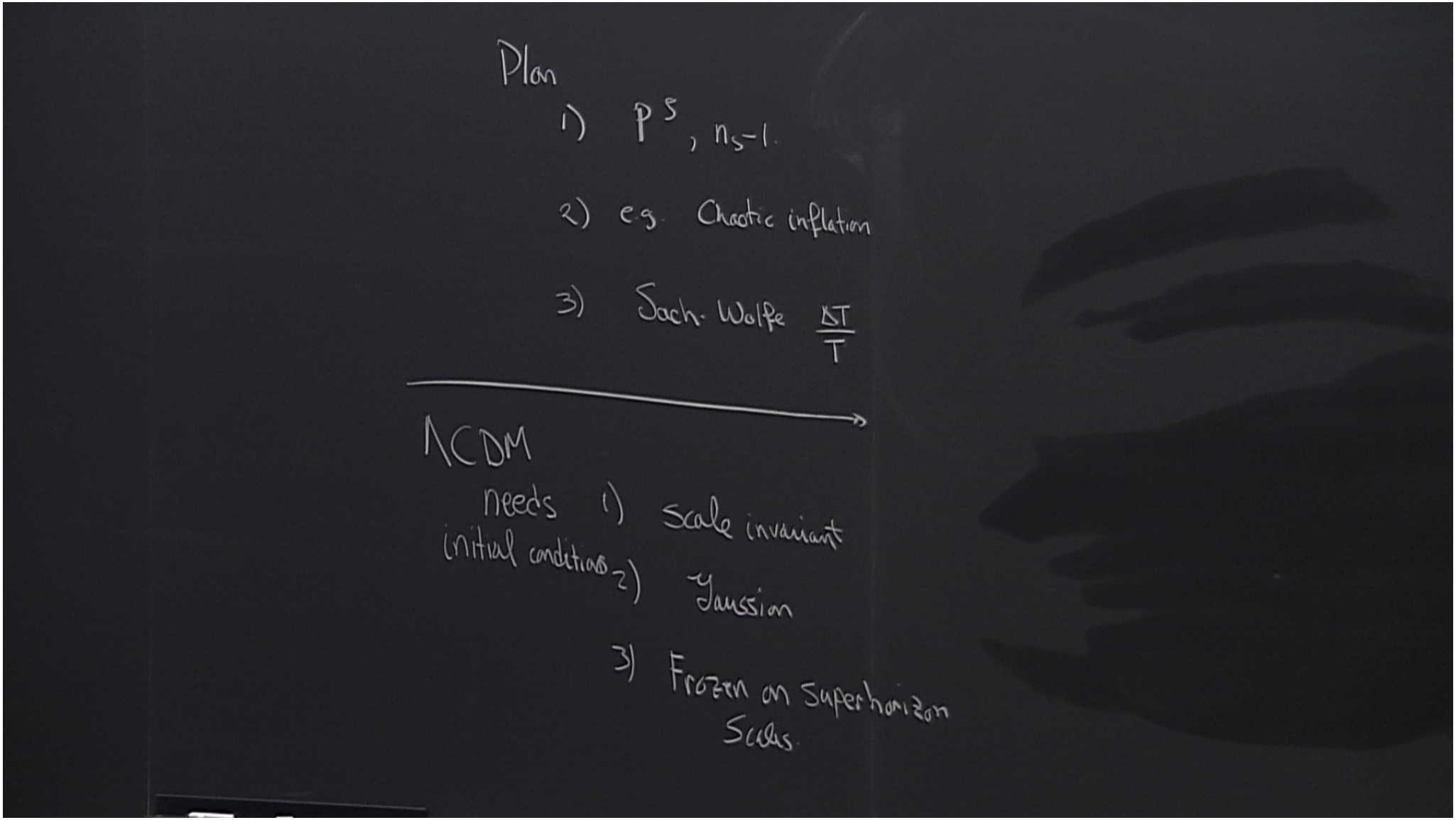
Title: Explorations in Cosmology - Lecture 8

Date: Mar 21, 2012 10:15 AM

URL: http://pirsa.org/12030032

Abstract:




Plan
1) P^S, $n_S - 1$.
2) e.g. Chaotic inflation
3) Sach. Wolfe $\frac{\Delta T}{T}$
ΛCDM
needs 1) Scale invariant
initial conditions 2) Gaussian
3) Frozen on Superhorizon
Scales.

$$S_2 = \int d\eta\, d^3x \left(\epsilon M_p^2 a^2 \left(\zeta'^2 - (\partial\zeta)^2 \right) \right)$$

ADM.

$$S_2 = \int d\eta\, d^3x \left(\epsilon M_p^2 a^2 \left(\zeta'^2 - (\partial\zeta)^2 \right) \right)$$

expanded to 2nd ordr.

$$\chi_k \Rightarrow v_k = \frac{1}{k^{3/2}\eta}$$

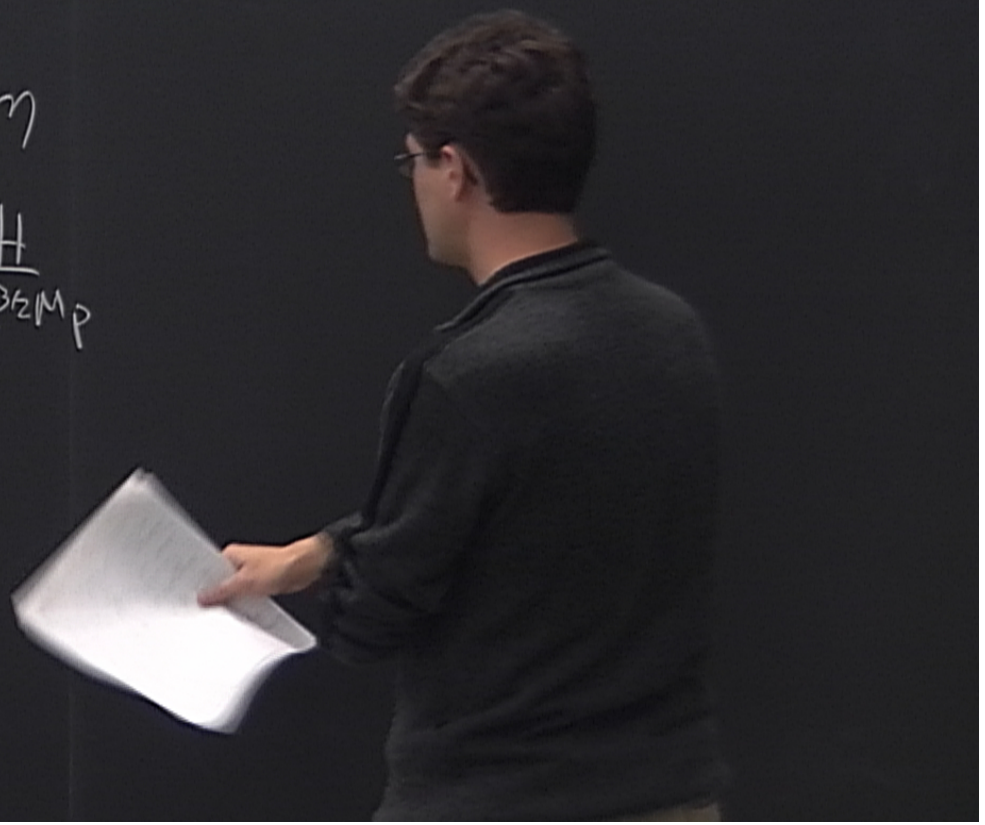
$$k\eta \ll 1$$

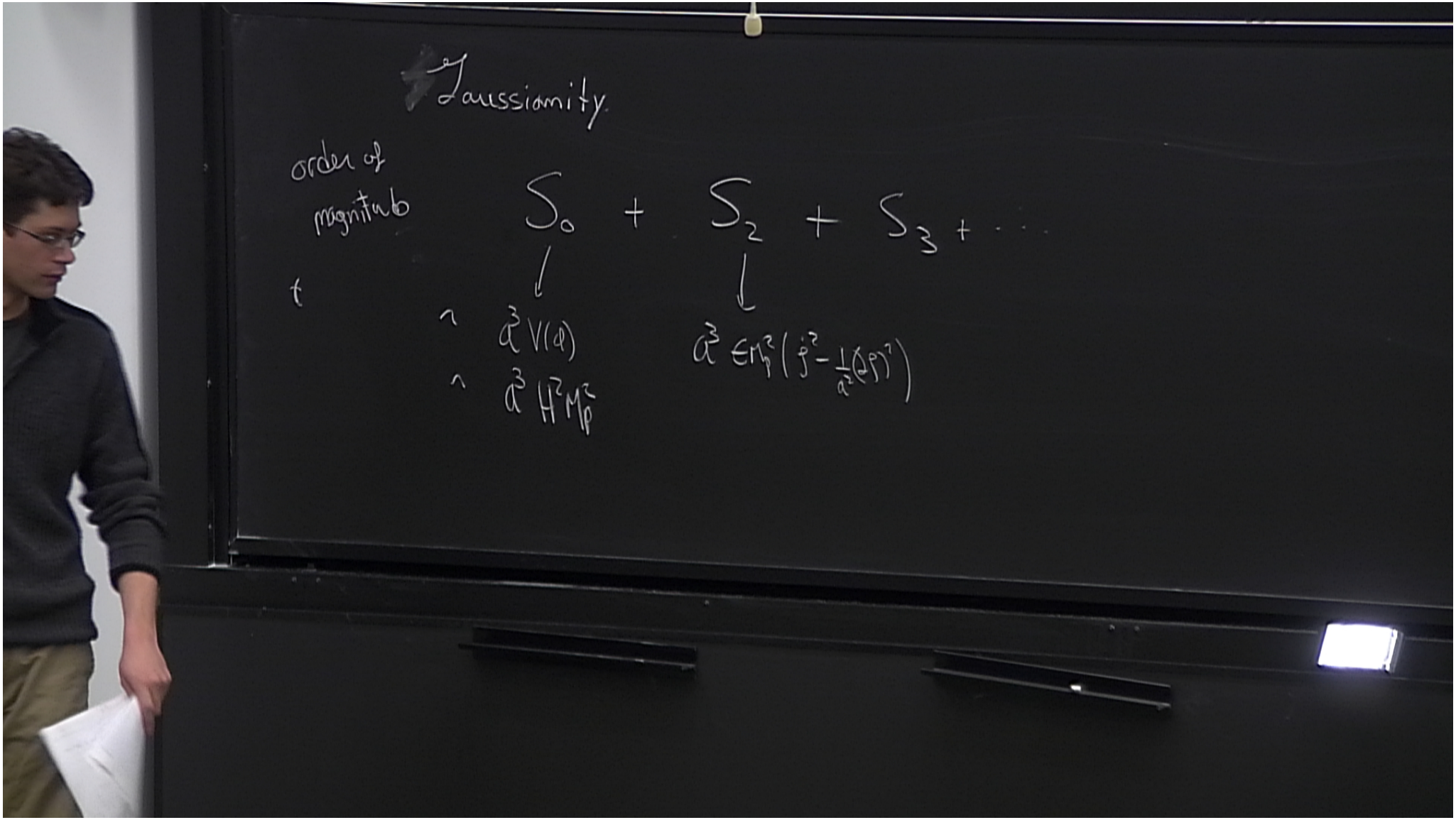
$$\zeta_k \Rightarrow u_k = \frac{H}{\sqrt{2}\epsilon k^{3/2} M_p}$$

