Title: Explorations in Cosmology - Lecture 9

Date: Mar 22, 2012 10:15 AM

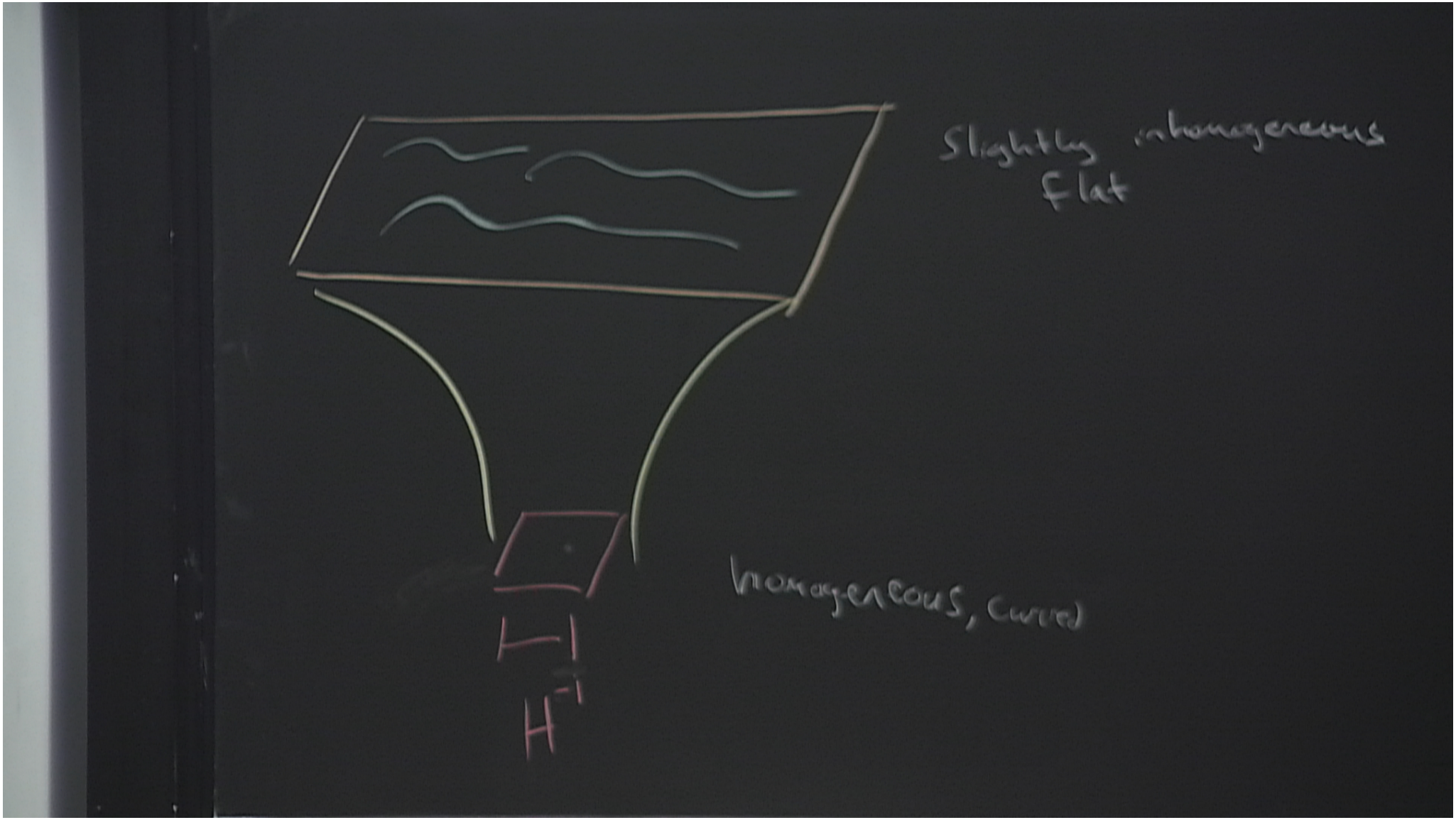
URL: http://pirsa.org/12030033

Abstract:



Preview

1. Slow-roll eternal inflation (today)
2. Vacuum decay in QFT (3)
3. False vacuum eternal inflation
4. Quantum Cosmology
5. Problems w/ inflation + alternative(s)


Slightly inhomogeneous
flat
Inhomogeneous, curved
H
H^-1

Source of Fluctuations

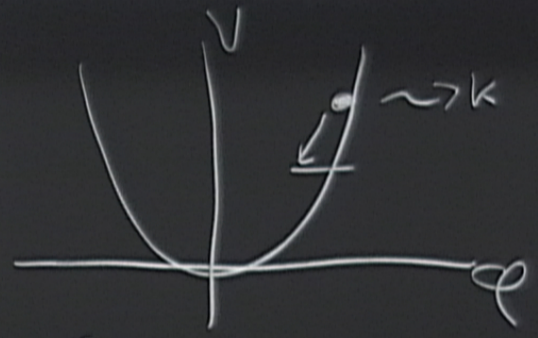
QFT in curved Spacetime

φ, χ → fluctuations

Source of accelerated expansion

$V(\varphi) \rightarrow \varepsilon_V = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1 \quad , \quad \eta_V = M_P^2 \frac{V''}{V} \ll$

$$N_e = \int H(\varphi(t))\, dt \gtrsim 60$$



$\sim k$

tensors

Perterbations

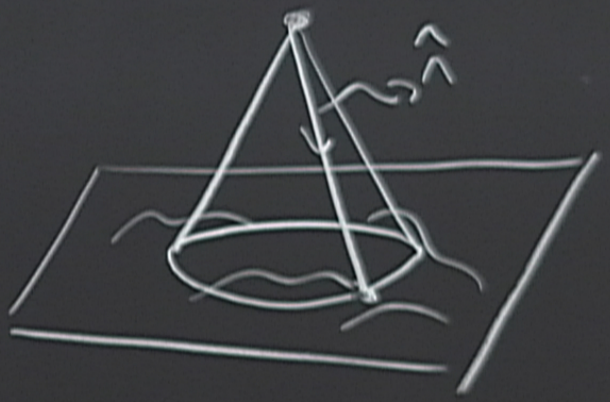
$$\text{Amplitude} - P^S = \frac{1}{12\pi^2 M_p^6} \frac{V^3}{V'^2}$$

φ is at some place on potential corresponds to some k

$$\text{tilt} - n_s - 1 = -6\varepsilon_V + 2\eta_V$$

tensors - $r = \frac{P_{tensors}}{P_{scalars}} = 16\,\epsilon_V$

CMB



Surface of last Scattering

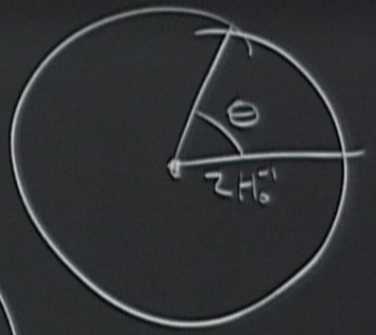
$$\frac{\delta T}{T}(\hat{n}) = -\frac{1}{5}\,\varphi(\hat{n})$$

large scales

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \qquad \theta \gtrsim 1^\circ \qquad \ell(\ell$$

