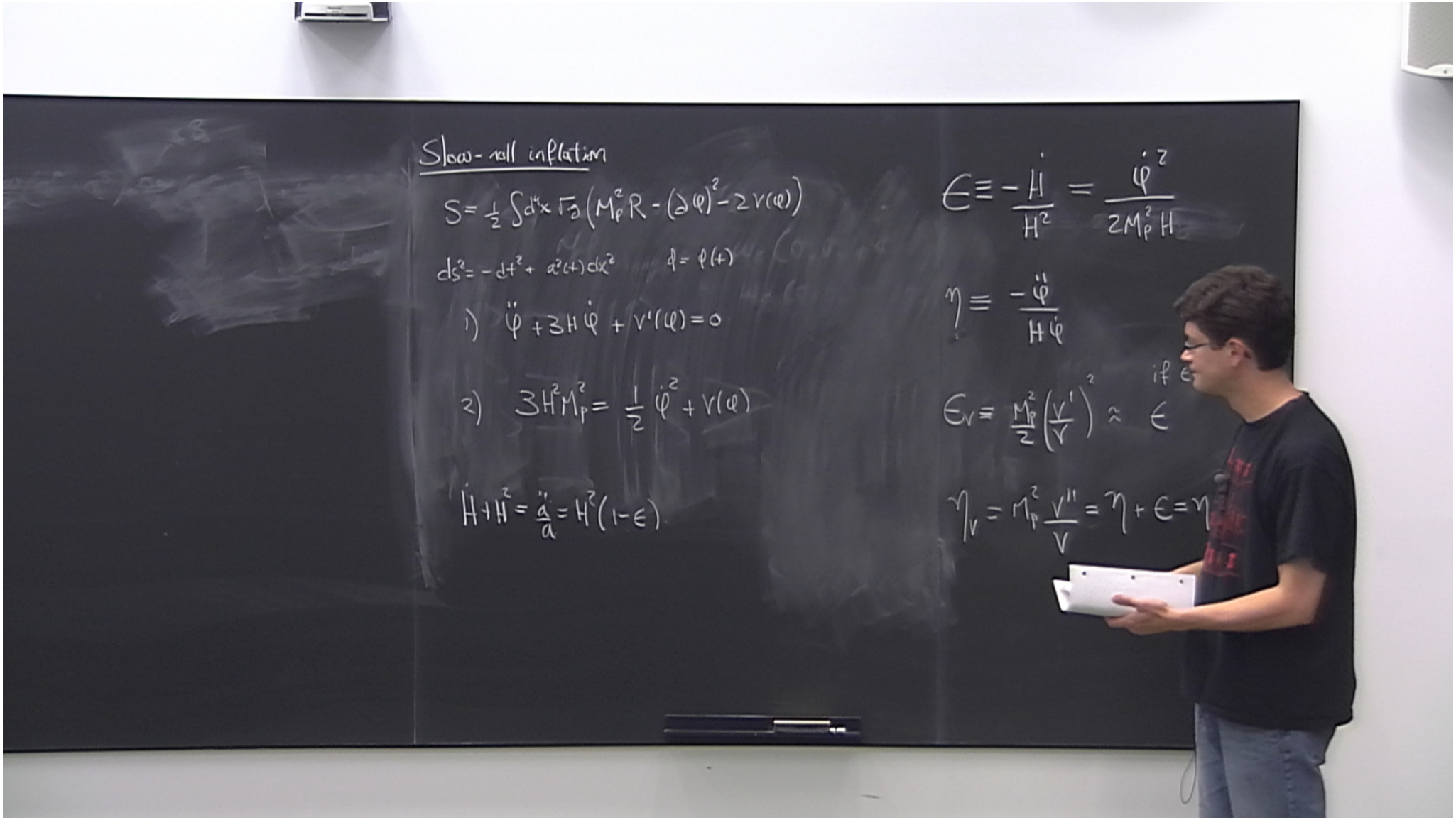
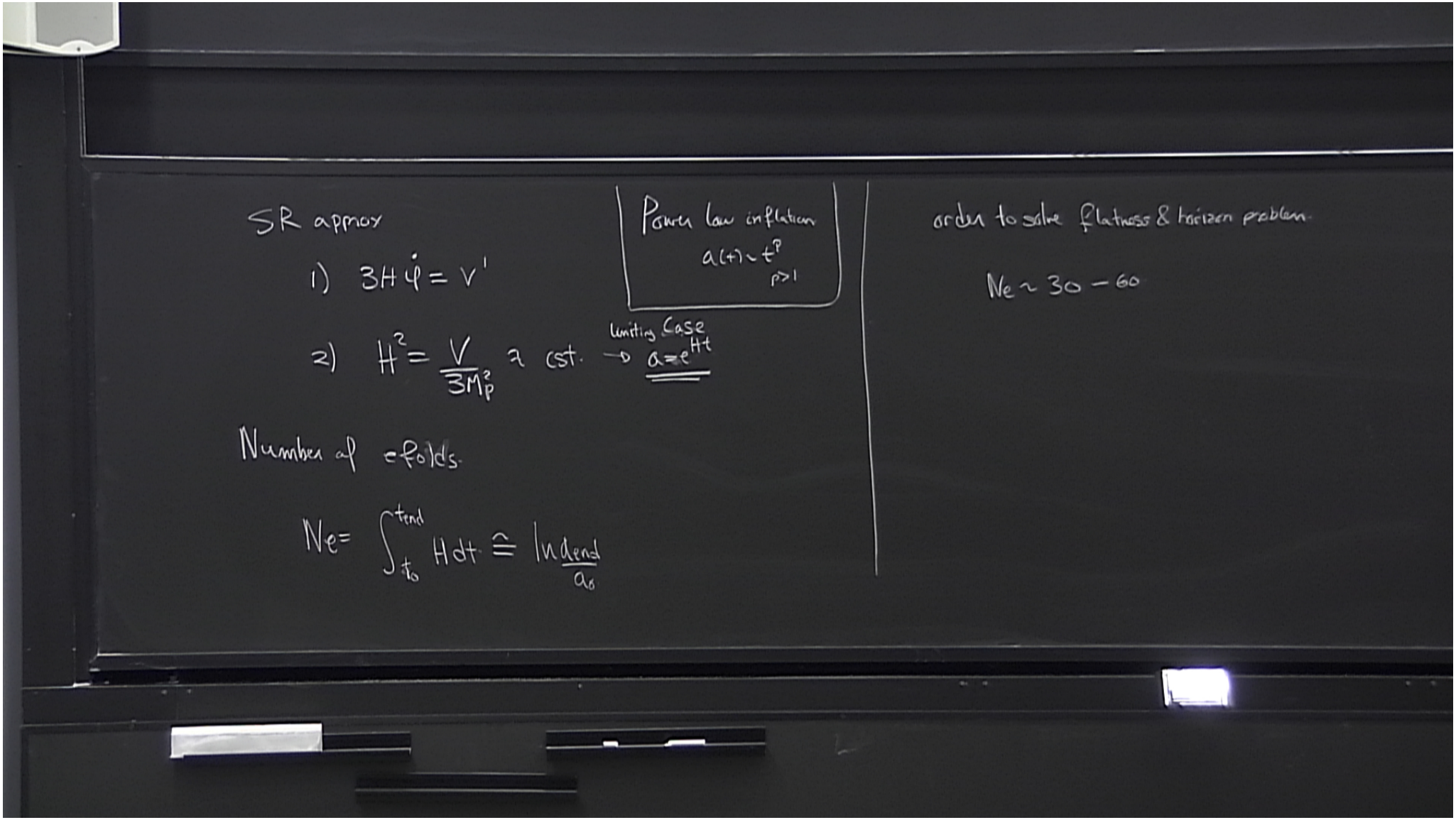
Title: Explorations in Cosmology - Lecture 7

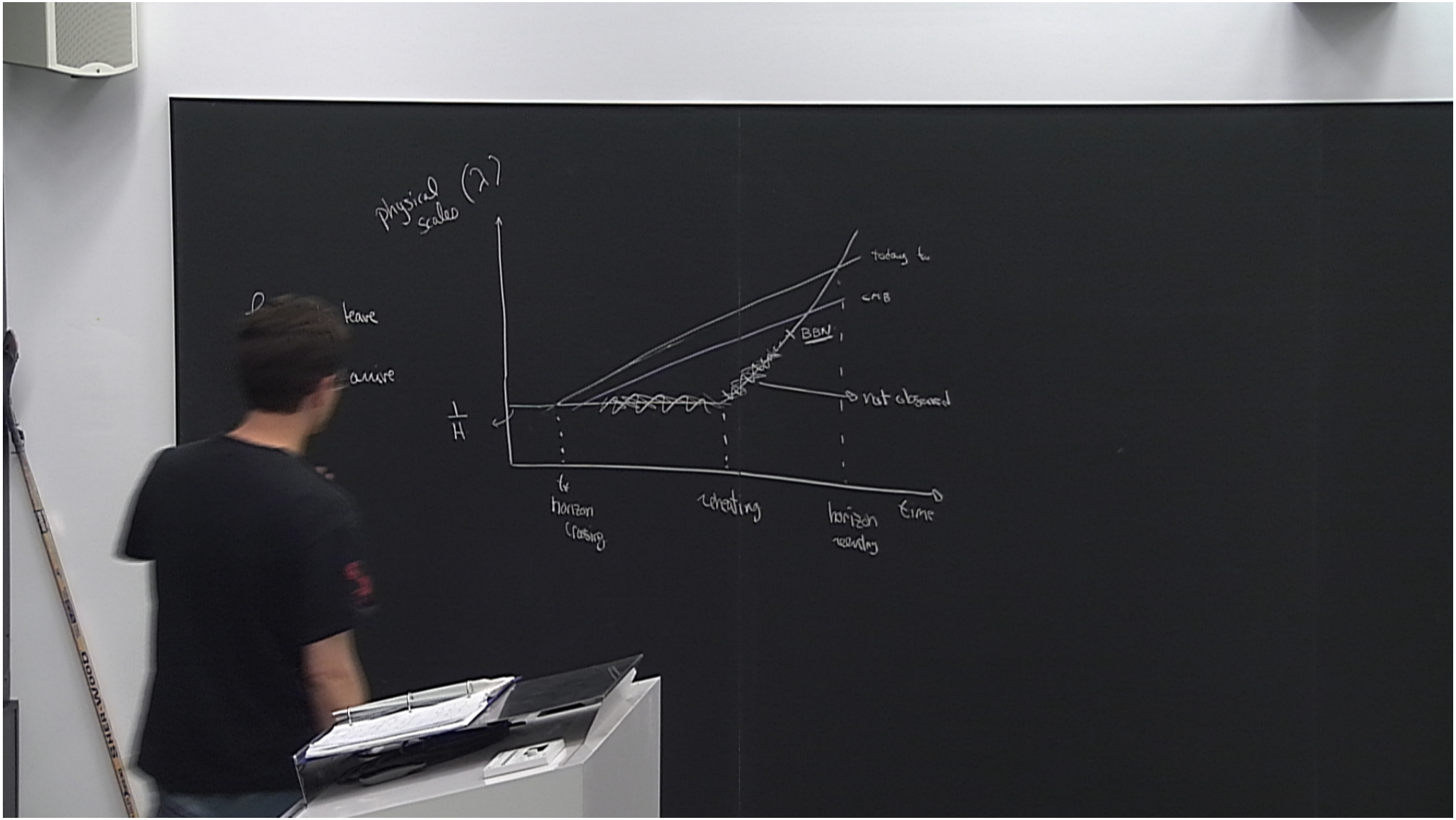
Date: Mar 20, 2012 10:15 AM

URL: http://pirsa.org/12030031

Abstract:


Slow-roll inflation
$S = \frac{1}{2}\int d^4x \sqrt{-g}\left(M_p^2 R - (\partial\varphi)^2 - 2V(\varphi)\right)$
$ds^2 = -dt^2 + a^2(t)\,dx^2$ $\quad \varphi = \varphi(t)$
1) $\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$
2) $3H^2M_p^2 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$\dot{H} + H^2 = \frac{\ddot{a}}{a} = H^2(1-\epsilon)$
$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H}$
$\eta \equiv -\frac{\ddot{\varphi}}{H\dot{\varphi}}$
$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \approx \epsilon$ if ϵ
$\eta_V = M_p^2\frac{V''}{V} = \eta + \epsilon = \eta$


SR approx
1) $3H\dot{\varphi} = V'$
2) $H^2 = \frac{V}{3M_p^2}$ ≈ cst. → limiting case $a = e^{Ht}$
Power law inflation
$a(t) \sim t^p$
$p > 1$
Number of efolds
$N_e = \int_{t_0}^{t_{end}} H dt \cong \ln \frac{a_{end}}{a_0}$
order to solve flatness & horizon problem
$N_e \sim 30 - 60$


Physical scales (λ)
today t_0
CMB
BBN
not observed
H⁻¹
leave
t_k horizon crossing
reentering
horizon reentry
Time

Power law inflation

$a(t) \sim t^p$

$p > 1$

iting Case

$a = e^{Ht}$

order to solve flatness & horizon problem.

$N_e \sim 3$

e.g Chaot ation $V(\varphi) = \frac{m^2\varphi^2}{2}$

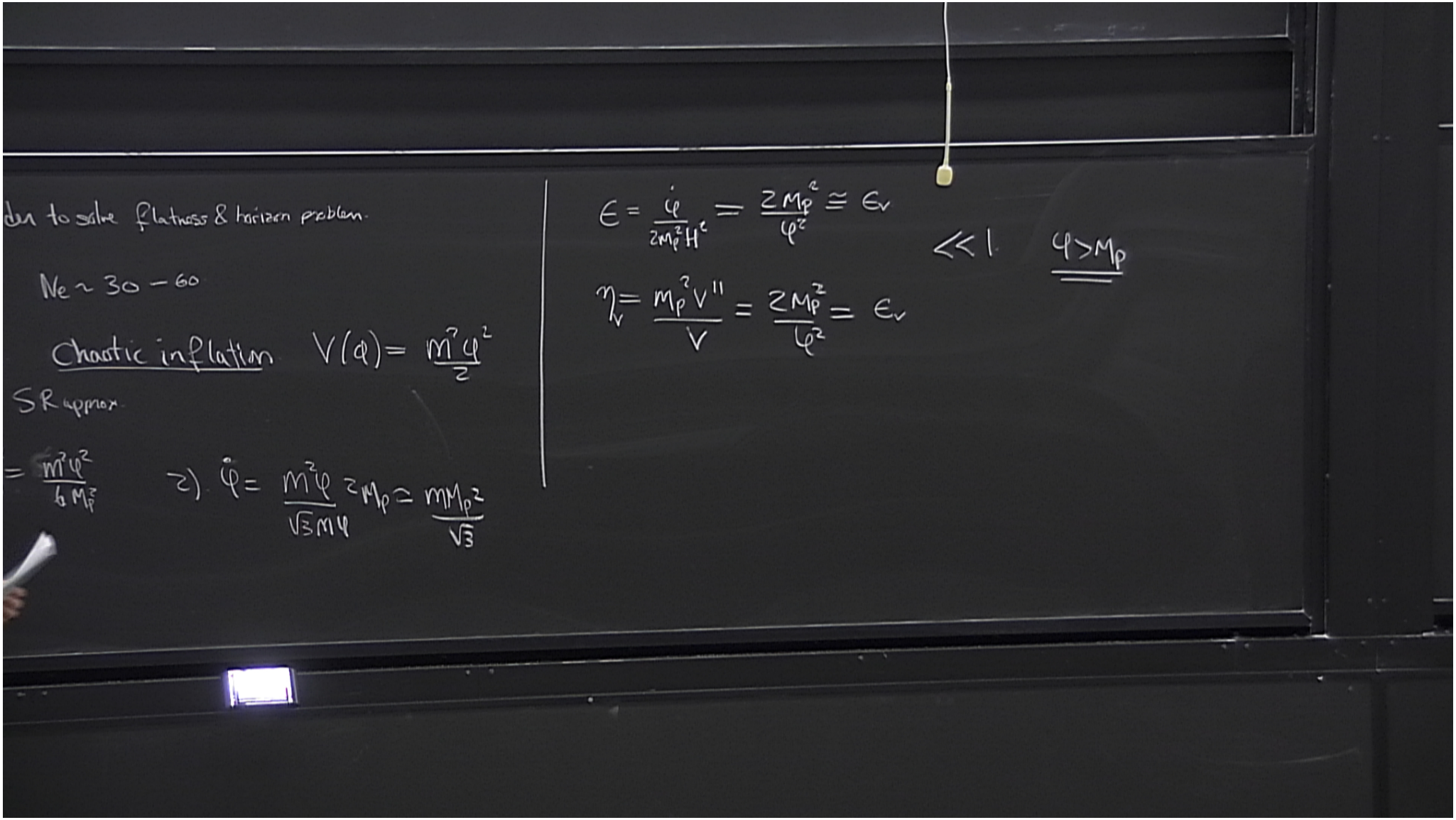
SR approx.