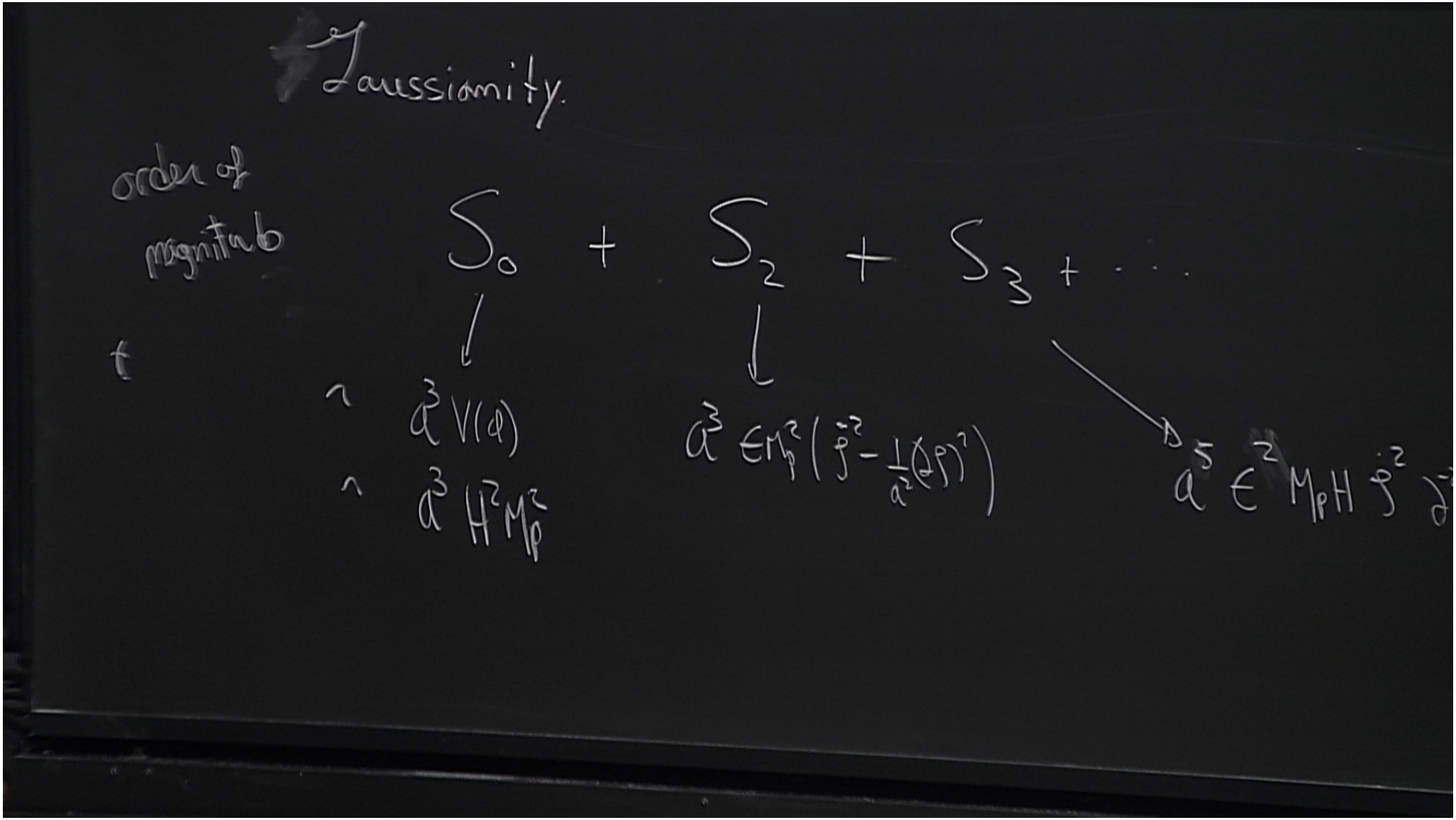
inflation

$\frac{\Delta T}{T}$

Superhorizon




Gaussianity
order of magnitude
$S_0 + S_2 + S_3 + \cdots$
$\sim a^3 V(\phi)$
$\sim a^3 H^2 M_p^2$
$a^3 \epsilon M_p^2 \left(\dot{\phi}^2 - \frac{1}{a^2} (\partial\phi)^2 \right)$


Gaussianity.
order of magnitude
$S_0 + S_2 + S_3 + \cdots$
$a^3 V(q)$
$a^3 H^2 M_p^2$
$a^3 \epsilon M_p^2 (\dot{\pi}^2 - \frac{1}{a^2}(\partial\pi)^2)$
$a^5 \epsilon^2 M_p H \dot{\pi}^2$

$$\langle \zeta_k^2 \rangle = \frac{H_*^2}{2\epsilon_* k^3 M_p^2}$$

$$\sigma \sim \langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\epsilon M_p}$$

$$t \sim \frac{1}{H} \rightsquigarrow \partial_t \sim H$$

$$\Delta x \sim \frac{1}{aH} \rightsquigarrow \frac{1}{a}\partial_x \sim H$$

$$\langle \zeta^2(x) \rangle = \int \frac{dk}{k} P^\zeta = \int \frac{dk}{k} \frac{H_*^2}{8\pi^2 \epsilon_* M_p^2}$$

Gaussianity

order of magnitude

$S_0 + S_2 + S_3 + \cdots$

$\sim a^3 V(\phi)$

$\sim a^3 H^2 M_p^2$

$a^3 \epsilon M_p^2 \left(\dot{\vec{\pi}}^2 - \frac{1}{a^2} (\vec{\partial}\pi)^2 \right)$

$a^3 H^4$

$a^5 \epsilon^2 M_p H \dot{\pi}^2 \partial^{-2}(\dot{\pi})$

$a^3 \epsilon^{1/2} H^5 / M_p$

t

if $\epsilon < 1$ & if $H < M_p$

$$S_3 < S_2 < S_0$$

ζ is almost free field. Interactions are small.

$\langle \zeta^2 \rangle$ is good enough $\Rightarrow$ Gaussian

$H \quad \zeta^2 \quad \mathcal{F}^{-1}(\zeta)$

M_p

$$\langle \zeta_k^2 \rangle = \frac{H_*^2}{2\epsilon_* k^3 M_p^2}$$

$$\delta \sim \langle \zeta^2 \rangle^{1/2} \sim \frac{H}{\sqrt{\epsilon} M_p}$$

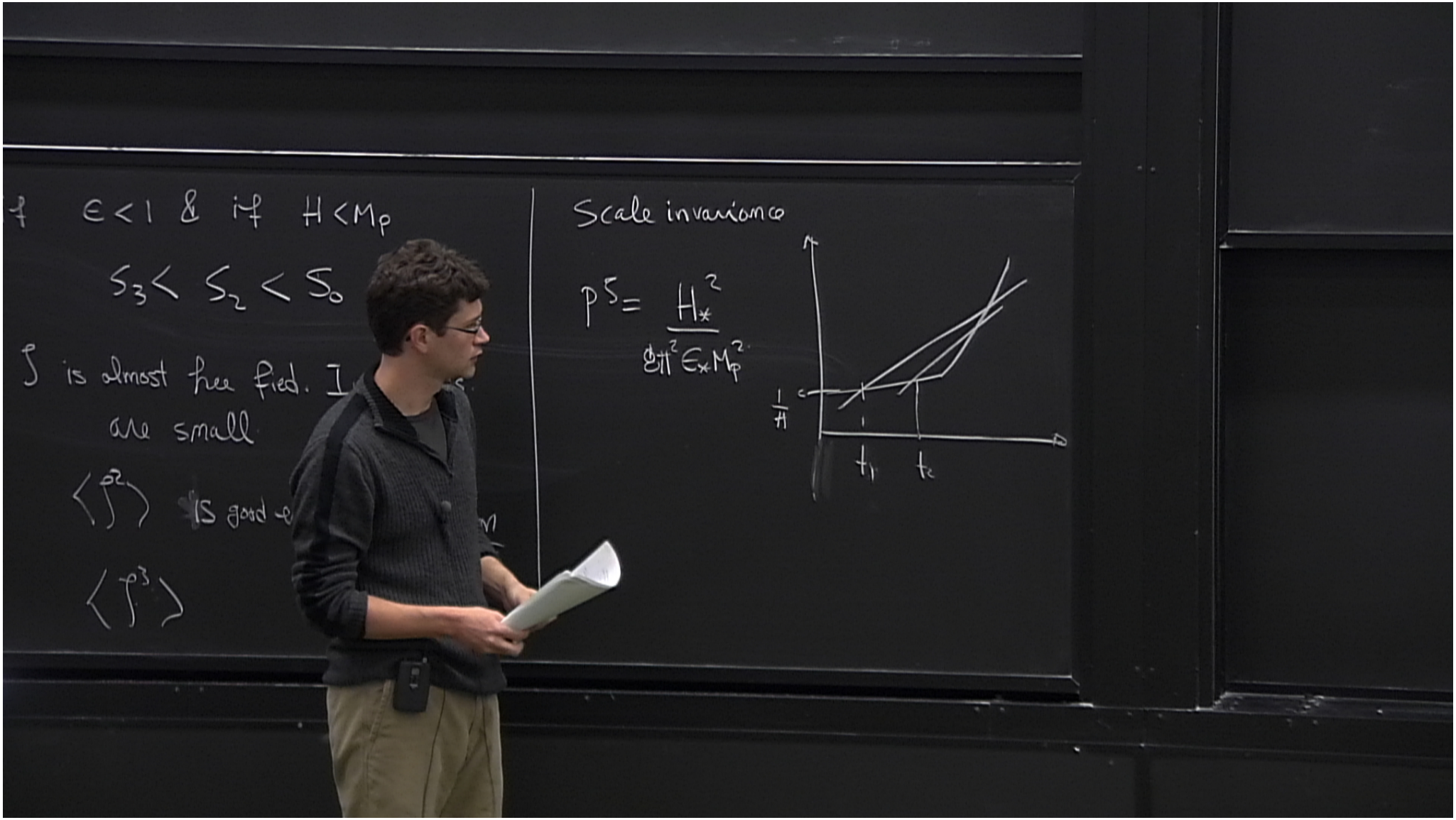
$$t \sim \frac{1}{H} \rightsquigarrow \partial_t \sim H$$

$$\Delta x \sim \frac{1}{aH} \rightsquigarrow \frac{1}{a}\partial_x \sim H$$

$$\langle \zeta^2(x) \rangle = \int \frac{dk}{k} P^S = \int \frac{dk}{k} \frac{H_*^2}{8\pi^2 \epsilon_* M_p^2}$$

