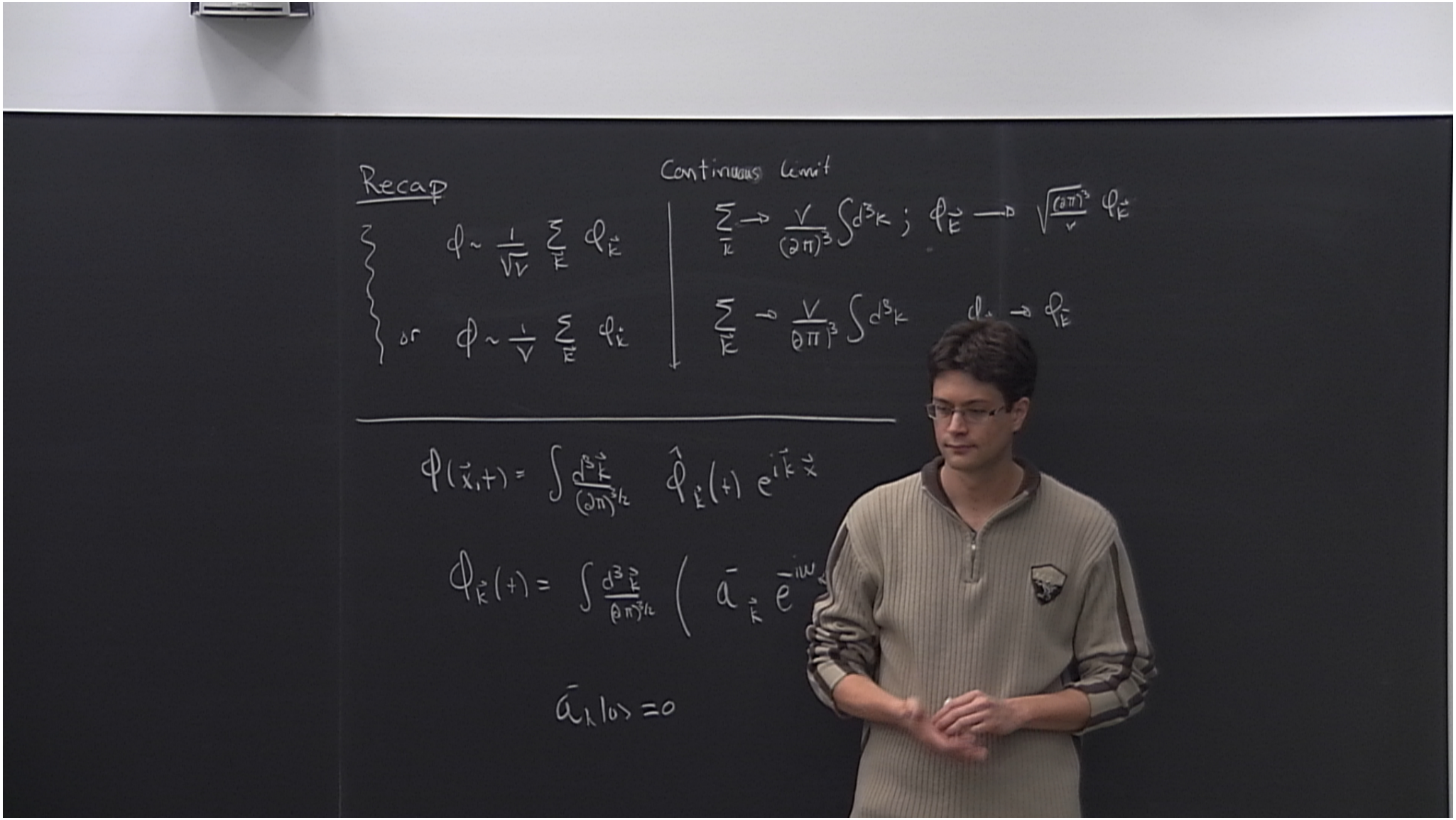
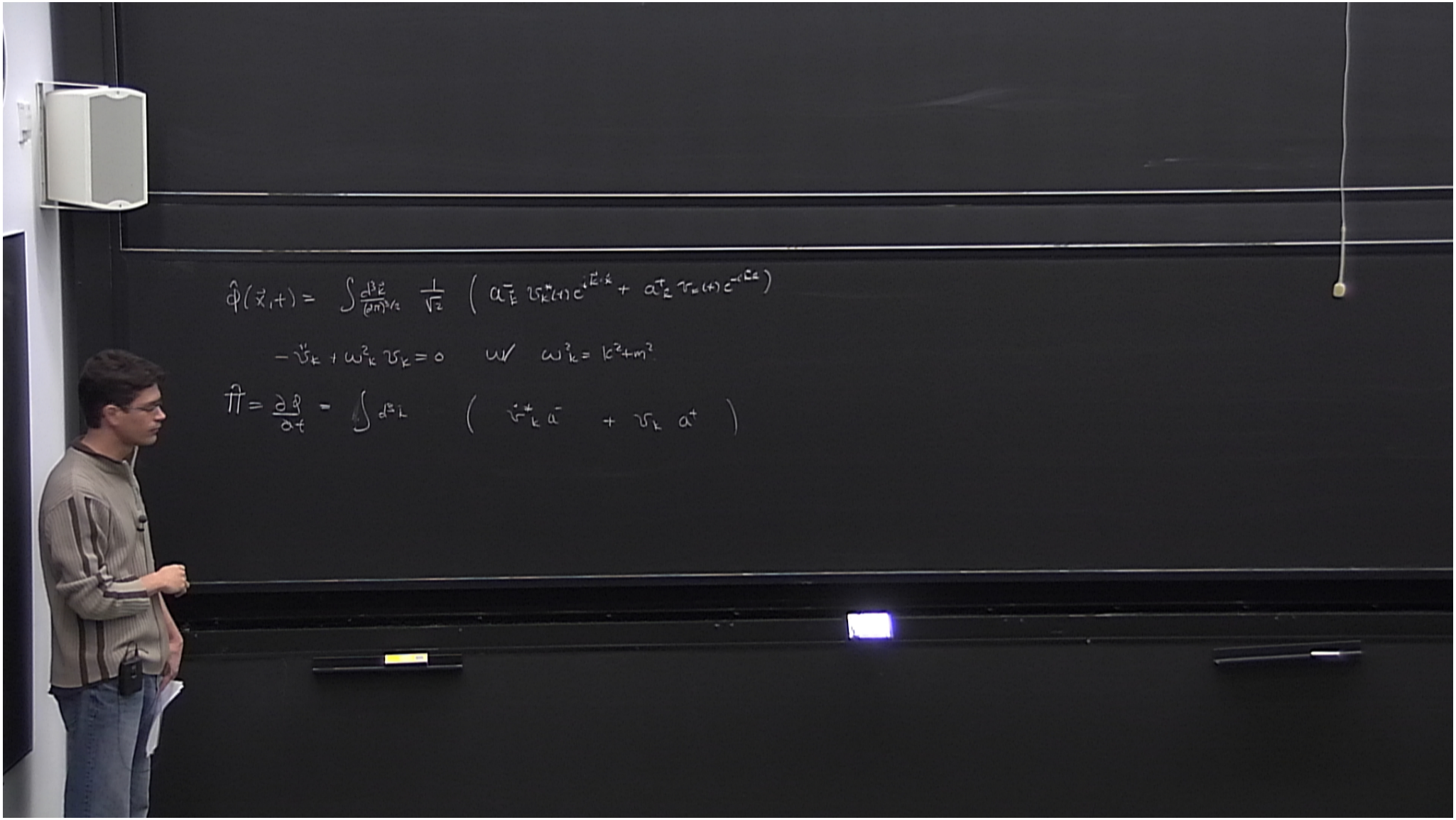
Title: Explorations in Cosmology - Lecture 3

Date: Mar 14, 2012 10:15 AM

URL: http://pirsa.org/12030025

Abstract:


Recap
Continuous Limit
or


$\hat{\phi}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2}} \left(a^-_{\vec{k}} v_k^*(t) e^{i\vec{k}\cdot\vec{x}} + a^+_{\vec{k}} v_k(t) e^{-i\vec{k}\cdot\vec{x}} \right)$
$-\ddot{v}_k + \omega_k^2 v_k = 0 \quad \omega^2_k = k^2 + m^2$
$\hat{\pi} = \frac{\partial \hat{\phi}}{\partial t} = \int d^3k \left(\dot{v}^*_k a^- + \dot{v}_k a^+ \right)$

Recap

$$\phi \sim \frac{1}{\sqrt{V}} \sum_{\vec{k}} \phi_{\vec{k}}$$

or

$$\phi \sim \frac{1}{V} \sum_{\vec{k}} \phi_{\vec{k}}$$

Continuous limit

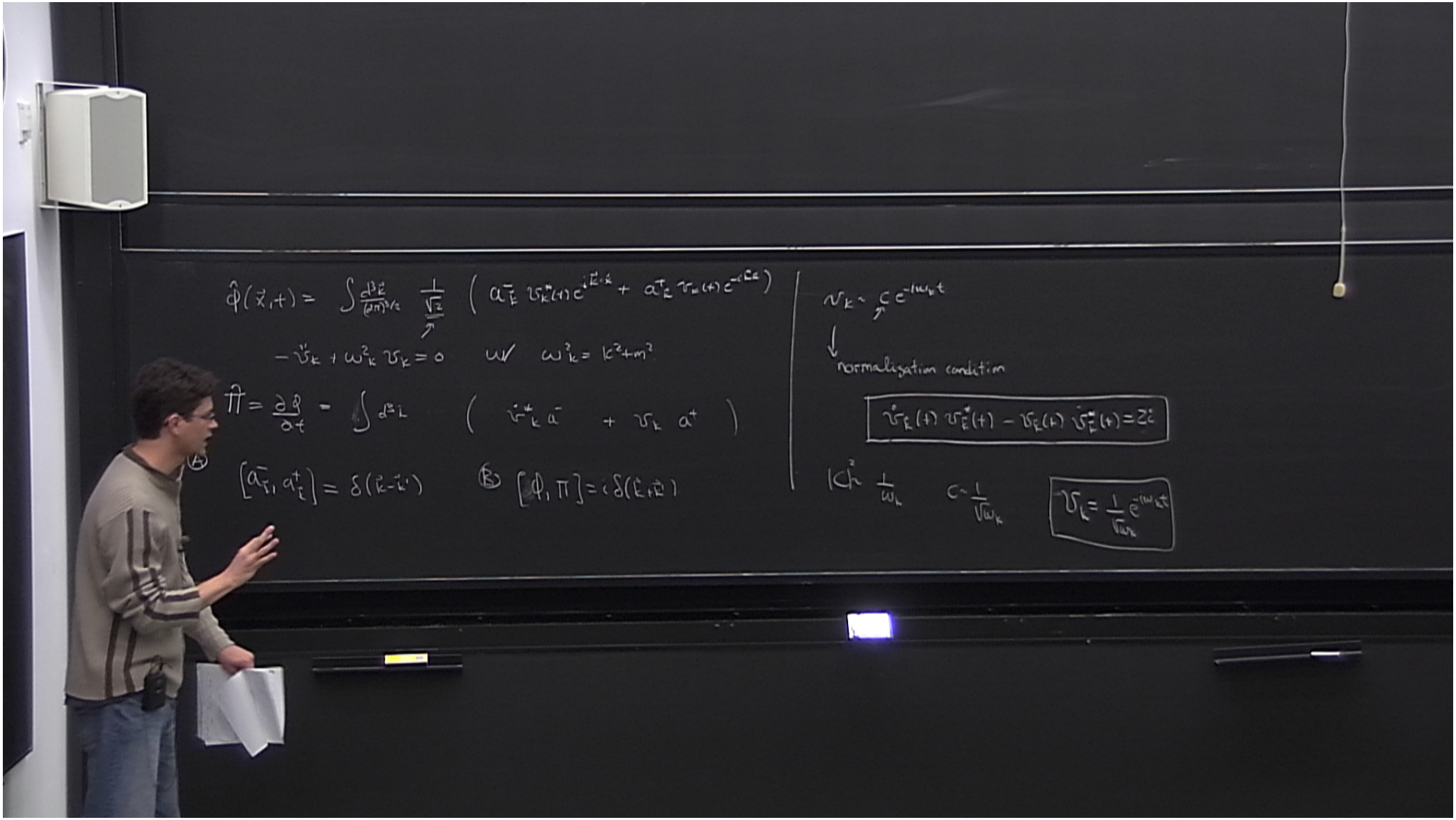
$$\sum_{\vec{k}} \to \frac{V}{(2\pi)^3} \int d^3k \; ; \quad \phi_{\vec{k}} \to \sqrt{\frac{(2\pi)^3}{V}}\, \phi_{\vec{k}}$$

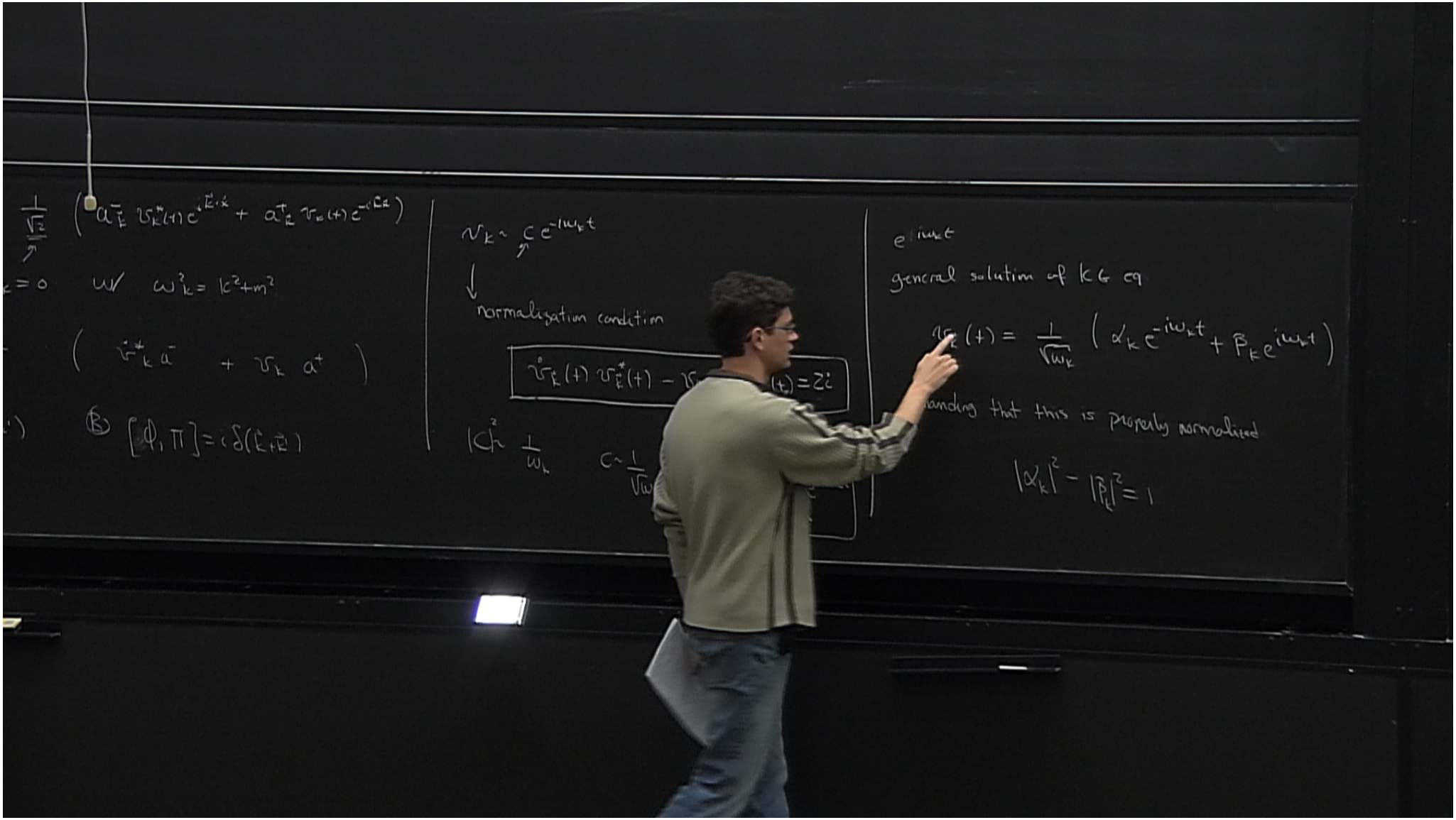
$$\sum_{\vec{k}} \to \frac{V}{(2\pi)^3} \int d^3k \; ; \quad \phi_{\vec{k}} \to \phi_{\vec{k}}$$

