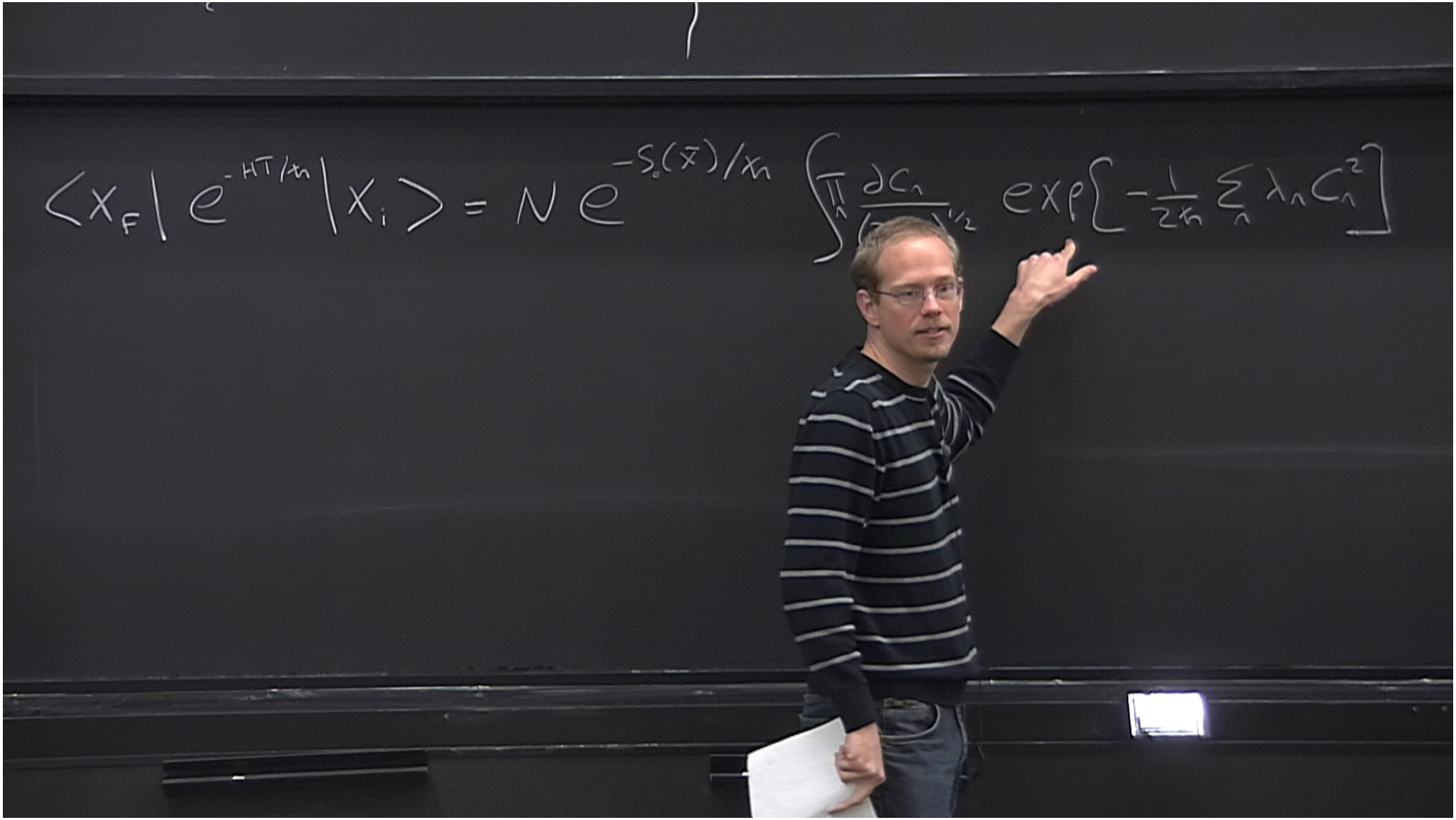
$$= \sum_{n,m} \lambda_n C_n C_m \int_{-T/2}^{T/2} X_n X_m \, d\tau \qquad \delta_{nm}$$

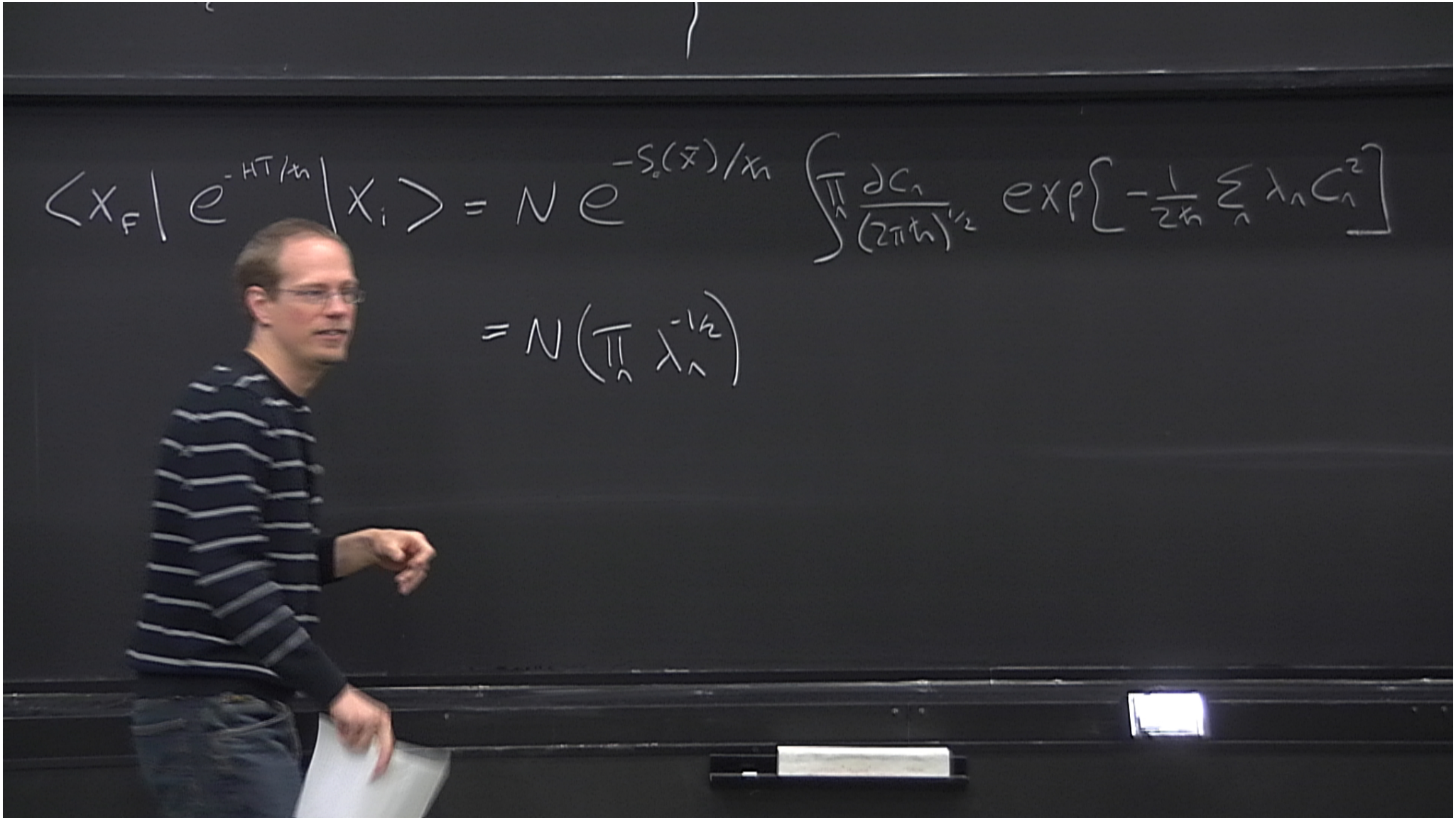

Define: $-\ddot{X}_n + V''(\bar{x}) X_n = \lambda_n X_n$
X_n - eigenfunctions
$\theta(\tau) = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} \left[\sum_n \lambda_n C_n X_n\right]\left[\sum_m C_m$
$= \sum_{n,m} \lambda_n C_n C_m \frac{}{\tau}$

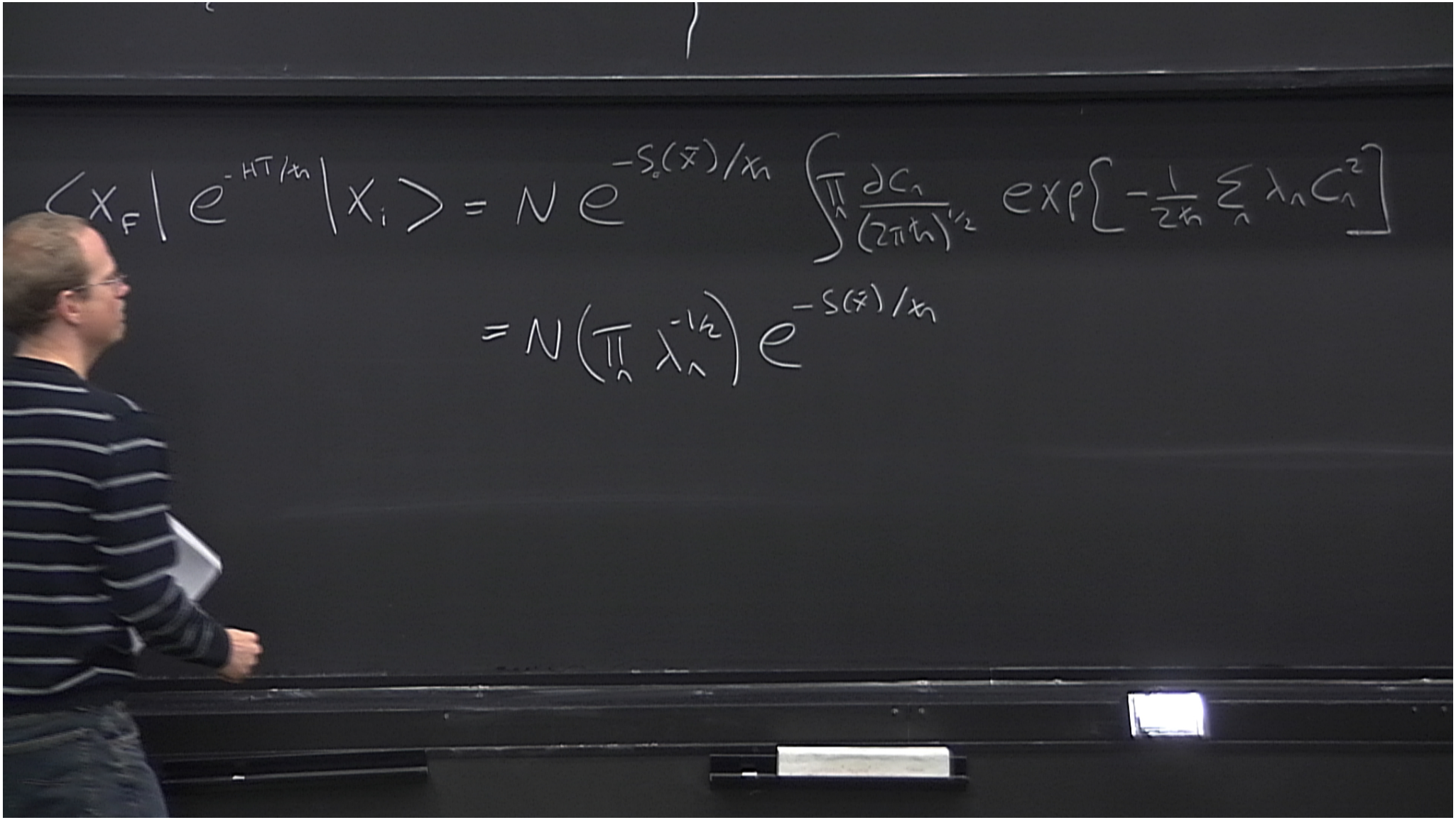

Define: $-\ddot{X}_n + V''(\bar{x}) X_n = \lambda_n X_n$
X_n - eigenfunctions
$\theta(\tau) = \frac{1}{2}\int_{-T/2}^{T/2} \left[\sum_n \lambda_n C_n X_n\right]\left[\sum_m C_m X_m\right] d\tau$
$= \sum_{n,m} \frac{\lambda_n C_n C_m}{2} \int_{-T/2}^{T/2} X_n X_m \, d\tau = \sum_n \frac{\lambda_n C_n^2}{2}$
δ_{nm}

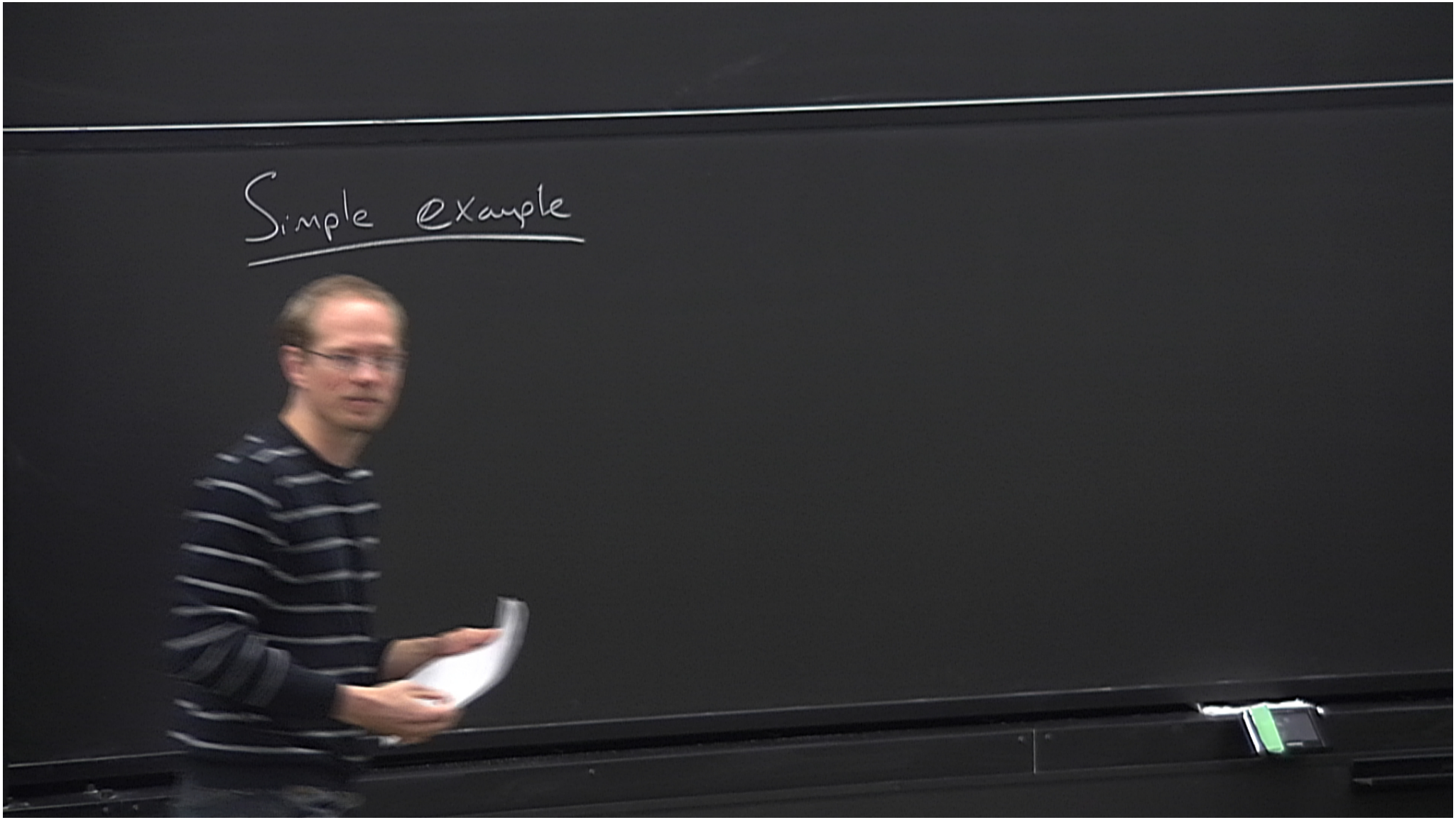

$\int_{-T/2}$
$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N e^{-S(\bar{x})/\hbar}$

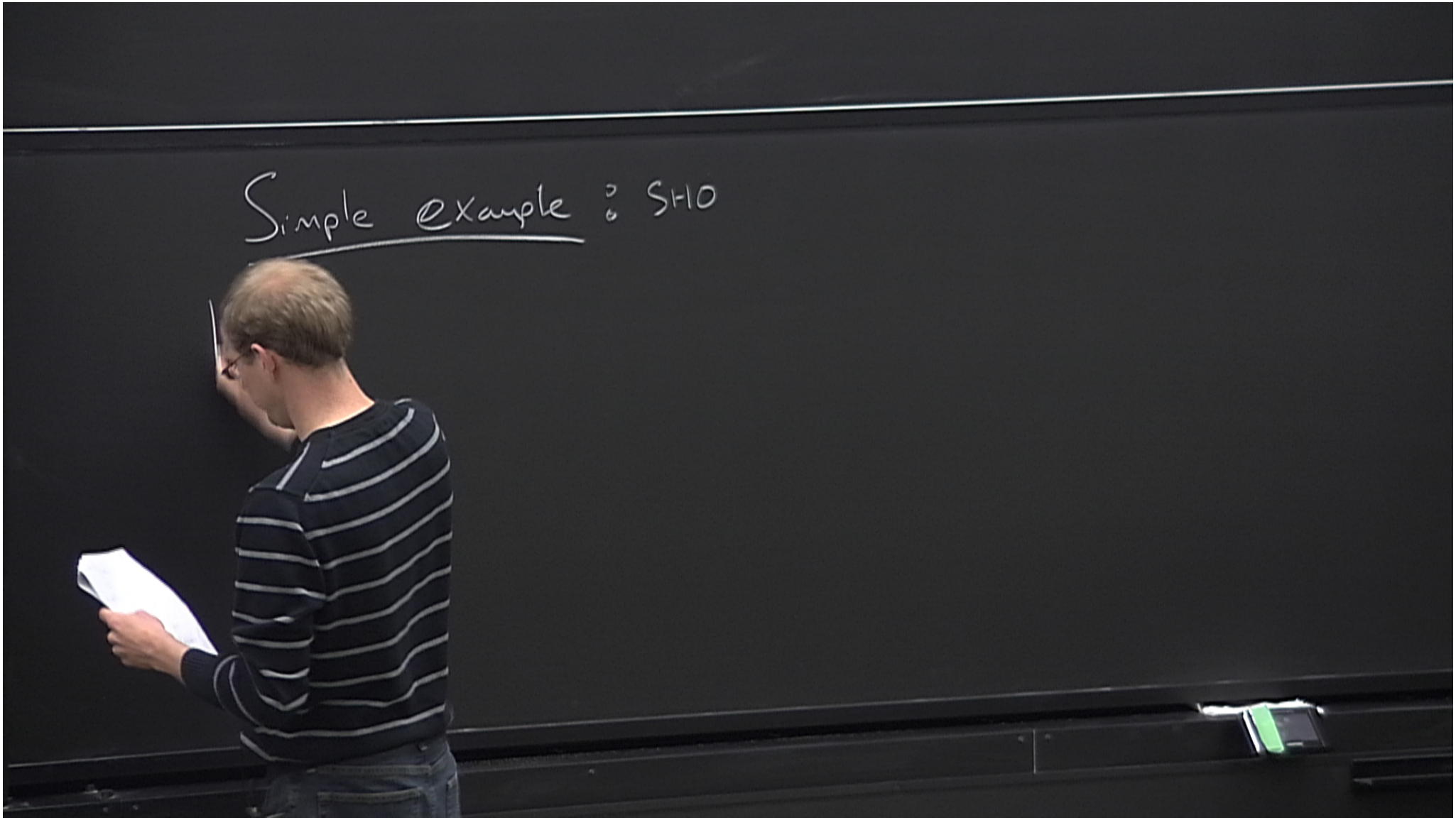

$\langle x_F | e^{-HT/\hbar} | x_i \rangle = N e^{-S_c(\bar{x})/\hbar}$

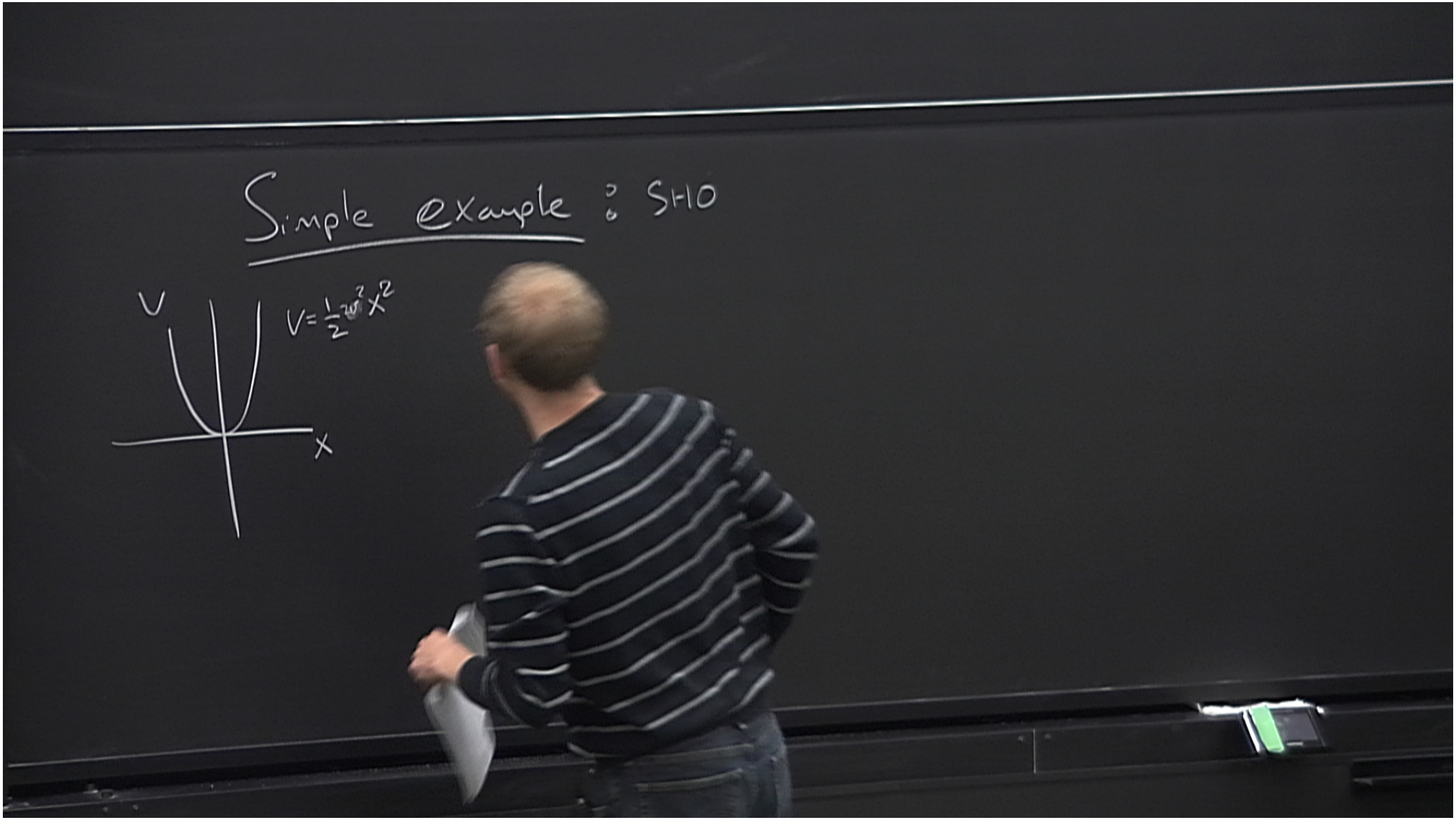

$\langle x_F | e^{-HT/\hbar} | x_i \rangle = N e^{-S_0(\bar{x})/\hbar} \int \prod_n \frac{dc_n}{(\quad)^{1/2}} \exp\left[-\frac{1}{2\hbar} \sum_n \lambda_n c_n^2\right]$

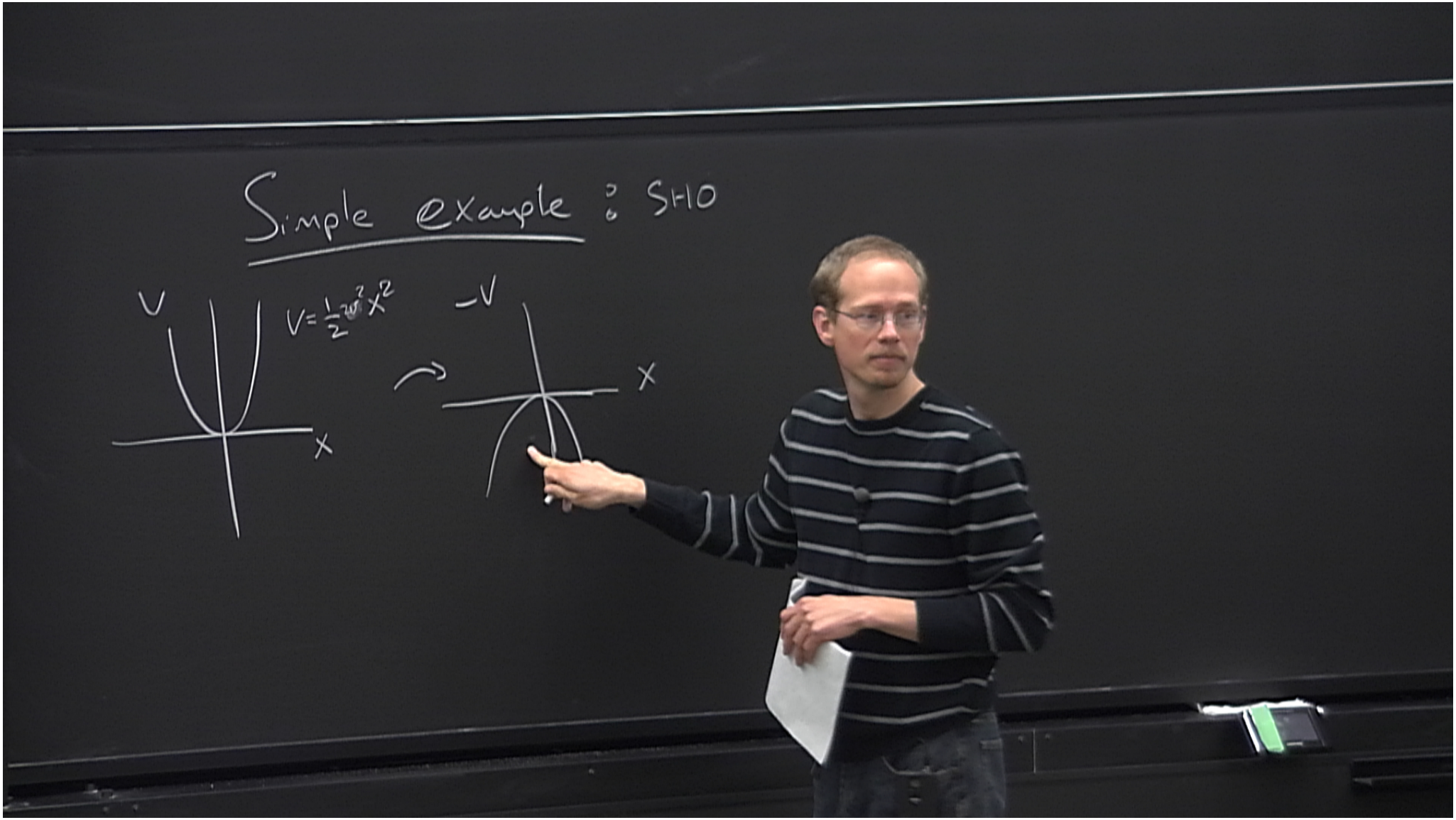

$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N e^{-S_0(\bar{x})/\hbar} \int \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}} \exp\left[-\frac{1}{2\hbar}\sum_n \lambda_n c_n^2\right]$
$= N\left(\prod_n \lambda_n^{-1/2}\right)$

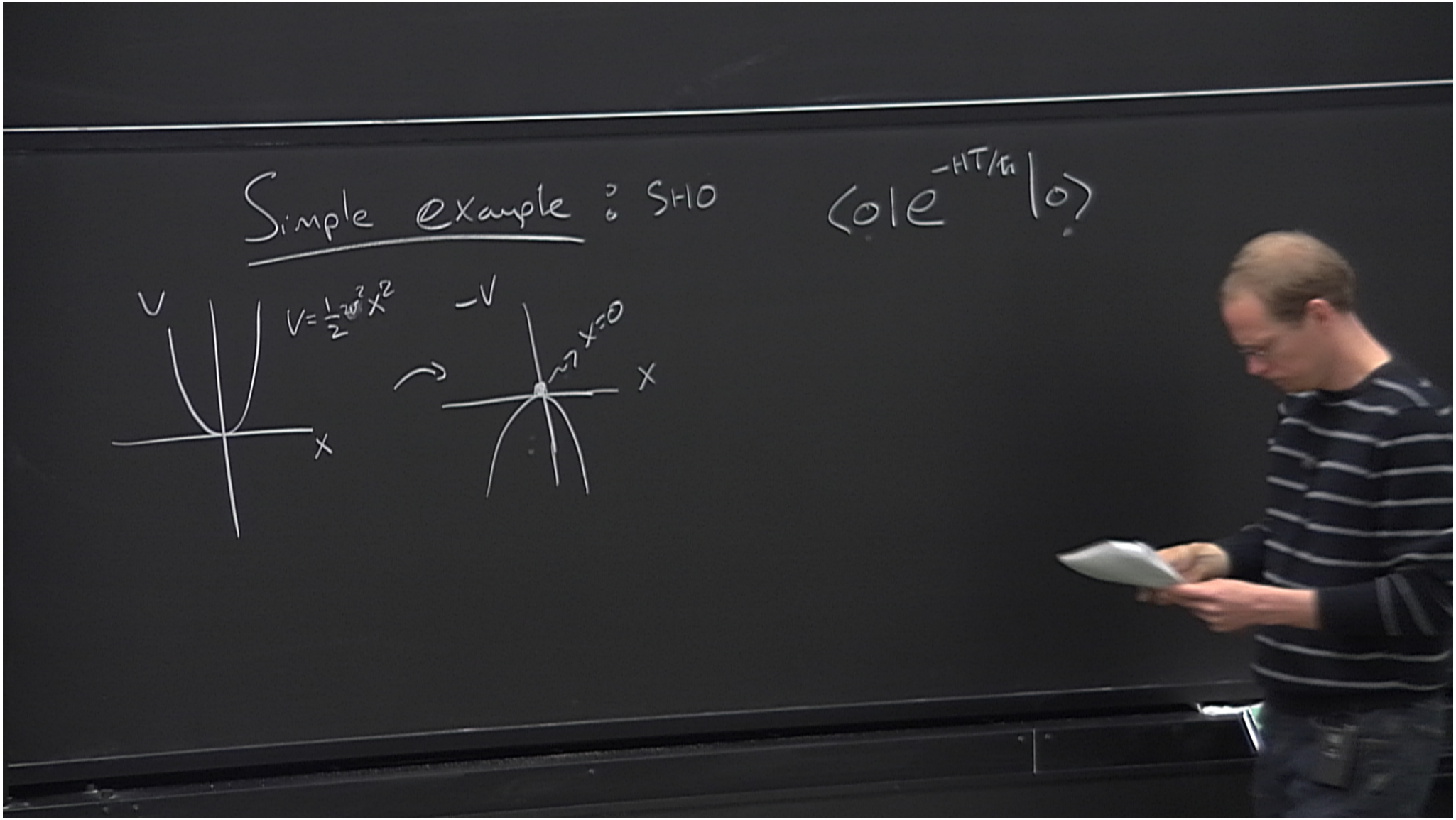

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Simple example


Simple example : SHO


Simple example : SHO
V
x
$V = \frac{1}{2}\omega^2 x^2$

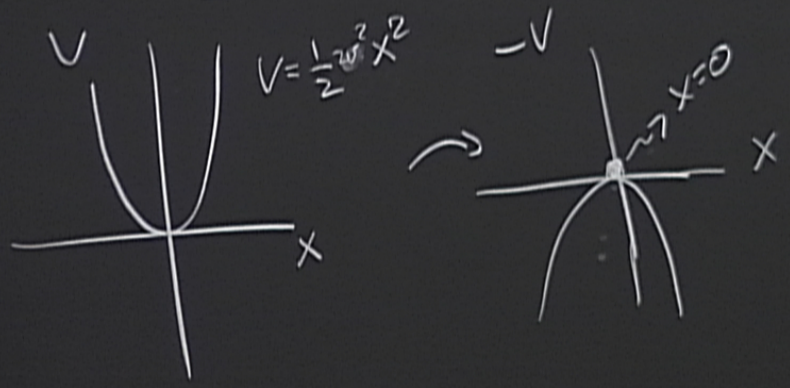

Simple example : SHO
V
$V=\frac{1}{2}\omega^2 x^2$
x
$\rightsquigarrow$
$-V$
x


Simple example : SHO
$\langle 0|e^{-HT/\hbar}|0\rangle$
V
$V=\frac{1}{2}\omega^2 x^2$
x
$-V$
$\omega \to \infty$
$x=0$
x

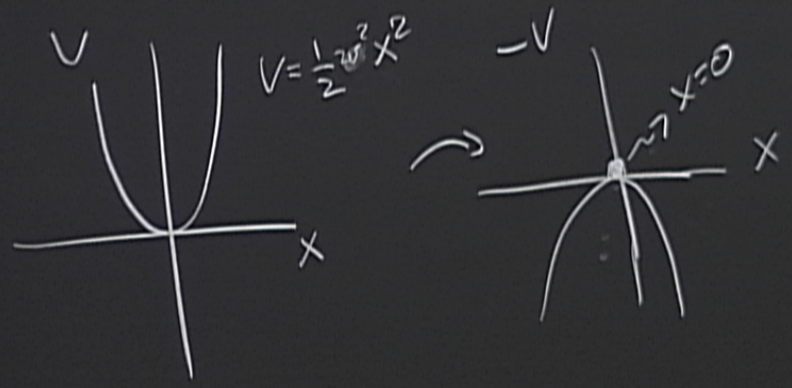
Simple example : SHO

$\langle 0 | e^{-HT/\hbar} | 0 \rangle$

$\bar{x} = 0$



Simple example : SHO



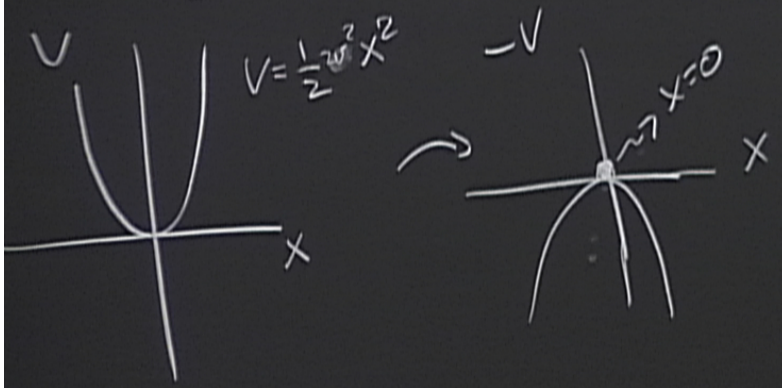
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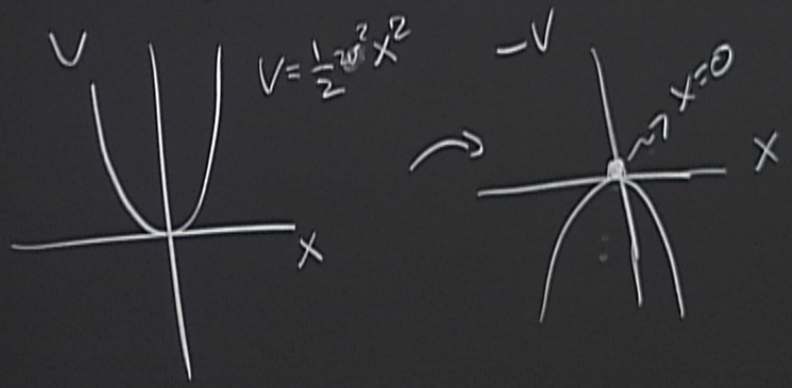
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$\bar{x} = 0 \,, \quad \omega^2 = V''(0)$




