Title: Explorations in Cosmology - Lecture 6

Date: Mar 19, 2012 10:15 AM

URL: http://pirsa.org/12030030

Abstract:

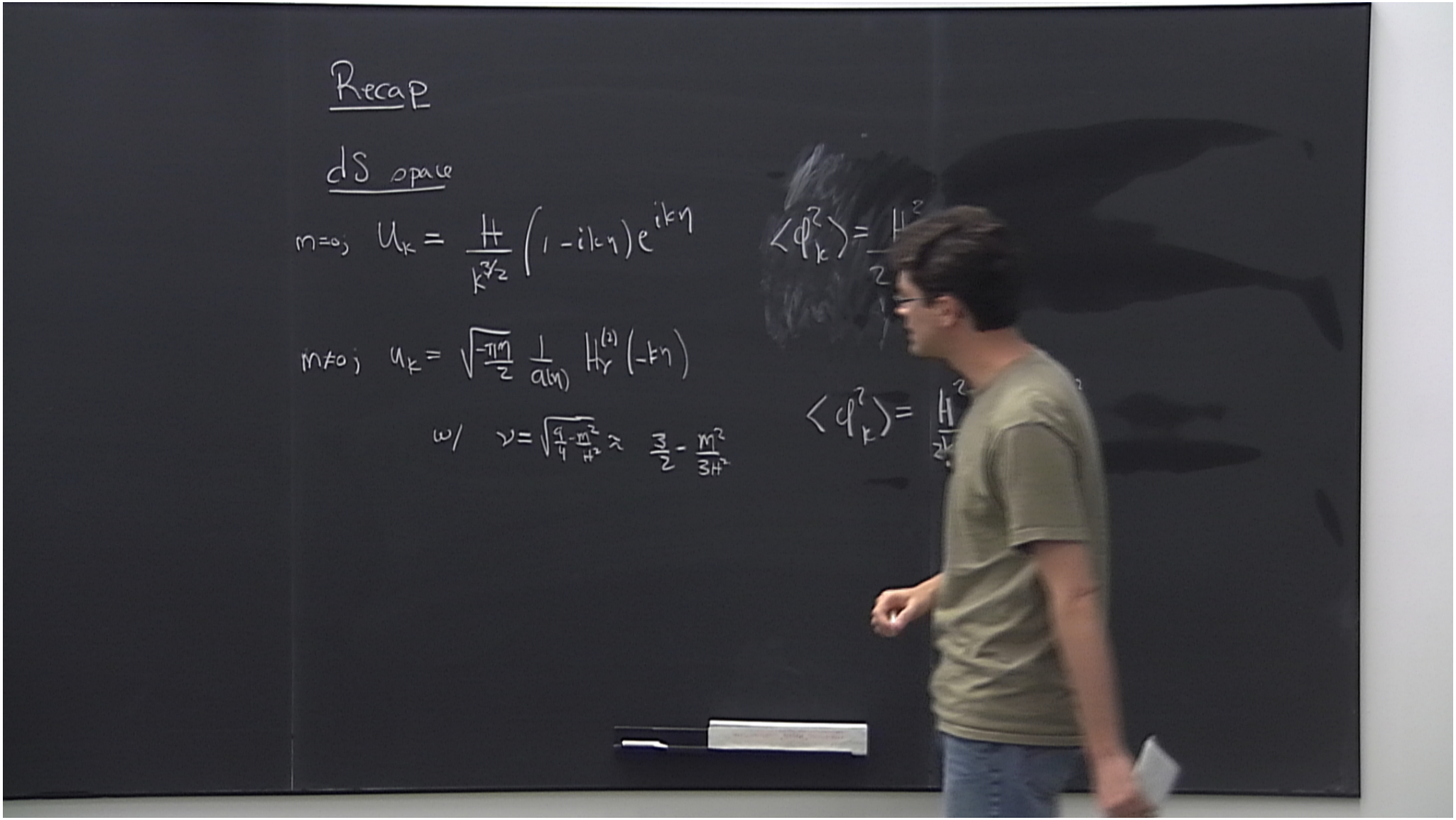
Recap

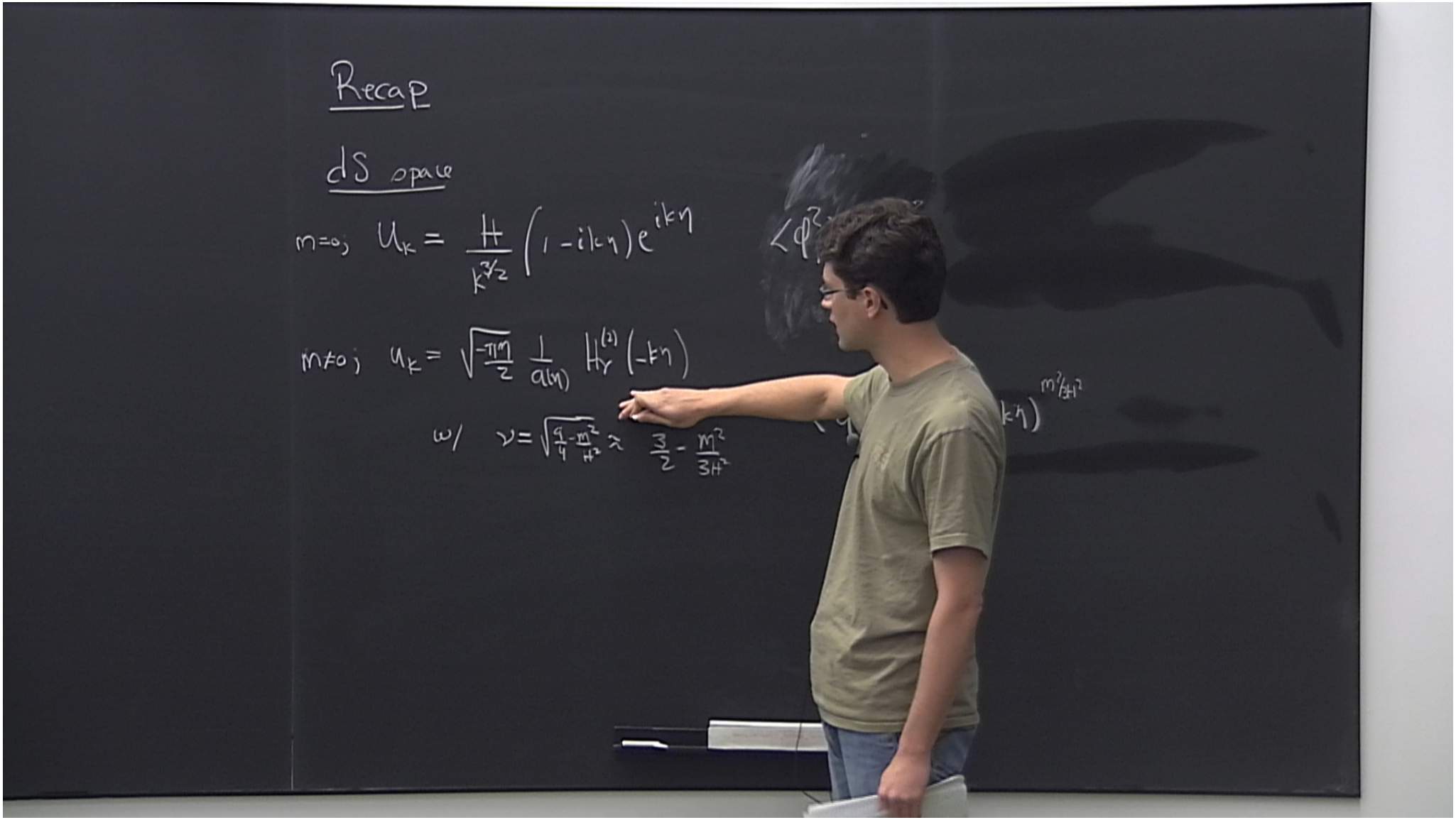
dS

$m=0$; $u_k = \frac{H}{k^{3/2}}\left(1 - ik\eta\right)e^{ik\eta}$ $\qquad \langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}$

$m \neq 0$; $u_k = \sqrt{\frac{-\pi\eta}{2}}\frac{1}{a(\eta)} H_\nu^{(2)}(-k\eta)$

w/ $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \approx \frac{3}{2} - \frac{m^2}{3H^2}$ $\qquad \langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}(-k\eta)^{m^2/3H^2}$


Recap
dS space
$m=0;\ u_k = \frac{H}{k^{3/2}}\left(1-ik\eta\right)e^{ik\eta}$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2}$
$m\neq 0;\ u_k = \sqrt{\frac{-\pi\eta}{2}}\frac{1}{a(\eta)}H_\nu^{(1)}(-k\eta)$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}$
$w/\ \ \nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \approx \frac{3}{2}-\frac{m^2}{3H^2}$


Recap
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$m=0;\ u_k = \frac{H}{k^{3/2}}\left(1-ik\eta\right)e^{ik\eta}$
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$\omega/\ \nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \approx \frac{3}{2}-\frac{m^2}{3H^2}$

Recap

dS space

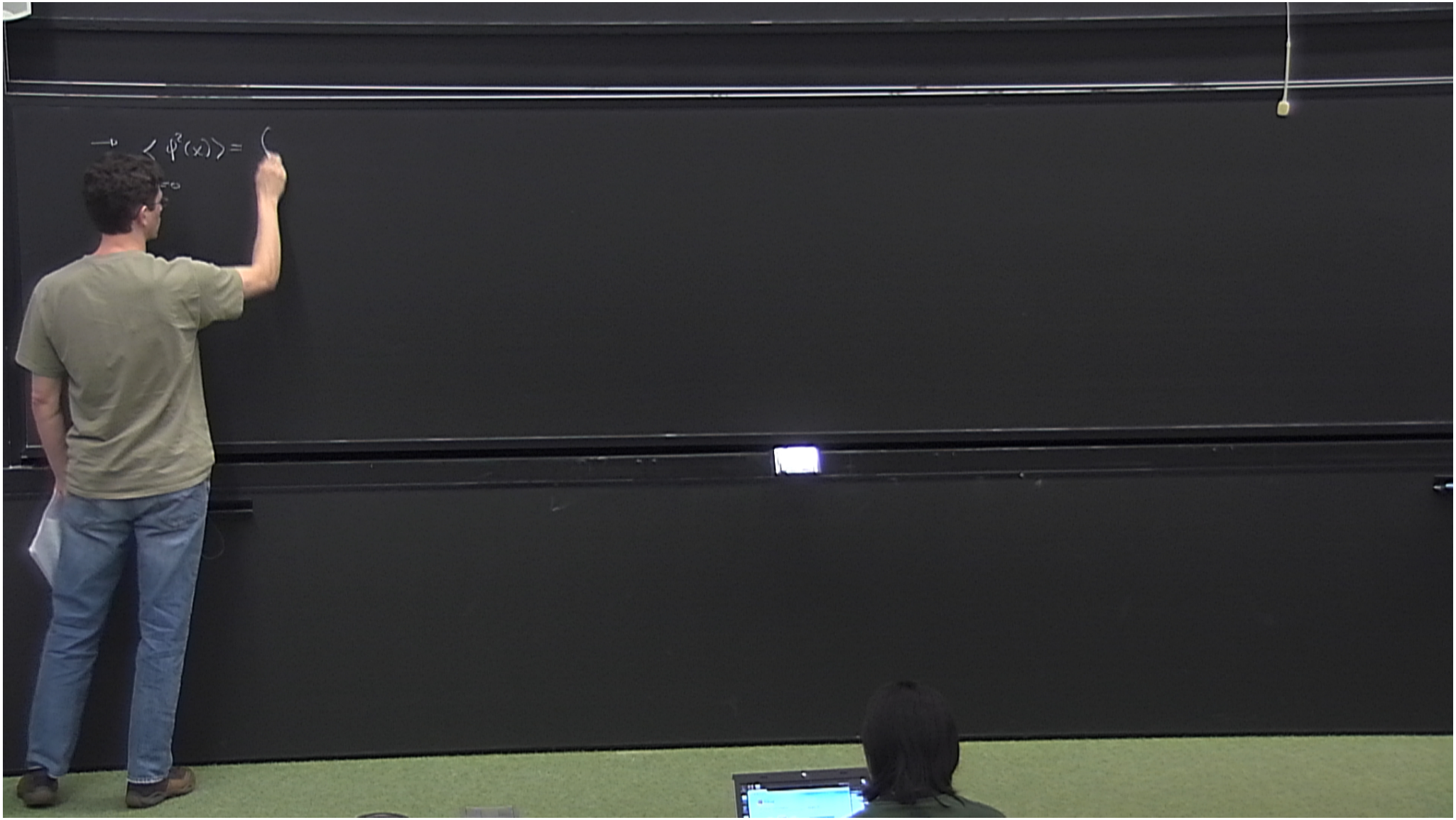
$m=0$; $u_k = \frac{H}{k^{3/2}}(1-ik\eta)e^{ik\eta}$ $\quad -k\eta \ll 1$

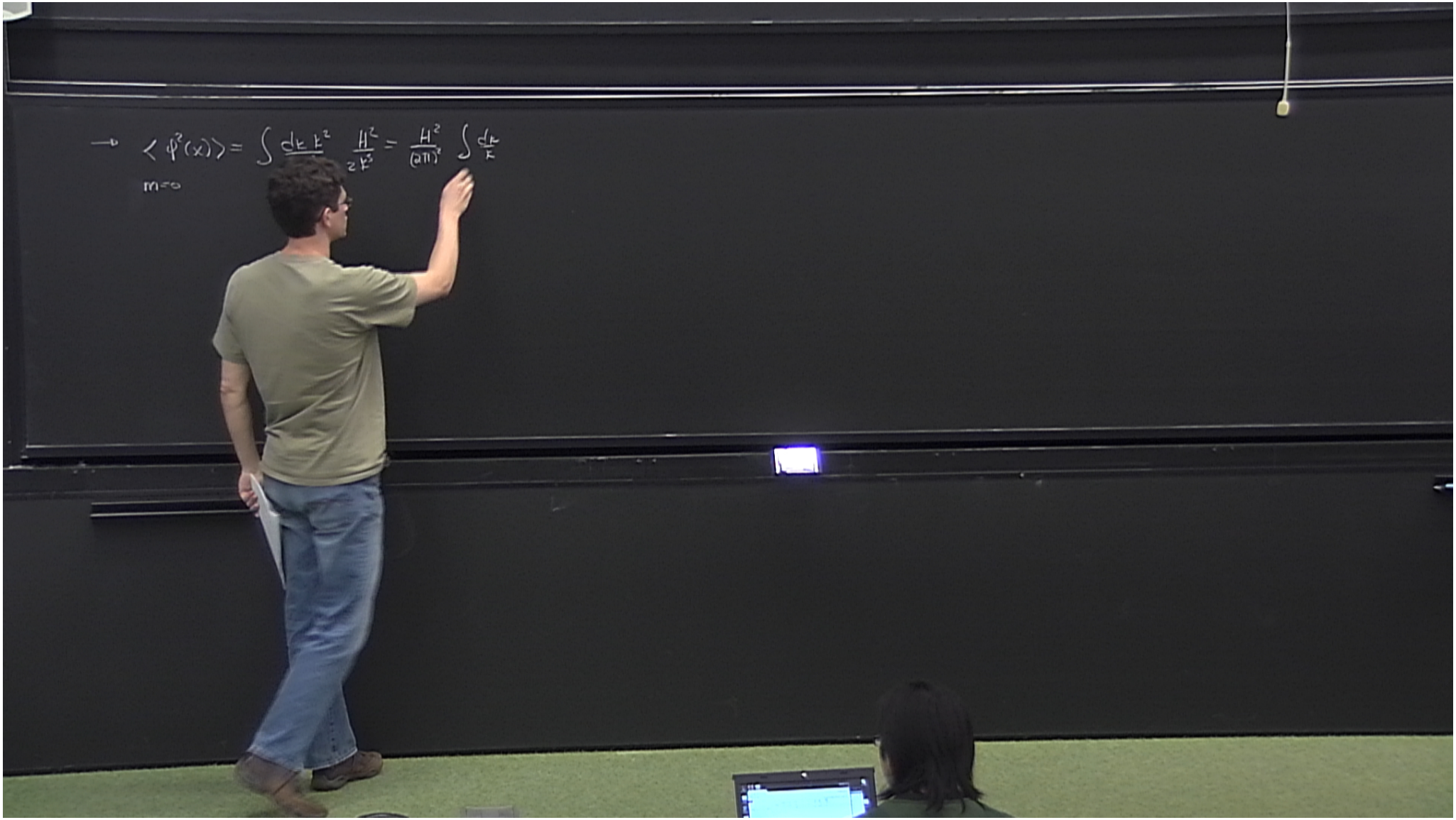
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}$

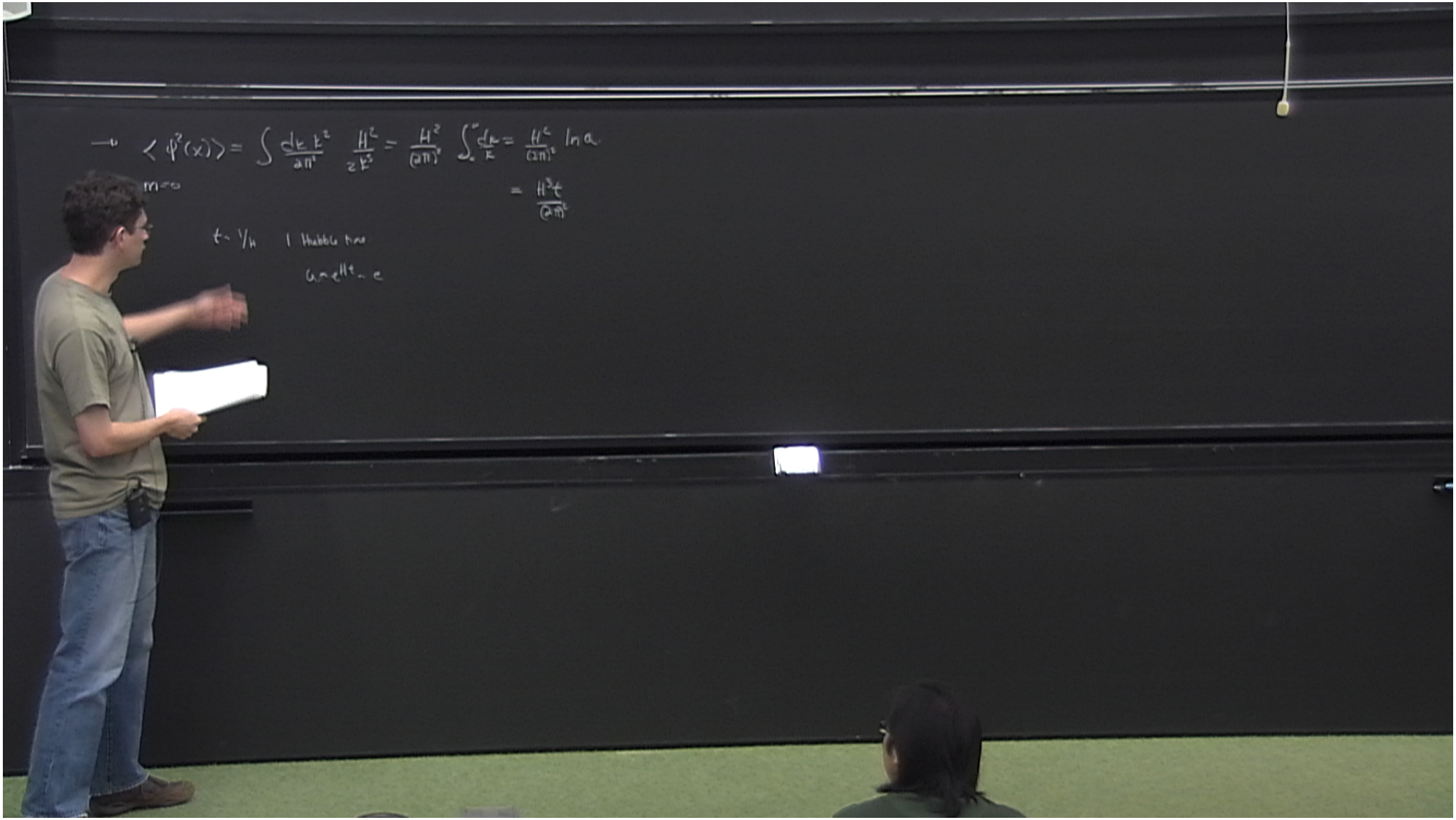
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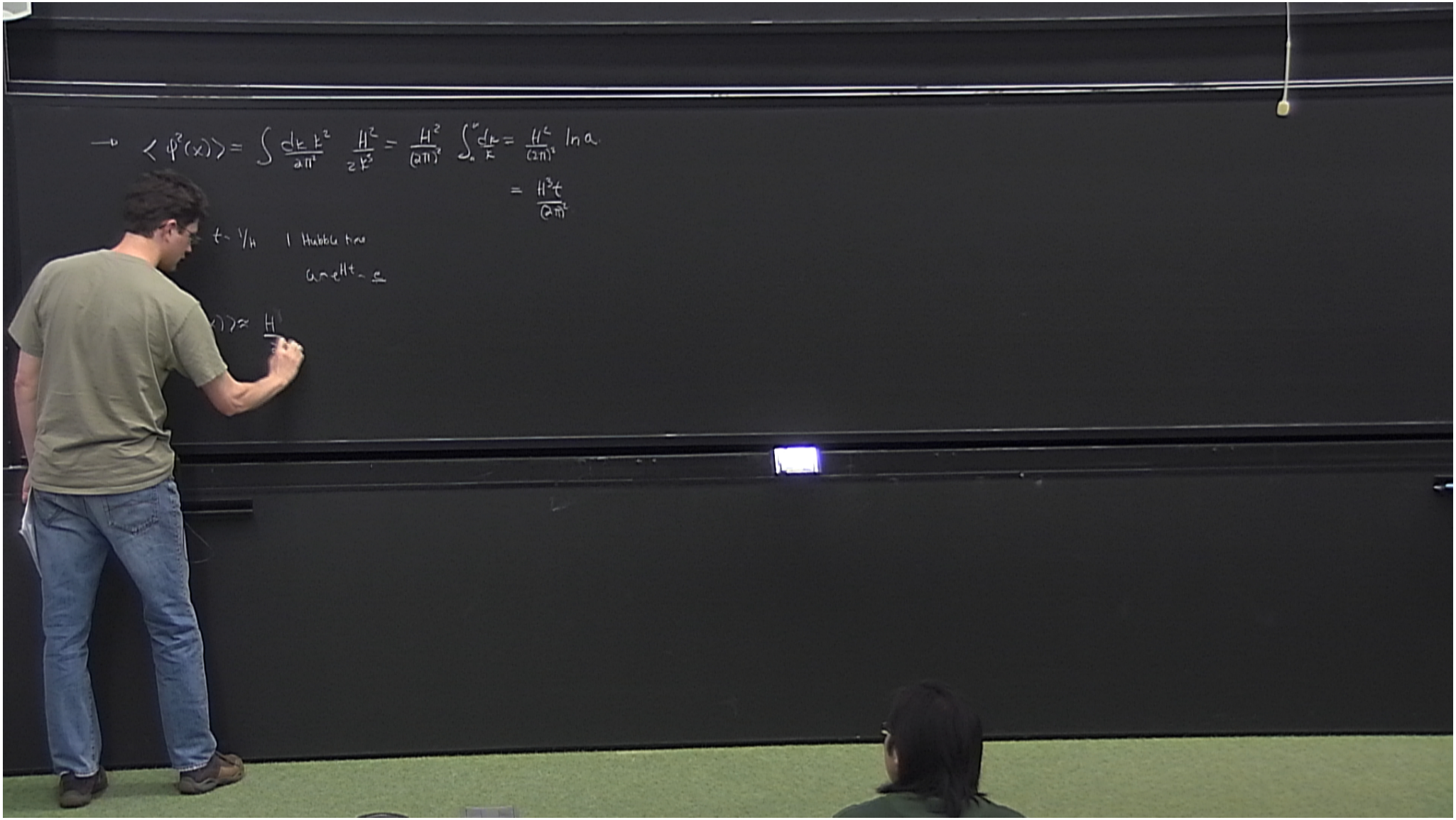
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}(-k\eta)^{m^2/H^2}$

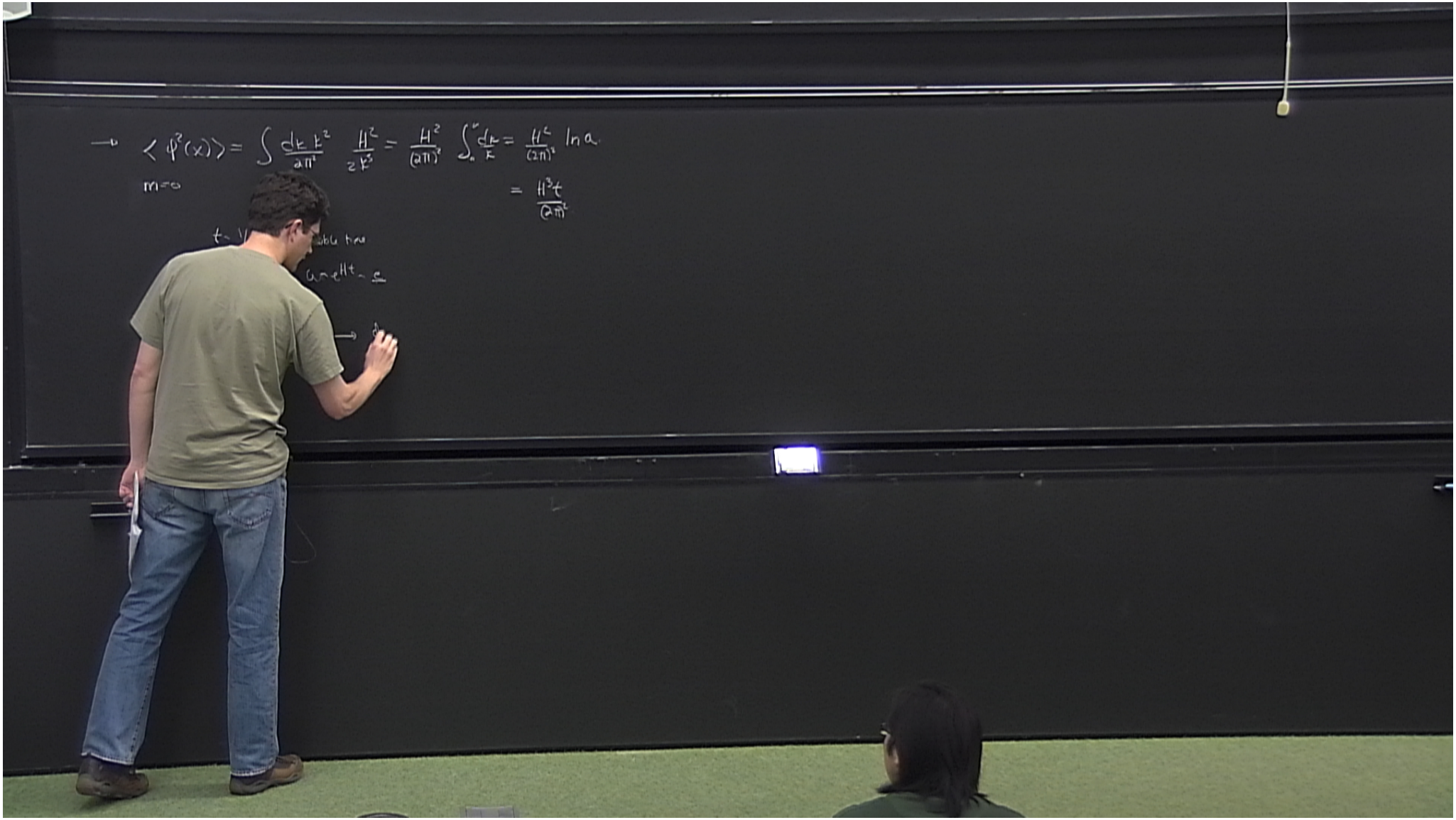
w/ $\nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \approx \frac{3}{2} - \frac{m^2}{3H^2}$

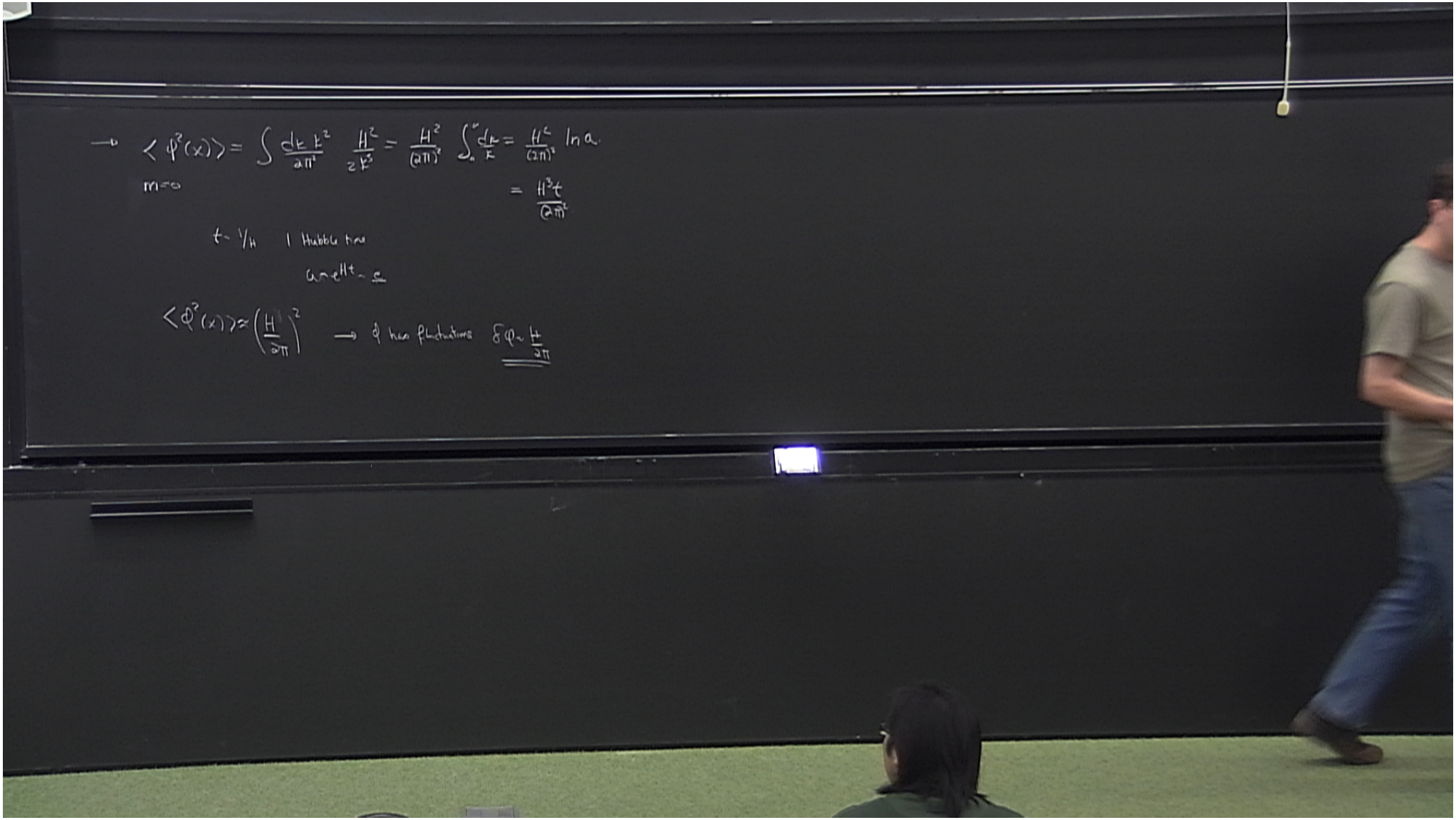

$\langle \psi^2(x) \rangle =$

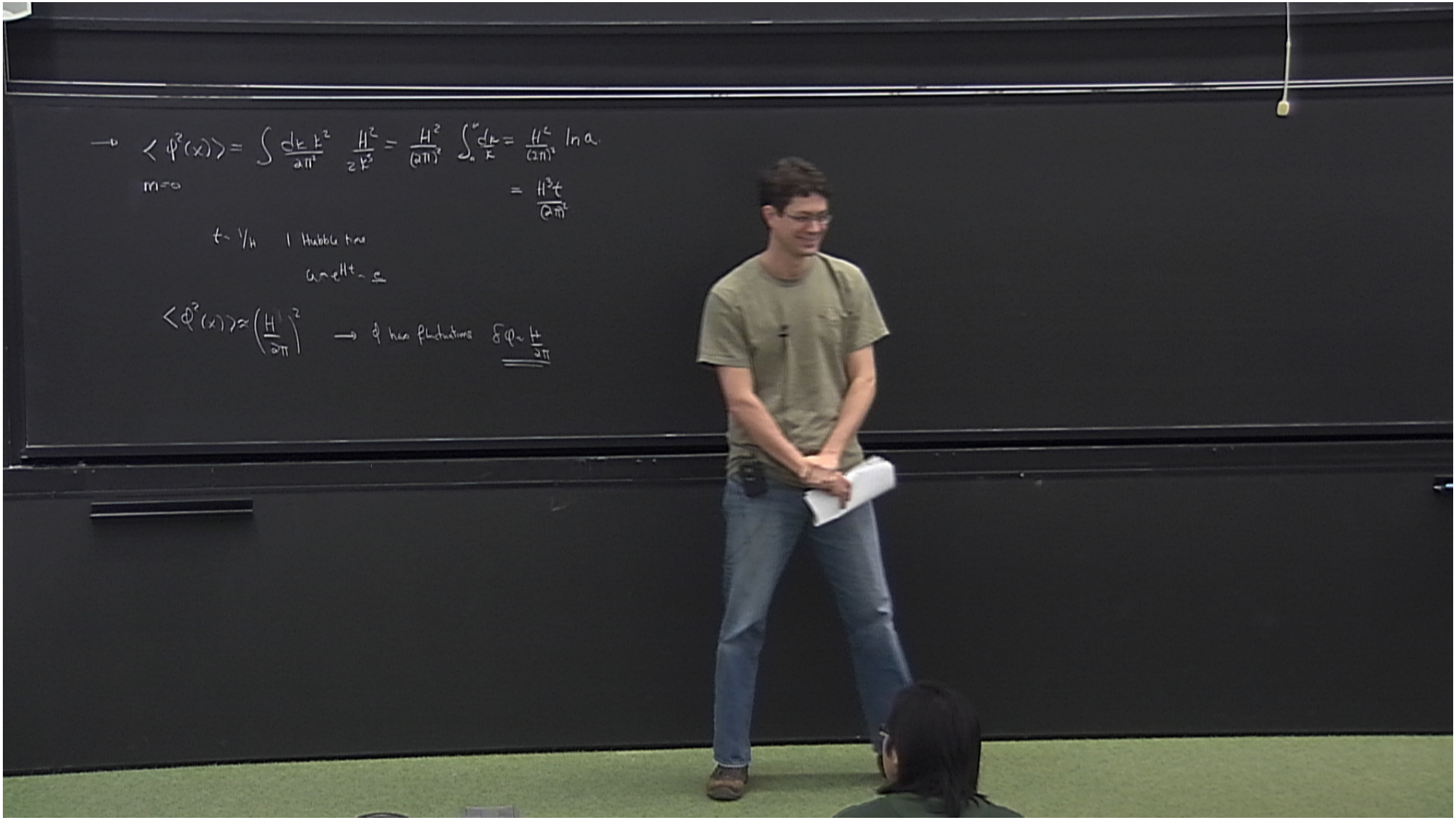

$\rightarrow \langle \varphi^2(x) \rangle = \int \frac{dk\, k^2}{2k^3} \frac{H^2}{(2\pi)^2} = \frac{H^2}{(2\pi)^2} \int \frac{dk}{k}$
m=0

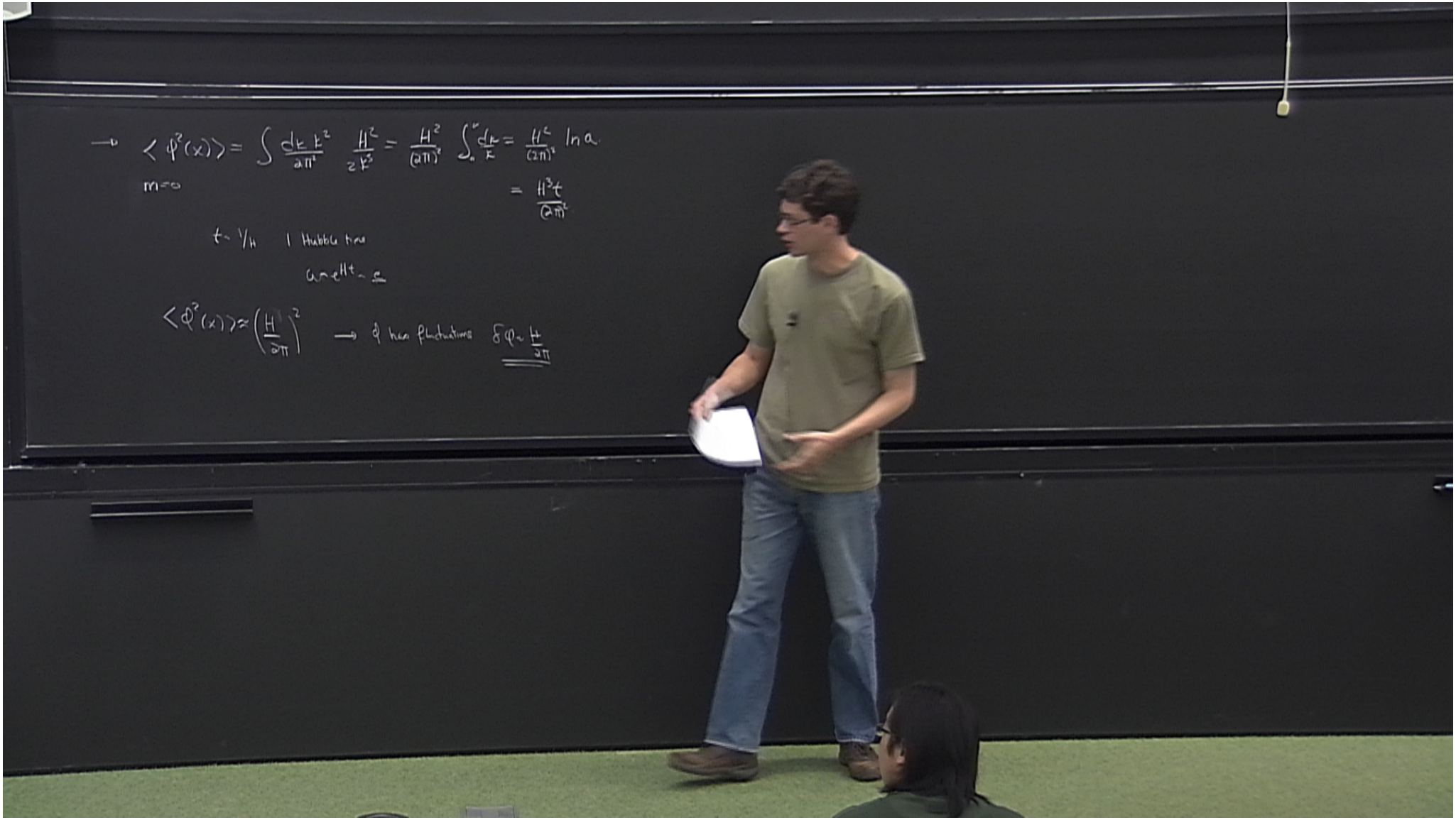

$\langle \varphi^2(x) \rangle = \int \frac{dk\, k^2}{2\pi^2} \frac{H^2}{2k^3} = \frac{H^2}{(2\pi)^2} \int_a^{\infty} \frac{dk}{k} = \frac{H^2}{(2\pi)^2} \ln a$
$= \frac{H^2 t}{(2\pi)^2}$
$m=0$
$t \sim 1/H$ 1 Hubble time

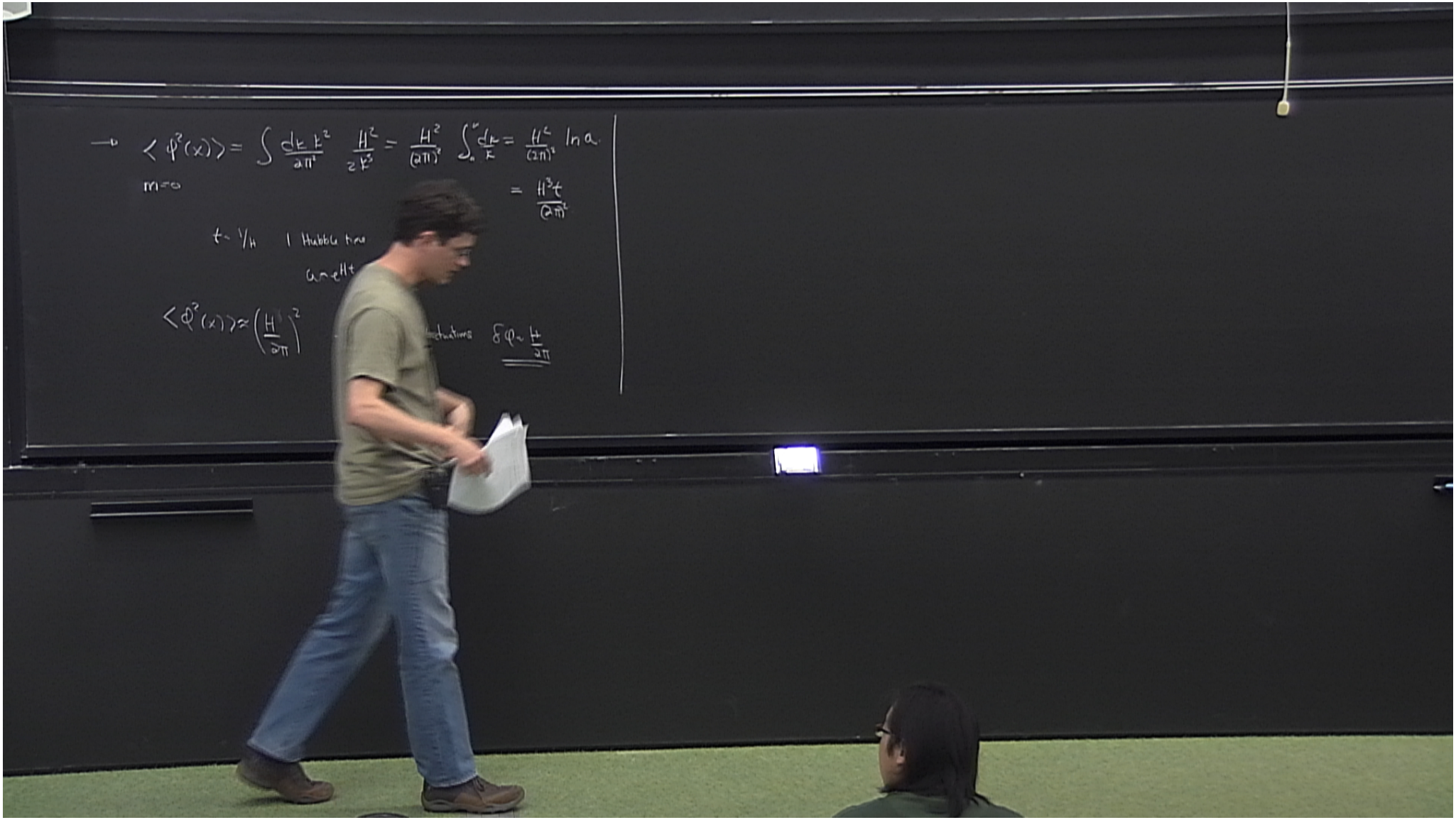

$\langle \varphi^2(x) \rangle = \int \frac{dk\, k^2}{2\pi^2} \frac{H^2}{2k^3} = \frac{H^2}{(2\pi)^2} \int_a^b \frac{dk}{k} = \frac{H^2}{(2\pi)^2} \ln a$
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$t \sim 1/H$ 1 Hubble time
$a \sim e^{Ht} \sim$

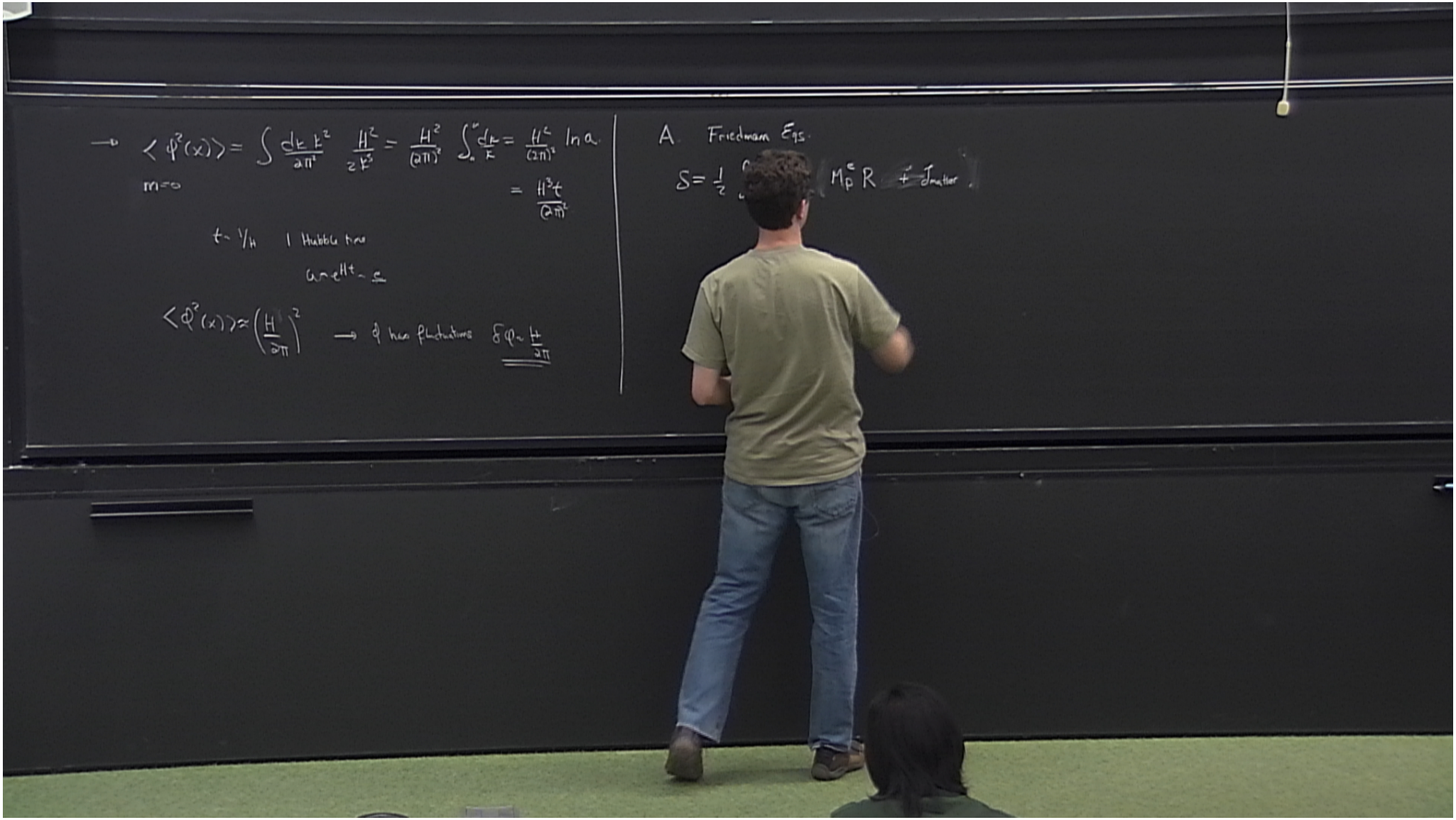

$\langle \varphi^2(x) \rangle = \int \frac{dk\, k^2}{2\pi^2} \frac{H^2}{2k^3} = \frac{H^2}{(2\pi)^2} \int_a^{a'} \frac{da}{a} = \frac{H^2}{(2\pi)^2} \ln a$
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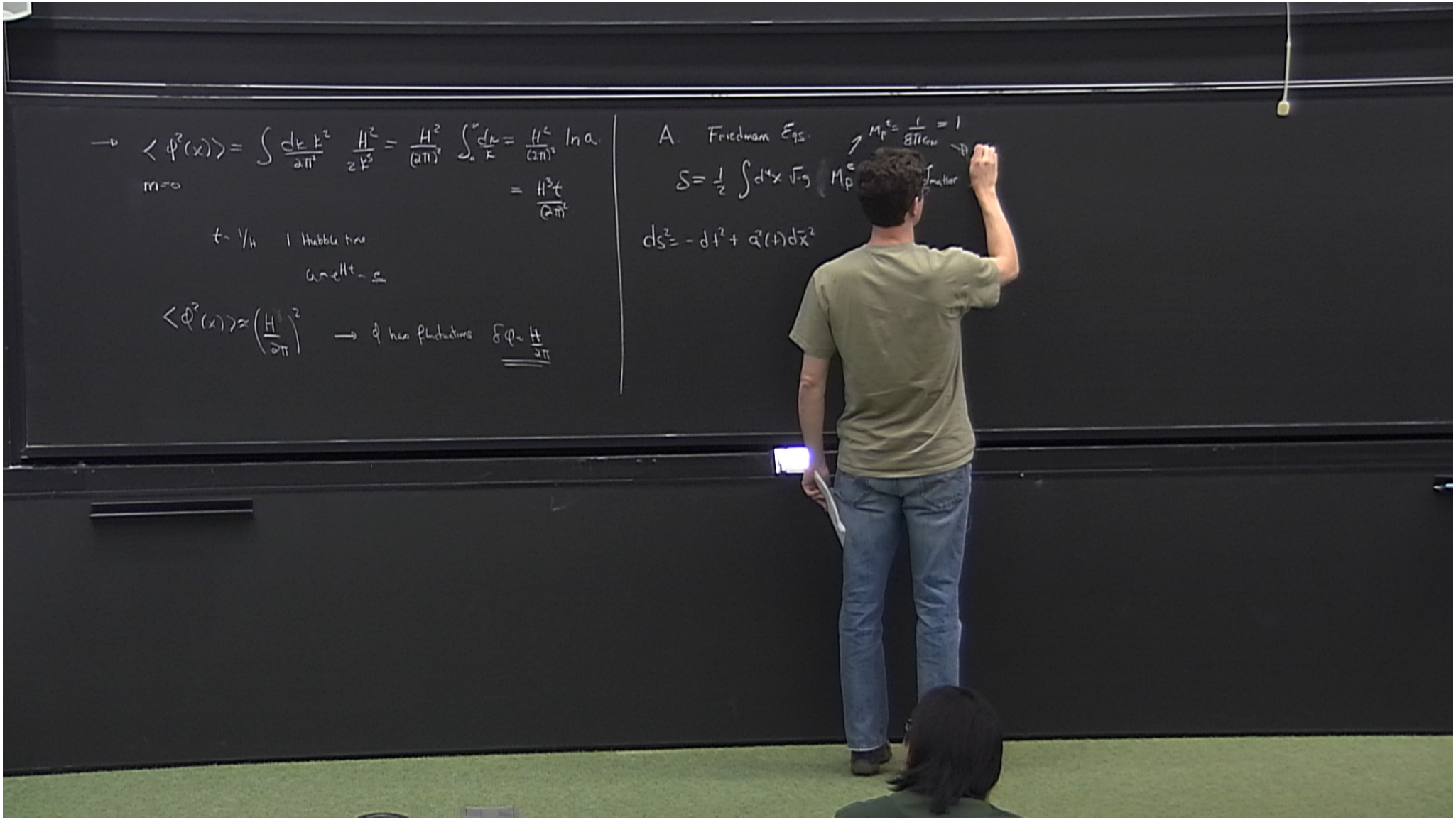

$\langle \phi^2(x) \rangle = \int \frac{dk\, k^2}{2\pi^2} \frac{H^2}{2k^3} = \frac{H^2}{(2\pi)^2} \int_a^a \frac{dk}{k} = \frac{H^2}{(2\pi)^2} \ln a$
$m=0$
$= \frac{H^3 t}{(2\pi)^2}$
$t \sim 1/H$ 1 Hubble time
$a \sim e^{Ht} \sim e$
$\langle \phi^2(x) \rangle \simeq \left(\frac{H}{2\pi}\right)^2$ → ϕ has fluctuations $\delta\phi \sim \frac{H}{2\pi}$

