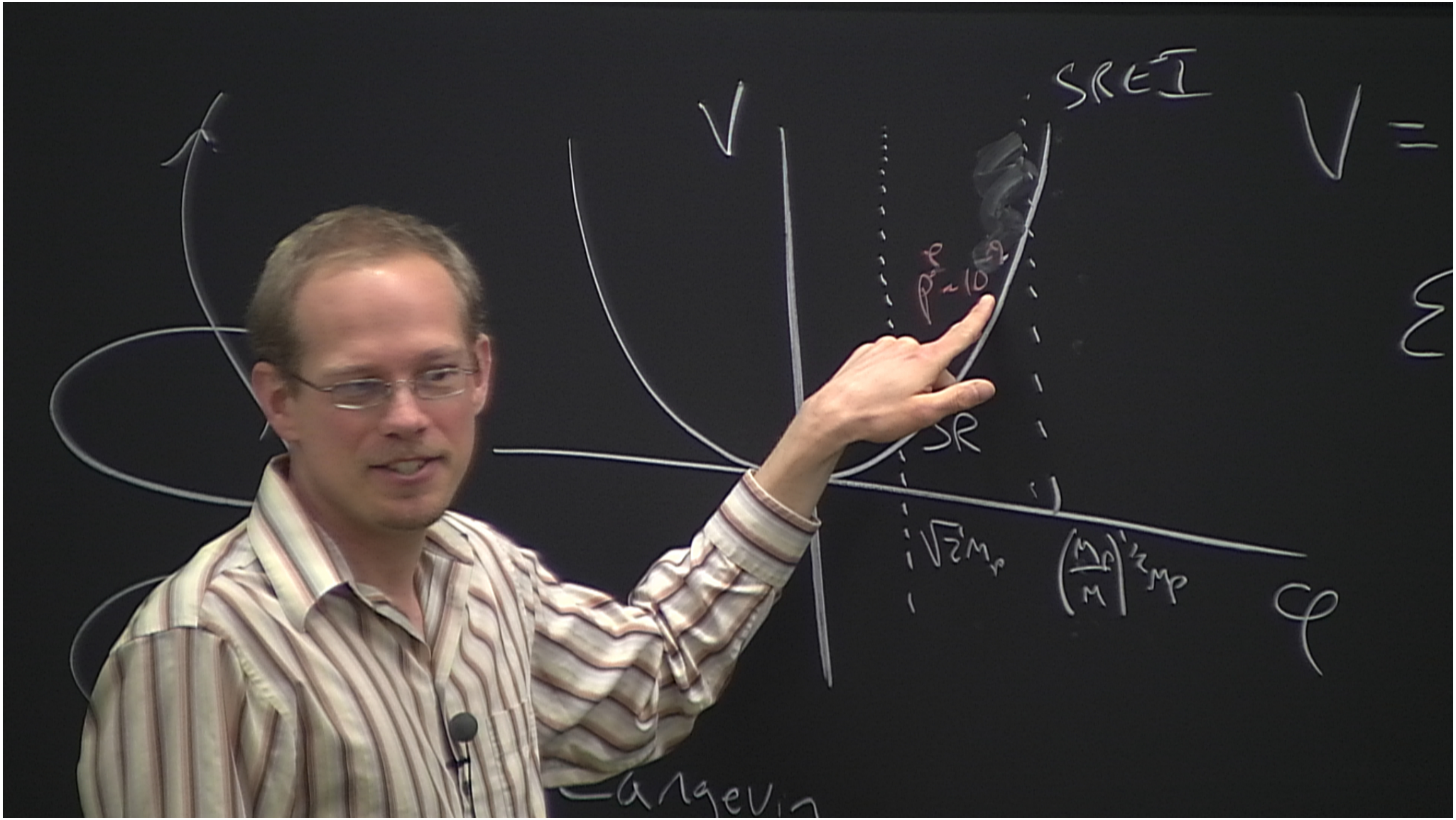
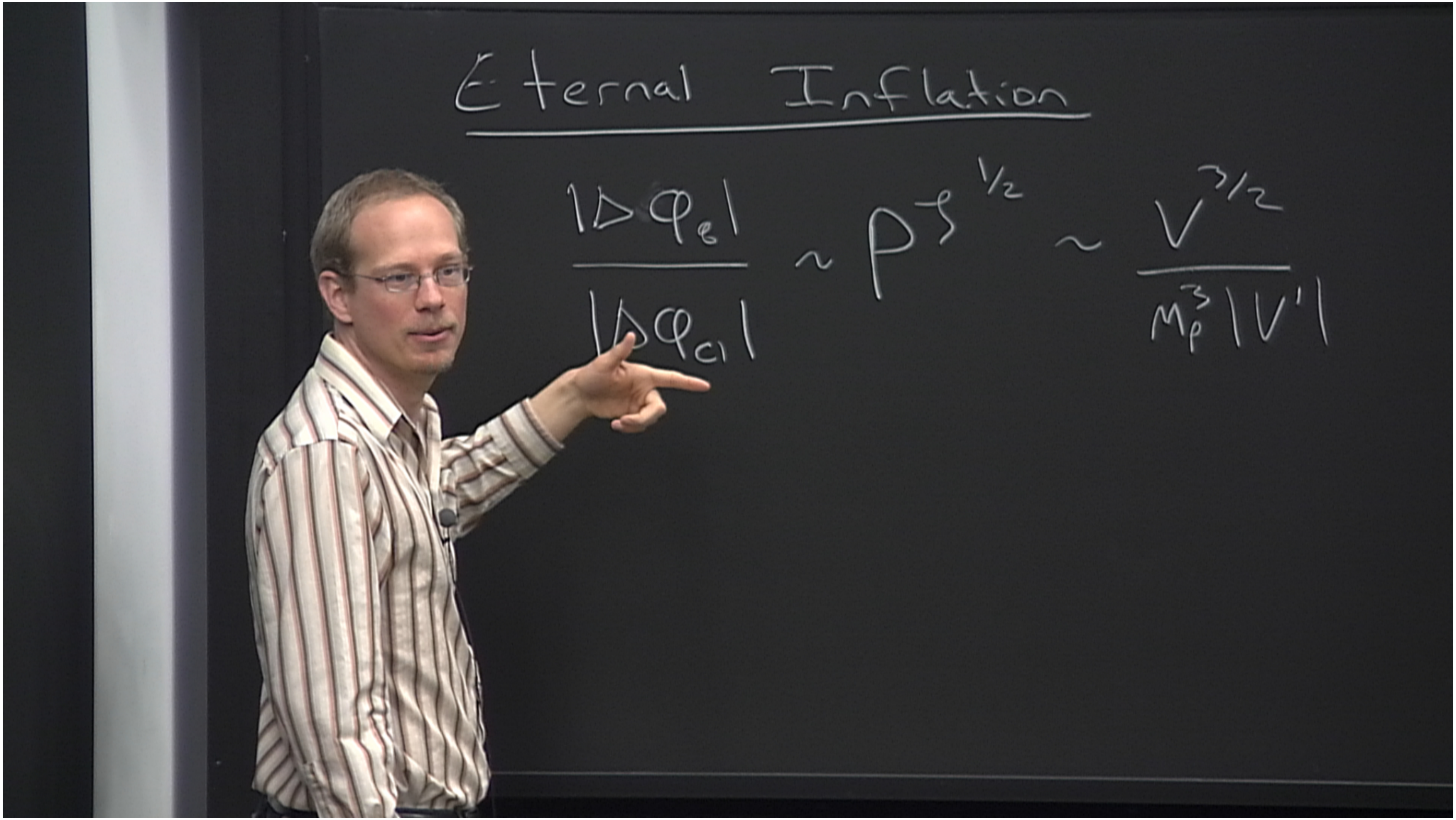
Title: Explorations in Cosmology - Lecture 10

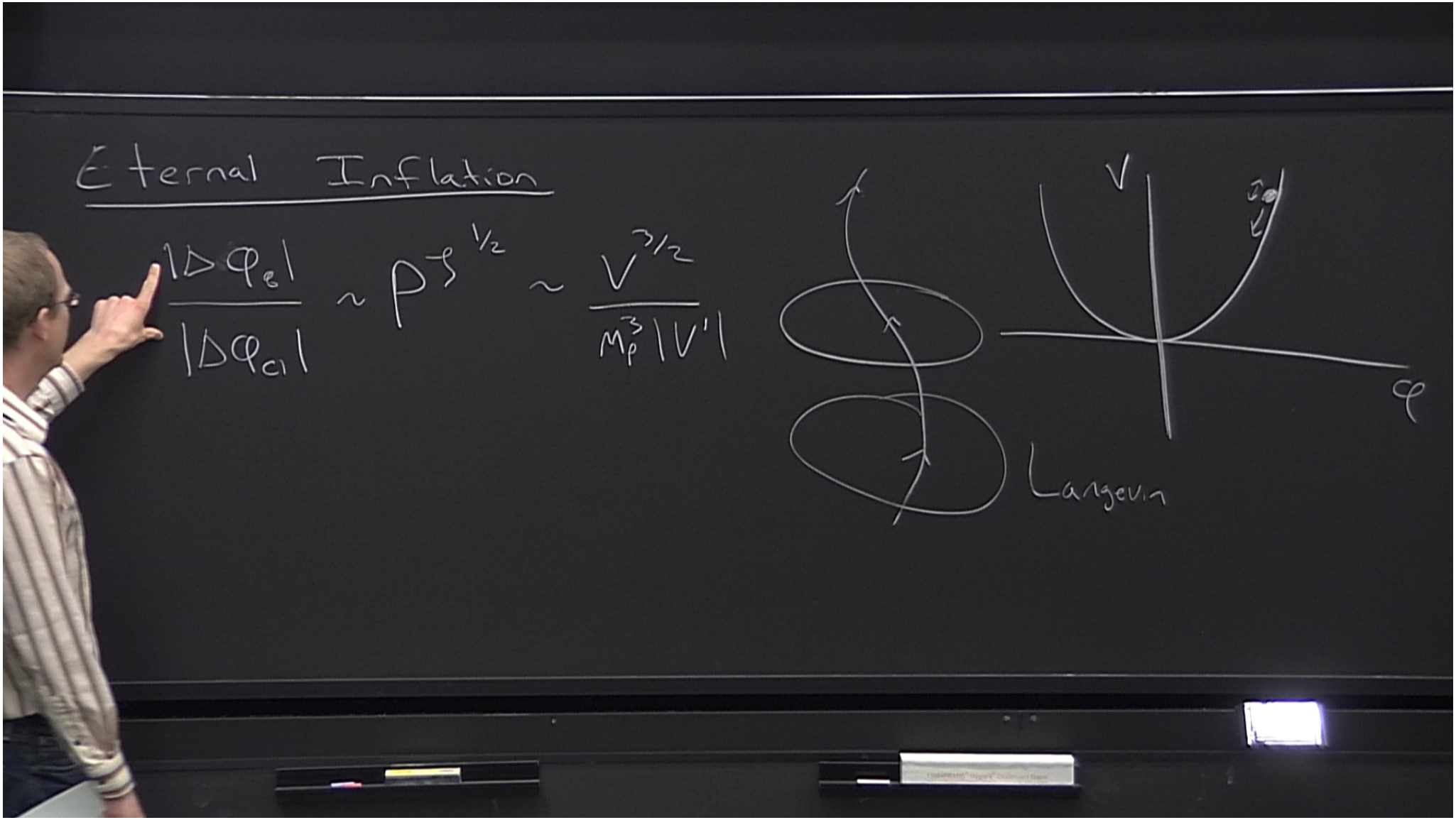
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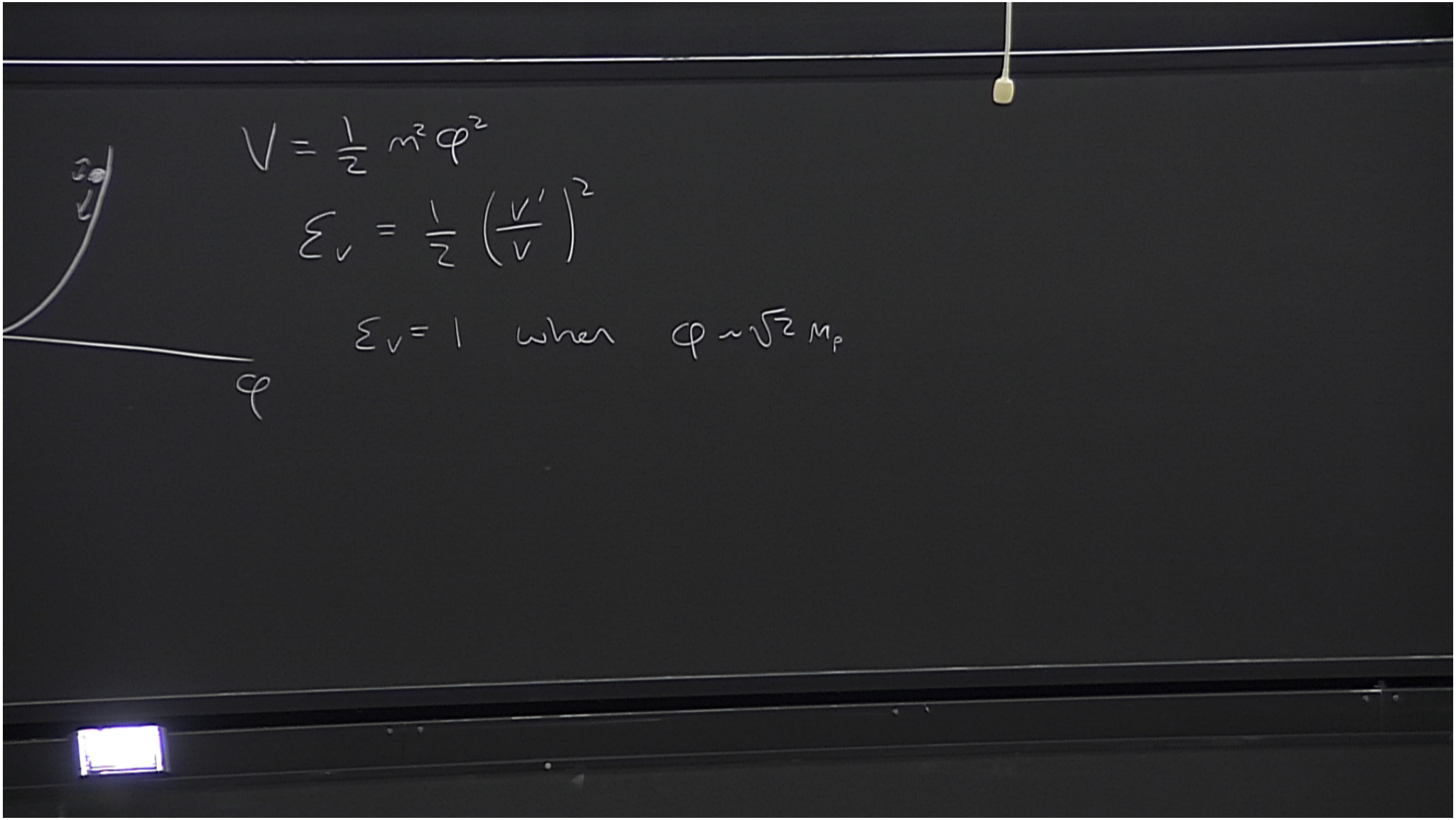
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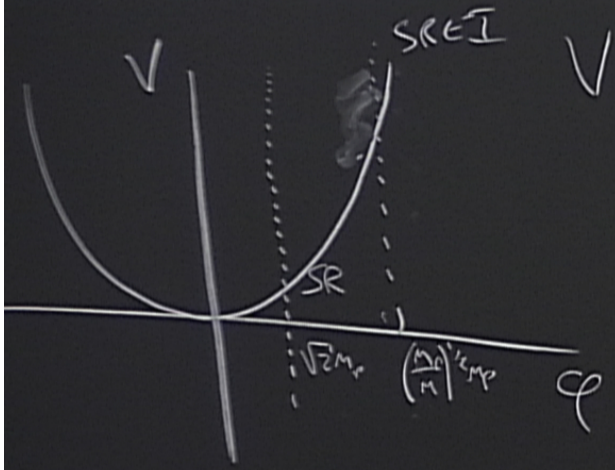
Abstract:


V
SREI
V =
SR
Langevin


Eternal Inflation
$\frac{|\Delta\varphi_e|}{|\Delta\varphi_{cl}|} \sim \rho\zeta^{1/2} \sim \frac{V^{3/2}}{M_p^3|V'|}$


Eternal Inflation
$\frac{|\Delta\varphi_e|}{|\Delta\varphi_{cl}|} \sim \rho^{1/2} \sim \frac{V^{3/2}}{M_p^3 |V'|}$
V
φ
Langevin


$V = \frac{1}{2} m^2 \varphi^2$
$\varepsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$
$\varepsilon_V = 1$ when $\varphi \sim \sqrt{2} M_p$
φ



$$V = \frac{1}{2} m^2 \varphi^2$$

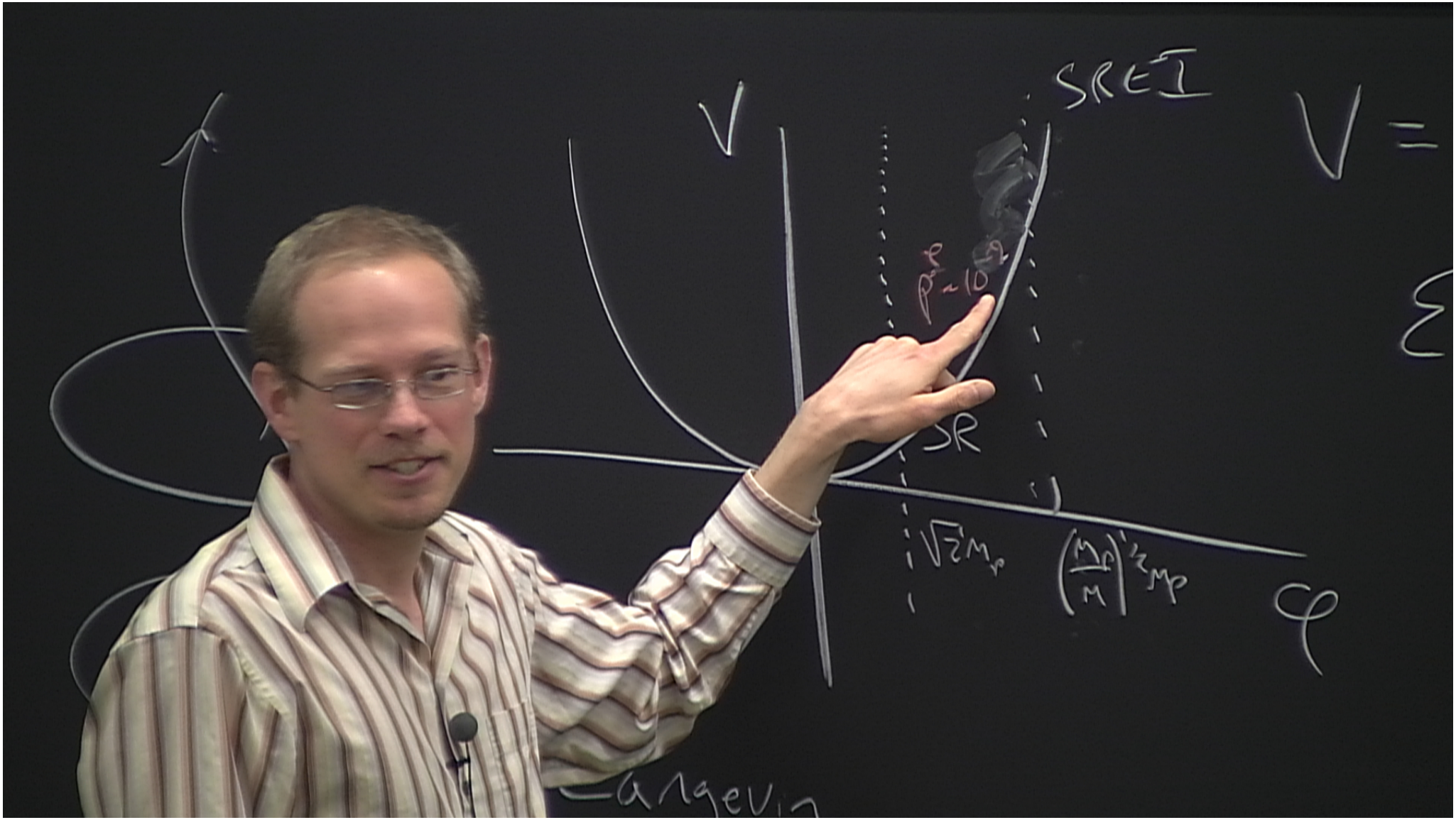
$$\varepsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

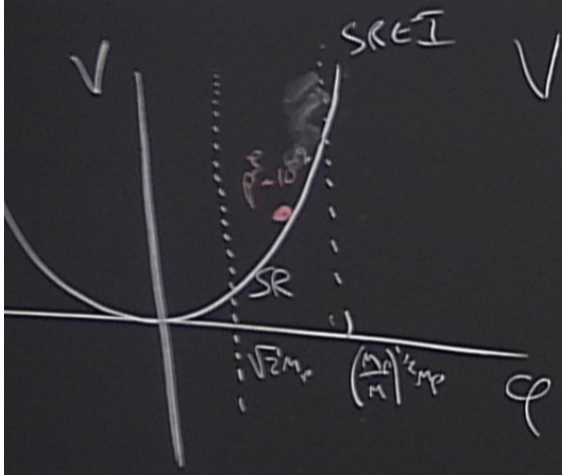
$\varepsilon_V = 1$ when $\varphi \sim \sqrt{2} M_p$

Langevin

$$\frac{V^{3/2}}{M_p^3 V'} = \frac{M^3 \varphi^3}{M_p^3 M^2 \varphi} = \frac{M \varphi_{EI}^2}{M_p^3} = 1 \Rightarrow \varphi_{EI} = \left(\frac{M_p^{3/2}}{M^{1/2}}\right) M_p$$

$M \ll M_p$


V
SREI
V =
SR
Langevin



$$V = \frac{1}{2} m^2 \varphi^2$$

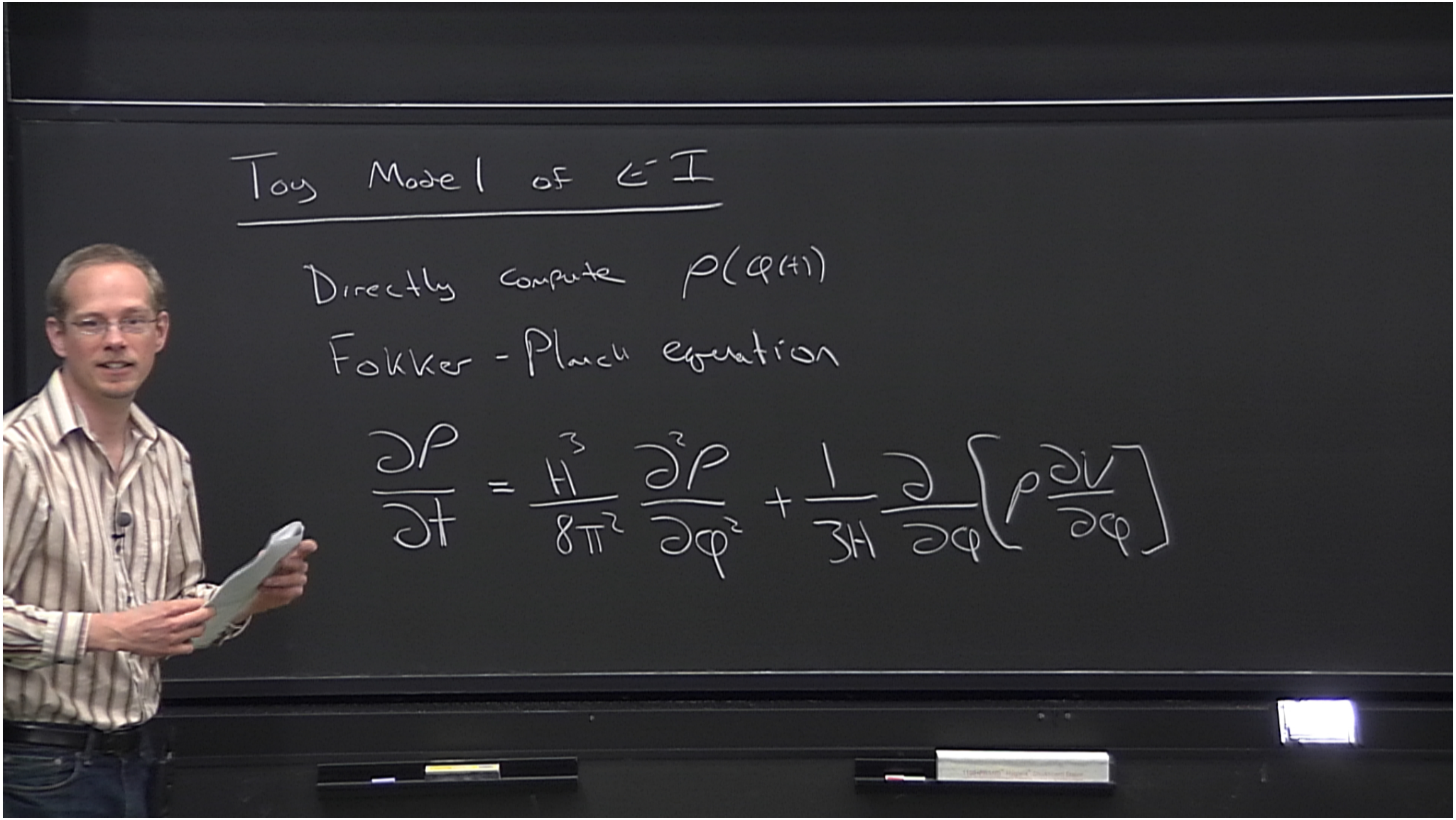
$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2$$

