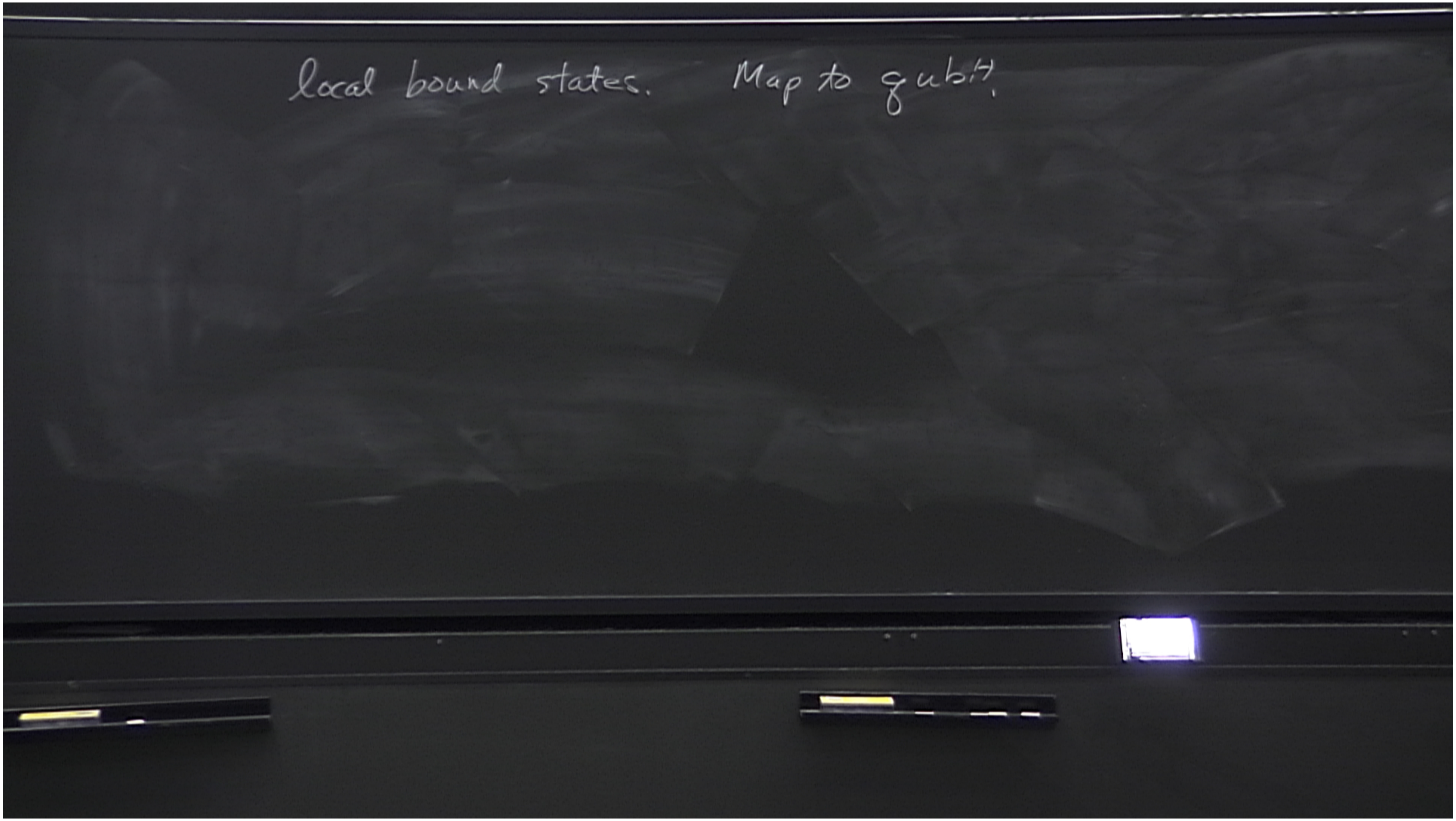
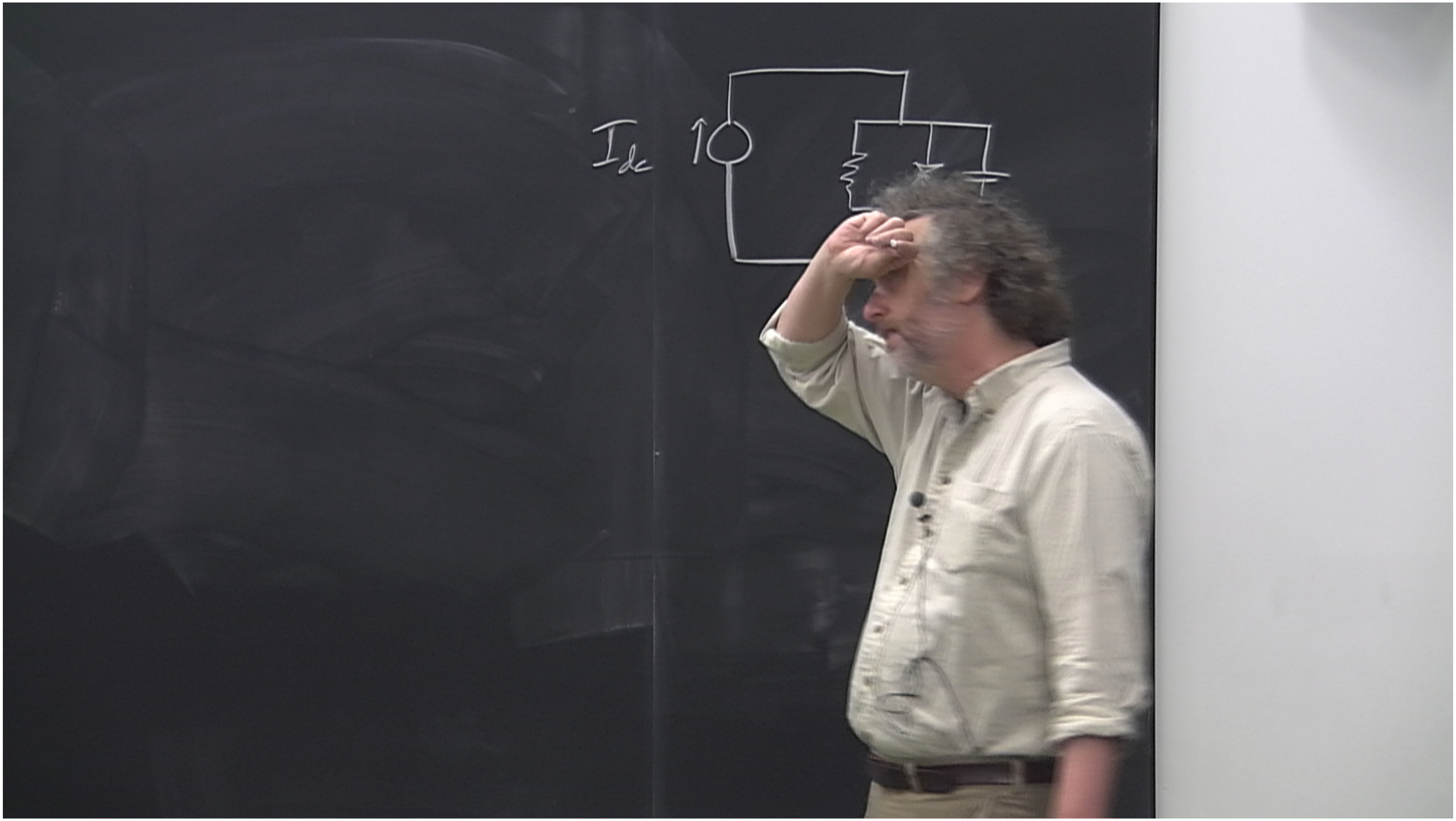
Title: Explorations in Quantum Information - Lecture 14

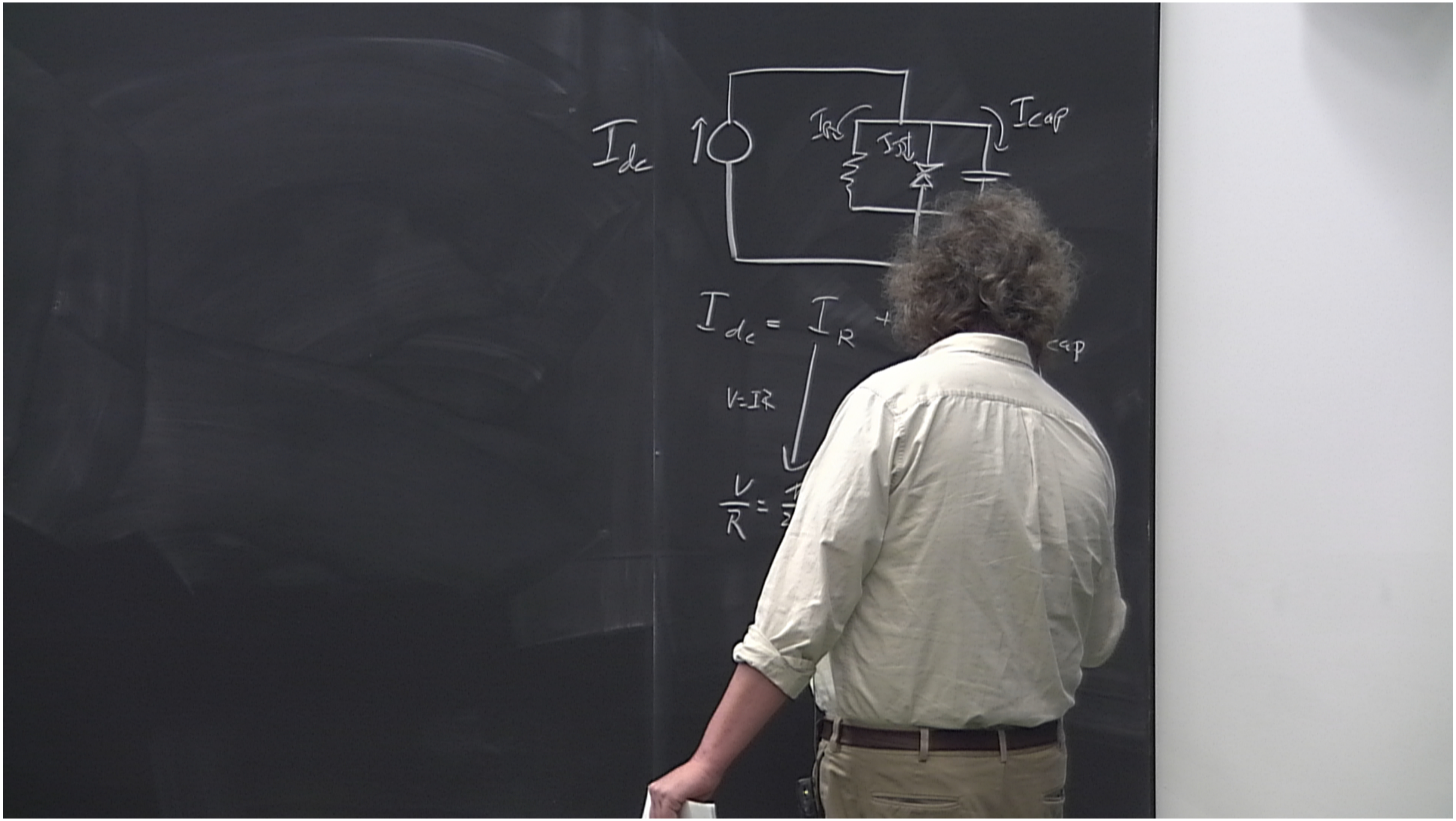
Date: Mar 29, 2012 02:00 PM

URL: http://pirsa.org/12030021

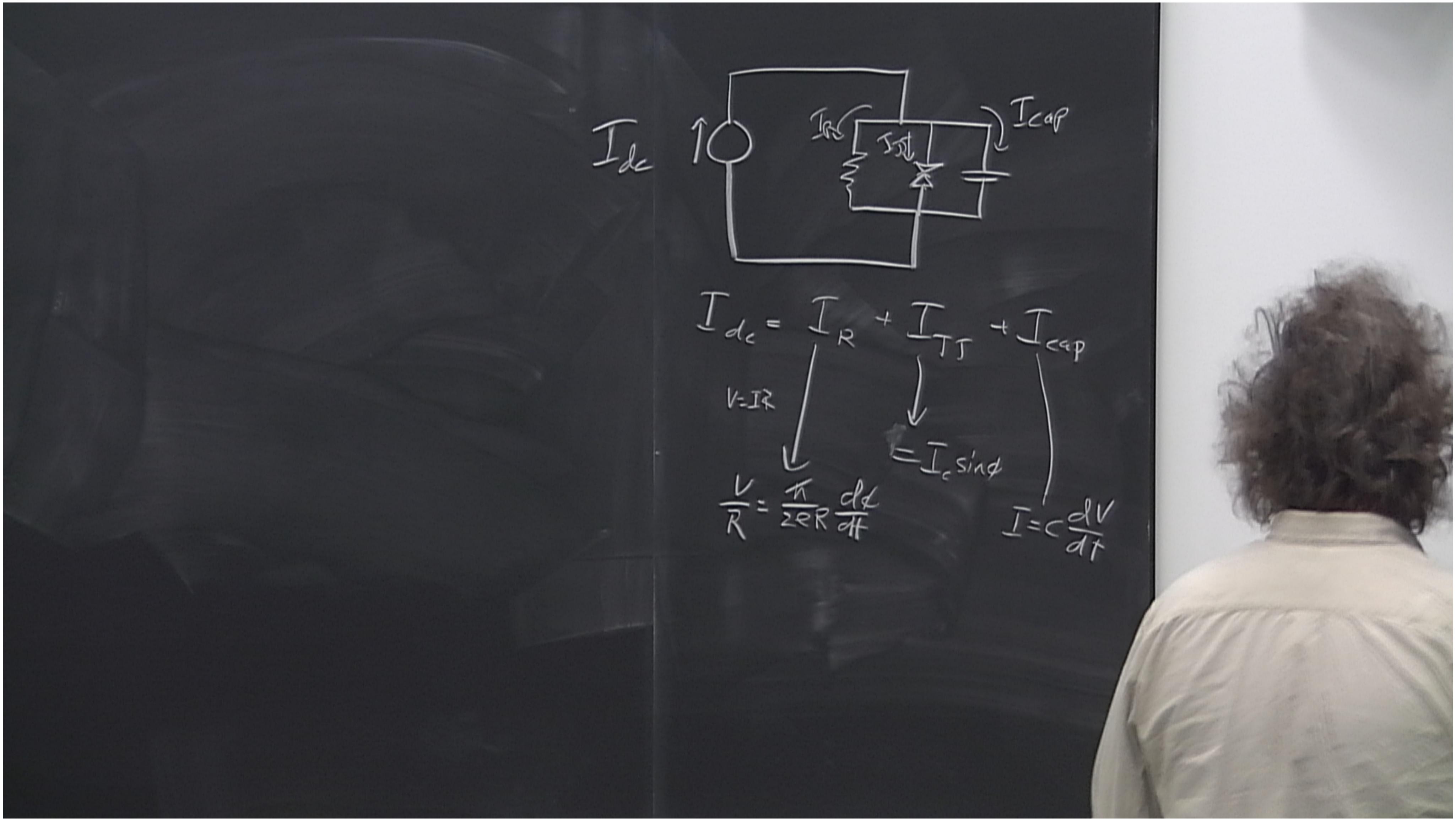
Abstract:


local bound states. Map to qubit.

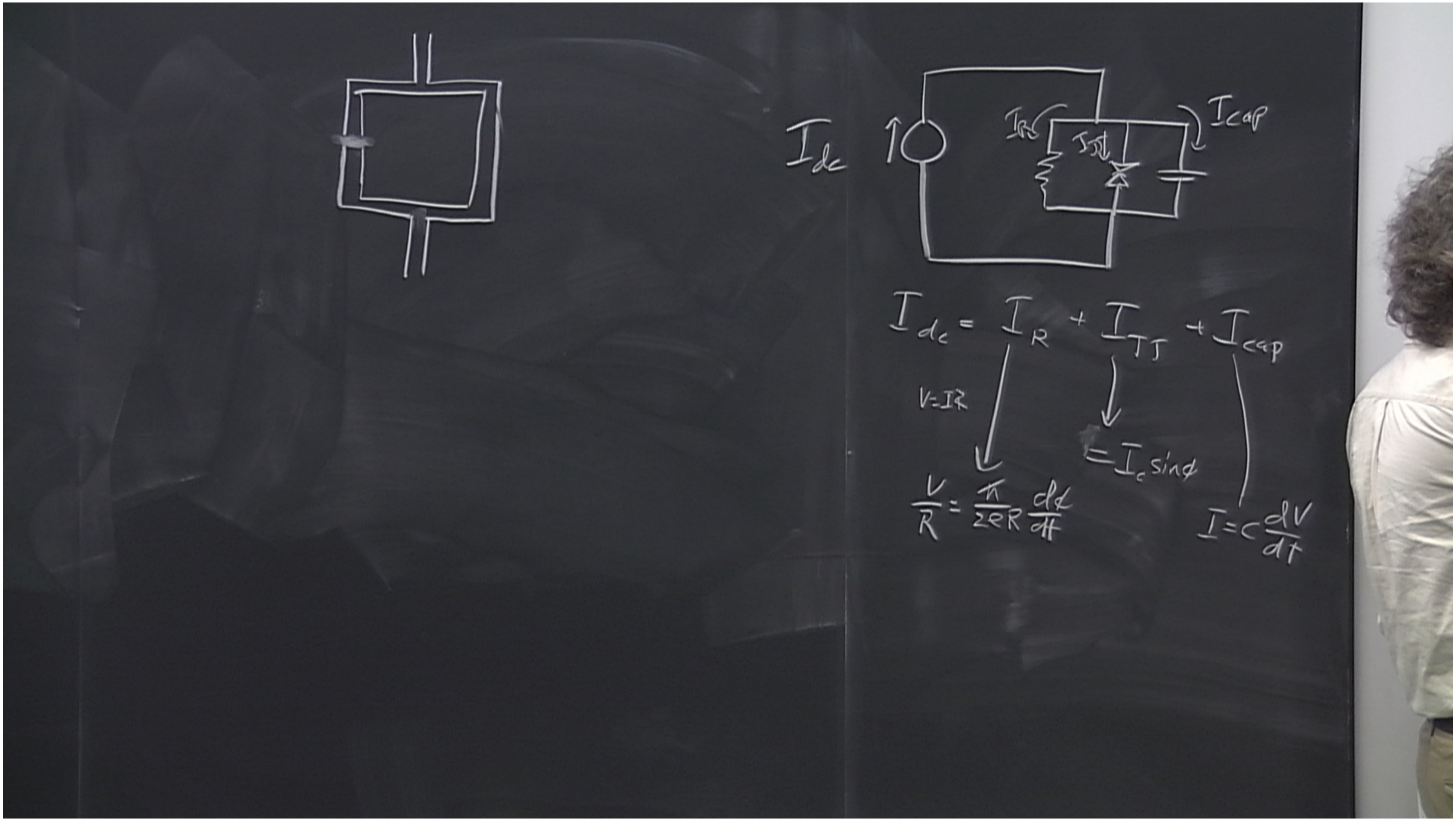

Idc

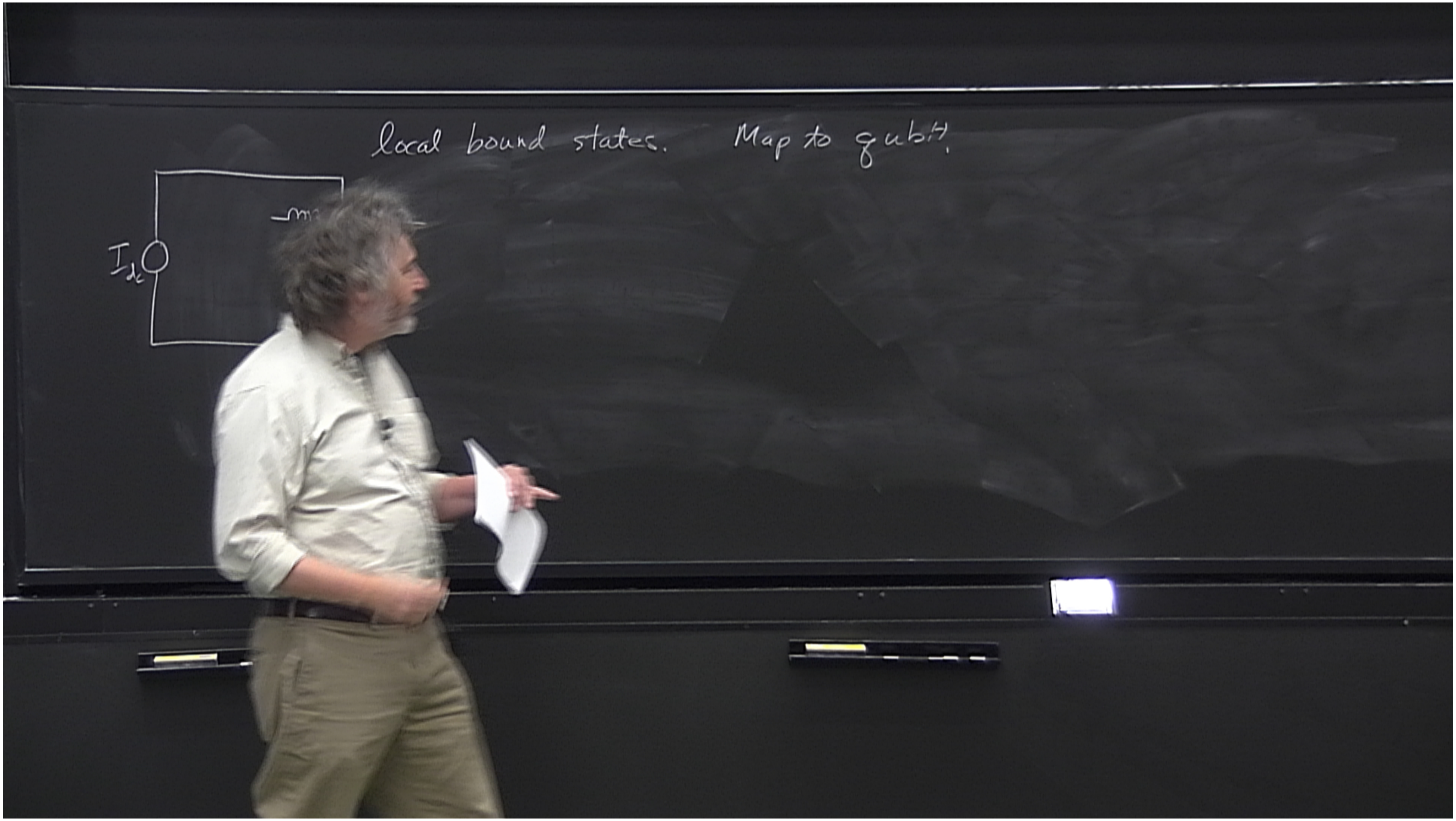


Idc
Icap
V=IR

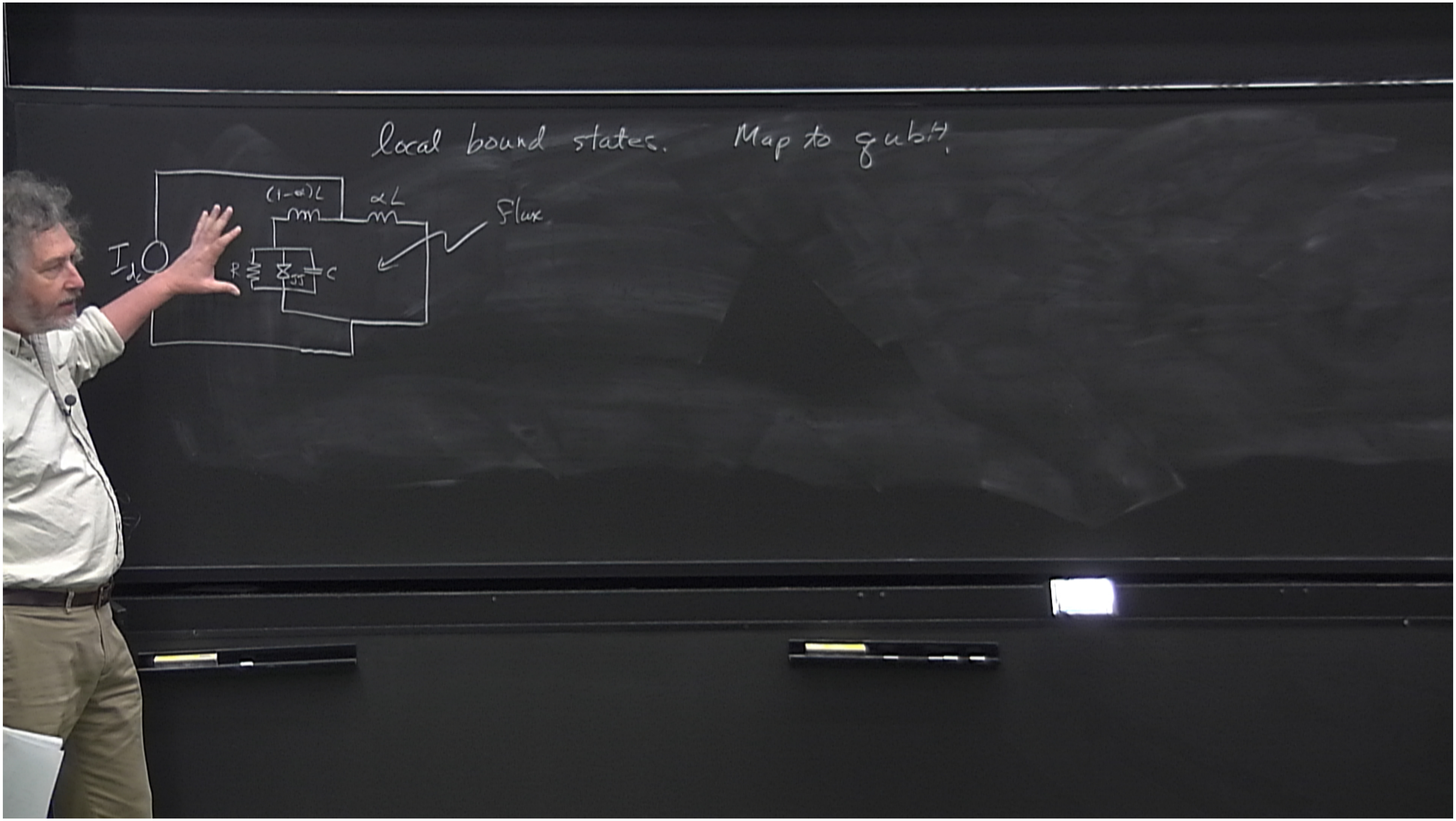


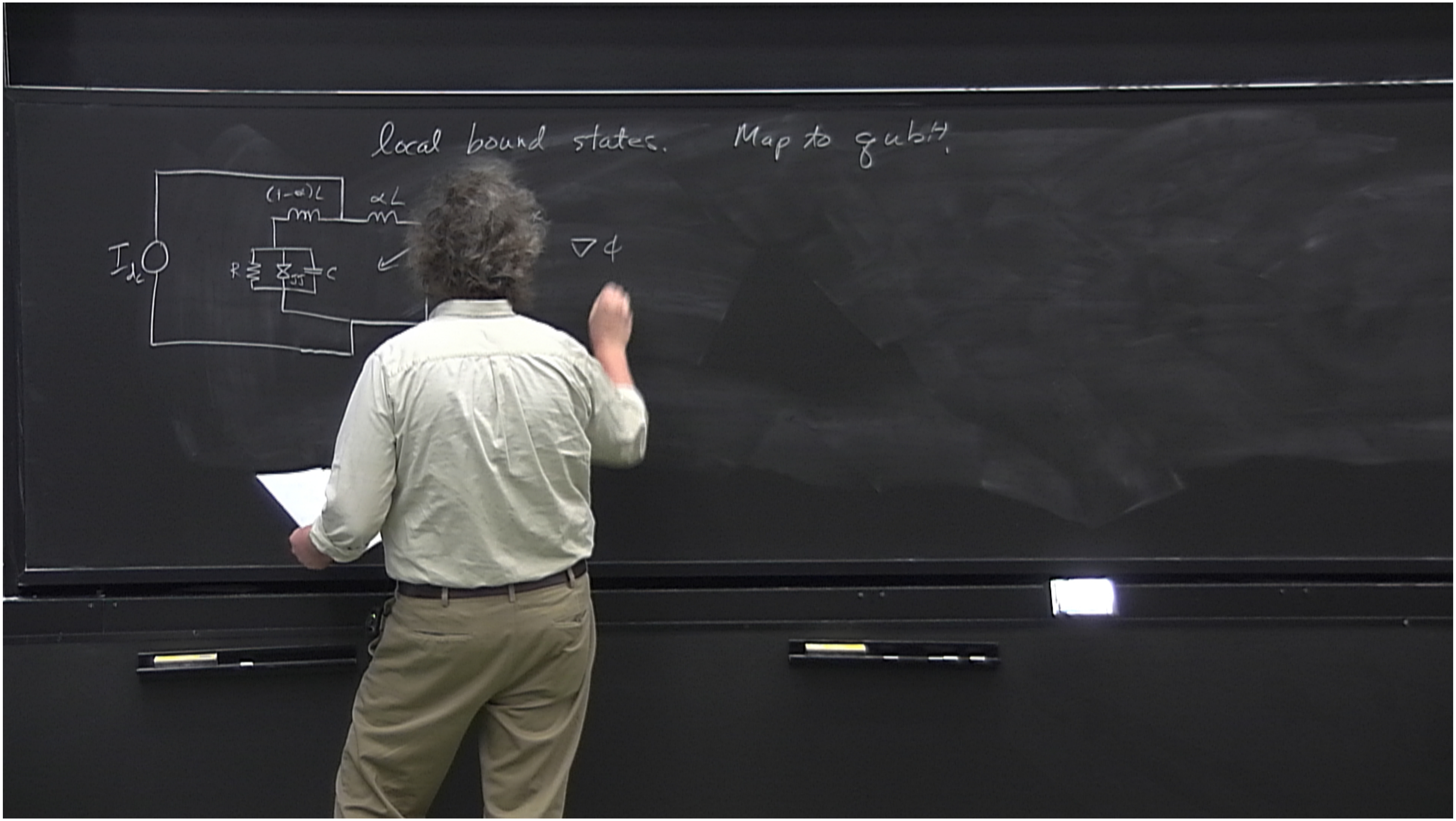
I_dc
I_cap
I_dc = I_R + I_JJ + I_cap
V=IR
= I_c sin φ
V/R = ħ/2eR dφ/dt
I = C dV/dt