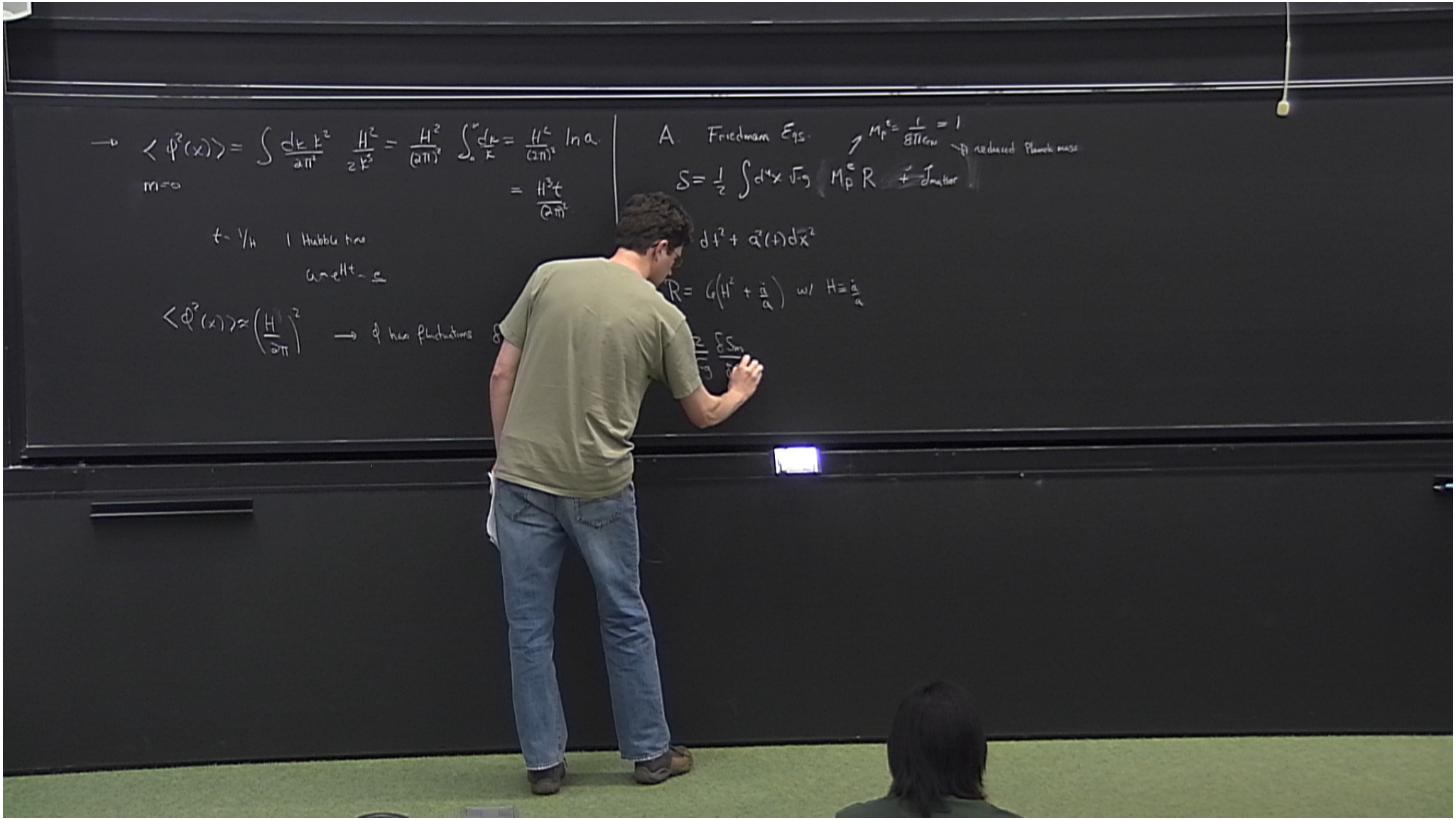

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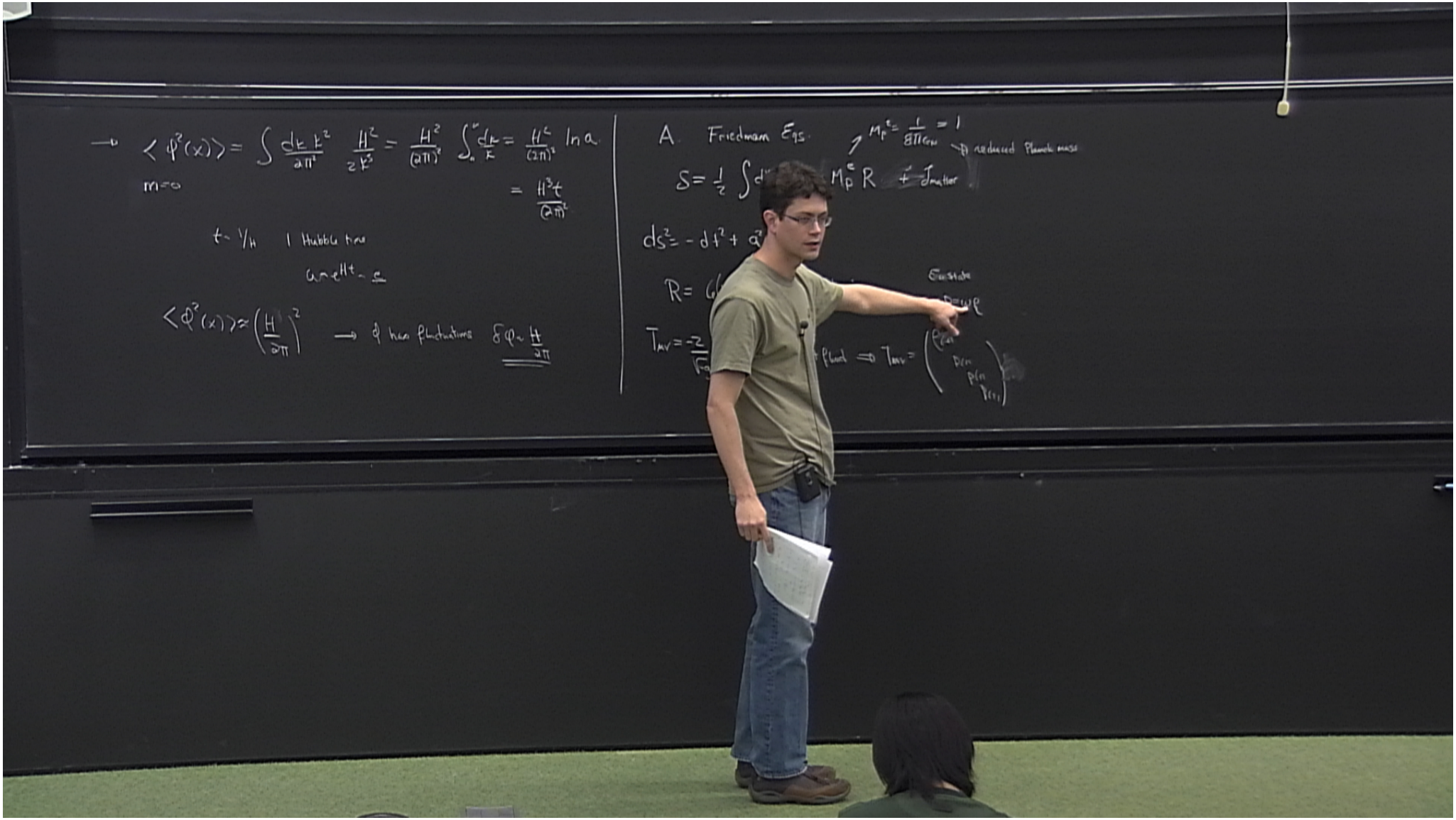

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$\langle \varphi^2(x) \rangle \approx \left(\frac{H}{2\pi}\right)^2$ → φ has fluctuations $\delta\varphi \sim \frac{H}{2\pi}$

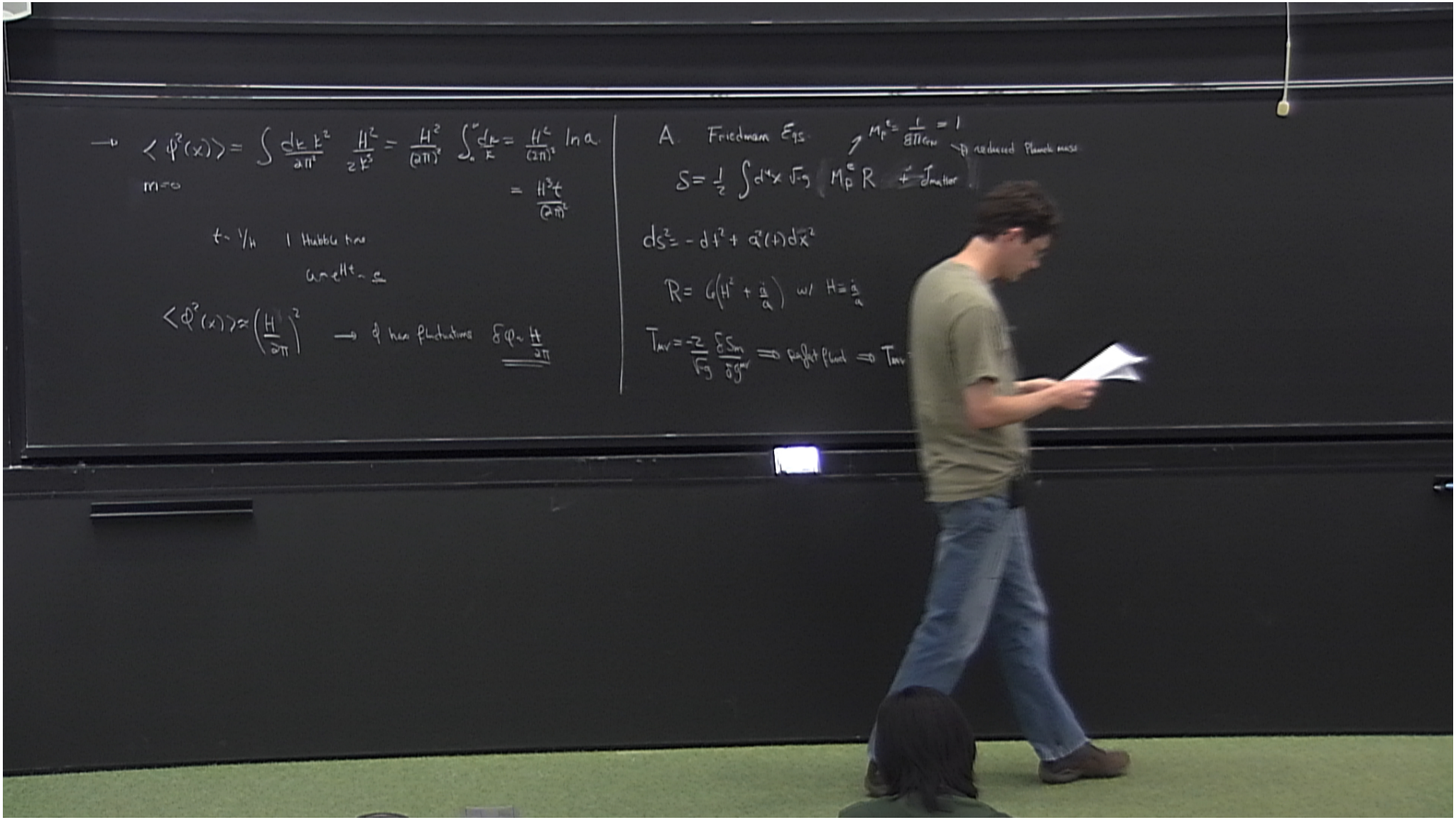

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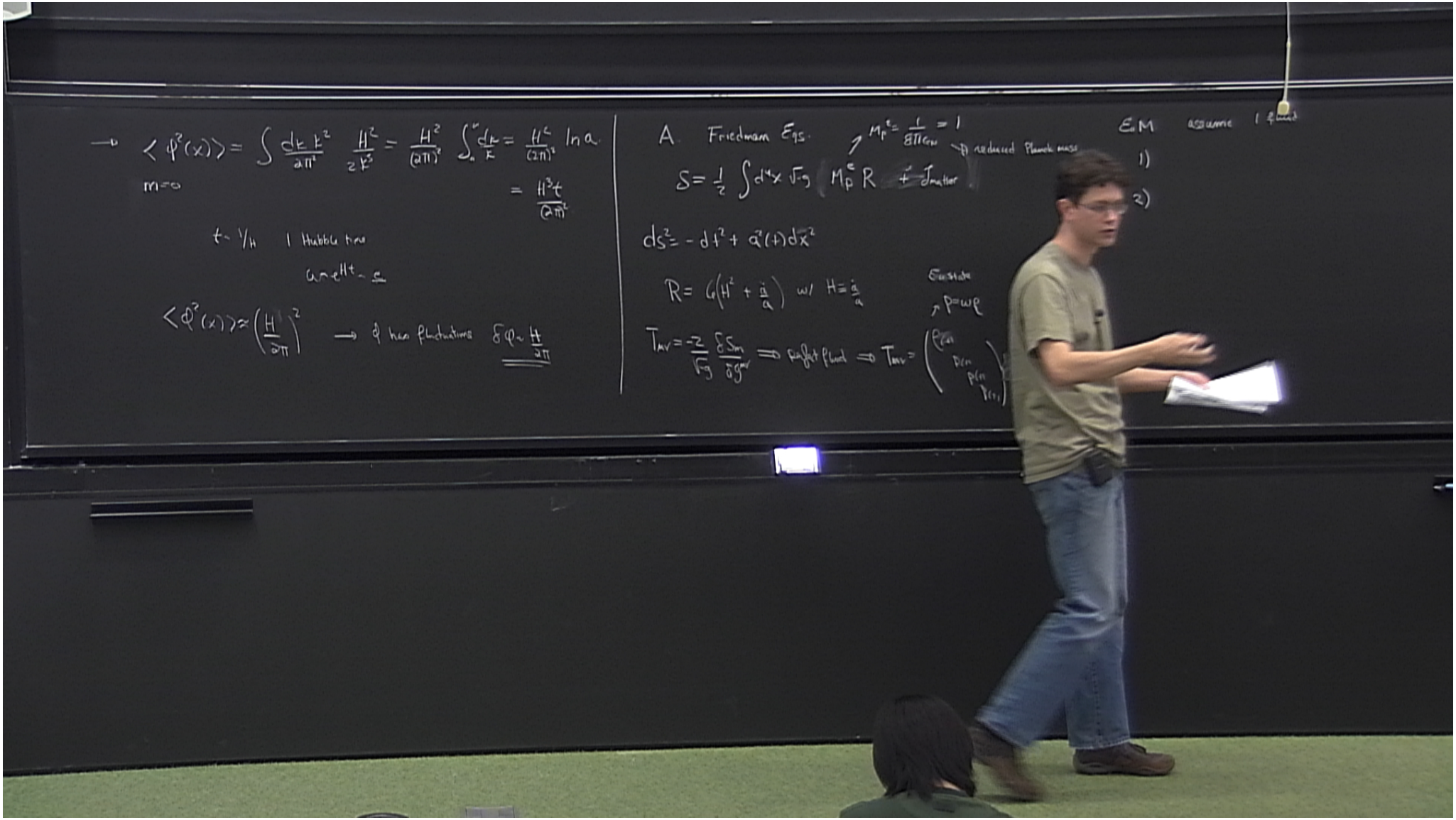

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A Friedman Eqs.
$S = \frac{1}{2}$ $M_p^2 R$ + S_{matter}

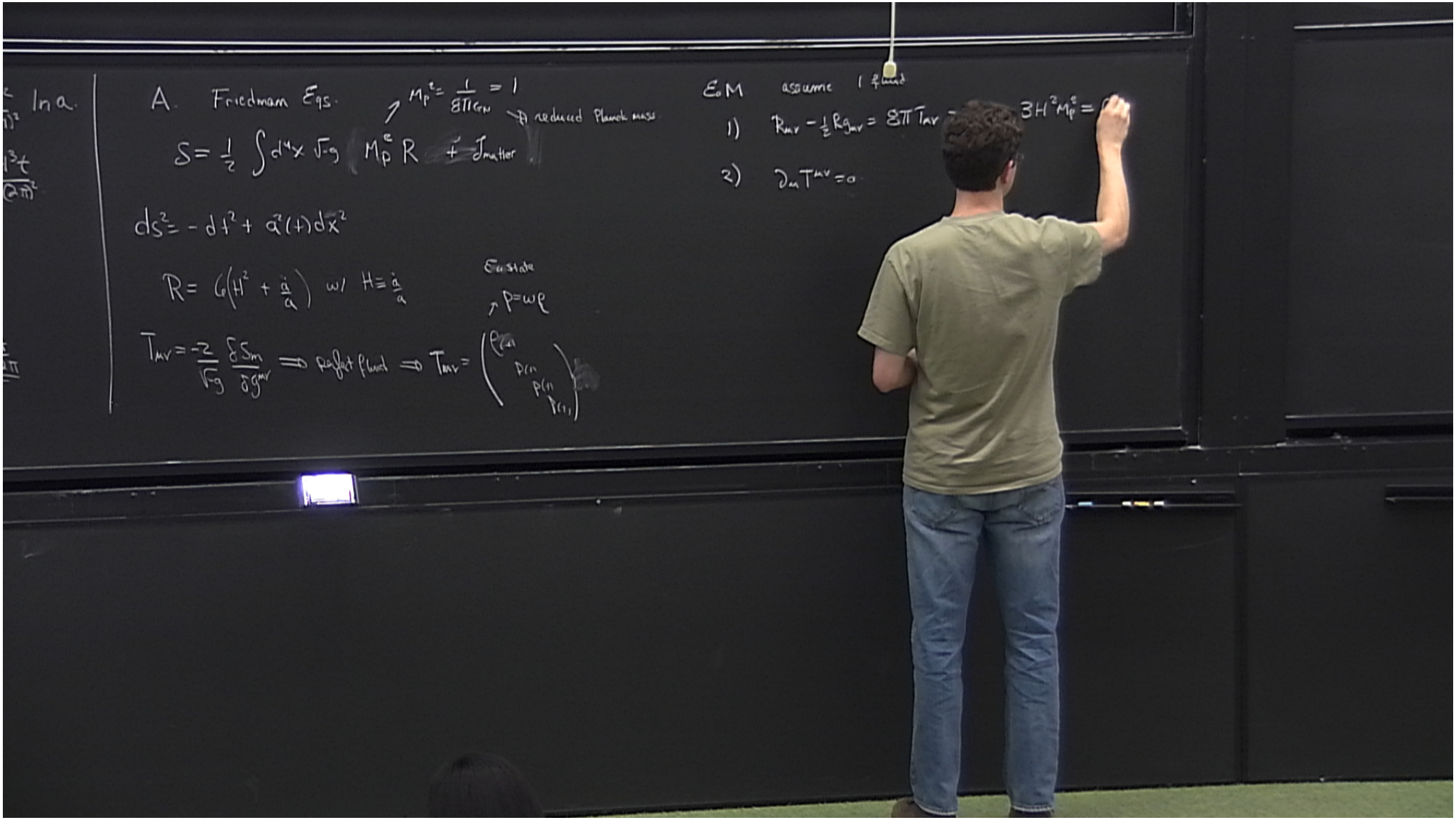

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$M_P^2 = \frac{1}{8\pi G_N} = 1$
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matter
$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$

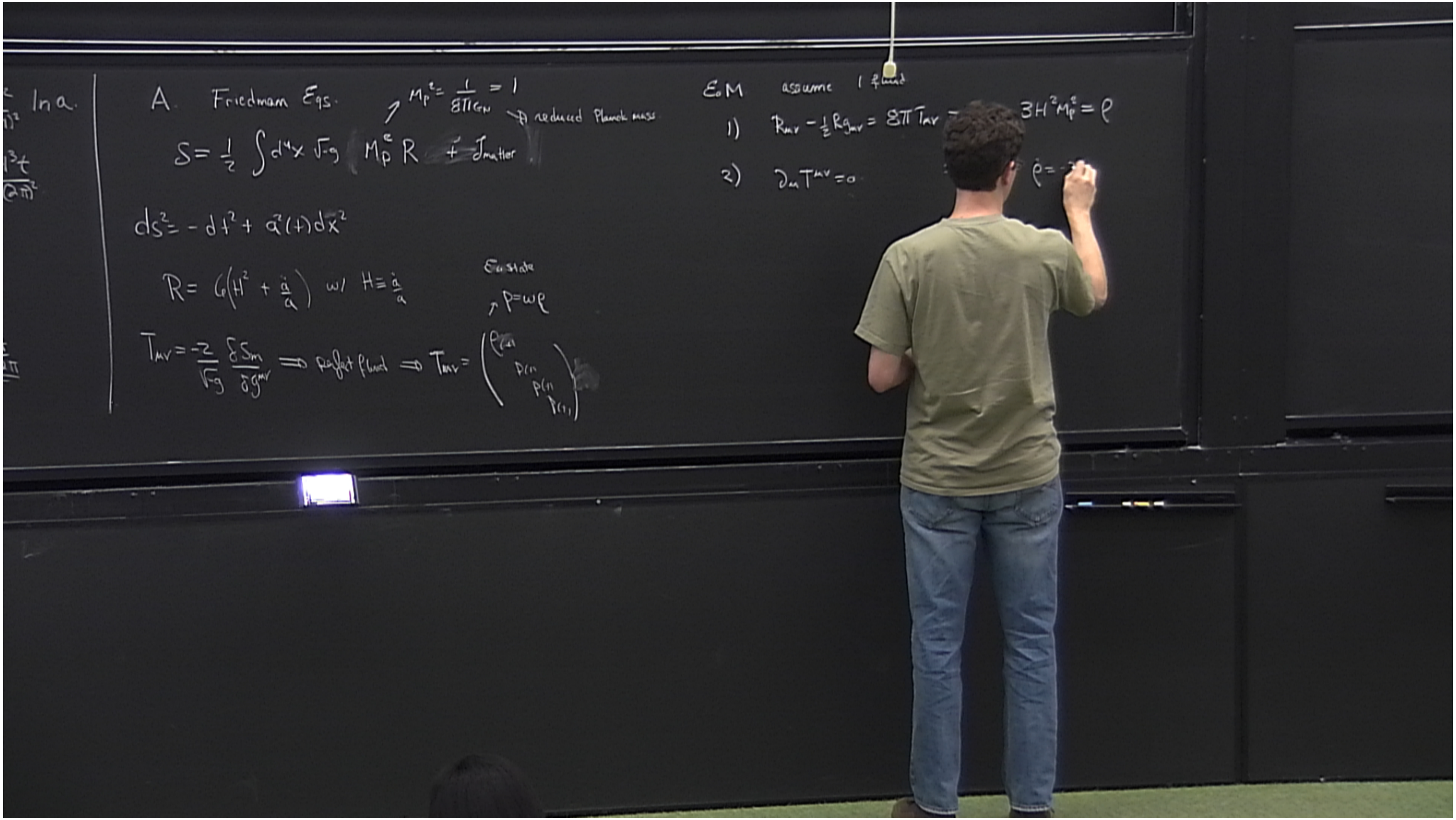

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$ds^2 = dt^2 + a^2(t) d\vec{x}^2$
$R = 6\left(H^2 + \frac{\ddot{a}}{a}\right)$ w/ $H \equiv \frac{\dot{a}}{a}$

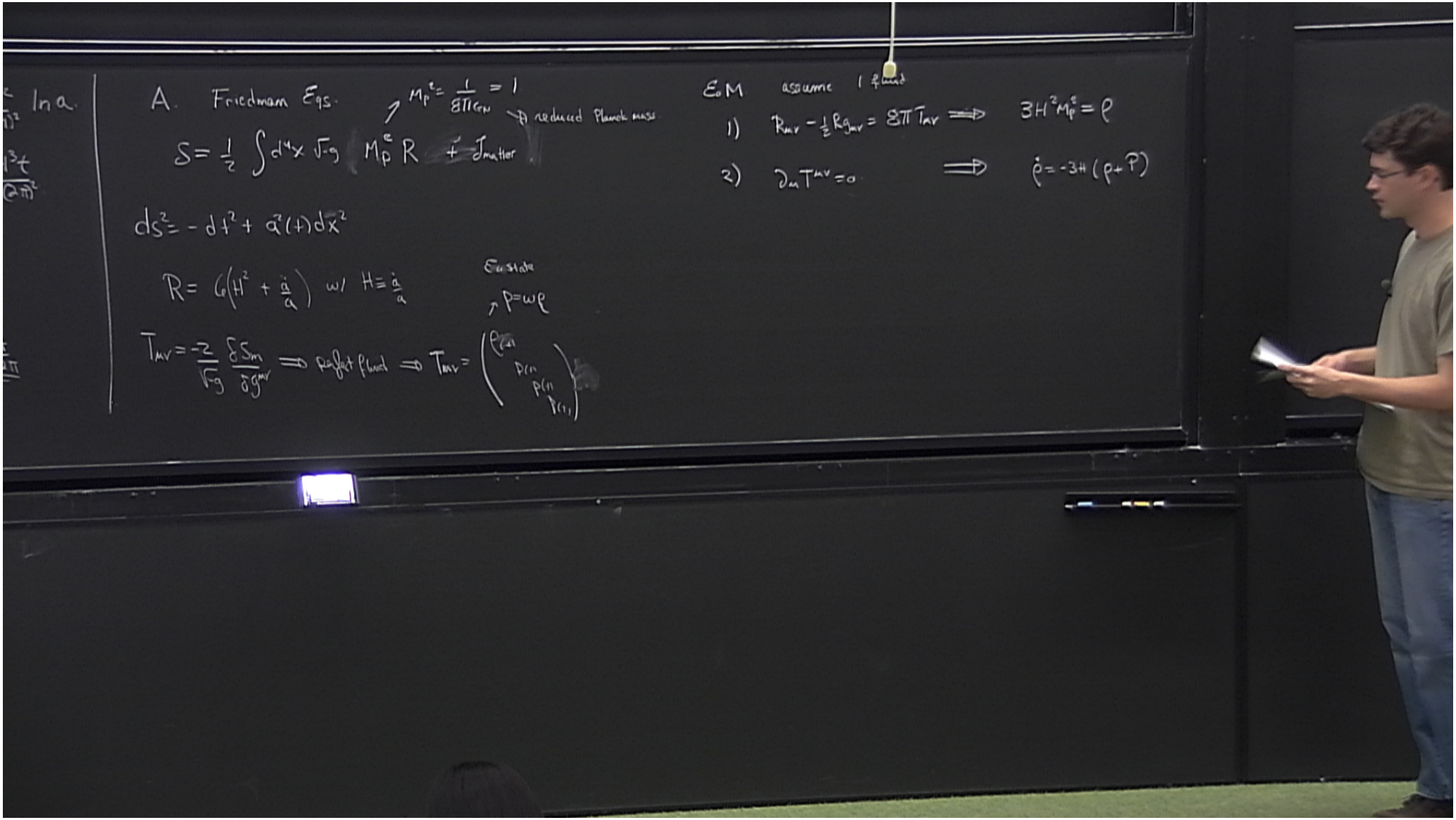

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A Friedman Eqs.
$M_p^2 = \frac{1}{8\pi G_N} = 1$ → reduced Planck mass
$S = \frac{1}{2} \int d$ $M_p^2 R$ + $\mathcal{L}_{matter}$
$ds^2 = -dt^2 + a^2$
$R = G$
$T_{\mu\nu} = \frac{-2}{\sqrt{g}}$ fluid ⇒ $T_{\mu\nu} =$

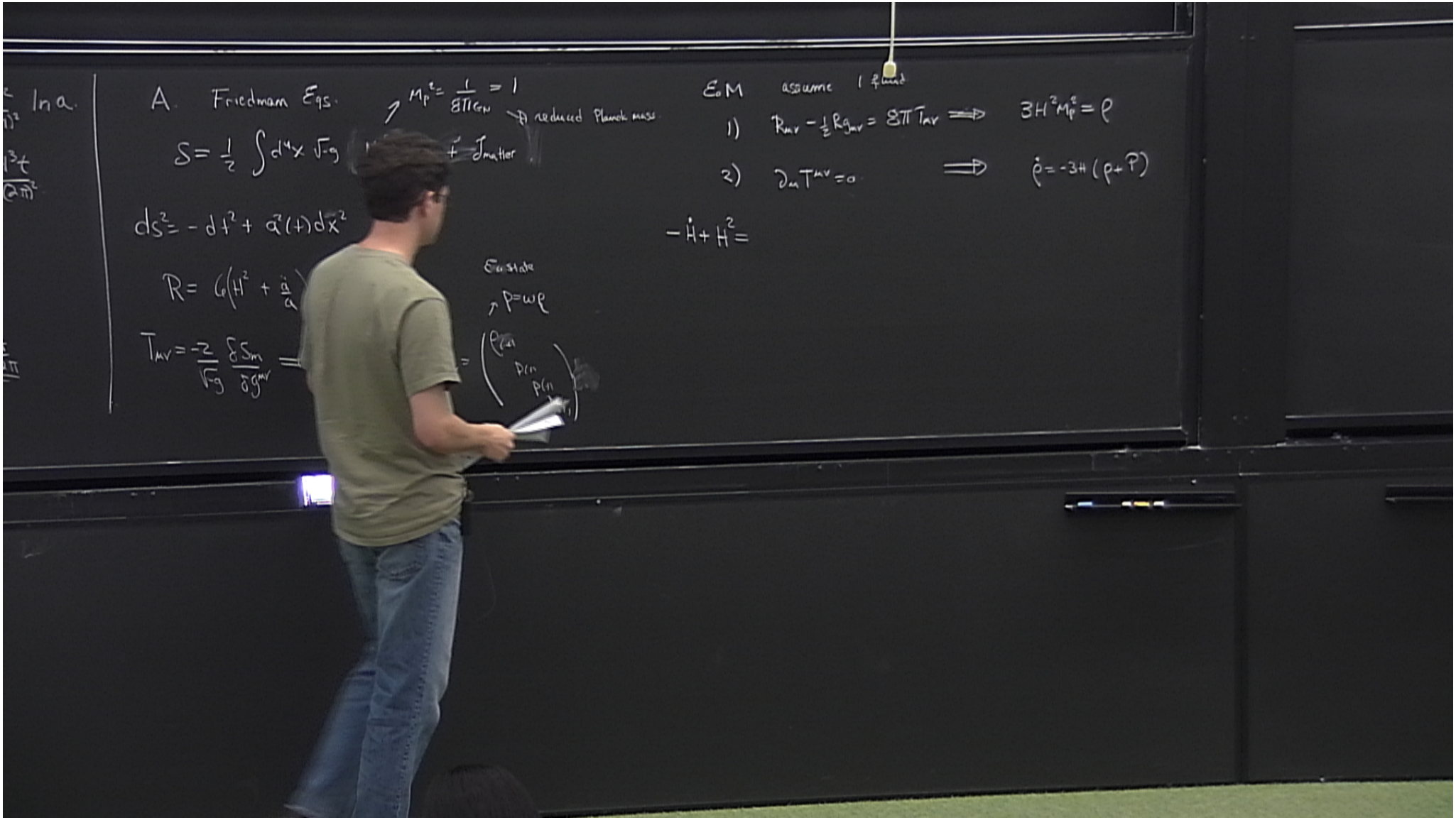

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$R = 6\left(H^2 + \frac{\ddot{a}}{a}\right)$ w/ $H \equiv \frac{\dot{a}}{a}$
$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$ ⇒ perfect fluid ⇒ $T_{\mu\nu}$

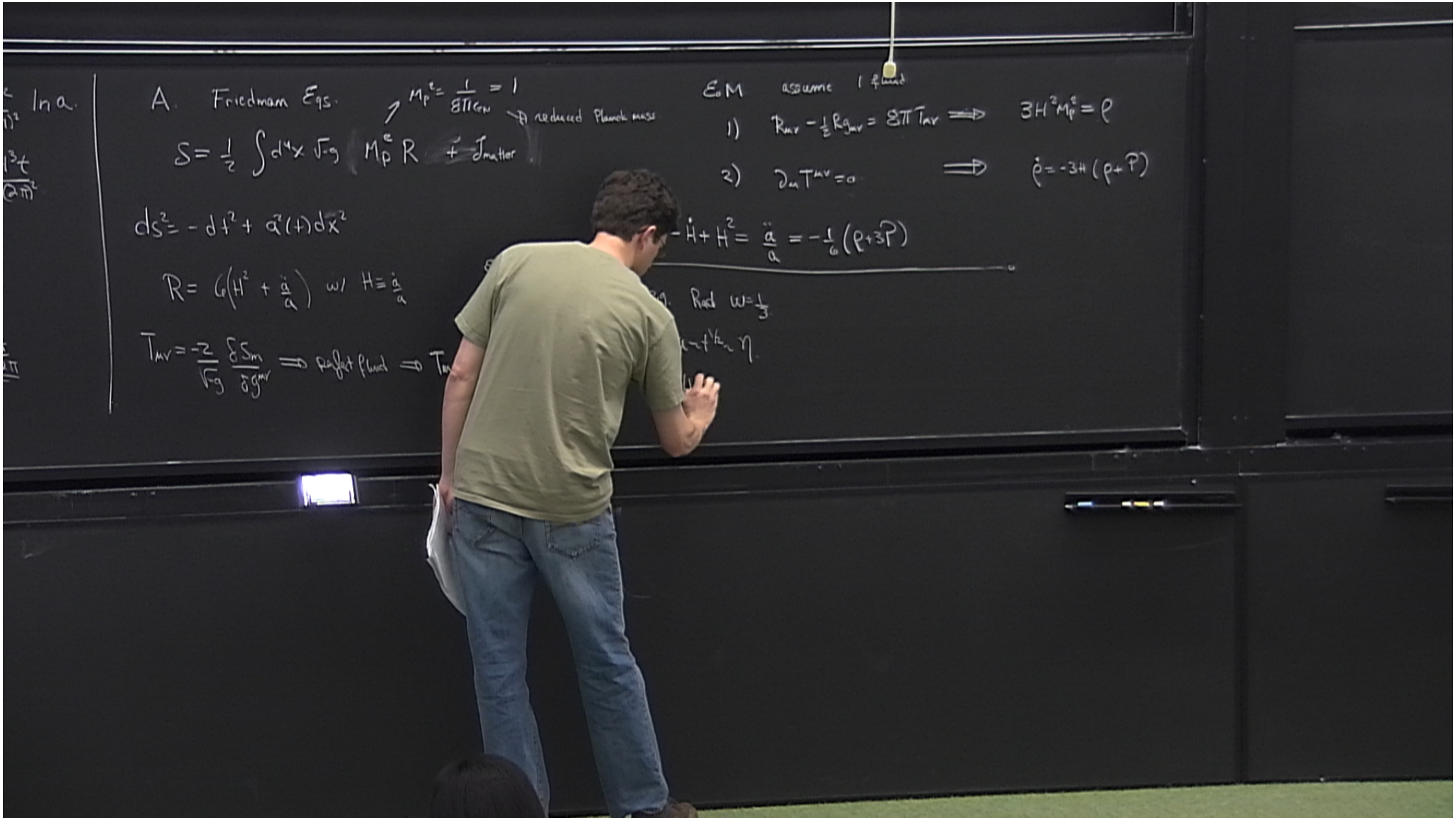

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$R = 6\left(H^2 + \frac{\dot{a}}{a}\right)$ w/ $H \equiv \frac{\dot{a}}{a}$
$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$ → perfect fluid → $T_{\mu\nu} =$
Eq. state
$p = w\rho$
E.M assume 1 fluid
1)
2)

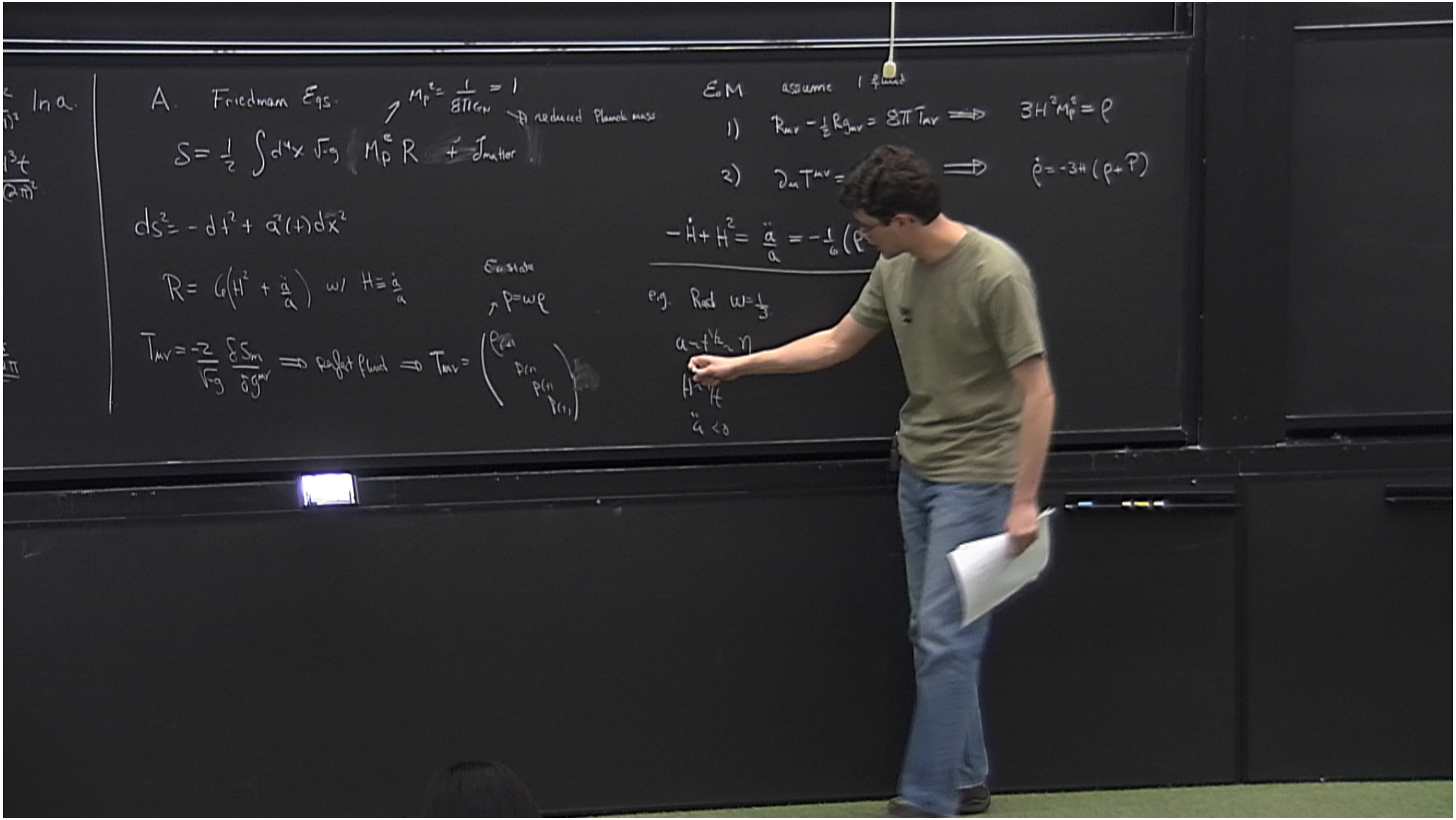

A. Friedmann Eqs.
reduced Planck mass
Einstein
perfect fluid
E.o.M
assume 1 fluid

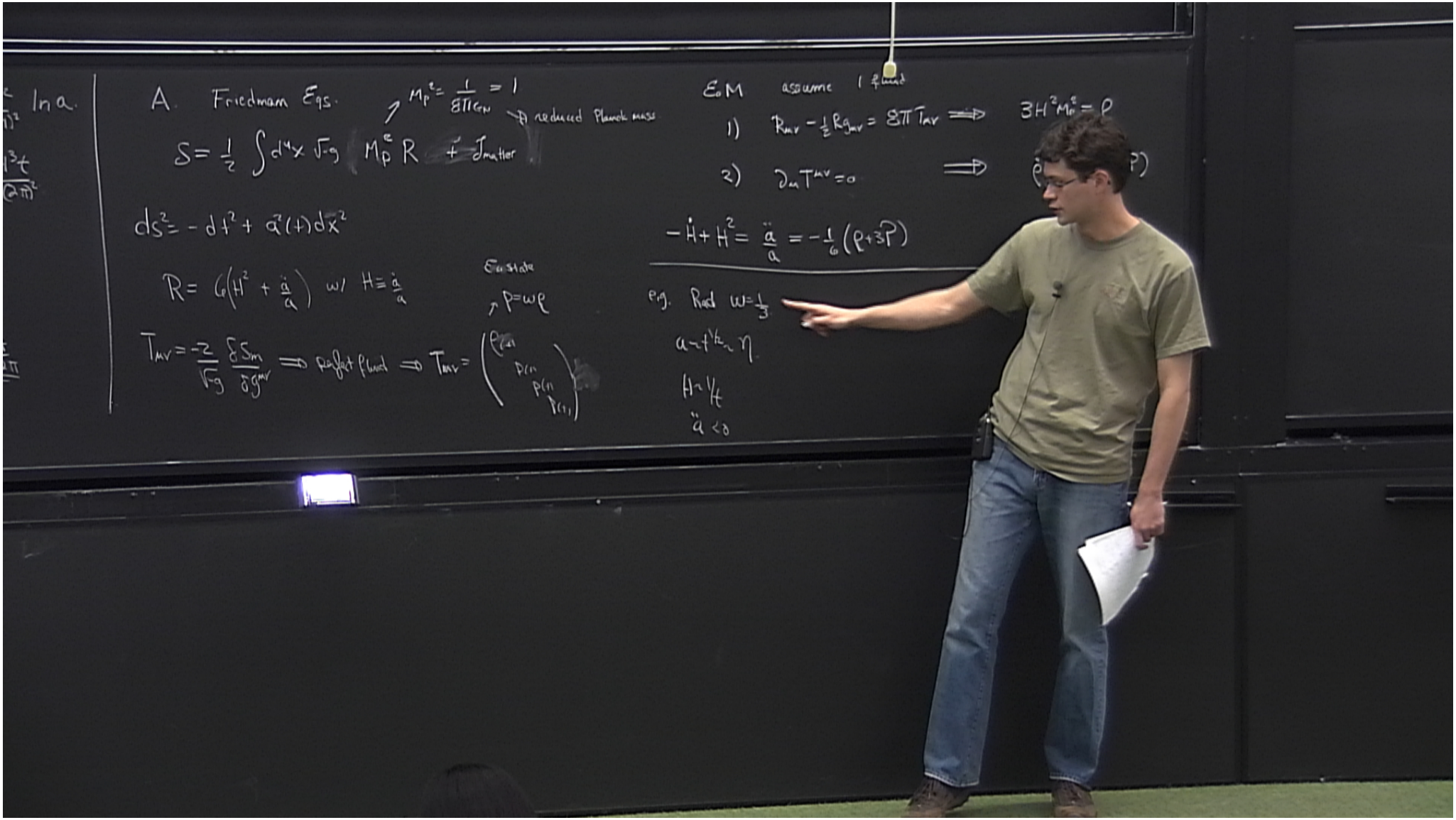

A. Friedmann Eqs.
$M_p^2 = \frac{1}{8\pi G_N} = 1$
reduced Planck mass
$S = \frac{1}{2}\int d^4x \sqrt{-g}\, M_p^2 R + \mathcal{L}_{matter}$
$ds^2 = -dt^2 + a^2(t)dx^2$
$R = 6\left(H^2 + \frac{\dot{a}}{a}\right)$ w/ $H \equiv \frac{\dot{a}}{a}$
Einstein
$p = w\rho$
$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}} \Rightarrow$ Perfect fluid $\Rightarrow T_{\mu\nu} =$
E.o.M assume 1 fluid
1) $R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi T_{\mu\nu}$
$3H^2 M_p^2 = \rho$
2) $\partial_\mu T^{\mu\nu} = 0$

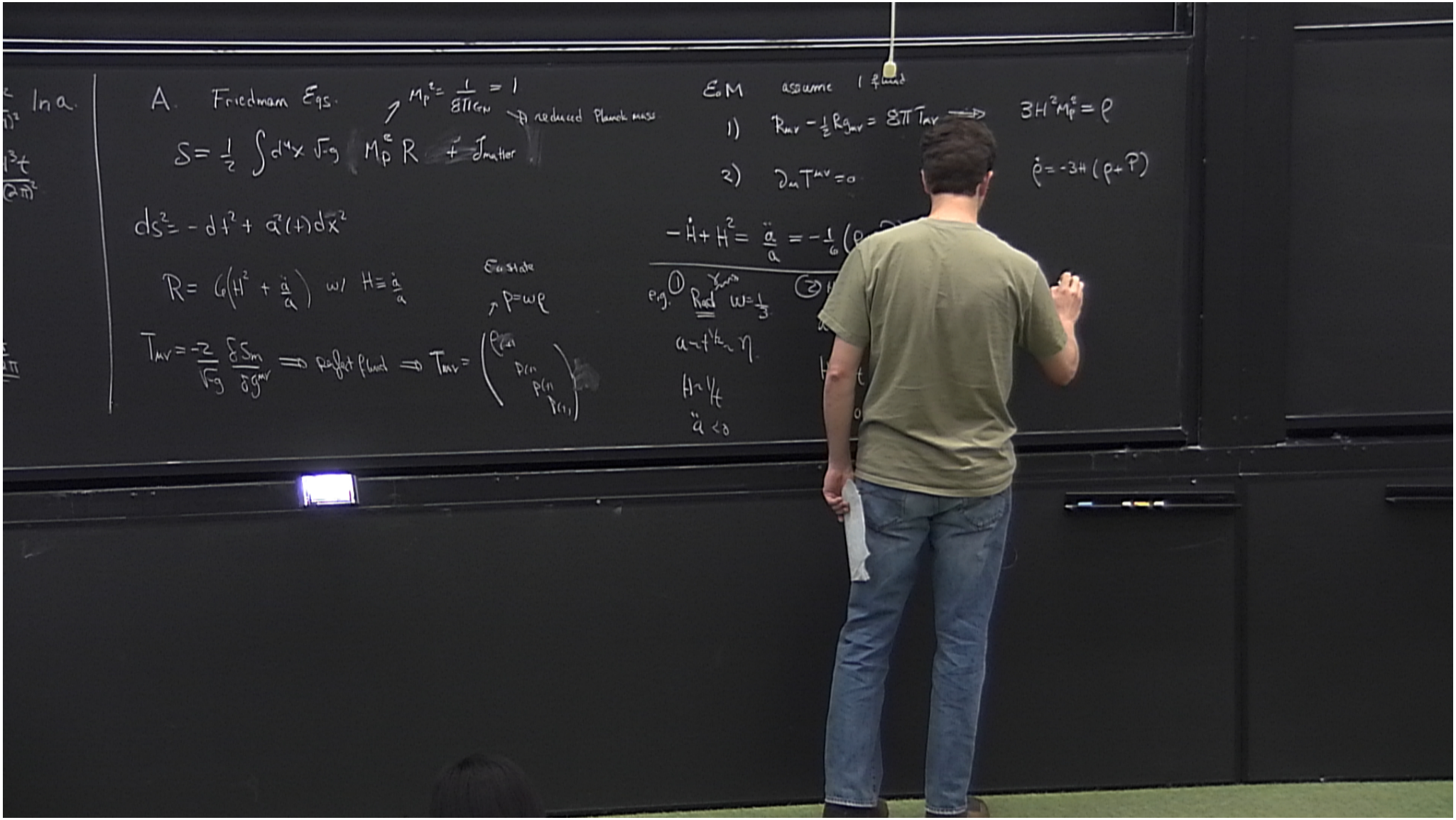

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$ds^2 = -dt^2 + a^2(t)dx^2$
$R = 6\left(H^2 + \frac{\dot{a}}{a}\right)$ w/ $H \equiv \frac{\dot{a}}{a}$
$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}} \Rightarrow$ perfect fluid $\Rightarrow T_{\mu\nu} =$
Eos state
$p = w\rho$
E.o.M assume 1 fluid
1) $R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = 8\pi T_{\mu\nu} \Rightarrow 3H^2 M_p^2 = \rho$
2) $\partial_\mu T^{\mu\nu} = 0 \Rightarrow \dot{\rho} = -3H(\rho + P)$

