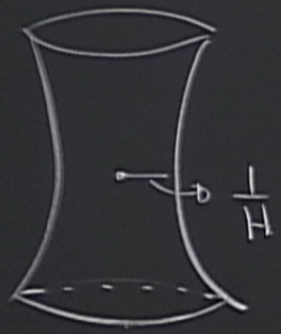
Title: Explorations in Cosmology - Lecture 5

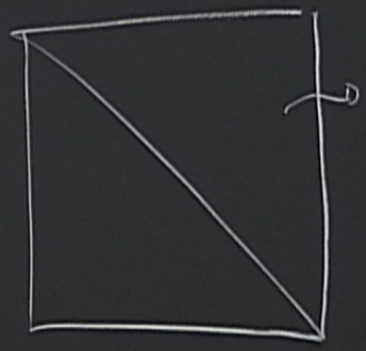
Date: Mar 16, 2012 10:15 AM

URL: http://pirsa.org/12030027

Abstract:


$g_s = g^2_{YM}$
$L^4/\alpha'^2 = \lambda$
10D coordinates
$G_{\mu\nu}(X)\,\partial_\alpha X^\mu\,\partial_\beta X^\nu$
$\Omega \to L\Omega$
$x \to Lx$
(... dimensionless ...
flat slicing





$$ds^2 = -\frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$$

$m=0$

$$v_k'' + \left(k^2 - \frac{2}{\eta^2}\right)v_k = 0$$

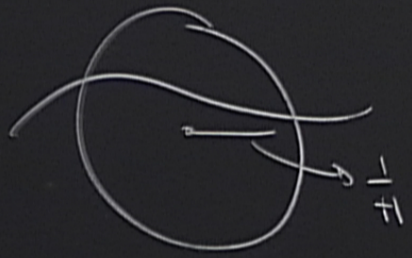
$$v_k = \frac{1}{k^{3/2}\eta}\left(1 - ik\eta\right)e^{ik\eta}$$

↓

Bunch-Davies Vacuum

super-horizon

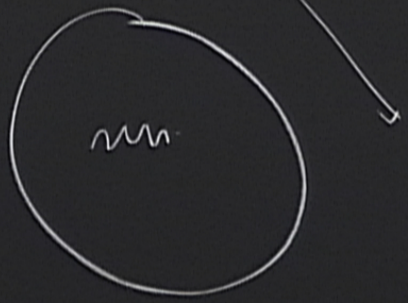
$-k\eta \ll 1$



sub-horizon

$-k\eta \gg 1$

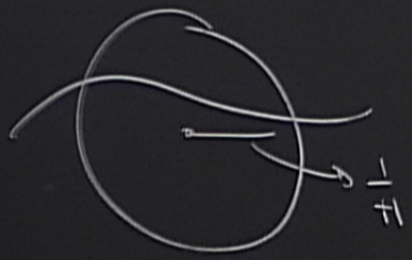
$\ell_{phys} = \frac{1}{H(-k\eta)}$



$v_k \sim \frac{1}{k} e^{ik\eta}$

super-horizon

$-k\eta \ll 1$



sub-horizon

$-k\eta \gg 1$ → $L_{phys} = \frac{1}{H(-k\eta)}$



$v_k \sim \frac{1}{k} e^{ik\eta}$

fixed k → $\eta \to -\infty$

*

$\eta = 0$

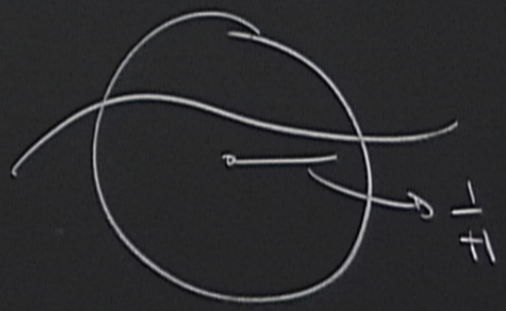
$$v_k'' + \left(k^2 - \frac{2}{\eta^2}\right) v_k = 0$$

