Title: Explorations in Cosmology - Lecture 2

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Collection: 11/12 PSI - Explorations in Cosmology

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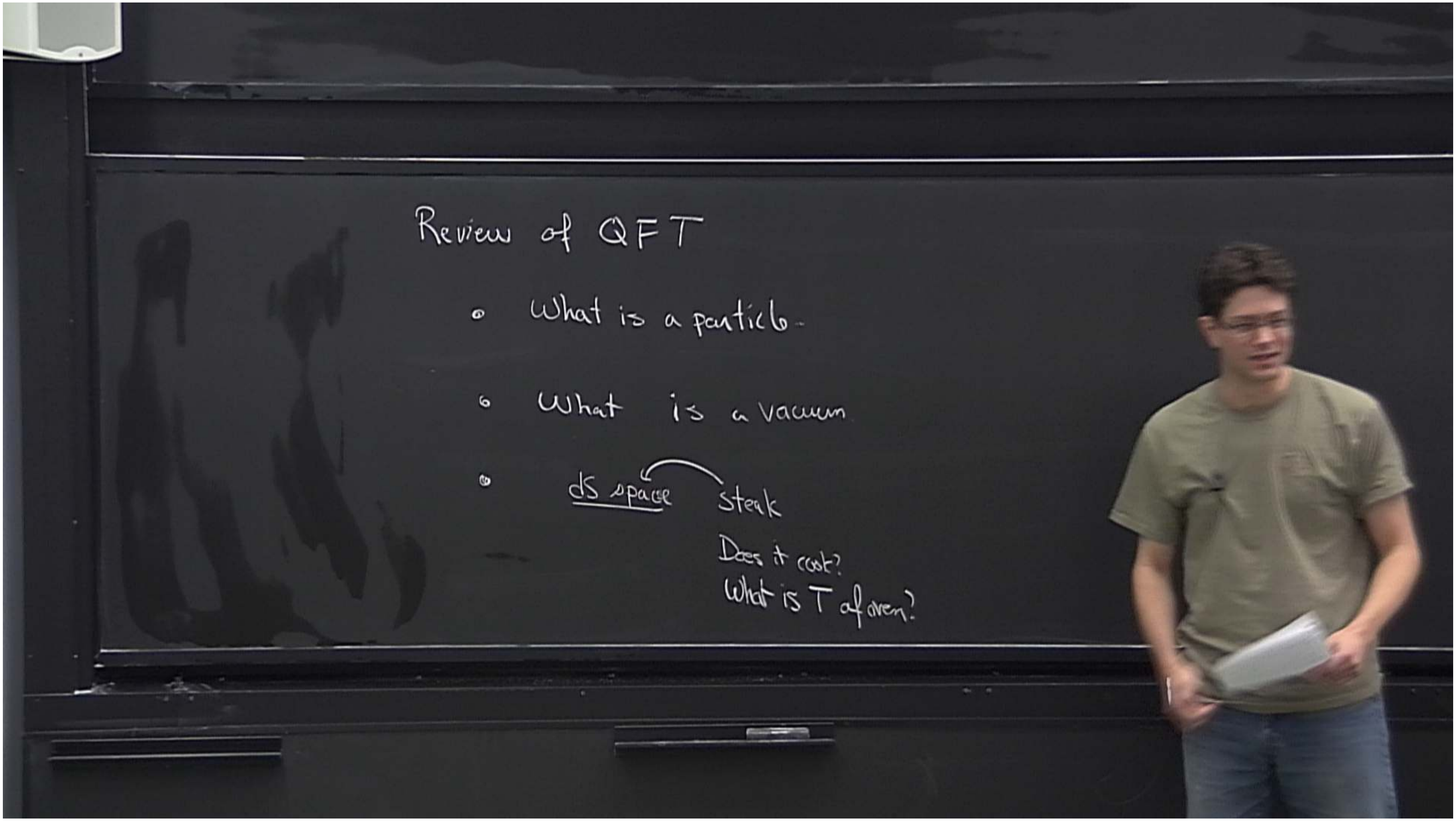
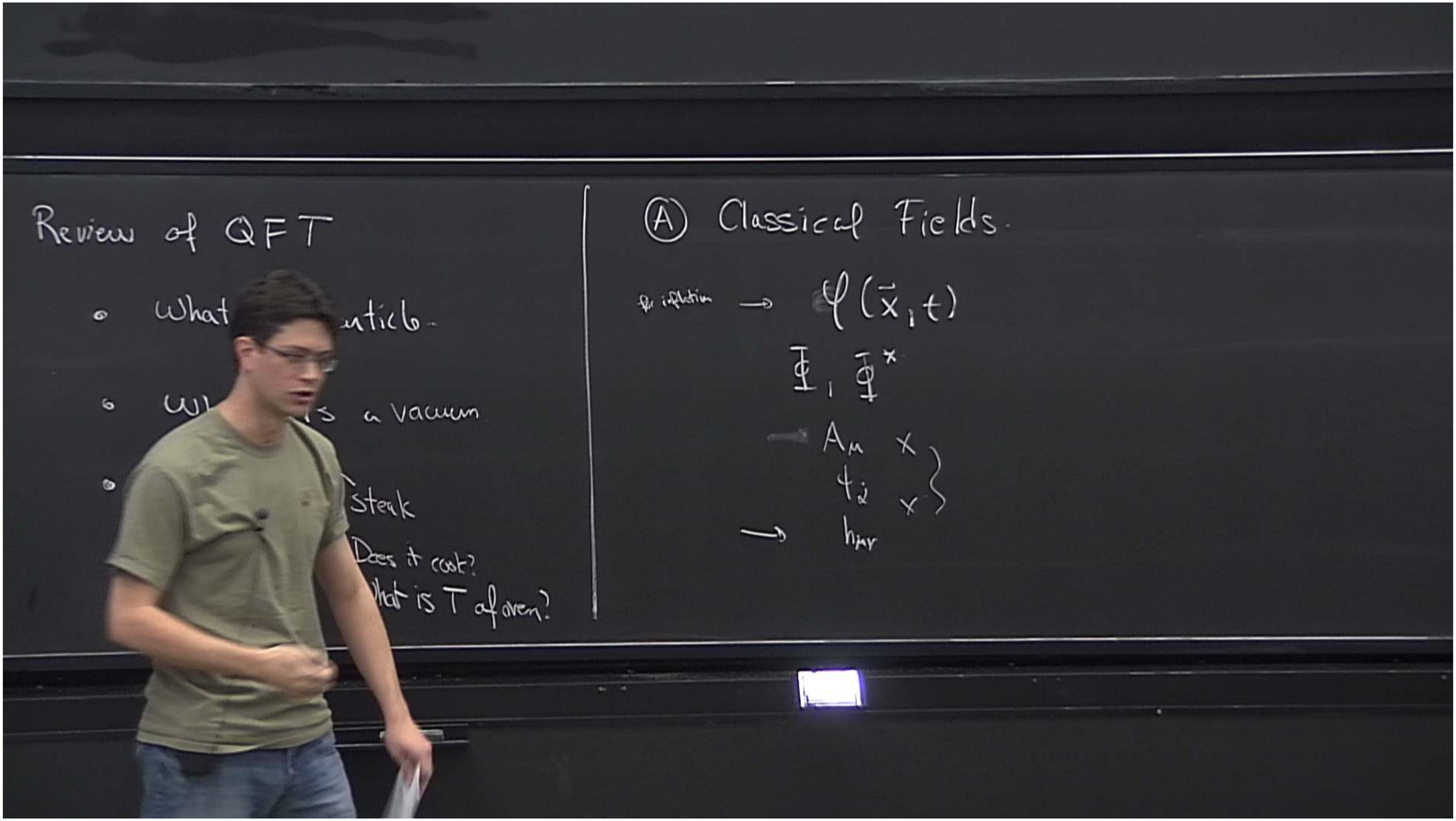
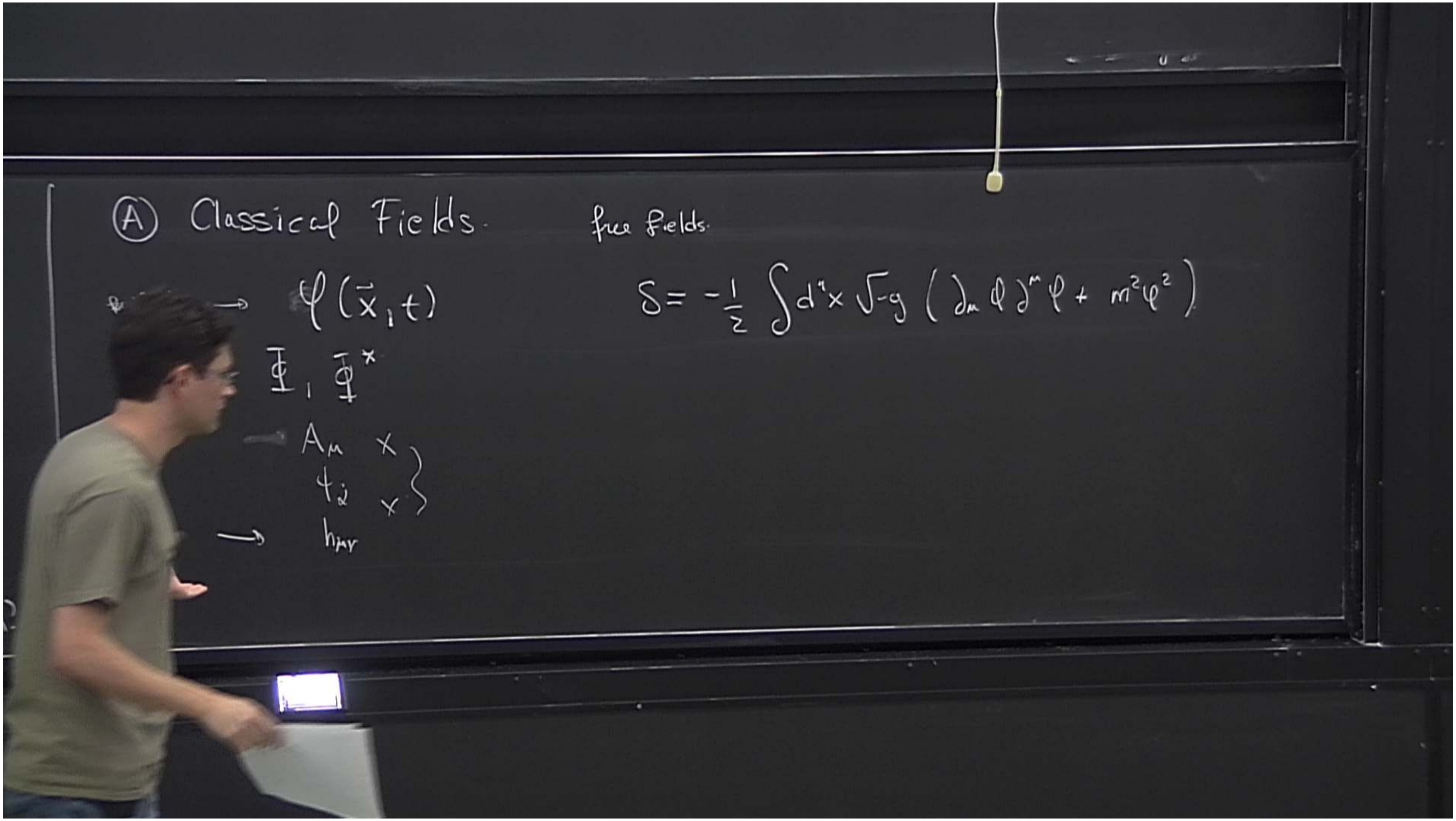
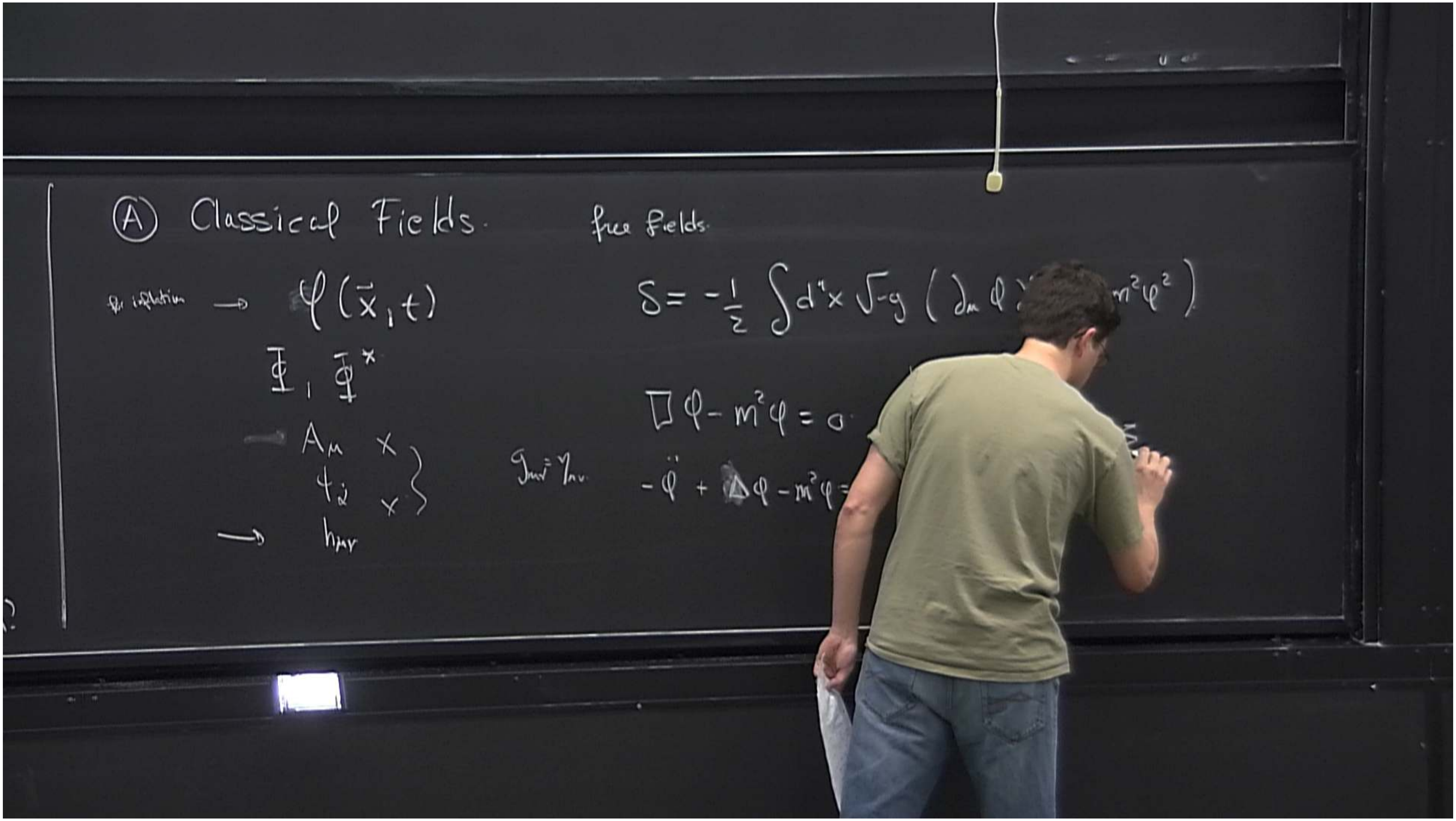
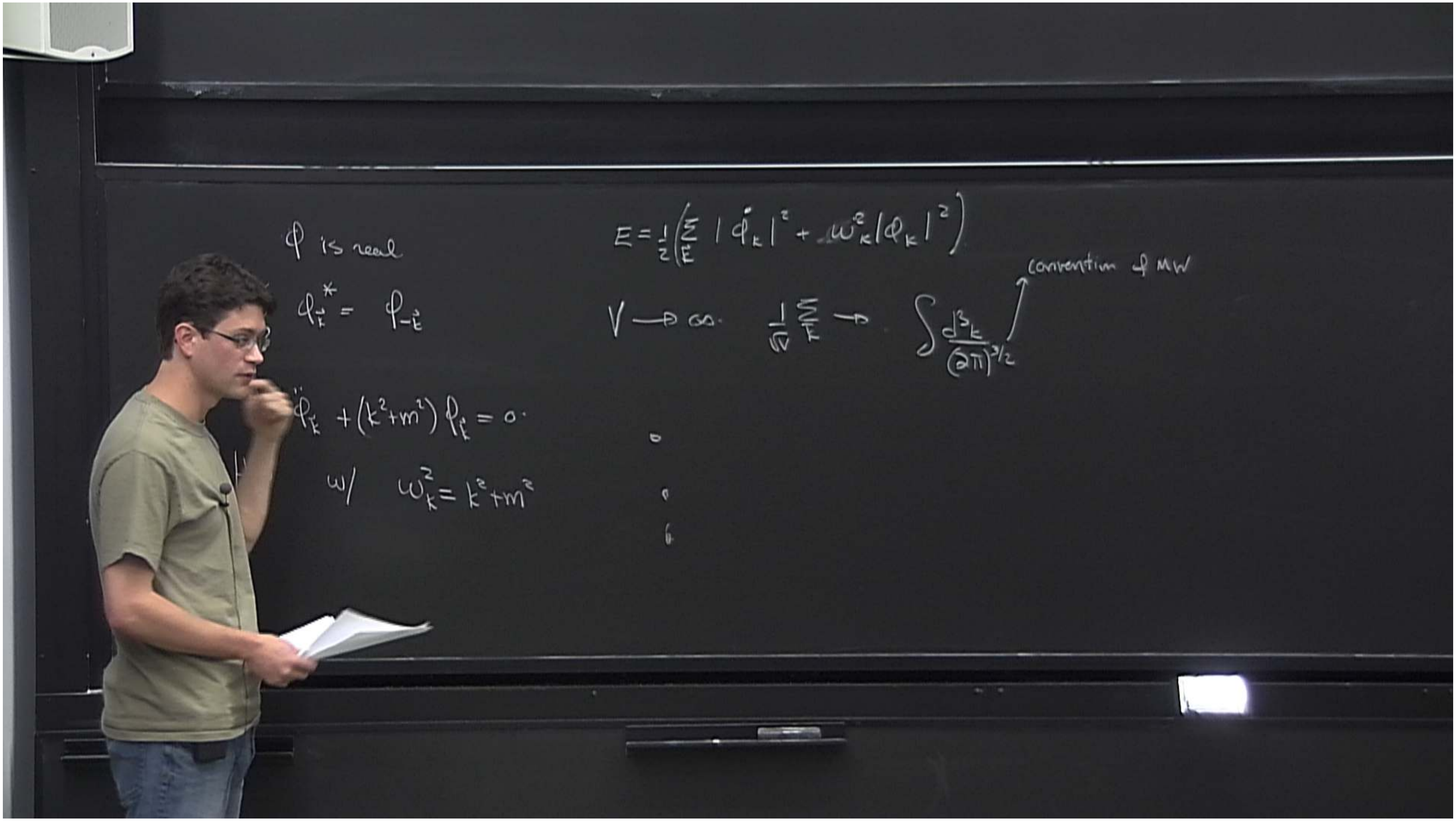
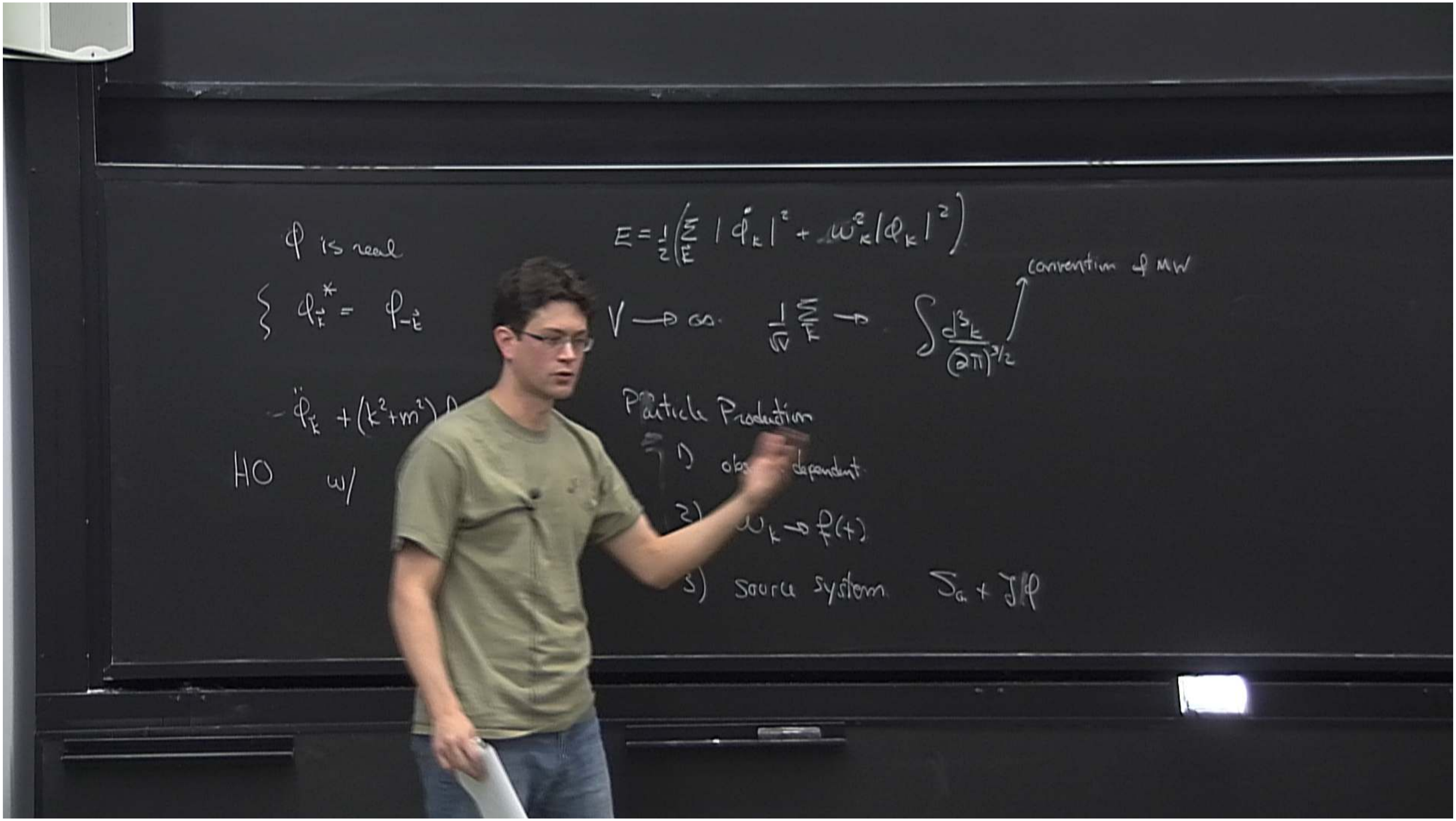

