Title: Explorations in Cosmology - Lecture 4

Date: Mar 15, 2012 10:15 AM

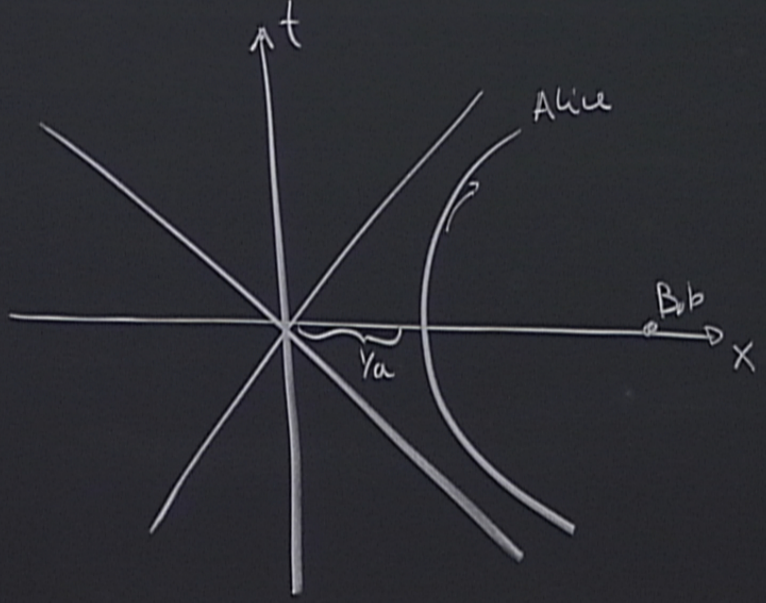
URL: http://pirsa.org/12030026

Abstract:


Unruh Effect
t
Alice
Ya

Recap.

Unruh Effect



Alice lives in Rindler wedge.
w/ coord $\tilde{u}, \tilde{v}$

Bob sees all of flat space
w/ u, v.

$u = \frac{1}{a} e^{a\tilde{u}}$ $\quad v = \frac{1}{a} e^{a\tilde{v}}$

Modes for Alice $e^{-i\omega\tilde{u}}$ $\quad e^{i\omega\tilde{u}}$

b^- $\quad b^+$

Bob $e^{-i\omega u}$ $\quad e^{i\omega v}$

Alice lives in Rindler wedge
w/ coord $\hat{u}, \hat{v}$

Bob sees all of flat space
w/ u, v

$u = \frac{1}{a} e^{a\hat{u}}$ $\quad v = \frac{1}{a} e^{a\tilde{v}}$

$5^1 = 0$

Bob → x

Modes for Alice $\quad e^{-i\omega\hat{u}} \quad e^{i\omega\tilde{u}}$

$\bar{b}_\omega |0\rangle_A = 0$ $\quad\quad \bar{b} \quad\quad b^\dagger$

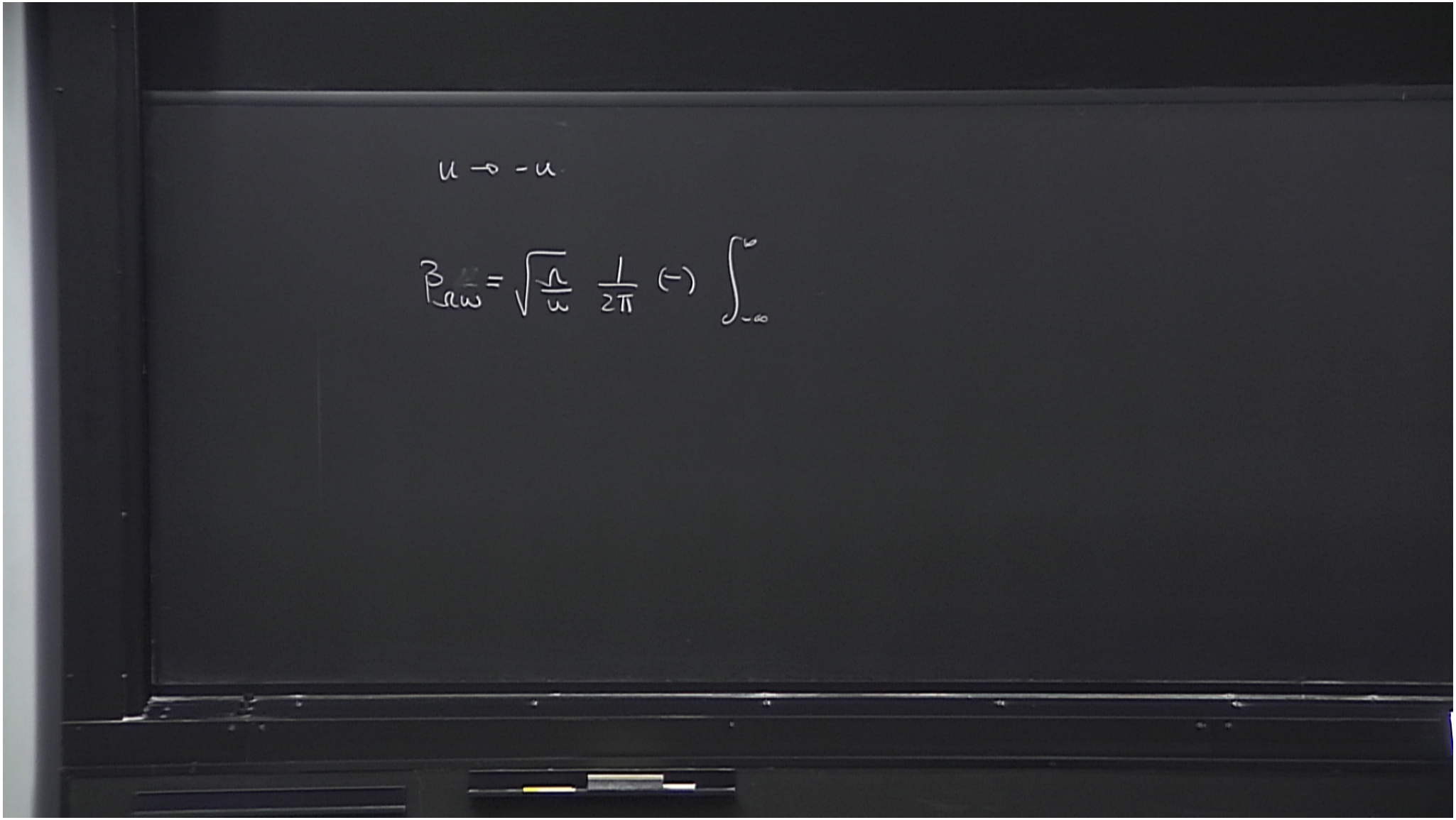
Bob $\quad e^{-i\omega u} \quad e^{i\omega v}$