$$v_k = \frac{1}{k^{3/2}\eta}(1 - ik\eta)e^{ik\eta}$$

Bunch-Davies vacuum

super-horizon

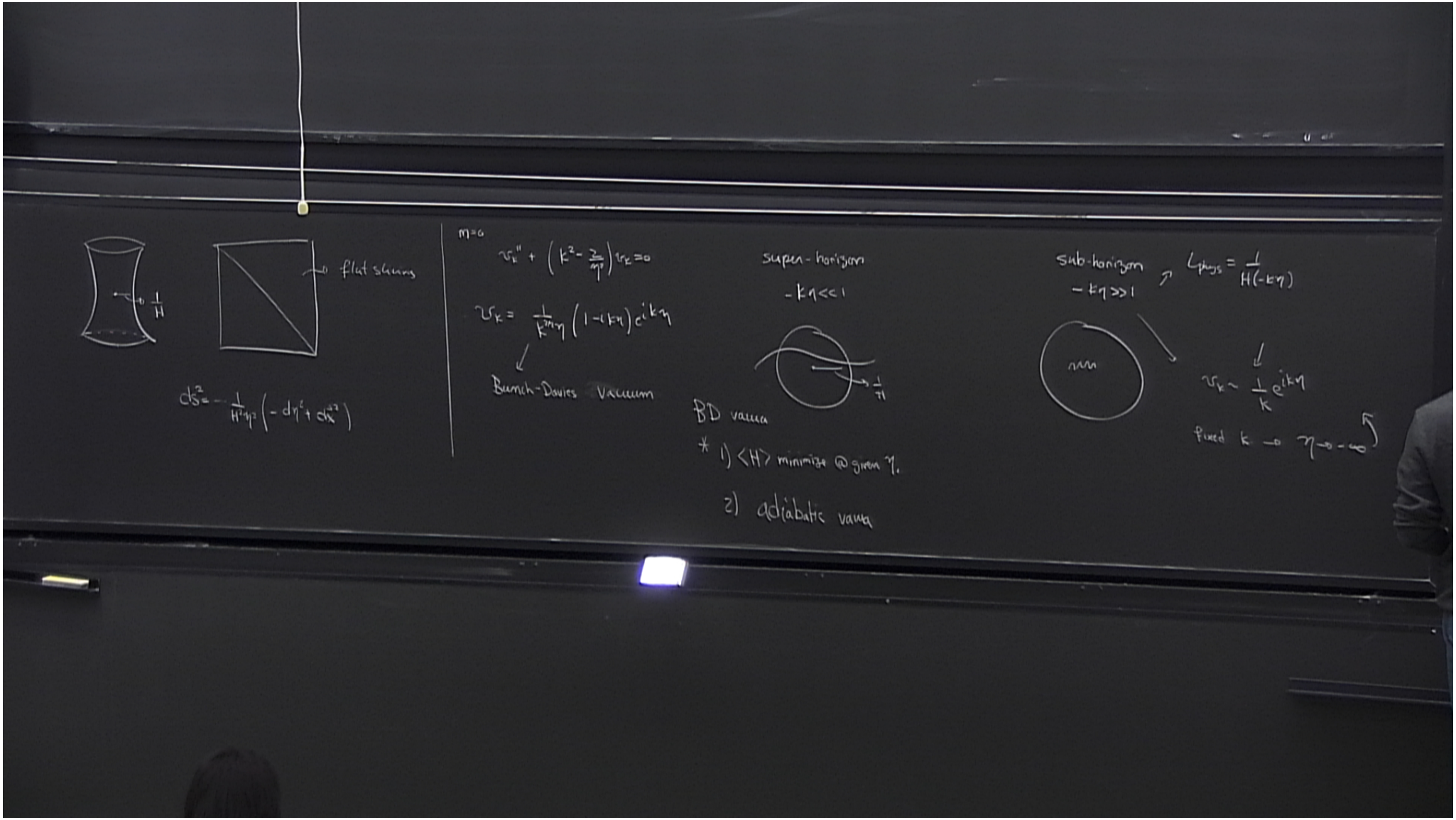
$-k\eta \ll 1$

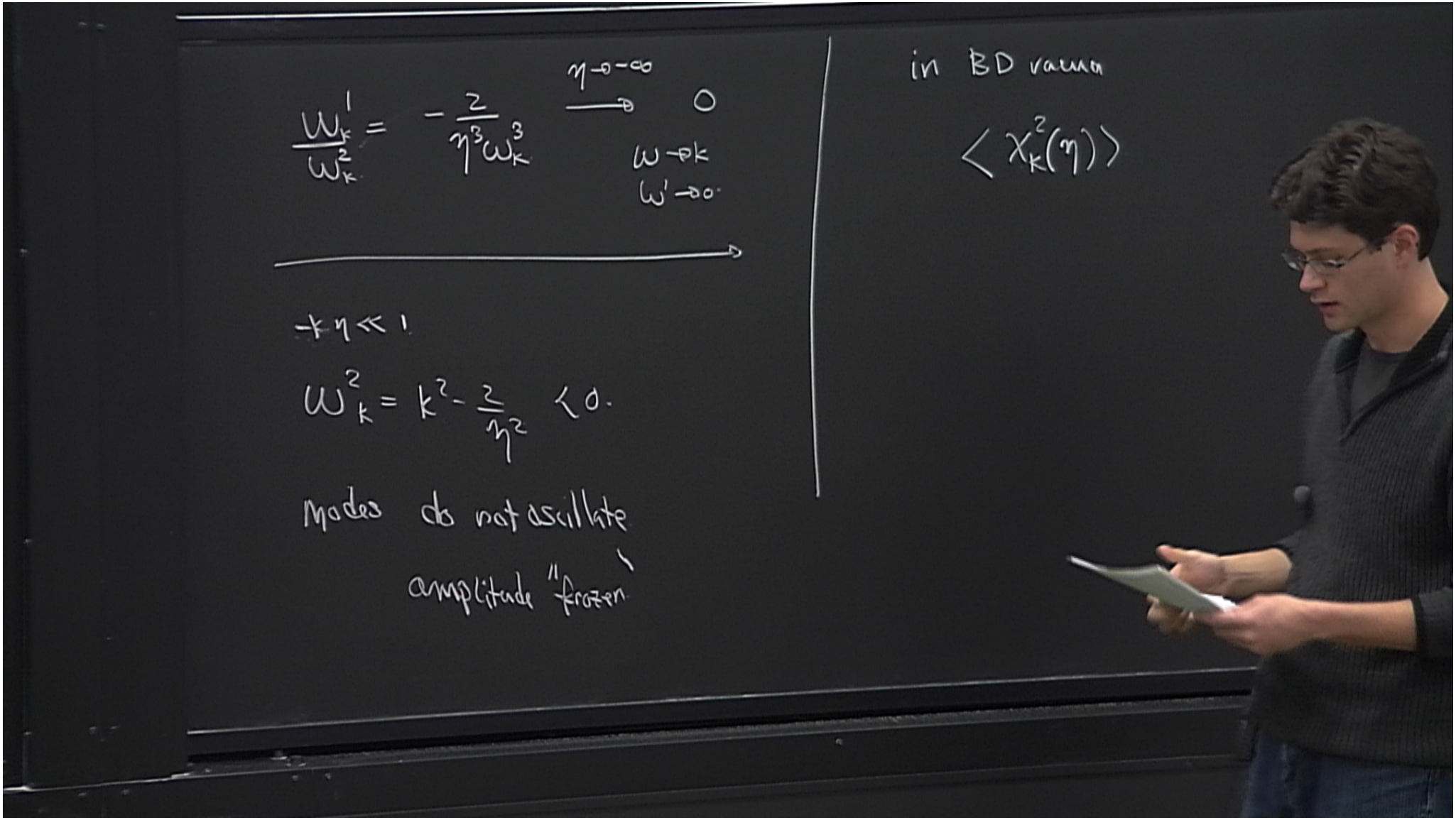


BD vacua

* 1) $\langle H \rangle$ minimize @ given η.

2) adiabatic vacua


flat slicing
$\frac{1}{H}$
$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$
m=0
$v_k'' + \left(k^2 - \frac{2}{\eta^2}\right)v_k = 0$
$v_k = \frac{1}{k^{3/2}\eta}\left(1 - ik\eta\right)e^{ik\eta}$
Bunch-Davies vacuum
super-horizon
$-k\eta \ll 1$
$\frac{1}{H}$
BD vacuum
* 1) $\langle H \rangle$ minimize @ given η.
2) adiabatic vacuum
sub-horizon
$-k\eta \gg 1$
$\ell_{phys} = \frac{1}{H(-k\eta)}$
$v_k \sim \frac{1}{k}e^{ik\eta}$
fixed k → $\eta \to -\infty$


$\frac{\omega_k'}{\omega_k^2} = -\frac{2}{\eta^3 \omega_k^3}$
$\eta \to -\infty \longrightarrow 0$
$\omega \to k$
$\omega' \to 0$
$-k\eta \ll 1$
$\omega_k^2 = k^2 - \frac{2}{\eta^2} < 0$.
modes do not oscillate
amplitude "frozen"
in BD vacuum
$\langle \chi_k^2(\eta) \rangle$

$$\frac{\omega_k'}{\omega_k^2} = -\frac{2}{\eta^3 \omega_k^3} \xrightarrow{\eta \to -\infty} 0$$

$\omega \to k$

$\omega' \to 0$

$\to$ $+\eta \ll 1$

$$\omega_k^2 = k^2 - \frac{2}{\eta^2} < 0.$$

Modes do not oscillate

amplitude "frozen"

in BD vacuum

$$\langle \chi_k^2(\eta) \rangle = \frac{1}{2} v_k^*(\eta) v_k(\eta)$$

$$a|0\rangle_{BD} = 0 \qquad = \frac{1}{2k^3\eta^2}\left(1 + k^2\eta^2\right)$$

growing mode

flat space

$$\frac{\omega_k'}{\omega_k^2} = -\frac{2}{\eta^3 \omega_k^3}$$

$\eta \to -\infty \longrightarrow 0$

$\omega \to k$

$\omega' \to 0$

$\rightarrow$

$\star\ \eta \ll 1$

$$\omega_k^2 = k^2 - \frac{2}{\eta^2} < 0.$$

modes do not oscillate

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$$\langle \chi_k^2(\eta) \rangle = \frac{1}{2} v_k^*(\eta) v_k(\eta)$$