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle$$

$$\ell(\ell+1) C_\ell \simeq \frac{2\pi}{25} P^3 \left(k = \frac{\ell H_0}{2}\right)$$

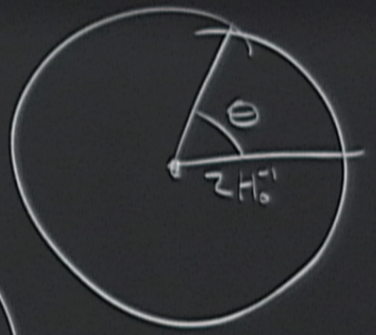


H_0 - present

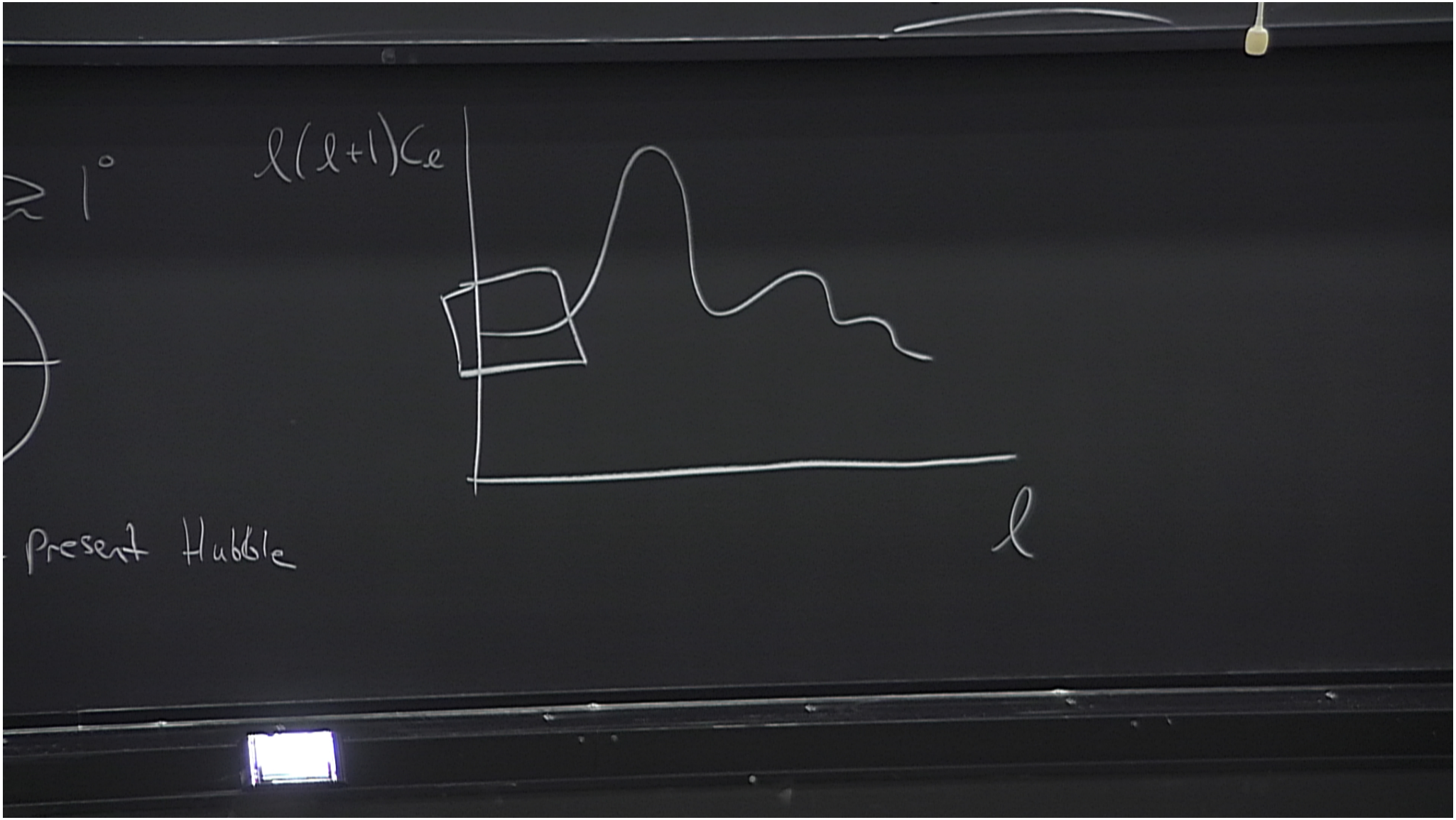
$$\frac{\delta T}{T}(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \qquad \theta \gtrsim 1^\circ \qquad l(l$$