$\uparrow$

$\hat{=} Gt$


ϵ<1 & if H<Mp
S3< S2 < S0
is almost free fied. I
are small
Scale invariance
$p^S = \frac{H_*^2}{(2\pi)^2 \epsilon_* M_p^2}$
H
1/H
t1
t2

$H < M_p$

$< S_0$

ed. Interactions.

enough $\Rightarrow$ Gaussian

Scale invariance

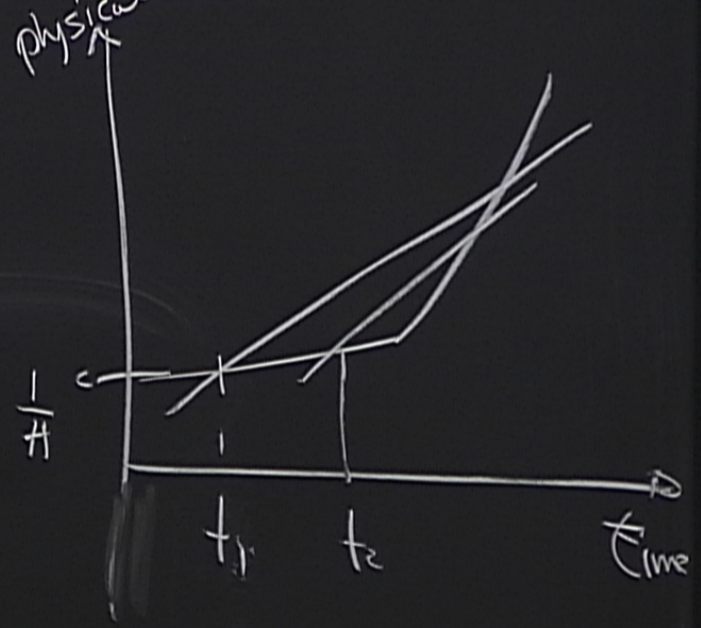
$$P^S = \frac{H_*^2}{8\pi^2 \epsilon_* M_p^2}$$

$t \to \infty$

H_* ↓

ϵ can go either way.

Very typical ϵ ↑

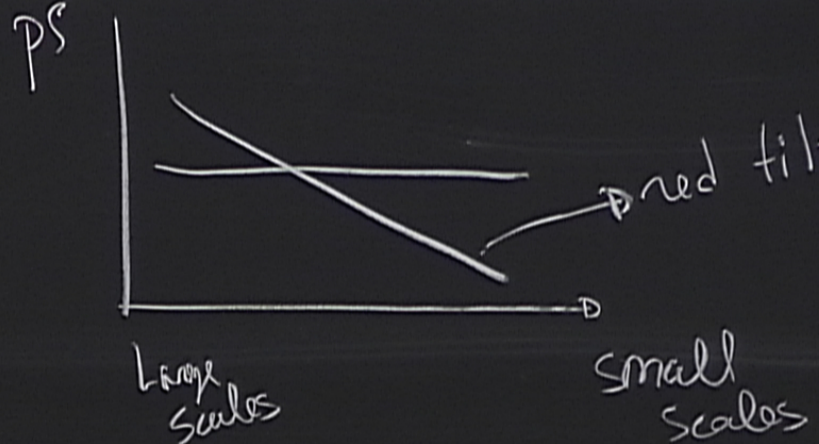


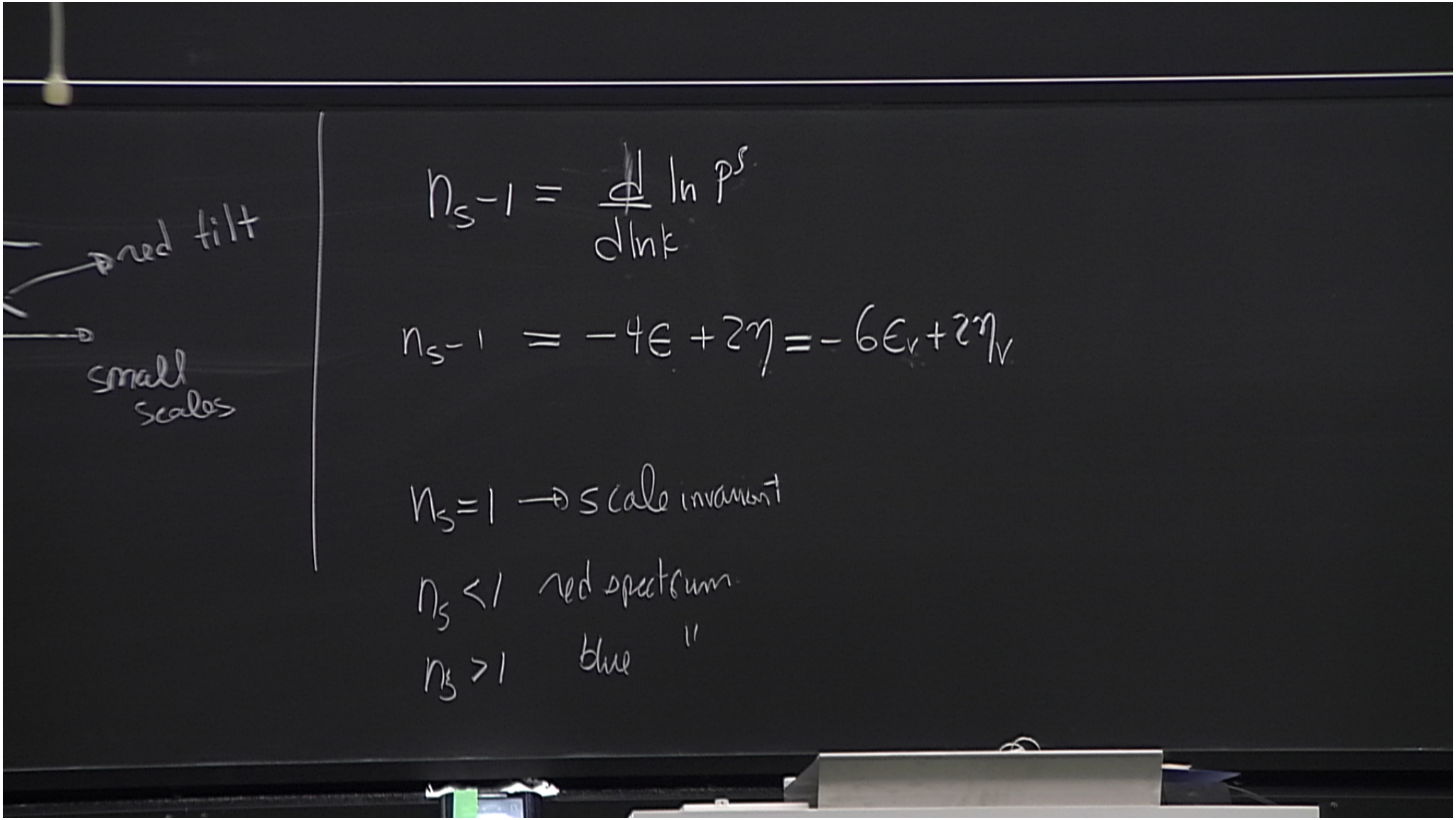
$\omega \to \infty$

many models $P^S \downarrow$

↳ more power on large scales.

red tilt




red tilt
small scales
$n_s - 1 = \frac{d \ln P^S}{d \ln k}$
$n_s - 1 = -4\epsilon + 2\eta = -6\epsilon_V + 2\eta_V$
$n_s = 1 \rightarrow$ scale invariant
$n_s < 1$ red spectrum
$n_s > 1$ blue "

$$n_s - 1 = \frac{d \ln P^S}{d \ln k}$$

$k = aH$

$d \ln k = d \ln a \approx H \, dt_*$ ($a \propto e^{Ht}$)

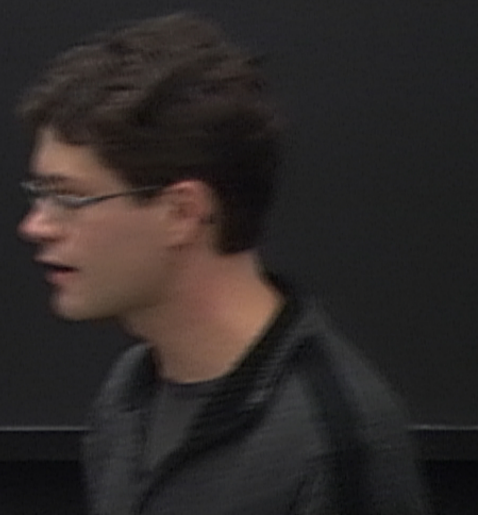
$$n_s - 1 = -4\epsilon + 2\eta = -6\epsilon_V + 2\eta_V$$