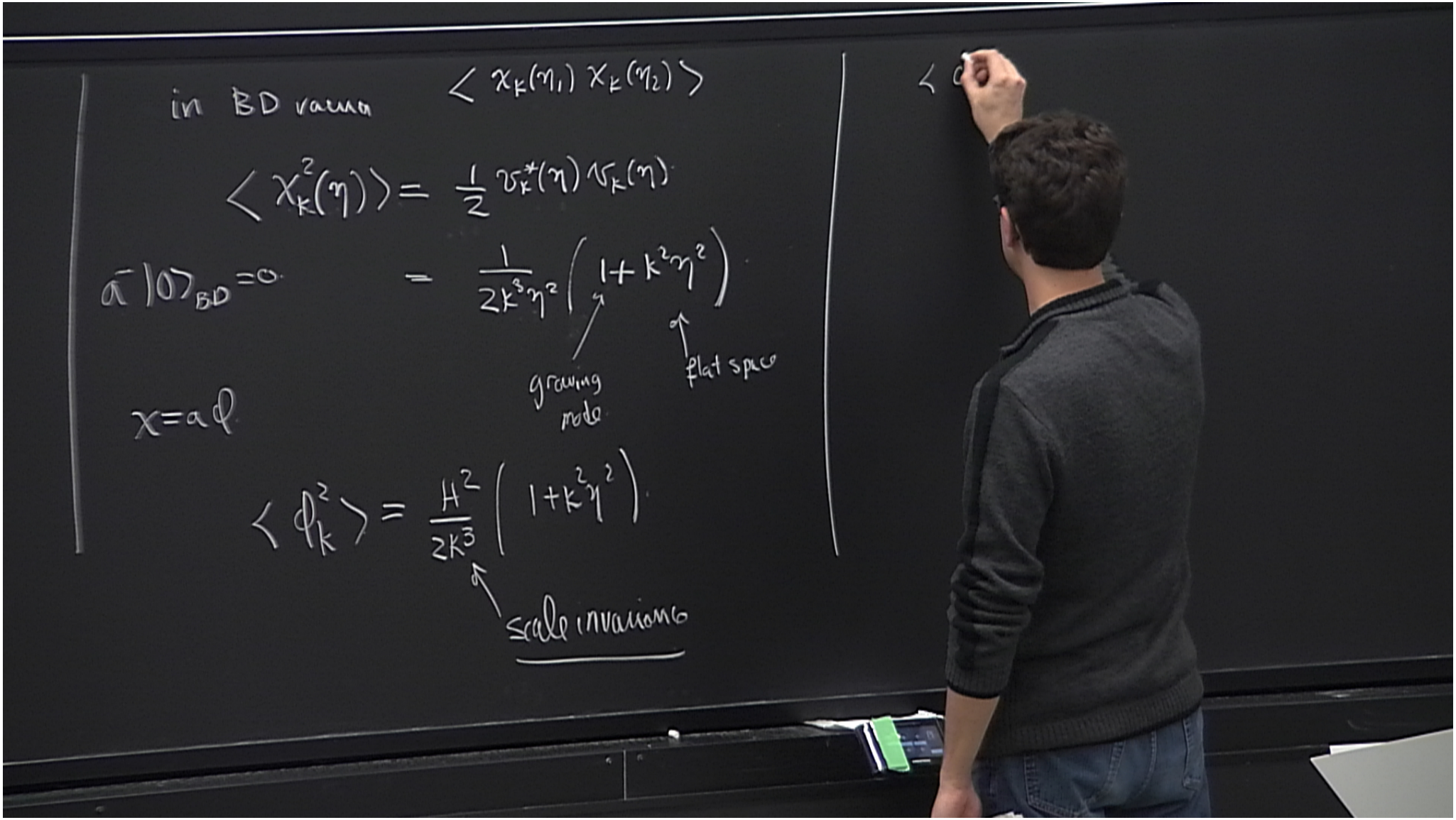
$$\hat{a}|0\rangle_{BD} = 0 \qquad = \frac{1}{2k^3\eta^2}\left(1 + k^2\eta^2\right)$$

growing mode

flat space

$\chi = a\phi$

$$\langle \phi_k^2 \rangle = \frac{H^2}{2k^3}\left(1 + k^2\eta^2\right)$$


in BD vacuum
$\langle \chi_k(\eta_1)\, \chi_k(\eta_2) \rangle$
$\langle \chi_k^2(\eta) \rangle = \frac{1}{2} v_k^*(\eta)\, v_k(\eta)$
$\hat{a}\,|0\rangle_{BD} = 0$
$= \frac{1}{2k^3\eta^2}\left(1 + k^2\eta^2\right)$
growing mode
flat space
$\chi = a\varphi$
$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3}\left(1 + k^2\eta^2\right)$
scale invariance

$$\langle \hat{q}(x)\,\hat{q}(y)\rangle = \int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2}\,u_k^*(\eta_1)\,u_k(\eta_2)\,e^{-i\vec{k}(\vec{x}-\vec{y})}$$

$$= \int \frac{dk\,k^2}{2\pi^2}\,\frac{1}{2}\,u_k^*\,u_k\,\frac{\sin kL}{kL} \quad \text{where } L = |\vec{x}-\vec{y}|$$

$L \to 0 \quad \eta_1 = \eta_2$

coincidence limit

$u_k \sim \frac{1}{\sqrt{\omega_k}}$

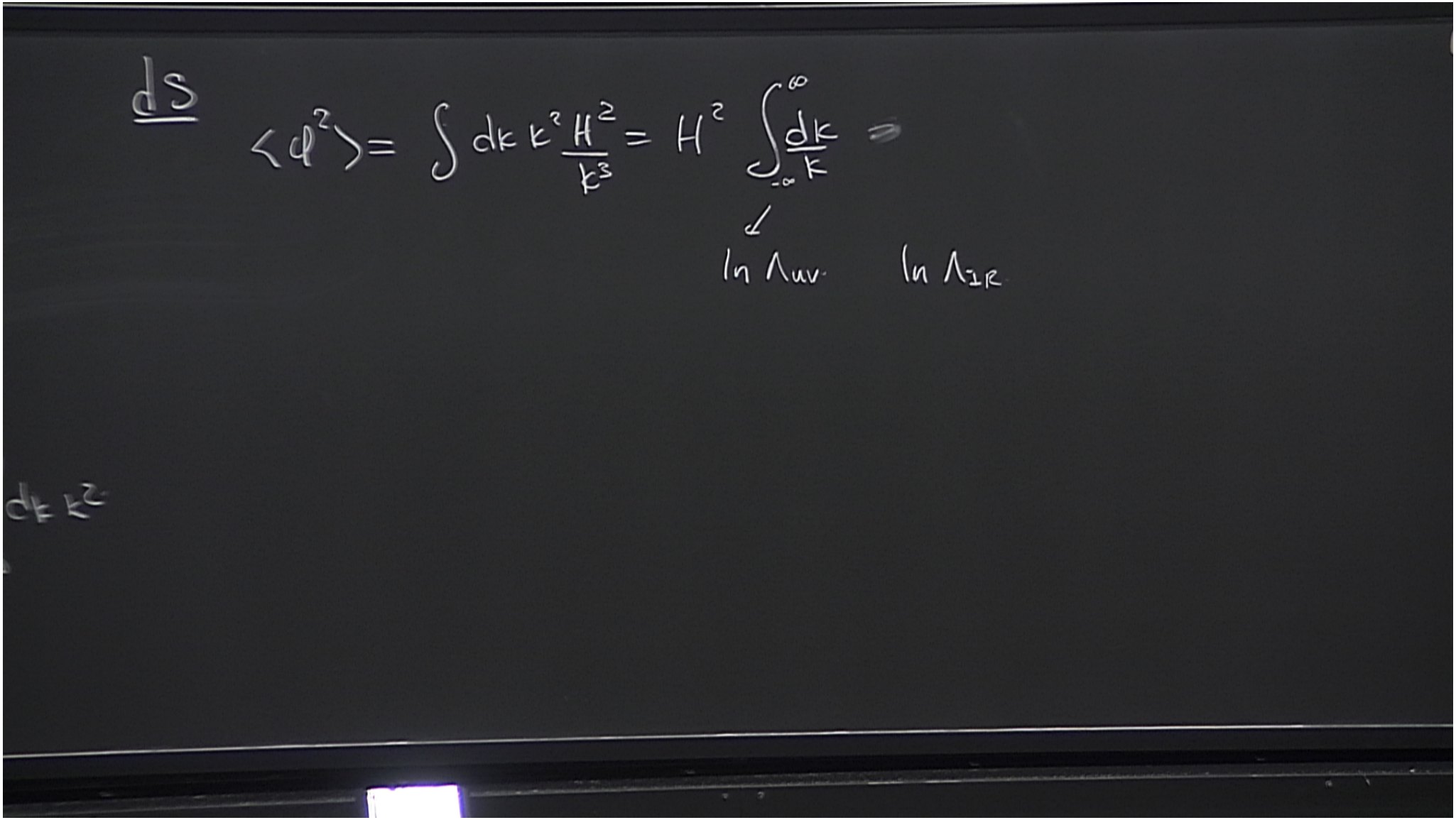
$$\langle \varphi^2(x) \rangle \cong \int dk\, k^2 |u_k|^2$$