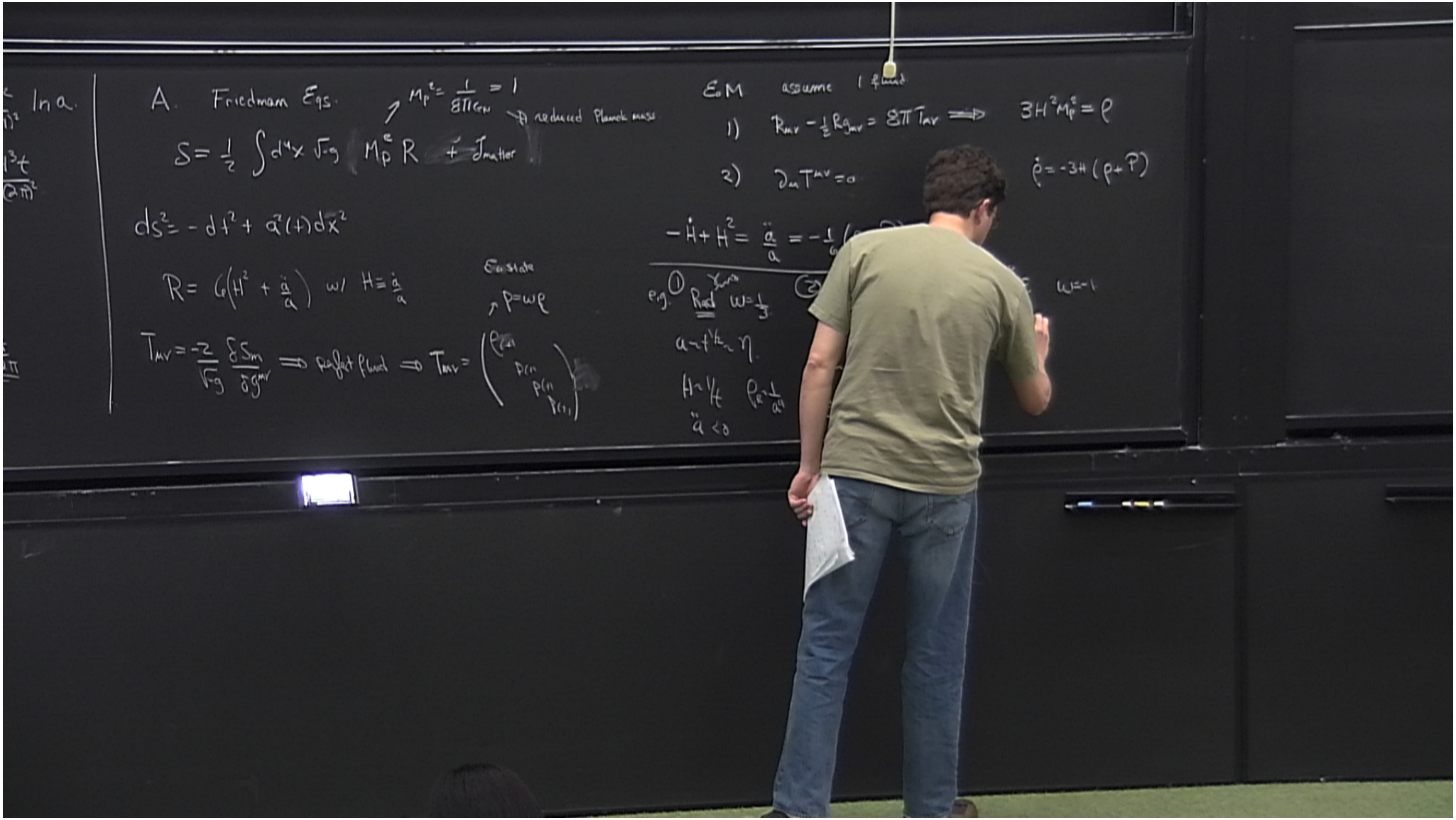

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume
Einstein
p=wρ

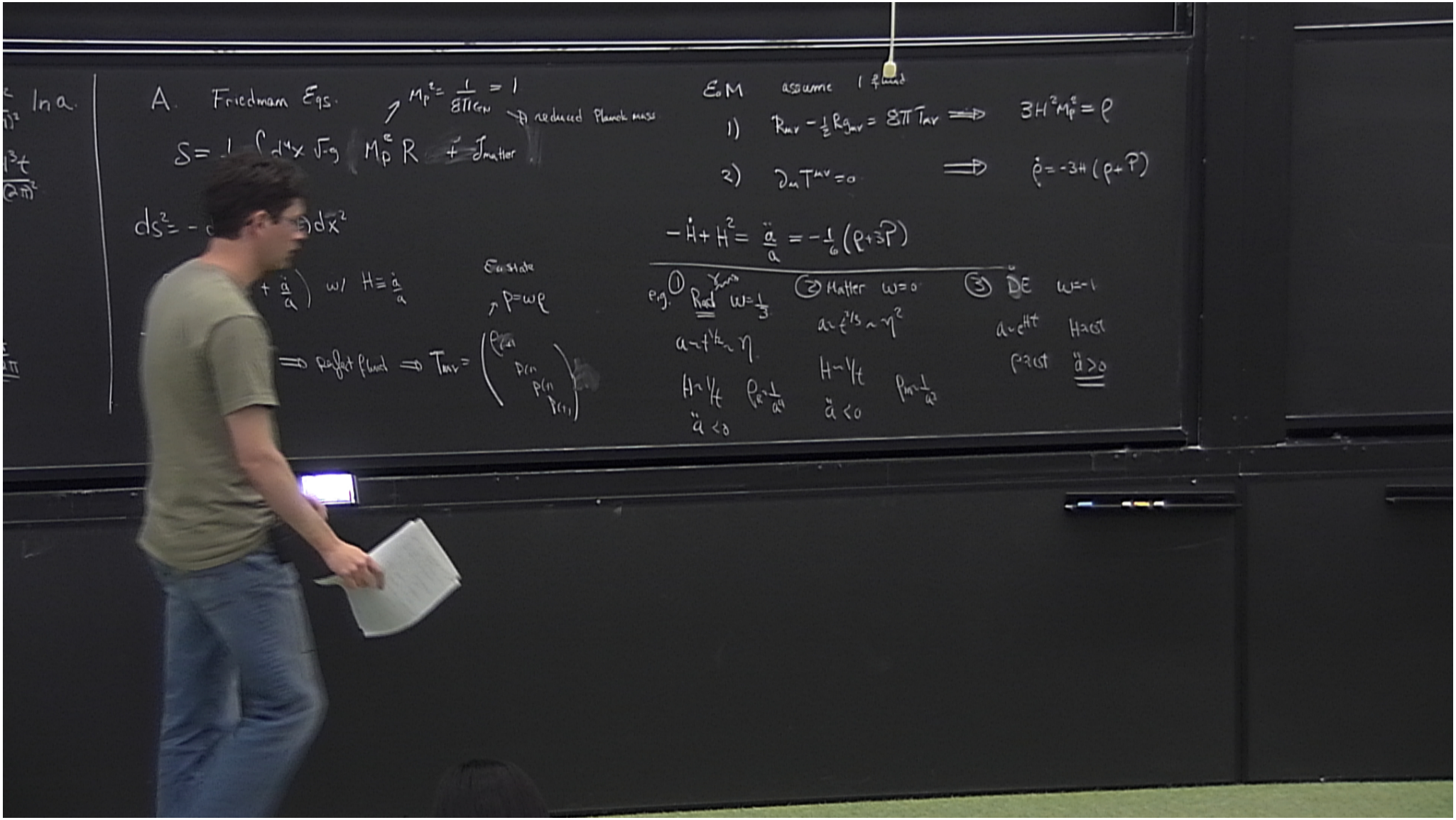

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume
1)
2)
Rad

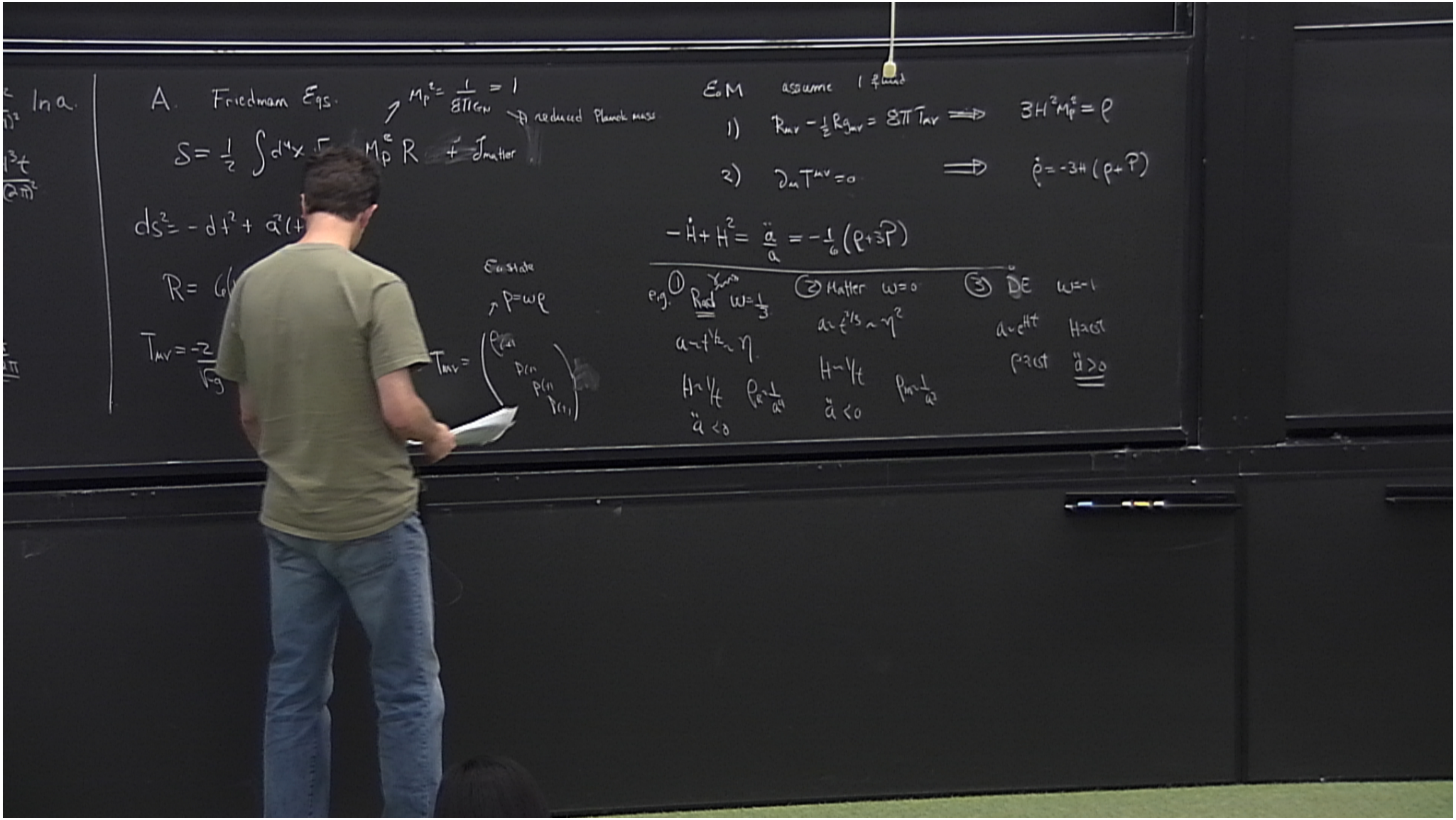

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume
Eq. State
Rad
Matter

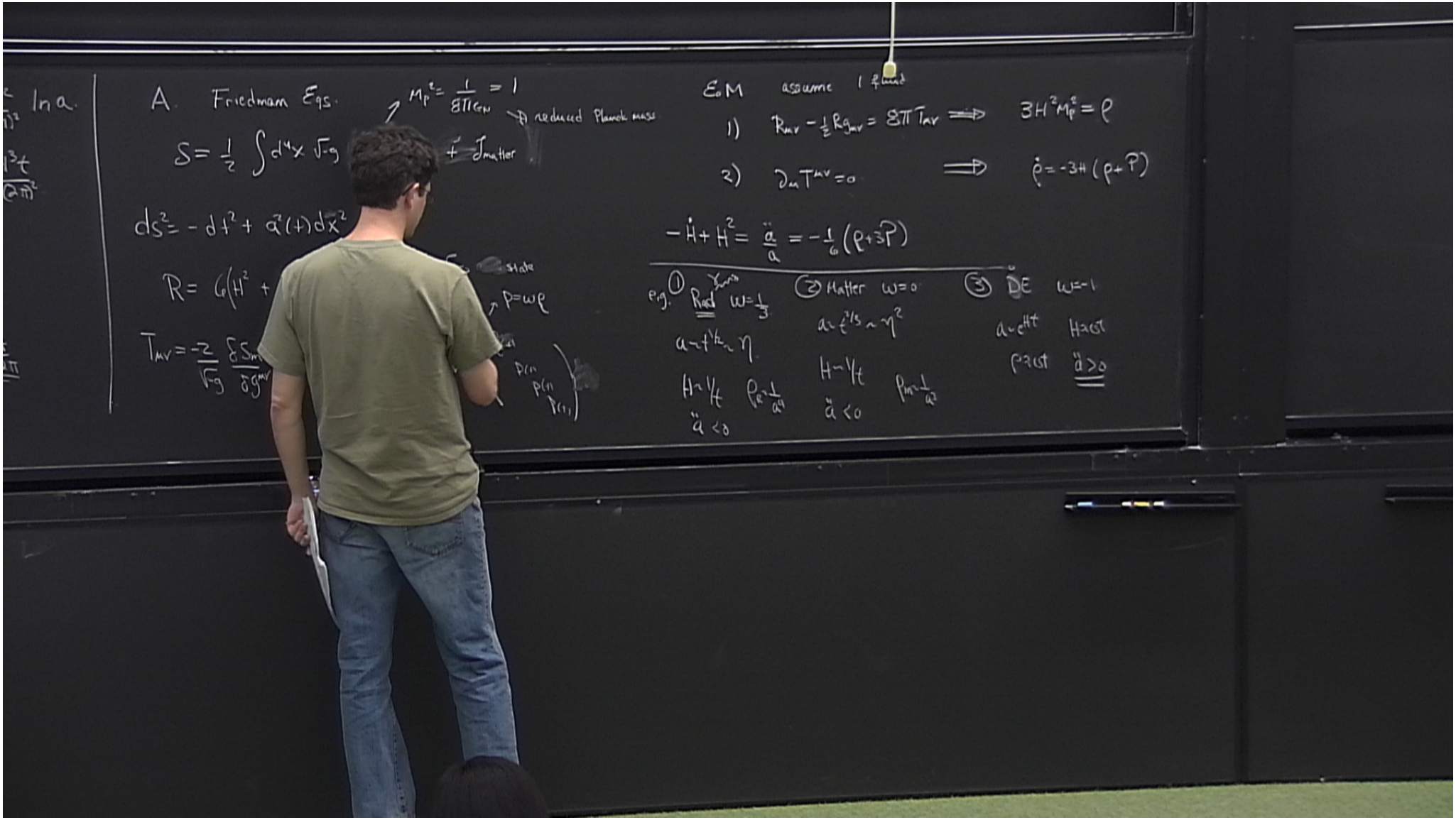

A. Friedmann Eqs.
reduced Planck mass
EoM
assume
Eq. State
Rad
a=t^1/2 ~ η
H=1/t
ä<0

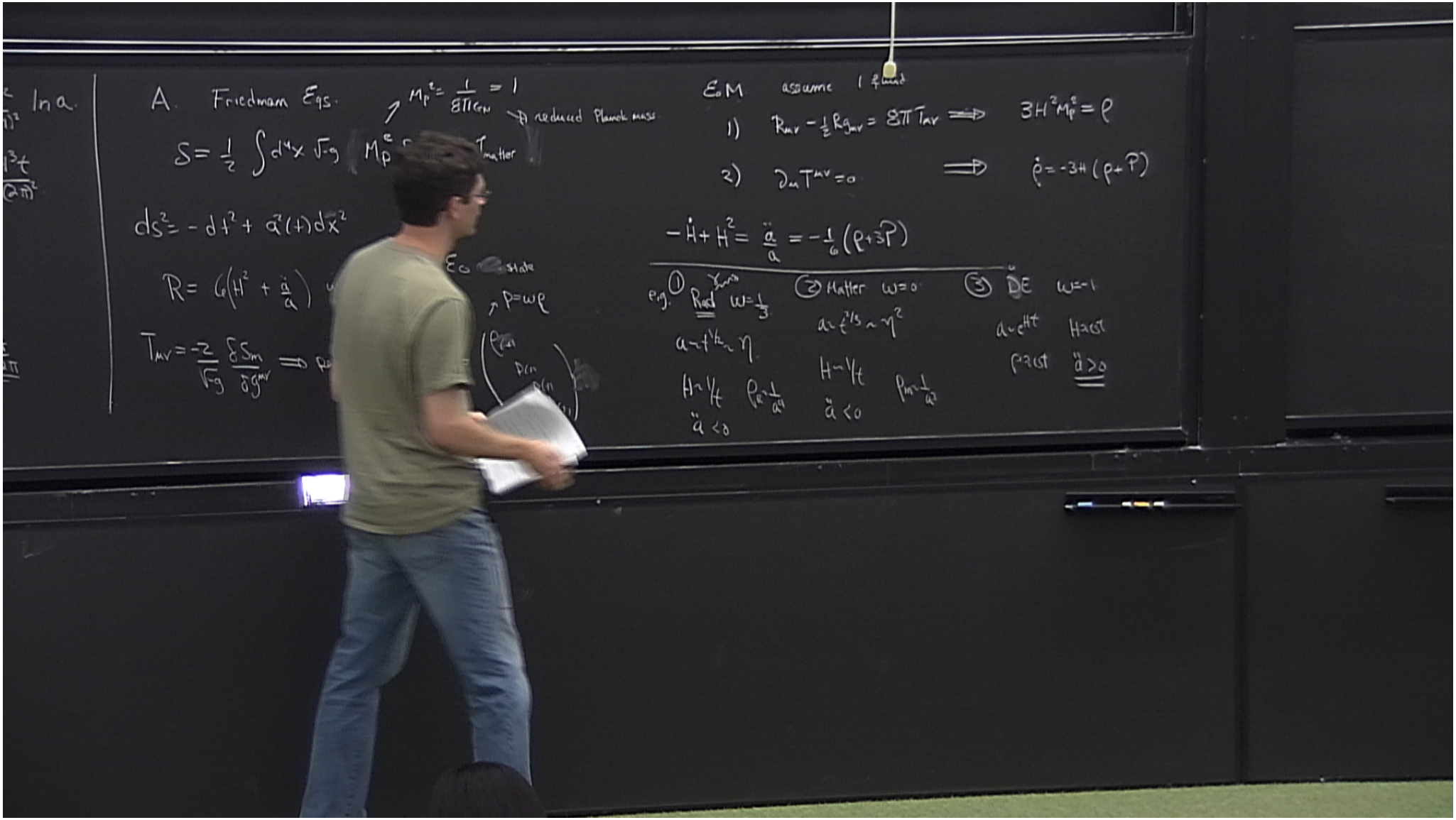

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume
Einstein
Rad
Pressure
Energy density
Flat
Curvature
Matter

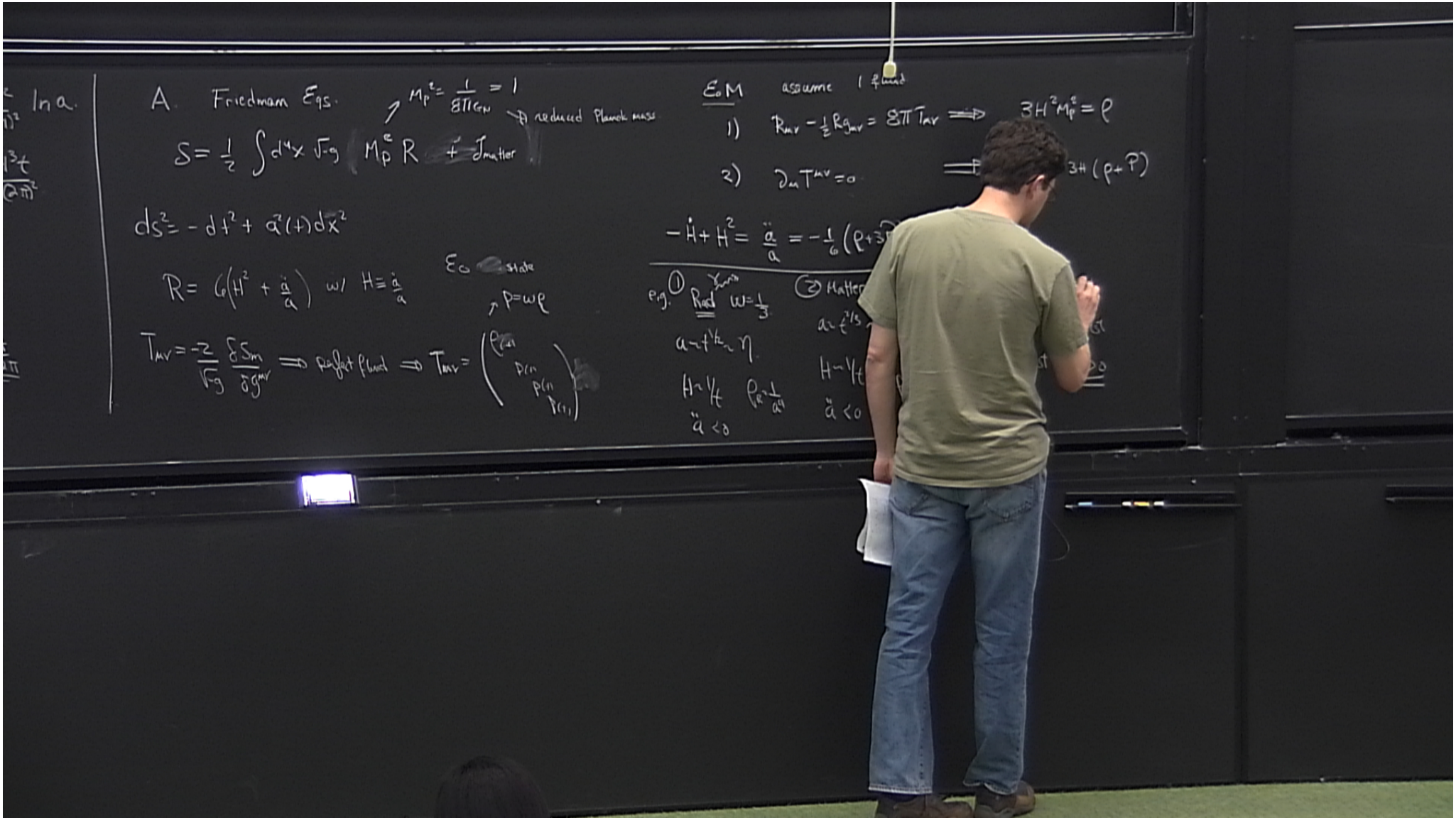

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume 1 fluid
Einstein
Rad
w=1/3
w=-1

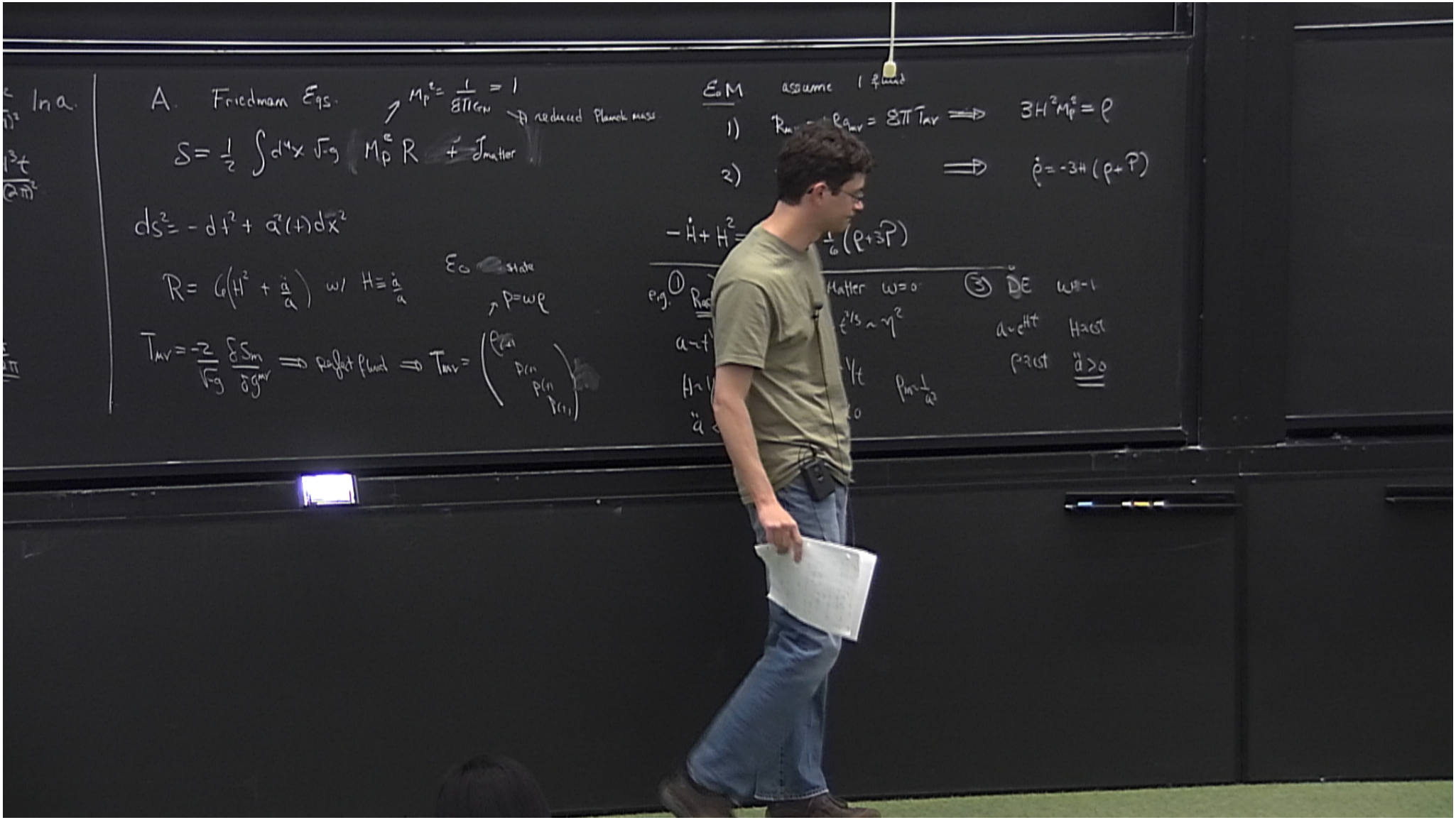

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume 1 fluid
Eq. state
Rad
Matter
DE
Pirsa: 12030030
Page 30/117

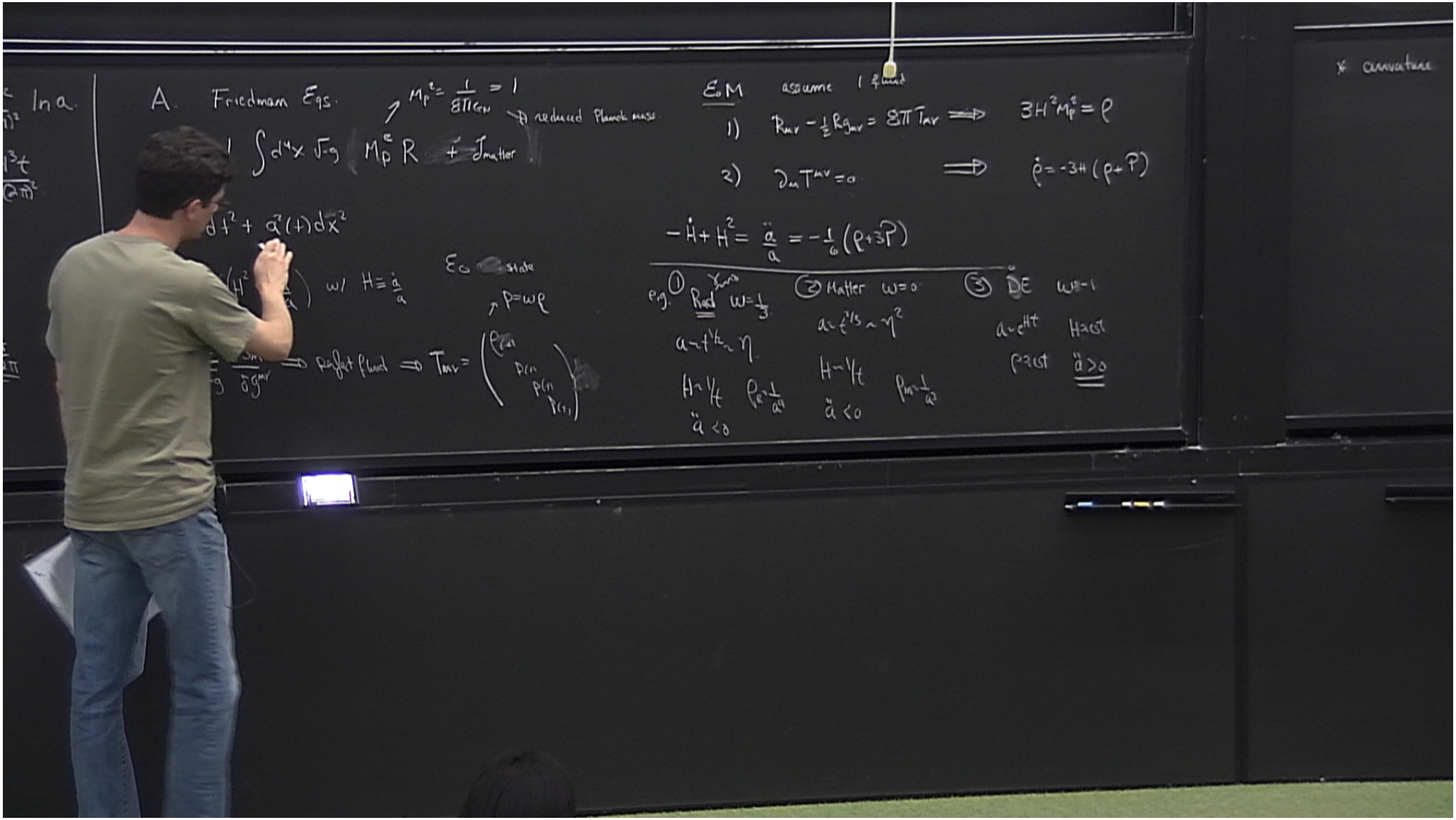

A. Friedmann Eqs.
reduced Planck mass
EoM
assume 1 fluid
Matter
DE
Rad

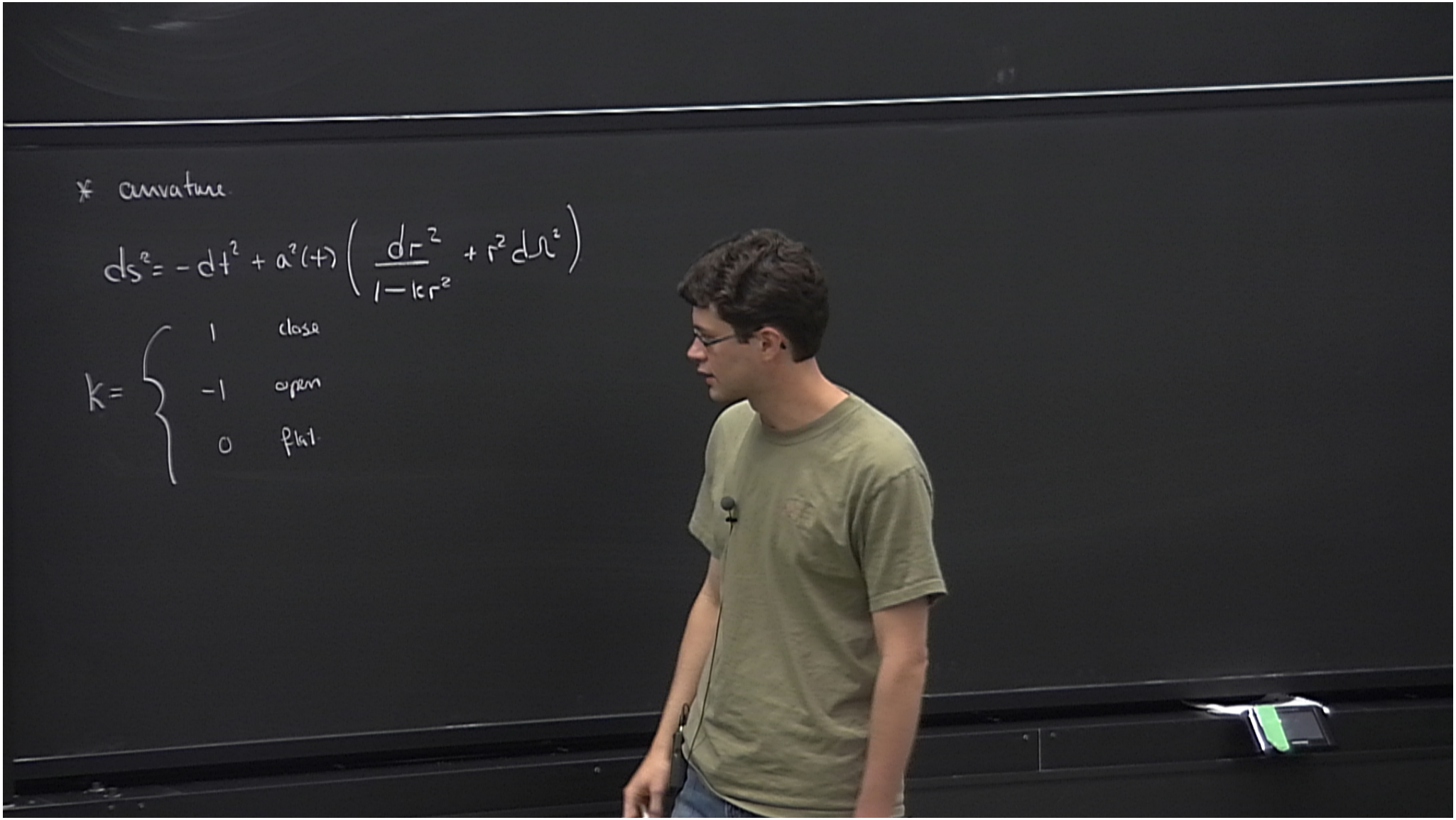

A. Friedmann Eqs.
reduced Planck mass
E.o.M assume 1 fluid
Rad
Matter
DE

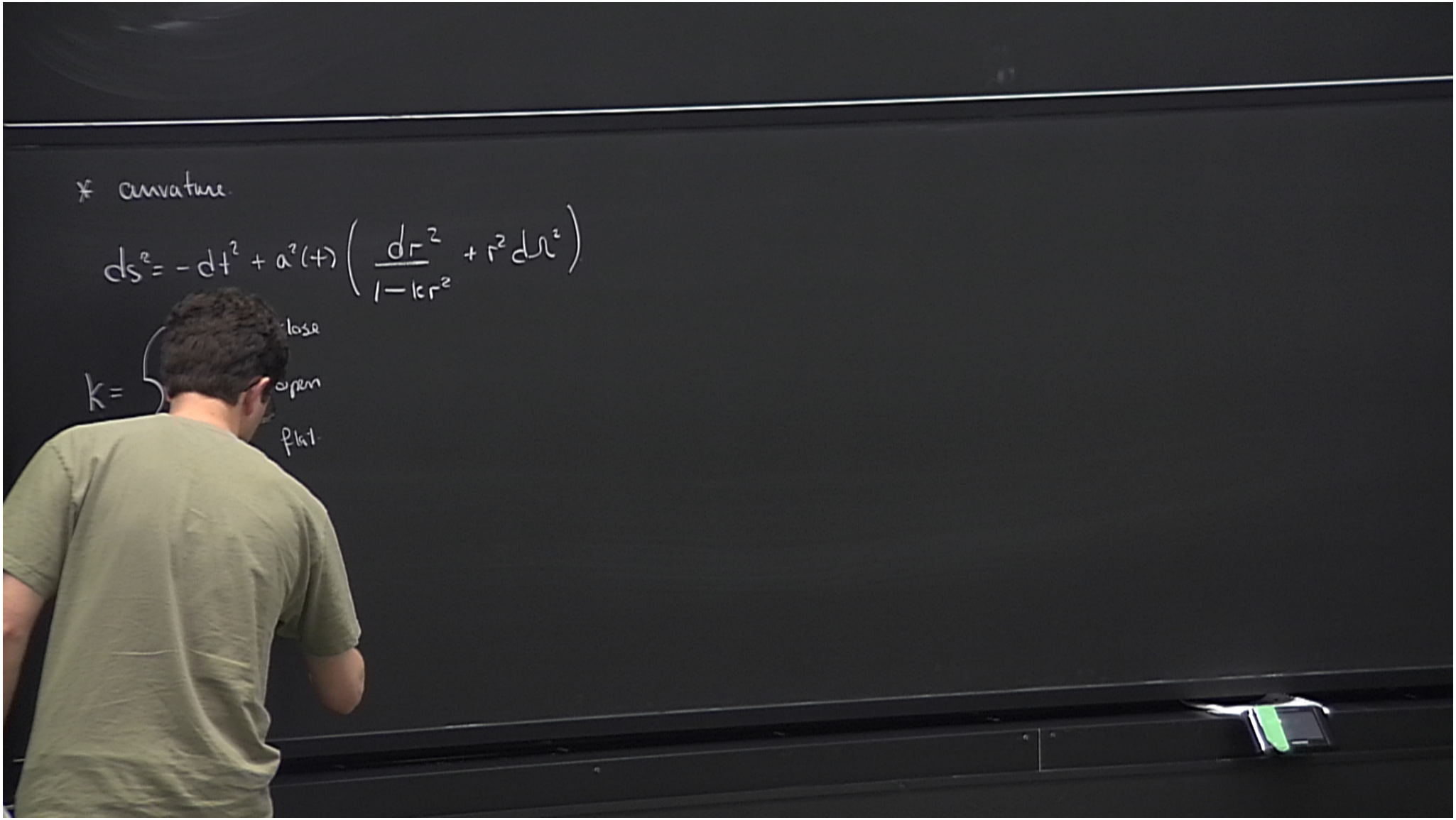

A. Friedmann Eqs.
reduced Planck mass
EoM
assume 1 fluid
Matter
DE
Rad

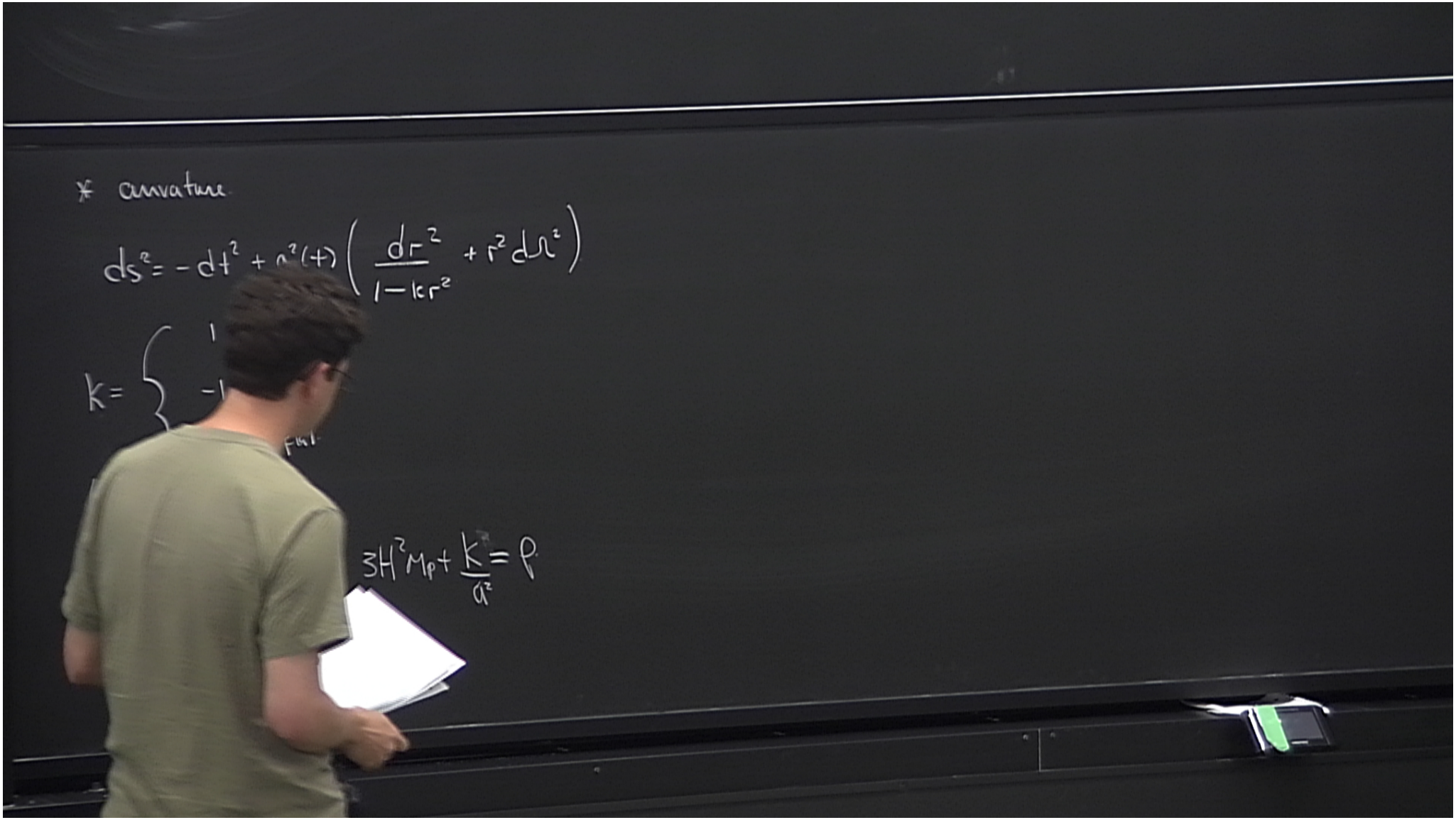

A. Friedmann Eqs.
reduced Planck mass
EoM
assume
Matter
Rad
Matter

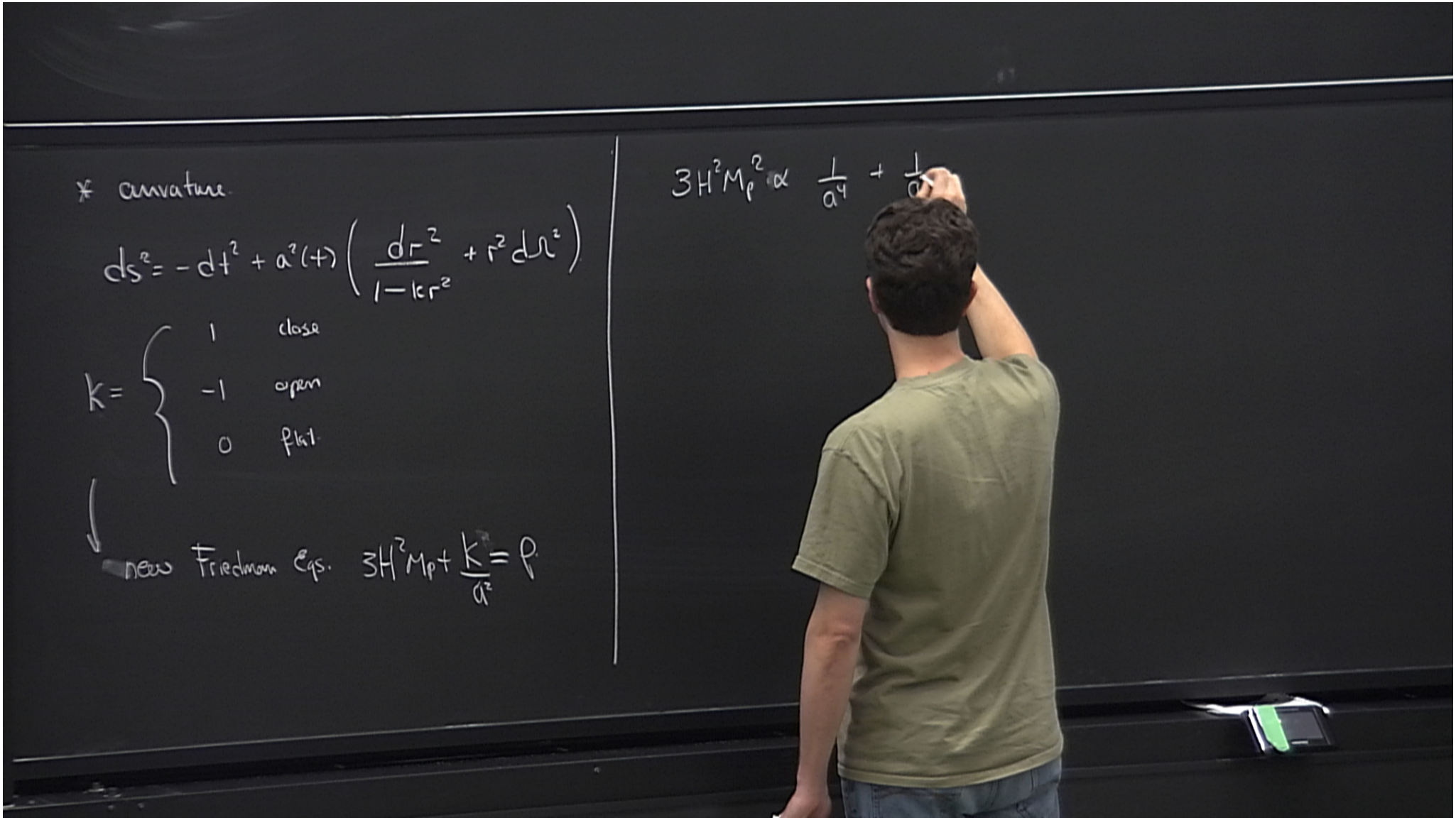

A. Friedmann Eqs.
reduced Planck mass
EoM
assume
matter
DE
Fig.

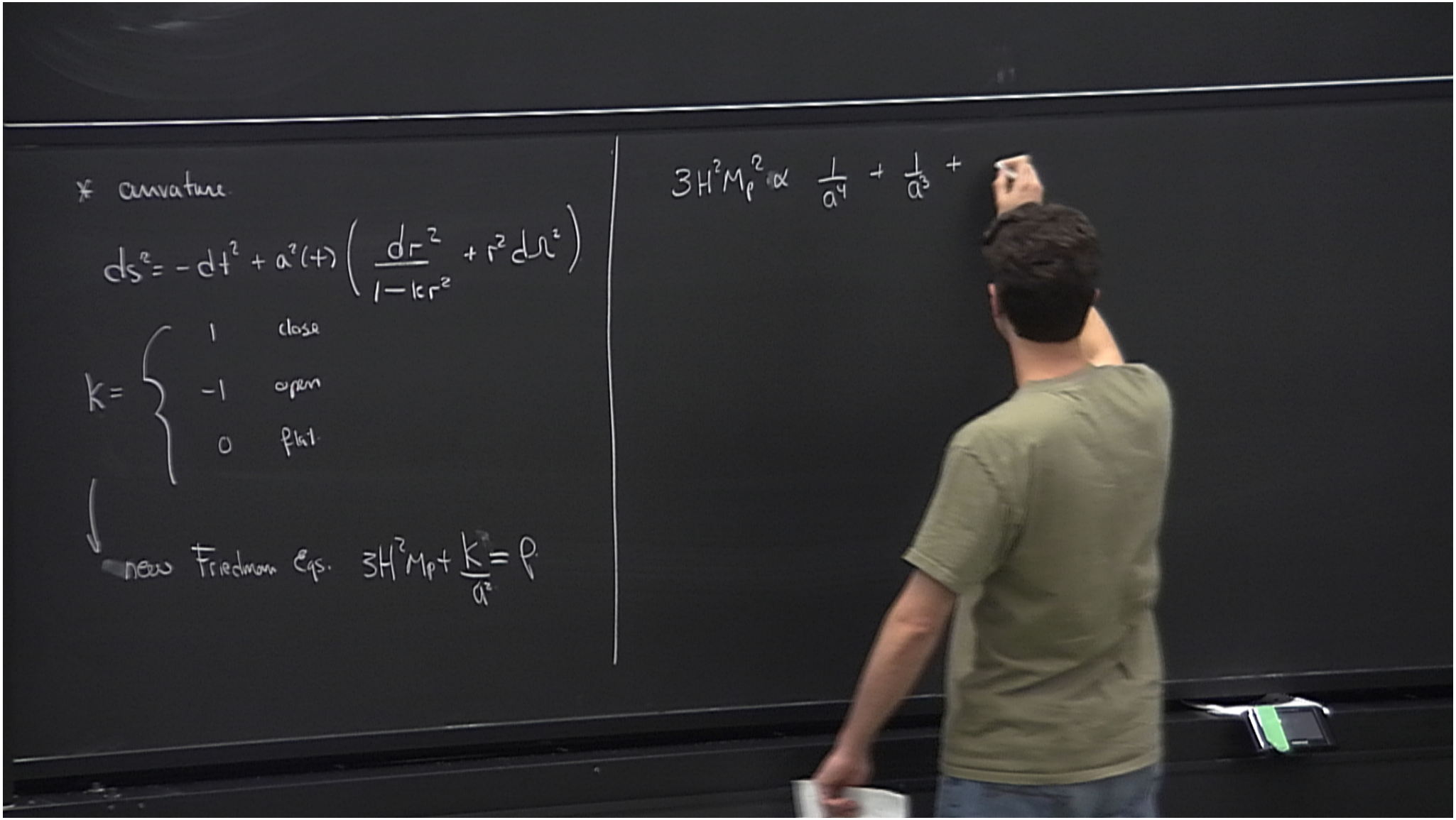

A. Friedmann Eqs.
reduced Planck mass
E.o.M
assume 1 fluid
Matter
DE
Pirsa: 12030030
Page 36/117

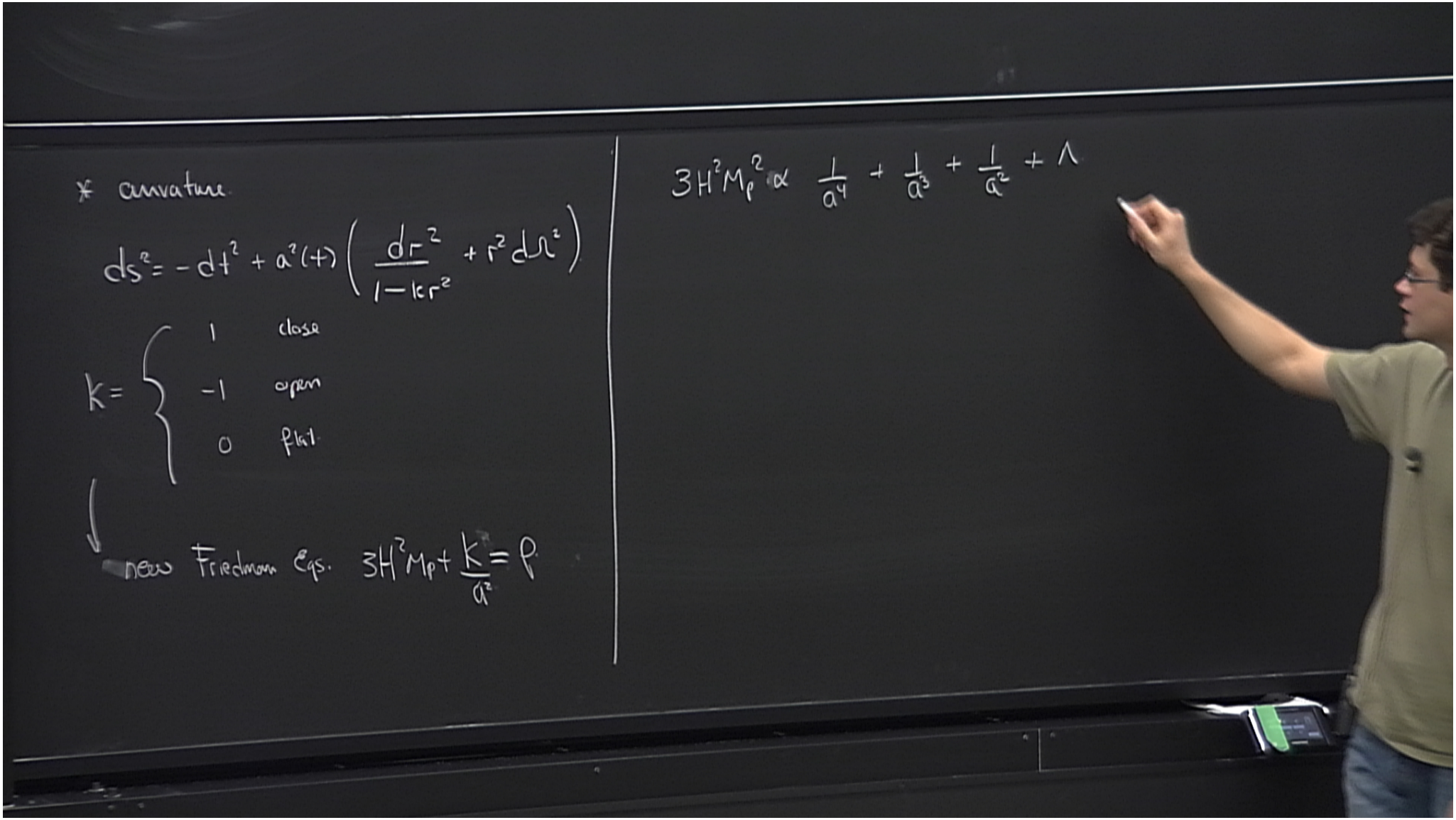

* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$


* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k =$
close
open
flat


* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k = \begin{cases} 1 \\ -1 \end{cases}$
FLRW
$3H^2 M_p + \frac{k}{a^2} = \rho$


* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$
new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$
$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a}$


* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
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* curvature
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$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$

* curvature

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

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↓

new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$

$$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$$

↑ rad ↑ matter ↑ curvature ↑ DE

* curvature

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

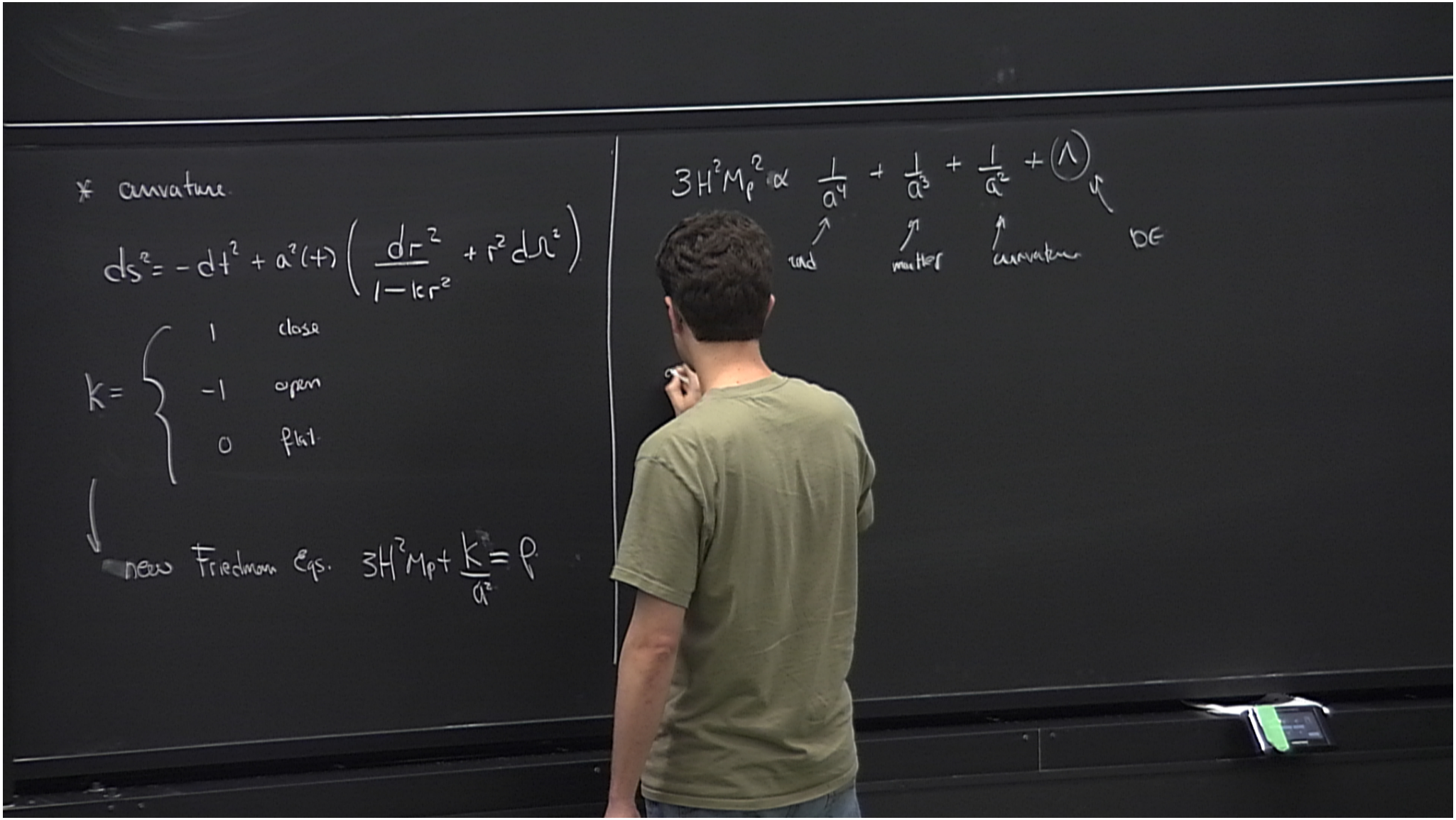
$$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$$

