Title: Explorations in Cosmology - Lecture 4

Date: Mar 15, 2012 10:15 AM

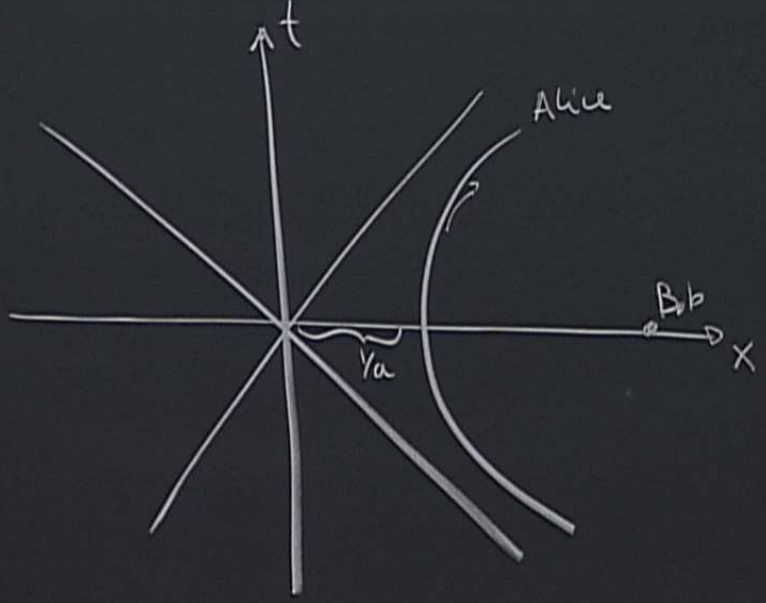
URL: http://pirsa.org/12030026

Abstract:

Unruh Effect
t
Alice
Ya

Recap.

Unruh Effect



Alice lives in Rindler wedge.
w/ coord $\tilde{u}, \tilde{v}$

Bob sees all of flat space
w/ u, v.

$u = \frac{1}{a} e^{a\tilde{u}}$ $\quad v = \frac{1}{a} e^{a\tilde{v}}$

Modes for Alice $e^{-i\omega\tilde{u}}$ $\quad e^{i\omega\tilde{u}}$

b^- $\quad b^+$

Bob $e^{-i\omega u}$ $\quad e^{i\omega v}$

Alice lives in Rindler wedge
w/ coord $\hat{u}, \hat{v}$

Bob sees all of flat space
w/ u, v

$u = \frac{1}{a} e^{a\hat{u}}$ $\quad v = \frac{1}{a} e^{a\hat{v}}$

$x^1 = 0$

Bob

x

Modes for Alice $\quad e^{-i\omega\hat{u}} \quad e^{i\omega\hat{u}}$

$b_\omega^- |0\rangle_A = 0$ $\quad\quad b^- \quad\quad b^+$

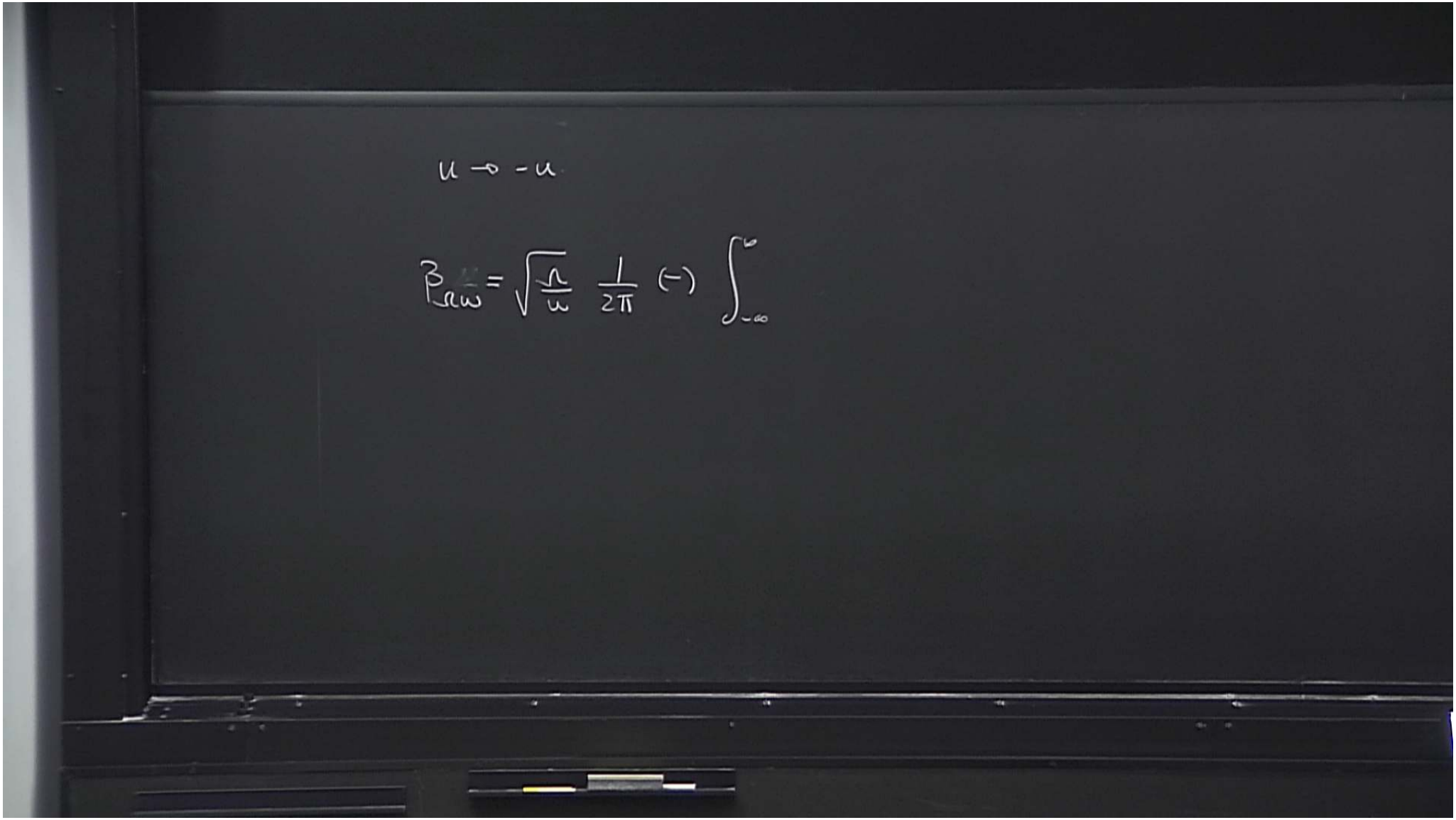
Bob $\quad e^{-i\omega u} \quad e^{i\omega v}$