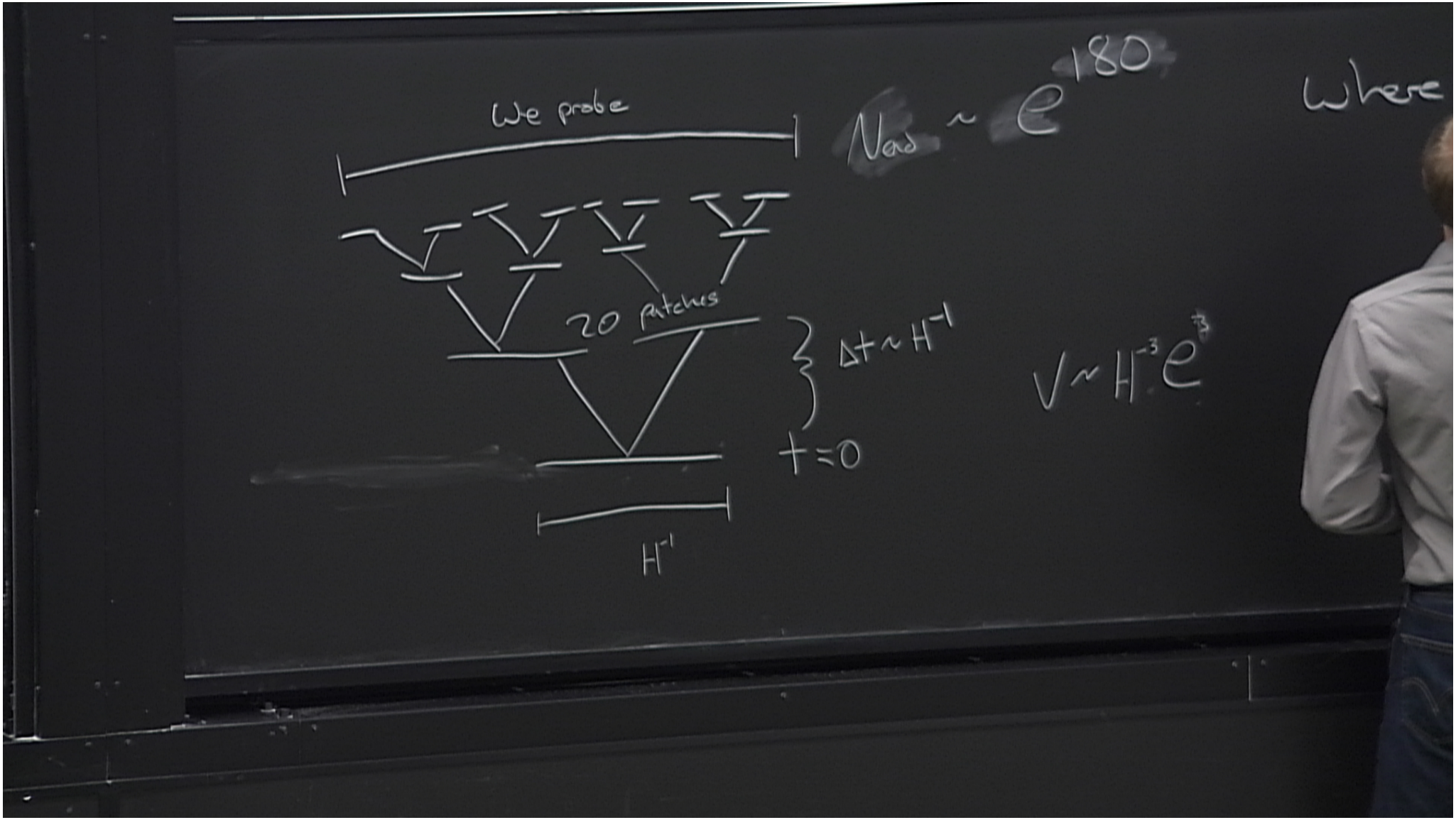
$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle$$

$$l(l+1) C_l \simeq \frac{2\pi}{25} P^3 \left(k = \frac{l H_0}{2}\right)$$



H_0 - present Hubble


≥ 1°
l(l+1)Cl
present Hubble
l

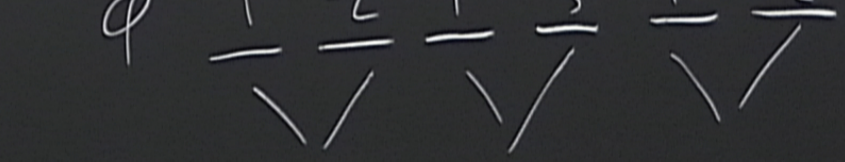

We probe
20 patches
$\Delta t \sim H^{-1}$
$t=0$
H^{-1}
$N_{obs} \sim e^{180}$
where
$V \sim H^{-3} e^{3}$

Interpretation

$\langle 0|\varphi^2(x)|0\rangle$ BD vacuum

$=$

$\langle \varphi^2(x) \rangle$



$\langle\ \rangle$ = expected statistics of φ on some spatial hypersurface

Some k

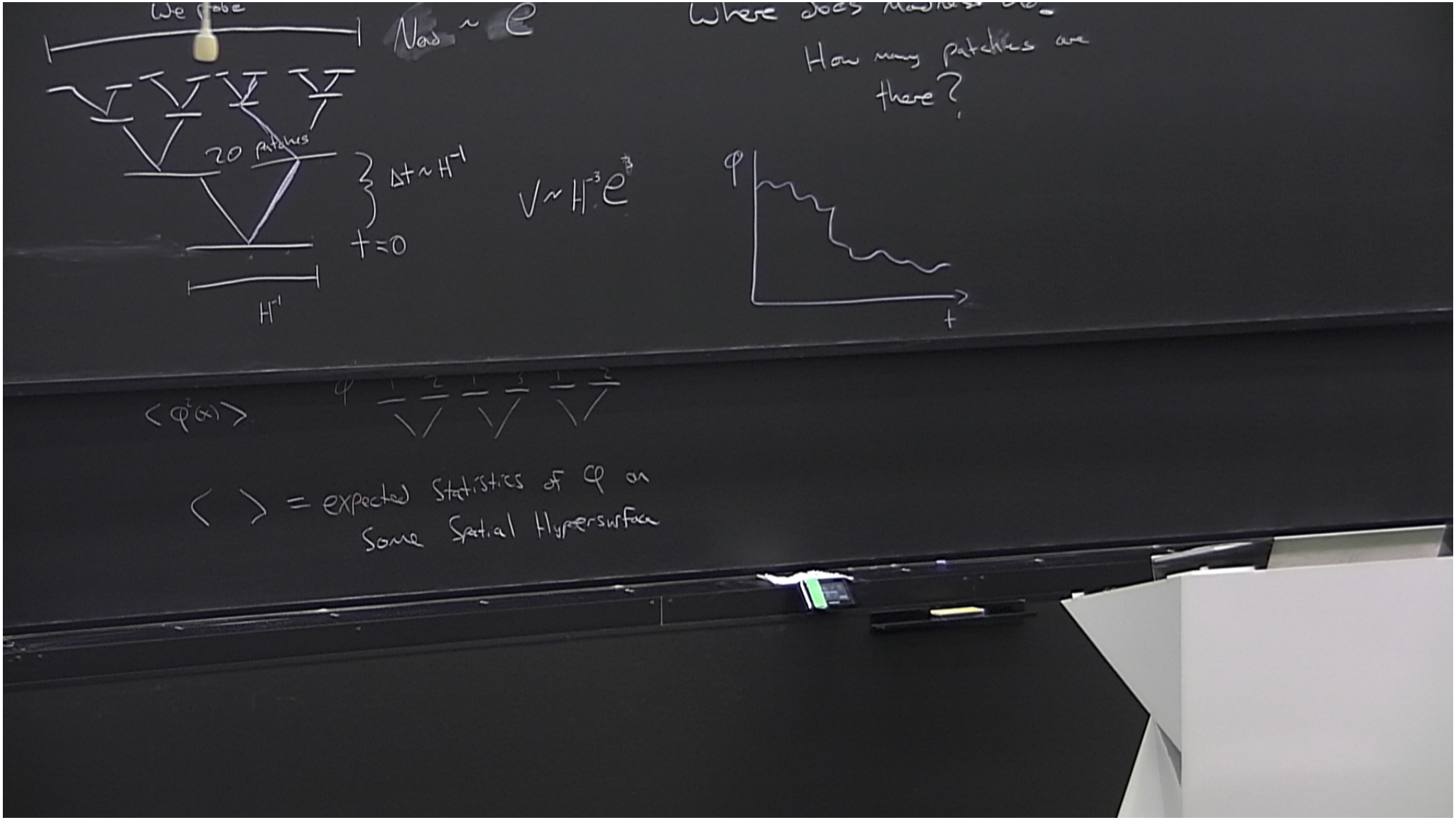
large Scales

Massless field in dS

$$\langle \varphi^2(x) \rangle = \int \frac{d^3k}{(2\pi)^3} |u_k|^2$$

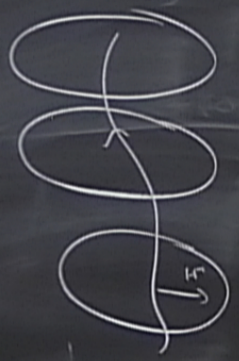
Contribution from $k \lesssim H$, $k \gtrsim \cancel{p_0} e^{-Ht}$ (with $p_0 \to H$)

$$= \int_{He^{-Ht}}^{H} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} = \frac{H^2}{2(2\pi)^3} (4\pi) \int_{He^{-Ht}}^{H} dk \frac{k^2}{k^3} = \frac{H^3 t}{4\pi^2}$$