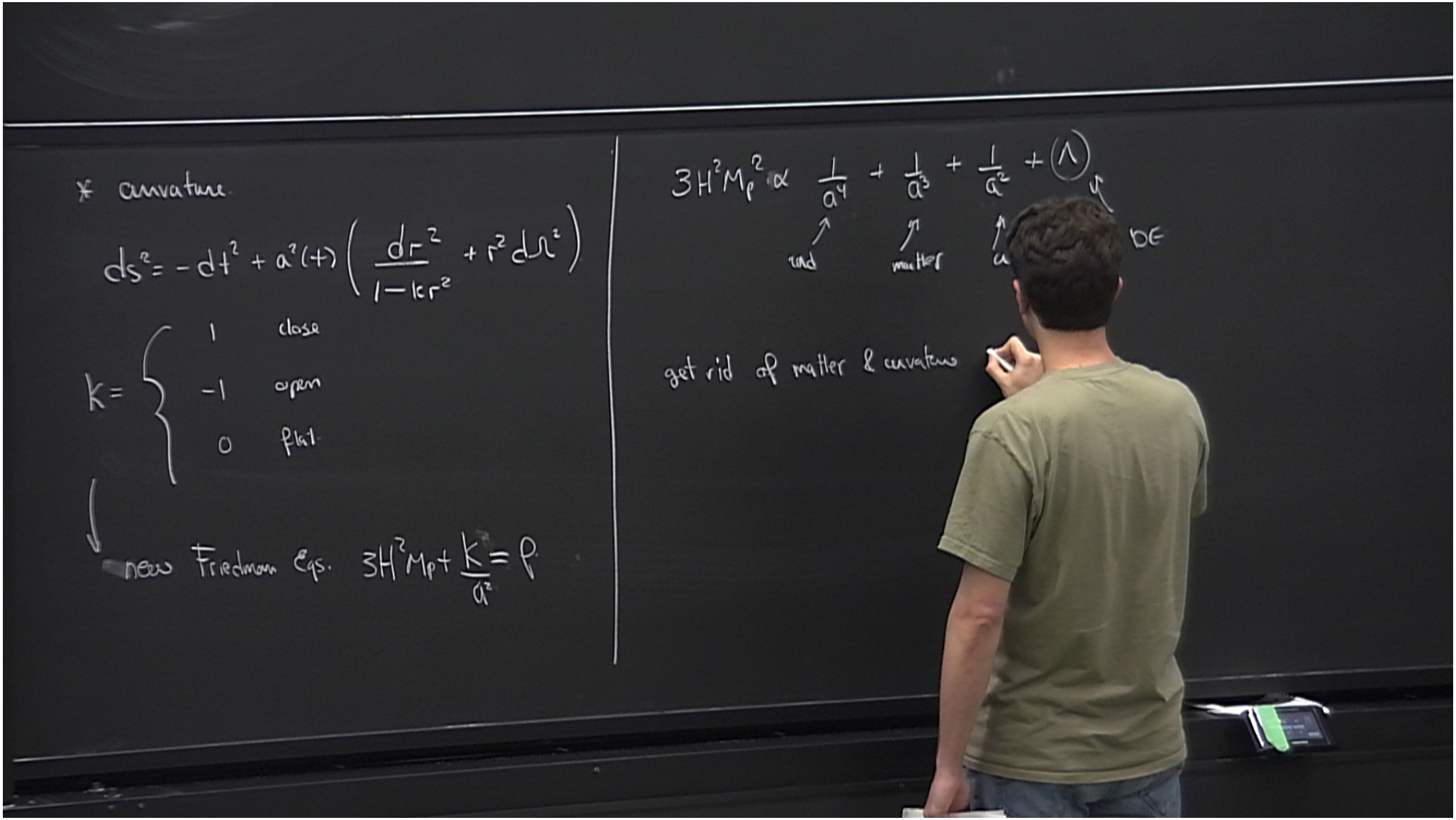
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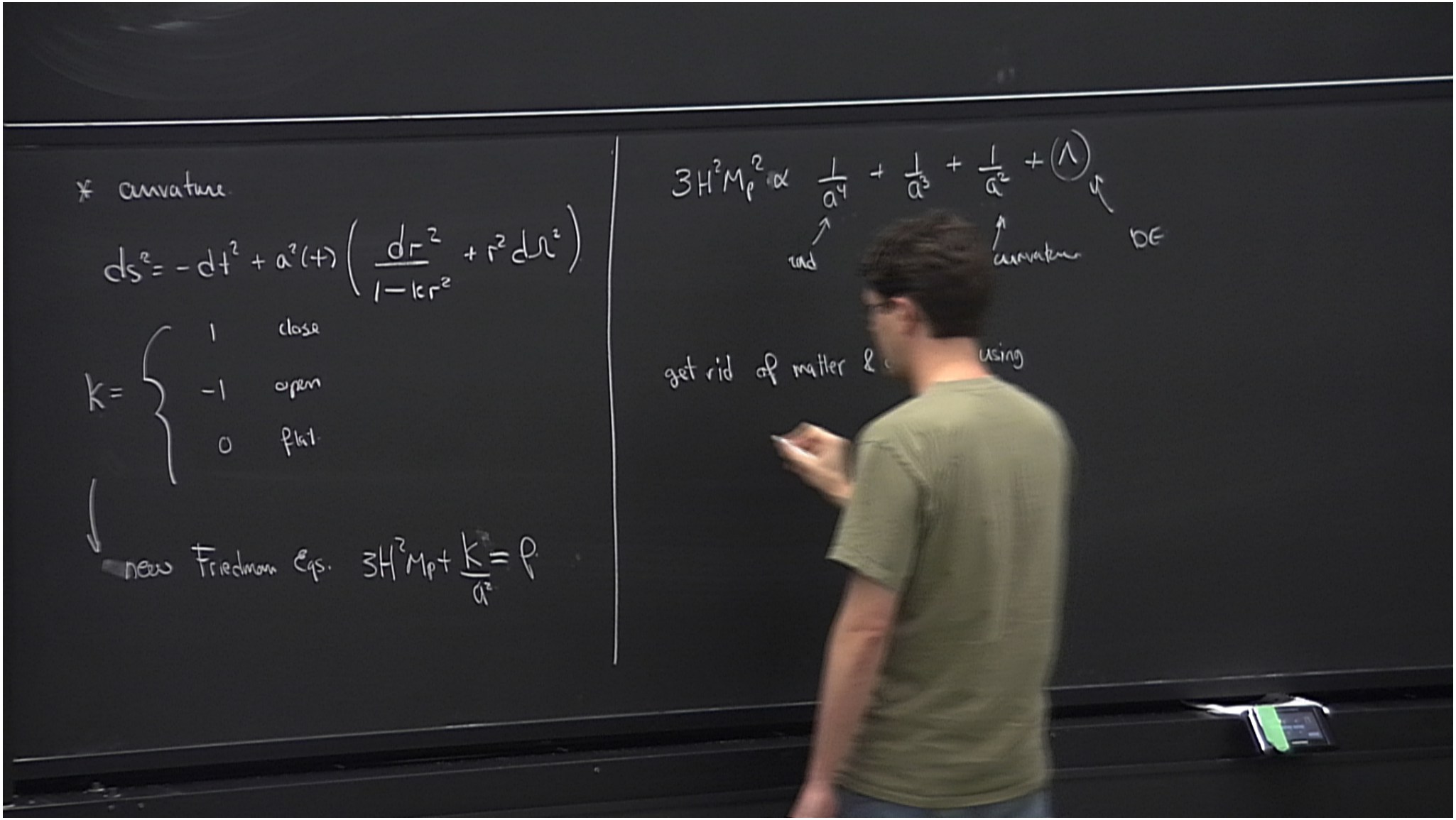
new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$

$$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$$

↑ rad ↑ matter ↑ curvature ↑ DE


* curvature
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rad
matter
curvature
DE


* curvature
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rad
matter
DE
get rid of matter & curvature


* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$
new Friedmann Eqs. $3H^2M_p + \frac{k}{a^2} = \rho$
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rad
curvature
DE
get rid of matter &
using

* curvature

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$$

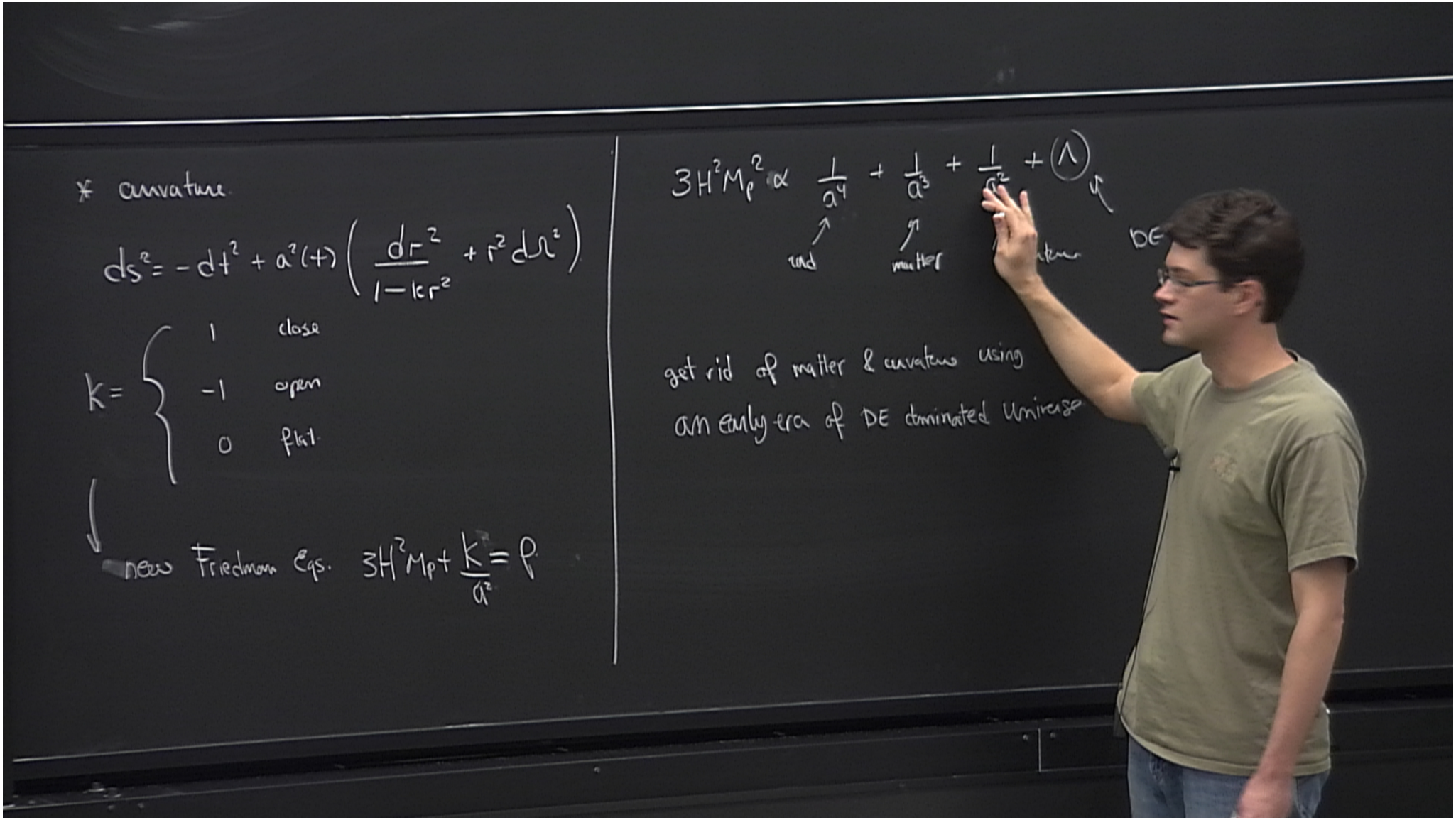
new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$

$$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$$

rad, matter, curvature, DE

get rid of matter & curvature using

an early era of DE dominated Universe


* curvature
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rad
matter
DE
get rid of matter & curvature using
an early era of DE dominated Universe

* curvature

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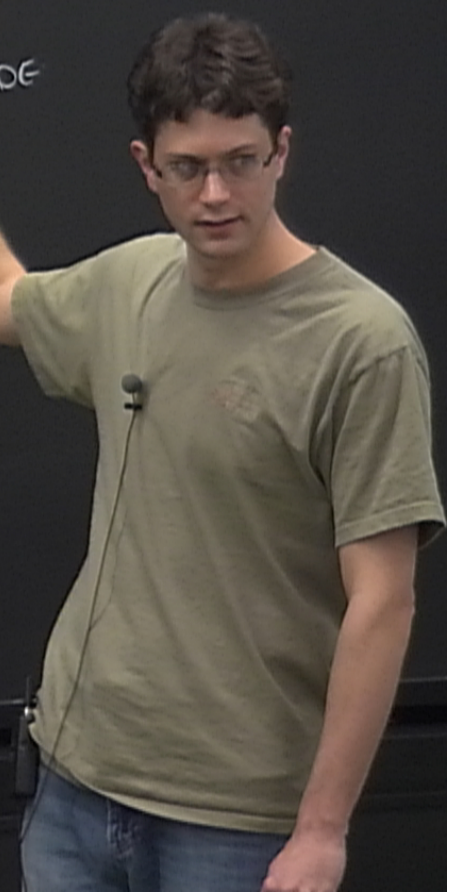
new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$

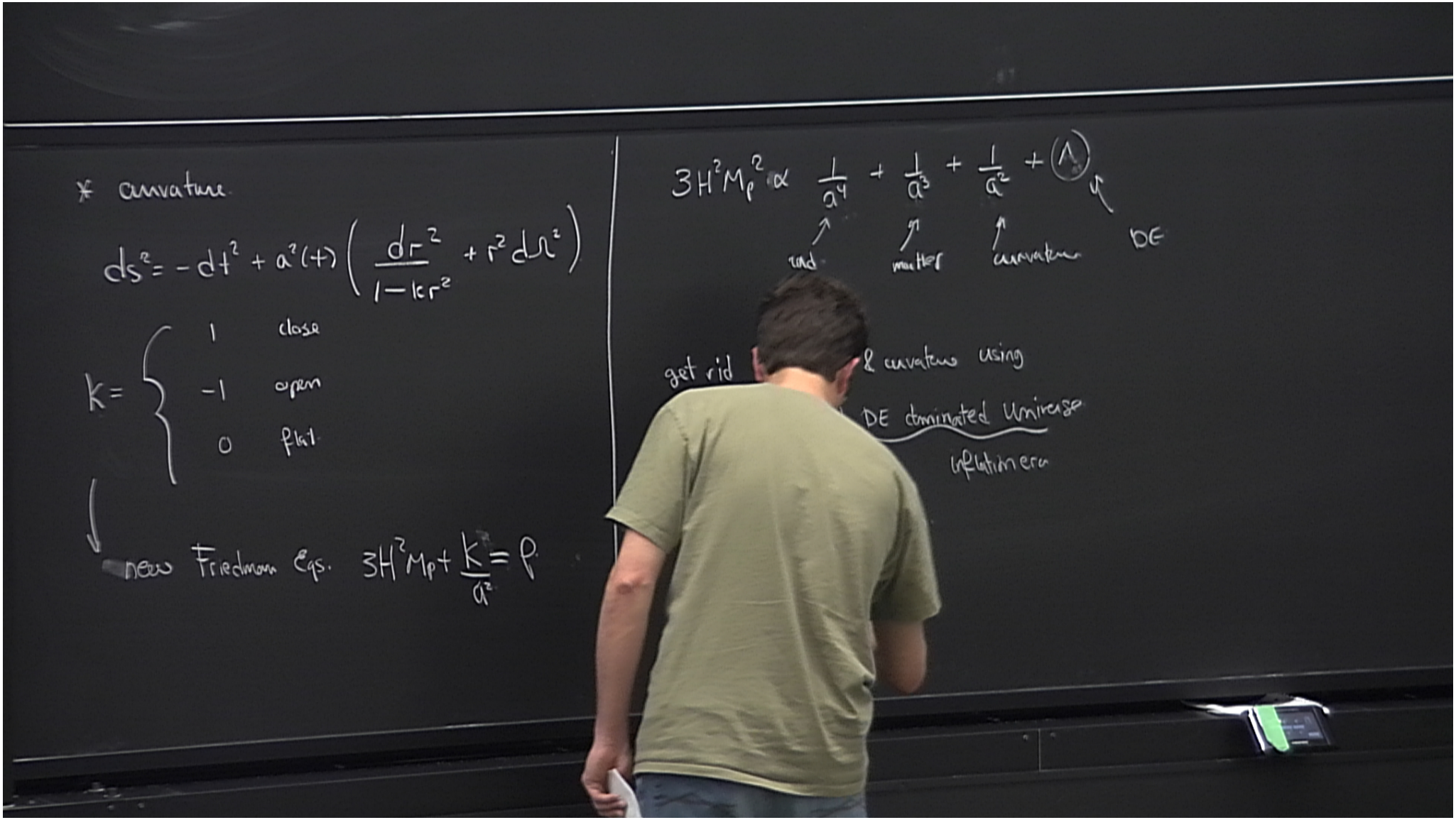
$$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$$

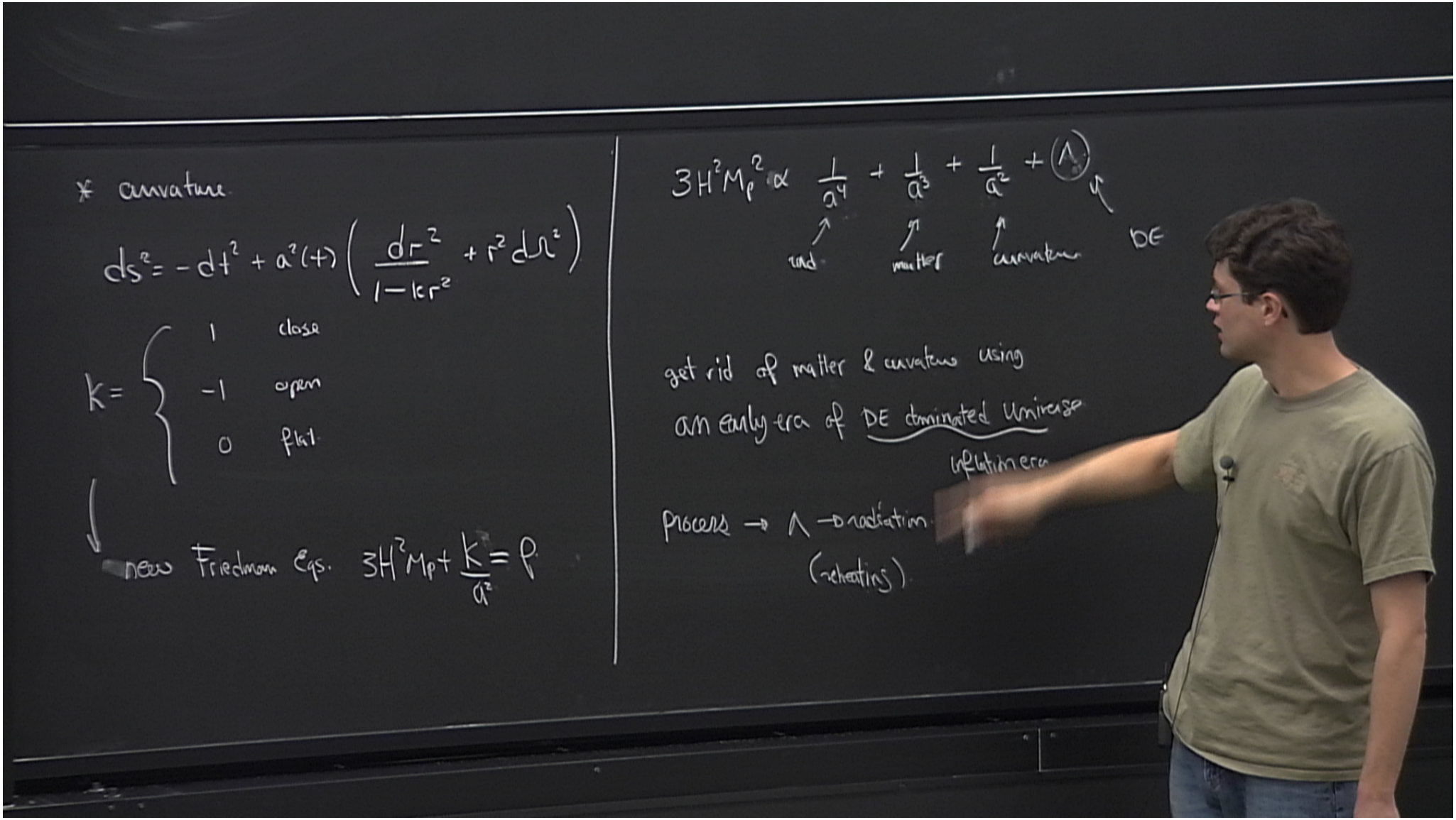
rad, matter, curvature, DE

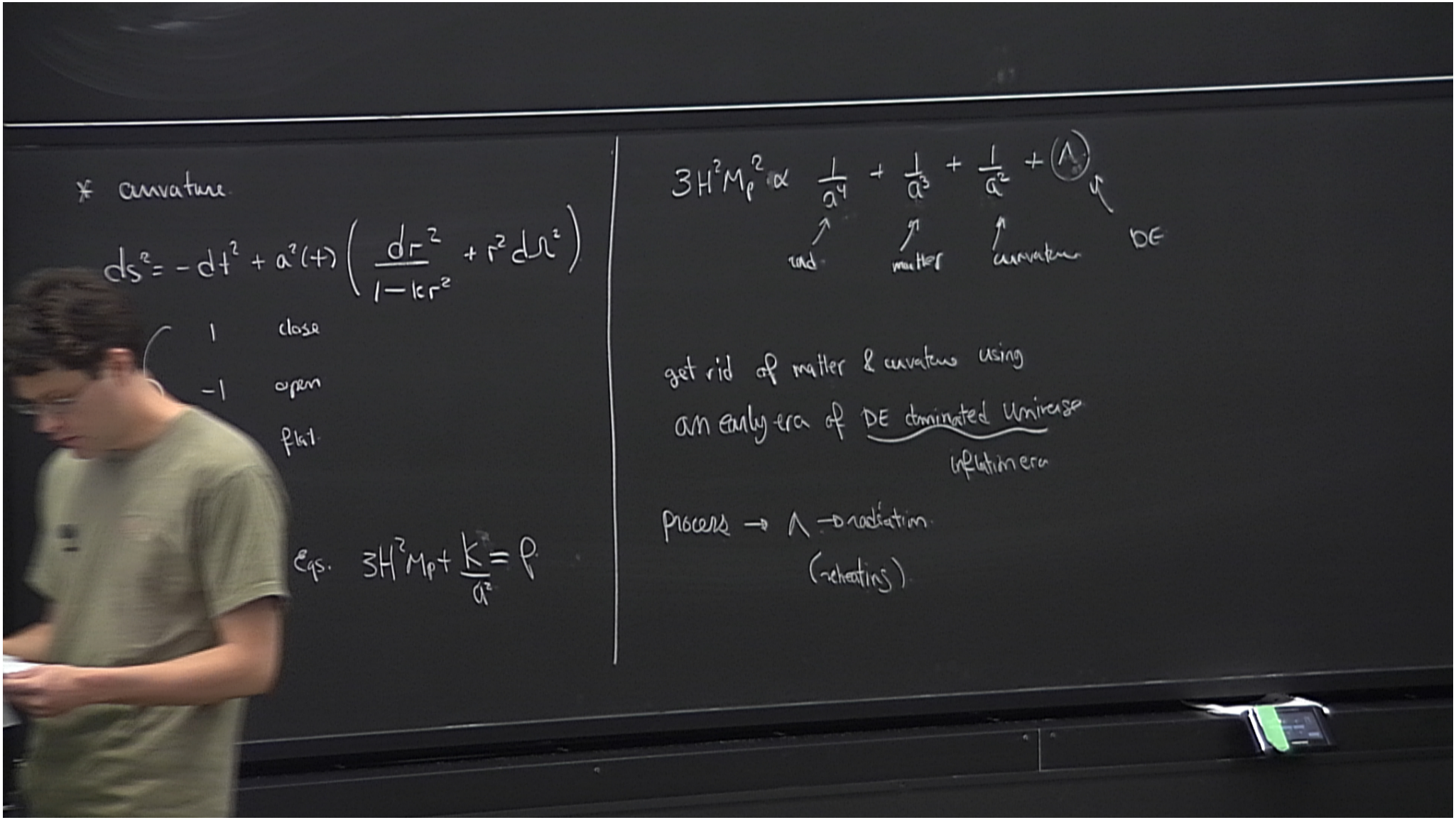
get rid of matter & curvature using

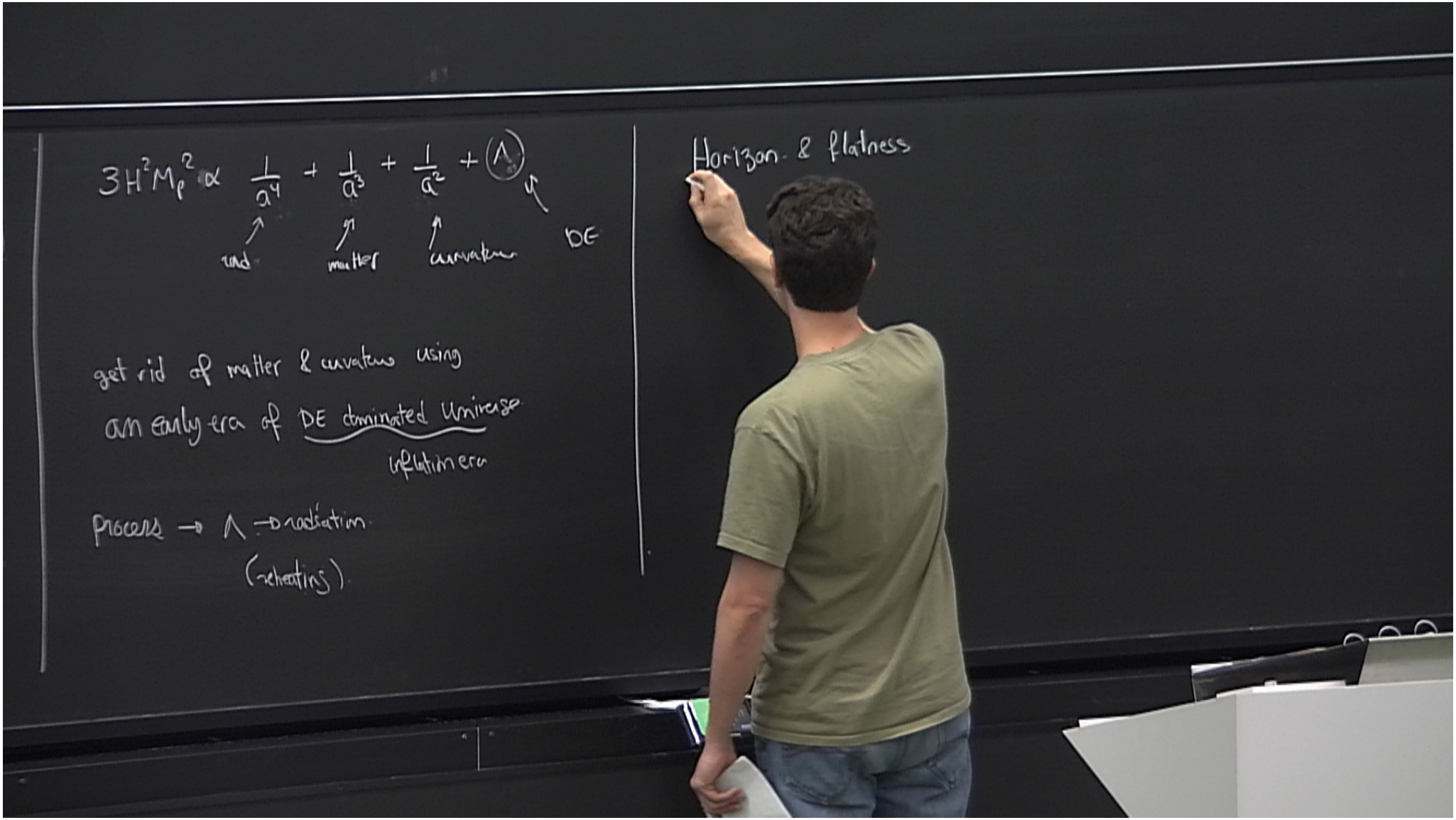
an early era of DE dominated Universe

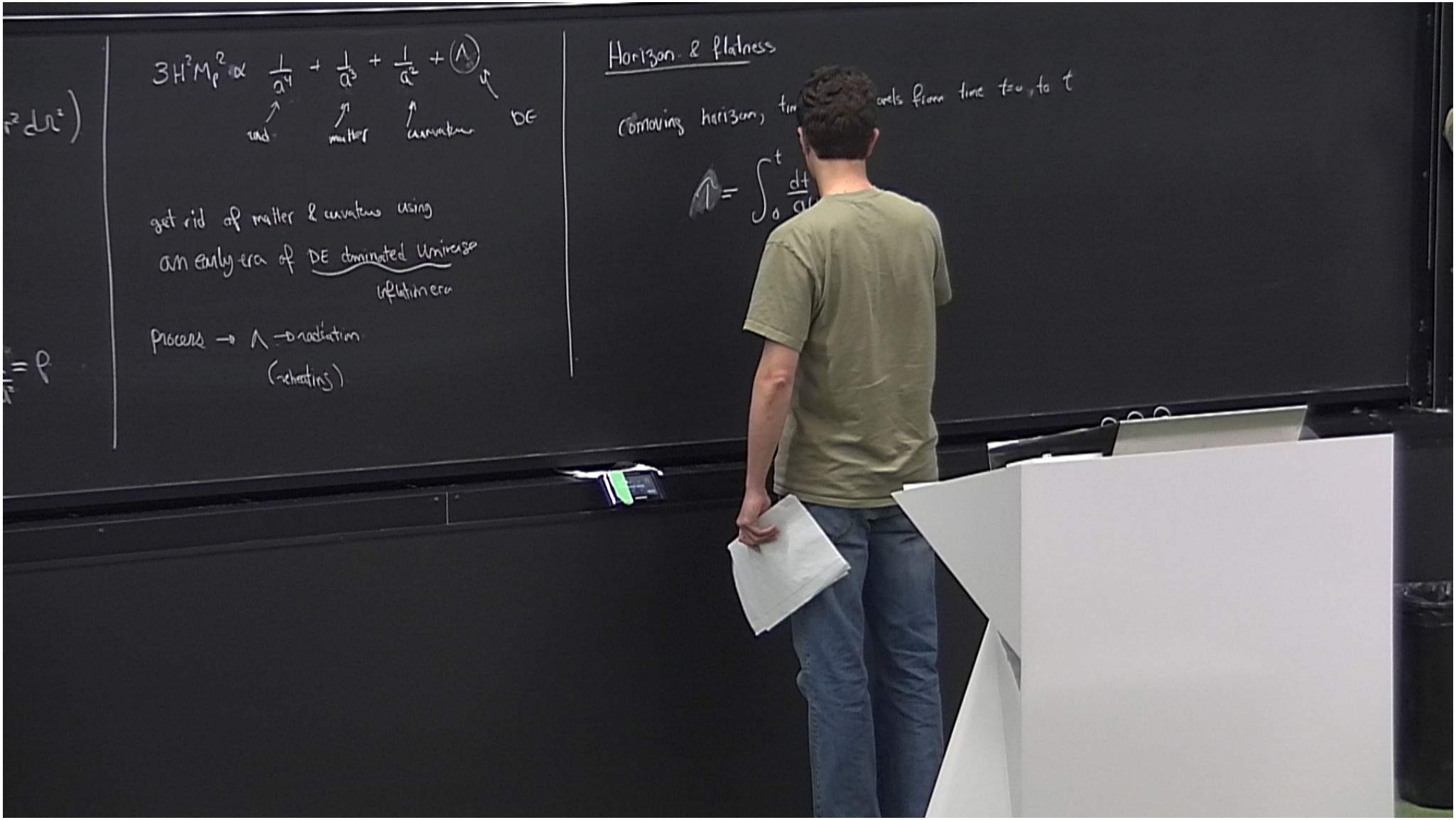


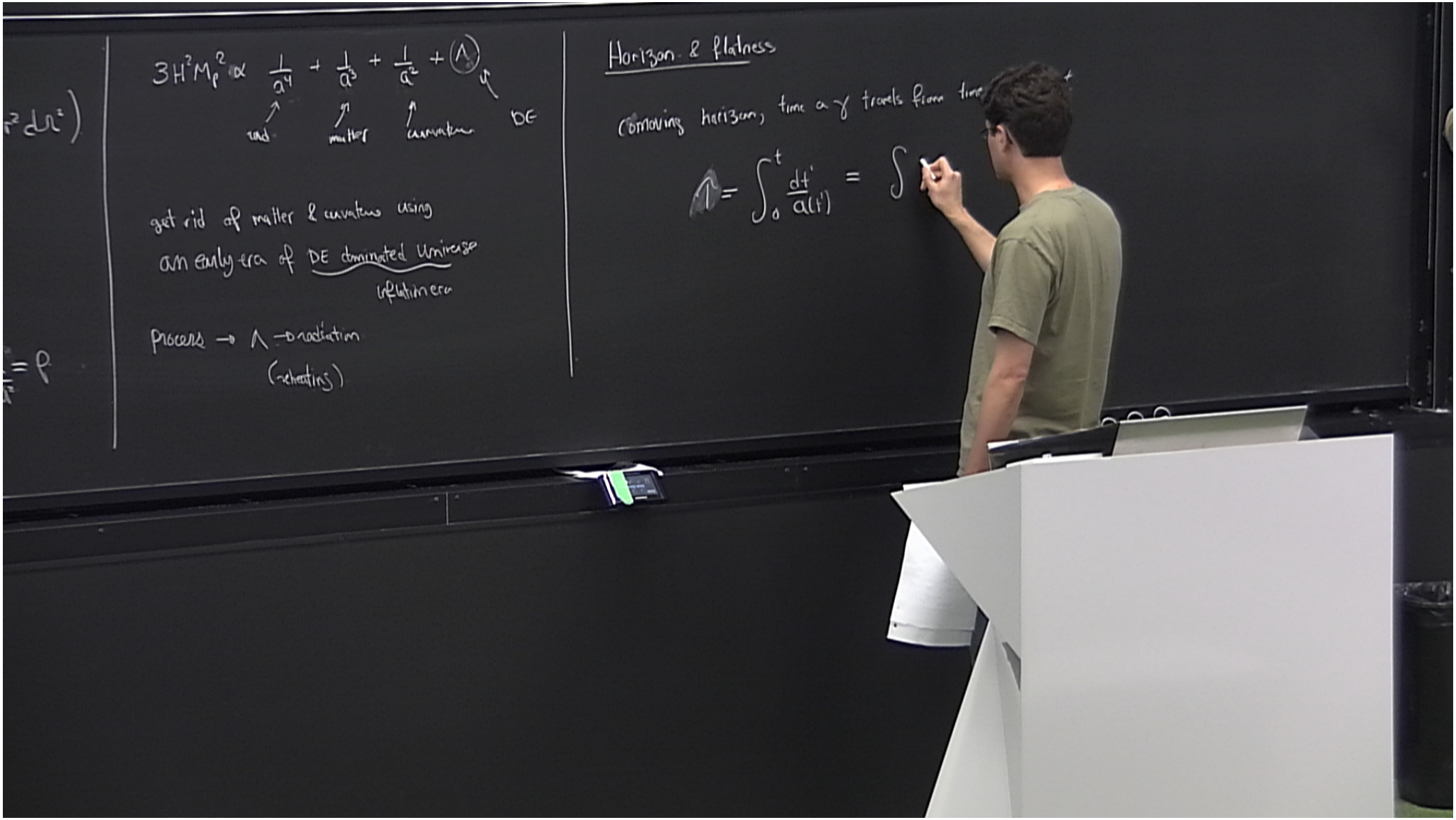

* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
$k = \begin{cases} 1 & \text{close} \\ -1 & \text{open} \\ 0 & \text{flat} \end{cases}$
new Friedmann Eqs. $3H^2 M_p + \frac{k}{a^2} = \rho$
$3H^2 M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid
& curvature using
DE dominated Universe
inflationary era

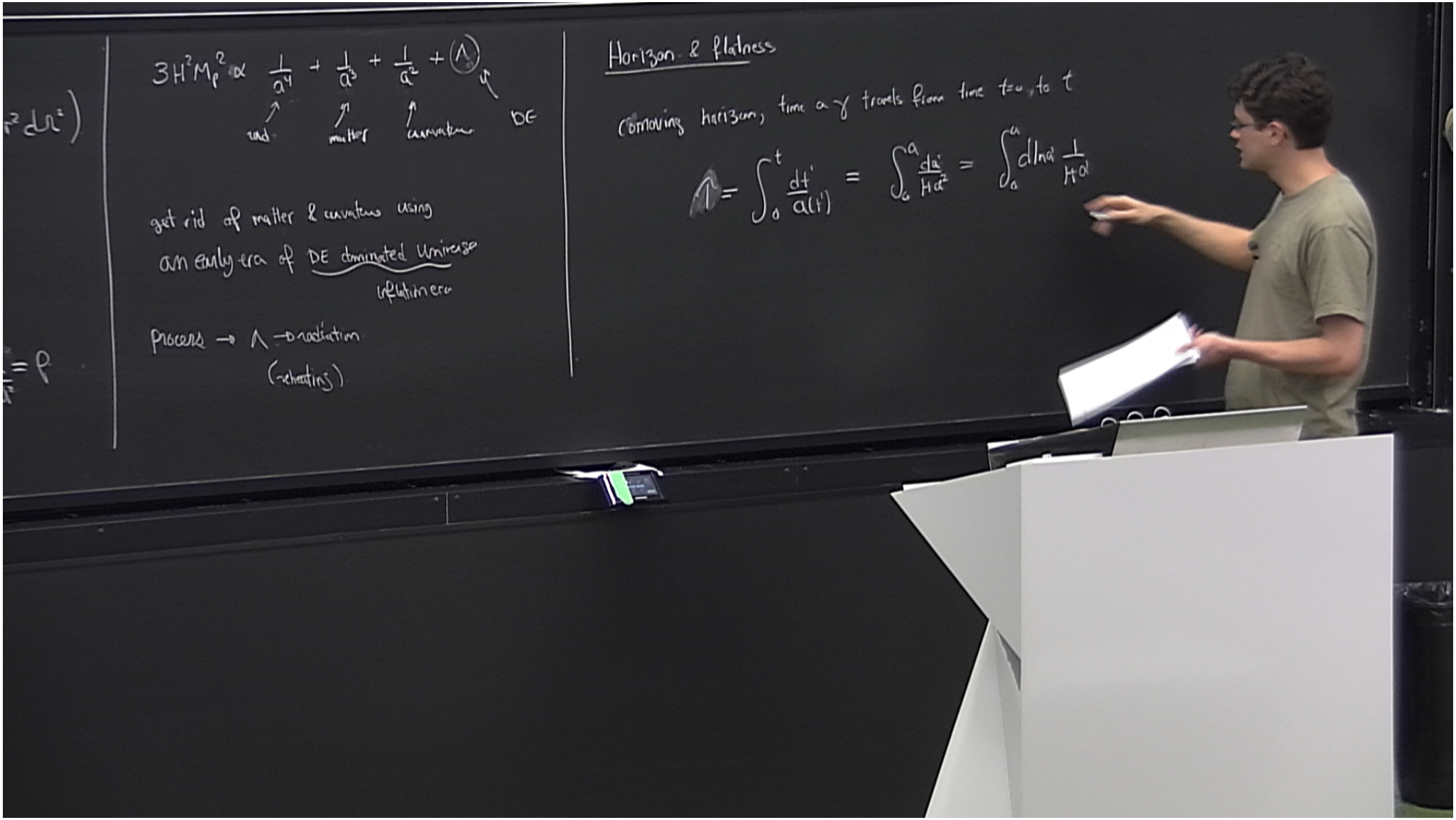

* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
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rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflation era
process → Λ → radiation
(reheating)

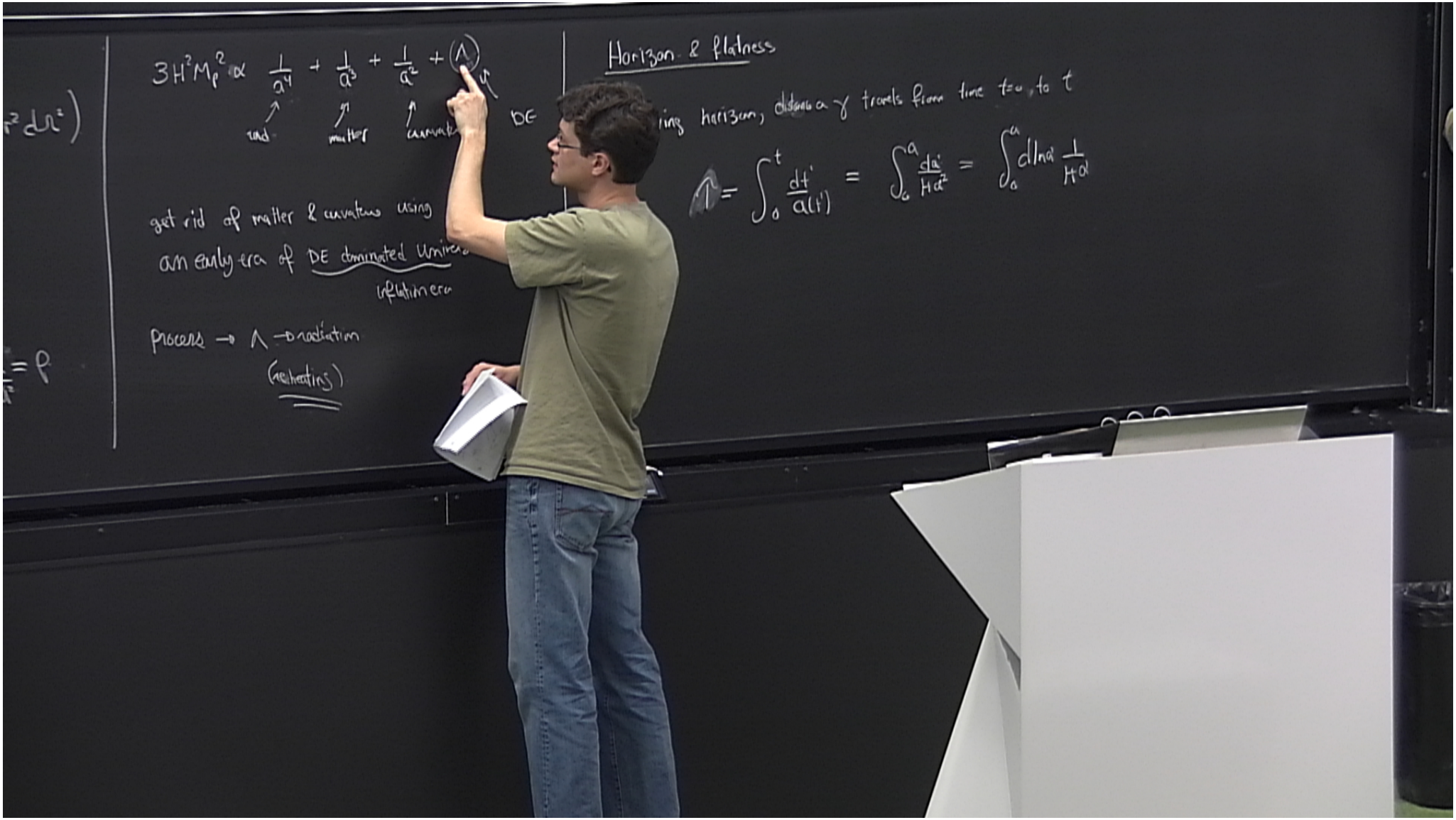

* curvature
$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$
1 close
-1 open
flat
Eqs. $3H^2M_p + \frac{k}{a^2} = \rho$
$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
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get rid of matter & curvature using
an early era of DE dominated Universe
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$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflation era
process → Λ → radiation
(reheating)
Horizon & flatness

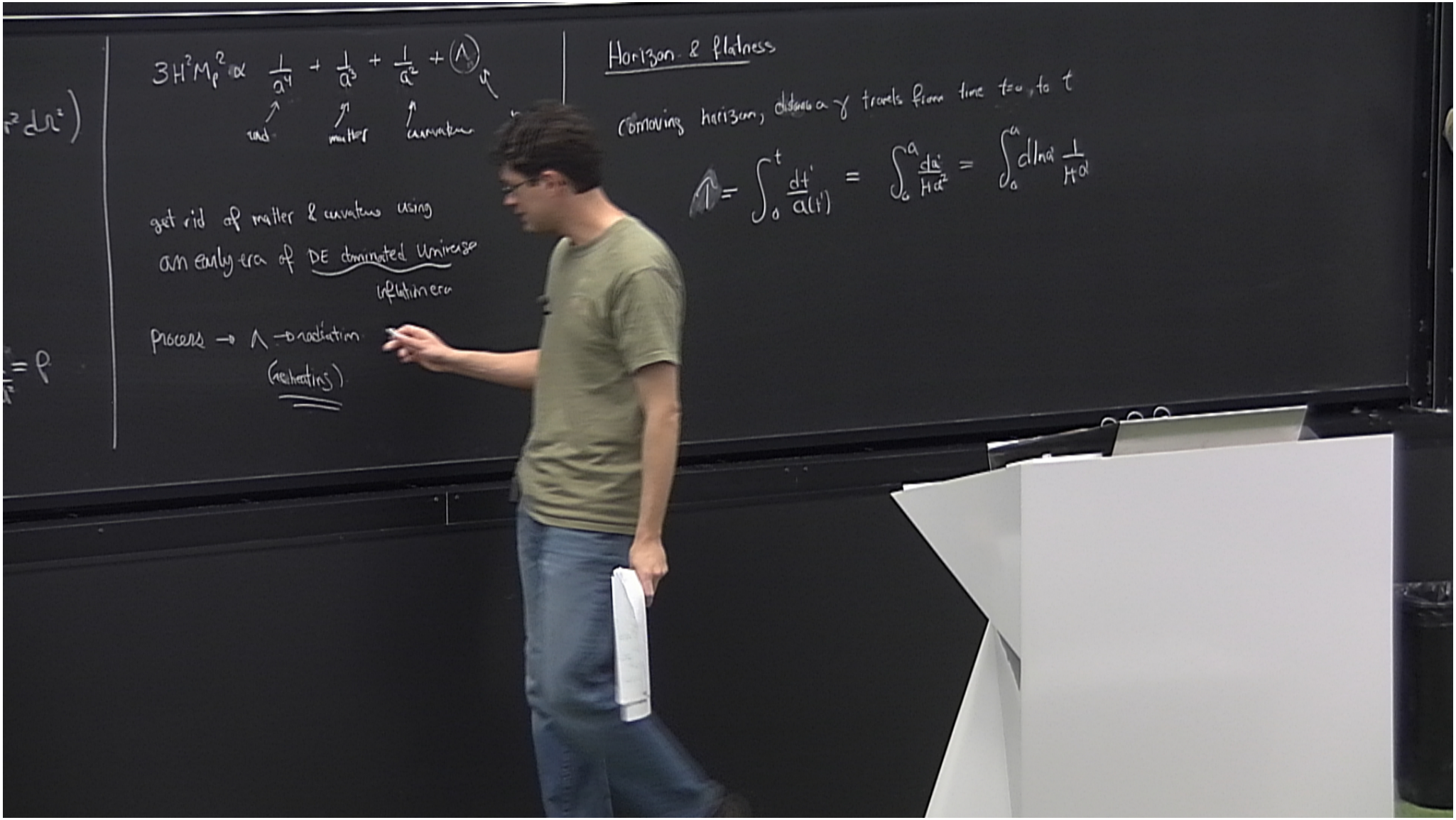

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflationary era
process → Λ → radiation
(reheating)
Horizon & flatness
(comoving horizon) time
$\eta = \int_0^t \frac{dt}{a}$

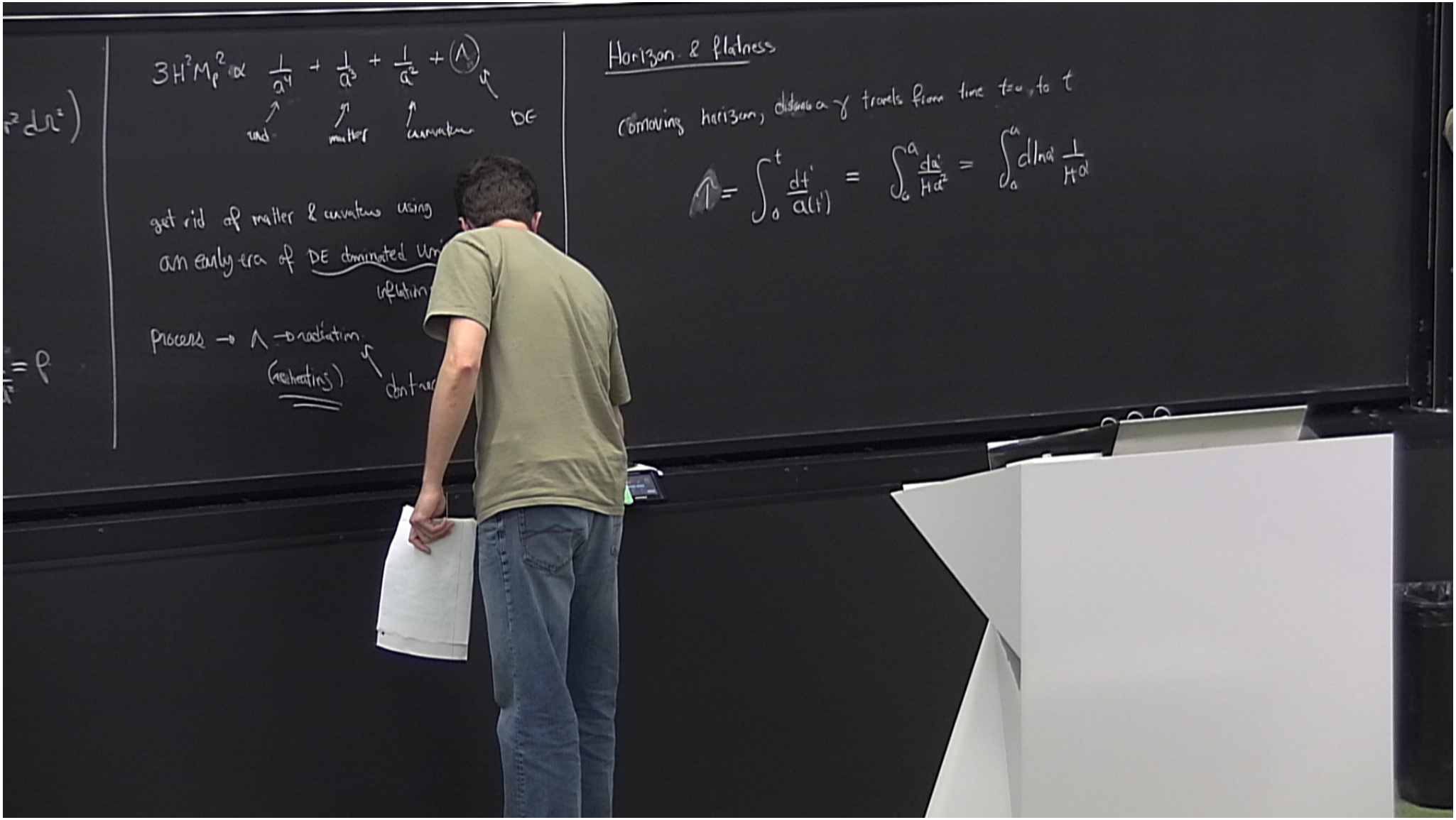

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflationary era
process → Λ → radiation
(reheating)
Horizon & flatness
comoving horizon, time a γ travels from time
$\tau = \int_0^t \frac{dt'}{a(t')} = \int$