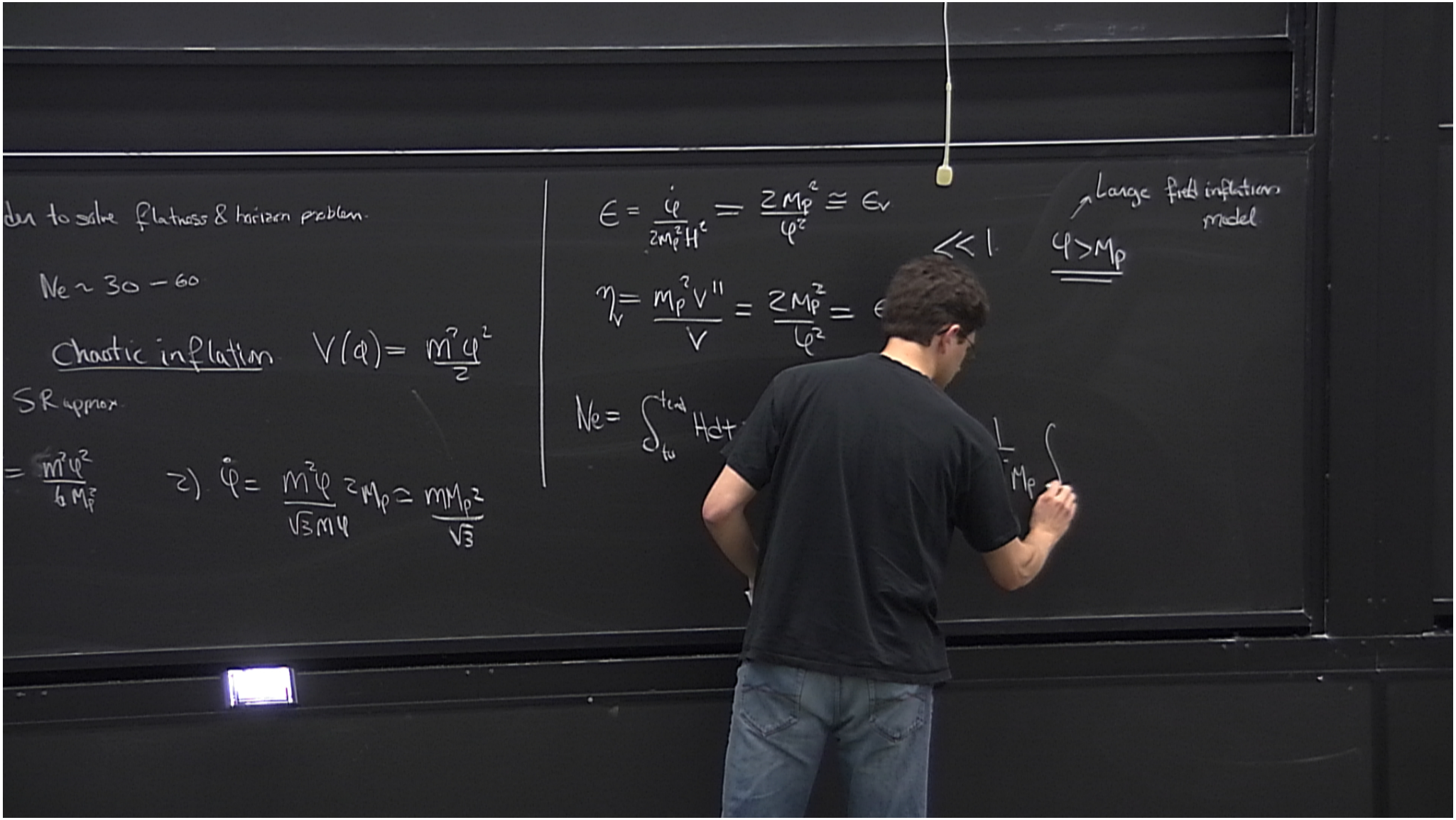
1) $H^2 = \frac{m^2\varphi^2}{6M_p^2}$

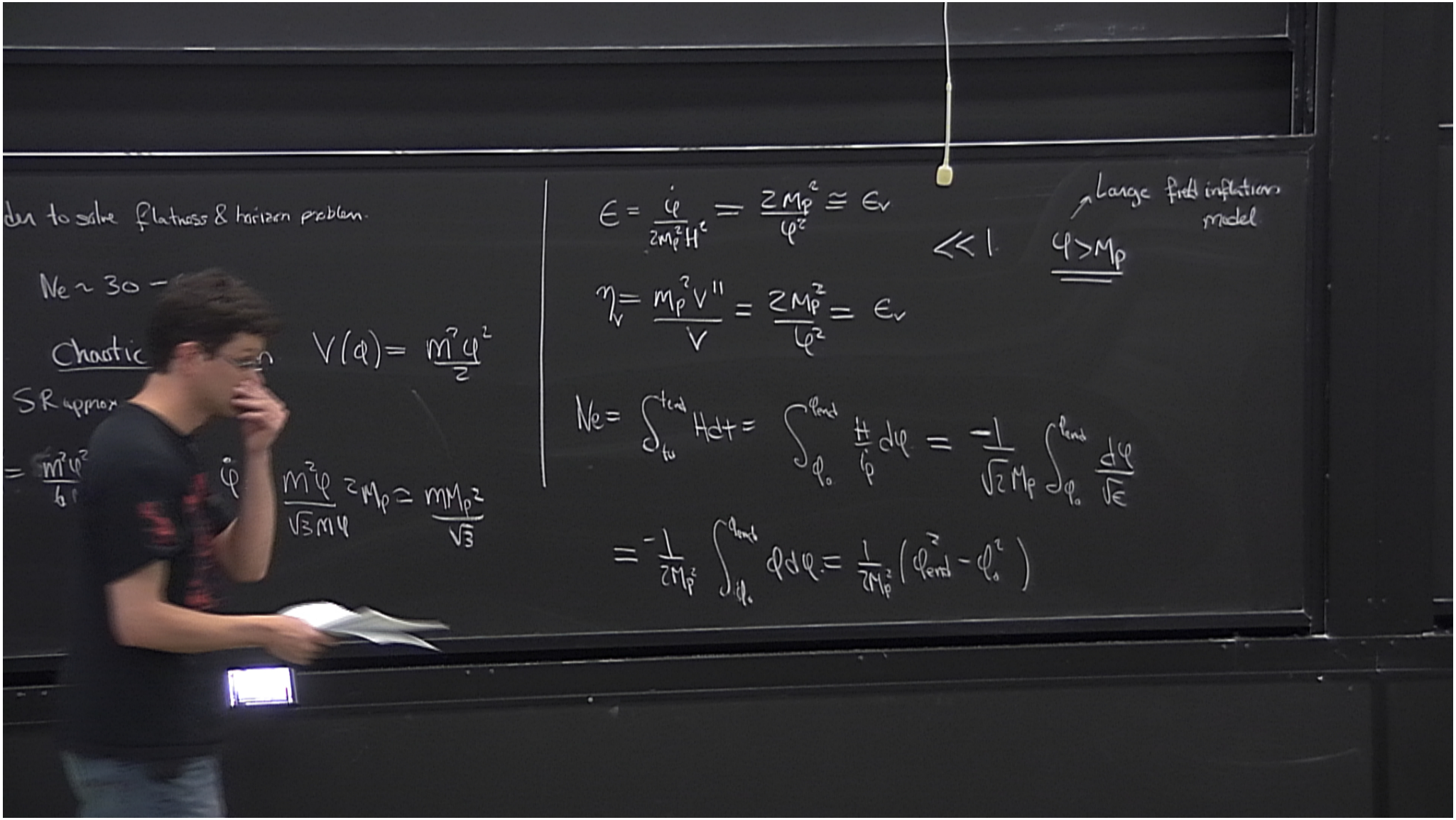
$\frac{m^2\varphi}{\sqrt{3}m\varphi} 2M_p \simeq \frac{mM_p^2}{\sqrt{3}}$

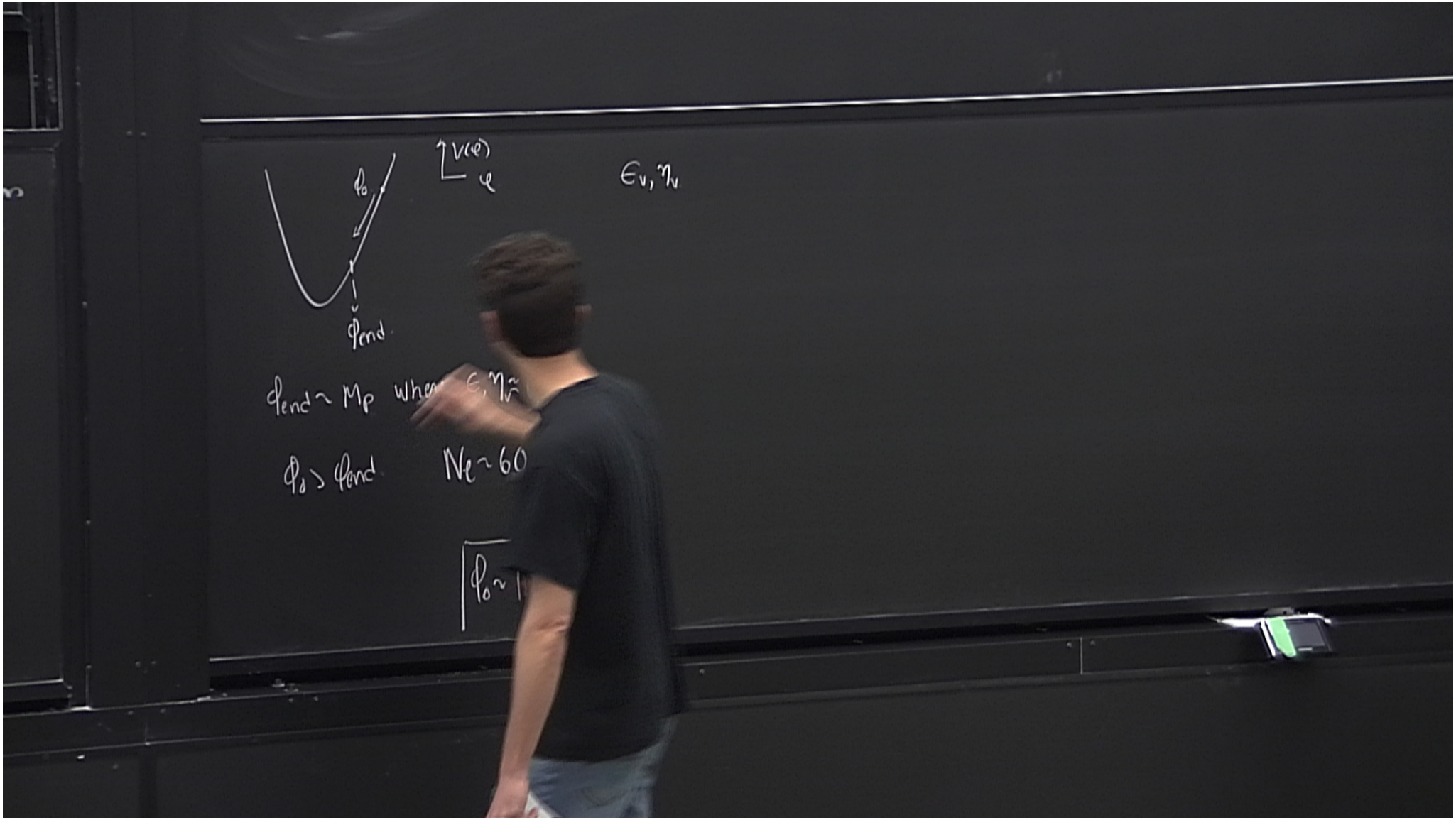
$\epsilon = \frac{\dot{\varphi}}{2M_p^2 H^2}$

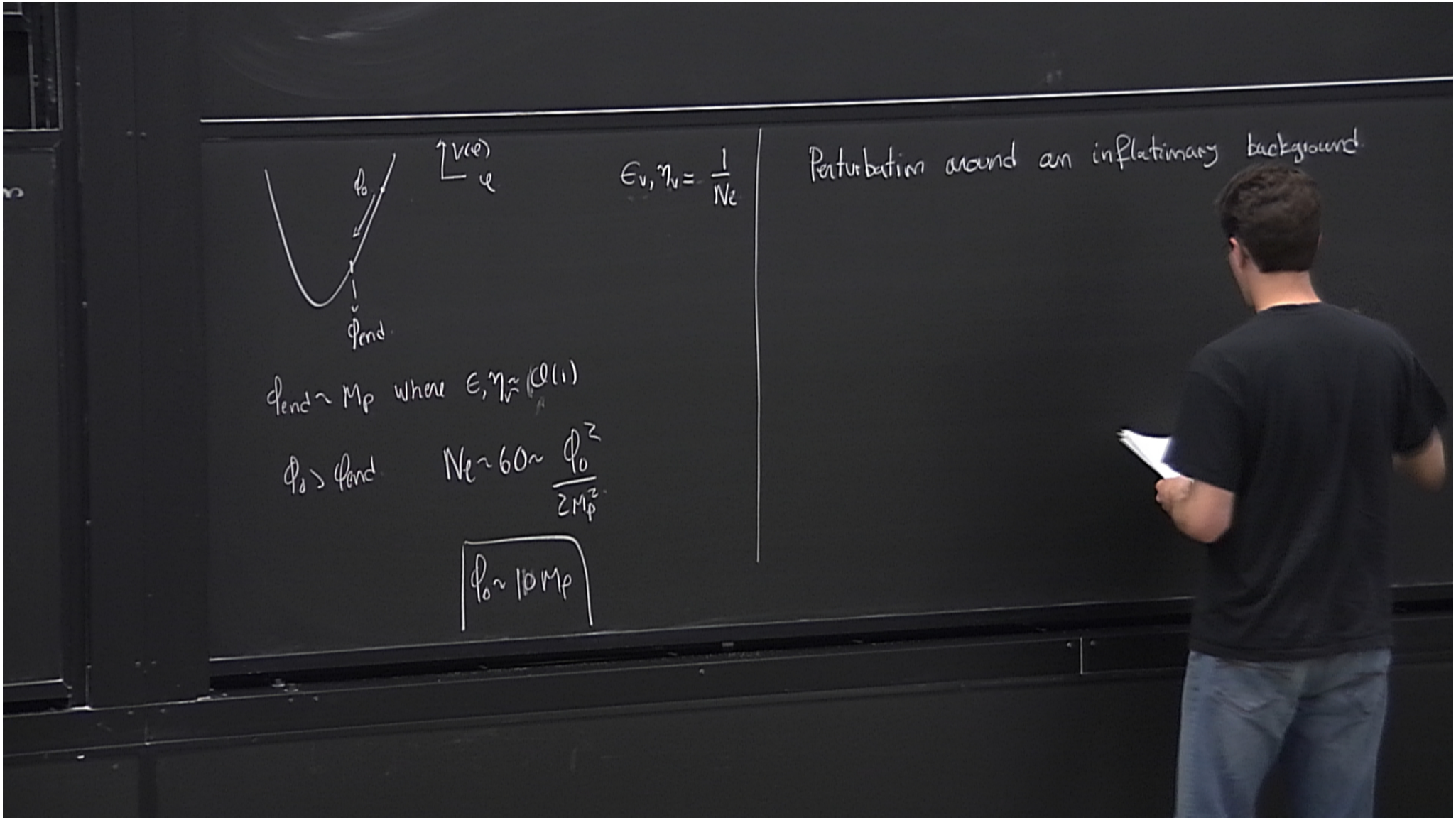
η

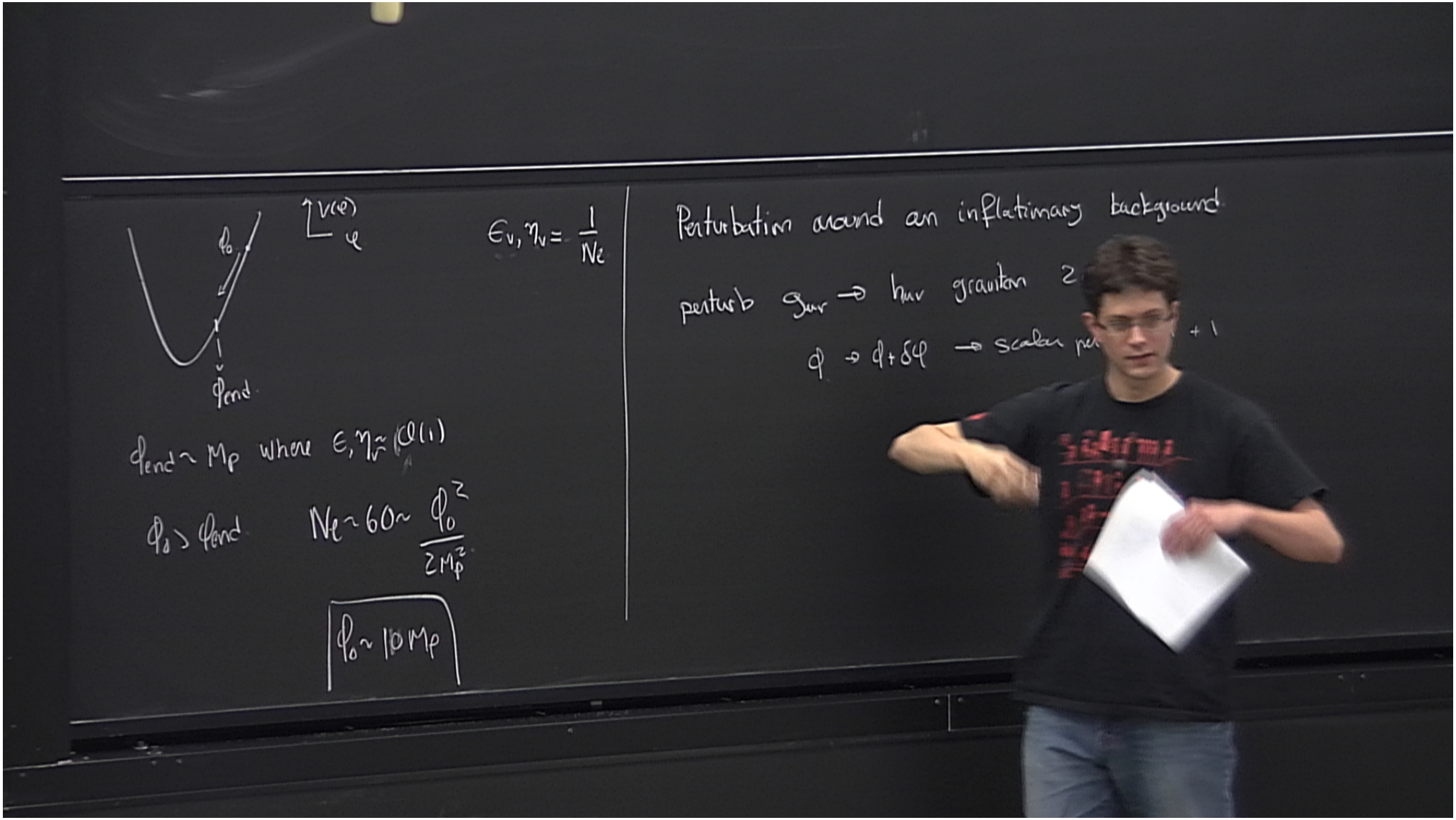

to solve flatness & horizon problem.
$N_e \sim 30 - 60$
Chaotic inflation $V(\varphi) = \frac{m^2\varphi^2}{2}$
SR approx
$= \frac{m^2\varphi^2}{6M_p^2}$
2) $\dot{\varphi} = \frac{m^2\varphi}{\sqrt{3}M\varphi} \; 2M_p \simeq \frac{mM_p^2}{\sqrt{3}}$
$\epsilon = \frac{\dot{\varphi}}{2M_p^2H^2} = \frac{2M_p^2}{\varphi^2} \cong \epsilon_v$
$\ll 1$ $\varphi > M_p$
$\eta_v = \frac{M_p^2 V''}{V} = \frac{2M_p^2}{\varphi^2} = \epsilon_v$