Review of QFT
What is a particle
What is a vacuum
dS space
steak
Does it cook?
What is T of oven?

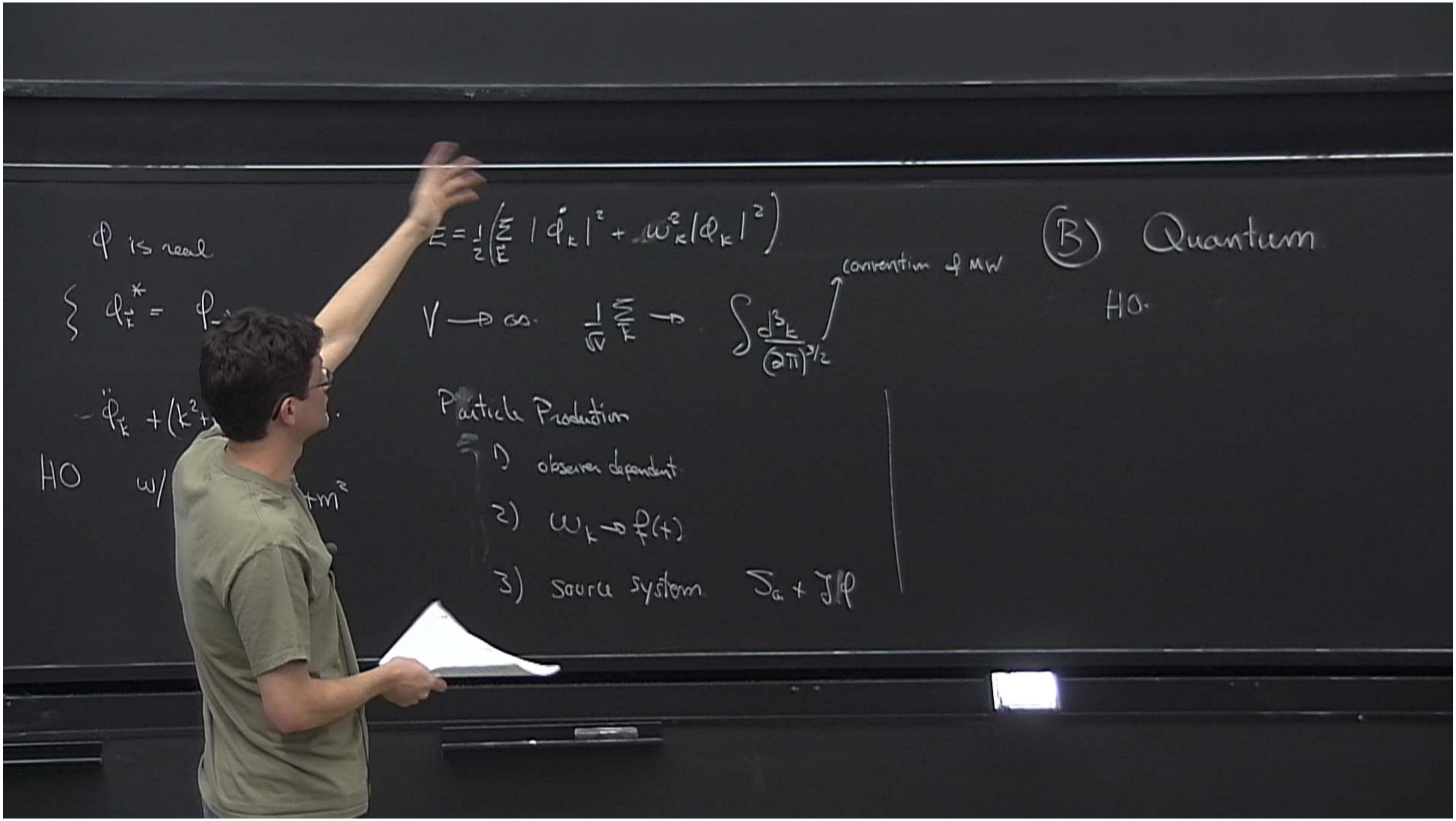

Review of QFT
What
a vacuum
Steak
Does it cook?
is T of oven?
(A) Classical Fields.
φ(x, t)
Φ, Φ*
Aμ
hμν