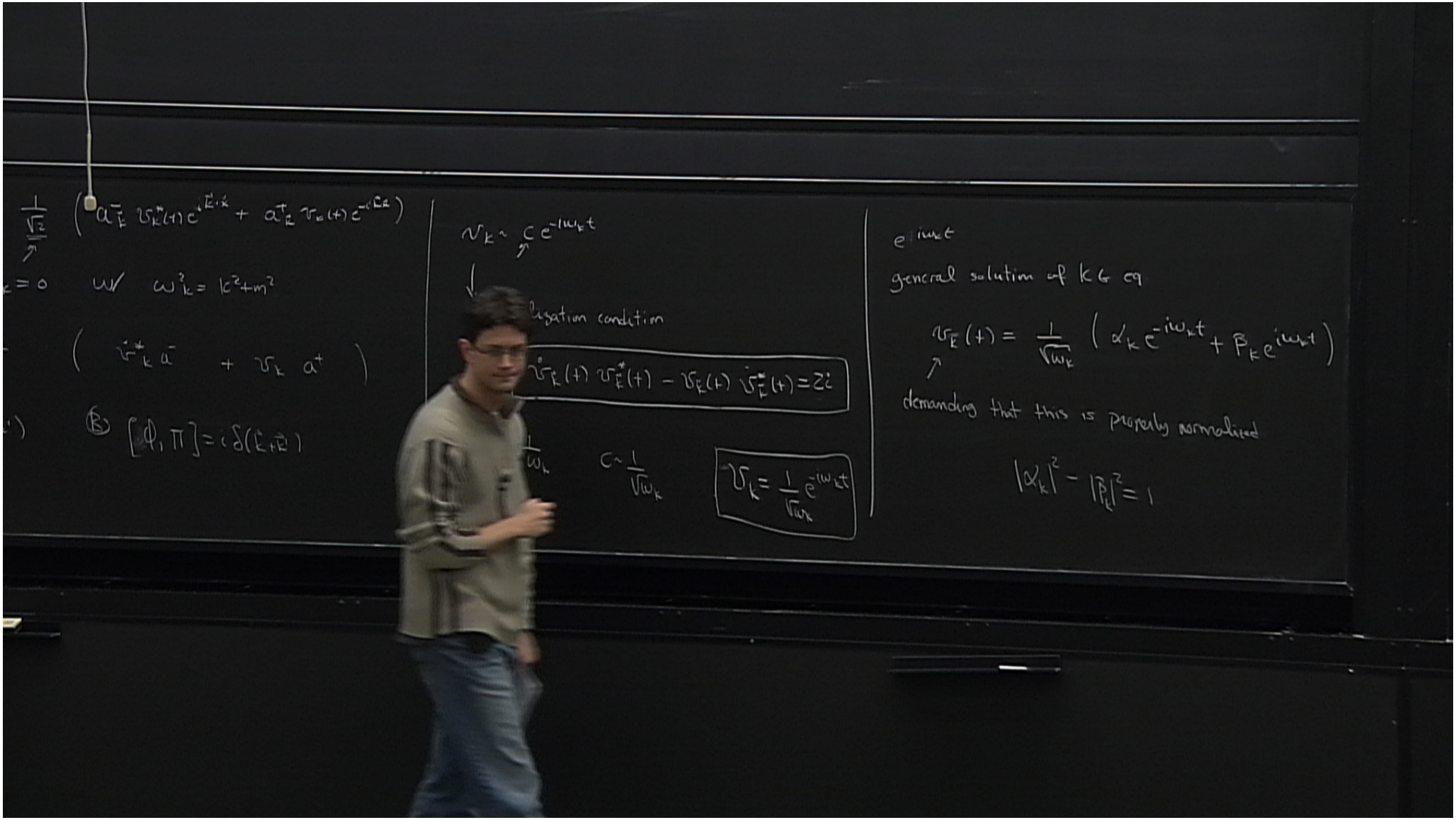
$$\phi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}}\, \hat{\phi}_{\vec{k}}(t)\, e^{i\vec{k}\cdot\vec{x}}$$

$$\phi_{\vec{k}}(t) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \left(a_{\vec{k}}\, e^{-i\omega_k t} e^{i\vec{k}\cdot\vec{x}} + a^\dagger_{\vec{k}}\, e^{i\omega_k t} e^{-i\vec{k}\cdot\vec{x}} \right) \frac{1}{\sqrt{2\omega_k}}$$

$$a_{\vec{k}} |0\rangle = 0$$


$\hat{\phi}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2}} \left(a_{\vec{k}}^- v_k^*(t) e^{i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^+ v_k(t) e^{-i\vec{k}\cdot\vec{x}} \right)$
$-\ddot{v}_k + \omega_k^2 v_k = 0 \quad \omega_k^2 = k^2 + m^2$
$\hat{\pi} = \frac{\partial \hat{\phi}}{\partial t} = \int d^3k \left(\dot{v}_k^* a^- + v_k a^+ \right)$
A
$[a_{\vec{k}}^-, a_{\vec{k}'}^+] = \delta(\vec{k}-\vec{k}')$
B
$[\phi, \pi] = i\delta(\vec{x}-\vec{x}')$
$v_k \sim \frac{c}{\sqrt{\omega_k}} e^{-i\omega_k t}$
normalisation condition
$\dot{v}_k(t) v_k^*(t) - v_k(t) \dot{v}_k^*(t) = 2i$
$|c|^2 \frac{1}{\omega_k} \quad c \sim \frac{1}{\sqrt{\omega_k}}$
$v_k = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$


normalisation condition
general solution of KG eq
$\omega_k^2 = k^2 + m^2$
$[\phi, \pi] = i\delta(\vec{k}+\vec{k}')$
$|\alpha_k|^2 - |\beta_k|^2 = 1$


$\frac{1}{\sqrt{2}}\left(a_{\vec{k}}\, v_k^*(t)\, e^{i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^\dagger\, v_k(t)\, e^{-i\vec{k}\cdot\vec{x}}\right)$
$\omega_k^2 = k^2 + m^2$
$\left(v_k^* a^- + v_k a^+\right)$
$[\phi, \pi] = i\delta(\vec{x}-\vec{x}')$
$v_k \sim c\, e^{-i\omega_k t}$
$v_k(t)\, v_k^*(t) - v_k(t)\, v_k^*(t) = 2i$
$c \sim \frac{1}{\sqrt{\omega_k}}$
$v_k = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$
$e^{i\omega t}$
general solution of KG eq
$v_k(t) = \frac{1}{\sqrt{\omega_k}}\left(\alpha_k e^{-i\omega_k t} + \beta_k e^{i\omega_k t}\right)$
demanding that this is properly normalized
$|\alpha_k|^2 - |\beta_k|^2 = 1$

$$\hat{\phi}(\vec{x},t) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2}} \left(a_{\vec{k}}^- v_k^*(t) e^{i\vec{k}\cdot\vec{x}} + a_{\vec{k}}^+ v_k(t) e^{-i\vec{k}\cdot\vec{x}} \right)$$

$$-\ddot{v}_k + \omega_k^2 v_k = 0 \quad \text{w/} \quad \omega_k^2 = k^2 + m^2$$

$$\hat{\pi} = \frac{\partial \hat{\phi}}{\partial t} = \int d^3k \left(\dot{v}_k^* a^- + v_k a^+ \right)$$

$$\left[a_{\vec{k}}^-, a_{\vec{k}'}^+ \right] = \delta(\vec{k}-\vec{k}') \qquad \text{(B)} \quad \left[\hat{\phi}, \hat{\pi} \right] = i\delta(\vec{x}-\vec{x}')$$

$v_k \sim c\, e^{-i\omega_k t}$

↓

normalisation condition

$$\dot{v}_k(t)\, v_k^*(t) - v_k(t)\, \dot{v}_k^*(t) = 2i$$

$|c|^2 \frac{1}{\omega_k} \qquad c \sim \frac{1}{\sqrt{\omega_k}}$

$$v_k = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$$

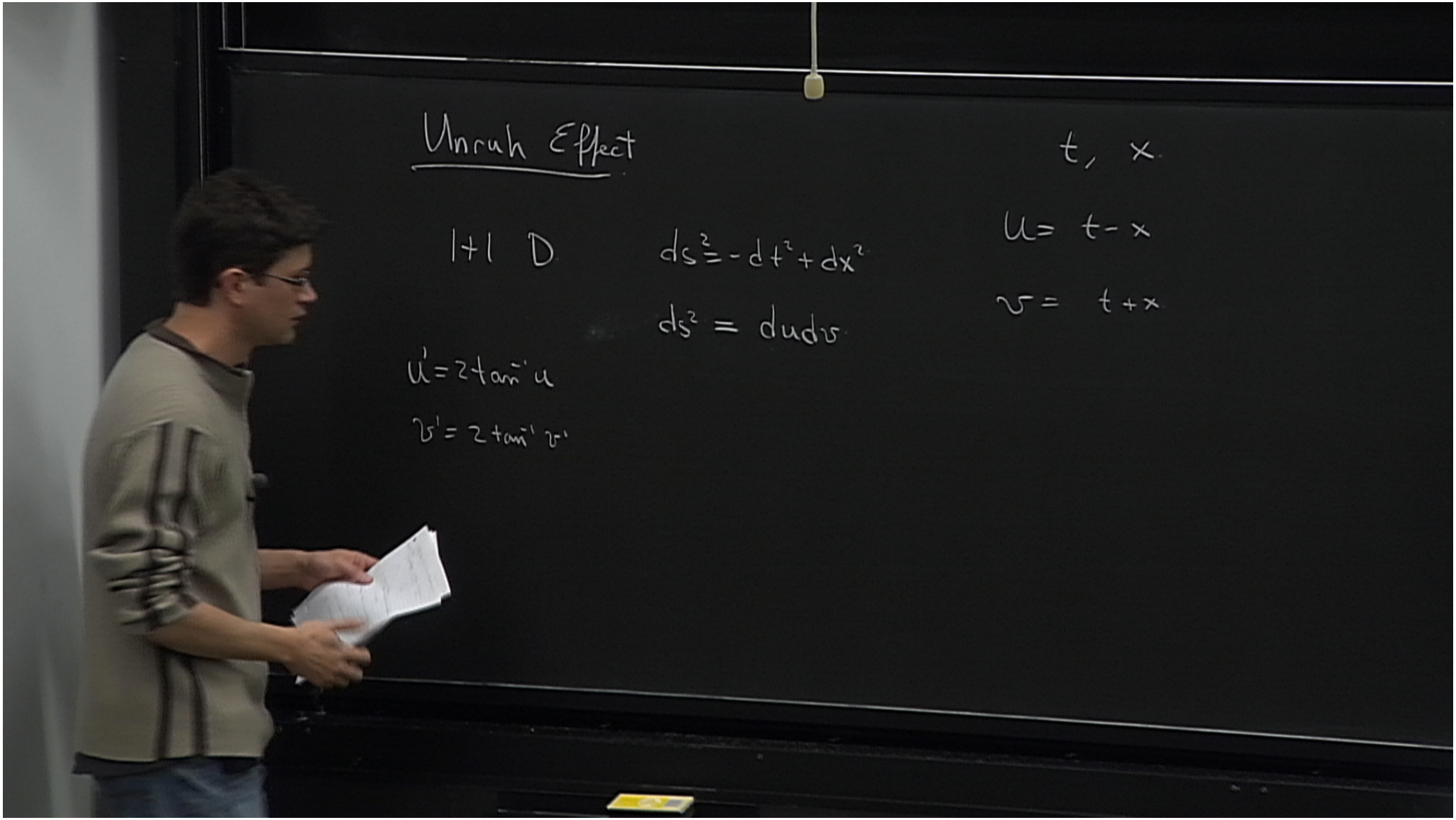
$e^{-i\omega_k t}$

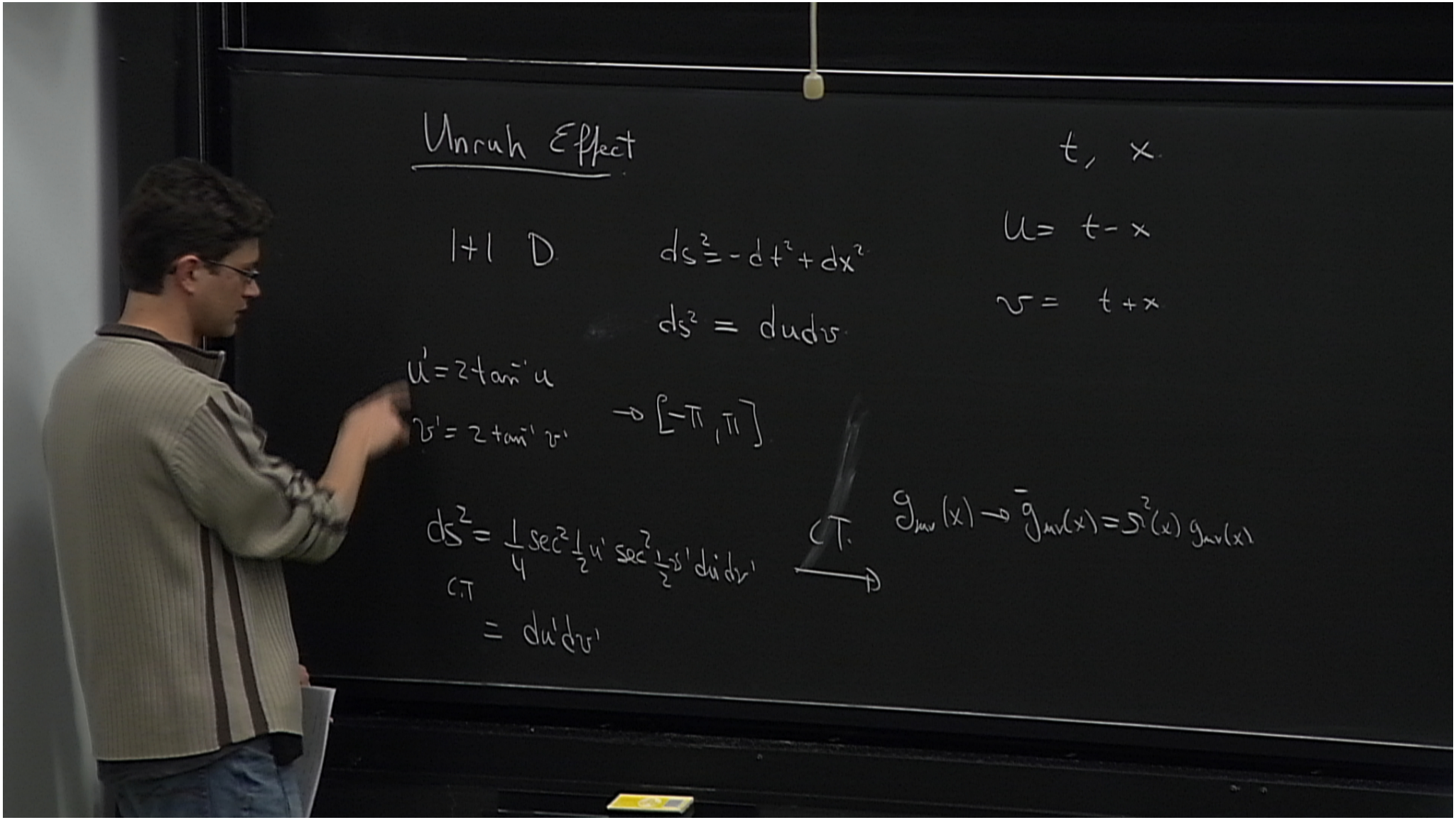
general solution of KG eq.