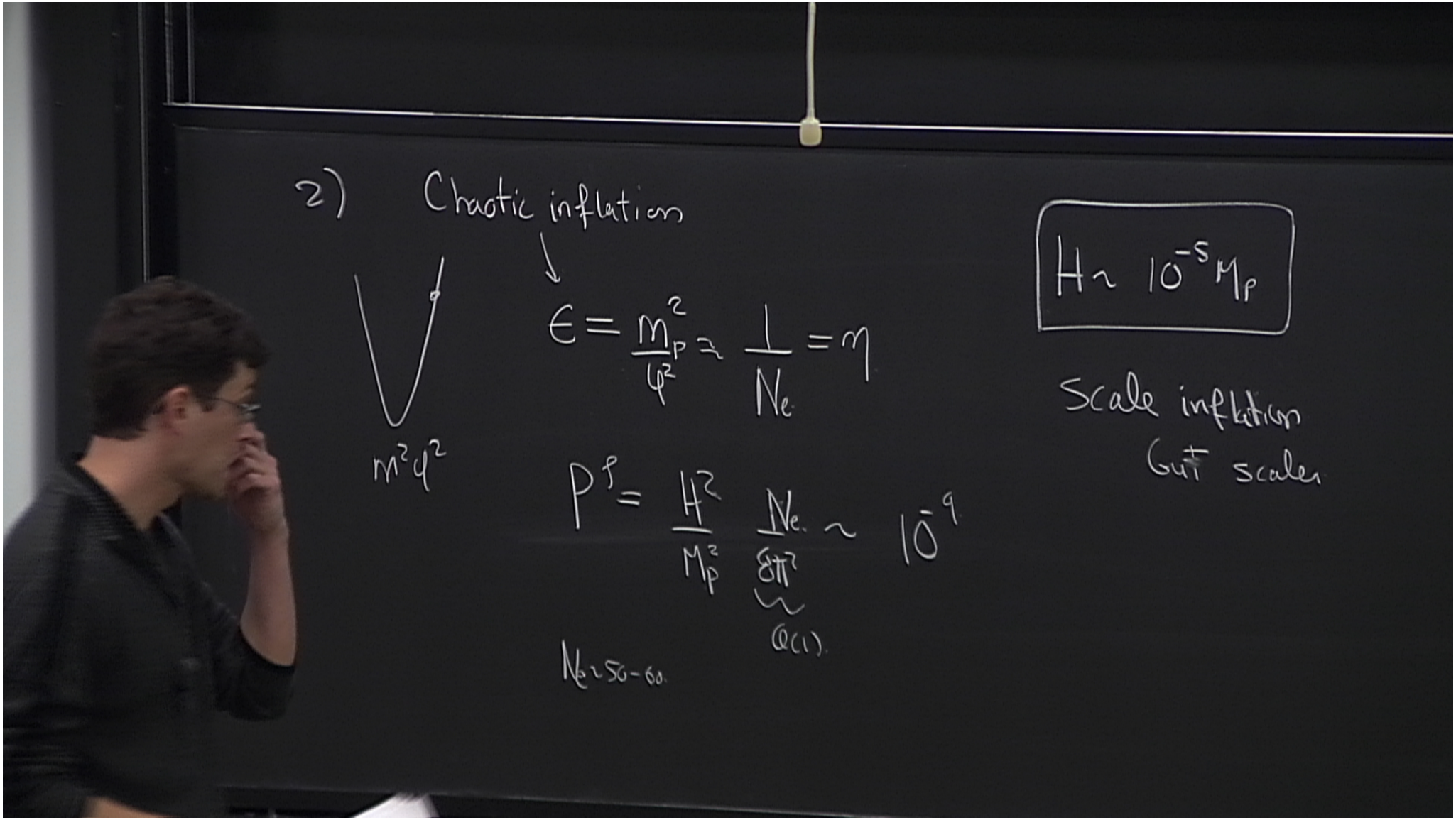
$$P^S \longleftrightarrow \left(\frac{\Delta T}{T}\right)^2 \sim 10^{-9}$$

$n_s = 1 \rightarrow$ scale invariant

$n_s < 1$ red spectrum

$n_s > 1$ blue "




2) Chaotic inflation
V
$m^2\varphi^2$
$\epsilon = \frac{m_p^2}{\varphi^2} \sim \frac{1}{N_e} = \eta$
$H \sim 10^{-5} M_p$
Scale inflation
Gut scales
$P^S = \frac{H^2}{M_p^2} \frac{N_e}{8\pi^2} \sim 10^{-9}$
$\mathcal{O}(1)$
$N_e \sim 50-60$

2) Chaotic inflation

V

$\epsilon_V = \epsilon = \frac{M_p^2}{4}\left(\frac{V'}{V}\right)^2 \simeq \frac{1}{N_e} = \eta_V$

$\frac{m^2\varphi^2}{V}$

$P_\zeta = \frac{H^2}{M_p^2}\frac{N_e}{8\pi^2} \sim 10^{-9}$

$8\pi^2 \sim O(1)$

$N_e \simeq 50\text{-}60$

$H \sim 10^{-5} M_p$

Scale inflation

GUT scales

fixes m

$n_s - 1 = -6\epsilon_V + 2\eta_V = -4\epsilon_V = -\frac{4}{N_e} \simeq 0.93$

2) Chaotic inflation

$V = \frac{m^2\varphi^2}{?}$

$\epsilon_V = \epsilon = \frac{M_p^2}{4}\left(\frac{V'}{V}\right)^2 \simeq \frac{1}{N_e} = \eta_V$

$P_S = \frac{H^2}{M_p^2}\frac{N_e}{8\pi^2} \sim 10^{-9}$

$8\pi^2 \sim O(1)$

$N_e \simeq 50-60.$

$H \sim 10^{-5} M_p$

Scale inflation

GUT scale

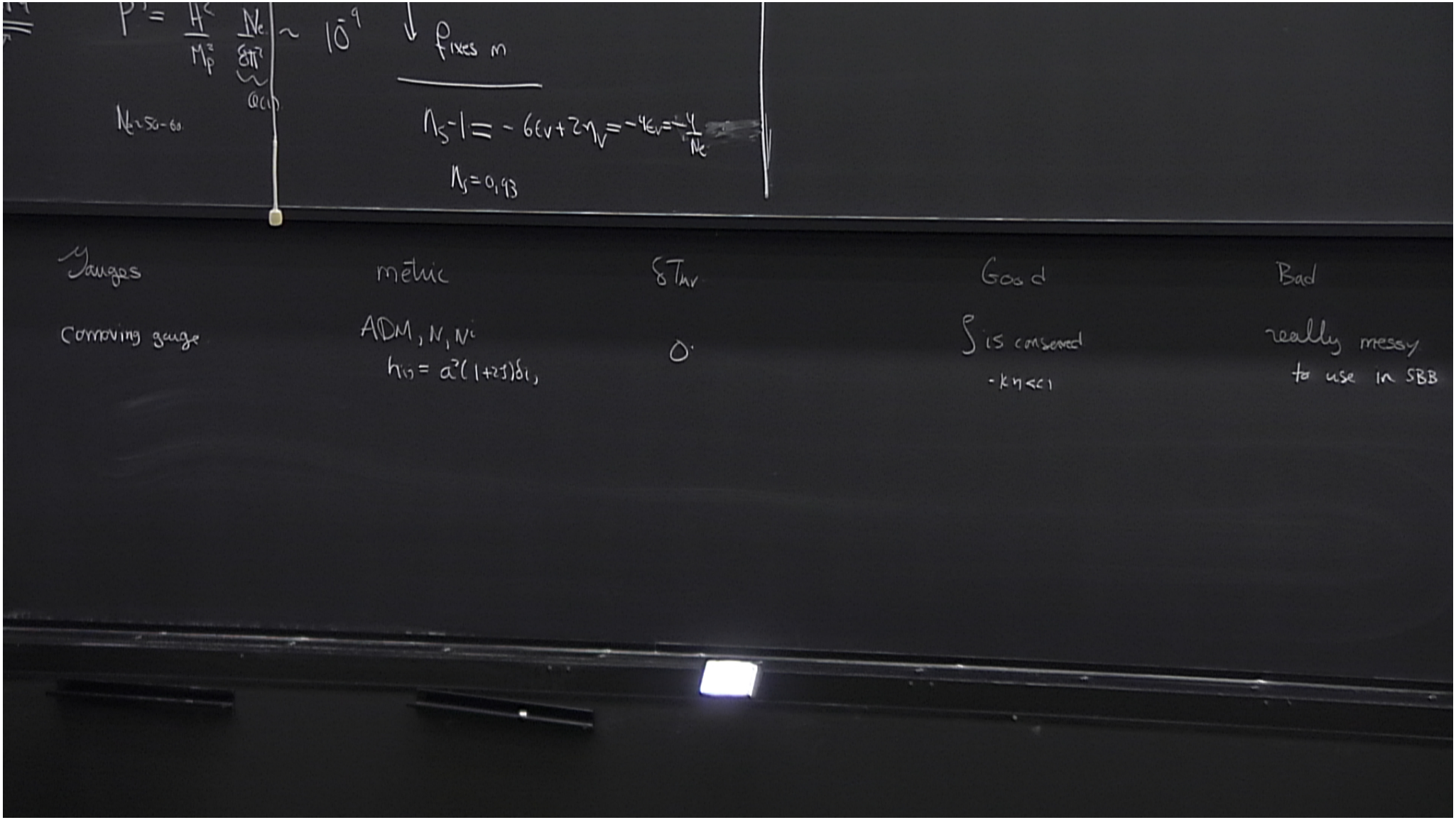
fixes m

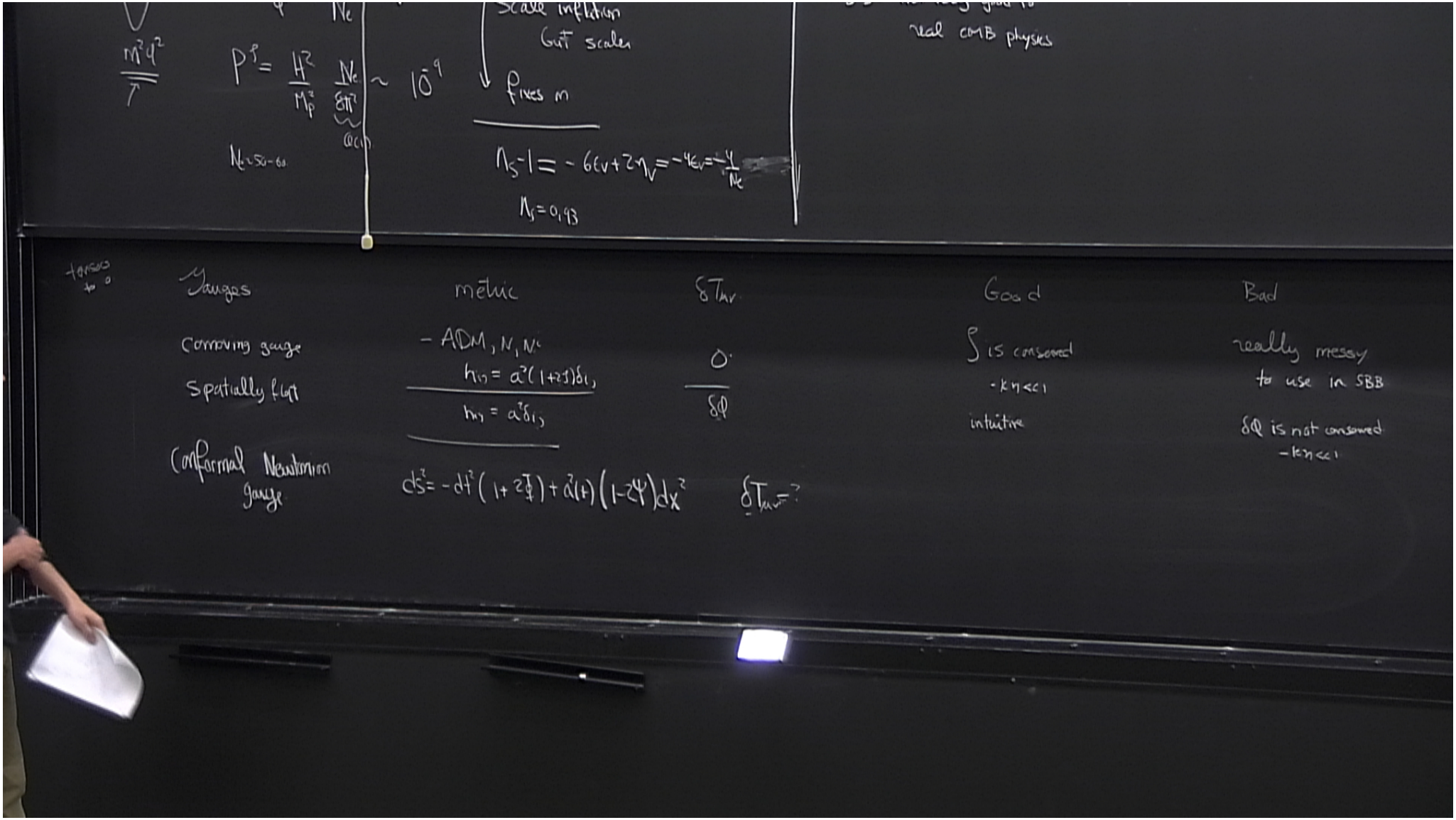
$n_S - 1 = -6\epsilon_V + 2\eta_V = -4\epsilon_V = -\frac{4}{N_e}$