Feynman + Hibbs

Simple example : SHO

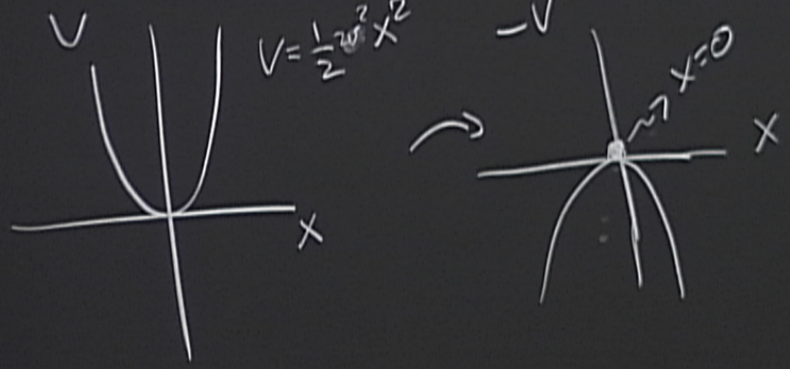


$$\langle 0|e^{-HT/\hbar}|0\rangle = N\left(\prod_n \lambda_n^{-1/2}\right)$$

$$\bar{x}=0\ ,\ \omega^2 = V''(0)$$

$$\rightarrow = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}$$

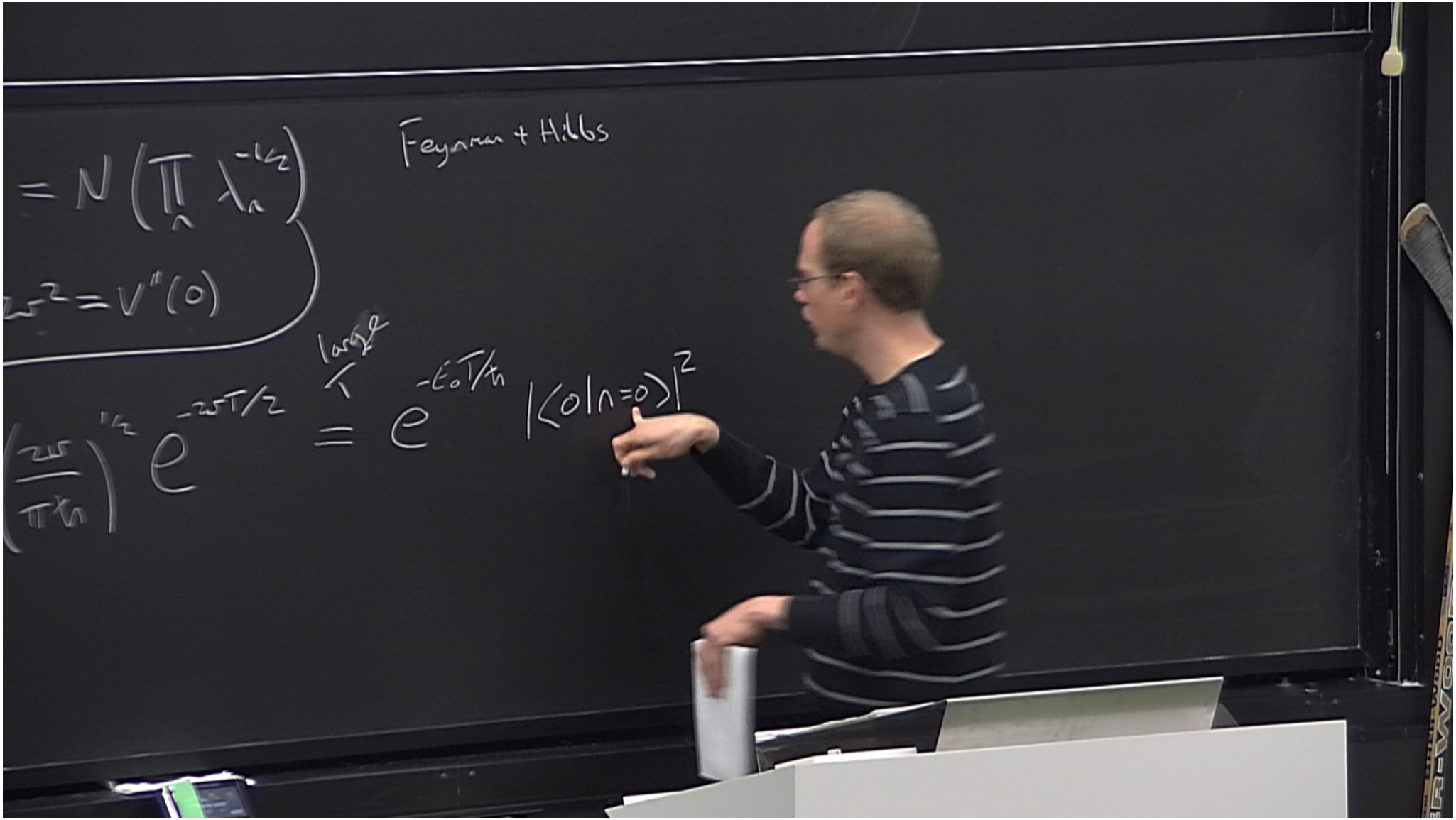
Simple example : SHO

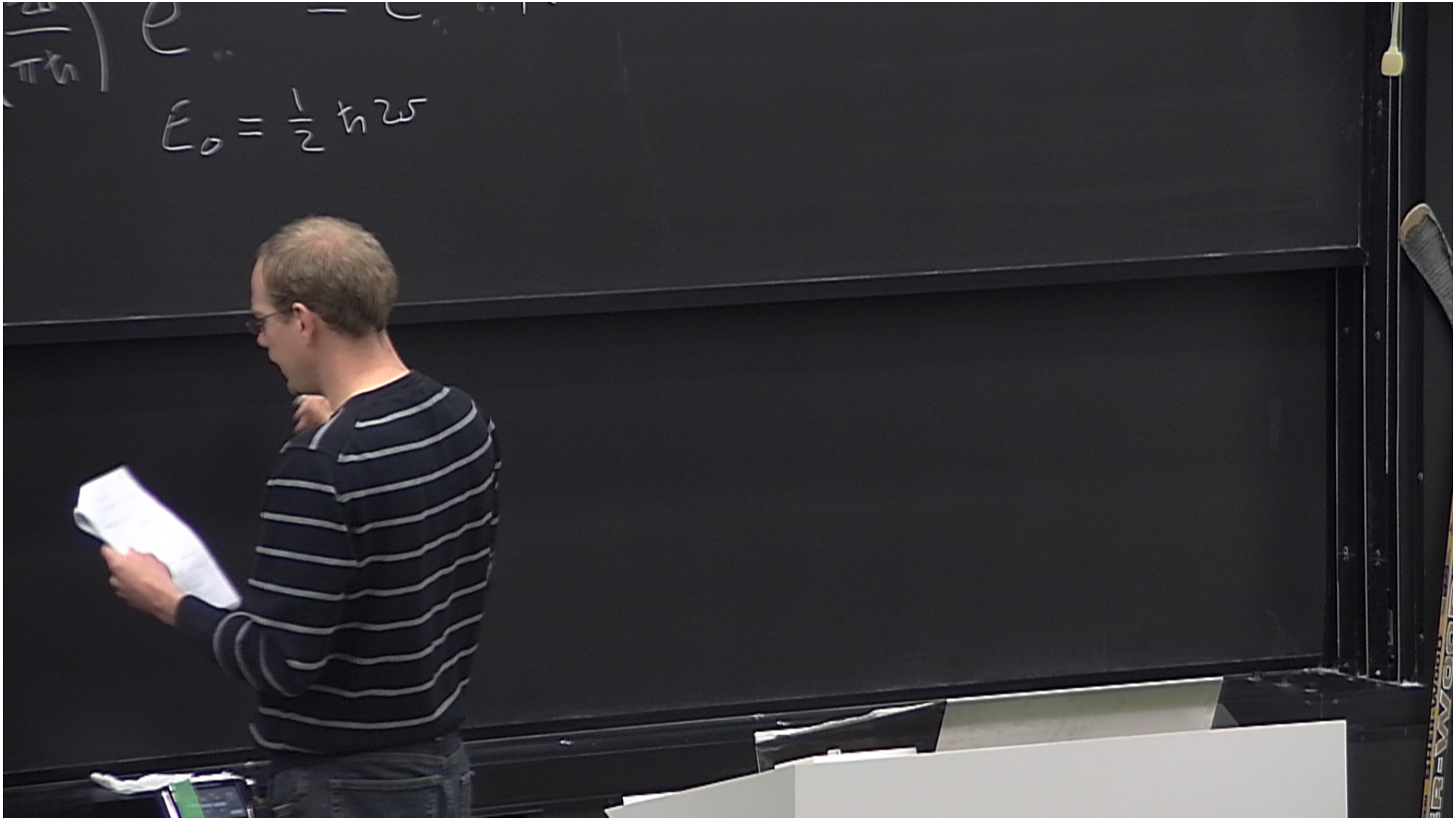


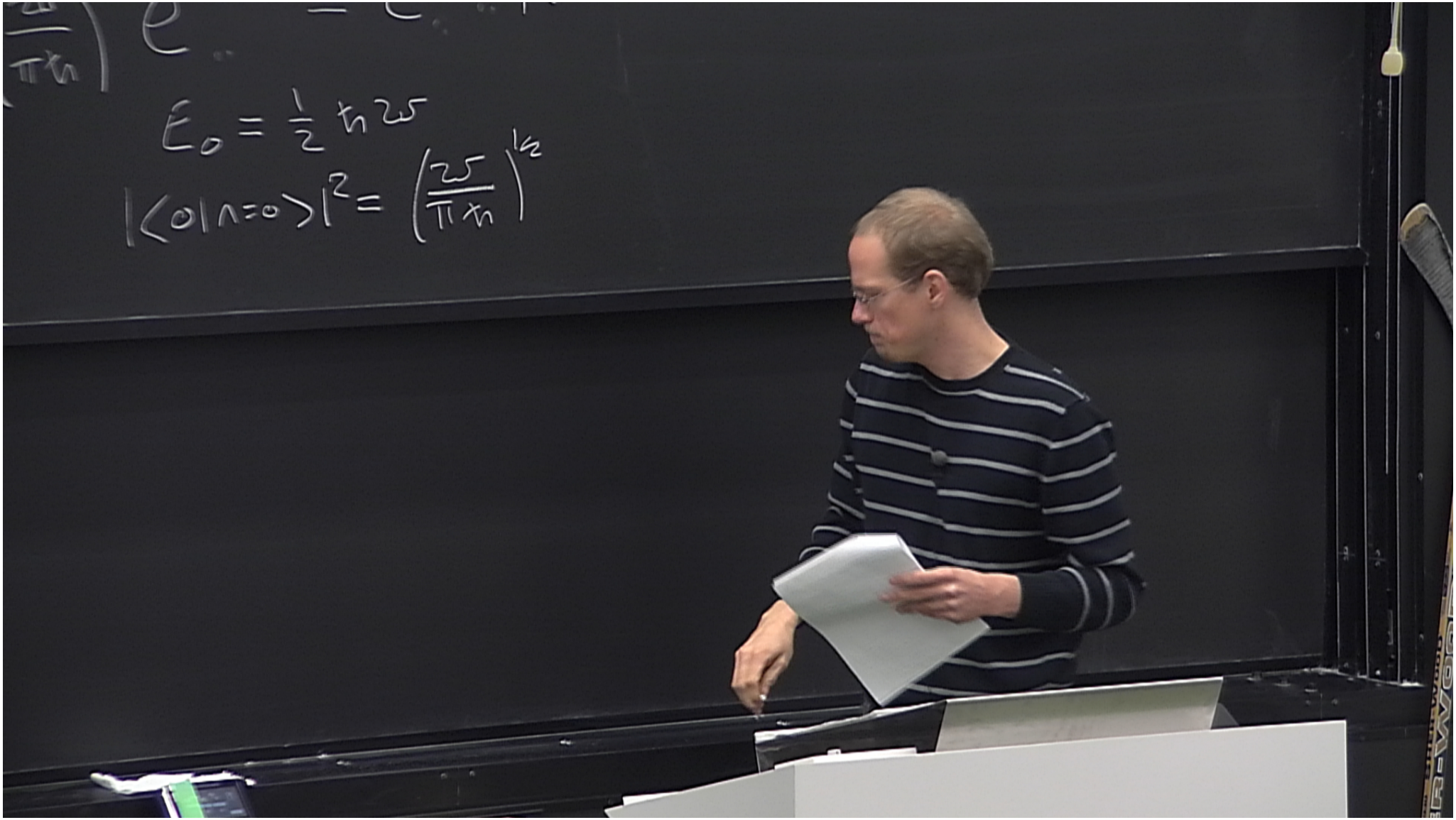
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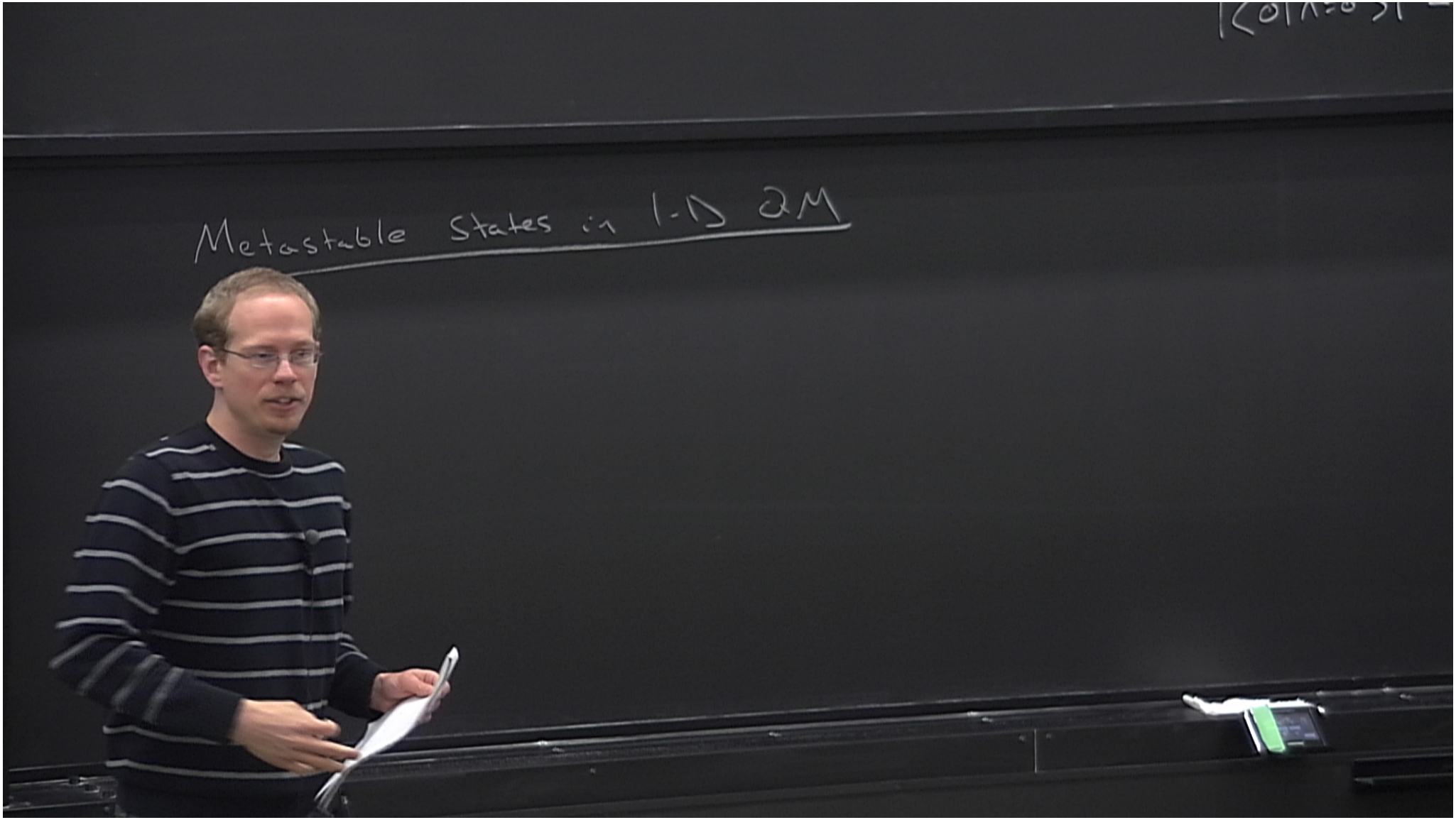
$\bar{x}=0\ ,\ \omega^2 = V''(0)$

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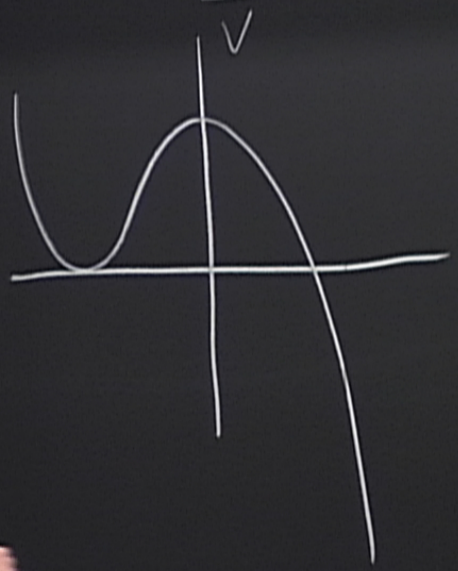

Feynman + Hibbs
$= N\left(\prod_n \lambda_n^{-1/2}\right)$
$\omega^2 = V''(0)$
$\left(\frac{2\pi}{\hbar T}\right)^{1/2} e^{-\omega T/2}$
large T
$= e^{-E_0 T/\hbar} |\langle 0|n=0\rangle|^2$

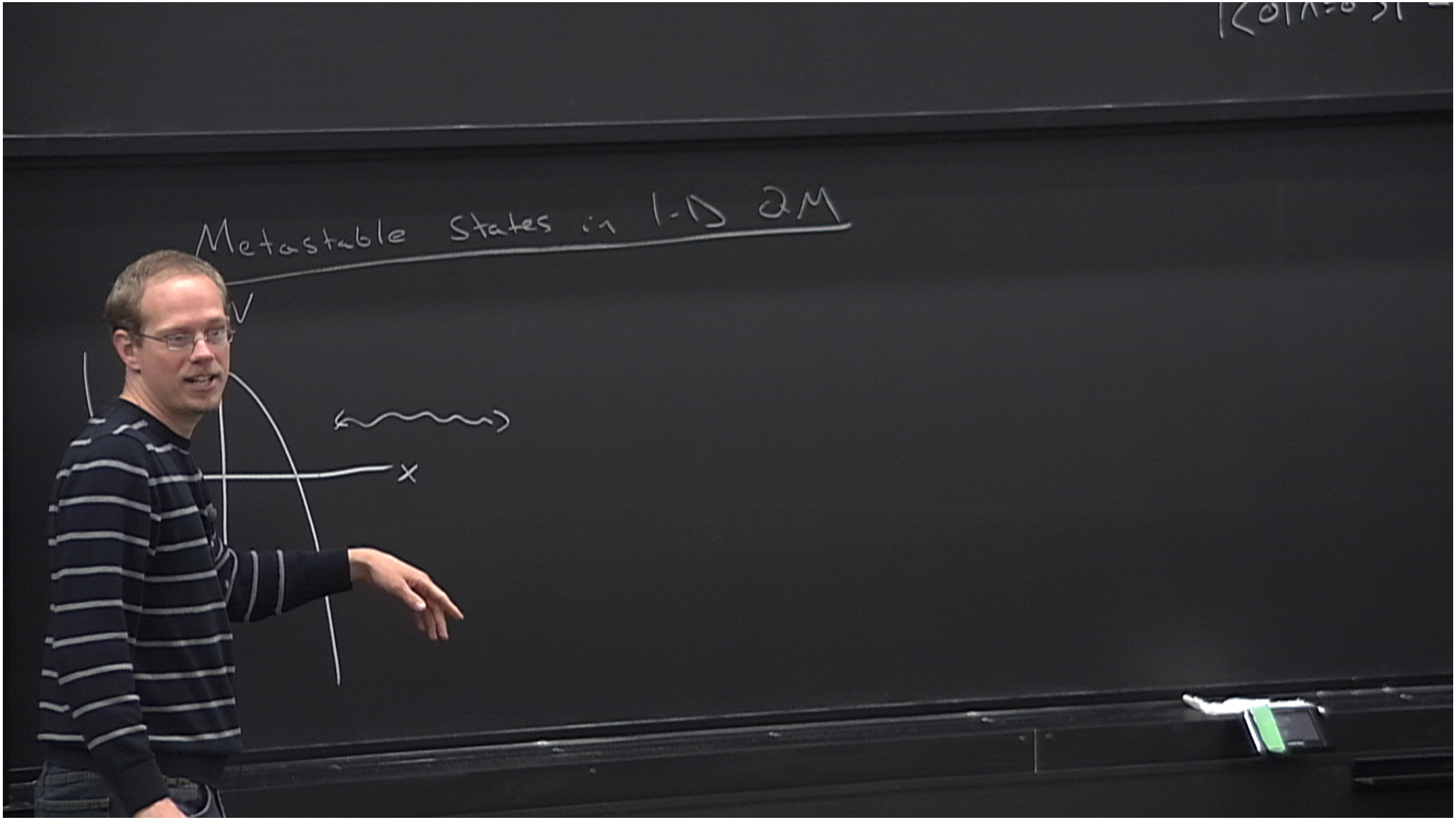

$E_0 = \frac{1}{2} \hbar \omega$

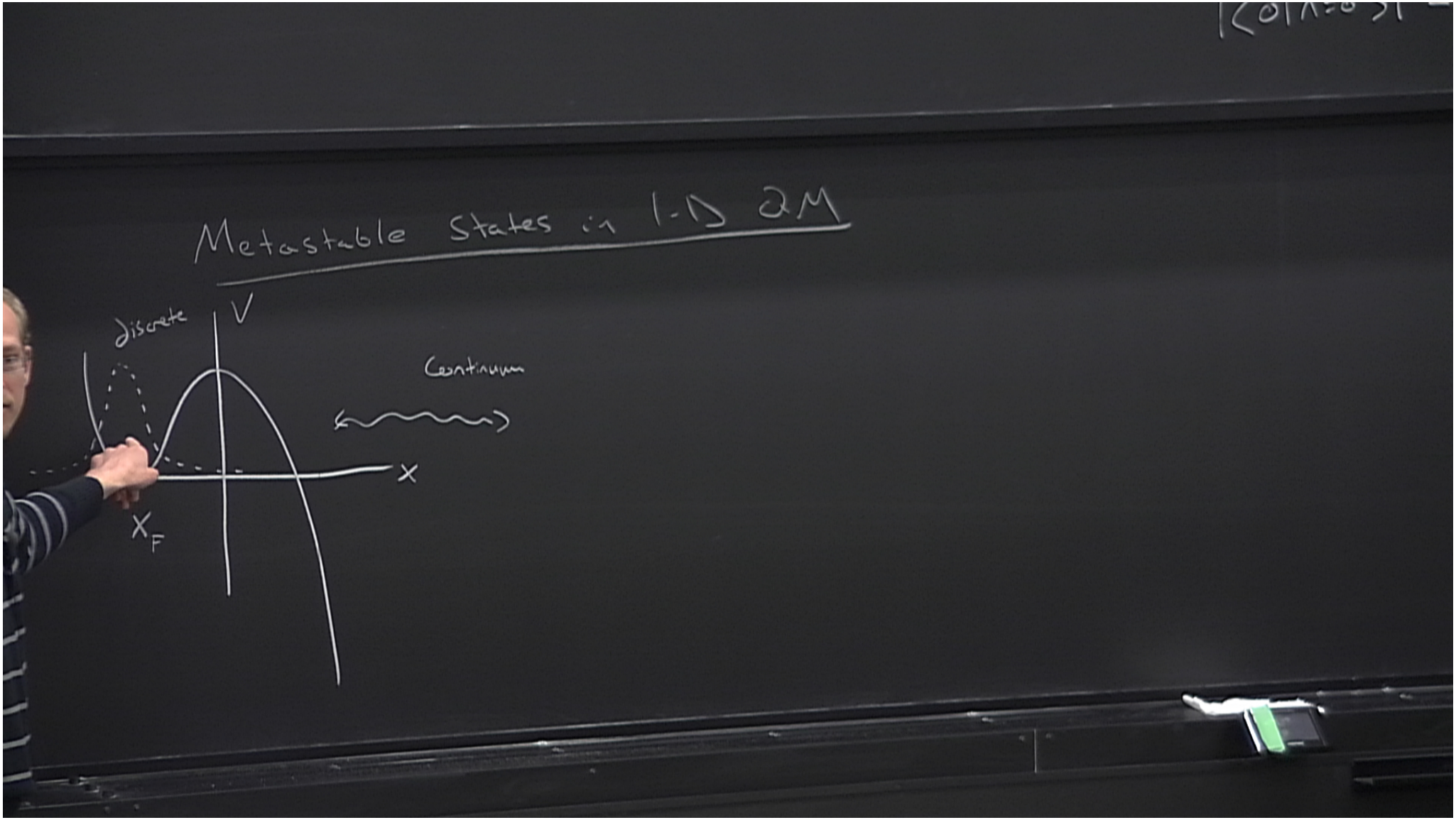

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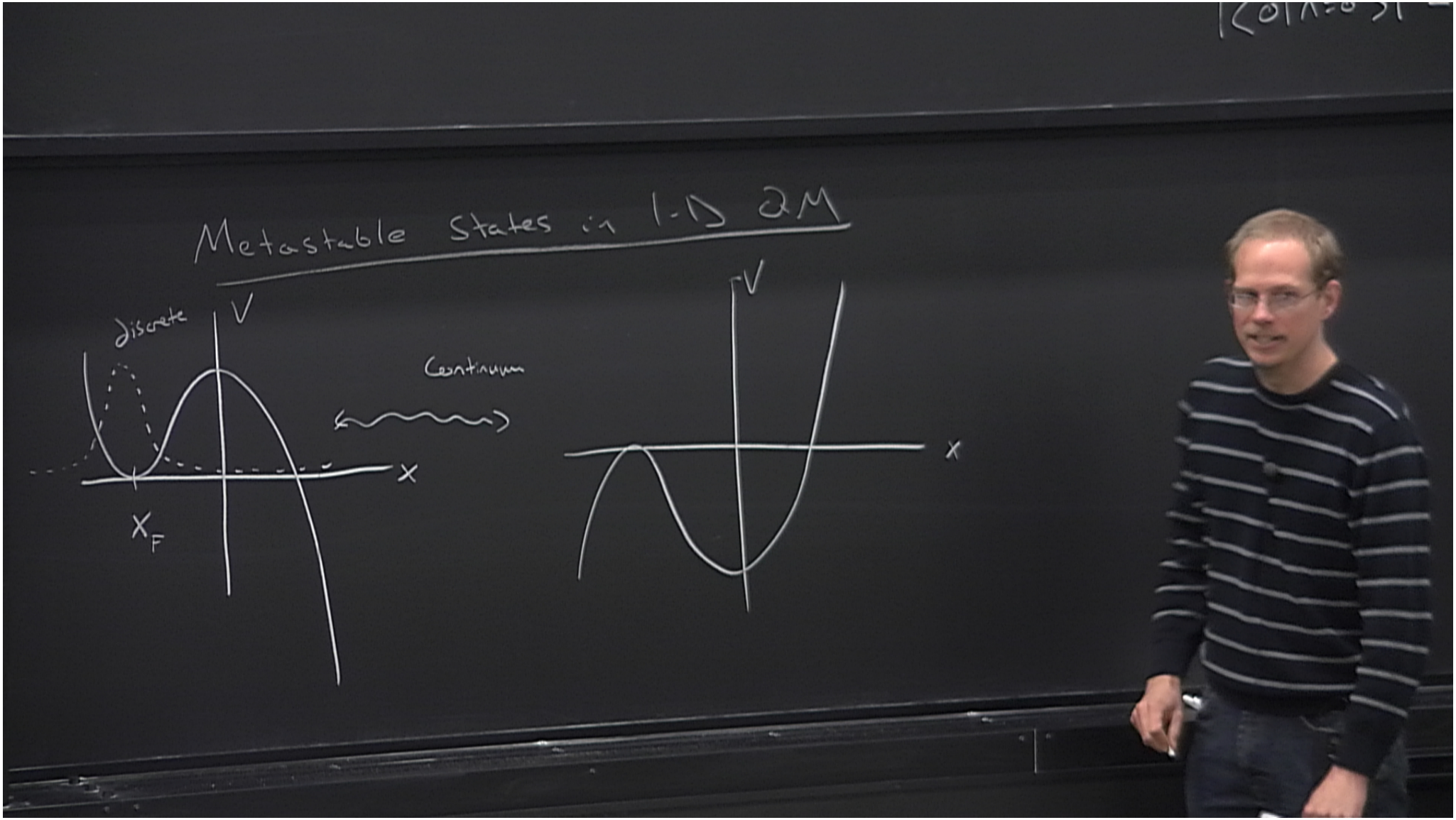

Metastable States in 1-D QM

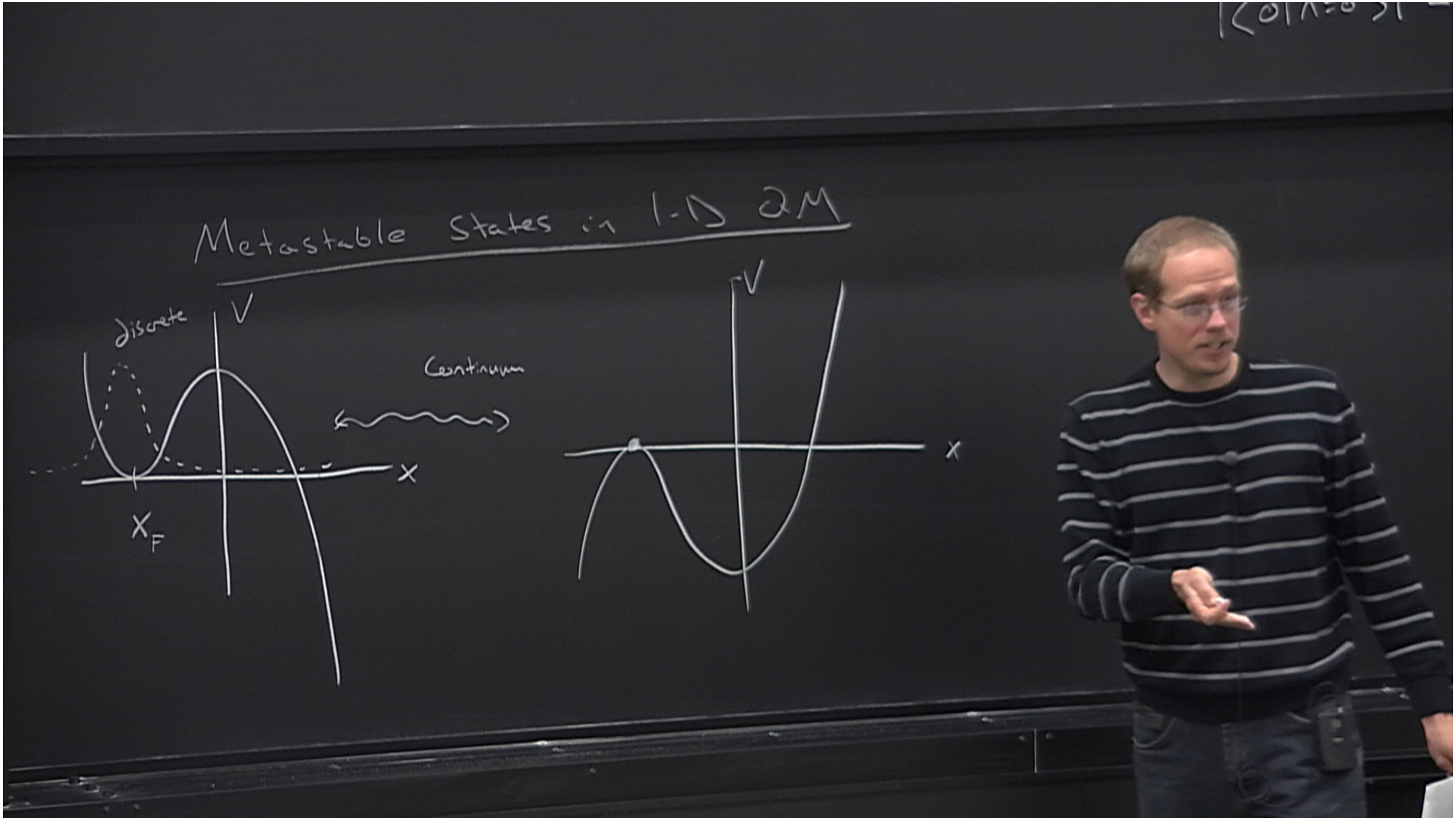
Metastable States in 1-D QM

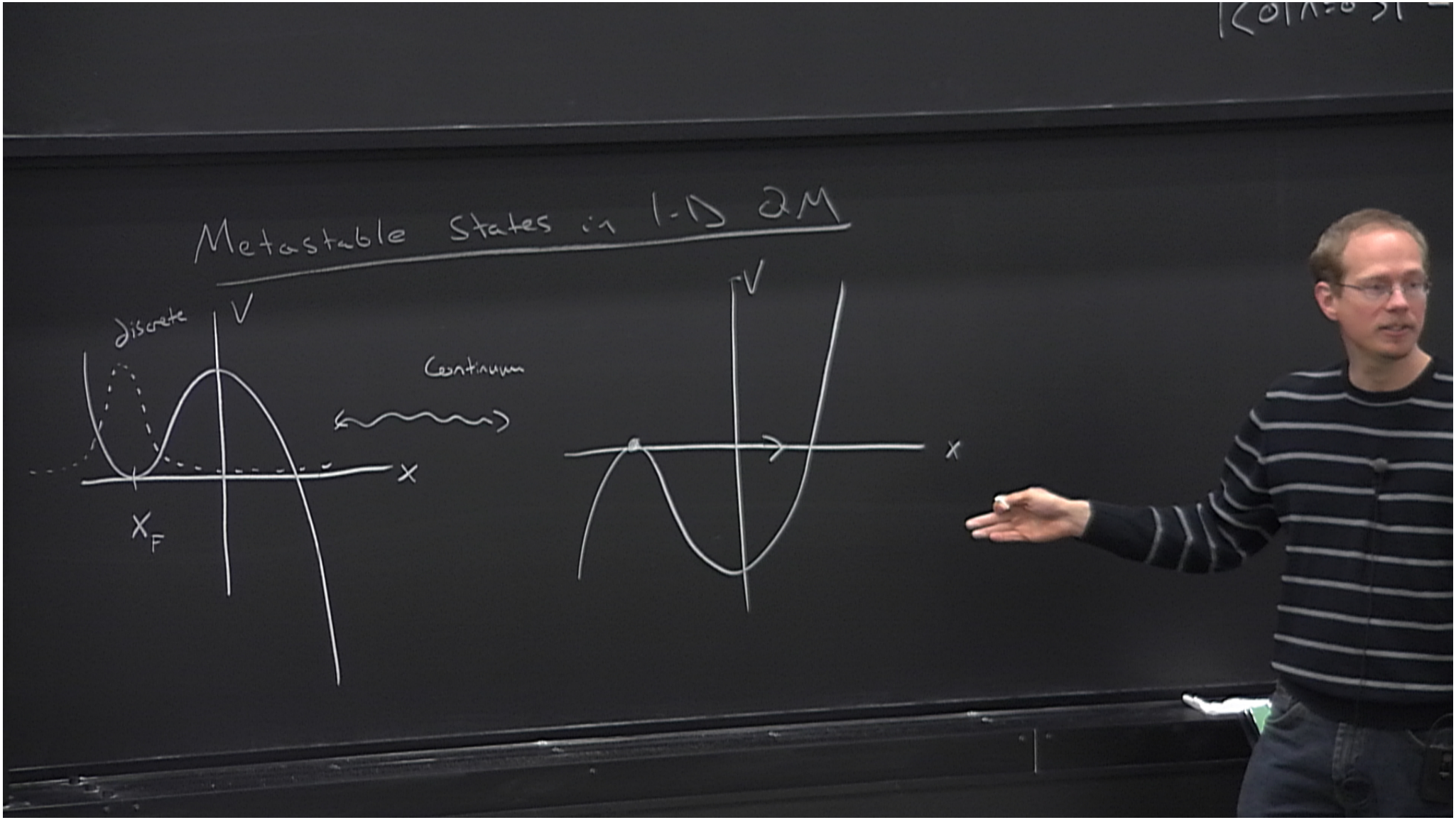


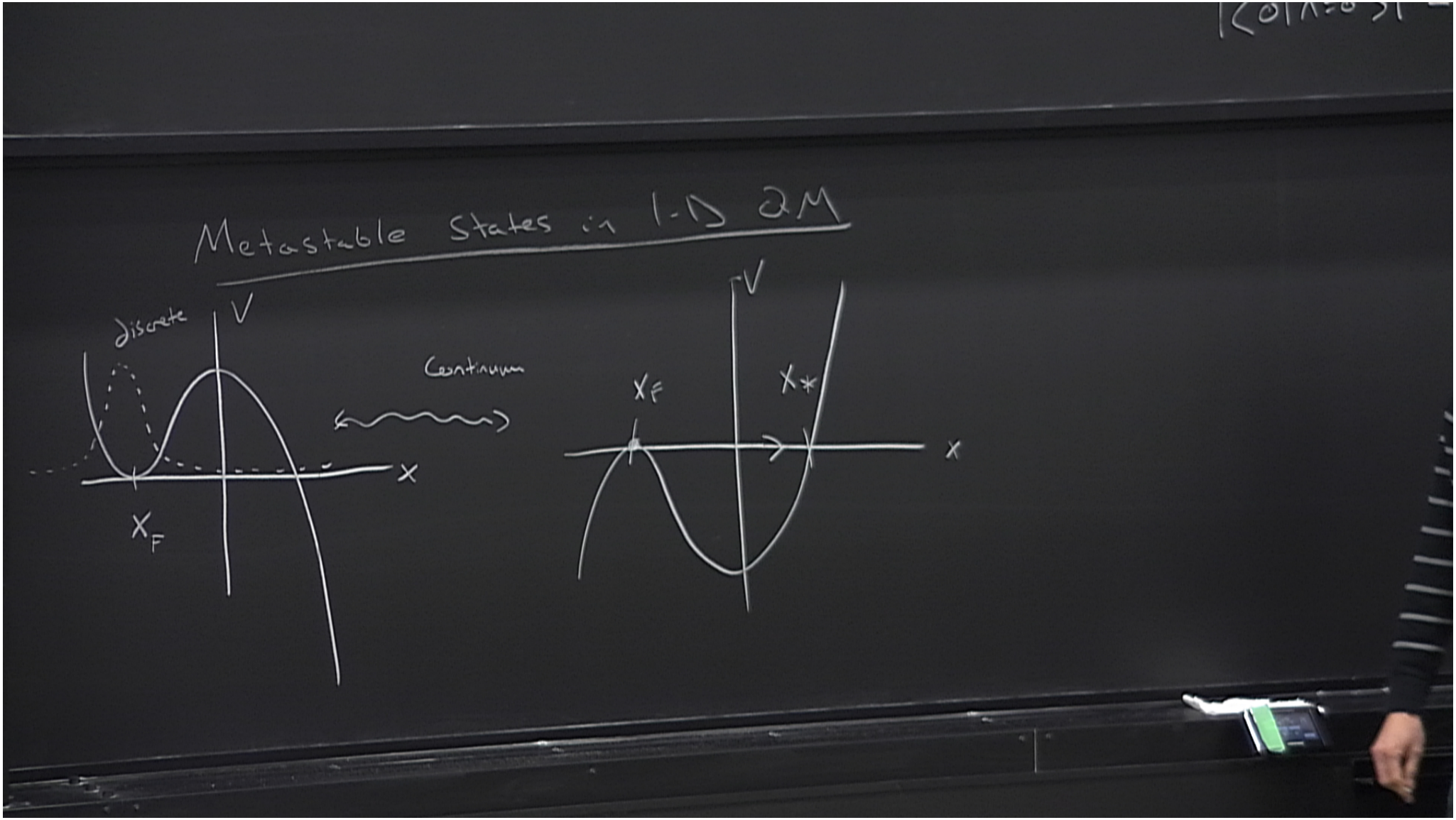

Metastable States in 1-D QM
V
x


Metastable States in 1-D QM
discrete
V
Continuum
X
X_F

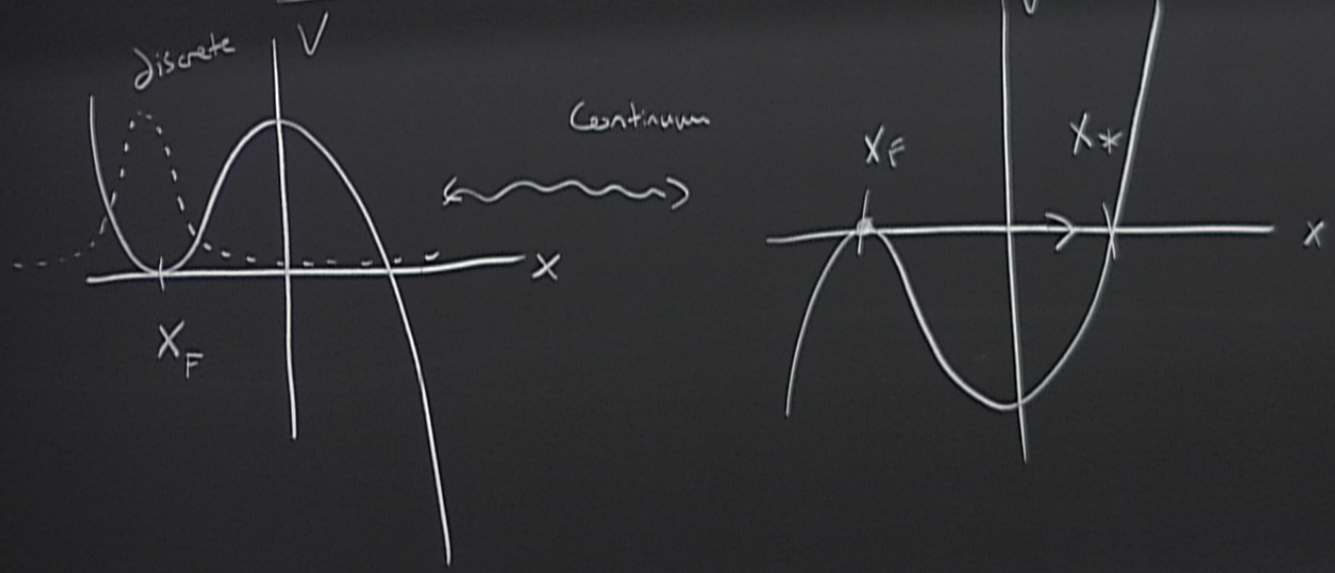

Metastable States in 1-D QM
discrete
V
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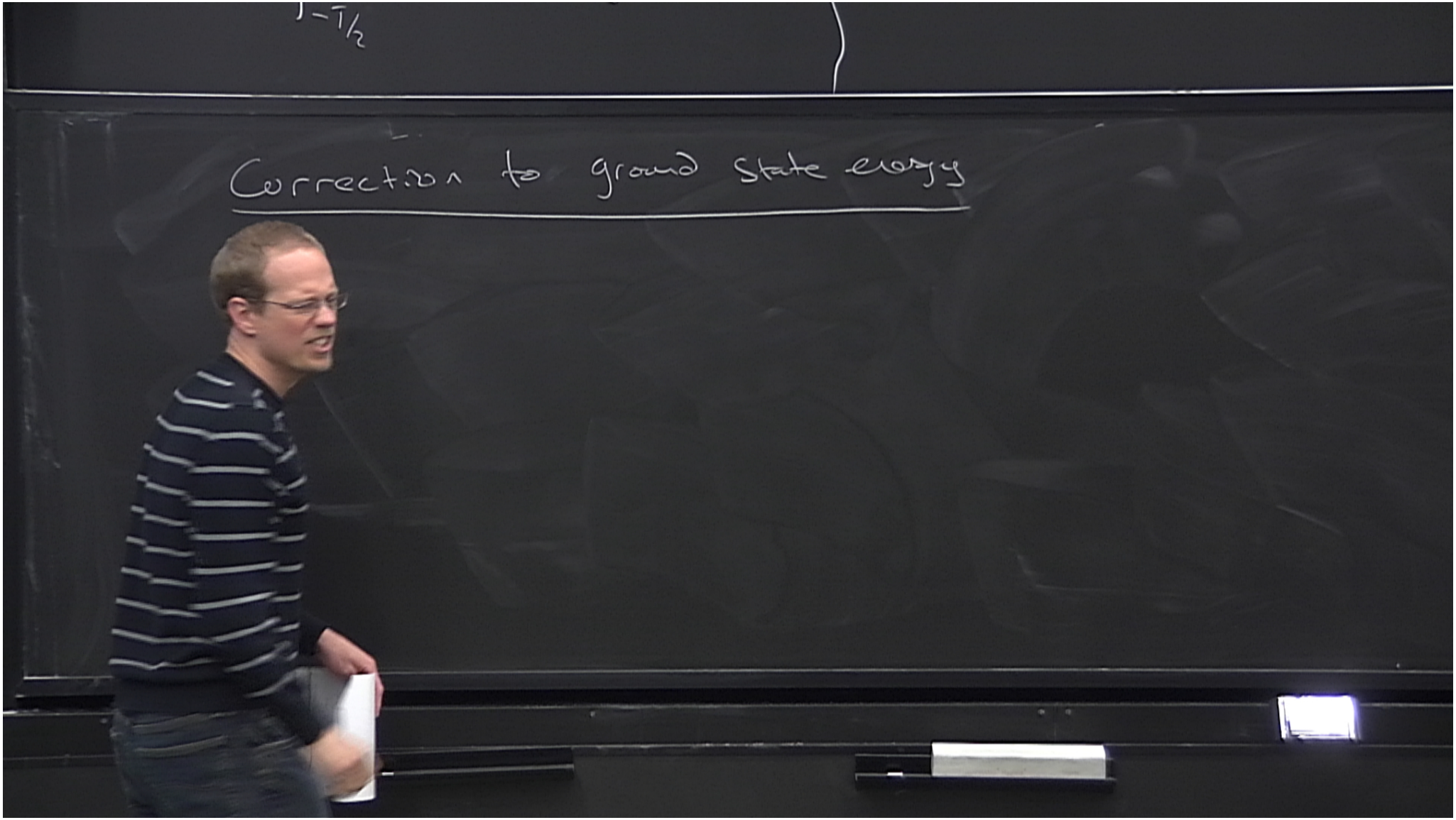