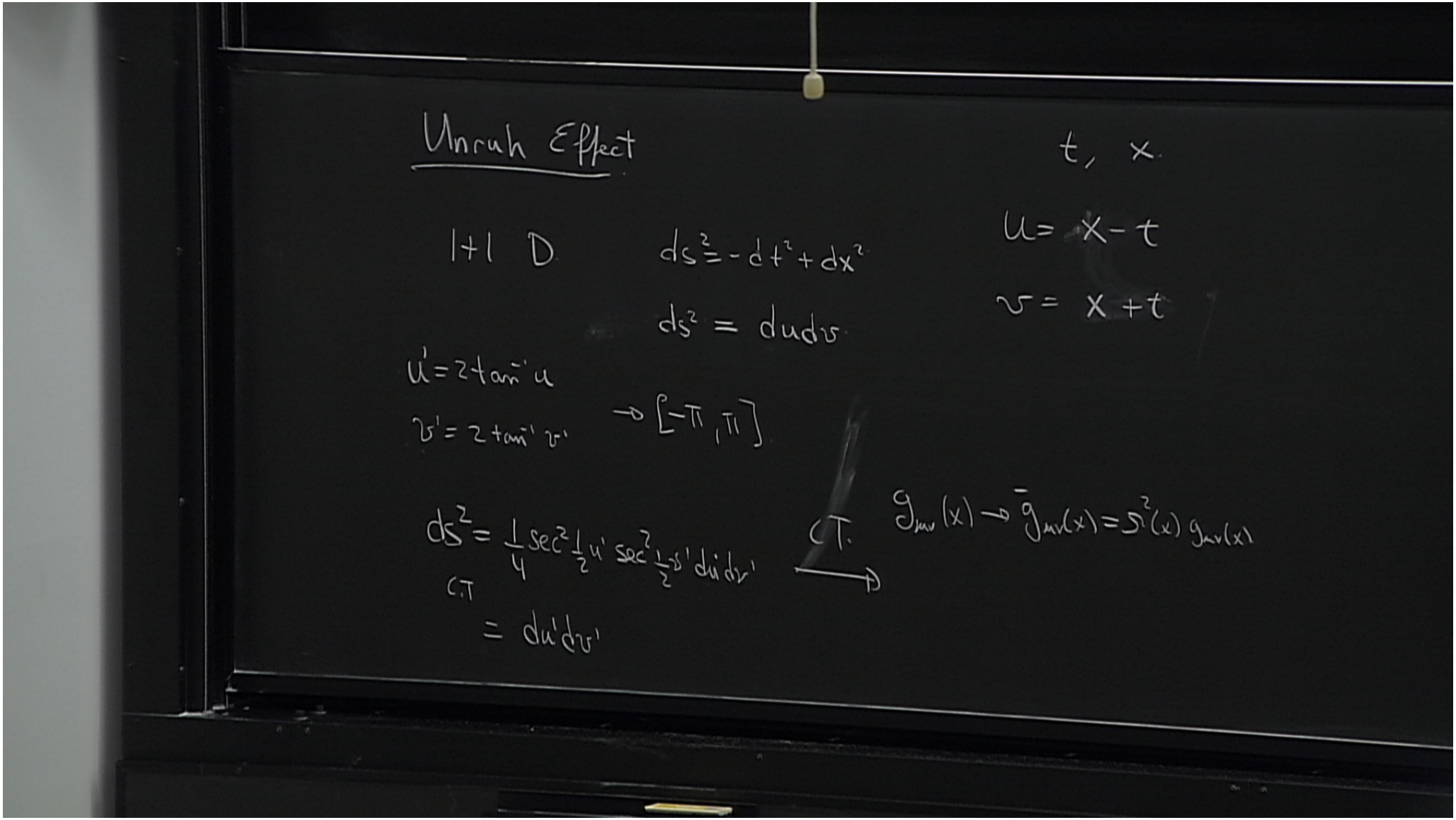
$$v_k(t) = \frac{1}{\sqrt{2\omega_k}} \left(\alpha_k e^{-i\omega_k t} + \beta_k e^{i\omega_k t} \right)$$

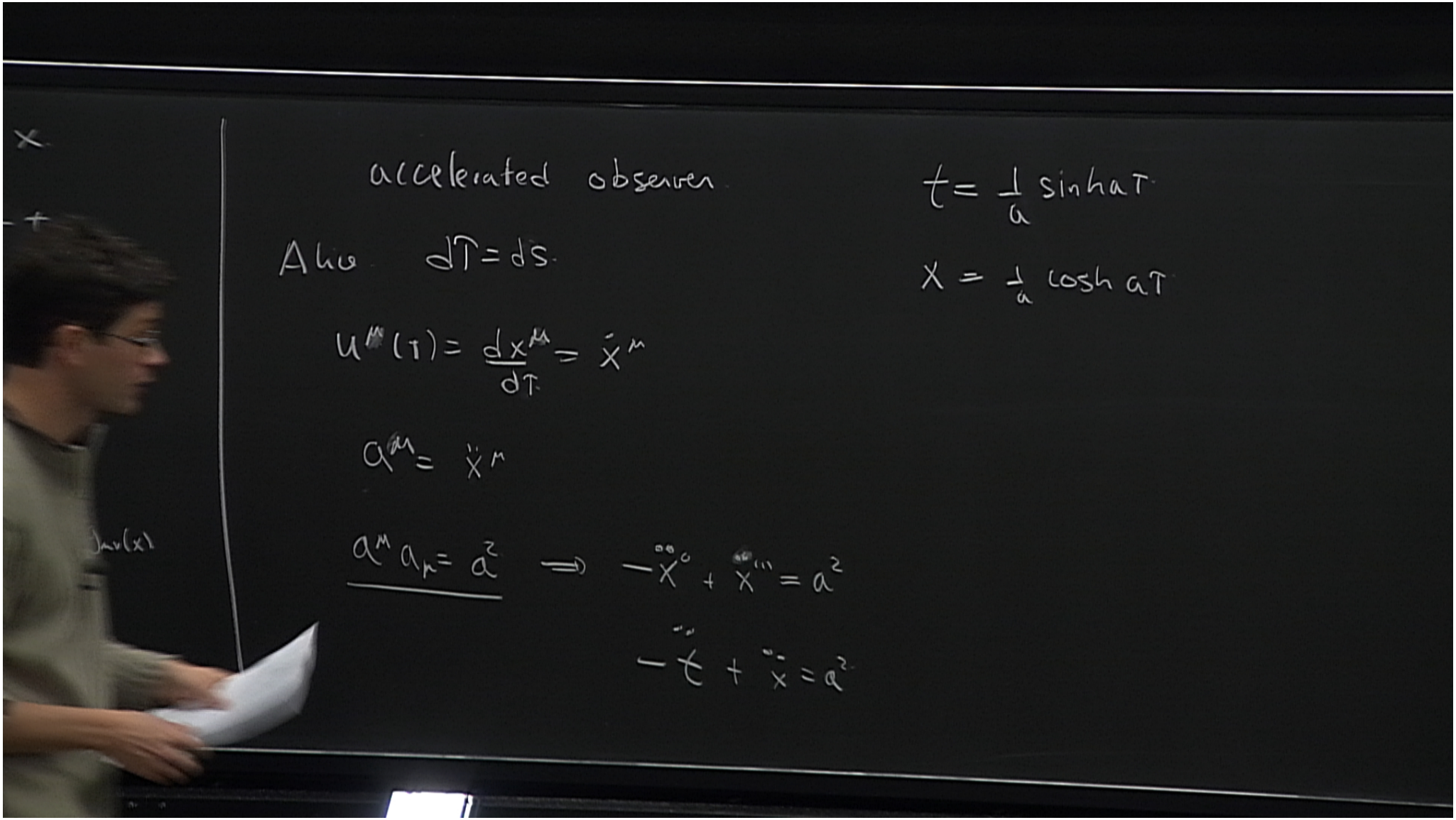
demanding that this is properly normalized.

$$|\alpha_k|^2 - |\beta_k|^2 = 1.$$


Unruh Effect
t, x
1+1 D
$ds^2 = -dt^2 + dx^2$
$u = t - x$
$v = t + x$
$ds^2 = du\,dv$
$u' = 2\tan^{-1} u$
$v' = 2\tan^{-1} v$


Unruh Effect
t, x
1+1 D
$ds^2 = -dt^2 + dx^2$
$u = t - x$
$v = t + x$
$ds^2 = du\,dv$
$u' = 2\tan^{-1} u$
$v' = 2\tan^{-1} v$
$\to [-\pi, \pi]$
$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u' \sec^2\frac{1}{2}v'\,du'\,dv'$
C.T.
$= du'\,dv'$
C.T.
$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) = \Omega^2(x)\, g_{\mu\nu}(x)$


Unruh Effect
t, x
1+1 D
$ds^2 = -dt^2 + dx^2$
$u = x - t$
$v = x + t$
$ds^2 = du\,dv$
$u' = 2\tan^{-1} u$
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$\to [-\pi, \pi]$
$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u' \sec^2\frac{1}{2}v'\,du'\,dv'$
C.T.
$= du'\,dv'$
C.T.
$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) = \Omega^2(x)\, g_{\mu\nu}(x)$


accelerated observer
Also $d\tau = ds$
$u^\mu(\tau) = \frac{dx^\mu}{d\tau} = \dot{x}^\mu$
$a^\mu = \ddot{x}^\mu$
$a^\mu a_\mu = a^2 \Rightarrow -\ddot{x}^0 + \ddot{x}^1 = a^2$
$-\ddot{t} + \ddot{x} = a^2$
$t = \frac{1}{a}\sinh a\tau$
$x = \frac{1}{a}\cosh a\tau$

accelerated observer

Also $d\tau = ds$

$$u^\mu(\tau) = \frac{dx^\mu}{d\tau} = \dot{x}^\mu$$

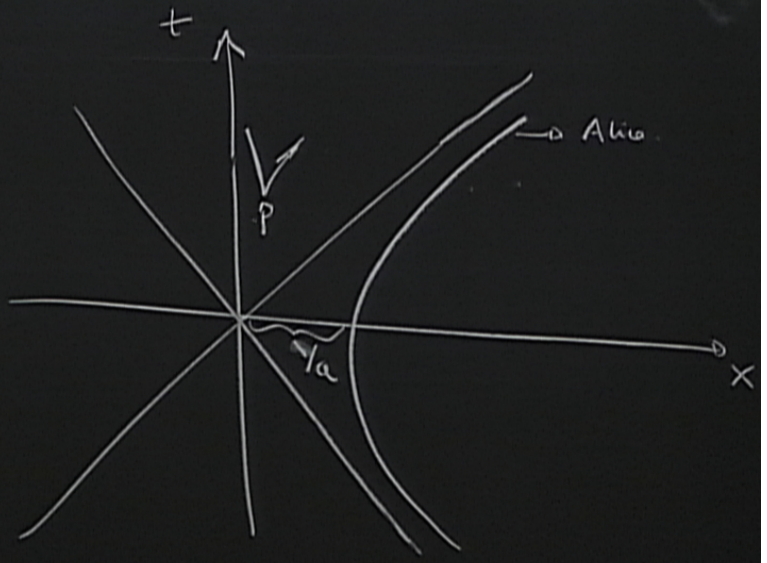
$$a^\mu = \ddot{x}^\mu$$

$$\underline{a^\mu a_\mu = a^2} \Rightarrow -\ddot{x}^0 + \ddot{x}^{(1)} = a^2$$

$$-\ddot{t} + \ddot{x} = a^2$$

$$t = \frac{1}{a}\sinh a\tau$$

$$x = \frac{1}{a}\cosh a\tau$$



accelerated observer

Alice $d\tau = ds$

$u^\mu(\tau) = \frac{dx^\mu}{d\tau} = \dot{x}^\mu$

$a^\mu = \ddot{x}^\mu$

$a^\mu a_\mu = a^2 \Rightarrow -\ddot{x}^0 + \ddot{x}^1 = a^2$

$-\ddot{t} + \ddot{x} = a^2$

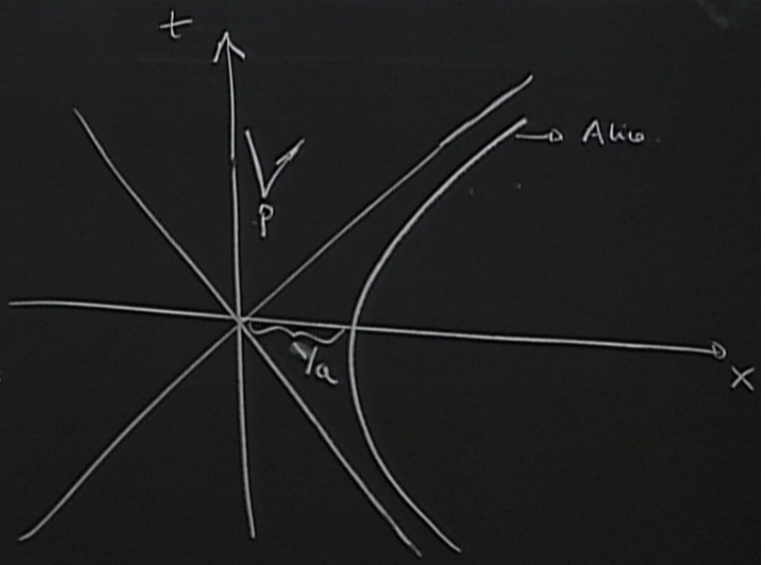
$t = \frac{1}{a}\sinh a\tau$

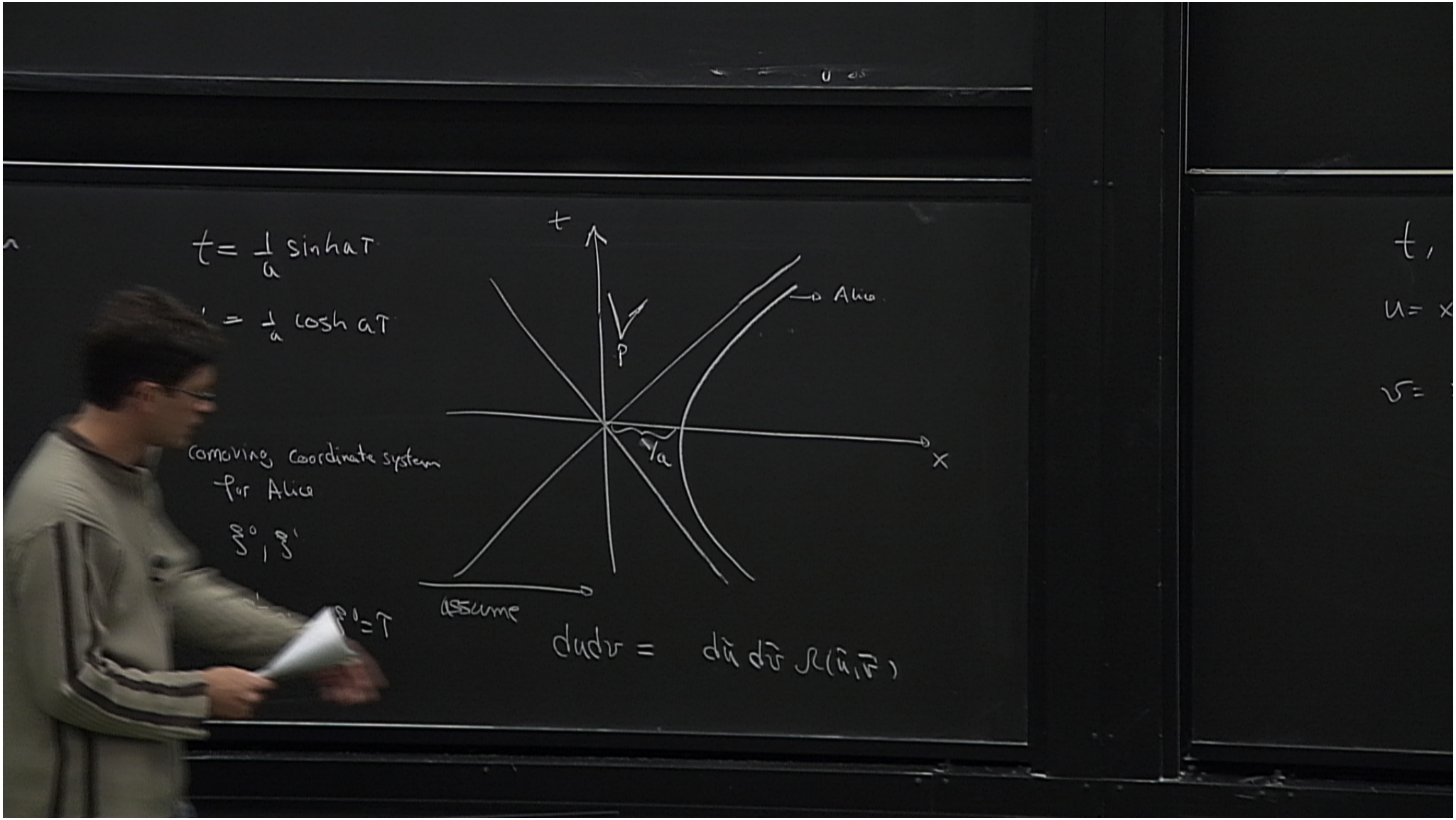
$x = \frac{1}{a}\cosh a\tau$

comoving coordinate system for Alice

ξ^0, ξ^1

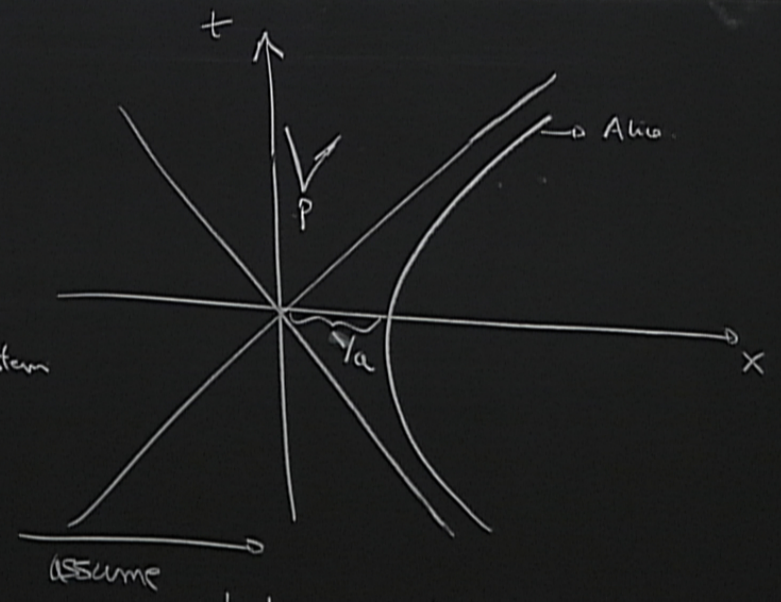
$\xi^1 = 0 \quad \xi^0 = \tau$




$t = \frac{1}{a}\sinh aT$
$= \frac{1}{a}\cosh aT$
comoving coordinate system
for Alice
ξ^0, ξ^1
$\xi^1 = T$
t
Alice
P
1/a
x
Assume
$dudv = d\tilde{u}\, d\tilde{v}\, \mathcal{R}(\tilde{u}, \tilde{v})$
t,
u= x
v=