$\epsilon_V = 1$ when $\varphi \sim \sqrt{2} M_p$

$$\frac{V^{3/2}}{M_p^3 V'} = \frac{M^3 \varphi^3}{M_p^3 M^2 \varphi} = \frac{M \varphi_{EI}^2}{M_p^3} = 1 \Rightarrow \varphi_{EI} = \left(\frac{M_p^{3/2}}{M^{1/2}}\right) M_p$$

$M \ll M_p$

Langevin


Toy Model of EI
Directly compute $p(\varphi(t))$
Fokker-Planck equation
$\frac{\partial p}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2 p}{\partial \varphi^2} + \frac{1}{3H}\frac{\partial}{\partial \varphi}\left[p\frac{\partial V}{\partial \varphi}\right]$

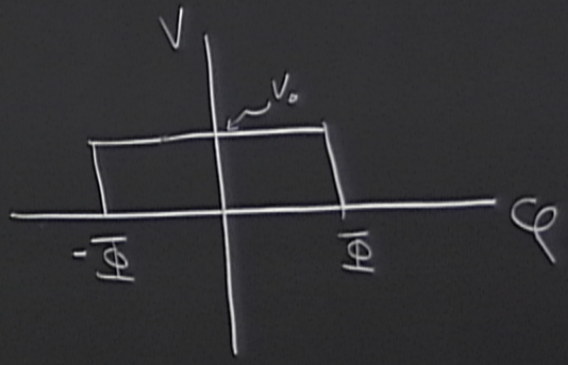
Toy Model of E-I

$$H^2 = \frac{V(\varphi)}{3M_p^2} \quad \text{(Slow-roll)}$$

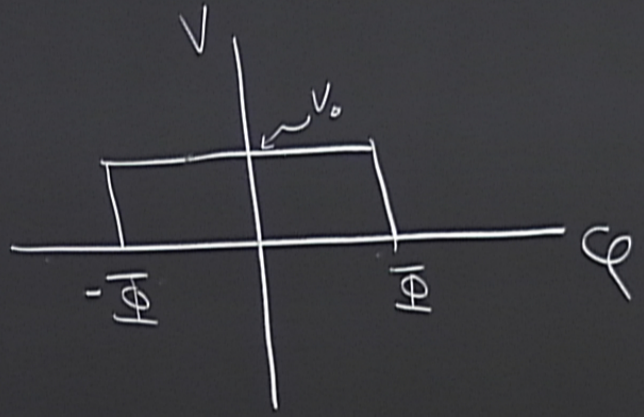
Directly compute $\rho(\varphi(t))$

Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = \frac{H(\varphi)^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \varphi^2} + \frac{1}{3H(\varphi)}\frac{\partial}{\partial \varphi}\left[\rho \frac{\partial V}{\partial \varphi}\right]$$



$= \frac{V(\varphi)}{3M_p^2}$ (Slow-roll)



$\left[\frac{\partial V}{\partial \varphi}\right]$

Infl. ends when $|\varphi| > \Phi$

$\rho(\varphi = \pm\Phi) = 0$

Solve by separation of variables

$\rho = f(t)\,g(\varphi)$

$$\frac{\partial \rho}{\partial t} = \frac{H^3(\varphi)}{8\pi^2}\frac{\partial^2 \rho}{\partial \varphi^2} + \frac{1}{3H(\varphi)}\frac{\partial}{\partial \varphi}\left[\rho \frac{\partial V}{\partial \varphi}\right]$$

$$\frac{1}{f}\frac{\partial f}{\partial t} = \frac{H^3}{8\pi^2}\frac{1}{g}\frac{\partial^2 g}{\partial \varphi^2} = C = \frac{-M^2 H^3}{32\Phi^2}$$

$$g = \cos\left[\frac{M\pi\varphi}{2\Phi}\right], \quad M = 1, 3, 5 \ldots$$

$$\frac{\partial f(t)}{\partial t} = -\frac{M^2 H^3}{32\Phi^2} f(t)$$

$$f = A_M \exp\left[-\frac{M^2 H^3}{32\Phi^2} t\right]$$

$$\rho = \sum_{M_{odd}} A_M \exp\left[-\frac{M^2 H^3}{32\Phi^2} t\right] G$$

$$\frac{M^2 H^3}{32\pi^2}$$

$$\frac{\partial f(t)}{\partial t} = -\frac{M^2 H^3}{32\pi^2} f(t)$$

$$f = A_M \exp\left[-\frac{M^2 H^3}{32\pi^2} t\right]$$

$$P = \sum_{\substack{M \\ \text{odd}}} A_M \exp\left[-\frac{M^2 H^3}{32\pi^2} t\right] \cos\left[\frac{M\pi\varphi}{2\pi}\right]$$