$a_\omega^- |0\rangle_B = 0$ $\quad\quad a^- \quad\quad a^+$

these modes are related by
a Bogoliubov transf

$$b_r^- = \int_0^\infty d\omega \left(\alpha_{r\omega} a_\omega^- - \beta_{r\omega} a_\omega^+ \right)$$

$$\begin{matrix} \alpha_{r\omega} \\ \beta_{r\omega} \end{matrix} = \left\{ \frac{1}{2\pi} \sqrt{\frac{r}{\omega}} \left(\begin{matrix} - \\ + \end{matrix} \right) \int_{-\infty}^{\infty} \frac{du}{u} e^{\mp i\omega u} e^{i\frac{r}{a} \ln a u} \right.$$


$u \to -u$
$\beta_{\omega} = \sqrt{\frac{\Lambda}{\omega}} \frac{1}{2\pi} (-) \int_{-\infty}$

$u \to -u$ $\qquad -i\ln(-1) = \pi$

$$\beta_{\omega u} = \sqrt{\frac{\Omega}{u}}\,\frac{1}{2\pi}\,(-)\int_{-\infty}^{\infty}\frac{du}{u}\,e^{-i\omega u}\,e^{i\frac{\Omega}{a}\ln|u|}\,e^{i\frac{\Omega}{a}\ln(-1)}$$

$$= \alpha_{\omega u}\,e^{-\pi\Omega/a}$$

$$\boxed{|\beta|^2 = |\alpha|^2\,e^{-2\pi\Omega/a}}$$ detailed balance.

Thermal s

$P_{nm} =$

Thermal system $\rightarrow \frac{1}{T}$ ($k_b = 1$)

$$\rho_{nm} = \langle \psi_n | e^{-\beta H} | \psi_m \rangle = |\alpha|^2 e^{-\beta E_m}$$

$$\rho_{mn} = \qquad\qquad = |\beta|^2 e^{-\beta E_n}$$

$$\rho_{nm} = \rho_{mn} \Rightarrow$$

Thermal system $\rightarrow \frac{1}{T}$ ($k_B \equiv 1$)

$$\rho_{nm} = \langle \psi_n | e^{-\beta H} | \psi_m \rangle = |\alpha|^2 e^{-\beta E_m}$$

$$\rho_{mn} = = |\beta|^2 e^{-\beta E_n}$$

$$\rho_{nm} = \rho_{mn} \Rightarrow |\beta|^2 = |\alpha|^2 e^{-\beta \Delta E}$$ ($\Delta \rightarrow \Omega$)

$E = \hbar\omega = \hbar$

$$\frac{2\pi}{a} = \frac{1}{T}; \quad \boxed{T = \frac{a}{2\pi}}$$

Ω

$E = \hbar\omega = \Lambda$

How many particles does Alice in Bob's vacuum

$$\hat{N}_\omega = \bar{b}_\omega b^+_\omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\omega = {}_B\langle 0|\, \bar{b}_\omega b^+_\omega\, |0\rangle_B$$