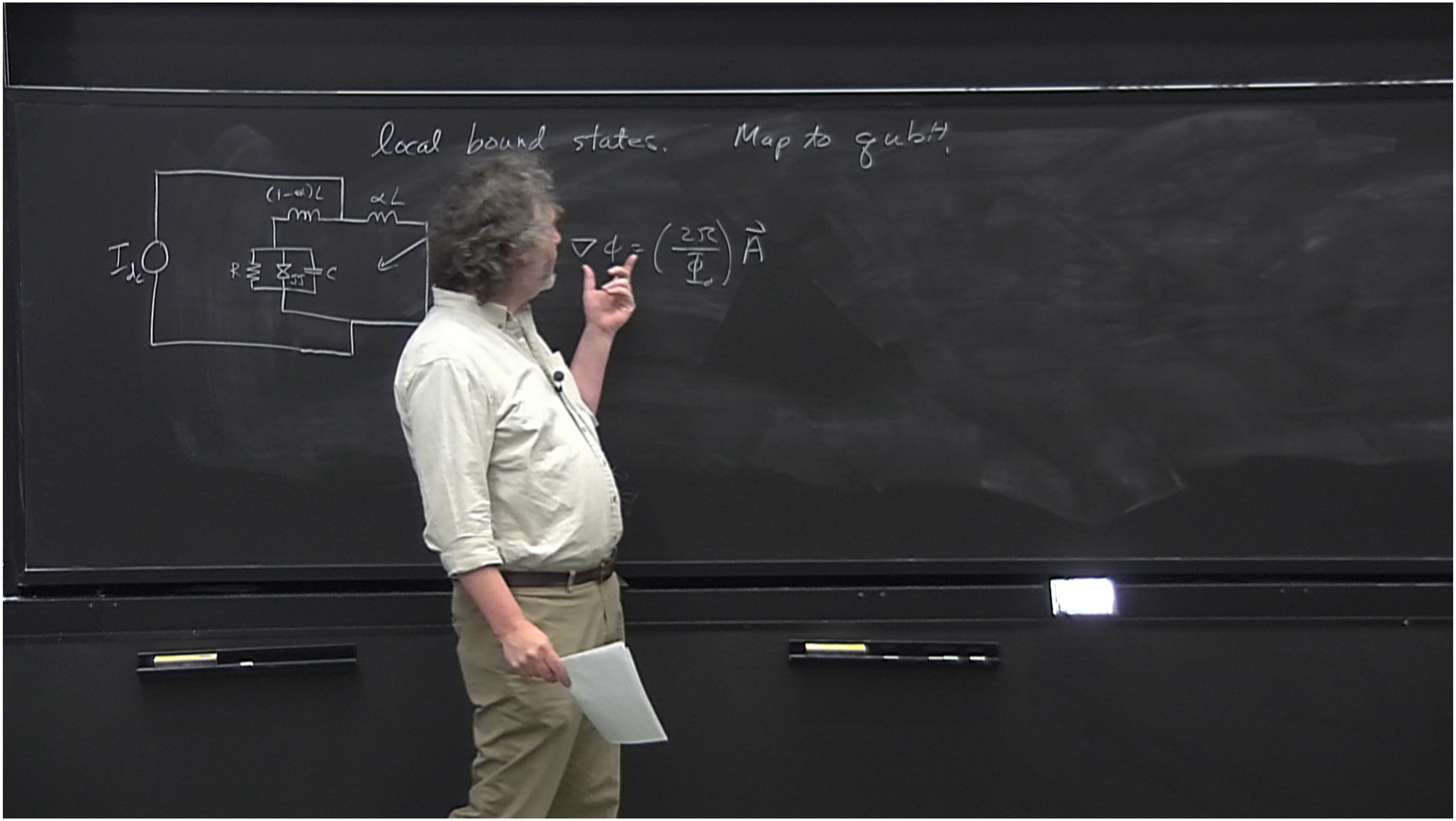

I_dc
I_R
I_JJ
I_cap
I_dc = I_R + I_JJ + I_cap
V=IR
= I_c sin φ
V/R = ħ/2eR dφ/dt
I = C dV/dt

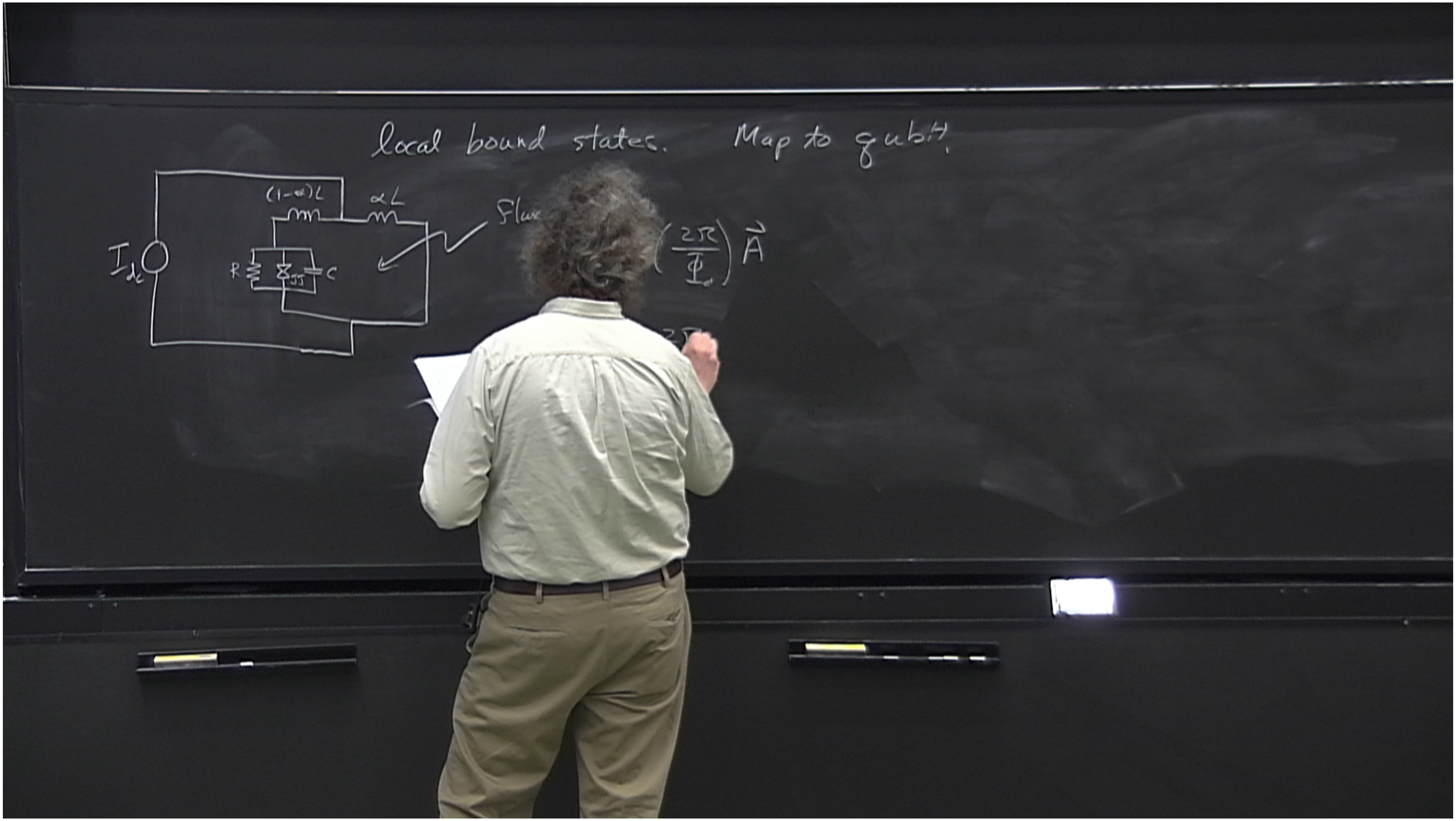

local bound states. Map to qubit.
Idc

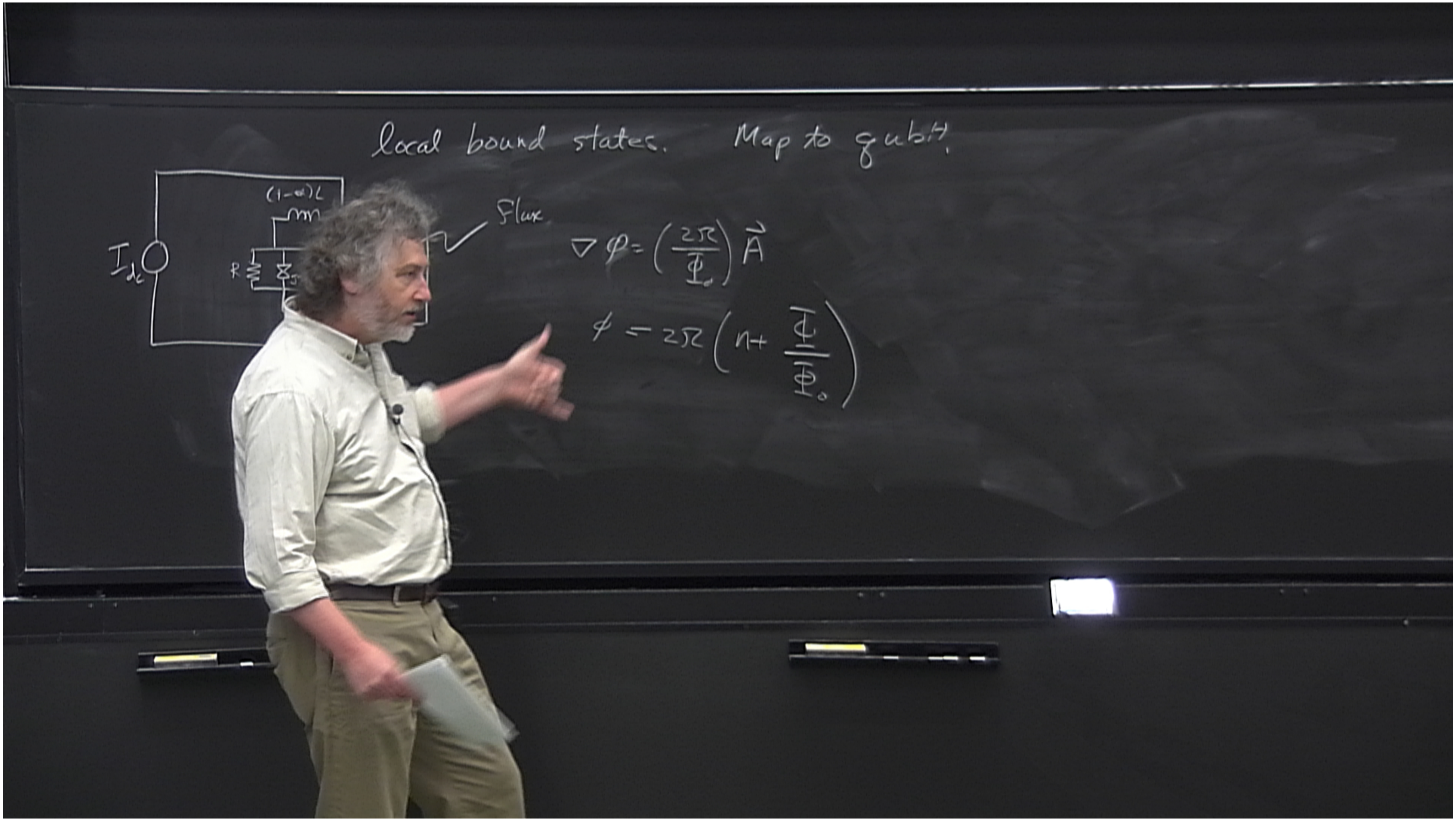

local bound states. Map to qubit.
(1-α)L
αL
flux
C
JJ

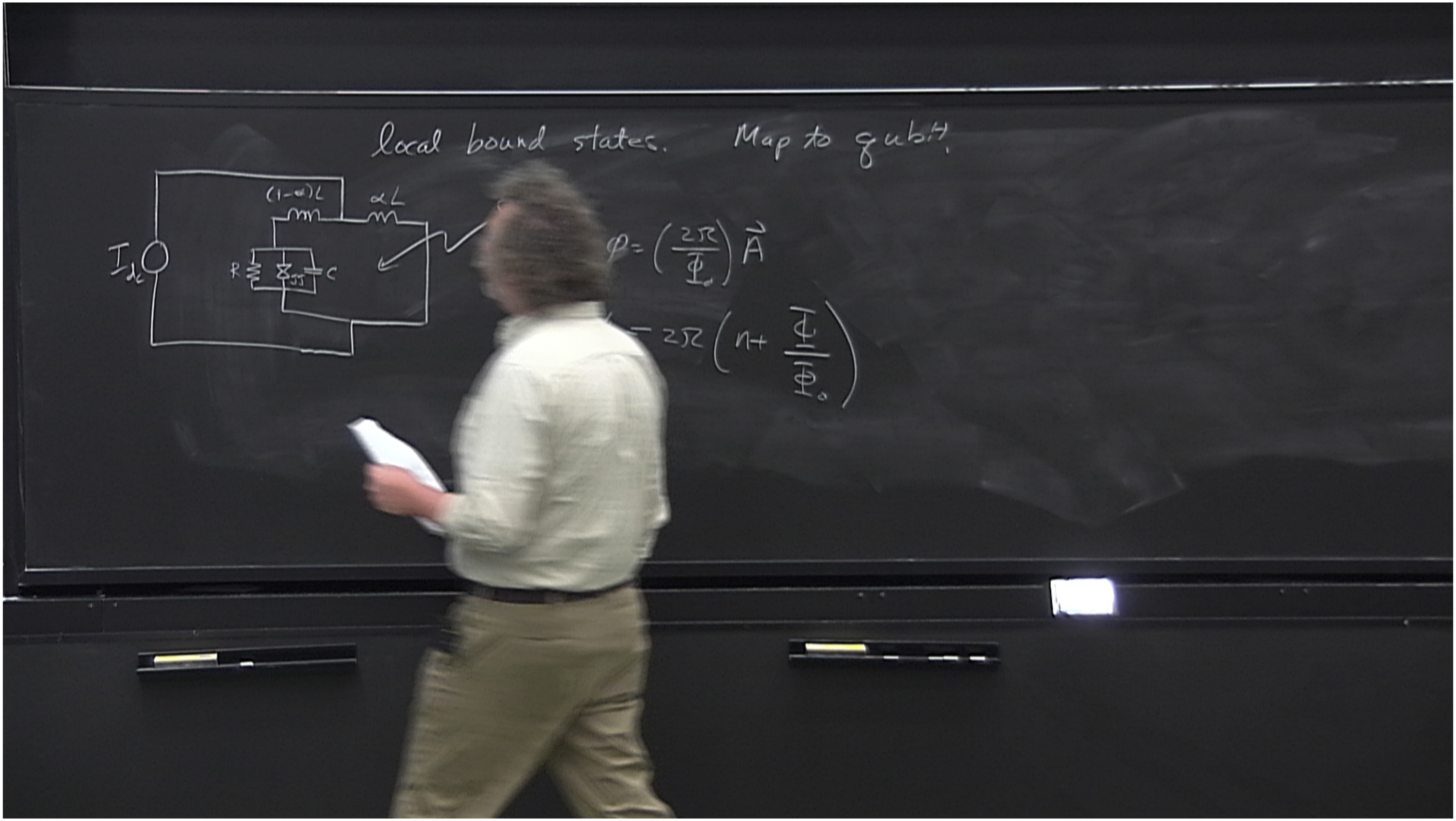

local bound states. Map to qubit.
(1-α)L
αL
flux
R
C
JJ

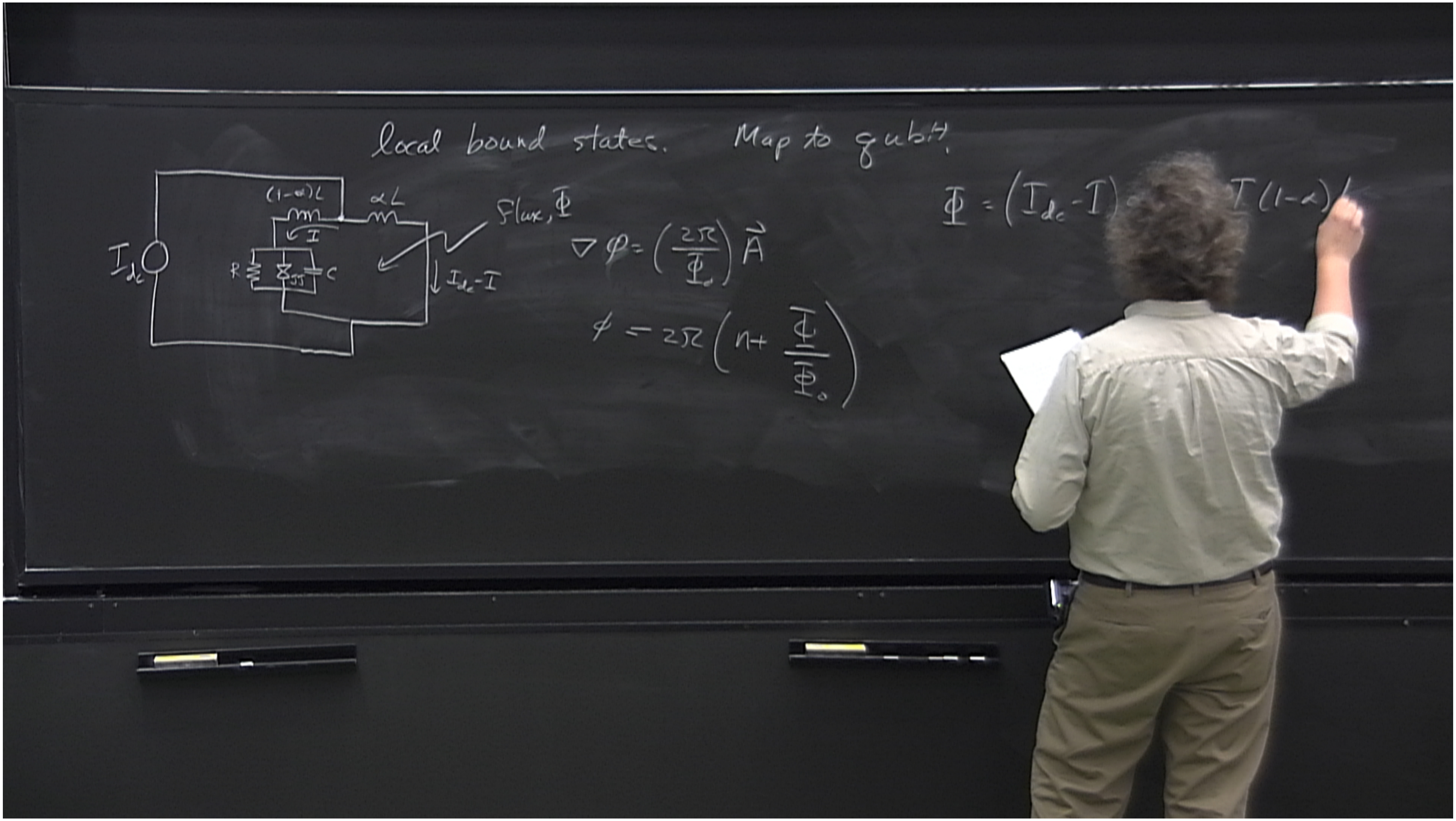

local bound states. Map to qubit.
(1−α)L
αL
R
JJ
C
∇φ

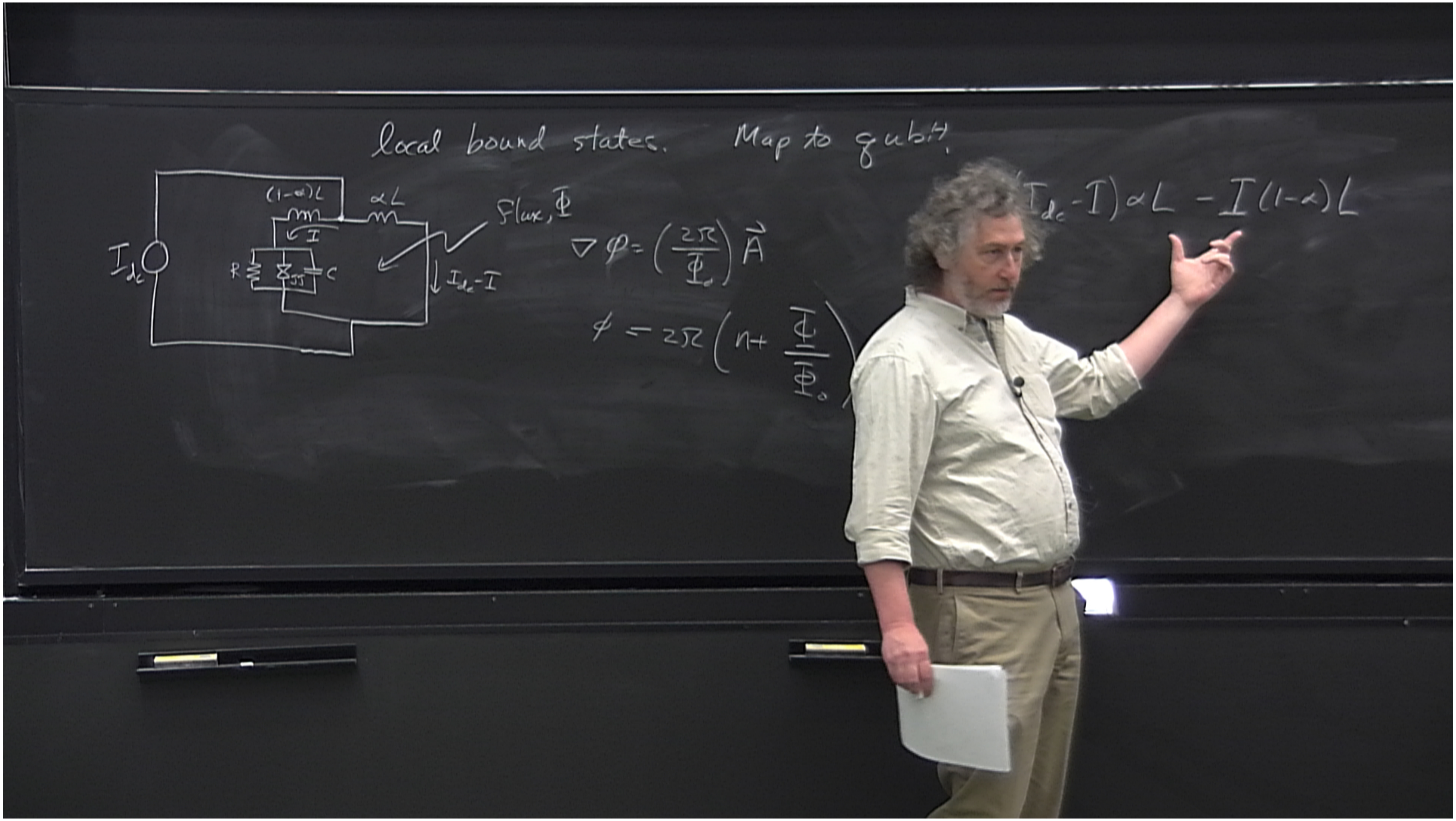

local bound states. Map to qubit.
(1-α)L
αL
I_{dc}
R
JJ
C
$\nabla\phi = \left(\frac{2\pi}{\Phi_0}\right)\vec{A}$

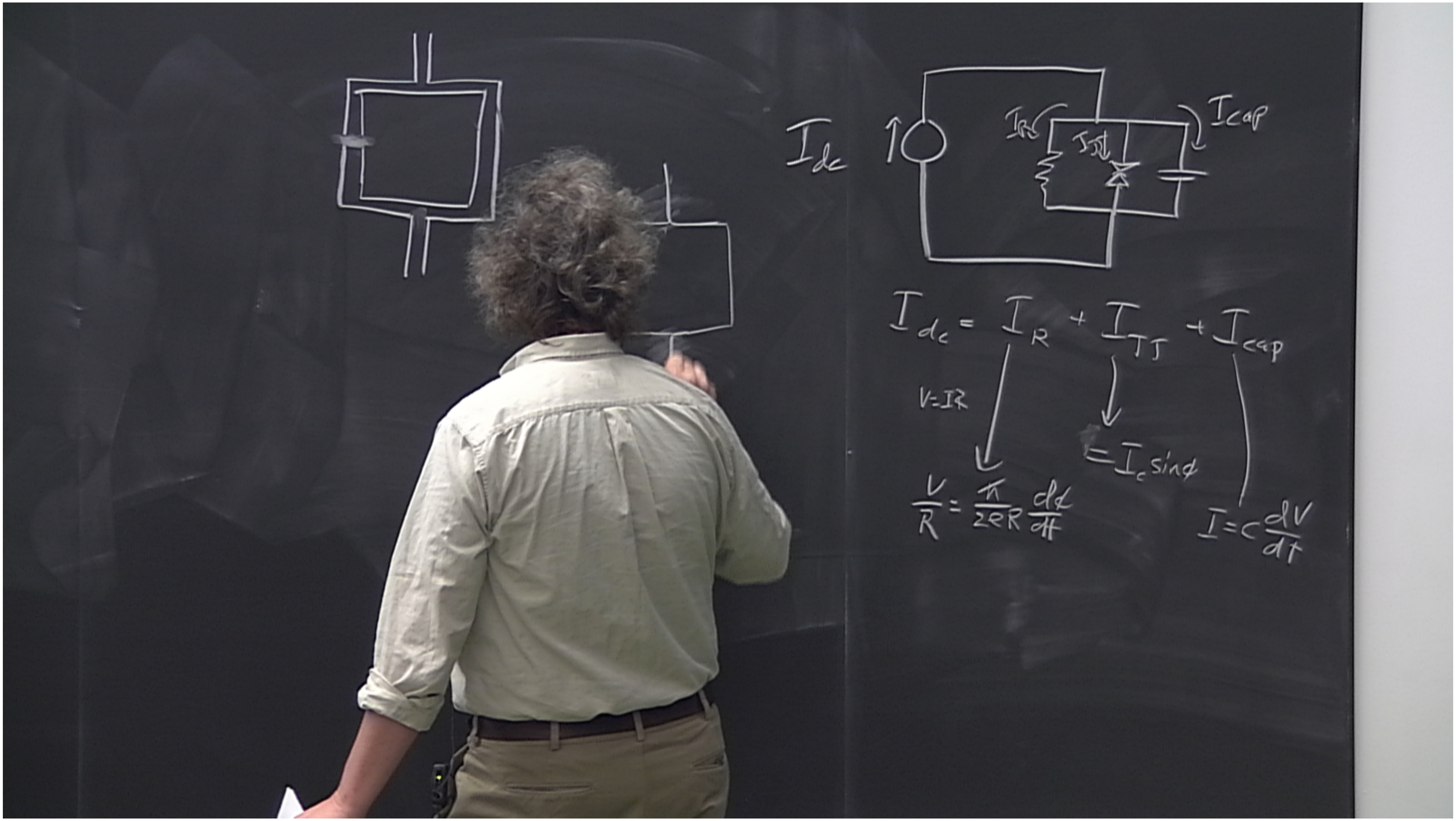

local bound states. Map to qubit.
(1-α)L
αL
Flux
I_dc
R
JJ
C


local bound states. Map to qubit.
(1−α)L
flux
I_dc
R
∇φ = (2π/Φ₀) A
φ = 2π(n + Φ/Φ₀)

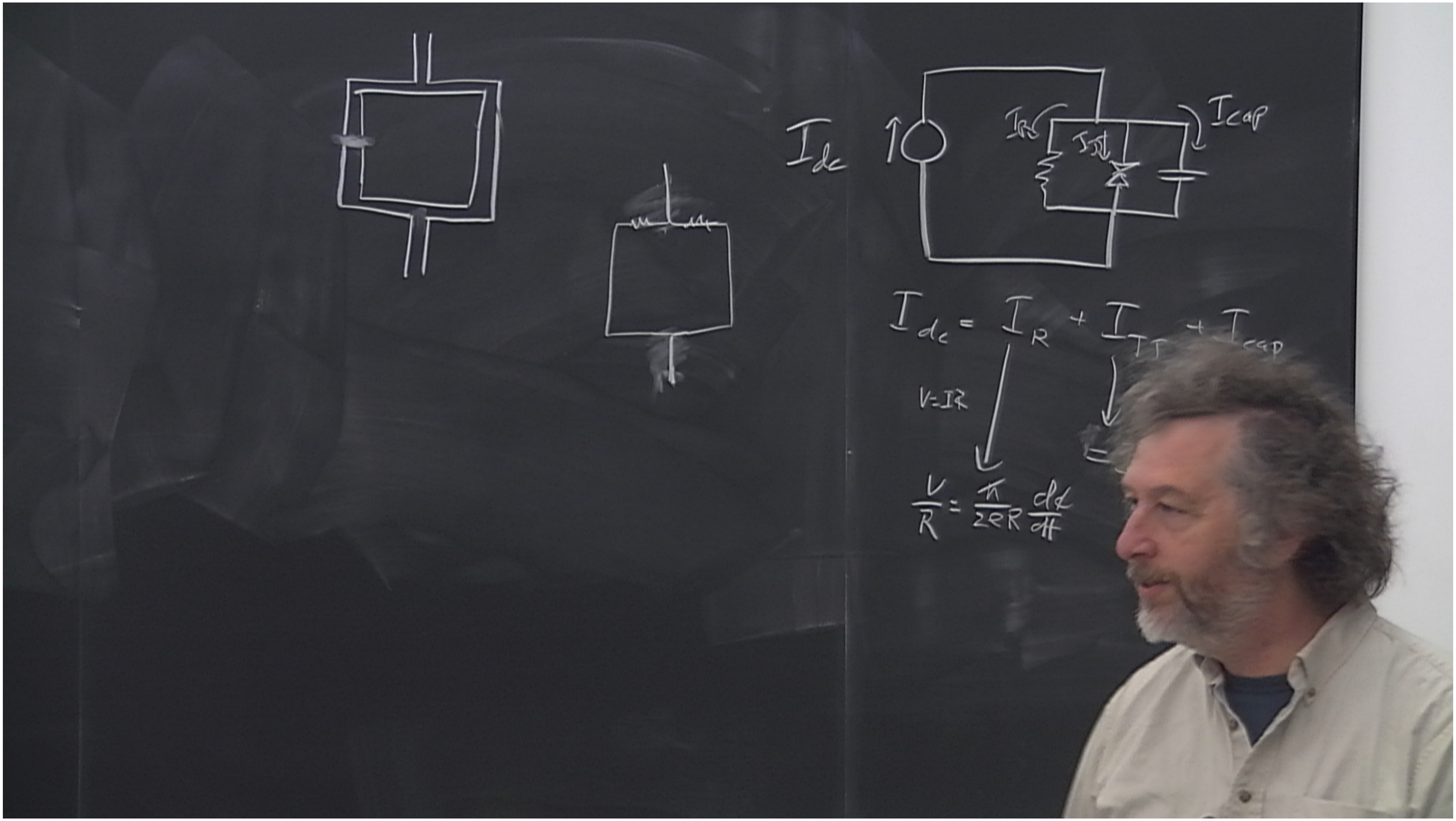

local bound states. Map to qubit.
(1−α)L
αL
I_{dc}
R
JJ
C
$\varphi = \left(\frac{2\pi}{\Phi_0}\right)\vec{A}$
$2\pi\left(n + \frac{\Phi}{\Phi_0}\right)$


local bound states.
Map to qubit.
(1-α)L
αL
flux, Φ
I
I_dc
R
JJ
C
I_dc - I
∇φ = (2π/Φ_0) A
φ = 2π(n + Φ/Φ_0)
Φ = (I_dc - I)
I(1-α)