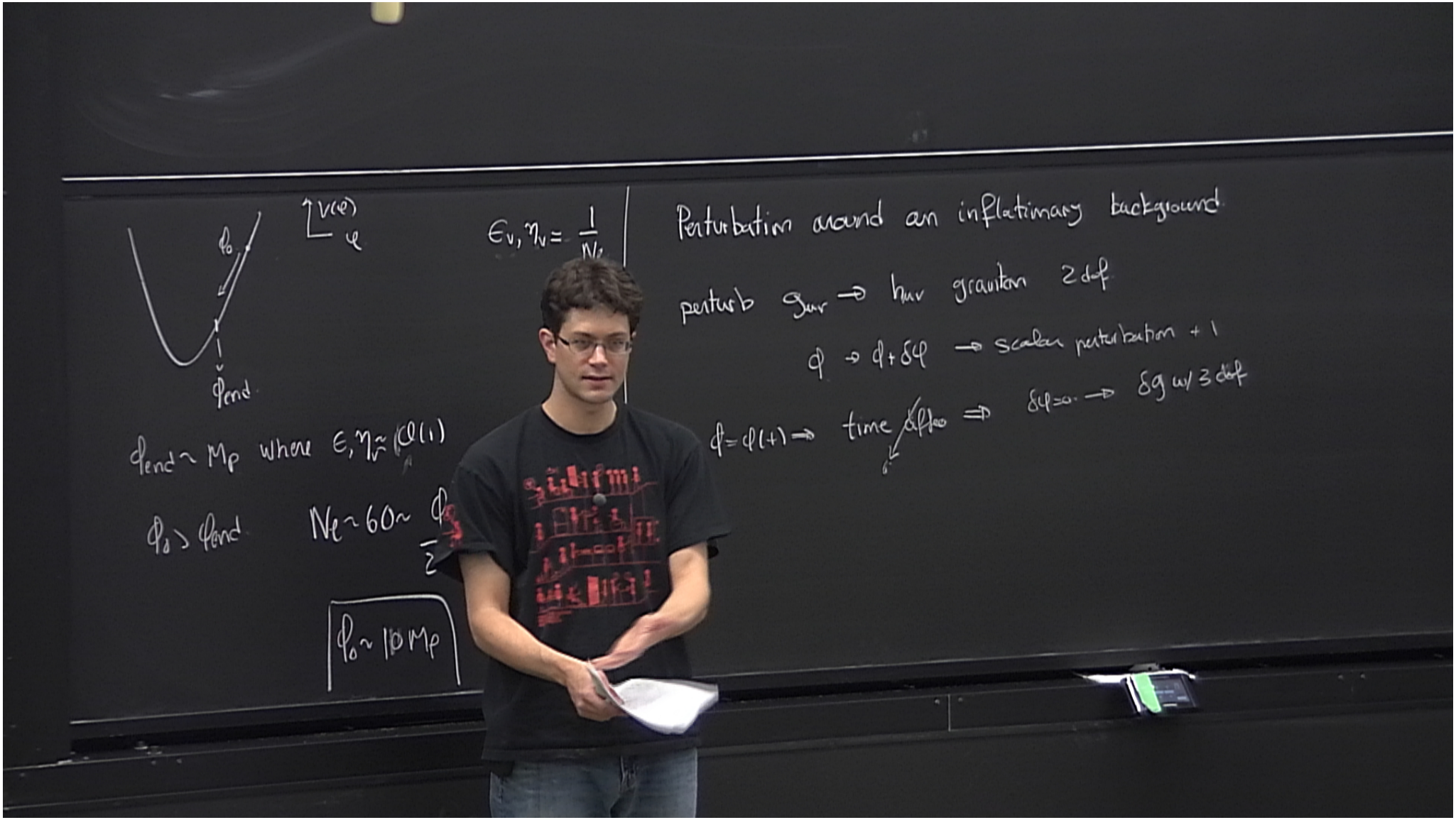

to solve flatness & horizon problem.
Ne ~ 30 − 60
Chaotic inflation V(φ) = m²φ²/2
SR approx
= m²φ²/6M_P²
2) φ̇ = m²φ/√3M_P ≃ mM_P²/√3
ε = φ̇/2M_P²H² = 2M_P²/φ² ≅ ε_V
<< 1
Large field inflation model
φ > M_P
η_V = M_P²V''/V = 2M_P²/φ² = ε
Ne = ∫ H dt
1/M_P ∫

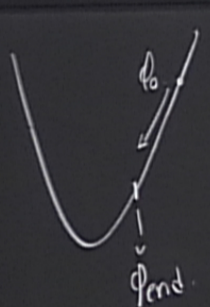

to solve flatness & horizon problem.
Ne ~ 30 –
Chaotic
SR approx
$V(\varphi) = \frac{m^2\varphi^2}{2}$
$\epsilon = \frac{\dot\varphi}{2M_p^2H^2} = \frac{2M_p^2}{\varphi^2} \equiv \epsilon_V$
$\ll 1$
Large field inflation model
$\varphi > M_p$
$\eta_V = \frac{M_p^2 V''}{V} = \frac{2M_p^2}{\varphi^2} = \epsilon_V$
$Ne = \int_{t_0}^{t_{end}} H dt = \int_{\varphi_0}^{\varphi_{end}} \frac{H}{\dot\varphi} d\varphi = \frac{-1}{\sqrt{2}M_p} \int_{\varphi_0}^{\varphi_{end}} \frac{d\varphi}{\sqrt{\epsilon}}$
$= \frac{-1}{2M_p^2} \int_{\varphi_0}^{\varphi_{end}} \varphi d\varphi = \frac{1}{2M_p^2}(\varphi_{end}^2 - \varphi_0^2)$


V(φ)
φ
ϵ_v, η_v
ϕ_0
ϕ_{end}
$\phi_{end} \sim M_P$ when $\epsilon, \eta \sim$
$\phi_0 > \phi_{end}$
$N_e \sim 60$
$\phi_0 \sim$


V(φ)
φ
φ0
φend
$\epsilon_v, \eta_v = \frac{1}{Ne}$
Perturbation around an inflationary background
$\phi_{end} \sim M_p$ where $\epsilon, \eta \simeq \mathcal{O}(1)$
$\phi_0 > \phi_{end}$
$Ne \sim 60 \sim \frac{\phi_0^2}{2M_p^2}$
$\phi_0 \sim 10 M_p$


V(φ)
φ
φ0
φend
$\epsilon_v, \eta_v = \frac{1}{Ne}$
$\phi_{end} \sim M_p$ where $\epsilon, \eta \sim \mathcal{O}(1)$
$\phi_0 > \phi_{end}$ $Ne \sim 60 \sim \frac{\phi_0^2}{2M_p^2}$
$\phi_0 \sim 10 M_p$
Perturbation around an inflationary background
perturb $g_{\mu\nu} \rightarrow h_{\mu\nu}$ graviton
$\phi \rightarrow \phi + \delta\phi$ → scalar


V(φ)
φ
φ0
φend
$\epsilon_v, \eta_v = \frac{1}{N_e}$
Perturbation around an inflationary background
perturb $g_{\mu\nu} \to h_{\mu\nu}$ graviton 2 dof
$\varphi \to \varphi + \delta\varphi$ → scalar perturbation + 1
$\varphi = \varphi(t) \Rightarrow$ time diffeo $\Rightarrow$ $\delta\varphi = 0$ → δg w/ 3 dof
$\varphi_{end} \sim M_p$ where $\epsilon, \eta \sim \mathcal{O}(1)$
$\varphi_0 > \varphi_{end}$ $N_e \sim 60 \sim$
$\varphi_0 \sim 10 M_p$



$\epsilon_v, \eta_v = \frac{1}{Ne}$

$\phi_{end} \sim M_p$ where $\epsilon, \eta \sim \mathcal{O}(1)$

$\phi_0 > \phi_{end}$ $\quad Ne \sim 60 \sim \frac{\phi_0^2}{2M_p^2}$

$\boxed{\phi_0 \sim 10 M_p}$

Perturbation around an inflationary background

perturb $g_{\mu\nu} \rightarrow h_{\mu\nu}$ graviton 2 dof

$\phi \rightarrow \phi + \delta\phi$ $\rightarrow$ scalar perturbation + 1

$\phi = \phi(t) \Rightarrow$ time ~~diffeo~~ $\Rightarrow$ $\delta\phi = 0$ $\rightarrow$ $\underline{\delta\phi \text{ w/ } 3 \text{ dof}}$

$\epsilon_v, \eta_v = \frac{1}{Ne}$

Perturbation around an inflationary background

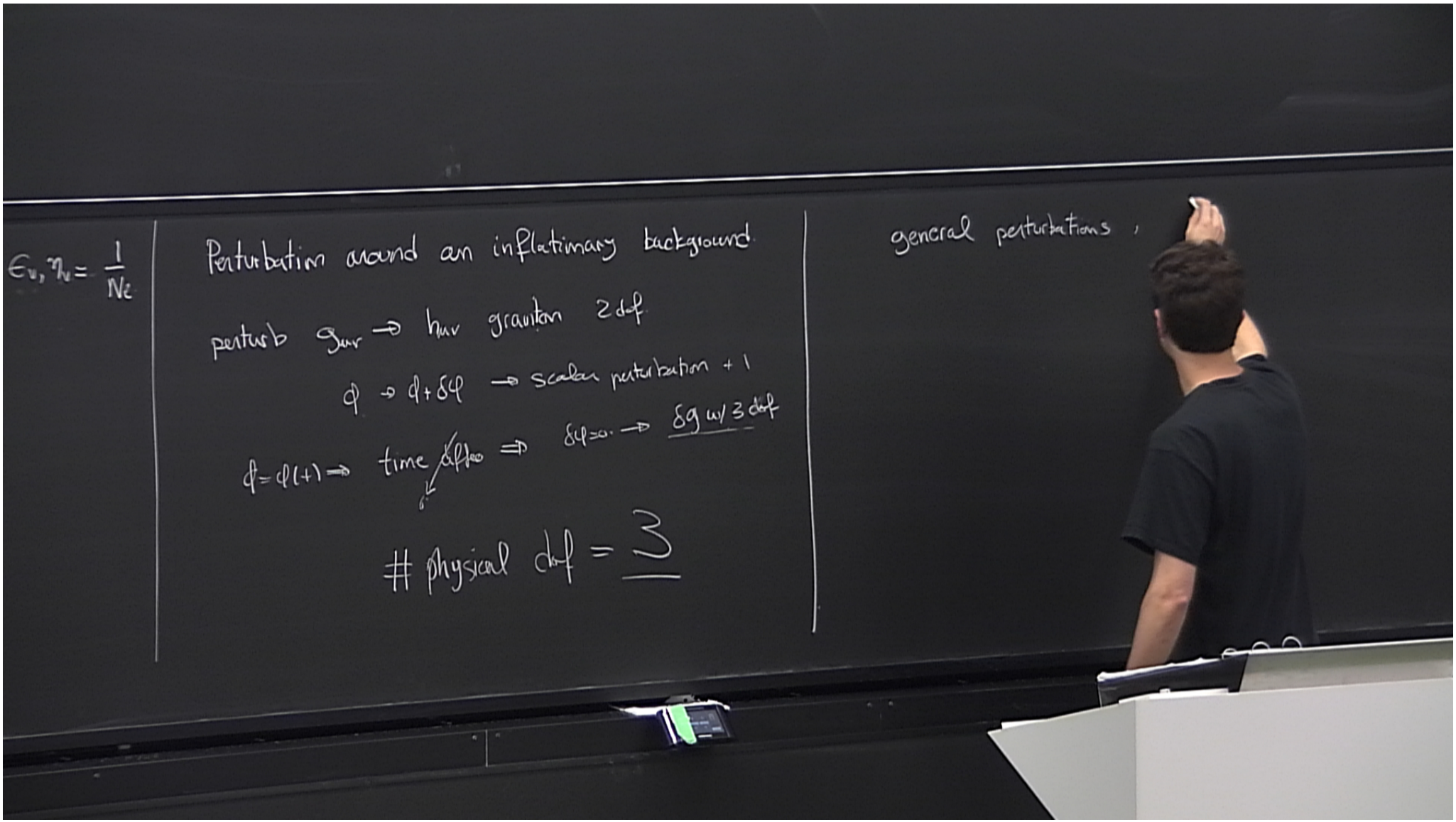
perturb $g_{\mu\nu} \rightarrow h_{\mu\nu}$ graviton 2 dof

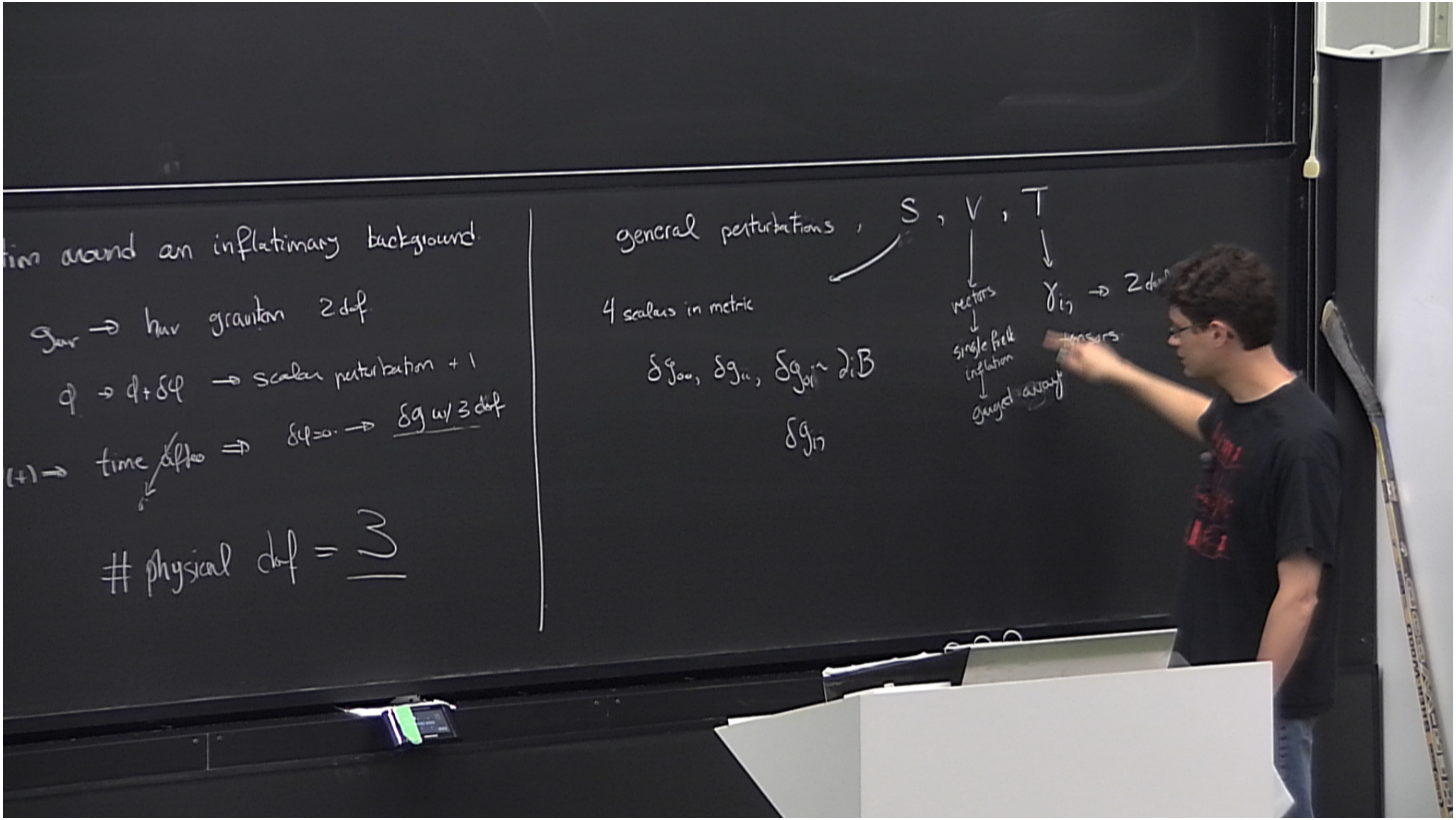
$\varphi \rightarrow \varphi + \delta\varphi$ → scalar perturbation + 1

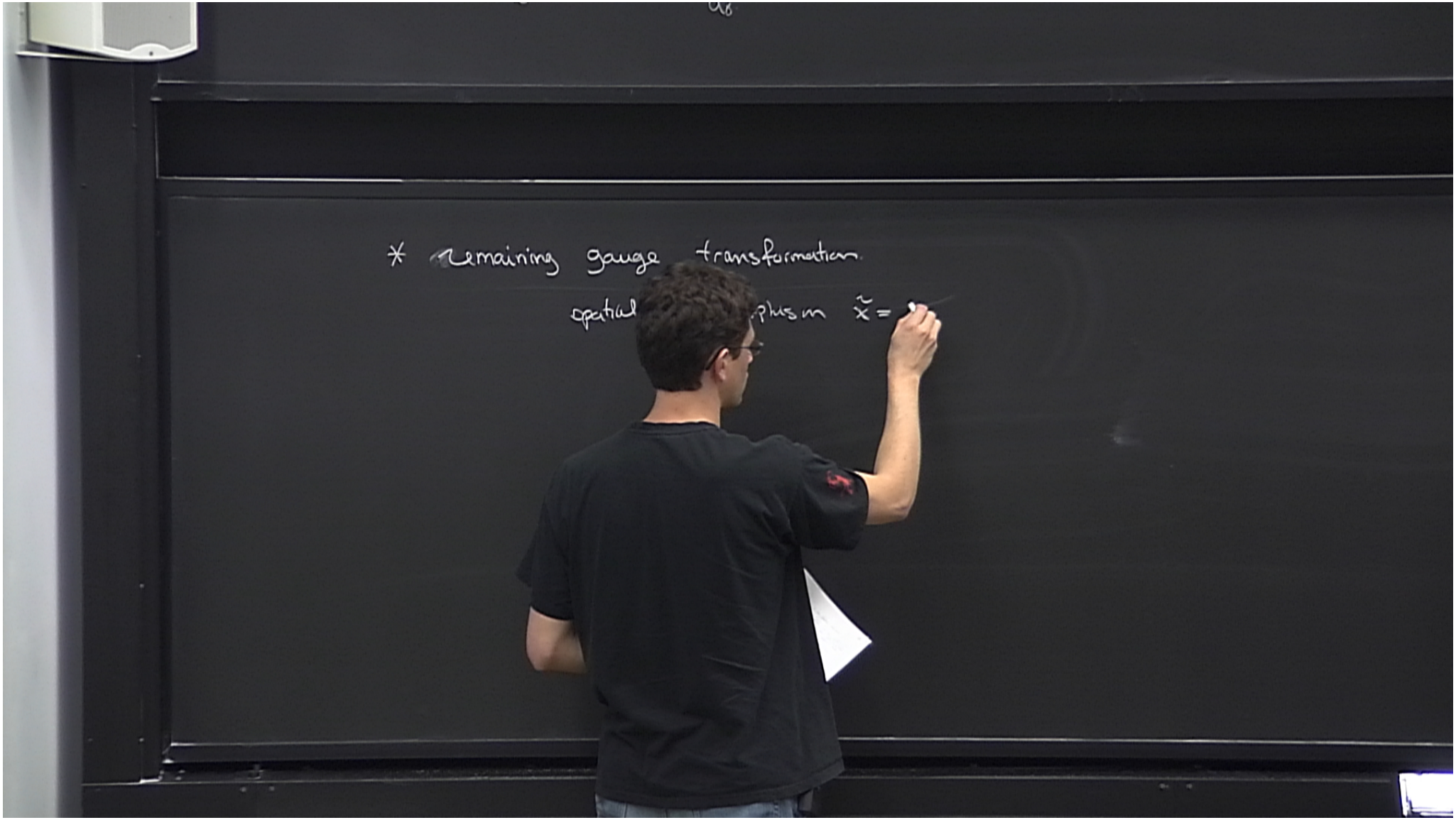
$\varphi = \varphi(t) \rightarrow$ time diffeo $\Rightarrow \delta\varphi = 0 \rightarrow$ δg w/ 3 dof