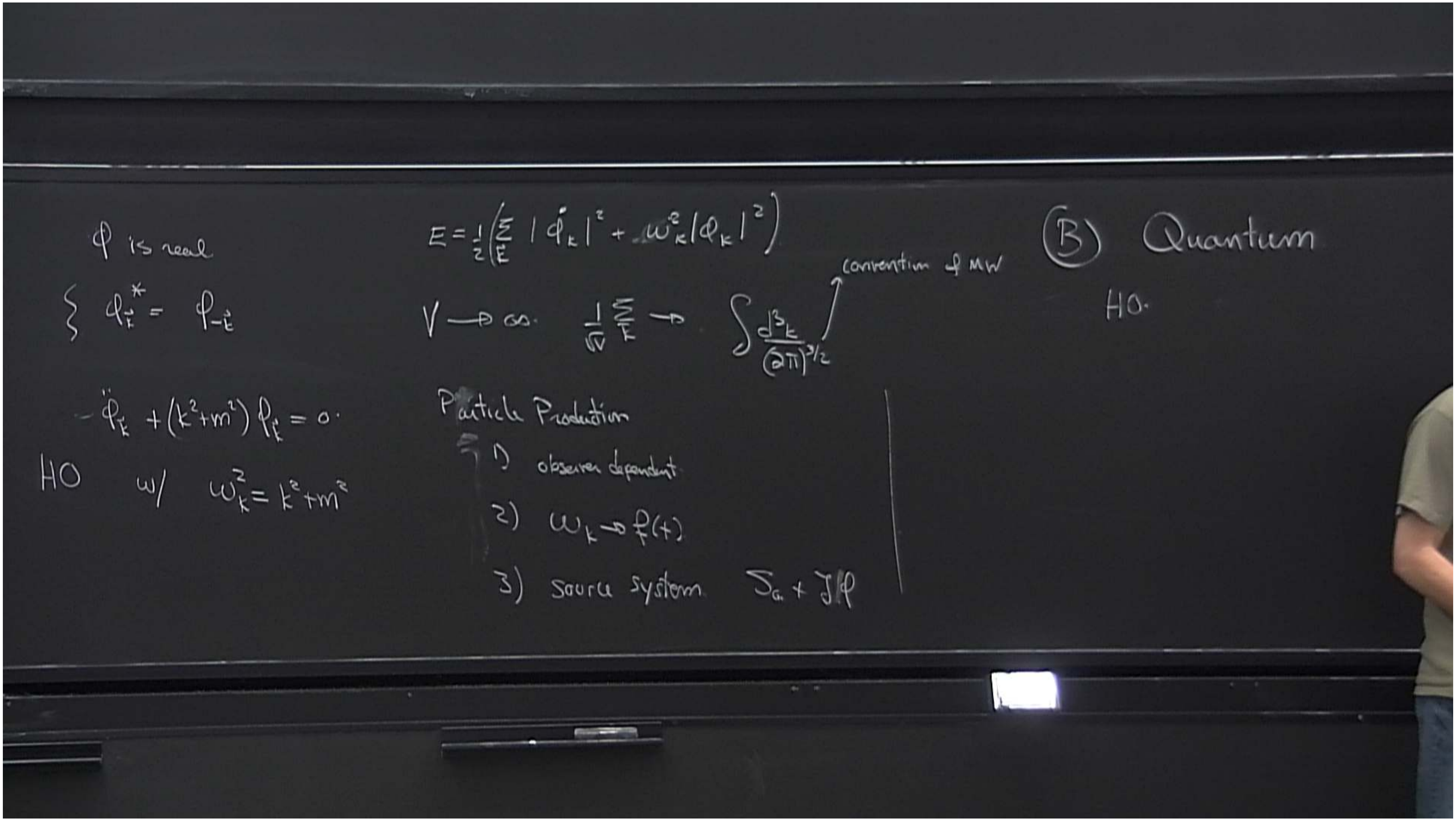

(A) Classical Fields.
free fields.
$\varphi(\vec{x}, t)$
Φ, Φ^*
A_μ ×
ψ_α ×
$h_{\mu\nu}$
$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left(\partial_\mu \varphi \partial^\mu \varphi + m^2\varphi^2\right)$

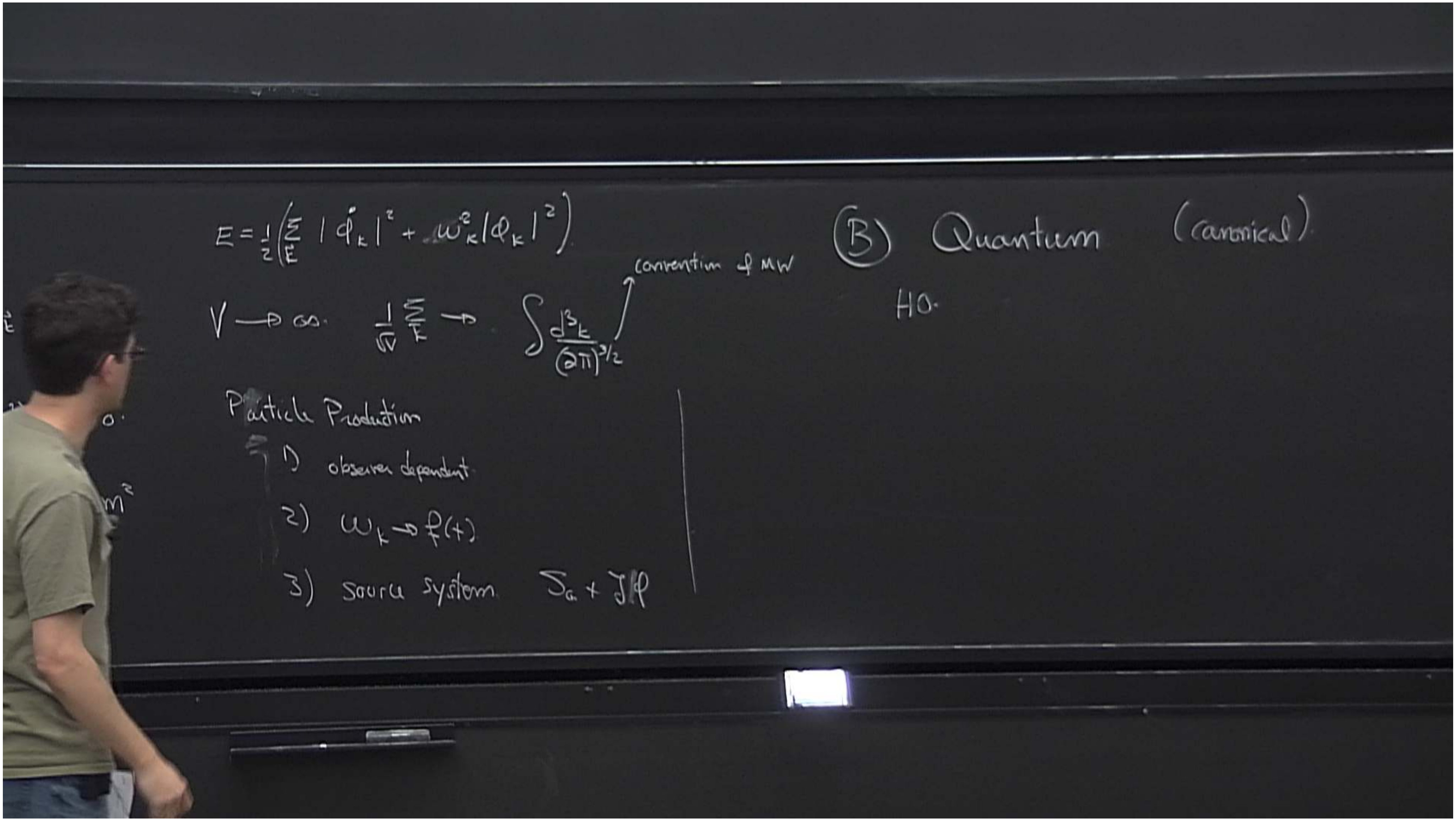

(A) Classical Fields.
free fields.
ex. inflation → φ(x⃗, t)
Φ, Φ*
A_μ ×
ψ_α ×
→ h_μν
g_μν = η_μν
S = −1/2 ∫d⁴x √−g (∂_μ φ ... m²φ²)
□φ − m²φ = 0
−φ̈ + Δφ − m²φ =

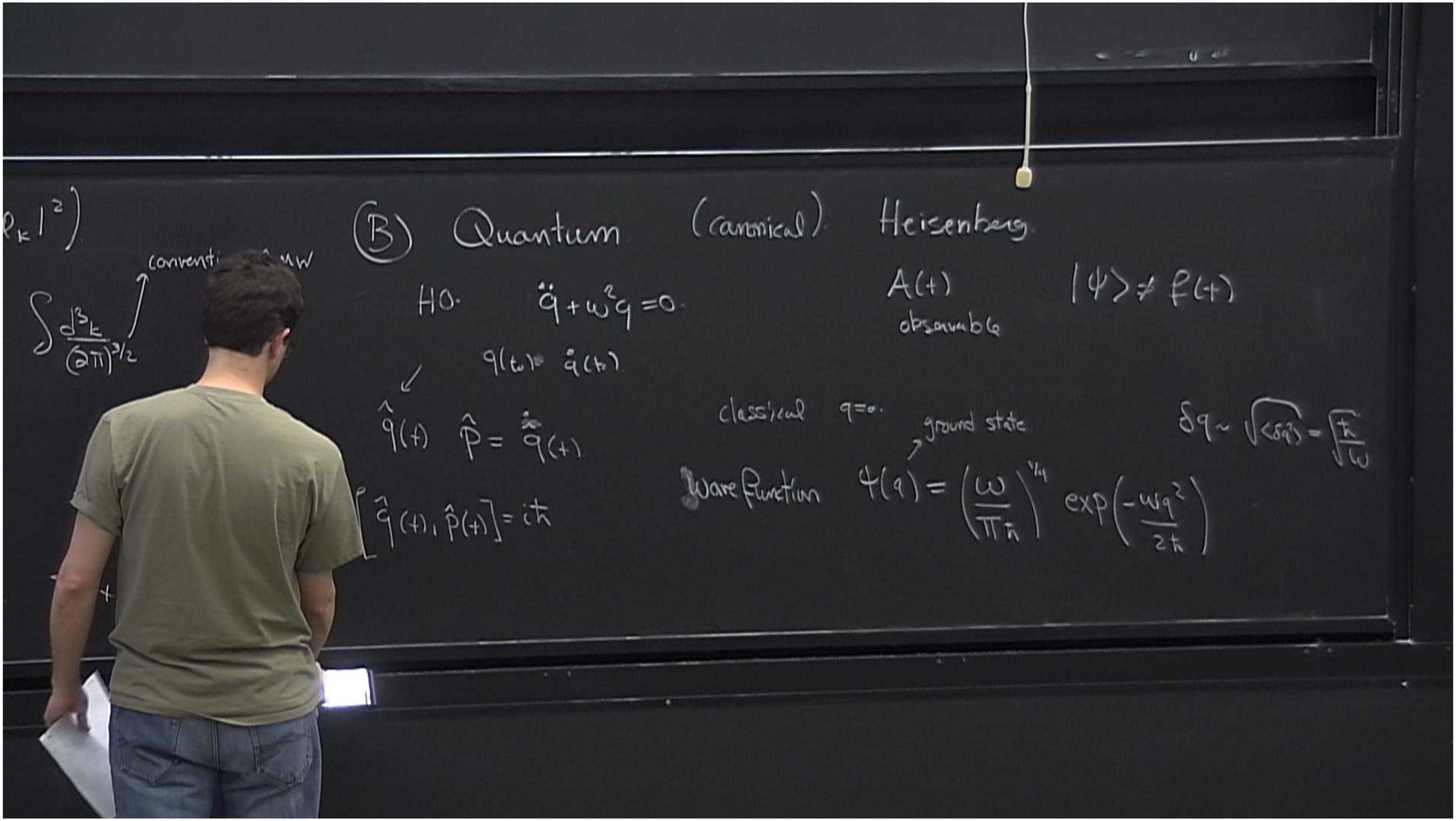

φ is real
φ*_k = φ_-k
φ̈_k + (k²+m²) φ_k = 0.
w/ ω²_k = k² + m²
E = 1/2 (Σ_k |φ̇_k|² + ω²_k |φ_k|²)
V → ∞. 1/√V Σ_k → ∫ d³k/(2π)^3/2
convention of MW

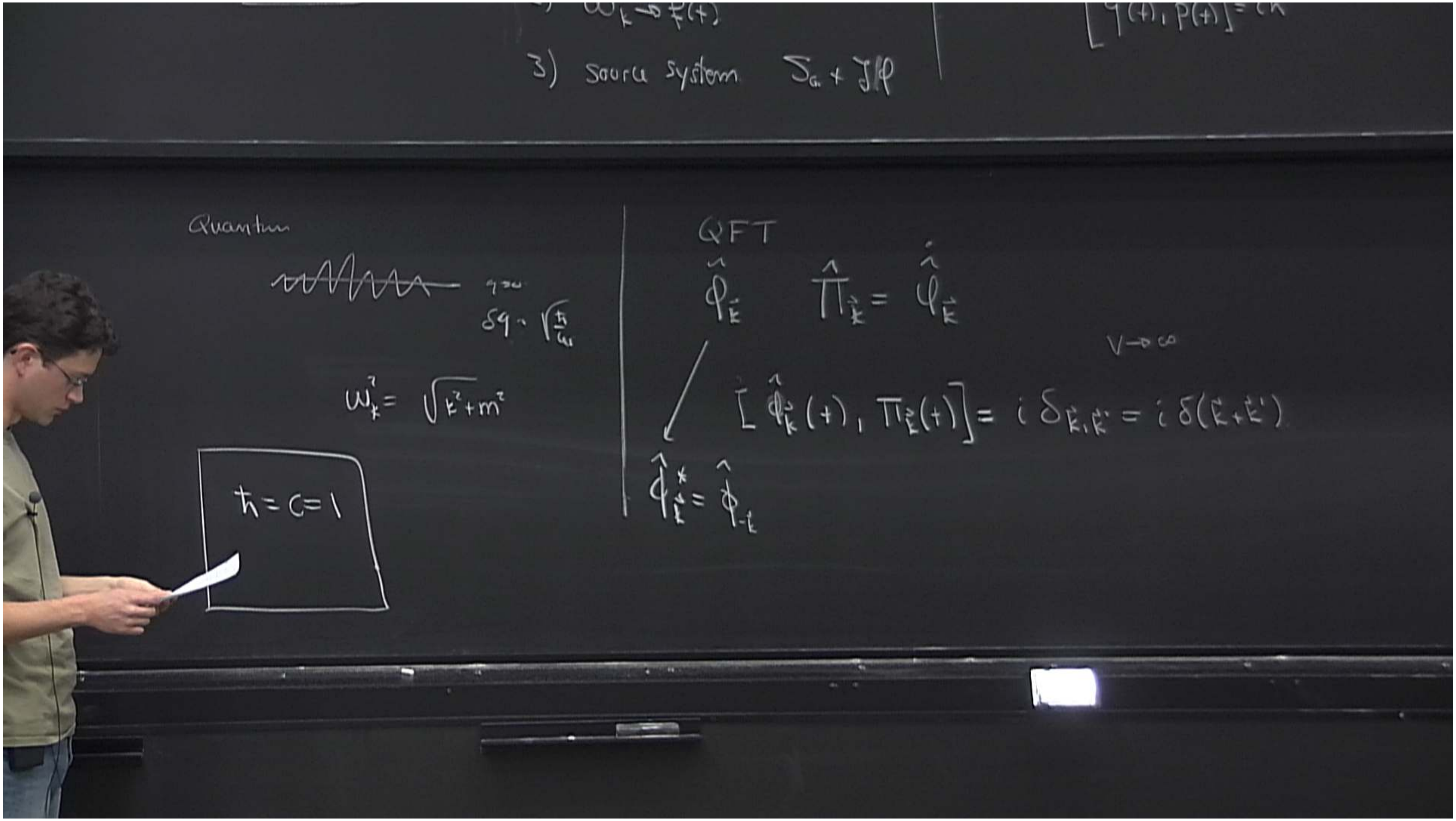

φ is real
E = ½(Σ |φ̇ₖ|² + ω²ₖ|φₖ|²)
convention of MW
V → ∞
Particle Production
1) observer dependent
2) ωₖ → f(t)
3) source system
HO w/