How many particles does Alice in Bob's vacua

$$\hat{N}_\Omega = b^+_\Omega b^-_\Omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\Omega = {}_B\langle 0|\, b^+_\Omega b^-_\Omega |0\rangle_B$$

Ω

$E = \hbar\Omega = \Omega$

$$= \langle 0| \int d\omega\, d\omega' \Big(\alpha$$

How many particles does Alice in Bob's vacuum

$$\hat{N}_R = b^+_R b^-_R$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_R = {}_B\langle 0|\, b^+_R b^-_R\, |0\rangle_B$$

$$= \Gamma[0] \int d\omega\, d\omega' \left(\alpha^*_{R\omega} a^+_\omega + \beta^* \bar{a}_\omega \right)\left(\alpha\, \bar{a}_{\omega'} - \beta\, a^+_{\omega'} \right) |0\rangle_B$$

R

$E = \hbar R = \hbar$

How many particles does Alice in Bob's vacuum

$$\hat{N}_\Omega = b^\dagger_\Omega b^-_\Omega$$

$$\bar{a}\,|0\rangle_B = 0$$

$$N_\Omega = {}_B\langle 0|\, b^\dagger_\Omega b^-_\Omega\, |0\rangle_B$$

Ω

$E = \hbar\Omega = \Lambda$

$$= {}_B\langle 0| \int d\omega\, d\omega' \left(\alpha^*_{\Omega\omega} a^\dagger_\omega + \beta^* \bar{a}_\omega\right)\left(\alpha\, a^-_{\omega'} - \beta\, a^\dagger_{\omega'}\right)|0\rangle_B$$

$$\bar{a}_\omega\, a^\dagger_{\omega'} = a^\dagger_{\omega'}\, \bar{a}_\omega + \delta(\omega'-\omega)$$

$$N_\Omega = \int d\omega \, |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{2\pi\Omega/a} - 1}$$

→ Planck spectrum.

$$T = \frac{a}{2\pi}$$

$$\int d\omega \left(|\alpha_{\Omega\omega}|^2 - |\beta_{\Omega\omega}|^2 \right) = \delta(0)$$

$$\int d\omega \, |\beta|^2 \left(e^{2\pi\Omega/a} - 1 \right) = \delta(0)$$

$$N_\Omega = \int d\omega \, |\beta_{\Omega\omega}|^2 = \frac{\delta(0)}{e^{2\pi\Omega/a} - 1}$$

→ Planck spectrum.

$$T = \frac{a}{2\pi}$$

$$\int d\omega \left(|\alpha_{\Omega\omega}|^2 - |\beta_{\Omega\omega}|^2 \right) = \delta(0)$$