Λ^2 quadratic divergences

Flat space $m \neq 0$

$$= \int dk\, k^2 \frac{1}{\sqrt{k^2+m^2}} \sim \int_m^\Lambda dk\, k + \int_0^m dk\, k^2$$

dS


ds
$\langle \phi^2 \rangle = \int dk\, k^2 \frac{H^2}{k^3} = H^2 \int_{-\infty}^{\infty} \frac{dk}{k} =$
$\ln \Lambda_{UV}$
$\ln \Lambda_{IR}$
$dk\, k^2$

ds

$$\langle q^2 \rangle = \int dk\, k^2 \frac{H^2}{k^3} = H^2 \int_{-\infty}^{\infty} \frac{dk}{k} =$$

$\ln \Lambda_{UV}$ $\ln \Lambda_{IR}$

UV : - dim regulation
- Pauli-Villard
- brute force Λ_{UV} → $p = \frac{k}{a} < \Lambda_{UV}$

$\propto k^2$

k

$\underline{ds}$

$$\langle \varphi^2 \rangle = \int dk\, k^2 \frac{H^2}{k^3} = H^2 \int_{-\infty}^{\infty} \frac{dk}{k} =$$

$\ln \Lambda_{UV}$ $\quad$ $\ln \Lambda_{IR}$

$\underline{\underline{UV}}$: - dim regulation
- Pauli-Villard
- brute force $\underline{\Lambda_{UV}} \rightarrow$ $\quad p = \frac{k}{a} < \underline{\Lambda_{UV}}$

$dk\, k^2$

$$k > \frac{1}{L}$$

L : critical size of inflating universe.

$\underline{ds}$ $\langle q^2 \rangle = \int dk\, k^2 \frac{H^2}{k^3} = H^2 \int_{-\infty}^{\infty} \frac{dk}{k} \approx H^2 \int_{1/L}^{\Lambda_{UV}\, a(\eta)} \frac{dk}{k} \approx H^2 \ln a + \underline{cst}$

$\ln \Lambda_{UV} \quad \ln \Lambda_{IR}$

$\underline{UV}$: - dim regulation
- Pauli-Villard
- brute force $\underline{\Lambda_{UV}} \Rightarrow \quad p = \frac{k}{a} < \underline{\Lambda_{UV}}$

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critical size of inflating universe.

dS

$$\langle \varphi^2 \rangle = \int dk\, k^2 \frac{H^2}{k^3} = H^2 \int_{-\infty}^{\infty} \frac{dk}{k} \sim H^2 \int_{1/L}^{\Lambda_{UV}\, a(\eta)} \frac{dk}{k} \approx H^2 \ln a + \text{cst}$$

$\ln \Lambda_{UV}$ $\quad \ln \Lambda_{IR}$

$$\boxed{\langle \varphi^2 \rangle = H^3 t}$$

$a \sim e^{Ht}$

$\sim \frac{1}{-H\eta}$

UV : - dim regulation
- Pauli-Villard
- brute force $\Lambda_{UV} \to$ $p = \frac{k}{a} < \Lambda_{UV}$

$k > \frac{1}{L}$

critical size of inflating universe.

$$H^2 \int_{-\infty}^{\infty} \frac{dk}{k} \approx H^2 \int_{1/L}^{\Lambda_{UV}\, a(\eta)} \frac{dk}{k} \approx H^2 \ln a + \text{cst}$$