$t = \frac{1}{a} \sinh aT$

$X = \frac{1}{a} \cosh aT$



comoving coordinate system for Alice

ξ^0, ξ^1

$\xi^1 = 0 \quad \xi^0 = T$

$dudv = d\tilde{u}\, d\tilde{v}\, \mathcal{R}(\tilde{u}, \tilde{v})$

$u = f(\tilde{u}) \quad v = f(\tilde{v})$

$+ \ddot{x} = a^2$

$+ \ddot{x} = a^2$

$t,$

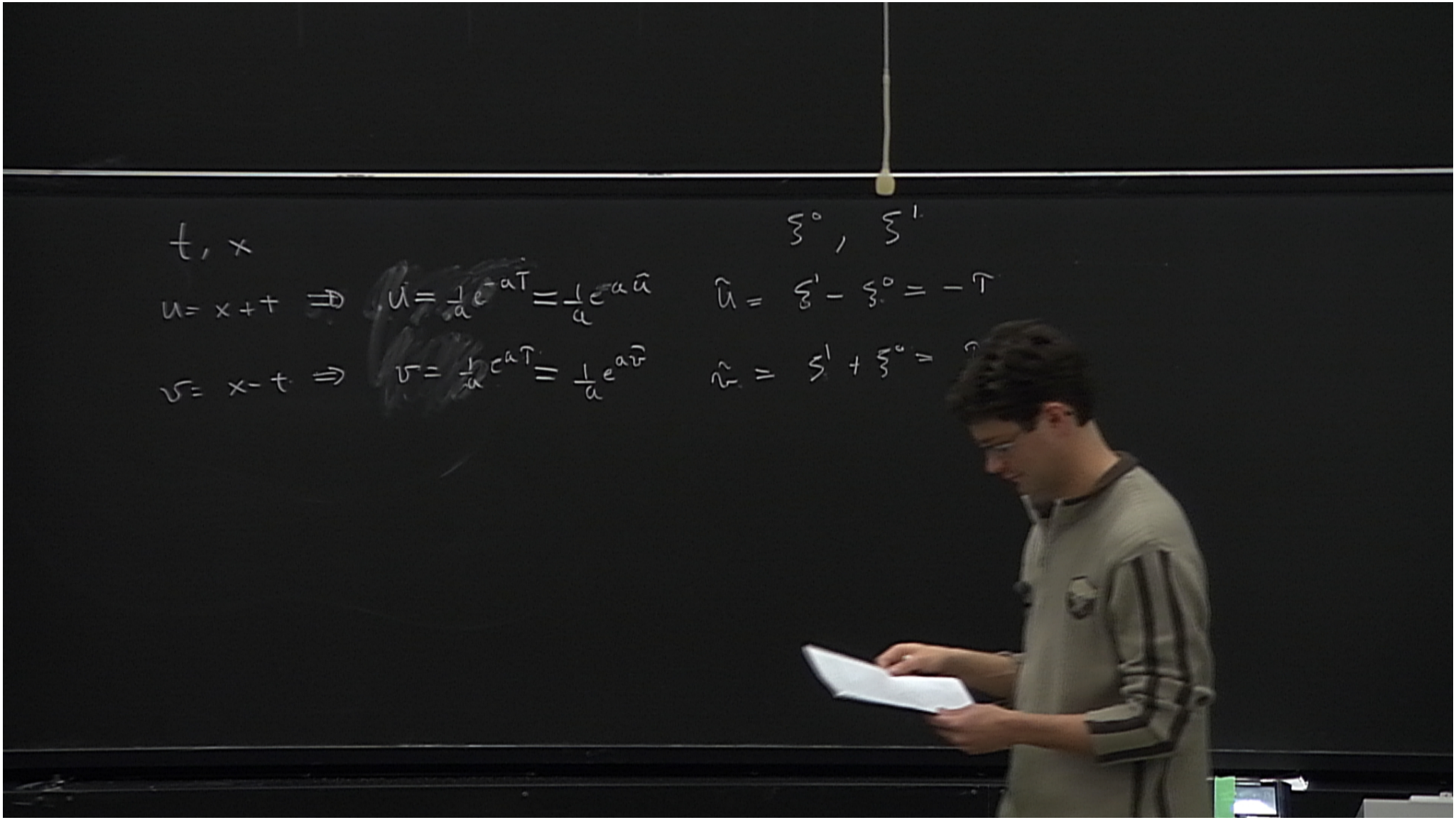
$u = x$

$v =$

t, x $\qquad\qquad$ ξ^0, ξ^1

$u = x + t \Rightarrow u = \frac{1}{a} e^{-aT} = \frac{1}{a} e^{a\hat{u}} \qquad \hat{u} = \xi^1 - \xi^0 = -T$

$v = x - t \Rightarrow v = \frac{1}{a} e^{aT} = \frac{1}{a} e^{a\hat{v}} \qquad \hat{v} = \xi^1 + \xi^0 = T$


t, x
ξ⁰, ξ¹
u = x + t ⇒ u = (1/a) e^{-aT} = (1/a) e^{a û}
û = ξ¹ − ξ⁰ = −T
v = x − t ⇒ v = (1/a) e^{aT} = (1/a) e^{a v̂}
v̂ = ξ¹ + ξ⁰ =

t, x $\qquad\qquad$ ξ^0, ξ^1

$u = x - t \Rightarrow U = \frac{1}{a} e^{-aT} = \frac{1}{a} e^{-a\hat{u}}$ $\qquad \hat{u} = \xi^1 - \xi^0 = -T$

$v = x + t \Rightarrow V = \frac{1}{a} e^{aT} = \frac{1}{a} e^{a\hat{v}}$ $\qquad \hat{v} = \xi^1 + \xi^0 = T$

$du = e^{a\tilde{u}} d\tilde{u}$ $\qquad dv = e^{a\tilde{v}} d\tilde{v}$

$$du\,dv = e^{a(\tilde{v}+\tilde{u})} d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{1^2} + d\xi^{1^2}\right)$$

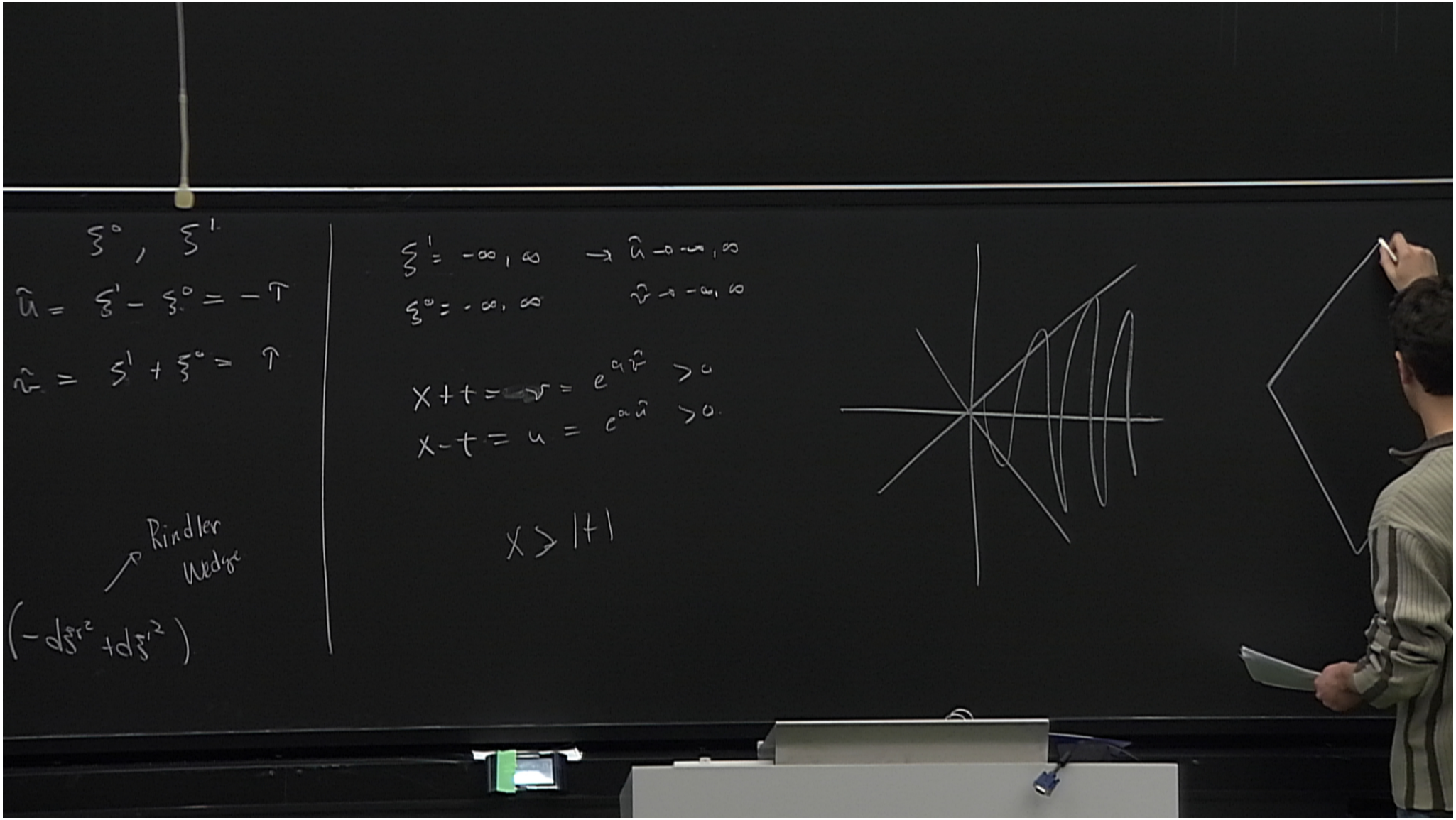
t, x $\qquad\qquad$ ξ^0, ξ^1

$u = x - t \Rightarrow U = \frac{1}{a} e^{-a\tilde{T}} = \frac{1}{a} e^{a\tilde{u}} \qquad \tilde{u} = \xi^1 - \xi^0 = -\tilde{T}$

$v = x + t \Rightarrow V = \frac{1}{a} e^{a\tilde{T}} = \frac{1}{a} e^{a\tilde{v}} \qquad \tilde{v} = \xi^1 + \xi^0 = \tilde{T}$

$du = e^{a\tilde{u}} d\tilde{u} \qquad dv = e^{a\tilde{v}} d\tilde{v}$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})} d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{1^2} + d\xi^{1^2}\right)$


ξ^0, ξ^1
$\hat{u} = \xi^1 - \xi^0 = -T$
$\hat{v} = \xi^1 + \xi^0 = T$
$\xi^1 = -\infty, \infty \rightarrow \hat{u} = -\infty, \infty$
$\xi^0 = -\infty, \infty \quad \hat{v} = -\infty, \infty$
$X + t = v = e^{a\hat{v}} > 0$
$X - t = u = e^{a\hat{u}} > 0$
$X > |t|$
Rindler Wedge
$(-d\xi^{1^2} + d\xi^{1^2})$

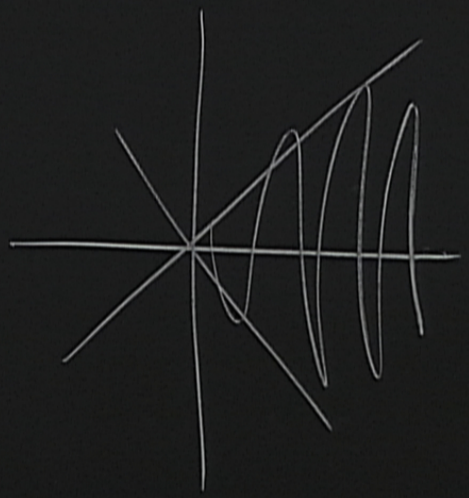
$\xi^1 = -\infty, \infty \quad \rightarrow \quad \hat{u} = -\infty, \infty$

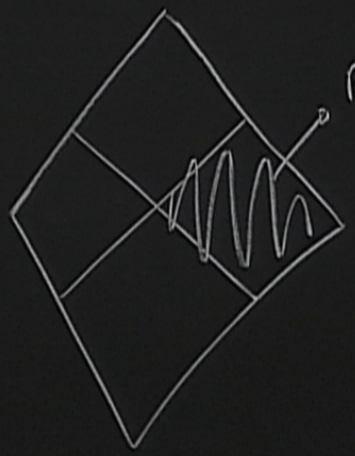
$\xi^0 = -\infty, \infty \quad \rightarrow \quad \hat{v} = -\infty, \infty$

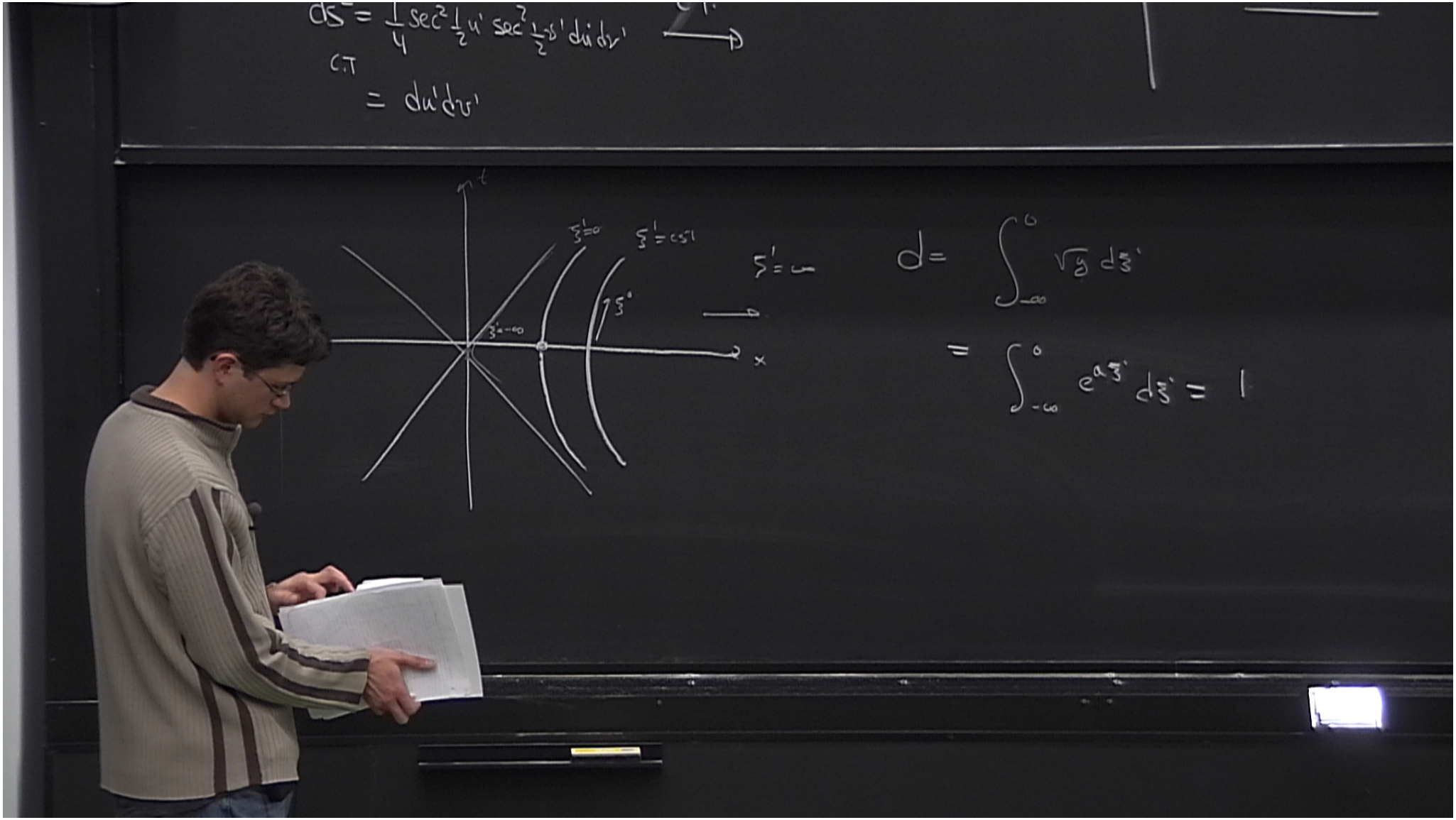
$x + t = v = e^{a\hat{v}} > 0$

$x - t = u = e^{a\hat{u}} > 0$

$x > |t|$






$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u'\sec^2\frac{1}{2}v'\,du'dv'$
$= du'dv'$
$\xi^1 = 0$
$\xi^1 = \text{const}$
$\xi^1 = -\infty$
ξ^1
$d = \int_{-\infty}^{0} \sqrt{g}\, d\xi^1$
$= \int_{-\infty}^{0} e^{a\xi^1} d\xi^1 = 1$