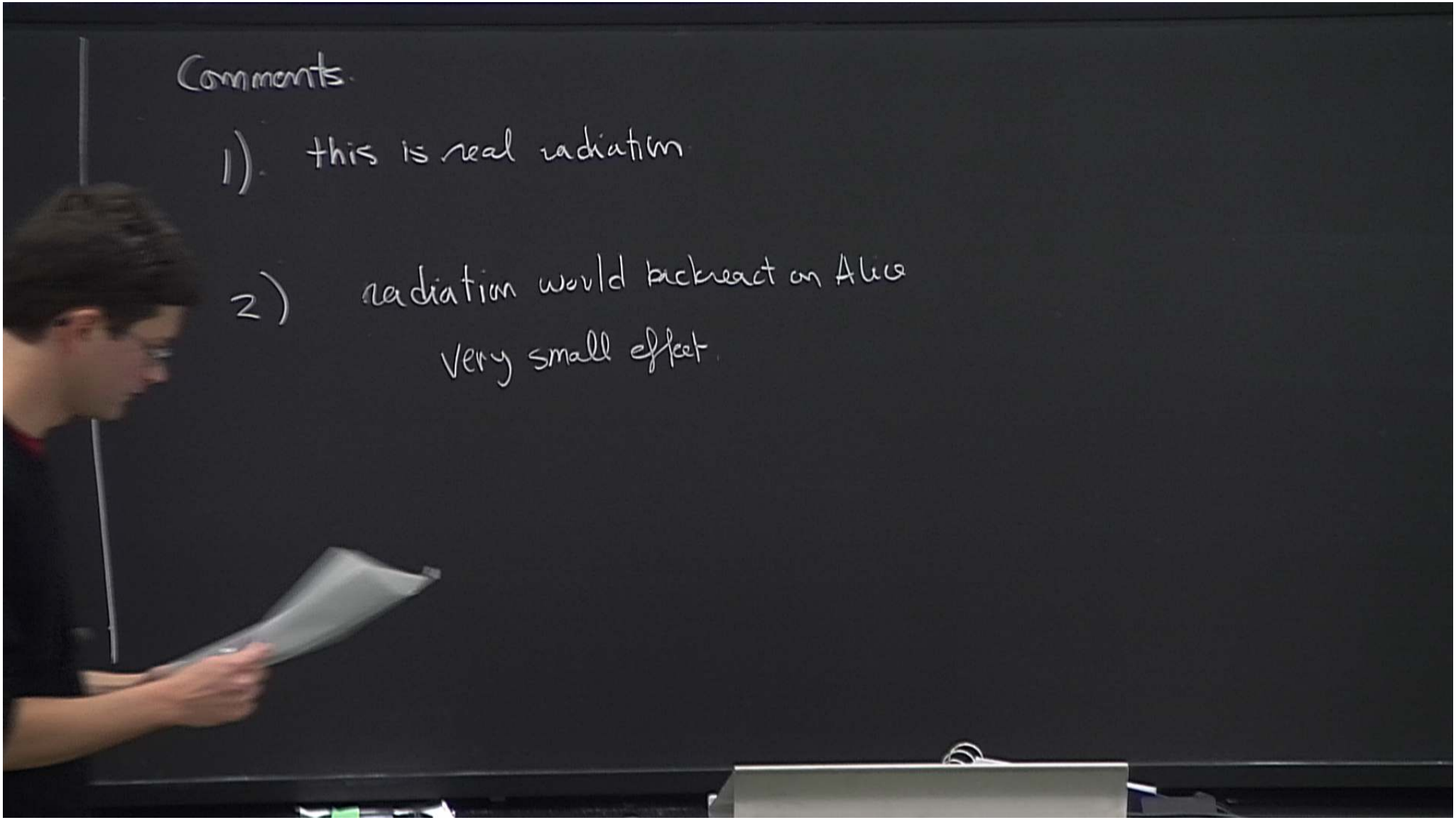
∞ volume

$\delta(0) \to V$

$$\int d\omega \, |\beta|^2 \left(e^{2\pi\Omega/a} - 1 \right) = \delta(0)$$

number density

$$n_\Omega = \frac{N_\Omega}{\delta(0)}$$


Comments
1) this is real radiation
2) radiation would backreact on Alice
very small effect

Comments

1) this is real radiation

2) radiation would backreact on Alice

very small effect

approximations

3) $m=0$, 1+1 D, constant a forever

Particle Production

1) $\omega_k(\eta)$ ✓

2) Driven HO

Consider homogeneous, isotropic

FRW: $ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$

$d\eta = \frac{dt}{a(t)}$ $\quad ds^2 = a^2(\eta)\,(-d\eta^2 + d\vec{x}^2)$

in 3+1D ϕ is not conformal

w/ $m^2_{eff} = m^2 a^2 - \frac{a''}{a^2}$

$$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left((\partial\phi)^2 + m^2\phi^2\right)$$

$$= -\frac{1}{2}\int d^4x\; a^2(\eta)\left(-\phi'^2 + (\nabla\phi)^2 + m^2\phi^2\right)$$

$$1 = \frac{d}{d\eta}$$

$$\chi = a(\eta)\,\phi$$

$$= -\frac{1}{2}\int d^4x\left(-\chi'^2 + (\nabla\chi)^2 + m^2_{eff}(\eta)\chi^2\right)$$

in 3+1D ϕ is not conformal

$$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left((\partial\phi)^2 + m^2\phi^2\right)$$

$$= -\frac{1}{2}\int d^4x\; a^2(\eta)\left(-\phi'^2 + (\nabla\phi)^2 + m^2\phi^2\right)$$

$$' = \frac{d}{d\eta}$$

$$\chi = a(\eta)\,\phi$$

$$= -\frac{1}{2}\int d^4x\left(-\chi'^2 + (\nabla\chi)^2 + m^2_{\text{eff}}(\eta)\chi^2\right)$$