$\ln \Lambda_{UV}$ $\quad$ $\ln \Lambda_{IR}$

$$\boxed{\langle \phi^2 \rangle = H^3 t}$$

$a \sim e^{Ht}$ $\qquad$ R

$\sim \frac{1}{-H\eta}$

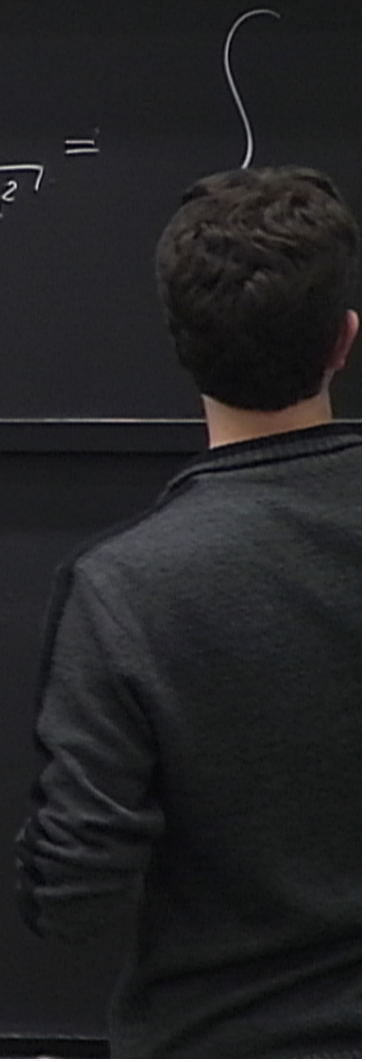
$p = \frac{k}{a} < \Lambda_{UV}$

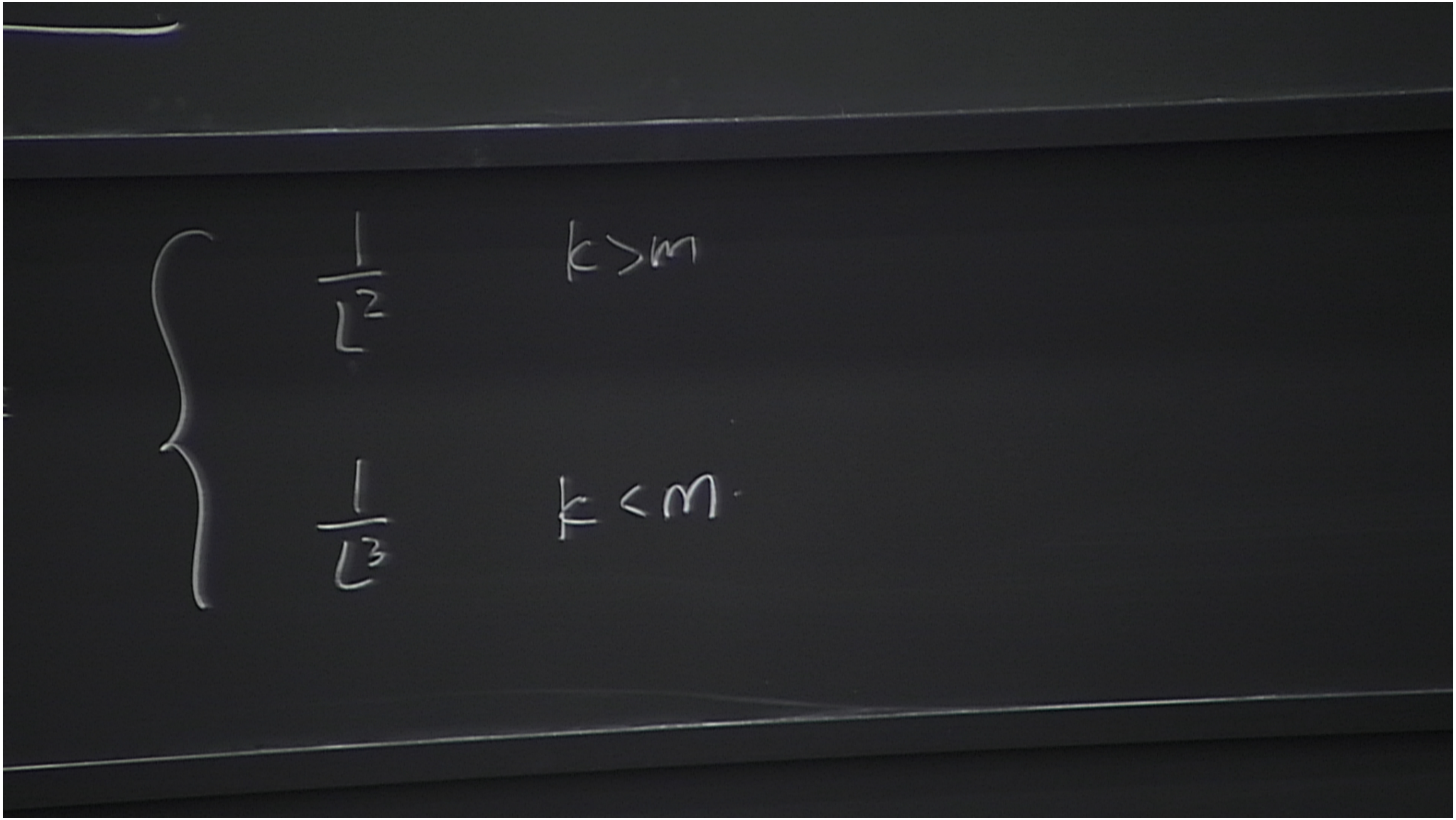
$\langle \phi \rangle = 0$ $\qquad$ $\langle \phi^2(x,t) \rangle = H^3 t$

Universe

beyond coincidence limit L finite $(\eta_1 = \eta_2)$ $kL \gg 1$

flat, $m \neq 0$ $\langle \phi(x)\phi(y)\rangle = \int_0^\infty \frac{dk\,k^2}{\sqrt{k^2+m^2}} \frac{\sin kL}{kL} = \int_0^{1/L} dk \frac{k^2}{\sqrt{k^2+m^2}} =$




$\frac{1}{L^2}$ $k > m$
$\frac{1}{L^3}$ $k < m$

beyond coincidence limit L finite $(\eta_1 = \eta_2)$ $kL \gg 1 \rightarrow \sin kL$ kills the integral

flat, $m \neq 0$ $\langle \phi(x)\phi(y)\rangle = \int_0^\infty \frac{dk\,k^2}{\sqrt{k^2+m^2}} \frac{\sin kL}{kL} = \int_0^{1/L} dk \frac{k^2}{\sqrt{k^2+m^2}} =$

ds.

$kL \ll 1$.

$\langle Q(x)Q(y)\rangle = H^2 \int_{1/L'}^{1/L} \frac{dk}{k} = H^2 \ln L/L' = H^2 \ln \frac{|\vec{x}-\vec{y}|}{L'}$

size of universe

$kL \gg 1 \rightarrow \sin kL$ kills the integral

$$\frac{\sin kL}{kL} = \int_0^{1/c} dk \, \frac{k^2}{\sqrt{k^2+m^2}} = \begin{cases} \frac{1}{L^2} & k > m \\ \frac{1}{L^3} & k < m \end{cases}$$

$$\langle \phi(\vec{x}) \phi(\vec{y}) \rangle = H^2 \left| \ln \left| \frac{\vec{x}-\vec{y}}{L} \right| \right| \rightarrow \xi : \text{correlation length } \infty$$

$\rightarrow$ critical phenomena

$\rightarrow$ scale invariance or conformal symmetry

beyond coincidence limit L finite $(\eta_1 = \eta_2)$ $kL \gg 1 \rightarrow \sin kL$ kills the integral

flat, $m \neq 0$ $\langle \phi(x)\phi(y)\rangle = \int_0^\infty \frac{dk\,k^2}{\sqrt{k^2+m^2}} \frac{\sin kL}{kL} = \int_0^{1/L} dk \frac{k^2}{\sqrt{k^2+m^2}} =$

$$\begin{cases} \frac{1}{L^2} & k > m \\ \frac{1}{L^3} & k < m \end{cases}$$

ds. $kL \ll 1$

$$\langle \phi(x)\phi(y)\rangle = H^2 \int_{1/L'}^{1/L} \frac{dk}{k} = H^2 \ln L/L' = H^2 \ln \left|\frac{\vec{x}-\vec{y}}{L'}\right|$$