$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u'\sec^2\frac{1}{2}v'\,du'dv'$
$= du'dv'$
$\xi^1 = 0$
$\xi^1 = \text{const}$
$\xi^1 = \infty$
$\xi^1 = -\infty$
$d = \int_{-\infty}^{0} \sqrt{g}\,d\xi^1$
$d = \int_{-\infty}^{0} e^{a\xi^1}\,d\xi^1 = \frac{1}{a}$
horizon

$-\ddot{t} + \ddot{x} = a^2$ $\quad \xi^0 = 0 \quad \xi^1 = T$

$du\,dv = d\tilde{u}\,d\tilde{v}\,f(\tilde{u},\tilde{v})$

$u = f(\tilde{u}) \quad v = f(\tilde{v})$

$= \frac{1}{a}$

horizon

Quantice ϕ $m=0$.

$$S = \frac{1}{2}\int \left[-(\partial_t \phi)^2 + (\partial_x \phi)^2\right] dt\,dx$$

$$= 2\int (\partial_u \phi\, \partial_v \phi)\, du\,dv \overset{\text{Pob}}{=} 2\int \left[\partial_{\tilde{u}} \phi\, \partial_{\tilde{v}} \phi\right] d\tilde{u}\,d\tilde{v} \overset{\text{Aku}}{}$$

in 1+1 D., $m=0$. ϕ is conformal symmetry.

$$\int g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi \sqrt{g}$$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})}\, d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{1,2} + d\xi^{1,2}\right)$

KG $\partial_u \partial_v \phi = 0$

$u = x + t$

$v = x - t$

$\phi = A(u) + B(v)$

Sols. $e^{-i\omega u}$ $+ e^{i\omega u}$ $\rightarrow$ $e^{-i\omega v}$ $e^{i\omega v}$

$$\hat{\phi} = \int_0^\infty d\omega \, \frac{1}{\sqrt{2\omega}} \left(a^{-,L}_\omega e^{-i\omega u} + a^{+,L}_\omega e^{i\omega u} \right) + \text{Left moving}$$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})}\, d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{r2} + d\xi^{r2}\right)$

KG $\partial_u \partial_v \phi = 0$

$u = x + t$

$v = x - t$

$\phi = A(u) + B(v)$

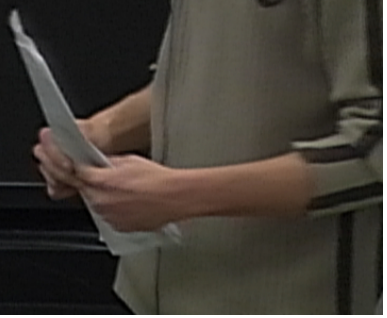
Sols. $e^{-i\omega u}$ $+ e^{i\omega u}$ $\quad e^{-i\omega v}$ $e^{i\omega v}$

$a_{\vec{k}}^{-} \equiv a_{k(\sin\theta)} = a_{k}^{-,L} + a^{-,R}$

$k = \omega$

$m = 0$

$$\hat{\phi} = \int_0^\infty d\omega \, \frac{1}{\sqrt{2\omega}} \left(a_\omega^{-,L} e^{-i\omega u} + a_\omega^{+,L} e^{i\omega u} \right) + \text{right-moving}$$



$du = e^{a\tilde{u}} d\tilde{u}$ $dv = e^{a\tilde{v}} d\tilde{v}$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})} d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}(-d\xi^{1^2} + d\xi^{1^2})$ → Rindler wedge

$x > |t|$

KG $\partial_u \partial_v \phi = 0$

$u = x + t$

$v = x - t$

$\phi = A(u) + B(v)$

Sols. $e^{-i\omega u}$ $+ e^{i\omega u}$ → $e^{-i\omega v}$ $e^{i\omega v}$

$a_{\vec{k}}^- = a_{k(\text{conv})}^- = a_k^{-,L} + a_k^{-,R}$

$k = \omega$

$m = 0$

$\frac{d}{du} A^{\pm} = \pm i\omega A^{\pm}$

$\hat{\phi} = \int_0^\infty d\omega \frac{1}{\sqrt{2\omega}} \left(\underbrace{a_\omega^{-,L} e^{-i\omega u}}_{A} + a_\omega^{+,L} e^{i\omega u} \right) + \text{right-moving}$

$du = e^{a\tilde{u}} d\tilde{u}$ $dv = e^{a\tilde{v}} d\tilde{v}$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})}\, d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{0\,2} + d\xi^{1\,2}\right)$ → Rindler wedge

$x > |t|$

KG $\partial_u \partial_v \phi = 0$

$u = x + t$

$v = x - t$

$\phi = A(u) + B(v)$

Sols. $e^{-i\omega u}$ $+ e^{i\omega u}$ → $e^{-i\omega v}$ $e^{i\omega v}$

in $1+1$

$a_k^- = a_{k,\pm} = a_k^{-,L} + a_k^{-,R}$

$k = \omega$

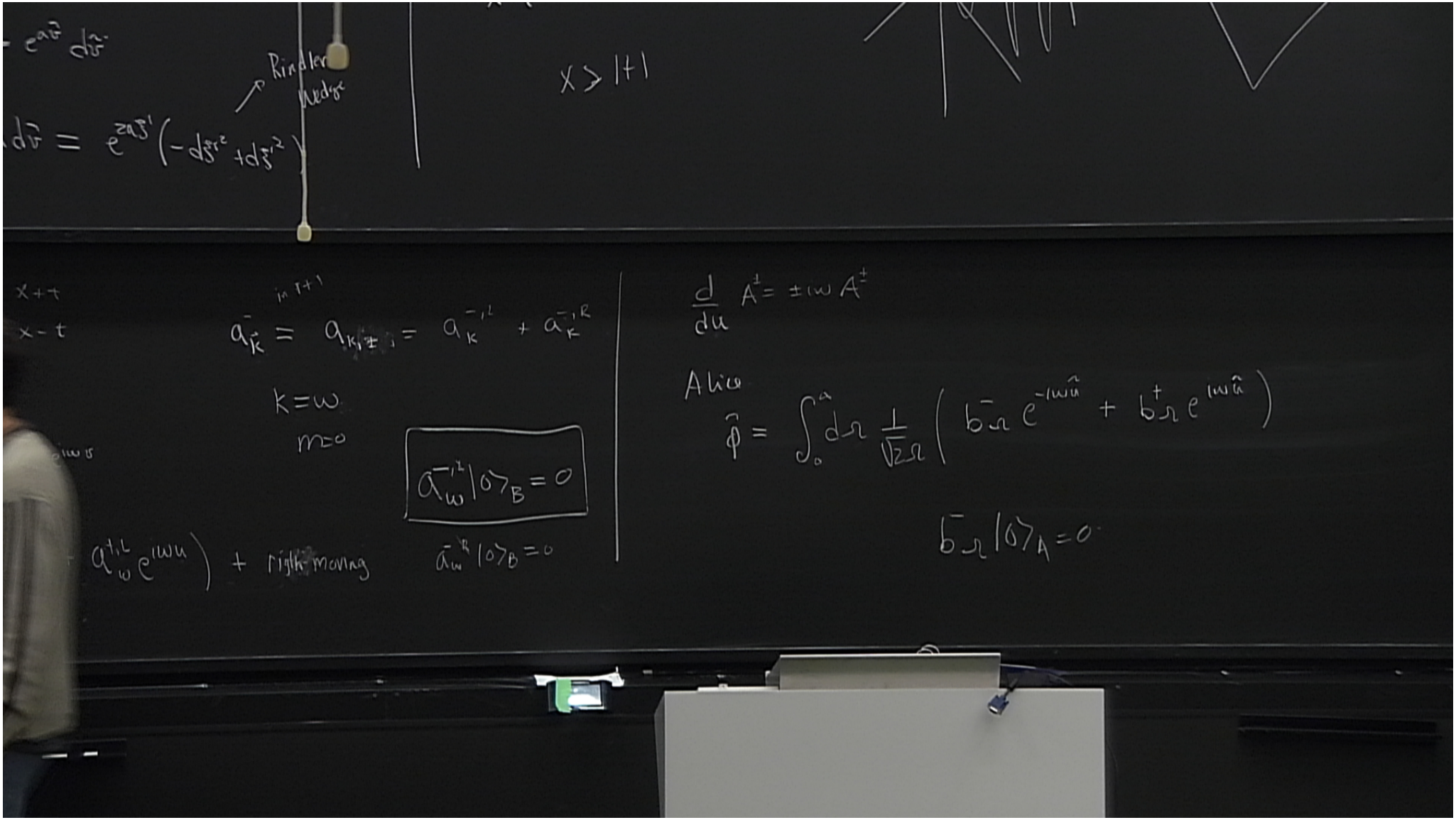
$m = 0$

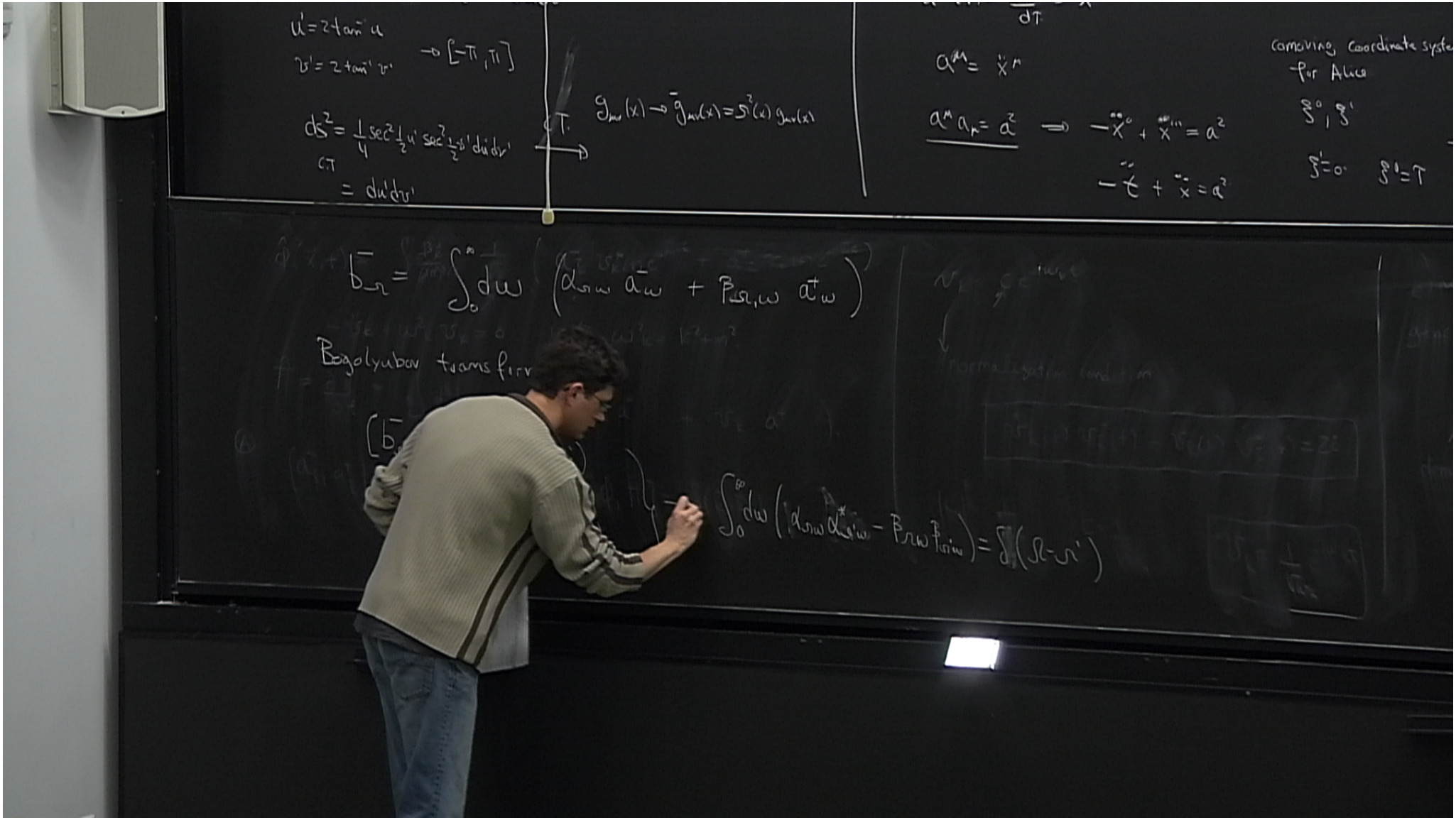
$a_\omega^{-,L}|0\rangle_B = 0$

$\bar{a}_\omega^{-,R}|0\rangle_B = 0$

$\hat{\phi} = \int_0^\infty d\omega\, \frac{1}{\sqrt{2\omega}} \left(\underbrace{a_\omega^{-,L} e^{-i\omega u}}_{A} + a_\omega^{+,L} e^{i\omega u} \right) + \text{right-moving}$

$\frac{d}{du} A^\pm = \pm i\omega A^\pm$


Rindler wedge
$X > |t|$
$x+t$
$x-t$
$a_k^- = a_{k,\pm} = a_k^{-,L} + a_k^{-,R}$
$k=\omega$
$m=0$
$a_\omega^{-,L}|0\rangle_B = 0$
$a_\omega^{-,R}|0\rangle_B = 0$
$\frac{d}{du} A^\pm = \pm i\omega A^\pm$
Alice
$\hat{\phi} = \int_0^\infty d\Omega \frac{1}{\sqrt{2\Omega}} \left(b_\Omega^- e^{-i\omega\hat{u}} + b_\Omega^+ e^{i\omega\hat{u}} \right)$
$b_\Omega^-|0\rangle_A = 0$
+ right-moving


$u' = 2\tan^{-1} u$
$v' = 2\tan^{-1} v$
$\to [-\pi, \pi]$
$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u' \sec^2\frac{1}{2}v' du' dv'$
$= du' dv'$
CT
$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) = \Omega^2(x)\, g_{\mu\nu}(x)$
$a^\mu = \ddot{x}^\mu$
$a^\mu a_\mu = a^2 \Rightarrow -\ddot{x}^0 + \ddot{x}^1 = a^2$
$-\ddot{t} + \ddot{x} = a^2$
comoving coordinate syste
for Alice
g^{00}, g^{11}
$g^{01} = 0$
$g^{00} = -1$
$\bar{b}_{-\omega} = \int_0^\infty d\omega \left(\alpha_{-\omega\omega} \bar{a}_\omega + \beta_{-\omega,\omega} a^\dagger_\omega \right)$
Bogolyubov transform
$\int_0^\infty d\omega \left(\alpha_{\omega\omega} \alpha^*_{\omega'\omega} - \beta_{\omega\omega} \beta^*_{\omega'\omega} \right) = \delta(\omega - \omega')$