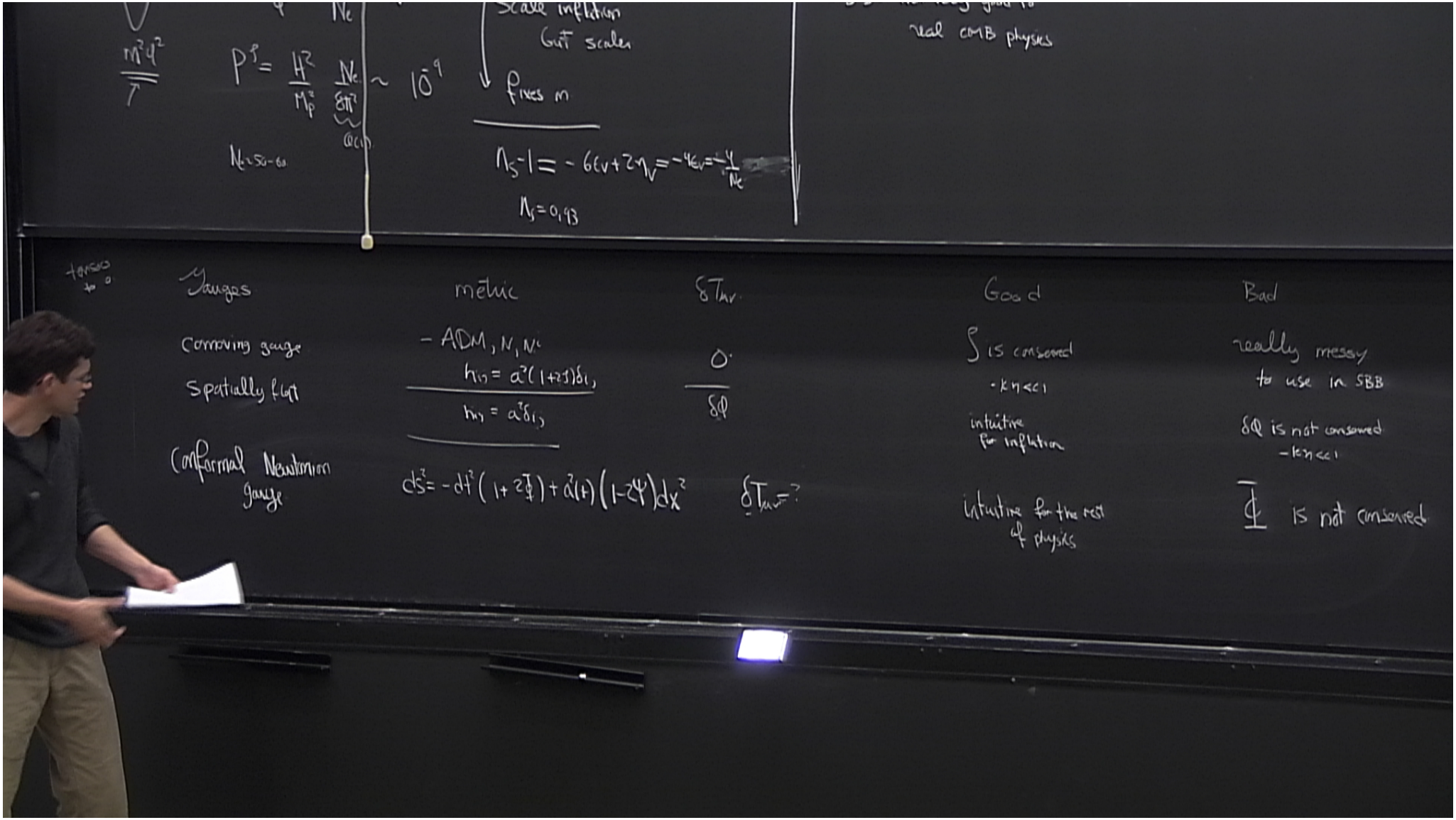
$n_S = 0,93$

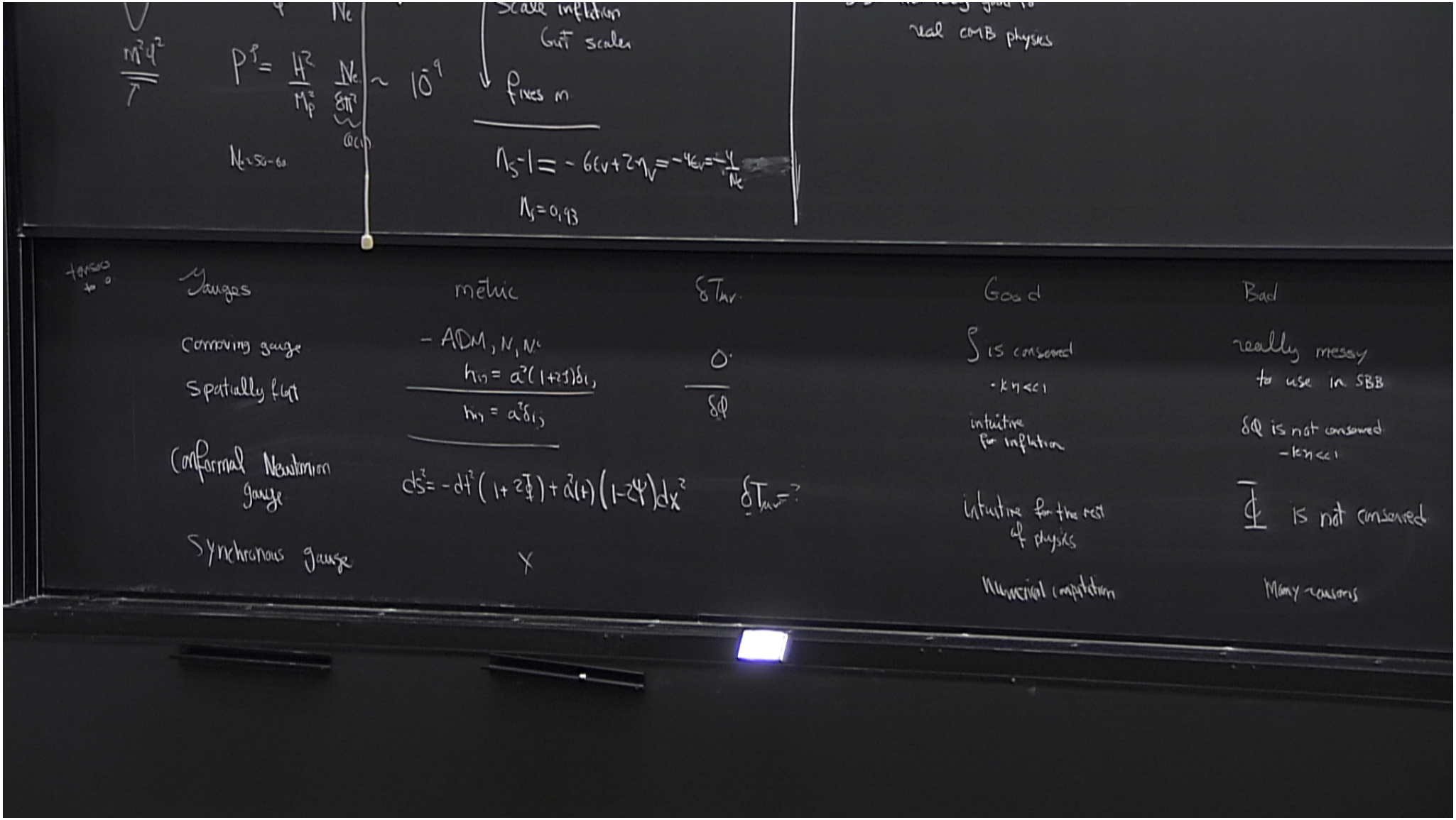
$P^S \rightarrow \left(\frac{\Delta T}{T}\right)^2$

$\int$ gauge not very good to real CMB physics


$P^2 = \frac{H^2}{M_p^2} \frac{N_e}{8\pi^2}$
$N_e \sim 50-60$
$\sim 10^{-9}$
ρ fixes m
$n_s - 1 = -6\epsilon_V + 2\eta_V = -4\epsilon_V = -\frac{4}{N_e}$
$n_s = 0,93$
Gauges
metric
$\delta T_{\mu\nu}$
Good
Bad
Comoving gauge
ADM, N, N^i
$h_{ij} = a^2(1+2\zeta)\delta_{ij}$
0
ζ is conserved
$-k\eta \ll 1$
really messy
to use in SBB