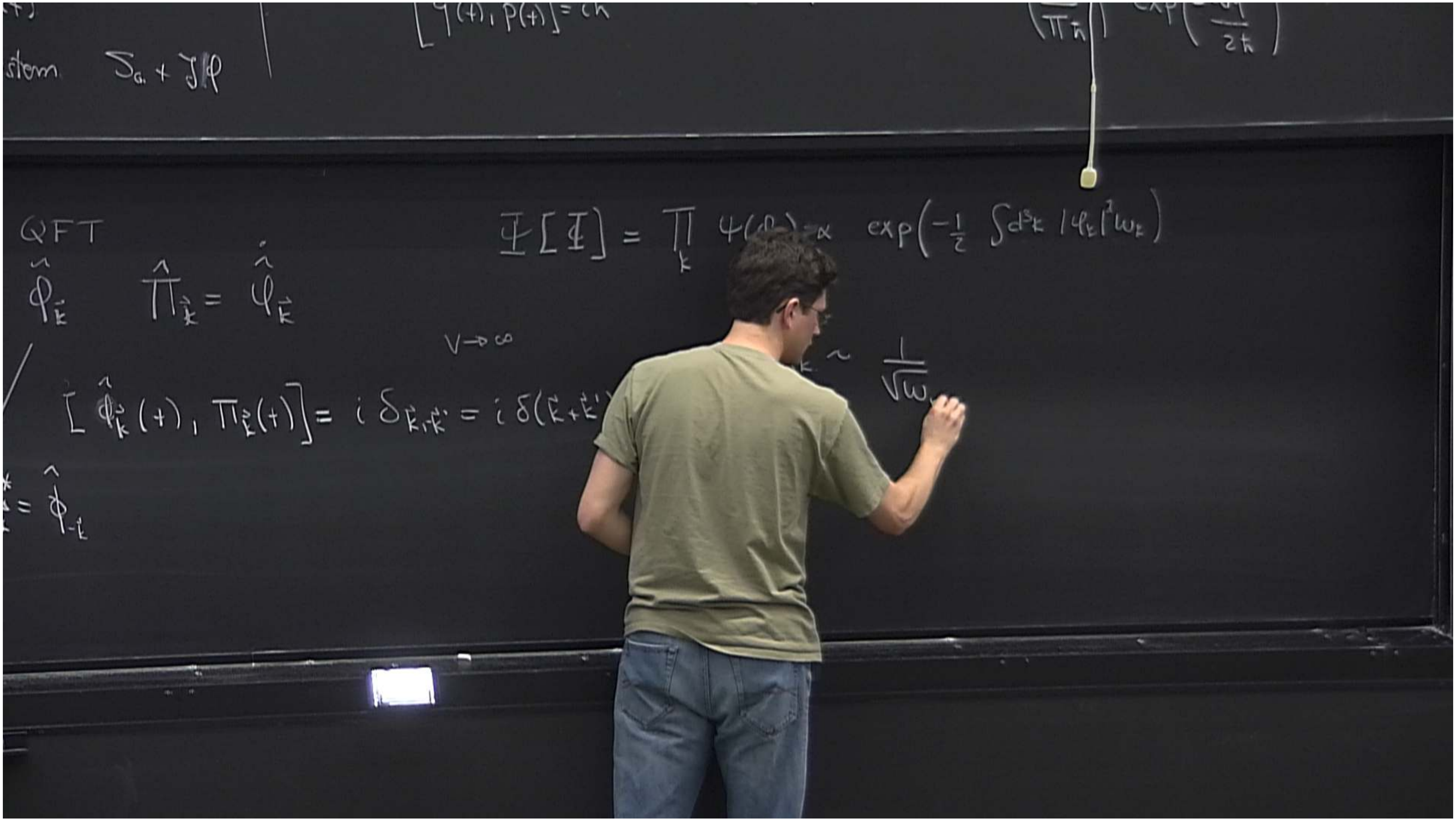

φ is real
HO
Particle Production
1) observer dependent
2)
3) source system
(B) Quantum
HO
convention of MW

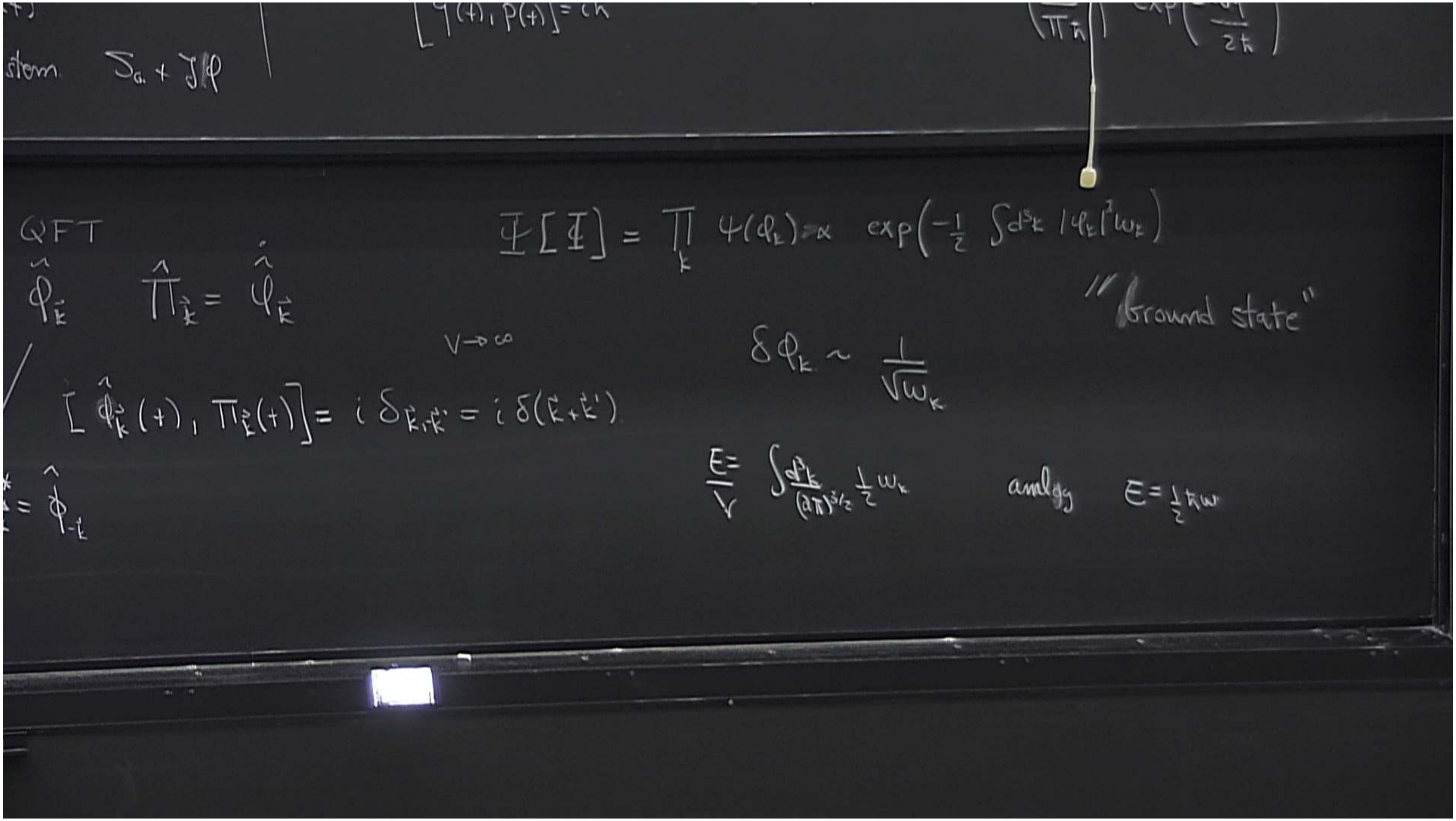

φ is real
$\{ \varphi_{\vec{k}}^* = \varphi_{-\vec{k}}$
$-\ddot{\varphi}_{\vec{k}} + (k^2+m^2)\varphi_{\vec{k}} = 0$
HO w/ $\omega_k^2 = k^2+m^2$
$E = \frac{1}{2}\left(\sum_k |\dot{\varphi}_k|^2 + \omega_k^2 |\varphi_k|^2\right)$
$V \to \infty$. $\frac{1}{\sqrt{V}}\sum_k \to \int \frac{d^3k}{(2\pi)^{3/2}}$
convention of MW
Particle Production
1) observer dependent
2) $\omega_k \to f(t)$
3) source system $S_a + J\varphi$
(B) Quantum
HO.


$E = \frac{1}{2}\left(\sum_k |\dot{\varphi}_k|^2 + \omega_k^2 |\varphi_k|^2\right)$
convention of MW
$V \to \infty$. $\frac{1}{\sqrt{V}}\sum_k \to \int \frac{d^3k}{(2\pi)^{3/2}}$
Particle Production
1) observer dependent
2) $\omega_k \to f(t)$
3) source system $S_a + J\varphi$
(B) Quantum (canonical)
H.O.

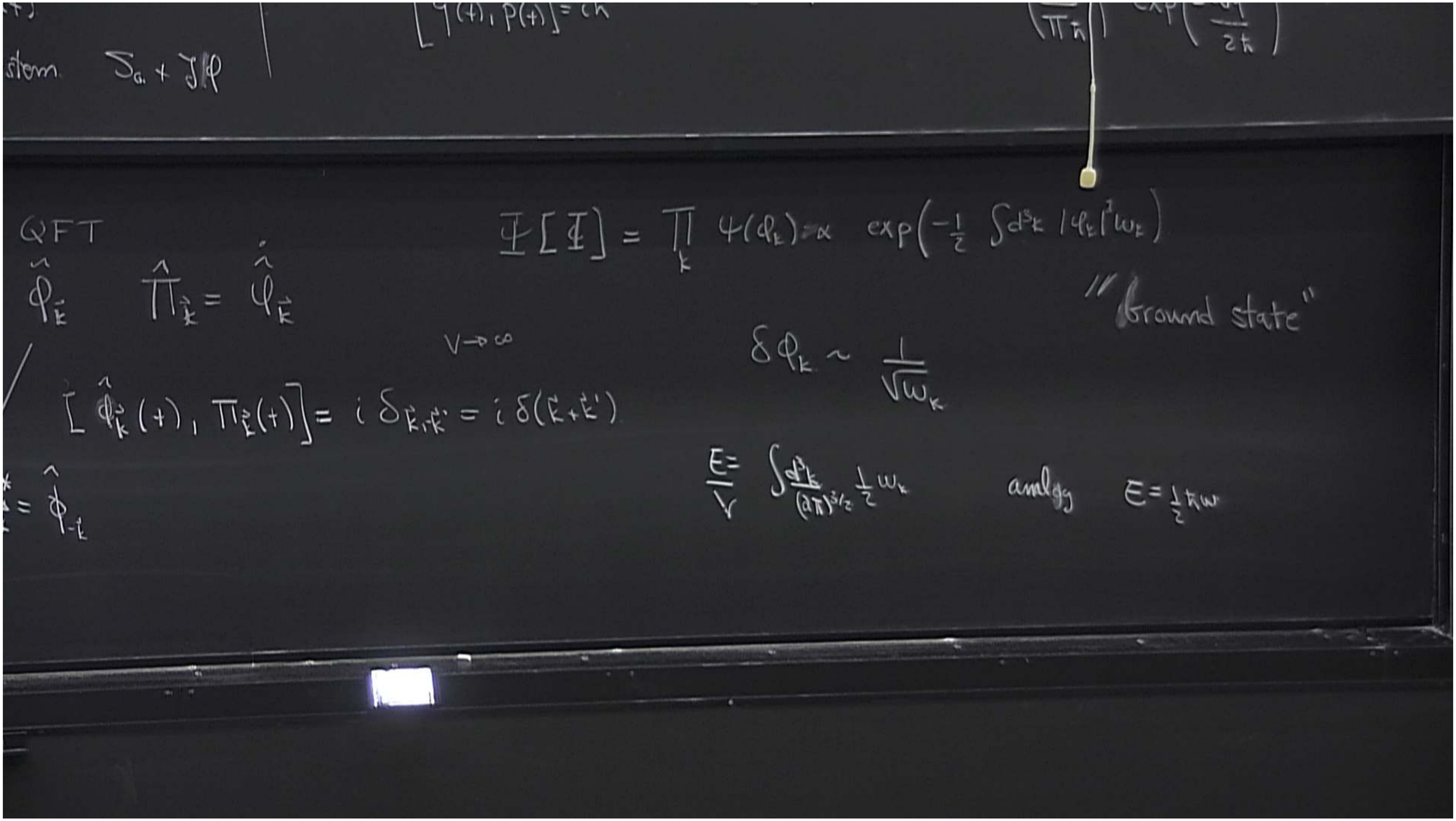

(B) Quantum (canonical) Heisenberg
H.O. q̈ + ω²q = 0
A(t)
observable
|ψ> ≠ f(t)
classical q=0
ground state
δq ~ √⟨δq²⟩ = √(ħ/ω)
wave function ψ(q) = (ω/πħ)^(1/4) exp(−ωq²/2ħ)
[q̂(t), p̂(t)] = iħ