Metastable States in 1-D QM
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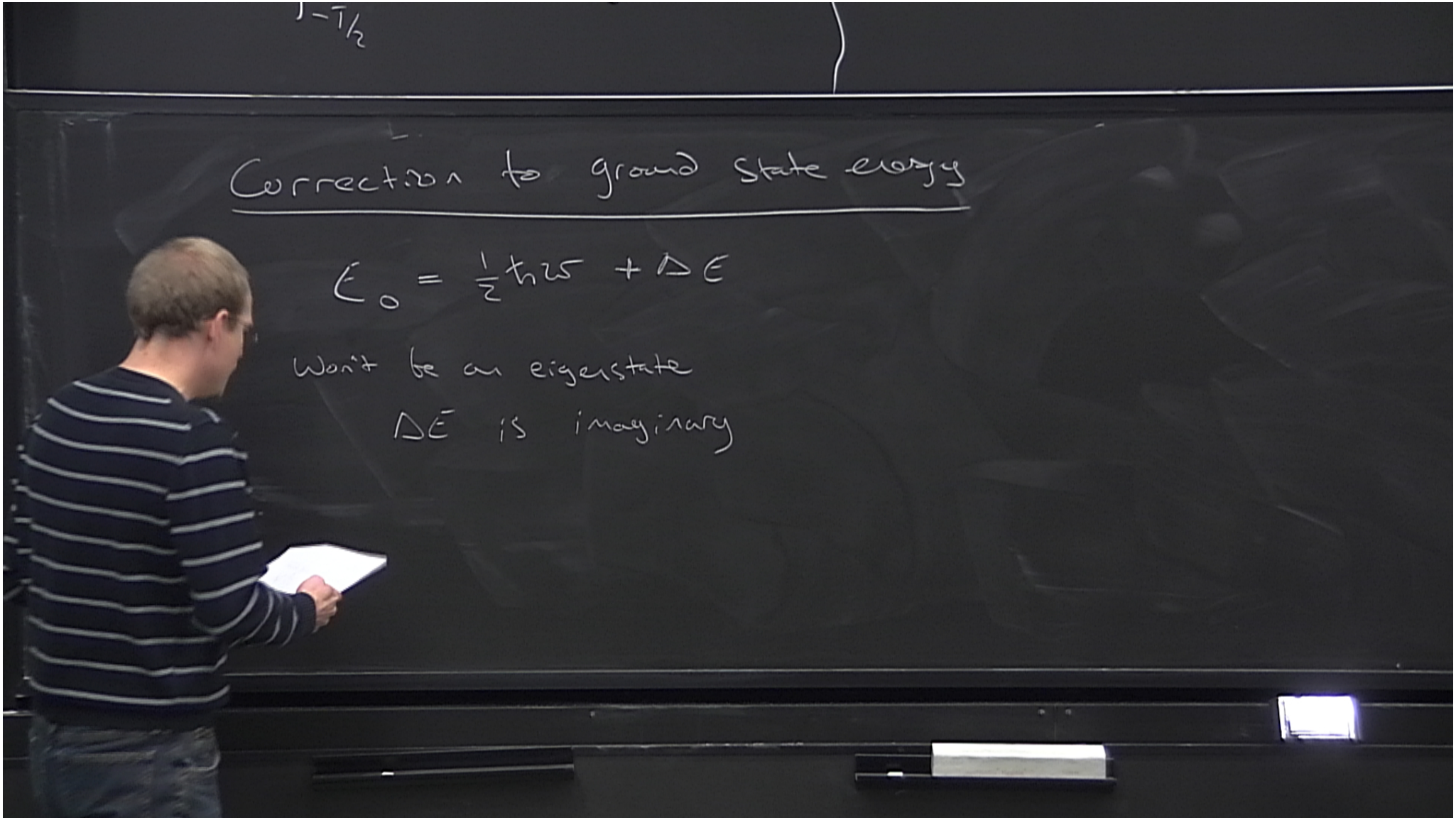

Metastable States in 1-D QM
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V
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Continuum
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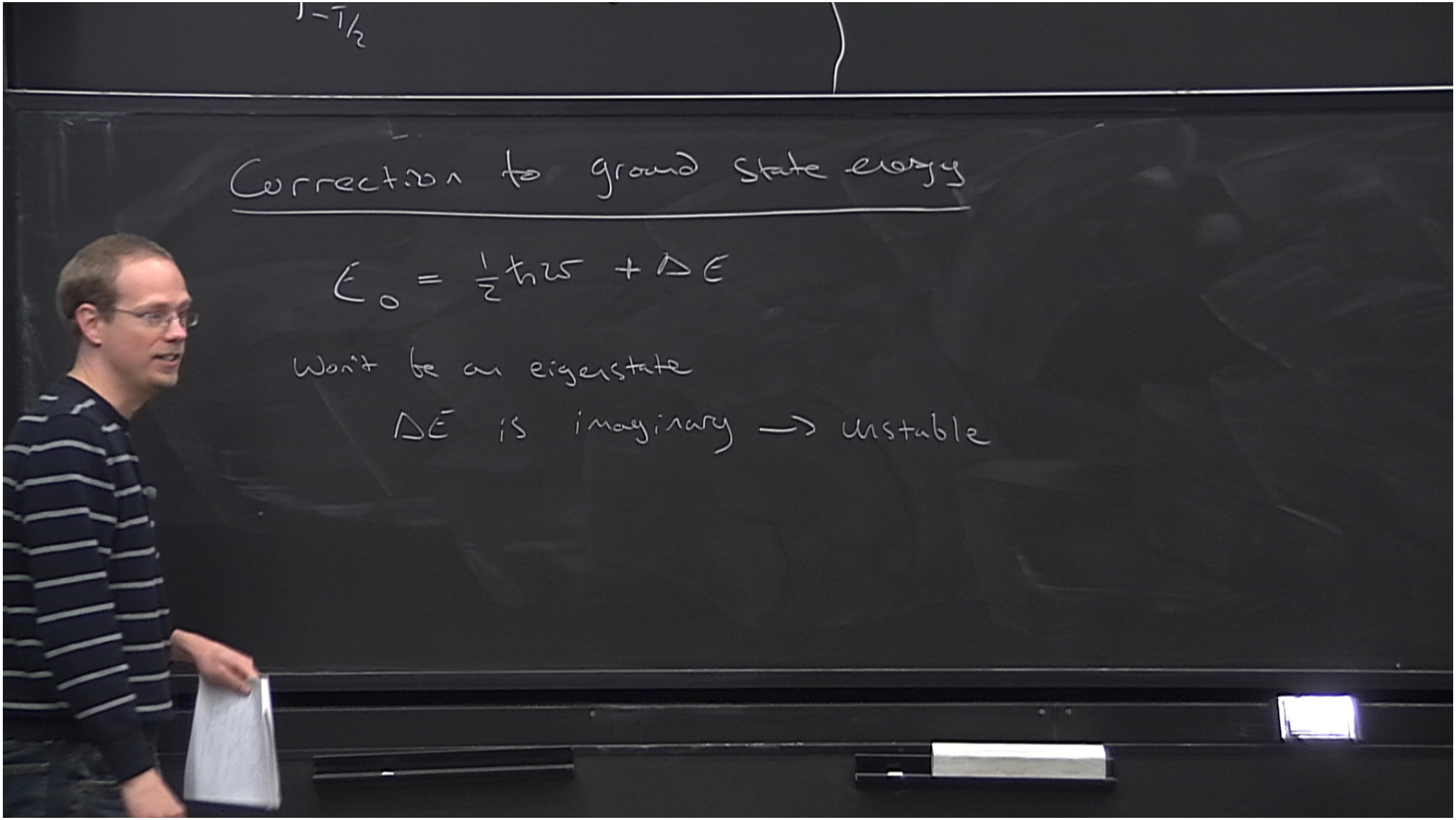

Metastable States in 1-D QM
discrete
V
x
x_F
Continuum
V
x_F
x_*
x

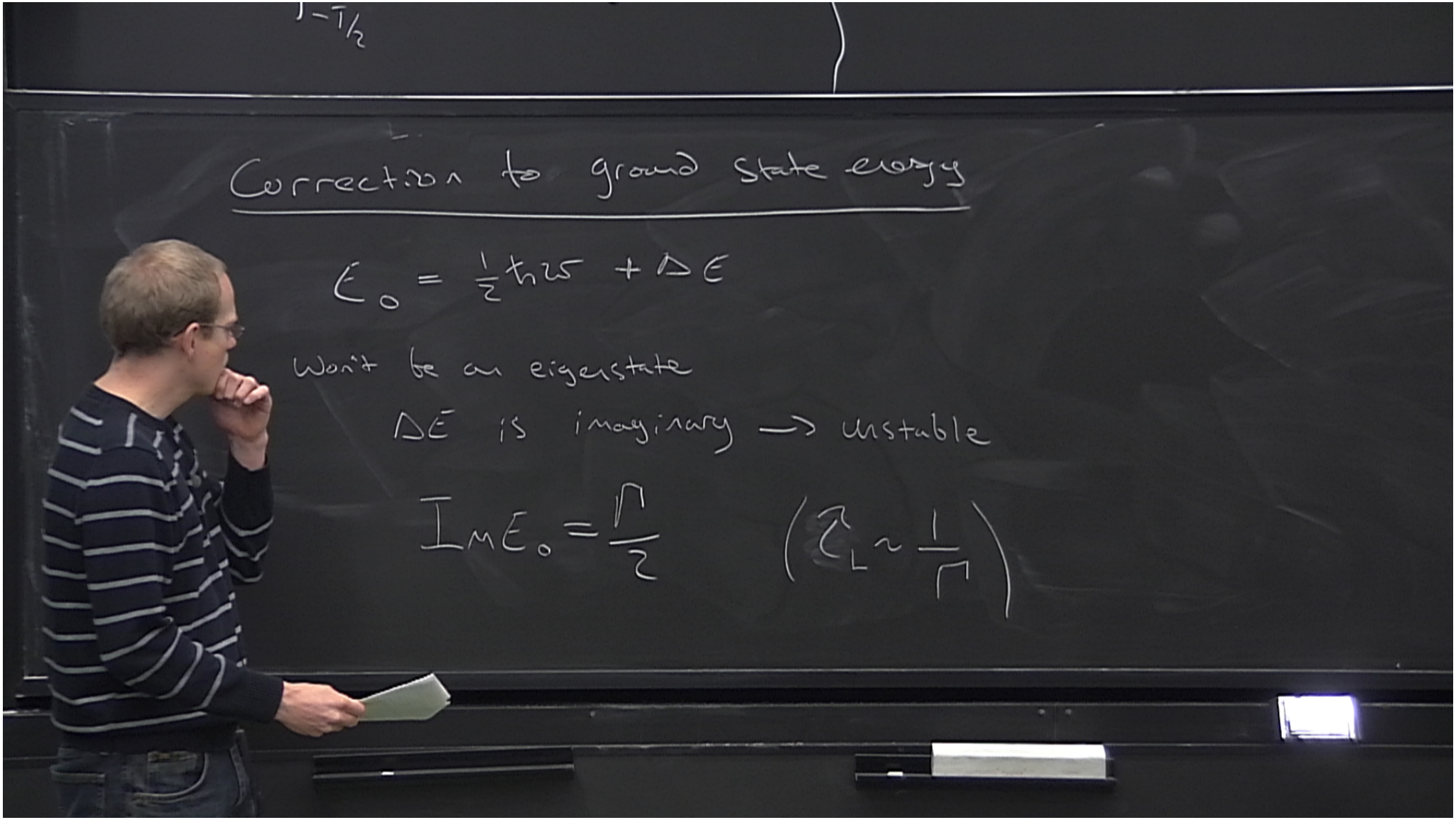
Metastable States in 1-D QM

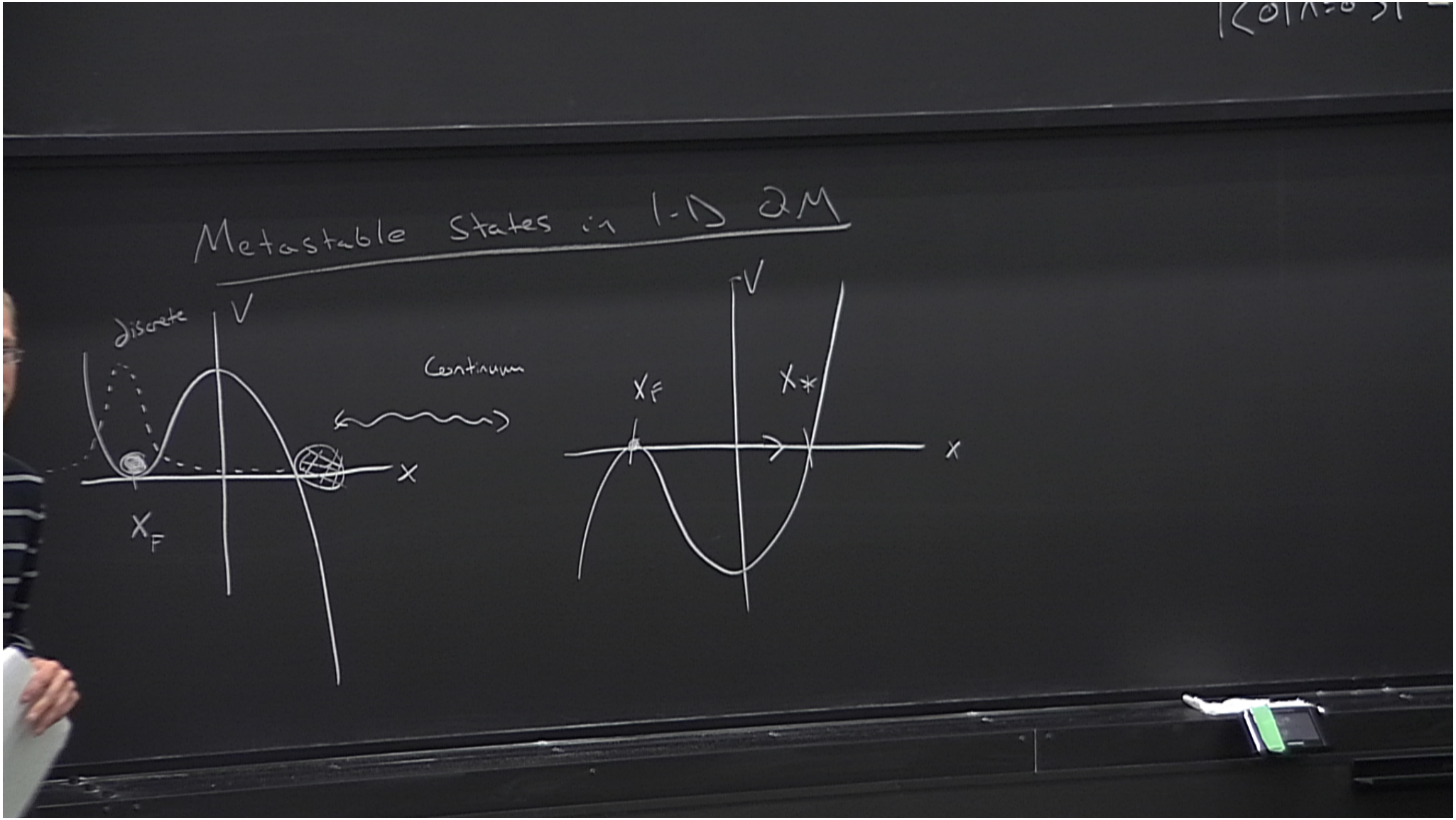


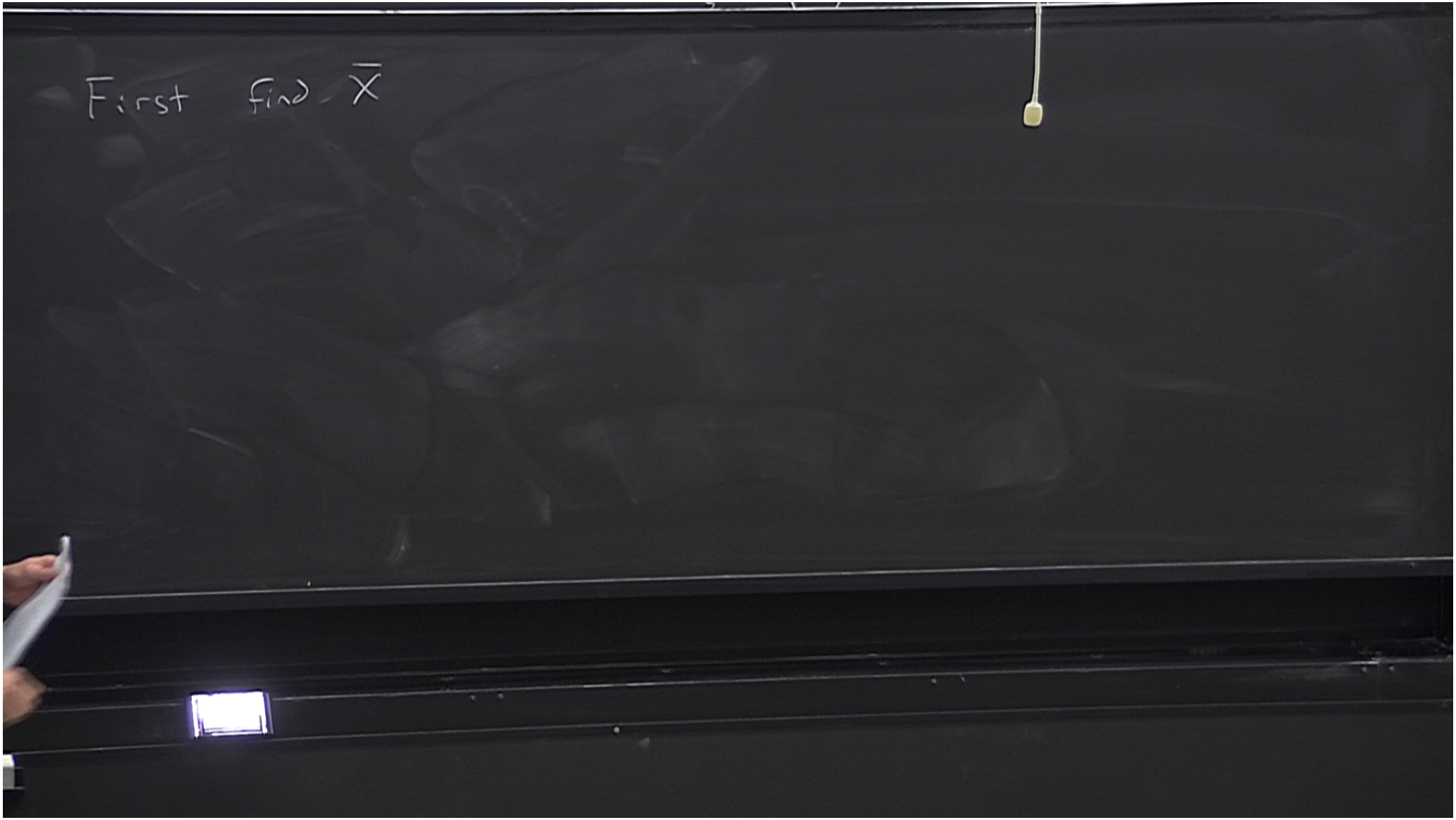

Correction to ground state energy


Correction to ground state energy
$E_0 = \frac{1}{2}\hbar\omega + \Delta E$
Won't be an eigenstate
ΔE is imaginary


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$E_0 = \frac{1}{2}\hbar\omega + \Delta E$
Won't be an eigenstate
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$\mathrm{Im} E_0 = \frac{\Gamma}{2}$ $\left(\tau_L \sim \frac{1}{\Gamma}\right)$

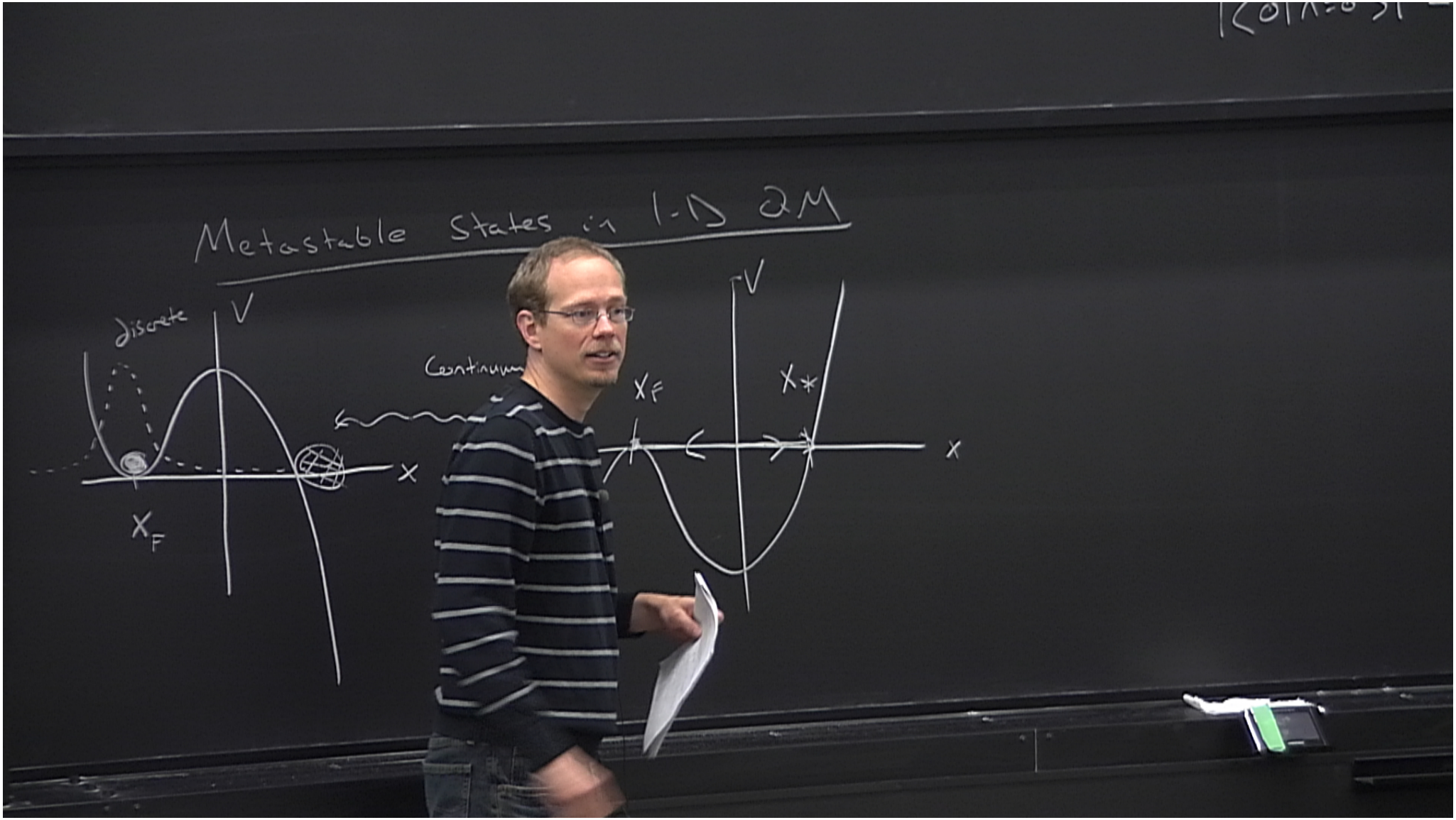

Metastable States in 1-D QM
discrete
V
Continuum
X
X_F
V
X_F
X_*
x


First find $\bar{X}$

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$$\langle x_F | e^{-HT/\hbar} | x_F \rangle$$

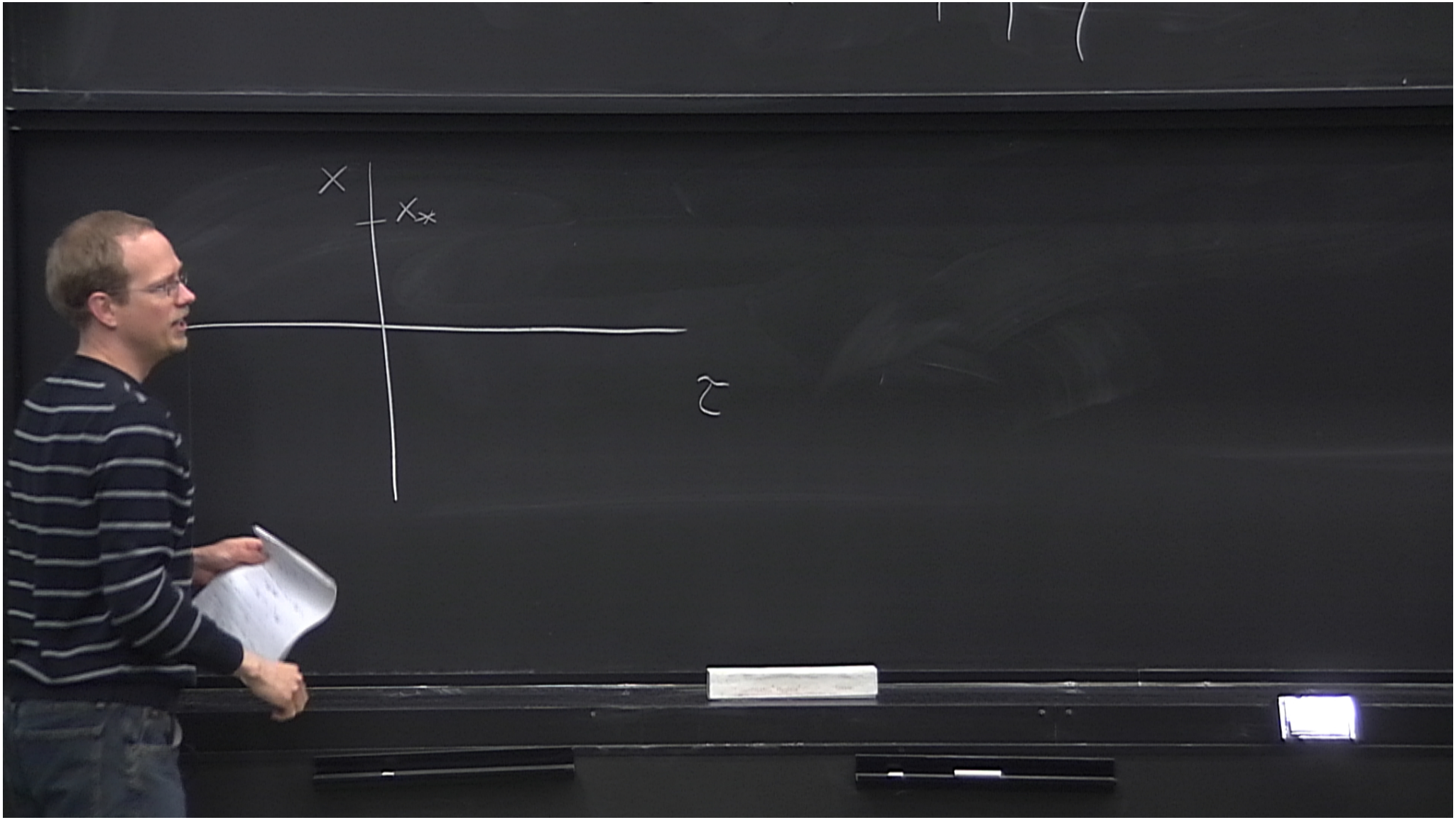
$$-\frac{\partial \bar{X}^2}{\partial \tau^2} + V'(\bar{x}) = 0 \quad \text{w/} \quad \bar{X}(\pm T/2) = x_F$$

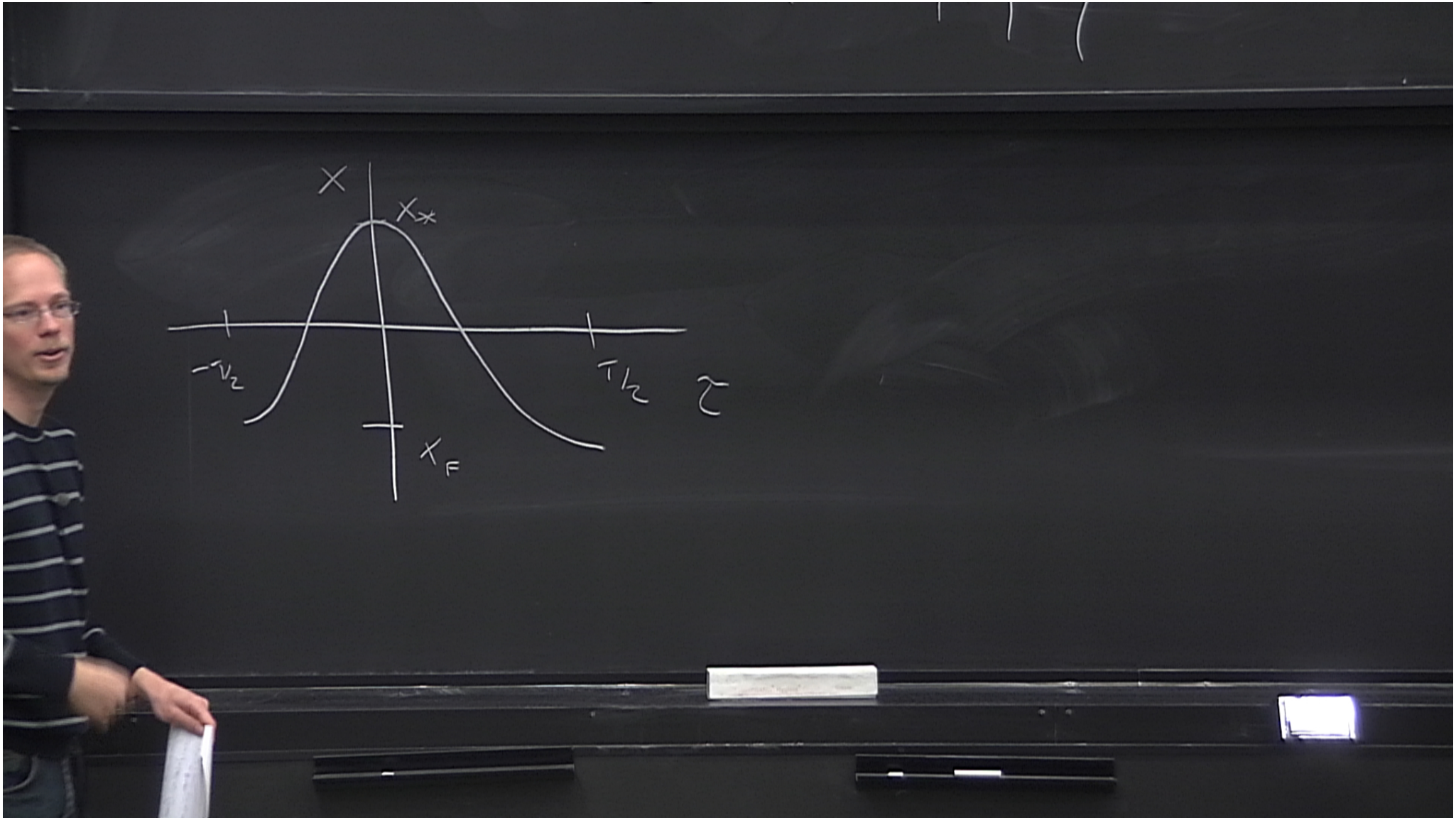

Metastable States in 1-D QM
discrete
V
x
x_F
Continuum
V
x_F
x_*
x

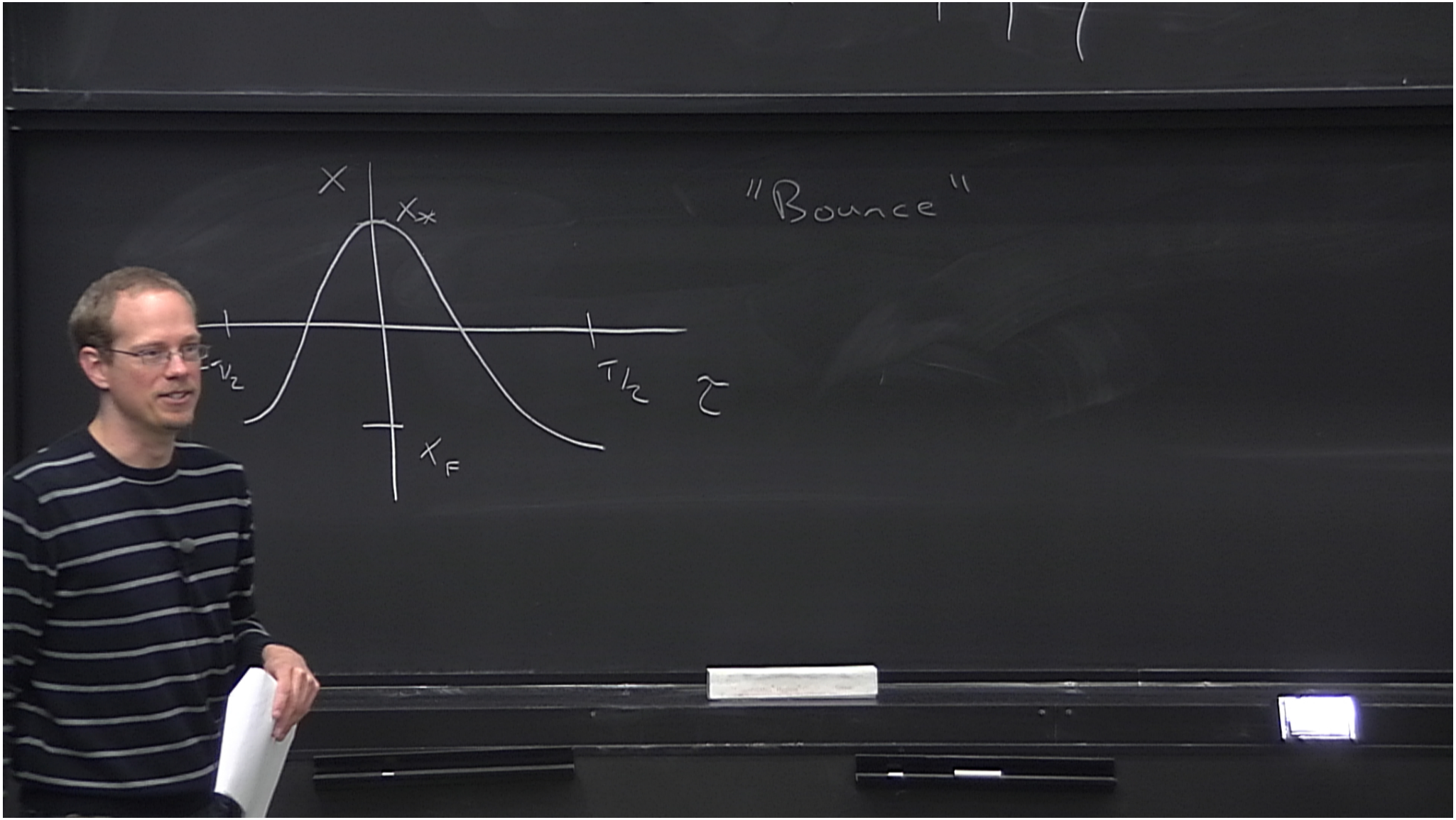
First find $\bar{X}$

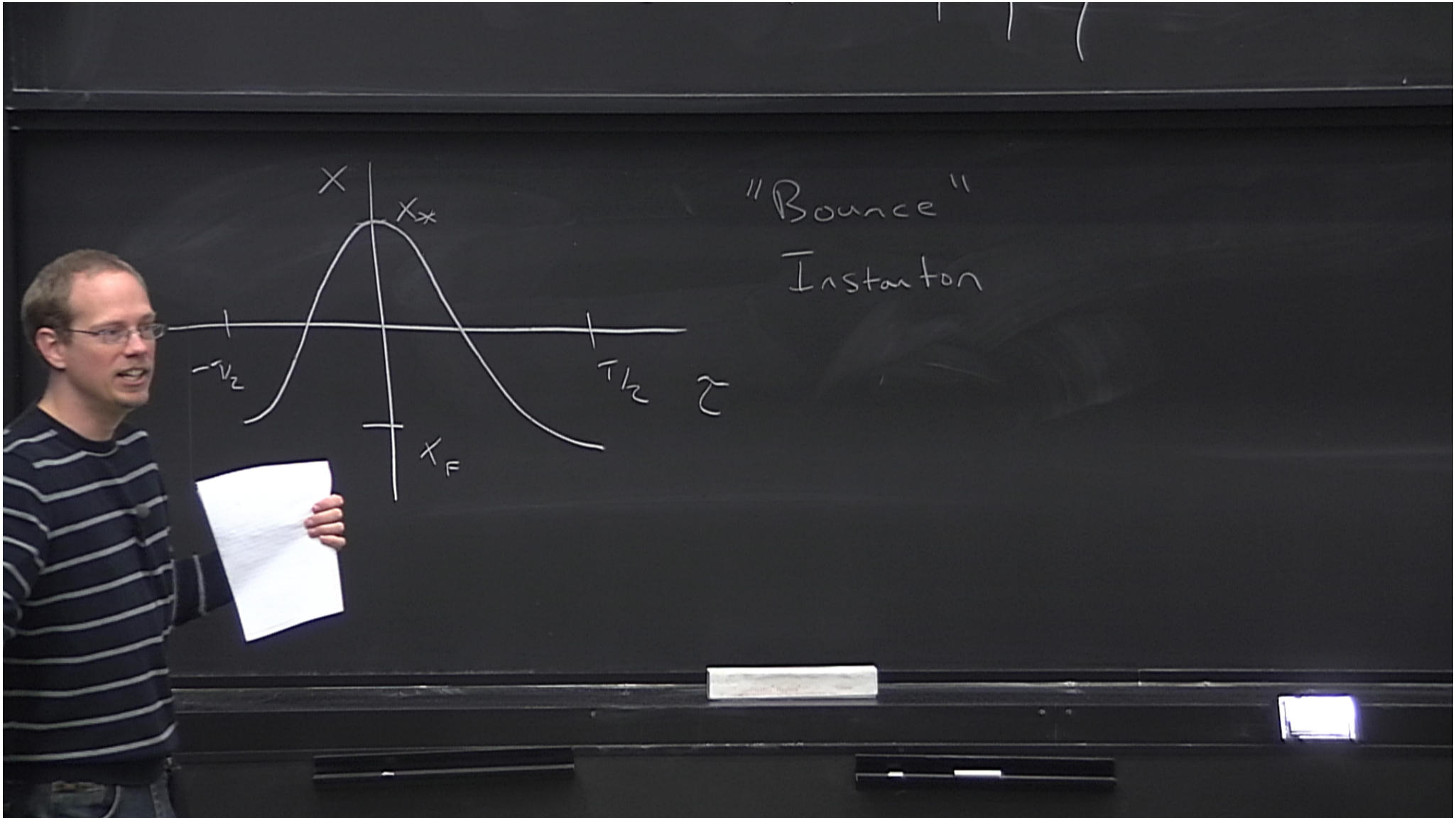
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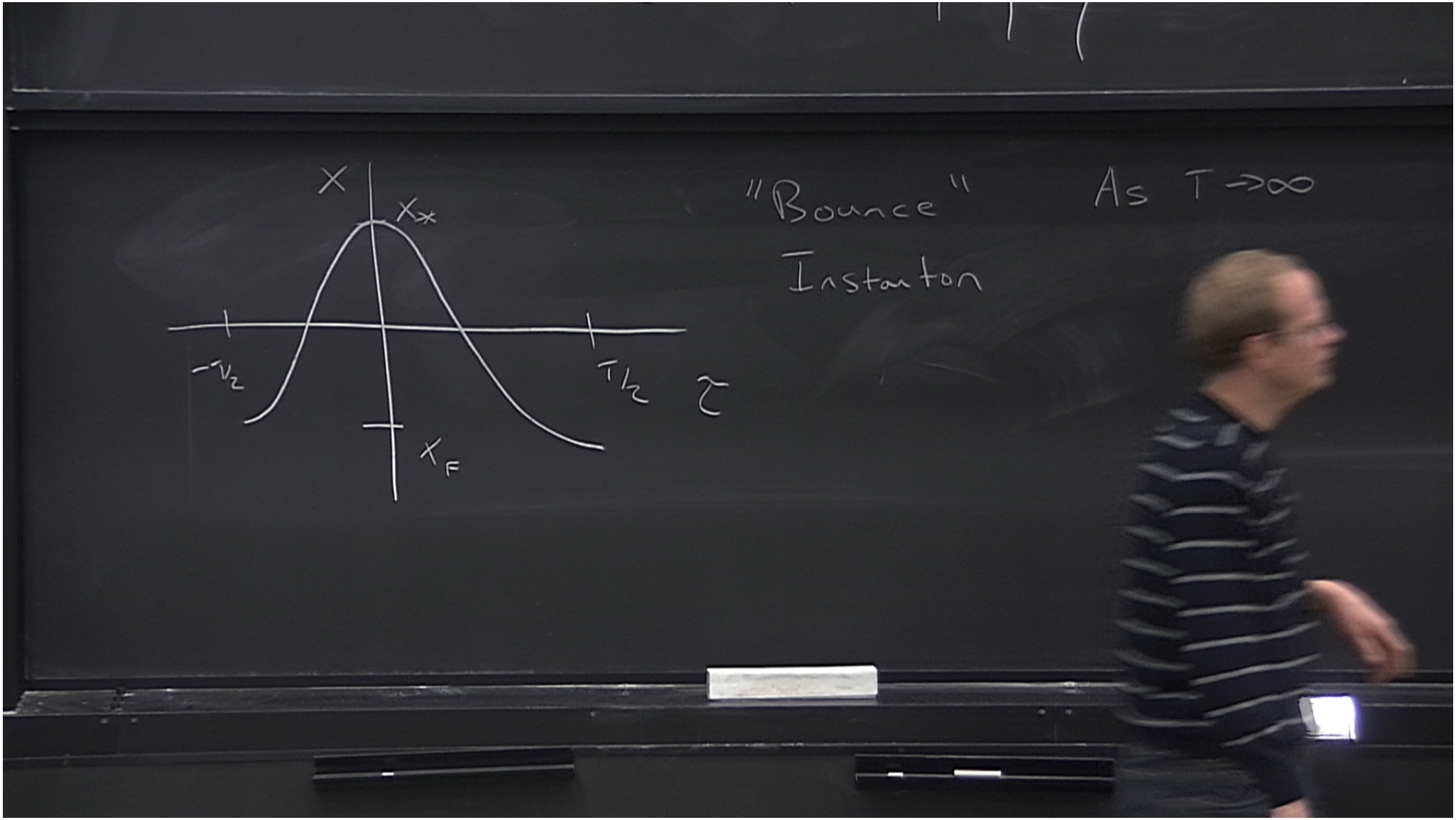
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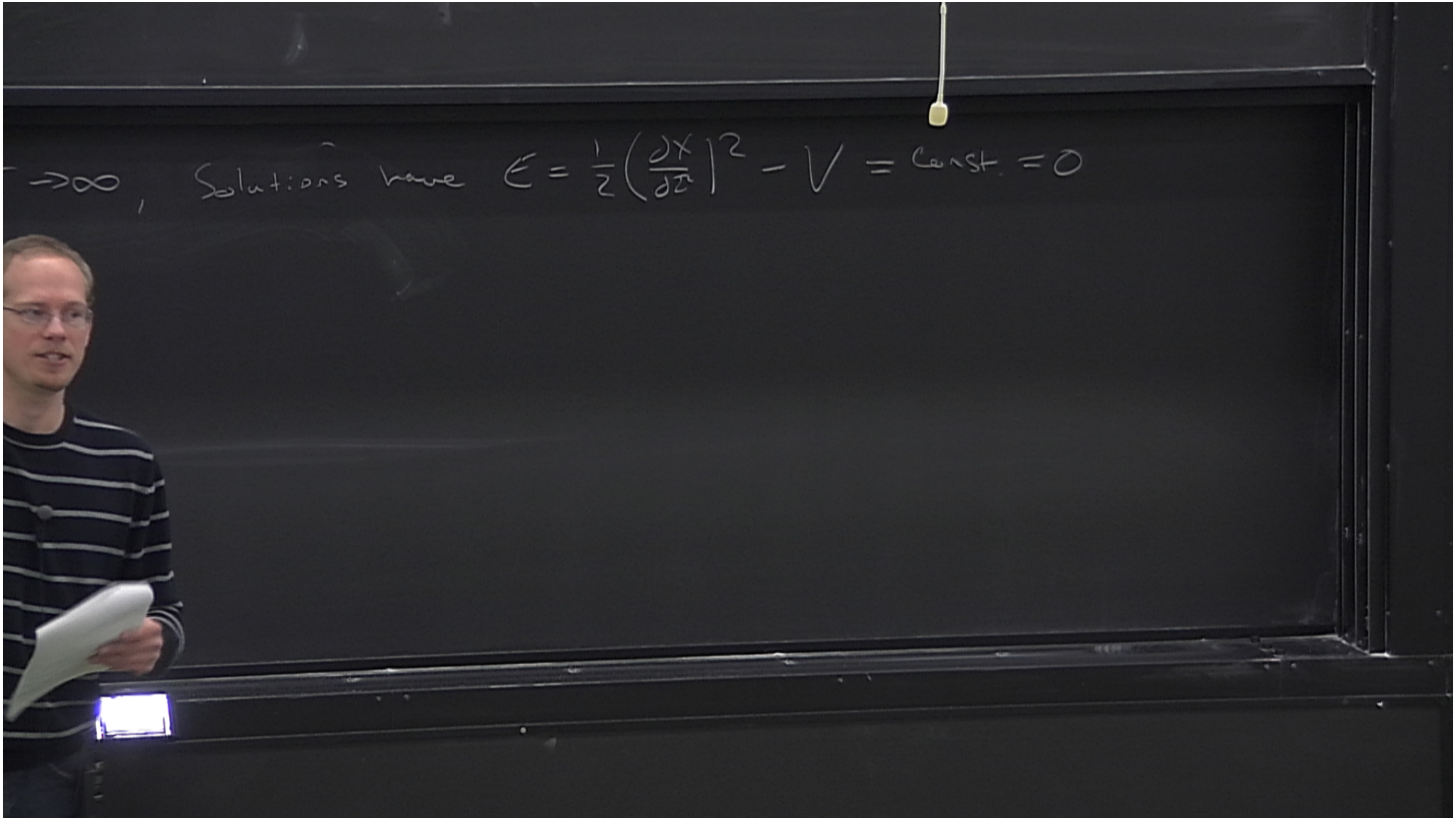