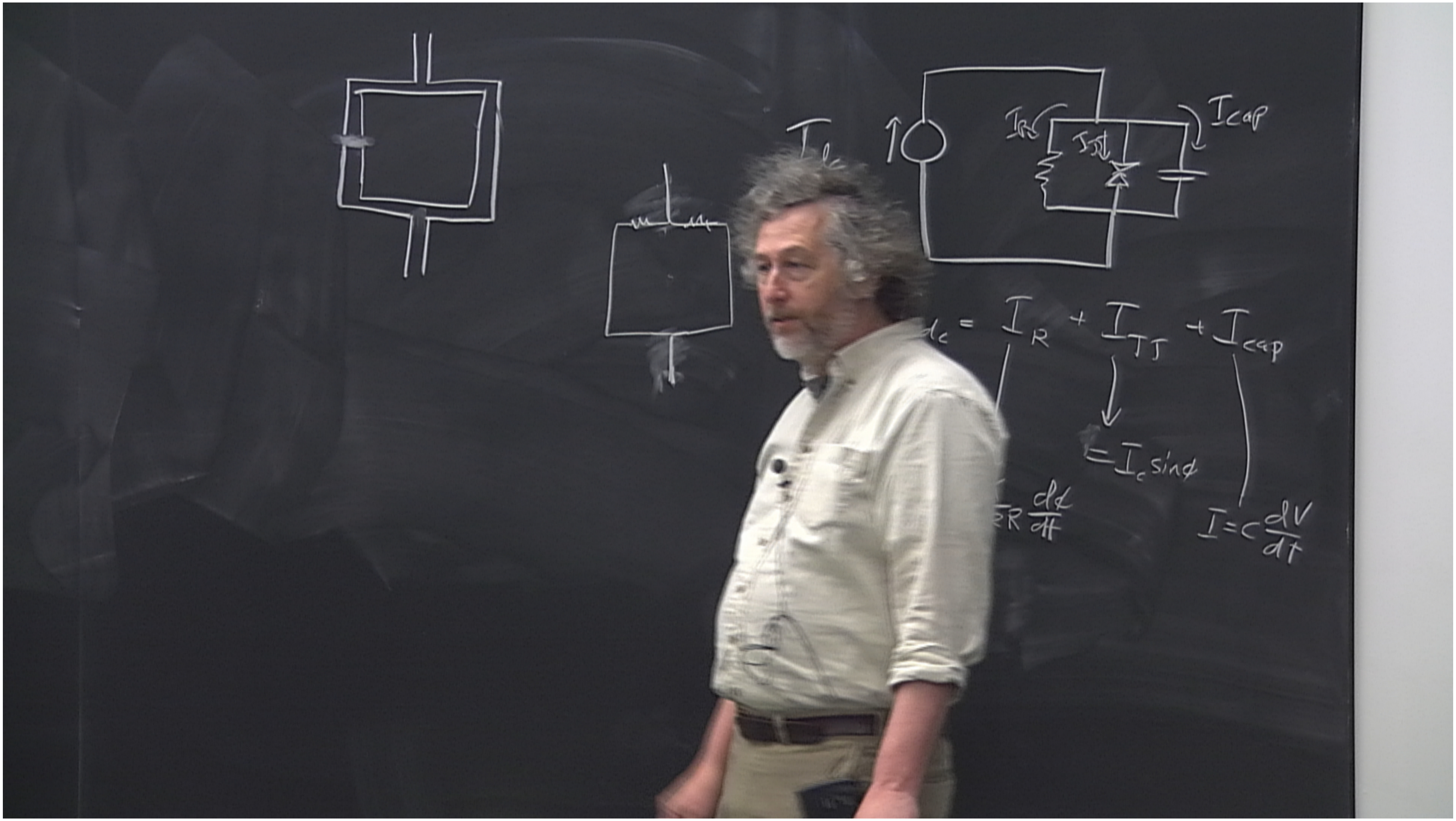

local bound states. Map to qubit
(1-α)L
αL
I
I_dc
R
JJ
C
flux, Φ
I_dc - I
∇φ = (2π/Φ₀) A
φ = 2π(n + Φ/Φ₀)
(I_dc - I)αL - I(1-α)L

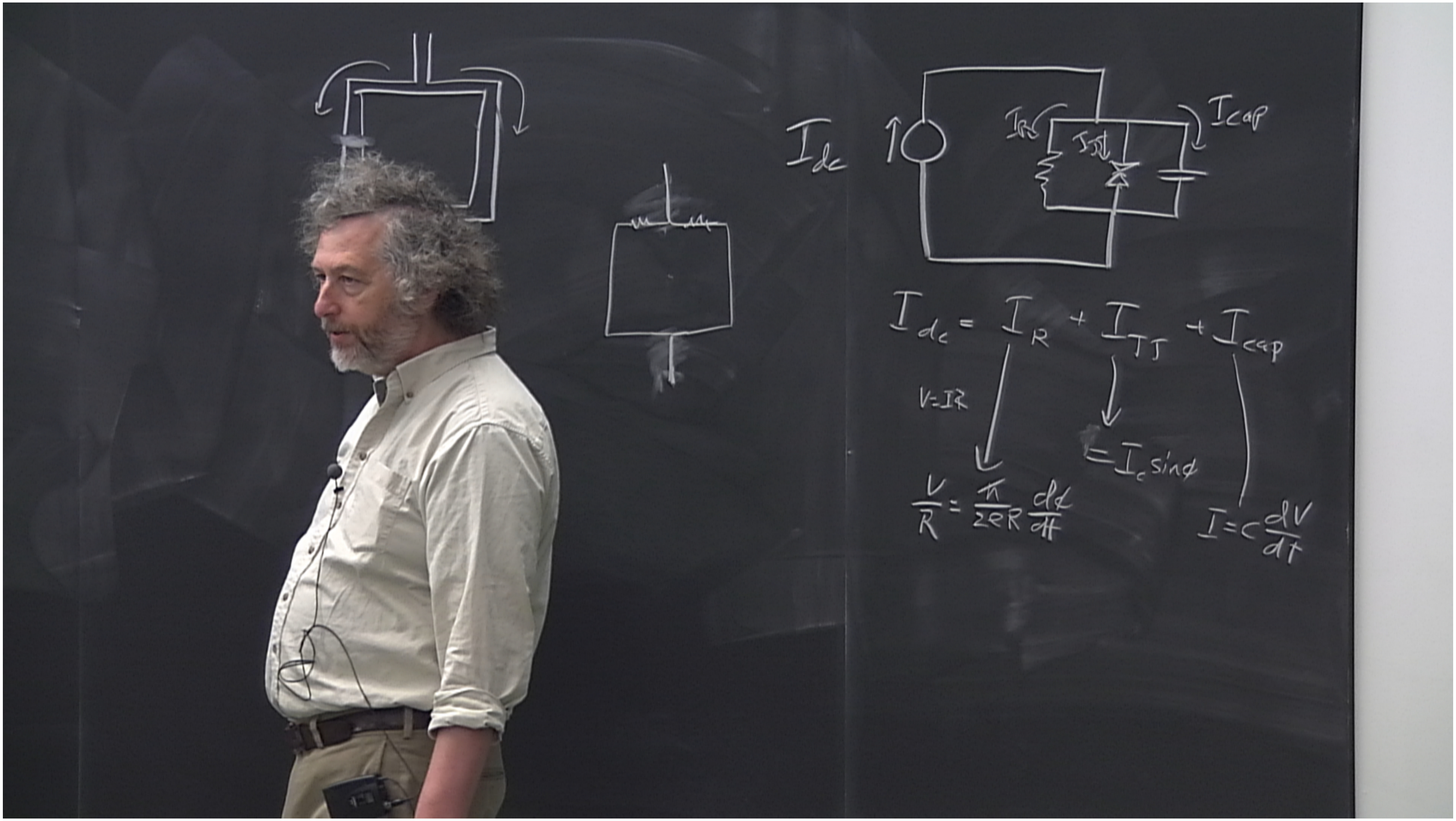


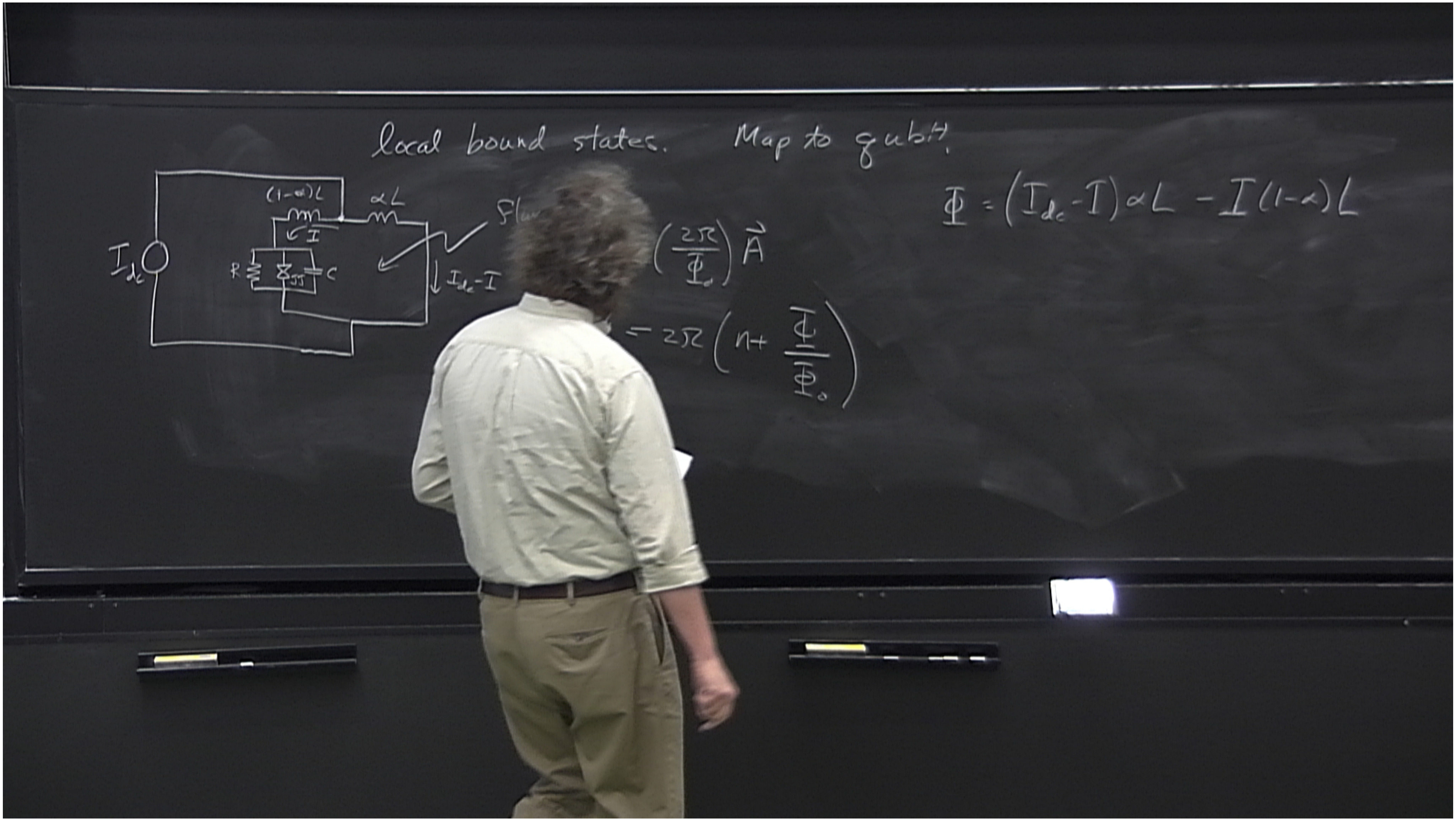
I_dc
I_R
I_JJ
I_cap
I_dc = I_R + I_JJ + I_cap
V=IR
= I_c sin φ
V/R = ħ/2eR dφ/dt
I = C dV/dt

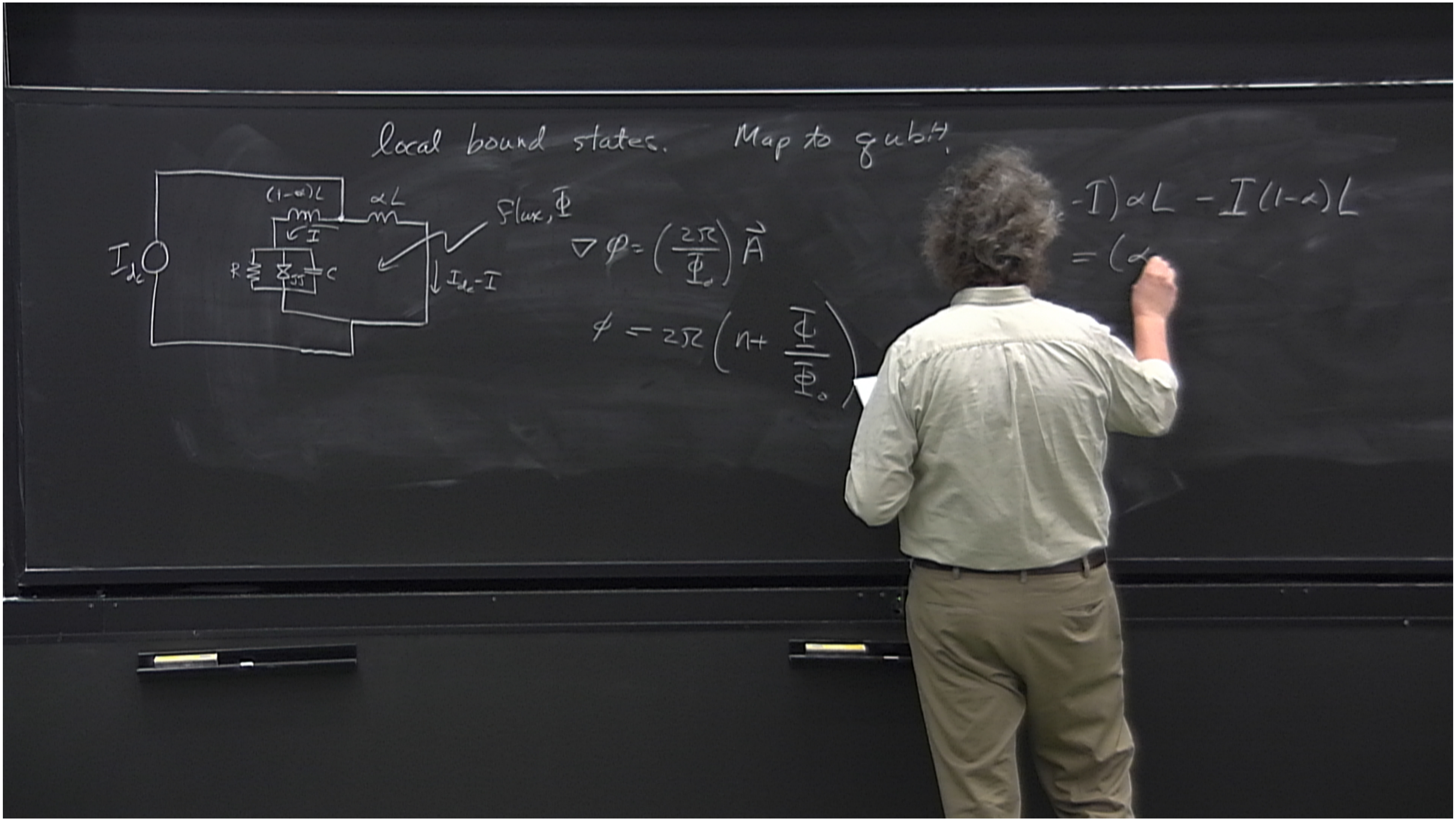


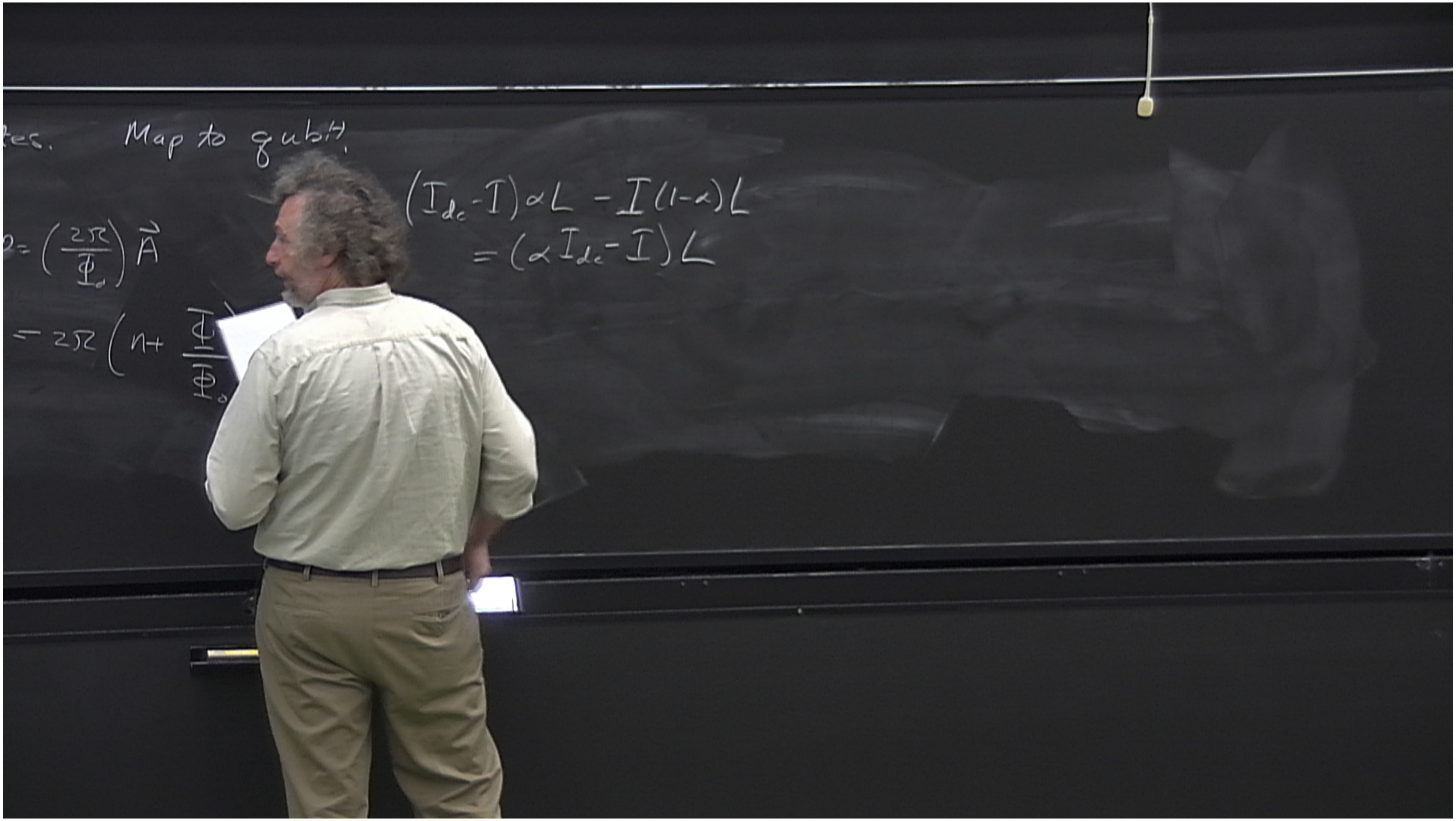
Idc
Icap
Idc = IR + IJJ + Icap
V=IR
V/R = ħ/2eR dφ/dt

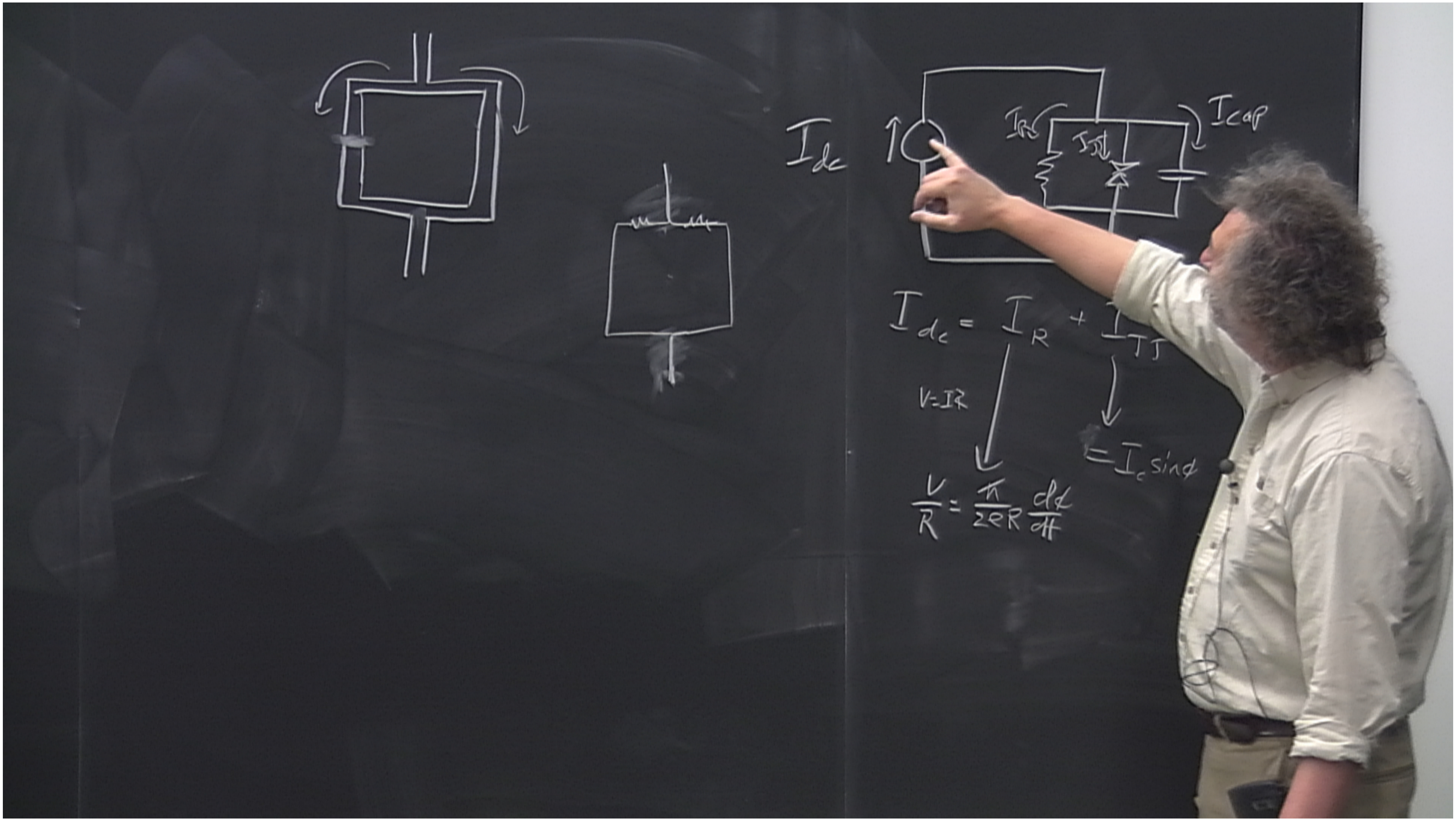

Icap
= IR + ITJ + Icap
= Ic sinφ
I = C dV/dt

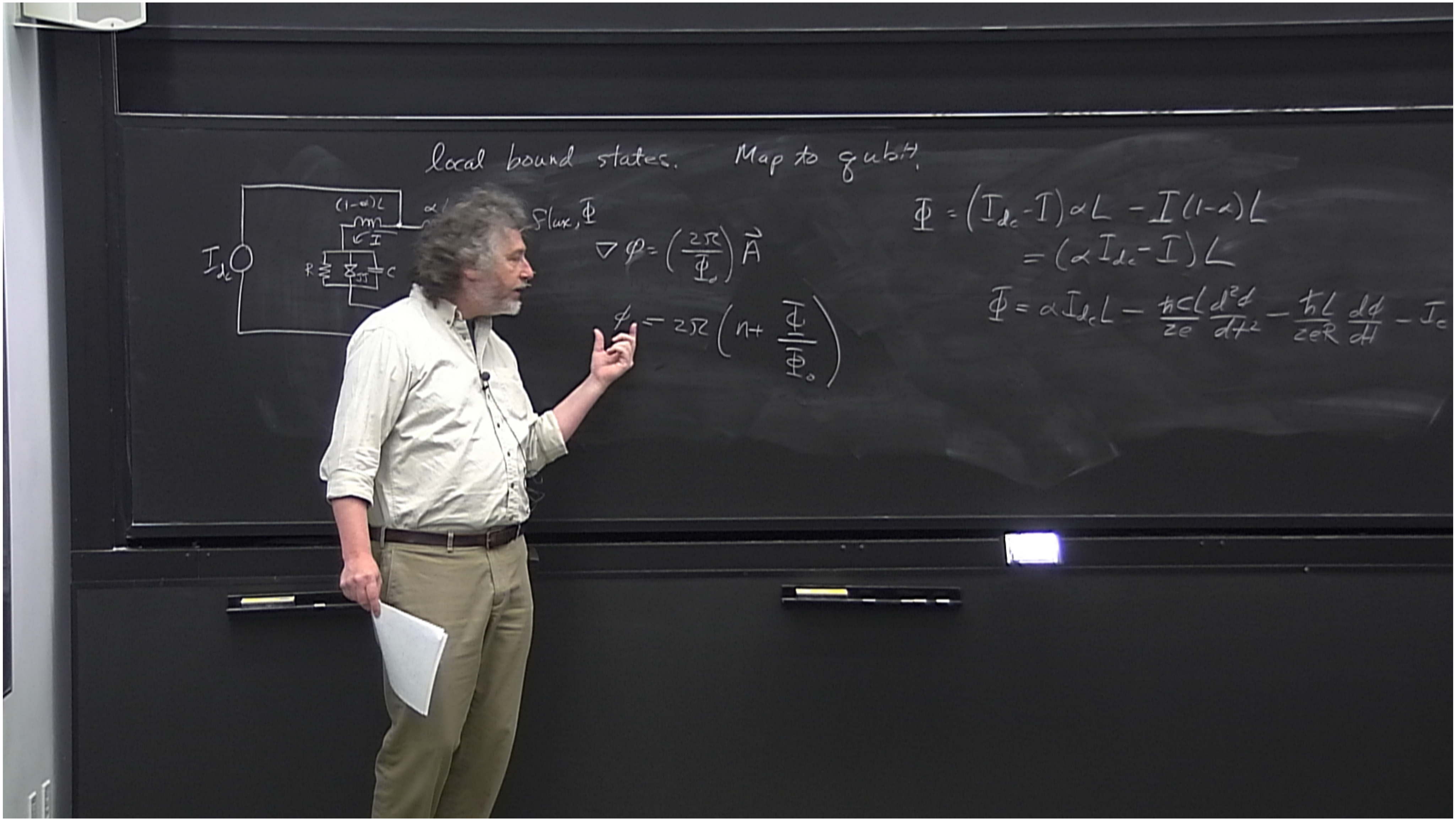

Idc
Icap
IR
IJJ
Idc = IR + IJJ + Icap
V=IR
= Ic sinφ
V/R = ħ/2eR dφ/dt
I=C dV/dt


local bound states. Map to qubit.
(1-α)L
αL
I
I_dc
R
JJ
C
I_dc - I
Φ = (I_dc - I) αL - I(1-α)L
(2π/Φ_0) A
= 2π(n + Φ/Φ_0)


local bound states. Map to qubit.
(1-α)L
αL
flux, Φ
I_dc
R
JJ
C
I
I_dc - I
∇φ = (2π/Φ₀) A
φ = 2π (n + Φ/Φ₀)
-I) αL - I(1-α)L
= (α


Map to qubit.
(I_dc - I) αL - I(1-α)L
= (αI_dc - I)L


I_dc
I_cap
I_dc = I_R + I_JJ
V=IR
= I_c sin φ
V/R = ħ/2eR dφ/dt


local bound states. Map to qubit.
$\Phi = (I_{dc} - I)\alpha L - I(1-\alpha)L$
$= (\alpha I_{dc} - I)L$
$\nabla\varphi = \left(\frac{2\pi}{\Phi_0}\right)\vec{A}$
$\phi = 2\pi\left(n + \frac{\Phi}{\Phi_0}\right)$
flux, Φ
$(1-\alpha)L$
I_{dc}
R
C
JJ

p to qubit.