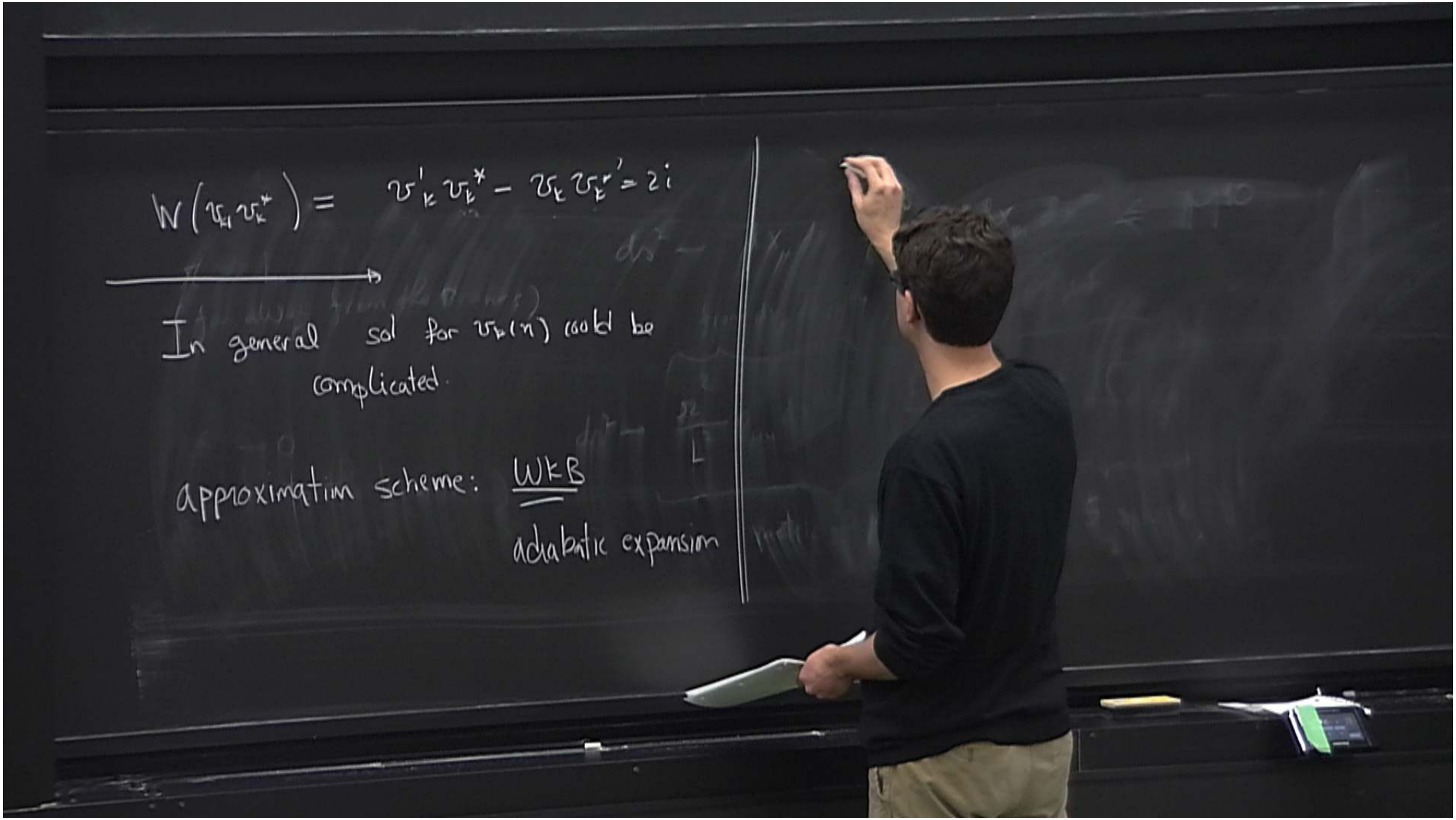
w/ $m^2_{\text{eff}} = m^2a^2 - \frac{a''}{a^2}$

→ exactly as in flat space

$$\chi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}}\frac{1}{\sqrt{2}}\left(\bar{a}_k v_k^* e^{-ikx} + a_k^\dagger v_k e^{ikx}\right)$$

$$v_k'' + \omega_k^2(\eta)\, v_k = 0.$$

w/ $\omega_k^2(\eta) = k^2 + m^2_{\text{eff}}(\eta)$


$W(v_k, v_k^*) = v_k' v_k^* - v_k v_k^{*\prime} = 2i$
In general sol for $v_k(\eta)$ could be complicated.
approximatim scheme: WKB
adiabatic expansion

$W(v_k, v_k^*) = v_k' v_k^* - v_k v_k^{*\prime} = 2i$

In general sol for $v_k(\eta)$ could be complicated.

approximation scheme: WKB

adiabatic expansion

$\frac{\dot{\omega}}{\omega^2} \ll 1$

$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta')\, d\eta'\right)$

$W(v_k, v_k^*) = v_k' v_k^* - v_k v_k^{*\prime} = 2i$

In general sol for $v_k(\eta)$ could be complicated.

approximatim scheme: WkB

adiabatic expansion

$\frac{\omega'}{\omega^2} \ll 1$

$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta')\, d\eta'\right)$

new modes

$u_k = \alpha_k v_k + \beta_k v_k^*$

$- v_k v_k^{*\prime} = 2i$

$\omega(\eta)$ could be

WKB

adiabatic expansion

$\frac{\omega'}{\omega^2} \ll 1.$

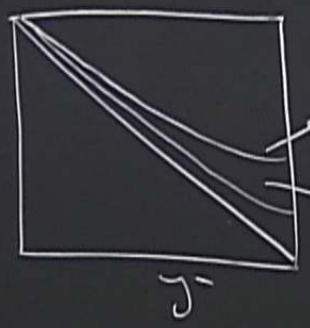
$v_k \propto \frac{1}{\sqrt{\omega_k(\eta)}} \exp\left(\pm i \int_{\eta_0}^{\eta} \omega_k(\eta)\, d\eta'' \right)$