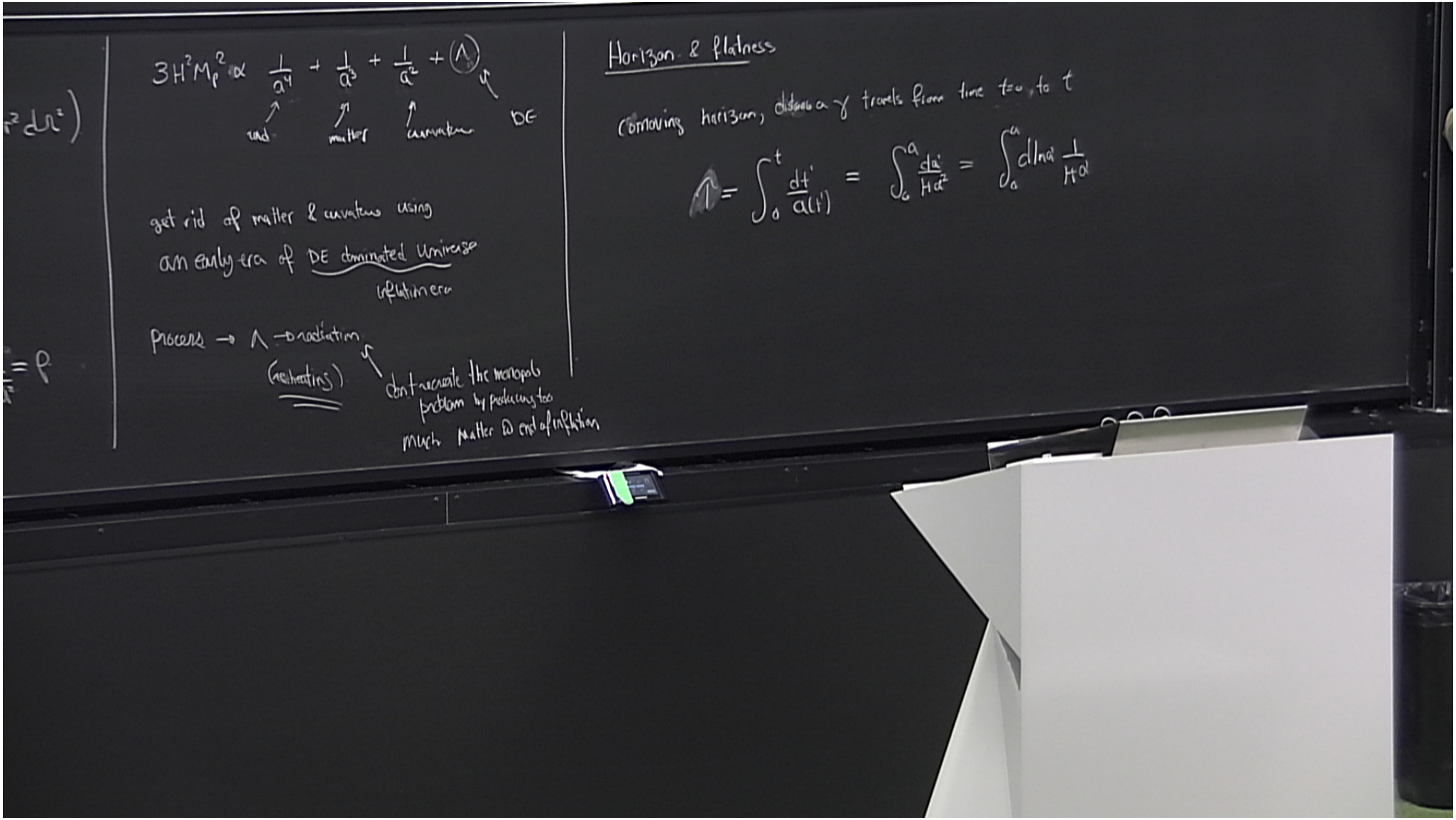

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflationary era
process → Λ → radiation
(reheating)
Horizon & flatness
Comoving horizon, time a γ travels from time $t=0$, to t
$\eta = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha'^2} = \int_0^a d\ln a' \frac{1}{Ha'}$

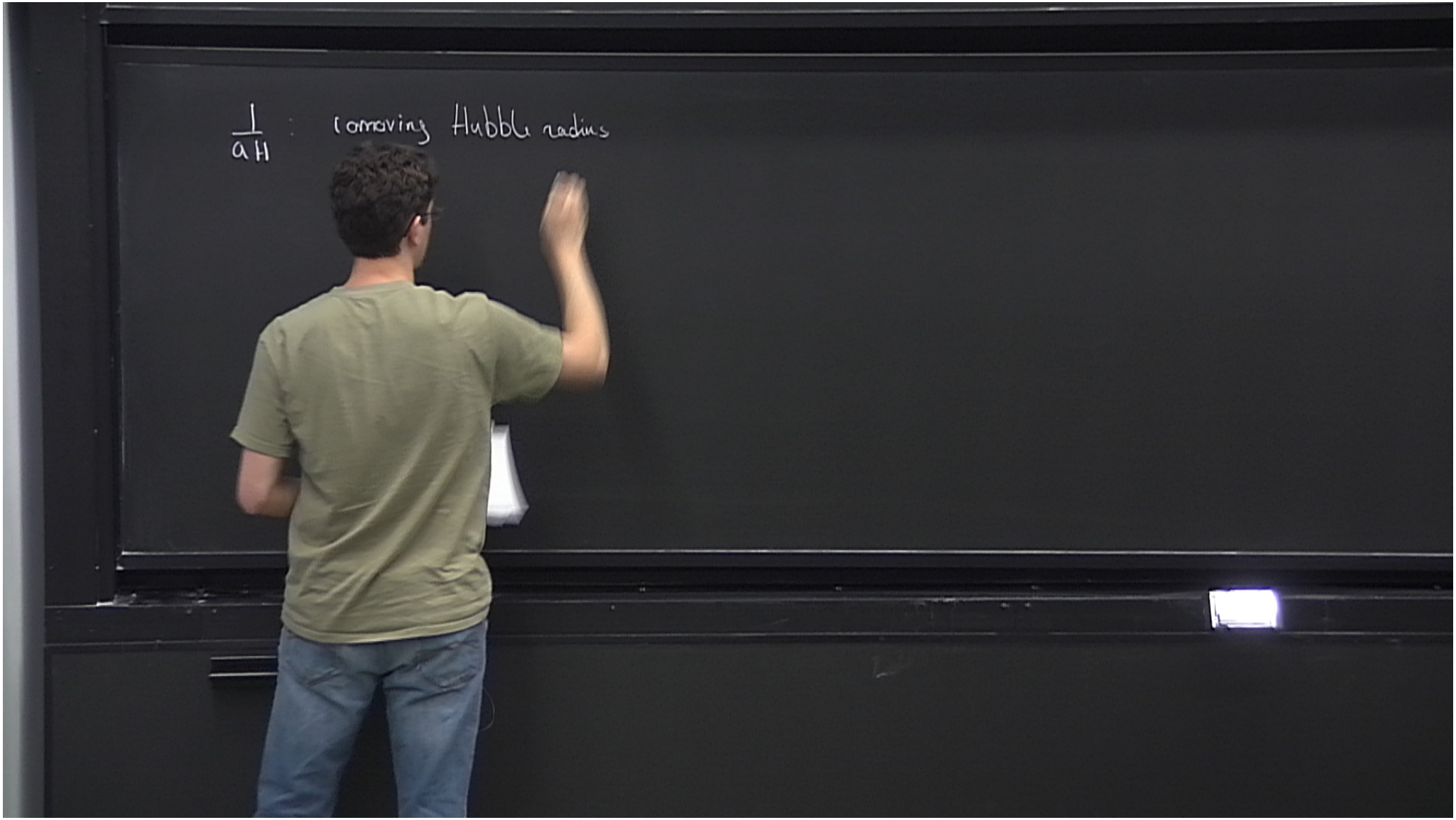

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
DE
Horizon & flatness
get rid of matter & curvature using
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process → Λ → radiation
(reheating)
$\chi = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha'^2} = \int_0^a d\ln a' \frac{1}{Ha'}$

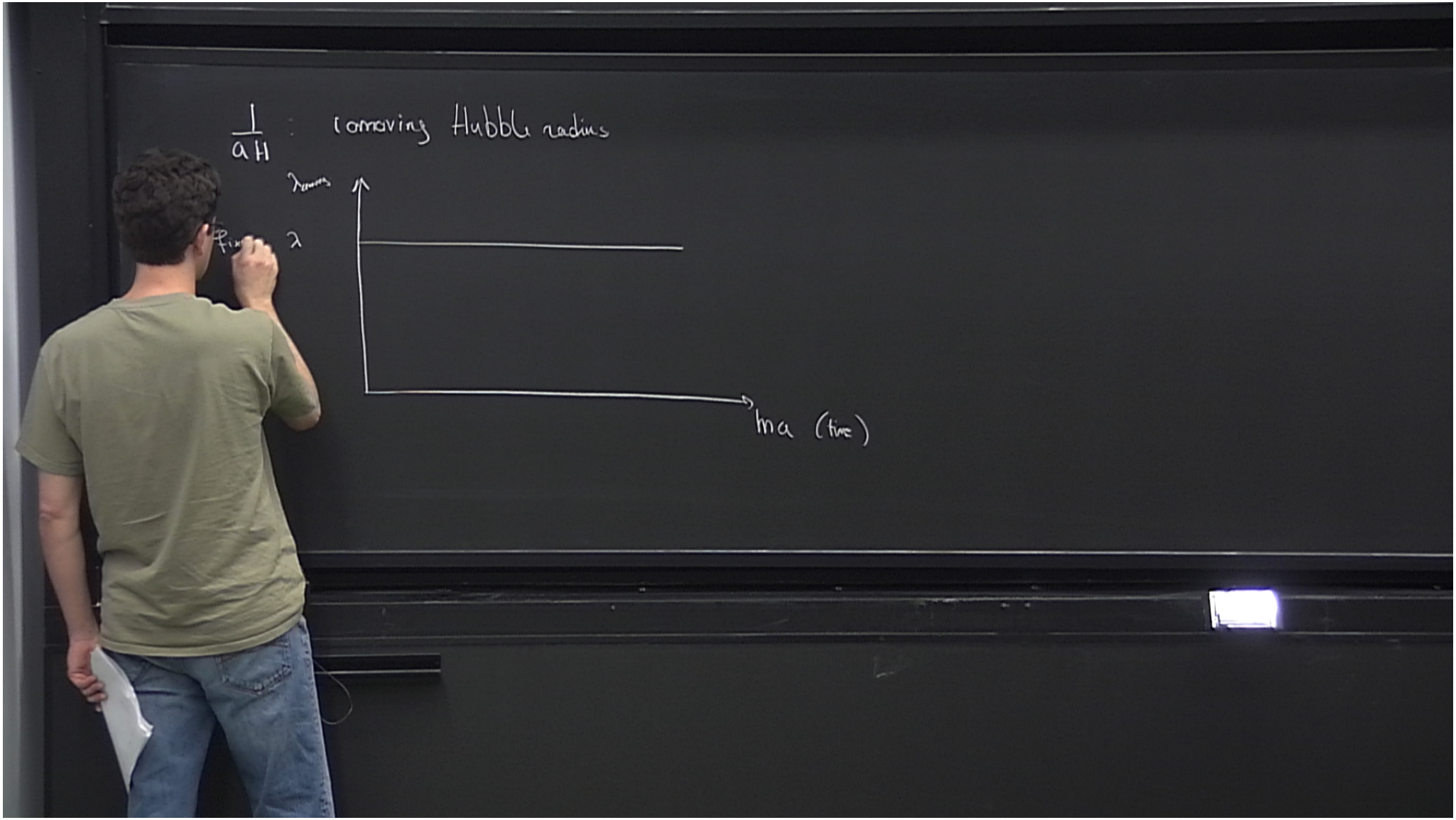

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
Horizon & Flatness
$\chi = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha'^2} = \int_0^a d\ln a' \frac{1}{Ha'}$
get rid of matter &
an early era of DE dominated
inflationary era
process → Λ → radiation
(reheating)

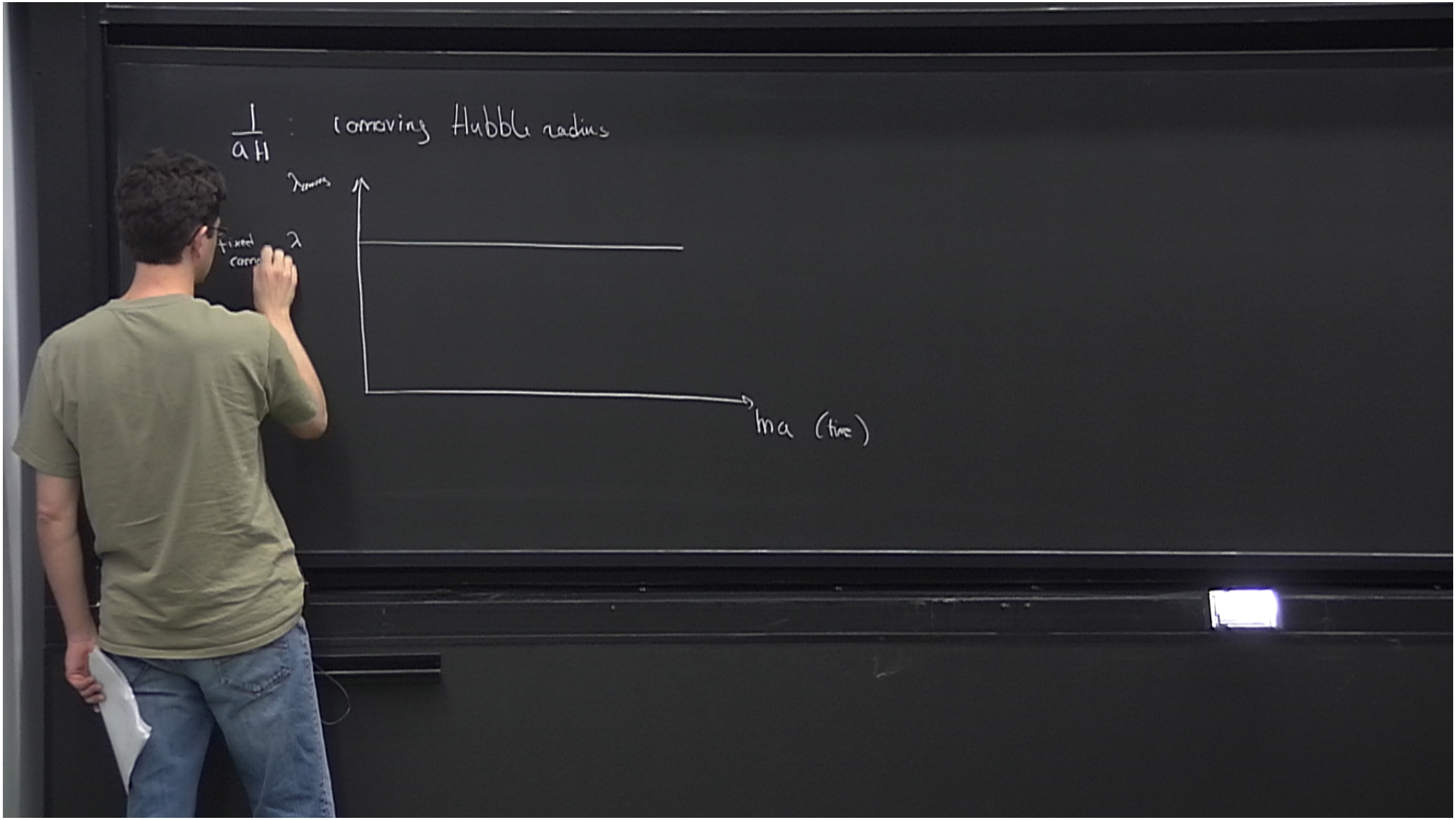

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
get rid of matter & curvature using
an early era of DE dominated Universe
inflationary era
process → Λ → radiation
(reheating)
Horizon & Flatness
comoving horizon, distance γ travels from time t_0 to t
$\Gamma = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha'^2} = \int_0^a d\ln a' \frac{1}{Ha}$

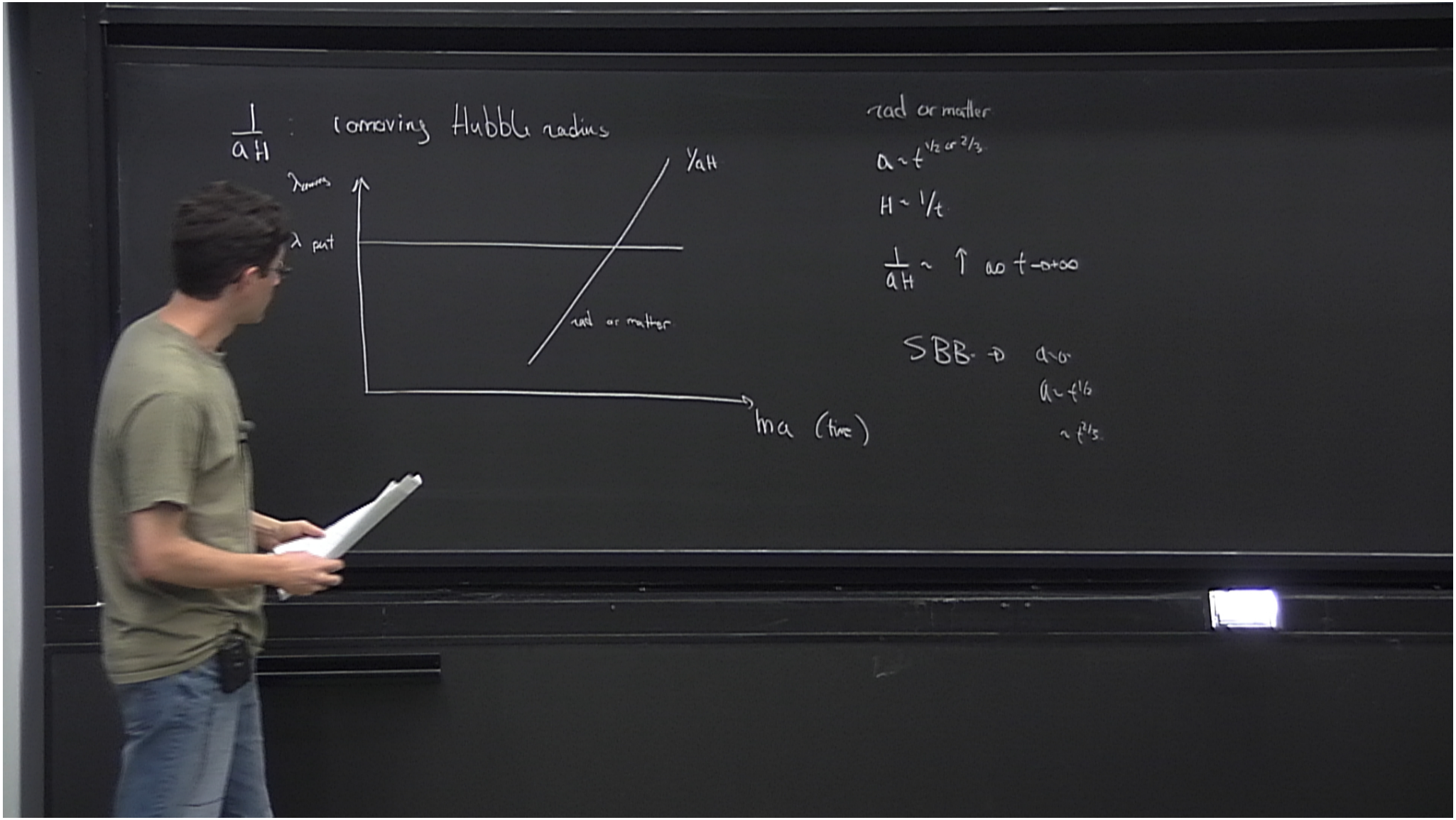

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated
inflation
process → Λ → radiation
(reheating)
Horizon & flatness
comoving horizon, distance γ travels from time t_{in} to t
$\chi = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha'^2} = \int_0^a d\ln a' \frac{1}{Ha'}$

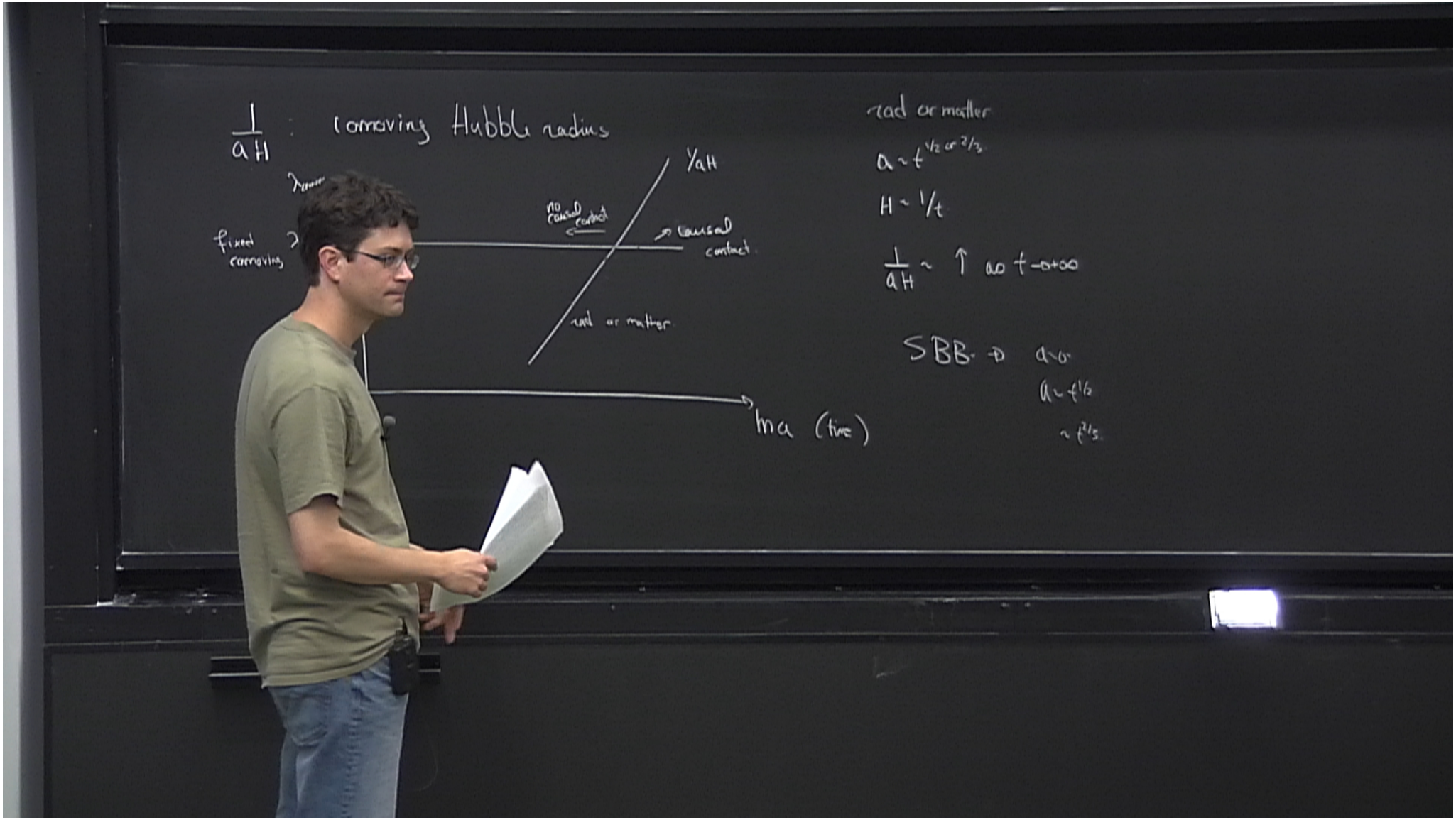

$3H^2M_p^2 \propto \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^2} + \Lambda$
rad
matter
curvature
DE
get rid of matter & curvature using
an early era of DE dominated Universe
inflationary era
process → Λ → reheating
(reheating)
don't recreate the monopole
problem by producing too
much matter @ end of inflation
Horizon & flatness
comoving horizon, distance γ travels from time $t=0$, to t
$$\chi = \int_0^t \frac{dt'}{a(t')} = \int_0^a \frac{da'}{Ha^2} = \int_0^a d\ln a' \frac{1}{Ha}$$

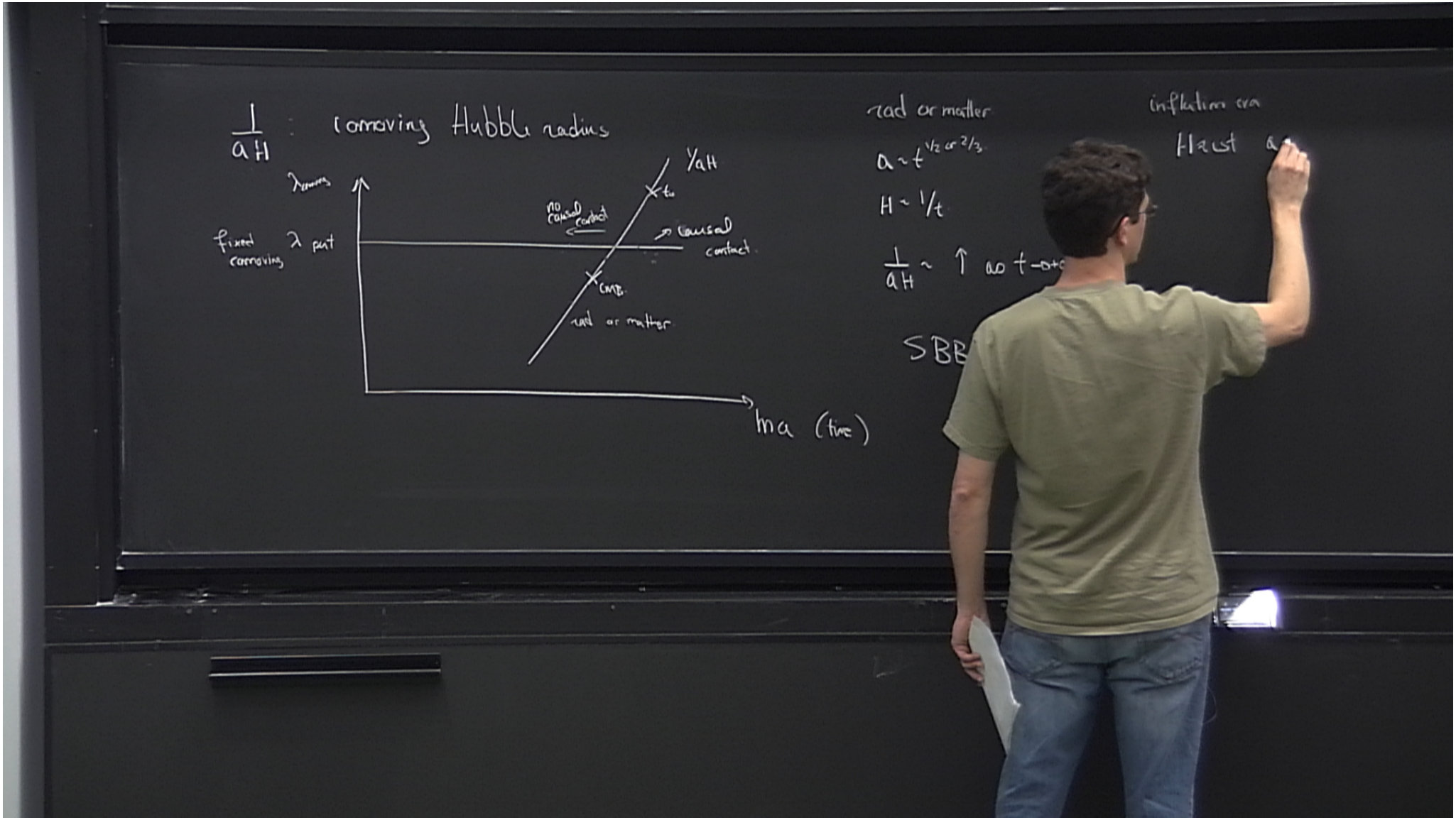

$\frac{1}{aH}$: comoving Hubble radius

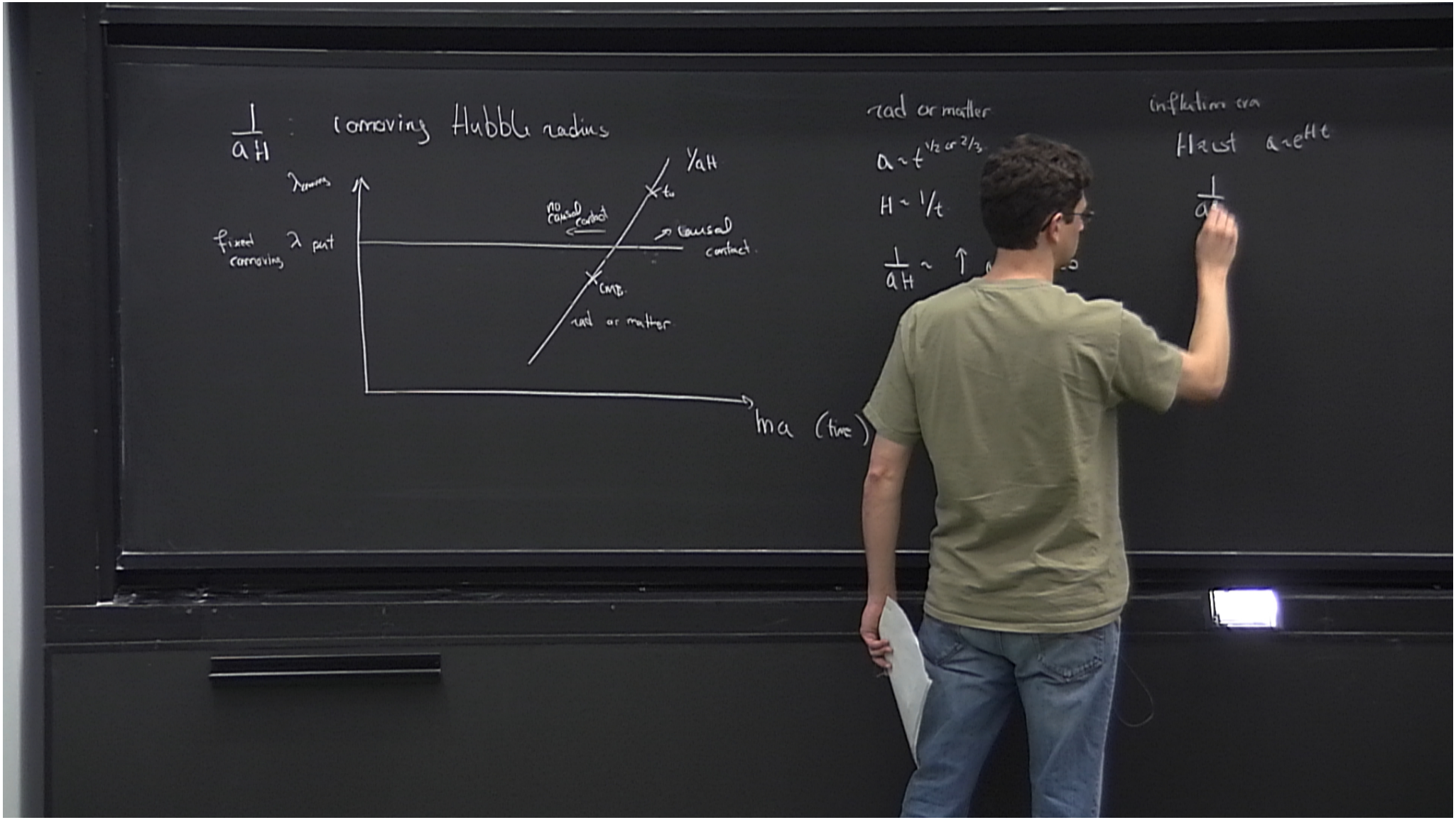

1/aH : comoving Hubble radius
λ
ln a (time)

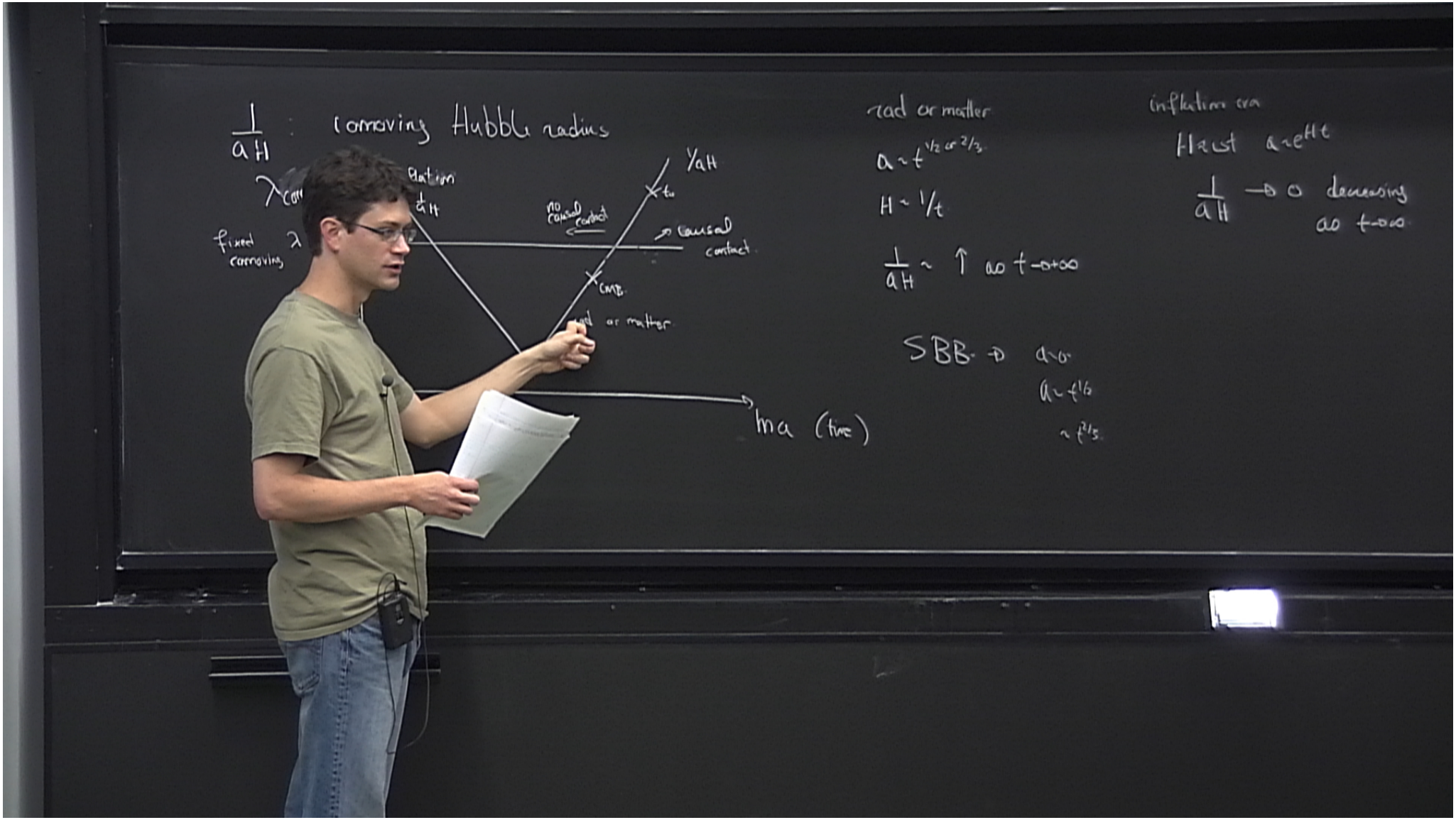

1/aH : comoving Hubble radius
fixed
ln a (time)

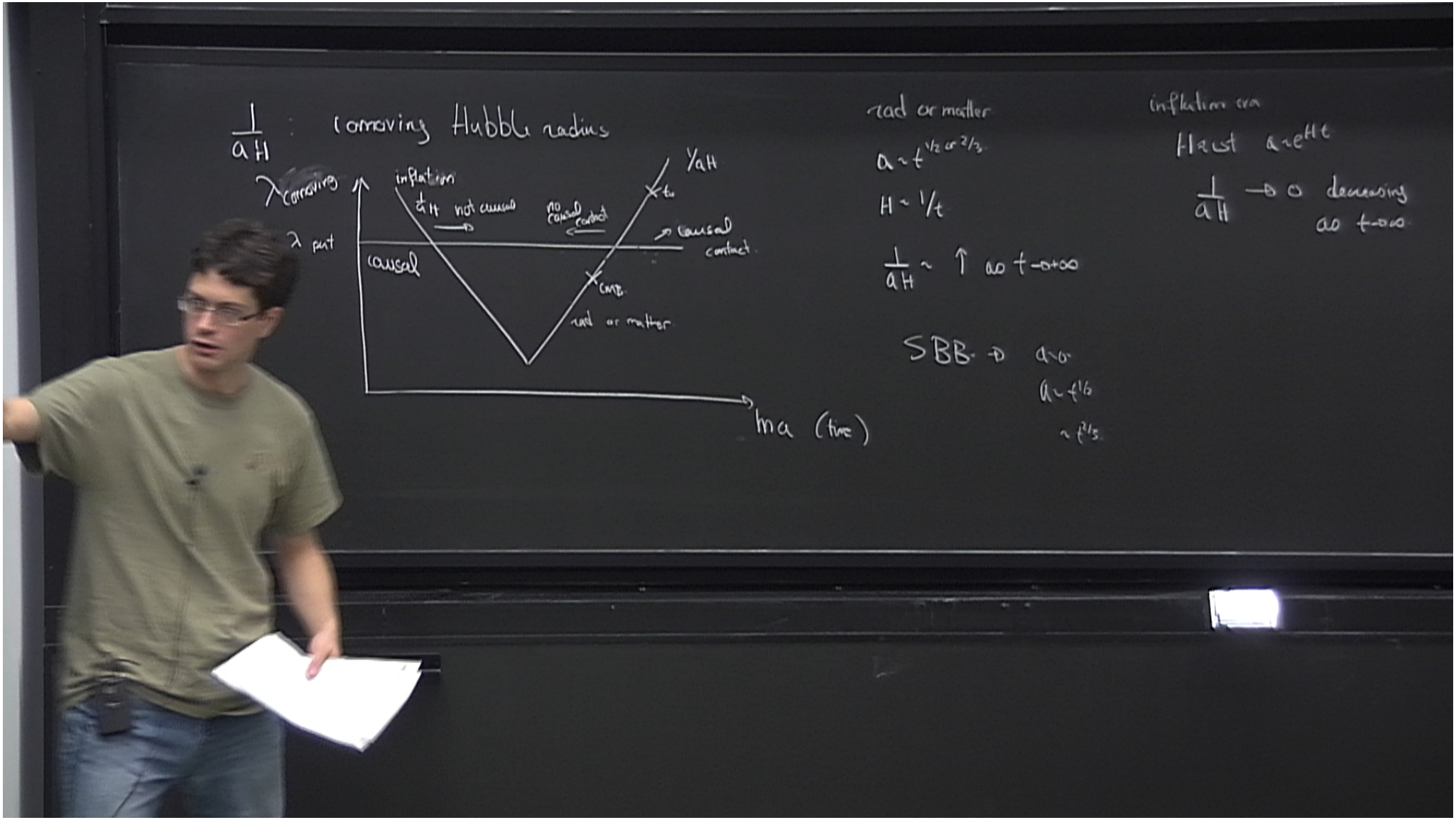

1/aH : comoving Hubble radius
λ comoving
λ phys
1/aH
rad or matter
ln a (time)
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
1/aH ~ ↑ as t → 0 → ∞
SBB → a → 0
a ~ t^1/2
~ t^2/3

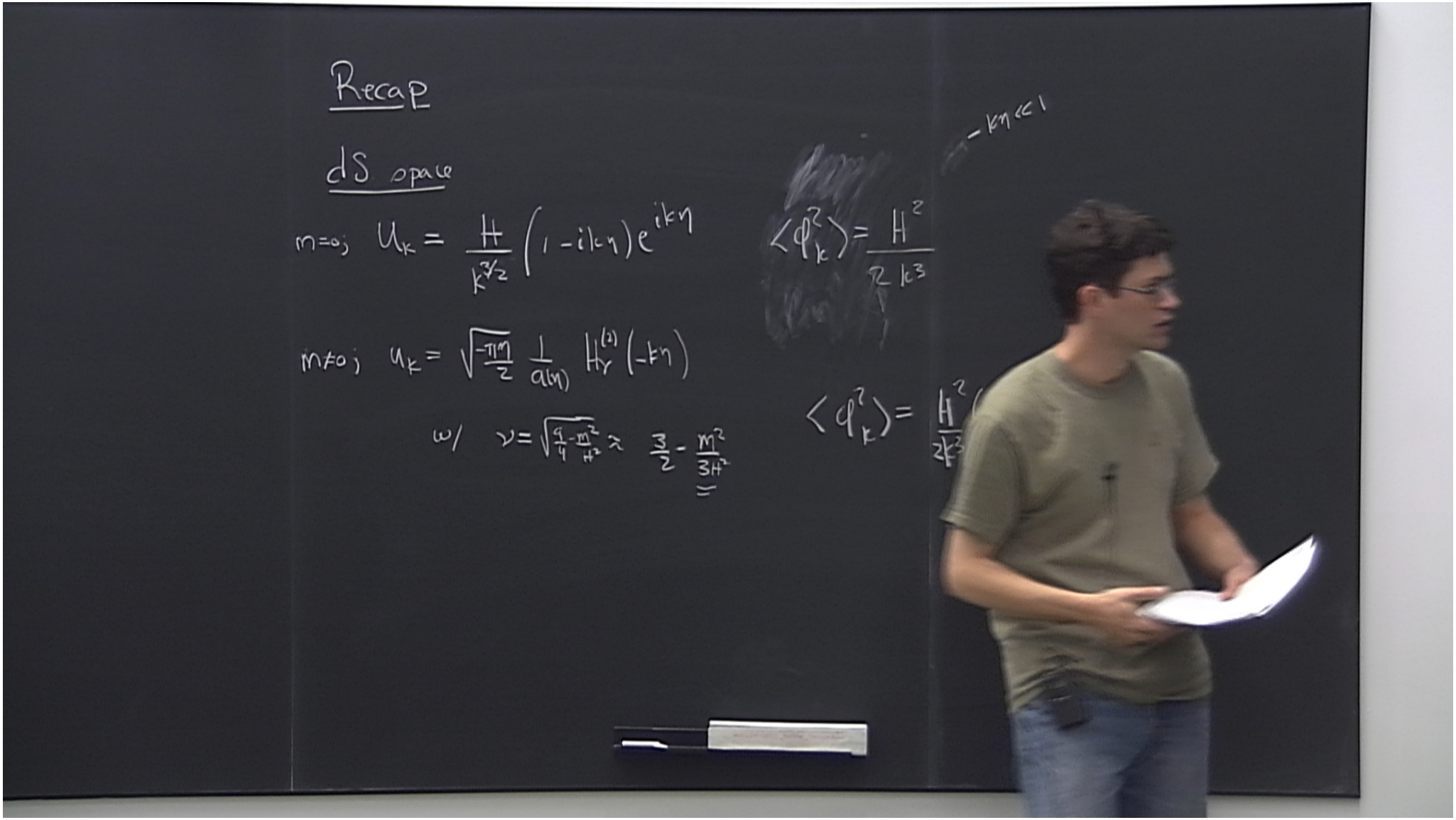

1/aH : comoving Hubble radius
1/aH
fixed comoving
no causal contact
causal contact
rad or matter
ln a (time)
rad or matter
a ~ t^{1/2 or 2/3}
H ~ 1/t
1/aH ~ ↑ as t → 0 → ∞
SBB → a → 0
a ~ t^{1/2}
~ t^{2/3}

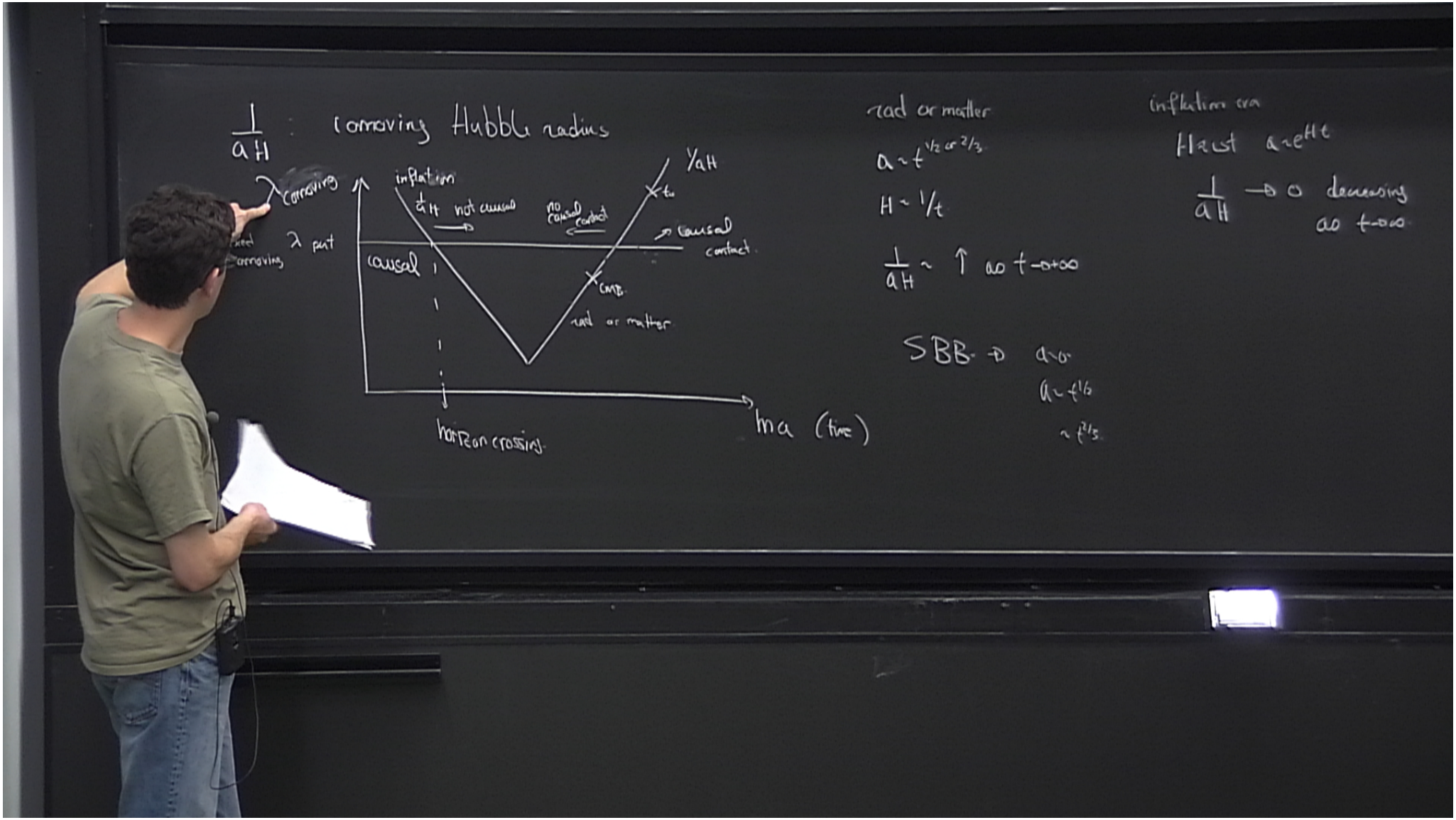

1/aH : comoving Hubble radius
λ comoving
fixed comoving λ put
no causal contact
causal contact
1/aH
t_in
CMB
rad or matter
ln a (time)
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
1/aH ~ ↑ as t →
inflation era
H ≈ cst

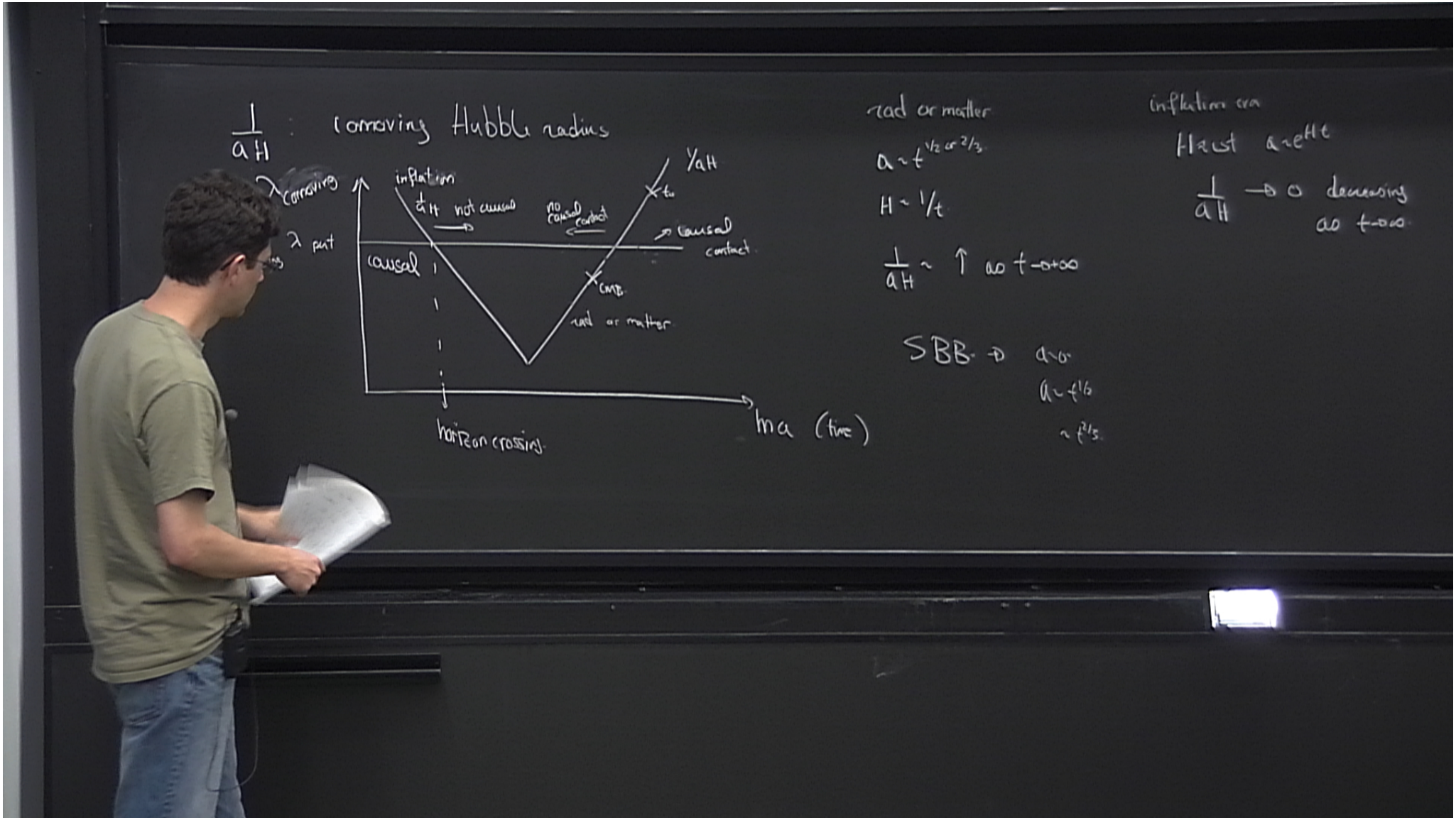

1/aH : comoving Hubble radius
λ proper
fixed comoving λ
1/aH
no causal contact
causal contact
t_eq
CMB
rad or matter
ln a (time)
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
inflation era
H = const
a ~ e^Ht

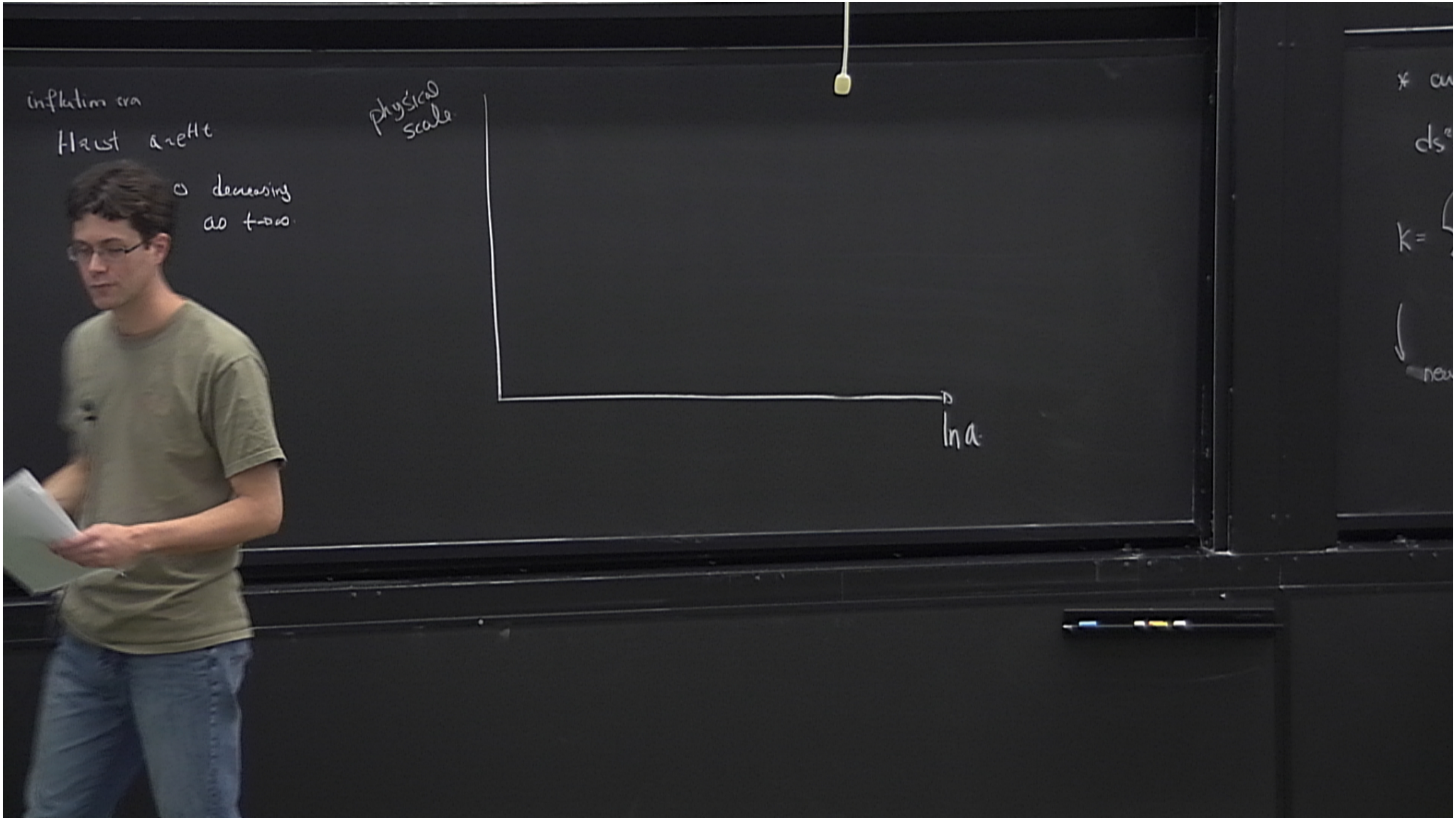

1/aH
comoving Hubble radius
fixed comoving λ
flat in 1/aH
no causal contact
causal contact
CMB
rad or matter
1/aH
ln a (time)
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
1/aH ~ ↑ as t→0→∞
SBB → a→0
a ~ t^1/2
~ t^2/3
inflation era
first a ~ e^Ht
1/aH → 0 decreasing
as t→∞