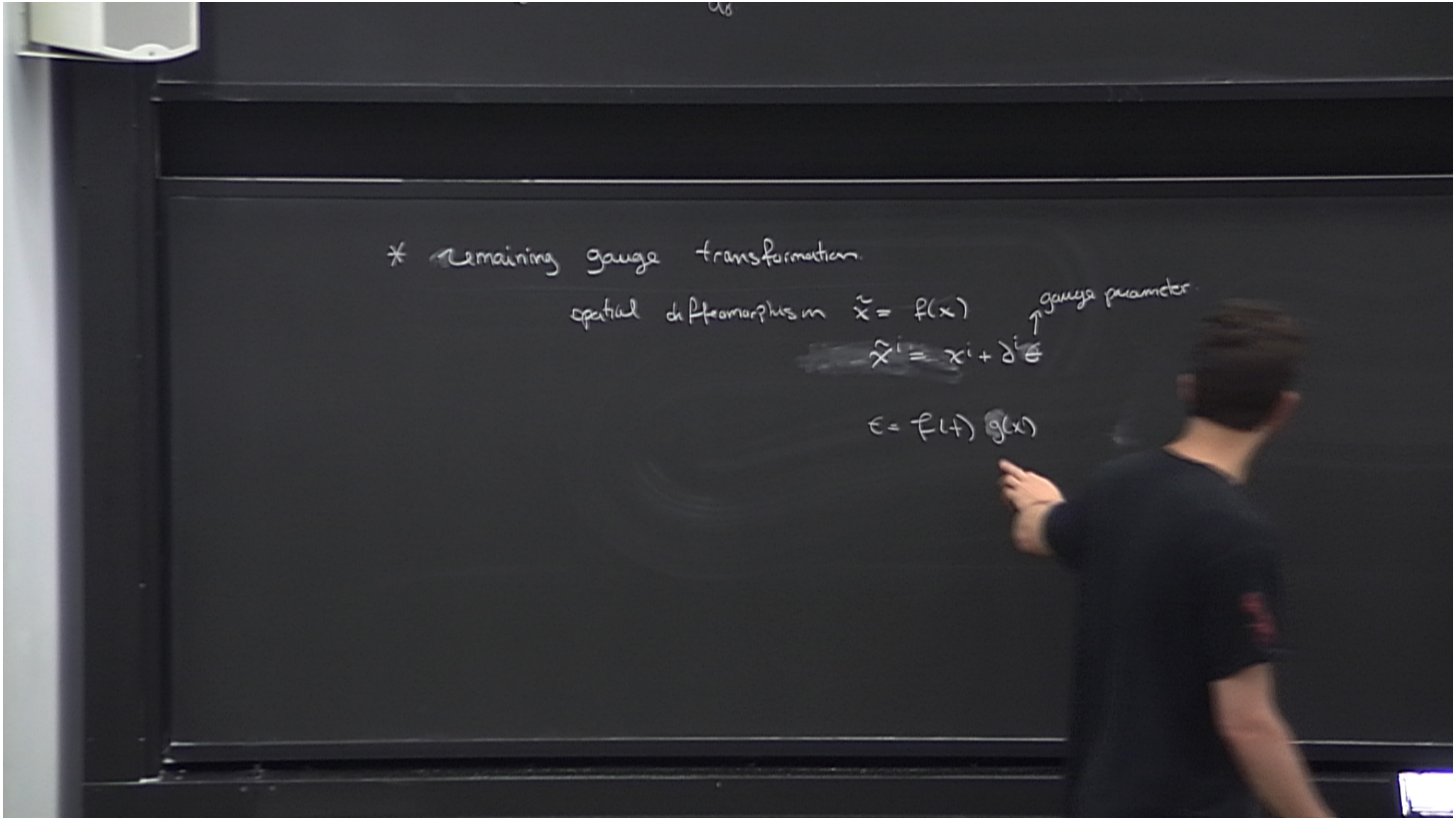
physical dof = 3

general perturbations,




tion around an inflationary background
$g_{\mu\nu}$ → $h_{\mu\nu}$ graviton 2 dof
φ → $\varphi + \delta\varphi$ → scalar perturbation + 1
(+) → time diffeo ⇒ $\delta\varphi = 0$ → δg w/ 3 dof
physical dof = 3
general perturbations, S, V, T
4 scalars in metric
δg_{00}, δg_{i0}, $\delta g_{ij} \sim 2_i B$
δg_{ij}
vectors
single field inflation
gauged away
γ_{ij} → 2 dof
tensors


* remaining gauge transformation
optical
plus m


* remaining gauge transformation
spatial diffeomorphism $\tilde{x} = f(x)$
gauge parameter
$\tilde{x}^i = x^i + \delta^i \epsilon$
$\epsilon = f(t)\, g(x)$

* remaining gauge transformation.

spatial diffeomorphism $\tilde{x} = f(x)$

gauge parameter

$\tilde{x}^i = x^i + \partial^i \epsilon$

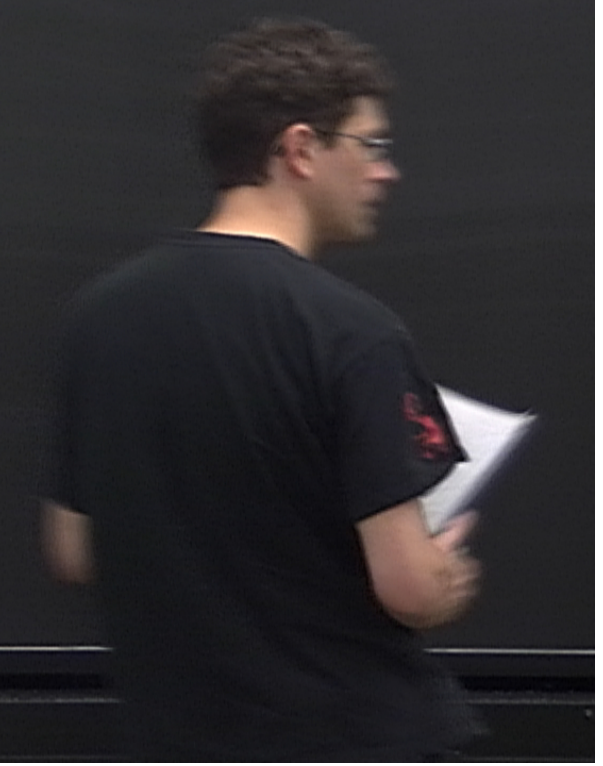
$\epsilon = f(t)\, g(x)$

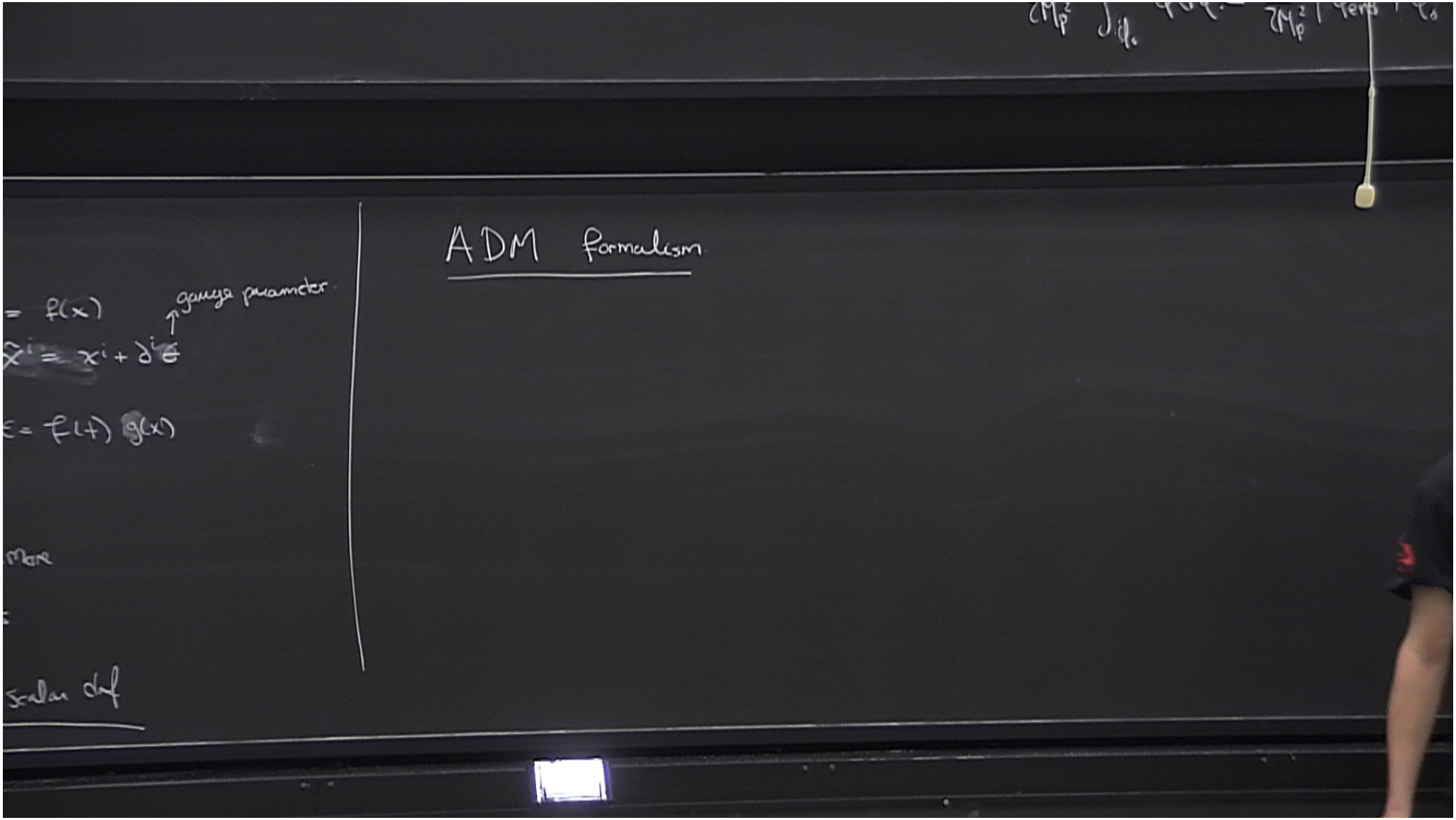
eliminate 2 scalars.

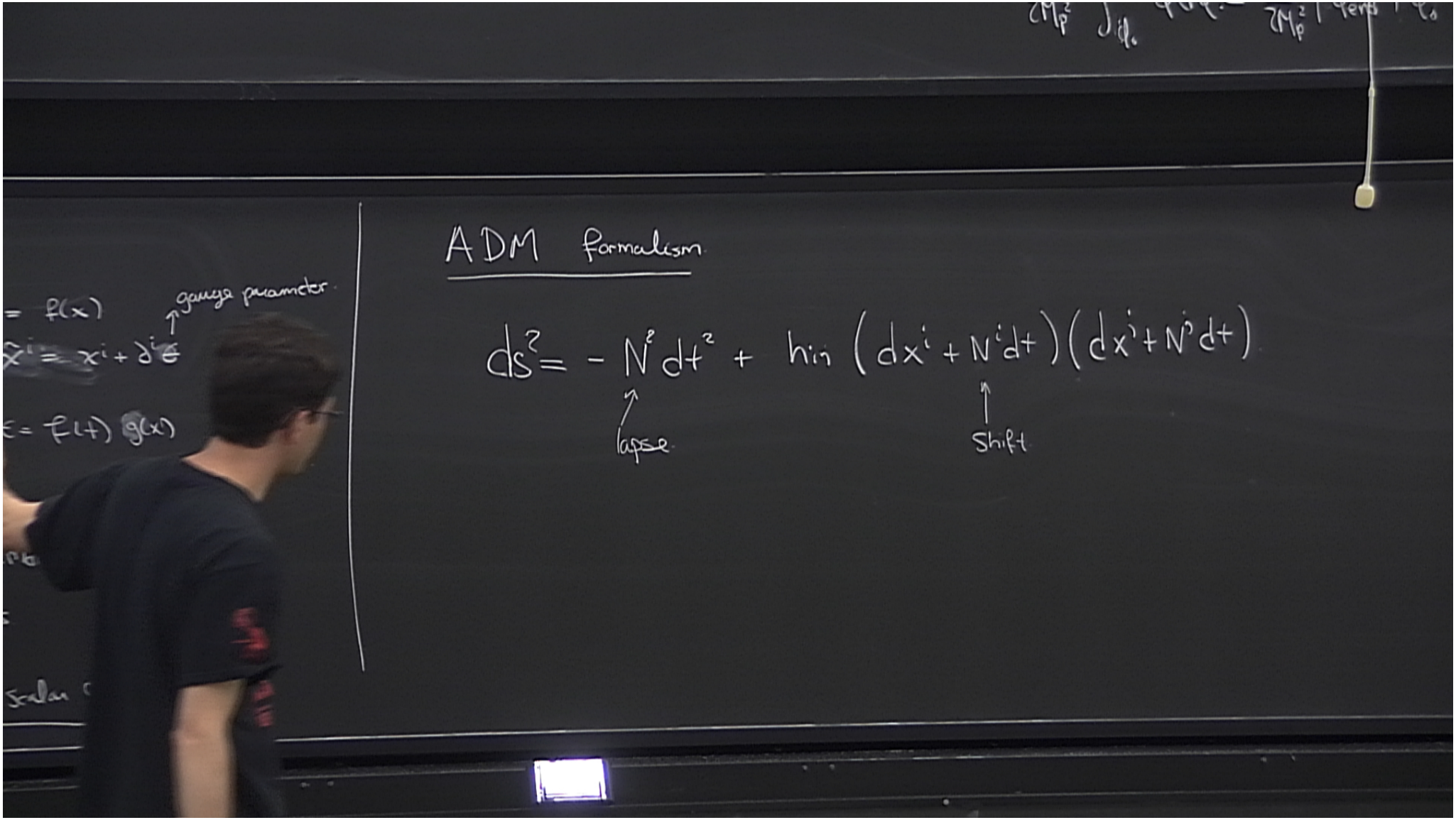
* use constraints in GR to eliminate 2 more