We probe
$N_{obs} \sim e^{}$
Where does
How many patches are there?
20 Patches
$\Delta t \sim H^{-1}$
$t=0$
H^{-1}
$V \sim H^{-3} e^{3}$
φ
t
$\langle \varphi^2(x) \rangle$
φ
$\langle\ \rangle$ = expected Statistics of φ on
Some Spatial Hypersurface

Many branches = Many histories

$$\varphi(t) = \int \frac{N(t)}{3H}\, dt$$

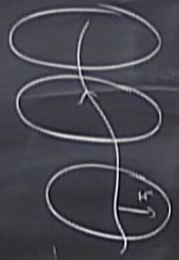


Langevin equation

$$\ddot{\phi} + 3H\dot{\phi} = -V' + N(t)$$

$$3H\dot{\varphi} = N(t) \sim \text{random Stochastic}$$

Many branches = Many histories



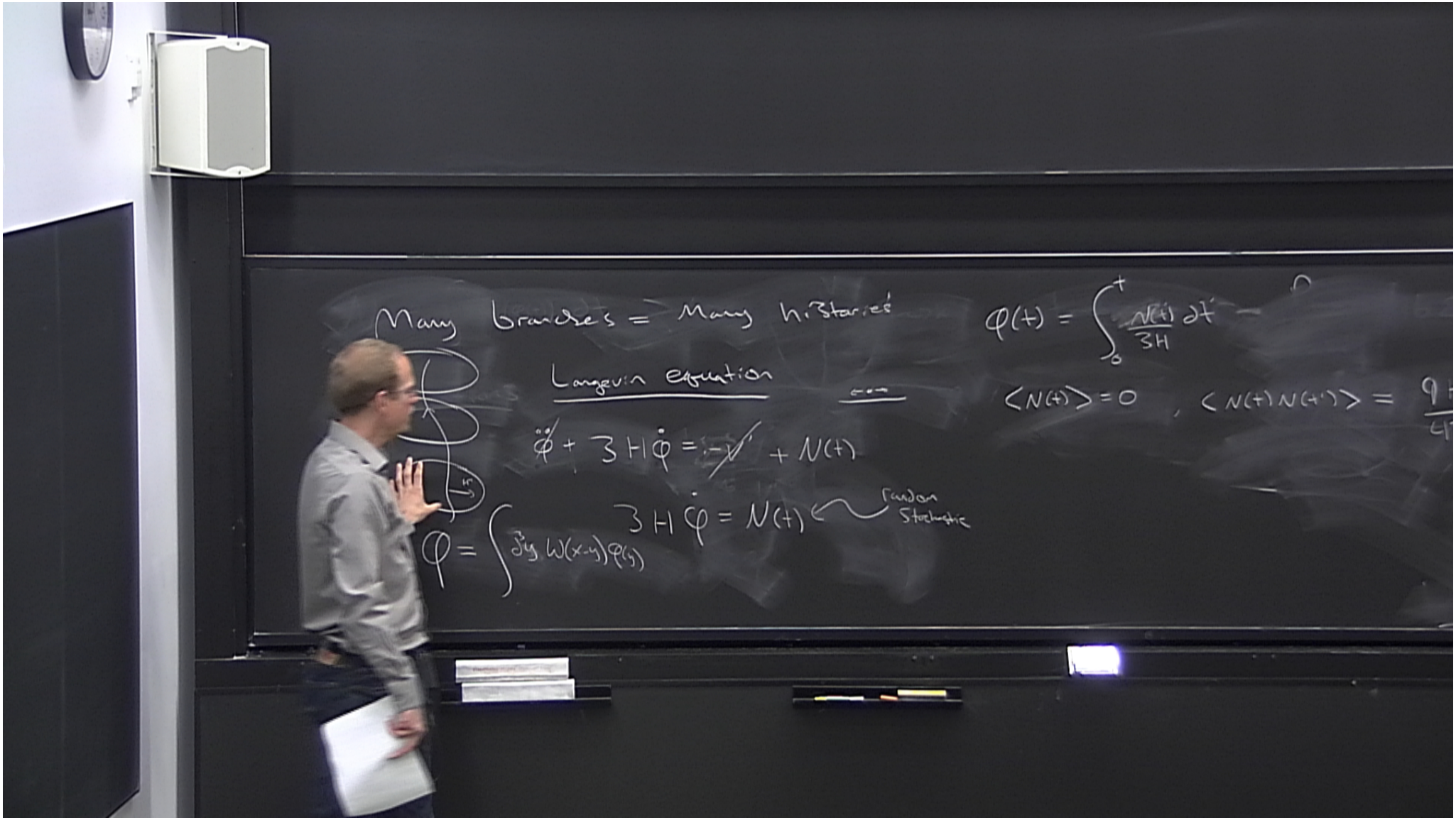
Langevin equation

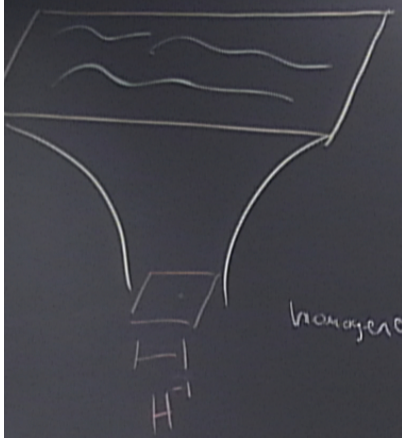
$$\ddot{\varphi} + 3H\dot{\varphi} = -V' + N(t)$$

$$3H\dot{\varphi} = N(t) \sim \text{random Stochastic}$$

$$\varphi(t) = \int_0^t \frac{N(t')}{3H}\, dt'$$

$$\langle N(t) \rangle = 0 \quad , \quad \langle N(t) N(t') \rangle = \frac{9H^5}{4\pi^2}\, \delta(t-t') \quad \text{(Markovian}$$