$\bar{a}_\omega |0\rangle_B = 0$ $\quad\quad \bar{a} \quad\quad a^\dagger$

these modes are related by
a Bogoliubov transf

$$\bar{b}_r = \int_0^\infty d\omega \left(\alpha_{r\omega} \bar{a}_\omega - \beta_{r\omega} a^\dagger_\omega \right)$$

$$\begin{matrix} \alpha_{r\omega} \\ \beta_{r\omega} \end{matrix} = \left\{ \frac{1}{2\pi} \sqrt{\frac{r}{\omega}} \left(\begin{matrix} - \\ + \end{matrix} \right) \int_{-\infty}^{\infty} \frac{du}{u} e^{\mp i\omega u} e^{i\frac{r}{a} \ln a u} \right.$$


$u \to -u$
$\beta_{\omega} = \sqrt{\frac{\Omega}{\omega}} \frac{1}{2\pi} (-) \int_{-\infty}$

$u \to -u$ $\qquad -i\ln(-1) = \pi$

$$\beta_{\omega\omega'} = \sqrt{\frac{\omega}{\omega'}}\frac{1}{2\pi}(-)\int_{-\infty}^{\infty}\frac{du}{u}\, e^{-i\omega u} e^{i\frac{\Omega}{a}\ln u} e^{i\frac{\Omega}{a}\ln(-1)}$$

$$= \alpha_{\omega\omega'} e^{-\pi\Omega/a}$$

$$\boxed{|\beta|^2 = |\alpha|^2 e^{-2\pi\Omega/a}}$$ detailed balance.

Thermal s

$P_{nm} =$

Thermal system $\frac{1}{T}$ ($k_b = 1$)

$$\rho_{nm} = \langle \psi_n | e^{-\beta H} | \psi_m \rangle = |\alpha|^2 e^{-\beta E_m}$$

$$\rho_{mn} = \qquad\qquad = |\beta|^2 e^{-\beta E_n}$$

$$\rho_{nm} = \rho_{mn} \Rightarrow$$

Thermal system $\rightarrow \frac{1}{T}$ ($k_B \equiv 1$)

$$\rho_{nnm} = \langle \psi_n | e^{-\beta H} | \psi_m \rangle = |\alpha|^2 e^{-\beta E_m}$$

$$\rho_{mn} = = |\beta|^2 e^{-\beta E_n}$$

$$\rho_{nm} = \rho_{mn} \Rightarrow |\beta|^2 = |\alpha|^2 e^{-\beta \Delta E} \rightarrow \Omega \qquad E = \hbar\omega = \Lambda$$

$$\frac{2\pi}{a} = \frac{1}{T}; \quad \boxed{T = \frac{a}{\partial \pi}}$$

Ω

$E = \hbar\omega = \Lambda$

How many particles does Alice in Bob's vacuum

$$\hat{N}_\omega = b^-_\omega b^+_\omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\omega = {}_B\langle 0|\, b^-_\omega b^+_\omega\, |0\rangle_B$$

How many particles does Alice in Bob's vacuum

$$\hat{N}_\Omega = b^+_\Omega b^-_\Omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\Omega = {}_B\langle 0|\, b^+_\Omega b^-_\Omega\, |0\rangle_B$$