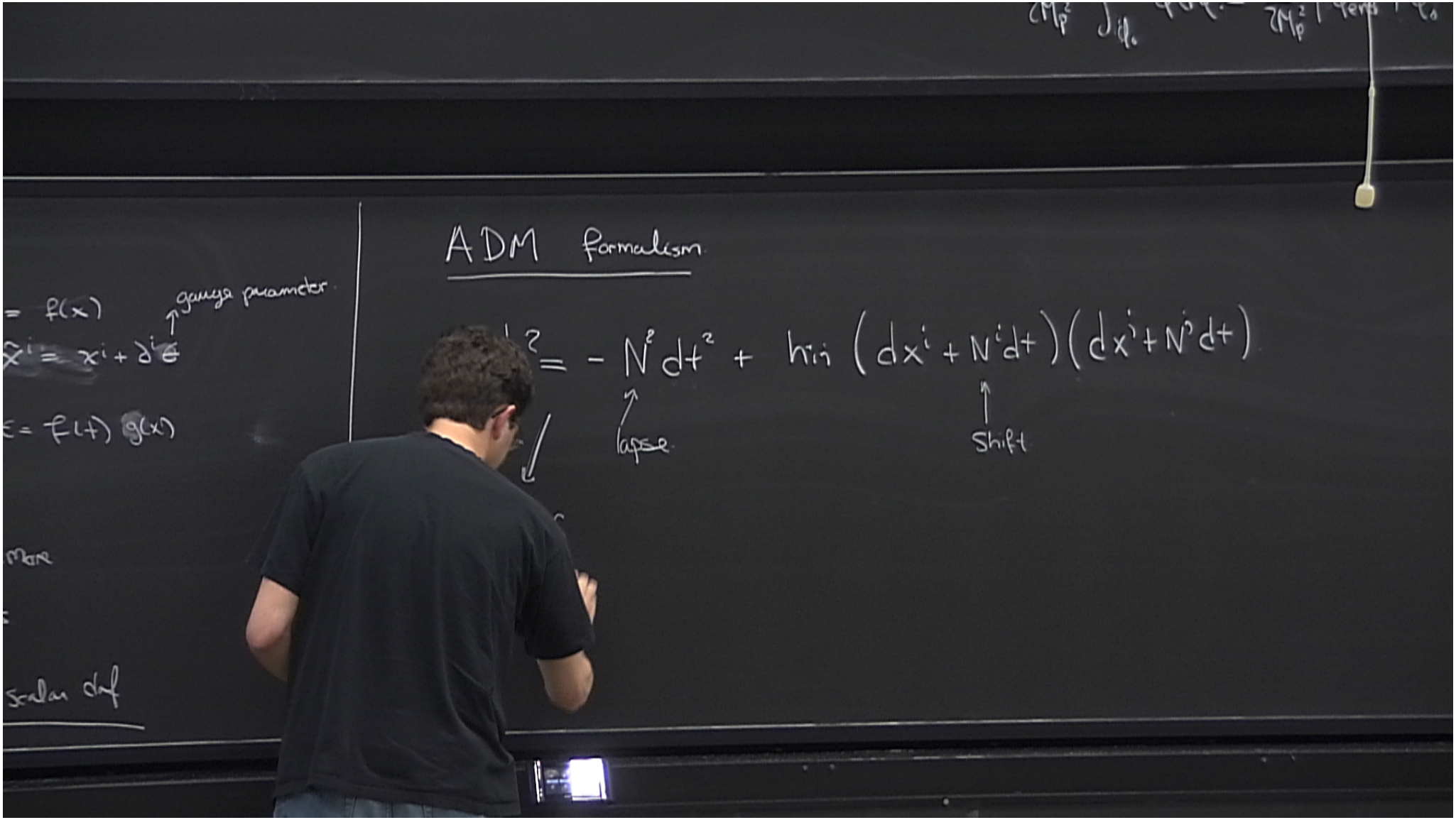
eliminate 2 more scalars

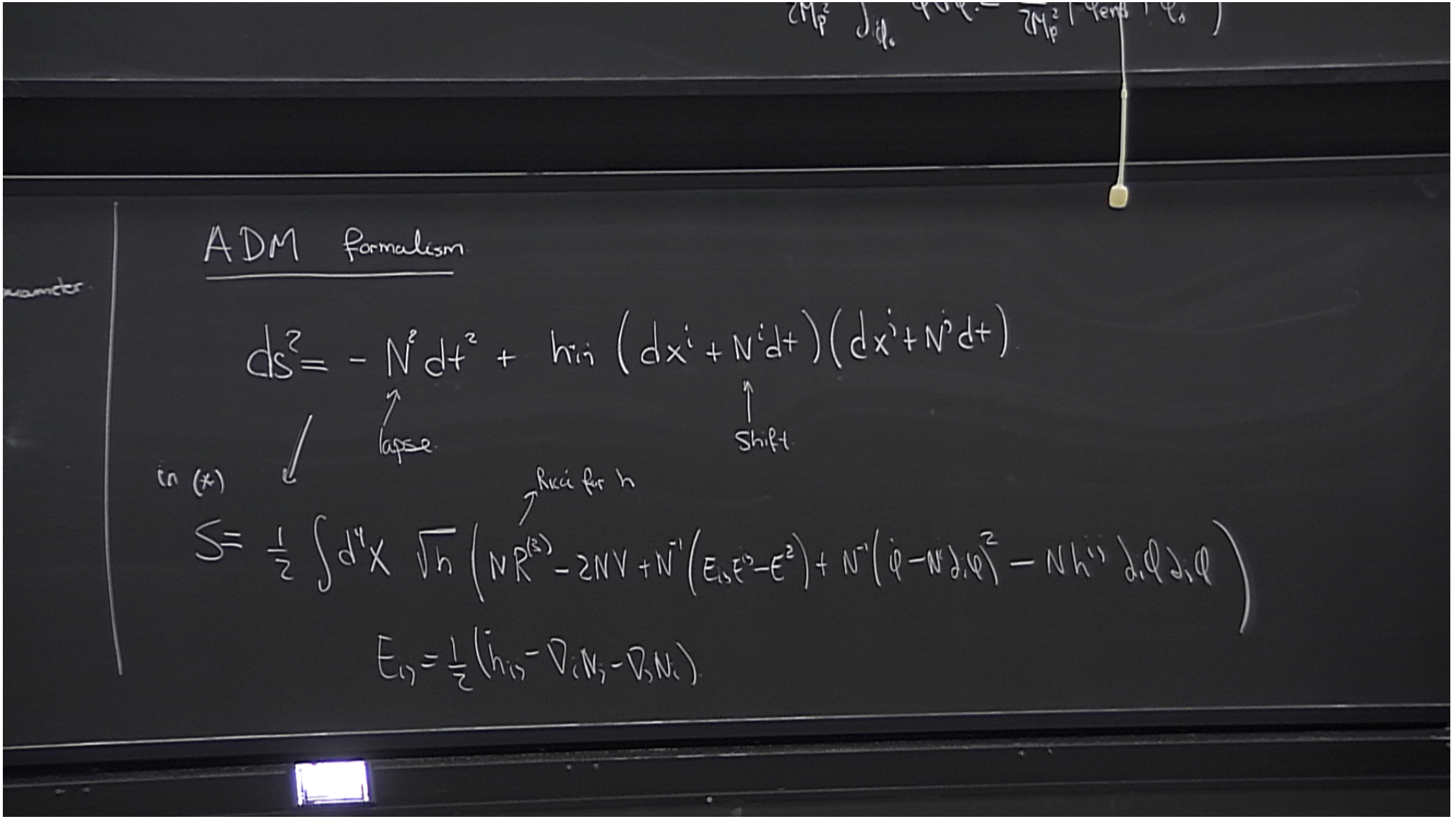
$5 - 2 - 2 = 1$ scalar dof

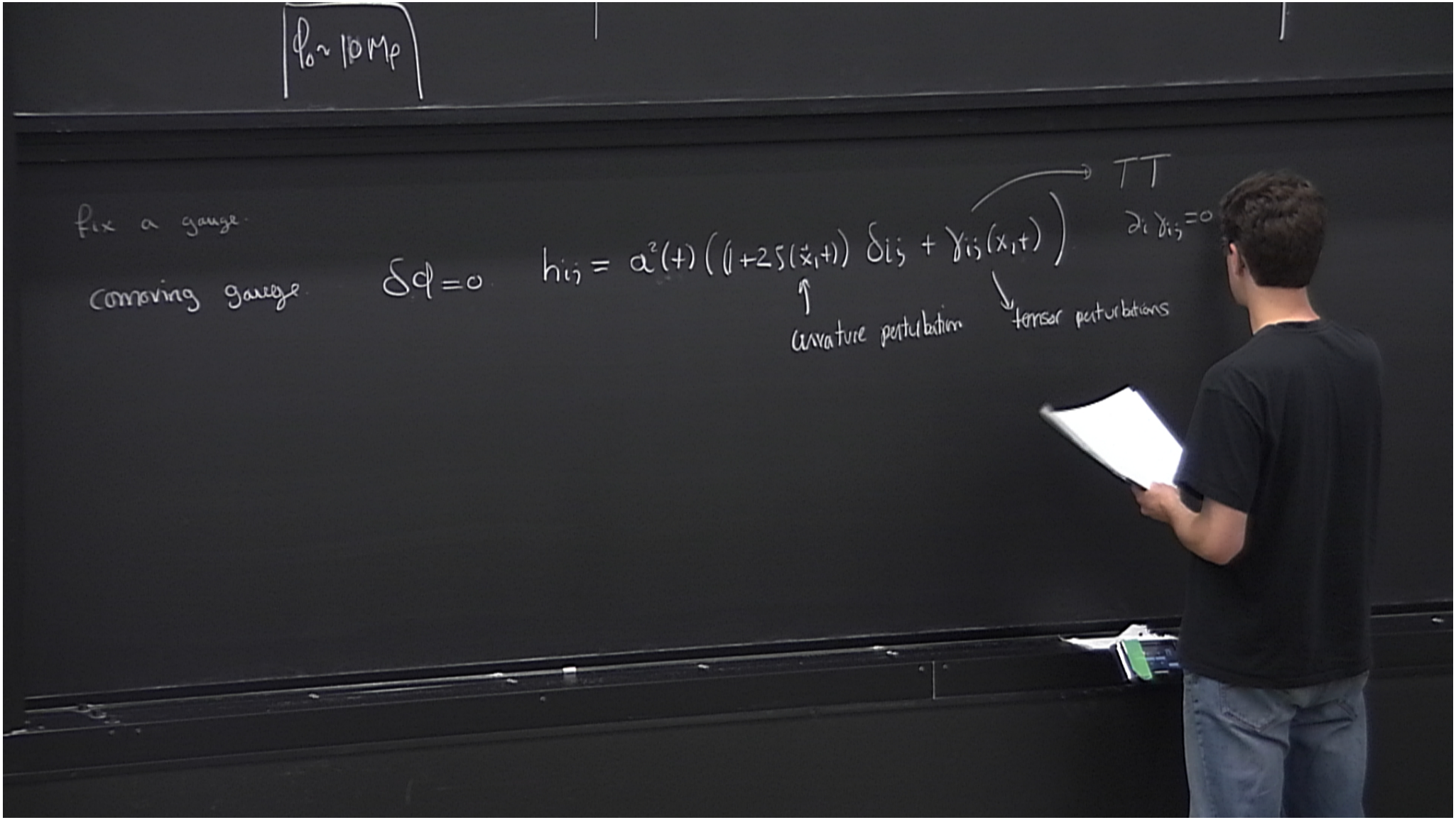



ADM formalism
gauge parameter
Scalar dof


ADM formalism
$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
lapse
Shift
gauge parameter
$= f(x)$
$\tilde{x}^i = x^i + \partial^i \epsilon$
$\epsilon = f(t)\, g(x)$


ADM formalism
gauge parameter
$= f(x)$
$\tilde{x}^i = x^i + \partial^i \epsilon$
$\epsilon = f(t)\, g(x)$
$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$
lapse
Shift
Scalar def


ADM formalism
$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$
lapse
Shift
in (*)
Ricci for h
$S = \frac{1}{2}\int d^4x \sqrt{h}\left(N R^{(3)} - 2N\Lambda + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + N^{-1}\left(\dot{\varphi} - N^i\partial_i\varphi\right)^2 - N h^{ij}\partial_i\varphi\partial_j\varphi\right)$
$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$


$\rho_0 \sim 10\,M_P$
fix a gauge
comoving gauge $\delta\phi = 0$
$h_{ij} = a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$
curvature perturbation
tensor perturbations
TT
$\partial_i \gamma_{ij} = 0$

gauge parameter

ADM formalism

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

N: lapse, N^i: shift

in (*)

$$S = \frac{1}{2}\int d^4x \sqrt{h}\left(N R^{(3)} - 2N\Lambda + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + N^{-1}\left(\dot{\varphi} - N^i\partial_i\varphi\right)^2 - N h^{ij}\partial_i\varphi\,\partial_j\varphi\right)$$

($R^{(3)}$: Ricci for h)

$$E_{ij} = \frac{1}{2}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right) \qquad \left(E^i_j\right)^2$$

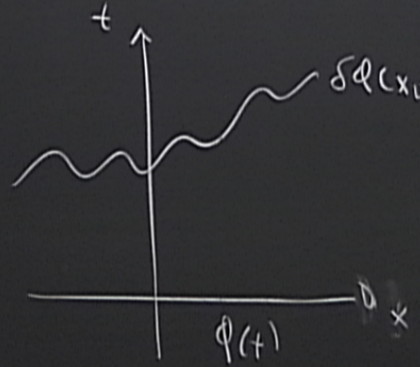
$\phi_0 \sim 10 M_p$

fix a gauge.

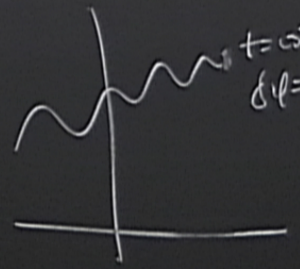
comoving gauge $\delta\phi = 0$ $h_{ij} = a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$ TT $\partial_i \gamma_{ij} = 0$

curvature perturbation

tensor perturbations



comoving gauge

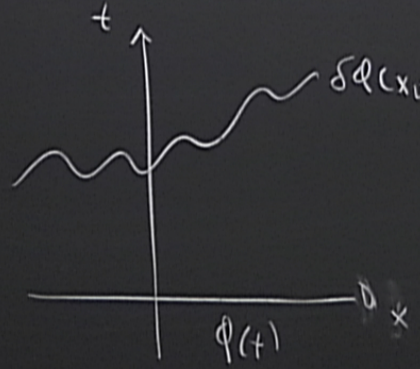


$\phi_0 \sim 10 M_p$

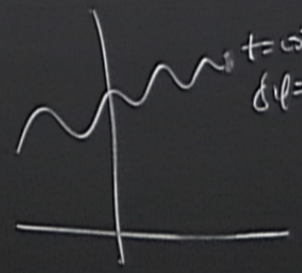
fix a gauge

comoving gauge $\delta\phi = 0$ $h_{ij} = a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$ $\gamma_{ij} \to$ TT, $\partial_i \gamma_{ij} = 0$

ζ: curvature perturbation. γ_{ij}: tensor perturbations



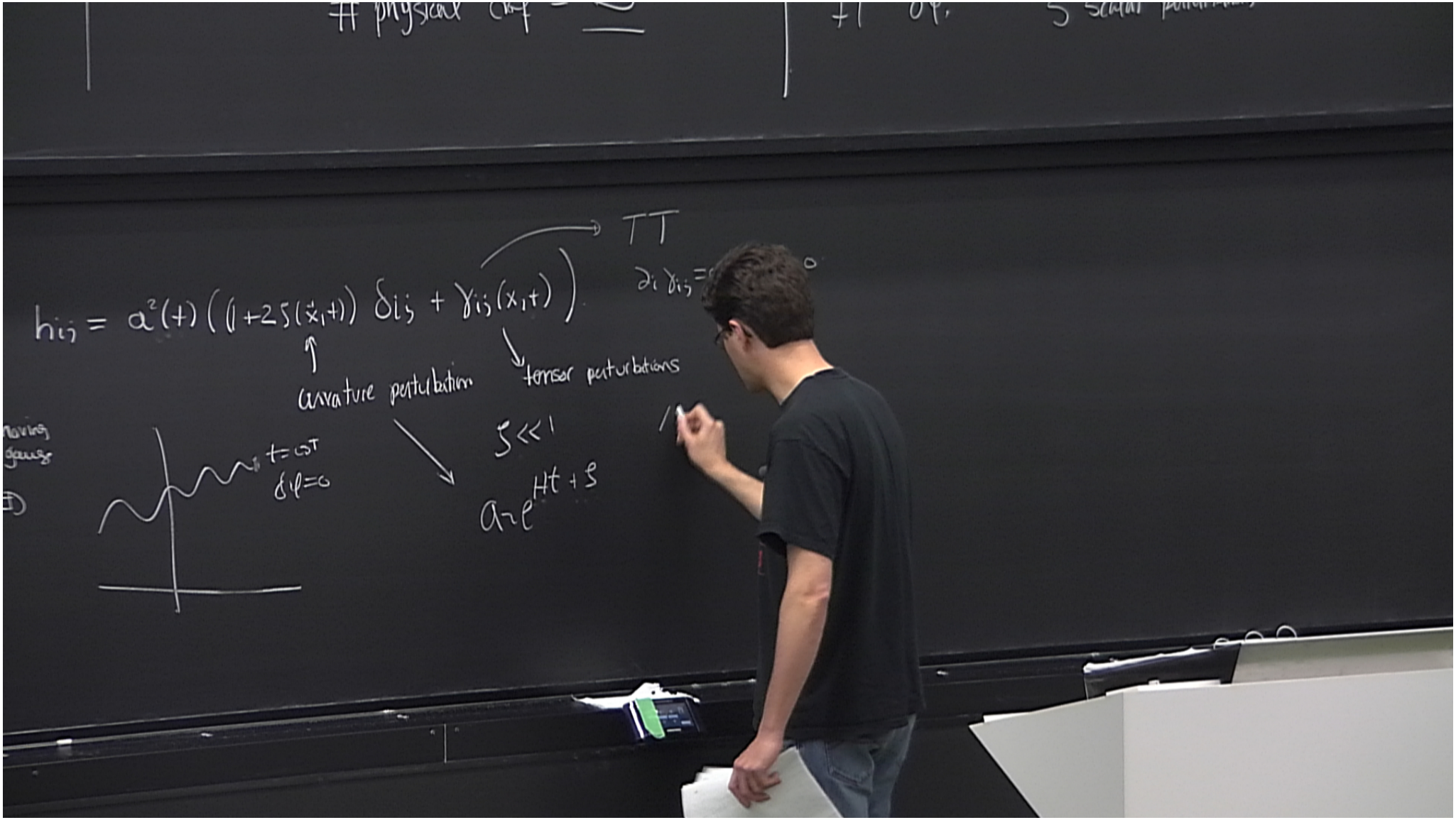
comoving gauge $\Rightarrow$

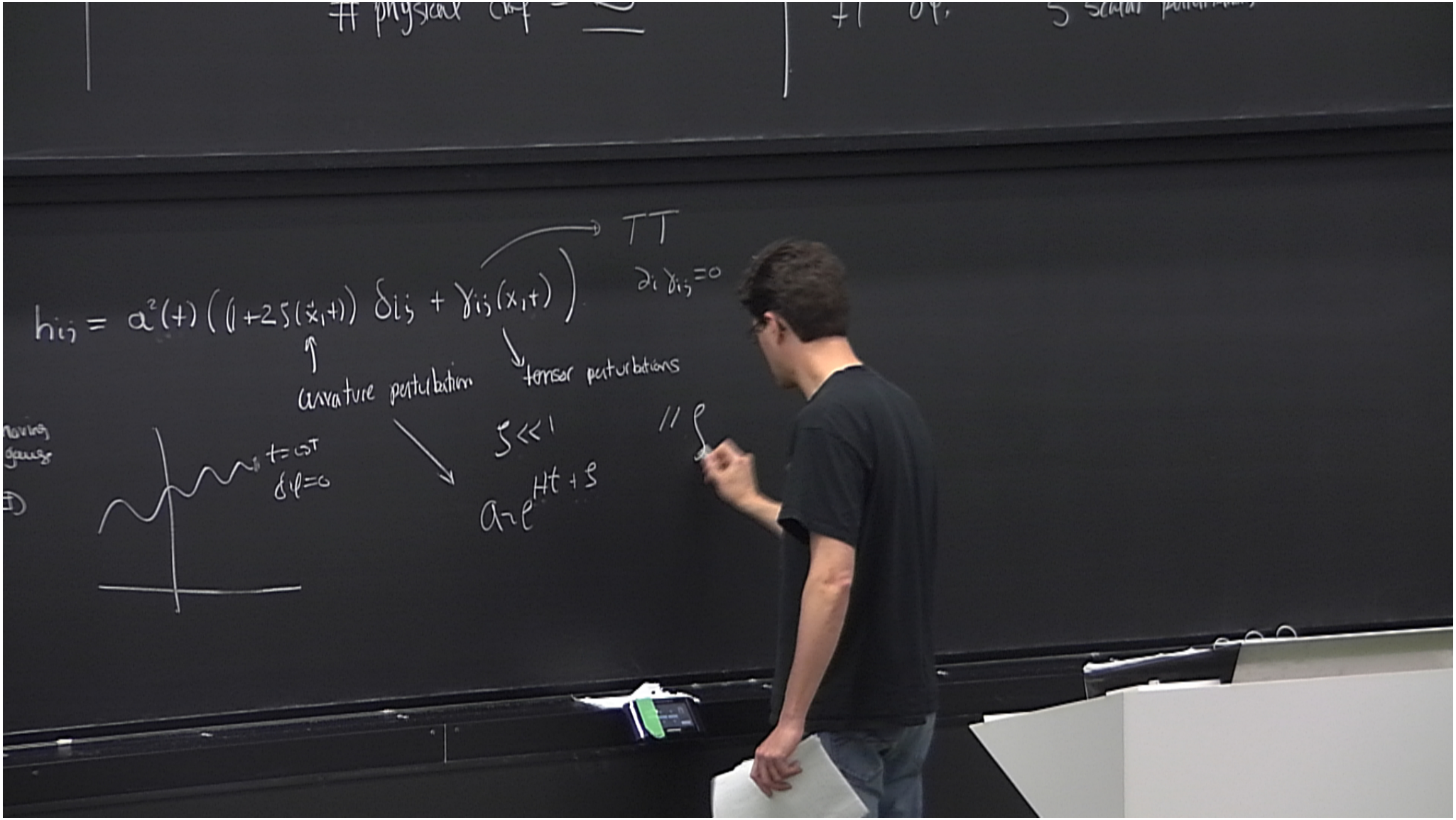


$5 - 2 - 2 = 1$ scalar dof

$\delta\phi = 0$ $\quad \gamma = 0$ forget about tensor

S


TT
$h_{ij} = a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$
curvature perturbation
tensor perturbations
$\zeta \ll 1$
$a \sim e^{Ht+\zeta}$
$t = t_0$
$\delta\varphi = 0$