new modes

$u_k = \alpha_k v_k + \beta_k v_k^*$

$|\alpha_k|^2 - |\beta|^2 = 1.$

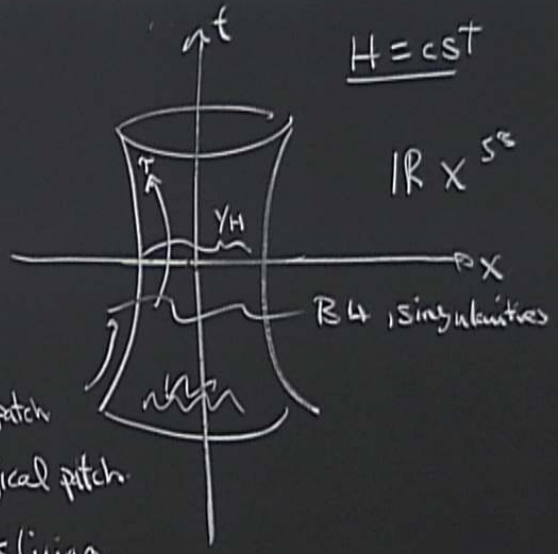
dS space



Poincaré patch

Cosmological patch

Flat slicing



$H = cst$

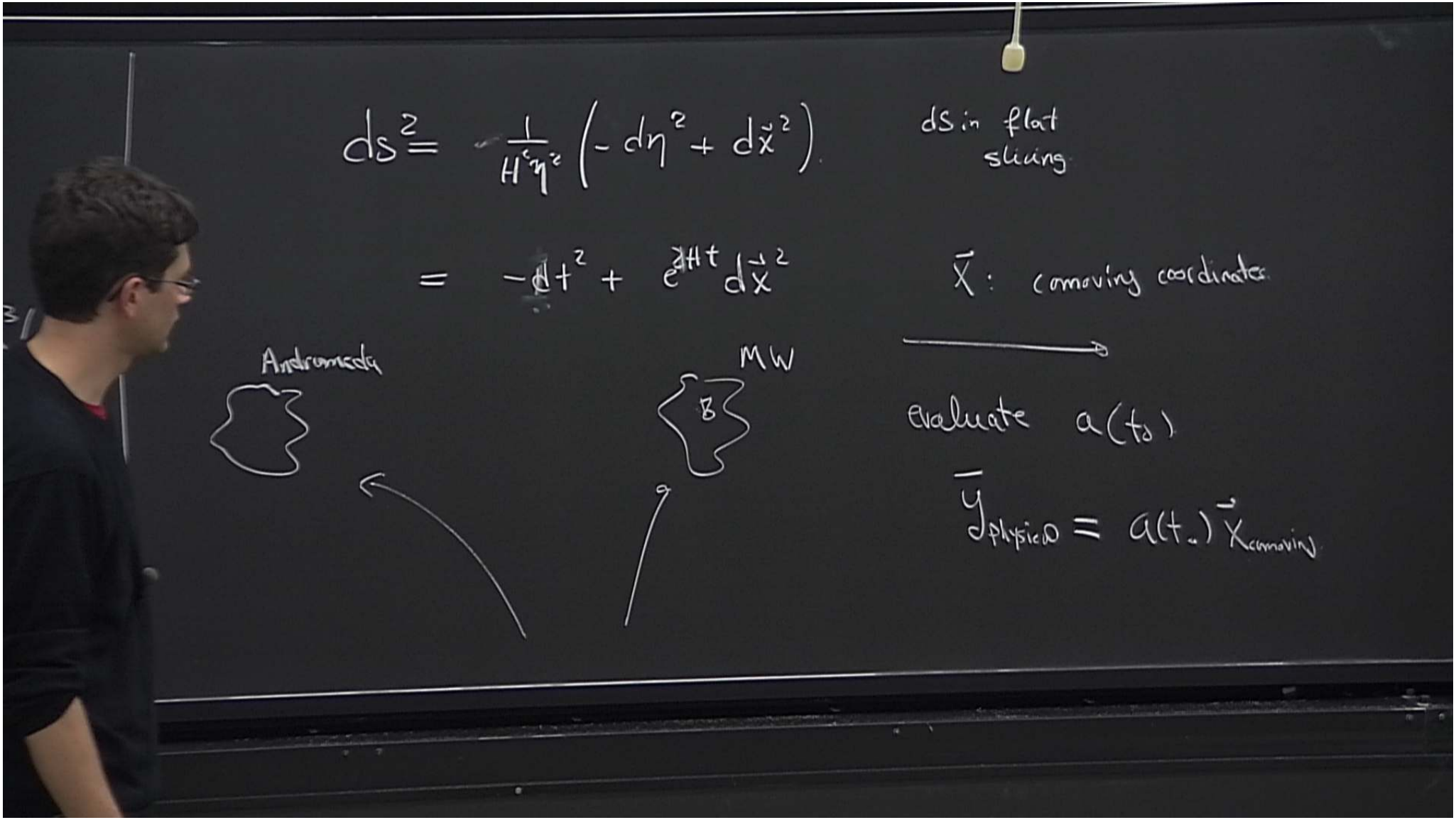
$\mathbb{R} \times S^3$

global dS

historical trivial

de Sitter $\mathbb{R} \times S^3/\mathbb{Z}_2$

$\mathbb{R} \times \mathbb{CP}_1$