$m^2 q^2$
$P^S = \frac{H^2}{M_p^2} \frac{N_e}{8\pi^2}$
$N_e \sim 50-60$
$\sim 10^{-9}$
Scale inflation
GUT scales
fixes m
$n_S - 1 = -6\epsilon_V + 2\eta_V = -4\epsilon_V = -\frac{4}{N_e}$
$n_S = 0,93$
real CMB physics
tensors to 0
Gauges
Comoving gauge
Spatially flat
Conformal Newtonian gauge
metric
- ADM, N, N^i
$h_{ij} = a^2(1+2\zeta)\delta_{ij}$
$h_{ij} = a^2\delta_{ij}$
$ds^2 = -dt^2(1+2\Phi) + a^2(t)(1-2\Psi)dx^2$
$\delta T_{\mu\nu}$
0
$\delta\rho$
$\delta T_{\mu\nu} = ?$
Good
ζ is conserved
$-k\eta \ll 1$
intuitive
Bad
really messy
to use in SBB
$\delta\rho$ is not conserved
$-k\eta \ll 1$


Ne
Scale inflation
GUT scales
real CMB physics
$\frac{m^2\phi^2}{?}$
$P^S = \frac{H^2}{M_p^2}\frac{Ne}{8\pi^2}$
$\sim 10^{-9}$
fixes m
$N_e \sim 50-60$
$n_S - 1 = -6\epsilon_V + 2\eta_V = -\frac{4}{Ne} - \frac{1}{Ne}$
$n_S = 0,93$
tensors to 0
Gauges
Comoving gauge
Spatially flat
Conformal Newtonian gauge
metric
- ADM, N, N^i
$h_{ij} = a^2(1+2\zeta)\delta_{ij}$
$h_{ij} = a^2\delta_{ij}$
$ds^2 = -dt^2(1+2\Phi) + a^2(t)(1-2\Psi)dx^2$
$\delta T_{\mu\nu}$
0
$\delta\rho$
$\delta T_{\mu\nu} = ?$
Good
ζ is conserved
$-k\eta \ll 1$
intuitive for inflation
intuitive for the rest of physics
Bad
really messy to use in SBB
$\delta\rho$ is not conserved
$-k\eta \ll 1$
Φ is not conserved


Scale inflation
GUT scales
real CMB physics
$\frac{m^2\phi^2}{?}$
$P^S = \frac{H^2}{M_p^2}\frac{N_e}{8\pi^2}$
$\sim 10^{-9}$
fixes m
$N_e \sim 50-60$
$n_S - 1 = -6\epsilon_V + 2\eta_V = -\frac{4}{N_e} - \frac{1}{N_e}$
$n_S = 0,93$
tensors to 0
Gauges
Comoving gauge
Spatially flat
Conformal Newtonian gauge
Synchronous gauge
metric
- ADM, N, N^i
$h_{ij} = a^2(1+2\zeta)\delta_{ij}$
$h_{ij} = a^2\delta_{ij}$
$ds^2 = -dt^2(1+2\Phi) + a^2(t)(1-2\Psi)dx^2$
X
$\delta T_{\mu\nu}$
0
$\delta\rho$
$\delta T_{\mu\nu} = ?$
Good
ζ is conserved
$-k\eta \ll 1$
intuitive for inflation
intuitive for the rest of physics
Numerical computation
Bad
really messy to use in SBB
$\delta\rho$ is not conserved
$-k\eta \ll 1$
Φ is not conserved
Many variables

comoving gauge to spatially flat

$$h_{ij} = e^{2Ht + 2\zeta}\,\delta_{ij} \qquad h_{ij} = e^{2H\tilde{t}}\,\delta_{ij}$$

$$\phi = \phi(t) \qquad \phi = \phi(\tilde{t}) + \delta\phi$$

$$\tilde{t} = t + \frac{\zeta}{H}$$

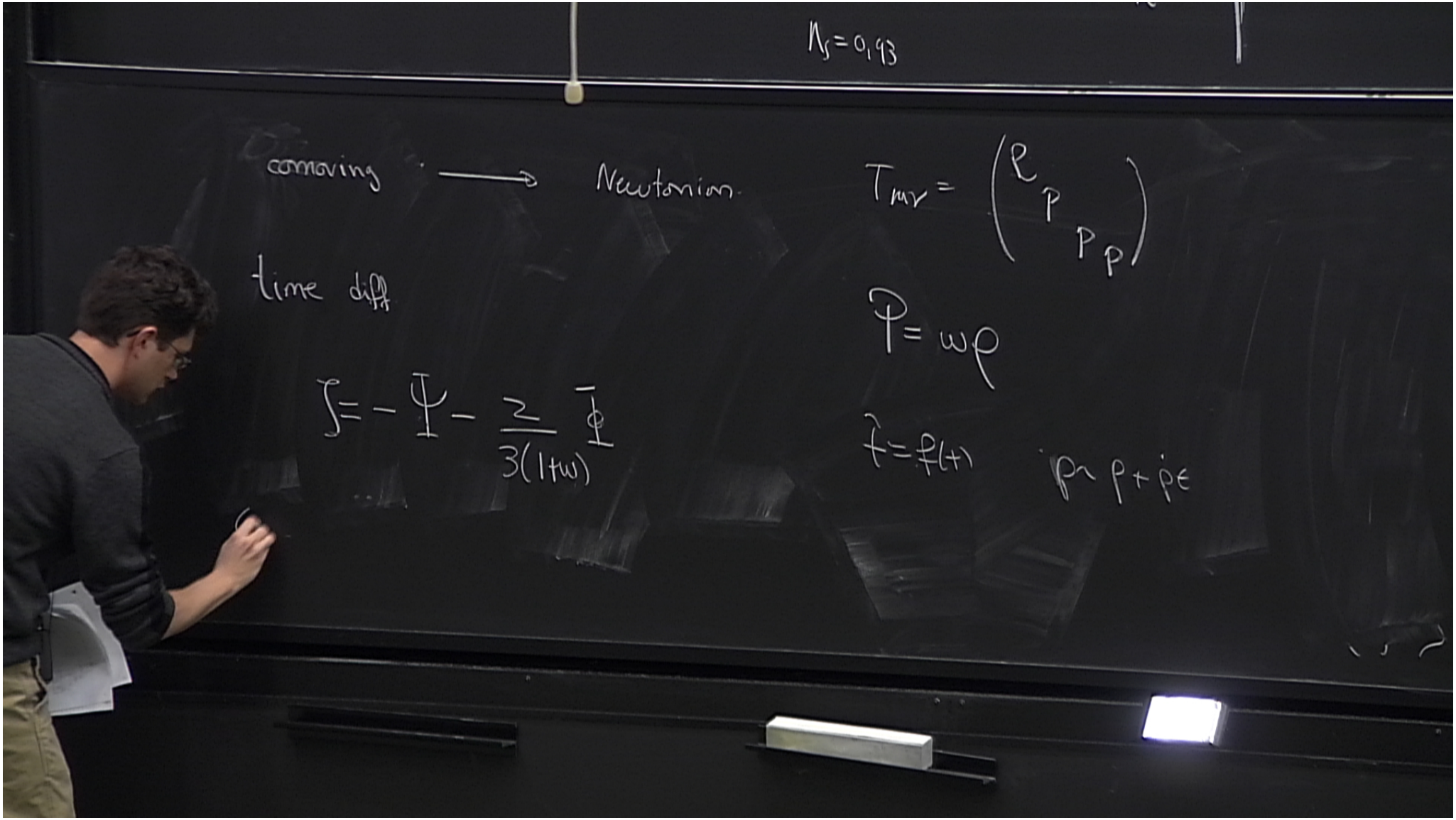
comoving gauge to spatially flat

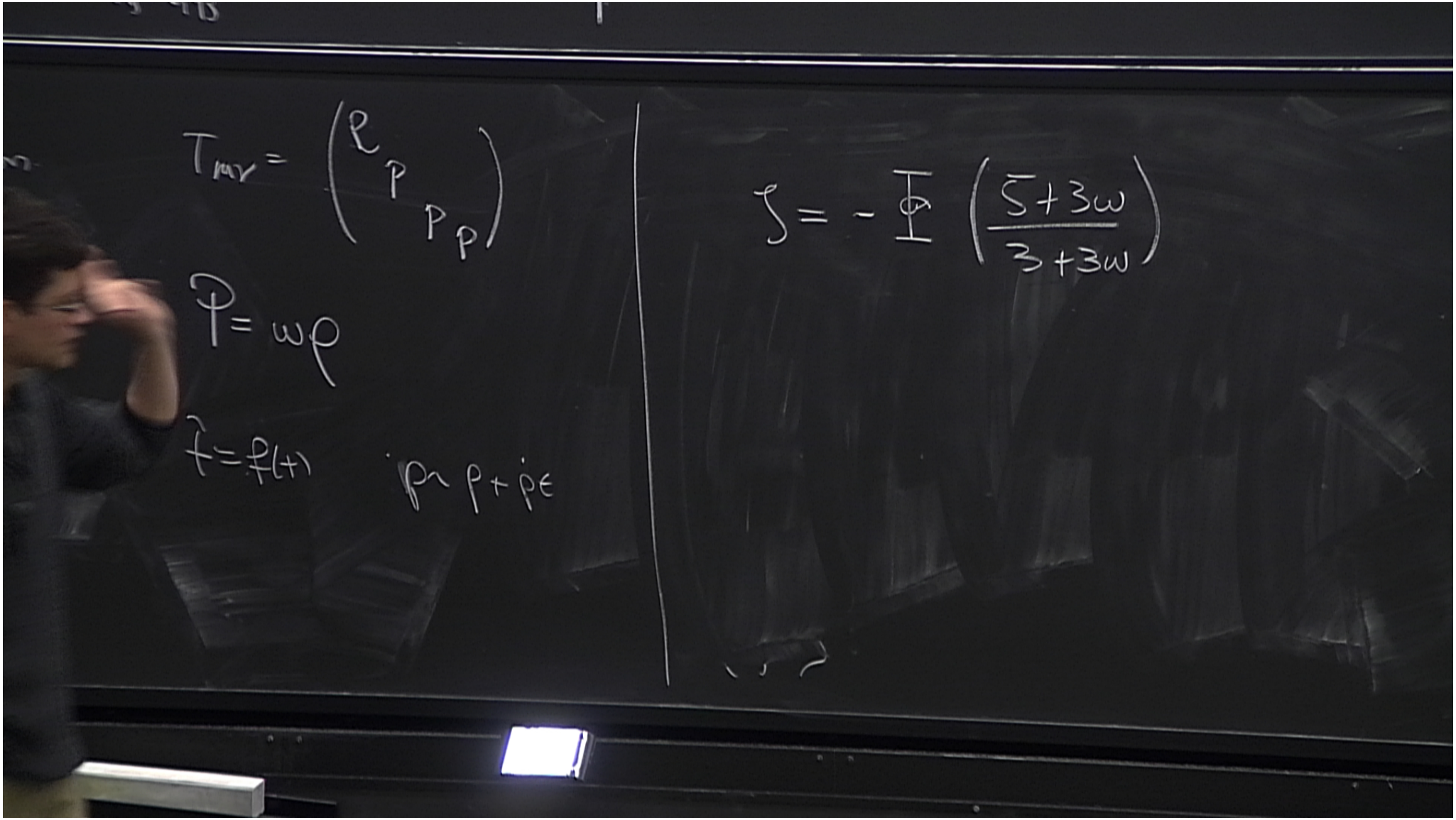
$$h_{ij} = e^{2Ht + 2\zeta}\,\delta_{ij} \qquad h_{ij} = e^{2H\tilde{t}}\,\delta_{ij}$$