Ω

$E = \hbar\omega = \Lambda$

$= \Gamma[0] \int d\omega\, d\omega' \left(\alpha \right.$

How many particles does Alice in Bob's vacuum

$$\hat{N}_R = b^+_R b^-_R$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_R = {}_B\langle 0|\, b^+_R b^-_R\, |0\rangle_B$$

R

$E = \hbar R = R$

$$= \Gamma[0] \int d\omega\, d\omega' \left(\alpha^*_{R\omega} a^+_\omega + \beta^* \bar{a}_\omega \right)\left(\alpha \bar{a}_{\omega'} - \beta a^+_{\omega'} \right) |0\rangle_B$$

How many particles does Alice in Bob's vacuum

$$\hat{N}_\Omega = b^\dagger_\Omega b^-_\Omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\Omega = {}_B\langle 0|\, b^\dagger_\Omega b^-_\Omega\, |0\rangle_B$$

Ω

$E = \hbar\Omega = \Lambda$

$$= \Gamma(0)\int d\omega\, d\omega' \left(\alpha^*_{\Omega\omega} a^\dagger_\omega + \beta^* \bar{a}_\omega\right)\left(\alpha\, a^-_{\omega'} - \beta\, a^\dagger_{\omega'}\right)|0\rangle_B$$

$$\bar{a}_\omega\, a^\dagger_{\omega'} = a^\dagger_{\omega'}\,\bar{a}_\omega + \delta(\omega'-\omega)$$

$$N_\Omega = \int d\omega \, |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{2\pi\Omega/a} - 1}$$

→ Planck spectrum.

$$T = \frac{a}{2\pi}$$

$$\int d\omega \left(|\alpha_{\Omega\omega}|^2 - |\beta_{\Omega\omega}|^2 \right) = \delta(0)$$

$$\int d\omega \, |\beta|^2 \left(e^{2\pi\Omega/a} - 1 \right) = \delta(0)$$

$$N_\Omega = \int d\omega \, |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{2\pi\Omega/a} - 1}$$

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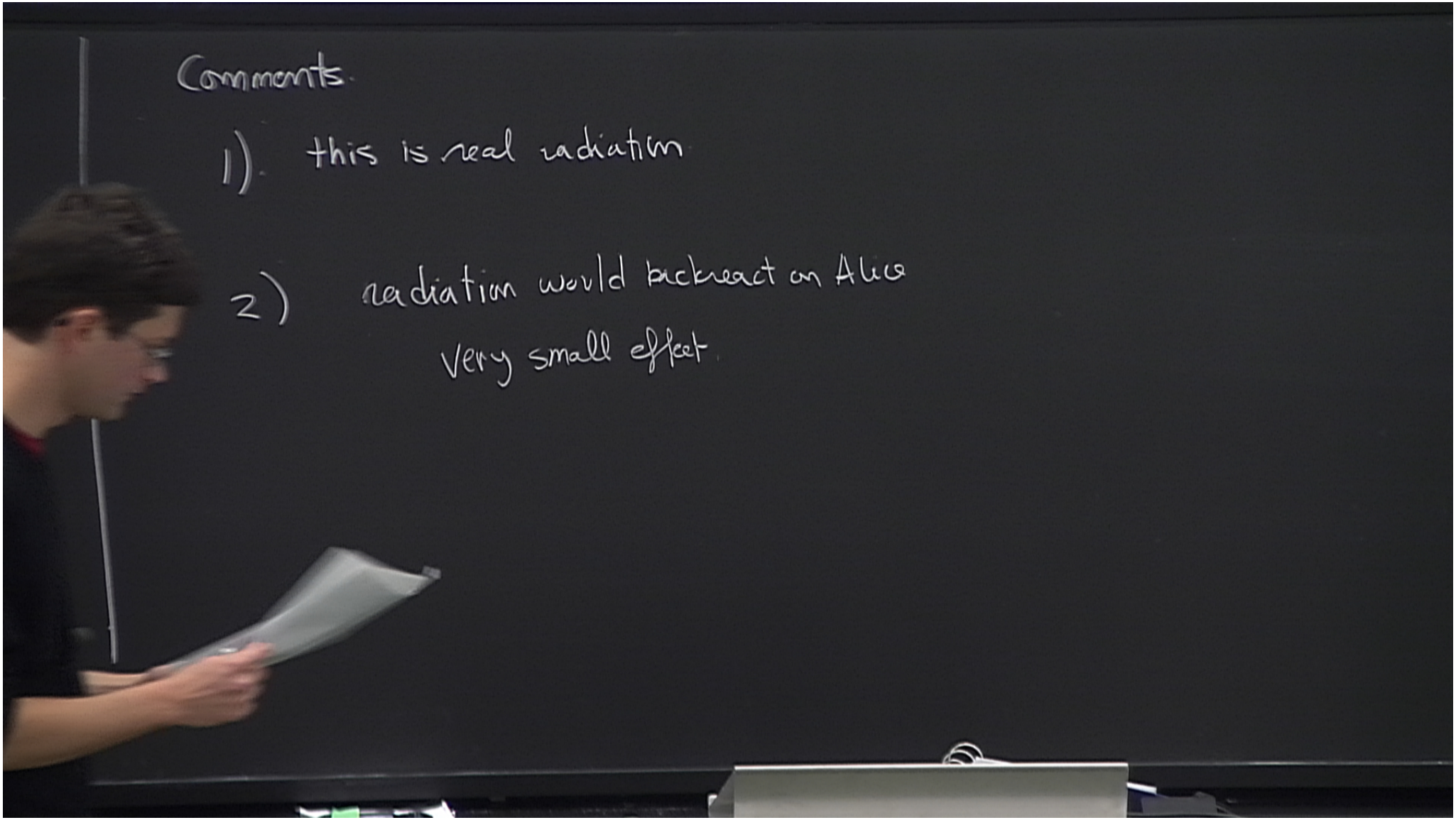
∞ volume