Many branches = Many histories
Langevin equation
$\varphi(t) = \int_0^t \frac{N(t)}{3H} dt$
$\langle N(t) \rangle = 0$
$\ddot{\varphi} + 3H\dot{\varphi} = -V' + N(t)$
$3H\dot{\varphi} = N(t)$
Random Stochastic
$\varphi = \int \frac{d^3y}{} W(x-y)\varphi(y)$



Massless field in dS

$$\langle \varphi^2(x) \rangle = \int \frac{d^3k}{(2\pi)^3} |u_k|^2$$

Contribution from $k \lesssim H$, $k \gtrsim \overrightarrow{H} e^{-Ht}$

$$= \int_{He^{-Ht}}^{H} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} = \frac{H^2}{2(2\pi)^3}(4\pi) \int_{He^{-Ht}}^{H} dk \frac{k^2}{k^3} = \frac{H^3 t}{4\pi^2}$$

Many branches = Many histories

$$\varphi(t) = \int_0^t \frac{N(t)}{3H} dt$$

Langevin equation

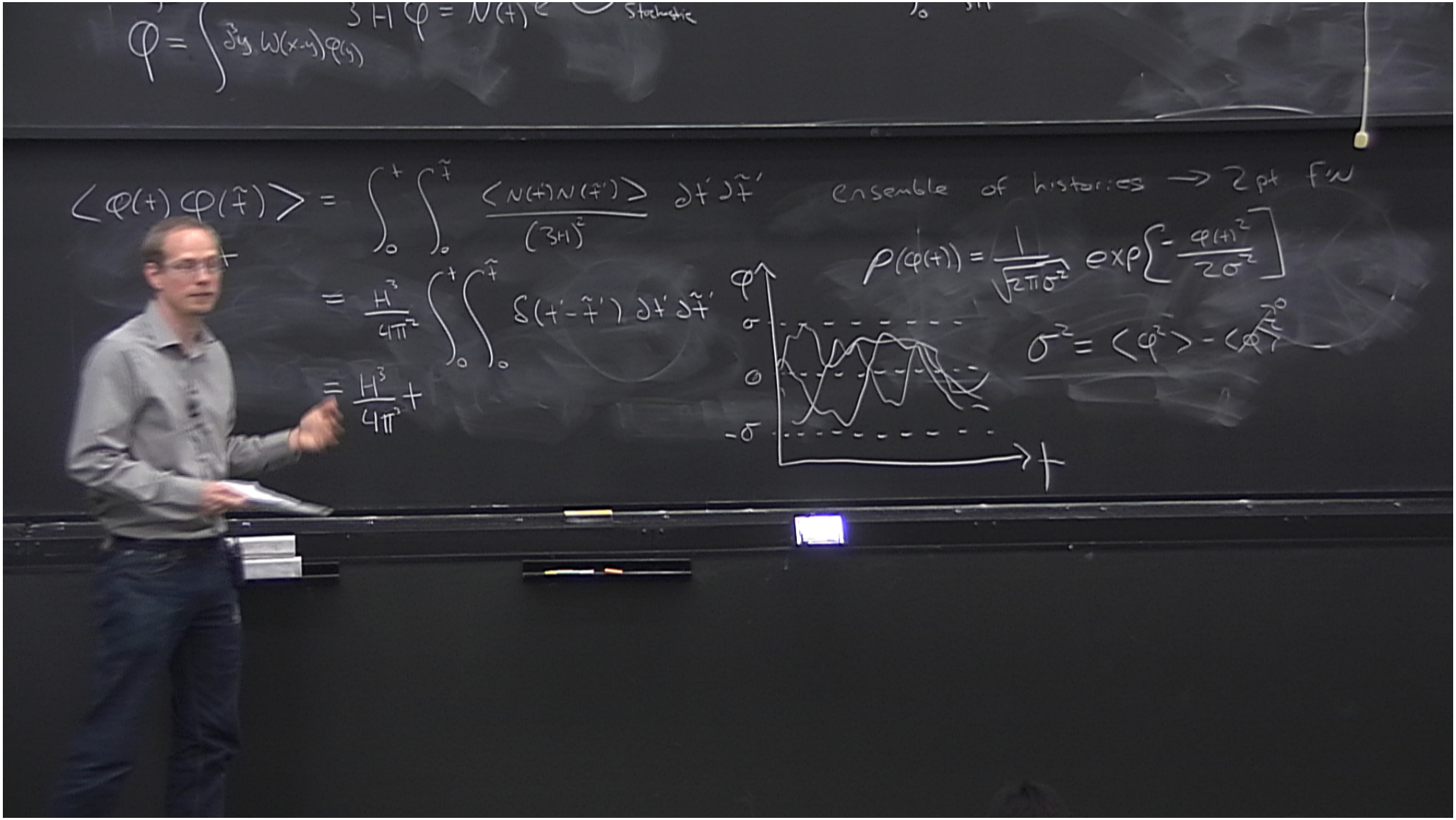
$$\ddot{\varphi} + 3H\dot{\varphi} = -V' + N(t)$$

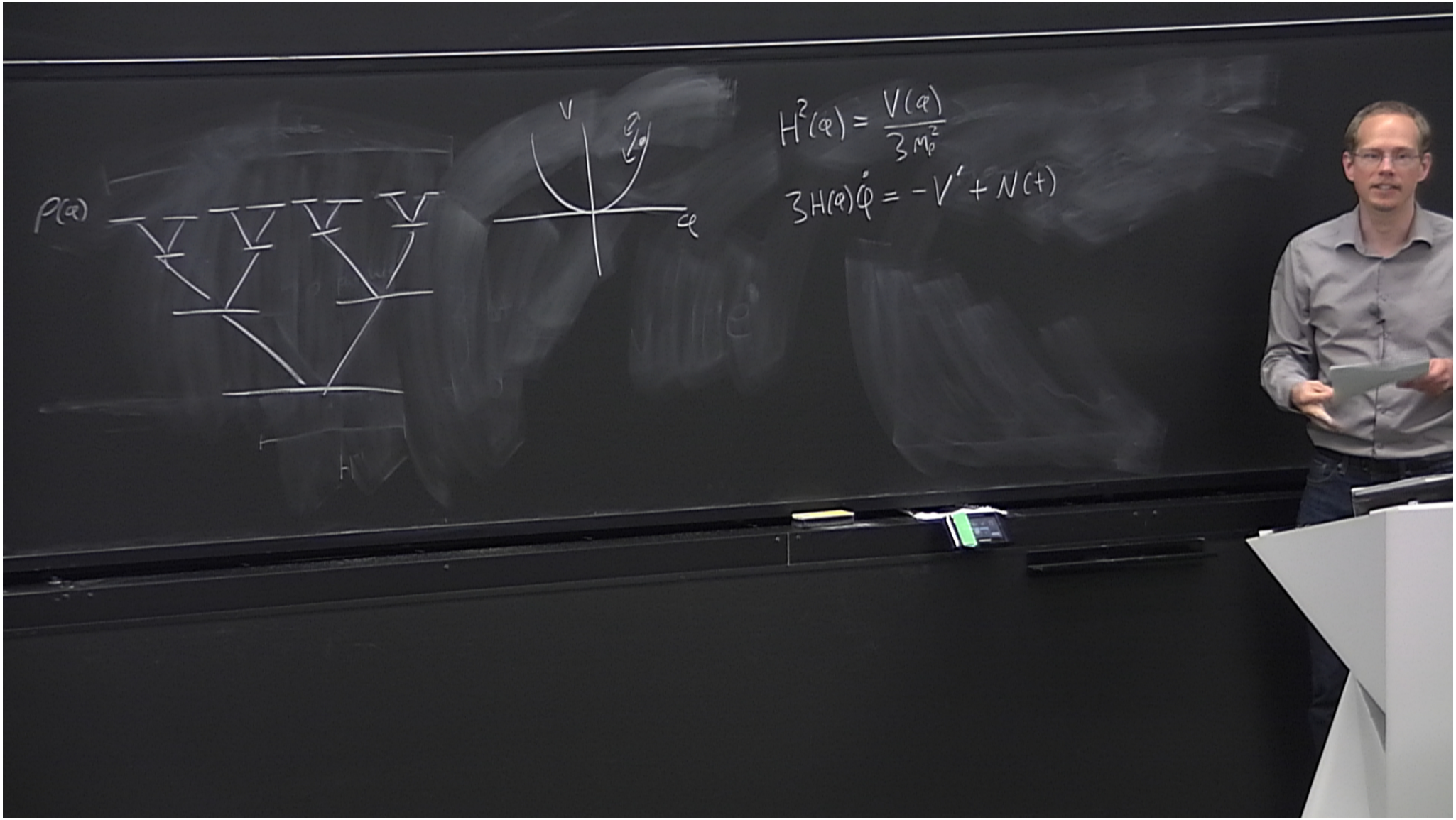
$$3H\dot{\varphi} = N(t) \sim \text{random Stochastic}$$