size of universe

→ ξ: correlation length ∞

→ critical phenomena

→ scale invariant or conformal symmetry

Logic fixed k

$H_\nu^{(1)} = J_\nu + i Y_\nu$

$H_\nu^{(2)} = (H_\nu^{(1)})^* = J - i Y$

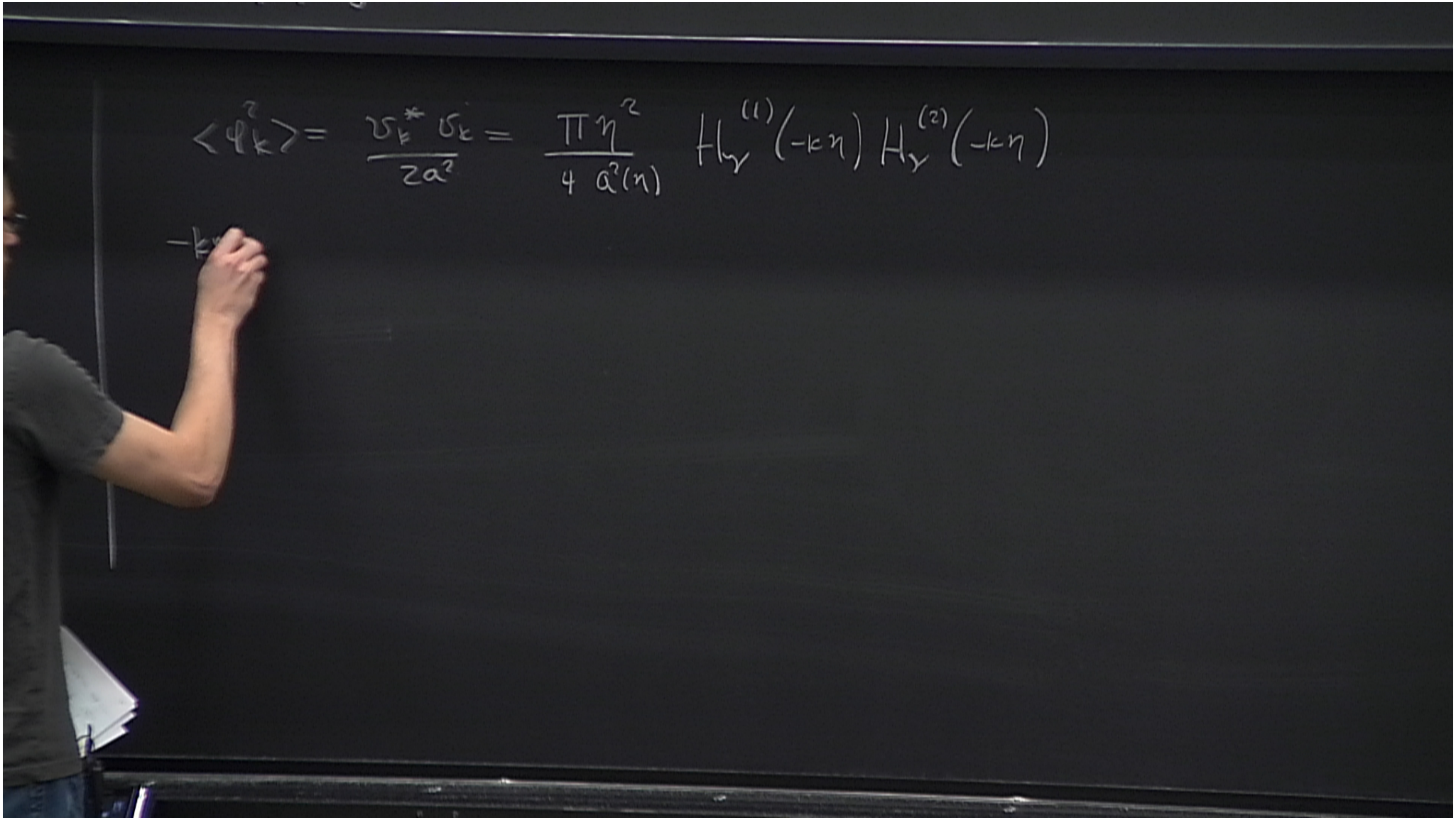
$-\infty \longrightarrow \eta_* = -\frac{1}{k} \longrightarrow$ amplitude is "frozen" $\langle q_k^2 \rangle = cst$

in flat space ↓ horizon crossing.

massive case

where $\nu = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$: order parameter

$$v_k'' + v_k\left(k^2 + \frac{m^2}{H^2} - \frac{2}{\eta^2}\right) = 0 \longrightarrow v_k = \sqrt{-\frac{\pi\eta}{2}}\, H_\nu^{(2)}(-k\eta) \Rightarrow e^{ik\eta} \quad (-k\eta \gg 1)$$


$\langle \hat{q}_k^2 \rangle = \frac{v_k^* v_k}{2a^2} = \frac{\pi \eta^2}{4 a^2(\eta)} H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta)$
$-k$

$$\langle \hat{\varphi}_k^2 \rangle = \frac{v_k^* v_k}{2a^2} = \frac{\pi \eta^2}{4 a^2(\eta)} H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta)$$

$-k\eta \ll 1$, $\nu = \frac{3}{2} - \frac{m^2}{3H^2}$ expand ν $m \ll H$.

$=$

Logic **fixed k**

$H_\nu^{(1)} = J_\nu + i Y_\nu$

$H_\nu^{(2)} = (H_\nu^{(1)})^* = J - i Y$

$-\infty \longrightarrow \eta_* = -\frac{1}{k} \longrightarrow$ amplitude is "frozen" $\langle q_k^2 \rangle = cst$

in flat space ↓ horizon crossing.

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massive case

$v_k'' + v_k\left(k^2 + \frac{m^2}{H^2} - \frac{2}{\eta^2}\right) = 0 \longrightarrow v_k = \sqrt{-\frac{\pi\eta}{2}} H_\nu^{(2)}(-k\eta) \Rightarrow e^{ik\eta}$ $-k\eta \gg 1$

$H_{3/2}^{(2)} \propto (1 - ik\eta) e^{ik\eta}$.

$$\langle \hat{\varphi}_k^2 \rangle = v_k^* v_k = \frac{\pi \eta^2}{4 a^2(\eta)} H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta)$$

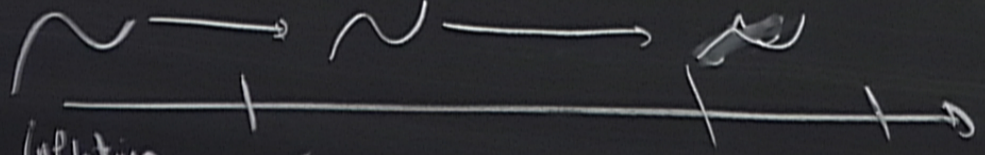
$-k\eta \ll 1$, $\nu = \frac{3}{2} - \underbrace{\frac{m^2}{3H^2}}_{\delta} + \cdots$ expand ν $m \ll H$ $\delta \ll 1$

$$\langle \hat{\varphi}_k^2 \rangle = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$$

$$\langle \hat{q}_k^2 \rangle = \frac{v_k^* v_k}{2a^2} = \frac{\pi \eta^2}{4 a^2(\eta)} H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta)$$

$-k\eta \ll 1$, $\nu = \frac{3}{2} - \underbrace{\frac{m^2}{3H^2}}_{\delta} + \cdots$ expand ν $m \ll H$. $\delta \ll 1$.

$$\Rightarrow \langle \hat{q}_k^2 \rangle = \frac{H^2}{2k^3} (-k\eta)^{2\delta} \quad \eta \to 0.$$



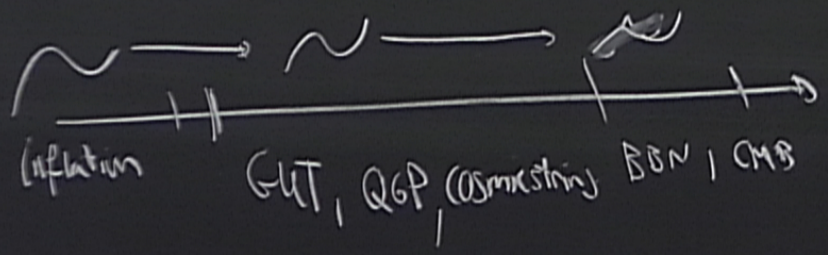
critical size of inflating universe.

$$\langle \varphi_k^2 \rangle = \frac{v_k^* v_k}{2a^2} = \frac{\pi \eta^2}{4 a^2(\eta)} H_\nu^{(1)}(-k\eta) H_\nu^{(2)}(-k\eta)$$