$\delta(0) \to V$

$$\int d\omega \, |\beta|^2 \left(e^{2\pi\Omega/a} - 1 \right) = \delta(0)$$

number density

$$n_\Omega = \frac{N_\Omega}{\delta(0)}$$


Comments
1) this is real radiation
2) radiation would backreact on Alice
very small effect

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approximations

3) $m=0$; 1+1 D; constant a forever

Particle Production

1) $\omega_k(\eta)$ ✓

2) Driven HO

Consider homogeneous, isotropic

FRW: $ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$

$d\eta = \frac{dt}{a(t)}$ $\quad ds^2 = a^2(\eta)\,(-d\eta^2 + d\vec{x}^2)$

in 3+1D ϕ is not conformal

w/ $m^2_{eff} = m^2 a^2 - \frac{a''}{a^2}$

$$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left((\partial\phi)^2 + m^2\phi^2\right)$$

$$= -\frac{1}{2}\int d^4x\; a^2(\eta)\left(-\phi'^2 + (\nabla\phi)^2 + m^2\phi^2\right)$$

$$1 = \frac{d}{d\eta}$$

$$\chi = a(\eta)\,\phi$$

$$= -\frac{1}{2}\int d^4x\left(-\chi'^2 + (\nabla\chi)^2 + m^2_{eff}(\eta)\chi^2\right)$$



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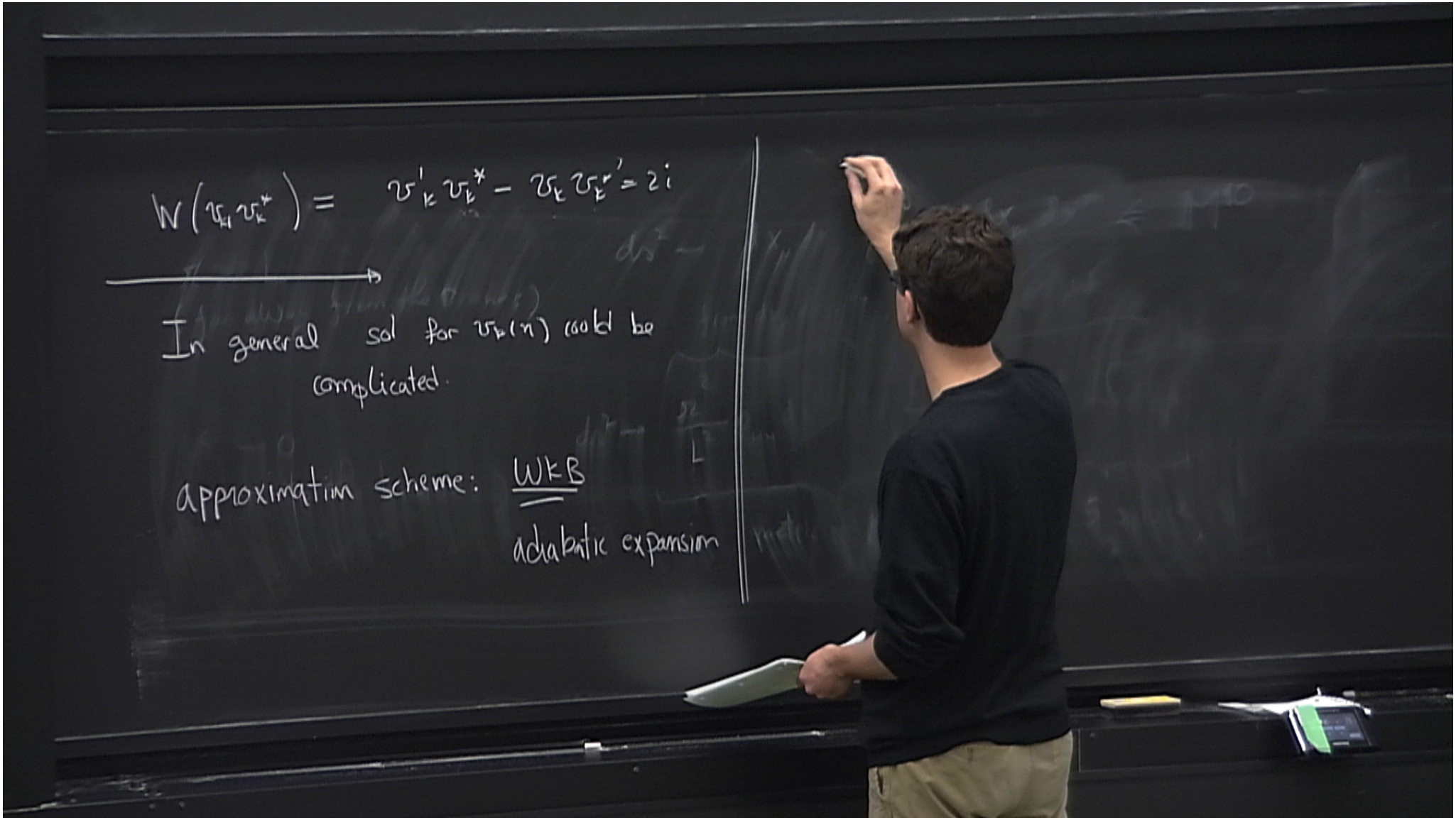
w/ $m^2_{\text{eff}} = m^2a^2 - \frac{a''}{a^2}$