$u' = 2\tan^{-1} u \to [-\pi, \pi]$

$v' = 2\tan^{-1} v'$

$ds^2 = \frac{1}{4}\sec^2\frac{1}{2}u' \sec^2\frac{1}{2}v' \, du' dv'$

$= du' dv'$

CT

$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) = \Omega^2(x)\, g_{\mu\nu}(x)$

dT

$a^\mu = \ddot{x}^\mu$

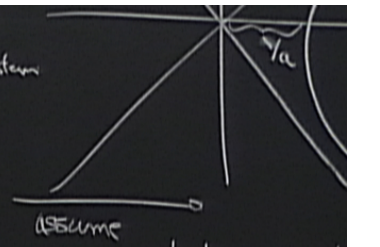
$a^\mu a_\mu = a^2 \Rightarrow -\ddot{x}^0 + \ddot{x}^1 = a^2$

$-\ddot{t} + \ddot{x} = a^2$

Comoving coordinate system for Alice

ξ^0, ξ^1

$\xi^1 = 0 \quad \xi^1 = T$



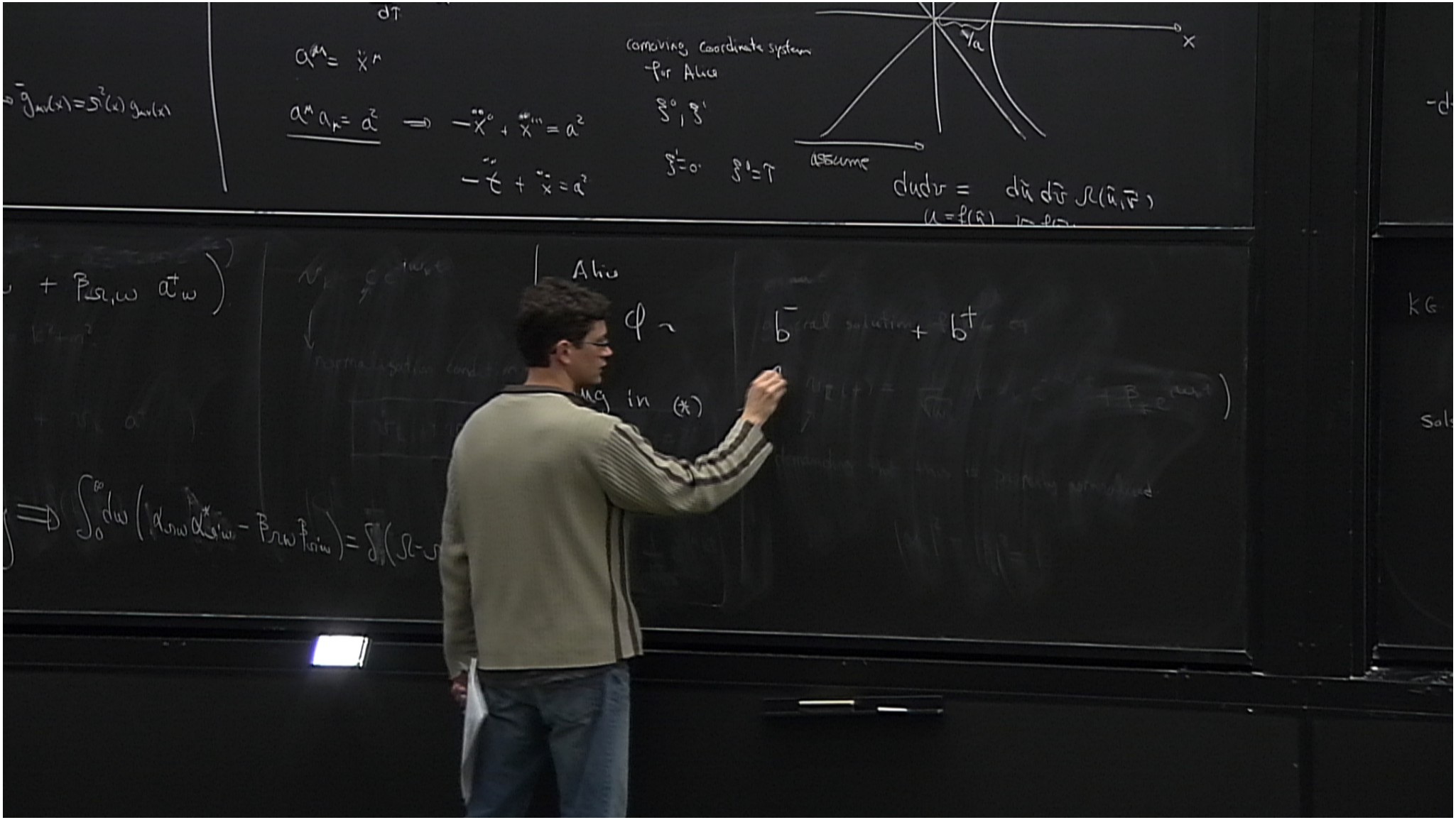
$du dv = d$

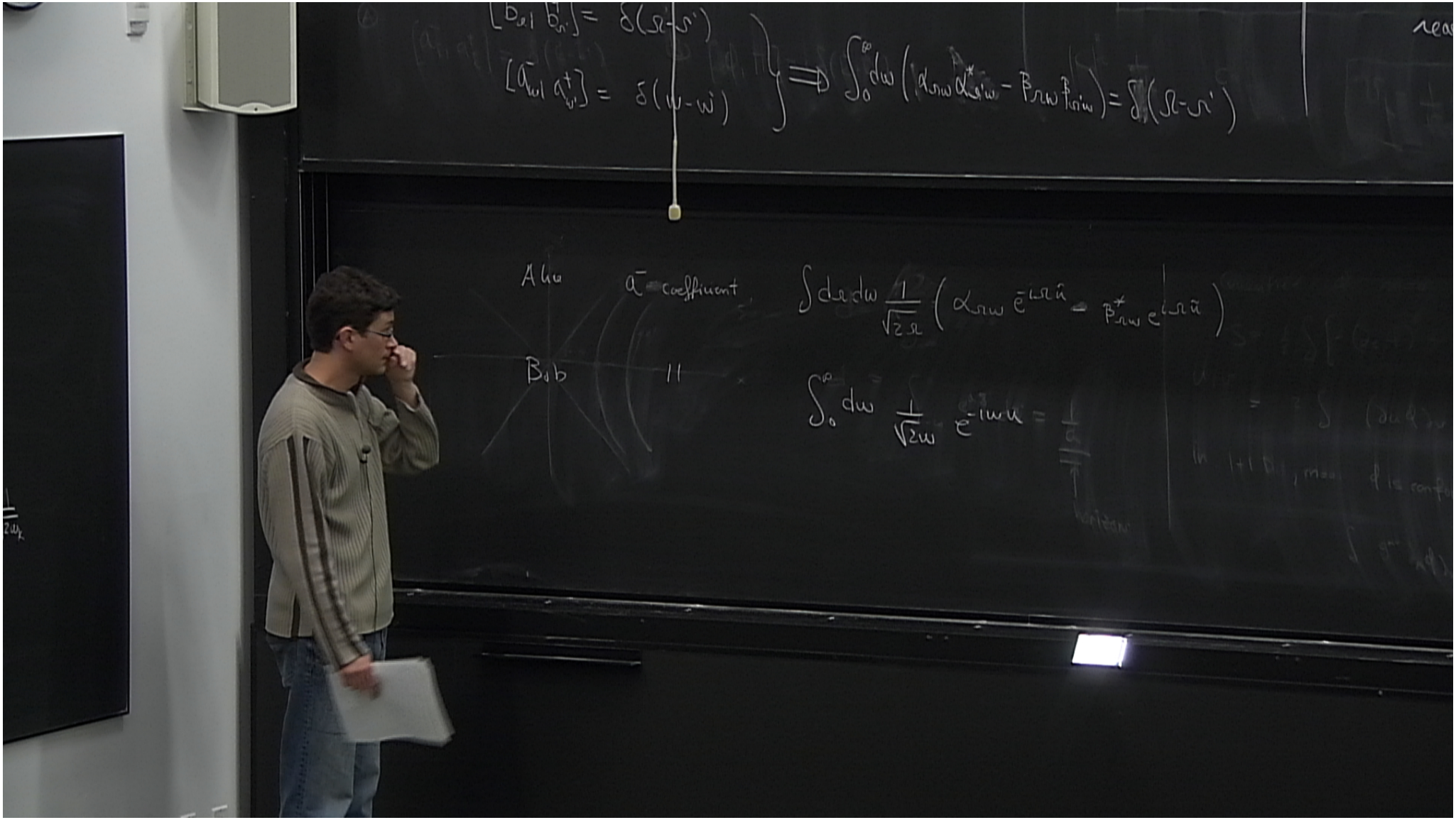
$u = f(\eta)$

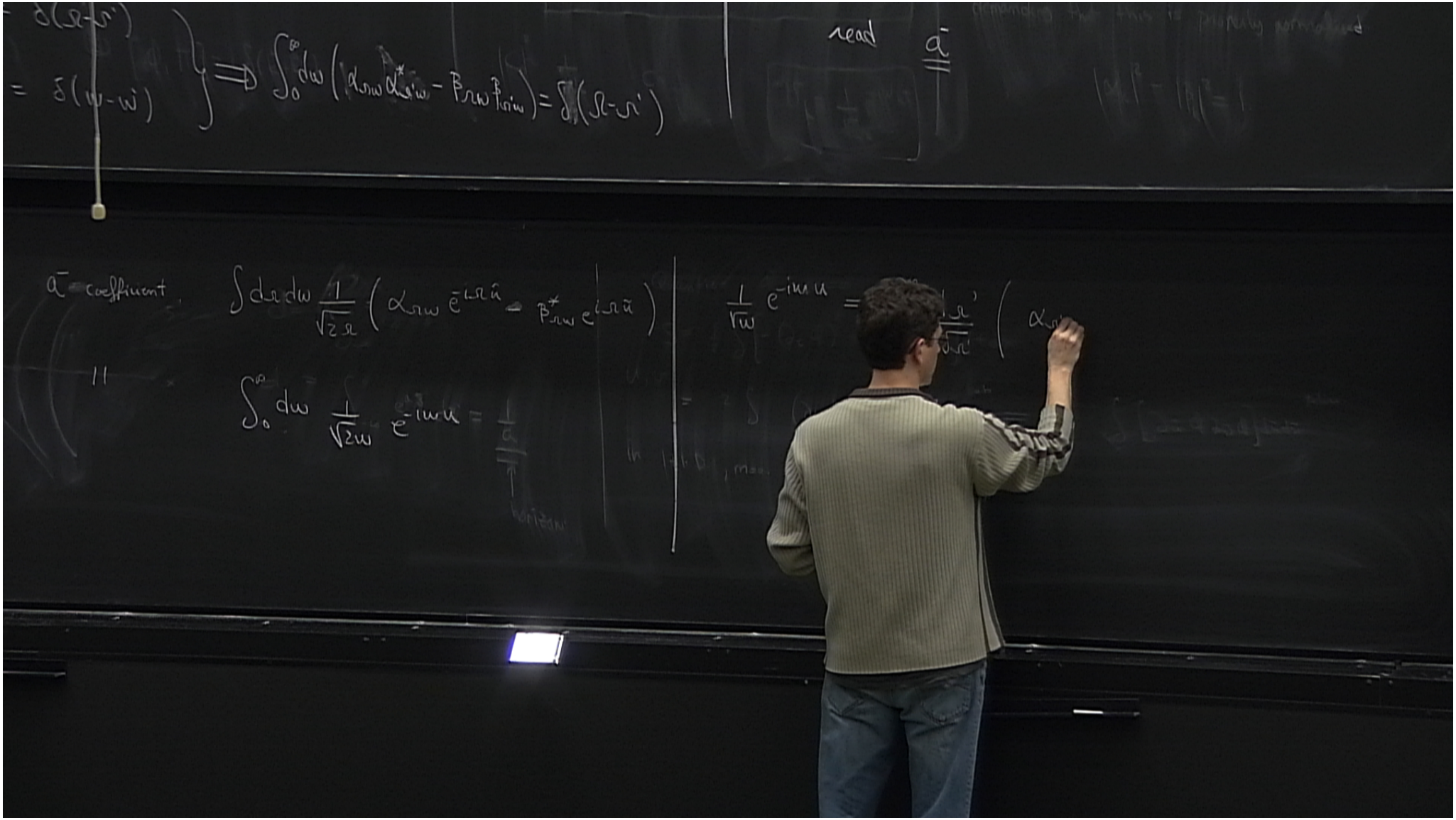
$$\bar{b}_\Omega = \int_0^\infty d\omega \left(\alpha_{\Omega\omega}\, \bar{a}_\omega + \beta_{\Omega,\omega}\, a^\dagger_\omega \right)$$