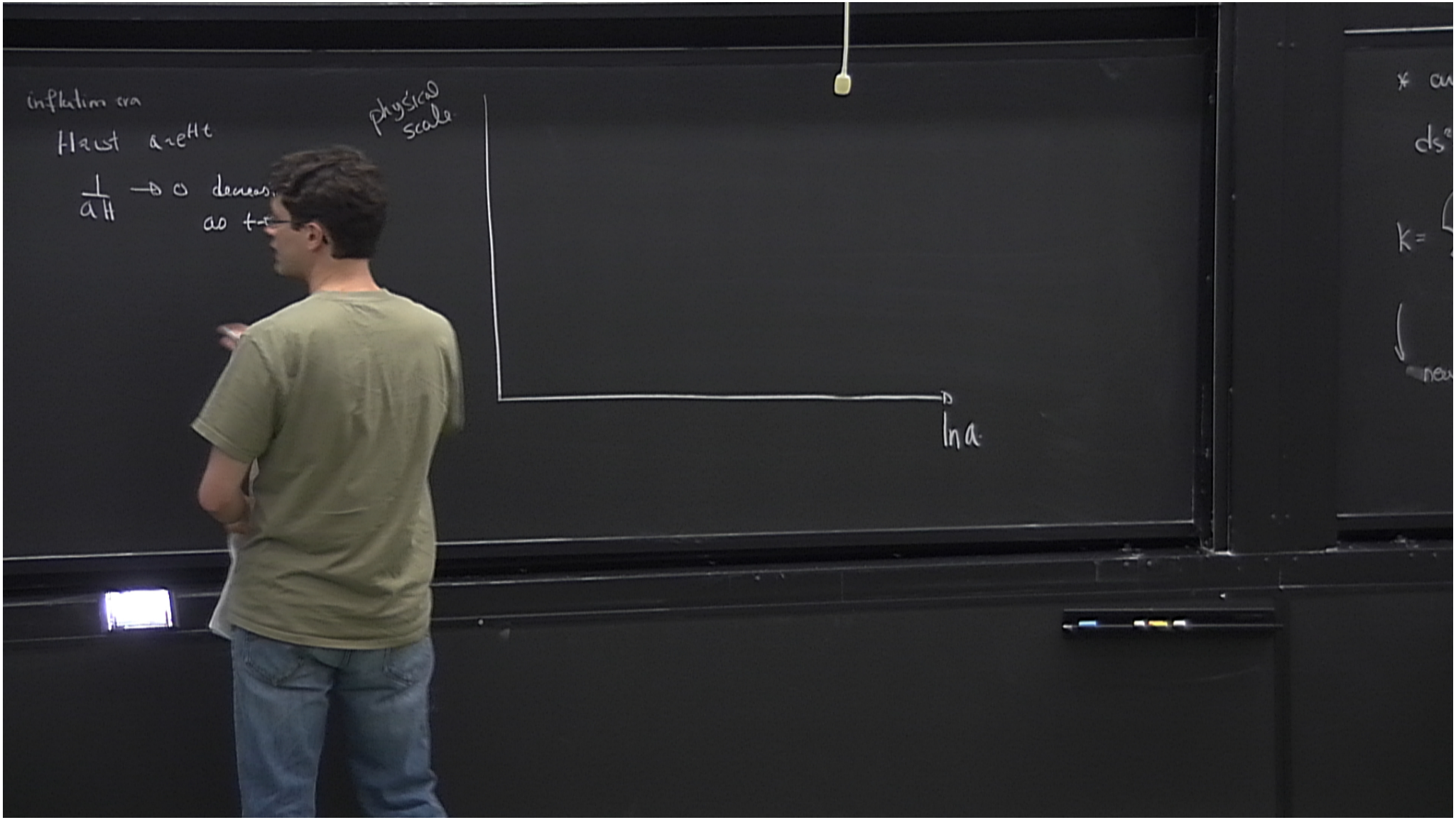

1/aH : comoving Hubble radius
λ comoving
λ phys
inflation
1/aH not caused
no caused contact
1/aH
caused contact
caused
CMB
rad or matter
ln a (time)
rad or matter
a ~ t^{1/2 or 2/3}
H ~ 1/t
1/aH ~ ↑ as t→0→∞
SBB → a→0
a ~ t^{1/2}
~ t^{2/3}
inflation era
first a ~ e^{Ht}
1/aH → 0 decreasing
as t→∞

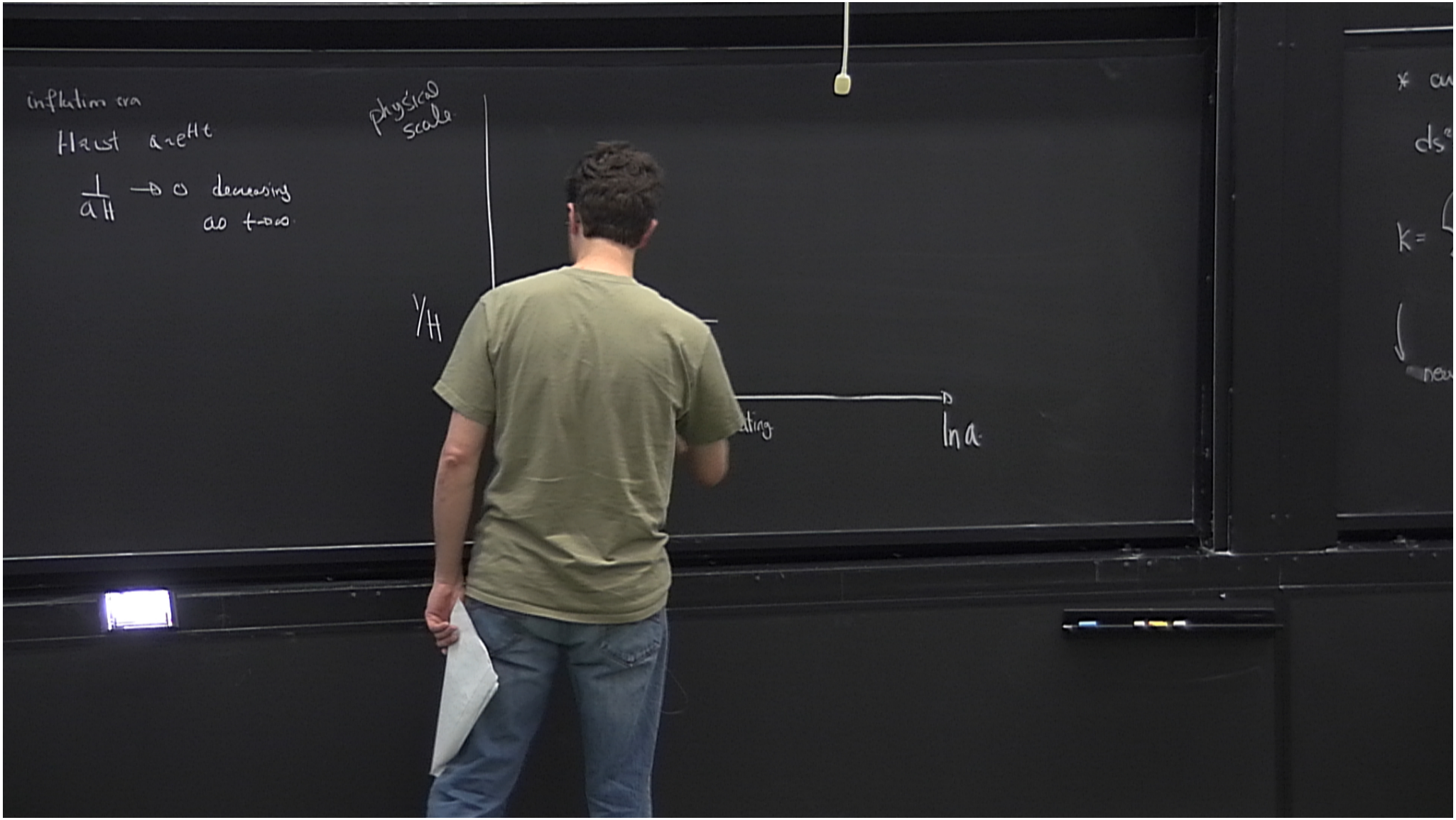

Recap
dS space
$m=0;\ u_k = \frac{H}{k^{3/2}}(1-ik\eta)e^{ik\eta}$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}$
$-k\eta \ll 1$
$m\neq 0;\ u_k = \sqrt{\frac{-\pi\eta}{2}}\frac{1}{a(\eta)}H_\nu^{(1)}(-k\eta)$
$\langle \varphi_k^2 \rangle =$
w/ $\nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \approx \frac{3}{2}-\frac{m^2}{3H^2}$

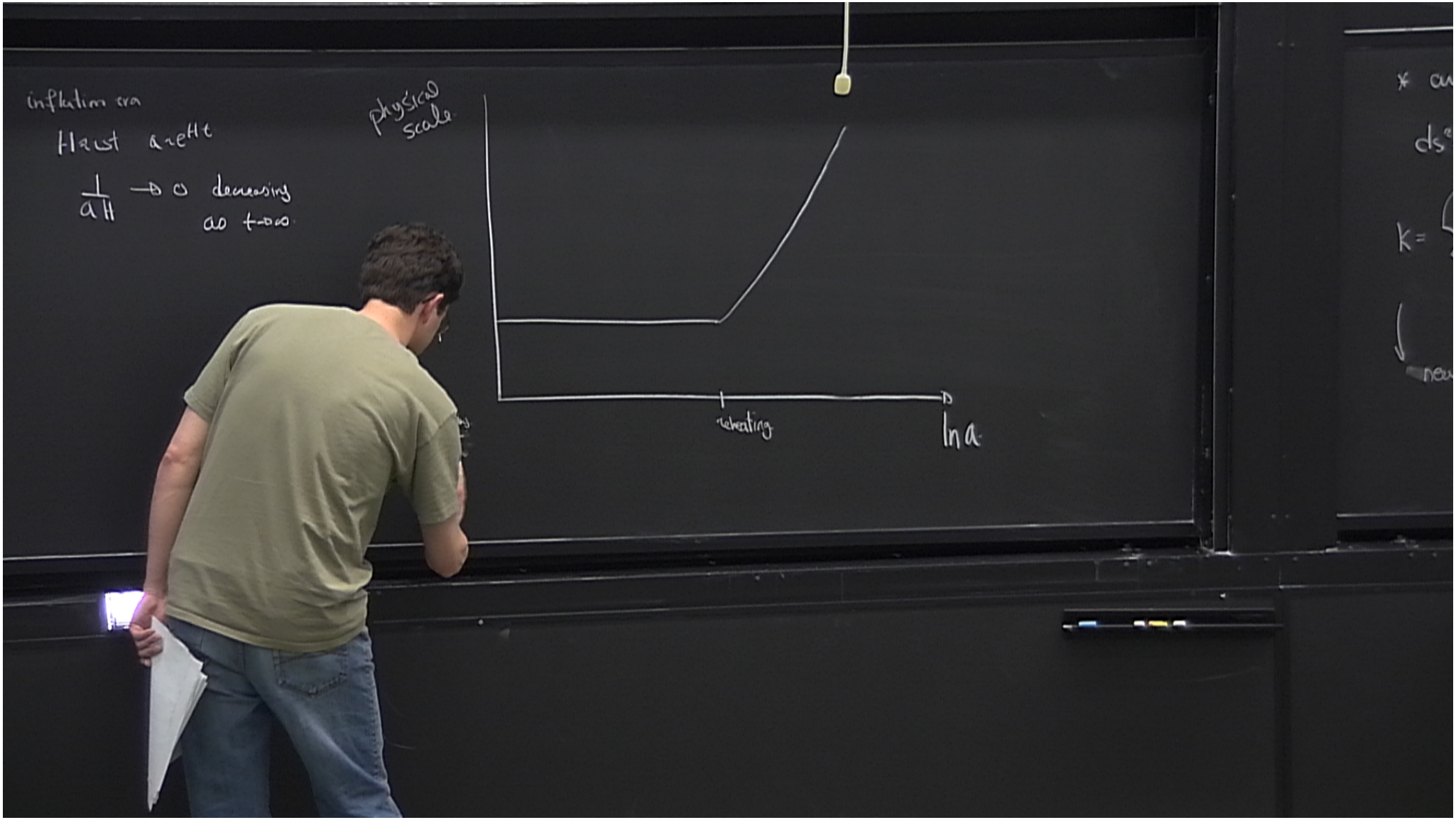

1/aH : comoving Hubble radius
comoving
λ part
inflation
1/aH not caused
no caused contact
1/aH
caused contact
caused
CMB
rad or matter
horizon crossing
ln a (time)
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
1/aH ~ ↑ as t→0→∞
SBB → a→0
a ~ t^1/2
~ t^2/3
inflation era
H=cst a~e^Ht
1/aH → 0 decreasing
as t→∞

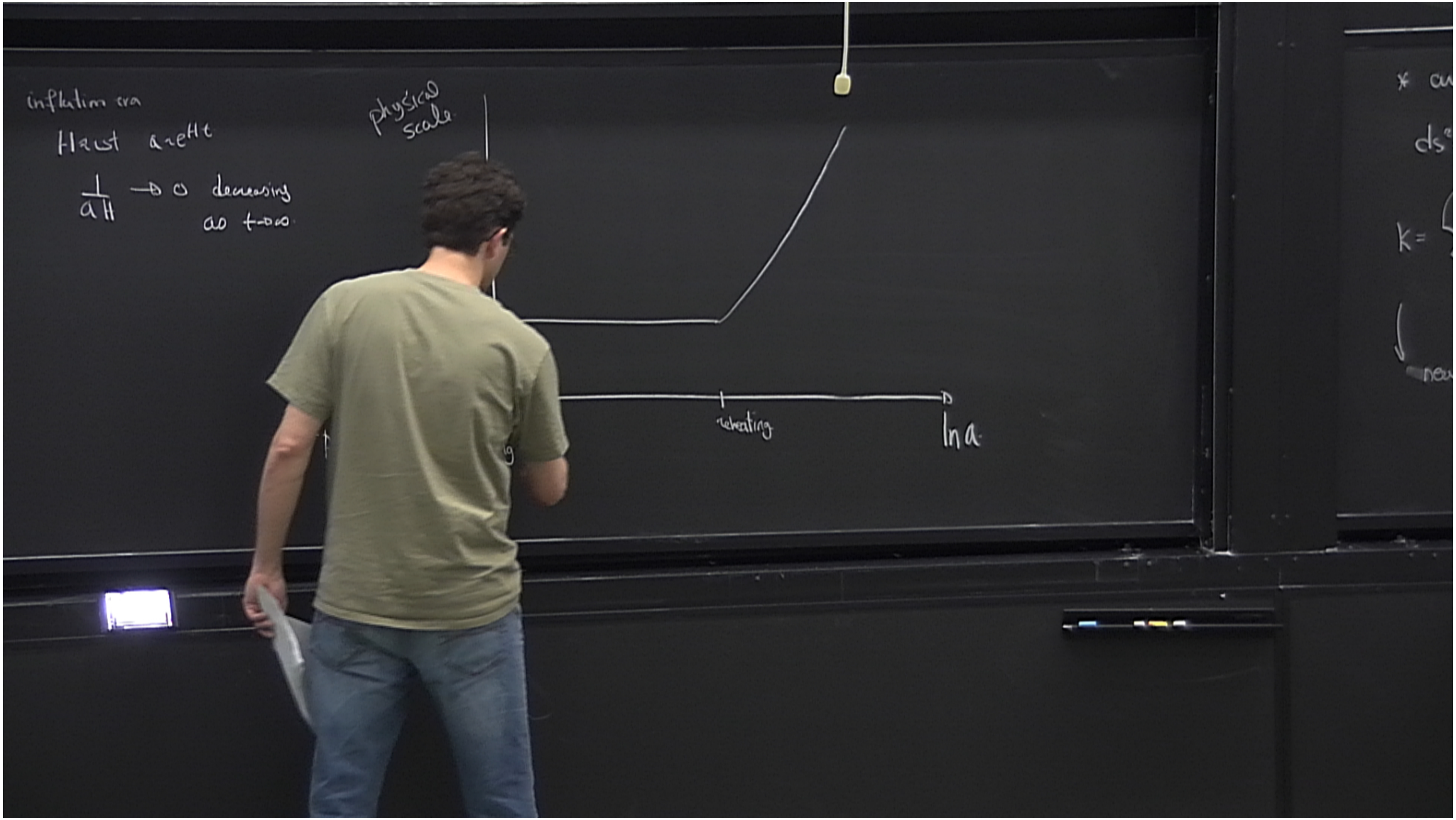

1/aH
comoving Hubble radius
λ comoving
λ phys
inflation
1/aH not caused
no caused contact
1/aH
caused contact
caused
CMB
rad or matter
horizon crossing
ln a (time)
rad or matter
a ~ t^{1/2 or 2/3}
H ~ 1/t
1/aH ~ ↑ as t → 0 → +∞
SBB → a → 0
a ~ t^{1/2}
~ t^{2/3}
inflation era
H ≈ cst a ~ e^{Ht}
1/aH → 0 decreasing
as t → ∞


inflation era
Hubble radius
decreasing
as t→∞
physical scale
ln a

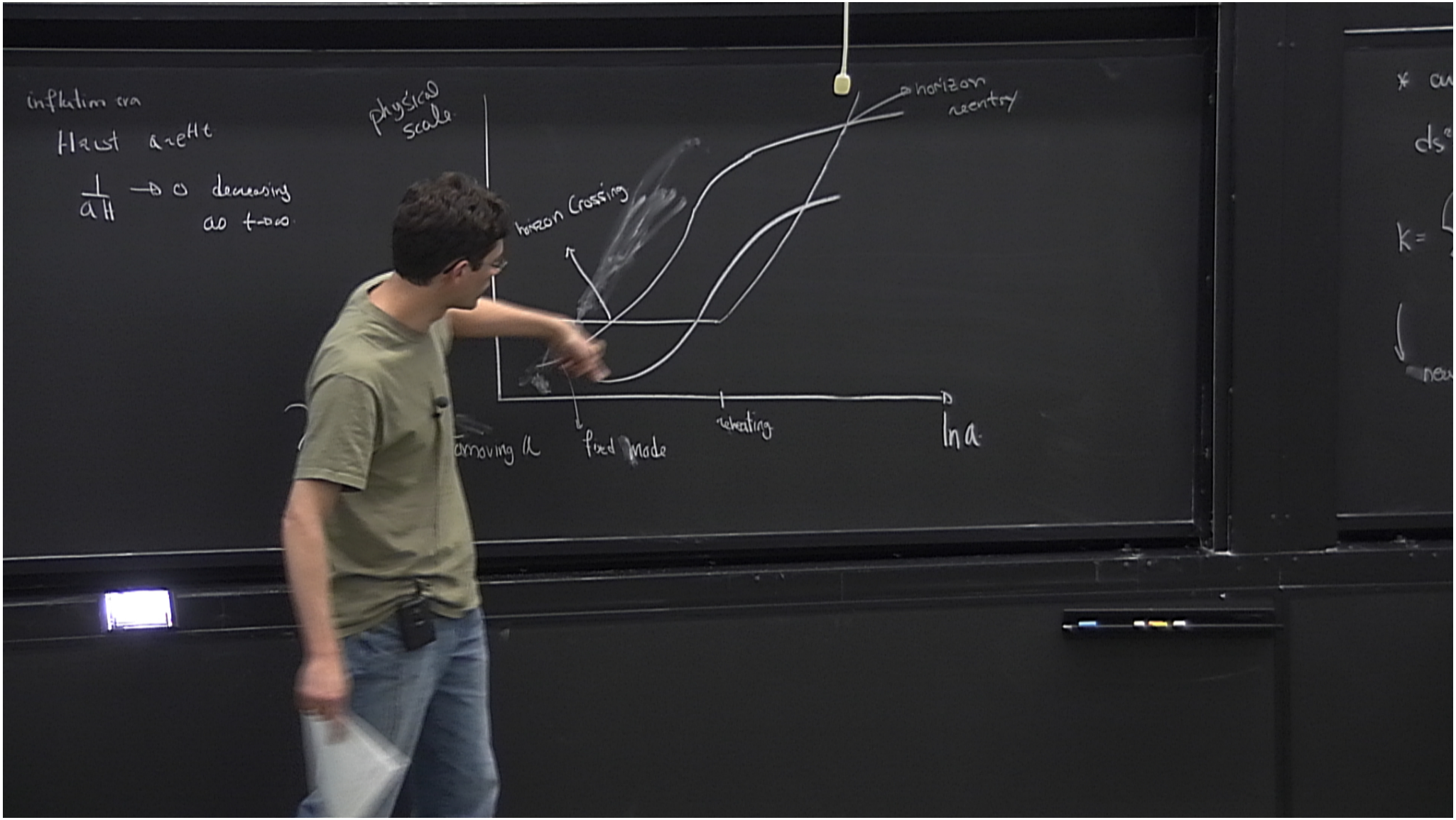

inflation era
Hubble radius
$\frac{1}{aH}$
physical scale
ln a
k =

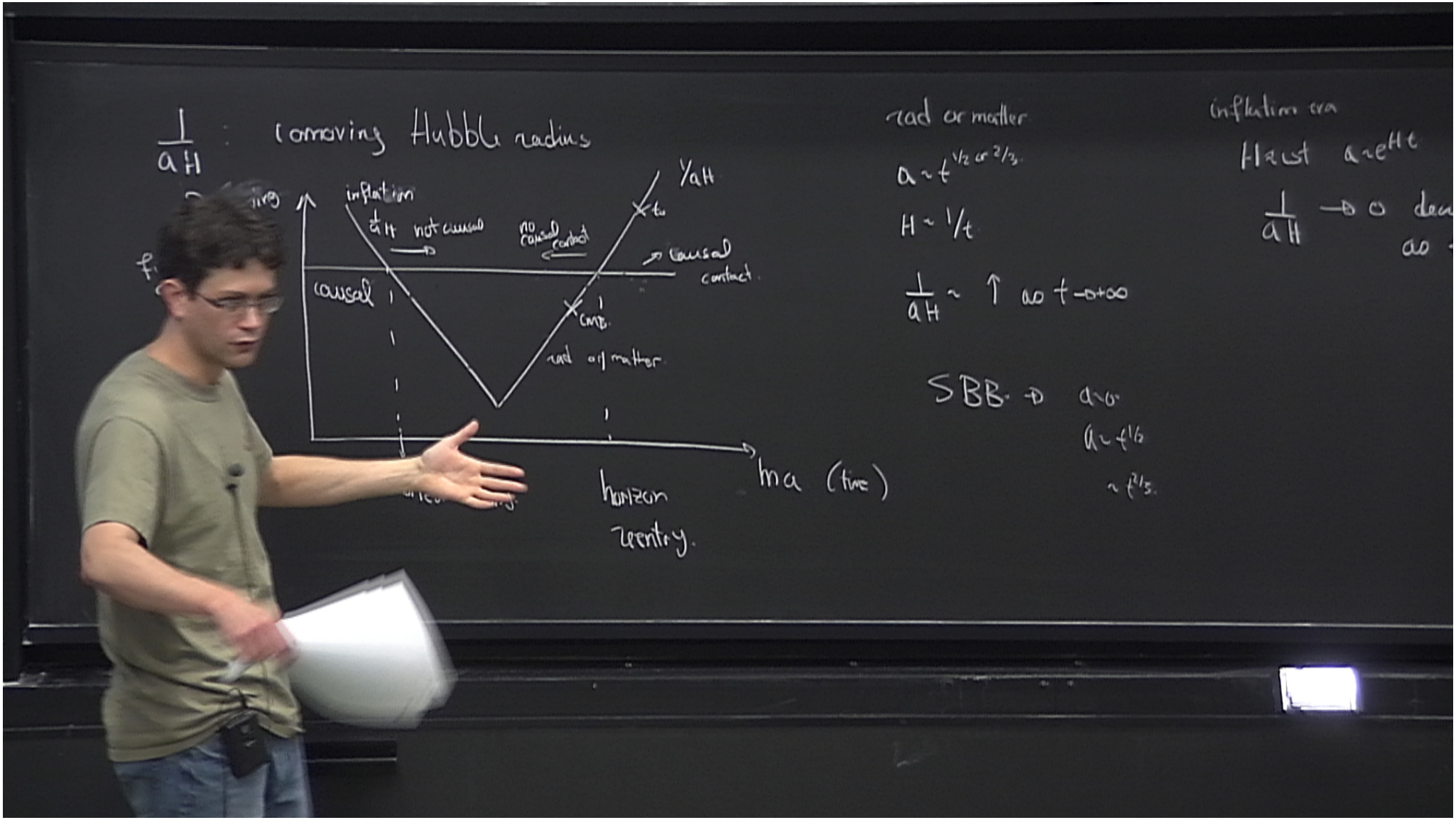

inflation era
Hard a=eHt
1/aH → 0 decreasing
as t→∞
physical scale
1/H
ln a

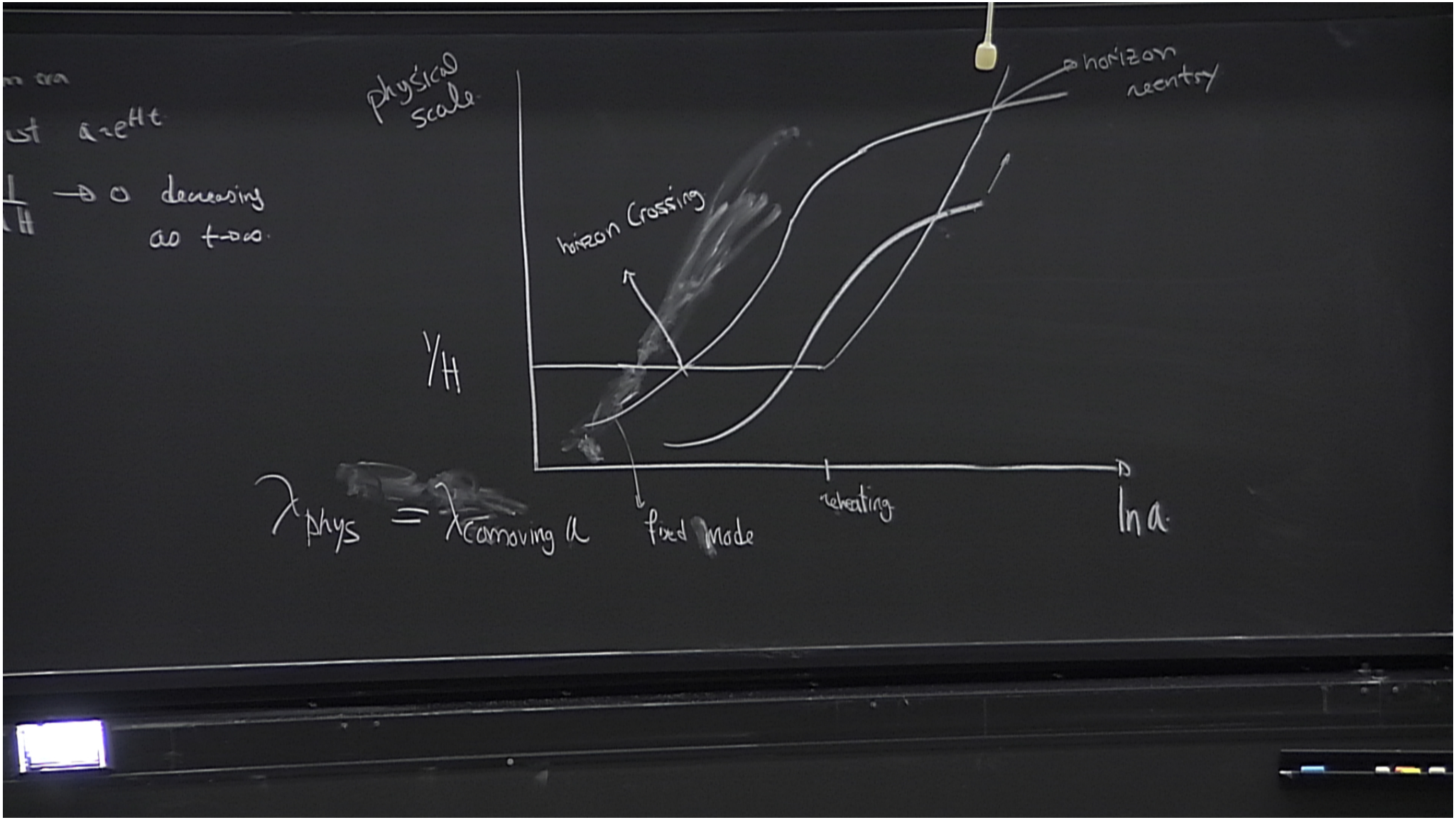

inflation era
Hard a~eHt
1/aH → 0 decreasing
as t→∞
physical scale
reheating
ln a

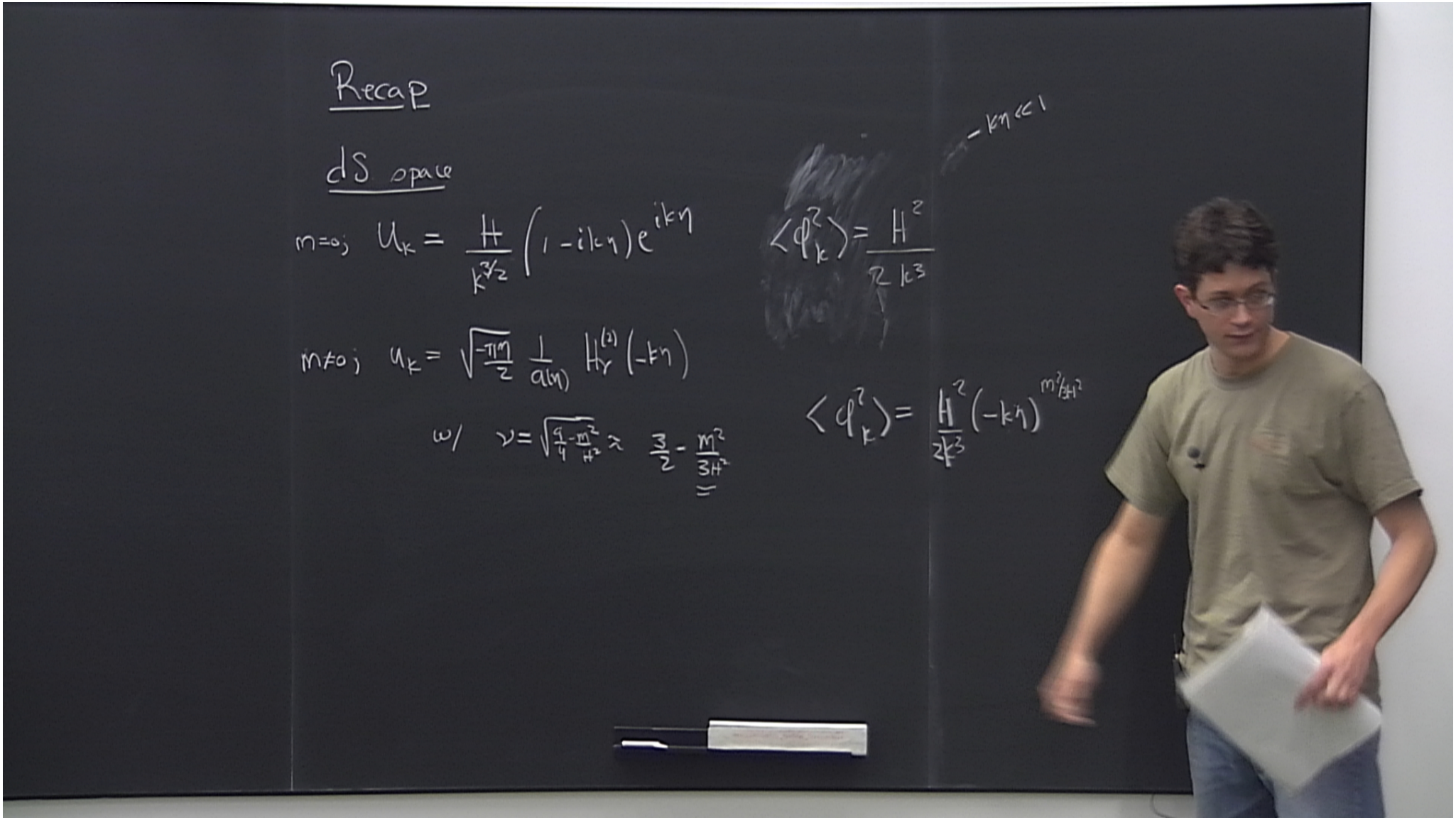

inflation era
Hard a~eHt
1/aH → 0 decreasing
as t→∞
physical scale
reheating
ln a

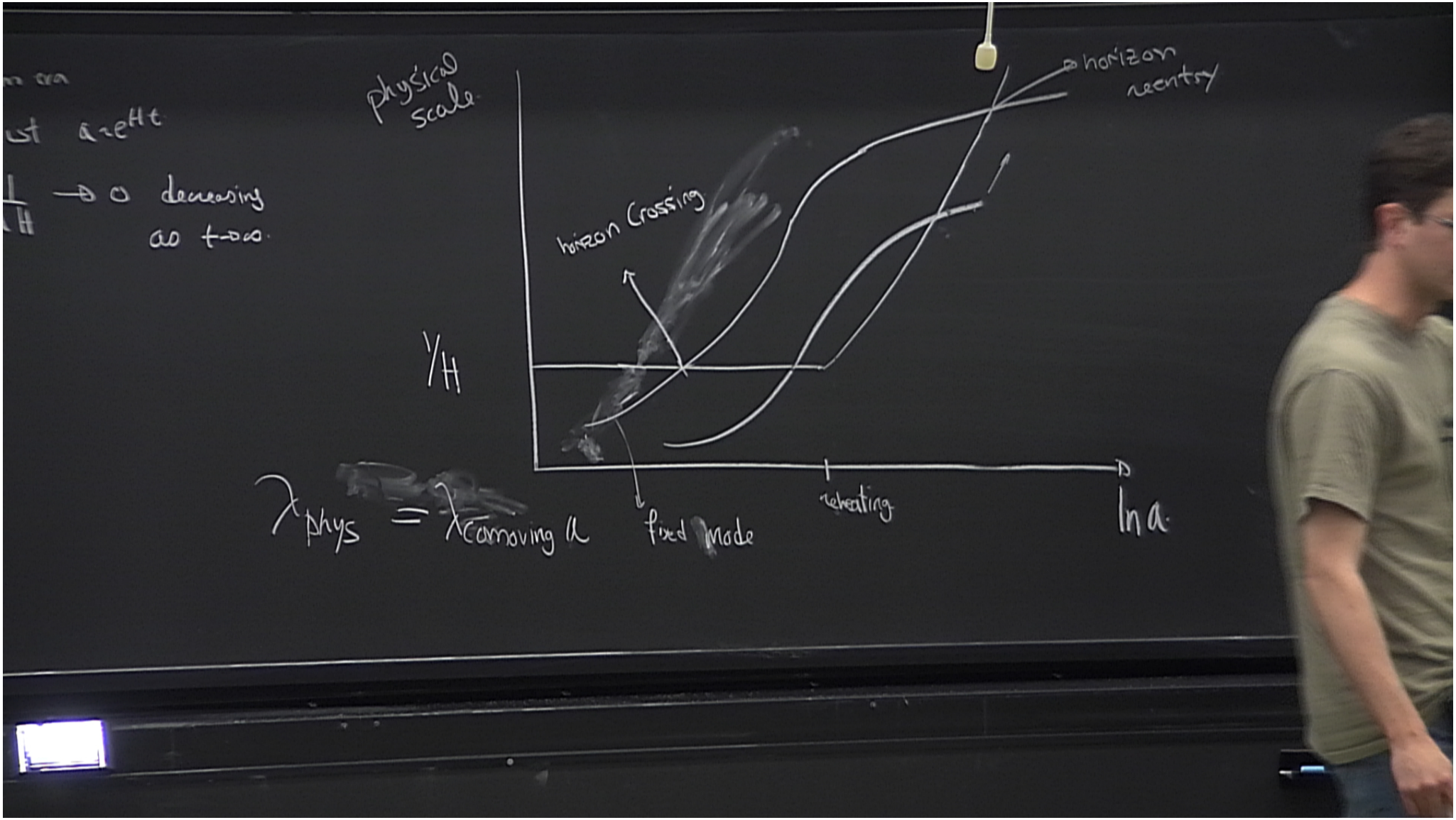

inflation era
Hubble radius
$\frac{1}{aH}$ → 0 decreasing as t→∞
physical scale
1/H
comoving k
reheating
ln a

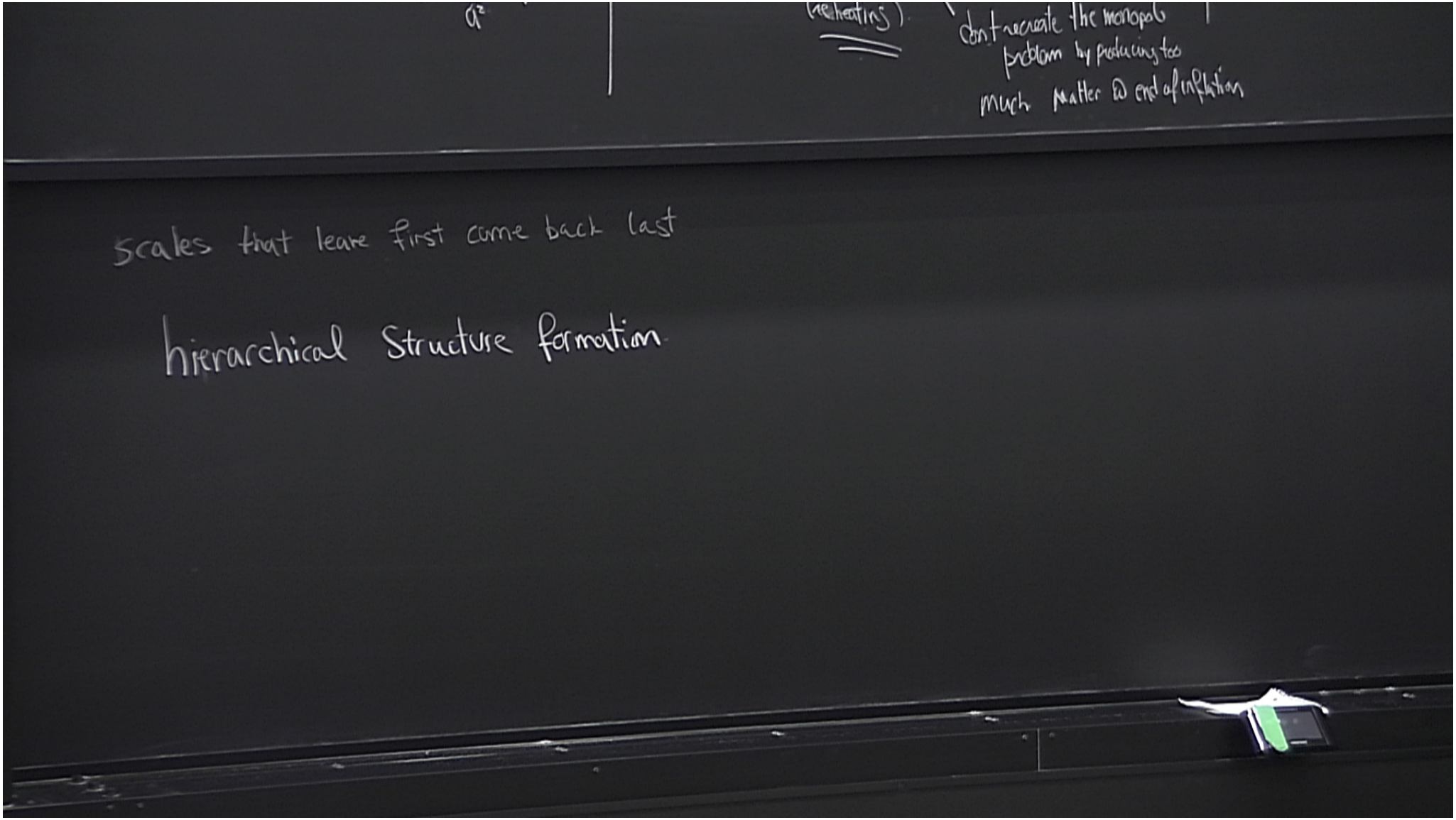

inflation era
H ≈ const a ∝ e^Ht
1/aH → 0 decreasing
as t→∞
physical scale
horizon crossing
horizon reentry
fixed mode
reheating
ln a

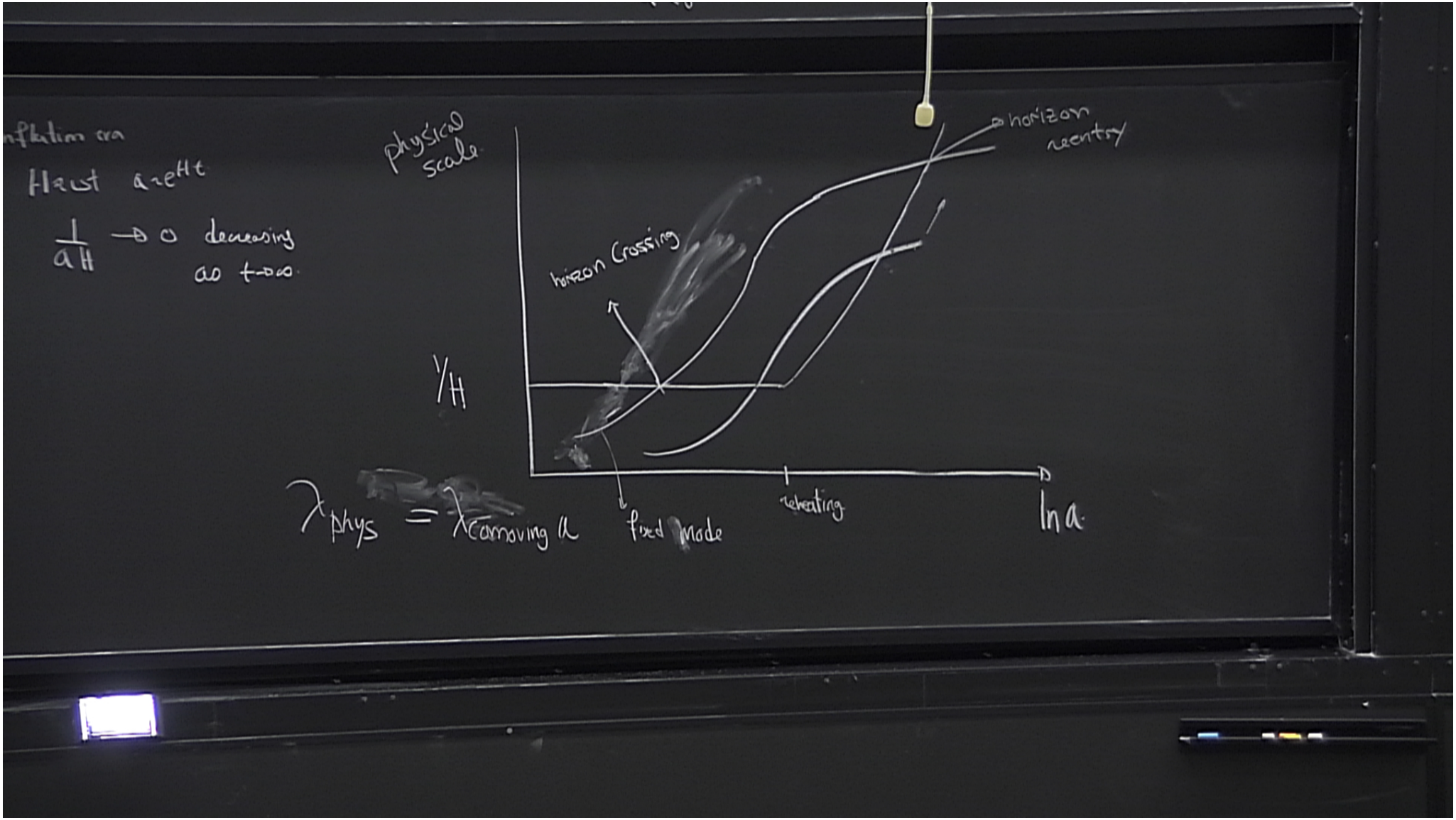

1/aH : comoving Hubble radius
inflation
1/aH not caused
no caused contact
caused contact
caused
1/aH
CMB
rad or matter
ln a (time)
horizon entry.
rad or matter
a ~ t^1/2 or 2/3
H ~ 1/t
1/aH ~ ↑ as t → 0 → +∞
SBB → a → 0
a ~ t^1/2
~ t^2/3
inflation era
Hard a ~ e^Ht
1/aH → 0 dec
as

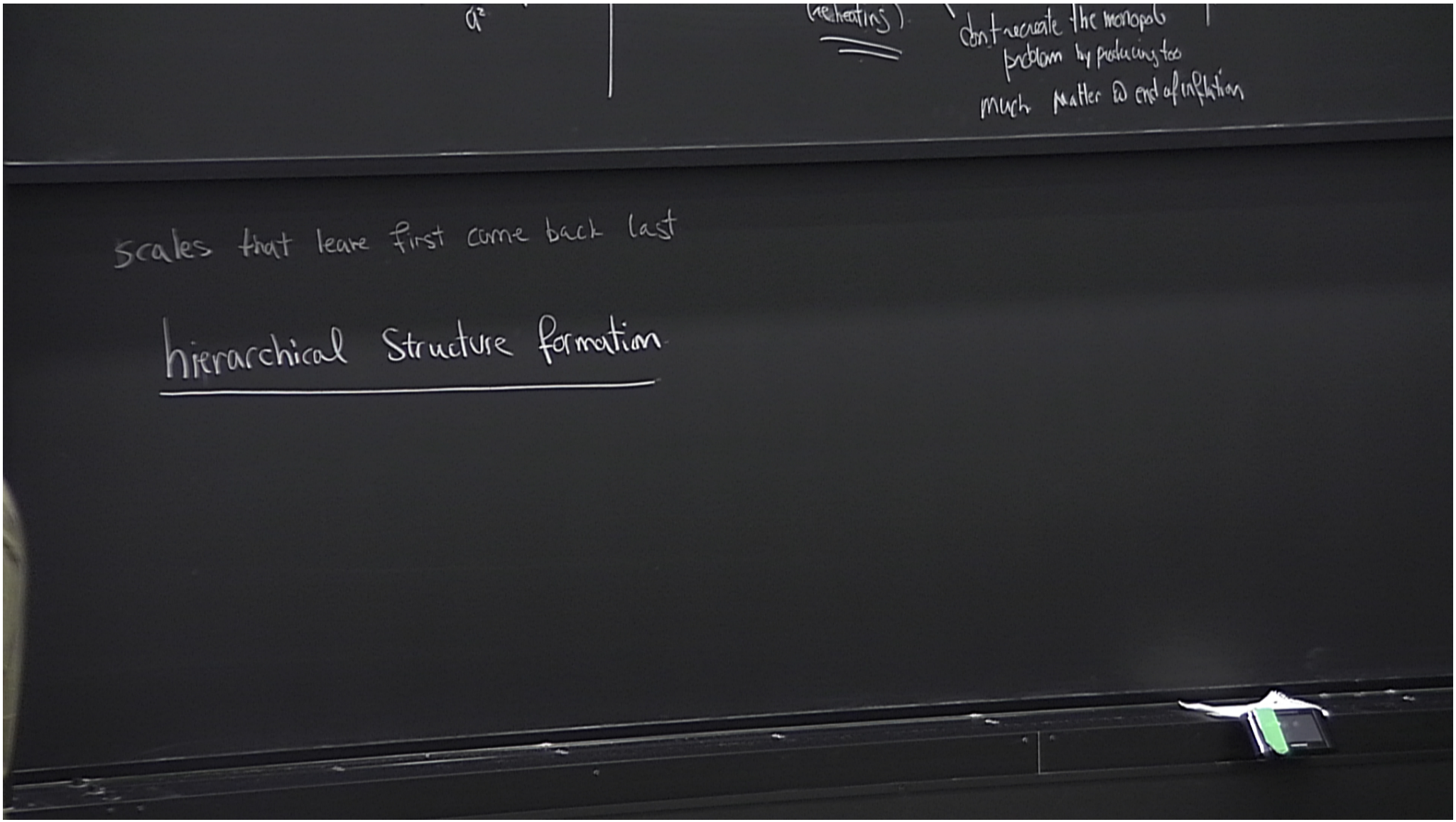

physical scale
horizon reentry
horizon Crossing
1/H
reheating
fixed mode
ln a

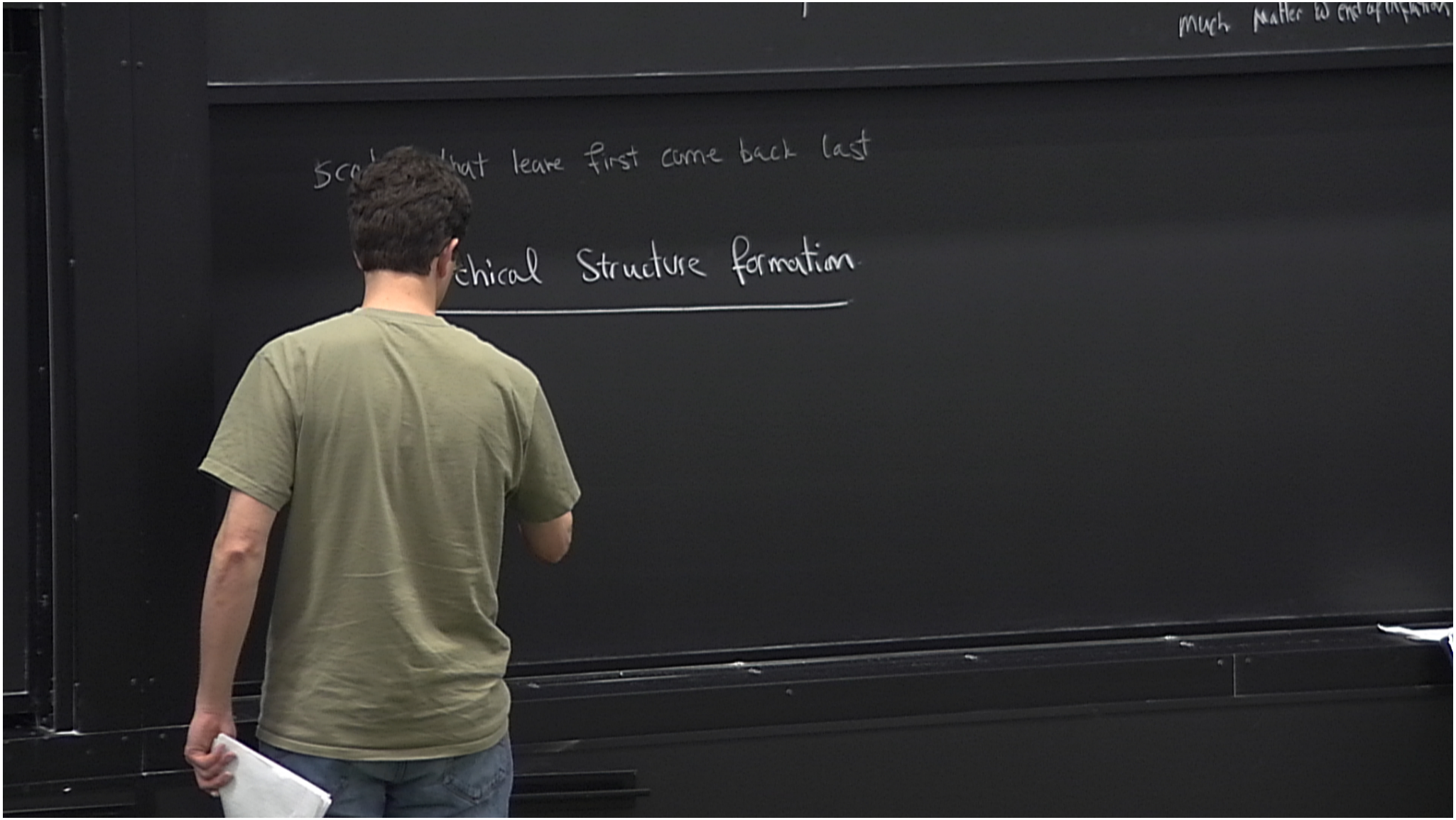

Recap
dS space
$m=0;\ u_k = \frac{H}{k^{3/2}}(1-ik\eta)e^{ik\eta}$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}$
$-k\eta \ll 1$
$m\neq 0;\ u_k = \sqrt{\frac{-\pi\eta}{2}}\frac{1}{a(\eta)}H_\nu^{(1)}(-k\eta)$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}(-k\eta)^{m^2/3H^2}$
w/ $\nu = \sqrt{\frac{9}{4}-\frac{m^2}{H^2}} \approx \frac{3}{2}-\frac{m^2}{3H^2}$