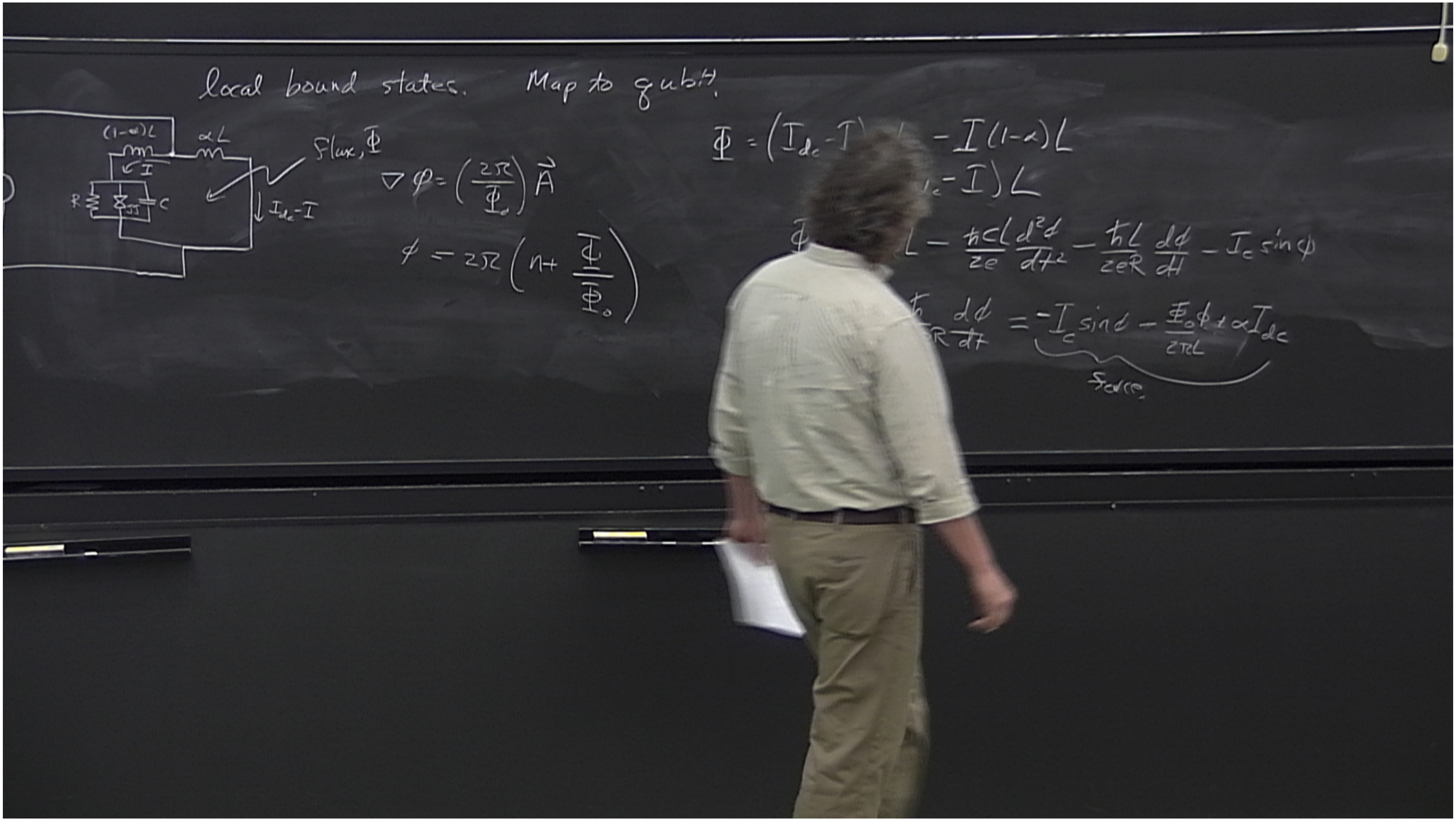
$$\Phi = (I_{dc} - I)\alpha L - I(1-\alpha)L$$

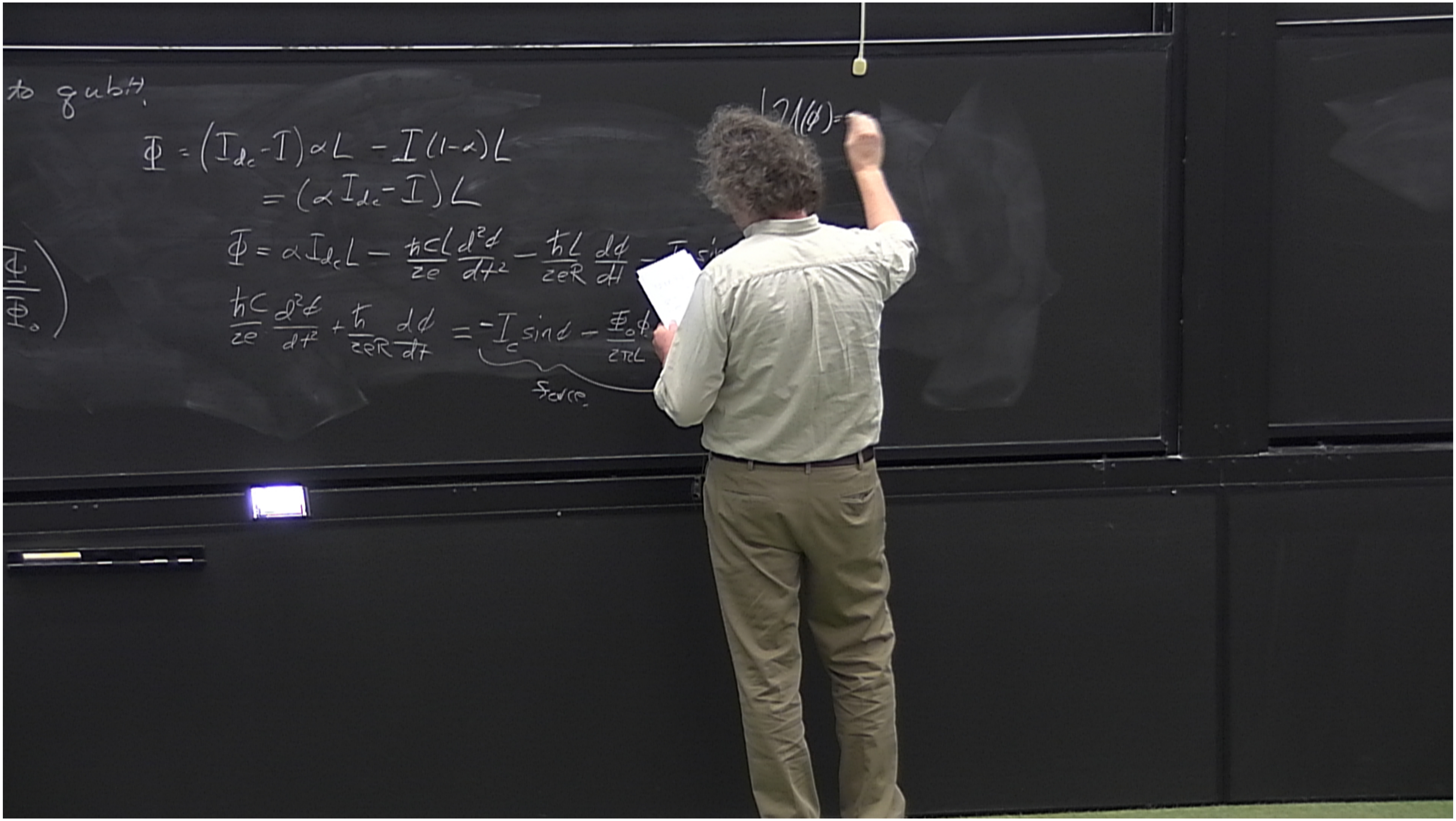
$$= (\alpha I_{dc} - I)L$$

$$\frac{\Phi}{\Phi_0}$$

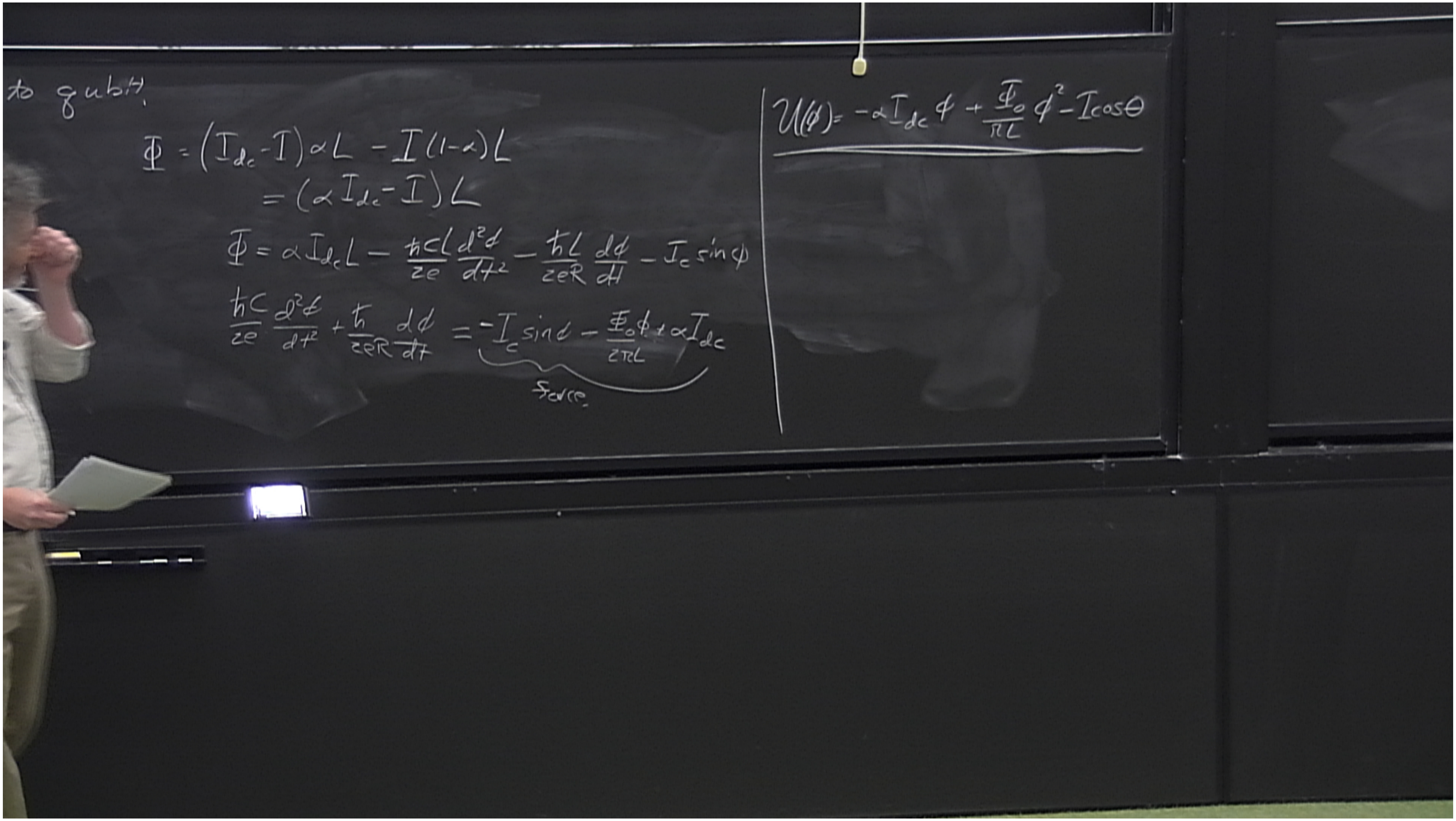
$$\Phi = \alpha I_{dc} L - \frac{\hbar C L}{2e}\frac{d^2\phi}{dt^2} - \frac{\hbar L}{2eR}\frac{d\phi}{dt} - I_c \sin\phi$$

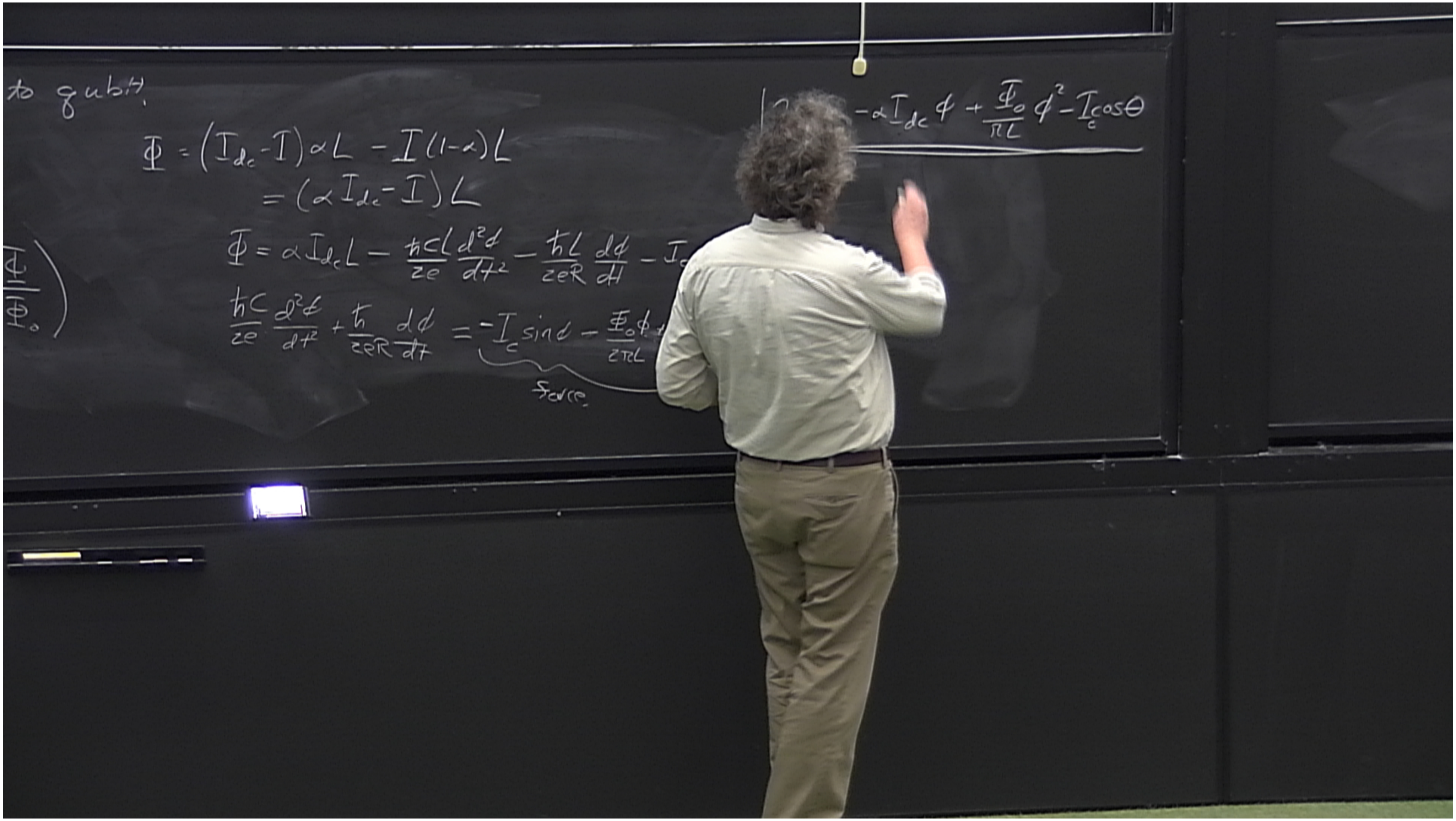
$$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} = \underbrace{-I_c \sin\phi - \frac{\Phi_0}{2\pi L} + \alpha I_{dc}}_{\text{force}}$$


local bound states. Map to qubit.
flux, Φ
force

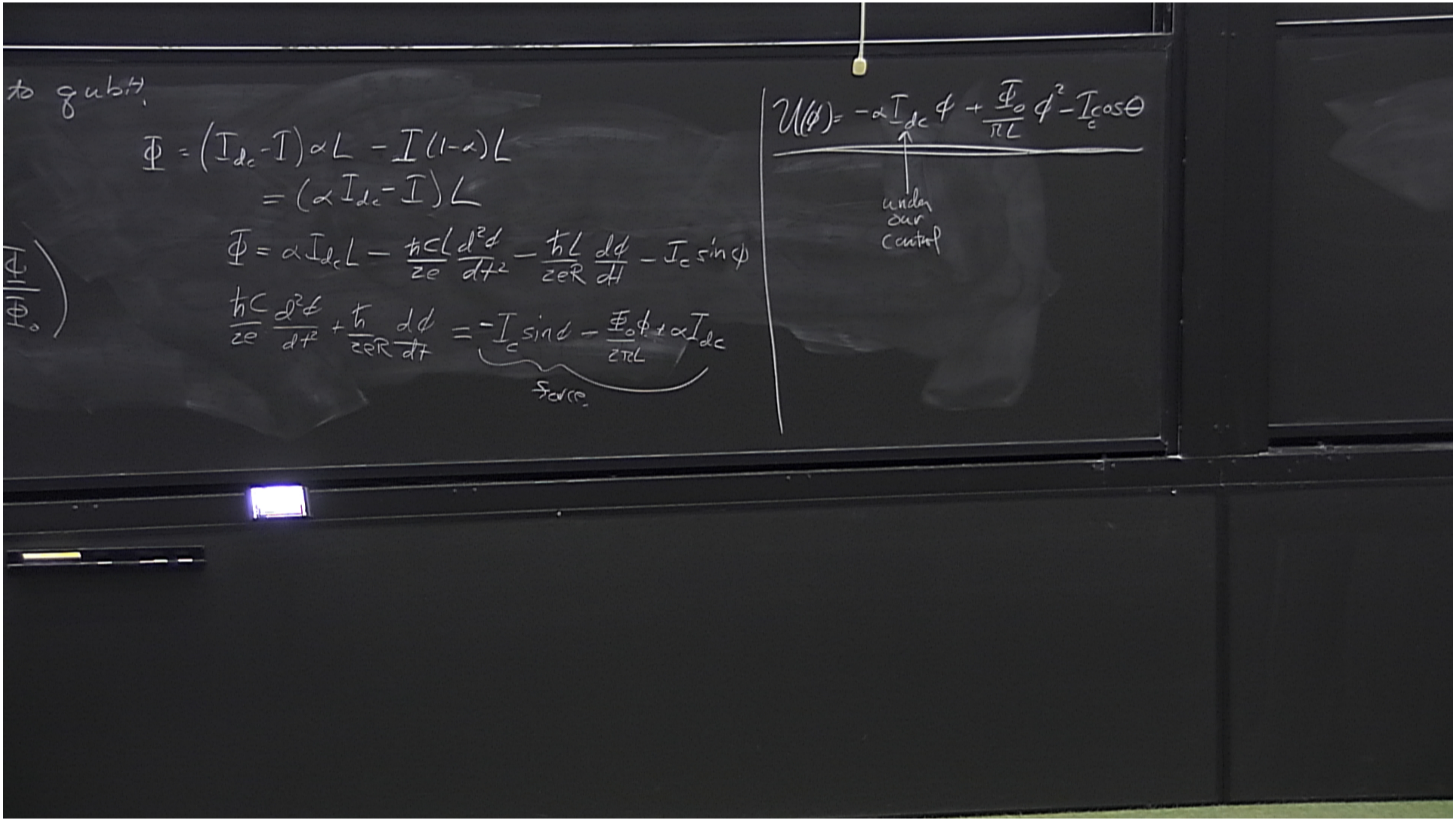


to qubit.
Φ = (I_dc − I) αL − I(1−α)L
= (α I_dc − I) L
Φ = α I_dc L − ħCL/2e d²φ/dt² − ħL/2eR dφ/dt
ħC/2e d²φ/dt² + ħ/2eR dφ/dt = −I_c sinφ − Φ₀φ/2πL
force
U(φ) =


to qubit.
Φ = (I_dc - I) αL - I(1-α)L
= (αI_dc - I)L
Φ = αI_dc L - ħCL/2e d²φ/dt² - ħL/2eR dφ/dt - I_c sin φ
ħC/2e d²φ/dt² + ħ/2eR dφ/dt = -I_c sin φ - Φ_0/2πL φ + αI_dc
force
U(φ) = -αI_dc φ + Φ_0/πL φ² - I_c cos θ



to qubit.
Φ = (I_dc − I) α L − I (1−α) L
= (α I_dc − I) L
force


to qubit.
$\Phi = (I_{dc} - I)\alpha L - I(1-\alpha)L$
$= (\alpha I_{dc} - I)L$
$\Phi = \alpha I_{dc} L - \frac{\hbar C L}{2e}\frac{d^2\phi}{dt^2} - \frac{\hbar L}{2eR}\frac{d\phi}{dt} - I_c \sin\phi$
$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} = -I_c \sin\phi - \frac{\Phi_0}{2\pi L}\phi + \alpha I_{dc}$
force
$U(\phi) = -\alpha I_{dc}\phi + \frac{\Phi_0}{\pi L}\phi^2 - I_c \cos\theta$
under our control

In[6]:= `Manipulate[Plot[Ur[ϕ, 1, Idc, α], {ϕ, 0, 8 π}], {Idc, 0, 3}, {α, 0, 1}]`

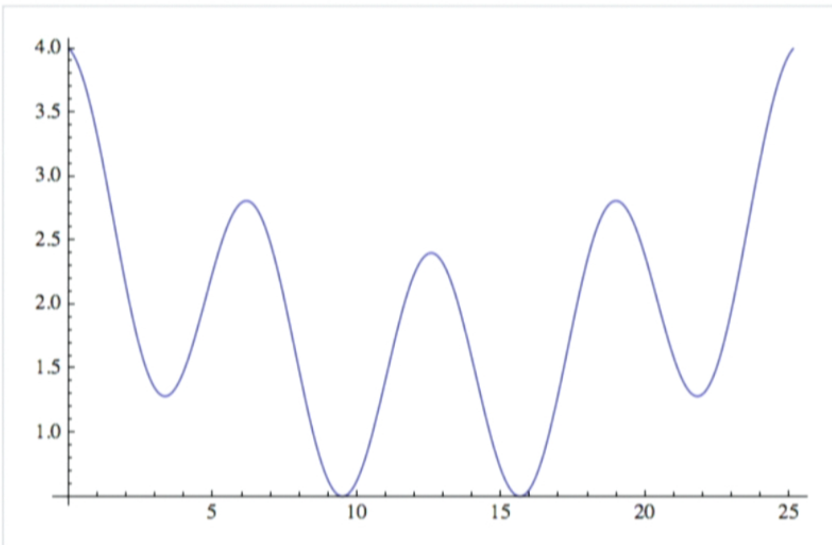
Idc

0.254192

α

0.010114

Out[6]=



- **numerical estimation of the eigenstructure**

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Phase qubit.nb

```
In[6]:= Manipulate[Plot[Ur[ϕ, 1, Idc, α], {ϕ, 0, 8 π}], {Idc, 0, 3}, {α, 0, 1}]
```

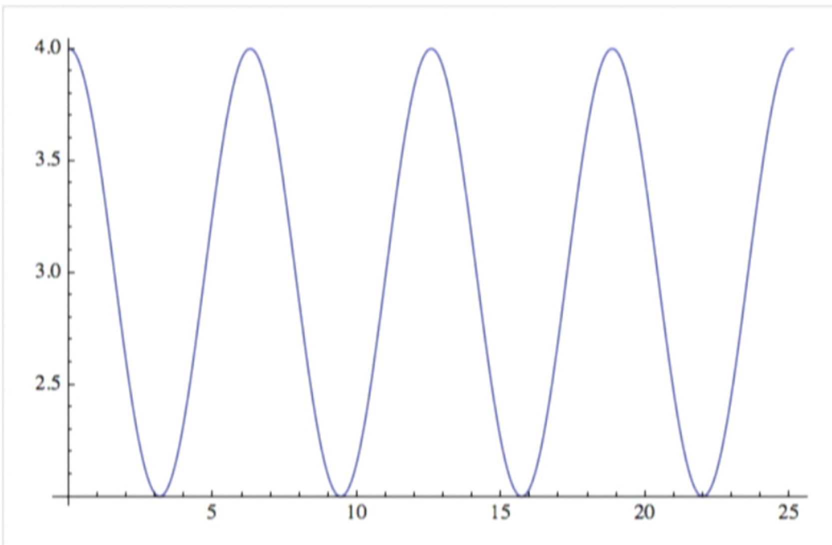
Idc

0.

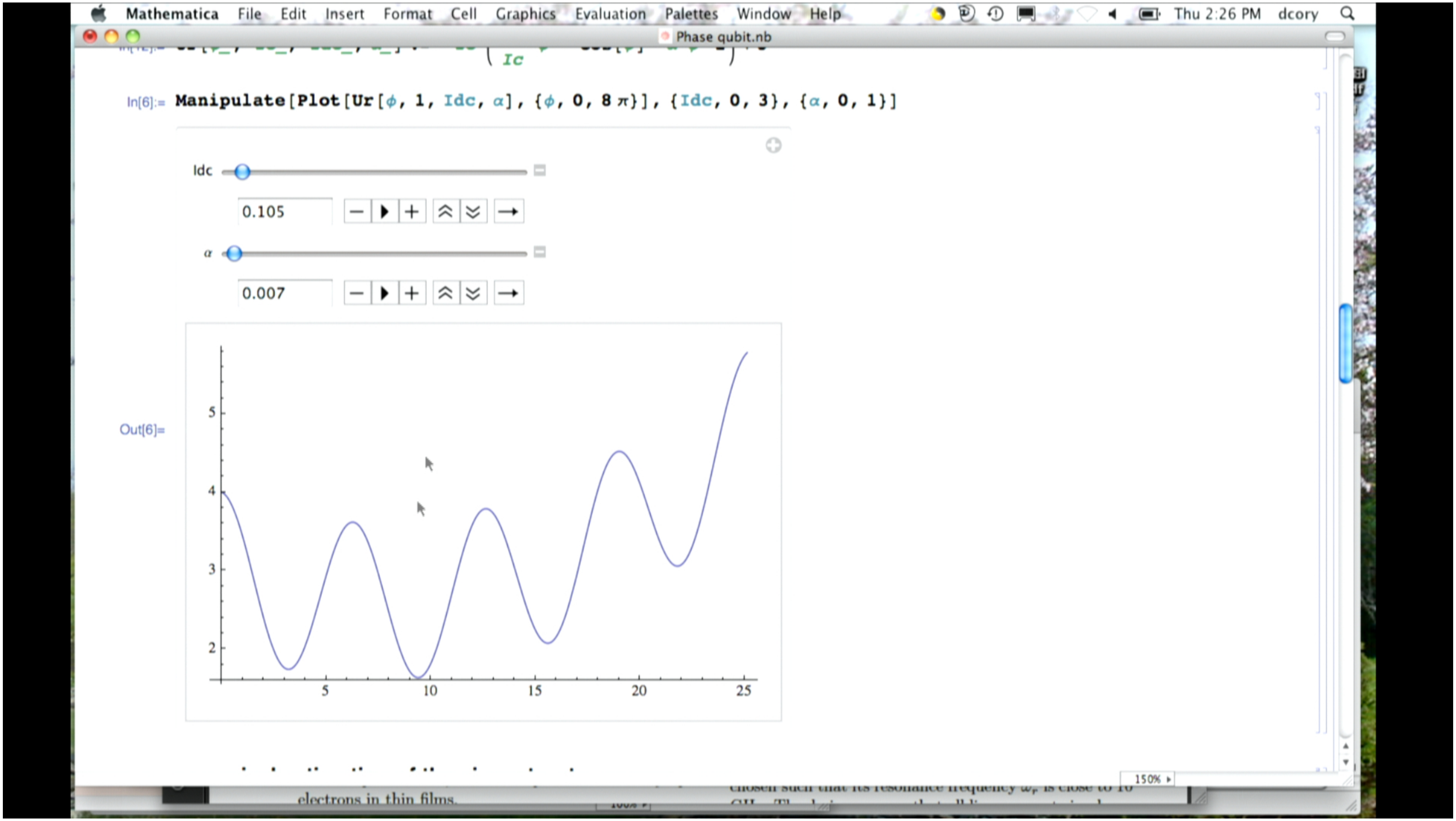
α

0.