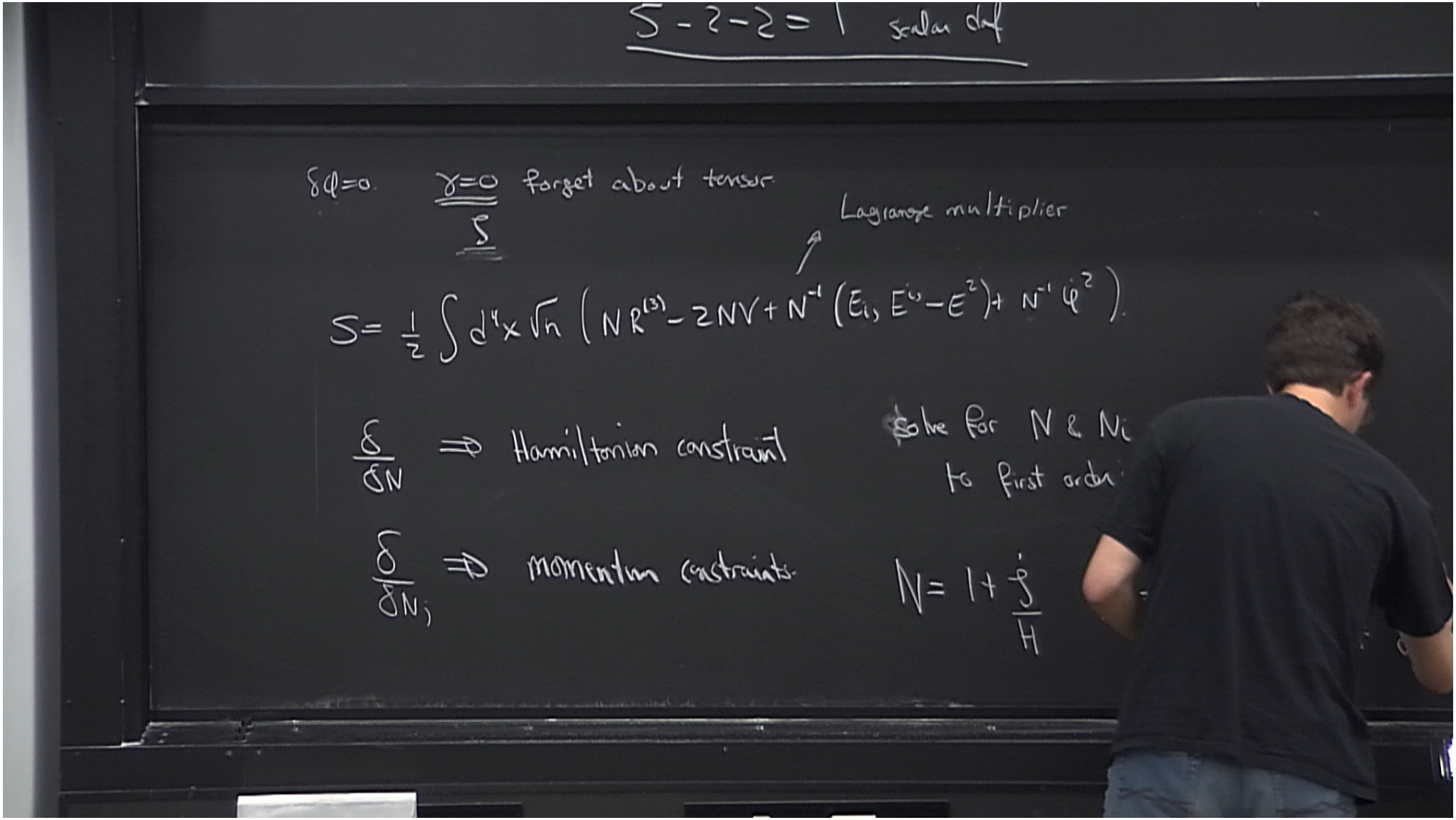

TT
$\partial_i \gamma_{ij} = 0$
$h_{ij} = a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$
curvature perturbation
tensor perturbations
$\zeta \ll 1$
$a = e^{Ht+\zeta}$
$t = t_0$
$\delta\varphi = 0$

$5 - 2 - 2 = 1$ scalar dof

$\delta\varphi = 0$ $\quad \gamma = 0$ forget about tensor

S

$$S = \frac{1}{2}\int d^4x \sqrt{h}\left(N R^{(3)} - 2NV + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + N^{-1}\dot{\varphi}^2\right).$$


5 - 2 - 2 = 1 scalar dof
δφ=0. γ=0 forget about tensor
S
Lagrange multiplier
S= 1/2 ∫ d⁴x √h (N R⁽³⁾ − 2NV + N⁻¹ (Eᵢⱼ Eⁱʲ − E²) + N⁻¹ φ̇²)
δ/δN ⇒ Hamiltonian constraint
Solve for N & Nᵢ
to first order
δ/δNᵢ ⇒ momentum constraints.
N = 1 + φ̇/H

$5 - 2 - 2 = 1$ scalar dof

$\delta\phi = 0$ $\quad \gamma = 0$ forget about tensor

ζ

Lagrange multiplier

$$S = \frac{1}{2}\int d^4x \sqrt{h}\left(N R^{(3)} - 2NV + N^{-1}\left(E_{ij}E^{ij} - E^2\right) + N^{-1}\dot{\phi}^2\right)$$

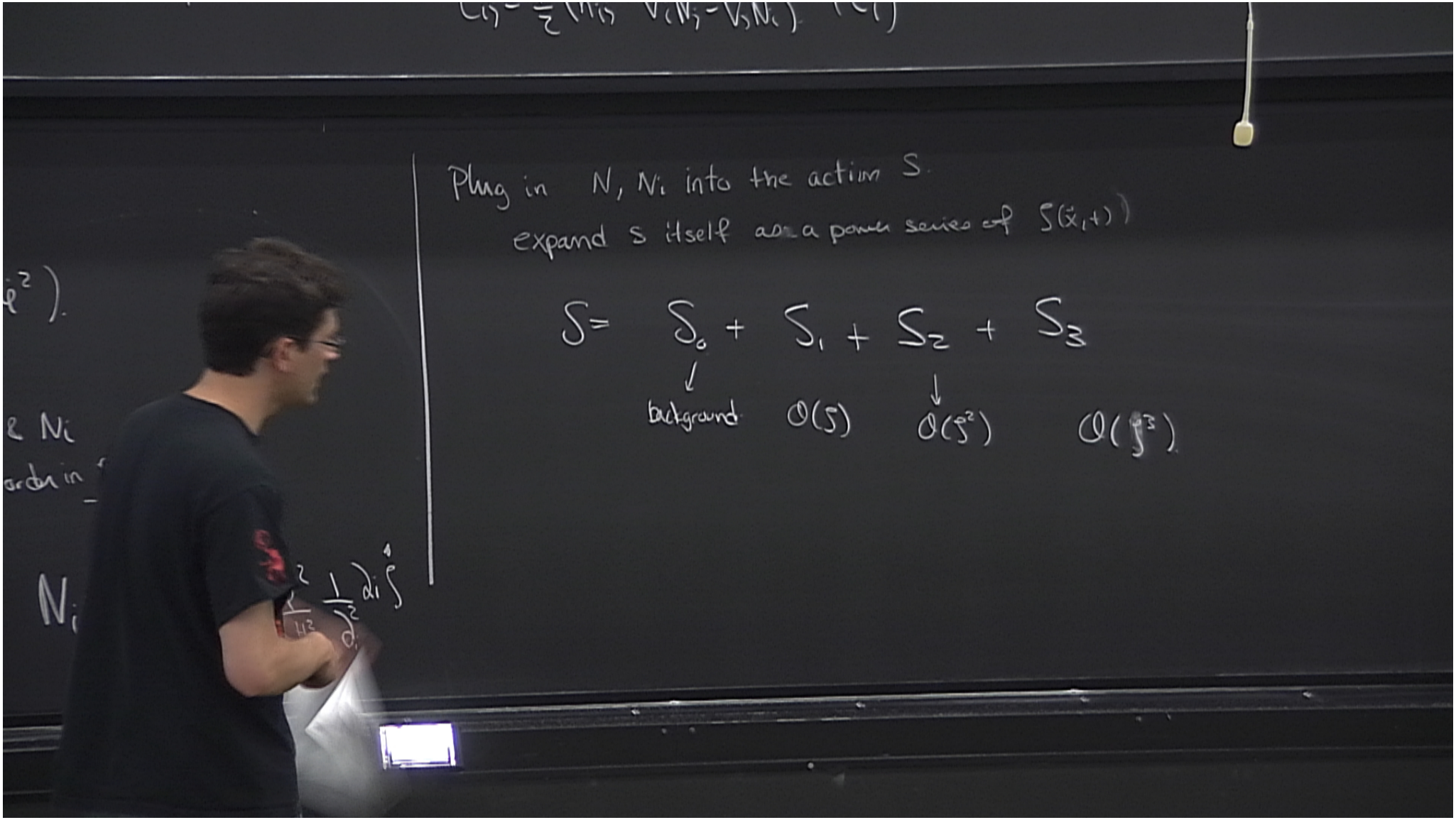
$\frac{\delta}{\delta N} \Rightarrow$ Hamiltonian constraint

$\frac{\delta}{\delta N_i} \Rightarrow$ momentum constraints.

Solve for N & N_i to first order in ζ

$$N = 1 + \frac{\dot{\zeta}}{H} \qquad N_i = -\frac{1}{a^2 H}\partial_i\zeta + \frac{\dot{\phi}^2}{2H^2}\frac{1}{\partial^2}\partial_i\dot{\zeta}$$

Plug in N_i


Plug in N, Ni into the action S
expand S itself as a power series of ζ(x,t)
S = S0 + S1 + S2 + S3
background
O(ζ)
O(ζ²)
O(ζ³)

Plug in N, N_i into the action S.

expand S itself as a power series of $\zeta(\vec{x},t)$

$$S = S_0 + S_1 + S_2 + S_3 + \cdots$$

S_0: background

S_1: $\mathcal{O}(\zeta)$

S_2: $\mathcal{O}(\zeta^2)$

S_3: $\mathcal{O}(\zeta^3)$

$$\int V_3 \int H^2 M_p^2 \dot{\zeta}^2$$

$$N_i = -\frac{1}{a^2 H}\partial_i \zeta + \frac{\dot{\zeta}^2}{2H^2}\frac{1}{\partial^2}\partial_i \dot{\zeta}$$

$\partial^2 = \partial_i \partial^i = \nabla$

& N_i

order in δ

$$N_i = -\frac{1}{a^2 H}\partial_i \zeta + \frac{\dot{\zeta}^2}{2H^2}\frac{1}{\partial^2}\partial_i \dot{\zeta}$$

Plug in N, N_i into the action S.