X
X_*
τ

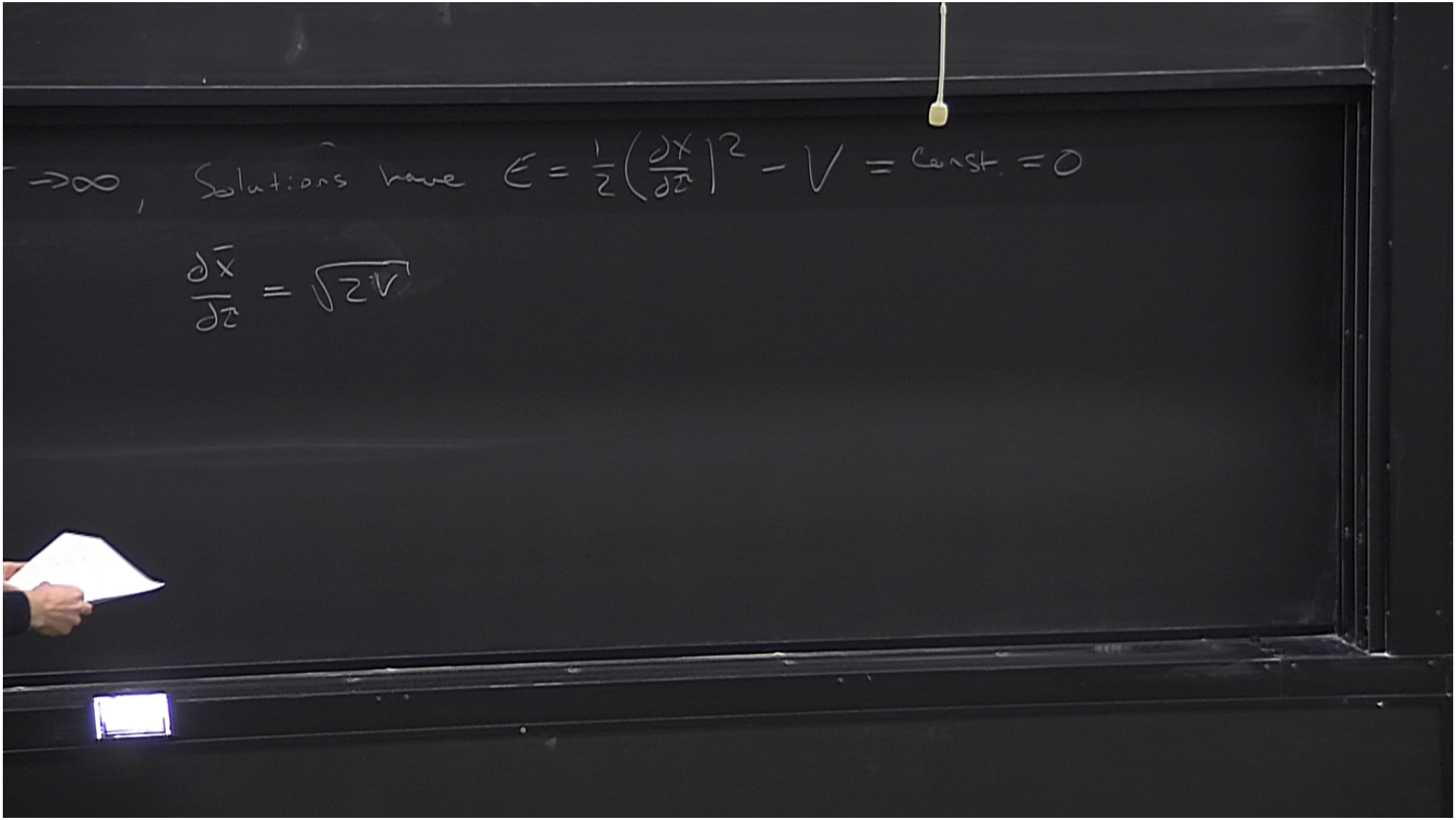

X
X_*
$-\tau_{1/2}$
$\tau_{1/2}$
τ
X_F

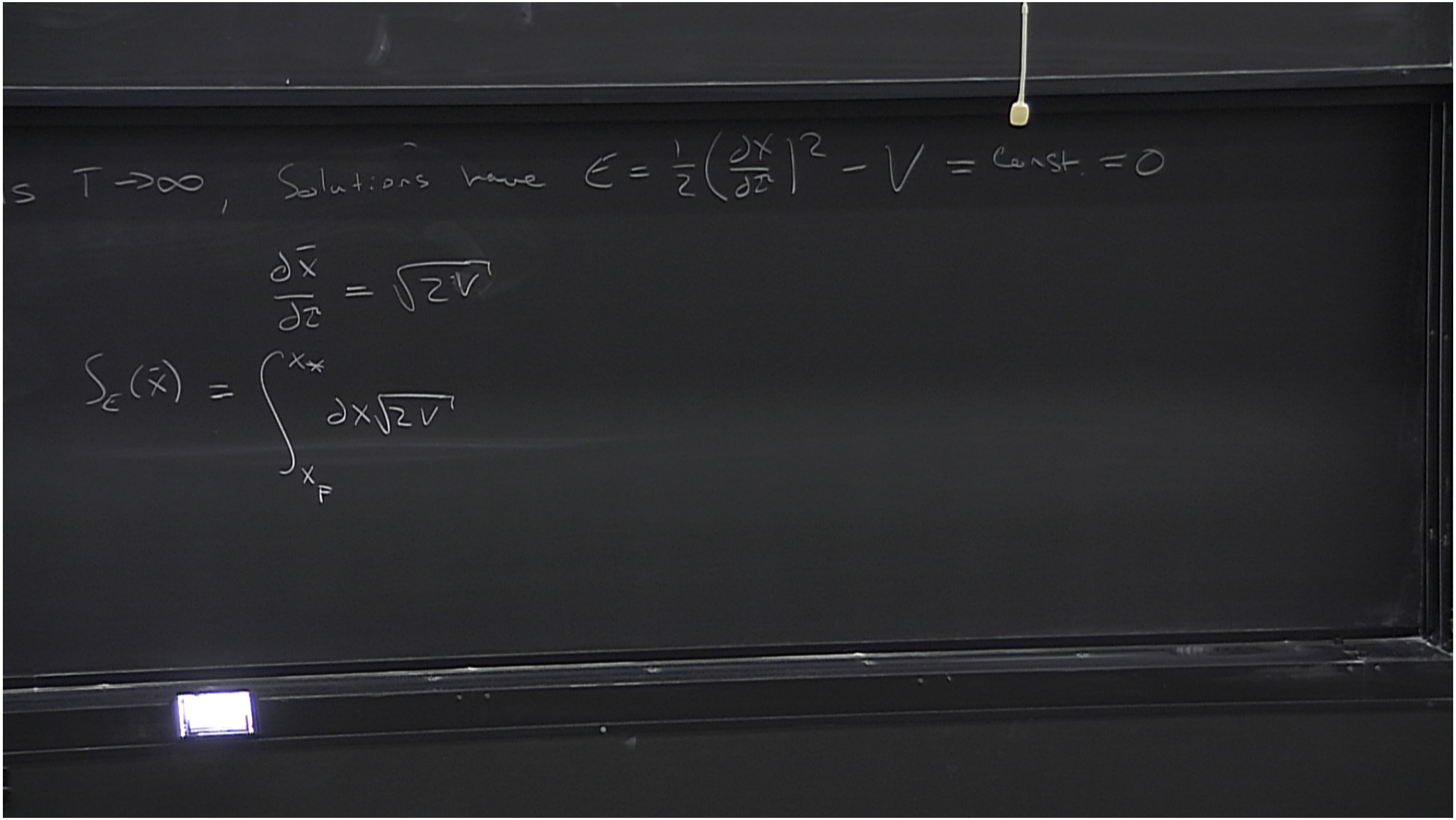

X
X_*
"Bounce"
$-\tau_2$
τ_2
τ
X_F

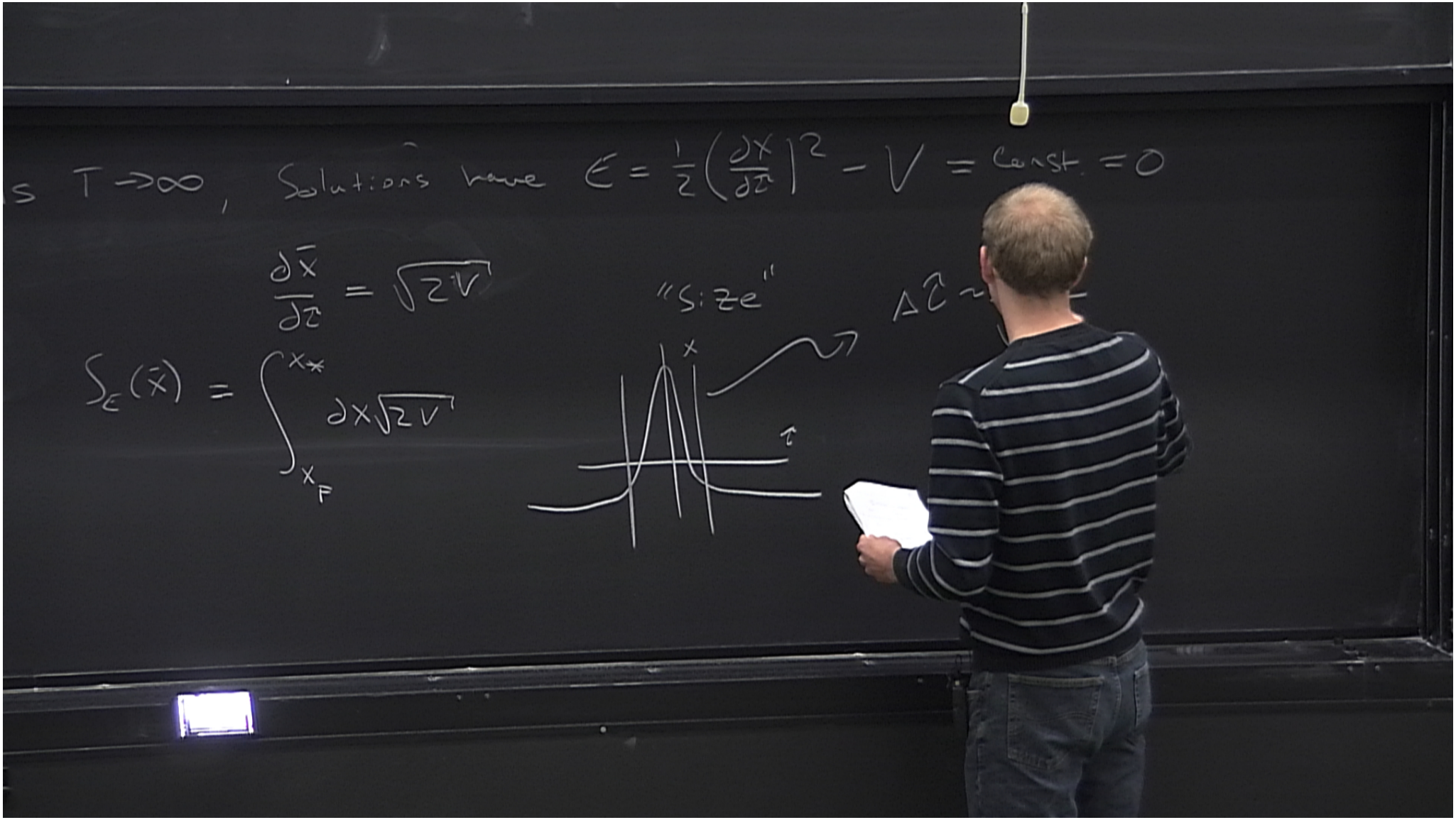

X
x_*
$-\tau_2$
τ_2
τ
x_F
"Bounce"
Instanton

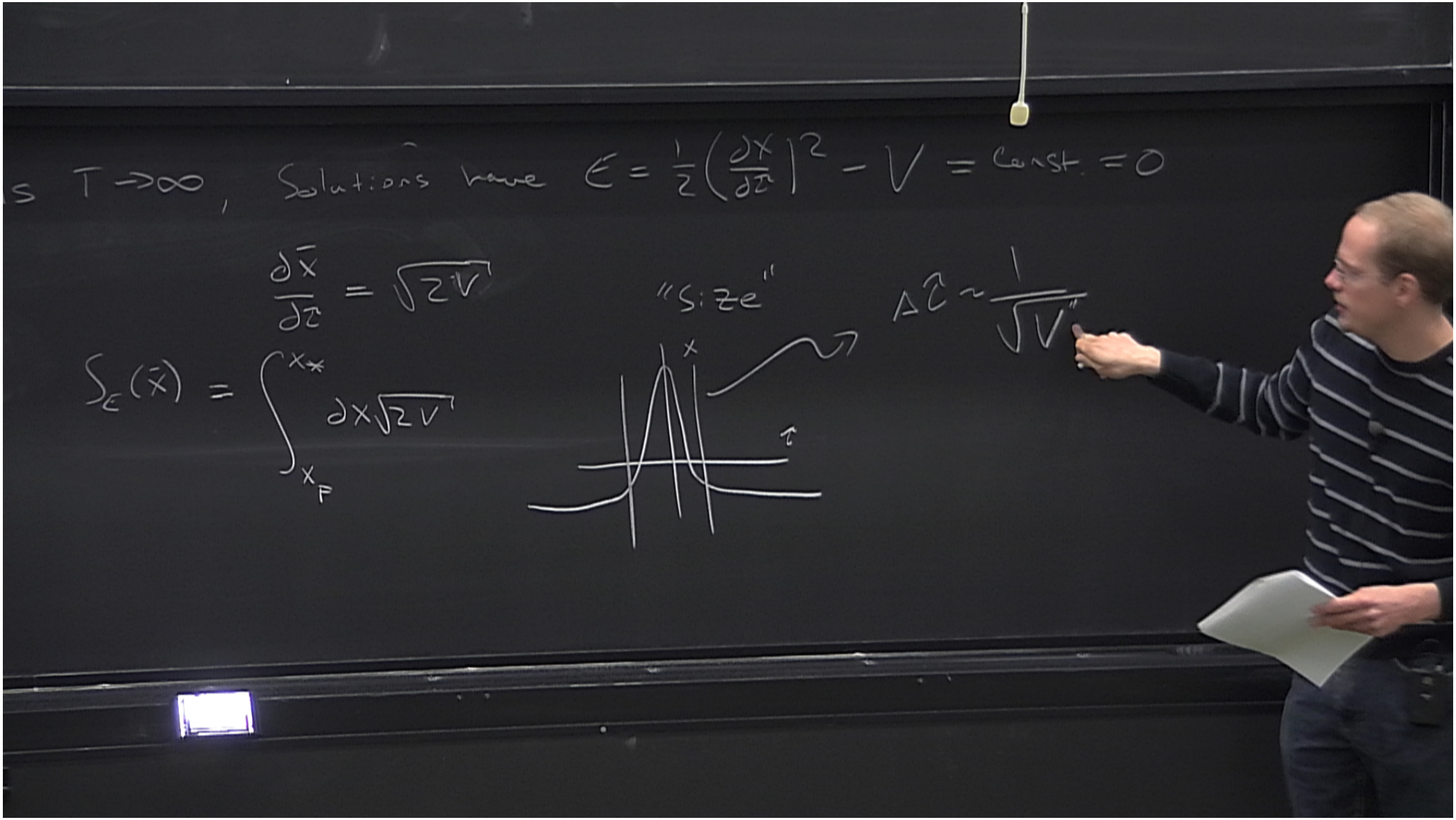

X
X_*
$-T_2$
T_2
τ
X_F
"Bounce"
Instanton
As $T \to \infty$


$\to\infty$, Solutions have $\epsilon = \frac{1}{2}\left(\frac{\partial x}{\partial \tau}\right)^2 - V = \text{const.} = 0$


$\to\infty$, Solutions have $\mathcal{E} = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 - V = \text{const.} = 0$
$\frac{dx}{dt} = \sqrt{2V}$


s $T \to \infty$, Solutions have $\epsilon = \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 - V = \text{Const.} = 0$
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$S_E(\bar{x}) = \int_{x_F}^{x_*} dx\sqrt{2V}$


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$\frac{d\bar{x}}{d\tau} = \sqrt{2V}$
"Size"
$\Delta\tau \sim$
$S_E(\bar{x}) = \int_{x_F}^{x_*} dx\sqrt{2V}$
x
τ

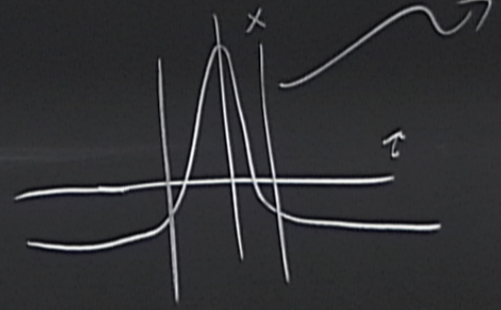

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$S_E(\bar{x}) = \int_{x_F}^{x_*} dx\sqrt{2V}$
"Size"
$\Delta\tau \sim \frac{1}{\sqrt{V''}}$
x
τ

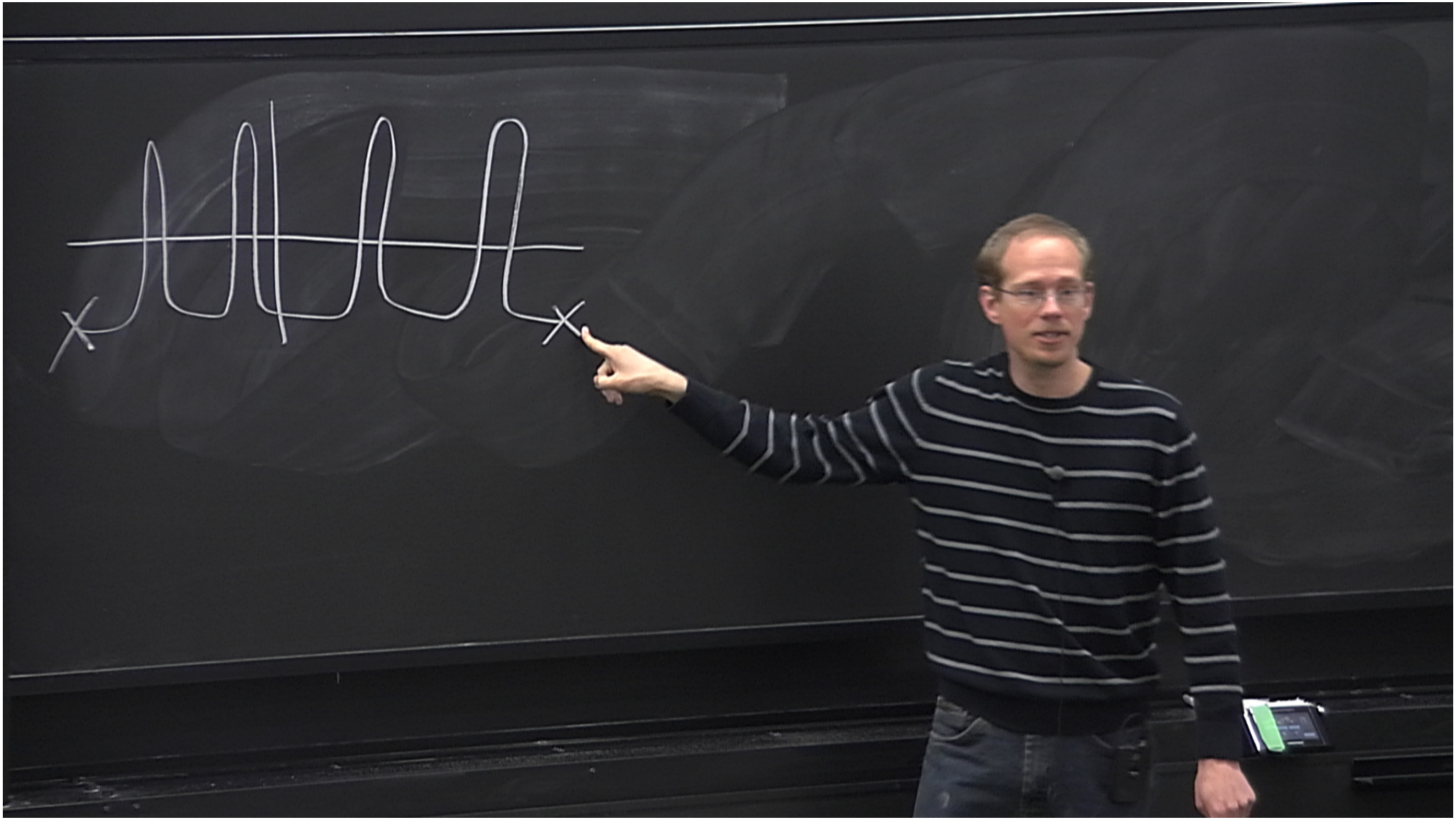
s $T \to \infty$, Solutions have $\mathcal{E} = \frac{1}{2}\left(\frac{dx}{d\bar{\tau}}\right)^2 - V = \text{Const.} = 0$

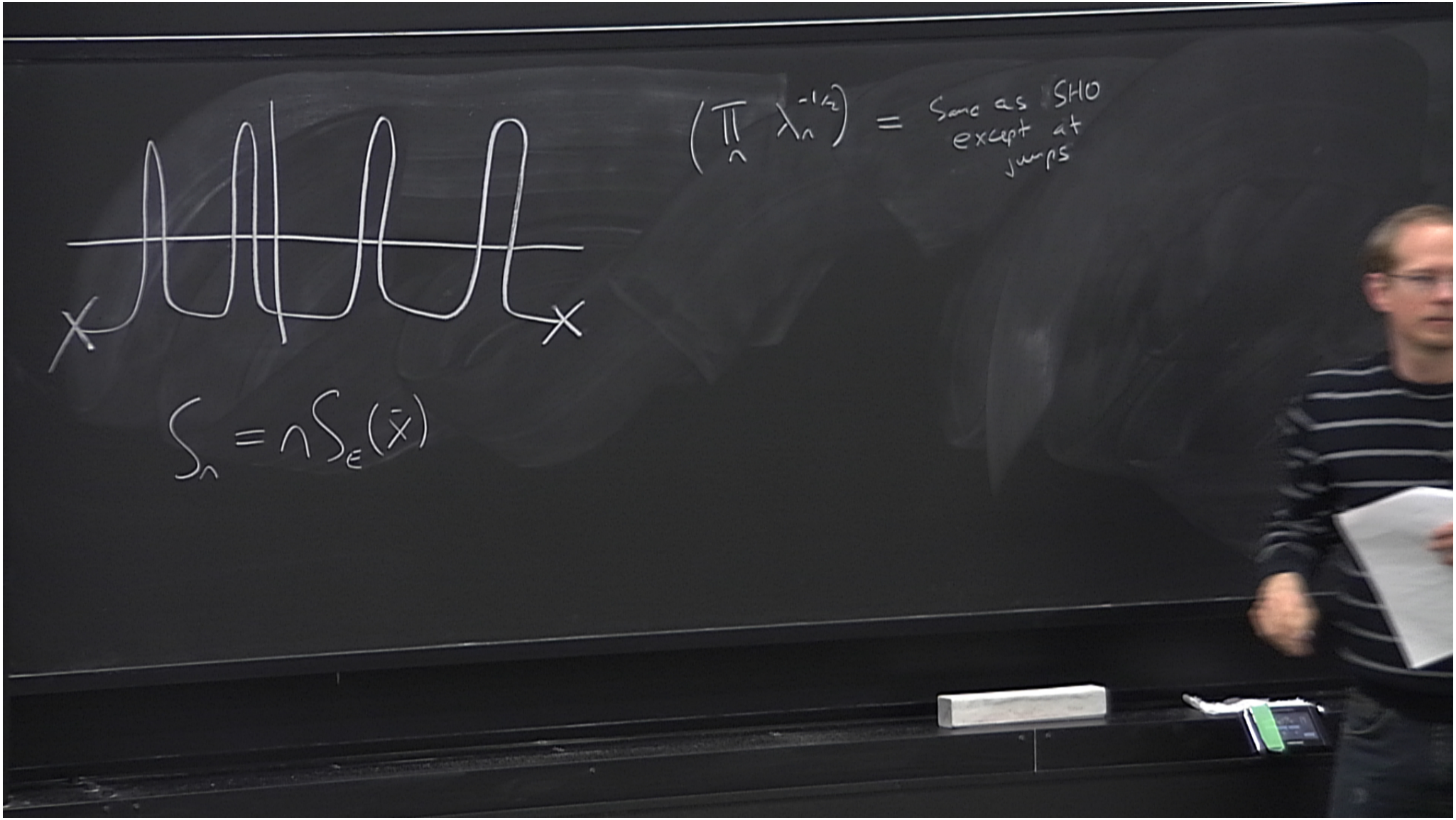
$$\frac{d\bar{x}}{d\bar{\tau}} = \sqrt{2V}$$

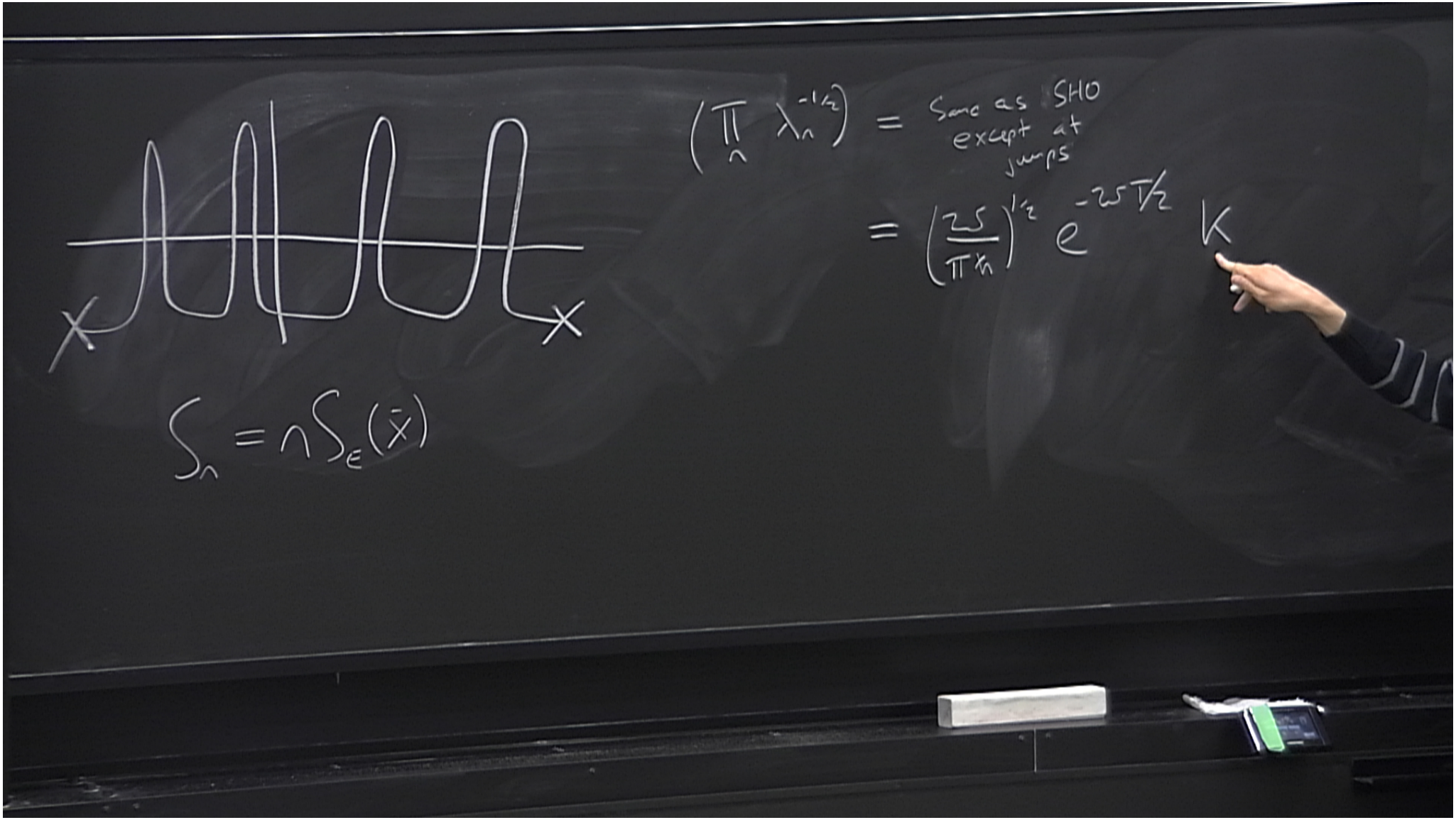
"size" $\Delta\bar{\tau} \sim \frac{1}{\sqrt{V''(x_{min})}}$

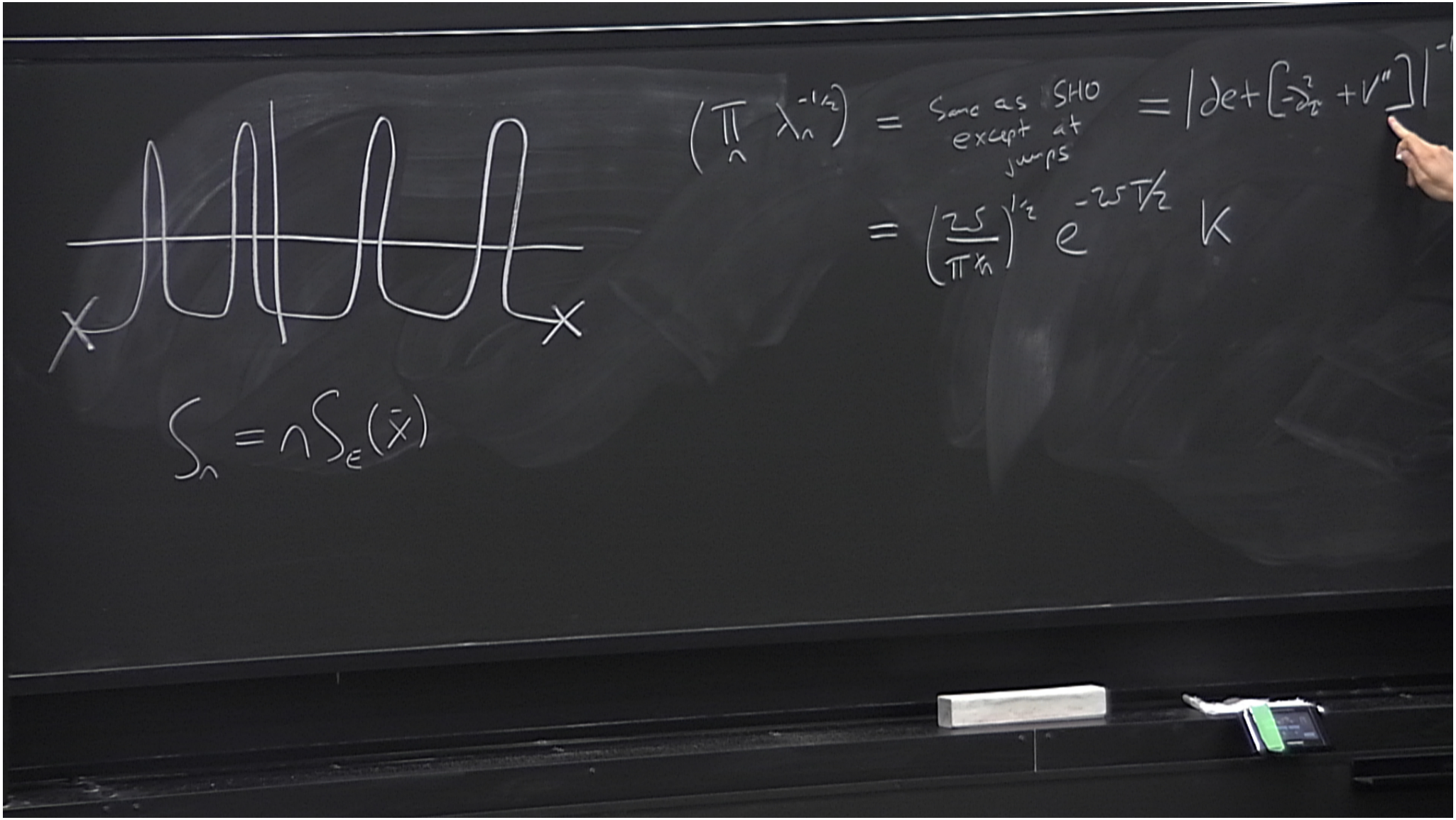
$$S_E(\bar{x}) = \int_{x_F}^{x_*} dx\sqrt{2V}$$