$$\phi = \phi(t) \qquad \phi = \phi(\tilde{t}) + \delta\phi \simeq \dot{\phi}\left(-\frac{1}{H}\right)\zeta$$

$$\tilde{t} = t + \frac{\zeta}{H}$$

$$\boxed{\zeta = -\frac{H}{\dot{\phi}}\,\delta\phi}$$

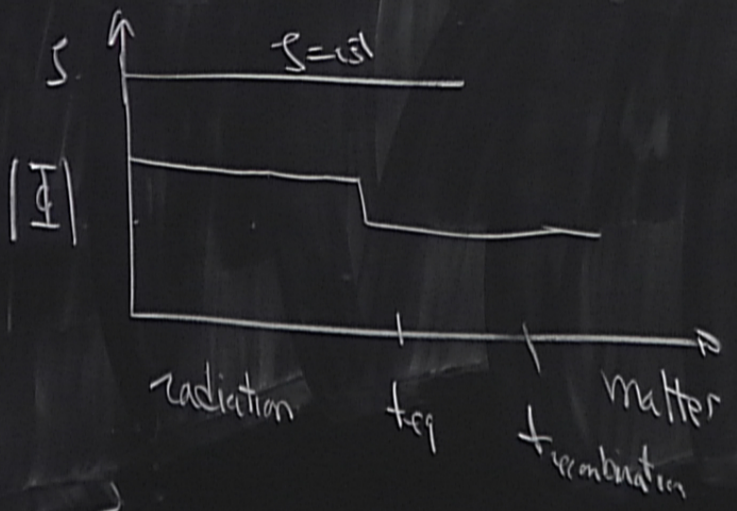

$n_s = 0.93$
comoving ⟶ Newtonian
$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$
time diff
$p = w\rho$
$\zeta = -\Psi - \frac{2}{3(1+w)}\frac{\dot{\Phi}}{\mathcal{H}}$
$\tilde{t} = f(t)$
$\rho \sim \rho + \dot{\rho}\epsilon$

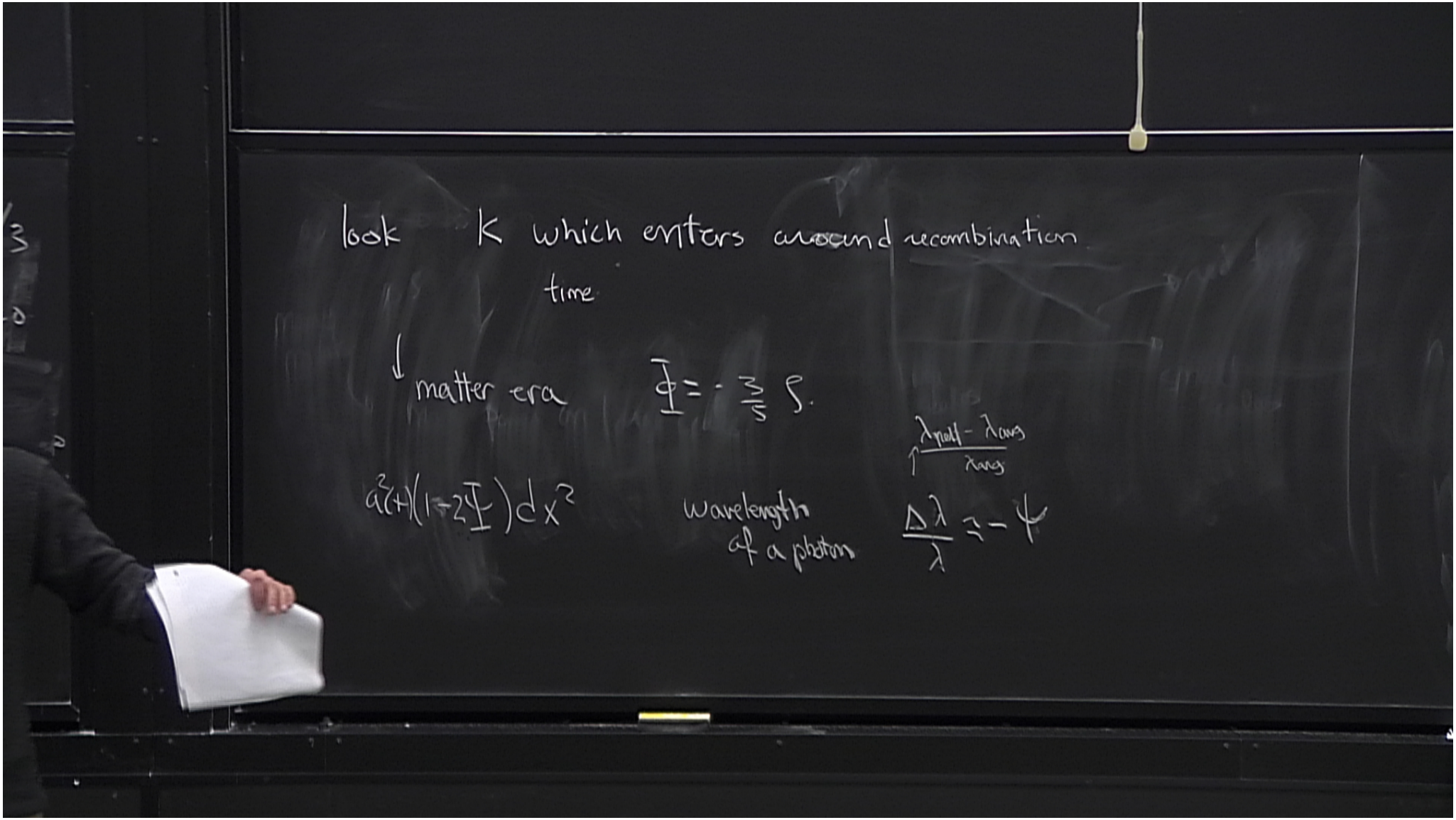

$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$
$p = w\rho$
$\dot{\rho} = f(t)$
$\rho \to \rho + \dot{\rho}\epsilon$
$\zeta = -\frac{5+3w}{3+3w}$

$$\zeta = -\Phi\left(\frac{5+3w}{3+3w}\right)$$

$\Phi =$ $-\frac{2}{3}\zeta$ · rad $w = 1/3$

$-\frac{3}{5}\zeta$ matter $w = 0$




look k which enters around recombination
time
matter era
$\dot{\Phi} = -\frac{3}{5}\Psi$.
$a^2(\tau)(1-2\Psi)dx^2$
wavelength of a photon
$\frac{\lambda_{now} - \lambda_{emit}}{\lambda_{emit}}$
$\frac{\Delta\lambda}{\lambda} \approx -\Psi$

look k which enters around recombination

time

↓ matter era $\Phi = -\frac{3}{5}\rho$.

$a^2(\tau)(1-2\Phi)dx^2$

$\frac{\lambda_{rec} - \lambda_{now}}{\lambda_{now}}$

wavelength of a photon

$\frac{\Delta\lambda}{\lambda} = -\psi$

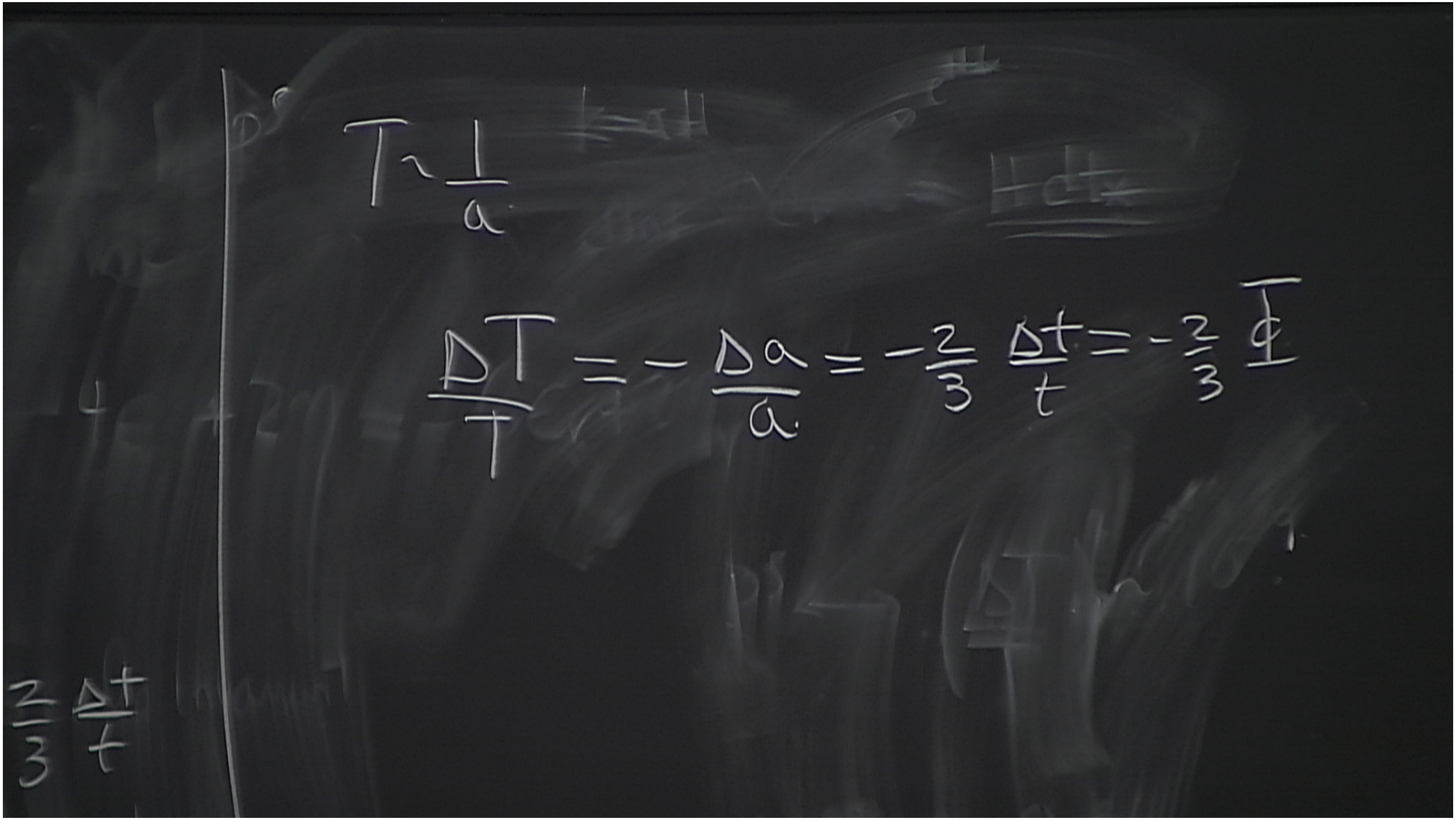
$\langle\Phi\rangle = \langle\psi\rangle = 0$