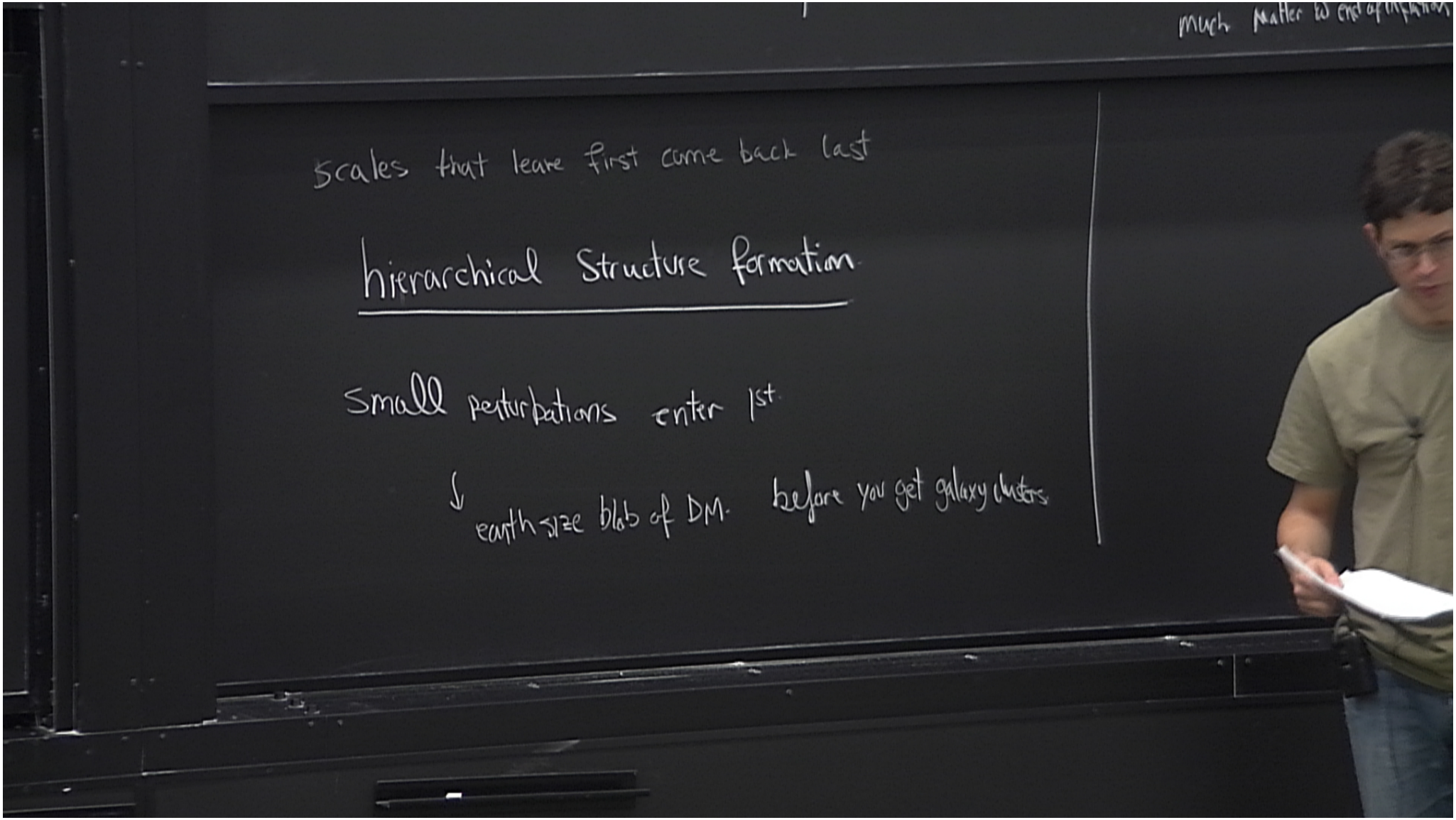

physical scale
horizon reentry
horizon Crossing
1/H
reheating
ln a
fixed mode

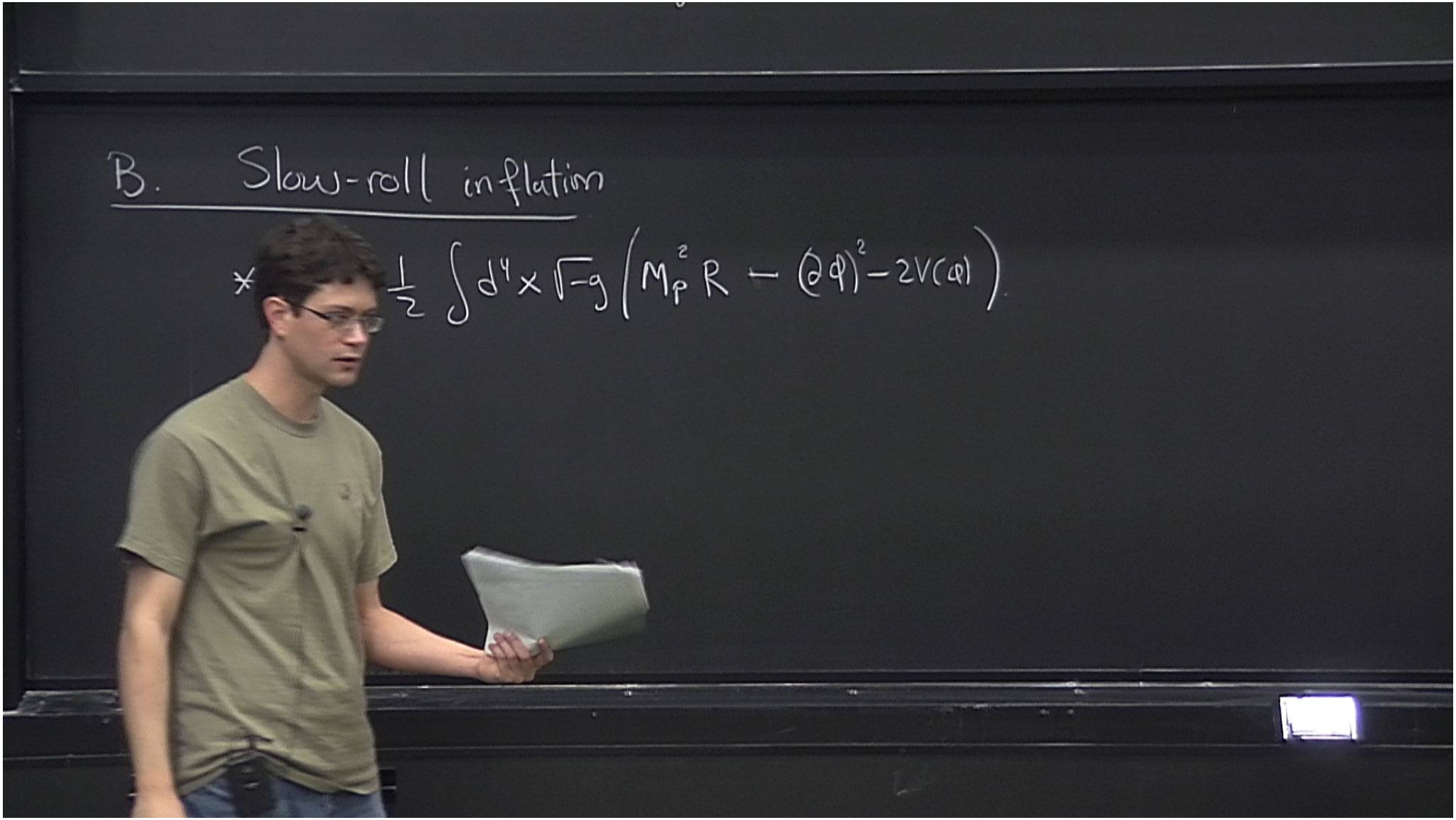

(reheating)
don't recreate the monopole problem by producing too much matter @ end of inflation
scales that leave first come back last
hierarchical structure formation


inflation era
H ≈ const a ≈ e^Ht
1/aH → 0 decreasing as t → ∞
physical scale
horizon Crossing
horizon reentry
1/H
λ_phys = λ_comoving a
fixed mode
reheating
ln a


(reheating)
don't recreate the monopole problem by producing too much matter @ end of inflation
scales that leave first come back last
hierarchical Structure formation

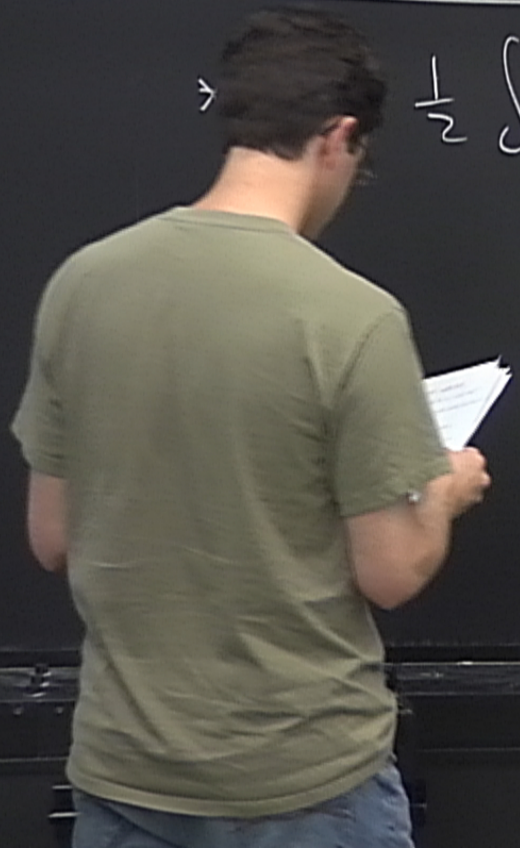

much
leave first come back last
Structure formation

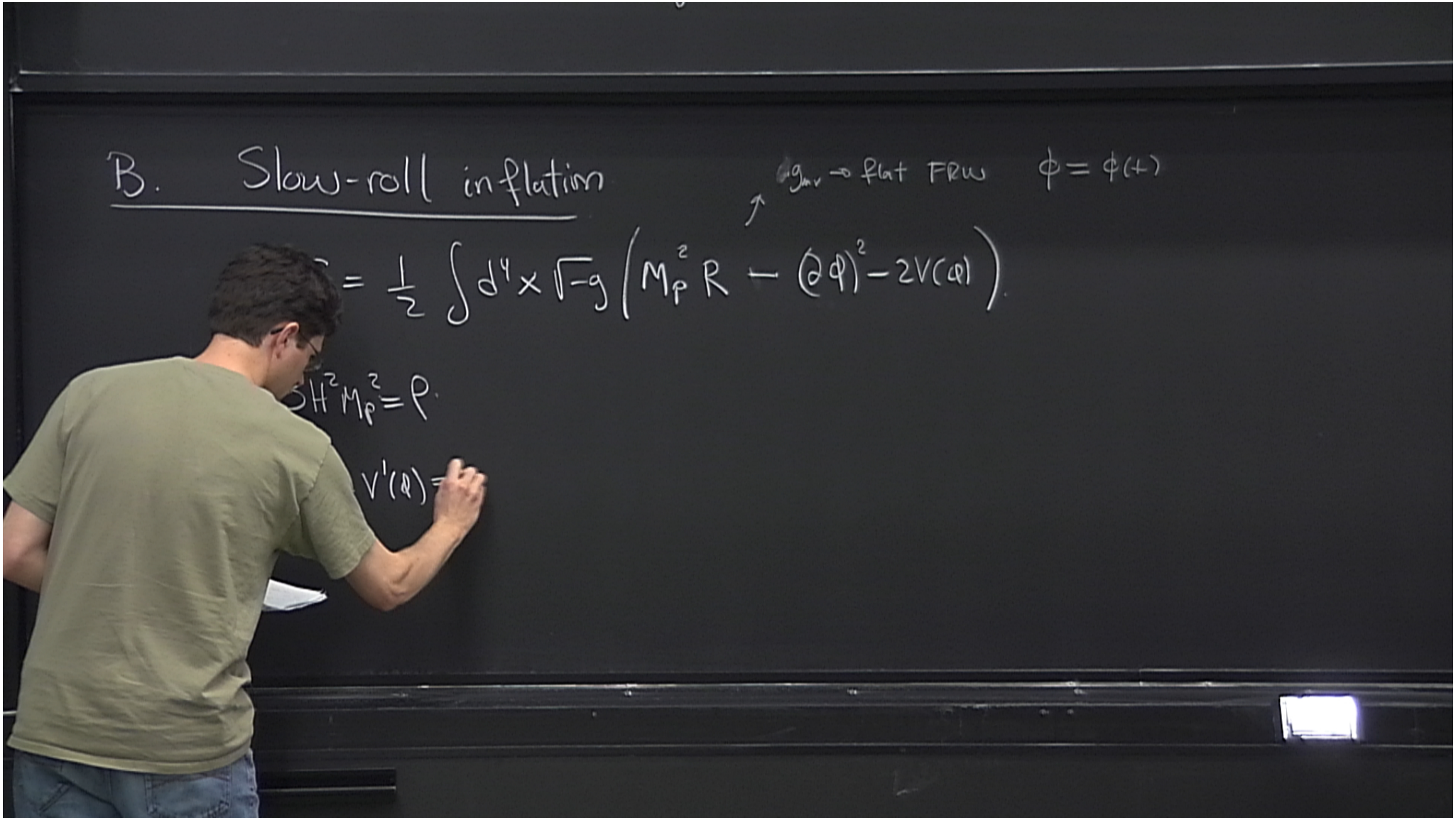

much matter
scales that leave first come back last
hierarchical Structure formation
small perturbations enter 1st.
earth size blob of DM. before you get galaxy clusters

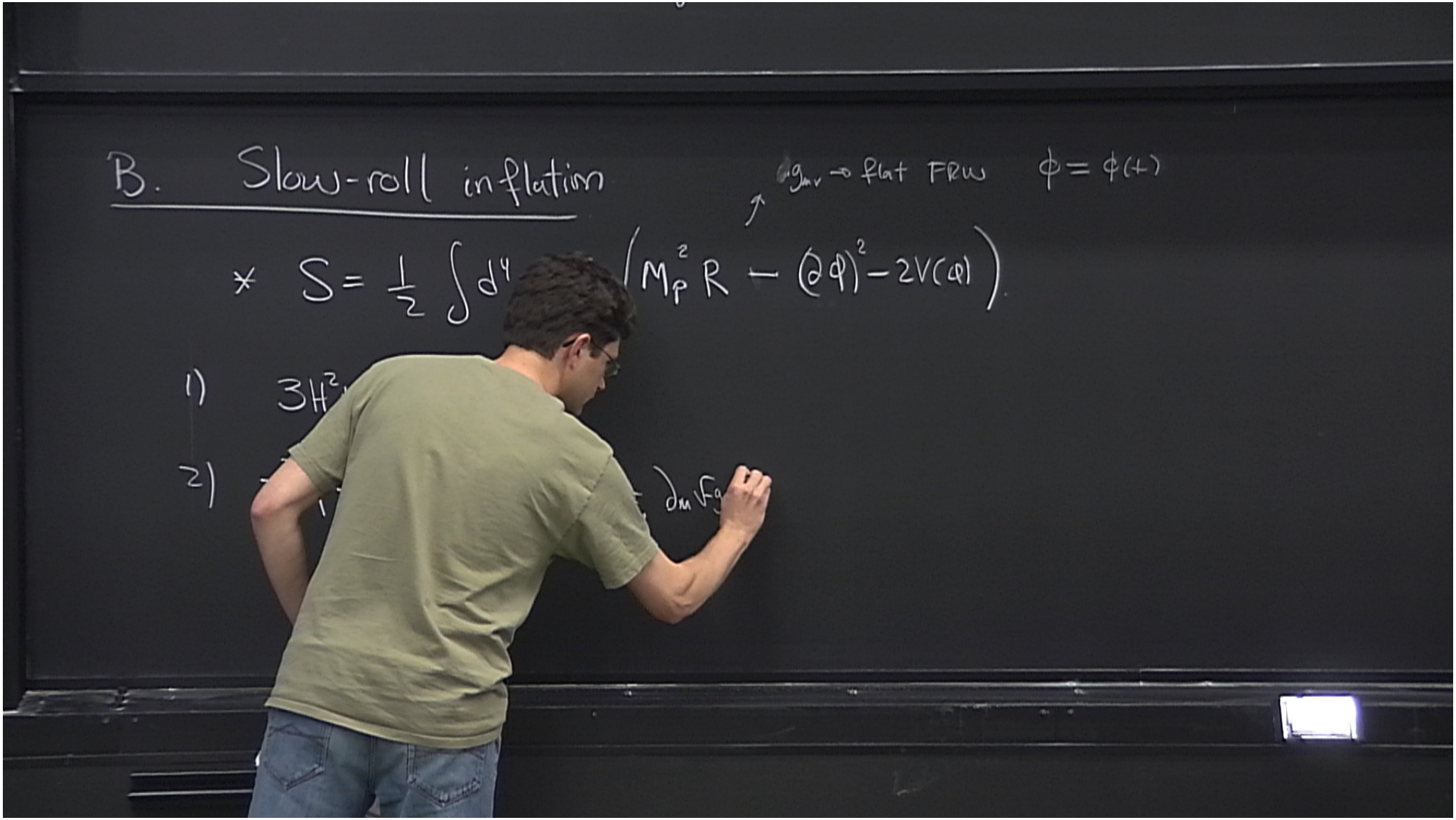

B. Slow-roll inflation
$* \quad \frac{1}{2}\int d^4x\sqrt{-g}\left(M_P^2 R - (\partial\varphi)^2 - 2V(\varphi)\right)$

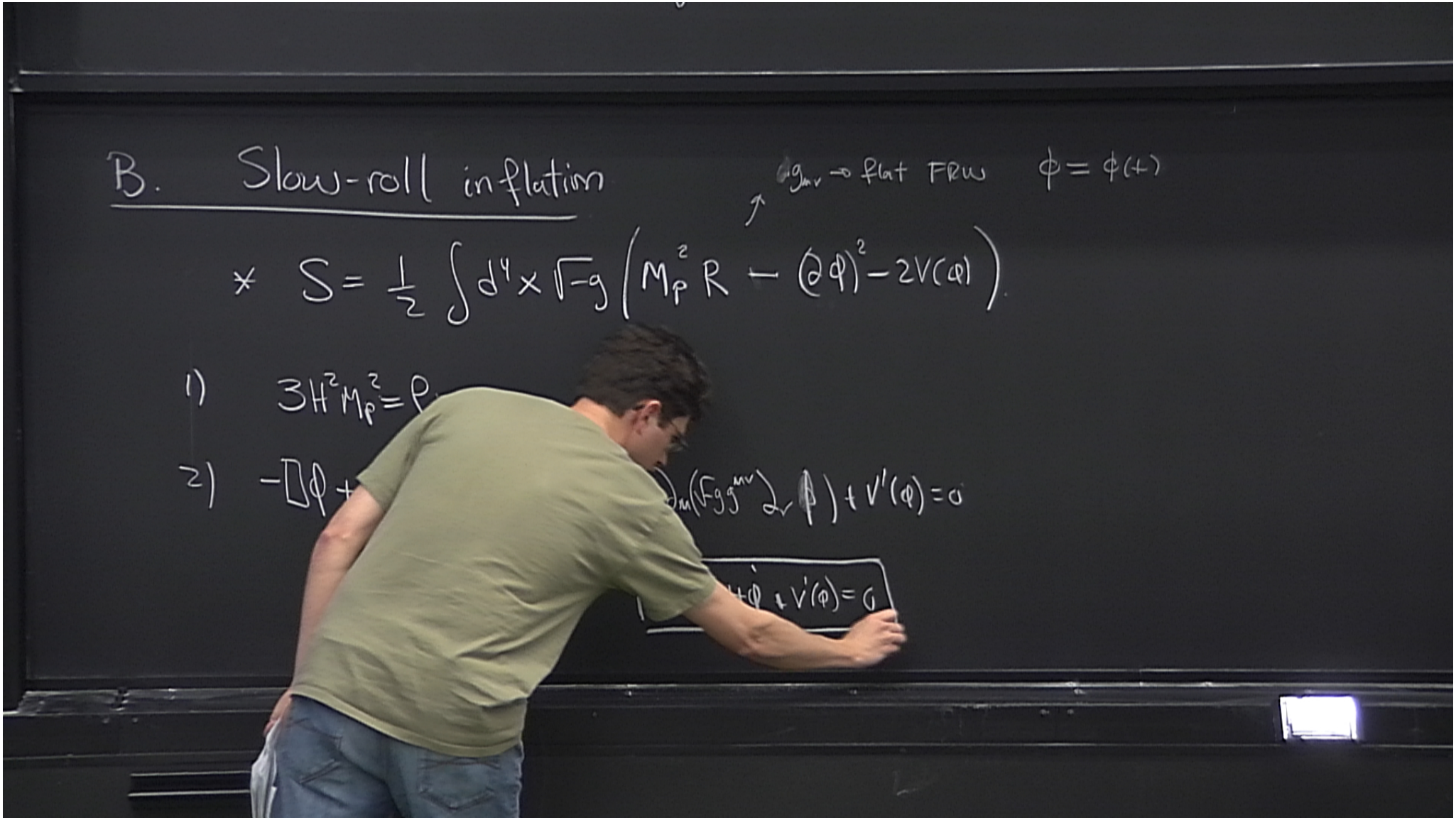
B. Slow-roll inflation

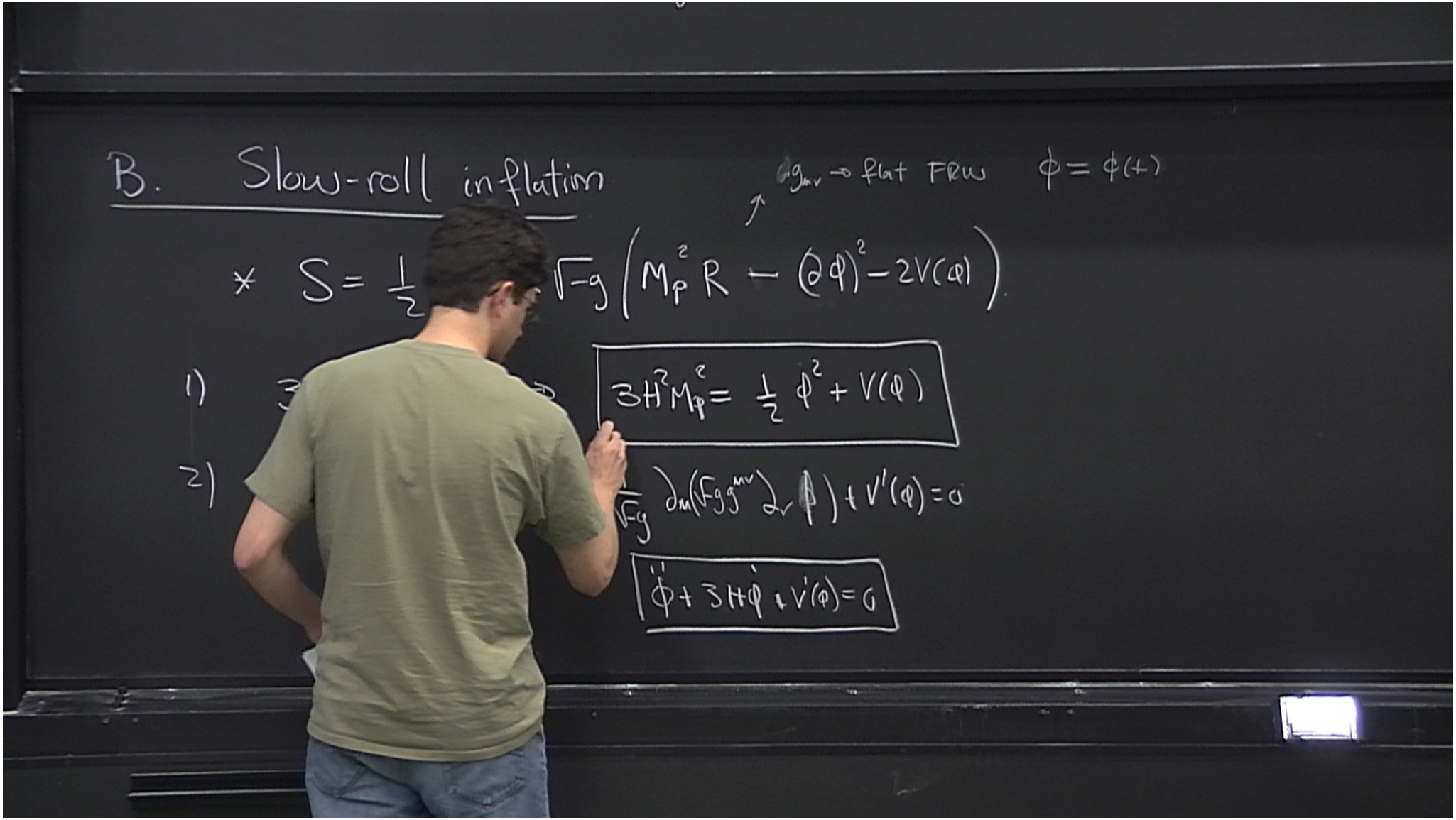
$$\Rightarrow \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2 R - (\partial\varphi)^2 - 2V(\varphi)\right)$$

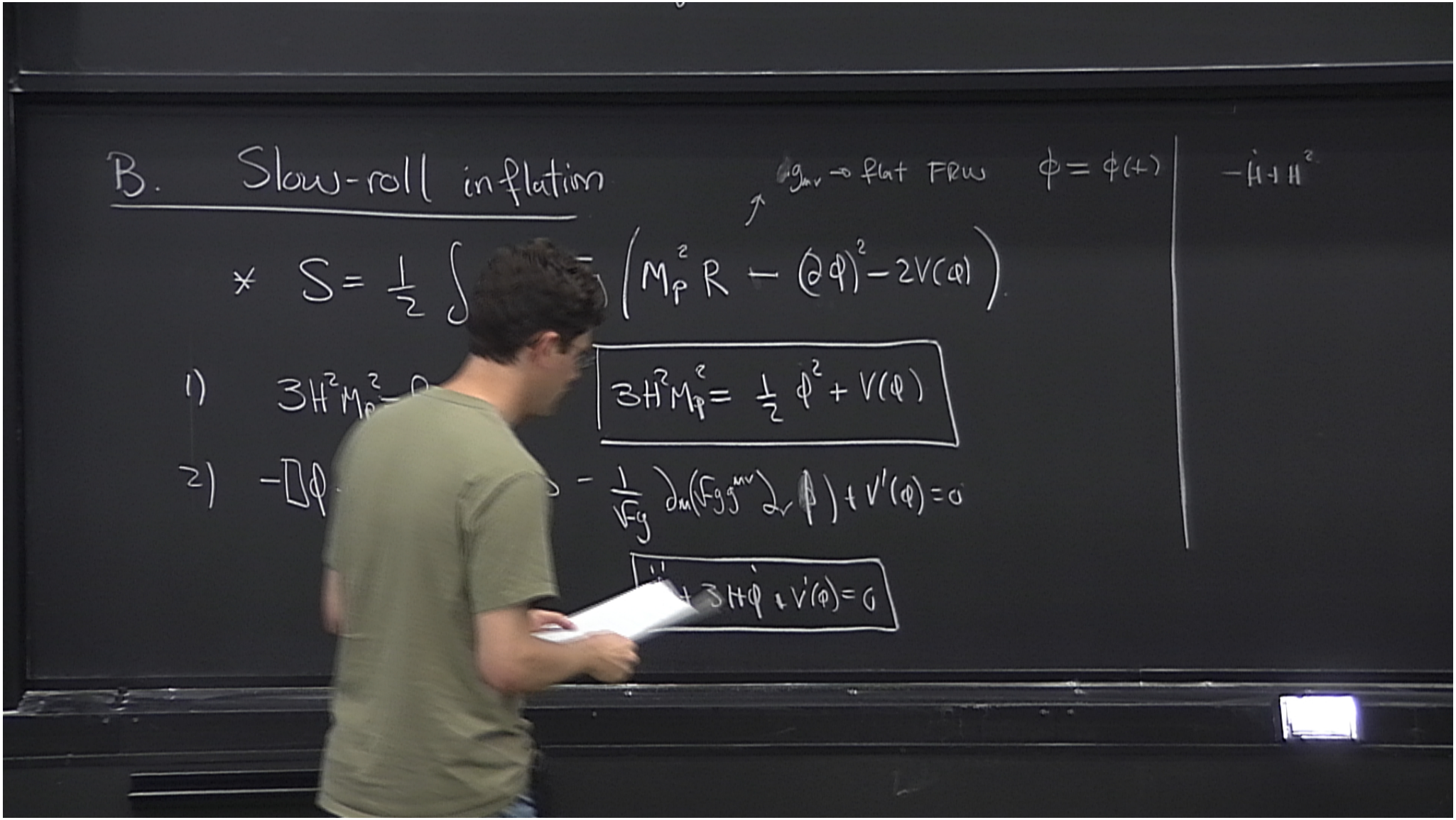


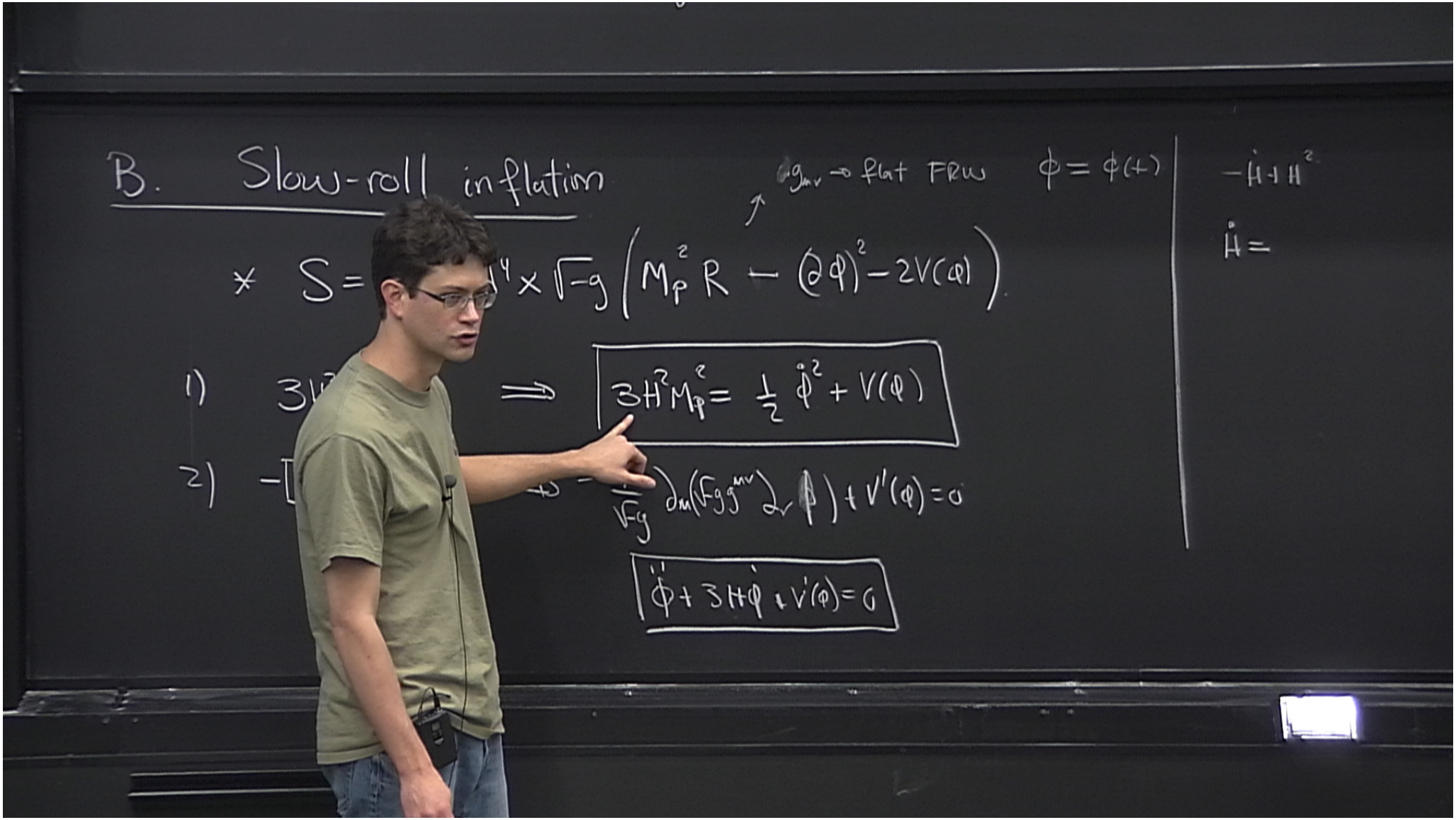

B. Slow-roll inflation
$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$= \frac{1}{2}\int d^4x\sqrt{-g}\left(M_p^2 R - (\partial\phi)^2 - 2V(\phi)\right)$
$H^2 M_p^2 = \rho$
$V'(\phi) =$

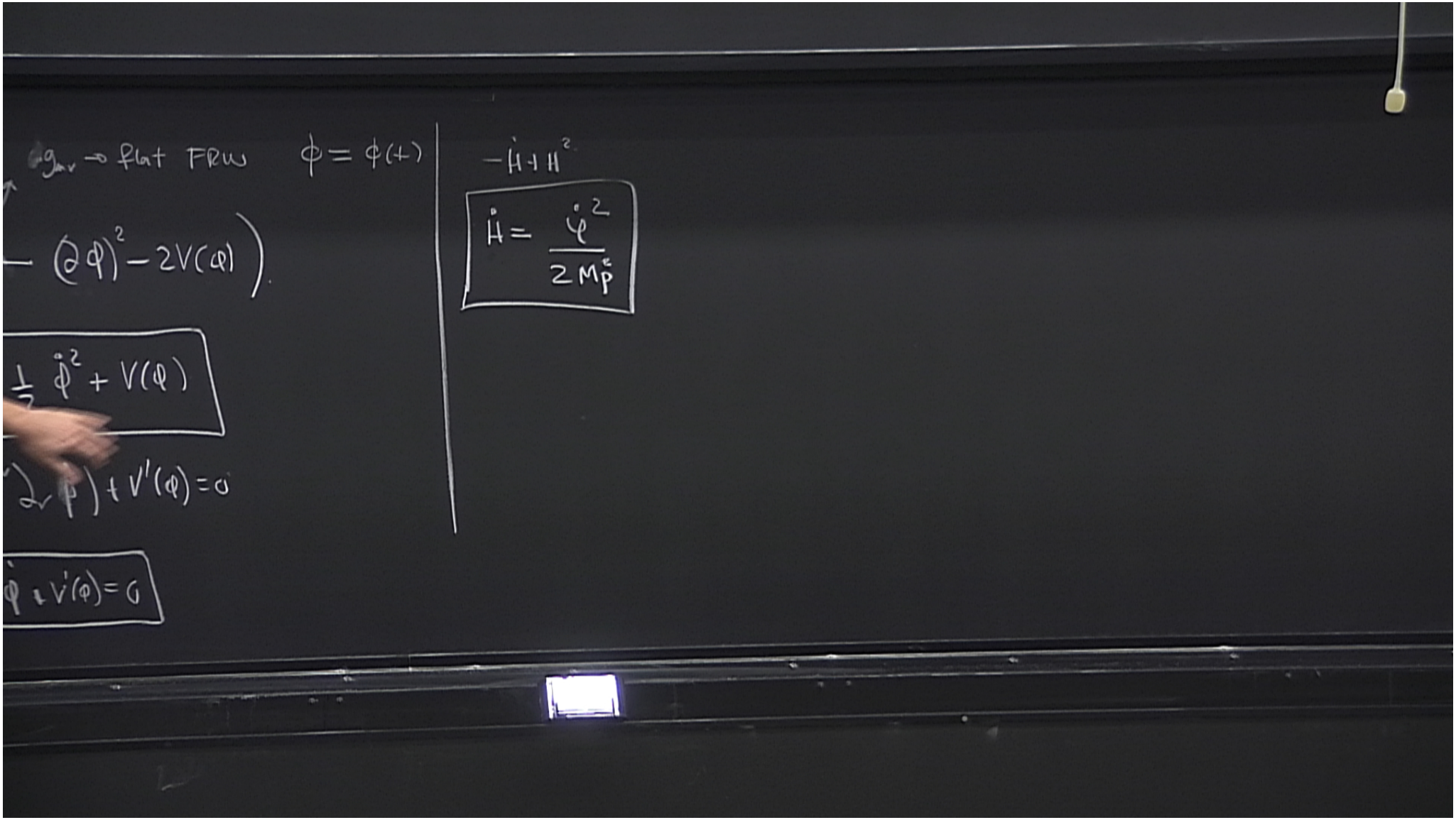

B. Slow-roll inflation
$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
* $S = \frac{1}{2}\int d^4$ $\left(M_P^2 R - (\partial\phi)^2 - 2V(\phi)\right)$
1) $3H^2$
2)

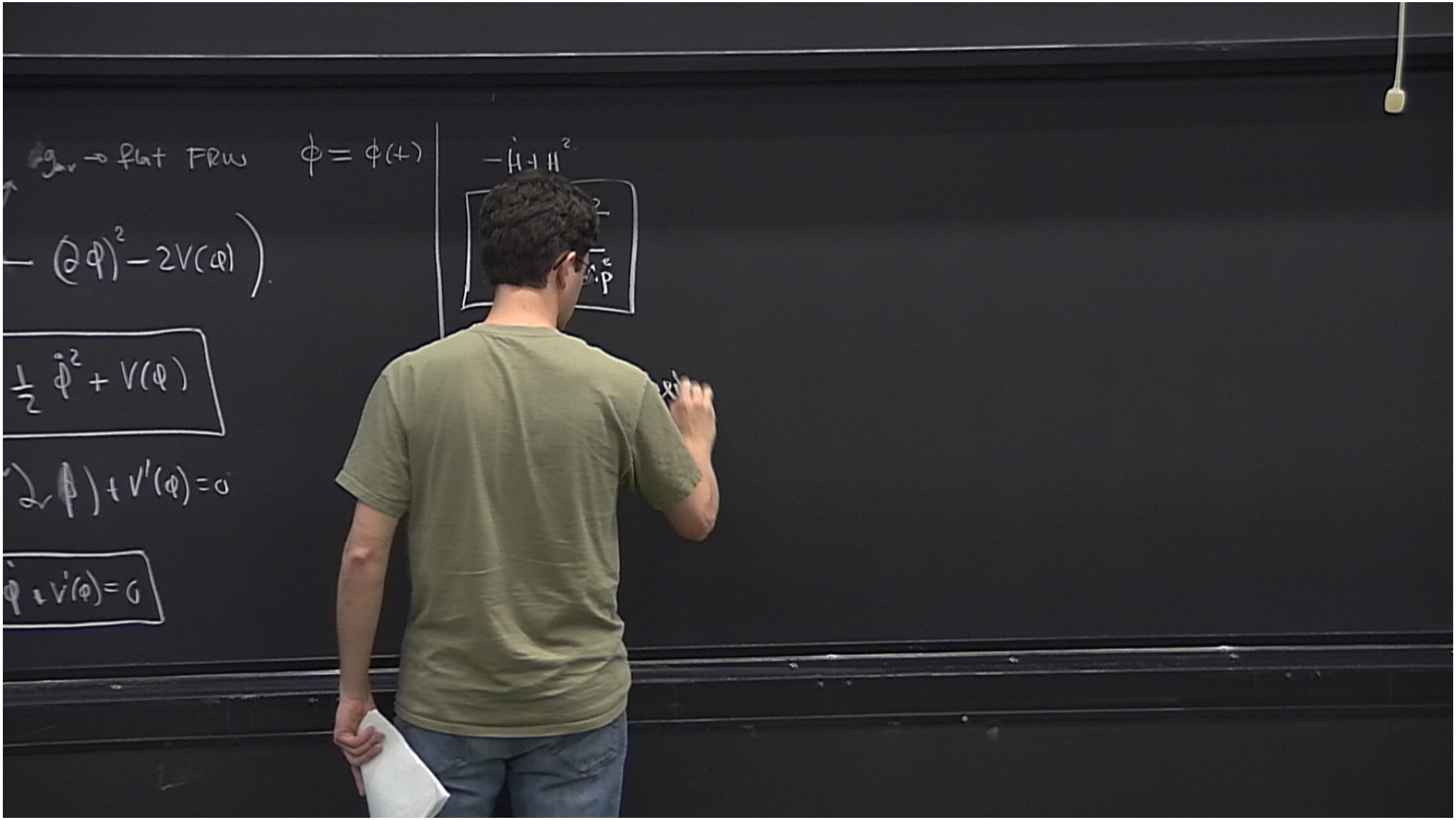

B. Slow-roll inflation
$g_{\mu\nu} \to$ flat FRW
$\phi = \phi(t)$
$* \; S = \frac{1}{2} \int d^4x \sqrt{-g} \left(M_p^2 R - (\partial\phi)^2 - 2V(\phi) \right)$
1) $3H^2 M_p^2 = \rho$
2) $-\Box\phi +$
$\partial_\mu(\sqrt{-g} g^{\mu\nu} 2\nu \phi) + V'(\phi) = 0$
$\ddot{\phi} + V'(\phi) = 0$


B. Slow-roll inflation
$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$* \; S = \frac{1}{2} \int \sqrt{-g} \left(M_p^2 R - (\partial\phi)^2 - 2V(\phi) \right)$
1)
$3H^2 M_p^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
2)
$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + V'(\phi) = 0$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$


B. Slow-roll inflation
$g_{\mu\nu} \to$ flat FRW
$\phi = \phi(t)$
$-\dot{H} + H^2$
$S = \frac{1}{2} \int \left(M_p^2 R - (\partial\phi)^2 - 2V(\phi) \right)$
1) $3H^2 M_p^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
2) $-\Box\phi - \frac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + V'(\phi) = 0$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$


B. Slow-roll inflation
$g_{\mu\nu} \to$ flat FRW
$\phi = \phi(t)$
$-\dot{H} + H^2$
$\dot{H} =$
$* \; S = \int d^4x \sqrt{-g} \left(M_p^2 R - (\partial\phi)^2 - 2V(\phi) \right)$
1) $3H^2 \Rightarrow$
$3H^2 M_p^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$
2) $-[$ $+ \frac{1}{\sqrt{-g}} \partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) + V'(\phi) = 0$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$


$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$-\dot{H}+H^2$
$\dot{H} = \frac{\dot{\varphi}^2}{2M_p^2}$
$-(\partial\varphi)^2 - 2V(\varphi)\Big)$
$\frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$\varphi) + V'(\varphi) = 0$
$\ddot{\varphi} + V'(\varphi) = 0$


flat FRW
$\phi = \phi(t)$
$-\dot{H} + H^2$
$(\partial\phi)^2 - 2V(\phi)$
$\frac{1}{2}\dot{\phi}^2 + V(\phi)$
$+ V'(\phi) = 0$
$\dot{\phi} + V'(\phi) = 0$

$g_{\mu\nu}$ → flat FRW $\phi = \phi(t)$

$-(\partial\phi)^2 - 2V(\phi)\Big)$

$\frac{1}{2}\dot{\phi}^2 + V(\phi)$

$\ddot{\phi} + V'(\phi) = 0$

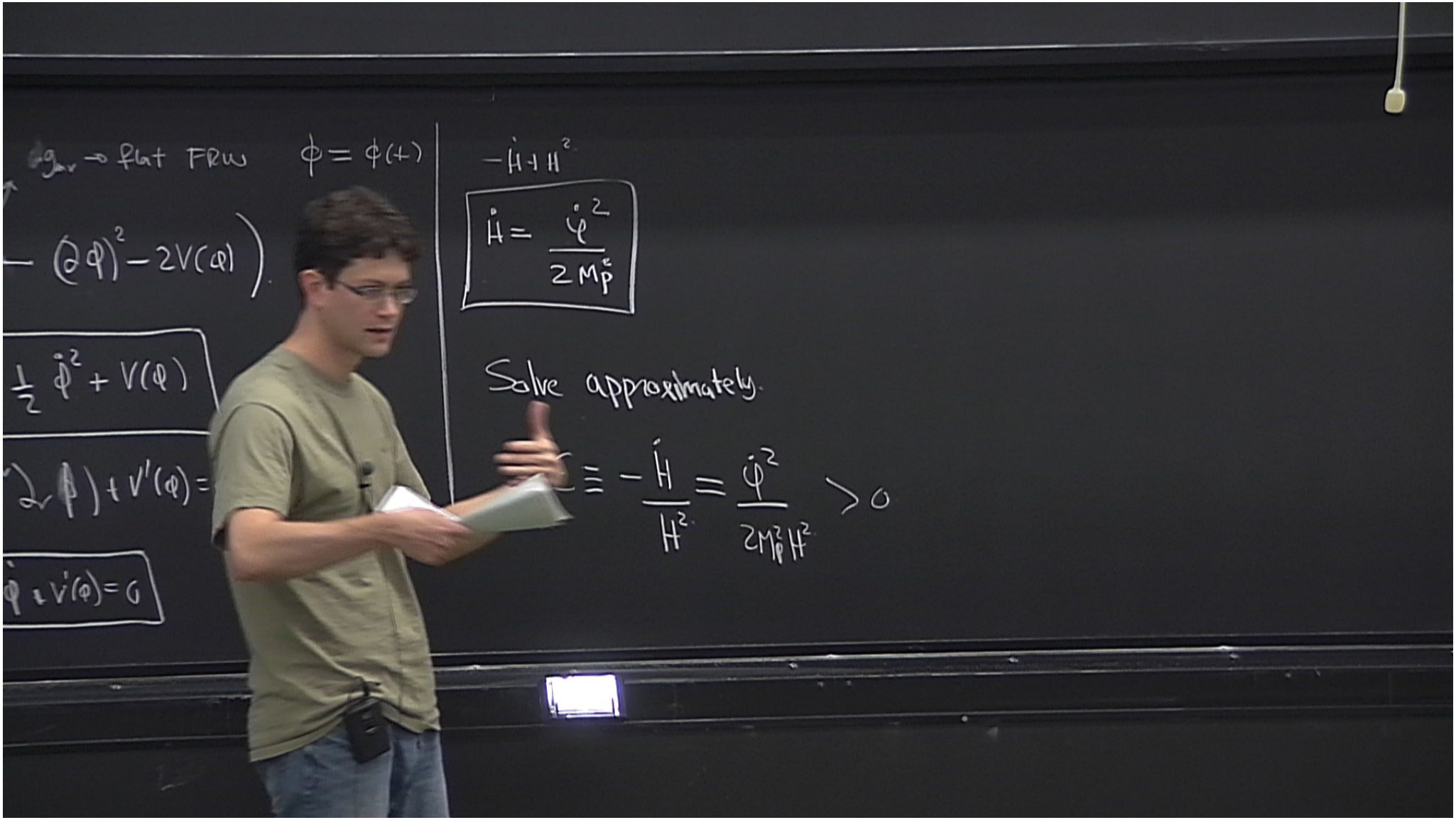
$\dot{\phi}\ddot{\phi} + V'(\phi)\dot{\phi} = 0$

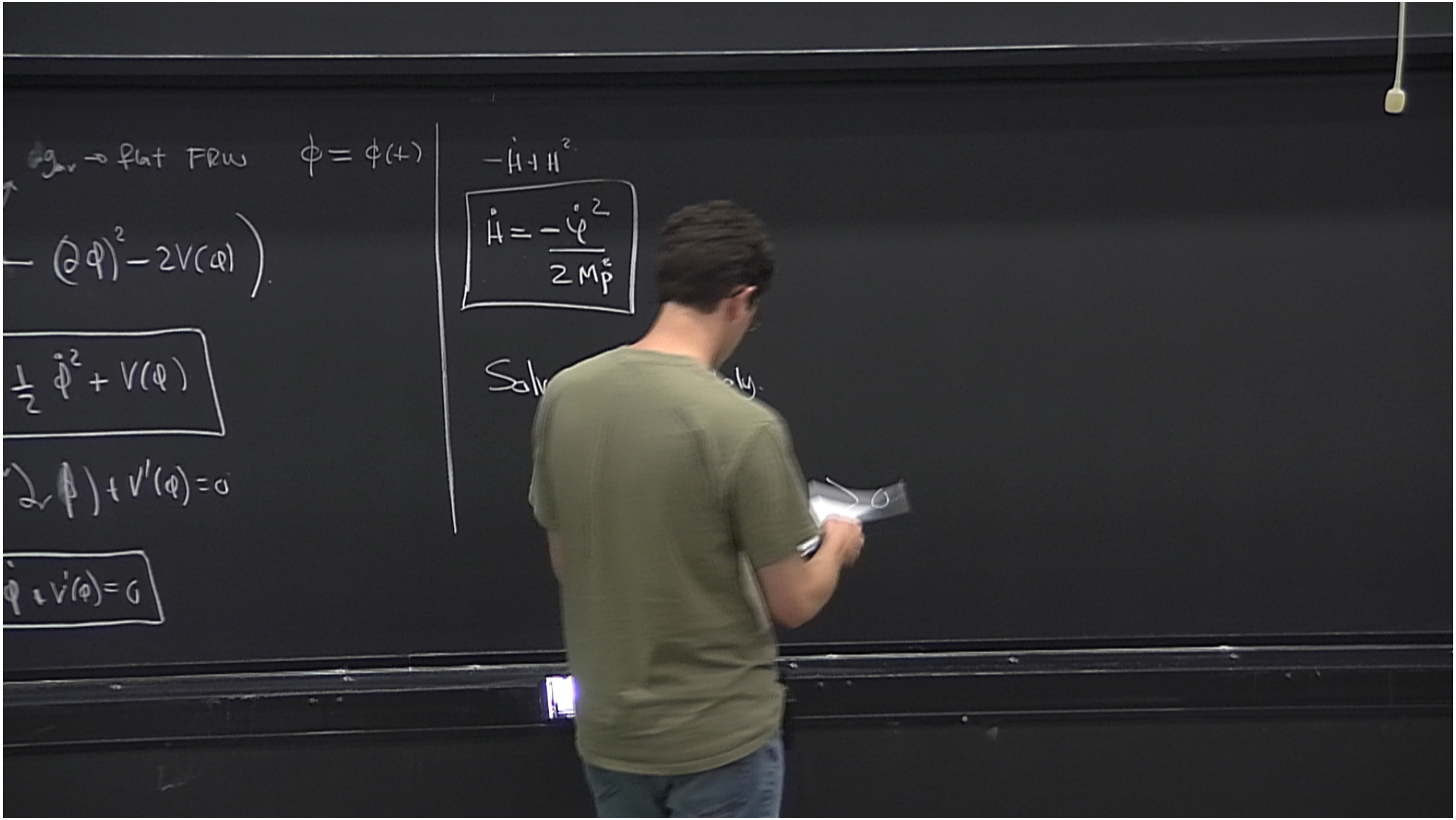
$-\dot{H} + H^2$

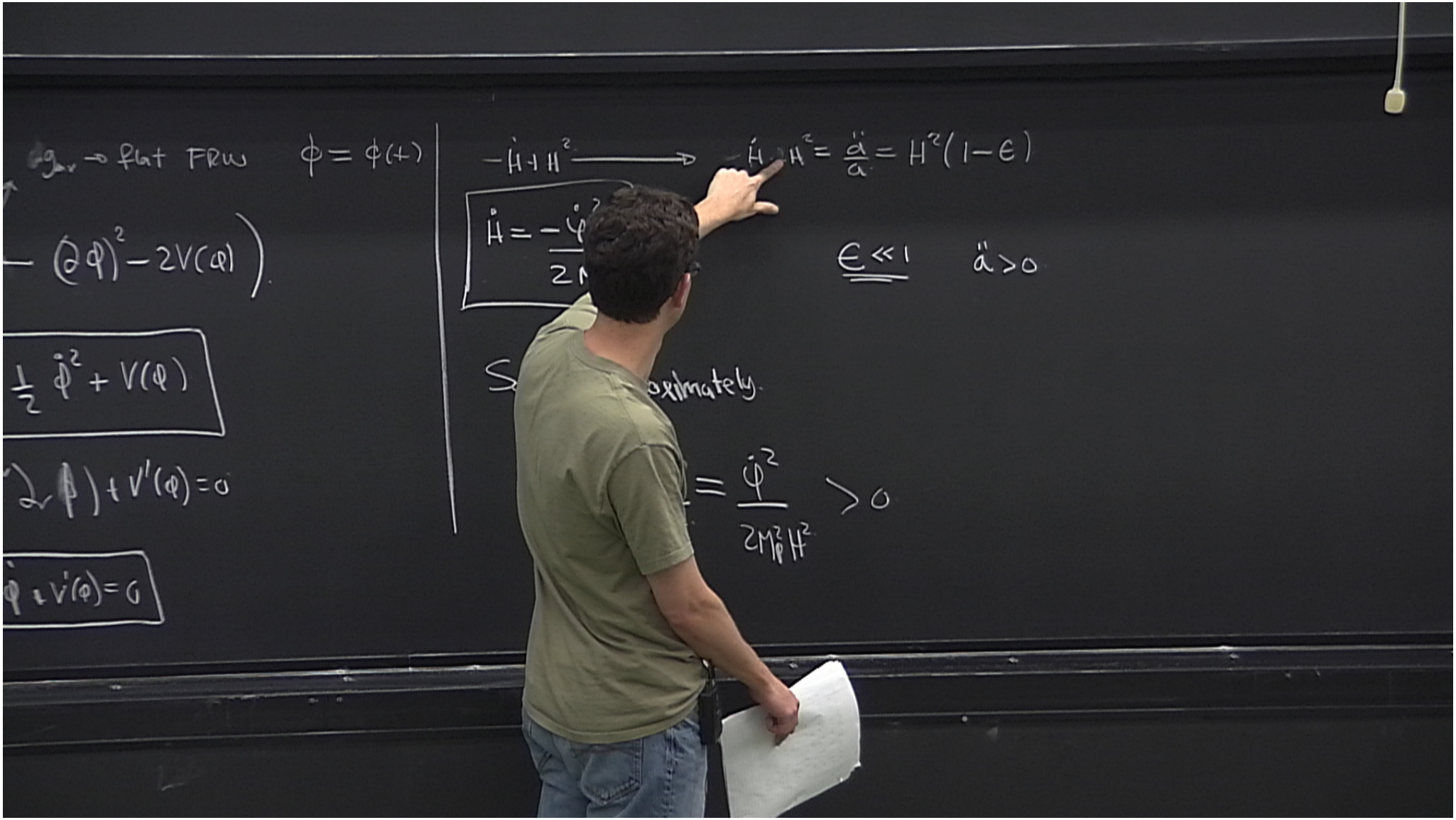
$$\dot{H} = \frac{\dot{\varphi}^2}{2M_p^2}$$

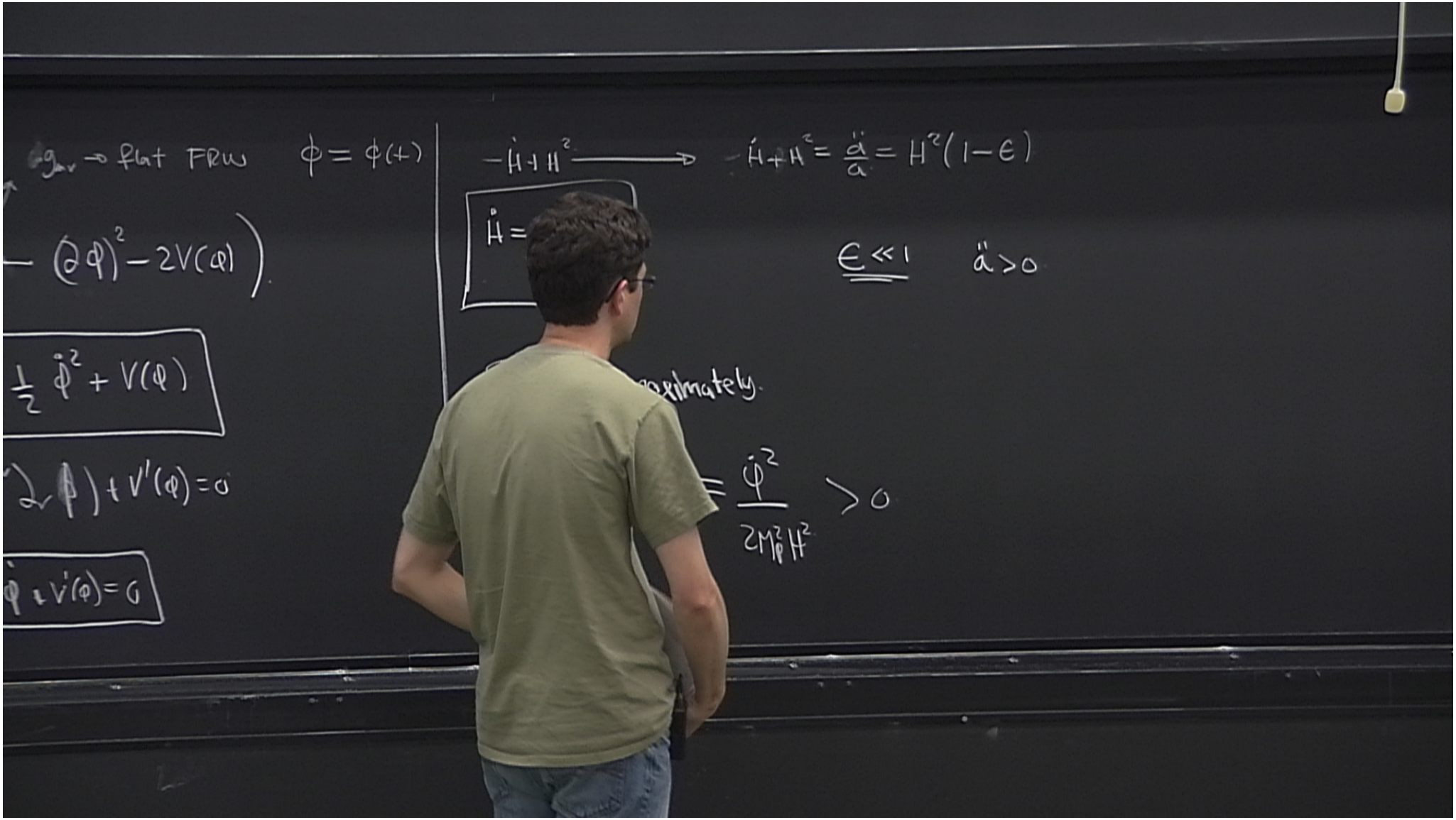
Solve approximately.