Out[6]=

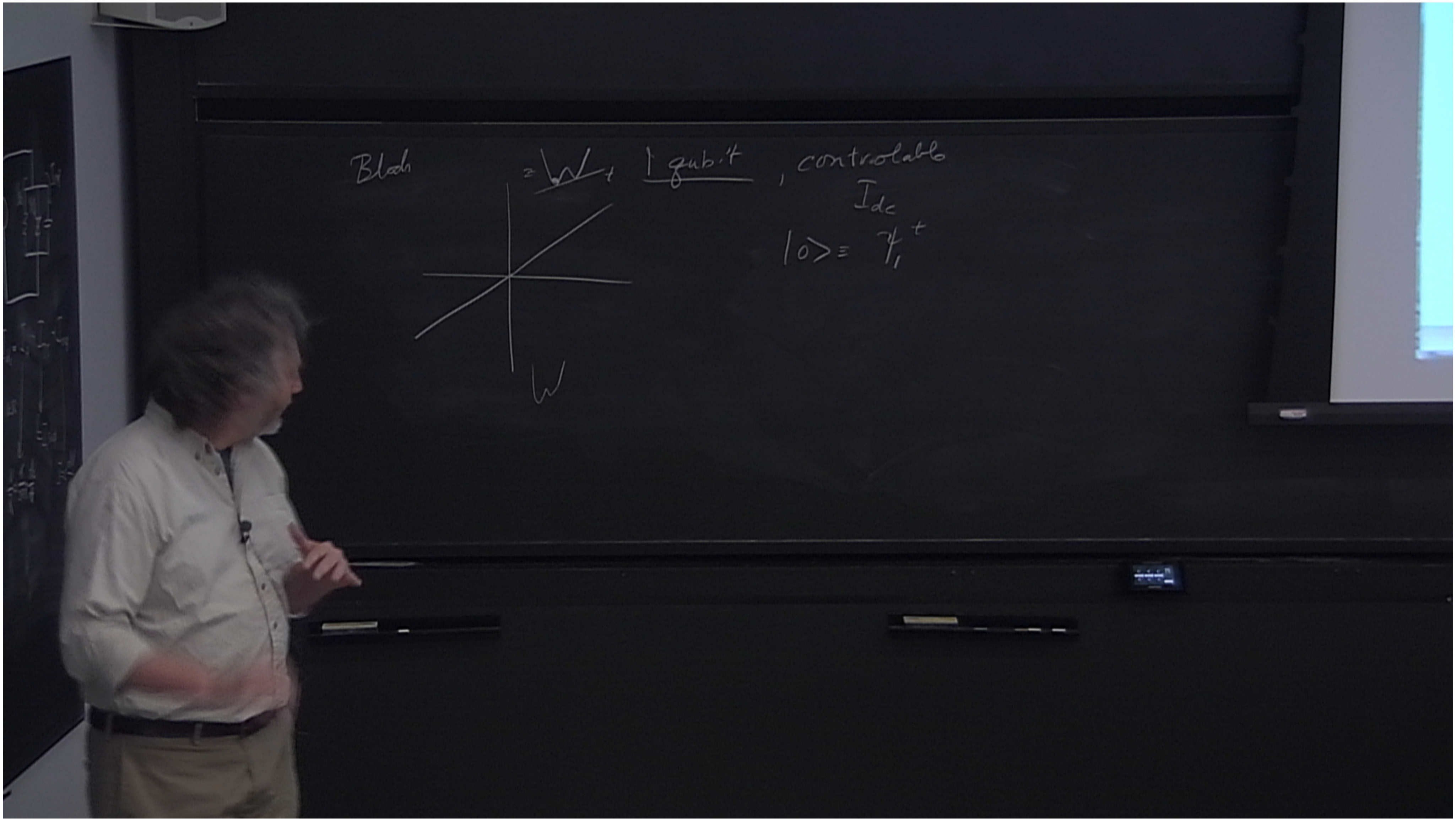


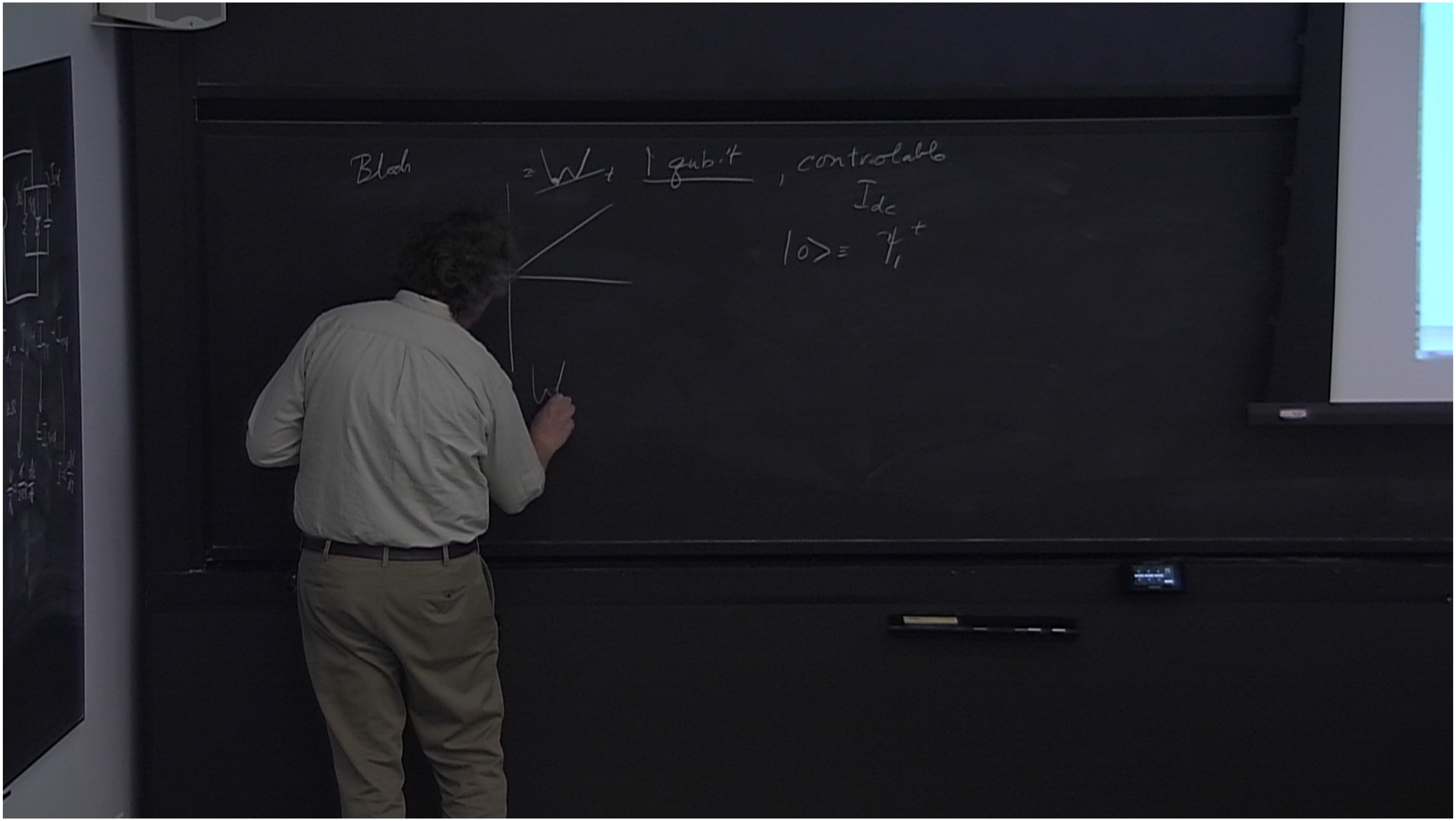
■ numerical estimation of the eigenstructure

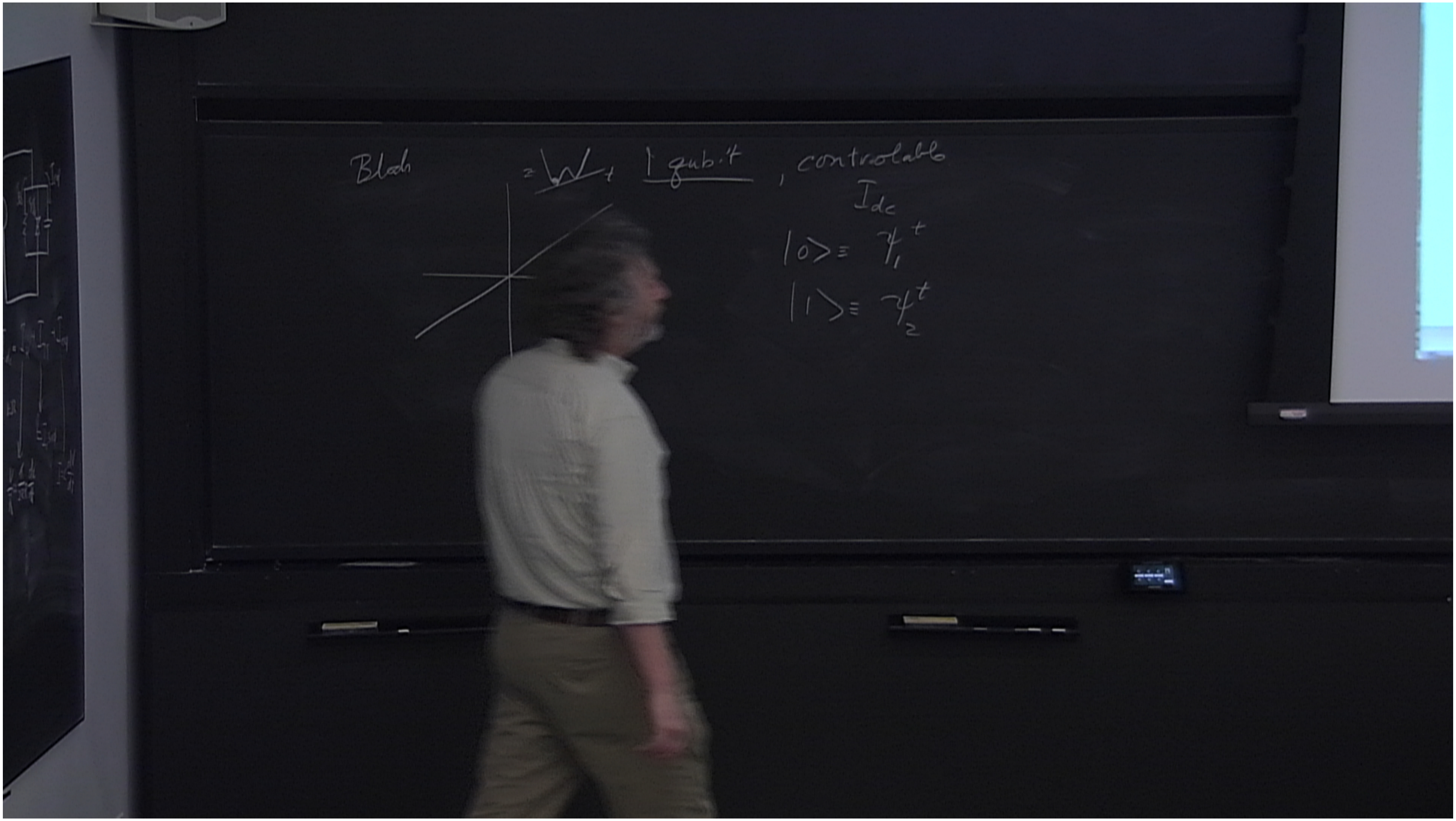

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 2:26 PM dcory
Phase qubit.nb
In[6]:= Manipulate[Plot[Ur[ϕ, 1, Idc, α], {ϕ, 0, 8 π}], {Idc, 0, 3}, {α, 0, 1}]
Idc
0.105
α
0.007
Out[6]=
2
3
4
5
5
10
15
20
25
150%


Bloch
z
1 qubit
, controlable
I_{dc}

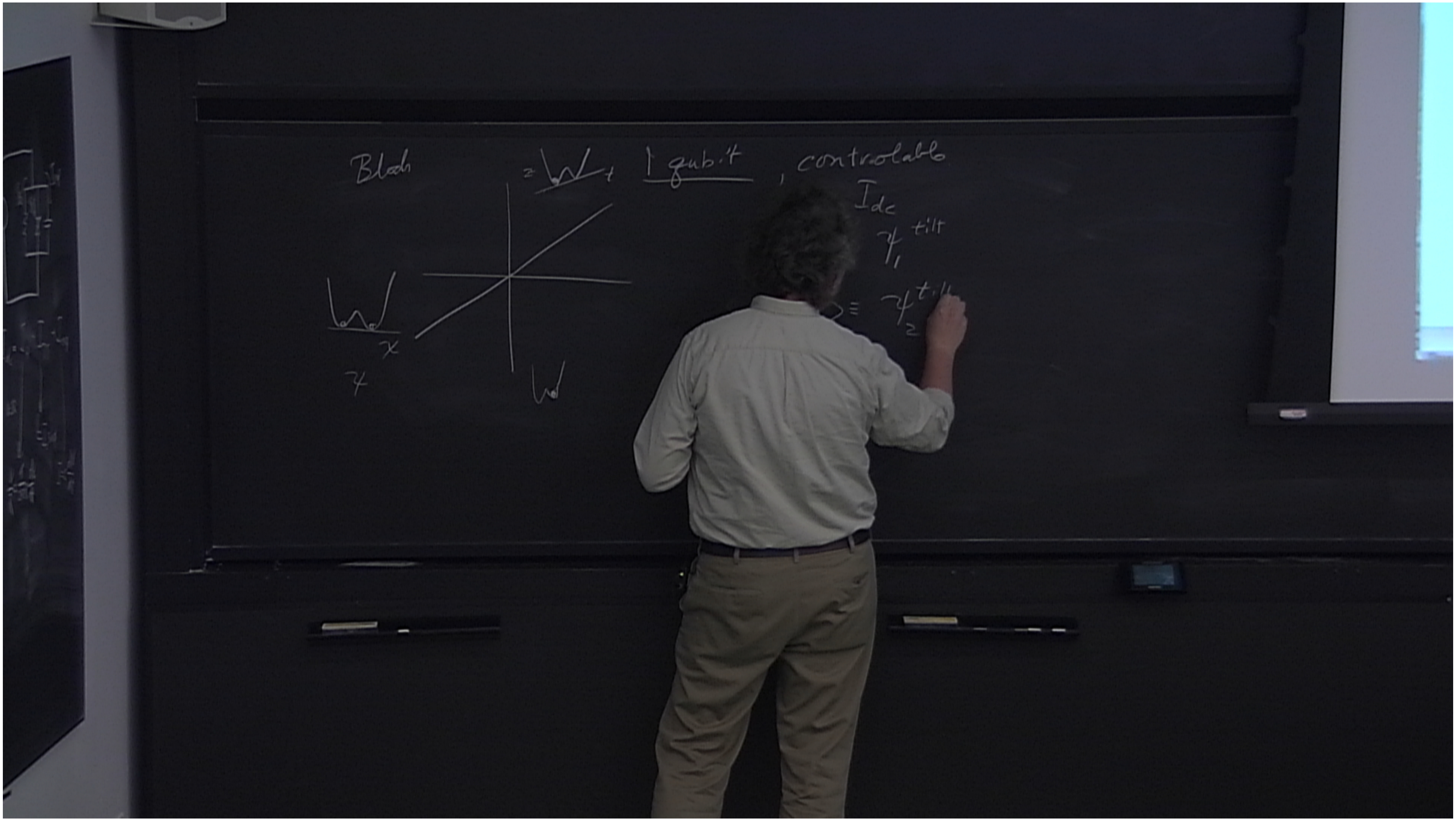

Bloch
= W
1 qubit
, controlable
I_{dc}

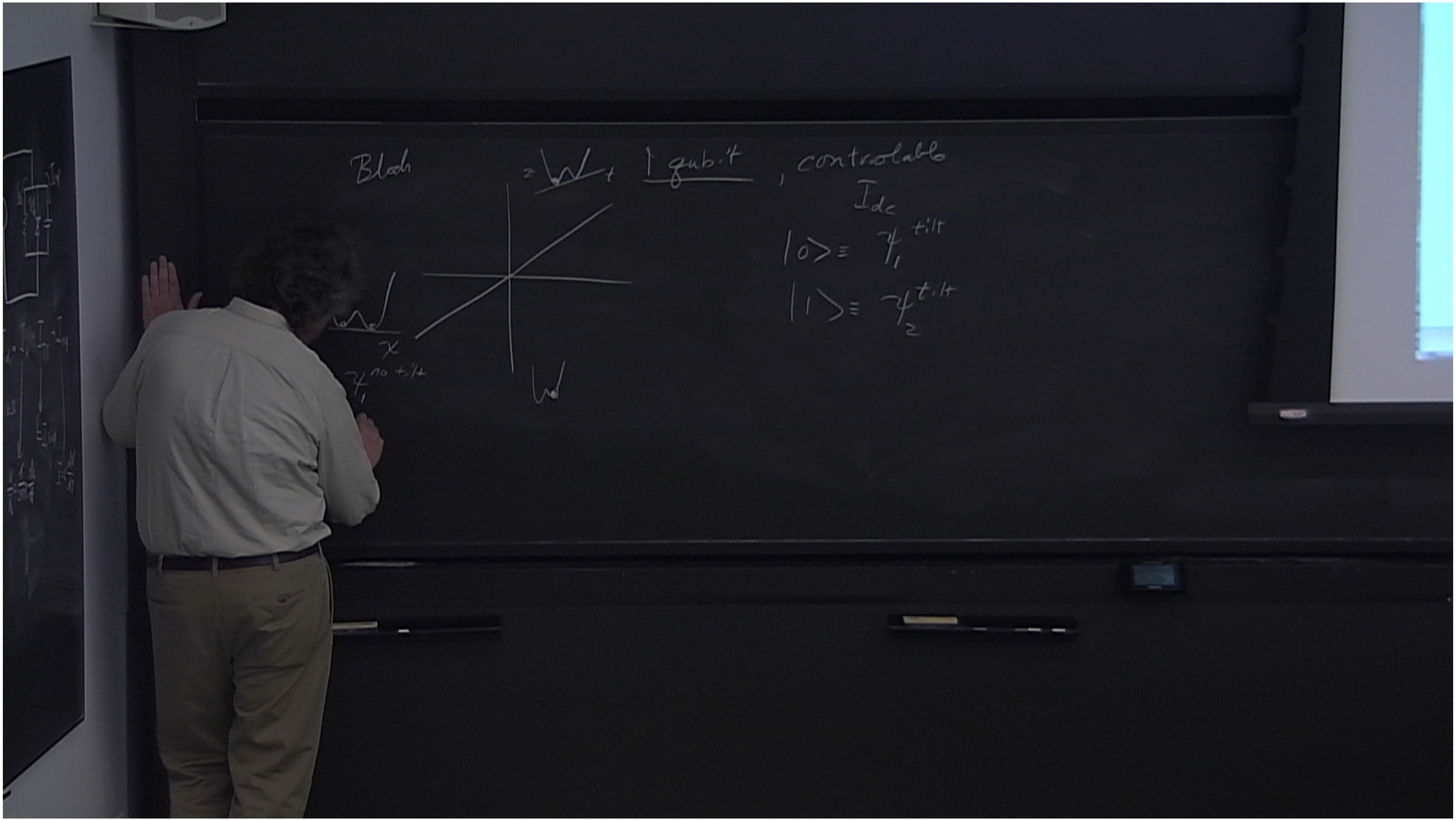

Bloch
1 qubit
, controlable
Idc

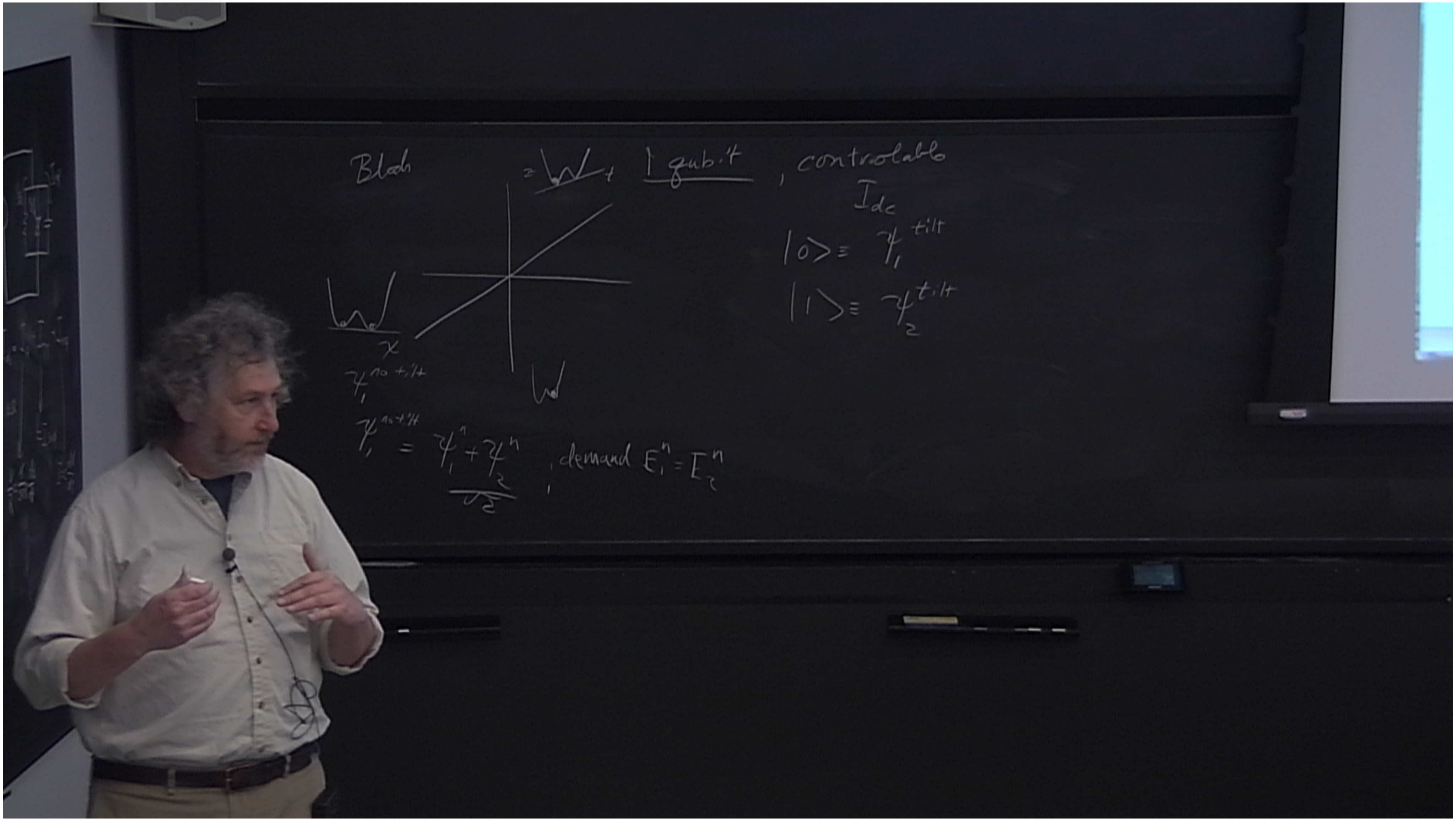

Bloch
1 qubit
, controlable
I_{dc}
$|0\rangle \equiv \psi_1^+$

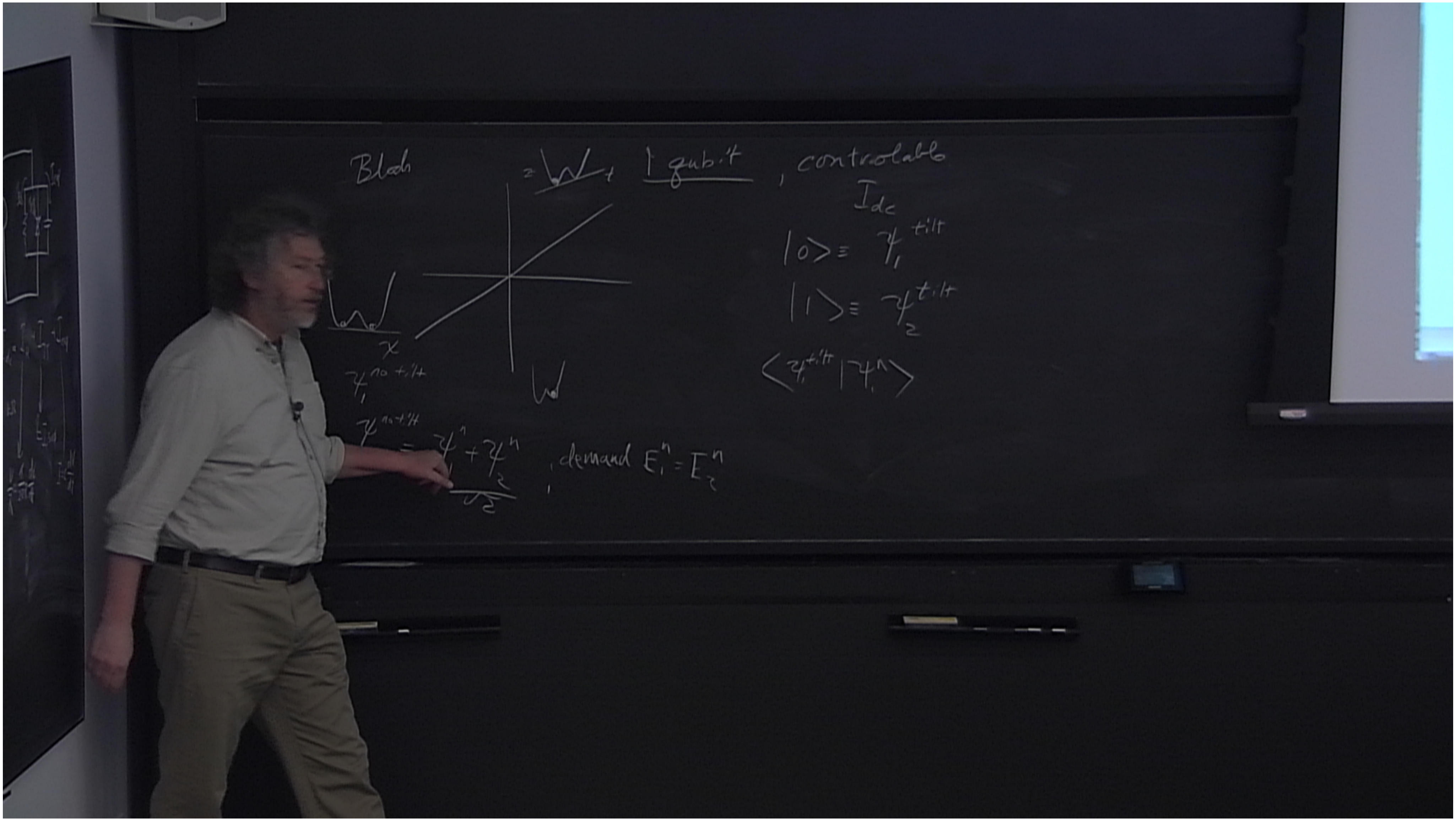


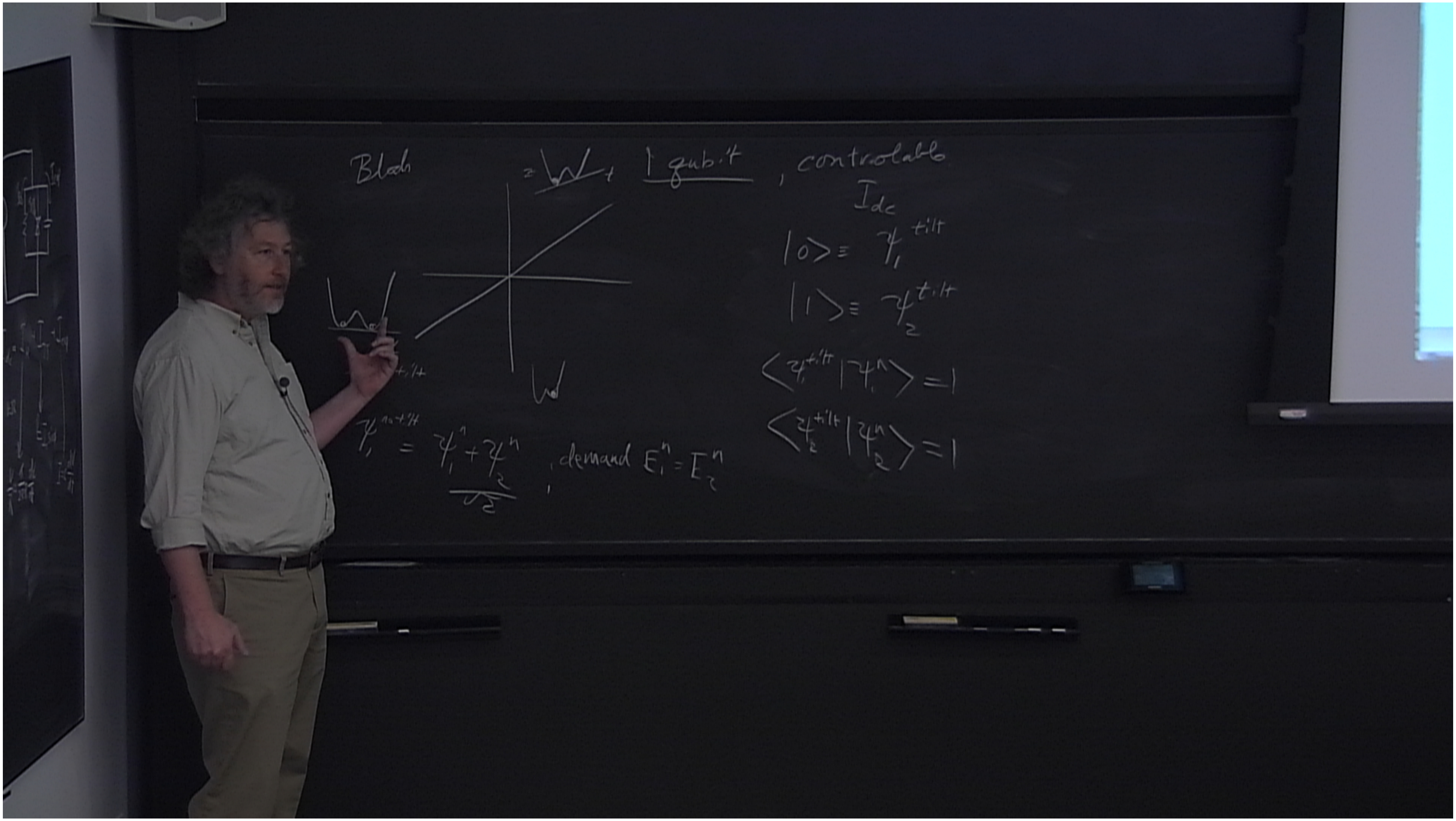
Bloch
1 qubit
, controllable
I_{dc}
$|0\rangle \equiv \psi_1^{+}$
$|1\rangle \equiv \psi_2^{+}$