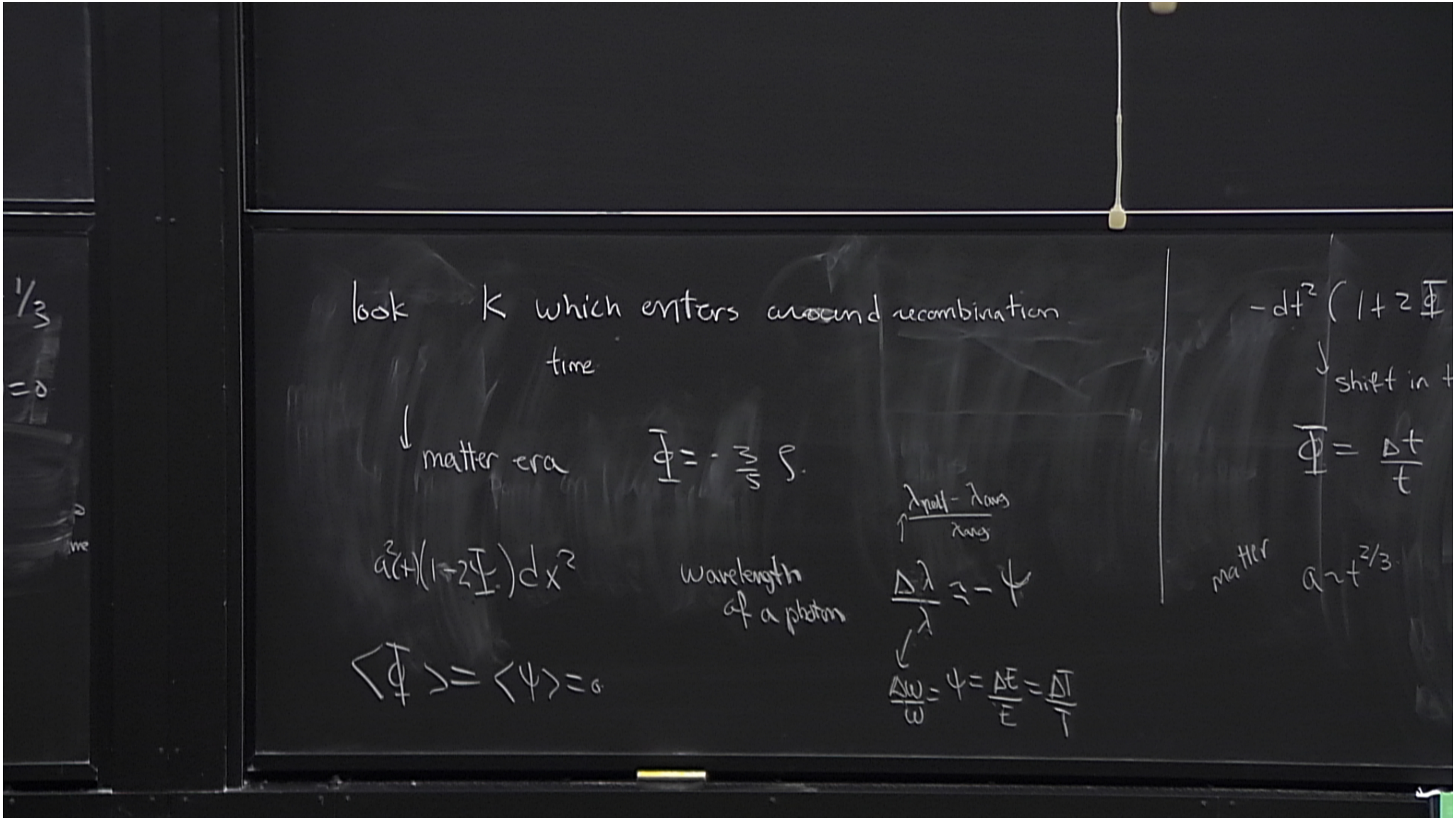
$-dt^2(1+2\Phi)$

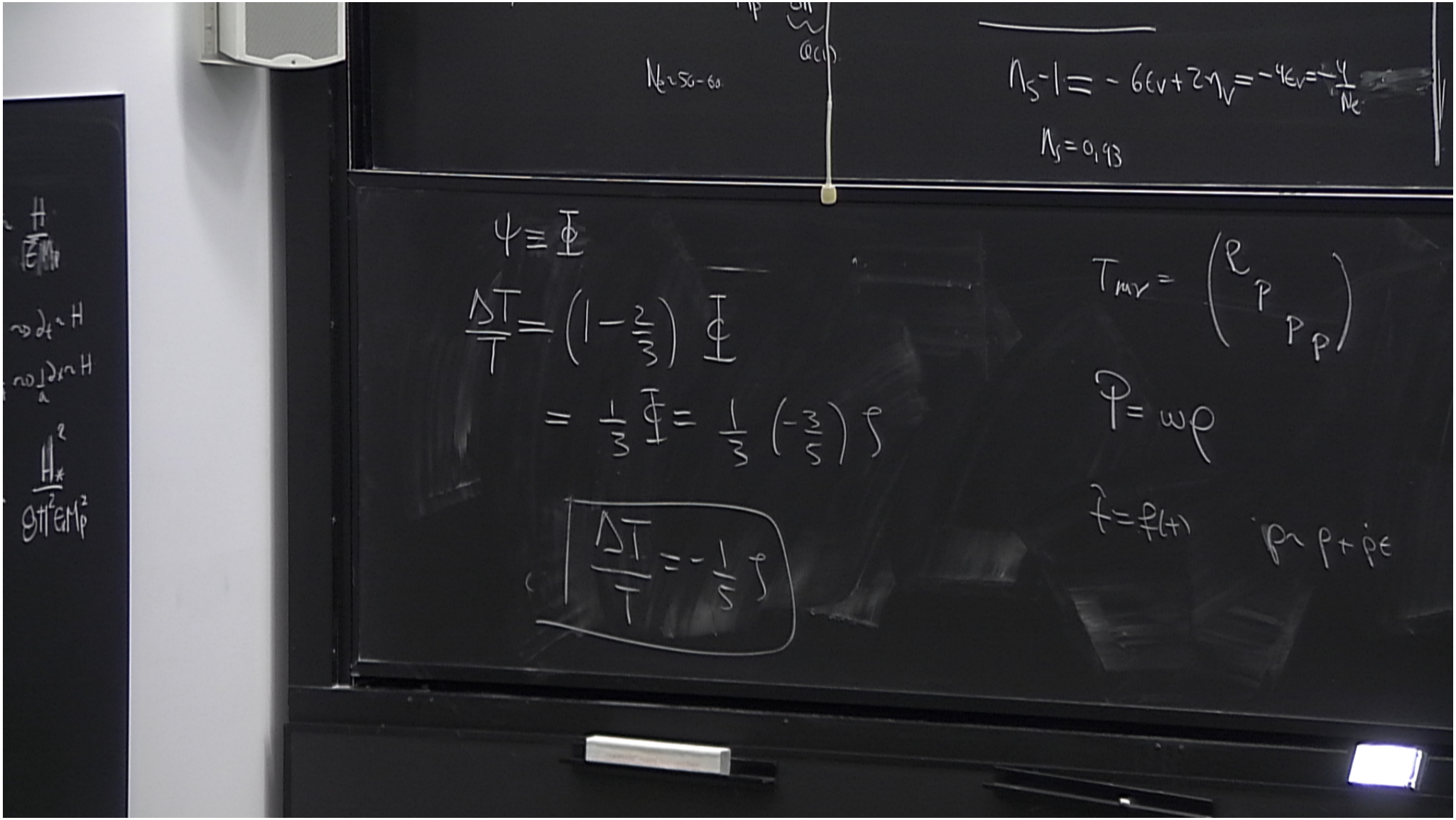
↓ shift in time

$\Phi = \frac{\Delta t}{t}$

matter $a \sim t^{2/3}$ $\frac{\Delta a}{a} = \frac{2}{3}\frac{\Delta t}{t}$


$T \sim \frac{1}{a}$
$\frac{\Delta T}{T} = -\frac{\Delta a}{a} = -\frac{2}{3}\frac{\Delta t}{t} = -\frac{2}{3}\frac{I}{C}$
$\frac{2}{3}\frac{\Delta t}{t}$


look K which enters around recombination
time
matter era
$\Phi = -\frac{3}{5}\delta$
$a^2(t)(1-2\Phi)dx^2$
wavelength of a photon
$\frac{\lambda_{now} - \lambda_{then}}{\lambda_{then}}$
$\frac{\Delta\lambda}{\lambda} \approx -\psi$
$\frac{\Delta\omega}{\omega} = \psi = \frac{\Delta E}{E} = \frac{\Delta T}{T}$
$\langle\Phi\rangle = \langle\psi\rangle = 0$
$-dt^2(1+2\Phi$
shift in t
$\Phi = \frac{\Delta t}{t}$
matter
$a \sim t^{2/3}$


$N_e \simeq 50-60$
$n_s - 1 = -6\epsilon_V + 2\eta_V = -4\epsilon_V = -\frac{4}{N_e}$
$n_s = 0,93$
$\Psi \equiv \Phi$
$\frac{\Delta T}{T} = \left(1 - \frac{2}{3}\right)\Phi$
$= \frac{1}{3}\Phi = \frac{1}{3}\left(-\frac{3}{5}\right)\zeta$
$\frac{\Delta T}{T} = -\frac{1}{5}\zeta$
$T_{\mu\nu} = \begin{pmatrix} \rho & & \\ & p & \\ & & p \end{pmatrix}$
$p = w\rho$
$\dot{f} = f(t)$
$\rho \sim \rho + \dot{\rho}\epsilon$