→ exactly as in flat space

$$\chi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2}}\left(\bar{a}_k v_k^* e^{-ikx} + a_k^\dagger v_k e^{ikx}\right)$$

$$v_k'' + \omega_k^2(\eta)\, v_k = 0.$$

w/ $\omega_k^2(\eta) = k^2 + m^2_{\text{eff}}(\eta)$


$W(v_k, v_k^*) = v_k' v_k^* - v_k v_k^{*\prime} = 2i$
In general sol for $v_k(\eta)$ could be complicated.
approximatim scheme: WKB
adiabatic expansion

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approximatim scheme: WkB

adiabatic expansion

$\frac{\dot{\omega}}{\omega^2} \ll 1$

$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta')\, d\eta'\right)$

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approximation scheme: WKB

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$$\frac{\omega'}{\omega^2} \ll 1$$

$$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta')\, d\eta'\right)$$

new modes

$$u_k = \alpha_k v_k + \beta_k v_k^*$$

$v_k v_k^{*\prime} = 2i$

$\omega(\eta)$ could be

WKB

adiabatic expansion

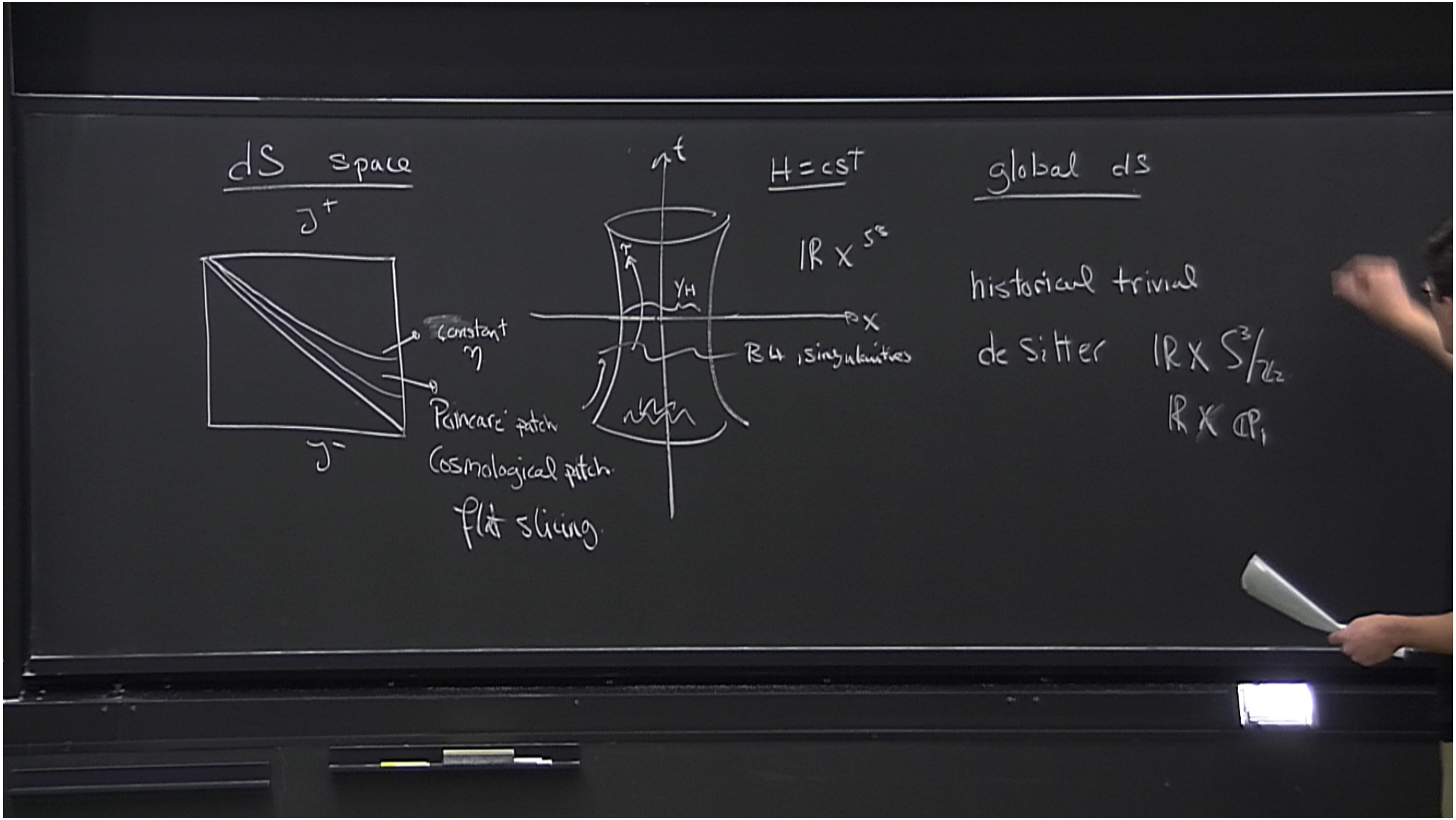
$\frac{\omega'}{\omega^2} \ll 1.$

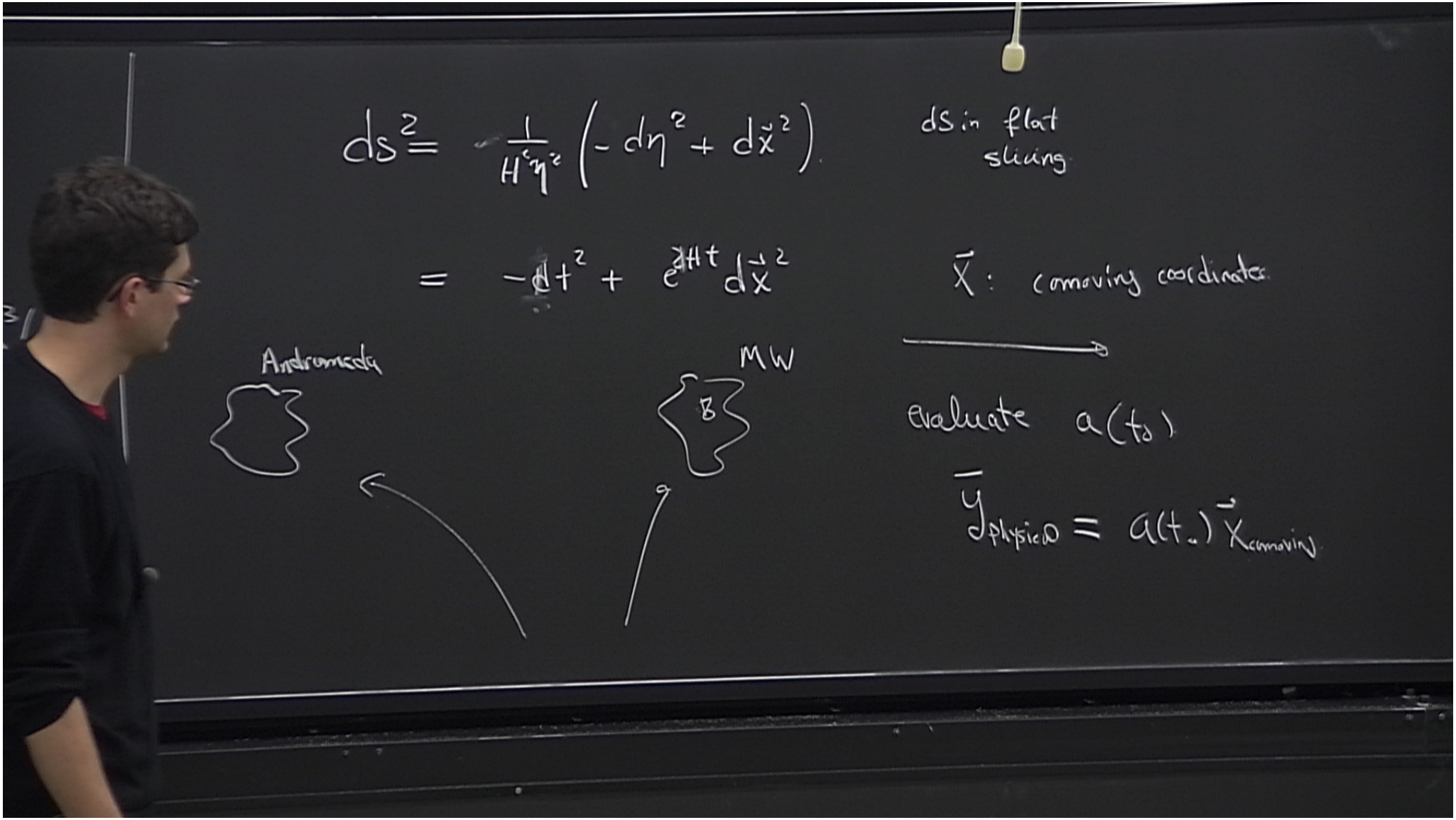
$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta)\, d\eta''\right)$