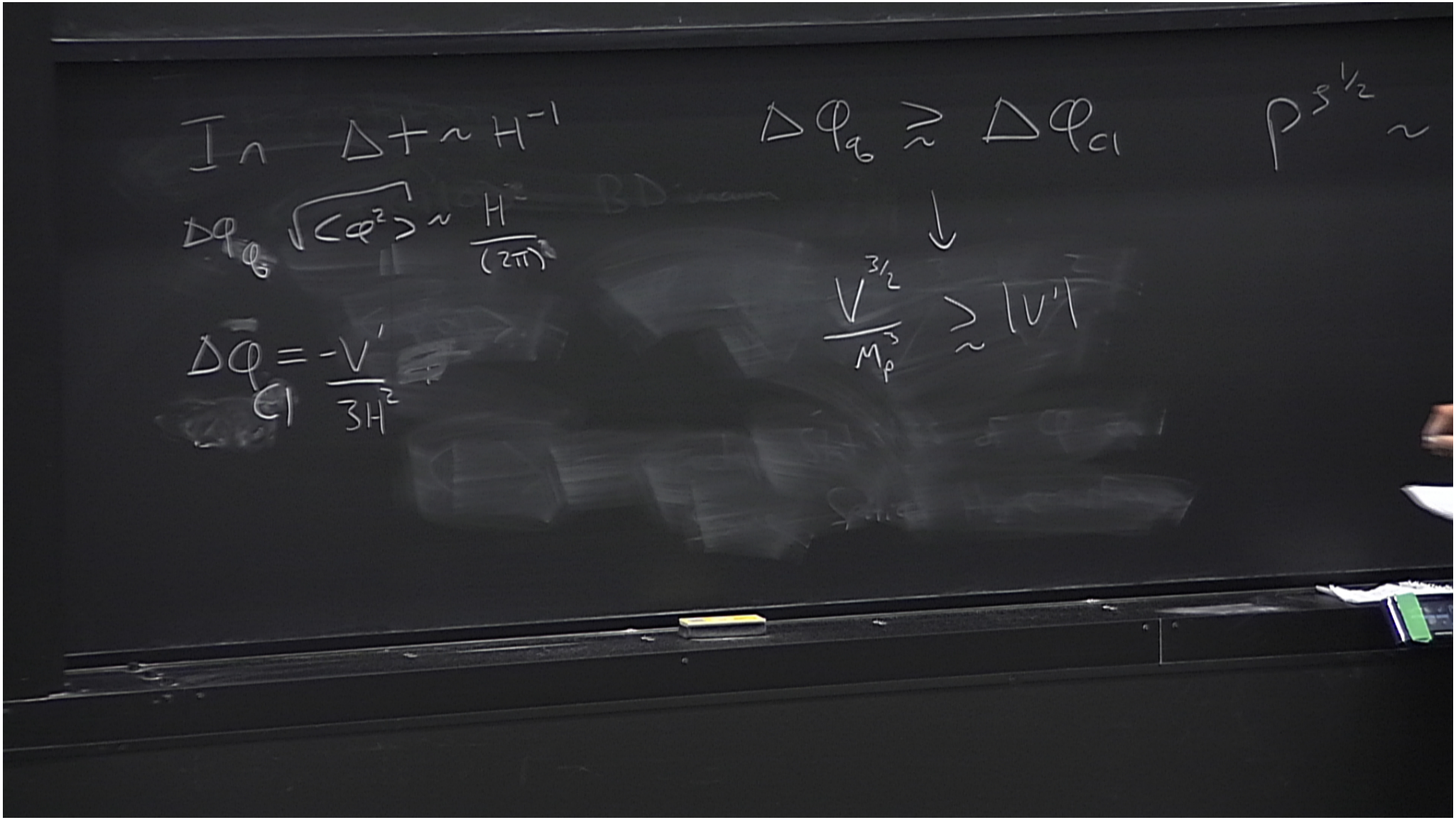
$$\varphi = \int d^3y \, W(x-y)\varphi(y)$$