$-k\eta \ll 1$, $\nu = \frac{3}{2} - \frac{m^2}{3H^2} + \cdots$ expand ν ($m \ll H$, $\delta \ll 1$)

$$\langle \varphi_k^2 \rangle = \frac{H^2}{2k^3} (-k\eta)^{2\delta}$$

$-k\eta \ll 1$, $\eta \to 0$



$$\langle \varphi^2(x) \rangle = \int_0^{1/\eta} dk \; H^2 (\eta)^{2\delta} \; k^{2\delta - 1}$$

$$\langle \varphi^2 \rangle \propto \frac{H^2}{\delta} = \frac{H^4}{m^2}$$

$m \ll H$

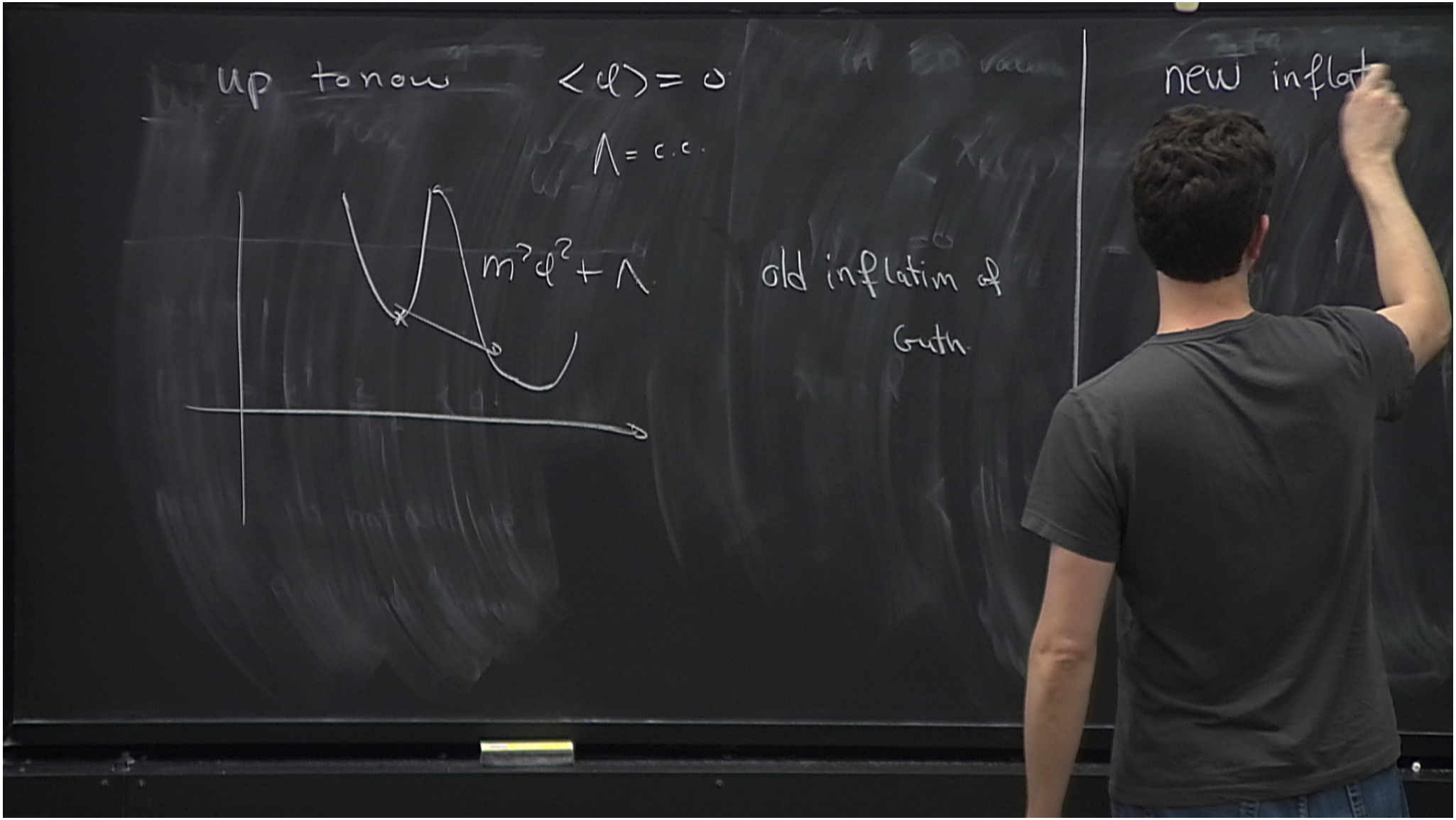
$$\langle \varphi^2(x) \rangle = \int_0^{-1/\eta} \frac{dk}{k} \, H^2 (-\eta)^{2\delta} \, k^{2\delta - 1}$$

$m \ll H$

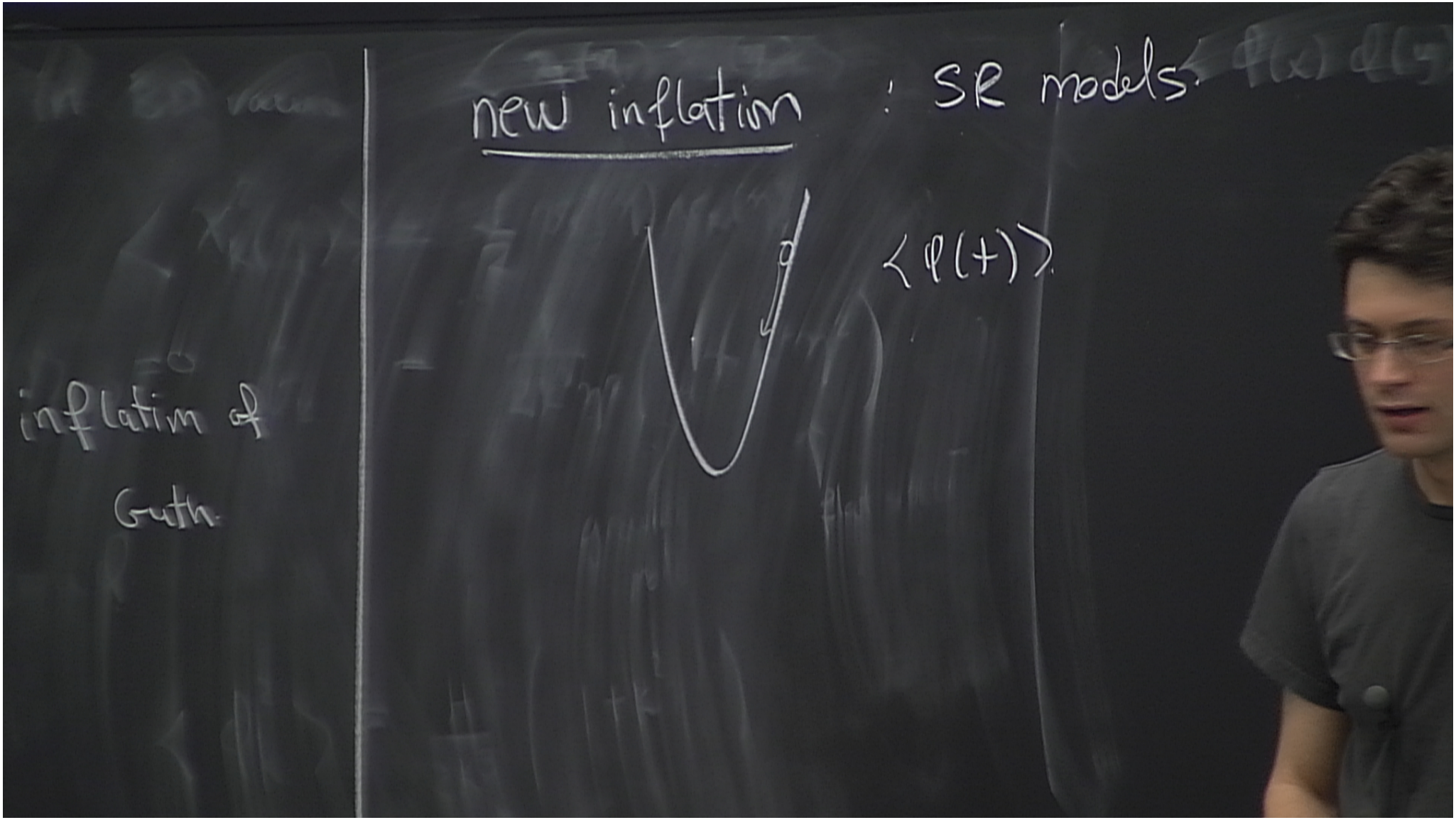
$\delta \ll 1$.