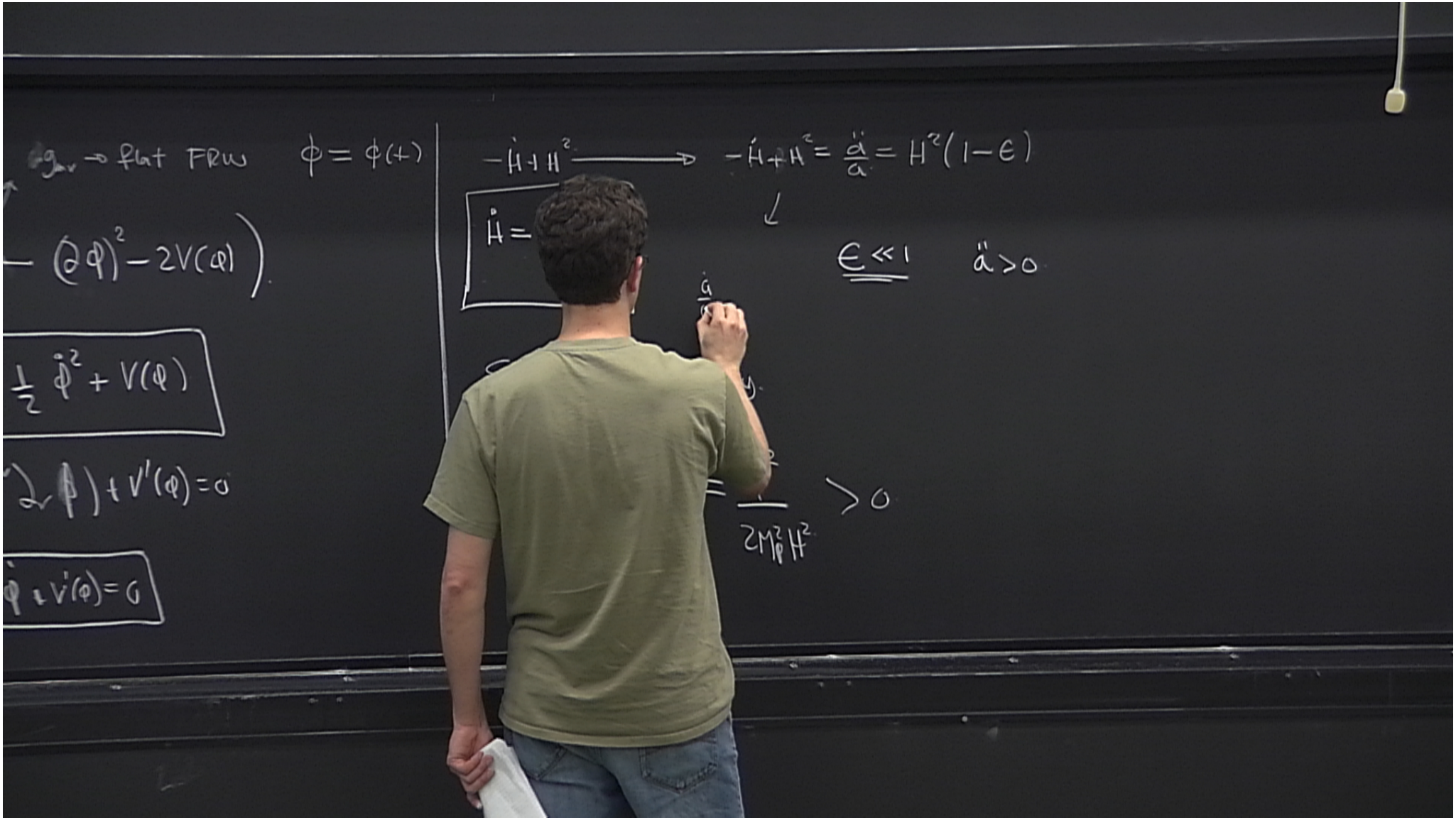
$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$$

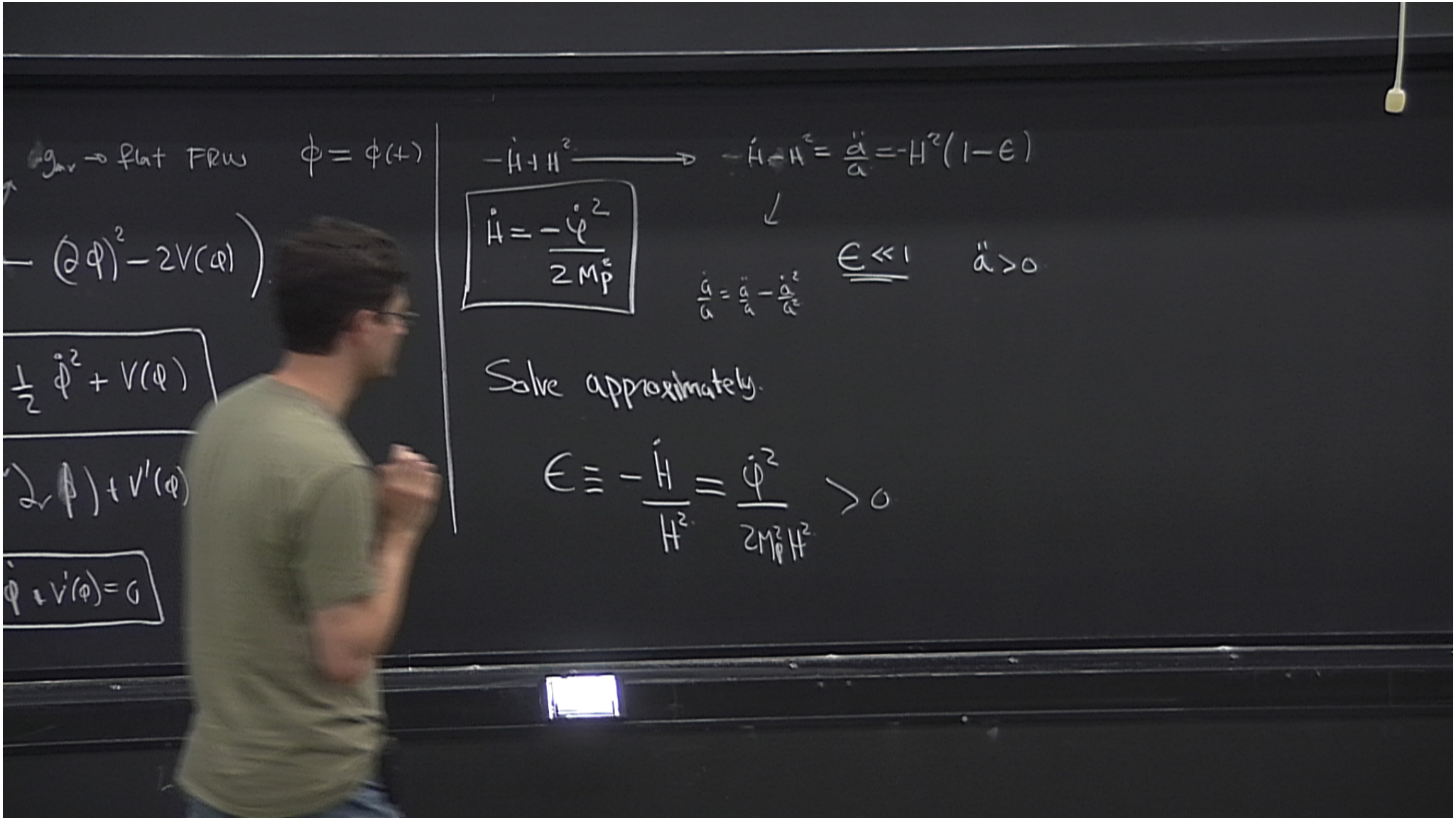

$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$-\dot{H} + H^2$
$\dot{H} = \frac{\dot{\varphi}^2}{2M_p^2}$
$(\partial\varphi)^2 - 2V(\varphi)$
$\frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$+ V'(\varphi) =$
$\dot{\varphi} + V'(\varphi) = 0$
Solve approximately.
$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$

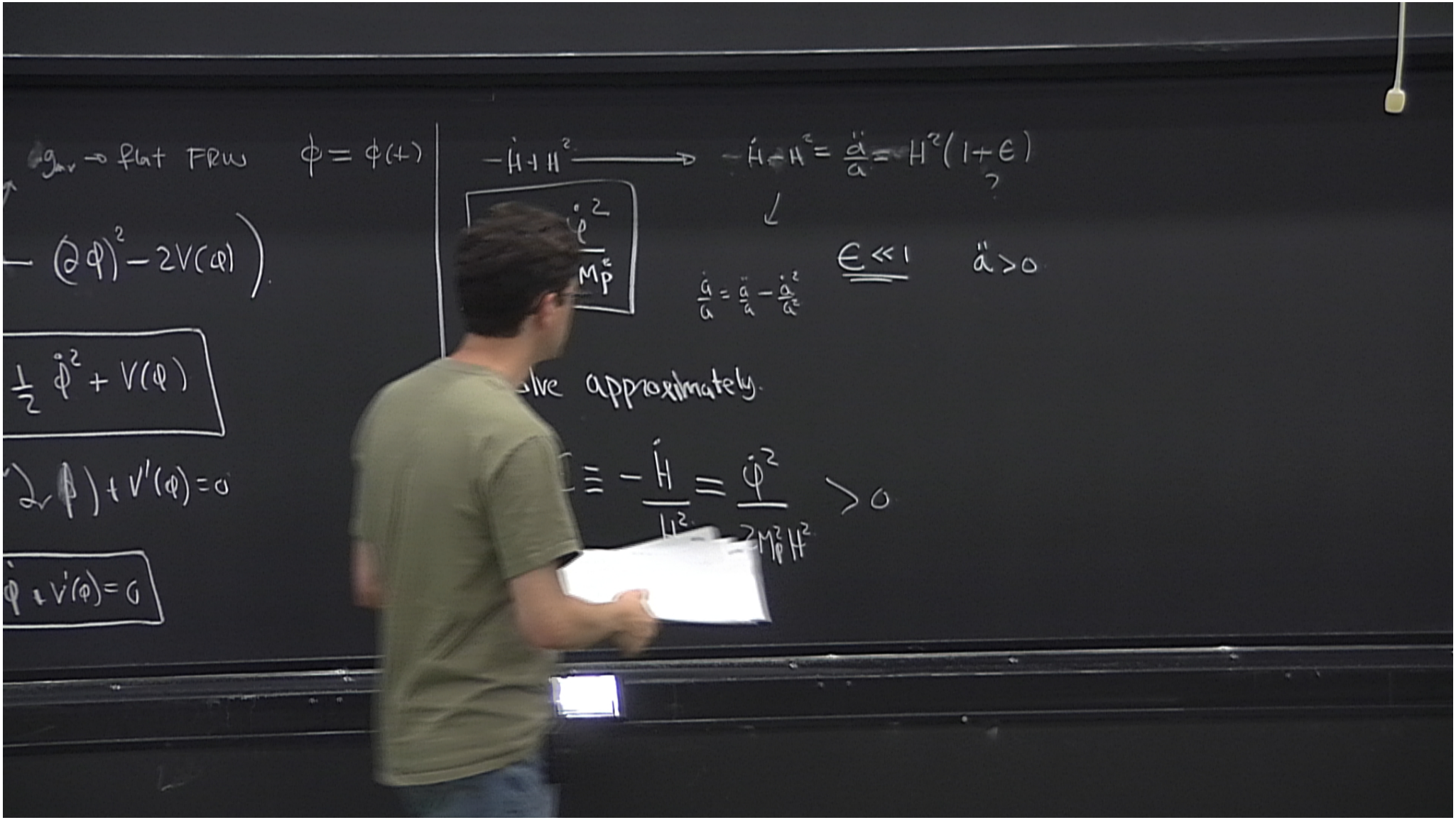

$g_{\mu\nu} \to$ flat FRW
$\phi = \phi(t)$
$-\dot{H} + H^2$
$\dot{H} = -\frac{\dot{\varphi}^2}{2M_p^2}$
$\frac{1}{2}\dot{\phi}^2 + V(\phi)$
$\ddot{\phi} + V'(\phi) = 0$
$\dot{\phi} + V'(\phi) = 0$

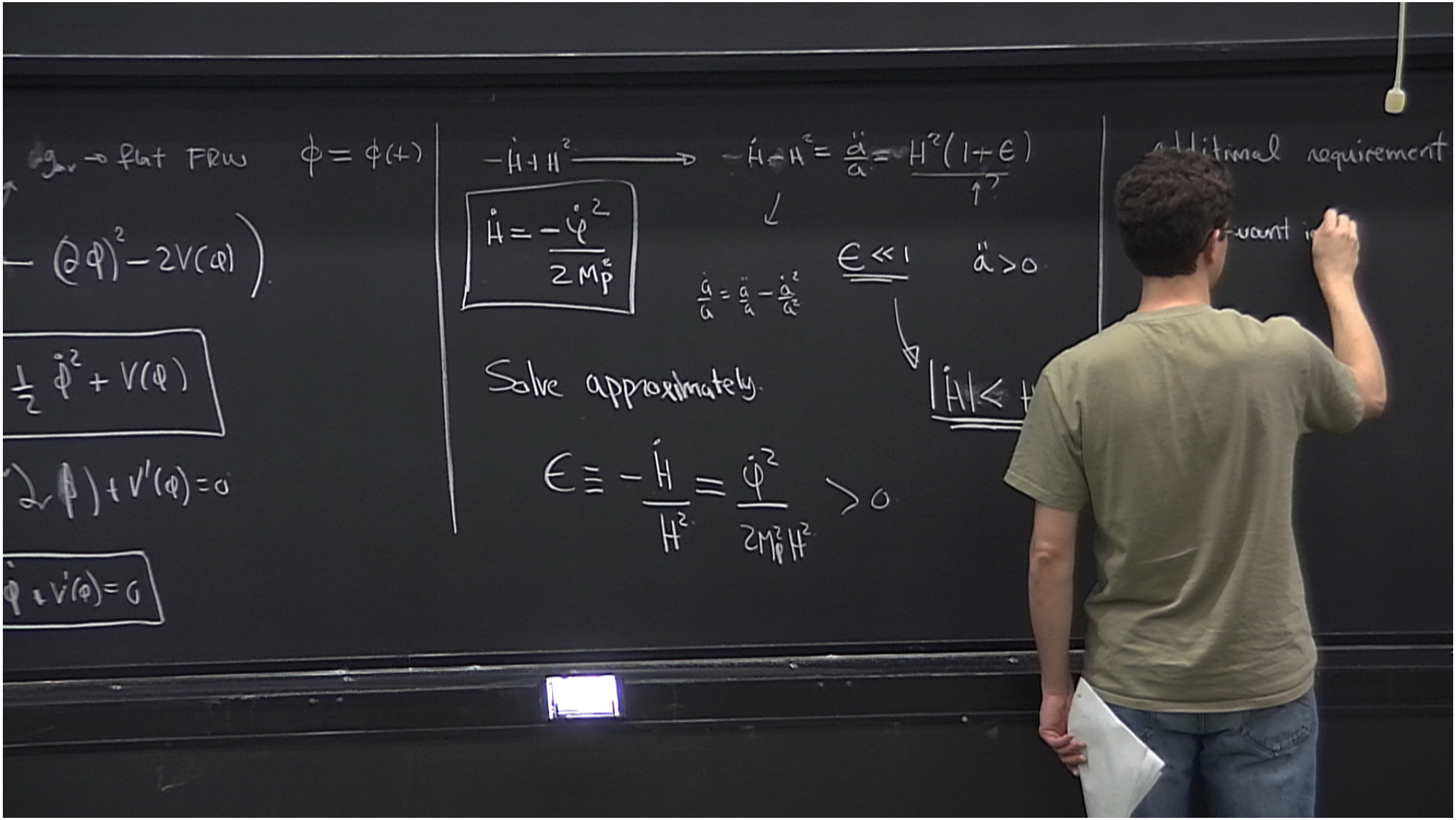

flat FRW
$\phi = \phi(t)$
$-\dot{H}+H^2 \longrightarrow \dot{H}+H^2 = \frac{\ddot{a}}{a} = H^2(1-\epsilon)$
$\epsilon \ll 1$
$\ddot{a} > 0$
$\frac{1}{2}\dot{\phi}^2 + V(\phi)$
$\ddot{\phi} + V'(\phi) = 0$
$\dot{\phi} + V'(\phi) = 0$
$= \frac{\dot{\phi}^2}{2M_p^2 H^2} > 0$

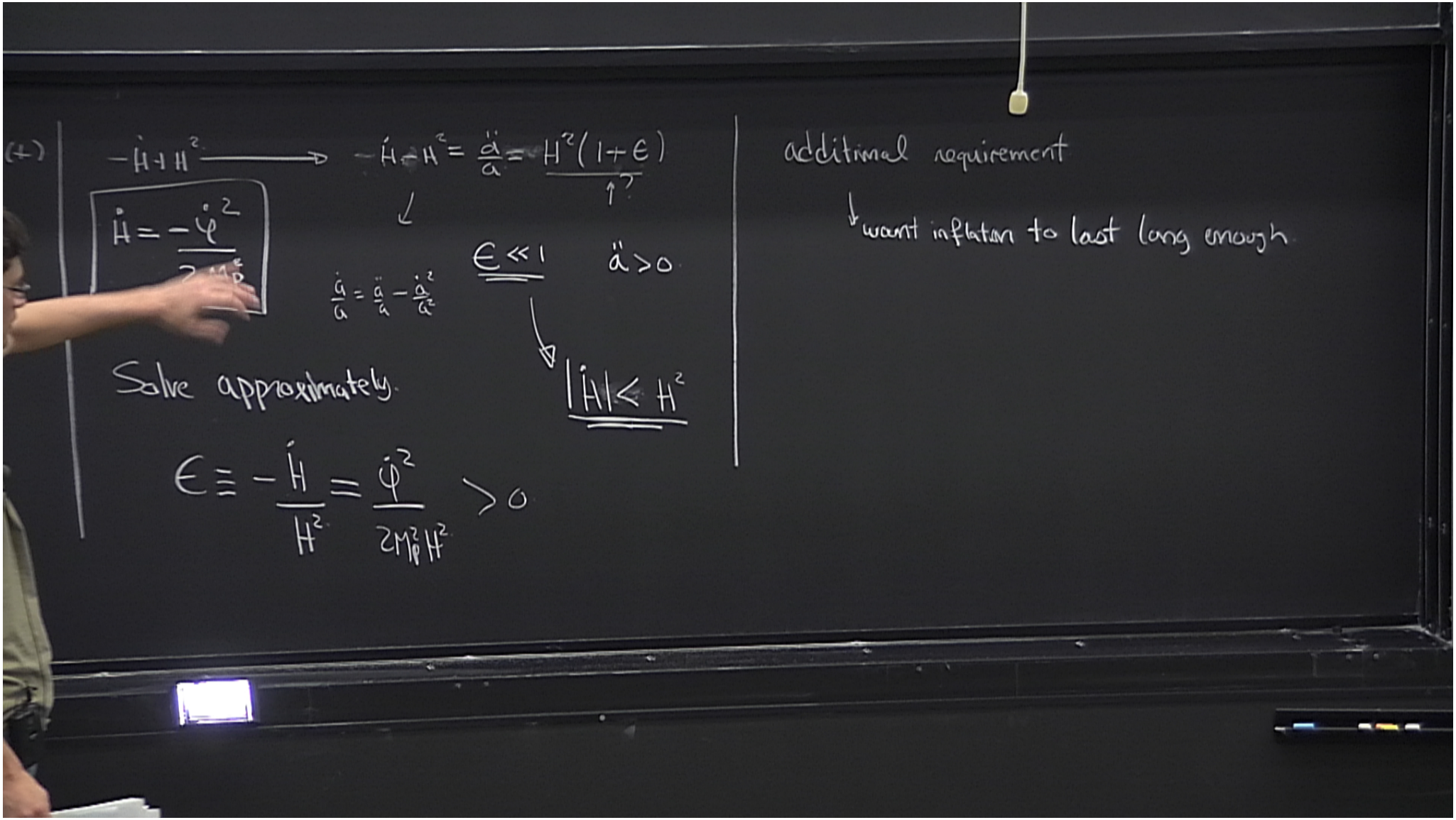

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$\equiv \frac{\dot{\phi}^2}{2M_p^2 H^2} > 0$

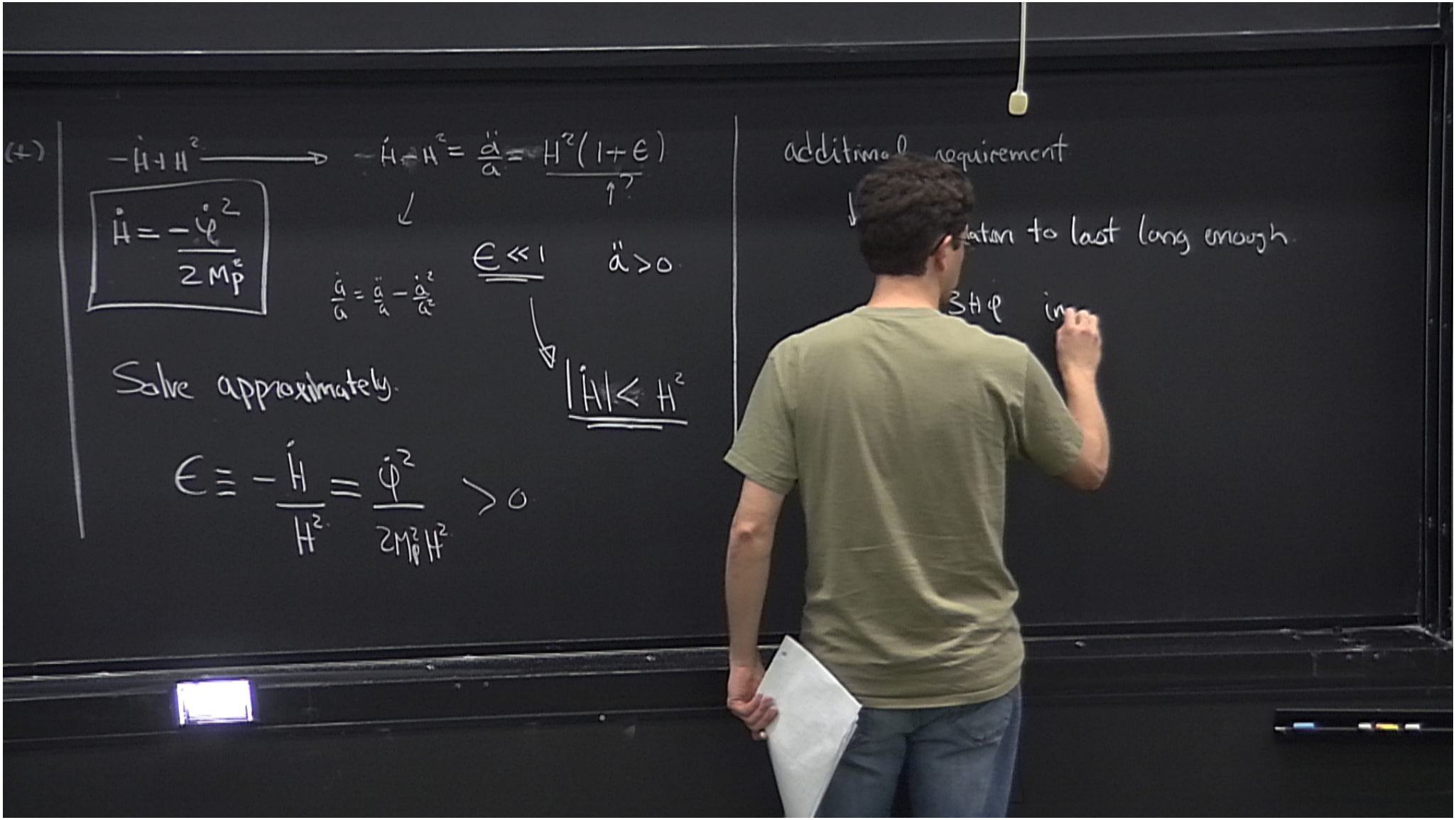

$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$-\dot{H}+H^2 \longrightarrow -\dot{H}+H^2 = \frac{\ddot{a}}{a} = H^2(1-\epsilon)$
$\epsilon \ll 1$
$\ddot{a} > 0$
$\frac{1}{2}\dot{\phi}^2 + V(\phi)$
$+V'(\phi)=0$
$\dot{\phi}+V'(\phi)=0$
> 0
$2M_p^2 H^2$


$g_{\mu\nu}$ → flat FRW $\phi = \phi(t)$
$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2 = \frac{\ddot{a}}{a} = -H^2(1-\epsilon)$
$\dot{H} = -\frac{\dot{\varphi}^2}{2M_p^2}$
$\frac{\ddot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$
$\epsilon \ll 1$ $\ddot{a} > 0$
$-(\partial\varphi)^2 - 2V(\varphi)$
$\frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$+V'(\varphi)$
$\dot{\varphi} + V'(\varphi) = 0$
Solve approximately.
$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$


$g_{\mu\nu}$ → flat FRW
$\phi = \phi(t)$
$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2 = \frac{\ddot{a}}{a} = H^2(1+\epsilon)$
$\epsilon \ll 1$
$\ddot{a} > 0$
$\frac{\dot{\phi}^2}{M_p^2}$
$\frac{\dot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$
$-(\partial\phi)^2 - 2V(\phi)$
$\frac{1}{2}\dot{\phi}^2 + V(\phi)$
$\partial\phi) + V'(\phi) = 0$
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flat FRW
$\phi = \phi(t)$
$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2 = \frac{\ddot{a}}{a} = H^2(1+\epsilon)$
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$\epsilon \ll 1$
$\ddot{a} > 0$
$\frac{\dot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$
Solve approximately.
$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$
$\frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$V'(\varphi) = 0$
$\dot{\varphi} + V'(\varphi) = 0$


(+)
$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2=\frac{\ddot{a}}{a}=H^2(1+\epsilon)$
$\dot{H}=-\frac{\dot{\varphi}^2}{2M_p^2}$
$\frac{\dot{a}}{a}=\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}$
$\epsilon \ll 1$ $\ddot{a}>0$
$|\dot{H}|\ll H^2$
Solve approximately.
$\epsilon \equiv -\frac{\dot{H}}{H^2}=\frac{\dot{\varphi}^2}{2M_p^2H^2}>0$
additional requirement
↓want inflaton to last long enough


(+)
$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2 = \frac{\ddot{a}}{a} = H^2(1+\epsilon)$
$\dot{H} = -\frac{\dot{\varphi}^2}{2M_p^2}$
$\frac{\ddot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$
$\epsilon \ll 1$
$\ddot{a} > 0$
$|\dot{H}| < H^2$
Solve approximately.
$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$
additional requirement
3Hφ

(+)

$-\dot{H}+H^2 \longrightarrow -\dot{H}-H^2 = \frac{\ddot{a}}{a} = H^2(1+\epsilon)$

$\dot{H} = -\frac{\dot{\varphi}^2}{2M_p^2}$

$\frac{\dot{a}}{a} = \frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}$

$\epsilon \ll 1 \qquad \ddot{a} > 0$

$|\dot{H}| \lesssim H^2$

Solve approximately.

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_p^2 H^2} > 0$$

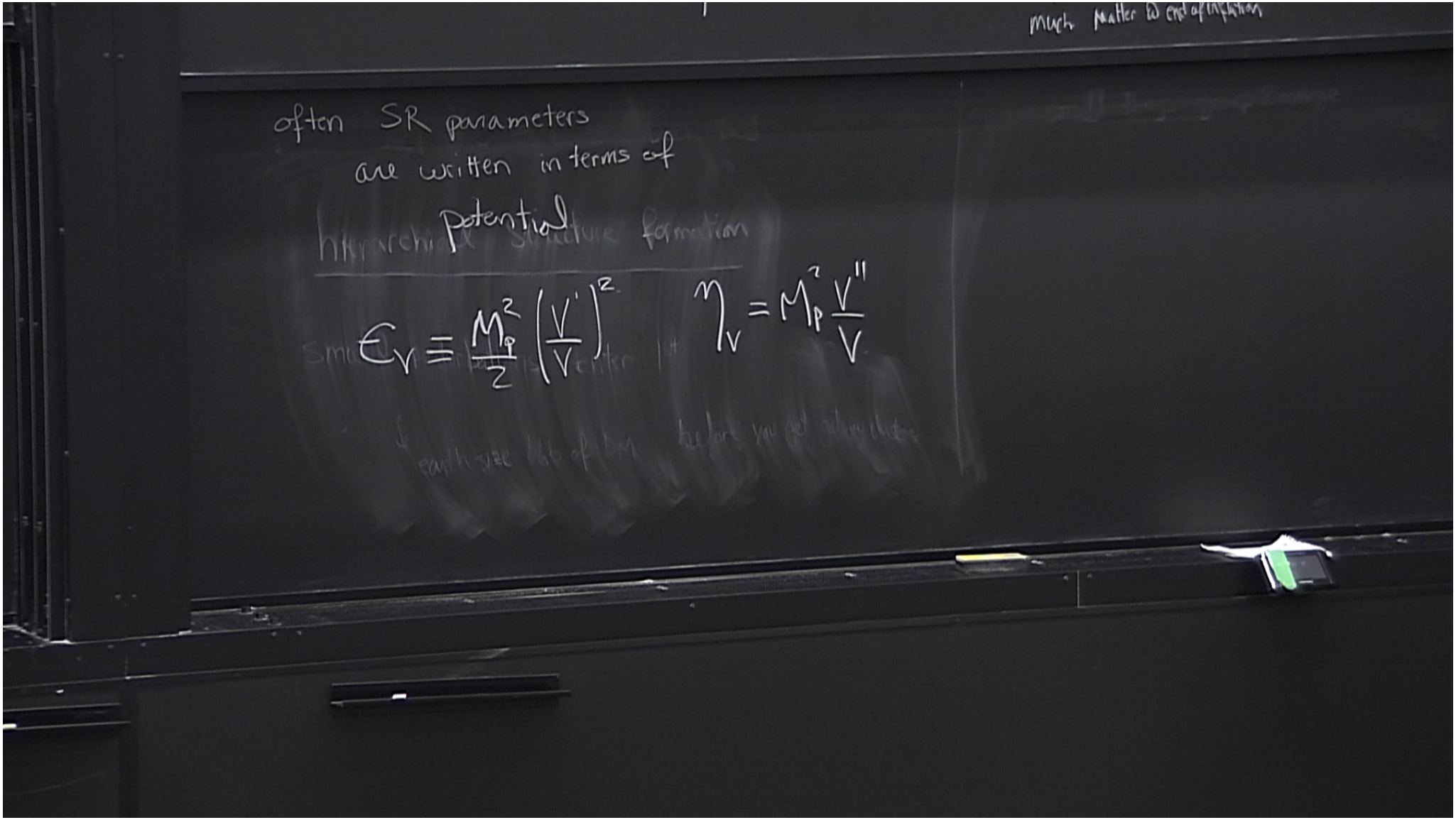
additional requirement

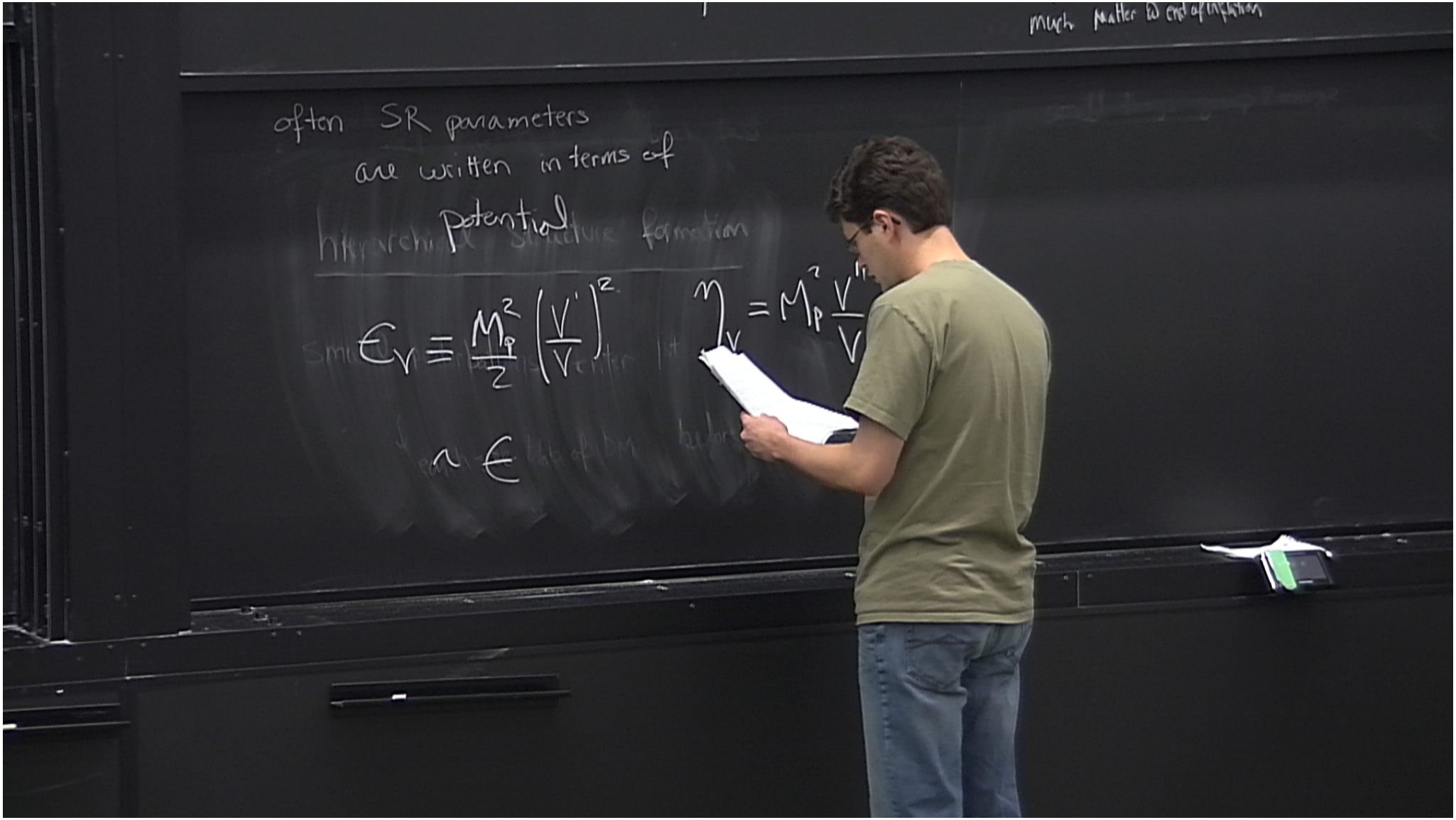
↓ want inflaton to last long enough

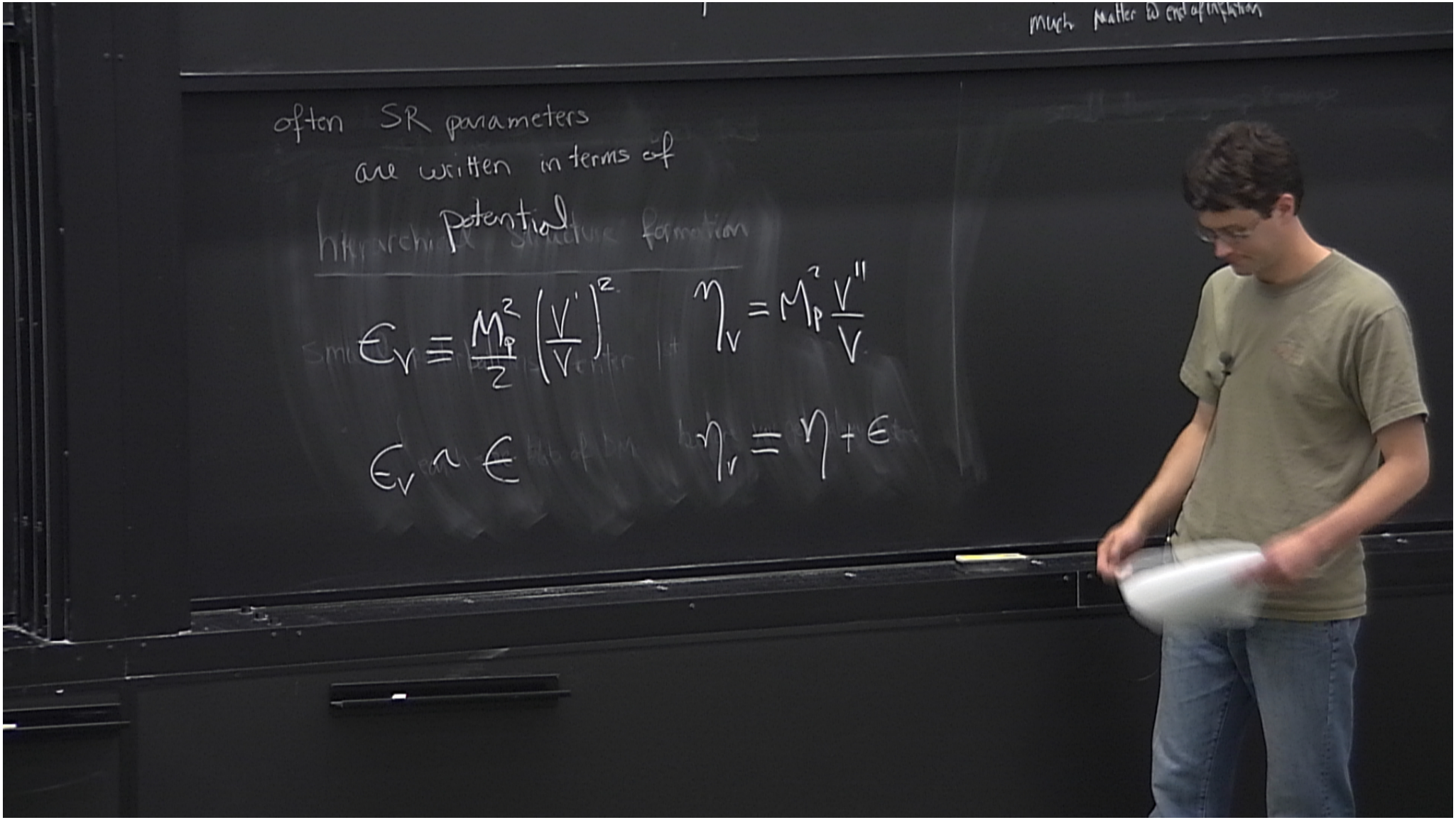
$\ddot{\varphi} < 3H\dot{\varphi}$ in EoM for φ

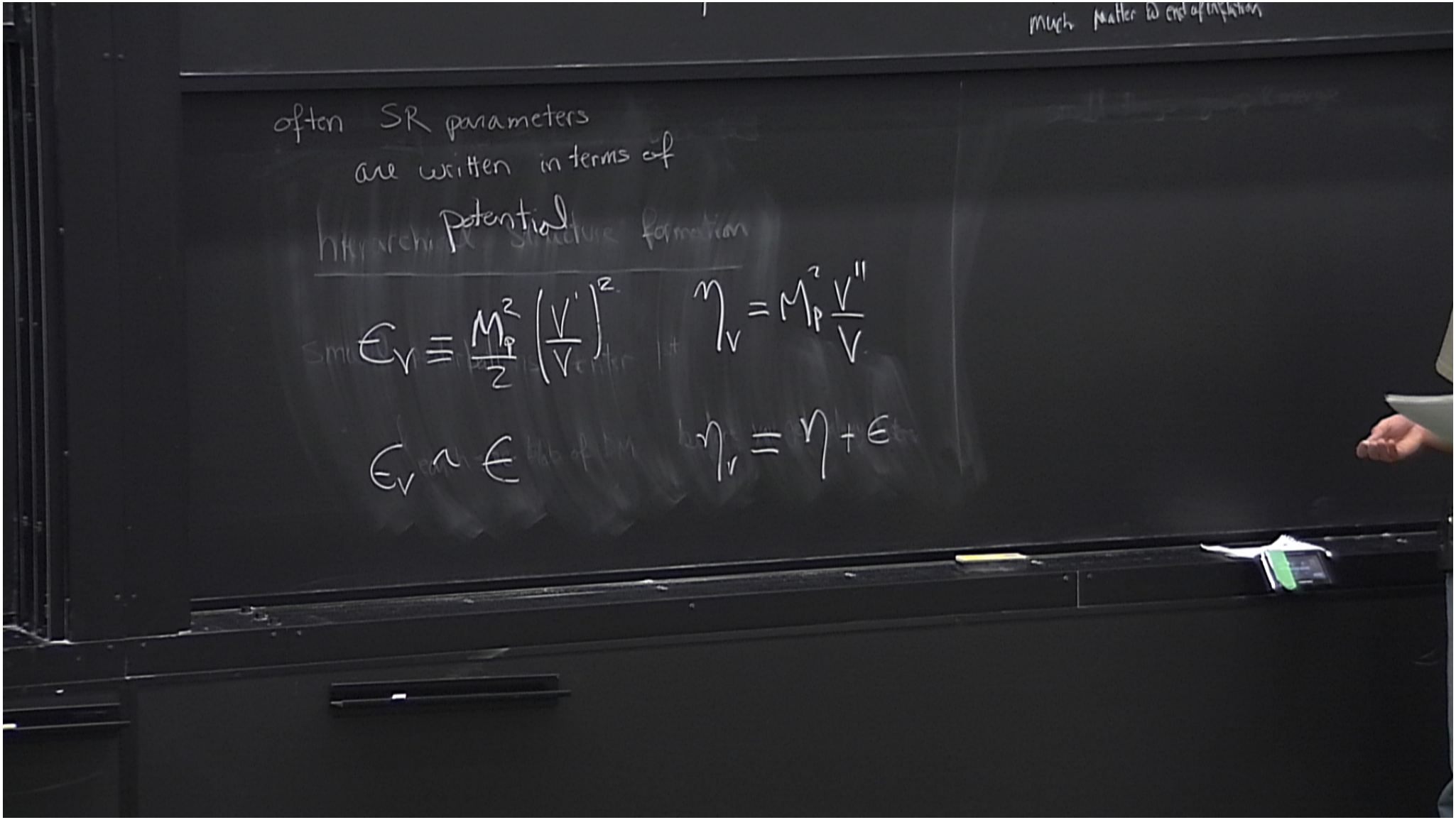
$$\eta = -\frac{\ddot{\varphi}}{H\dot{\varphi}} \qquad |\eta| \ll 1$$

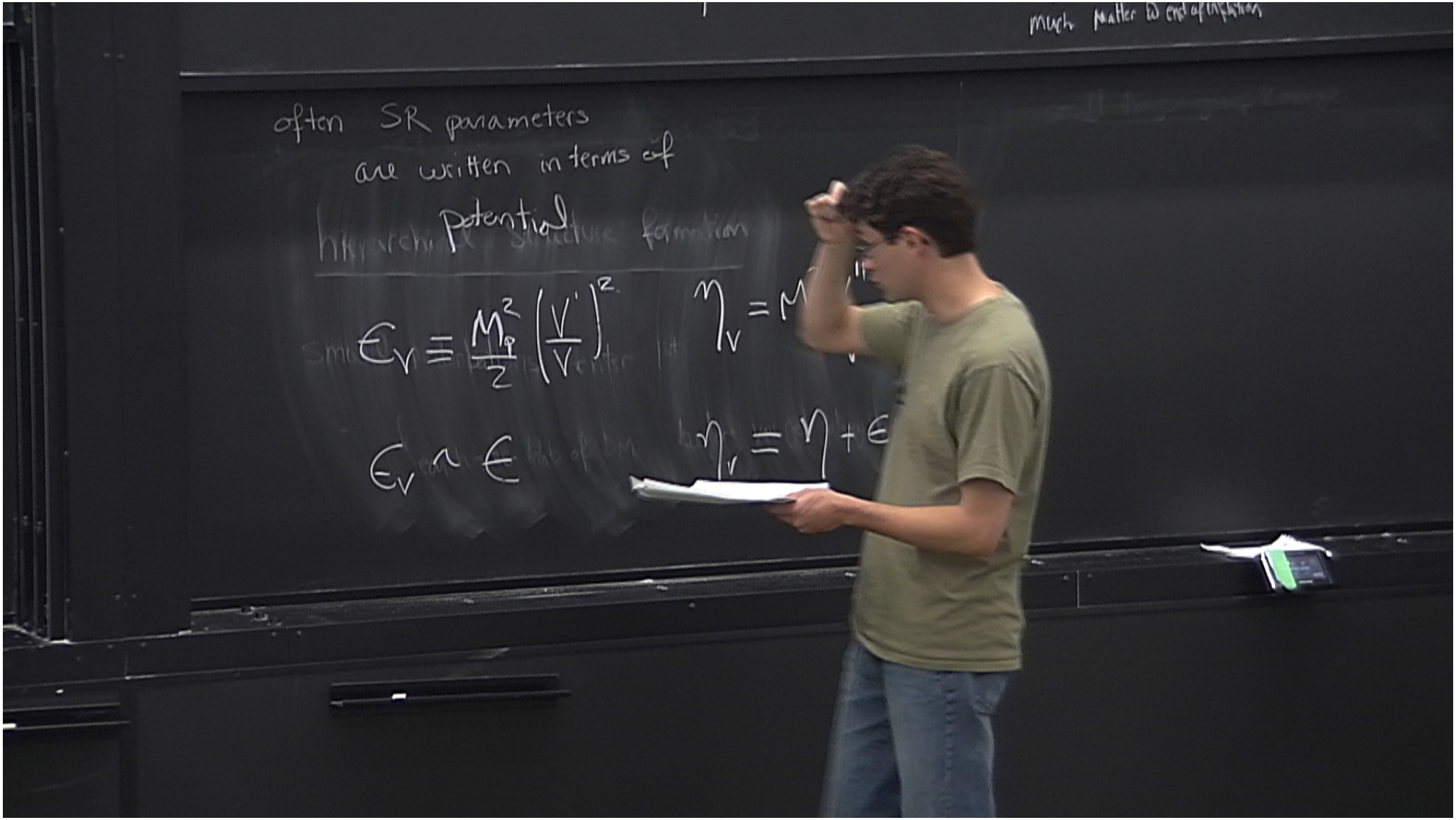
can have either signs


much matter @ end of inflation
often SR parameters
are written in terms of
potential
$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta_V = M_p^2 \frac{V''}{V}$


much matter @ end of inflation
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hierarchical structure formation
$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta_V = M_p^2 \frac{V''}{V}$
$\sim \epsilon$


much matter @ end of inflation
often SR parameters
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hierarchical structure formation
$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta_V = M_p^2 \frac{V''}{V}$
$\epsilon_V \sim \epsilon$
$\eta_V = \eta + \epsilon$


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often SR parameters
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$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta_V = M_p^2 \frac{V''}{V}$
$\epsilon_V \sim \epsilon$
$\eta_V = \eta + \epsilon$


much matter @ end of inflation
often SR parameters
are written in terms of
potential
$\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$
$\eta_V = M$
$\epsilon_V \sim \epsilon$
$\eta_V = \eta + \epsilon$