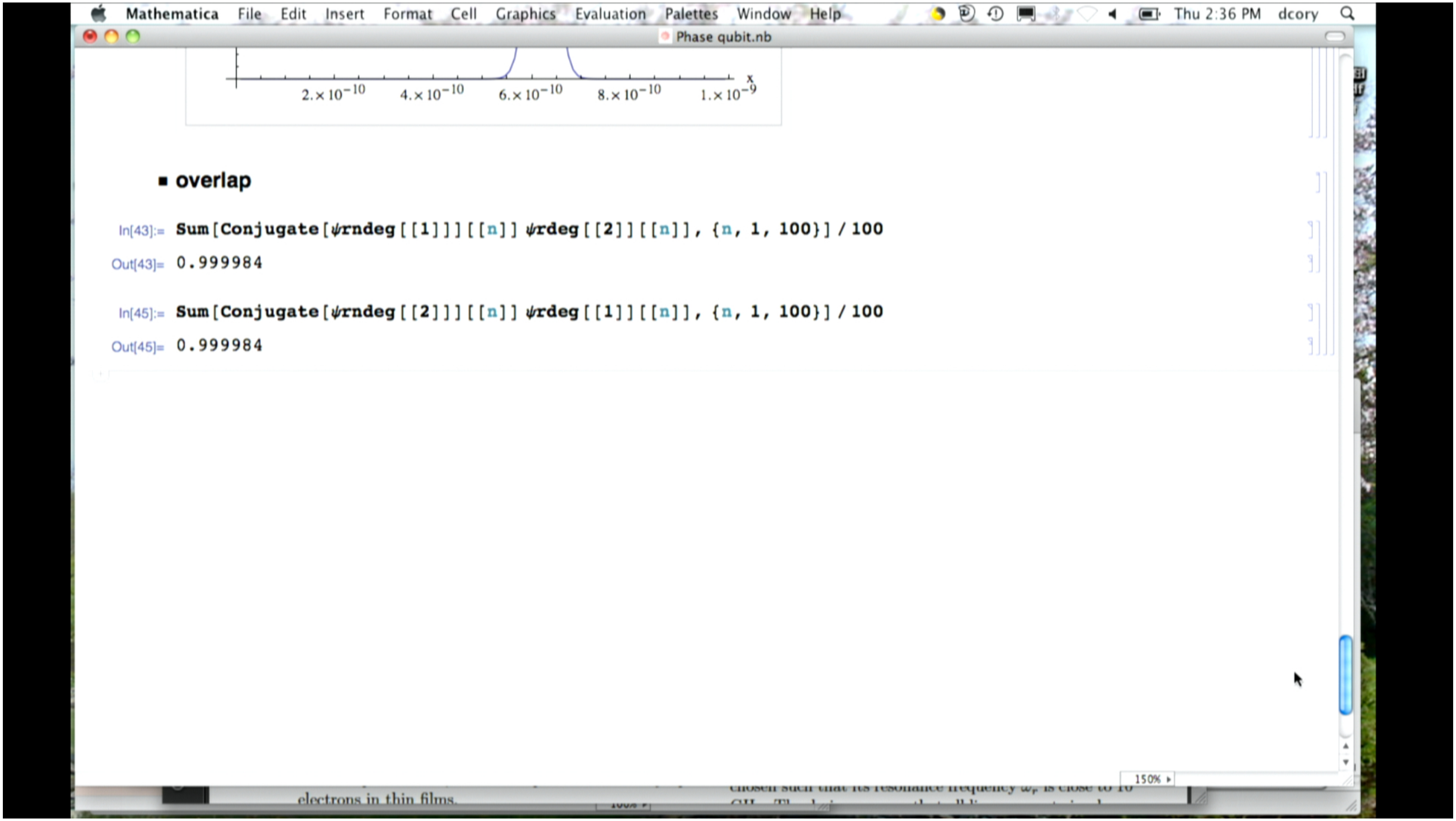

Bloch
1 qubit
controlable
tilt

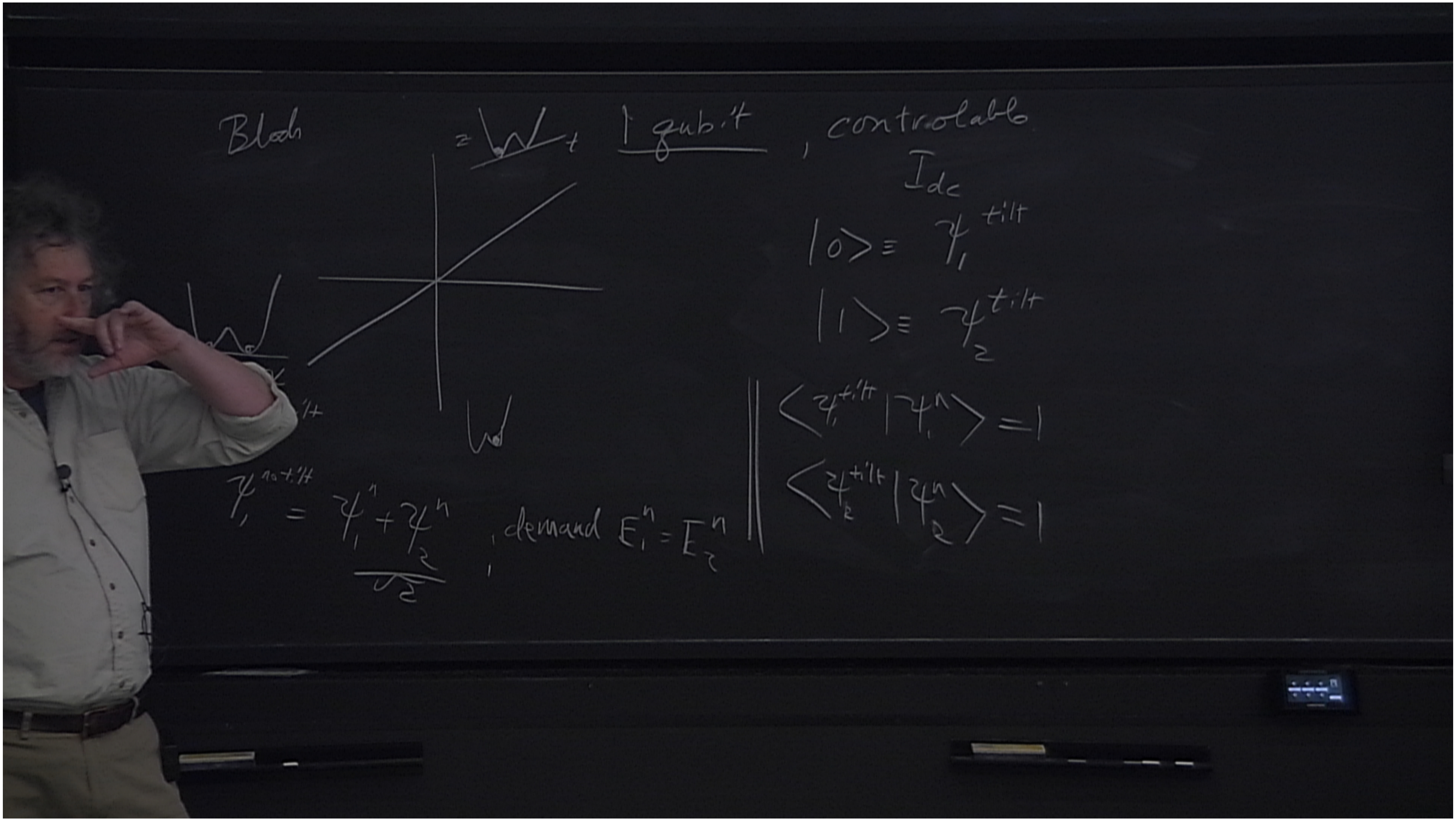

Bloch
1 qubit
, controlable
Idc
|0> ≡ ψ1 tilt
|1> ≡ ψ2 tilt
x
ψ1 no tilt

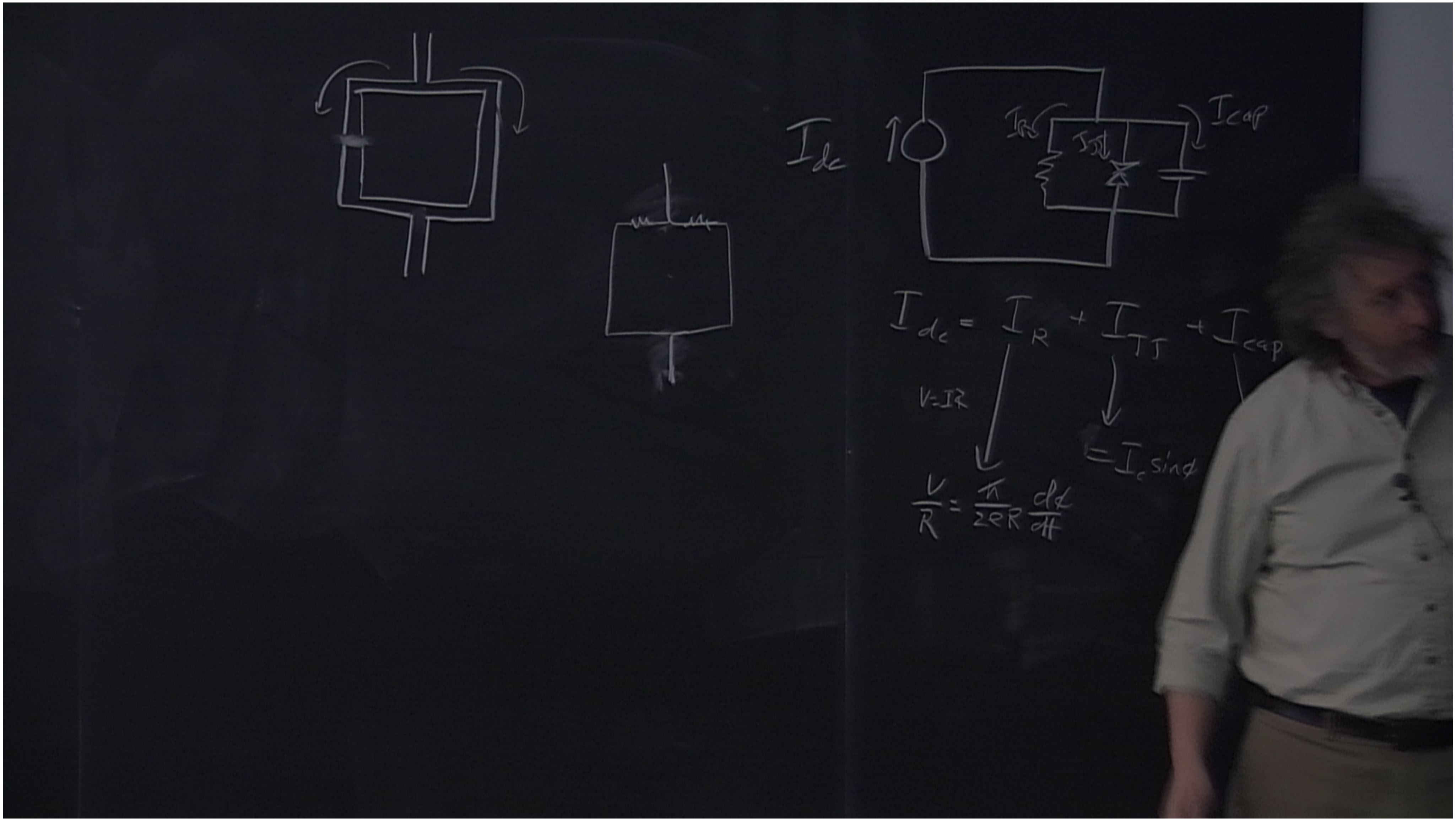

Bloch
1 qubit
, controlable
I_{dc}
$|0\rangle \equiv \psi_1^{tilt}$
$|1\rangle \equiv \psi_2^{tilt}$
x
$\psi_1^{no\ tilt}$
$\psi_1^{no\ tilt} = \frac{\psi_1^n + \psi_2^n}{\sqrt{2}}$, demand $E_1^n = E_2^n$

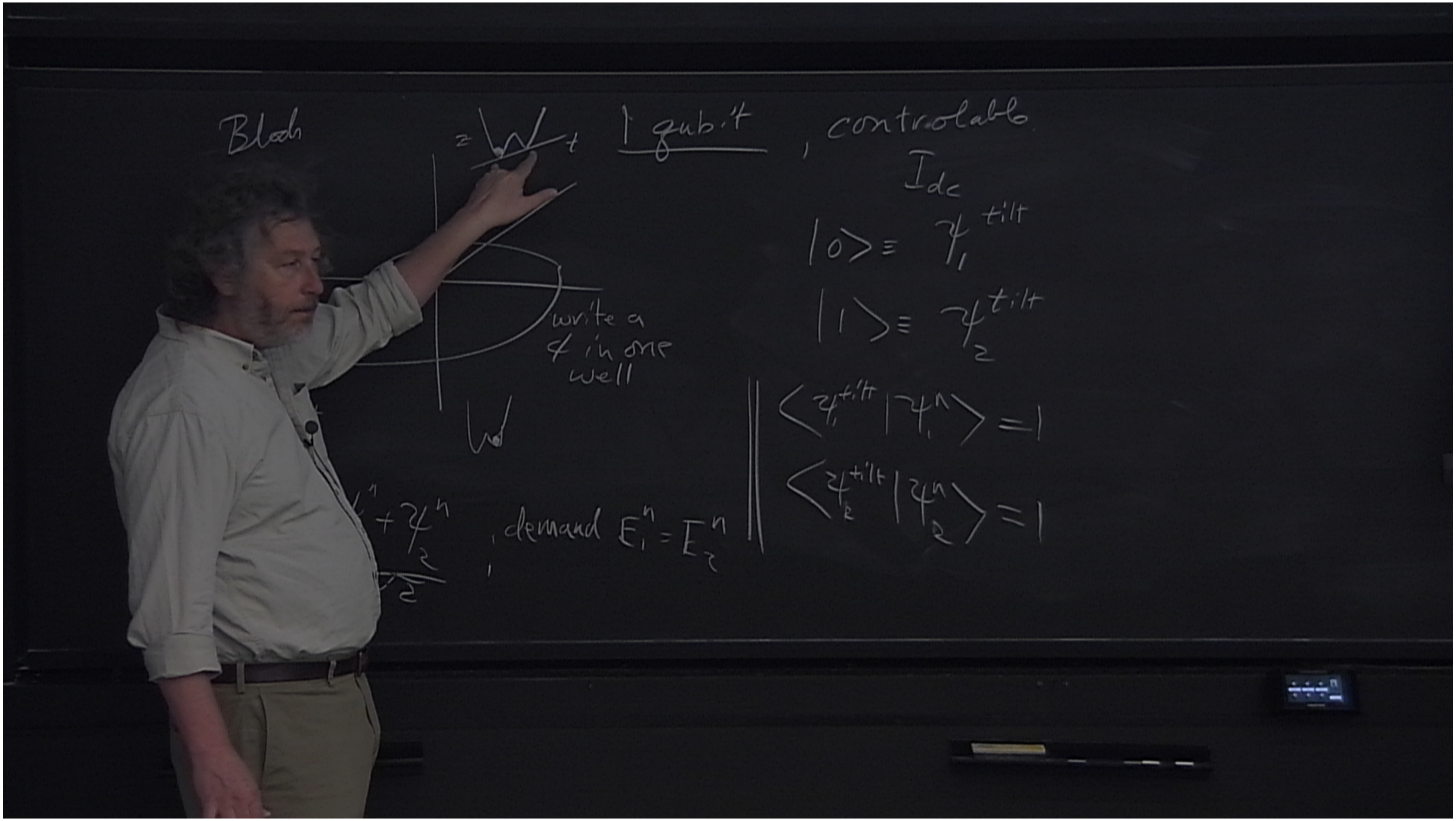

Bloch
1 qubit
, controllable
I_{dc}
$|0\rangle \equiv \psi_1^{tilt}$
$|1\rangle \equiv \psi_2^{tilt}$
$\langle \psi_1^{tilt} | \psi_1^{n} \rangle$
$\psi_1^{no\ tilt}$
$\psi^{no\text{-}tilt} = \frac{\psi_1^n + \psi_2^n}{\sqrt{2}}$
demand $E_1^n = E_2^n$

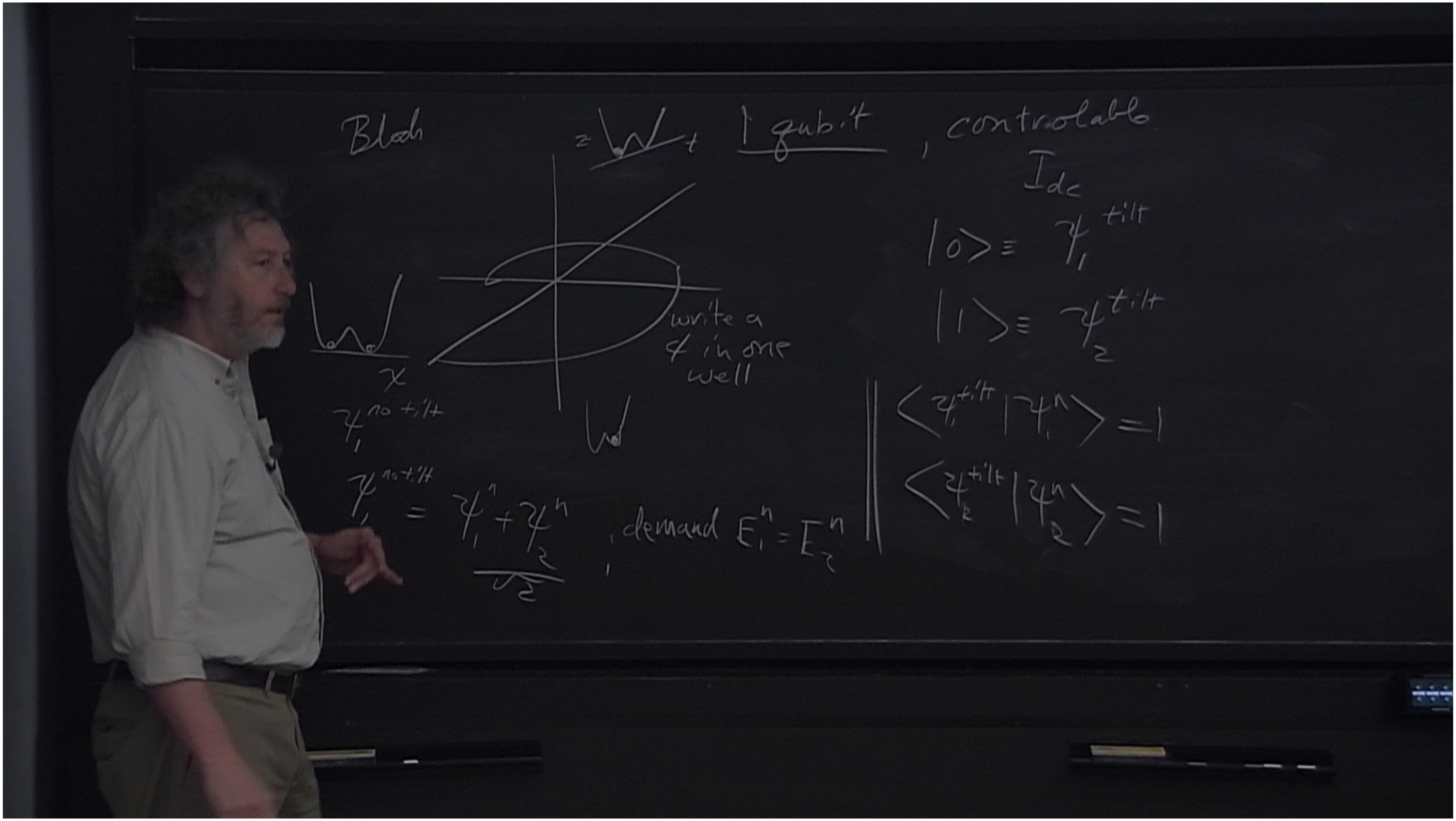

Bloch
1 qubit
, controlable
Idc
demand


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 2:36 PM dcory
Phase qubit.nb
2.× 10^-10 4.× 10^-10 6.× 10^-10 8.× 10^-10 1.× 10^-9
x
overlap
In[43]:= Sum[Conjugate[ψrndeg[[1]]][[n]] ψrdeg[[2]][[n]], {n, 1, 100}] / 100
Out[43]= 0.999984
In[45]:= Sum[Conjugate[ψrndeg[[2]]][[n]] ψrdeg[[1]][[n]], {n, 1, 100}] / 100
Out[45]= 0.999984
150%

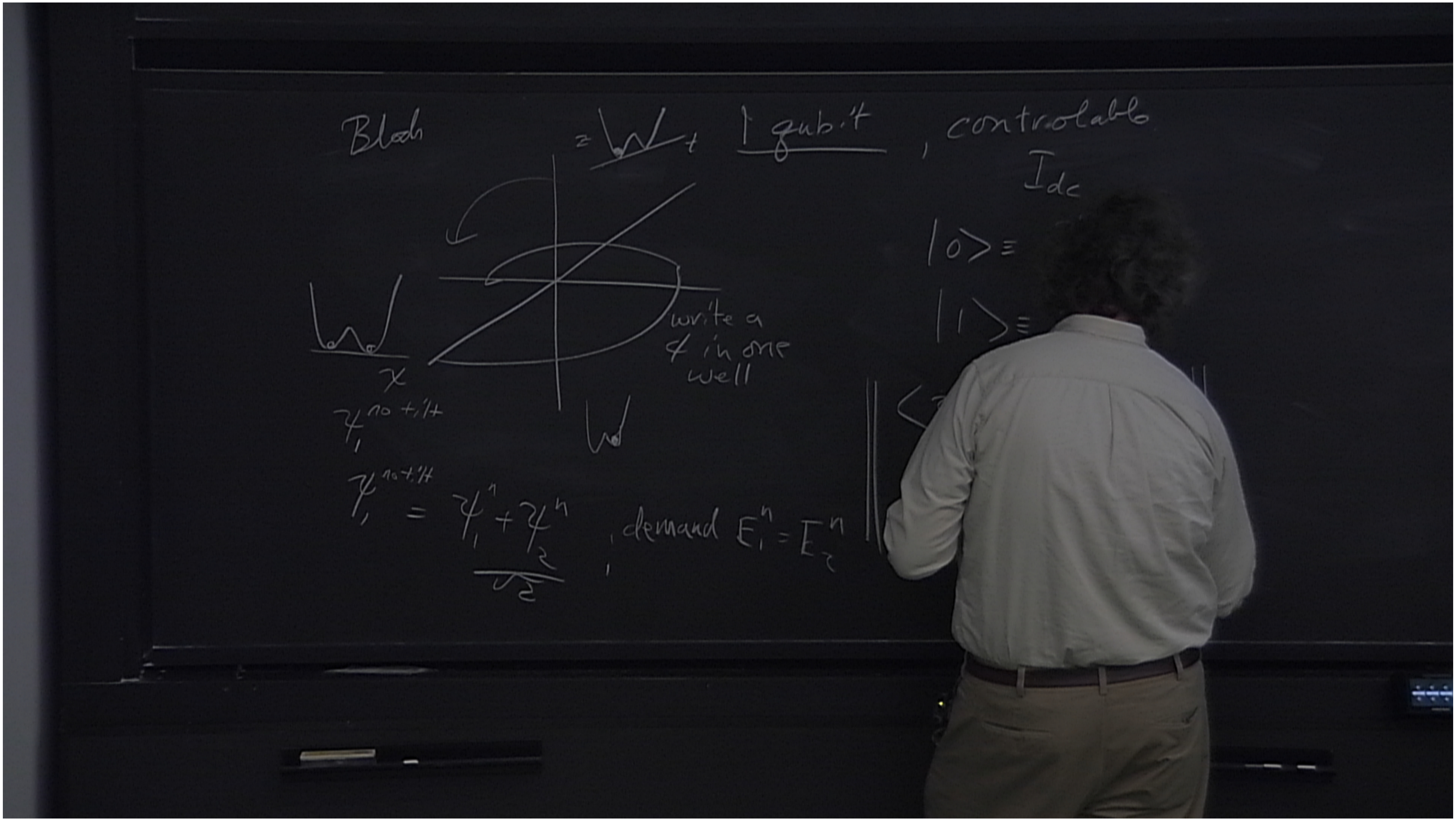

Bloch
1 qubit
, controlable
Idc
|0> ≡ ψ1 tilt
|1> ≡ ψ2 tilt
<ψ1 tilt | ψ1 n> = 1
<ψ2 tilt | ψ2 n> = 1
ψ1 non-tilt = (ψ1 n + ψ2 n) / √2
, demand E1 n = E2 n

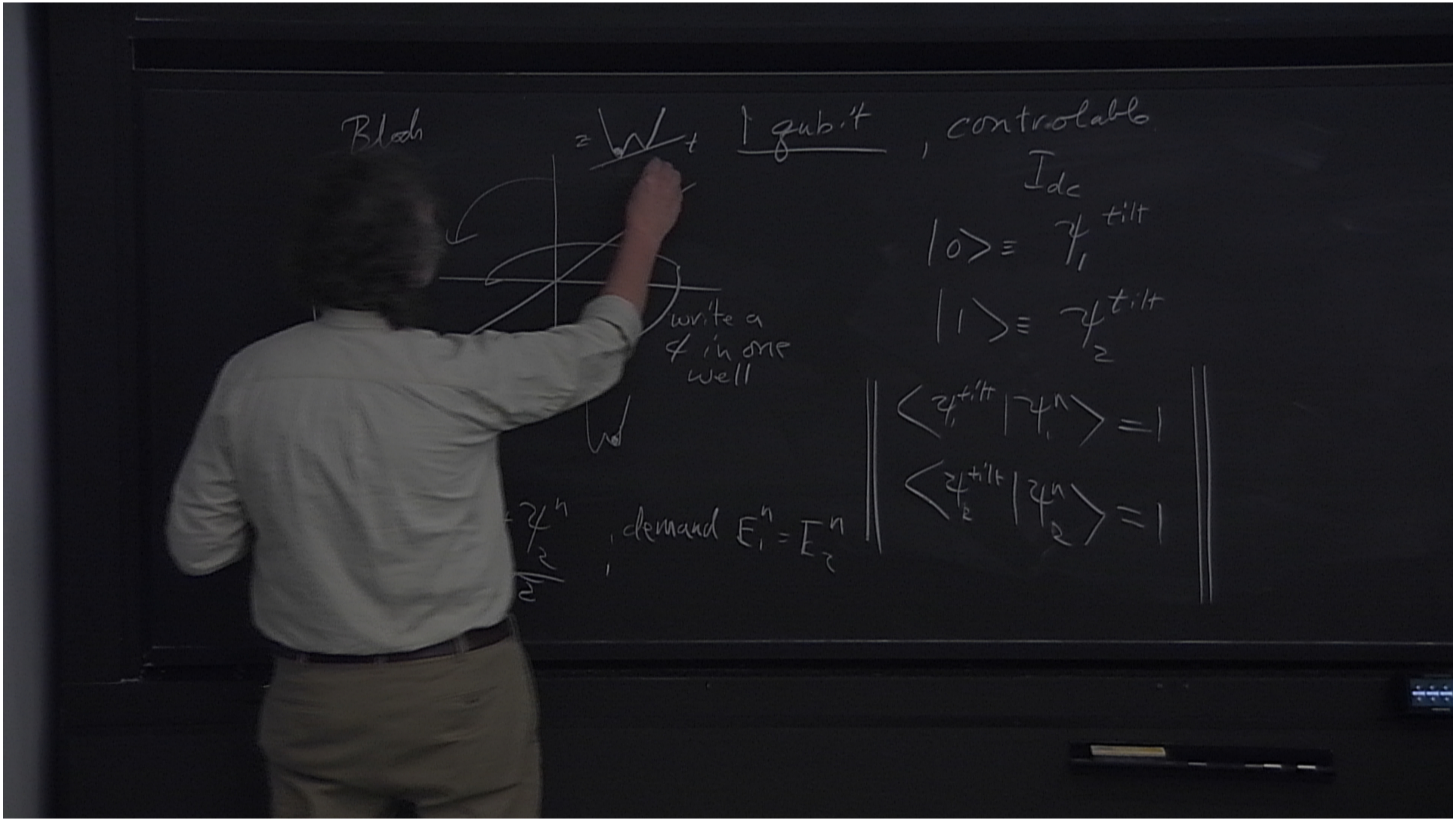

I_dc
I_cap
I_dc = I_R + I_JJ + I_cap
V=IR
= I_c sin φ
V/R = ħ/2eR dφ/dt

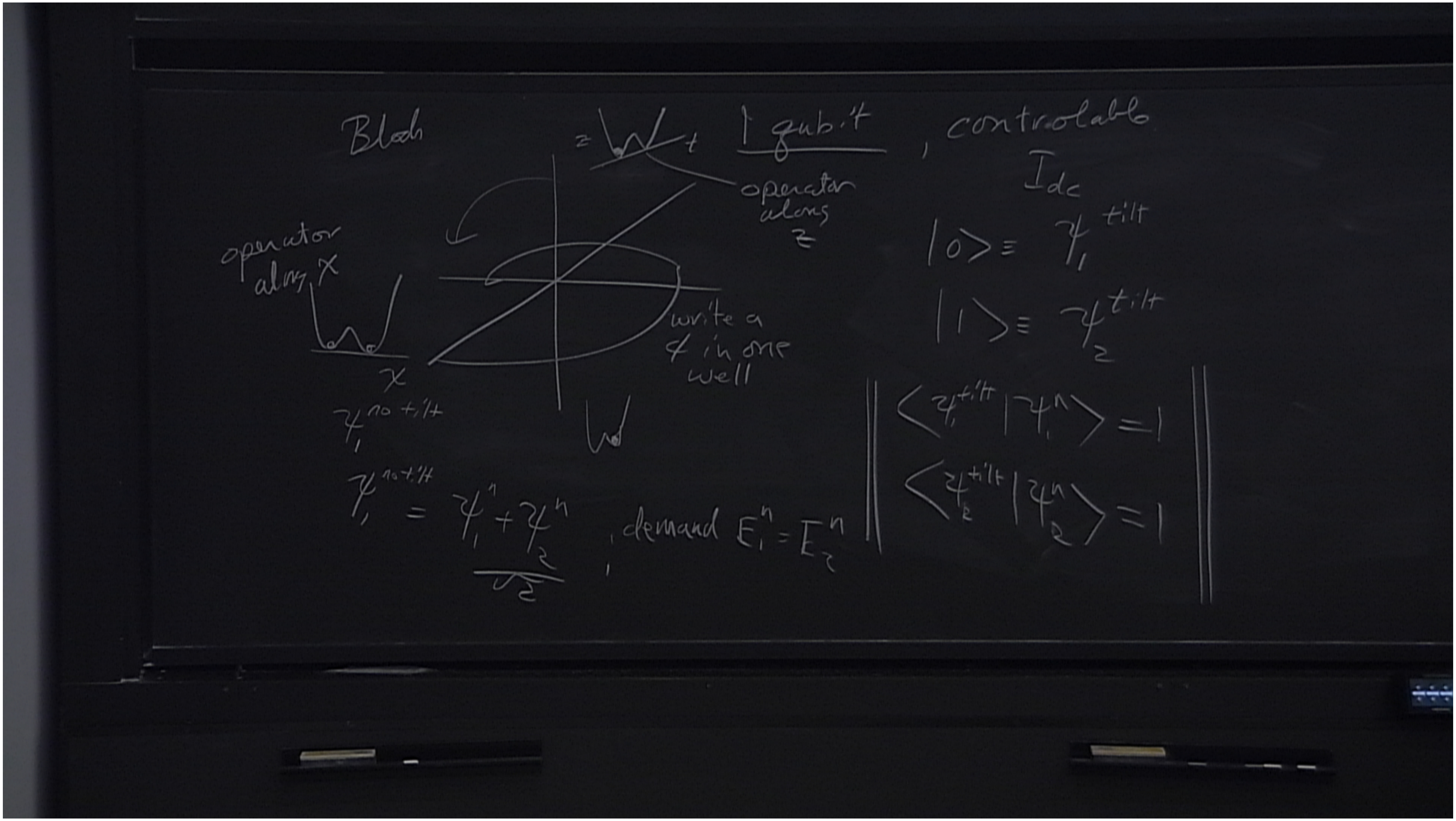

Bloch
1 qubit
, controlable
Idc
|0> ≡ ψ1 tilt
|1> ≡ ψ2 tilt
write a ψ in one well
<ψ1 tilt | ψ1 n> = 1
<ψ2 tilt | ψ2 n> = 1
demand E1 n = E2 n