3) source system $S_G + J\phi$
Quantum
$\delta q \sim \sqrt{\frac{\hbar}{\omega}}$
$\omega_k = \sqrt{k^2+m^2}$
$\hbar = c = 1$
QFT
$\hat\phi_{\vec k}$ $\hat\Pi_{\vec k} = \dot{\hat\phi}_{\vec k}$
$V \to \infty$
$[\hat\phi_{\vec k}(t), \Pi_{\vec k'}(t)] = i\,\delta_{\vec k, \vec k'} = i\,\delta(\vec k + \vec k')$
$\hat\phi^*_{\vec k} = \hat\phi_{-\vec k}$


QFT
V→∞



QFT
$\hat{\varphi}_{\vec{k}}$ $\hat{\Pi}_{\vec{k}} = \dot{\hat{\varphi}}_{\vec{k}}$
$V \to \infty$
$[\hat{\varphi}_{\vec{k}}(t), \Pi_{\vec{k}'}(t)] = i\delta_{\vec{k},\vec{k}'} = i\delta(\vec{k}+\vec{k}')$
$\hat{\phi}_{-\vec{k}}$
$\Psi[\Phi] = \prod_k \psi(\varphi_k) \propto \exp\left(-\frac{1}{2}\int d^3k\, |\varphi_k|^2 \omega_k\right)$
"Ground state"
$\delta\varphi_k \sim \frac{1}{\sqrt{\omega_k}}$
$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2}\omega_k$
analogy $E = \frac{1}{2}\hbar\omega$


QFT
$\Psi[\Phi] = \prod_k \psi(\phi_k) \propto \exp\left(-\frac{1}{2}\int d^3k\, |\phi_k|^2 \omega_k\right)$
$\hat{\phi}_{\vec{k}} \quad \hat{\Pi}_{\vec{k}} = \dot{\hat{\phi}}_{\vec{k}}$
"Ground state"
$V \to \infty$
$\delta\phi_k \sim \frac{1}{\sqrt{\omega_k}}$
$[\hat{\phi}_{\vec{k}}(t), \Pi_{\vec{k}'}(t)] = i\,\delta_{\vec{k},\vec{k}'} = i\,\delta(\vec{k}+\vec{k}')$
$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2}\omega_k$
analogy $E = \frac{1}{2}\hbar\omega$
$= \hat{\phi}_{-\vec{k}}$

$[q(t), p(t)] = i\hbar$

stem $S_G + J\varphi$

QFT

$\hat{\varphi}_{\vec{k}} \qquad \hat{\Pi}_{\vec{k}} = \dot{\hat{\varphi}}_{\vec{k}}$

$V \to \infty$

$[\hat{\varphi}_{\vec{k}}(t), \Pi_{\vec{k}'}(t)] = i\,\delta_{\vec{k},\vec{k}'} = i\,\delta(\vec{k}+\vec{k}')$

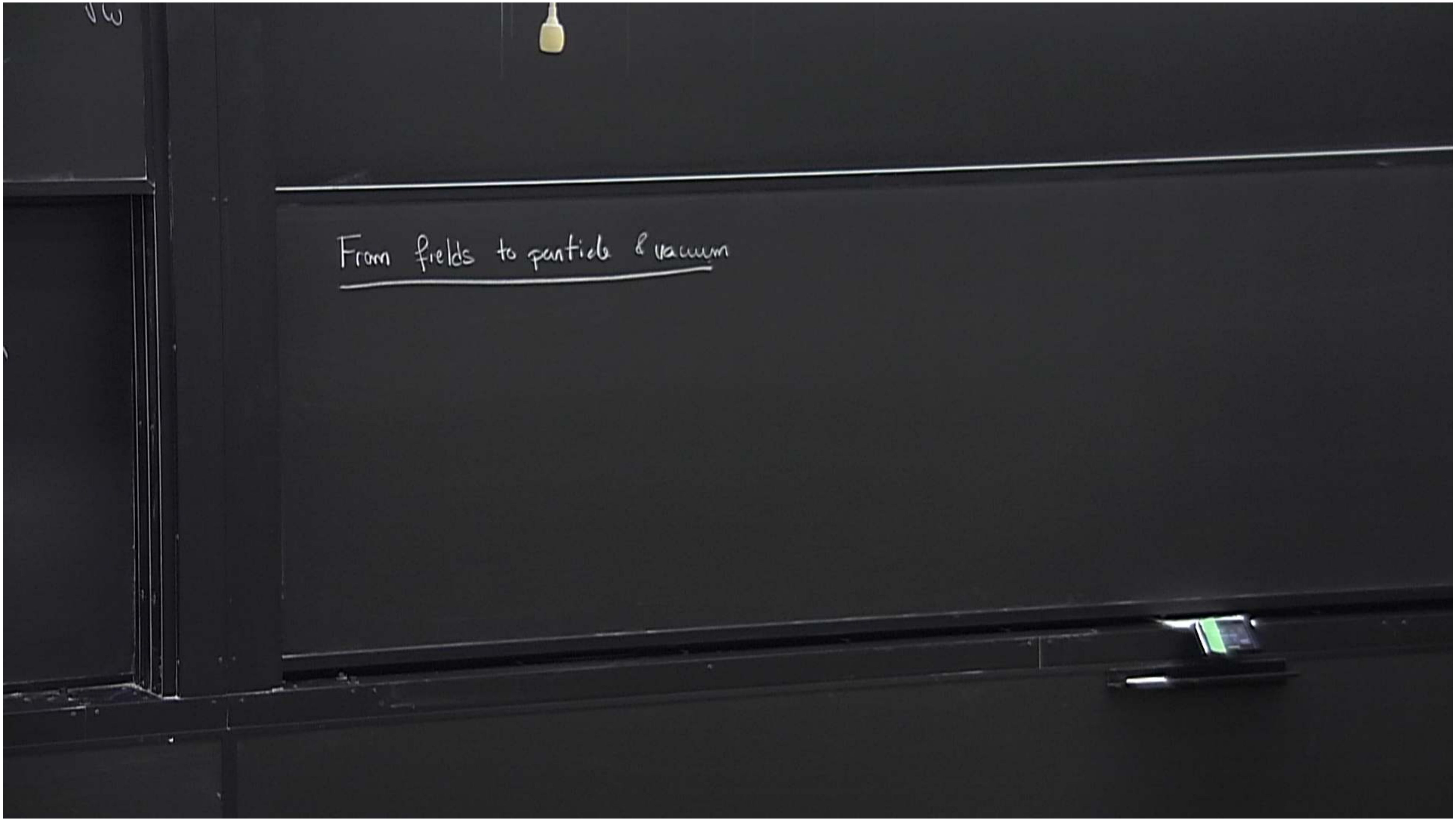
$\hat{\varphi}^*_{\vec{k}} = \hat{\varphi}_{-\vec{k}}$

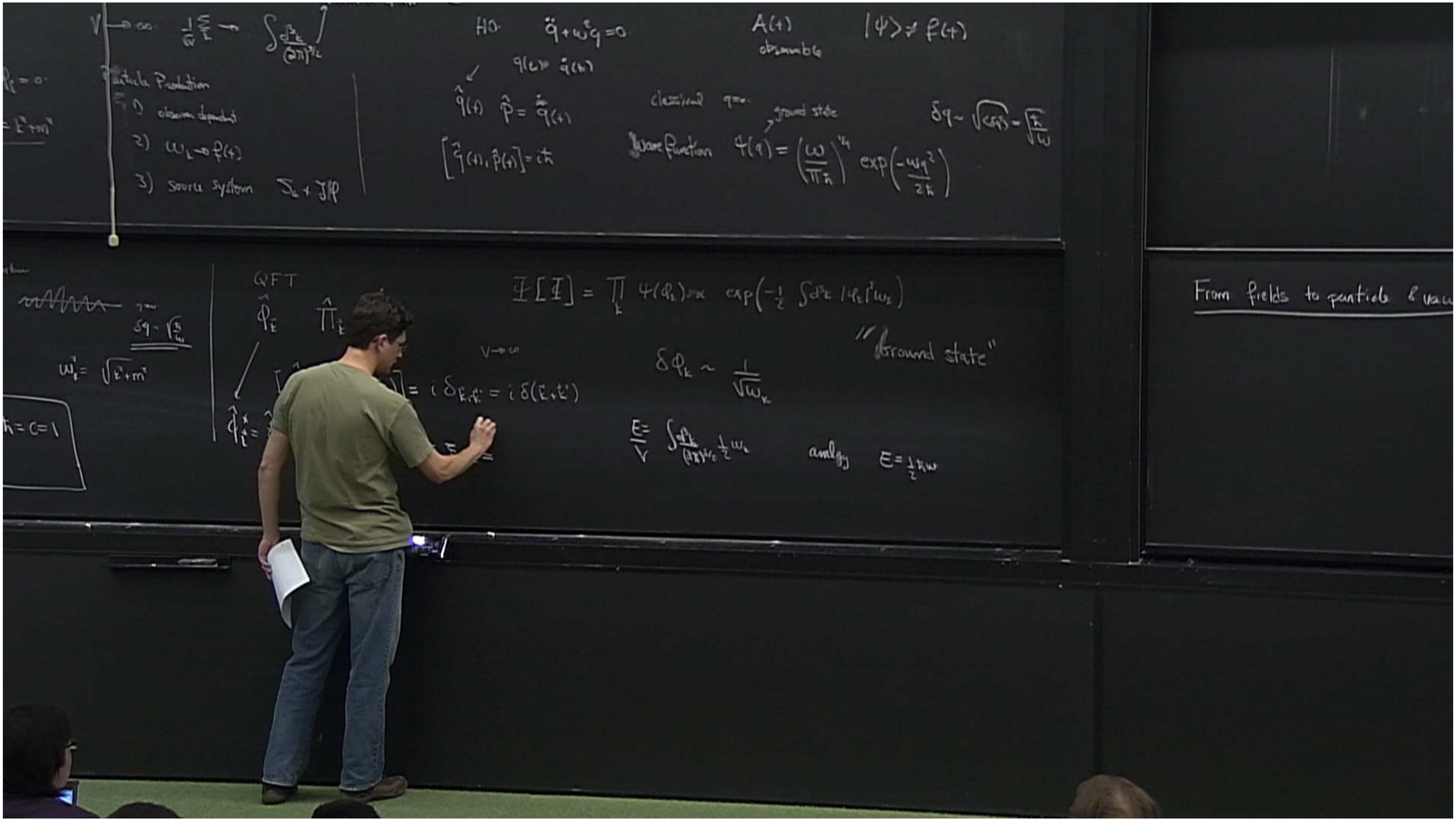
$\Psi[\Phi] = \prod_k \psi(\varphi_k) \propto \exp\left(-\frac{1}{2}\int d^3k\, |\varphi_k|^2 \omega_k\right)$

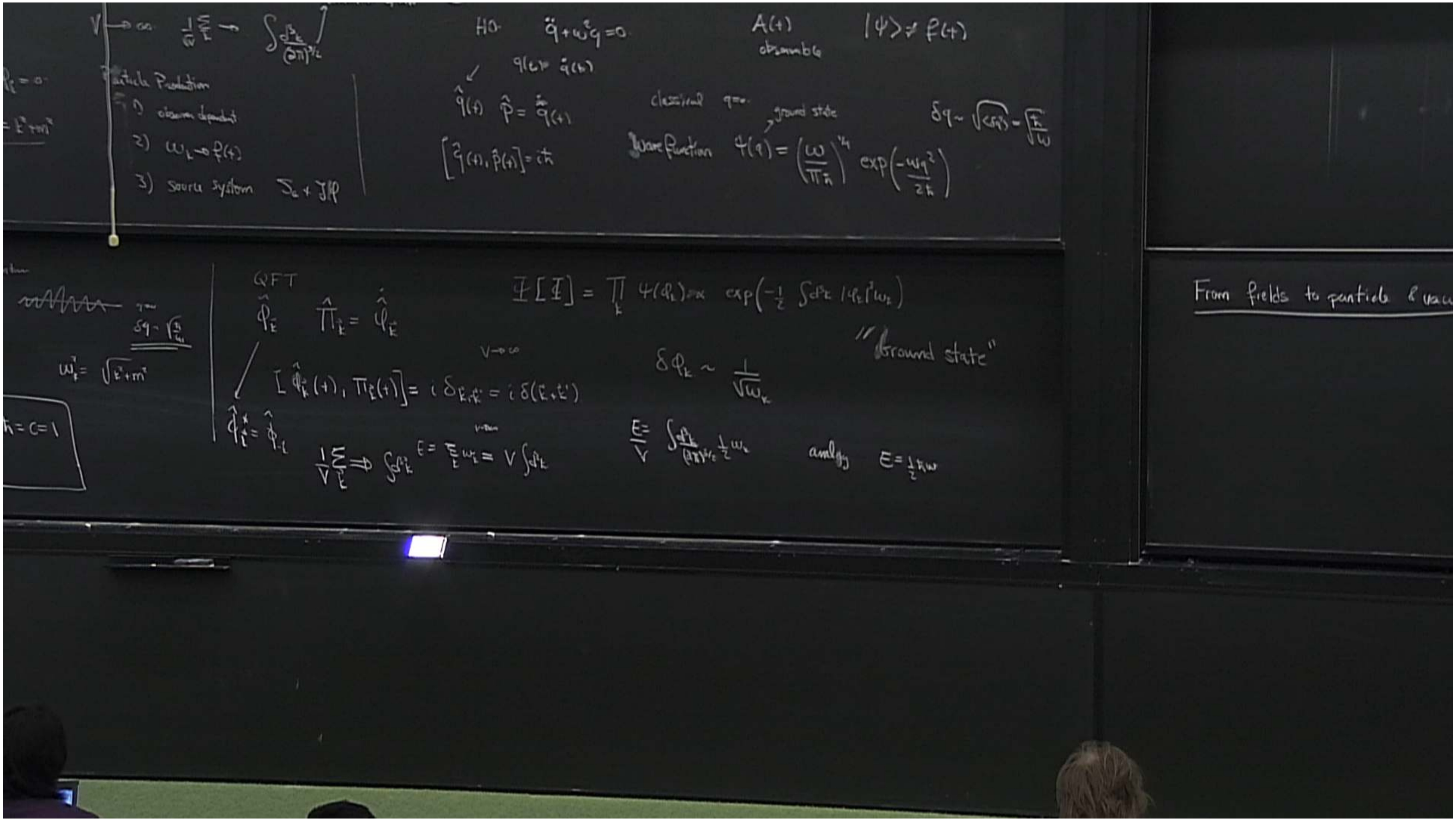
"Ground state"