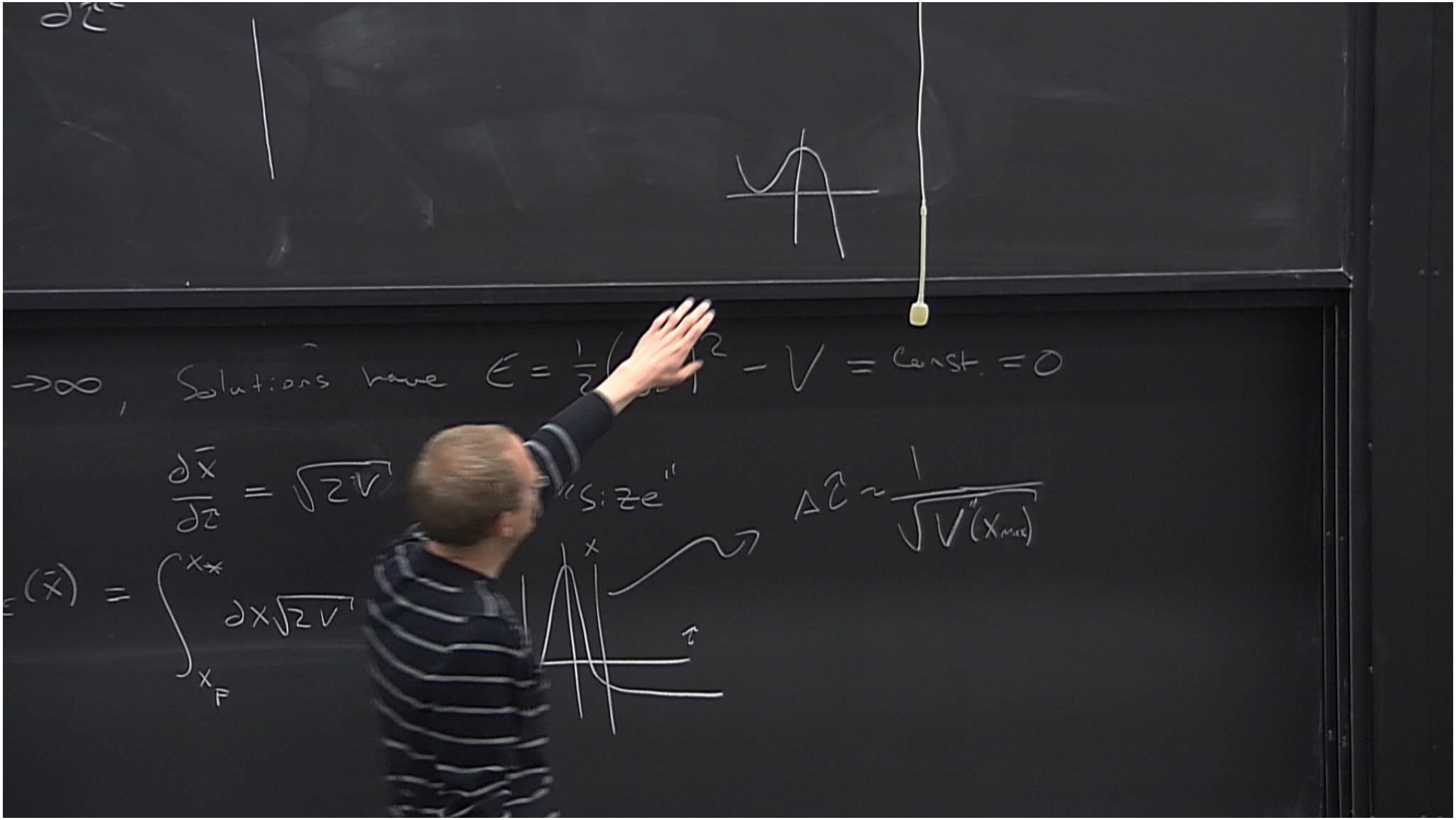


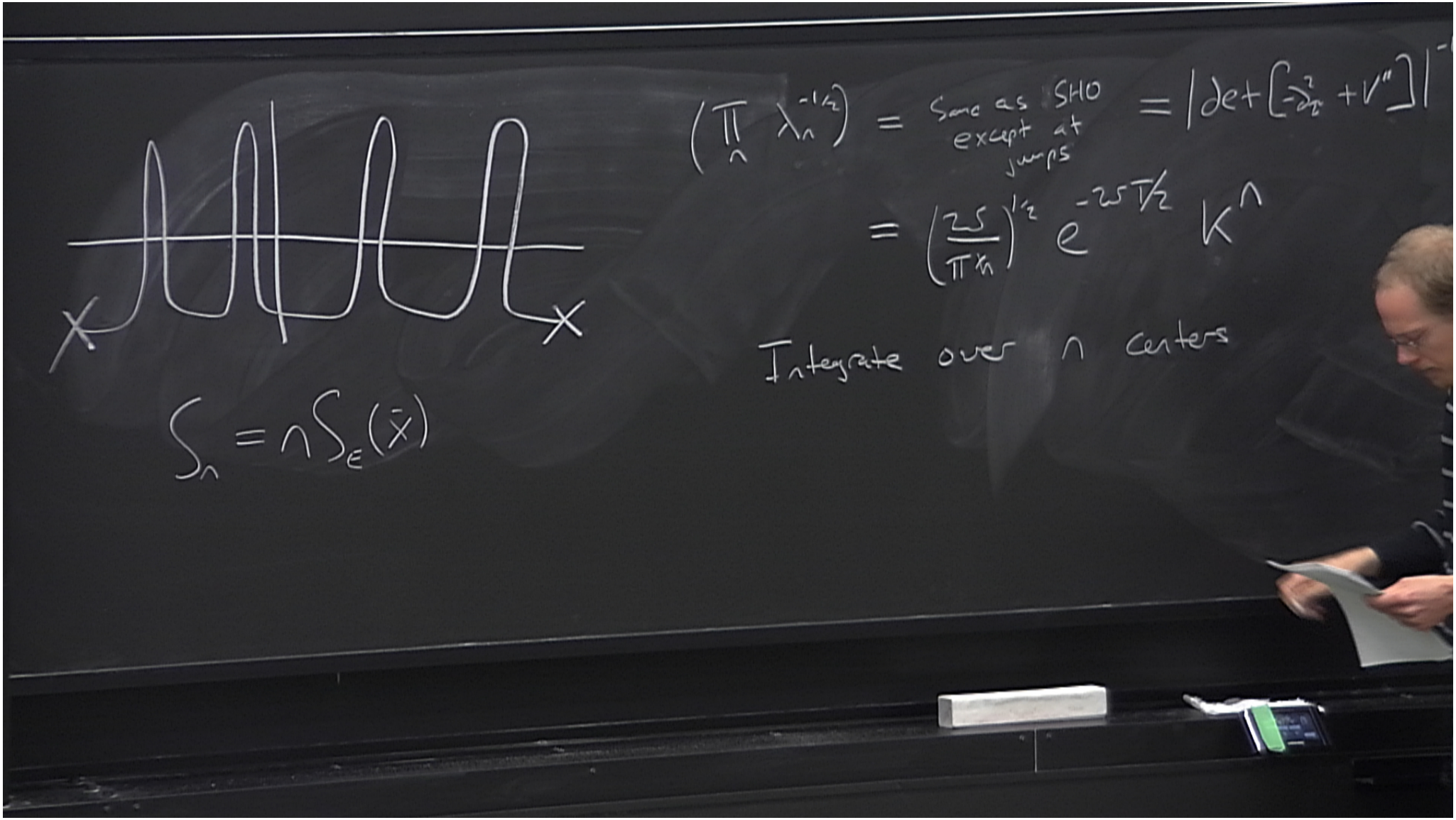


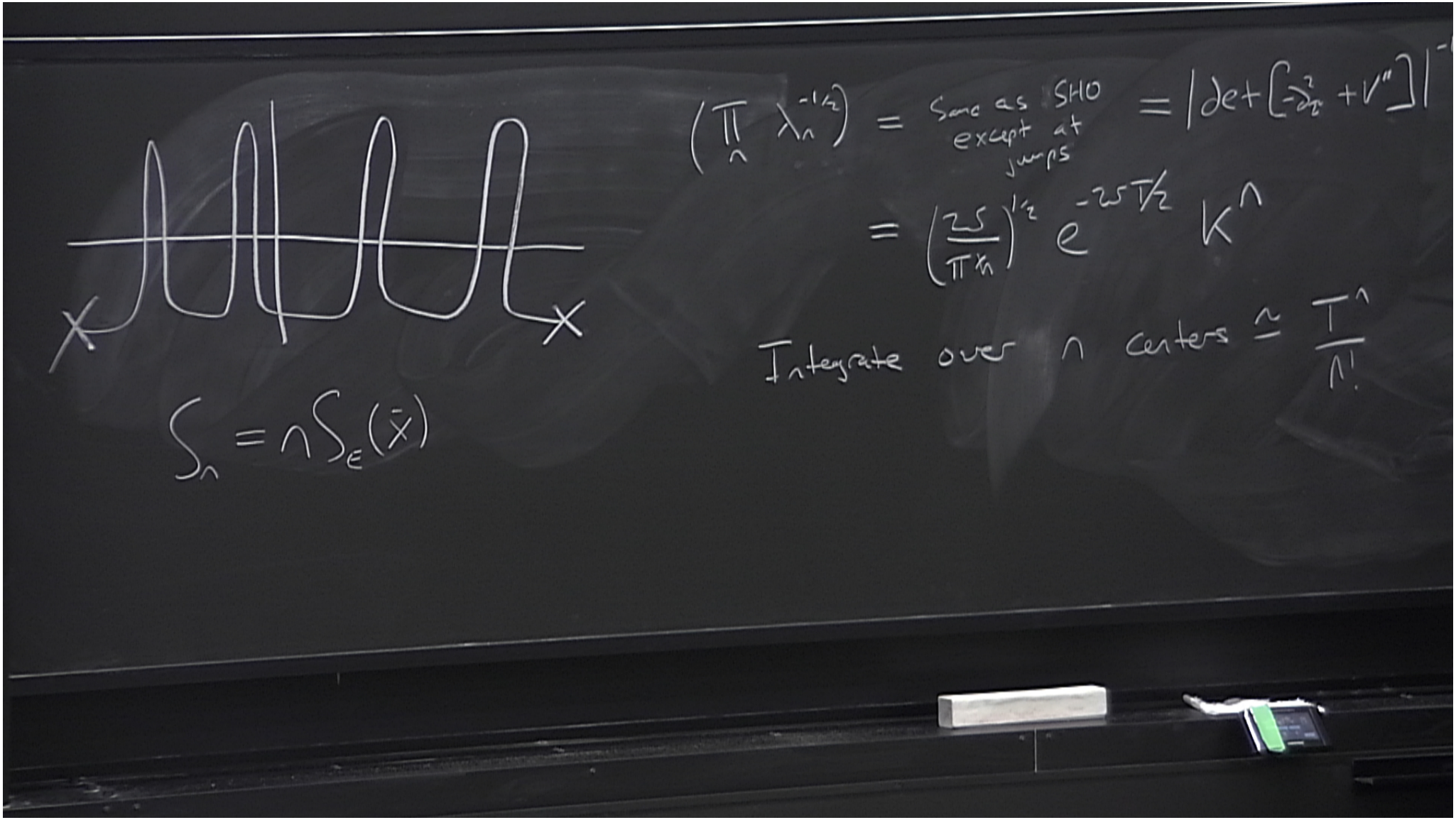

$\left(\prod_n \lambda_n^{-1/2}\right)$ = Same as SHO except at jumps
$S_n = n S_\epsilon(\bar{x})$

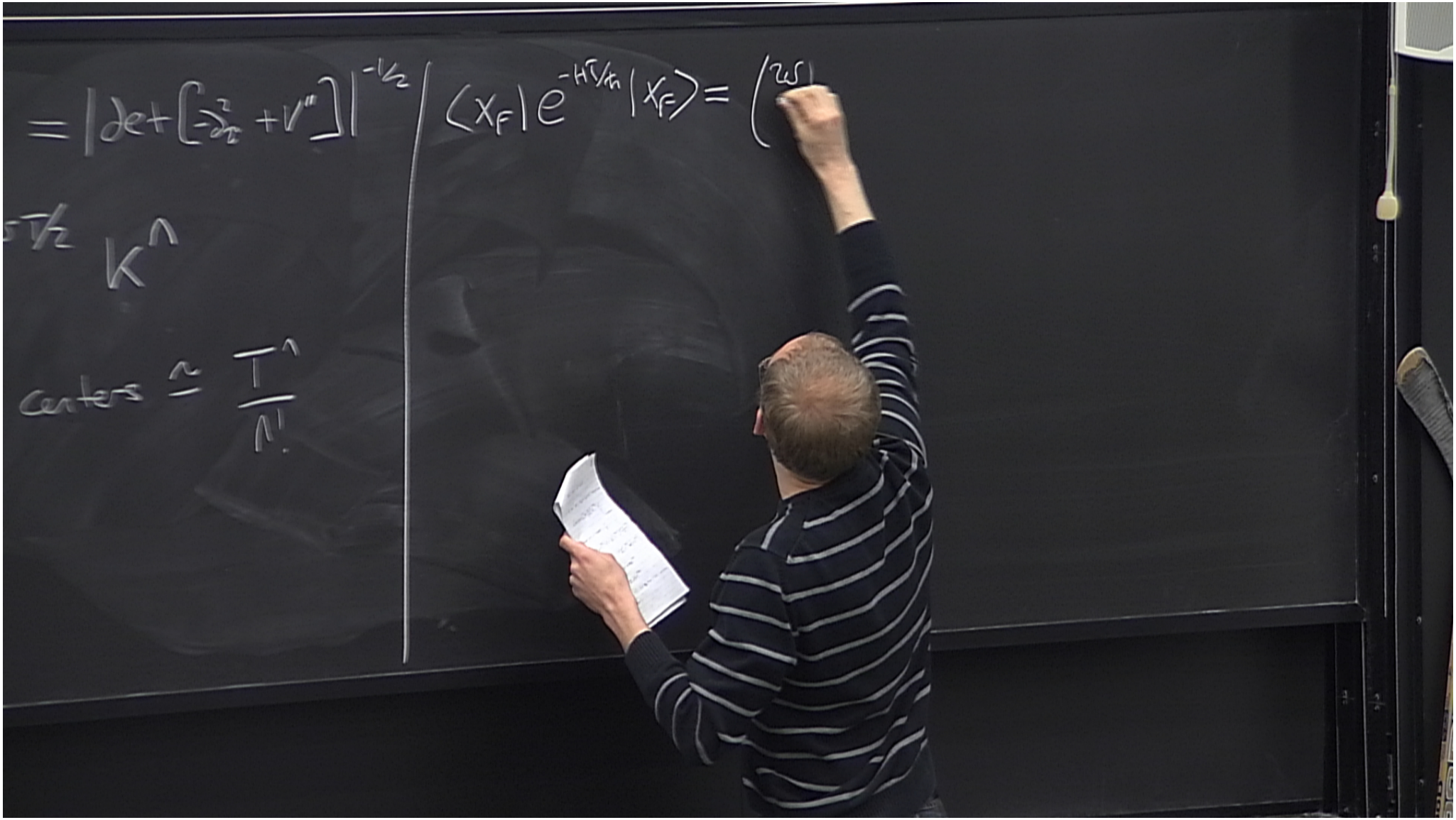

$\left(\prod_n \lambda_n^{-1/2}\right)$ = Same as SHO except at jumps
$= \left(\frac{2S}{\pi\hbar}\right)^{1/2} e^{-2ST/\hbar} K$
$S_n = n S_\epsilon(\bar{x})$

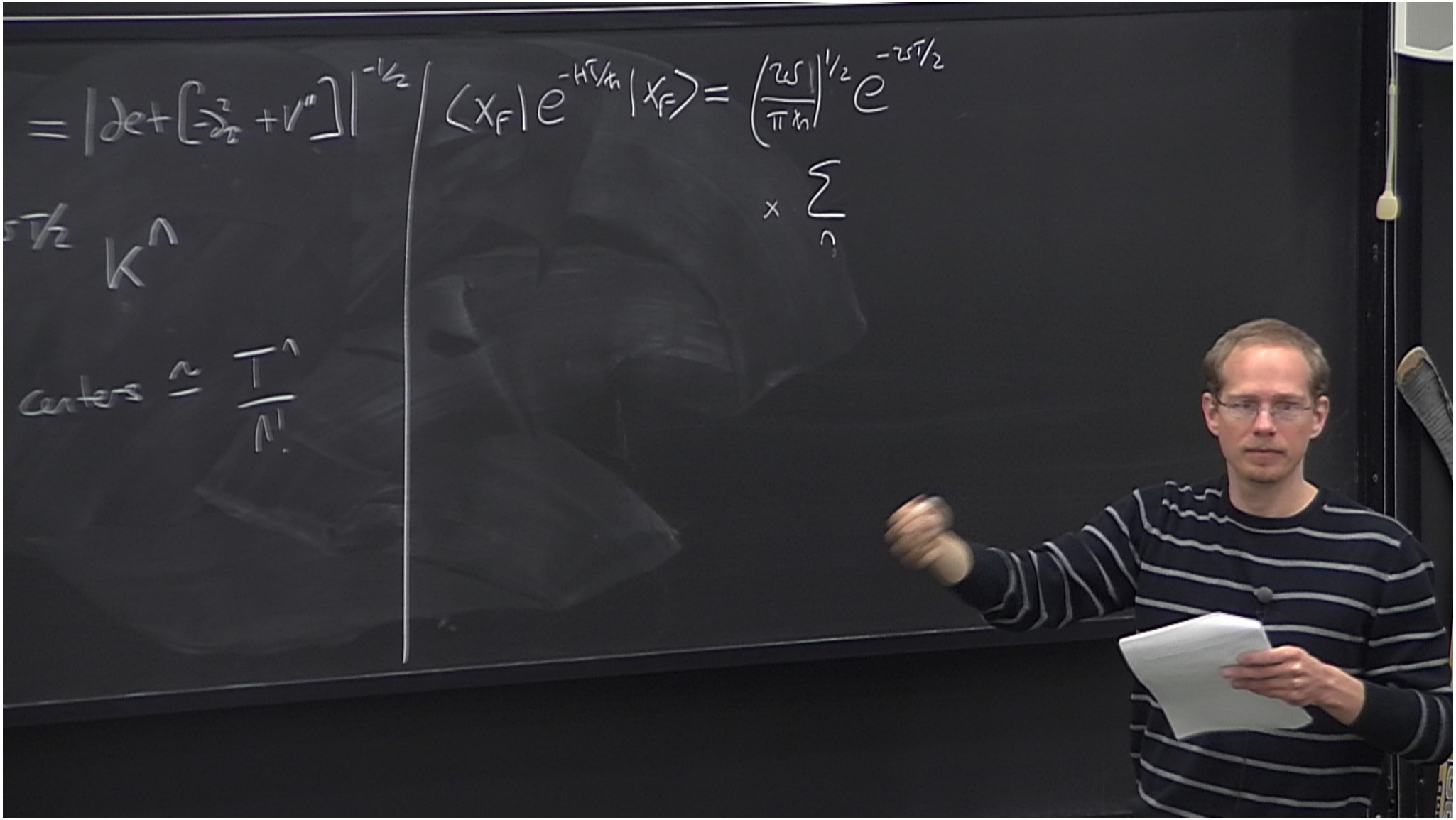

$\left(\prod_n \lambda_n^{-1/2}\right)$ = Same as SHO except at jumps $= |\det[-\partial_\tau^2 + V'']|^{-1/2}$
$= \left(\frac{2S}{\pi\hbar}\right)^{1/2} e^{-2ST/\hbar} K$
$S_n = n S_\epsilon(\bar{x})$

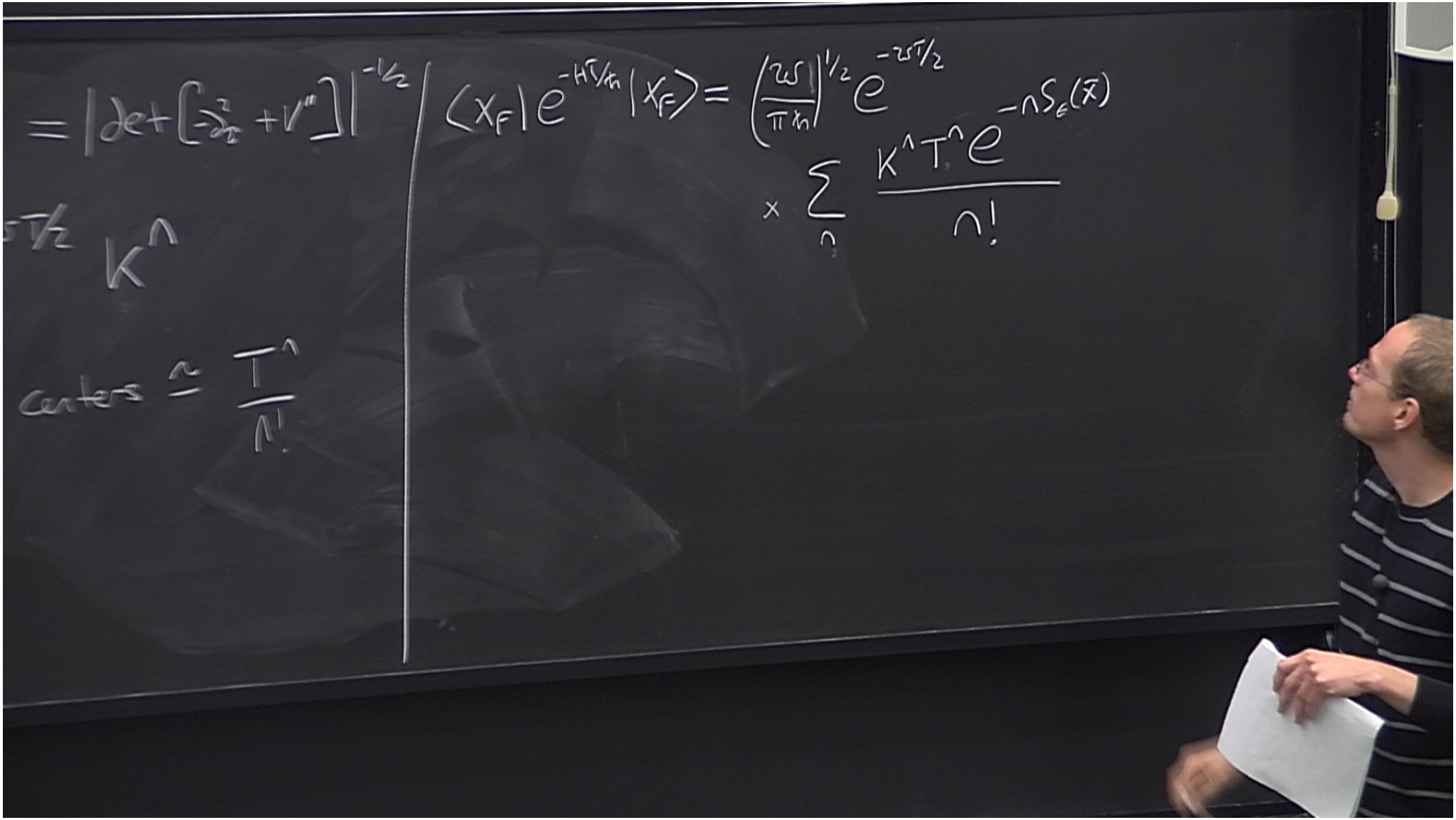

Solutions have
Const. = 0
"size"
x
t

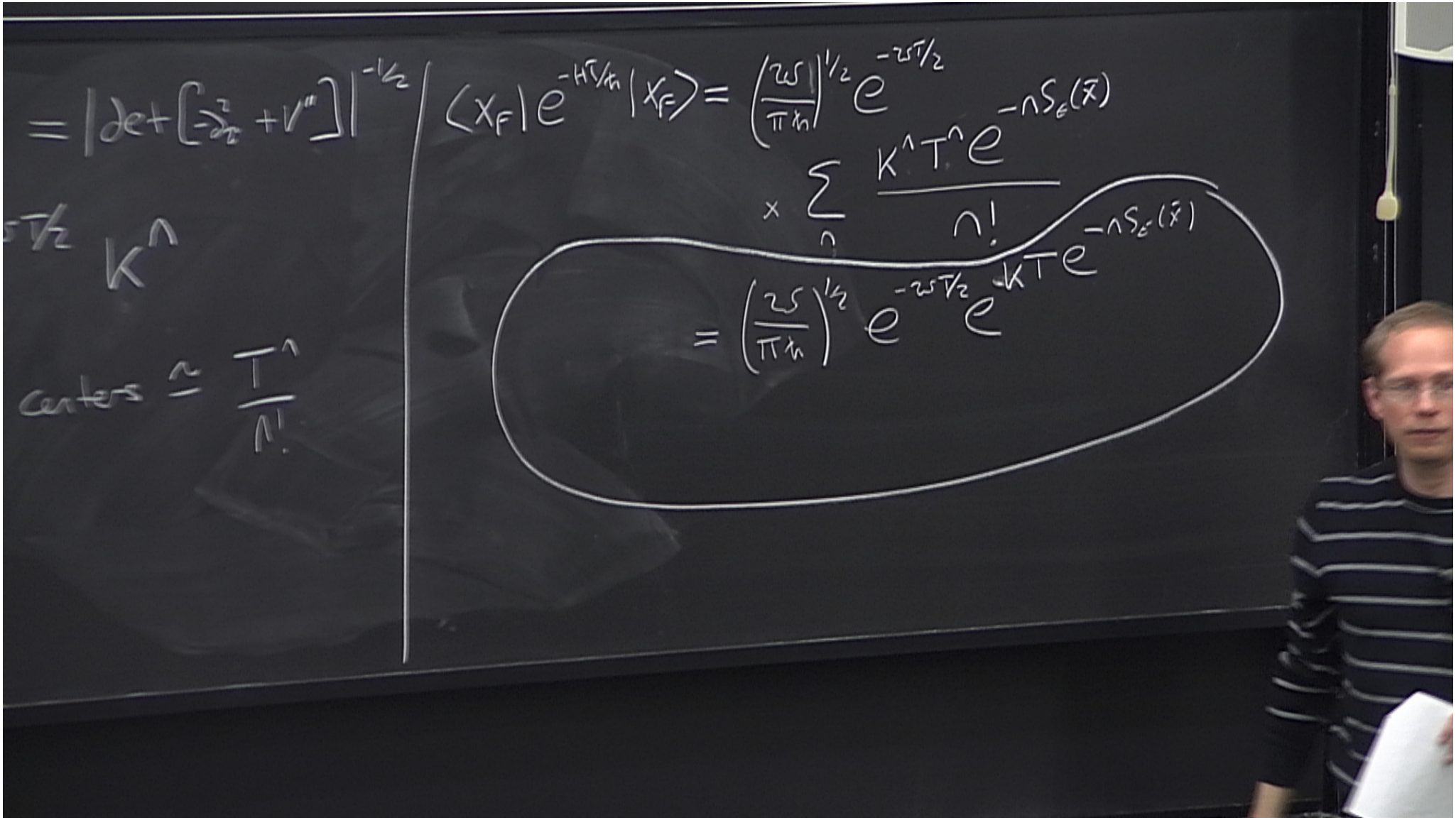

$\left(\prod_n \lambda_n^{-1/2}\right)$ = Same as SHO except at jumps $= |\det[-\partial_x^2 + V'']|^{-1/2}$
$= \left(\frac{2S}{\pi\hbar}\right)^{1/2} e^{-2ST/\hbar} K^n$
Integrate over n centers
$S_n = n S_\epsilon(\bar{x})$

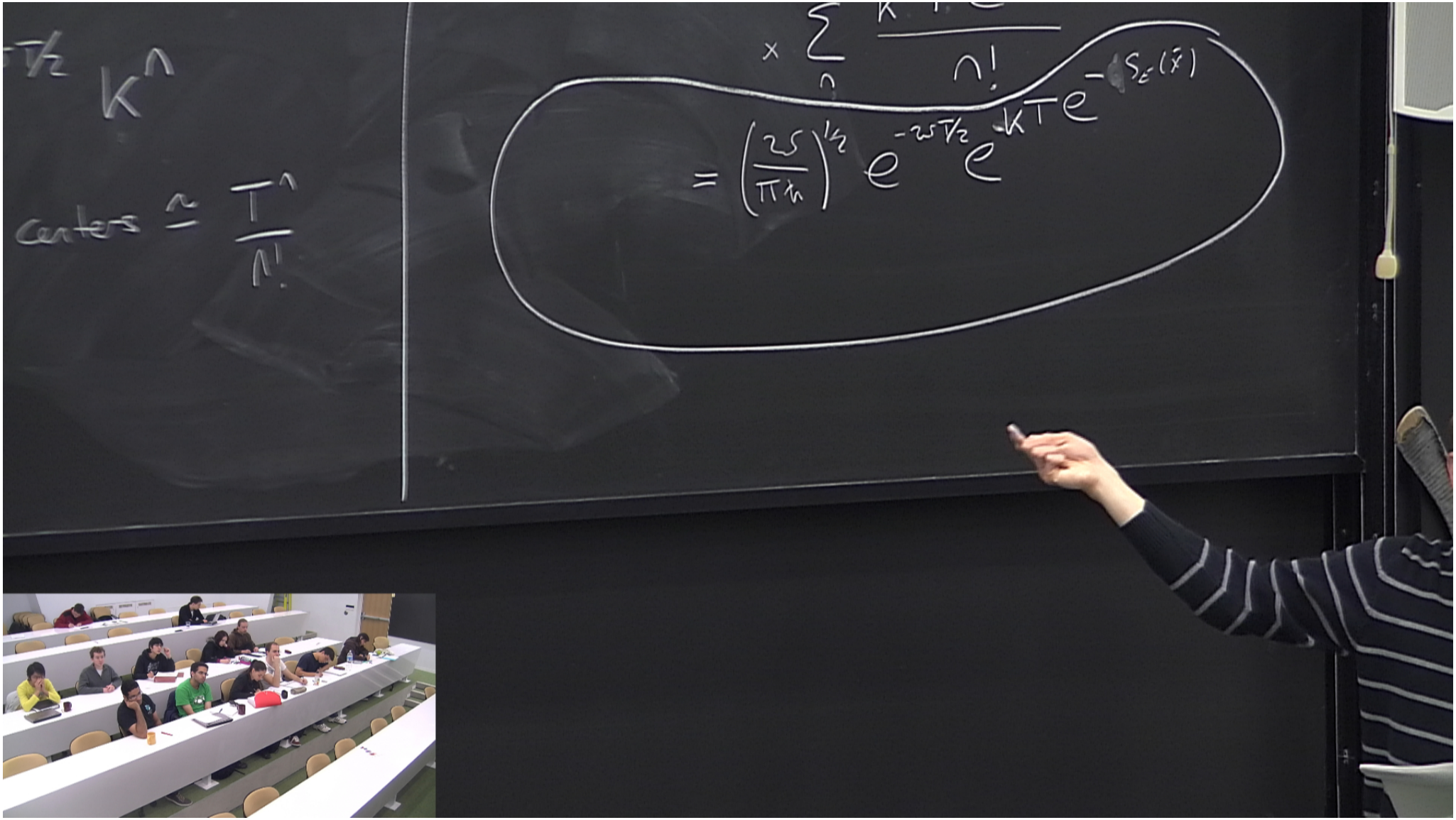

$\left(\prod_{n} \lambda_n^{-1/2}\right)$ = Same as SHO except at jumps $= |\det[-\partial_x^2 + V'']|^{-1}$
$= \left(\frac{2S}{\pi\hbar}\right)^{1/2} e^{-2ST/\hbar} K^n$
Integrate over n centers $\simeq \frac{T^n}{n!}$
$S_n = n S_\epsilon(\bar{x})$

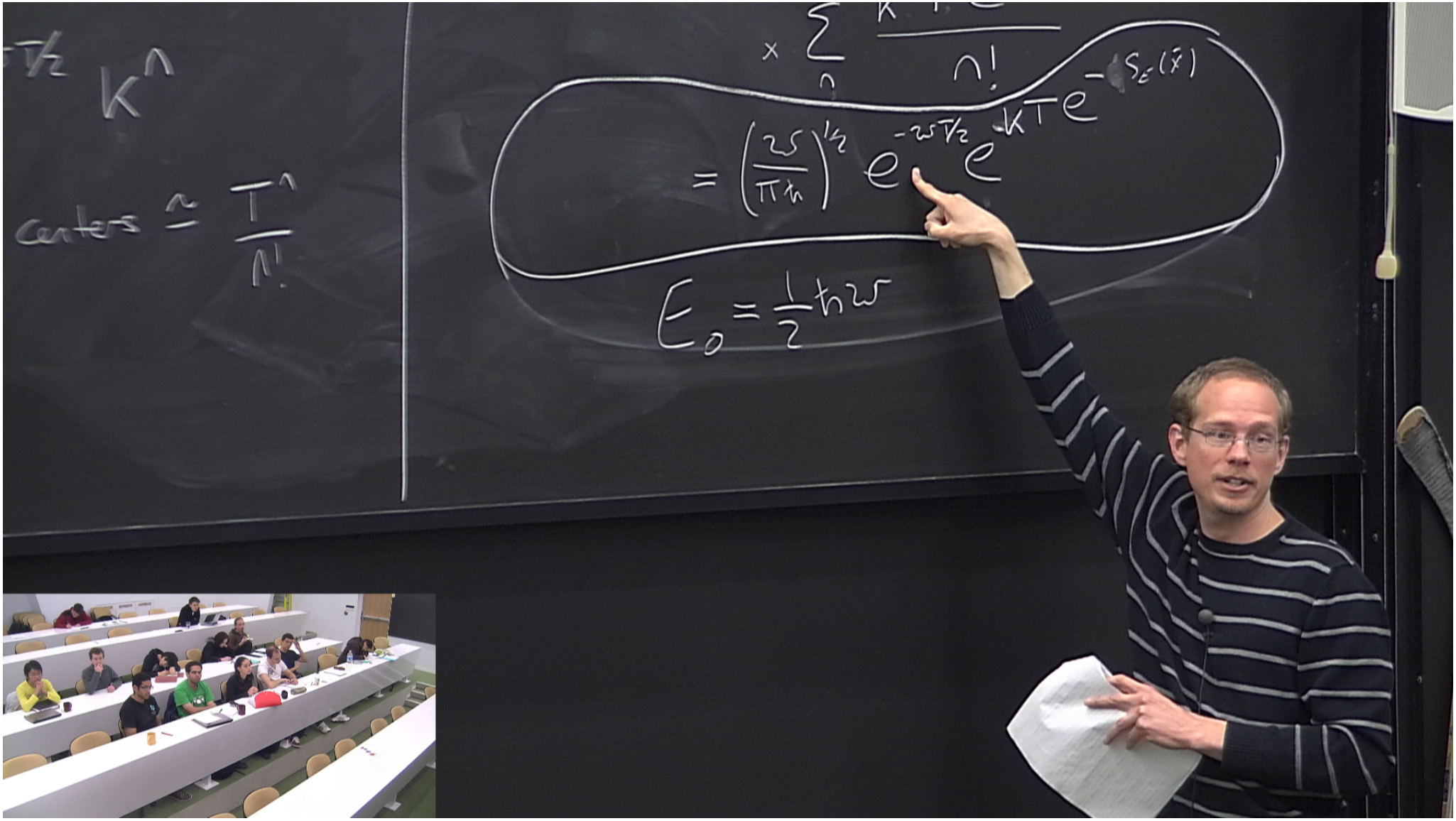

$= |\det[-\frac{\partial^2}{\partial x^2} + V'']|^{-1/2}$
$\langle x_F | e^{-HT/\hbar} | x_F \rangle = \left(\frac{2\pi}{}\right.$
$K^{\wedge}$
centers $\simeq \frac{T^{\wedge}}{\Lambda'}$

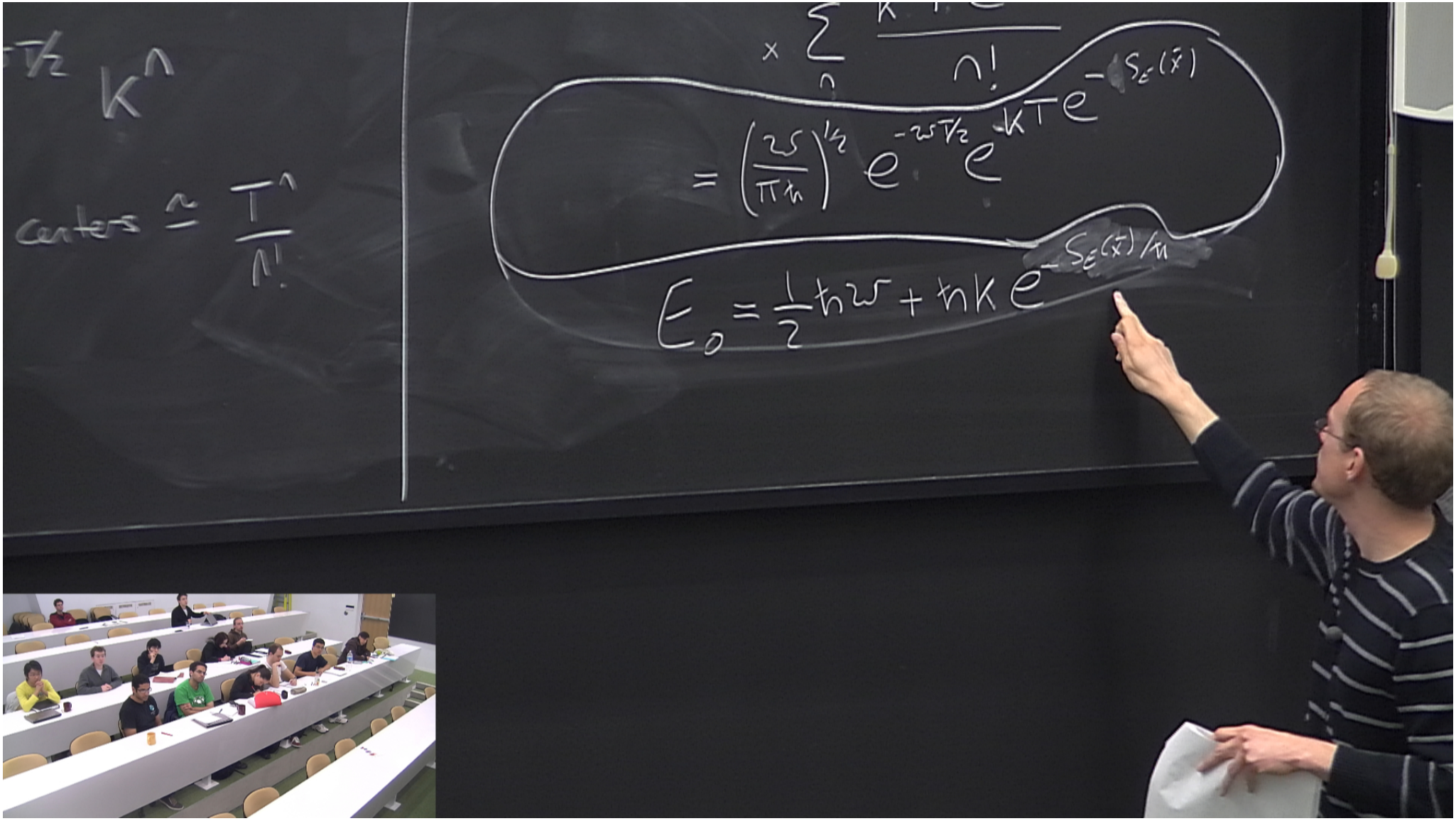

$= |\det[-\partial_\tau^2 + V'']|^{-1/2}$
$\langle x_F | e^{-HT/\hbar} | x_F \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}$
$\times \sum_n$
$K^\wedge$
centers $\simeq \frac{T^\wedge}{\Lambda'}$

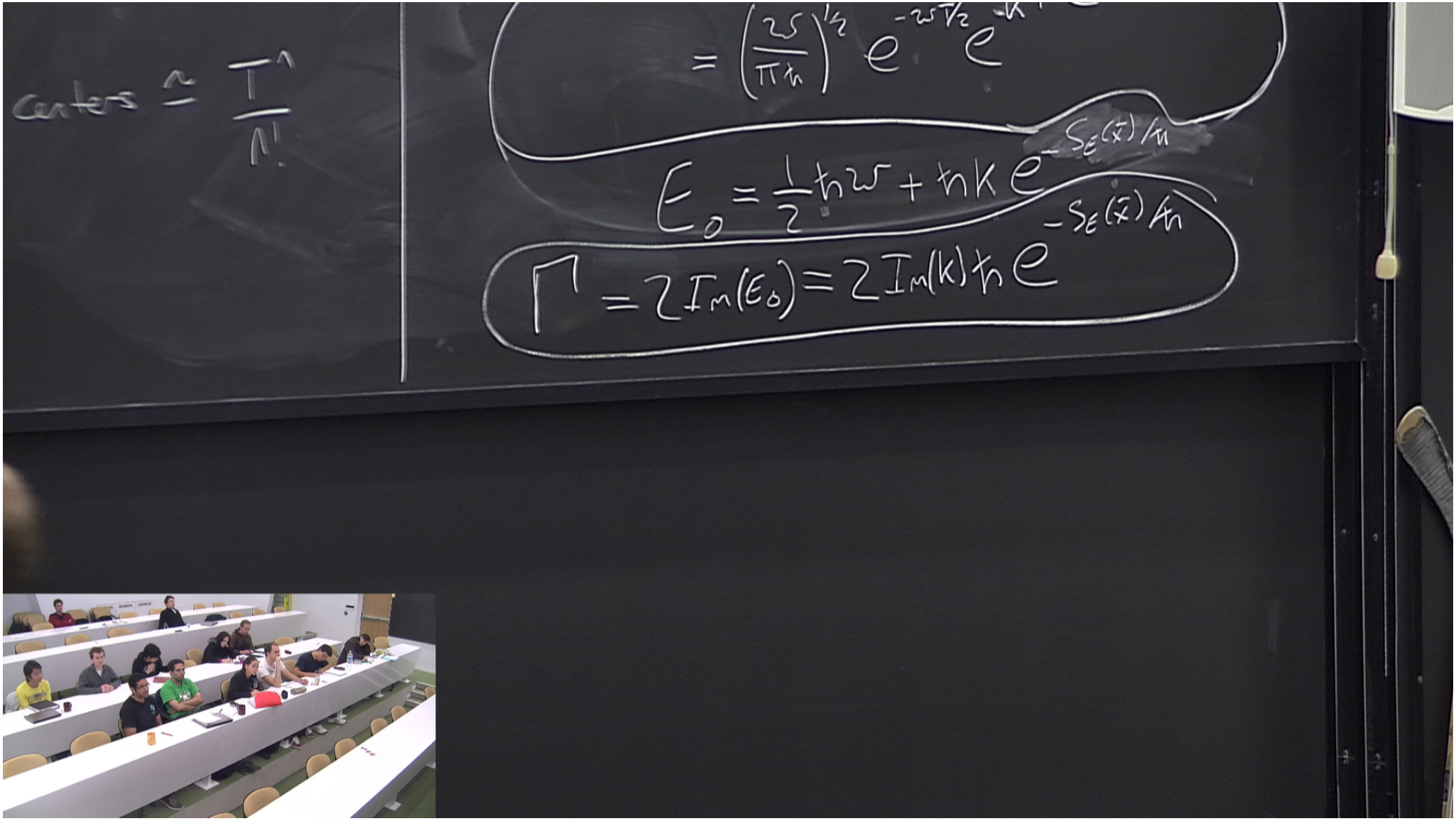

$= |\det[-\partial_\tau^2 + V'']|^{-1/2}$
$\langle x_F | e^{-HT/\hbar} | x_F \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}$
$\times \sum_n \frac{K^n T^n e^{-n S_E(\bar{x})}}{n!}$
K^n
centers $\simeq \frac{T^n}{n!}$