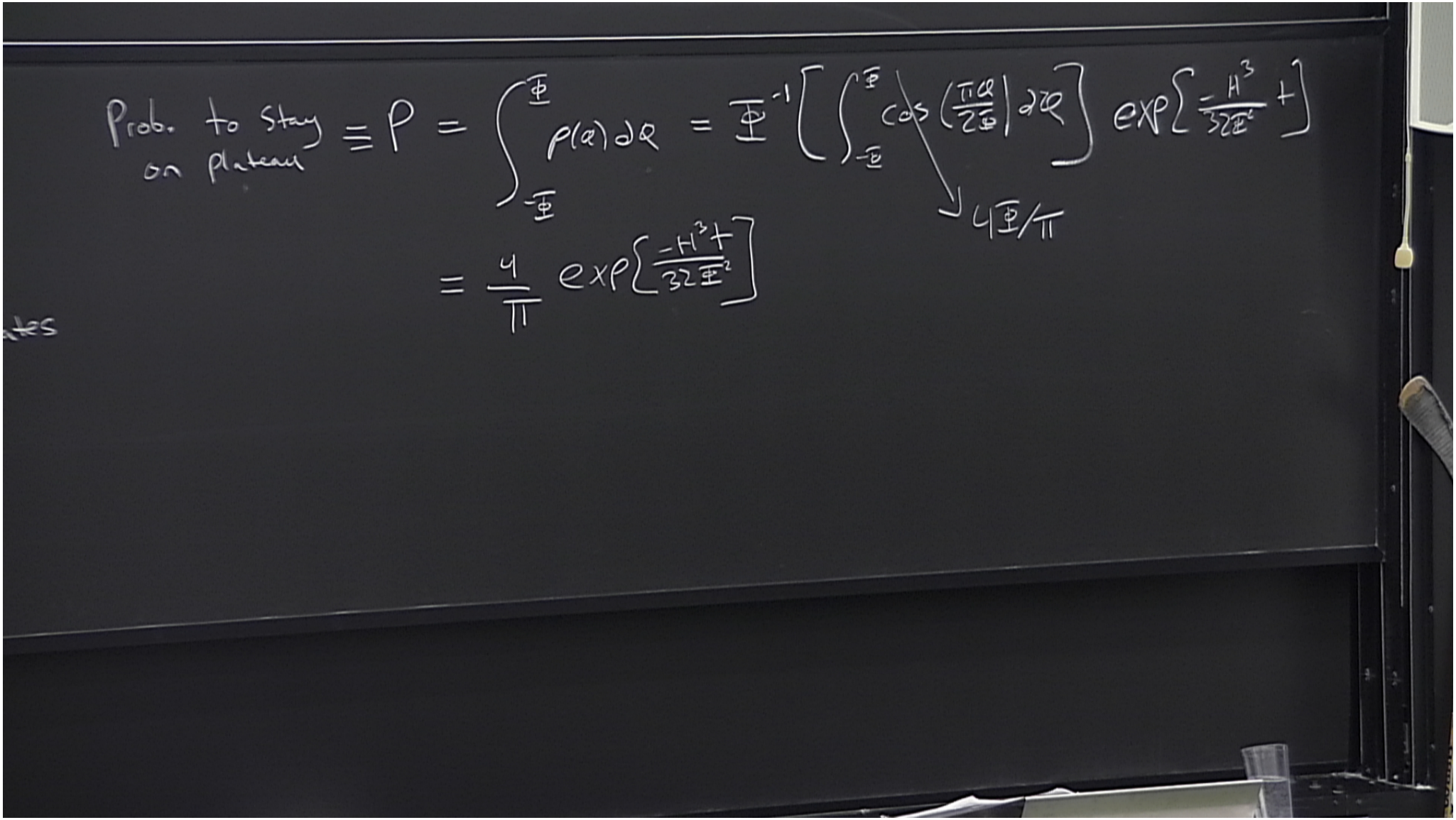
Assume $\rho(t=0) = S(\varphi)$

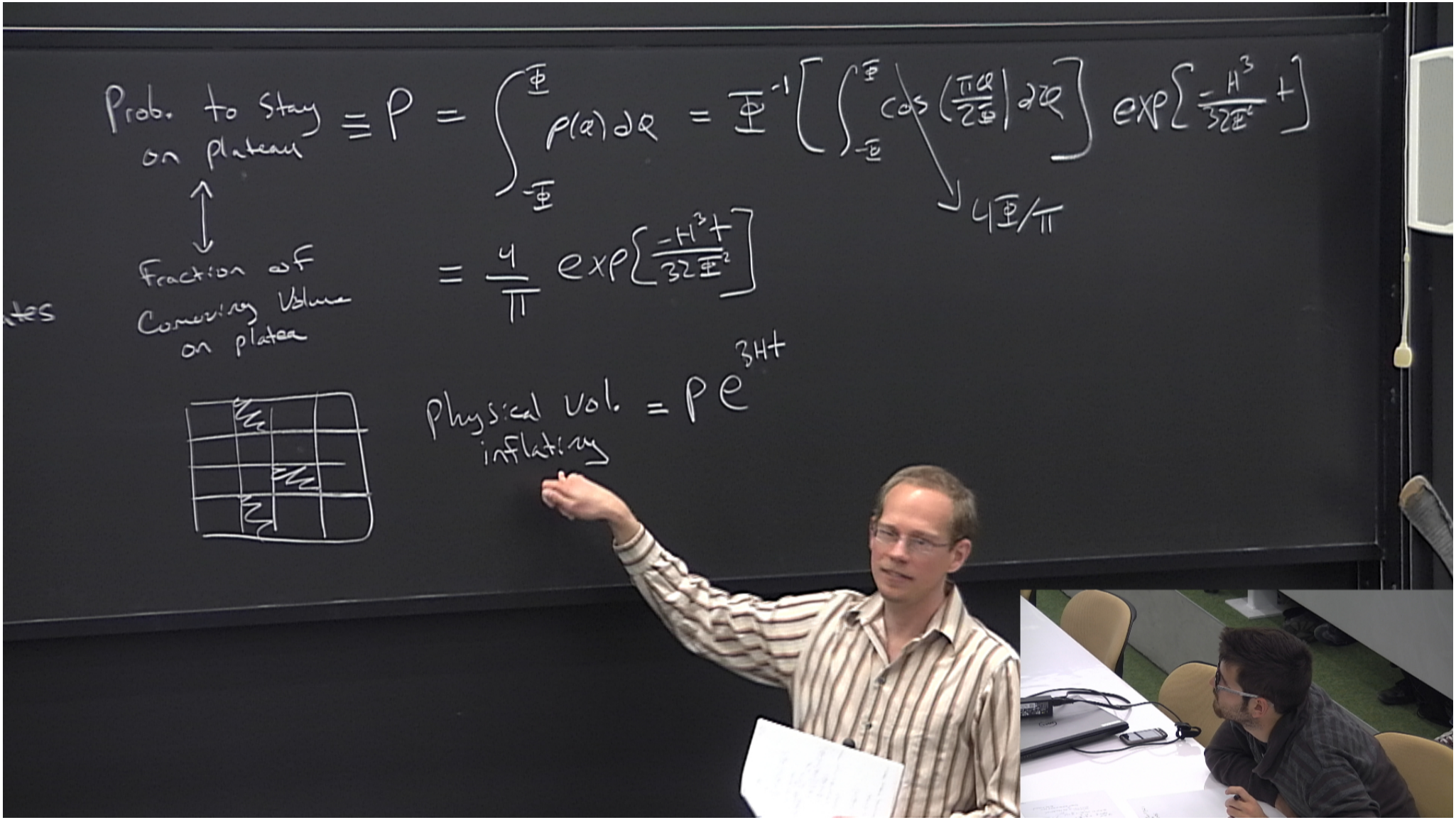
Prob. to
on pl

$\Rightarrow A_M = \bar{\Phi}^{-1}$

At large t, $M=1$ term dominates

$$\rho = \bar{\Phi}^{-1} \cos\left(\frac{\pi\varphi}{2\bar{\Phi}}\right) \exp\left[-\frac{H^3}{32\bar{\Phi}^2} t\right]$$

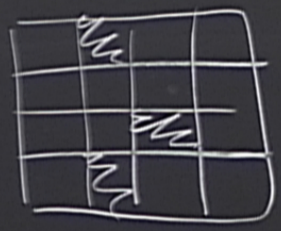

Prob. to stay on plateau $\equiv P = \int_{-\Phi}^{\Phi} \rho(q)\,dq = \Phi^{-1}\left[\int_{-\Phi}^{\Phi} \cos\left(\frac{\pi q}{2\Phi}\right) dq\right] \exp\left[\frac{-H^3}{32\Phi^2} + \right]$
$4\Phi/\pi$
$= \frac{4}{\pi} \exp\left[\frac{-H^3 +}{32\Phi^2}\right]$
tes


Prob. to stay on plateau ≡ P = ∫ ρ(α) dα = π⁻¹ [∫ cos(...) dα] exp[...]
4π/π
Fraction of Comoving Volume on plateau
= 4/π exp[...]
Physical Vol. = ρ e^{3Ht}
inflating

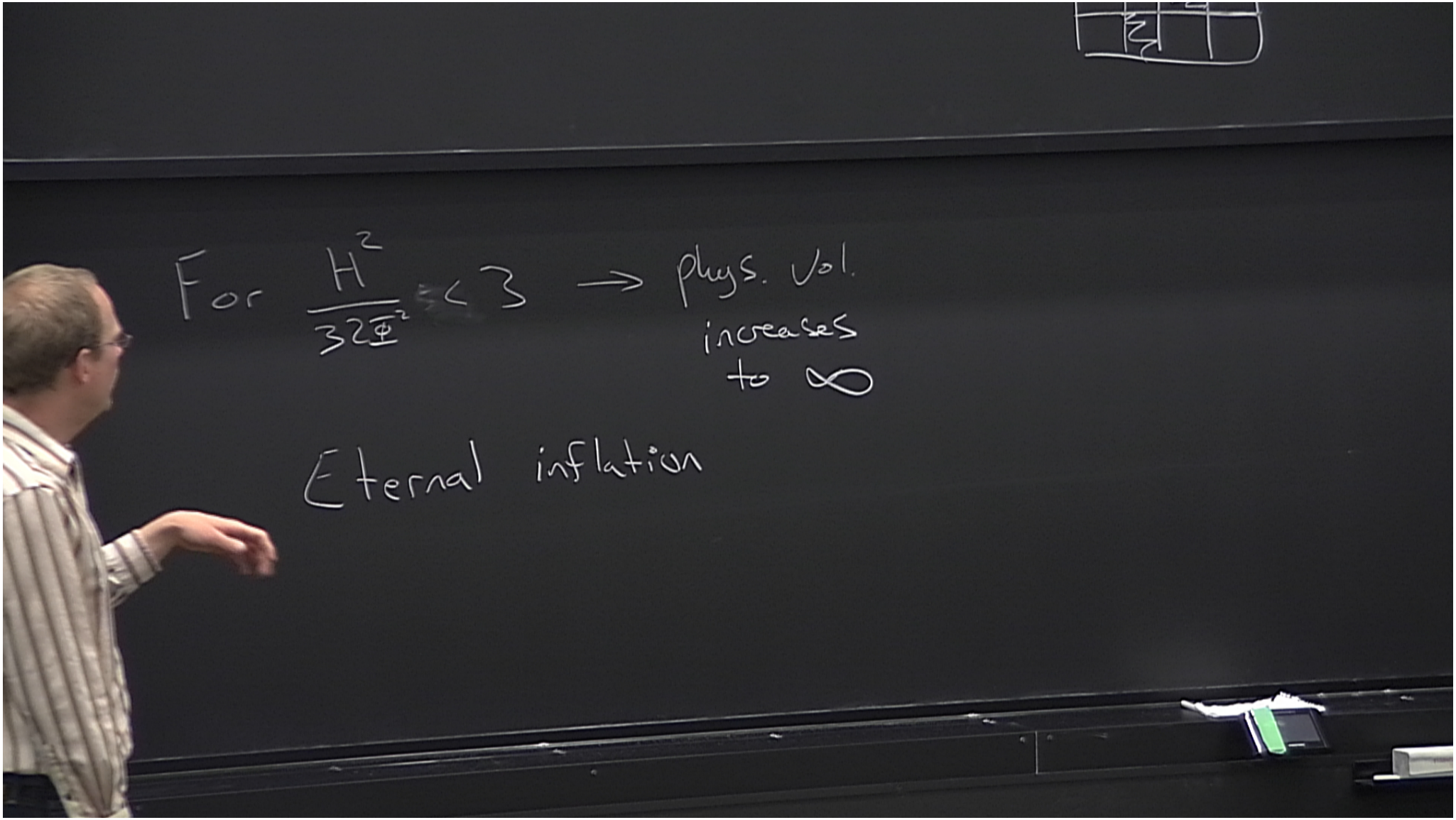
Prob. to stay on plateau $\equiv P = \int_{-\Phi}^{\Phi} \rho(\varphi)\, d\varphi = \Phi^{-1}\left[\left(\int_{-\Phi}^{\Phi} \cos\left(\frac{\pi\varphi}{2\Phi}\right) d\varphi\right]\right.$ $\exp\left[\frac{-H^3}{32\Phi^2}+\right]$

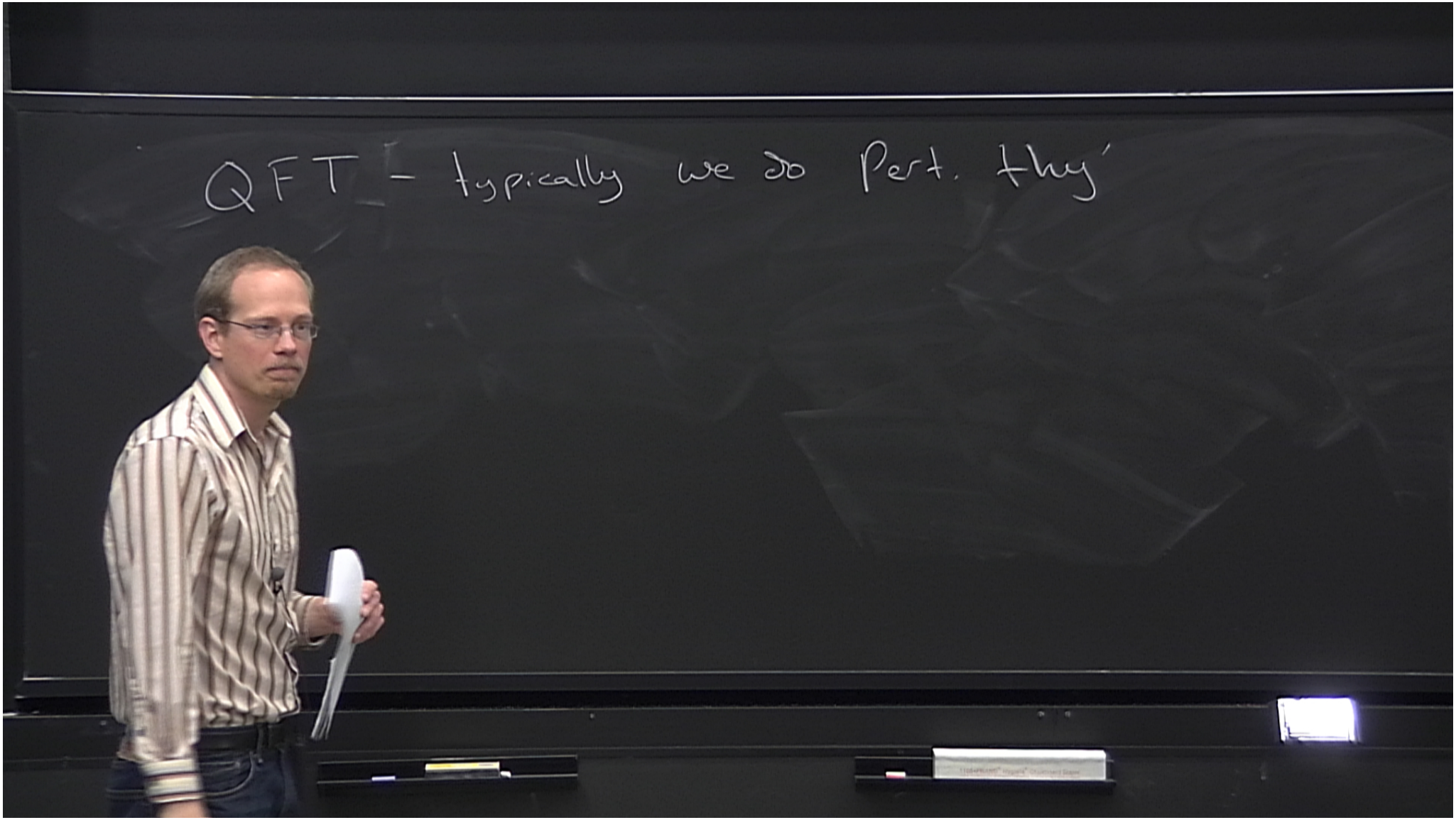
$\downarrow$ $4\Phi/\pi$

Fraction of Comoving Volume on plateau $= \frac{4}{\pi}\exp\left[\frac{-H^3+}{32\Phi^2}\right]$



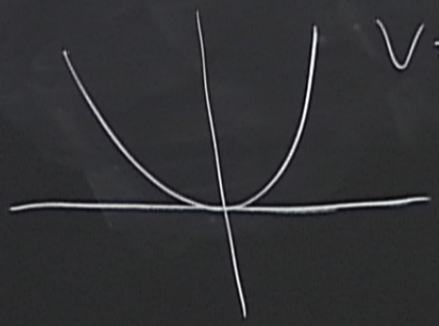
Physical Vol. inflating $= \rho e^{3Ht} = \frac{4}{\pi}\exp\left[\left(3-\frac{H^3}{32\Phi^2}\right)+\right]$


For $\frac{H^2}{32\pi^2} < 3 \rightarrow$ phys. vol. increases to ∞
Eternal inflation


QFT - typically we do Pert. thy'

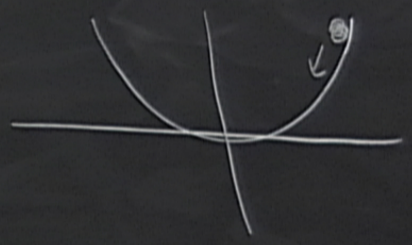
QFT - typically we do Pert. thy

e.g.