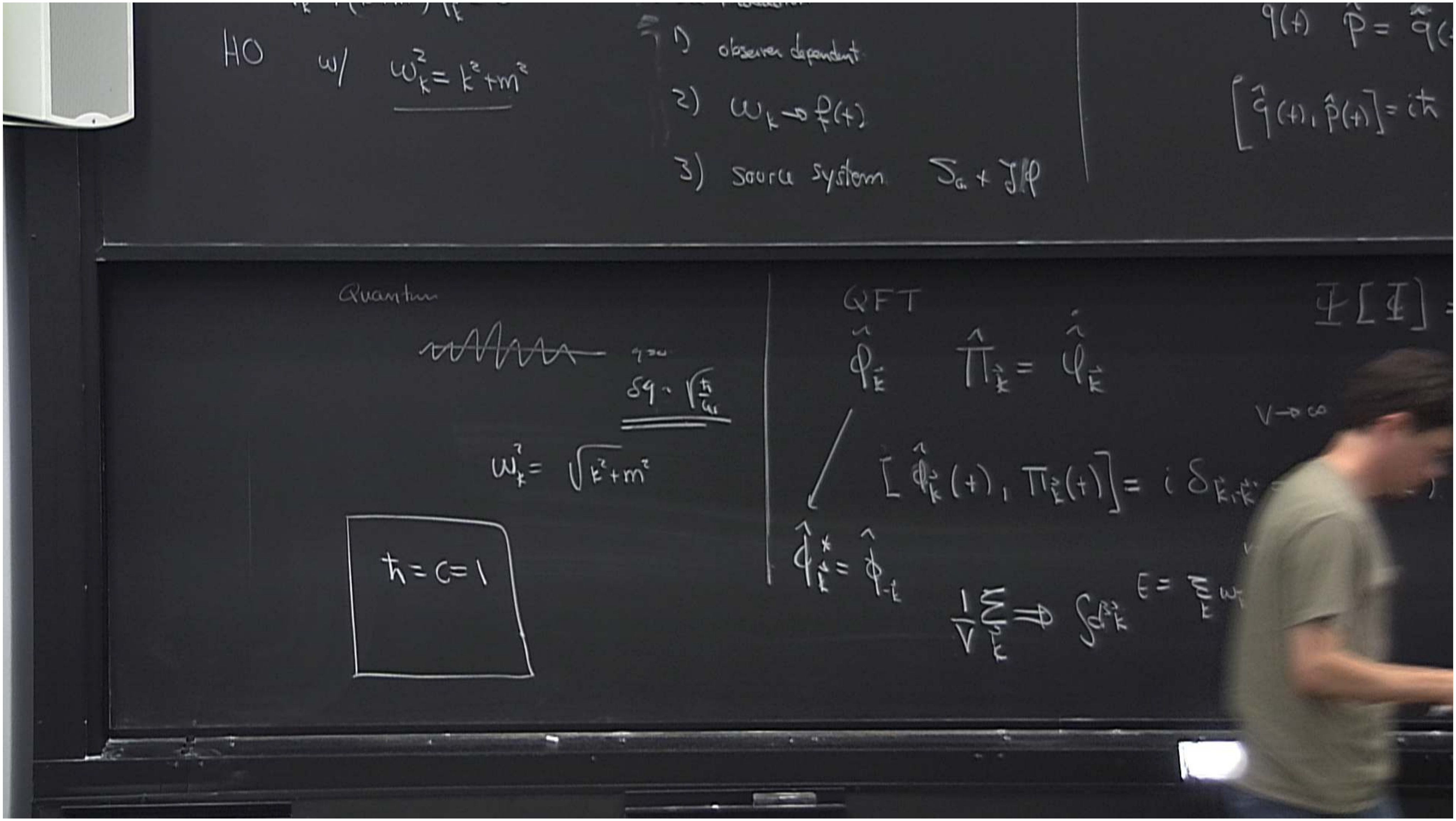
$\delta\varphi_k \sim \frac{1}{\sqrt{\omega_k}}$

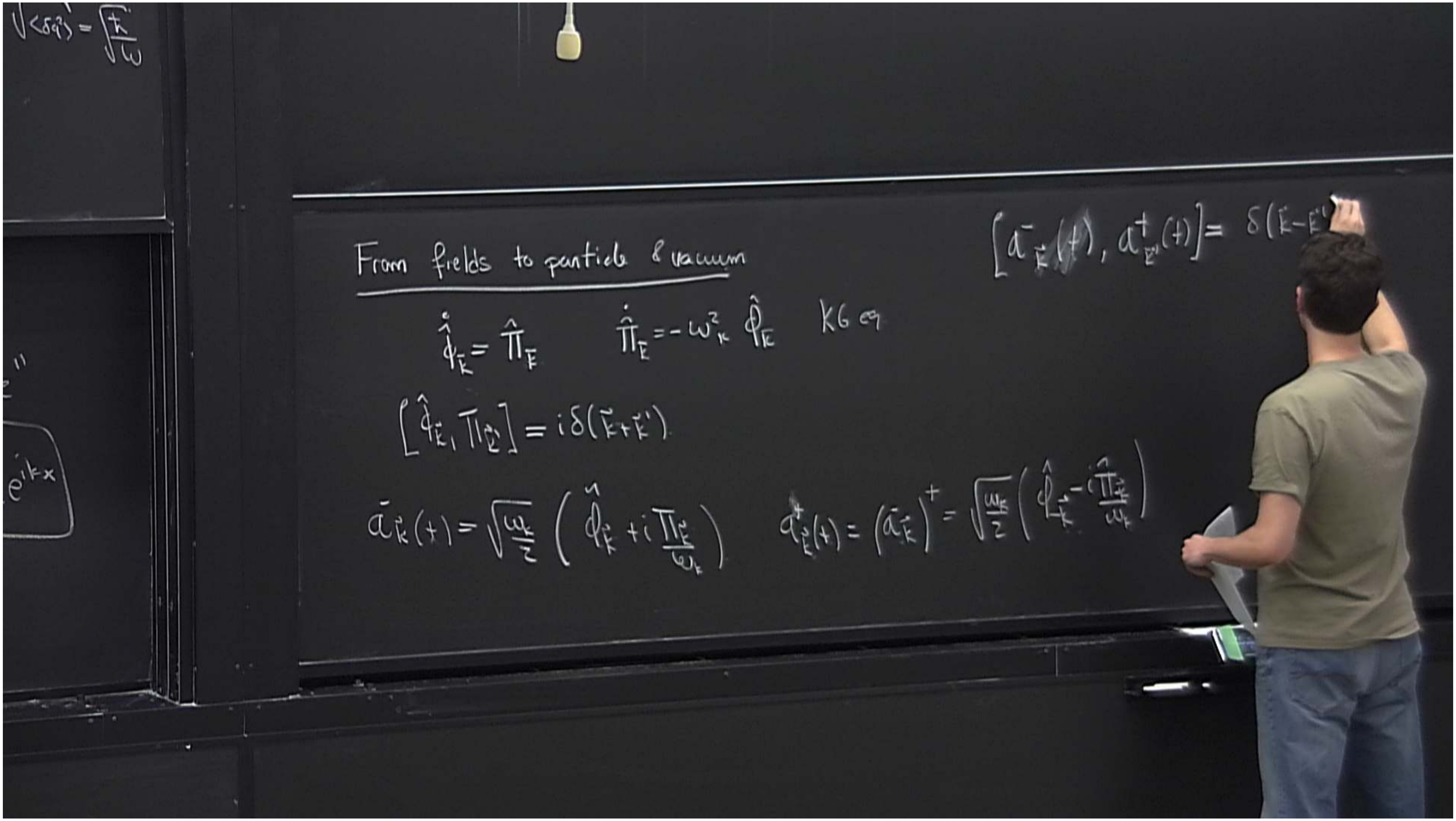
$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2}\omega_k$ analogy $E = \frac{1}{2}\hbar\omega$

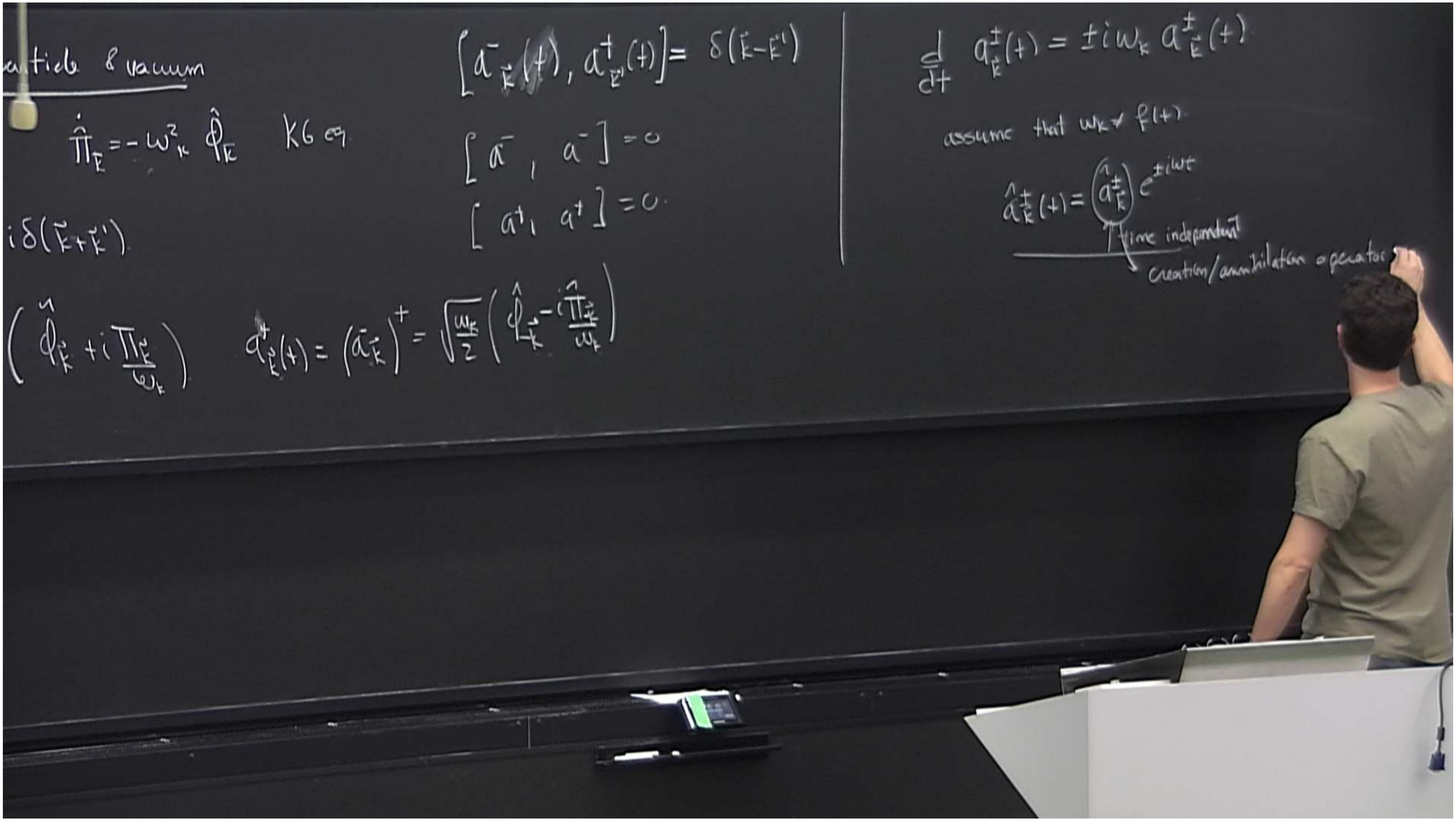

From fields to particle & vacuum

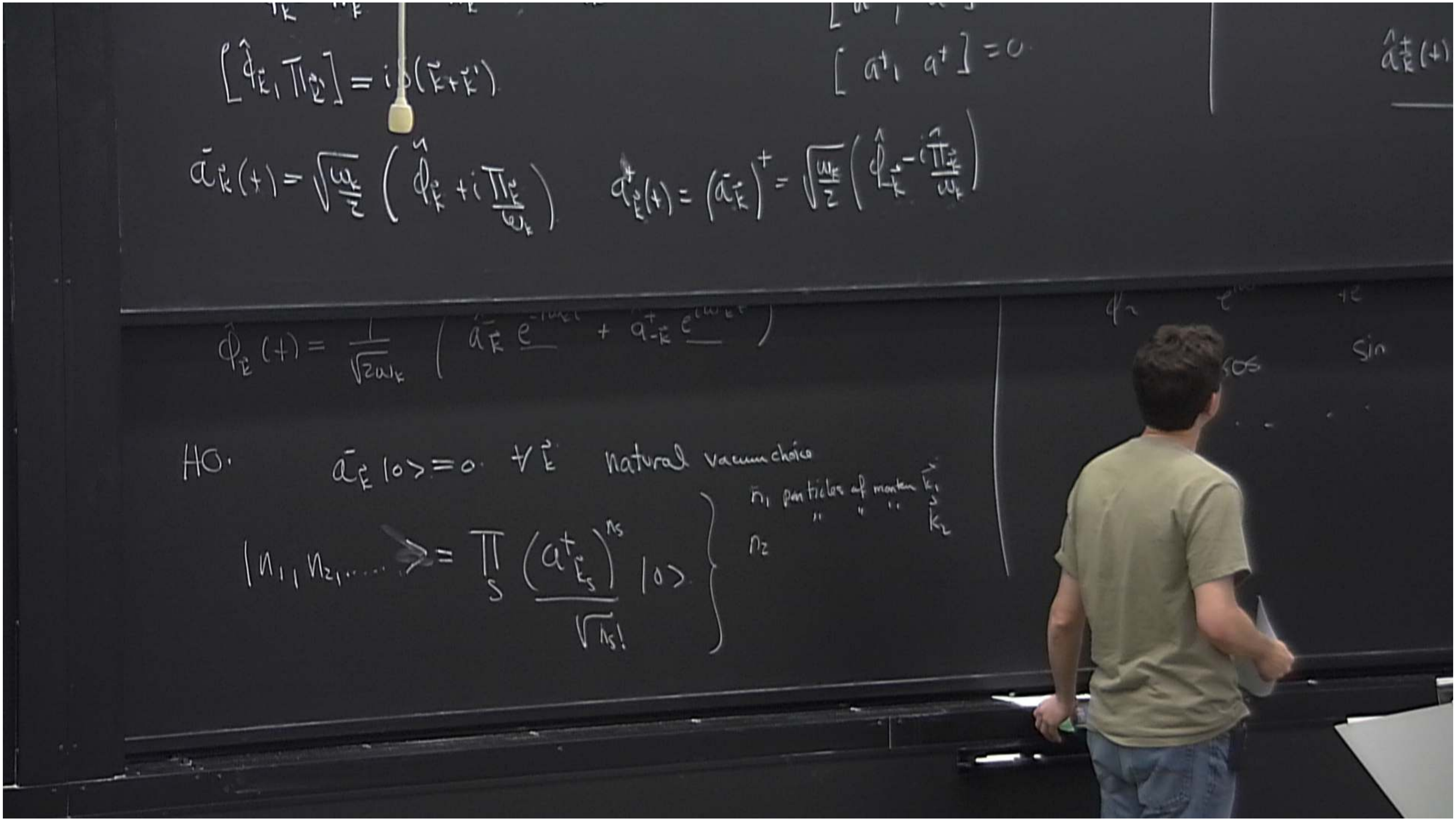

QFT
"Ground state"