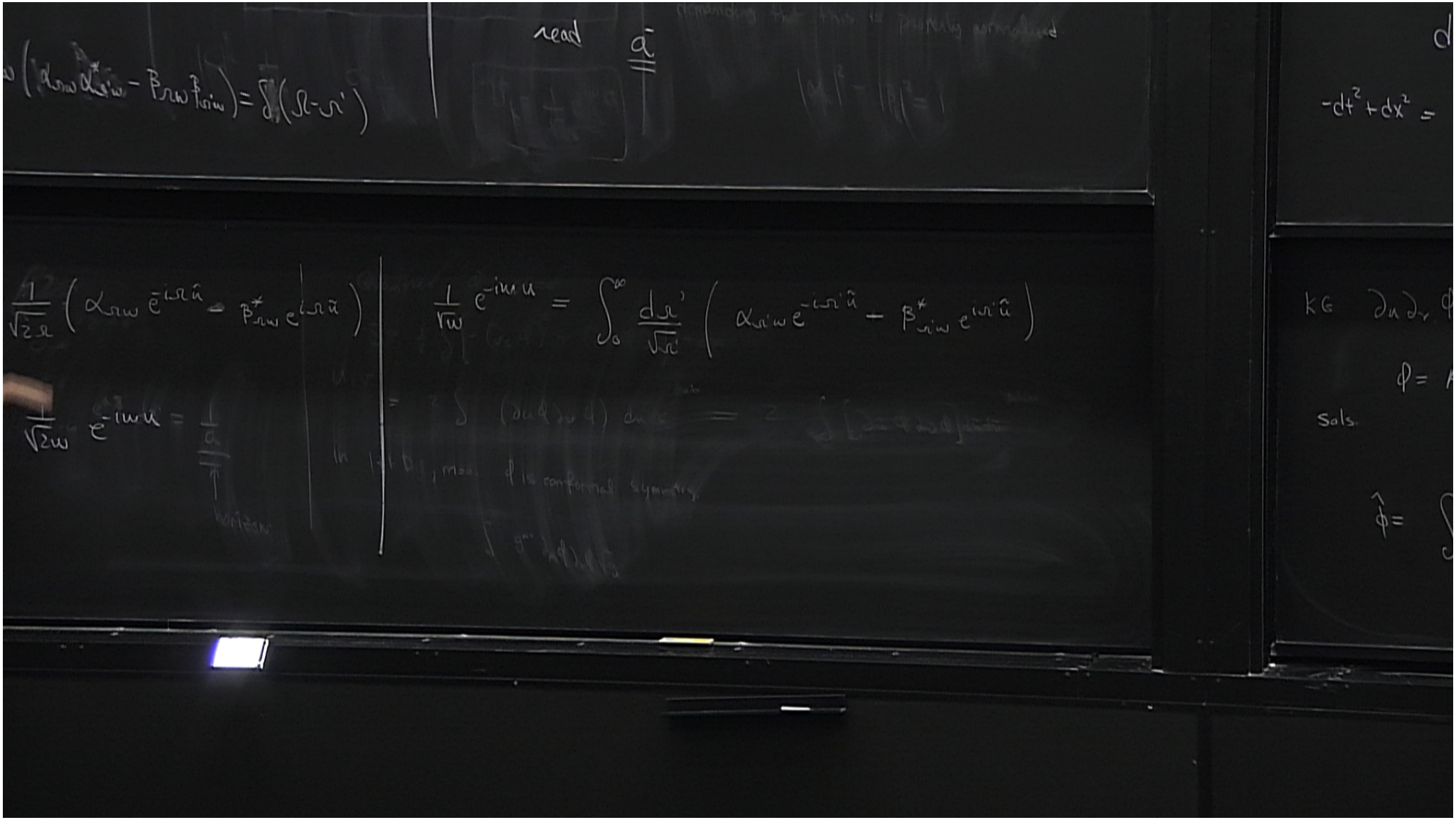
Bogolyubov transformation

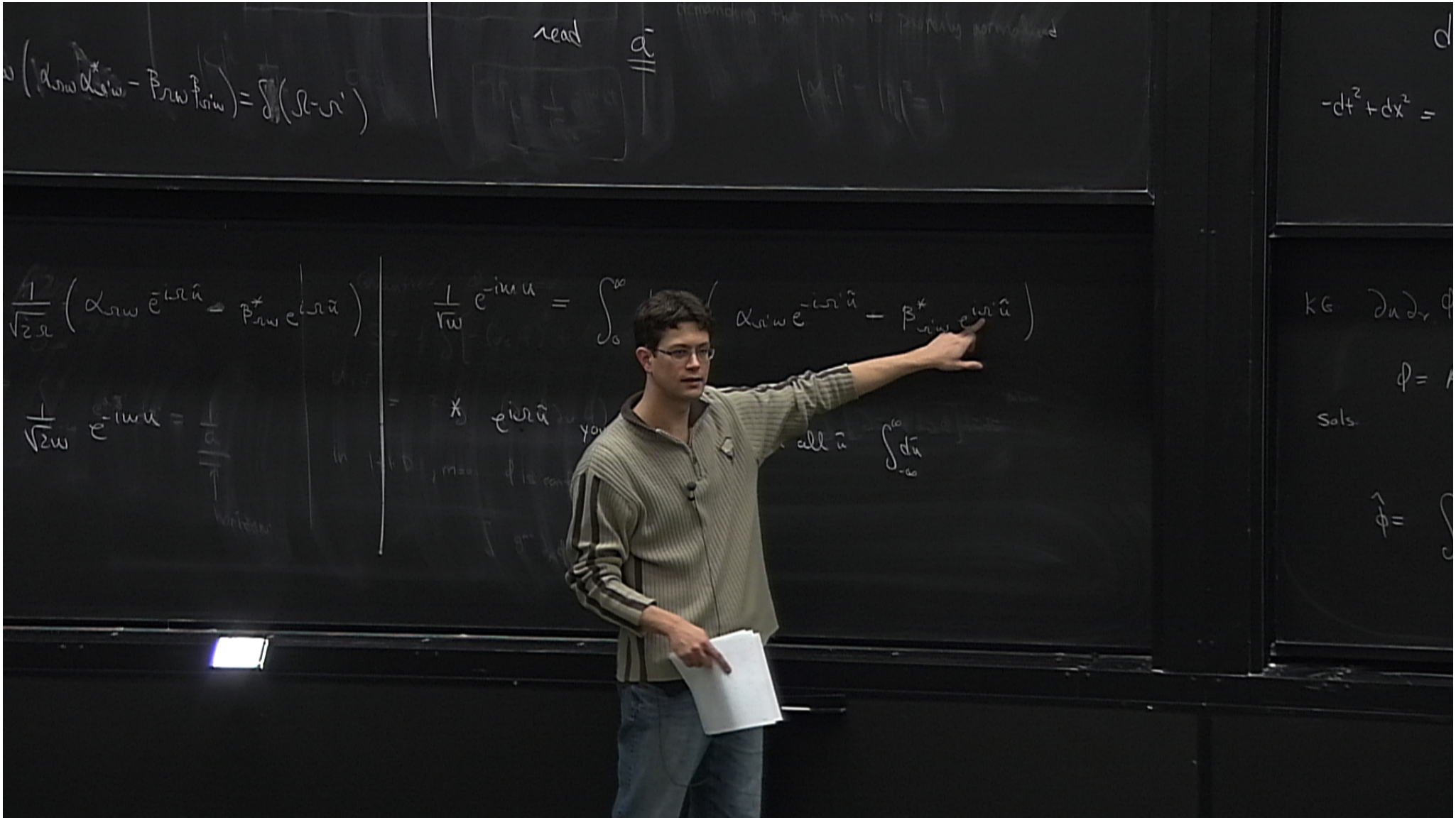
$$\left.\begin{aligned} [\bar{b}_\Omega, b^\dagger_{\Omega'}] &= \delta(\Omega - \Omega') \\ [\bar{a}_\omega, a^\dagger_{\omega'}] &= \delta(\omega - \omega') \end{aligned}\right\} \Rightarrow \int_0^\infty d\omega \left(\alpha_{\Omega\omega} \alpha^*_{\Omega'\omega} - \beta_{\Omega\omega} \beta^*_{\Omega'\omega} \right) = \delta(\Omega - \Omega')$$

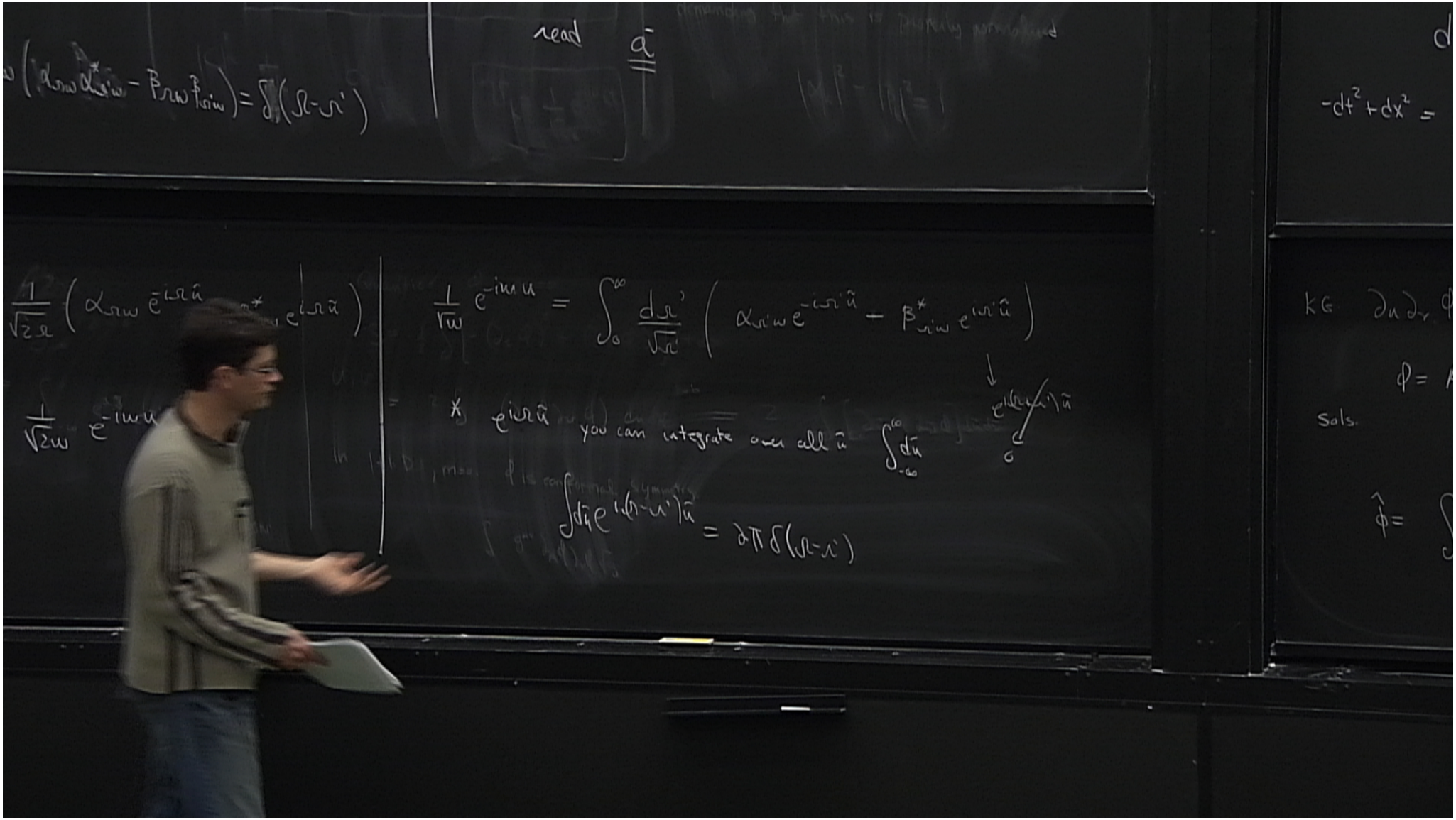

comoving coordinate system
for Alice
assume
Alice
KG

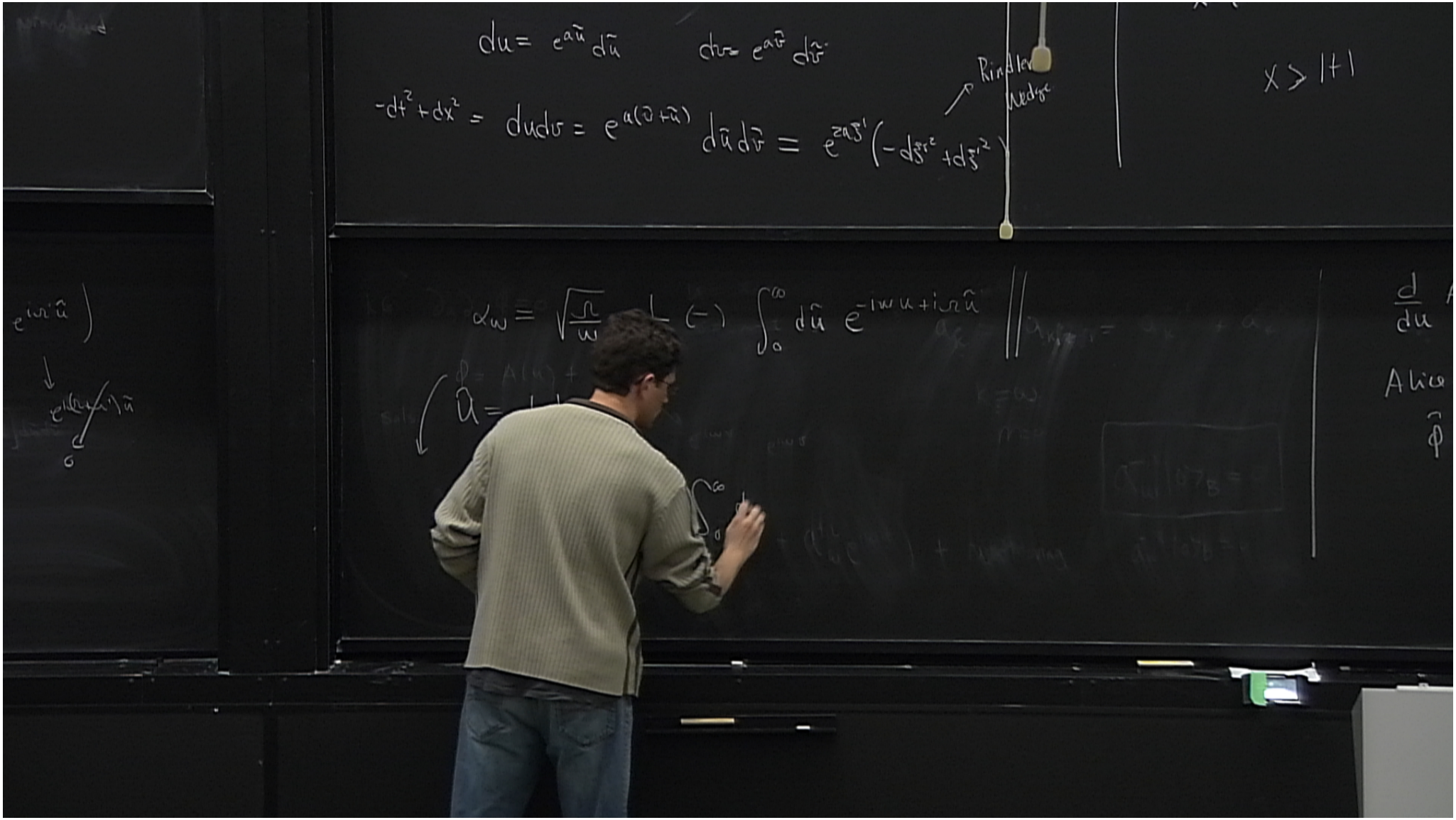

$[b_{\Omega}, b^{\dagger}_{\Omega'}] = \delta(\Omega-\Omega')$
$[a_{\omega}, a^{\dagger}_{\omega'}] = \delta(\omega-\omega')$
$\Rightarrow \int_0^\infty d\omega \left(\alpha_{\Omega\omega}\alpha^{*}_{\Omega'\omega} - \beta_{\Omega\omega}\beta^{*}_{\Omega'\omega}\right) = \delta(\Omega-\Omega')$
A ↔ a coefficient
B ↔ b "
$\int d\Omega\, d\omega \frac{1}{\sqrt{2\pi}} \left(\alpha_{\Omega\omega} e^{-i\omega u} - \beta^{*}_{\Omega\omega} e^{i\omega u}\right)$
$\int_0^\infty d\omega \frac{1}{\sqrt{2\omega}} e^{-i\omega u} =$