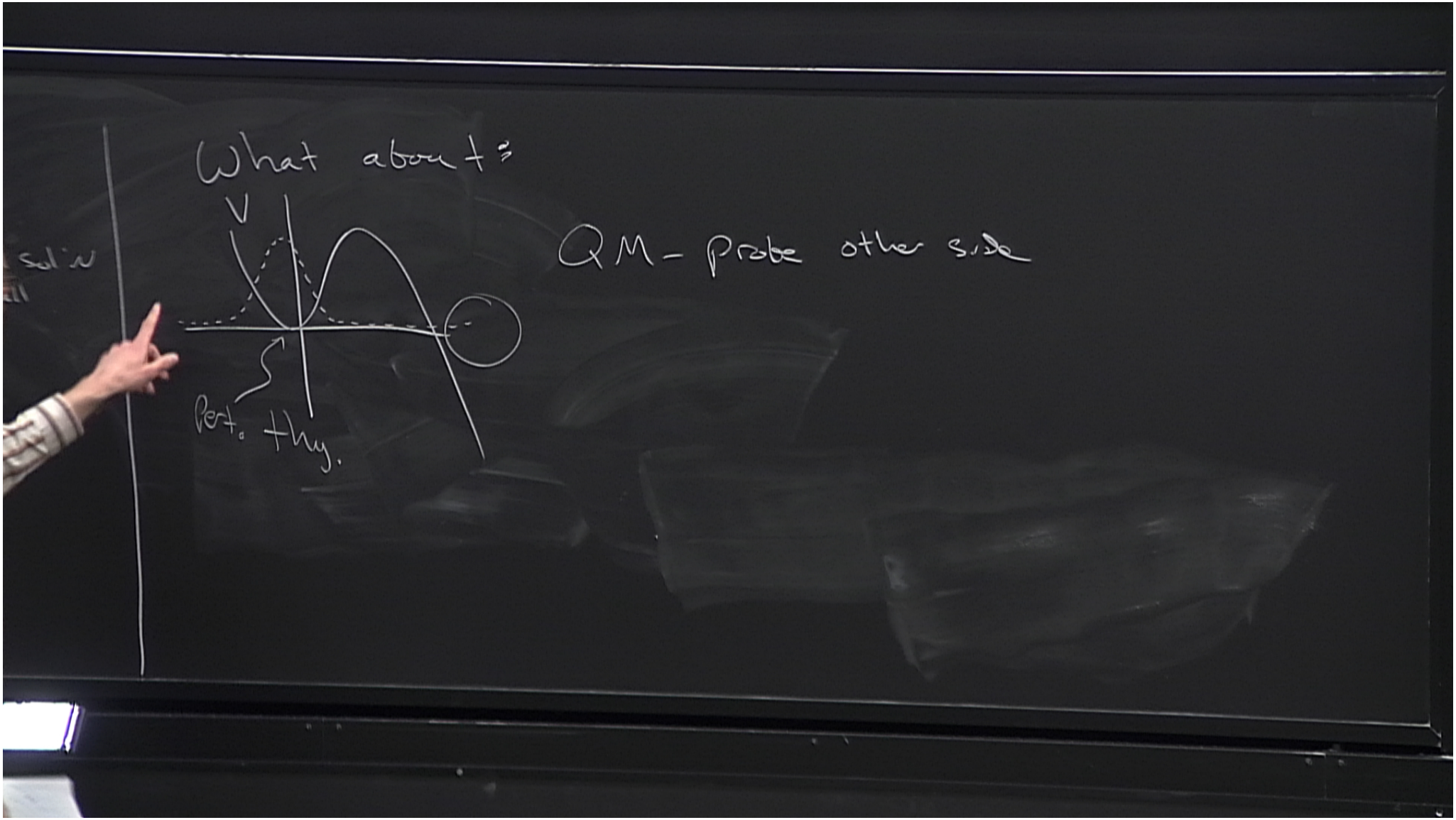


$V = \frac{1}{2} m^2 \varphi^2 + \text{Small } \varphi^4 \ldots$

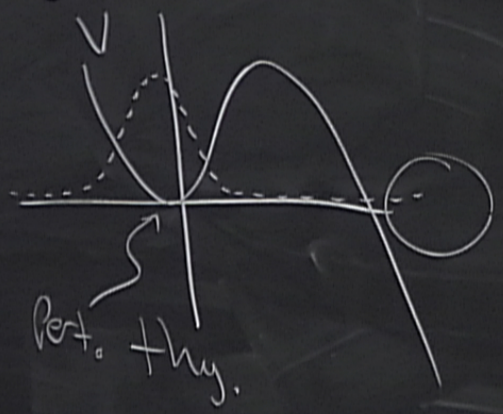
Inflation



Classical sol'n + Small


What about?
V
Pert. thy.
QM - probe other side

What about?

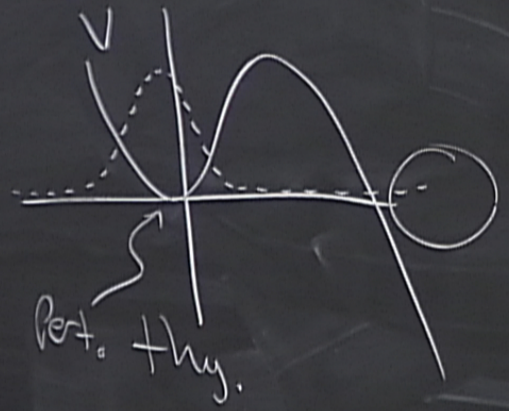


QM - probe other side

1-D QM - tunneling

QFT - Can't tunnel everywhere at once!

What about?



QM - probe other side

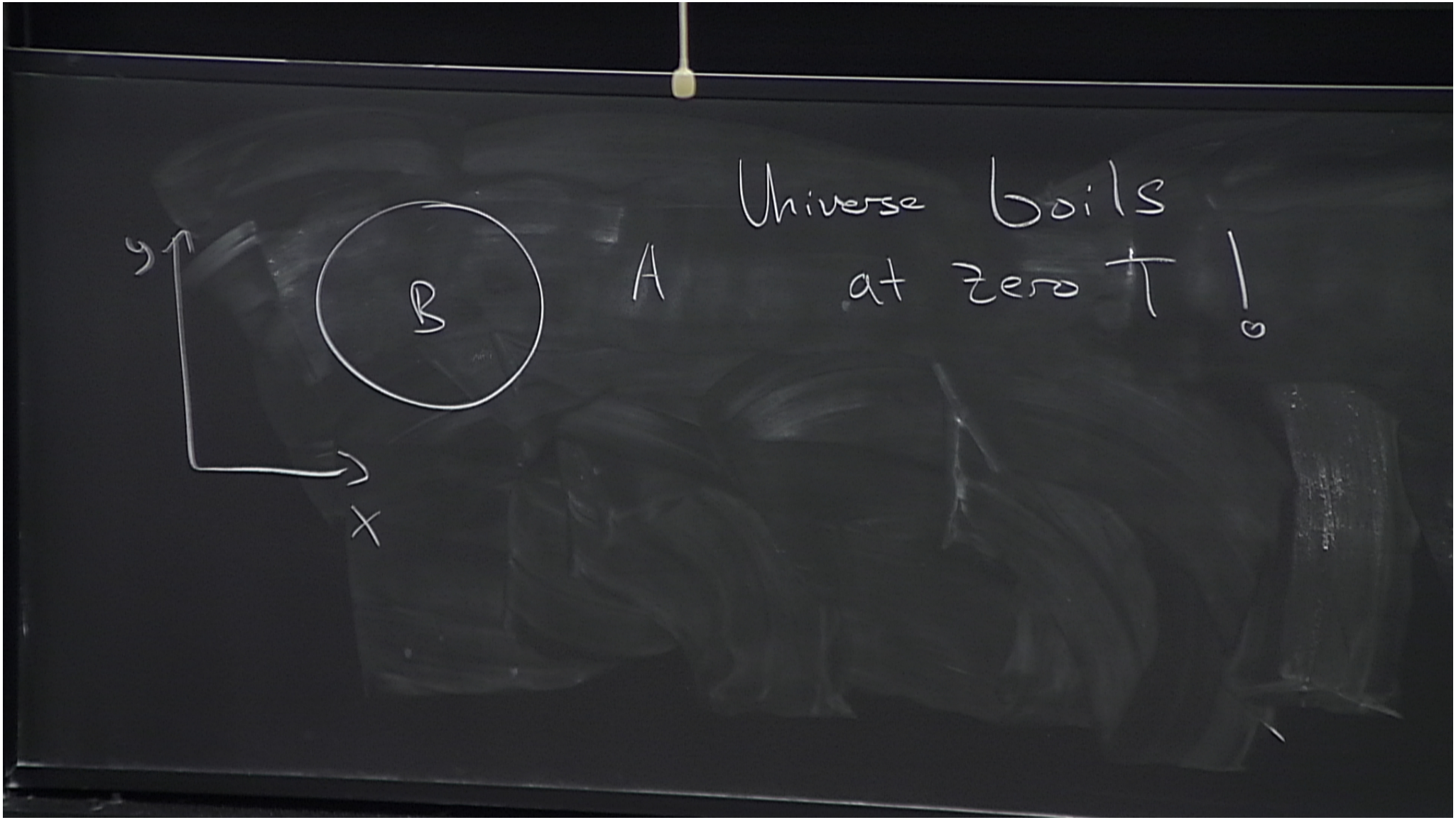
1-D QM - tunneling

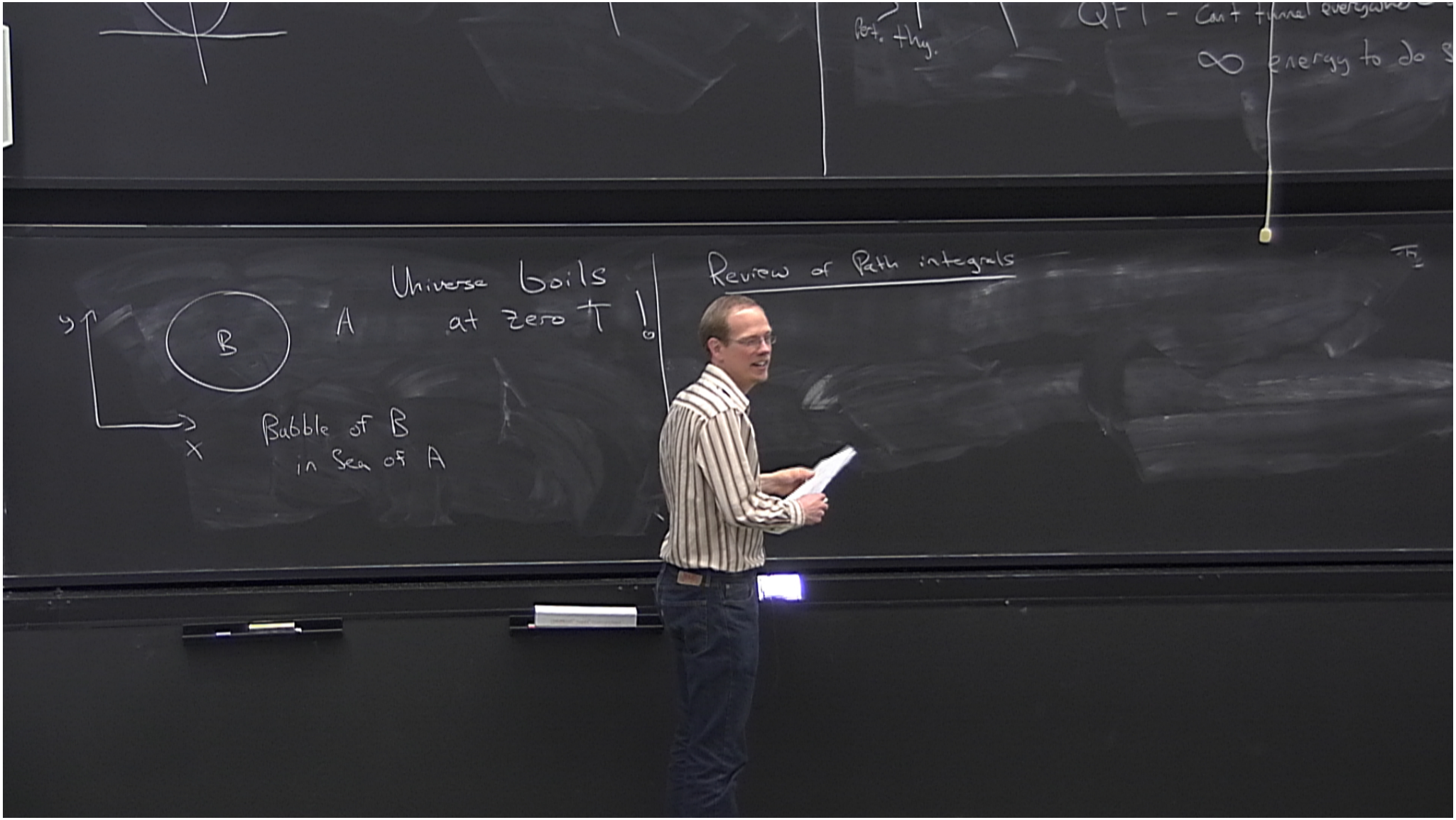
QFT - Can't tunnel everywhere at once!