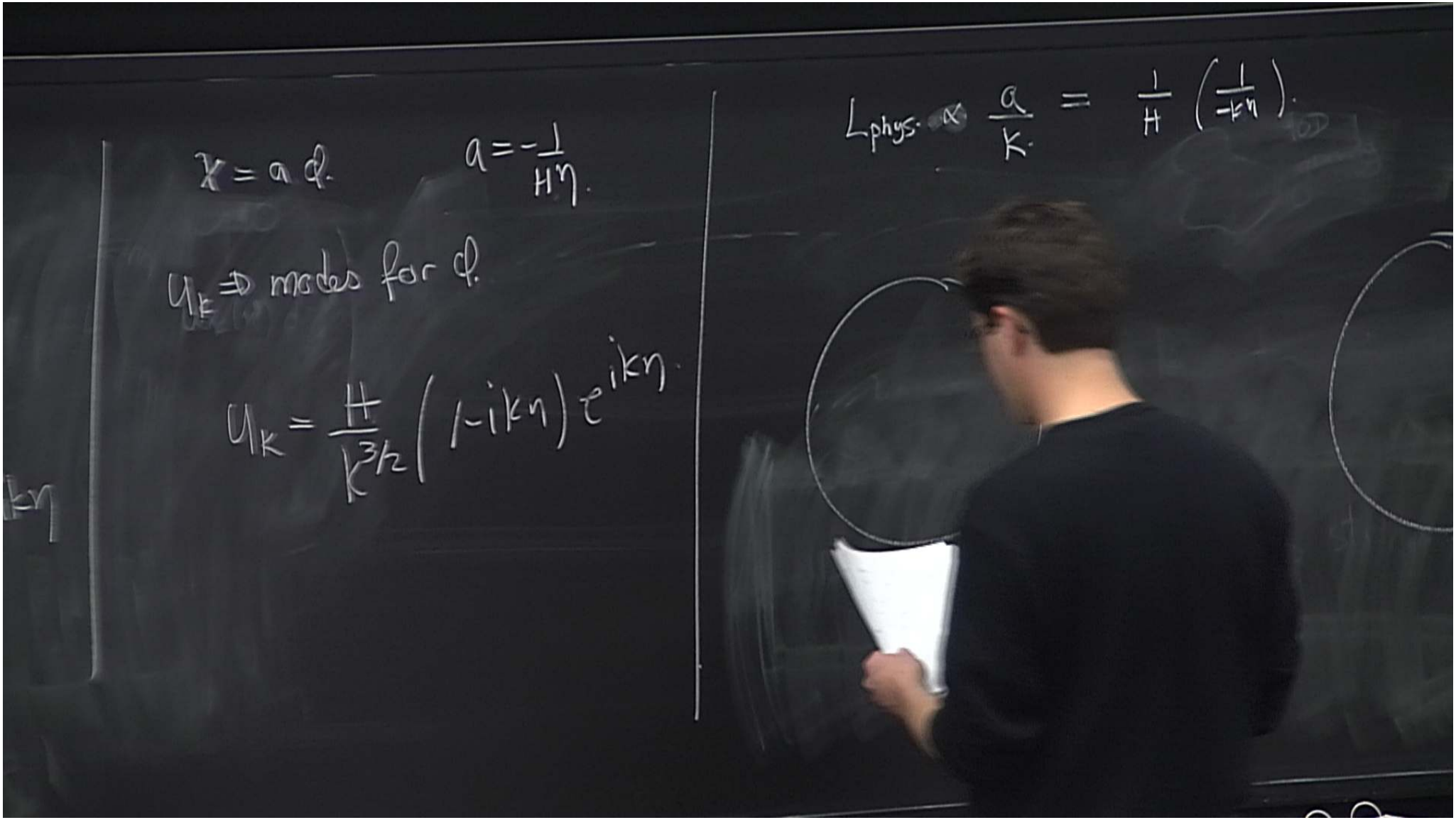

$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + d\vec{x}^2\right)$
dS in flat slicing
$= -dt^2 + e^{2Ht} d\vec{x}^2$
$\vec{x}$: comoving coordinates
Andromeda
MW
B
Evaluate $a(t_0)$
$\vec{y}_{physical} = a(t_0)\vec{x}_{comoving}$

Modes in dS

$$a = \frac{-1}{H\eta} \qquad \omega_k^2(\eta) = k^2 + \left(\frac{m^2}{H^2} - 2\right)\frac{1}{\eta^2}$$

$m = 0$.