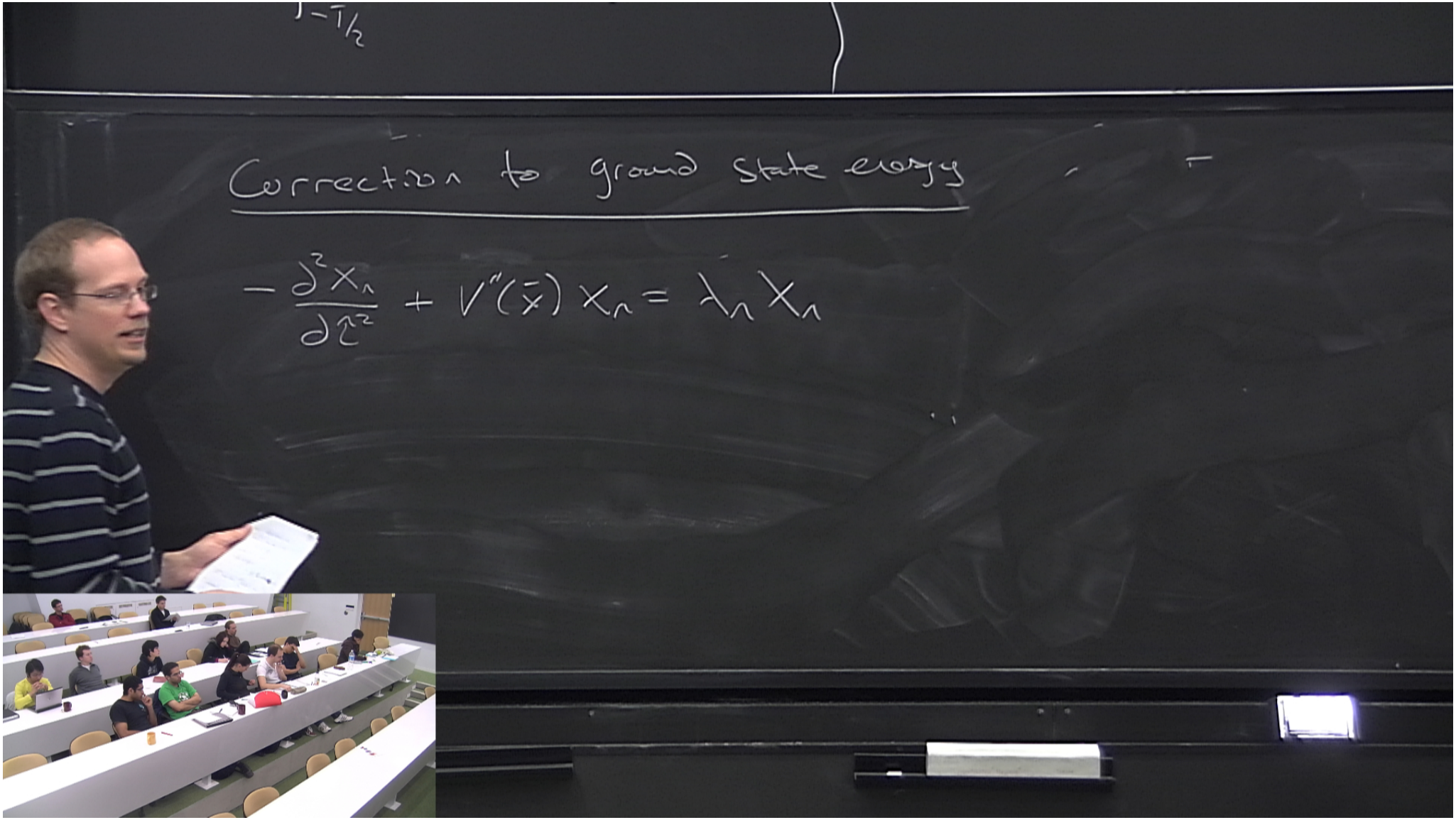

$= |\det[-\partial_\tau^2 + V'']|^{-1/2}$
$\langle x_F | e^{-H\mathcal{T}/\hbar} | x_F \rangle = \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega\mathcal{T}/2}$
$\times \sum_n \frac{K^n T^n e^{-n S_E(\bar{x})}}{n!}$
$= \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega\mathcal{T}/2} e^{KT e^{-n S_E(\bar{x})}}$
K^n
centers $\simeq \frac{T^n}{n!}$

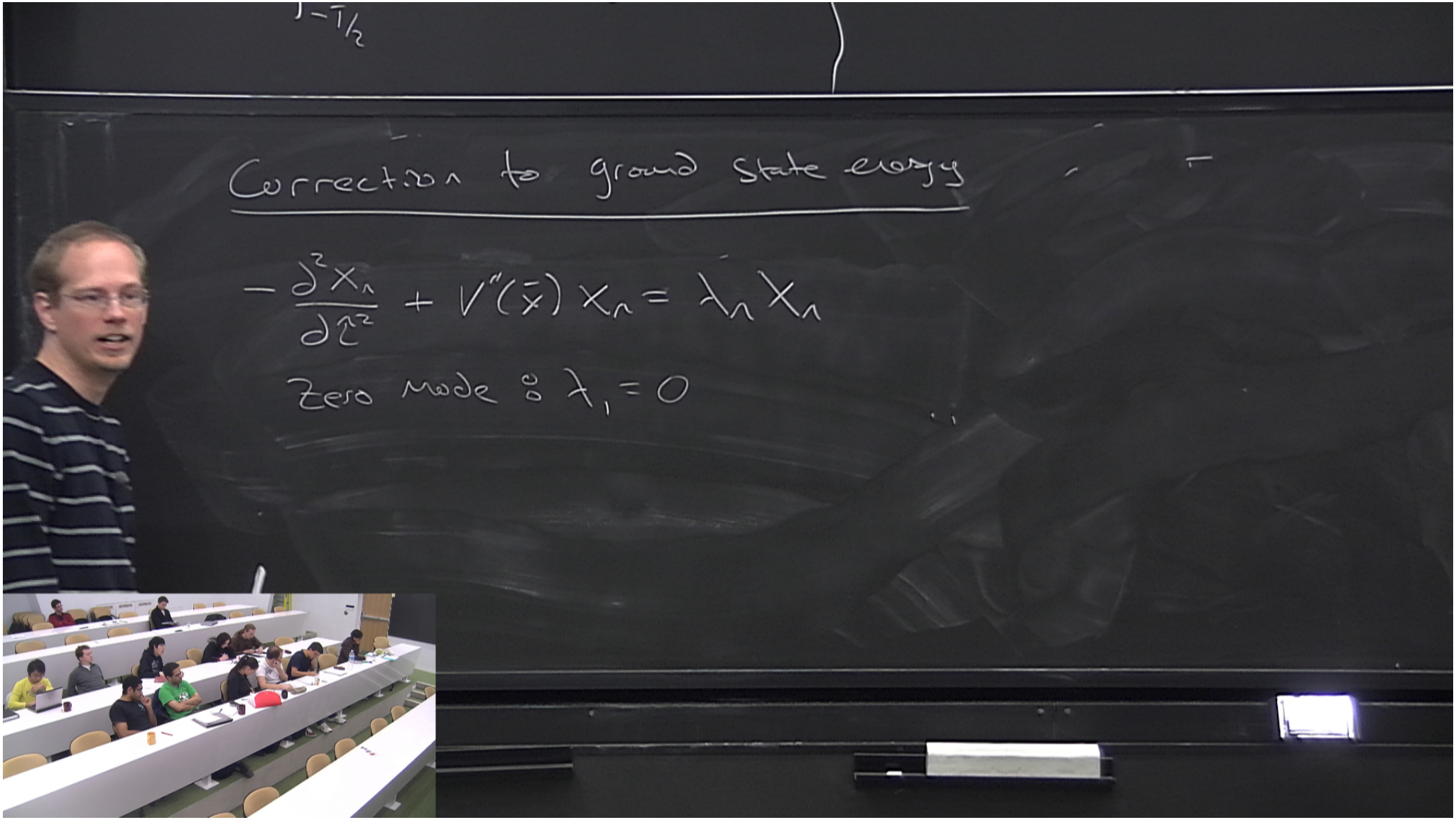

$\sum_n \frac{}{n!}$
$= \left(\frac{2\pi}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} e^{-kT} e^{-S_E(\vec{x})}$
$\frac{T^n}{n!}$

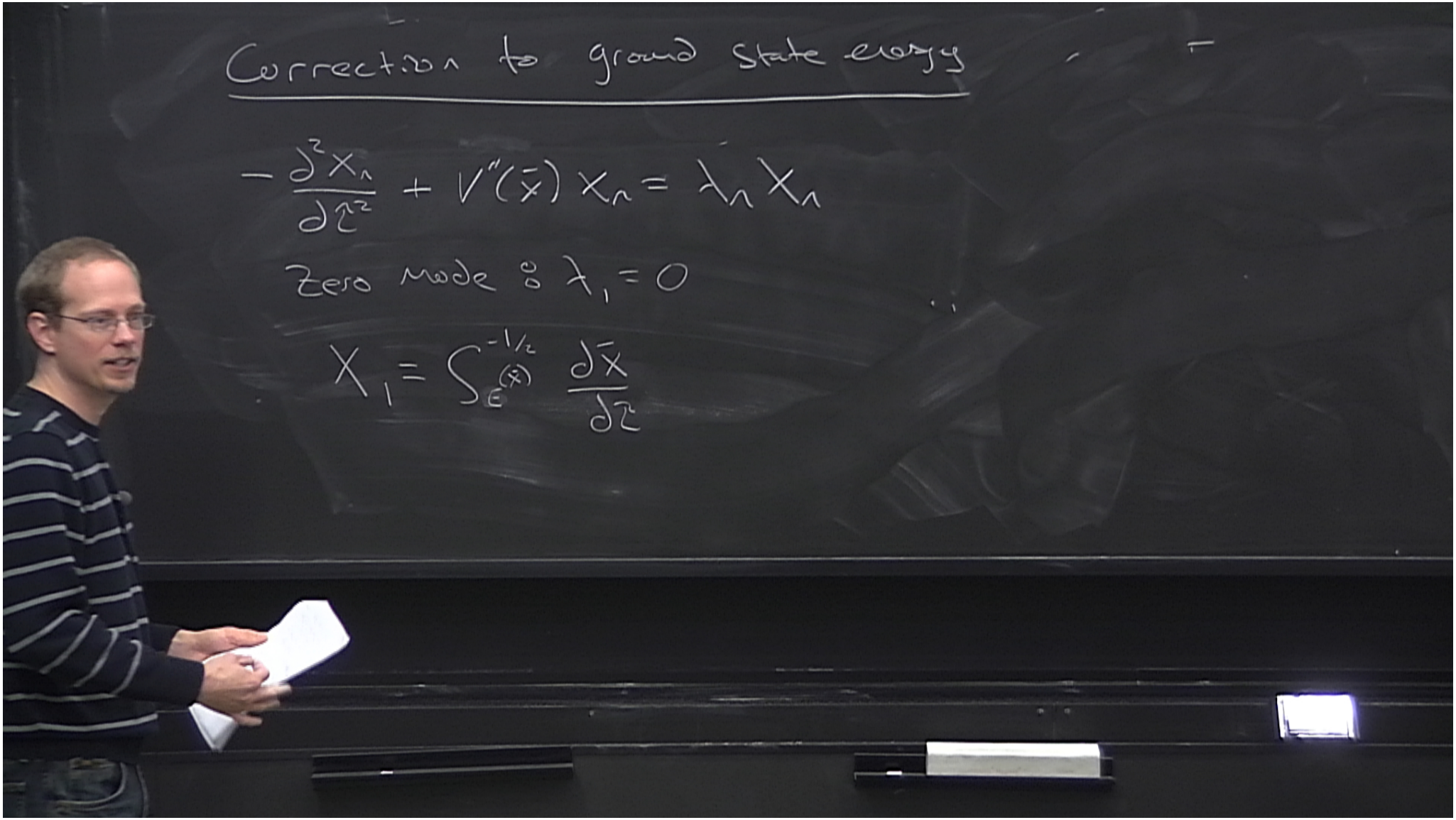

$E_0 = \frac{1}{2}\hbar\omega$

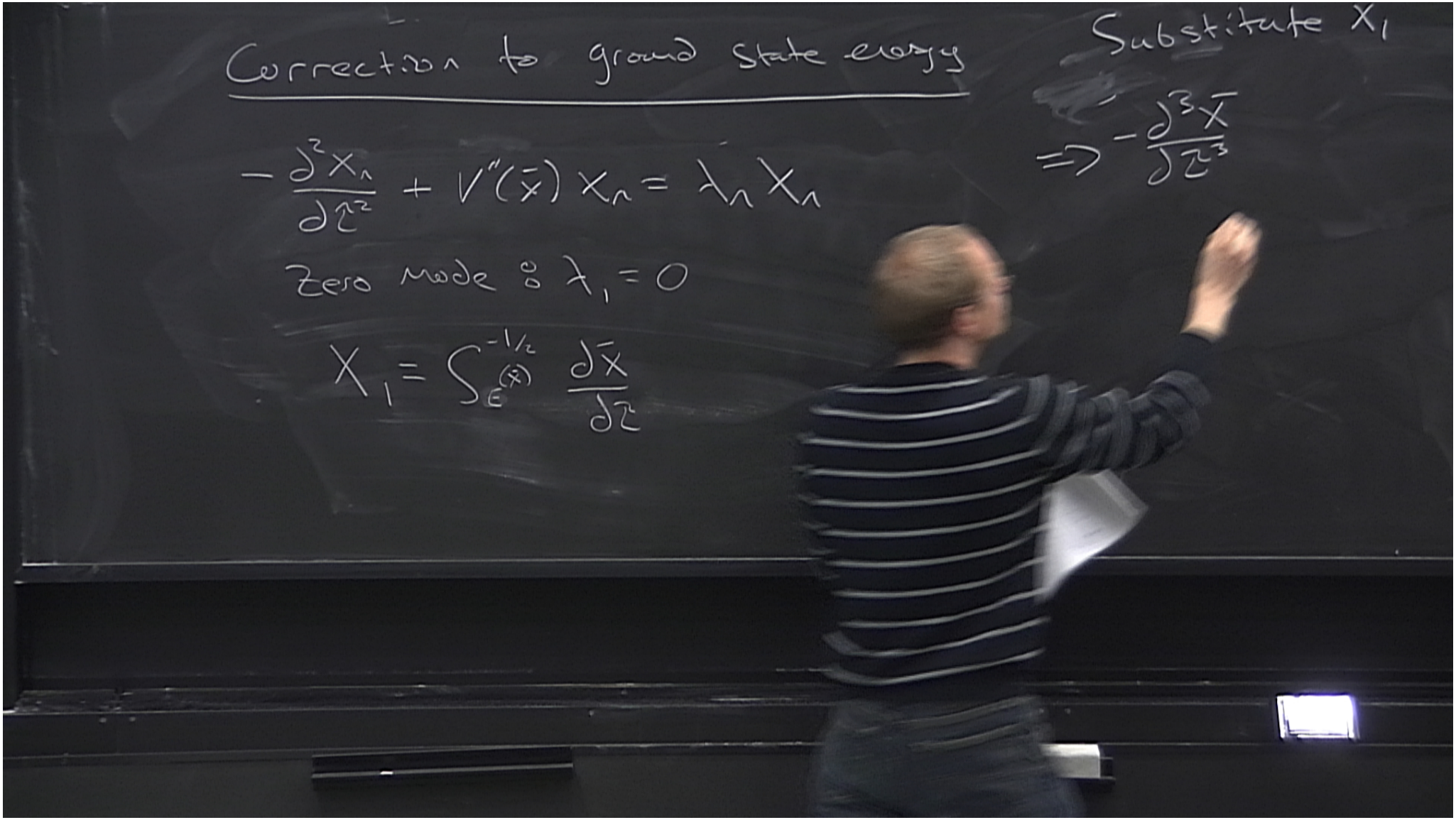

$\times \sum_n \frac{k^n}{n!}$
$= \left(\frac{2\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} e^{-kT} e^{-S_E(\bar{x})}$
$E_0 = \frac{1}{2}\hbar\omega + \hbar k e^{-S_E(\bar{x})/\hbar}$
$\frac{T^n}{n!}$

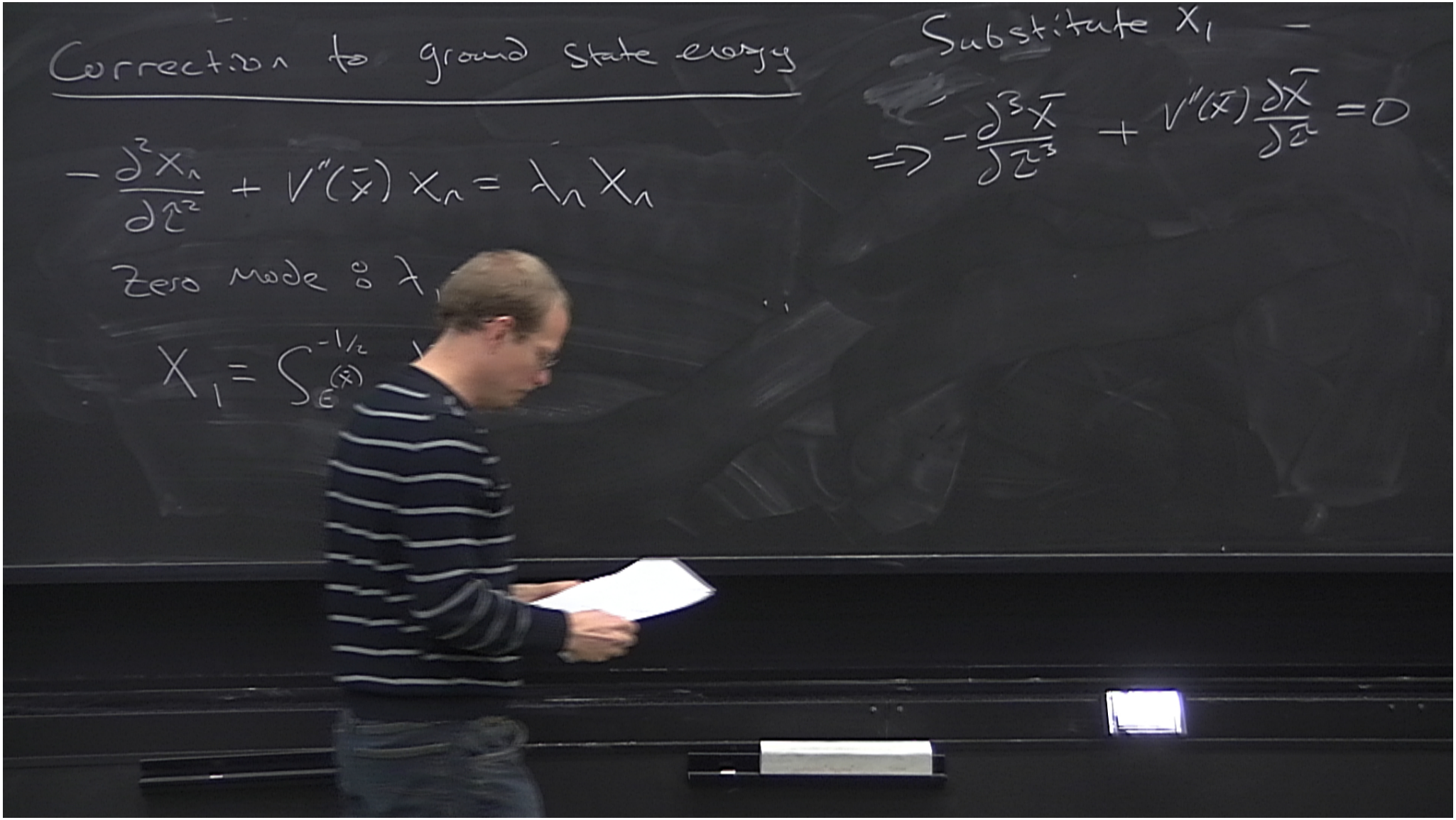

centers ≃ T^1 / 1!
= (2ω/πħ)^{1/2} e^{-ωx²/2ħ} e^{ikx}
E_0 = 1/2 ħω + ħk e^{-S_E(x̄)/ħ}
Γ = 2 Im(E_0) = 2 Im(k) ħ e^{-S_E(x̄)/ħ}

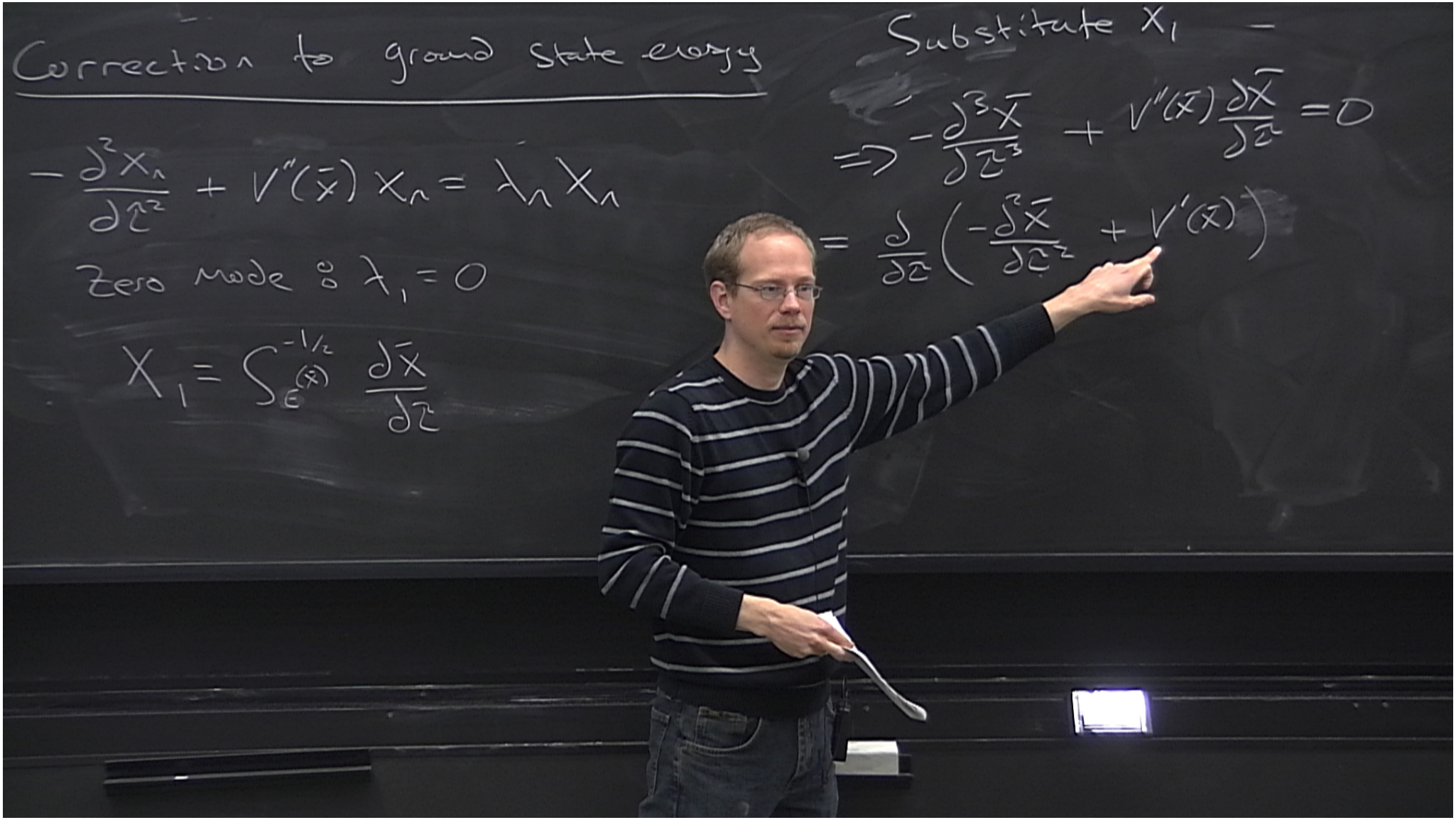

Correction to ground state energy
$-\frac{\partial^2 \chi_n}{\partial \tilde{x}^2} + V''(\tilde{x})\chi_n = \lambda_n \chi_n$

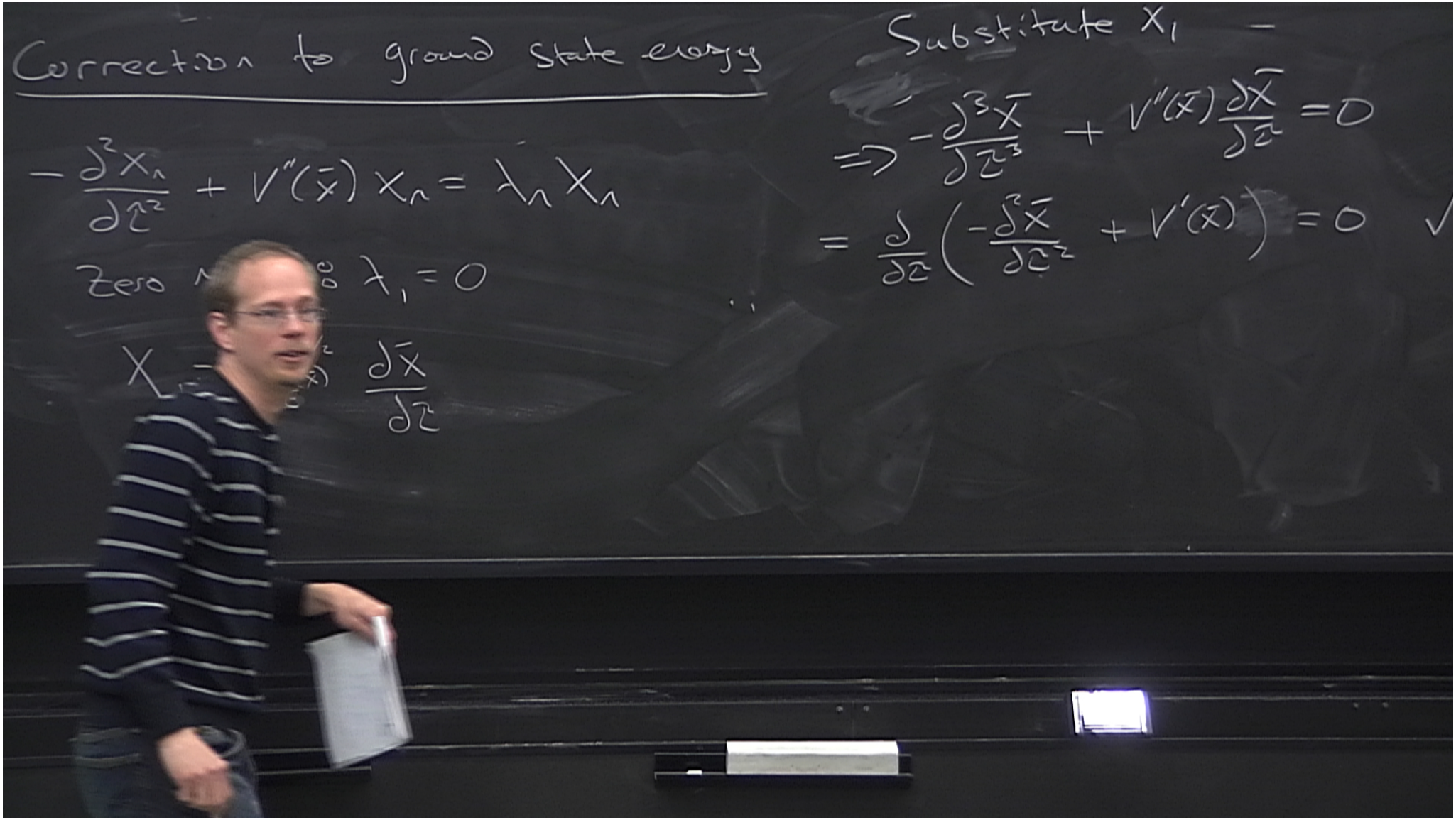

Correction to ground state energy
$-\frac{\partial^2 \chi_n}{\partial \tau^2} + V''(\bar{x})\chi_n = \lambda_n \chi_n$
zero mode : $\lambda_1 = 0$

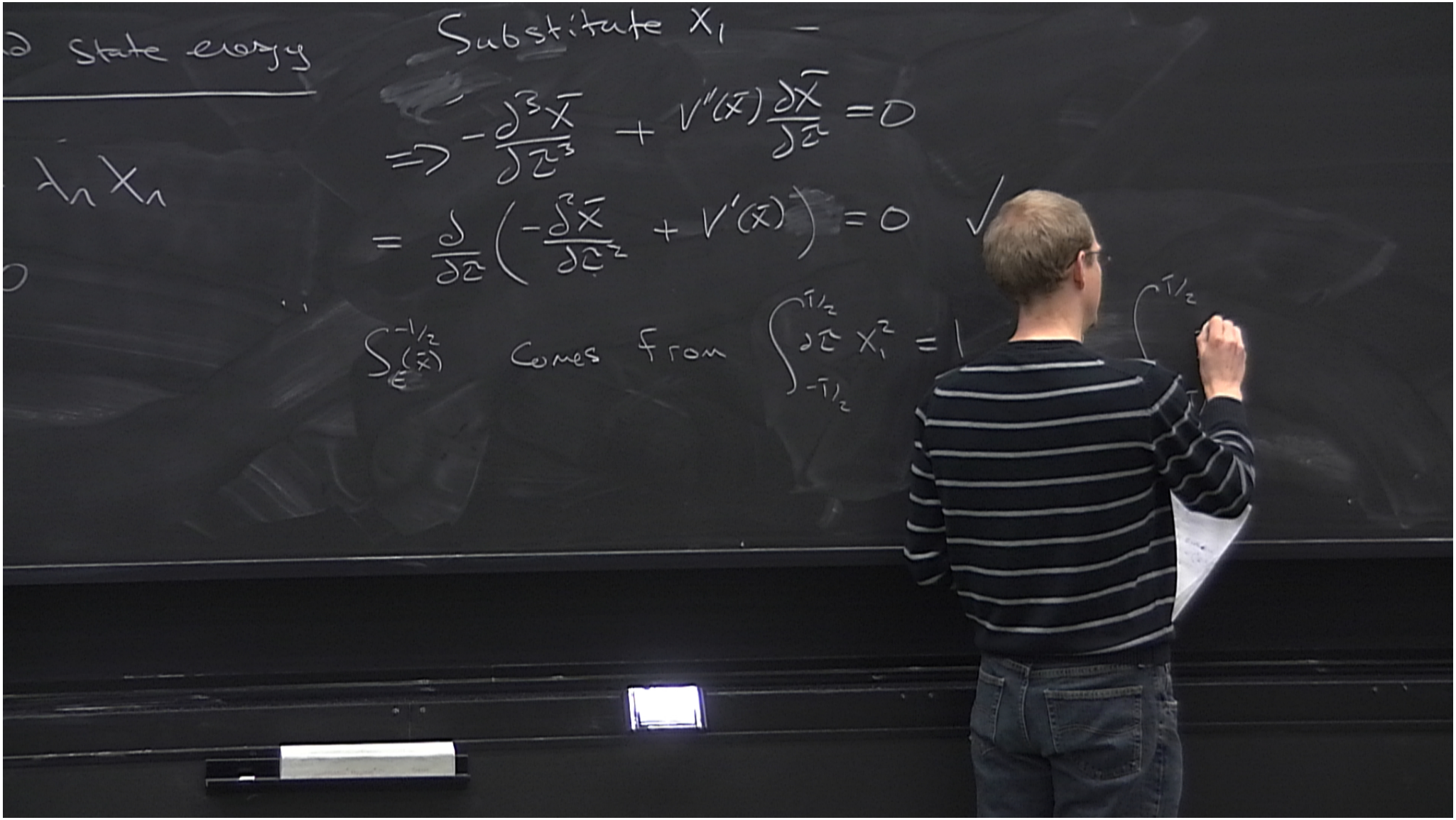

Correction to ground state energy
$-\frac{d^2 X_n}{d\tau^2} + V''(\bar{x}) X_n = \lambda_n X_n$
Zero mode : $\lambda_1 = 0$
$X_1 = S_E^{(\bar{x})^{-1/2}} \frac{d\bar{x}}{d\tau}$

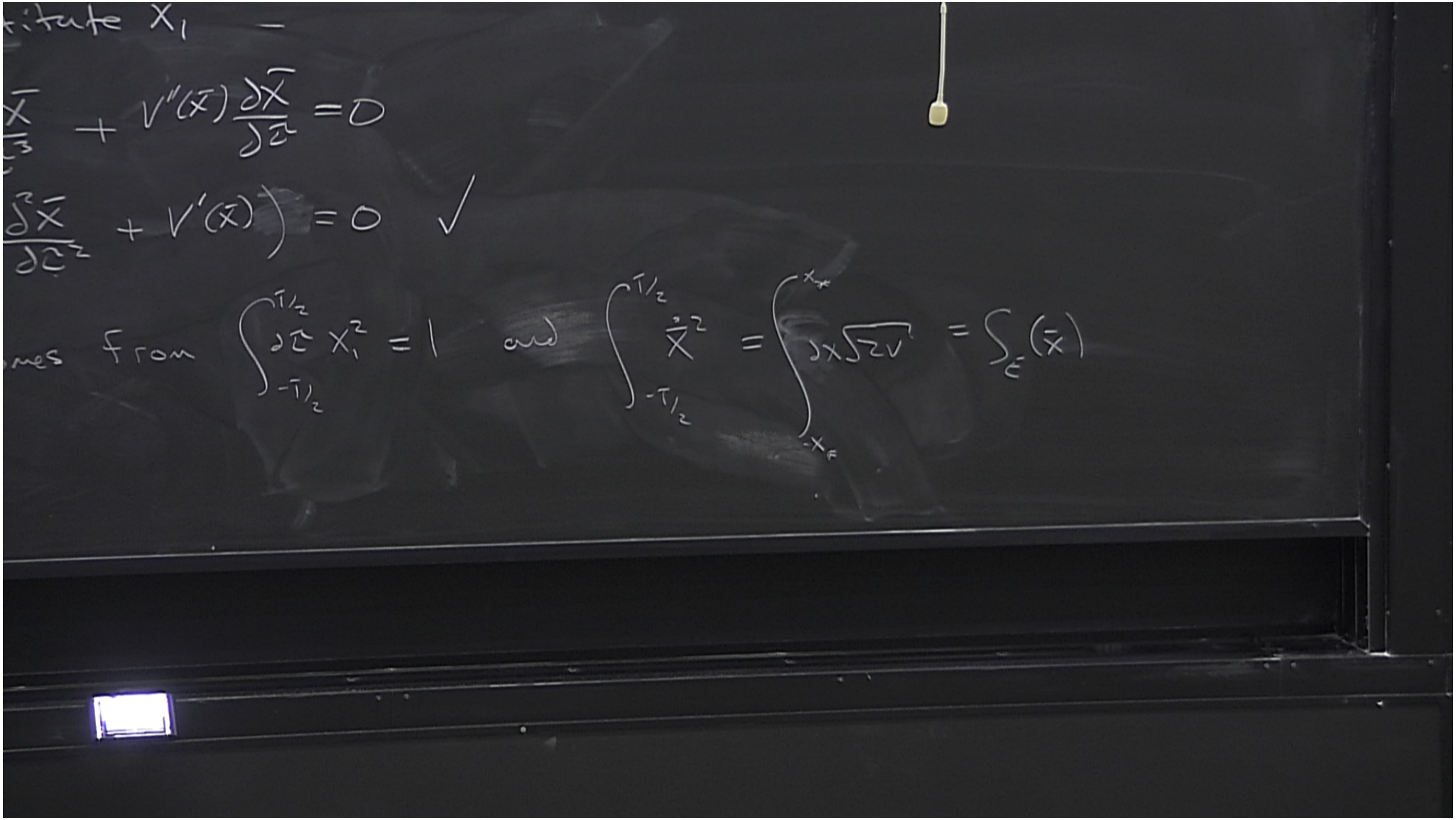

Correction to ground state energy
$-\frac{d^2 X_n}{d\bar{z}^2} + V''(\bar{x}) X_n = \lambda_n X_n$
Zero mode : $\lambda_1 = 0$
$X_1 = S_E^{(\bar{x})^{-1/2}} \frac{d\bar{x}}{d\bar{z}}$
Substitute X_1
$\Rightarrow -\frac{d^3 \bar{x}}{d\bar{z}^3}$

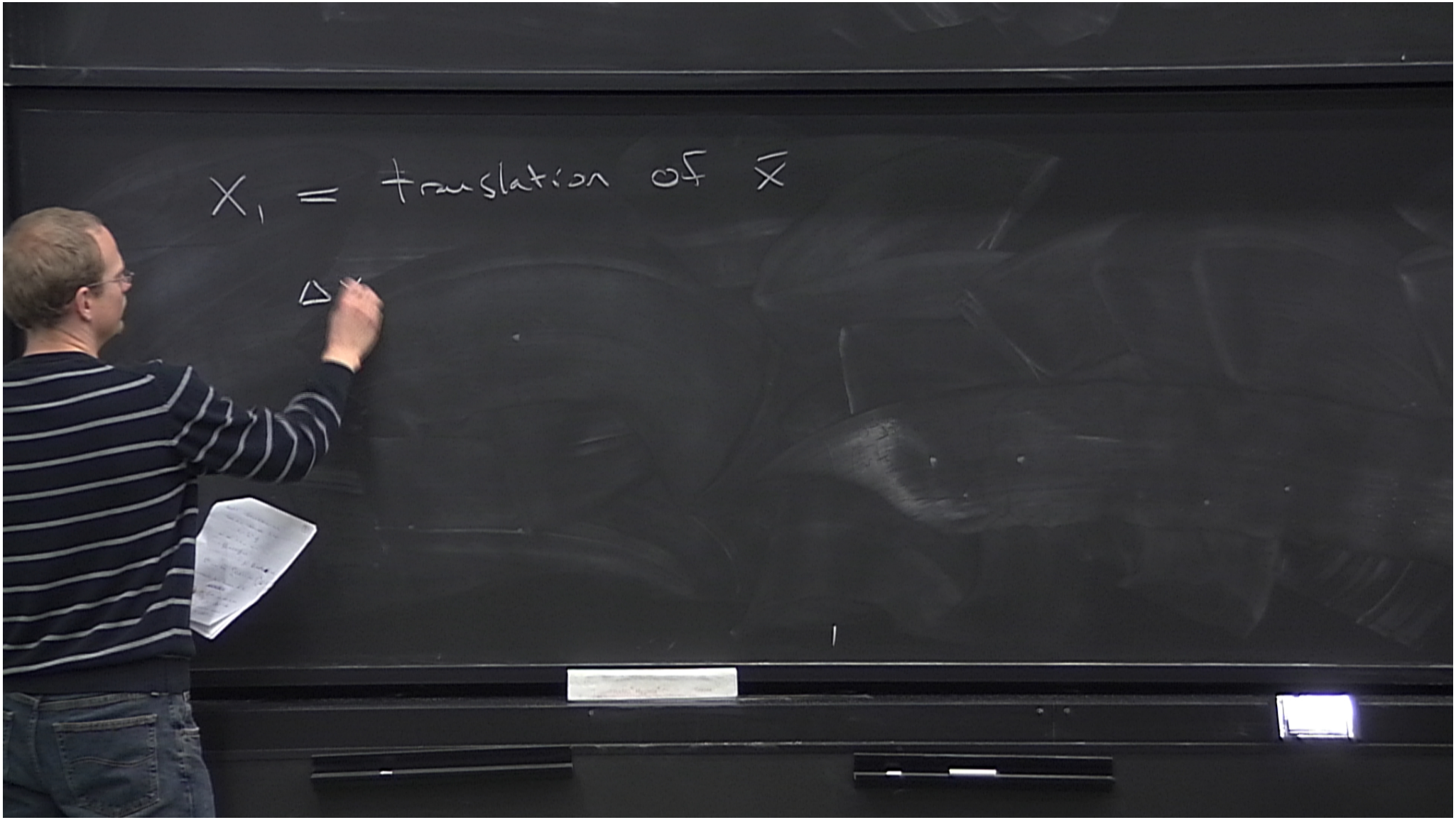

Correction to ground state energy
$-\frac{d^2 X_n}{d\tilde{x}^2} + V''(\bar{x}) X_n = \lambda_n X_n$
Zero mode : λ
$X_1 = S_\epsilon^{-1/2}$
Substitute X_1 –
$\Rightarrow -\frac{d^3 \bar{x}}{d\tilde{x}^3} + V''(\bar{x}) \frac{d\bar{x}}{d\tilde{x}} = 0$