∞ energy to do so!

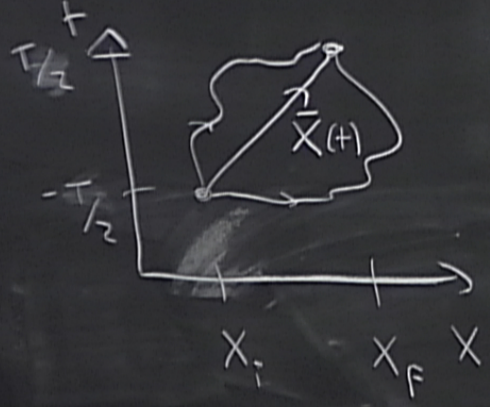
(Slow-roll)

Infl. ends when $|\varphi| > \frac{M_p}{5}$


Universe boils at zero T!
A
B
y
x

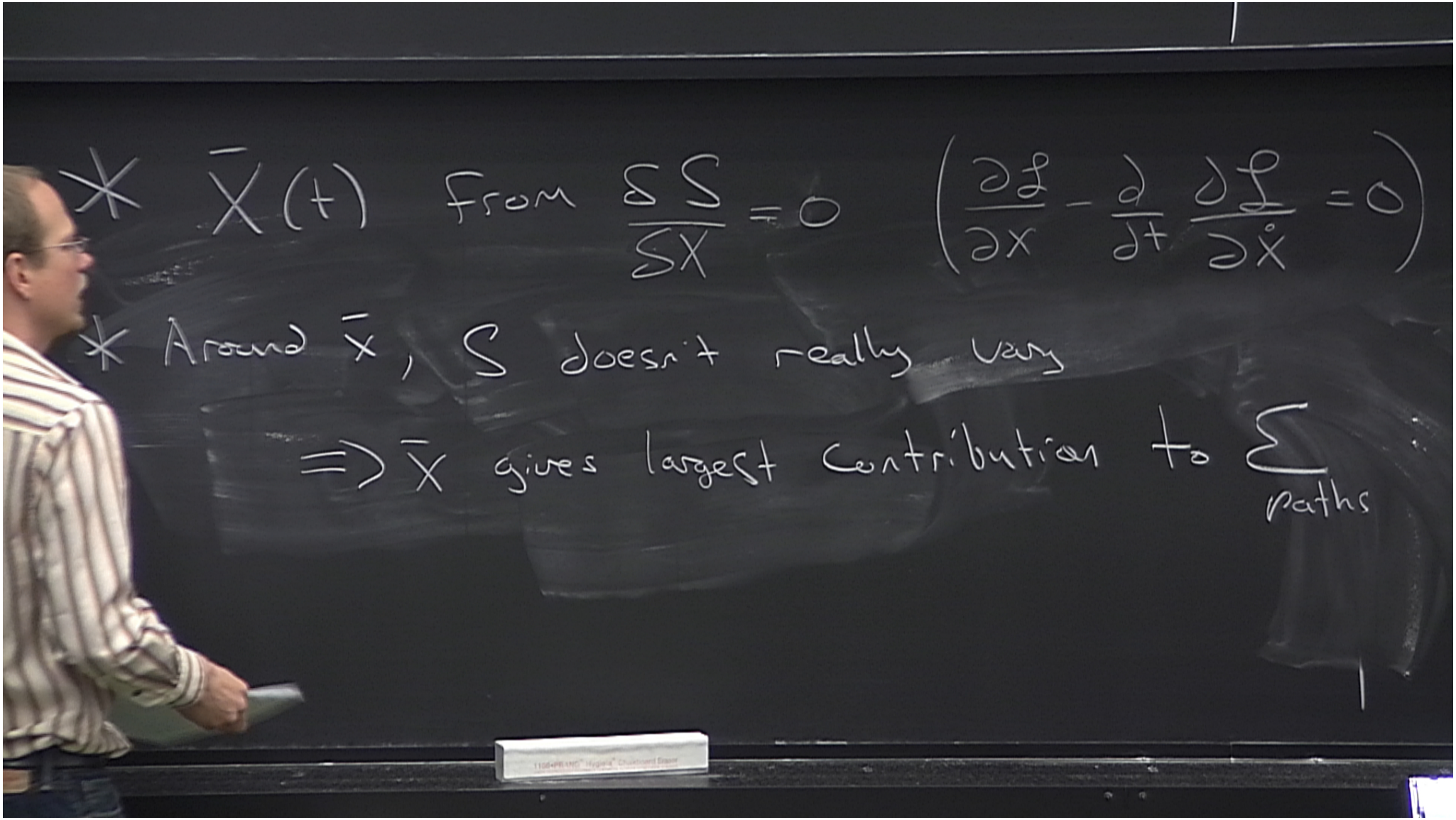

Pert. thy.
QFT - Can't finite energy
∞ energy to do
Universe boils at zero T !
A
B
y
x
Bubble of B in Sea of A
Review of Path integrals

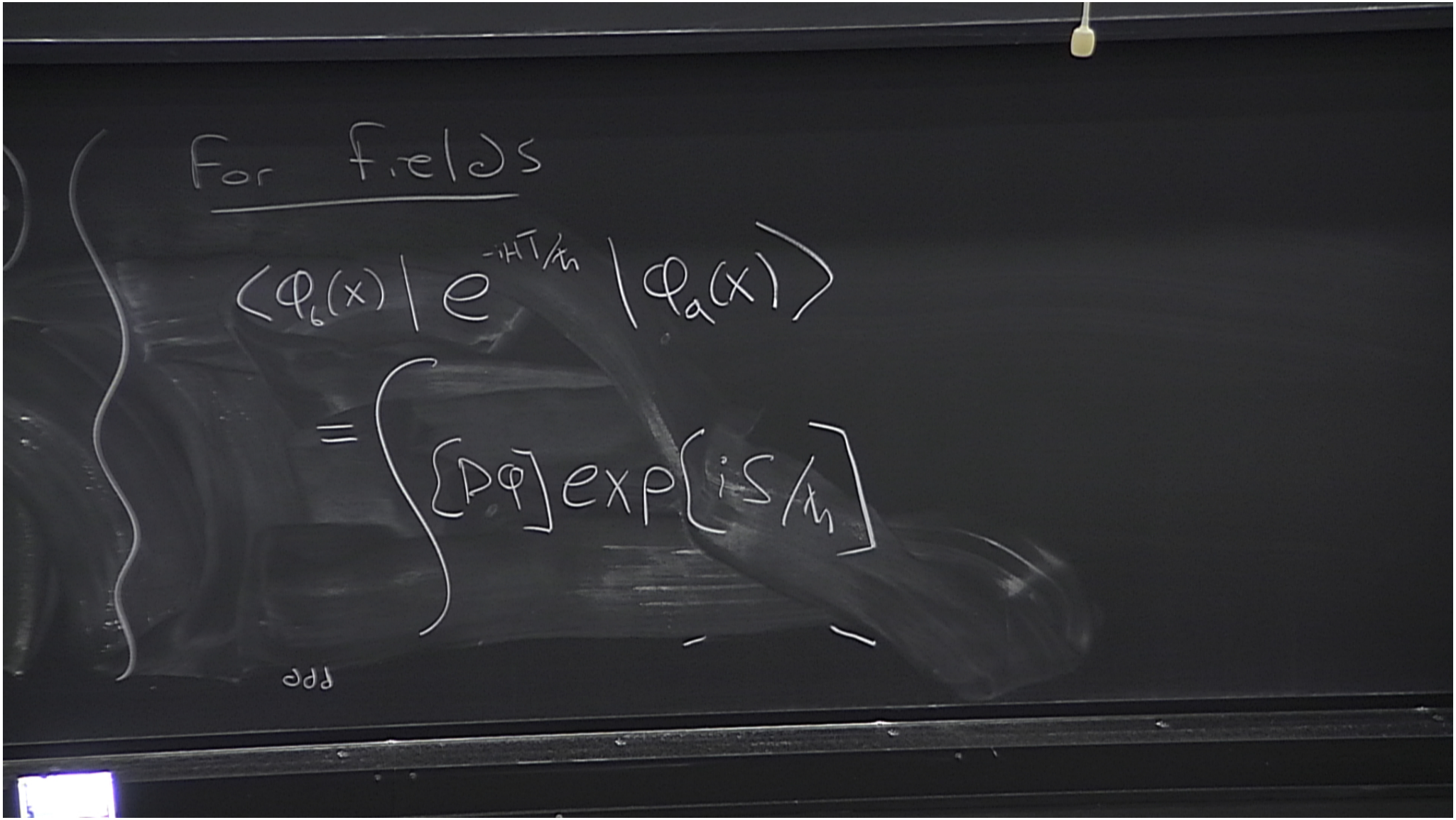
Review of Path integrals (1-D QM)