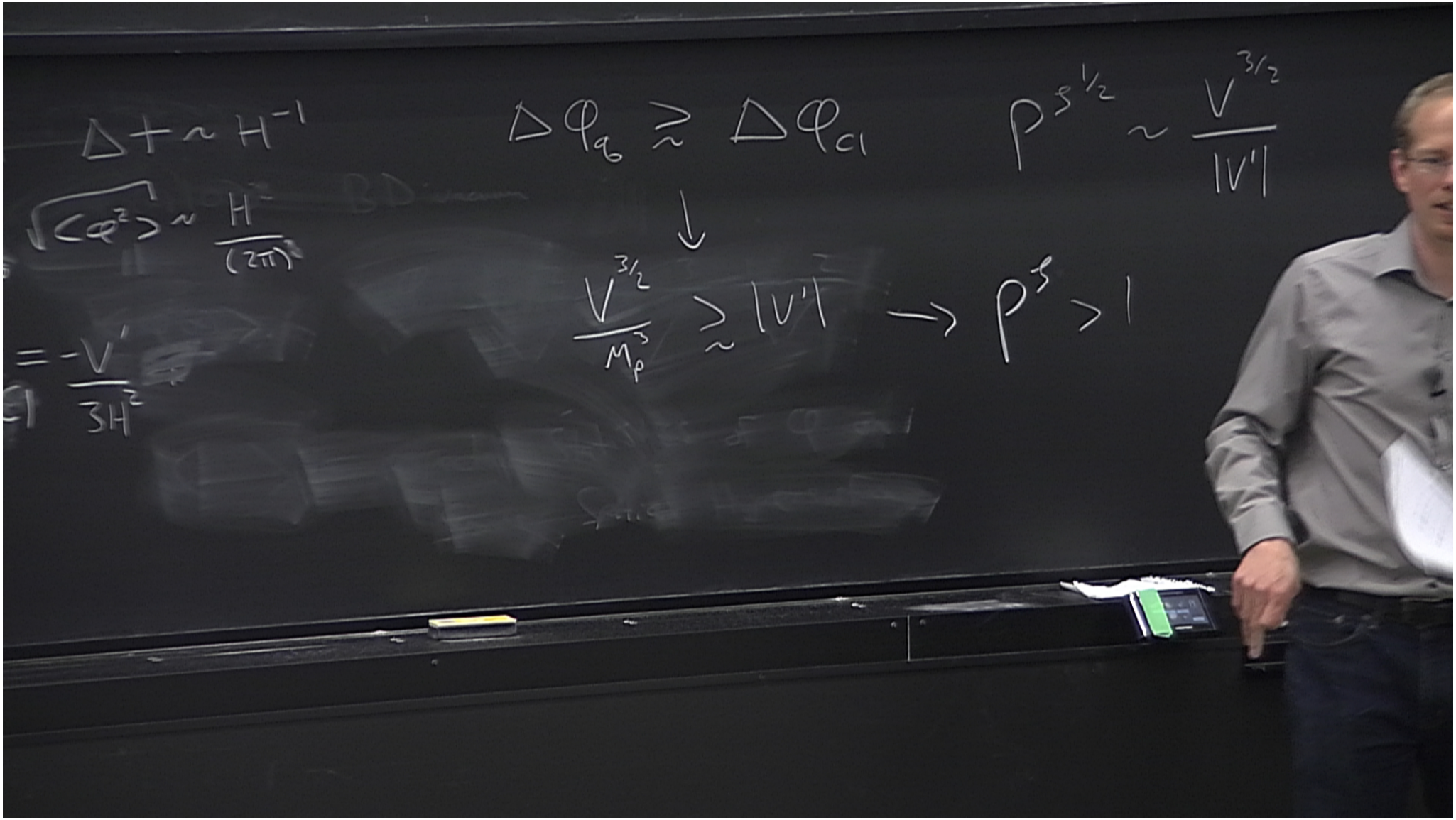
$$\langle N(t) \rangle = 0 \quad , \quad \langle N(t)N(t') \rangle = \frac{9H^5}{4\pi^2} \delta(t-t') \quad (\text{Markovian})$$

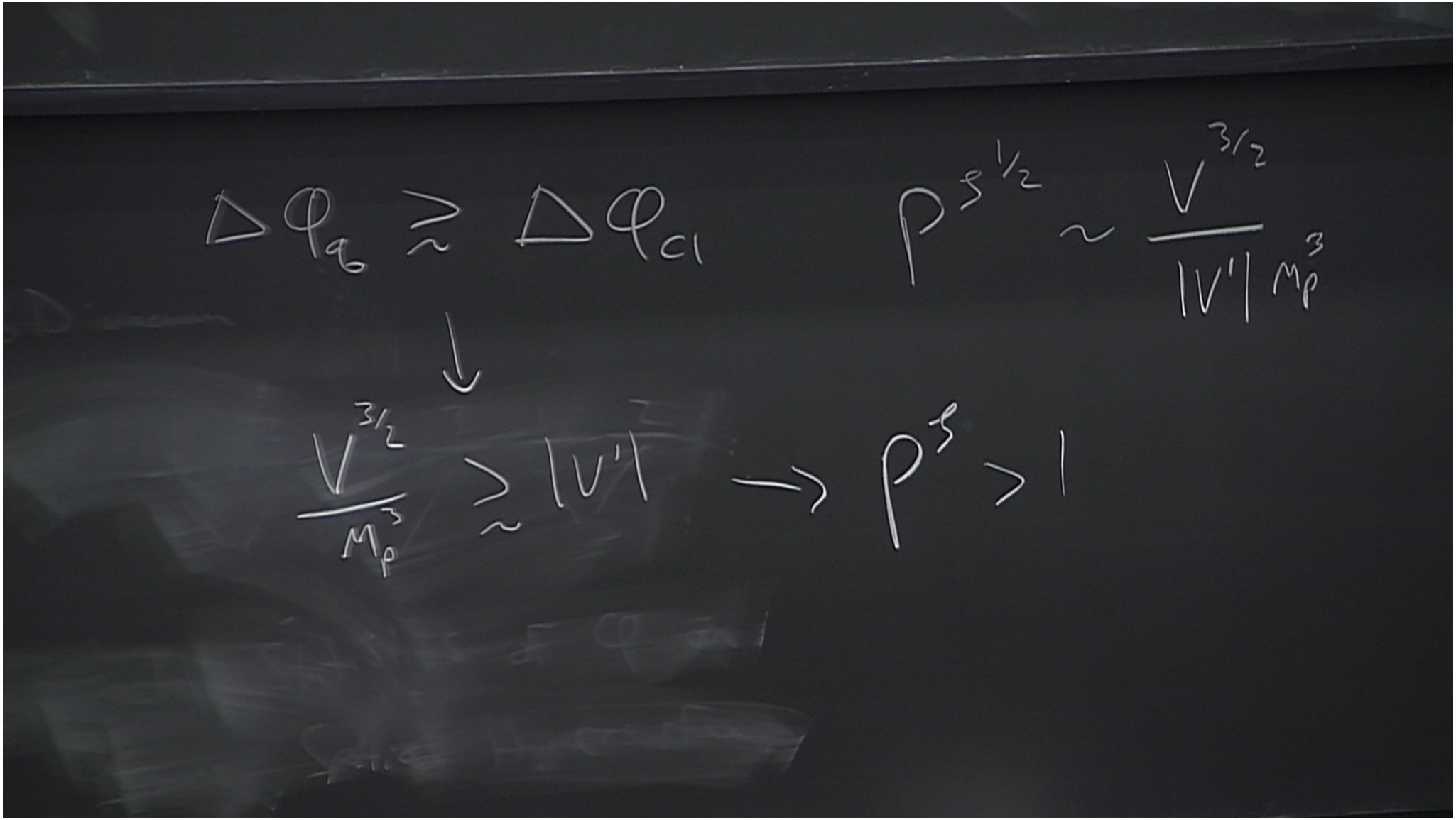
$$\langle \varphi(t) \rangle = \int_0^t \frac{\langle N(t') \rangle}{3H} dt = 0$$