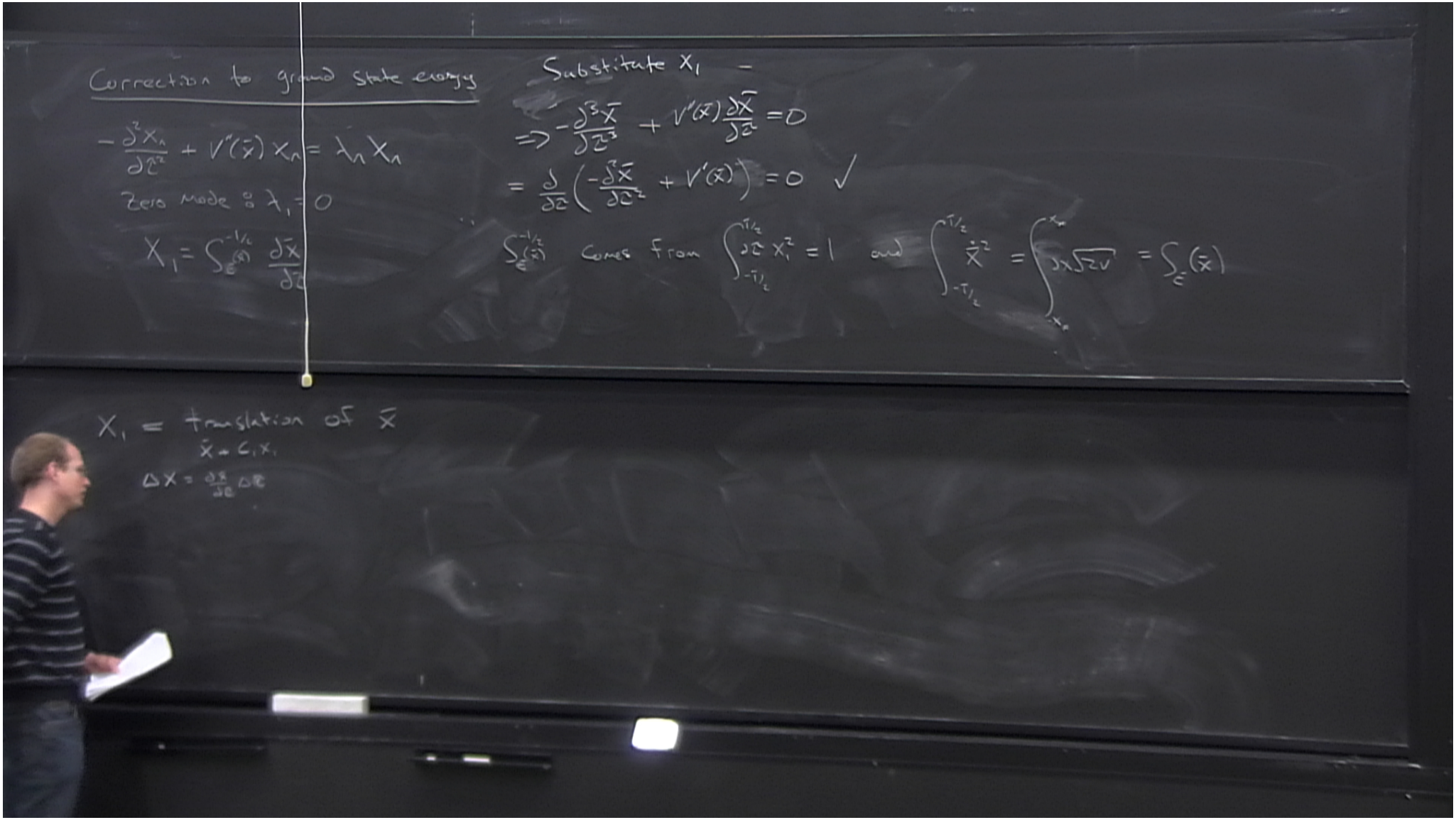

Correction to ground state energy
$-\frac{d^2 x_n}{d\tau^2} + V''(\bar{x})\, x_n = \lambda_n x_n$
Zero mode $\Rightarrow \lambda_1 = 0$
$x_1 = S_E^{(\bar{x})^{-1/2}} \frac{d\bar{x}}{d\tau}$
Substitute x_1 –
$\Rightarrow -\frac{d^3\bar{x}}{d\tau^3} + V''(\bar{x})\frac{d\bar{x}}{d\tau} = 0$
$= \frac{d}{d\tau}\left(-\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x})\right)$

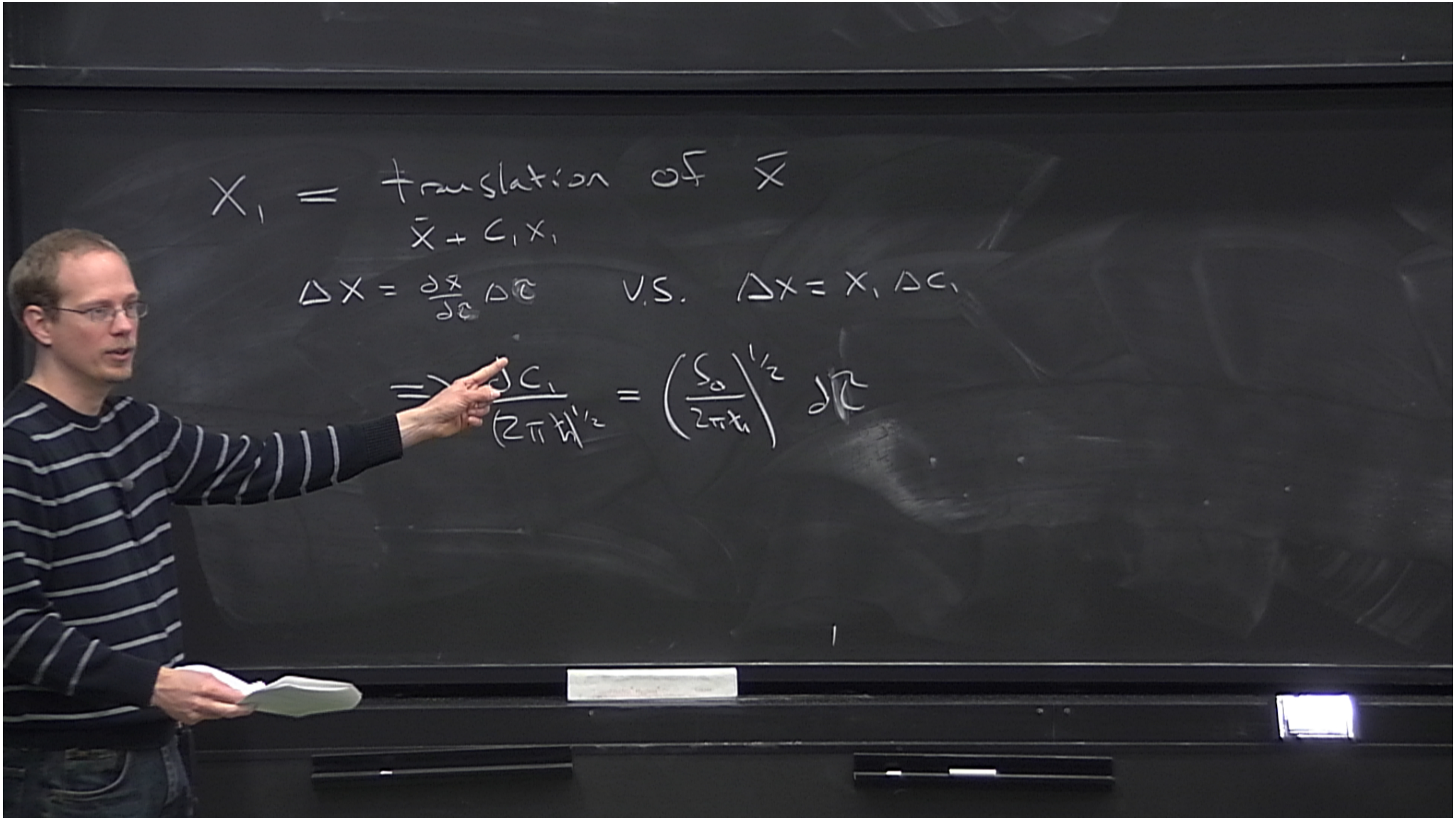

Correction to ground state energy
$-\frac{\partial^2 X_1}{\partial \bar{x}^2} + V''(\bar{x}) X_1 = \lambda_1 X_1$
zero
$\lambda_1 = 0$
$\frac{d\bar{x}}{d\bar{x}}$
Substitute X_1
$\Rightarrow -\frac{\partial^3 \bar{x}}{\partial \bar{x}^3} + V''(\bar{x}) \frac{\partial \bar{x}}{\partial \bar{x}} = 0$
$= \frac{\partial}{\partial \bar{x}} \left(-\frac{\partial^2 \bar{x}}{\partial \bar{x}^2} + V'(\bar{x}) \right) = 0$ ✓

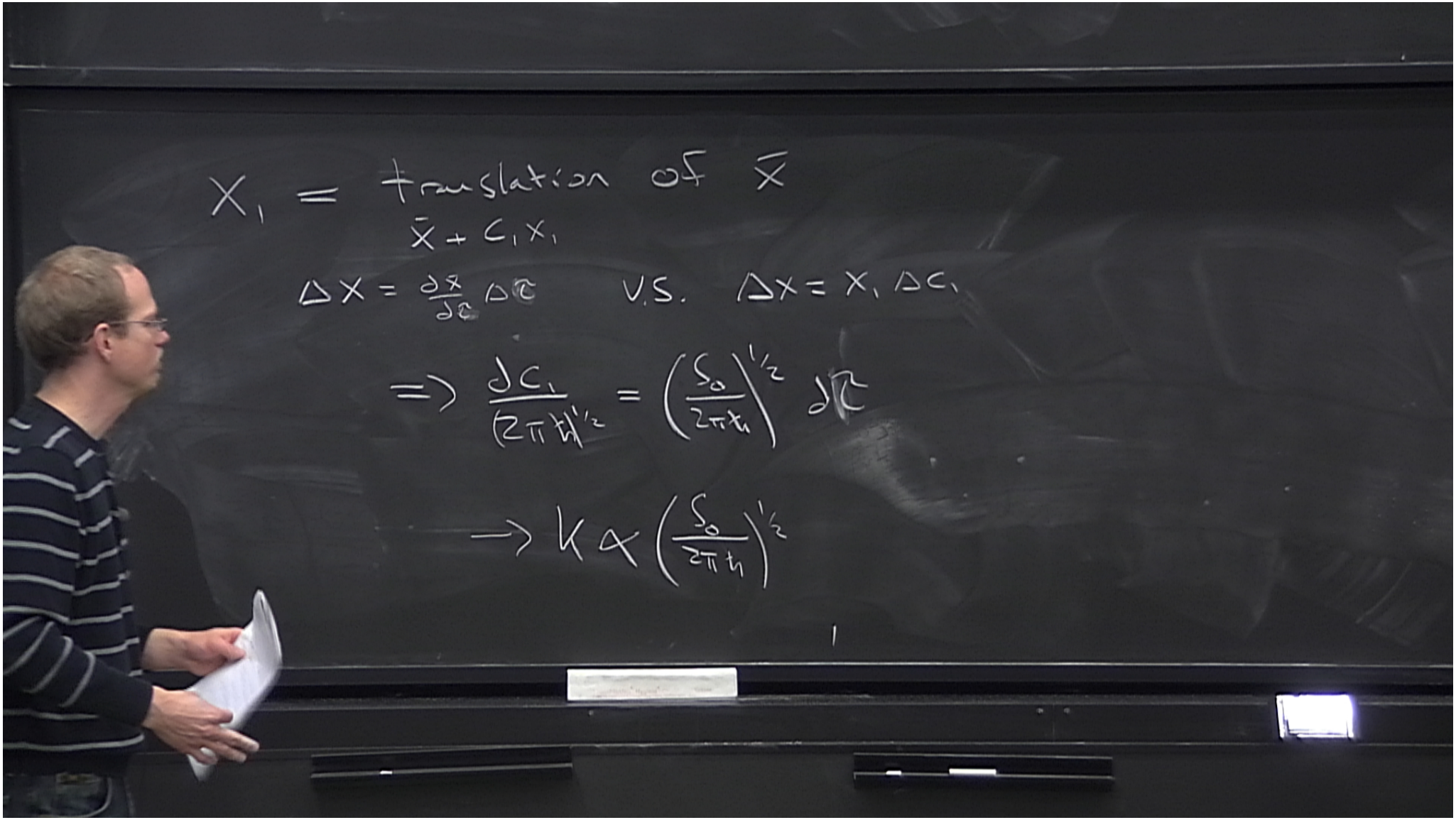

State energy
Substitute X_1
$\lambda_1 X_1$
$\Rightarrow -\frac{\partial^3 \bar{X}}{\partial \bar{z}^3} + V''(\bar{x})\frac{\partial \bar{X}}{\partial \bar{z}} = 0$
$= \frac{\partial}{\partial \bar{z}}\left(-\frac{\partial^2 \bar{X}}{\partial \bar{z}^2} + V'(\bar{x})\right) = 0$ ✓
$S_\epsilon^{-1/2}(\bar{x})$ Comes from $\int_{-\bar{l}/2}^{\bar{l}/2} d\bar{z}\, X_1^2 = 1$

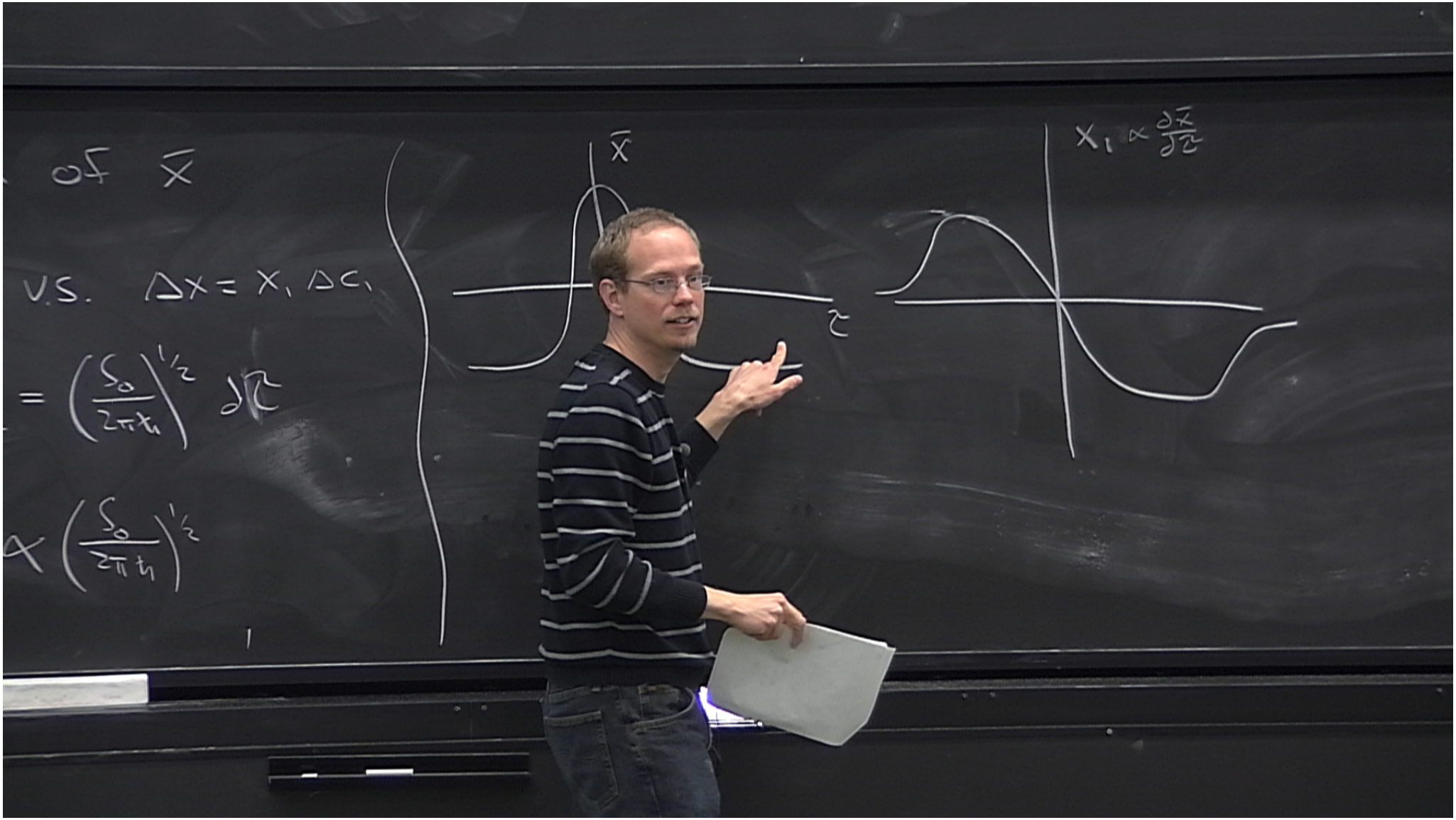

itute x_1 –
$\frac{\bar{x}}{\bar{t}^3} + V''(\bar{x})\frac{d\bar{x}}{d\bar{t}} = 0$
$\frac{d^2\bar{x}}{d\bar{t}^2} + V'(\bar{x})\Big) = 0$ ✓
mes from $\int_{-\bar{T}/2}^{\bar{T}/2} d\bar{t}\, x_1^2 = 1$ and $\int_{-\bar{T}/2}^{\bar{T}/2} \dot{\bar{x}}^2 = \int_{-x_F}^{x_F} dx\sqrt{2V} = S_{\bar{E}}(\bar{x})$

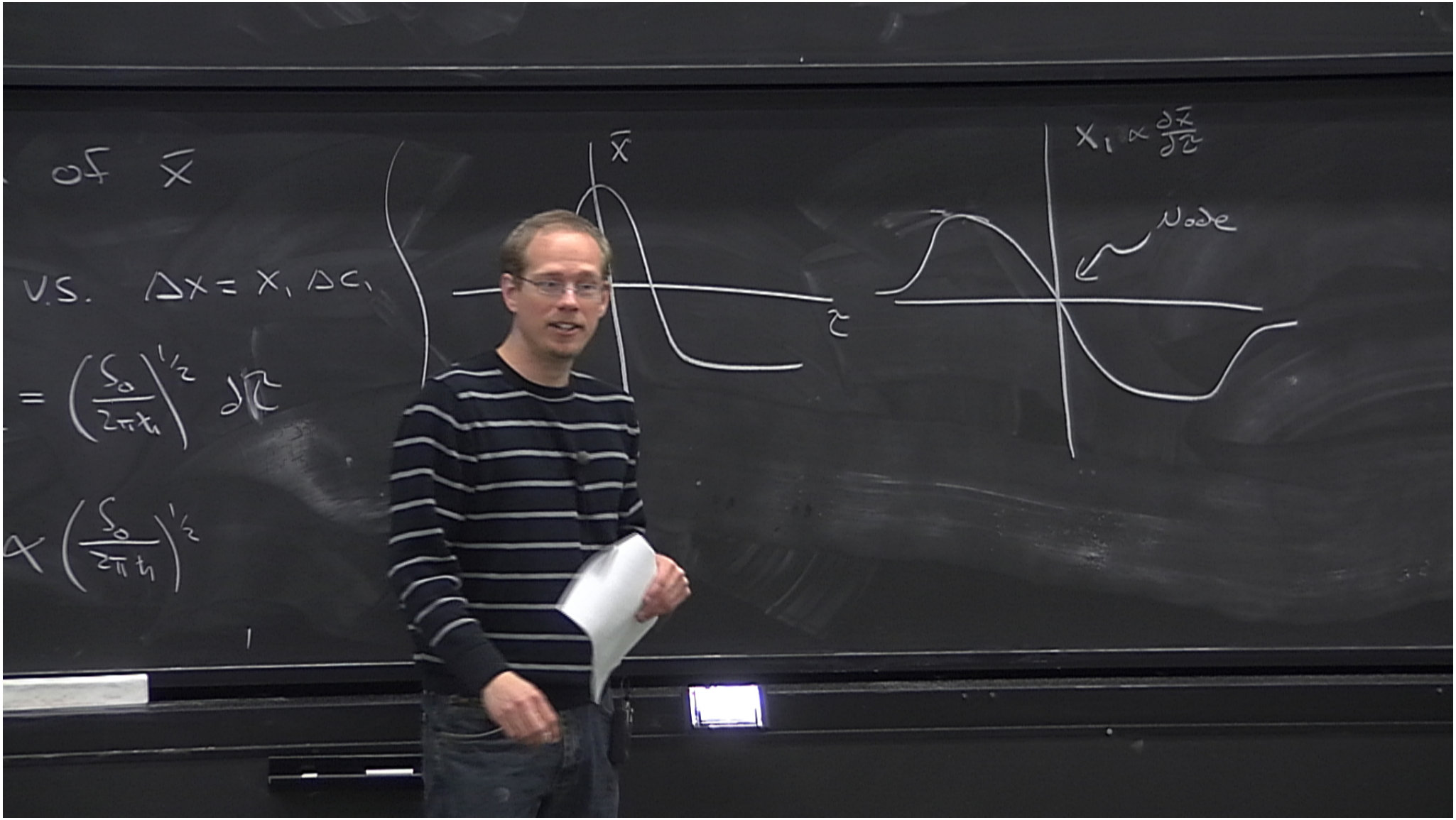

X_1 = translation of $\bar{x}$
Δx

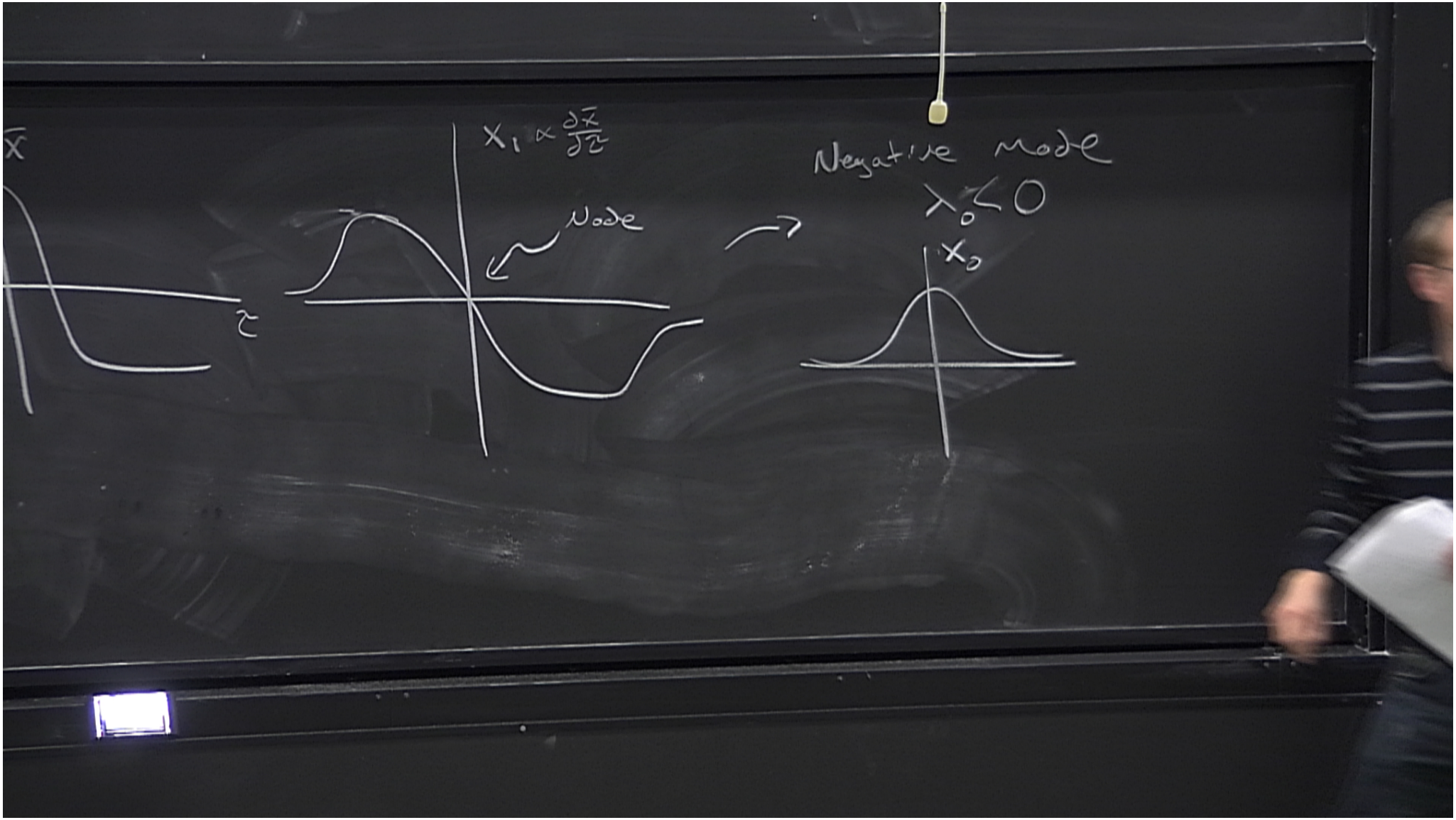

Correction to ground state energy
$-\frac{d^2 x_n}{d\tau^2} + V''(\bar{x})\, x_n = \lambda_n x_n$
Zero mode $\Rightarrow \lambda_1 = 0$
$x_1 = \zeta_\epsilon^{-1/2}(\bar{x})\, \frac{d\bar{x}}{d\tau}$
Substitute x_1 –
$\Rightarrow -\frac{d^3 \bar{x}}{d\tau^3} + V''(\bar{x})\frac{d\bar{x}}{d\tau} = 0$
$= \frac{d}{d\tau}\left(-\frac{d^2\bar{x}}{d\tau^2} + V'(\bar{x})\right) = 0$ ✓
$\zeta_\epsilon^{-1/2}(\bar{x})$ comes from $\int_{-T/2}^{T/2} d\tau\, x_1^2 = 1$ and $\int_{-T/2}^{T/2} \dot{\bar{x}}^2 = \int_{x_a}^{x_b} dx\sqrt{2V} = \zeta_\epsilon(\bar{x})$
x_1 = translation of $\bar{x}$
$\bar{x} \to \epsilon x_1$
$\Delta x = \frac{dx}{d\epsilon}\Delta\epsilon$

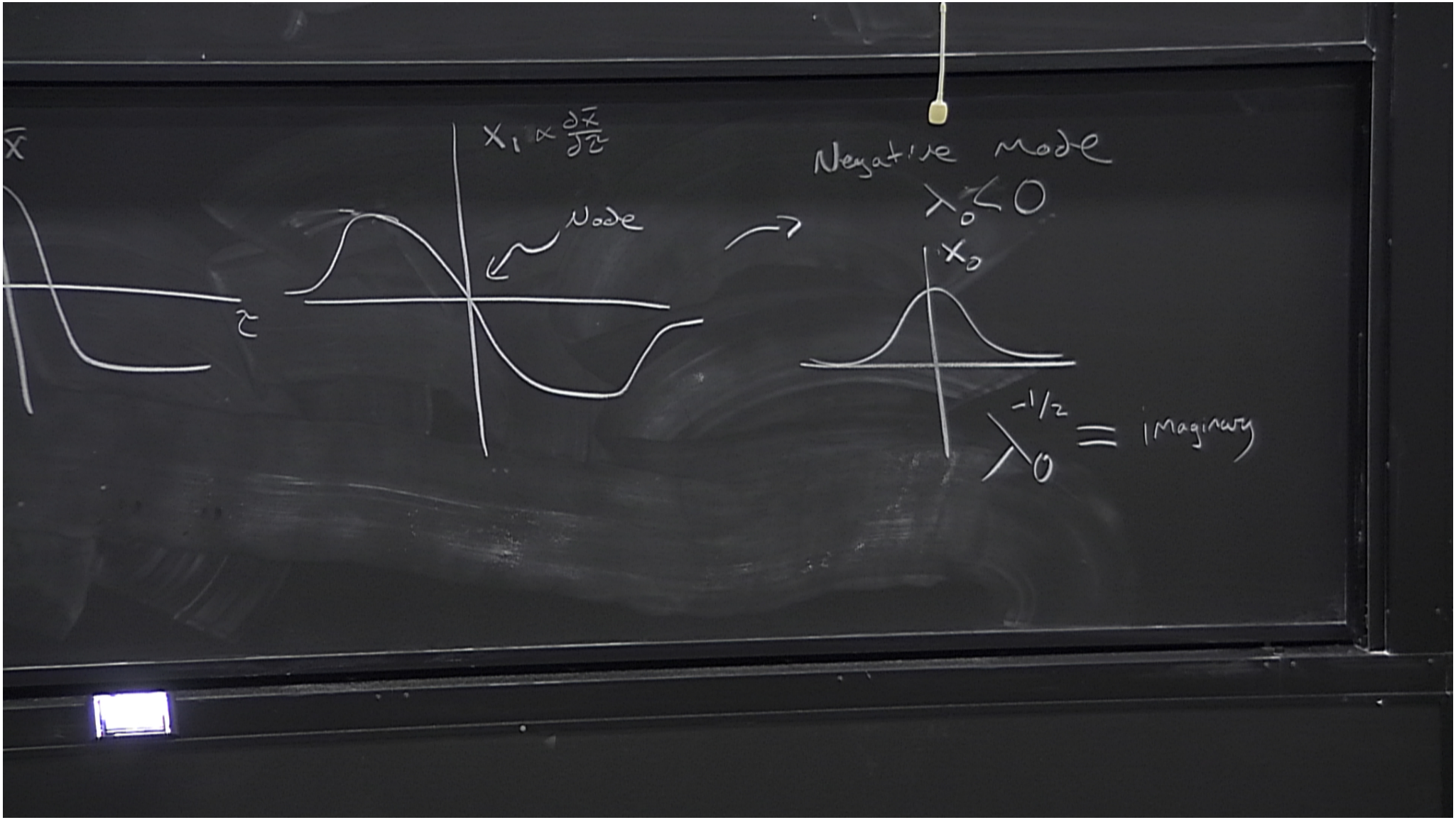

X_1 = translation of $\bar{x}$
$\bar{x} + c_1 x_1$
$\Delta x = \frac{\partial x}{\partial c} \Delta c$ V.S. $\Delta x = x_1 \Delta c_1$
$\Rightarrow \frac{\Delta c_1}{(2\pi\hbar)^{1/2}} = \left(\frac{\rho_0}{2\pi\hbar}\right)^{1/2} dc$

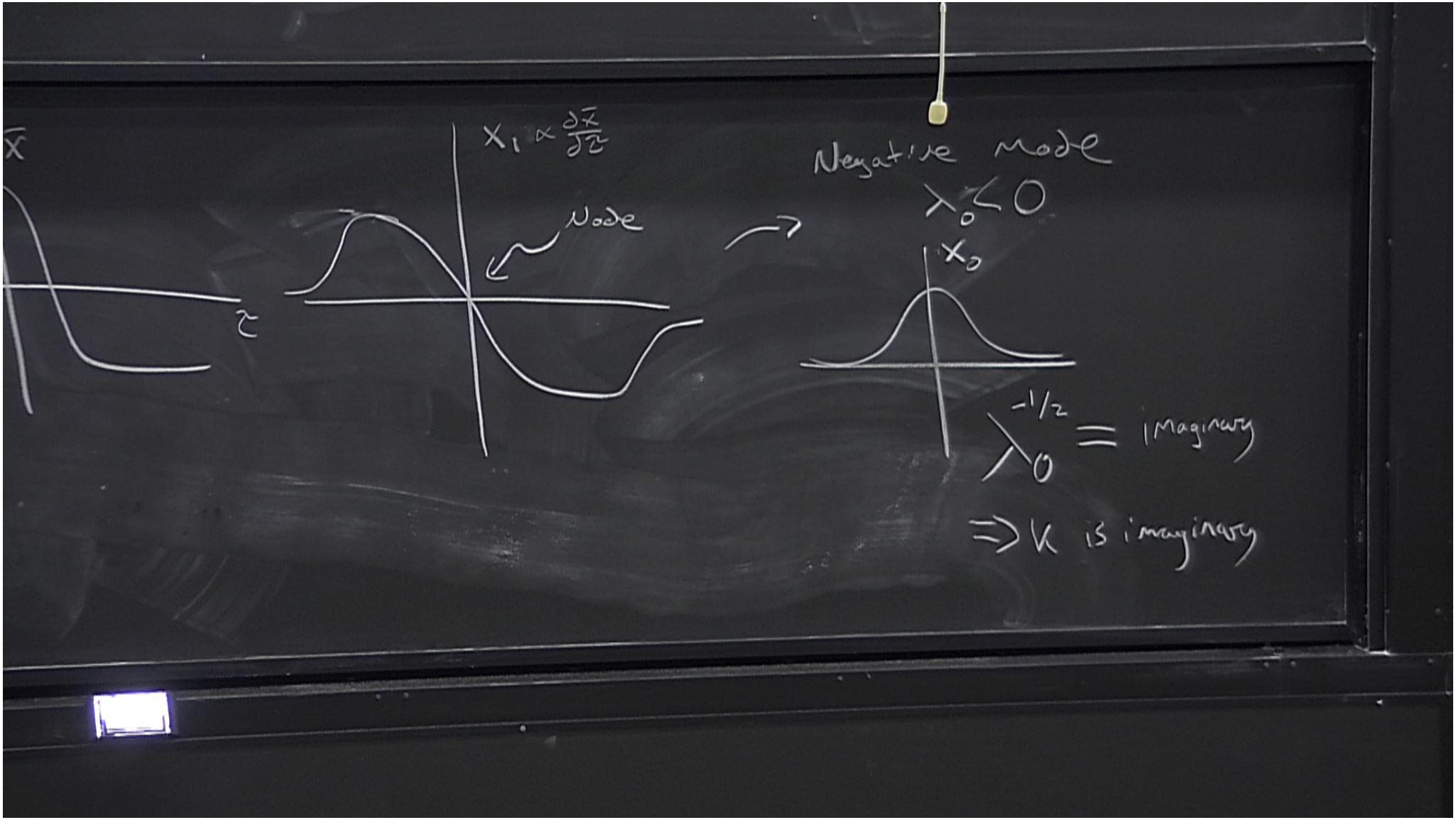

X_1 = translation of $\bar{x}$
$\bar{x} + c_1 x_1$
$\Delta x = \frac{\partial x}{\partial c} \Delta c$ V.S. $\Delta x = x_1 \Delta c_1$
$\Rightarrow \frac{dc_1}{(2\pi\hbar)^{1/2}} = \left(\frac{S_0}{2\pi\hbar}\right)^{1/2} d\tau$
$\rightarrow k \propto \left(\frac{S_0}{2\pi\hbar}\right)^{1/2}$

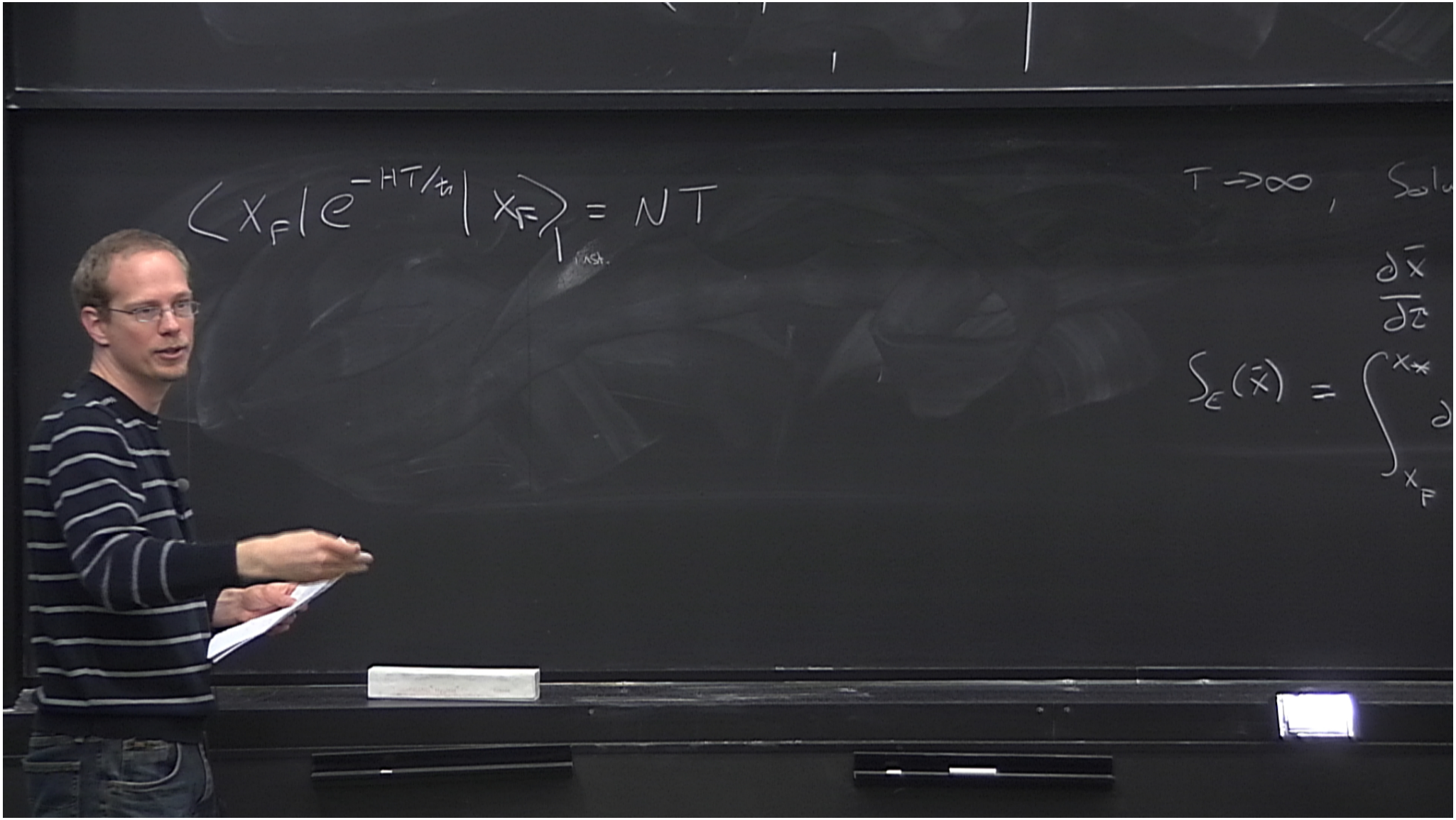

of $\bar{x}$
V.S. $\Delta x = X_1 \Delta c_1$
$= \left(\frac{S_0}{2\pi t_1}\right)^{1/2} d\xi$
$\propto \left(\frac{S_0}{2\pi t_1}\right)^{1/2}$
$\bar{x}$
ξ
$X_1 \propto \frac{\partial \bar{x}}{\partial \xi}$

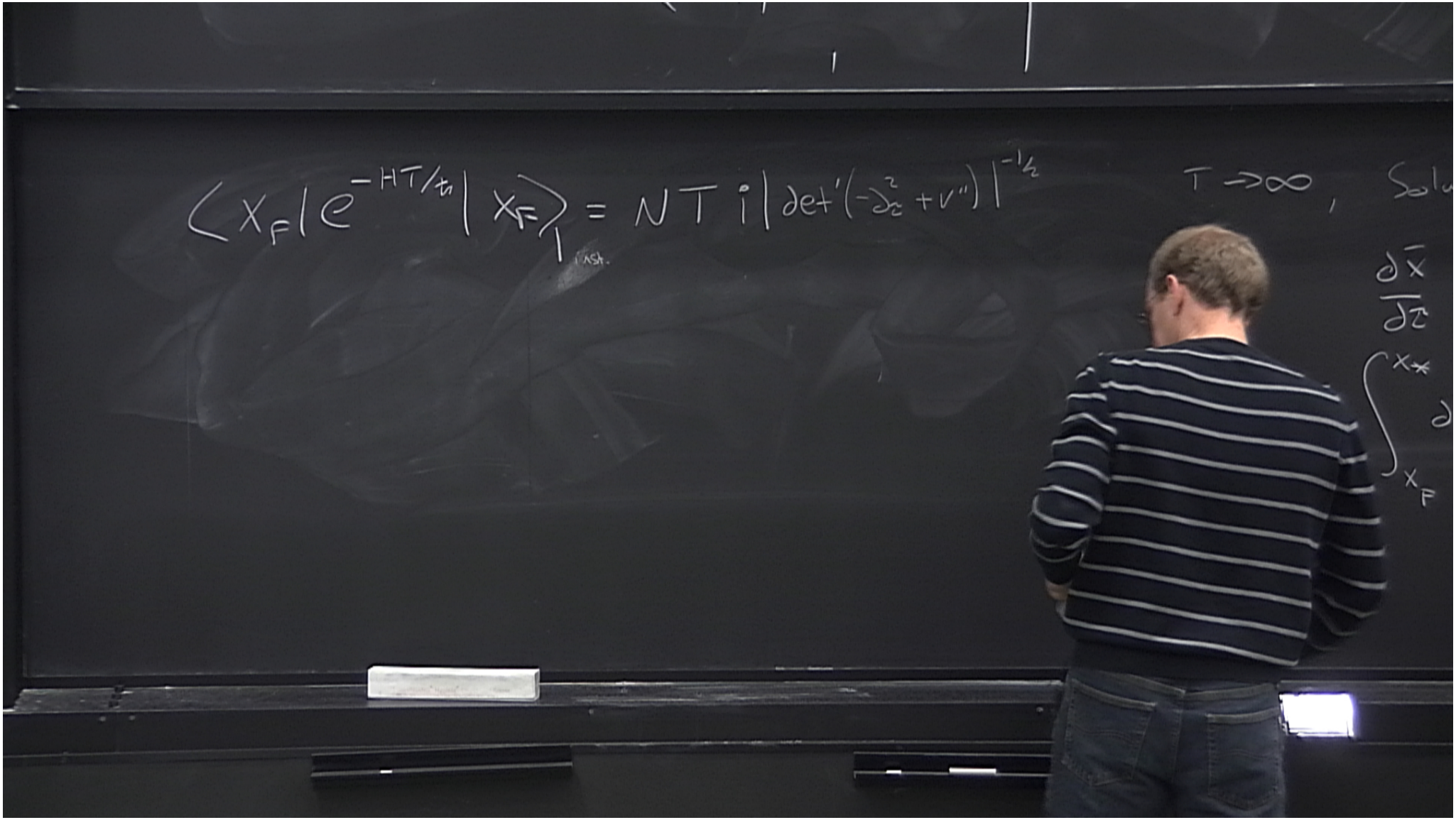

of $\bar{x}$
vs. $\Delta x = X_1 \Delta c_1$
$= \left(\frac{S_0}{2\pi t_1}\right)^{1/2} d\xi$
$\propto \left(\frac{S_0}{2\pi t_1}\right)^{1/2}$
$\bar{x}$
ξ
$X_1 \propto \frac{\partial \bar{x}}{\partial \xi}$
Node