Particle Production
1) observer dependent
3) source system
HO
observable
classical
ground state
Wave function
QFT
"Ground state"
analogy
From fields to particle & vacu


HO w/ ω_k² = k² + m²
1) observer dependent
2) ω_k → f(t)
3) source system S_G + Jφ
[q̂(t), p̂(t)] = iħ
Quantum
δq ~ √(ħ/ω)
ω_k = √(k² + m²)
ħ = c = 1
QFT
φ̂_k Π̂_k = φ̇_k
V → ∞
[φ̂_k(t), Π_k'(t)] = i δ_{k,k'}
φ̂*_k = φ̂_{-k}
(1/V) Σ_k ⇒ ∫d³k
E = Σ_k ω


From fields to particle & vacuum
KG eq


particle & vacuum
KG eq
assume that ωk ≠ f(t)
time independent
creation/annihilation operator


[a†, a†] = 0
HO.
a_k |0> = 0 ∀ k natural vacuum choice
n₁ particles of momentum k₁
n₂ " " " k₂
sin
cos

$$\bar{a}_{\vec{k}}(t) = \sqrt{\frac{\omega_k}{2}}\left(\hat{\phi}_{\vec{k}} + i\frac{\hat{\Pi}_{\vec{k}}}{\omega_k}\right) \qquad a^{+}_{\vec{k}}(t) = \left(\bar{a}_{\vec{k}}\right)^{+} = \sqrt{\frac{\omega_k}{2}}\left(\hat{\phi}_{-\vec{k}} - i\frac{\hat{\Pi}_{-\vec{k}}}{\omega_k}\right)$$

$$\hat{\phi}_{\vec{k}}(t) = \frac{1}{\sqrt{2\omega_k}}\left(\hat{a}_{\vec{k}}\, e^{-i\omega_k t} + \hat{a}^{+}_{-\vec{k}}\, e^{i\omega_k t}\right)$$

HO. $\bar{a}_{\vec{k}}|0\rangle = 0 \quad \forall \vec{k}$ natural vacuum choice

$$|n_1, n_2, \ldots\rangle = \prod_s \frac{\left(a^{+}_{\vec{k}_s}\right)^{n_s}}{\sqrt{n_s!}}|0\rangle$$