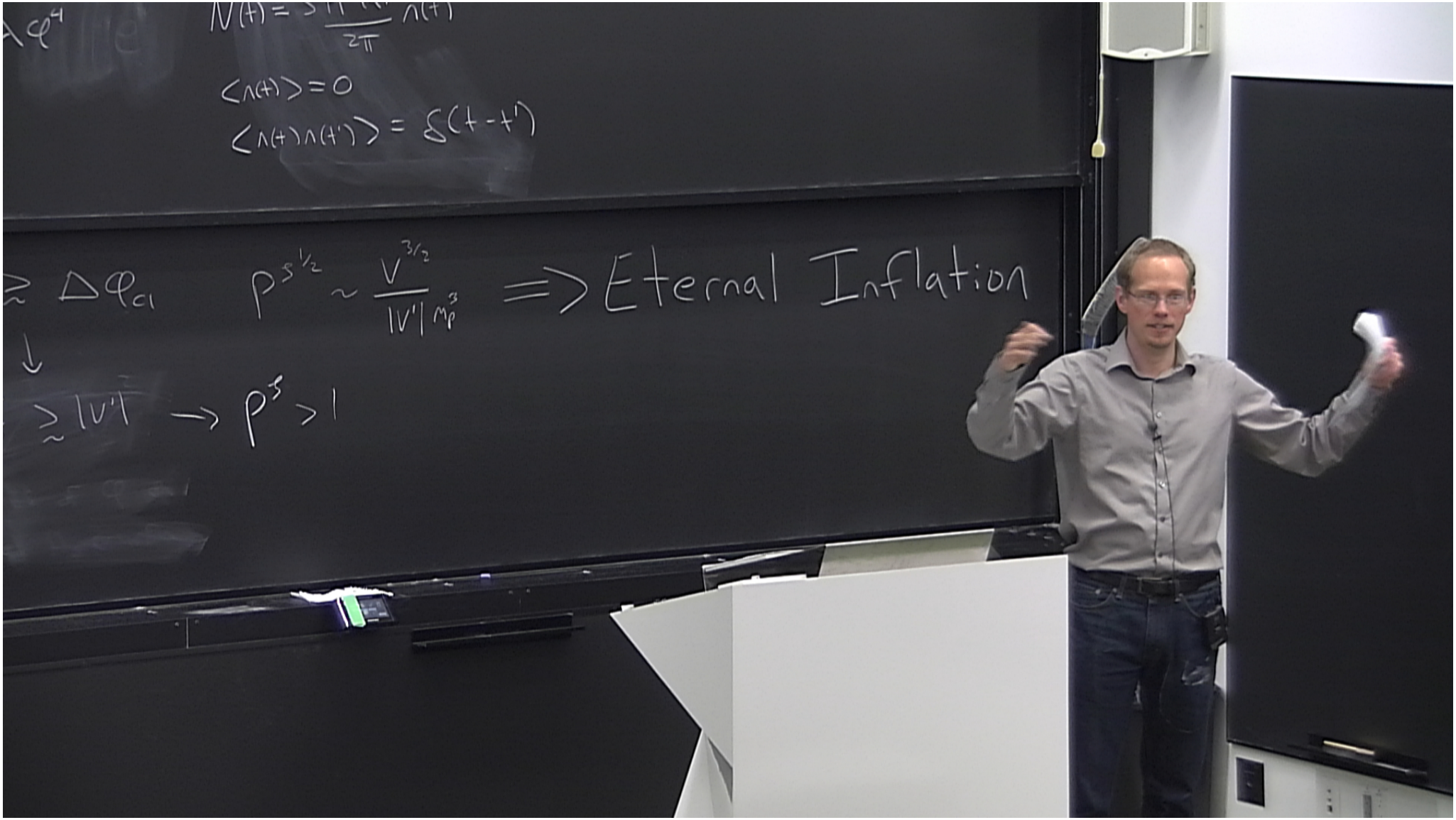

$\varphi = \int d^3y\, W(x-y)\varphi(y)$
$3H\dot{\varphi} = N(t)$
stochastic
$\langle \varphi(t)\varphi(\tilde{t})\rangle = \int_0^t \int_0^{\tilde{t}} \frac{\langle N(t')N(\tilde{t}')\rangle}{(3H)^2}\, dt'\, d\tilde{t}'$
ensemble of histories → 2pt fn
$= \frac{H^3}{4\pi^2} \int_0^t \int_0^{\tilde{t}} \delta(t'-\tilde{t}')\, dt'\, d\tilde{t}'$
$= \frac{H^3}{4\pi^2} t$
$P(\varphi(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\varphi(t)^2}{2\sigma^2}\right]$
$\sigma^2 = \langle \varphi^2 \rangle - \langle \varphi \rangle^2$
φ
σ
0
$-\sigma$
t


ρ(φ)
V
φ
$H^2(\varphi) = \frac{V(\varphi)}{3M_p^2}$
$3H(\varphi)\dot{\varphi} = -V' + N(t)$


In $\Delta t \sim H^{-1}$
$\Delta\varphi_{\text{fluct}} \sqrt{\langle\varphi^2\rangle} \sim \frac{H}{(2\pi)}$
$\Delta\varphi = \frac{-V'}{3H^2}$
ϵ_1
$\Delta\varphi_{\text{fluct}} \gtrsim \Delta\varphi_{\text{cl}}$
$\frac{V^{3/2}}{M_p^3} \gtrsim |V'|$
$P^{s^{1/2}} \sim$


$\Delta t \sim H^{-1}$
$\Delta\varphi_q \gtrsim \Delta\varphi_{cl}$
$\rho^{s^{1/2}} \sim \frac{V^{3/2}}{|V'|}$
$\sqrt{\langle\varphi^2\rangle} \sim \frac{H}{(2\pi)}$
$= -\frac{V'}{3H^2}$
$\frac{V^{3/2}}{M_p^3} \gtrsim |V'| \rightarrow \rho^s > 1$


$\Delta\varphi_b \gtrsim \Delta\varphi_{cl}$
$p^{S^{1/2}} \sim \frac{V^{3/2}}{|V'| M_p^3}$
$\frac{V^{3/2}}{M_p^3} \gtrsim |V'|$
$\rightarrow p^S > 1$


$\Delta\varphi^4$
$N(t) = \frac{\Lambda(t)}{2\pi}$
$\langle \Lambda(t) \rangle = 0$
$\langle \Lambda(t) \Lambda(t') \rangle = \delta(t-t')$
$\gtrsim \Delta\varphi_{cl}$
$p^{5/2} \sim \frac{V^{3/2}}{|V'| M_p^3}$
$\Rightarrow$ Eternal Inflation
$\gtrsim |v'|$
$\rightarrow p^5 > 1$