$$\langle \varphi^2 \rangle \propto \frac{H^2}{\delta} = \frac{H^4}{m^2}$$

$$m \ll H$$

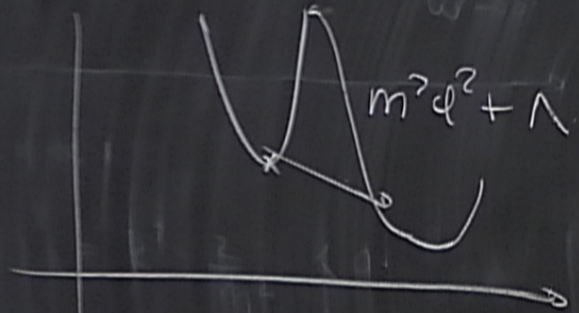

up to now
$\langle \varphi \rangle = 0$
Λ = c.c.
$m^2\varphi^2 + \Lambda$
old inflation of Guth
new inflat


new inflation : SR models.
inflation of
Guth.


new inflation : SR models.
<φ(t)>
inflation of
Guth.

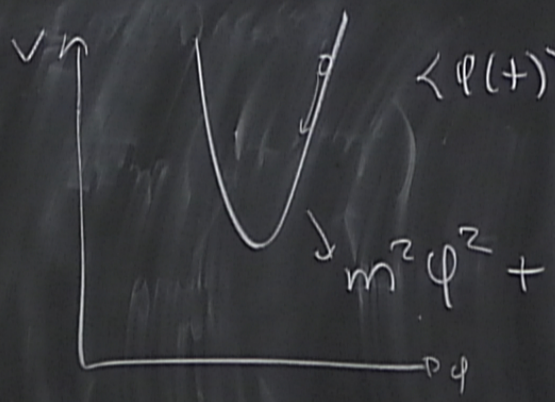
up to now $\langle \varphi \rangle = 0$

Λ = c.c.



old inflation of Guth

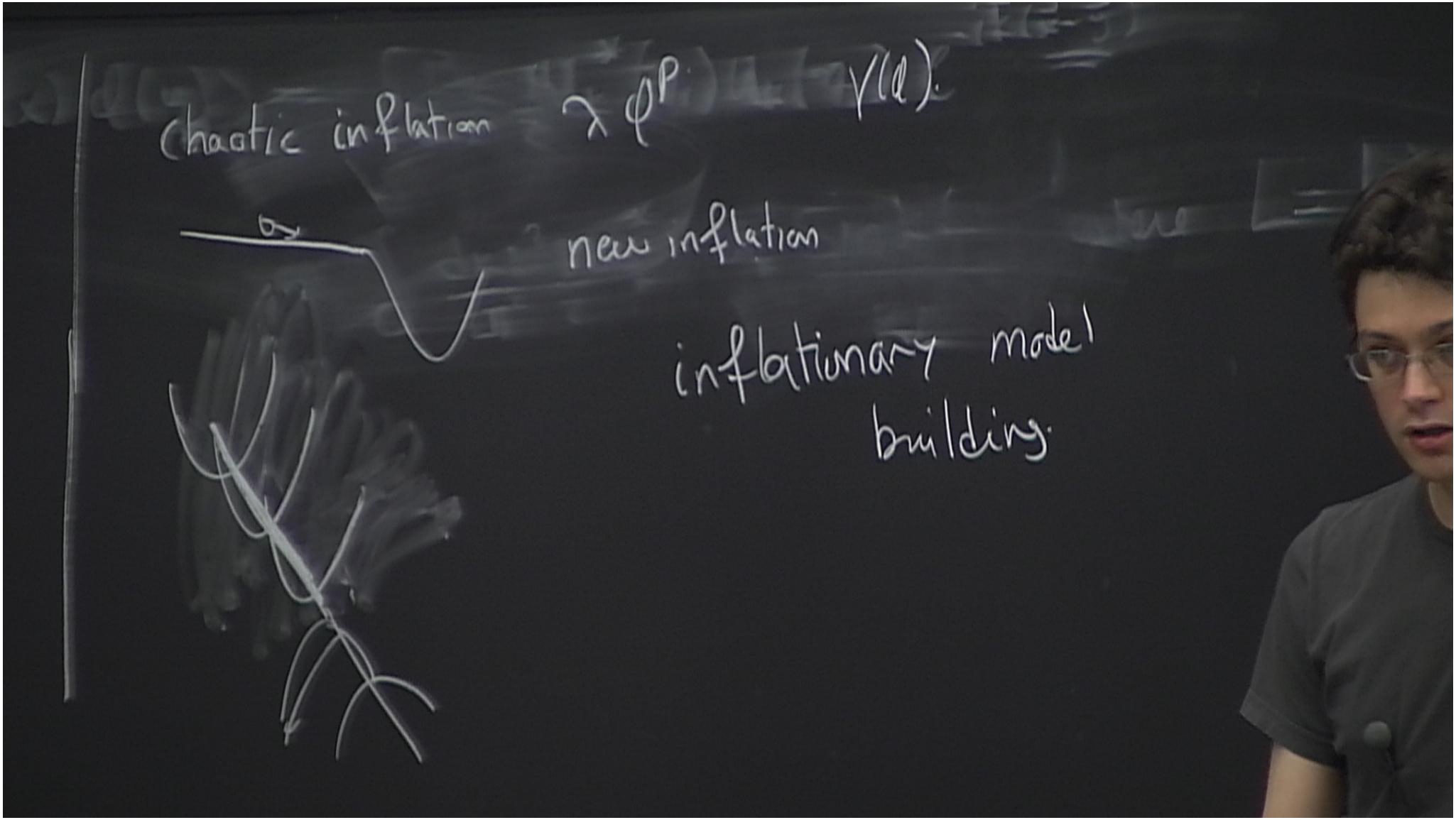
new inflation : SR models



Chaotic inflation $\lambda\phi^p$



new inflation


chaotic inflation $\lambda\phi^p$ $V(\phi)$.
new inflation
inflationary model building.


new inflation : SR models
chaotic inflation
new inflation
inflationary model building