$$v_k'' + \left(\frac{-2}{\eta^2} + k^2\right) v_k = 0. \qquad v_k = \frac{1}{k^{3/2}\eta}\left(1 - ik\eta\right) e^{ik\eta}$$


$\chi = a\,\delta\phi$
$a = -\frac{1}{H\eta}$
$u_k \Rightarrow$ modes for $\delta\phi$
$u_k = \frac{H}{k^{3/2}}(1 - ik\eta)\,e^{ik\eta}$
$L_{phys.} \propto \frac{a}{k} = \frac{1}{H}\left(\frac{1}{-k\eta}\right)$

$(1-ik\eta)\, e^{ik\eta}$

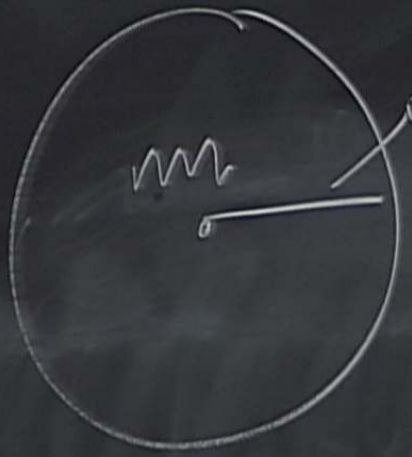
$\chi = a\,\delta\phi$. $\quad a = -\frac{1}{H\eta}$.

$u_k \Rightarrow$ modes for $\delta\phi$.

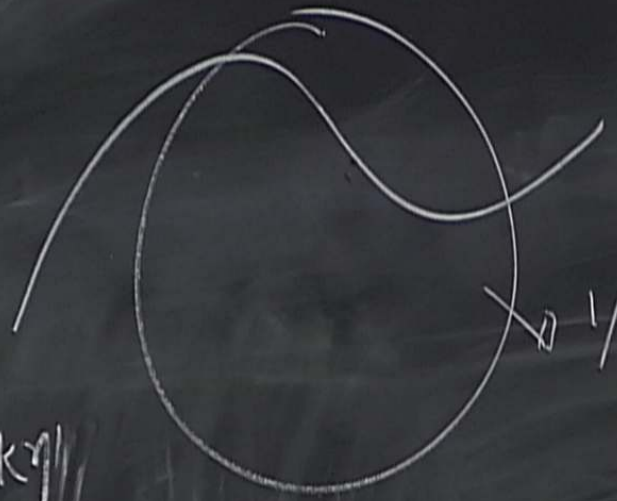
$$u_k = \frac{H}{k^{3/2}}(1-ik\eta)\, e^{ik\eta}.$$

$L_{phys.} \propto \frac{a}{k} = \frac{1}{H}\left(\frac{1}{-\dot{\eta}}\right)$

Sub-hubble modes



super-Hubble modes



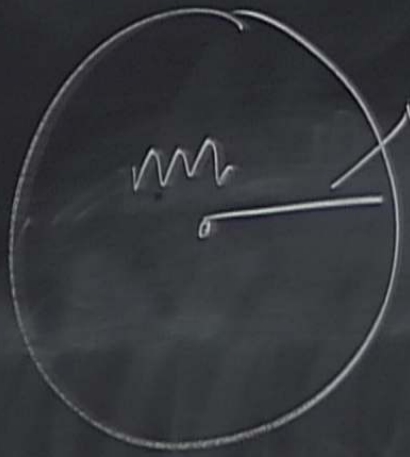
$v_k = \frac{1}{\sqrt{k}} e^{-ik\eta}$

$-k\eta \gg 1$

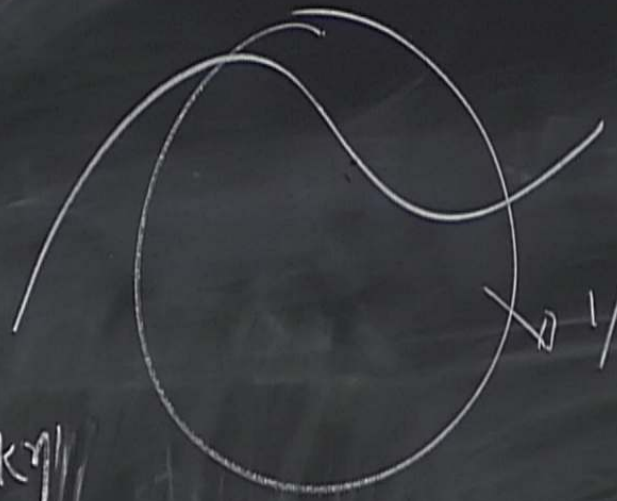
$-k\eta \ll 1$

$L_{phys.} \propto \frac{a}{k} = \frac{1}{H}\left(\frac{1}{-\dot{\eta}}\right)$

Sub-hubble modes



super-Hubble modes



$v_k = \frac{1}{\sqrt{k}} e^{-ik\eta}$

$-k\eta \gg 1$

$-k\eta \ll 1$