expand S itself as a power series of $\zeta(\vec{x},t)$

$$S = S_0 + S_1 + S_2 + S_3 + \cdots$$

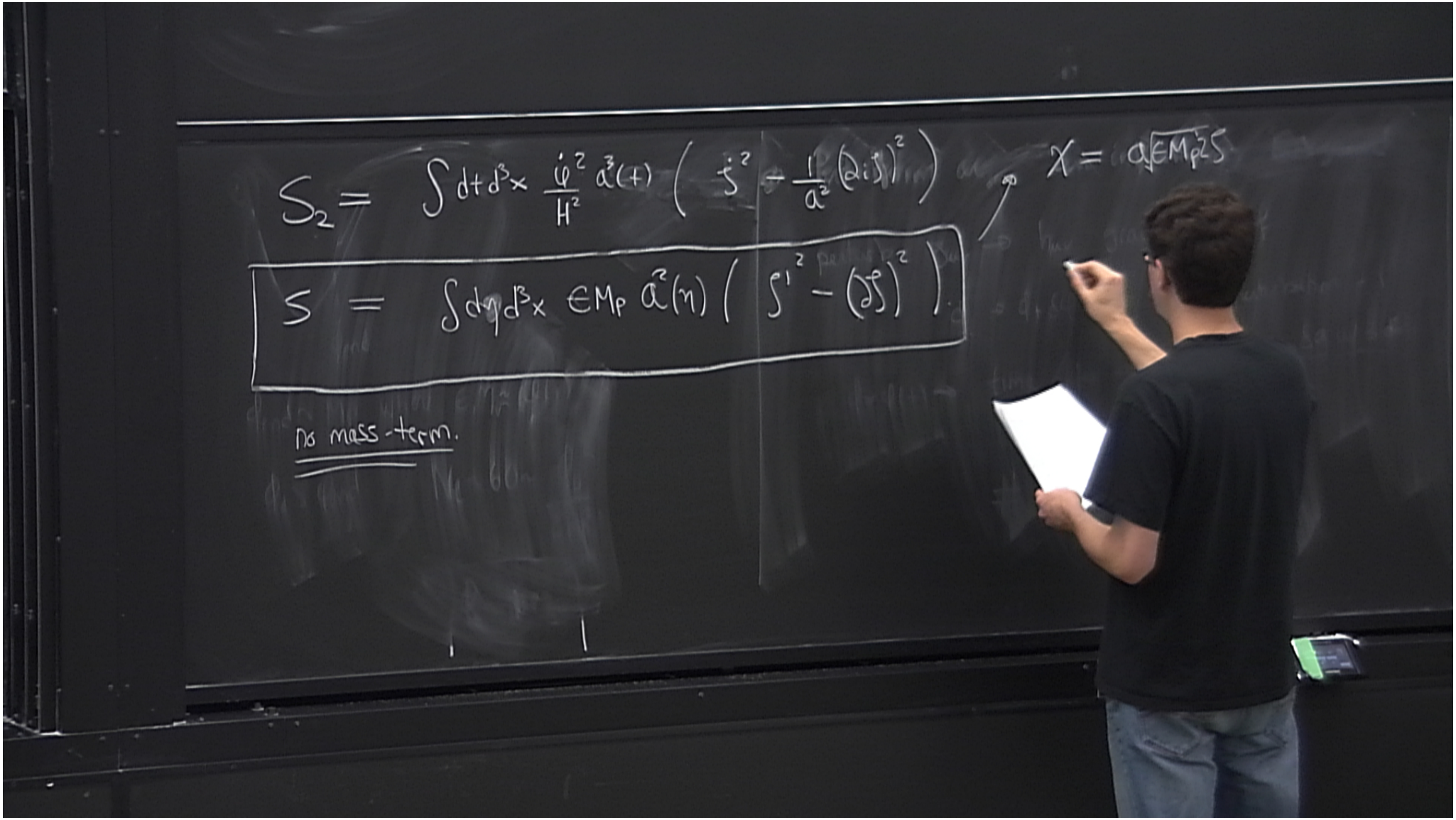
S_0: background

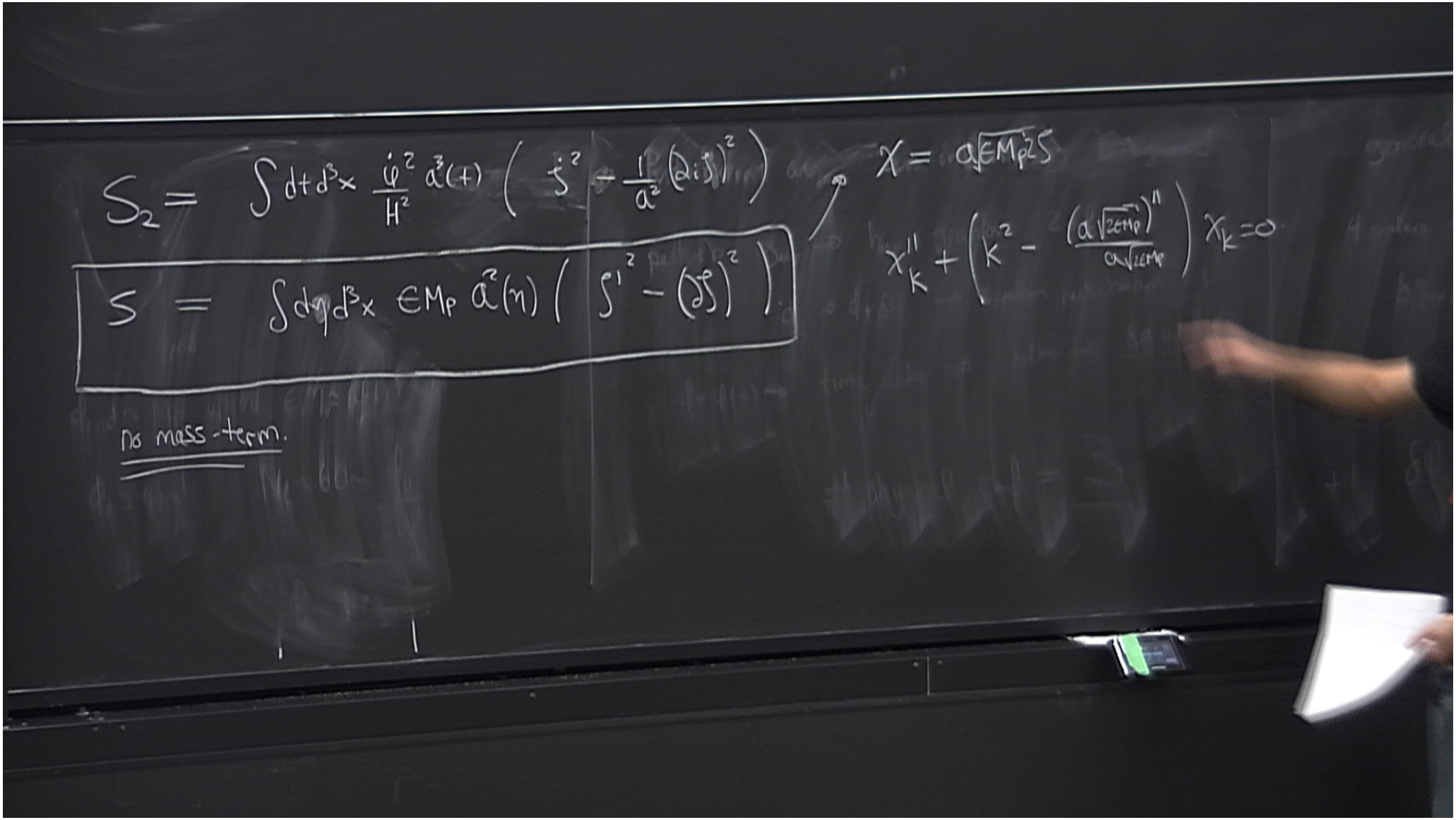
S_1: $\mathcal{O}(\zeta)$

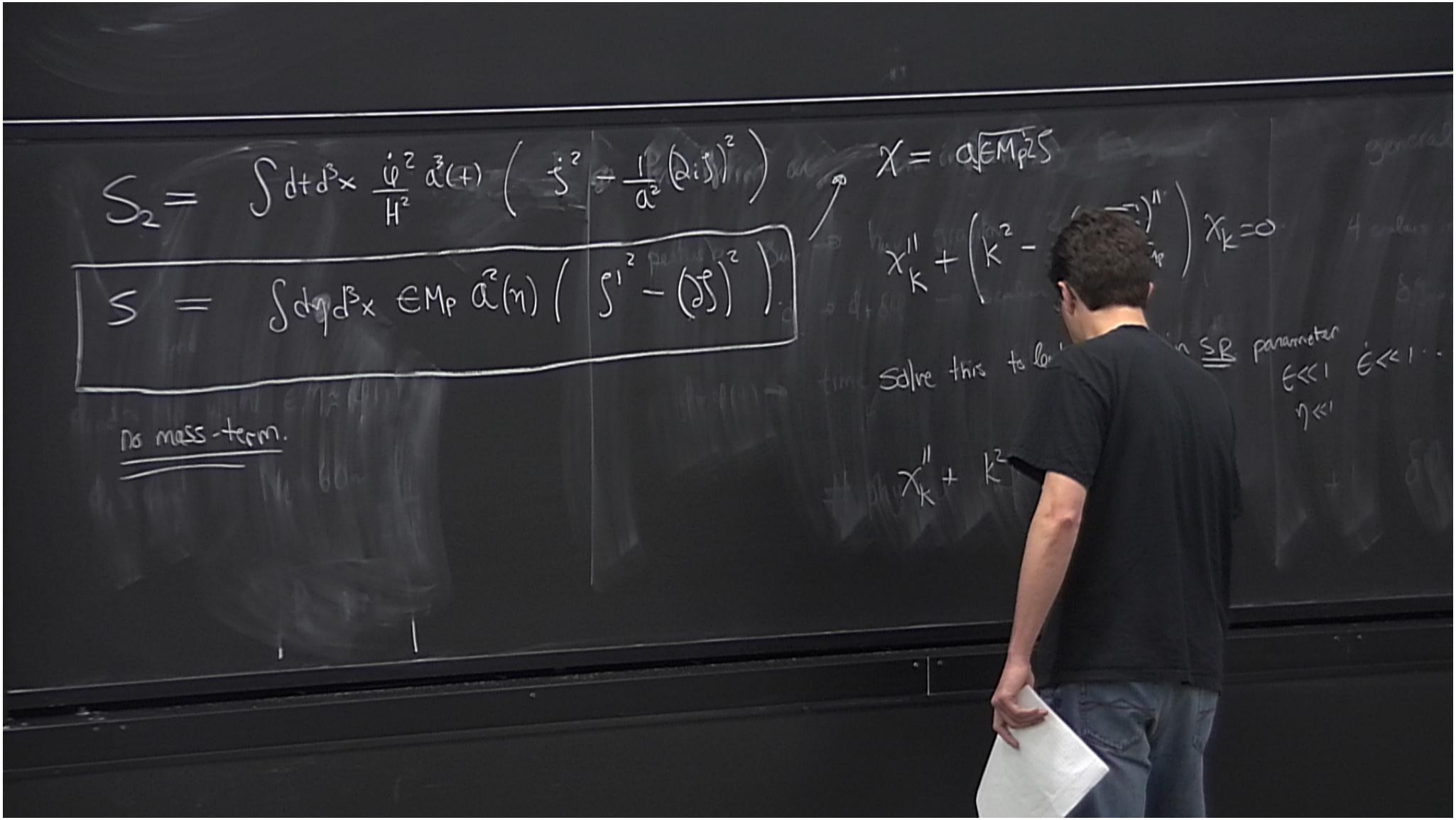
S_2: $\mathcal{O}(\zeta^2)$

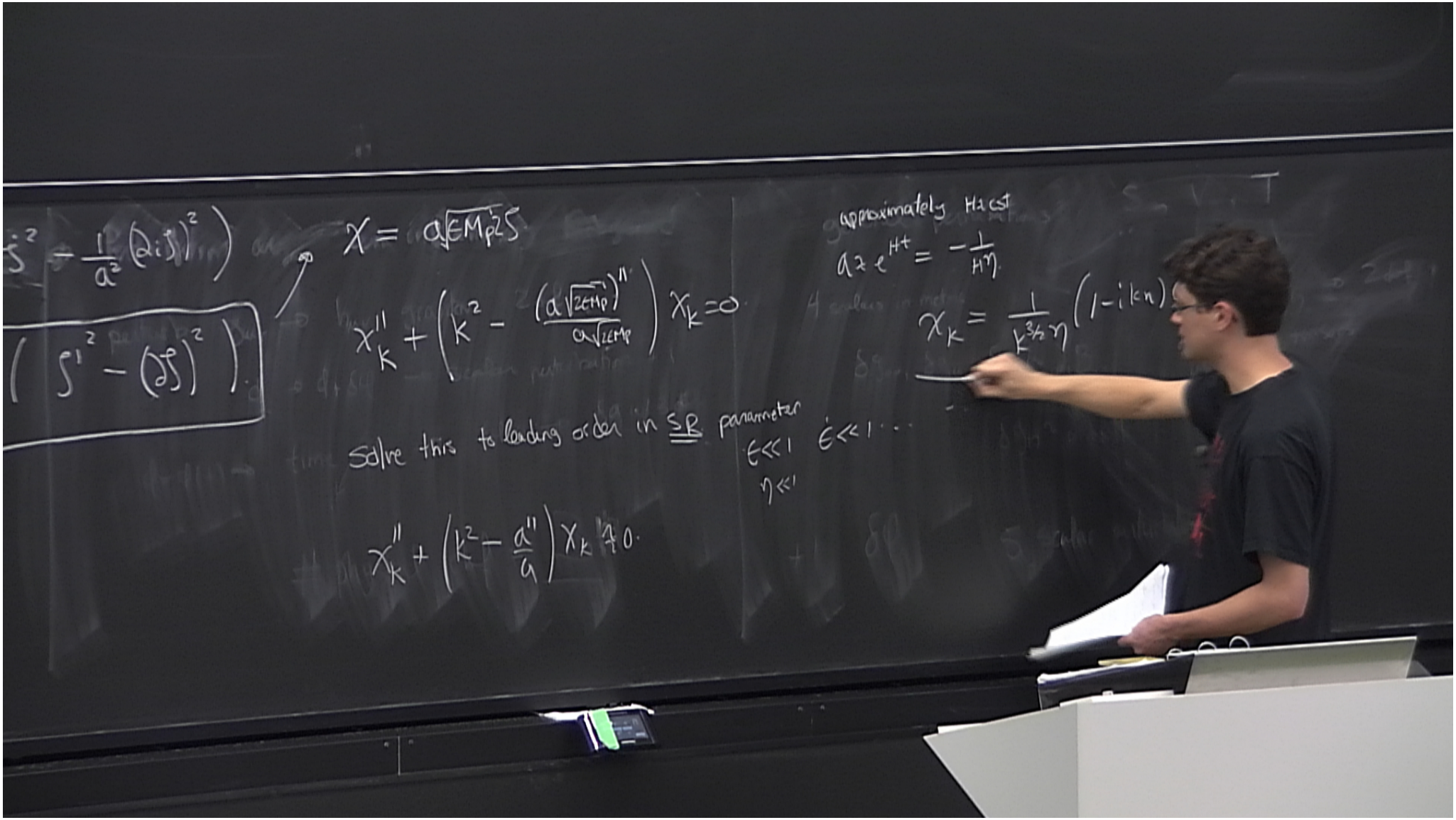
S_3: $\mathcal{O}(\zeta^3)$

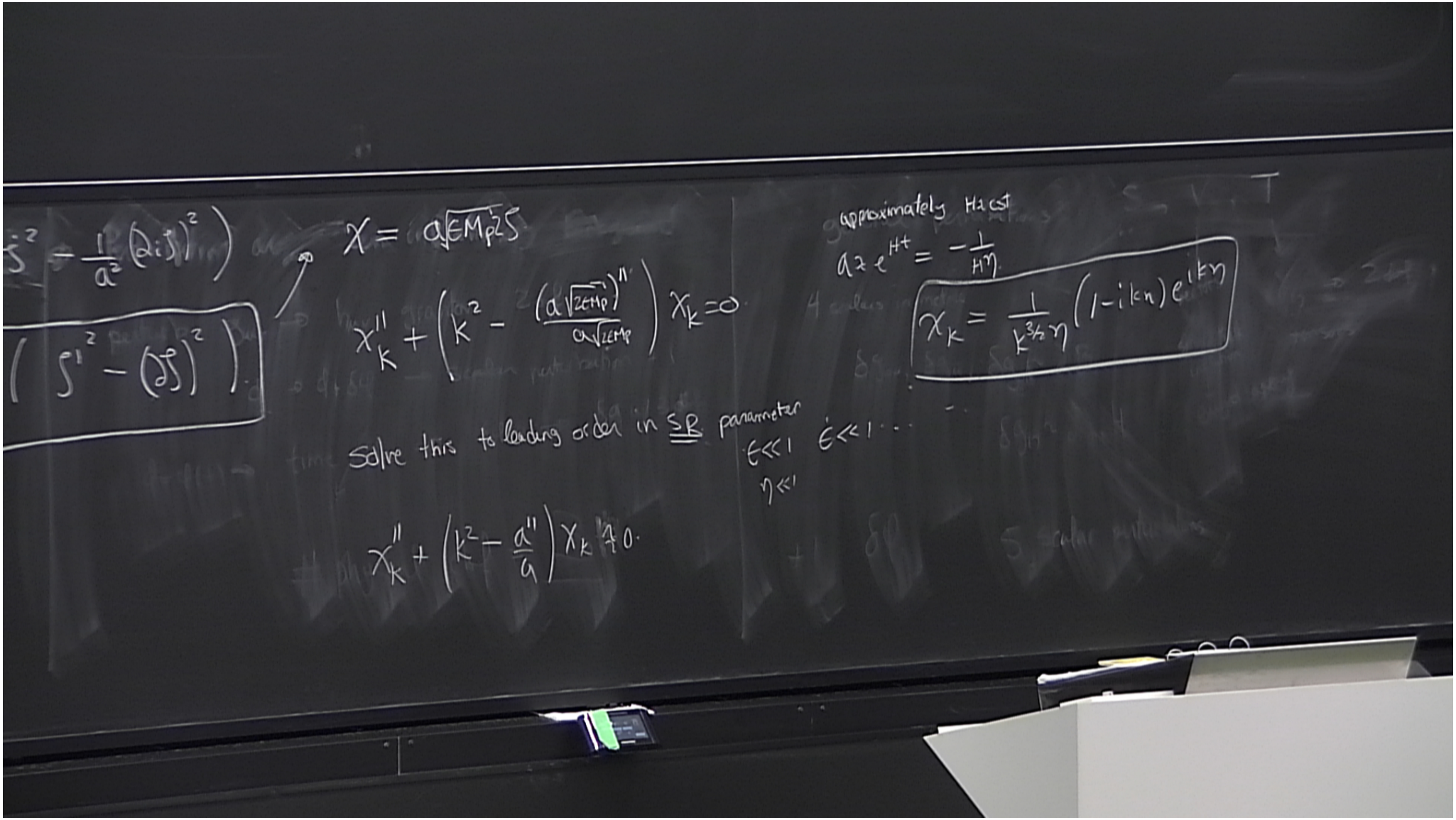
$$\int V \to \int H^2 M_p^2$$


$S_2 = \int dt\, d^3x\, \frac{\dot{\varphi}^2}{H^2} a^3(t) \left(\dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 \right)$
$\chi = a \sqrt{\epsilon M_p^2}\, \zeta$
$S = \int d\eta\, d^3x\, \epsilon M_p\, a^2(\eta) \left(\zeta'^2 - (\partial \zeta)^2 \right)$
no mass-term.


$S_2 = \int d\tau d^3x \, \frac{\dot{\varphi}^2}{H^2} a^3(t) \left(\dot{\zeta}^2 - \frac{1}{a^2} (\partial_i \zeta)^2 \right)$
$\chi = a\sqrt{2\epsilon M_p^2}\zeta$
$S = \int d\eta d^3x \, \epsilon M_p \, a^2(\eta) \left(\zeta'^2 - (\partial\zeta)^2 \right)$
$\chi_k'' + \left(k^2 - \frac{(a\sqrt{2\epsilon M_p})''}{a\sqrt{2\epsilon M_p}} \right) \chi_k = 0$
No mass-term.


$S_2 = \int d\tau d^3x \frac{\dot{\varphi}^2}{H^2} a^3(\tau) \left(\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i \zeta)^2 \right)$
$S = \int d\eta d^3x \, \epsilon M_p \, a^2(\eta) \left(\zeta'^2 - (\partial\zeta)^2 \right)$
No mass-term.
$\chi = a\sqrt{\epsilon} M_p \zeta$
$\chi_k'' + \left(k^2 - \quad \right) \chi_k = 0$
Solve this to
SR parameter
$\epsilon \ll 1 \quad \dot{\epsilon} \ll 1 \ldots$
$\eta \ll 1$
$\chi_k'' + k^2$


$\chi = a\sqrt{\epsilon M_p^2}\,S$
$\chi_k'' + \left(k^2 - \frac{(a\sqrt{2\epsilon M_p})''}{a\sqrt{2\epsilon M_p}}\right)\chi_k = 0$
Solve this to leading order in SR parameter $\epsilon \ll 1$ $\dot{\epsilon} \ll 1 \ldots$ $\eta \ll 1$
$\chi_k'' + \left(k^2 - \frac{a''}{a}\right)\chi_k \neq 0$
approximately H=cst
$a \propto e^{Ht} = -\frac{1}{H\eta}$
$\chi_k = \frac{1}{k^{3/2}\eta}\left(1 - i(\epsilon\eta)\right)$


$\chi = a\sqrt{\epsilon M_p^2}\,\zeta$
$\chi_k'' + \left(k^2 - \frac{(a\sqrt{2\epsilon M_p})''}{a\sqrt{2\epsilon M_p}}\right)\chi_k = 0$
Solve this to leading order in SR parameter $\epsilon \ll 1$ $\epsilon \ll 1$ ... $\eta \ll 1$
$\chi_k'' + \left(k^2 - \frac{a''}{a}\right)\chi_k \neq 0$
approximately H=cst
$a \propto e^{Ht} = -\frac{1}{H\eta}$
$\chi_k = \frac{1}{k^{3/2}\eta}(1 - ik\eta)e^{ik\eta}$

$5 - 2 - 2 = 1$ scalar dof

$-k\eta \ll 1$

$v_k = \frac{1}{k^{3/2}\eta}$

$\zeta_k \Longrightarrow u_k = \frac{H}{\sqrt{2\epsilon m_p}\, k^{3/2}}$

$$\int dt\, d^3x\, \frac{\dot\varphi^2}{H^2}\, a^3(t) \left(\dot\zeta^2 - \frac{1}{a^2}(\partial_i \zeta)^2 \right)$$

$$\int d\eta\, d^3x\, \epsilon M_p^2\, a^2(\eta) \left(\zeta'^2 - (\partial\zeta)^2 \right)$$

$$v = a\sqrt{\epsilon 2}\, M_p\, \zeta$$

$$v_k'' + \left(k^2 - \frac{(a\sqrt{2\epsilon M_p})''}{a\sqrt{2\epsilon M_p}} \right) v_k = 0$$