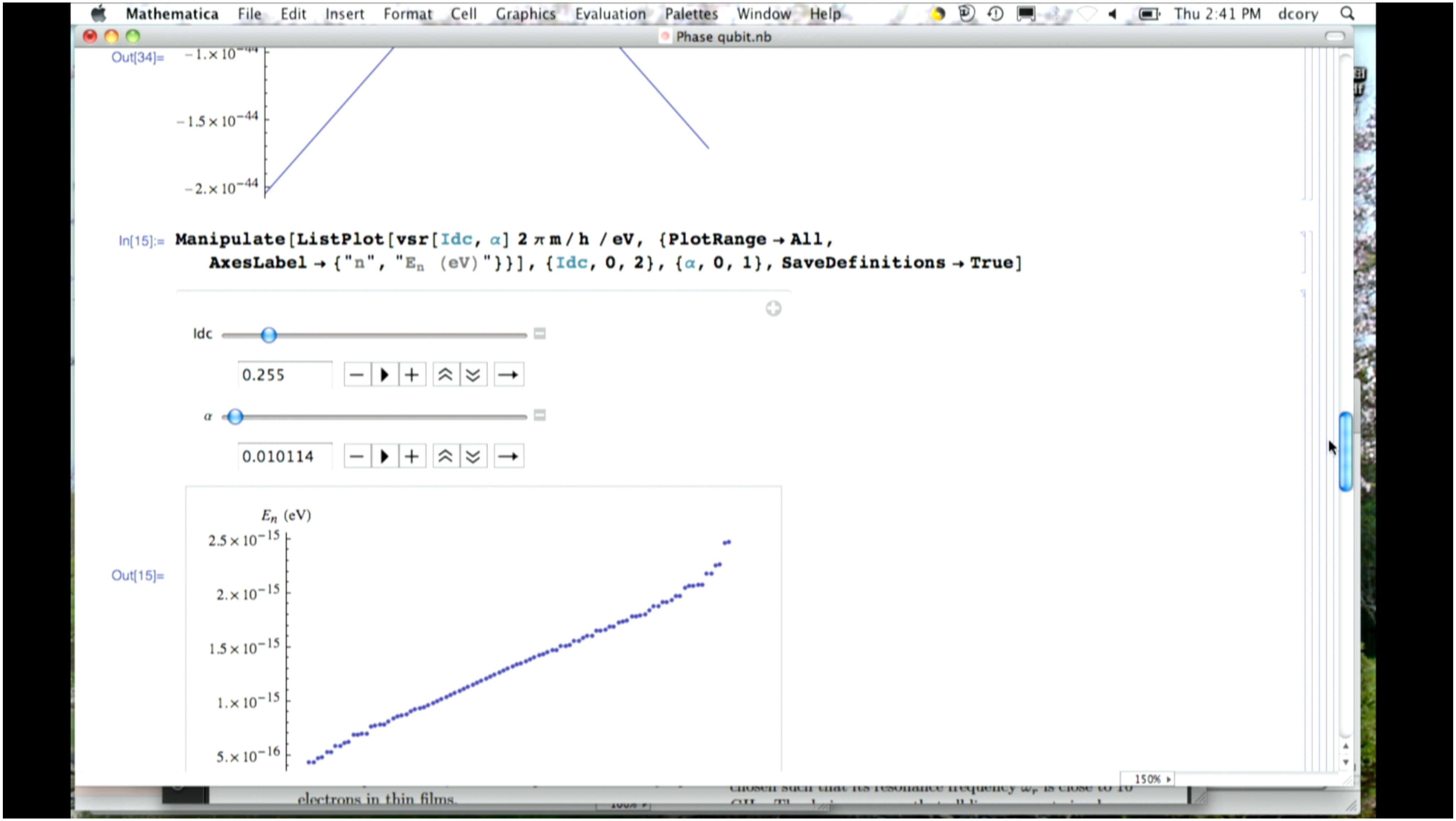


Bloch
1 qubit
, controlable
I_{dc}
$|0\rangle \equiv \psi_1^{tilt}$
$|1\rangle \equiv \psi_2^{tilt}$
write a
ψ in one
well
x
$\psi_1^{no\ tilt}$
$\psi_1^{no\ tilt} = \frac{\psi_1^n + \psi_2^n}{\sqrt{2}}$
, demand $E_1^n = E_2^n$
$\langle \psi_1^{tilt} | \psi_1^n \rangle = 1$
$\langle \psi_2^{tilt} | \psi_2^n \rangle = 1$


Bloch
1 qubit
, controlable
|0> ≡
|1> ≡
write a
ψ in one
well
demand


Bloch
1 qubit
, controlable
I_{dc}
$|0\rangle \equiv \psi_1^{tilt}$
$|1\rangle \equiv \psi_2^{tilt}$
write a ψ in one well
$\langle \psi_1^{tilt} | \psi_1^{n} \rangle = 1$
$\langle \psi_2^{tilt} | \psi_2^{n} \rangle = 1$
, demand $E_1^n = E_2^n$


Bloch
1 qubit
, controlable
I_dc
operator along z
operator along x
write a ψ in one well
|0> ≡ ψ_1^tilt
|1> ≡ ψ_2^tilt
ψ_1^no tilt
ψ_1^no tilt = (ψ_1^n + ψ_2^n)/√2
, demand E_1^n = E_2^n
<ψ_1^tilt | ψ_1^n> = 1
<ψ_2^tilt | ψ_2^n> = 1


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 2:41 PM dcory
Phase qubit.nb
Out[34]=
-1.5 × 10^-44
-2. × 10^-44
In[15]:= Manipulate[ListPlot[vsr[Idc, α] 2 π m / h / eV, {PlotRange → All,
AxesLabel → {"n", "En (eV)"}}], {Idc, 0, 2}, {α, 0, 1}, SaveDefinitions → True]
Idc
0.255
α
0.010114
En (eV)
2.5 × 10^-15
2. × 10^-15
1.5 × 10^-15
1. × 10^-15
5. × 10^-16
Out[15]=
150%

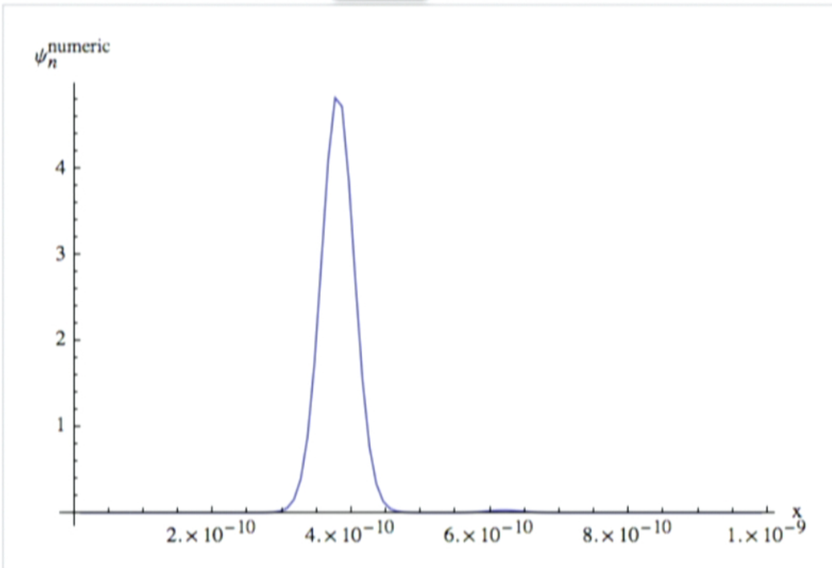
```
{AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
SaveDefinitions → True]
```

n

1

Step Forward

Out[36]=



- **non degenerate**

In[37]:=
```
vsr[0.255, 0.010114][[1]] - vsr[0.255, 0.010114][[2]]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 2:42 PM dcory

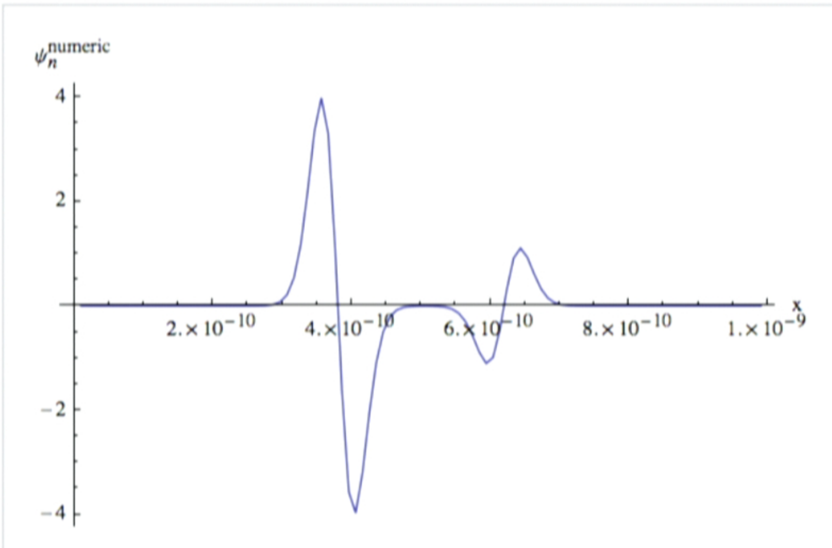
Phase qubit.nb

```
{AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
SaveDefinitions → True]
```

n

3

Out[36]=



- **non degenerate**

In[37]:=
```
vsr[0.255, 0.010114][[1]] - vsr[0.255, 0.010114][[2]]
```

150%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 2:43 PM dcory

Phase qubit.nb

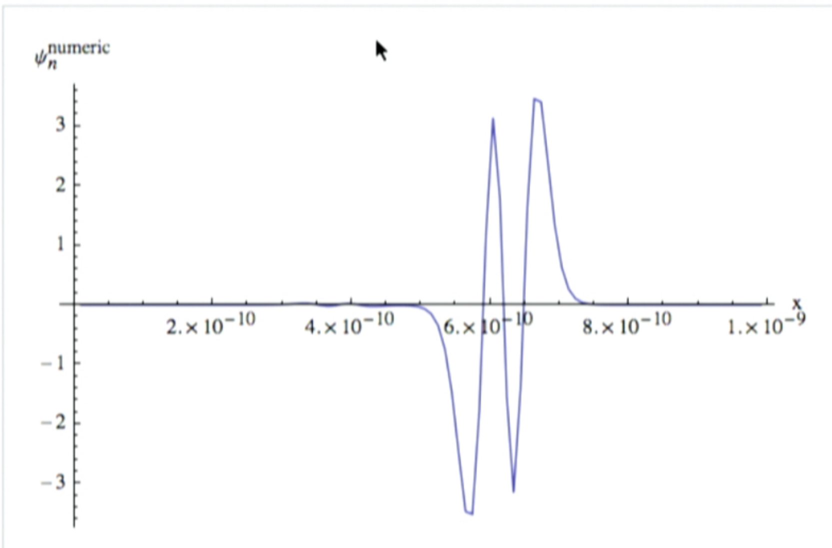
```
In[38]:= ψrndeg = Table[Eigenvectors[fdHr[0.255, 0.010114]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];