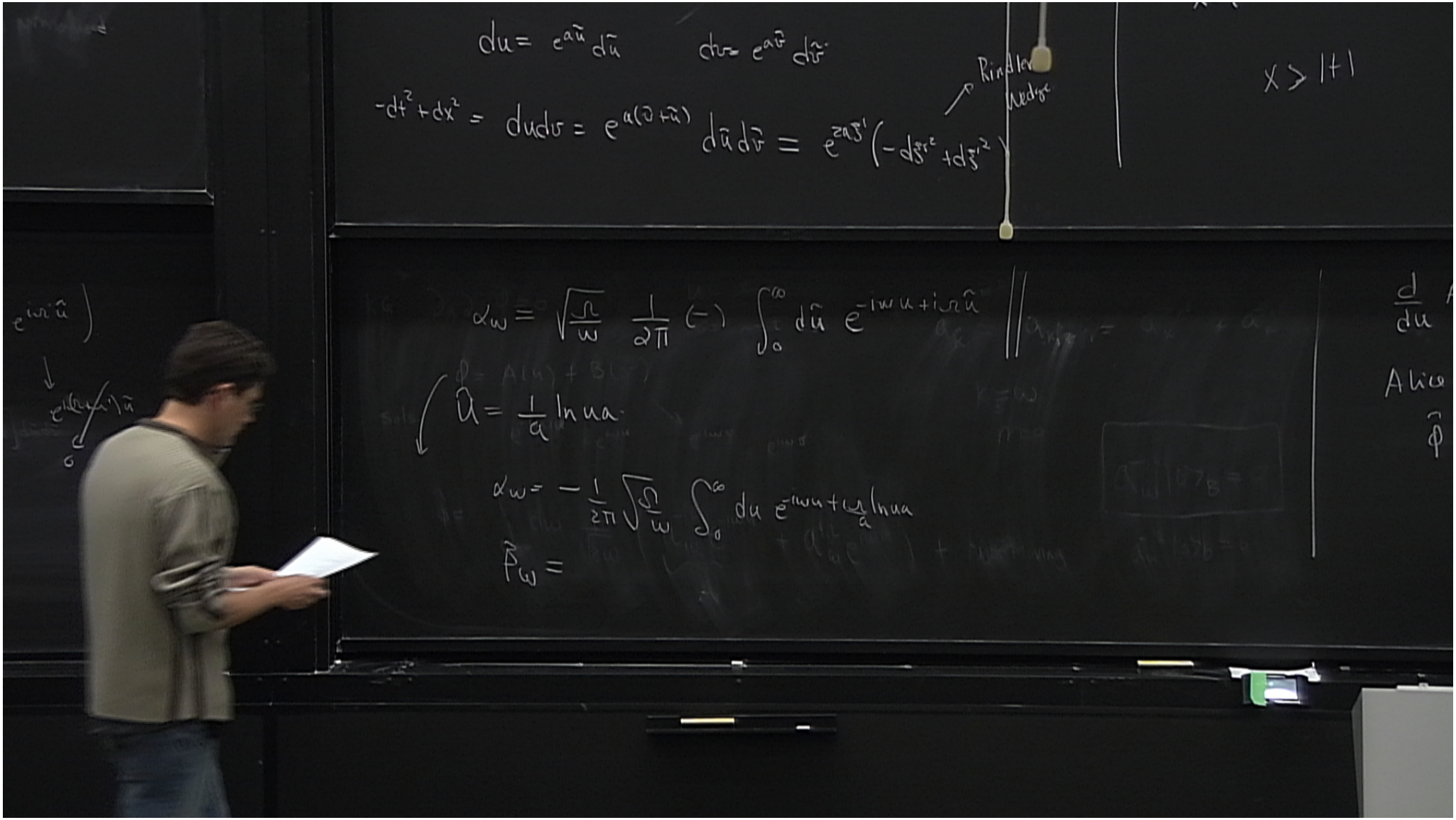

read
coefficient

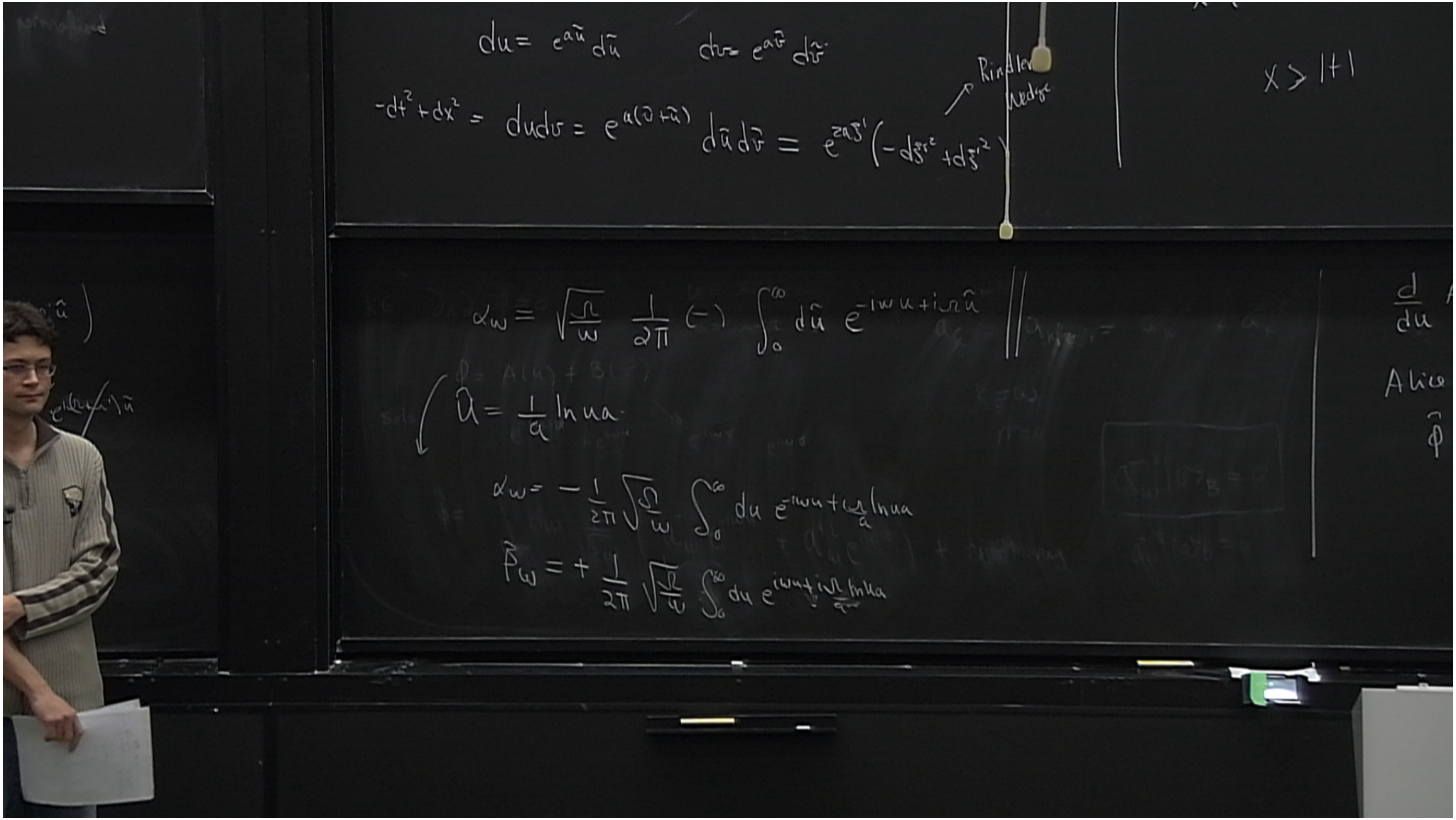



read
$\hat{a}$
$-dt^2 + dx^2 =$
$\frac{1}{\sqrt{2\omega}} e^{-i\omega u} =$
$\frac{1}{\sqrt{2\omega}} \left(\alpha_{\omega\omega'} e^{-i\omega' \hat{u}} - \beta^*_{\omega\omega'} e^{i\omega' \hat{u}} \right)$
$\frac{1}{\sqrt{2\omega}} e^{-i\omega u} =$
$\int_{-\infty}^{\infty} d\hat{u}$
$\hat{\phi} =$


read
$\frac{1}{\sqrt{2\omega}}\left(\alpha_{\omega}\, e^{-i\omega\hat{u}} - \beta^{*}_{\omega}\, e^{i\omega\hat{u}}\right)$
$\frac{1}{\sqrt{2\omega}}\, e^{-i\omega u} = \int_0^\infty \frac{d\omega'}{\sqrt{2\omega'}}\left(\alpha_{\omega\omega'} e^{-i\omega'\hat{u}} - \beta^{*}_{\omega\omega'} e^{i\omega'\hat{u}}\right)$
$* e^{i\omega\hat{u}}$
you can integrate over all $\hat{u}$
$\int_{-\infty}^{\infty} d\hat{u}$
$\int d\hat{u}\, e^{i(\omega-\omega')\hat{u}} = 2\pi\delta(\omega-\omega')$
$-dt^2 + dx^2 =$
KG
Sols.


$du = e^{a\tilde{u}} d\tilde{u}$
$dv = e^{a\tilde{v}} d\tilde{v}$
$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})}\, d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{r2} + d\xi^{1^2}\right)$
Rindler wedge
$x > |t|$
$\alpha_\omega = \sqrt{\frac{\Omega}{\omega}} \frac{1}{2\pi} (-) \int_0^\infty d\tilde{u}\, e^{-i\omega u + i\omega\tilde{u}}$
$\frac{d}{du}$
$e^{i\omega\tilde{u}}$


$du = e^{a\tilde{u}} d\tilde{u}$
$dv = e^{a\tilde{v}} d\tilde{v}$
$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})}\, d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{r2} + d\xi^{1^2}\right)$
Rindler
$x > |t|$
$\alpha_\omega = \sqrt{\frac{\Omega}{\omega}}\ \frac{1}{2\pi}(-) \int_0^\infty d\tilde{u}\ e^{-i\omega u + i\Omega\tilde{u}}$
$\tilde{u} = \frac{1}{a} \ln ua$
$\alpha_\omega = -\frac{1}{2\pi}\sqrt{\frac{\Omega}{\omega}} \int_0^\infty du\ e^{-i\omega u + i\frac{\Omega}{a}\ln ua}$
$\frac{d}{du}$


$du = e^{a\tilde{u}} d\tilde{u}$
$dv = e^{a\tilde{v}} d\tilde{v}$
$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})} d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}(-d\xi^{r2} + d\xi^{1^2})$
Rindler wedge
$x > |t|$
$\alpha_\omega = \sqrt{\frac{\Omega}{\omega}} \frac{1}{2\pi} (-) \int_0^\infty d\tilde{u}\, e^{-i\omega u + i\Omega\tilde{u}}$
$\tilde{u} = \frac{1}{a} \ln ua$
$\alpha_\omega = -\frac{1}{2\pi}\sqrt{\frac{\Omega}{\omega}} \int_0^\infty du\, e^{-i\omega u + i\frac{\Omega}{a}\ln ua}$
$\beta_\omega = +\frac{1}{2\pi}\sqrt{\frac{\Omega}{\omega}} \int_0^\infty du\, e^{i\omega u + i\frac{\Omega}{a}\ln ua}$
$\frac{d}{du}$

$du = e^{a\tilde{u}} d\tilde{u}$ $dv = e^{a\tilde{v}} d\tilde{v}$

$-dt^2 + dx^2 = du\,dv = e^{a(\tilde{v}+\tilde{u})} d\tilde{u}\,d\tilde{v} = e^{2a\xi^1}\left(-d\xi^{0\,2} + d\xi^{1\,2}\right)$ → Rindler wedge

$x > |t|$

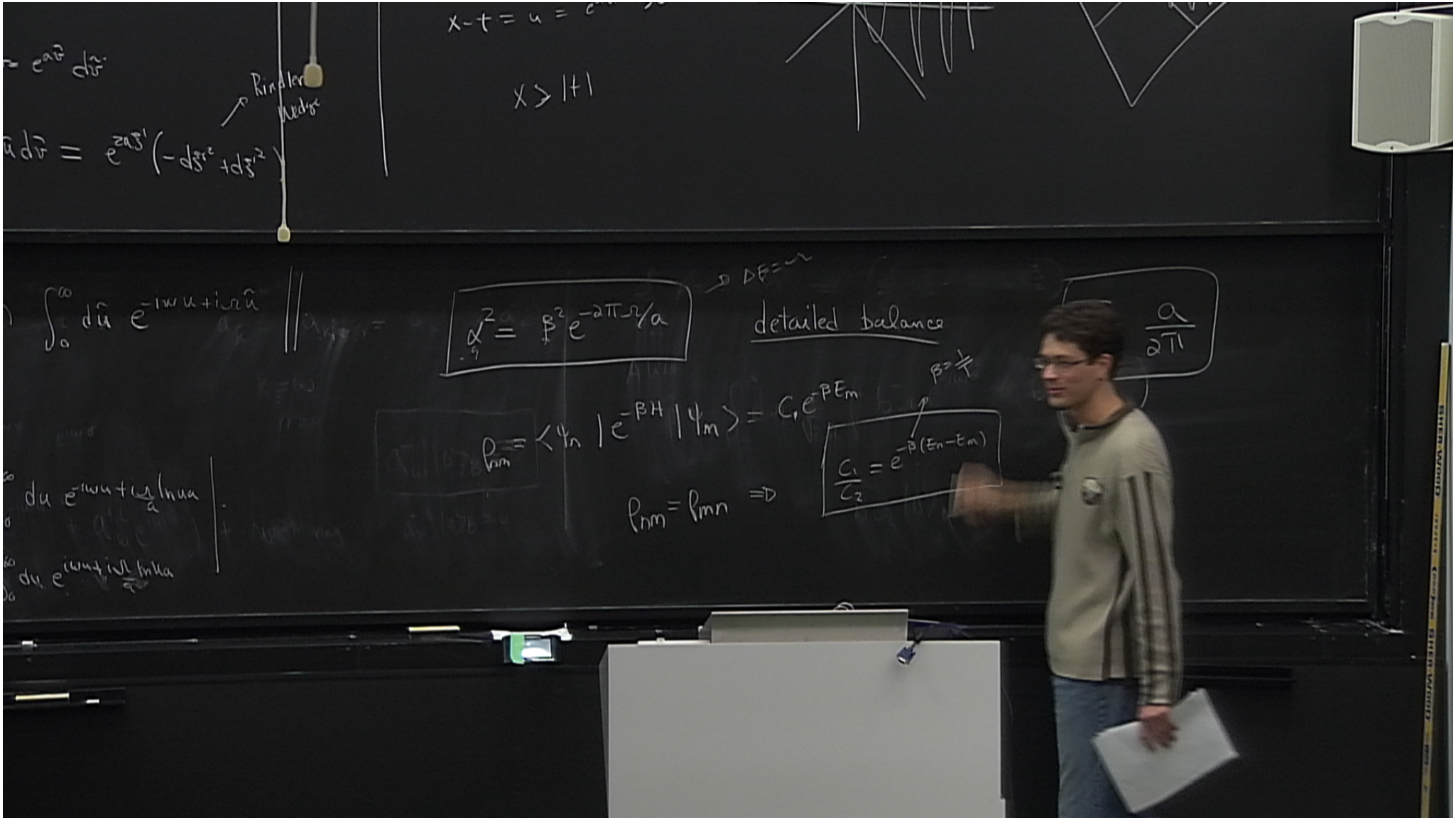
$\alpha_\omega = \sqrt{\frac{\Omega}{\omega}} \frac{1}{2\pi} (-) \int_0^\infty d\tilde{u}\, e^{-i\omega u + i\Omega\tilde{u}}$ ||

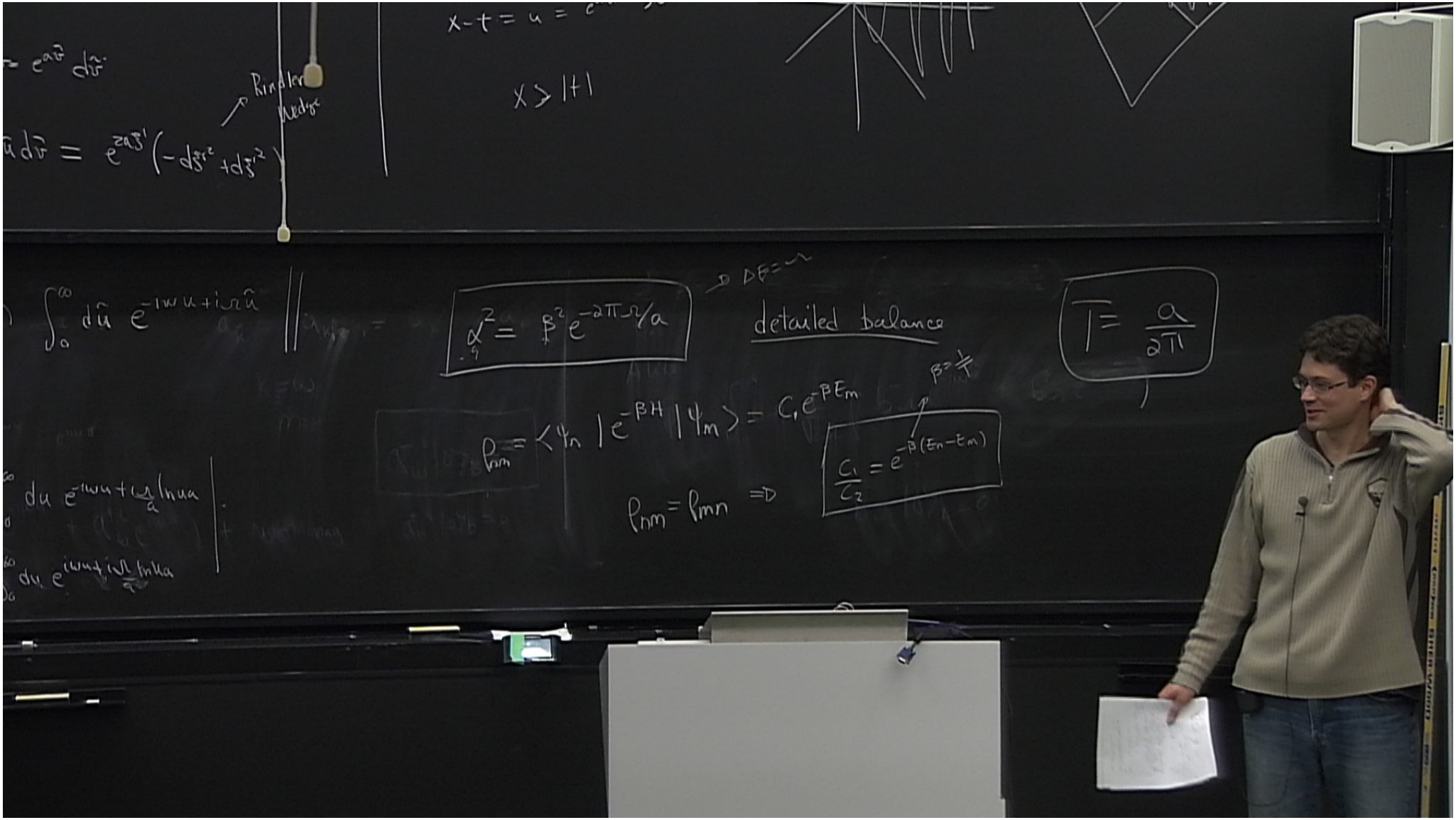
$\alpha^2 = \beta^2 e^{-2\pi\Omega/a}$ detailed balance

$\tilde{u} = \frac{1}{a} \ln ua$

$\alpha_\omega = -\frac{1}{2\pi}\sqrt{\frac{\Omega}{\omega}} \int_0^\infty du\, e^{-i\omega u + i\frac{\Omega}{a}\ln ua}$

$\beta_\omega = +\frac{1}{2\pi}\sqrt{\frac{\Omega}{\omega}} \int_0^\infty du\, e^{i\omega u + i\frac{\Omega}{a}\ln ka}$


Rindler
detailed balance


Rindler
detailed balance
$\alpha^2 = \beta^2 e^{-2\pi\omega/a}$
$T = \frac{a}{2\pi}$
$\rho_{nm} = \langle \psi_n | e^{-\beta H} | \psi_m \rangle = C_1 e^{-\beta E_n}$
$\rho_{nm} = \rho_{mn} \Rightarrow$
$\frac{C_1}{C_2} = e^{-\beta(E_n - E_m)}$
$\beta = \frac{1}{T}$