new modes

$u_k = \alpha_k v_k + \beta_k v_k^*$

$|\alpha_k|^2 - |\beta|^2 = 1.$


dS space
J^+
J^-
constant η
Poincaré patch
Cosmological patch
flat slicing
t
T_F
Y_H
x
R4, singularities
H=cst
$\mathbb{R} \times S^3$
global dS
historical trivial
de Sitter
$\mathbb{R} \times S^3/\mathbb{Z}_2$
$\mathbb{R} \times \mathbb{CP}_1$

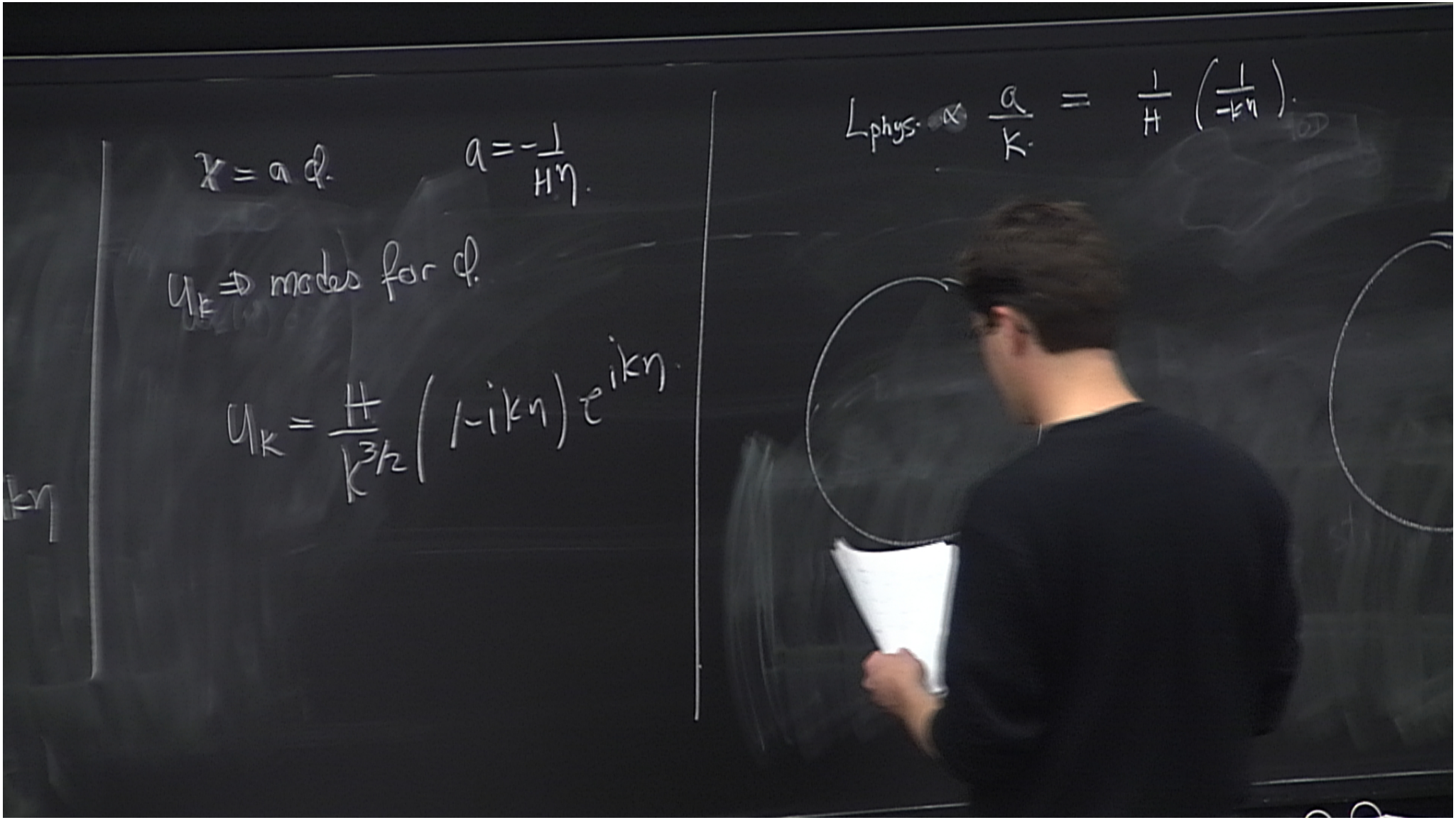

$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$
ds in flat slicing
$= -dt^2 + e^{2Ht} d\vec{x}^2$
$\vec{x}$: comoving coordinates
Andromeda
MW
B
Evaluate $a(t_0)$
$\vec{y}_{physical} = a(t_0)\vec{x}_{comoving}$