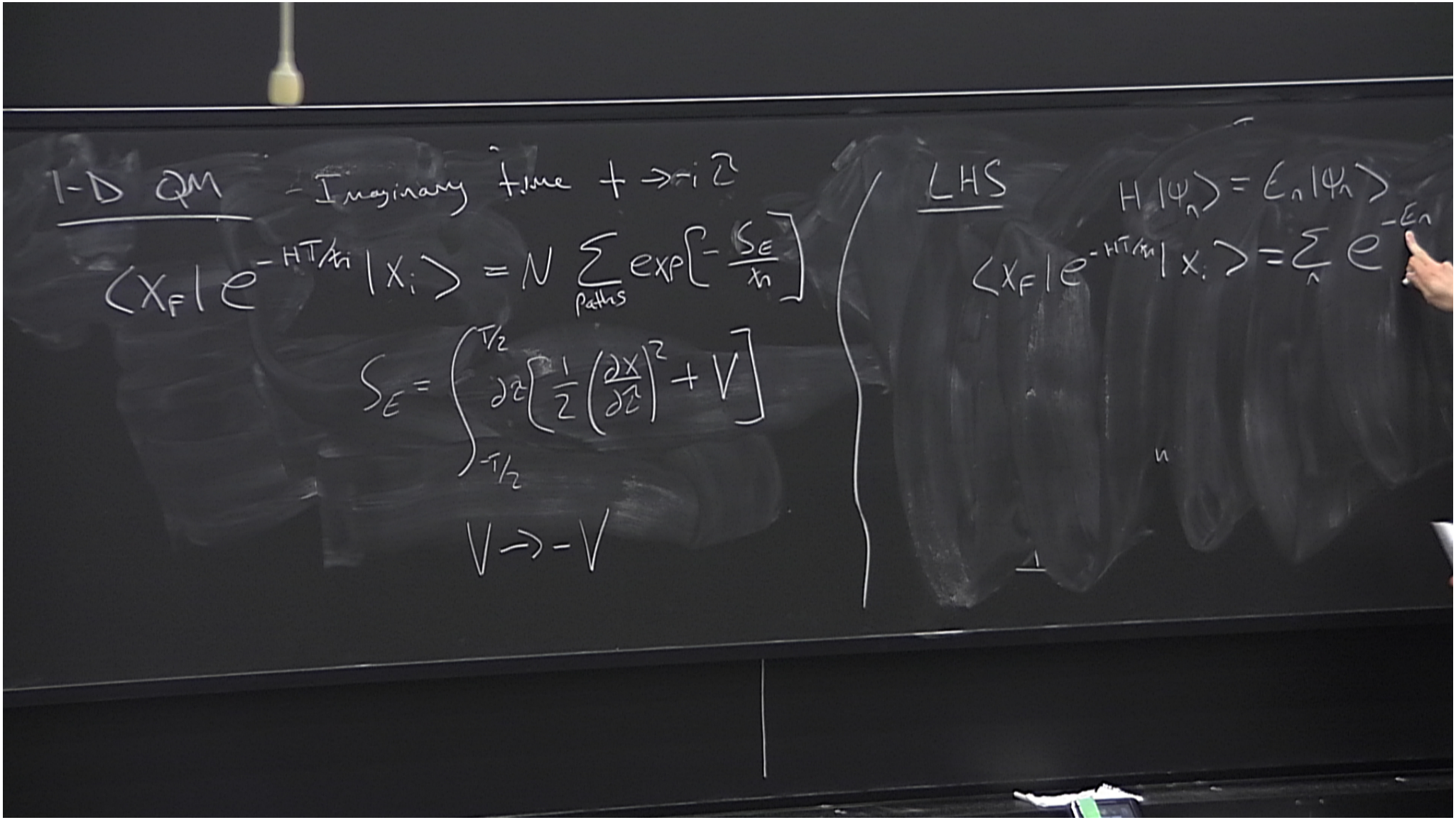


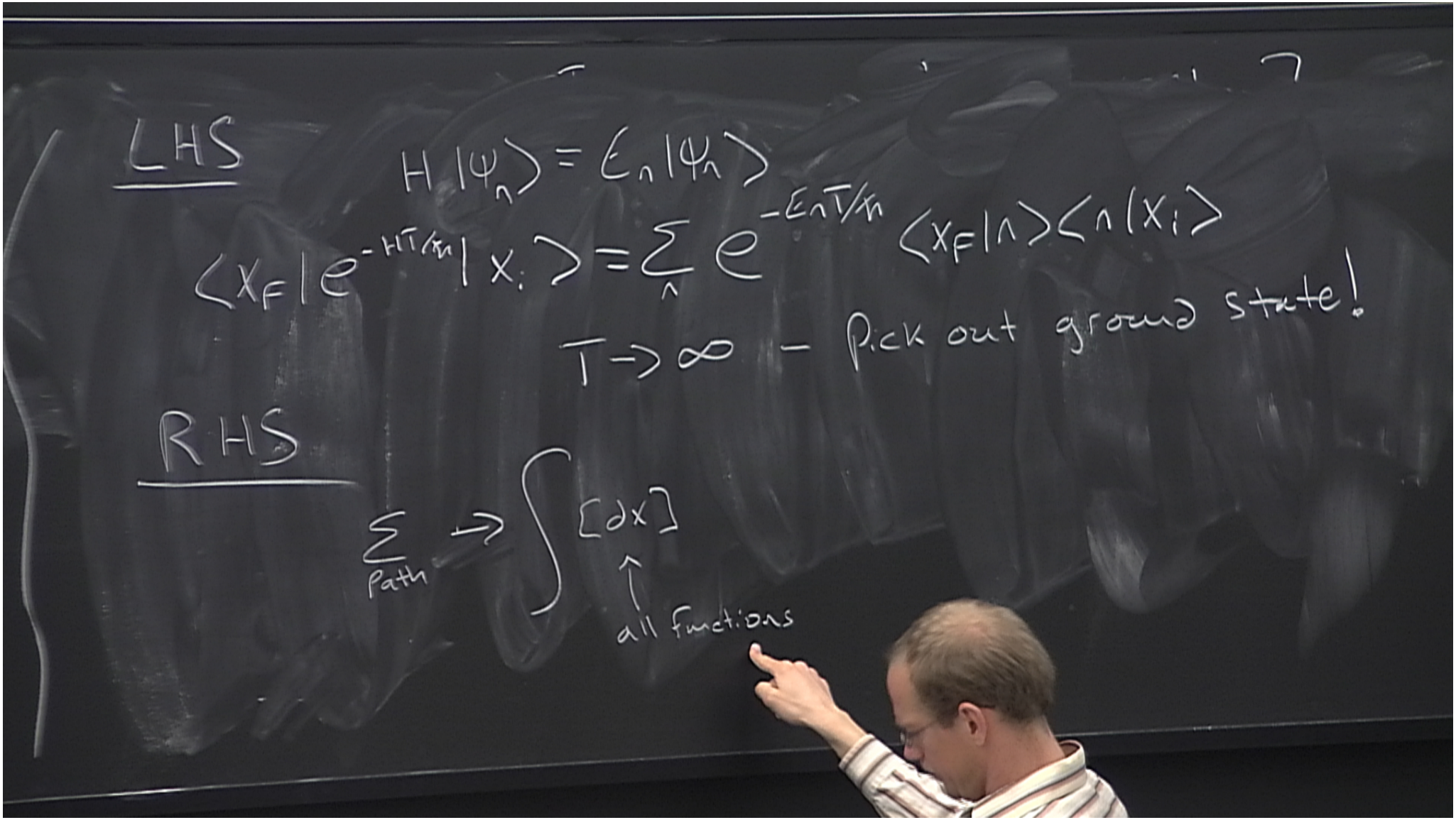
$$\langle x_f | e^{iHT/\hbar} | x_i \rangle = N \sum_{\text{Paths}} e^{iS[x(t)]/\hbar}$$

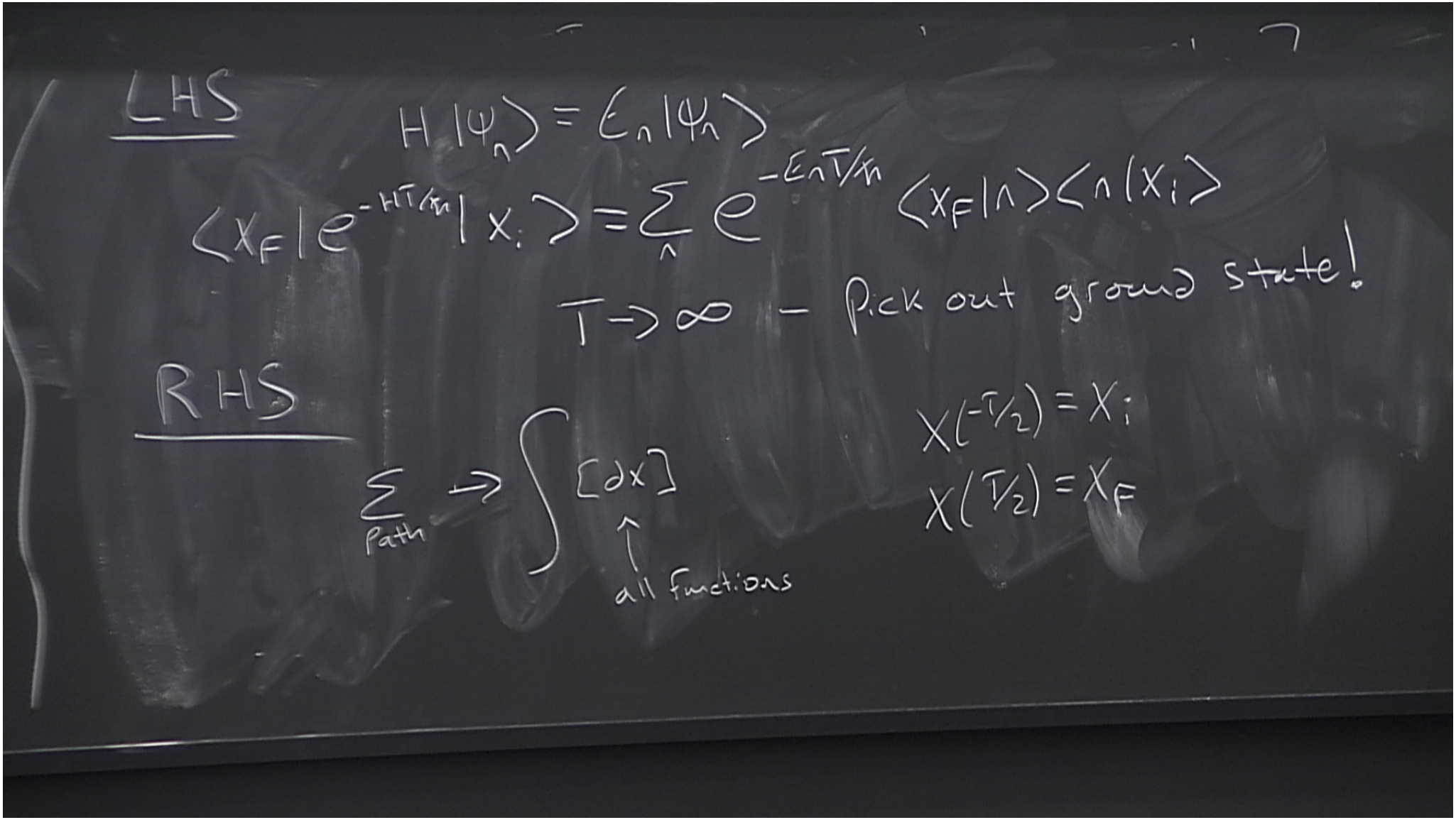
$$S = \int_{-T/2}^{T/2} \left(\frac{1}{2}\left(\frac{dx}{dt}\right)^2 - V \right) dt$$