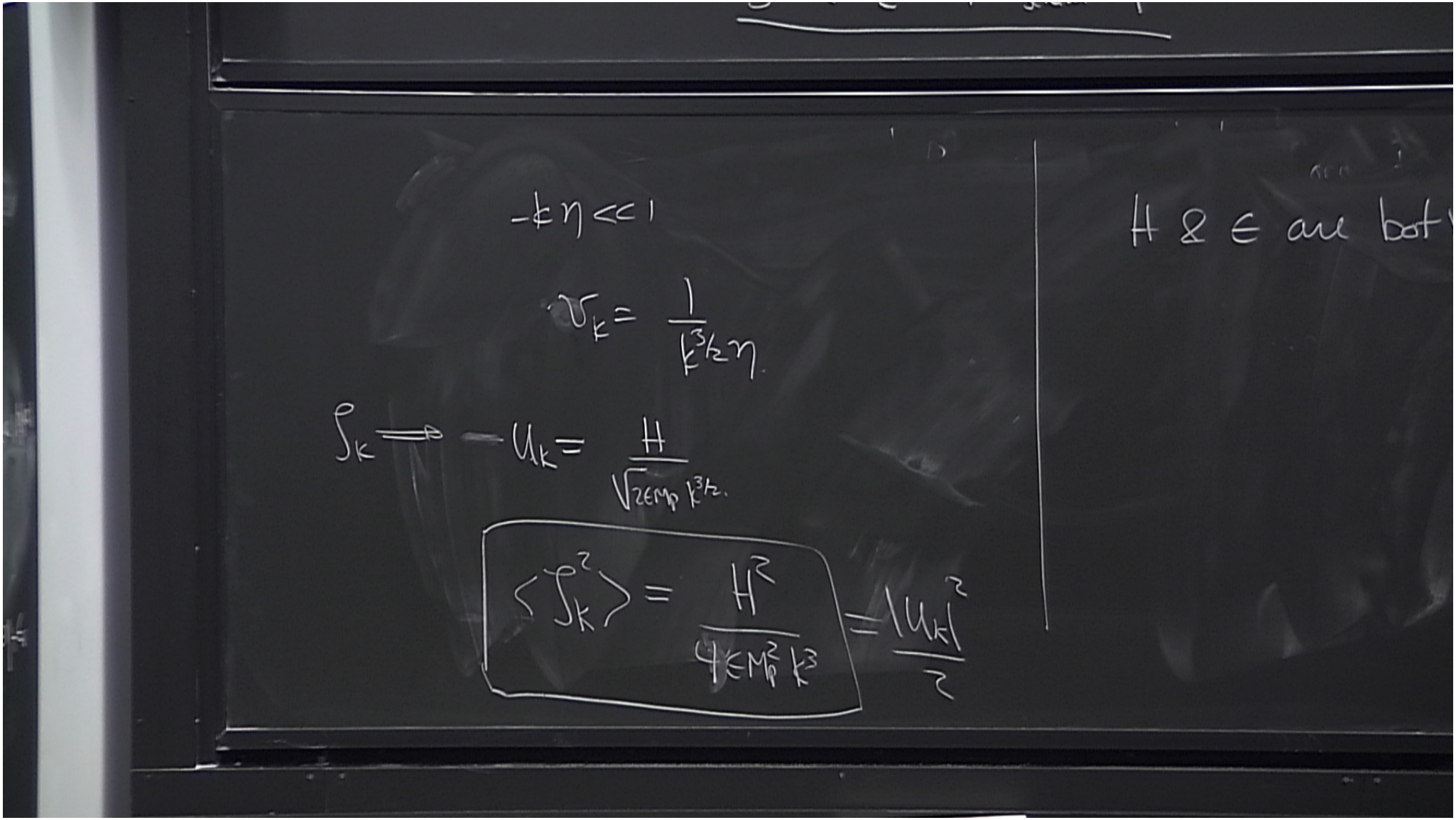
Solve this to leading order in SR parameter $\epsilon \ll 1$ $\dot\epsilon \ll 1$ ... $\eta \ll 1$

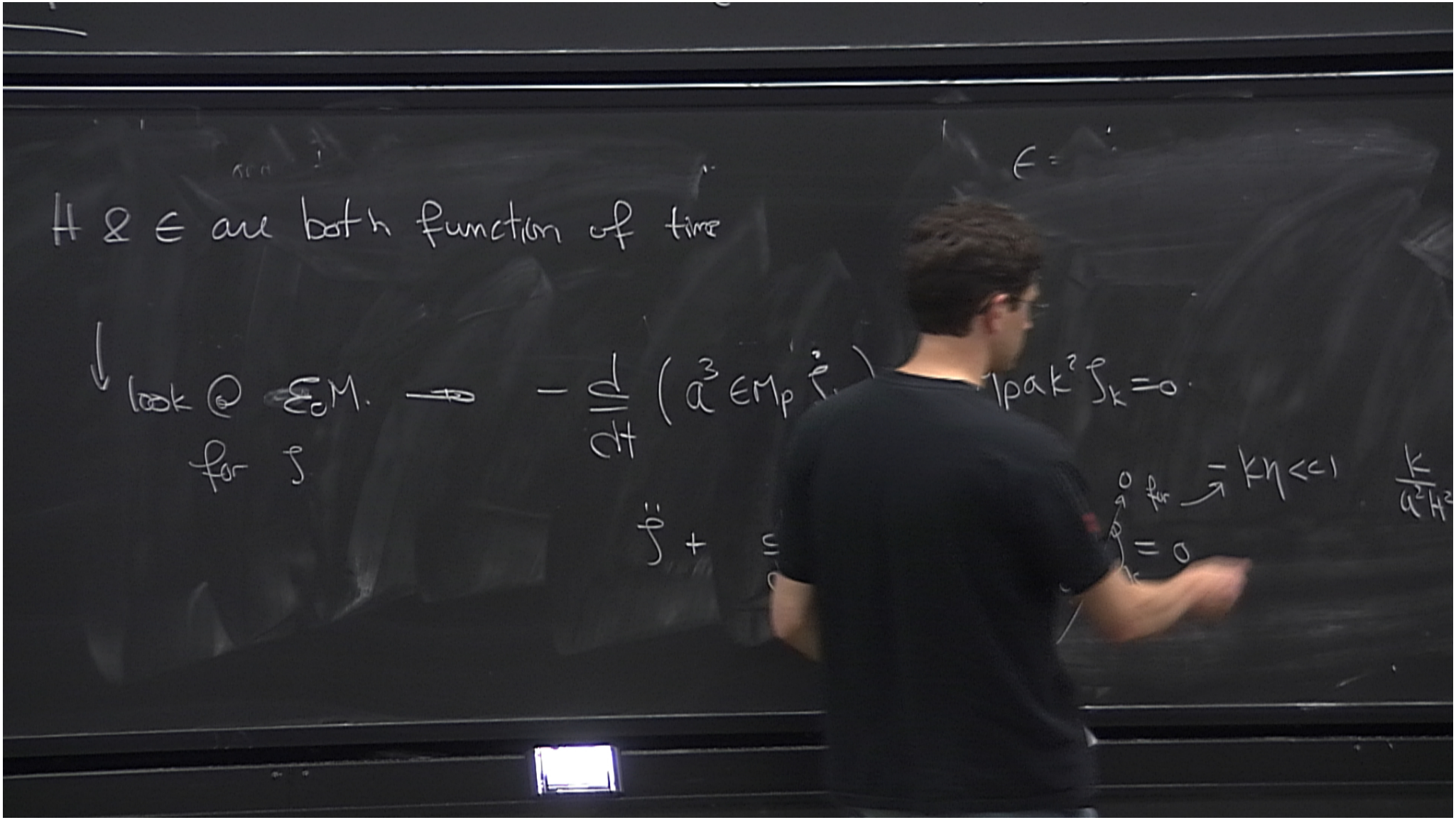
$$v_k'' + \left(k^2 - \frac{a''}{a} \right) v_k \neq 0.$$

approximately Hubble const

$$a \propto e^{Ht} = -\frac{1}{H\eta}$$

$$v_k = \frac{1}{k^3}$$


$-k\eta \ll 1$
$v_k = \frac{1}{k^{3/2}\eta}$
$\zeta_k \Longrightarrow v_k = \frac{H}{\sqrt{2\epsilon M_p k^{3/2}}}$
$\langle \zeta_k^2 \rangle = \frac{H^2}{4\epsilon M_p^2 k^3} = \frac{|v_k|^2}{z}$
H & ϵ are bot


H & ϵ are both function of time
look @ EoM.
for ζ
ϵ =
kη << 1

$\epsilon =$

of time

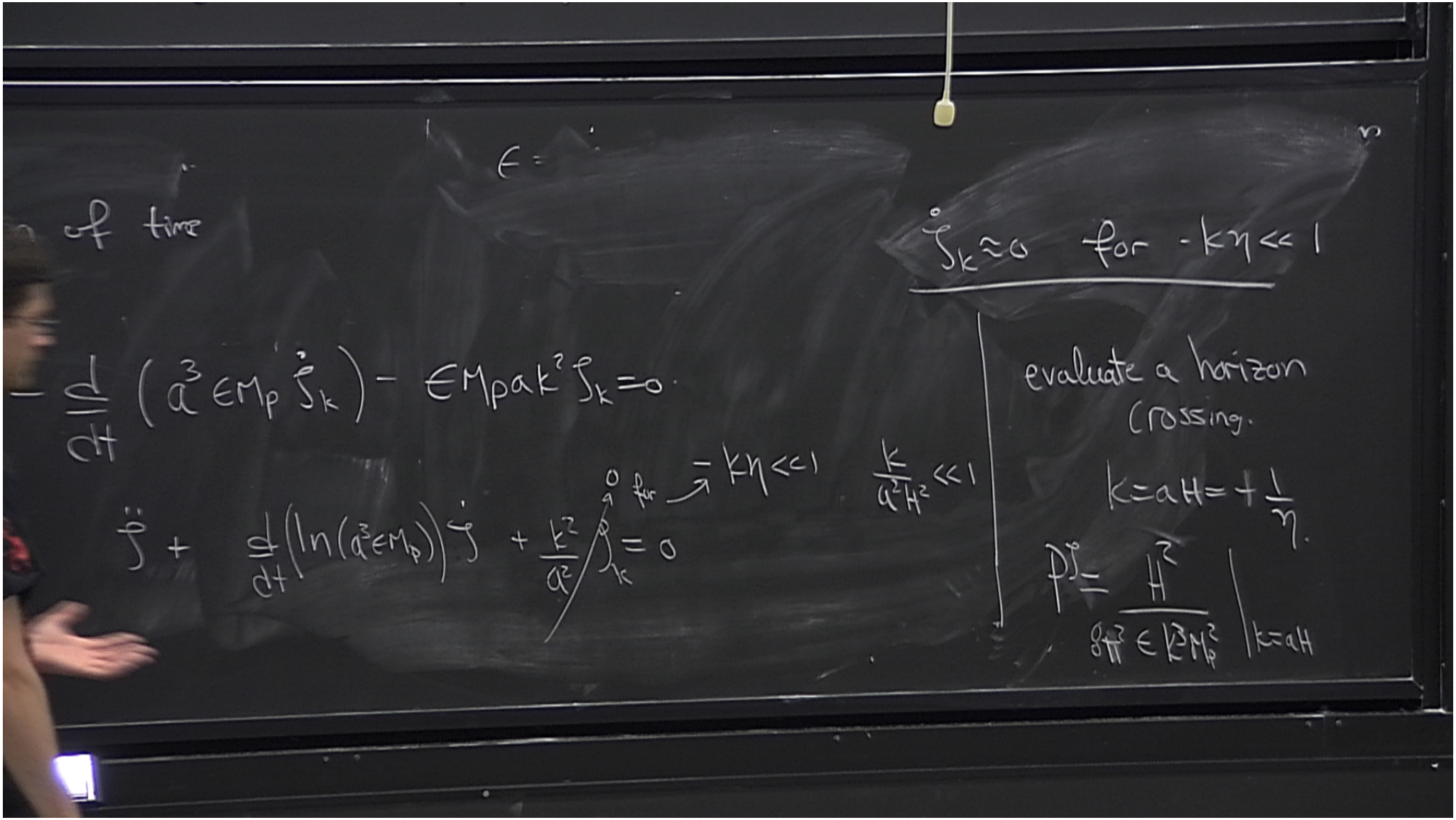
$\dot{\zeta}_k \approx 0$ for $-k\eta \ll 1$

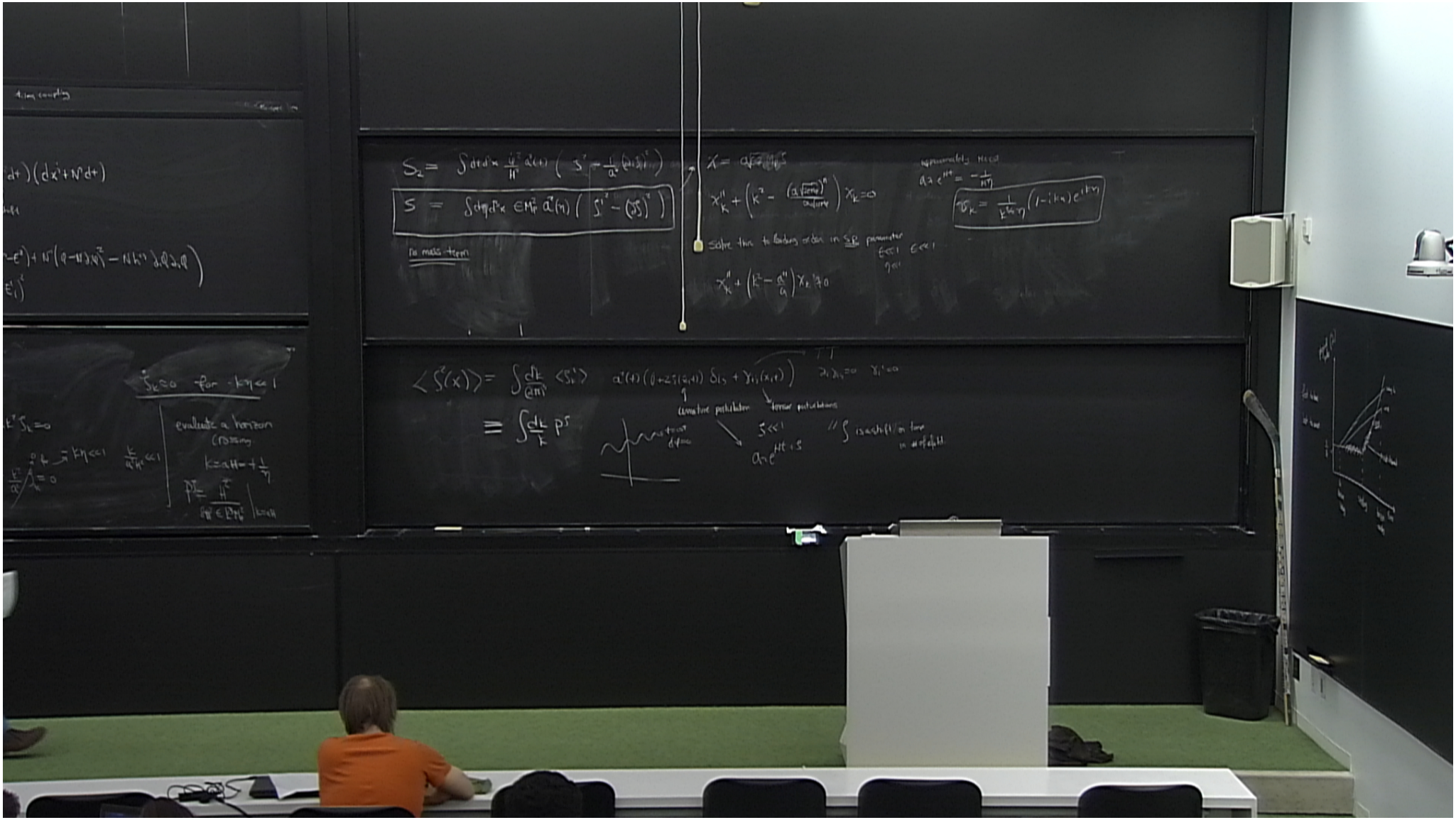
$$-\frac{d}{dt}\left(a^3 \epsilon M_p \dot{\zeta}_k\right) - \epsilon M_p a k^2 \zeta_k = 0$$

$$\ddot{\zeta} + \frac{d}{dt}\left(\ln(a^3 \epsilon M_p)\right)\dot{\zeta} + \frac{k^2}{a^2}\zeta_k = 0$$

$\frac{k}{a^2 H^2} \ll 1$ $\quad = k\eta \ll 1$ $\quad \to 0$ for

τ_0)


of time
$\epsilon =$
$\dot{\zeta}_k \approx 0$ for $-k\eta \ll 1$
$-\frac{d}{dt}\left(a^3 \epsilon M_p \dot{\zeta}_k\right) - \epsilon M_p a k^2 \zeta_k = 0$
$\ddot{\zeta} + \frac{d}{dt}\left(\ln(a^3 \epsilon M_p)\right)\dot{\zeta} + \frac{k^2}{a^2}\zeta_k = 0$
$\frac{k}{a^2 H^2} \ll 1$
evaluate a horizon crossing.
$k = aH = +\frac{1}{\eta}$.
$P_\zeta = \frac{H^2}{8\pi^2 \epsilon k^3 M_p^2}\Big|_{k=aH}$


$S_2 = \int dt\, d^3x \, \frac{a^3}{H^2} \dot{\zeta}^2 \left(1 - \frac{1}{a^2} (\partial_i \zeta)^2 \right)$
$S = \int d\eta\, d^3x \, \epsilon M_{Pl}^2 a^2(\eta) \left(\zeta'^2 - (\partial \zeta)^2 \right)$
no mass term
$\chi = a\zeta$
$\chi_k'' + \left(k^2 - \frac{(a\sqrt{\epsilon})''}{a\sqrt{\epsilon}} \right) \chi_k = 0$
solve this to leading order in SR parameter
$\chi_k'' + \left(k^2 - \frac{a''}{a} \right) \chi_k = 0$
$\zeta_k = \frac{1}{k^{3/2} a\sqrt{\epsilon}} (1 + ik\eta) e^{-ik\eta}$
$\langle \zeta^2(x) \rangle = \int \frac{dk}{(2\pi)^3} \langle \zeta_k^2 \rangle$
$\equiv \int \frac{dk}{k} P^2$
$a^2(\eta) (\delta_{ij} + 2\zeta(\eta, x)) \delta_{ij} + \gamma_{ij}(x, \eta))$
$\lambda_i \lambda_j = 0 \quad \gamma_i^i = 0$
tensor perturbations
scalar perturbations
$\zeta \ll 1$
$a \sim e^{Ht + \zeta}$
// ζ is constant on time in uniform
$\dot{\zeta}_k \to 0$ for $-k\eta \ll 1$
evaluate at horizon crossing
$k = aH = +\frac{1}{\eta}$
$P_\zeta^2 = \frac{H^2}{8\pi^2 \epsilon P_{Pl}^2}$

$$\langle \quad \rangle = \int \frac{d^3k}{(2\pi)^3} \langle \zeta_k^2 \rangle \qquad a^2(t)\left((1+2\zeta(\vec{x},t))\,\delta_{ij} + \gamma_{ij}(x,t)\right)$$

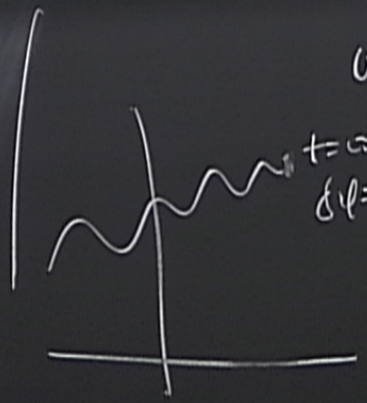
TT

$\partial_i \gamma_{ij} = 0 \qquad \gamma_i{}^i = 0$

curvature perturbation

tensor perturbations

$$\equiv \int \frac{dk}{k} P_\zeta$$



$\zeta \ll 1$

$a \sim e^{Ht + \zeta}$

// ζ is a shift in time in # of efold.