n_1 particles of momentum $\vec{k}_1$

n_2 " " " $\vec{k}_2$

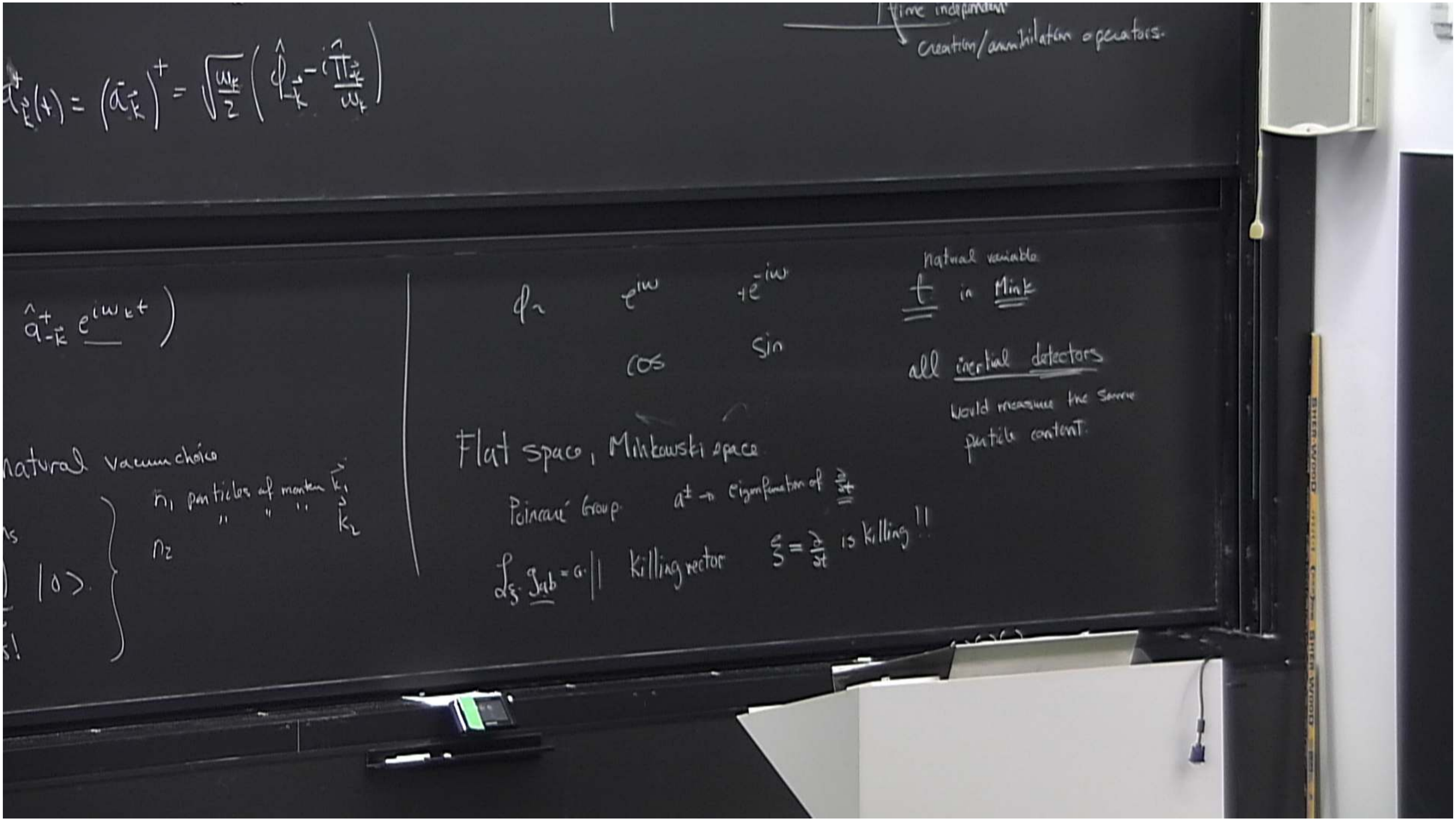
$\phi \sim e^{i\omega} + e^{-i\omega}$

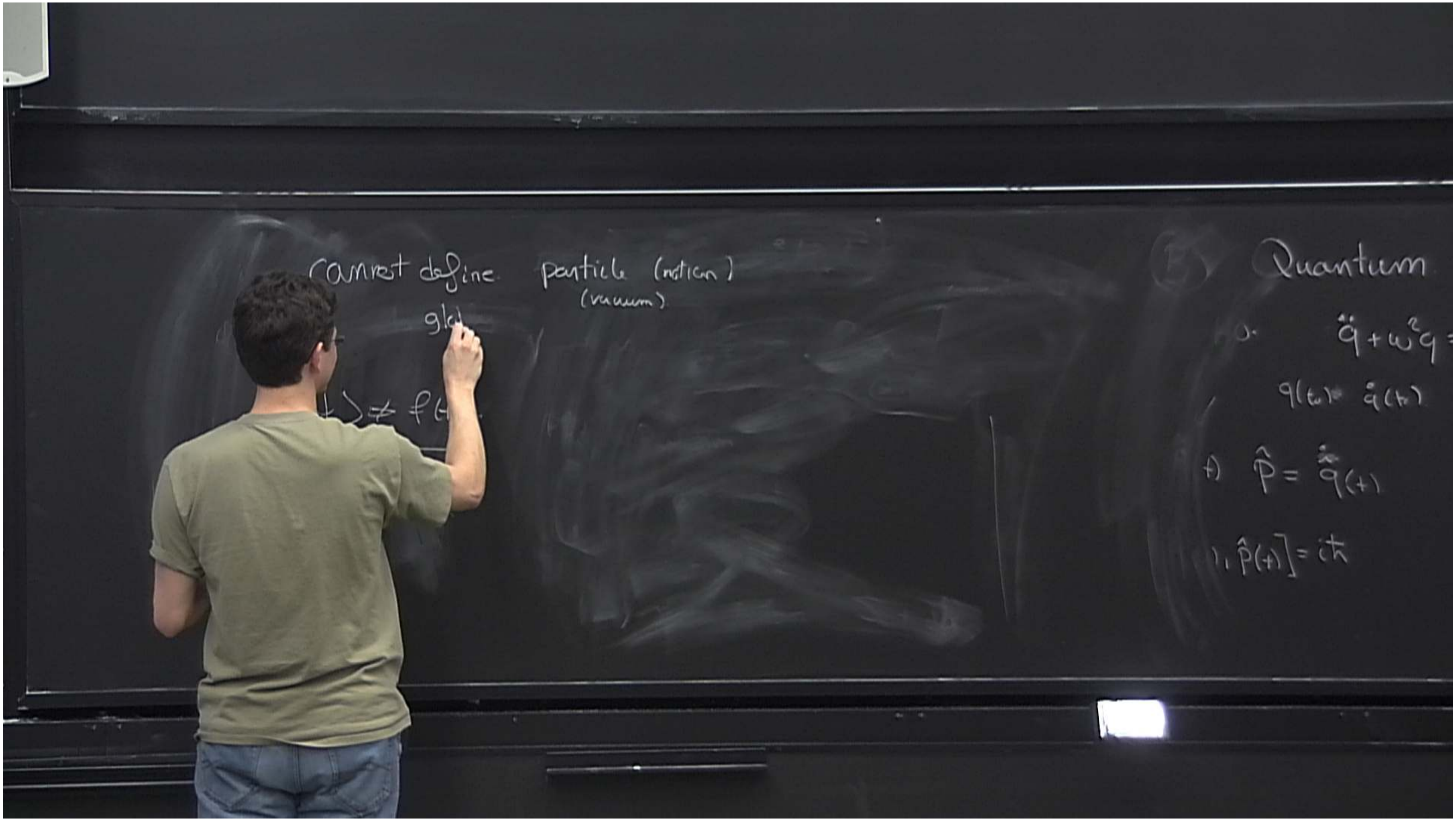
cos sin

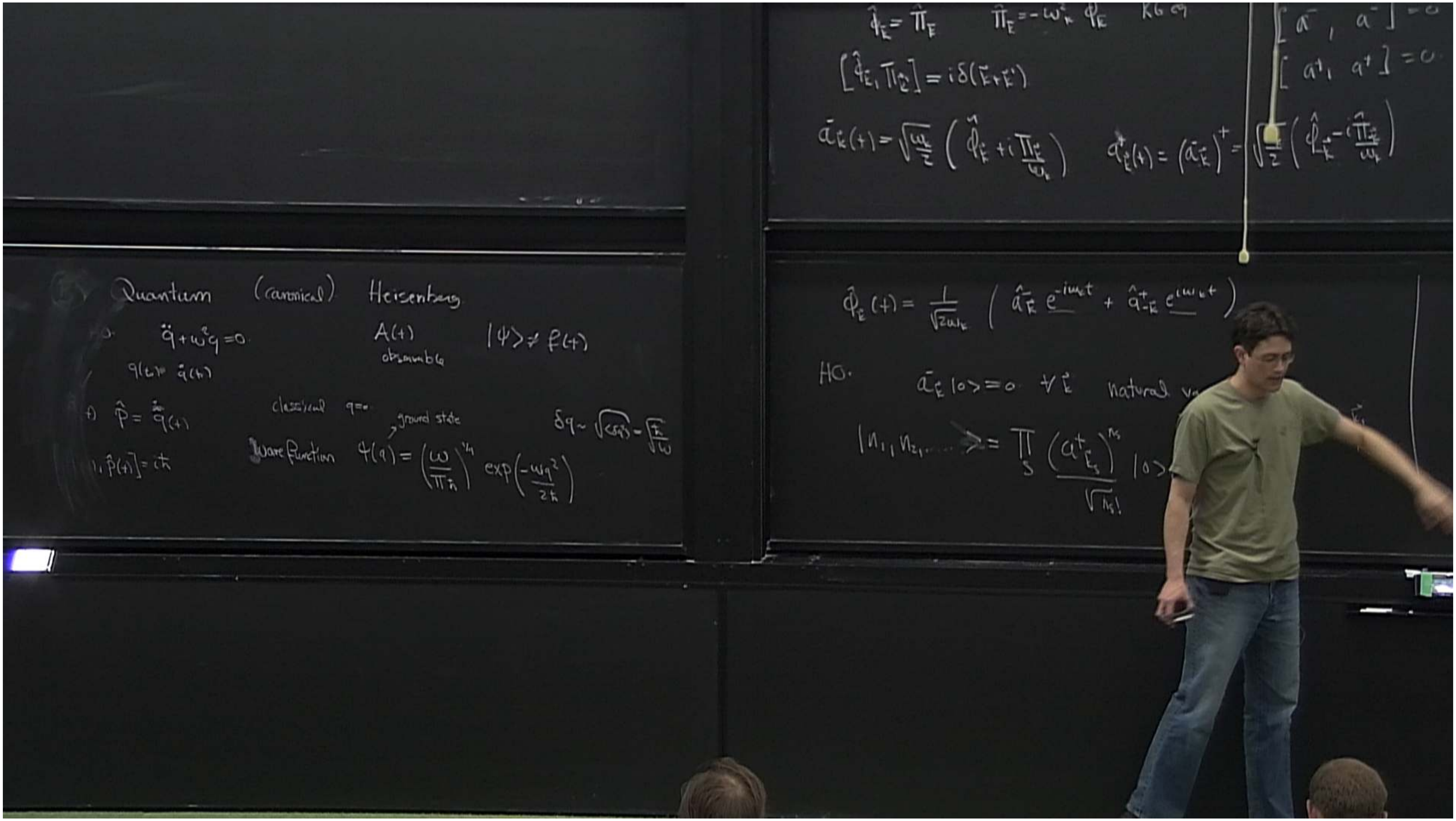
Flat space, Minkowski space

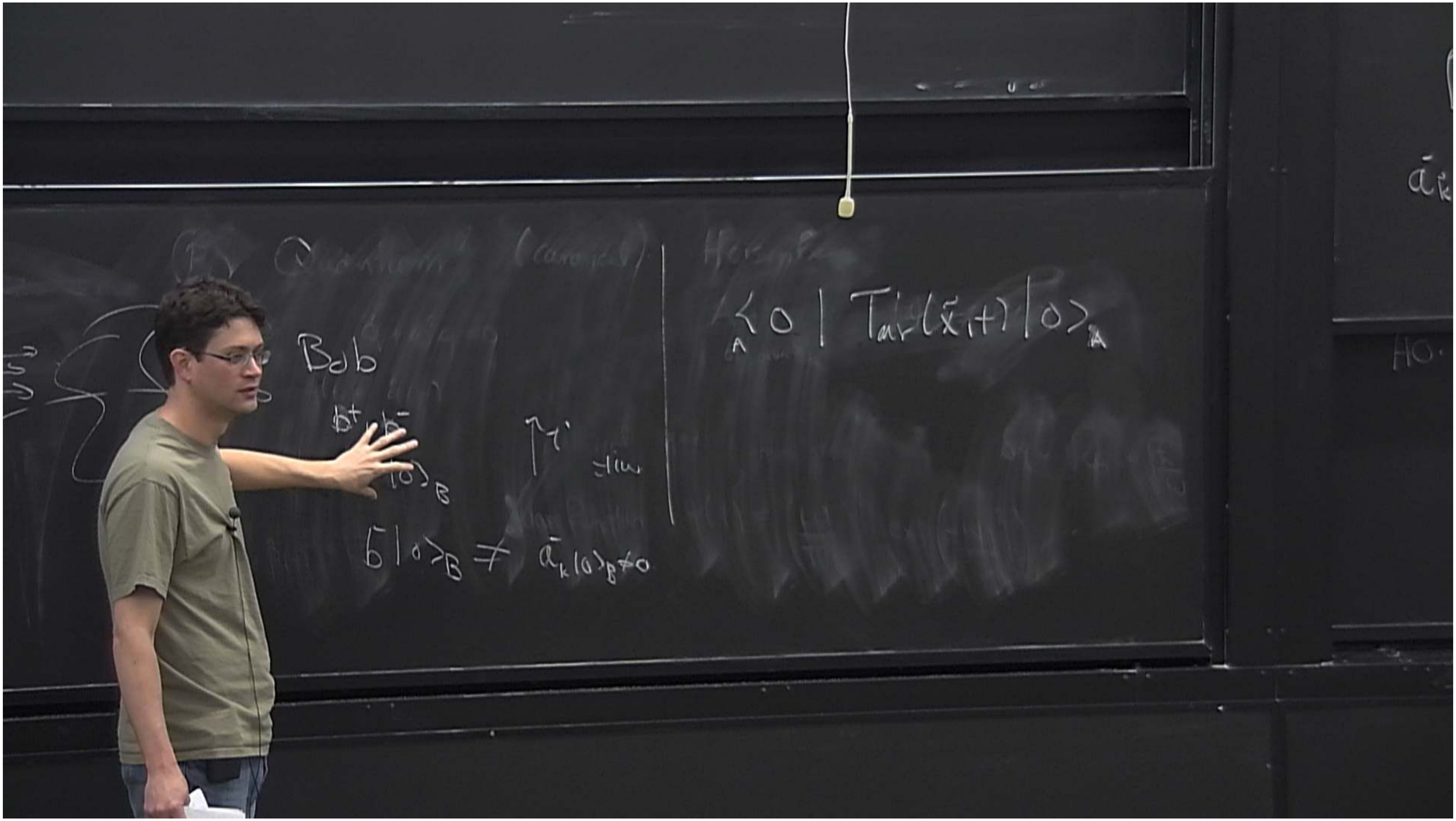
Poincaré Group $a^{\pm} \to$ eigenfunction of $\frac{\partial}{\partial t}$

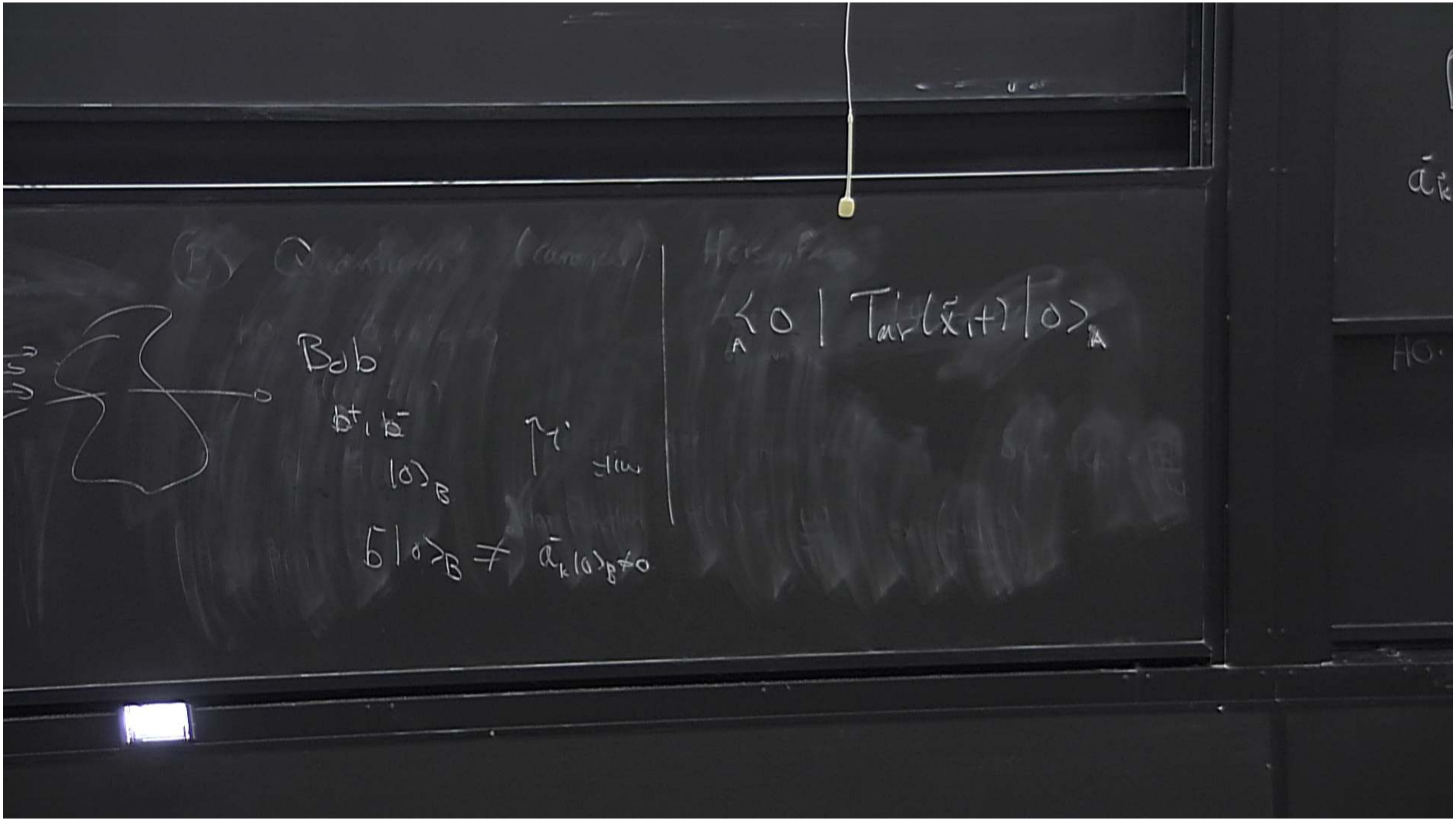
$\mathcal{L}_{\xi}\, g_{ab} = 0$ Killing vector $\xi = \frac{\partial}{\partial t}$ is killing

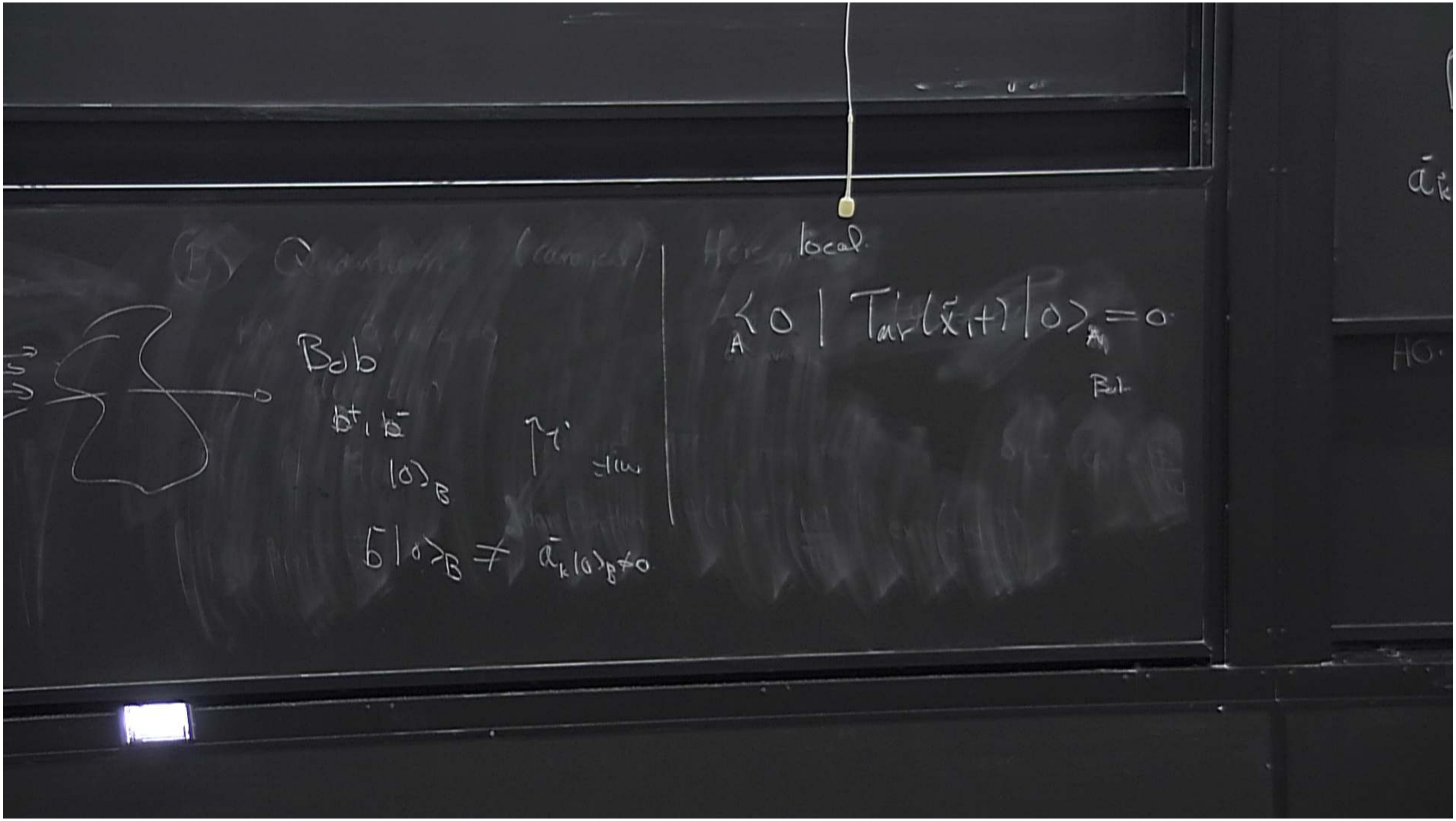

time independent
creation/annihilation operators.
natural variable
t in Mink
all inertial detectors
would measure the same
particle content.
cos
sin
natural vacuum choice
Flat space, Minkowski space
Poincaré Group
Killing vector
is killing !!


cannot define particle (notion)
(vacuum)
Quantum


Quantum (canonical) Heisenberg
observable
classical
ground state
wave function
HO.
natural


Bob
$|0\rangle_B$
$b|0\rangle_B \neq \bar{a}_k|0\rangle_B \neq 0$
${}_A\langle 0 | T\,\mathrm{any}(\bar{x},t) | 0 \rangle_A$


Bob
|0>_B
b|0>_B ≠ ā_k|0>_B ≠ 0
_A<0| T_μν(x̄, t) |0>_A


local.
Bob