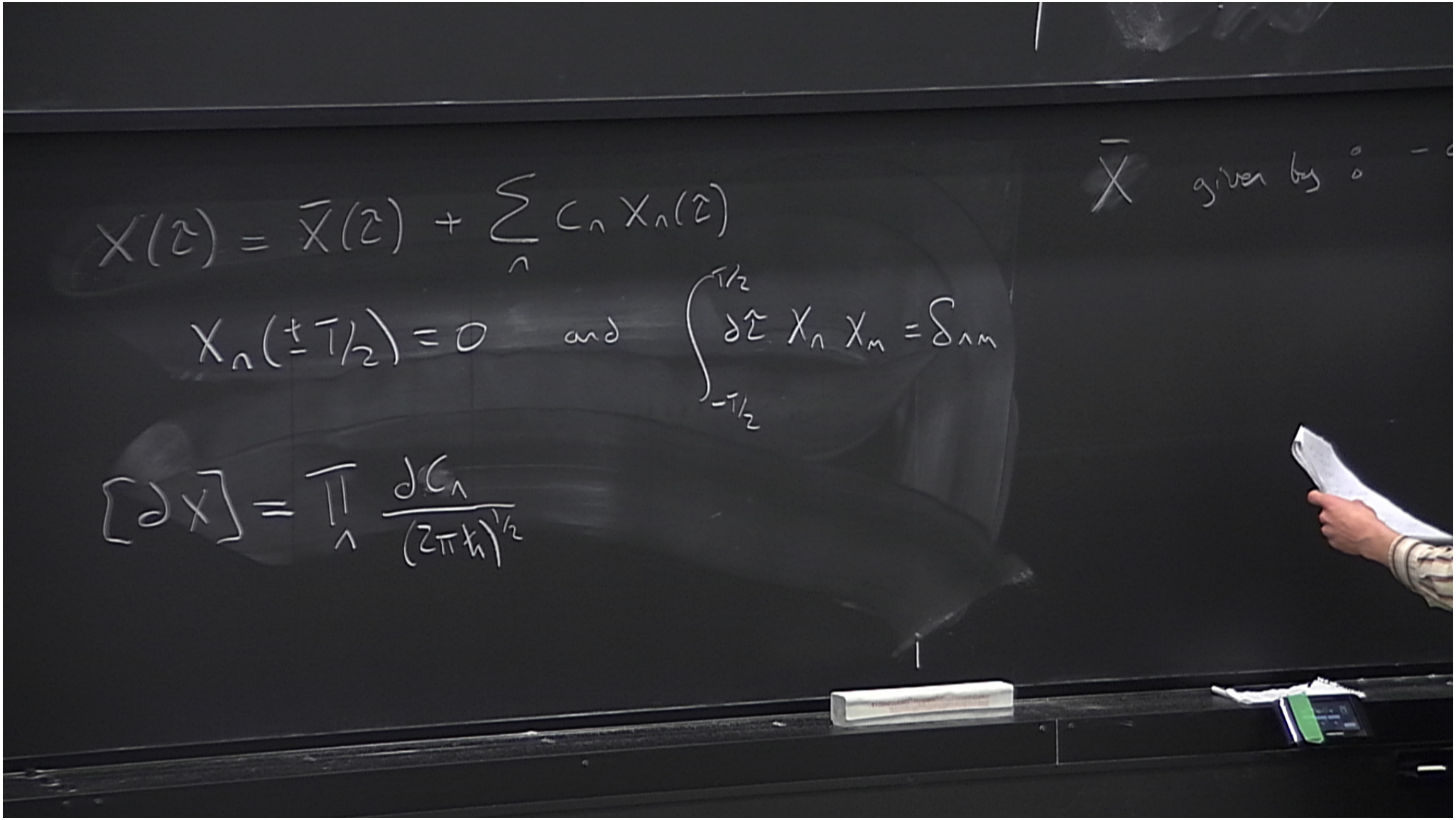

* $\bar{X}(t)$ from $\frac{\delta S}{\delta X} = 0$ $\left(\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = 0\right)$
* Around $\bar{x}$, S doesn't really vary
$\Rightarrow \bar{X}$ gives largest contribution to $\sum_{\text{paths}}$

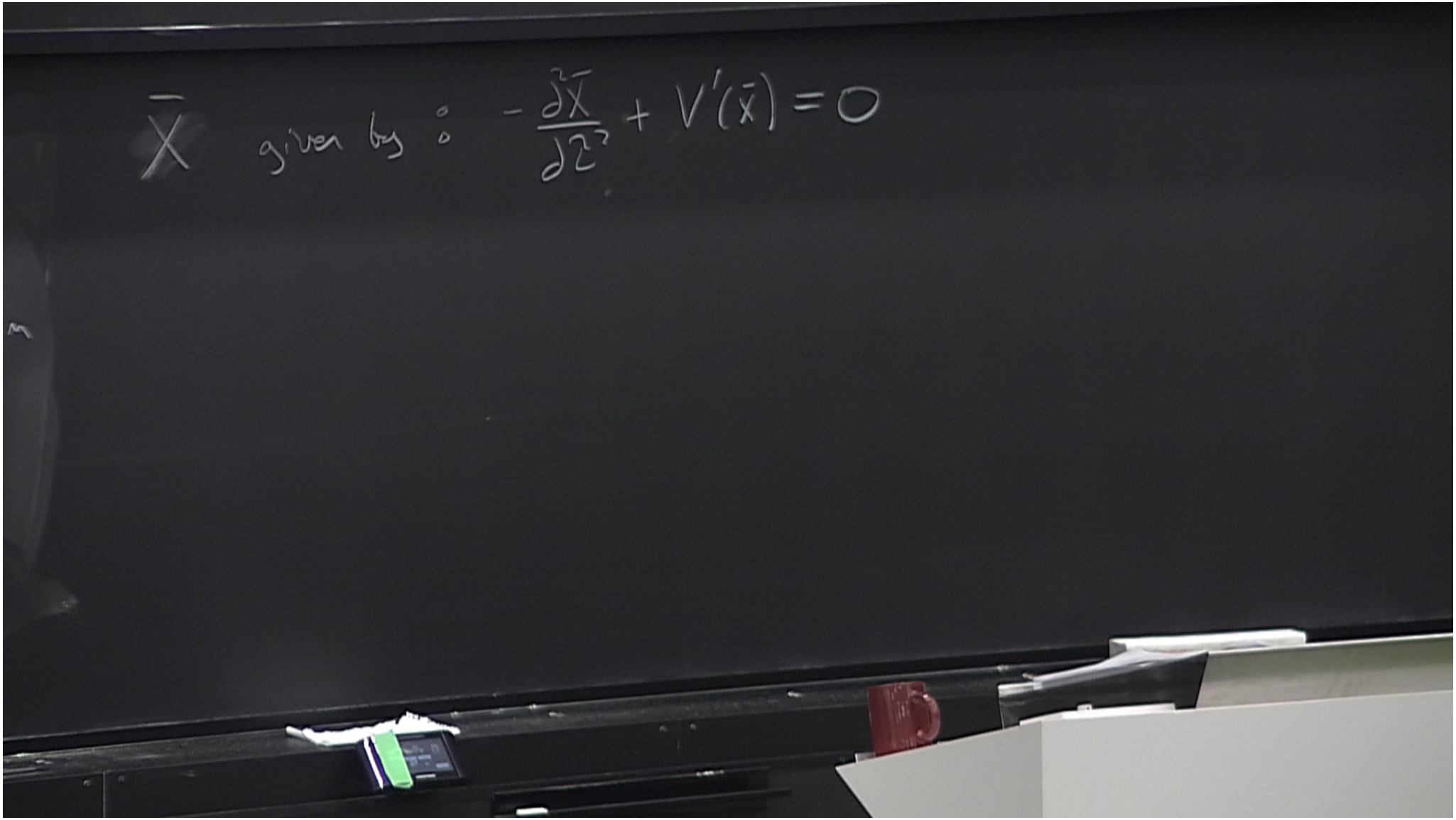

For fields
$\langle \varphi_b(x) | e^{-iHT/\hbar} | \varphi_a(x) \rangle$
$= \int [D\varphi] \exp[iS/\hbar]$

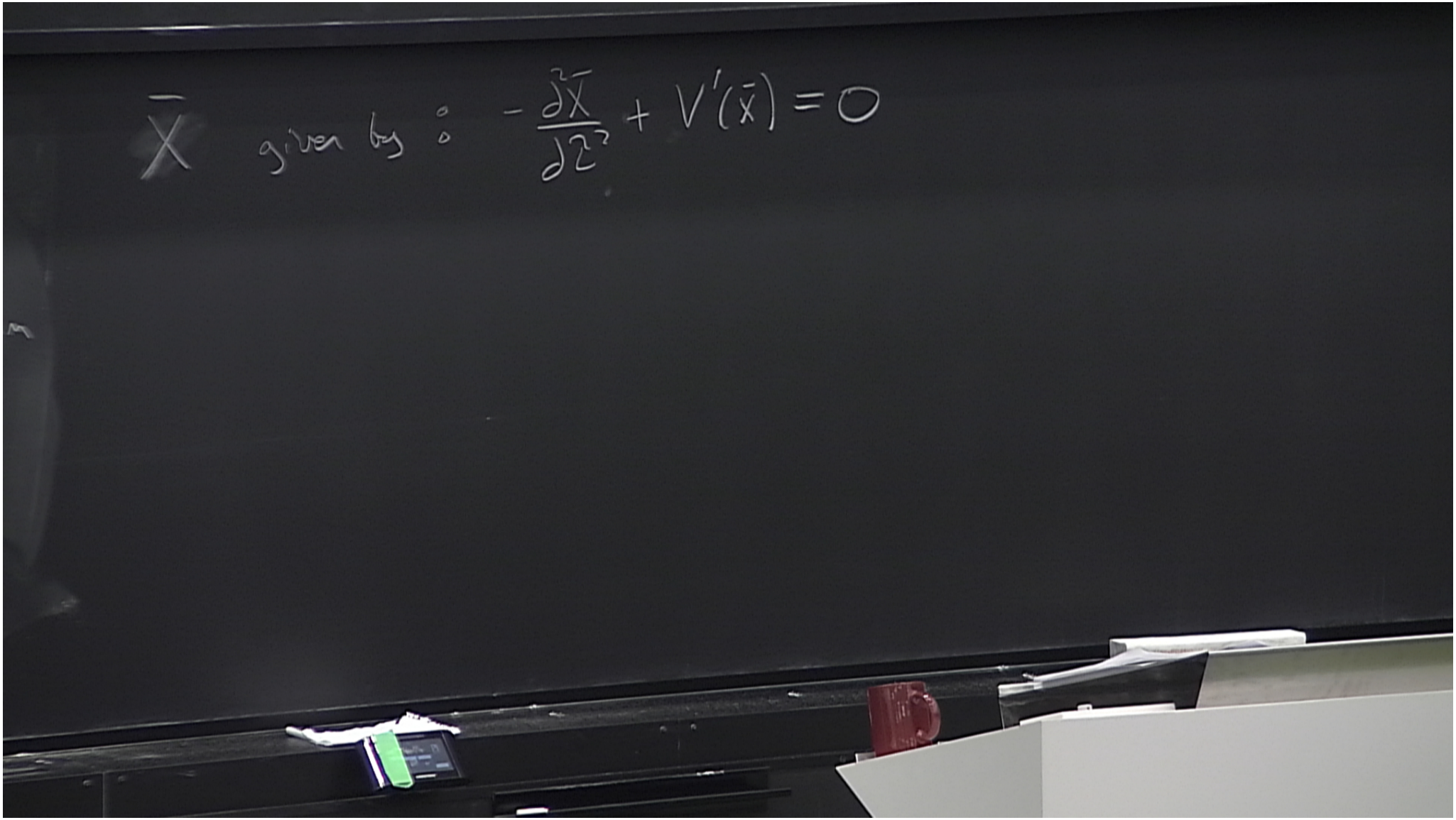

1-D QM - Imaginary time $t \to -i\tau$
$\langle x_F | e^{-HT/\hbar} | x_i \rangle = N \sum_{\text{paths}} \exp\left[-\frac{S_E}{\hbar}\right]$
$S_E = \int_{-T/2}^{T/2} d\tau \left[\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + V\right]$
$V \to -V$
LHS
$H|\psi_n\rangle = E_n|\psi_n\rangle$
$\langle x_F | e^{-HT/\hbar} | x_i \rangle = \sum_n e^{-E_n}$


LHS
$H|\psi_n\rangle = E_n|\psi_n\rangle$
$\langle x_F|e^{-HT/\hbar}|x_i\rangle = \sum_n e^{-E_nT/\hbar}\langle x_F|n\rangle\langle n|x_i\rangle$
$T\to\infty$ – pick out ground state!
RHS
$\sum_{\text{Path}} \to \int [dx]$
all functions


LHS
$H|\psi_n\rangle = E_n|\psi_n\rangle$
$\langle x_F|e^{-HT/\hbar}|x_i\rangle = \sum_n e^{-E_n T/\hbar}\langle x_F|n\rangle\langle n|x_i\rangle$
$T\to\infty$ – pick out ground state!
RHS
$\sum_{\text{Path}} \to \int [dx]$
all functions
$x(-T/2) = x_i$
$x(T/2) = x_F$


$X(\tau) = \bar{X}(\tau) + \sum_n c_n X_n(\tau)$
$\bar{X}$ given by :
$X_n(\pm T/2) = 0$ and $\int_{-T/2}^{T/2} d\tau\, X_n X_m = \delta_{nm}$
$[\mathcal{D}X] = \prod_n \frac{dc_n}{(2\pi\hbar)^{1/2}}$


$\bar{X}$ given by : $-\frac{d^2\bar{x}}{dt^2} + V'(\bar{x}) = 0$


$\bar{X}$ given by : $-\frac{d^2\bar{x}}{dt^2} + V'(\bar{x}) = 0$

$$\langle x_F | e^{-HT/\hbar} | x_i \rangle = N \int \prod_n \frac{dx_n}{(2\pi\hbar)^{1/2}} e^{-S_E/\hbar}$$

Taylor expand around $\bar{x}$