Modes in dS

$$a = \frac{-1}{H\eta} \qquad \omega_k^2(\eta) = k^2 + \left(\frac{m^2}{H^2} - 2\right)\frac{1}{\eta^2}$$

$m = 0$.

$$v_k'' + \left(\frac{-2}{\eta^2} + k^2\right) v_k = 0. \qquad v_k = \frac{1}{k^{3/2}\eta}\left(1 - ik\eta\right) e^{ik\eta}$$


$\chi = a\,\delta\phi$
$a = -\frac{1}{H\eta}$
$u_k \Rightarrow$ modes for $\delta\phi$
$u_k = \frac{H}{k^{3/2}}(1 - ik\eta)\,e^{ik\eta}$
$L_{phys.} \propto \frac{a}{k} = \frac{1}{H}\left(\frac{1}{-k\eta}\right)$

$(1-ik\eta)\, e^{ik\eta}$

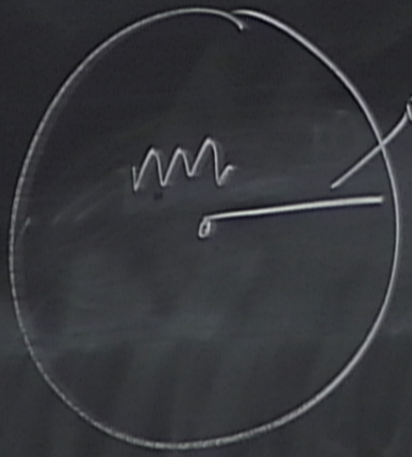
$\chi = a\,\delta\phi$. $\quad a = -\frac{1}{H\eta}$.

$u_k \Rightarrow$ modes for $\delta\phi$.

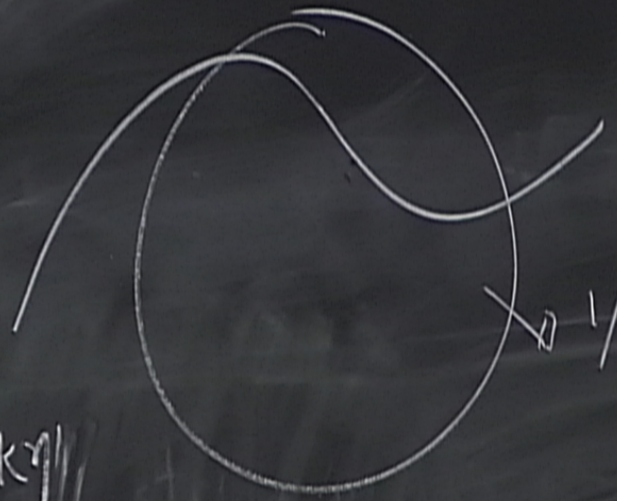
$$u_k = \frac{H}{k^{3/2}}(1-ik\eta)\, e^{ik\eta}.$$

$L_{phys.} \propto \frac{a}{k} = \frac{1}{H}\left(\frac{1}{-k\eta}\right)$

Sub-hubble modes



super-Hubble modes



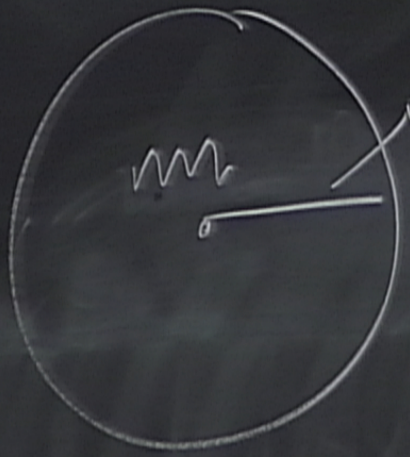
$v_k = \frac{1}{\sqrt{k}} e^{-ik\eta}$

$-k\eta \gg 1$

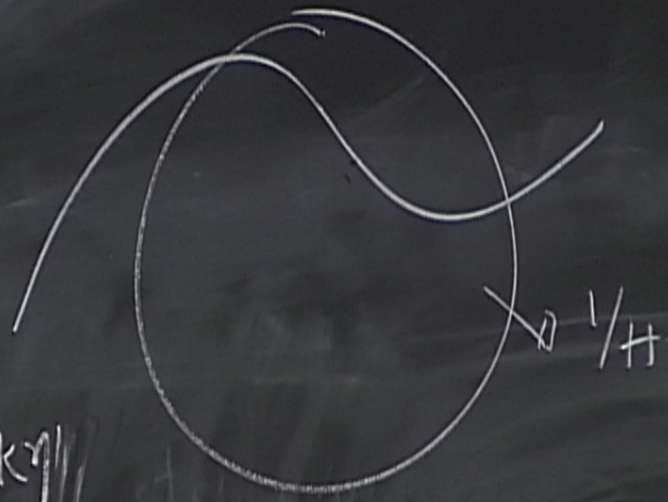
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