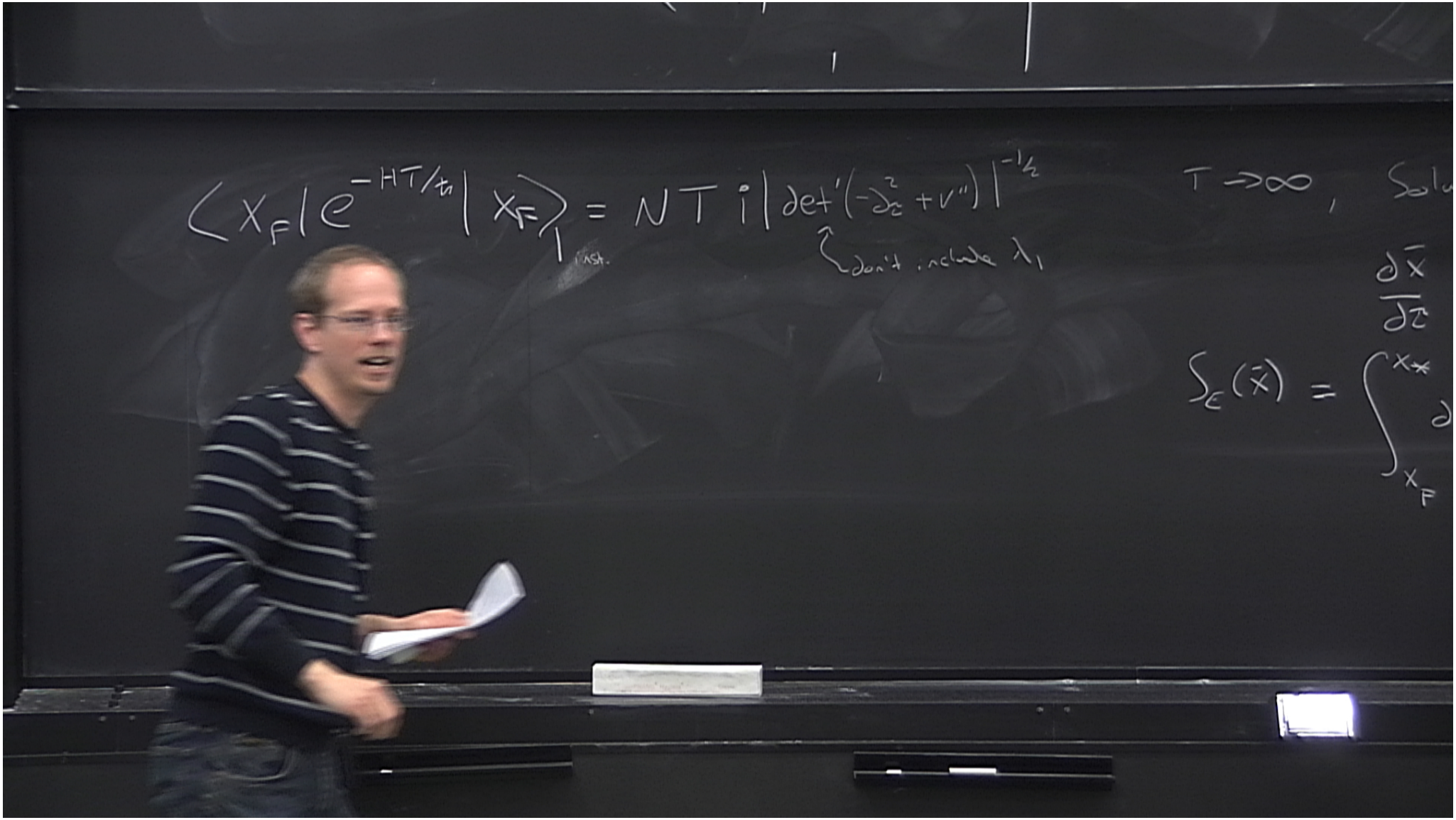

$\bar{x}$
τ
$x_1 \propto \frac{d\bar{x}}{d\tau}$
Node
Negative mode
$x_0 < 0$
x_0

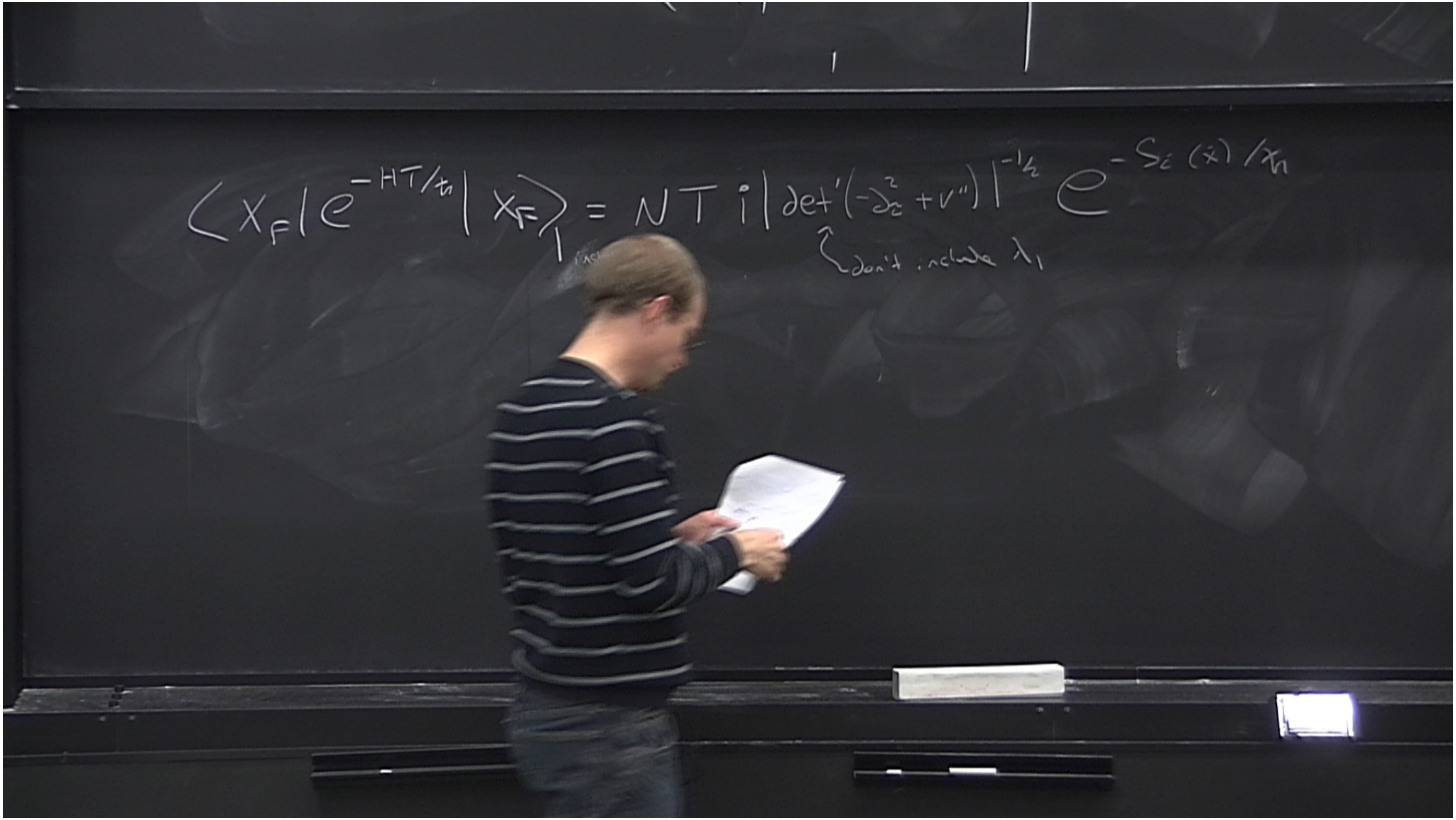

$\bar{x}$
τ
$x_1 \propto \frac{d\bar{x}}{d\tau}$
Node
Negative mode
$x_0 < 0$
x_0
$\lambda_0^{-1/2}$ = imaginary

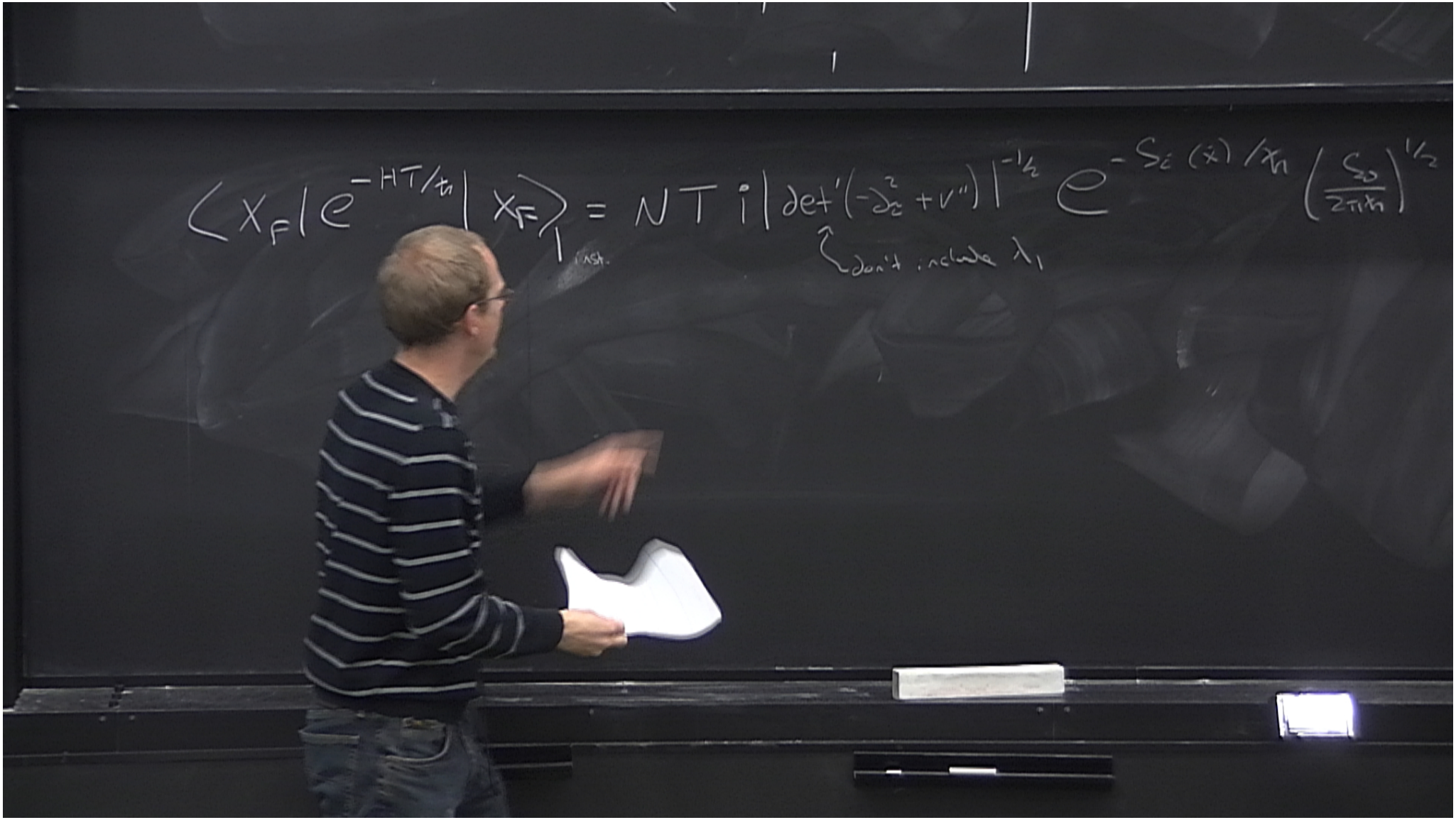

$\bar{x}$
τ
$x_1 \propto \frac{d\bar{x}}{d\tau}$
Node
Negative mode
$x_0 < 0$
x_0
$\lambda_0^{-1/2}$ = imaginary
$\Rightarrow k$ is imaginary

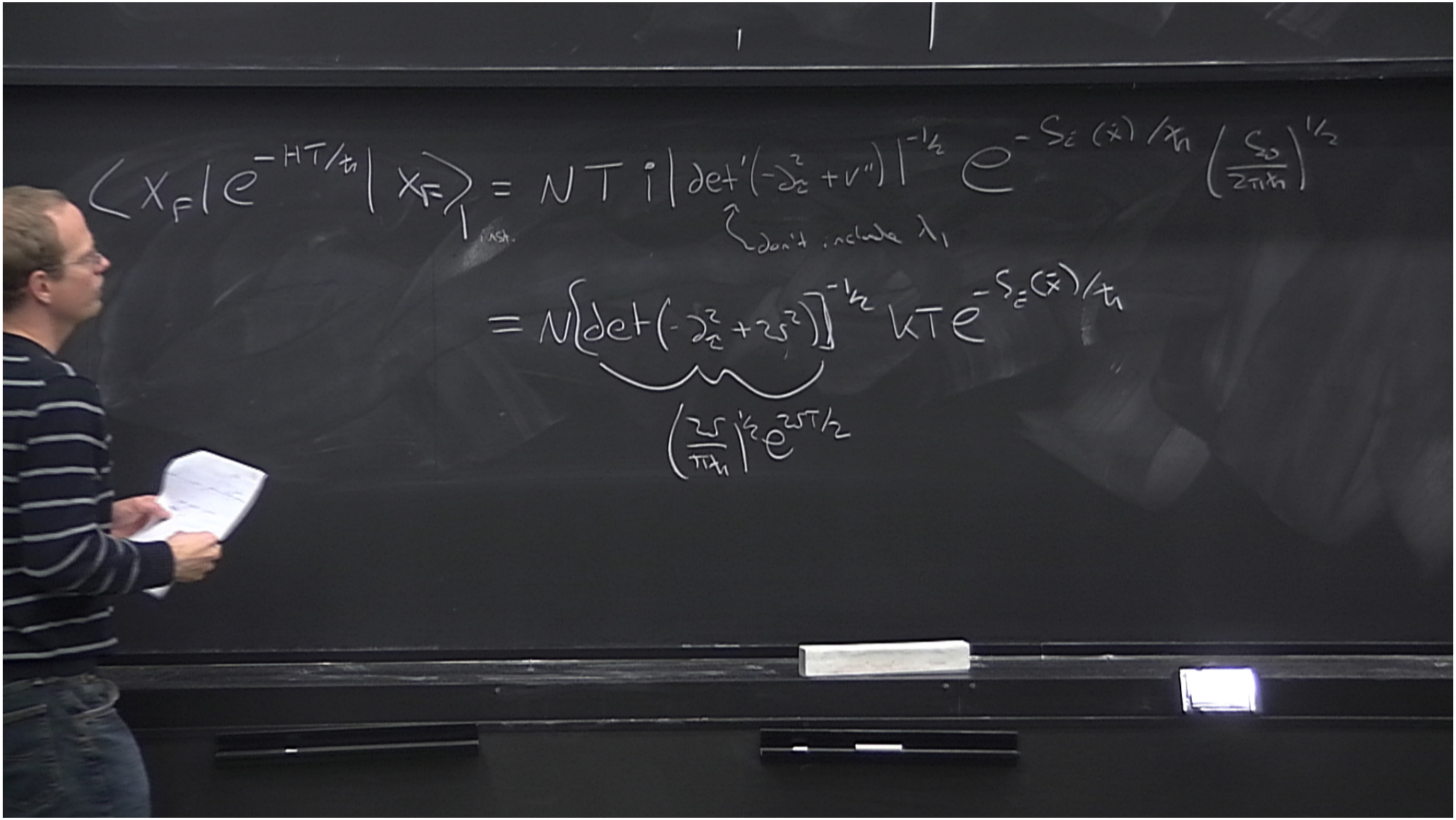

$\langle x_f | e^{-HT/\hbar} | x_f \rangle = NT$
$T \to \infty$,
$S_E(\bar{x}) =$

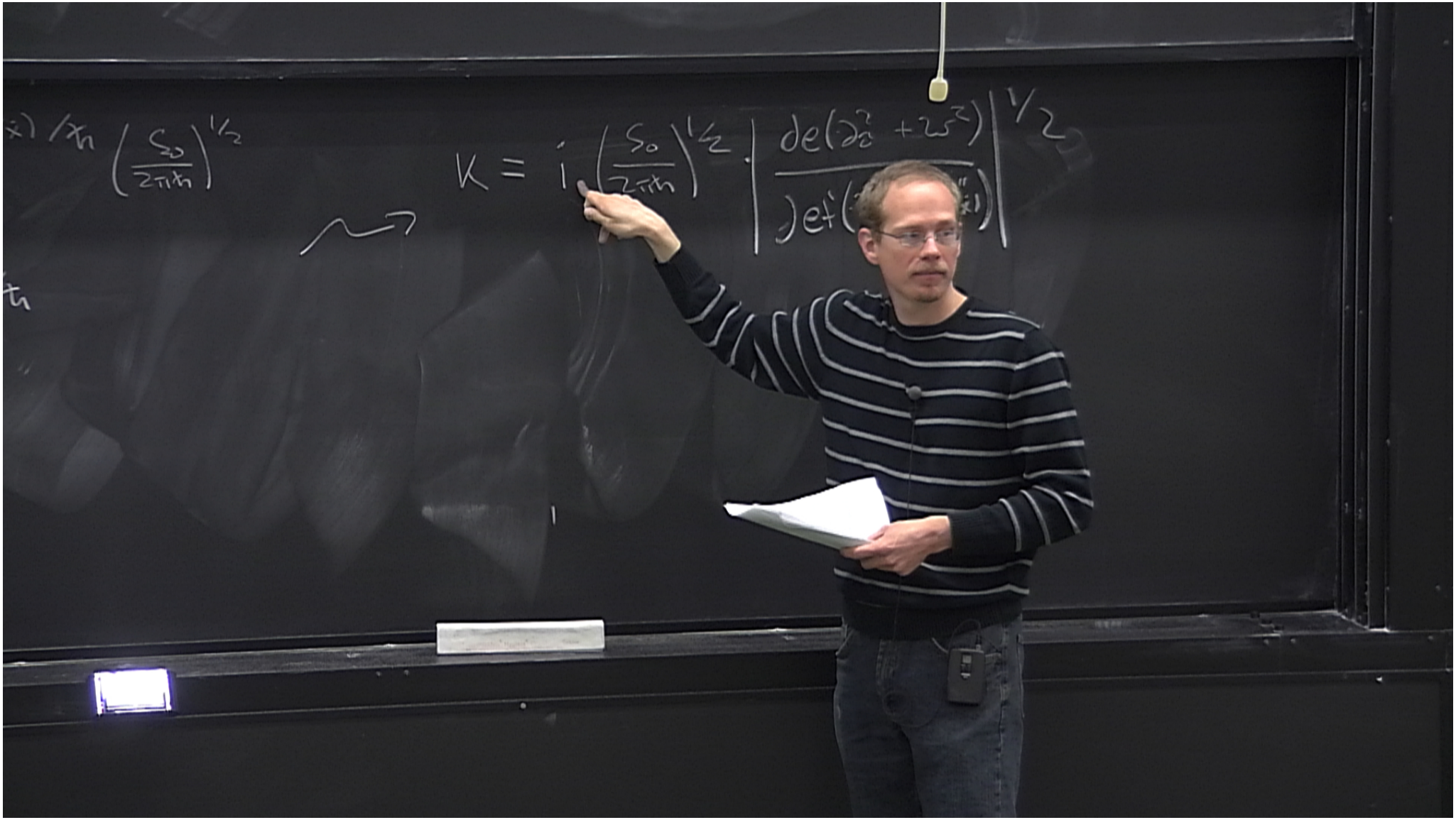

$\langle x_F | e^{-HT/\hbar} | x_F \rangle = N T i \left| \det'\left(-\partial_\tau^2 + V''\right)\right|^{-1/2}$
$T \to \infty$,
$\frac{d\bar{x}}{d\tau}$
x_*
x_F

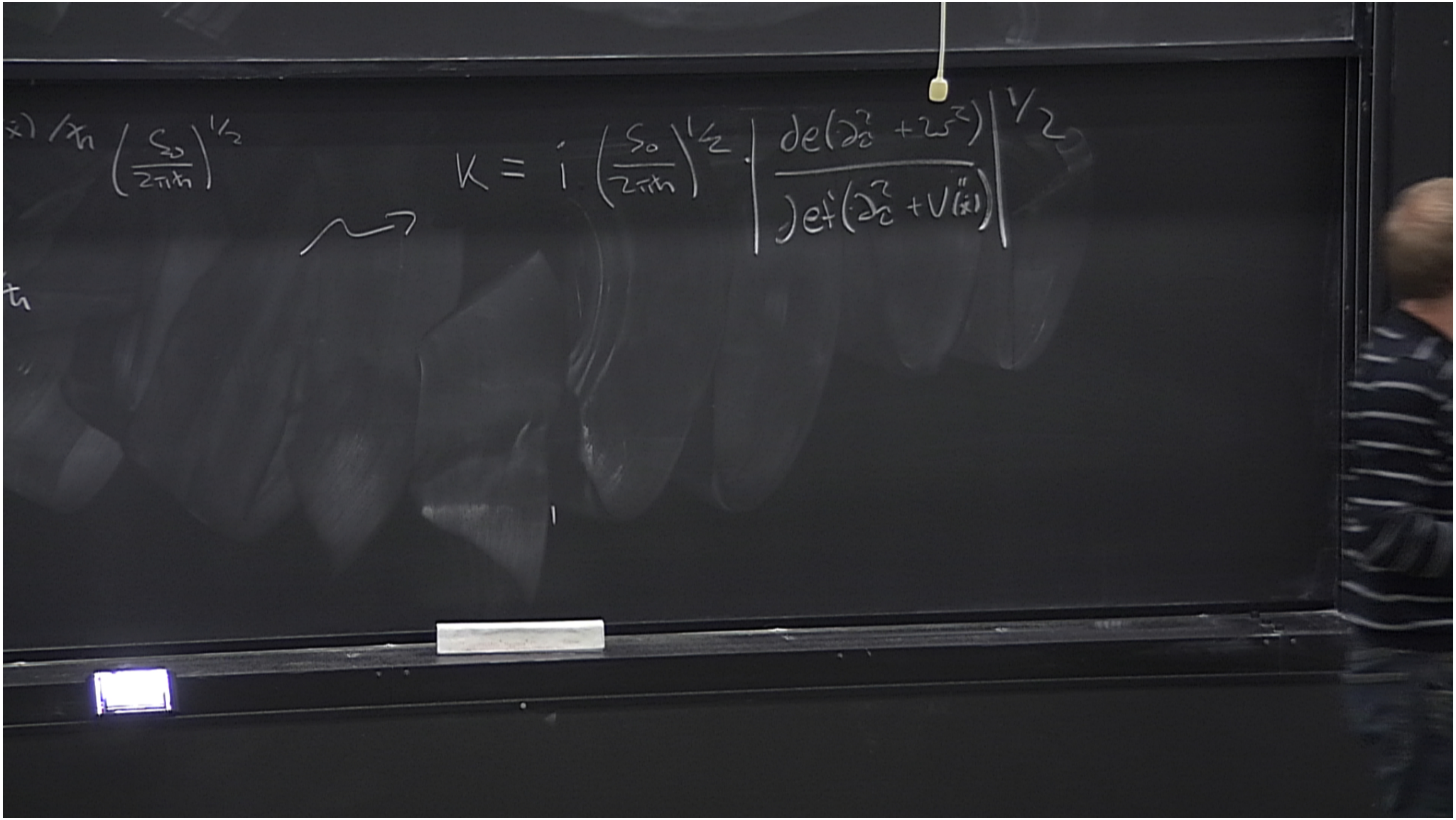

$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N T i \left| \det'\left(-\partial_\tau^2 + V''\right) \right|^{-1/2}$
{don't include λ_1
$T \to \infty$,
$S_E(\bar{x}) = \int_{x_i}^{x_f}$

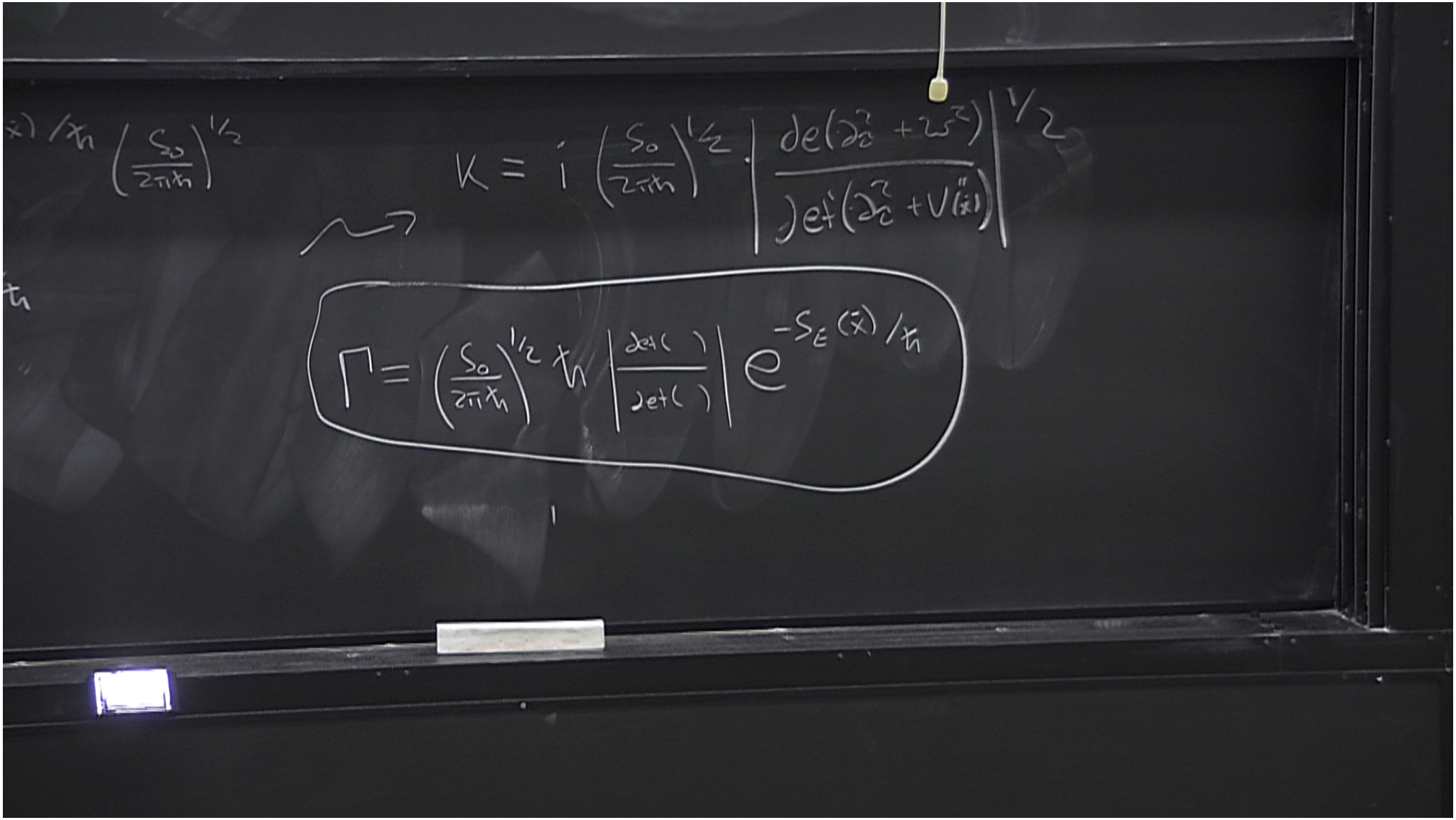

$\langle x_f | e^{-HT/\hbar} | x_i \rangle = NT \int |\det'(-\partial_\tau^2 + V'')|^{-1/2} e^{-S_c(x)/\hbar}$
don't include λ_1

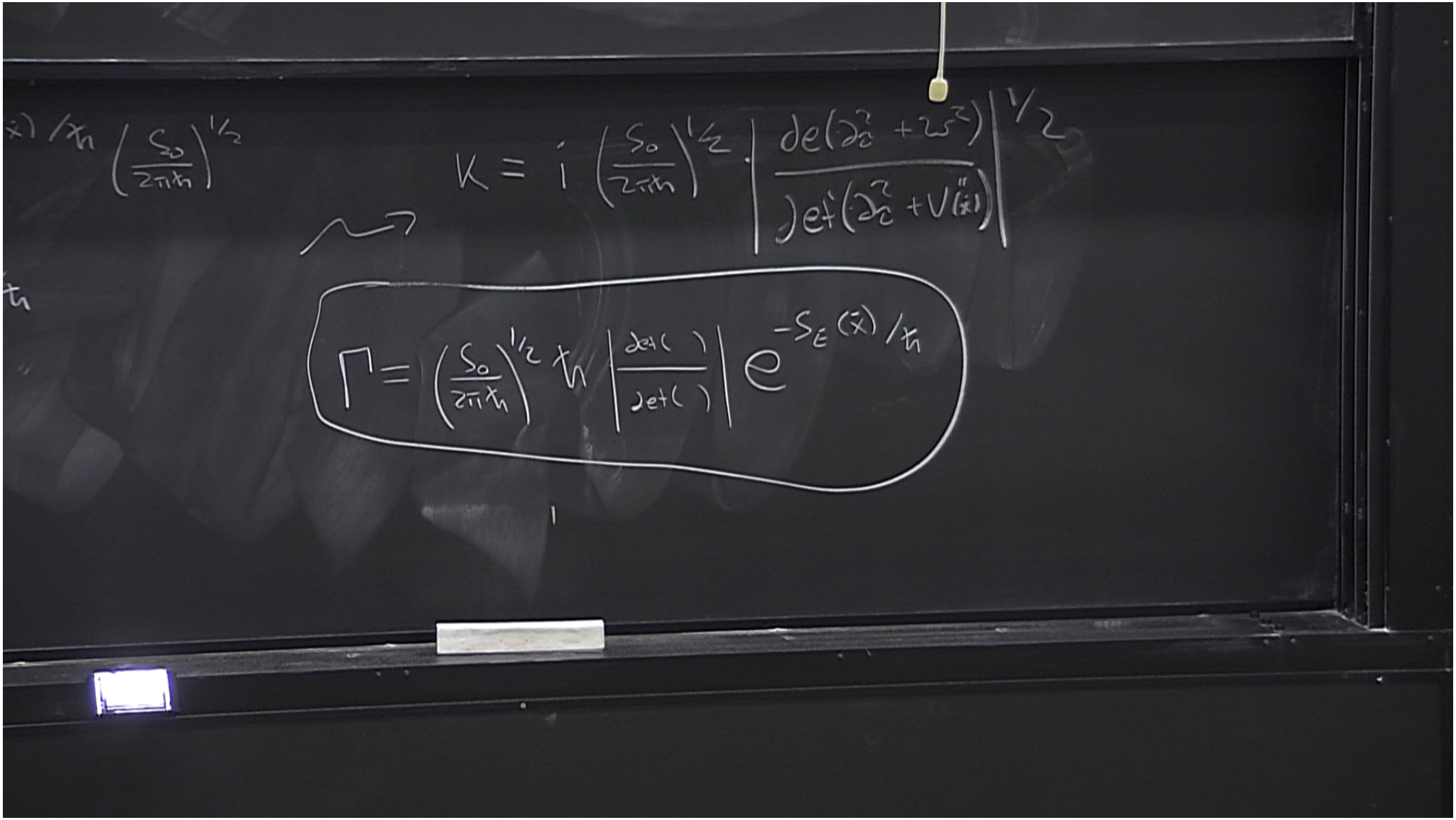

$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N T \, i \, |\det'(-\partial_\tau^2 + V'')|^{-1/2} \, e^{-S_c(x)/\hbar} \left(\frac{S_0}{2\pi\hbar}\right)^{1/2}$
inst.
{don't include λ_1


$\langle x_f | e^{-HT/\hbar} | x_i \rangle = NTi \left| \det'(-\partial_\tau^2 + V'') \right|^{-1/2} e^{-S_E(\bar{x})/\hbar} \left(\frac{S_0}{2\pi\hbar} \right)^{1/2}$
inst.
don't include λ_1
$= N \left[\det(-\partial_\tau^2 + \omega^2) \right]^{-1/2} kT e^{-S_E(\bar{x})/\hbar}$
$\left(\frac{\omega}{\pi\hbar} \right)^{1/2} e^{\omega T/2}$


$K = i \left(\frac{S_0}{2\pi i\hbar}\right)^{1/2} \cdot \left| \frac{\det(\partial_z^2 + \omega^2)}{\det(\quad)} \right|^{1/2}$


$\left(\frac{S_0}{2\pi i\hbar}\right)^{1/2}$
$K = i\left(\frac{S_0}{2\pi i\hbar}\right)^{1/2} \left|\frac{\det(\partial_\tau^2 + \omega^2)}{\det(\partial_\tau^2 + V''(x_1))}\right|^{1/2}$


$\left(\frac{S_0}{2\pi\hbar}\right)^{1/2}$
$K = i\left(\frac{S_0}{2\pi\hbar}\right)^{1/2} \left|\frac{\det(\partial_\tau^2 + \omega^2)}{\det(\partial_\tau^2 + V''(x))}\right|^{1/2}$
$\Gamma = \left(\frac{S_0}{2\pi\hbar}\right)^{1/2} \hbar \left|\frac{\det(\ )}{\det(\ )}\right| e^{-S_E(x)/\hbar}$


$K = i \left(\frac{S_0}{2\pi\hbar}\right)^{1/2} \left| \frac{\det(\partial_\tau^2 + \omega^2)}{\det(\partial_\tau^2 + V''(x))} \right|^{1/2}$
$\Gamma = \left(\frac{S_0}{2\pi\hbar}\right)^{1/2} \hbar \left| \frac{\det(\ )}{\det(\ )} \right| e^{-S_E(\bar{x})/\hbar}$

$\int d\tau$

$x_1 =$ translation of x

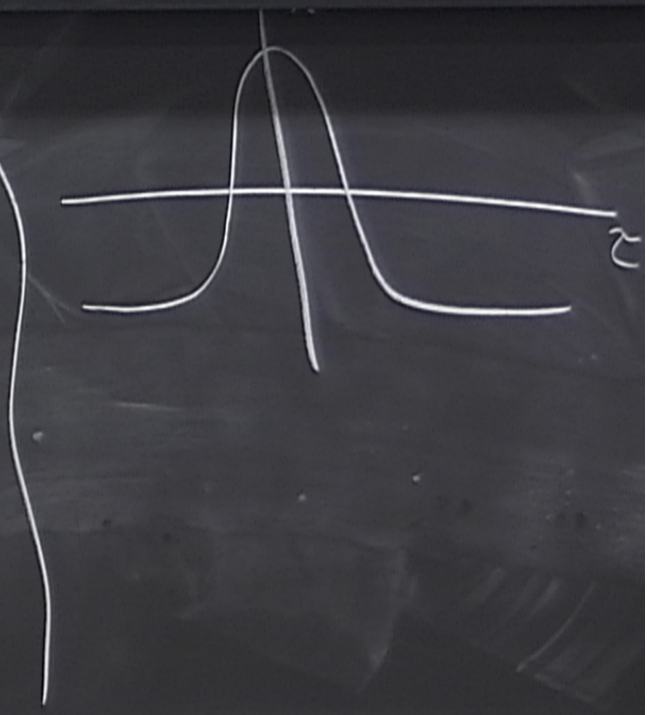
$\bar{x} + c_1 x_1$

$\Delta x = \frac{dx}{d\tau} \Delta\tau$ v.s. $\Delta x = x_1 \Delta c_1$

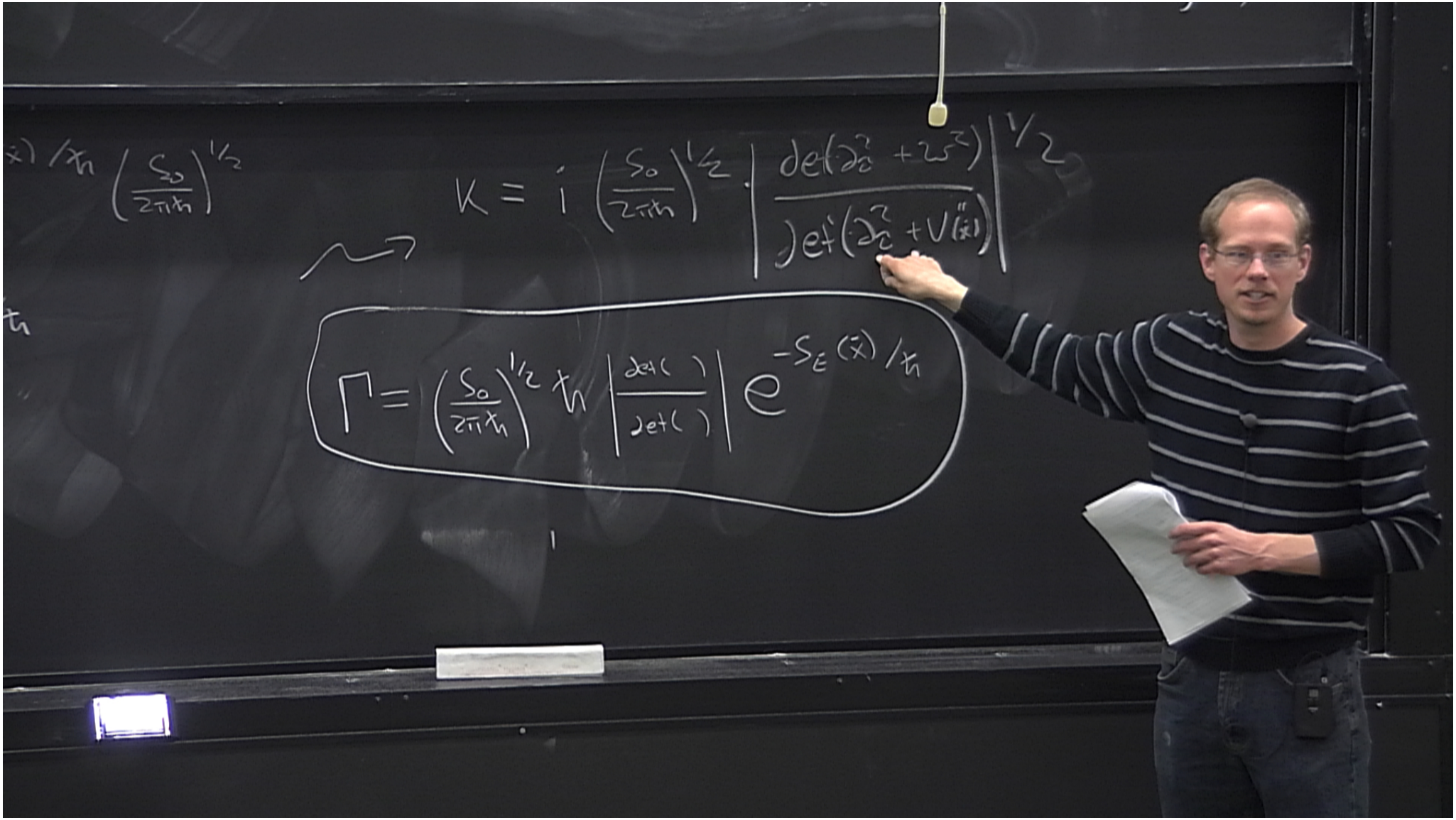
$\Rightarrow \frac{dc_1}{(2\pi\hbar)^{1/2}} = \left(\frac{S_0}{2\pi\hbar}\right)^{1/2} d\tau$

$\rightarrow K \propto \left(\frac{S_0}{2\pi\hbar}\right)^{1/2}$

$\bar{x} + c_1 x_1 - \bar{x} = c_1 x_1$



$\langle x_f | e^{-HT/\hbar} | x_i \rangle = N \int |\det'(-\partial_\tau^2 + V'')|^{-1/2} e^{-S_c(x)/\hbar} \left(\frac{S_0}{2\pi\hbar}\right)^{1/2}$


$\left(\frac{S_0}{2\pi i\hbar}\right)^{1/2}$
$K = i\left(\frac{S_0}{2\pi i\hbar}\right)^{1/2} \left|\frac{\det(\partial_\tau^2 + \omega^2)}{\det(\partial_\tau^2 + V''(x))}\right|^{1/2}$
$\Gamma = \left(\frac{S_0}{2\pi i\hbar}\right)^{1/2} \hbar \left|\frac{\det(\ )}{\det(\ )}\right| e^{-S_E(\bar{x})/\hbar}$