In[39]:= Manipulate[ListPlot[ Transpose[{x, ψrndeg[[n]]}],
   {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
  SaveDefinitions → True]
```

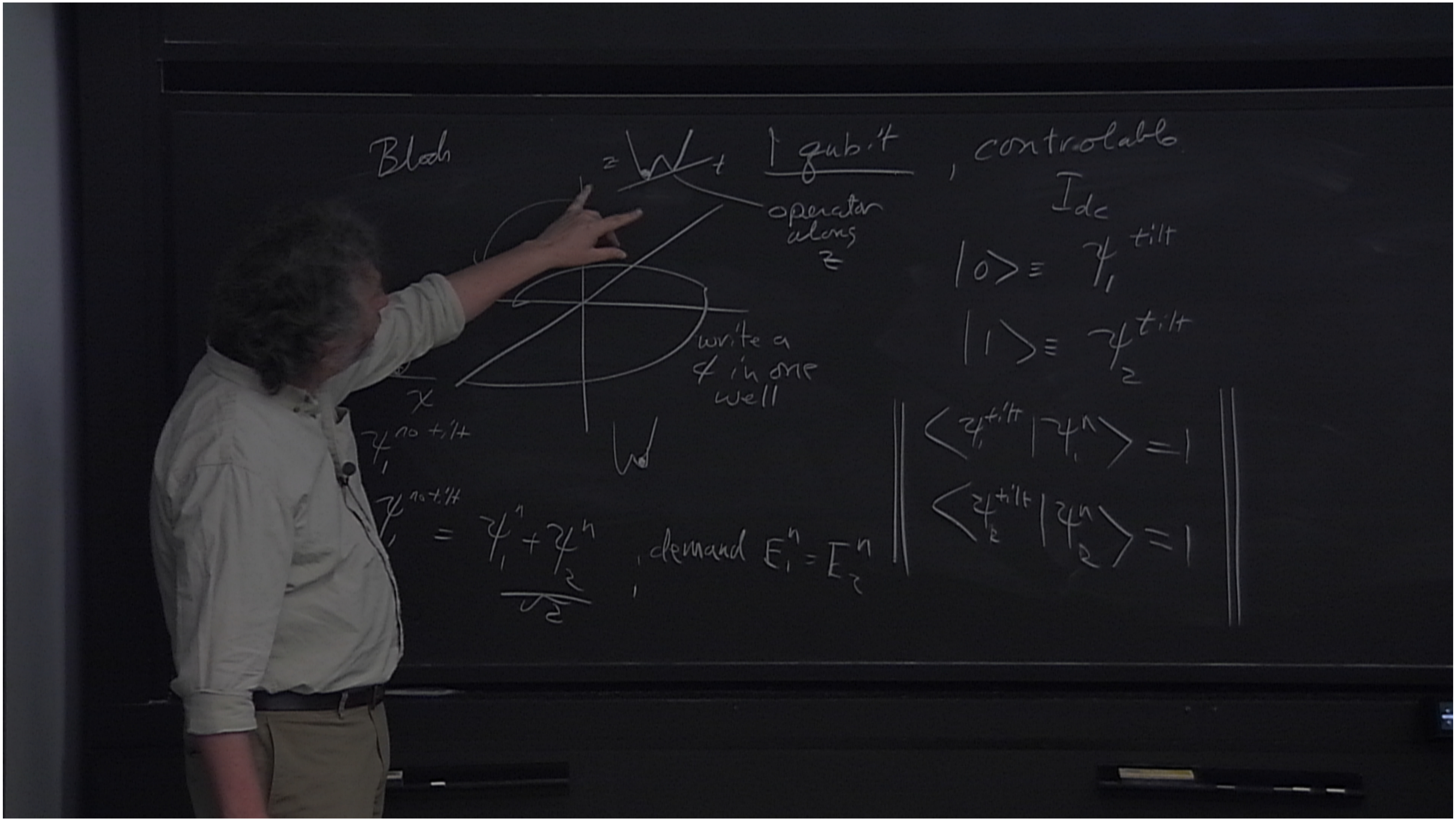
n

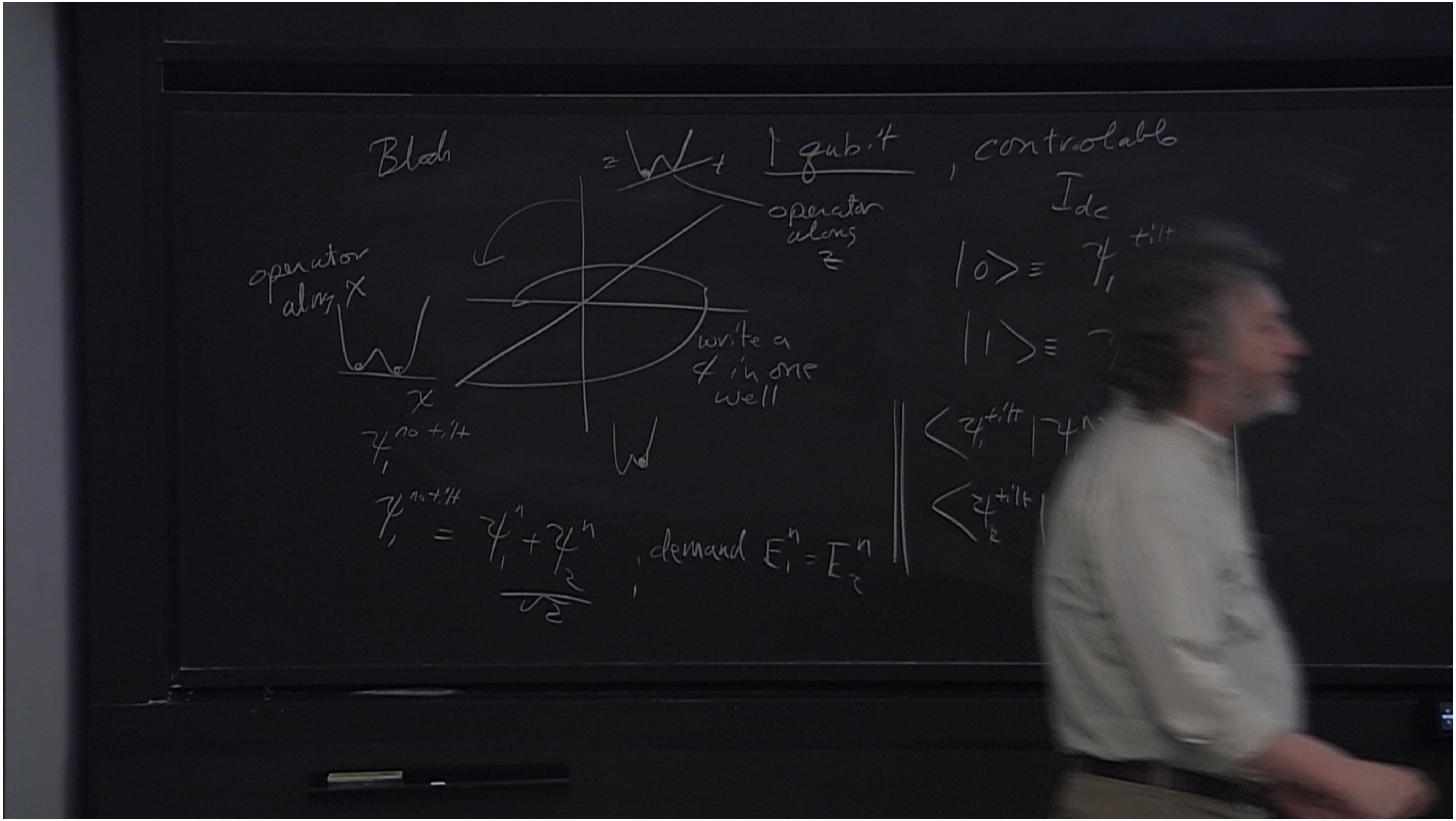
9

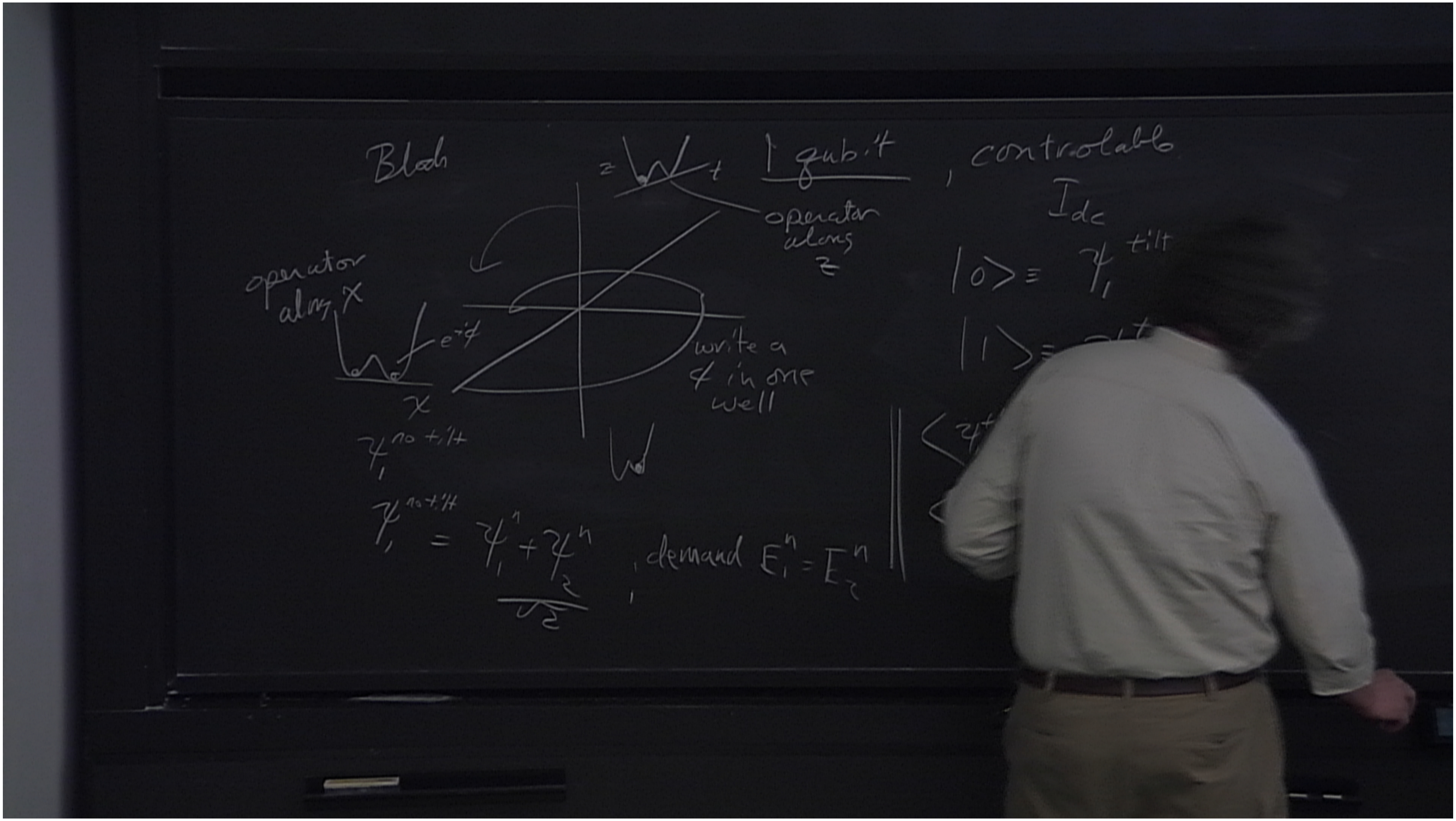
Out[39]=

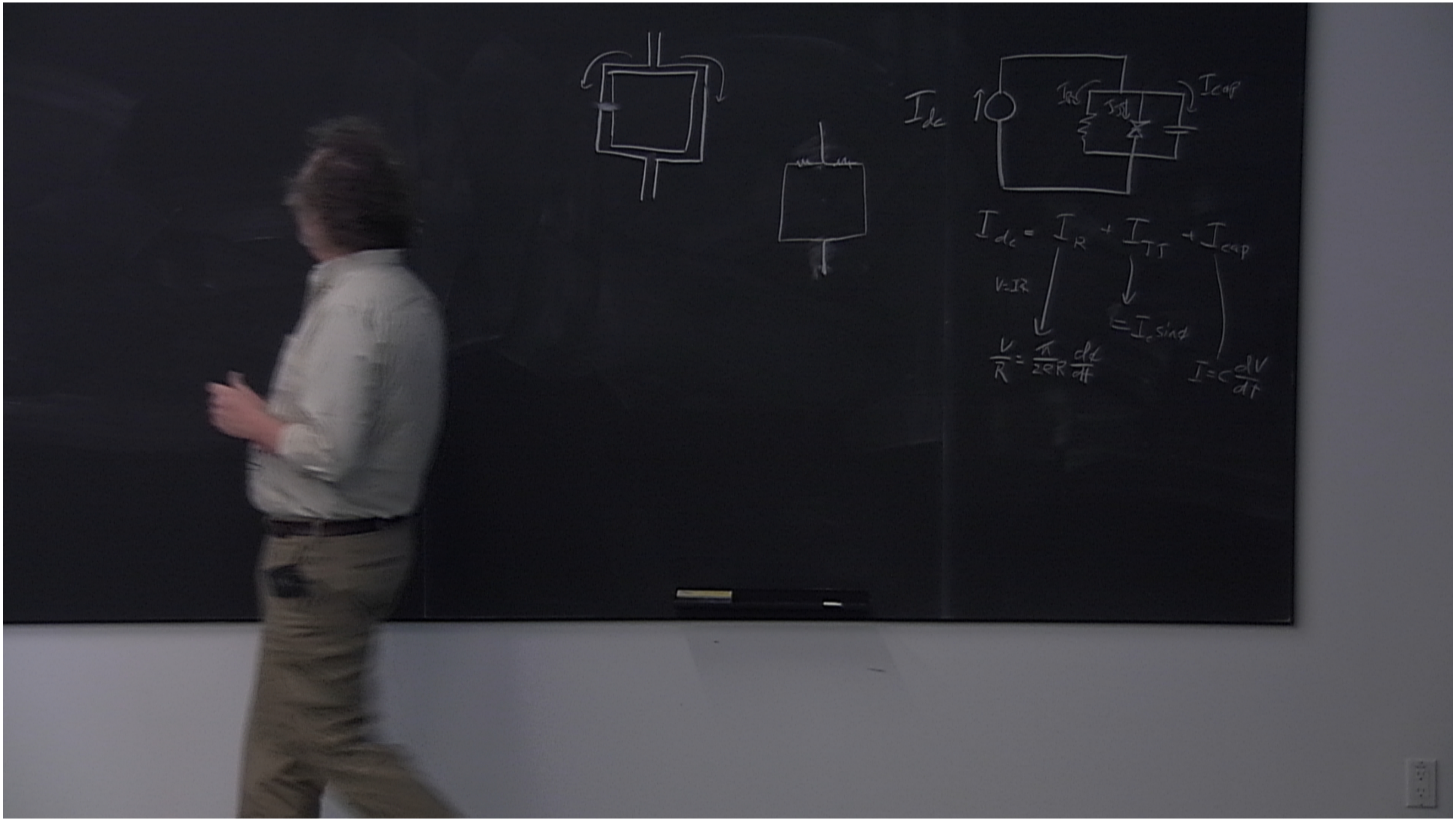


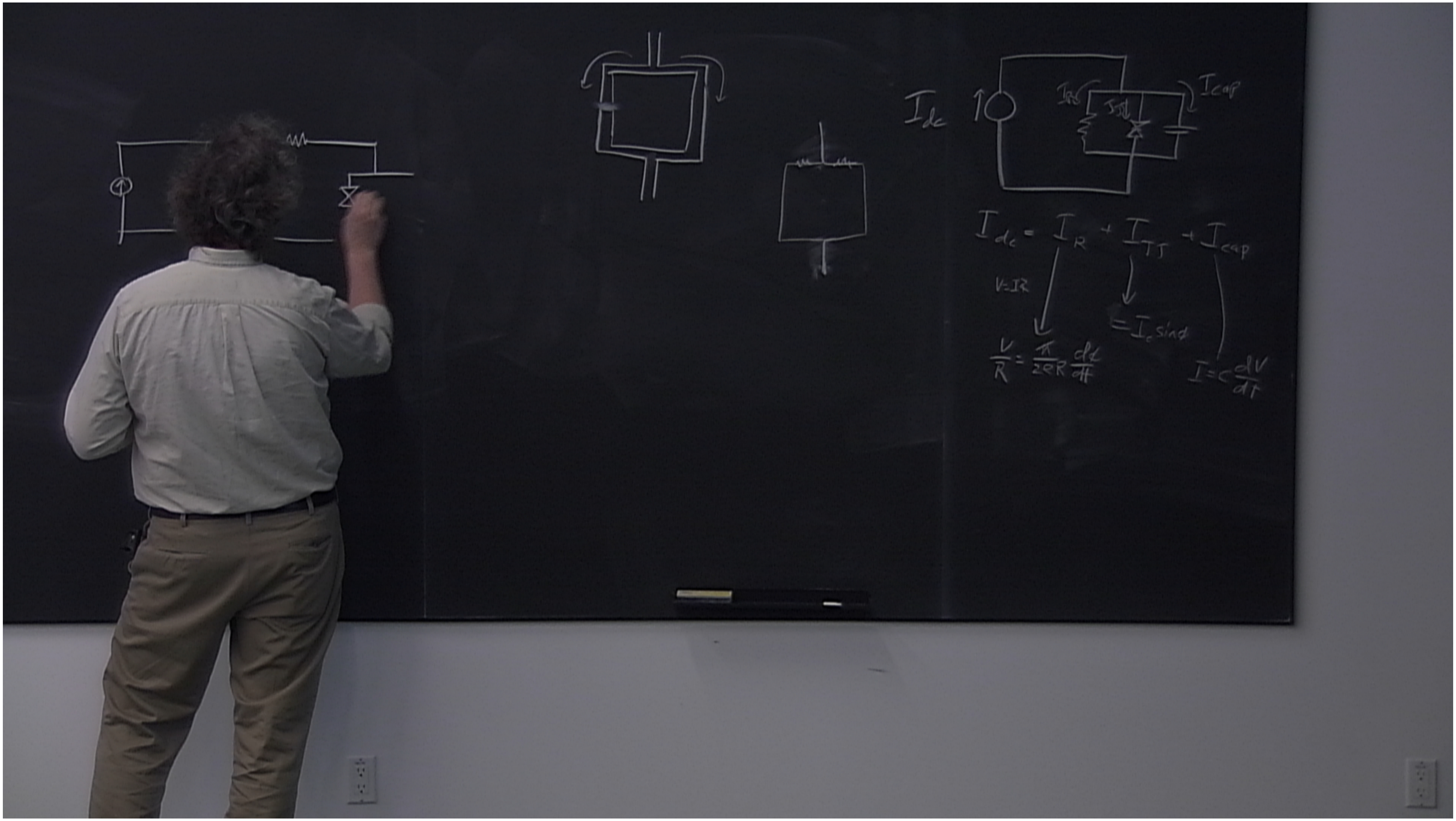
■ **overlap**


Bloch
1 qubit
, controlable
operator along z
I_{dc}
$|0\rangle \equiv \psi_1^{tilt}$
$|1\rangle \equiv \psi_2^{tilt}$
write a ψ in one well
$\psi_1^{no\ tilt}$
$\psi_1^{no\ tilt} = \frac{\psi_1^n + \psi_2^n}{\sqrt{2}}$
, demand $E_1^n = E_2^n$
$\langle \psi_1^{tilt} | \psi_1^n \rangle = 1$
$\langle \psi_2^{tilt} | \psi_2^n \rangle = 1$

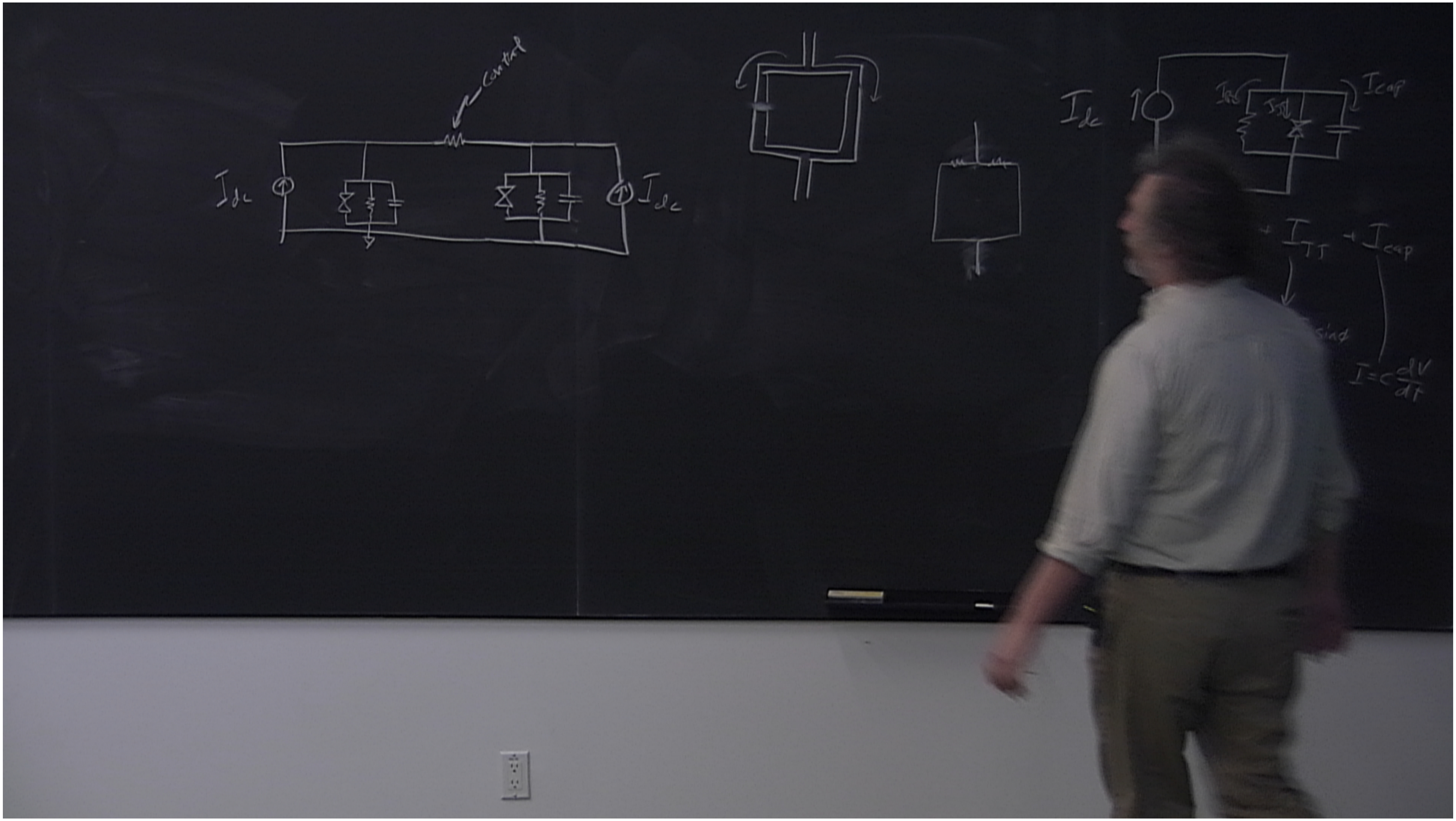

Bloch
1 qubit
controlable
operator along z
operator along x
write a ψ in one well
demand

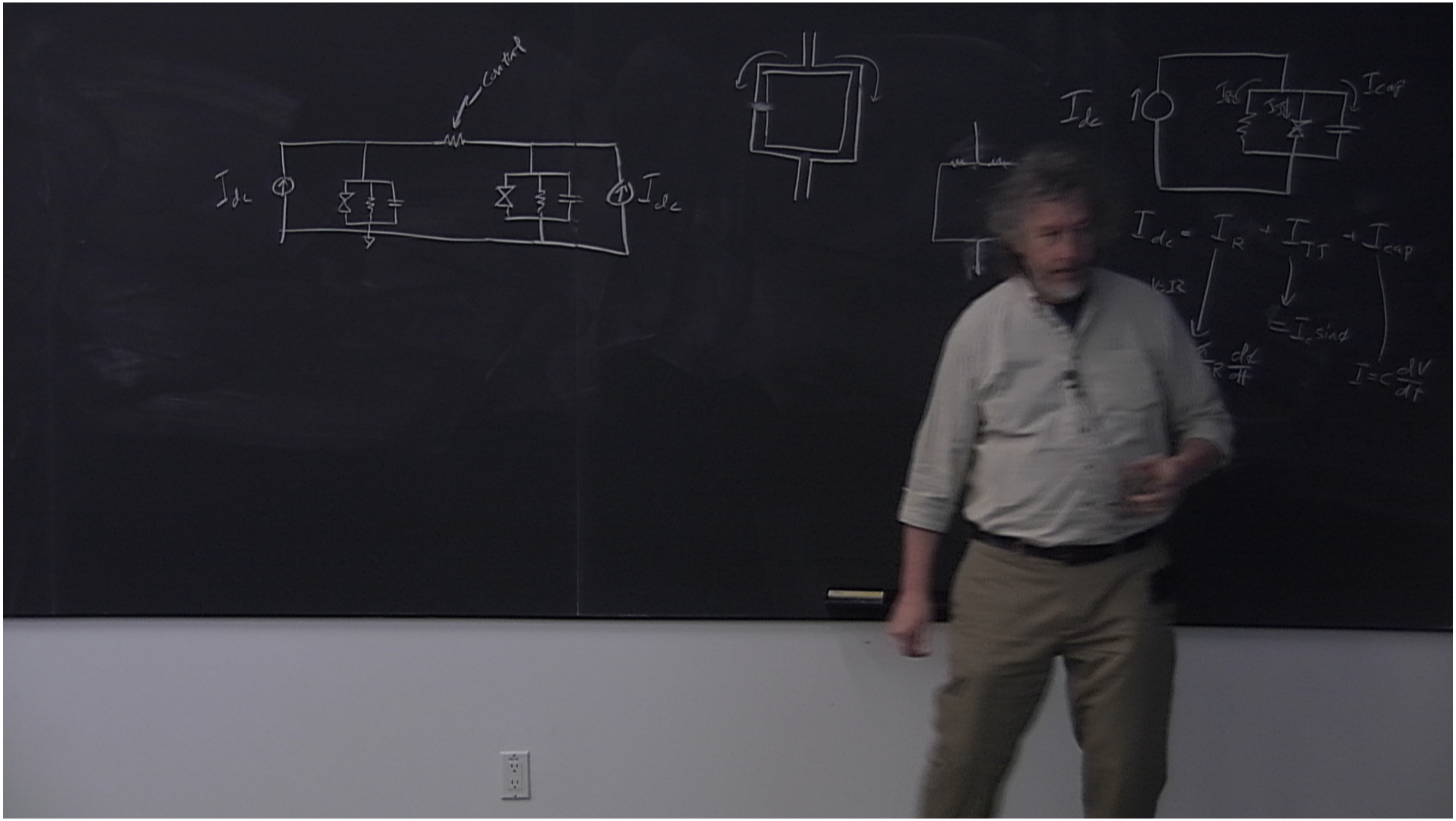

Bloch
1 qubit
, controlable
operator along z
operator along x
write a ψ in one well
demand


I_dc
I_cap
I_dc = I_R + I_JJ + I_cap
V=IR
= I_c sinφ
I = C dV/dt

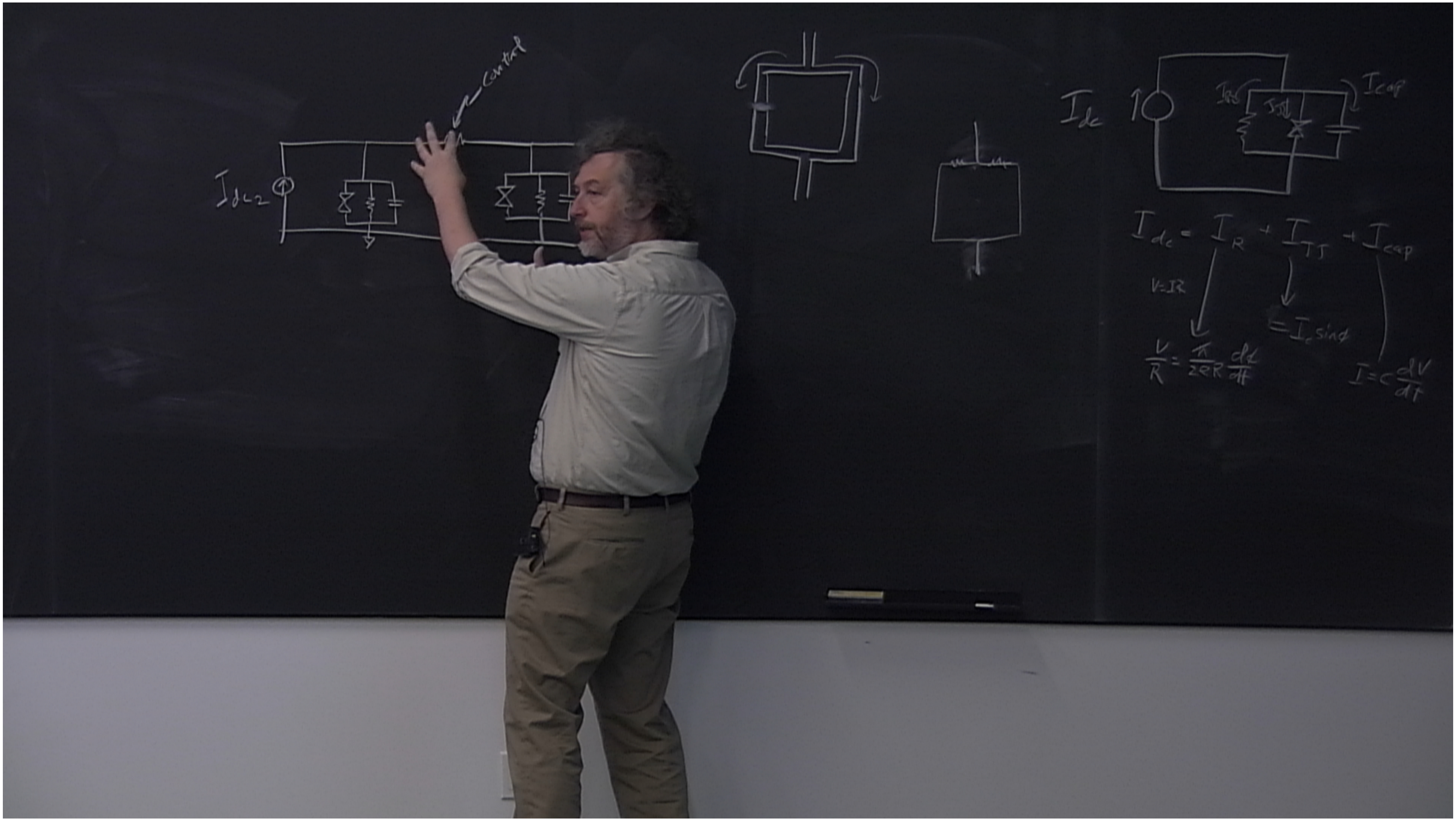


Idc
Icap
Idc = IR + ITJ + Icap
V=IR
= Ic sinφ
V/R = π/2eR dφ/dt
I=C dV/dt


control
Idc
Idc
Idc
Icap
I=C dV/dt



control
I_{dc}
I_{dc}
I_{dc}
I_{cap}
$I_{dc} = I_R + I_{JJ} + I_{cap}$
$= I_c \sin\alpha$
$I = C \frac{dV}{dt}$


control
Idc2
Idc
Icap
Idc = IR + IJJ + Icap
V=IR
= Ic sinδ
I = C dV/dt

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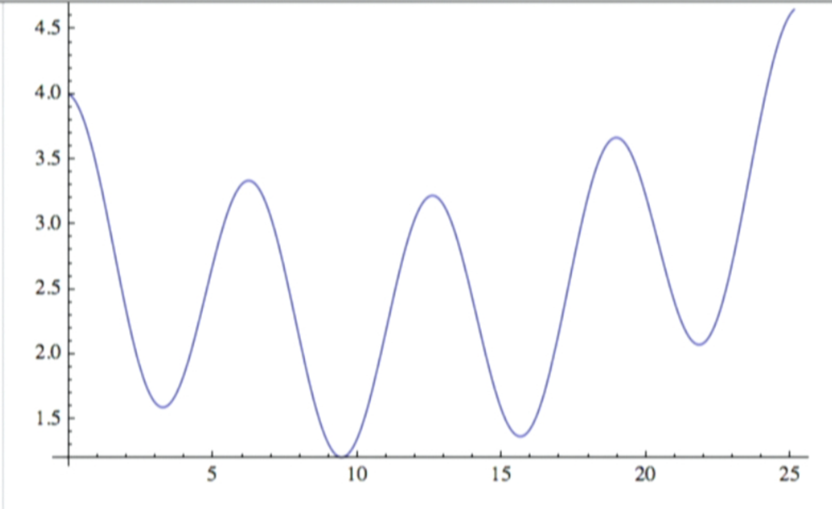
Phase qubit.nb

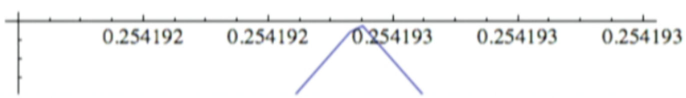
Out[6]=



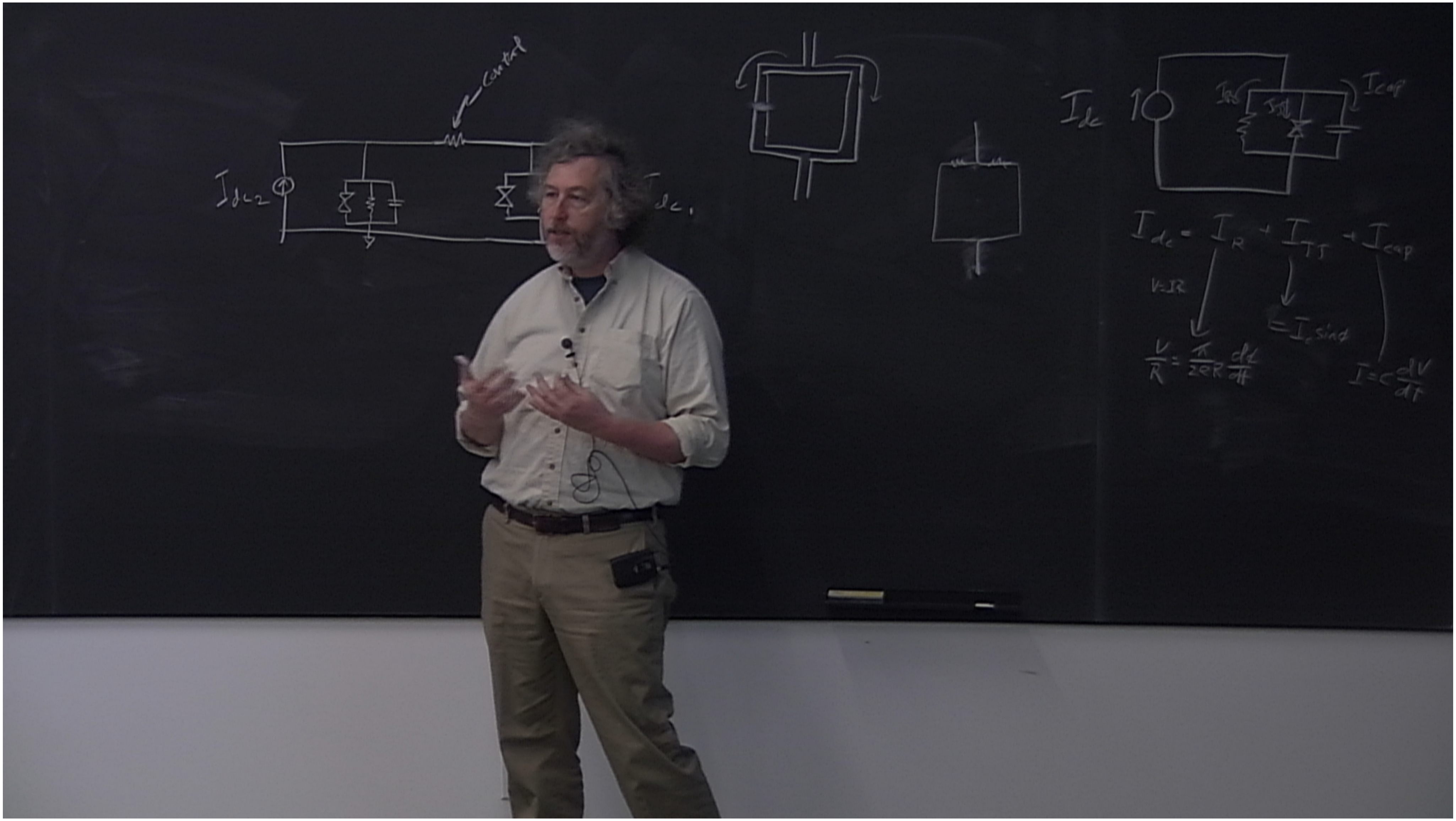
- **numerical estimation of the eigenstructure**

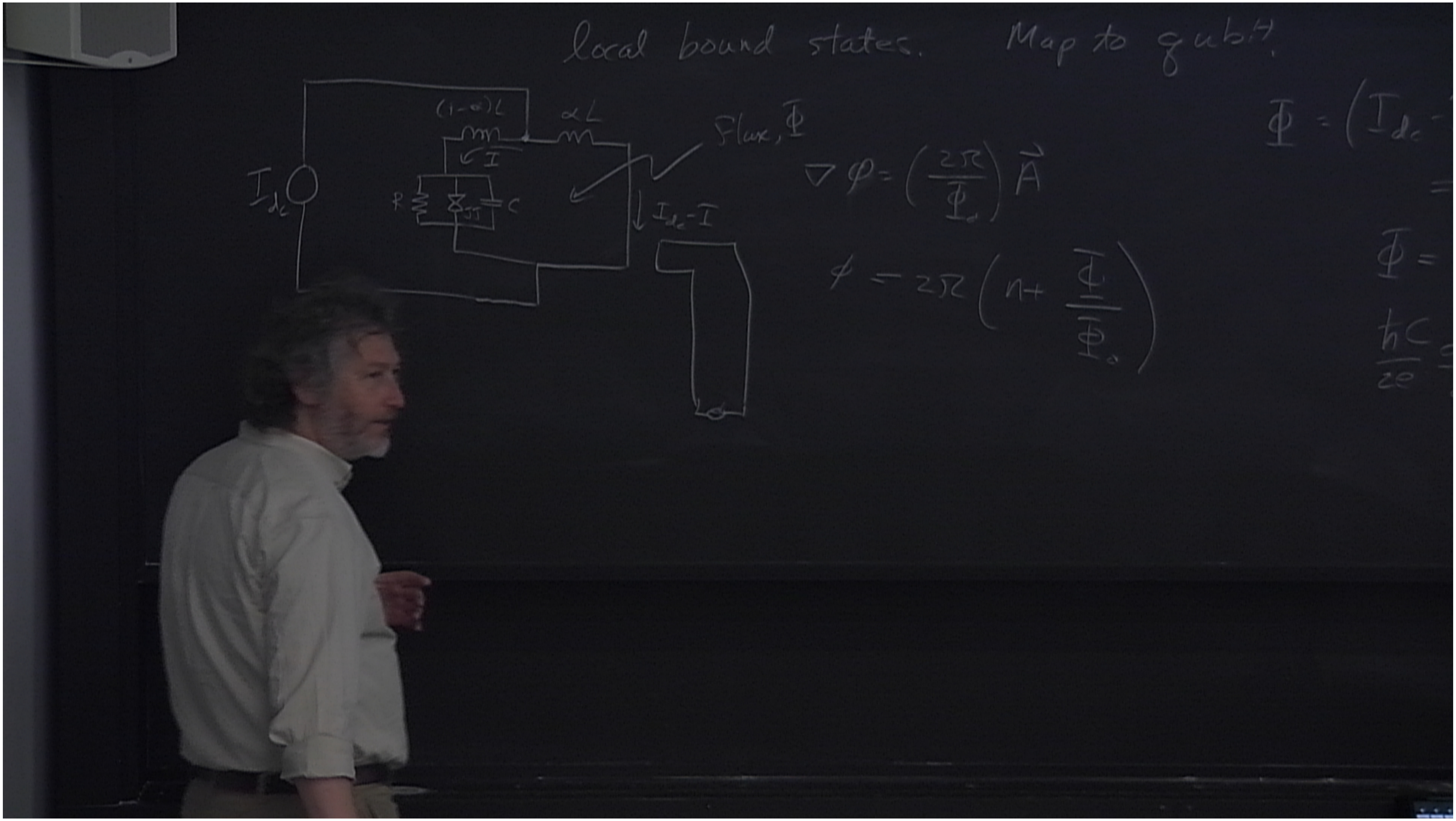
- **define finite difference Hamiltonian**

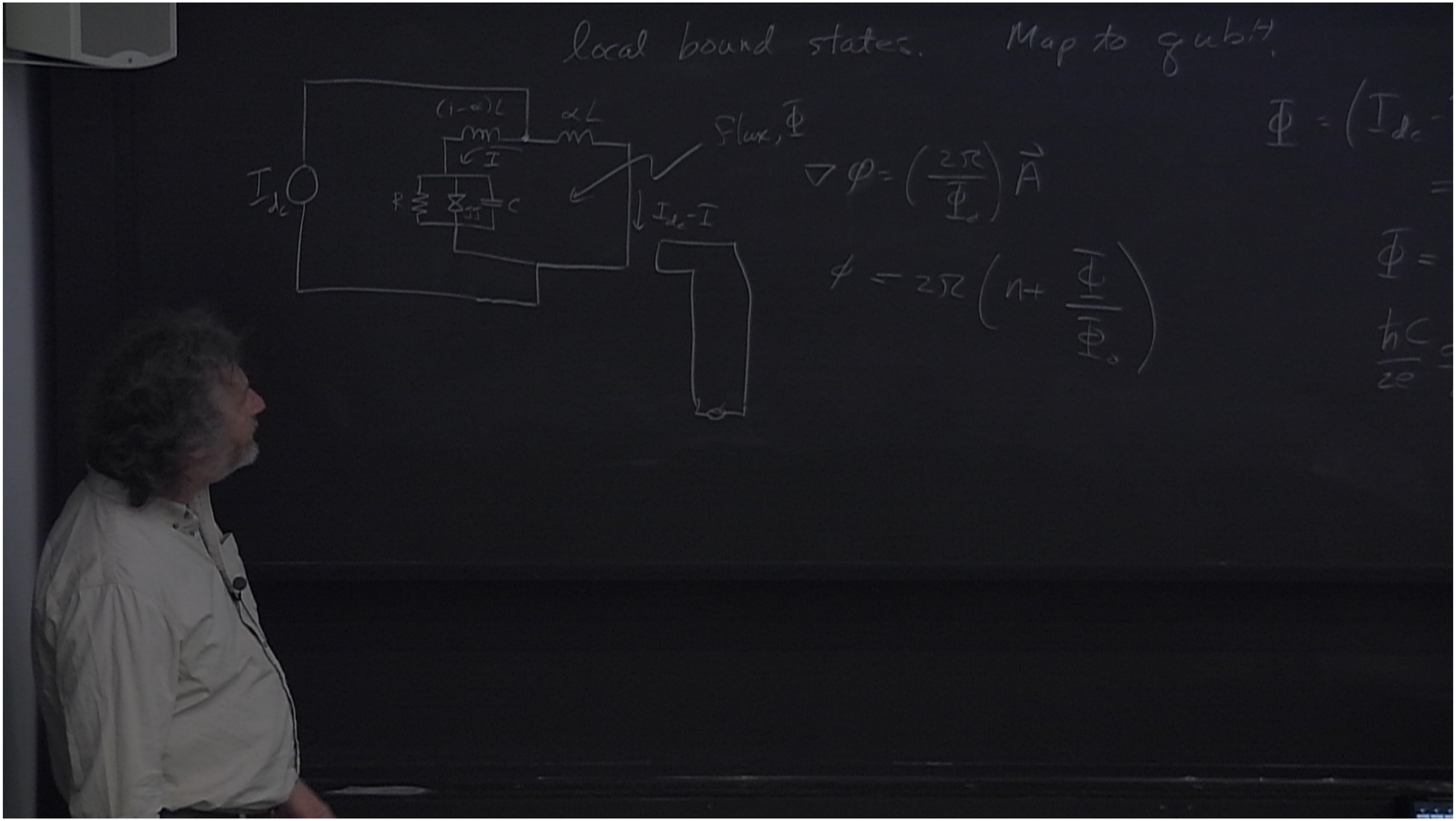
```
In[13]:= fdHr[Idc_, α_] :=
   t0 (DiagonalMatrix[Table[2 + Ur[(n - 1) (8 π / 99), 1, Idc, α], {n, 1, 100}]] +
      DiagonalMatrix[Table[-1, {99}], 1] + DiagonalMatrix[Table[-1, {99}], -1]);

In[14]:= vsr[Idc_, α_] := Reverse[Eigenvalues[fdHr[Idc, α]]];

In[34]:= Plot[vsr[Idc, 0.010114][[1]] - vsr[Idc, 0.010114][[2]], {Idc, 0.254192, 0.254193}]
```

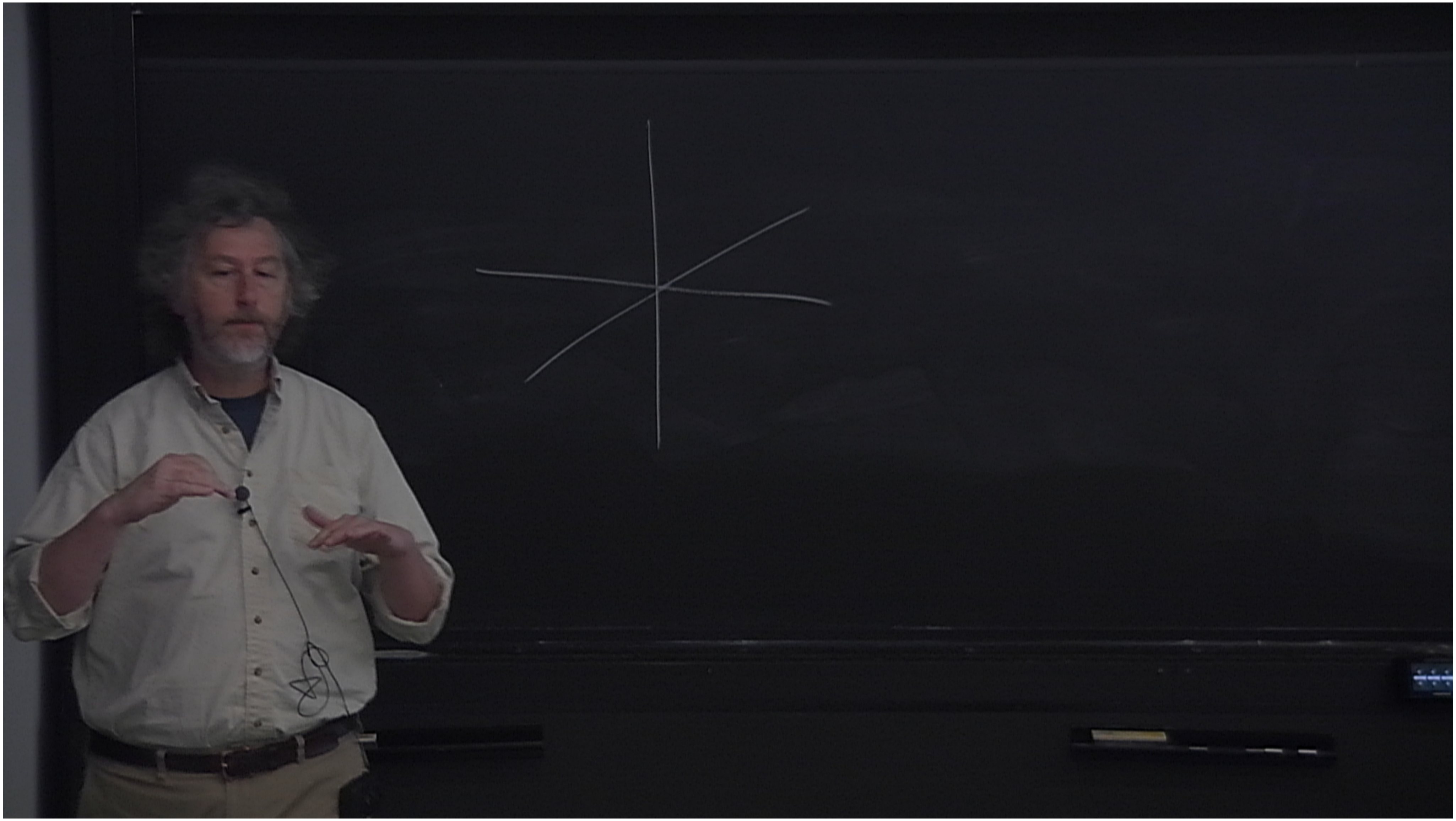


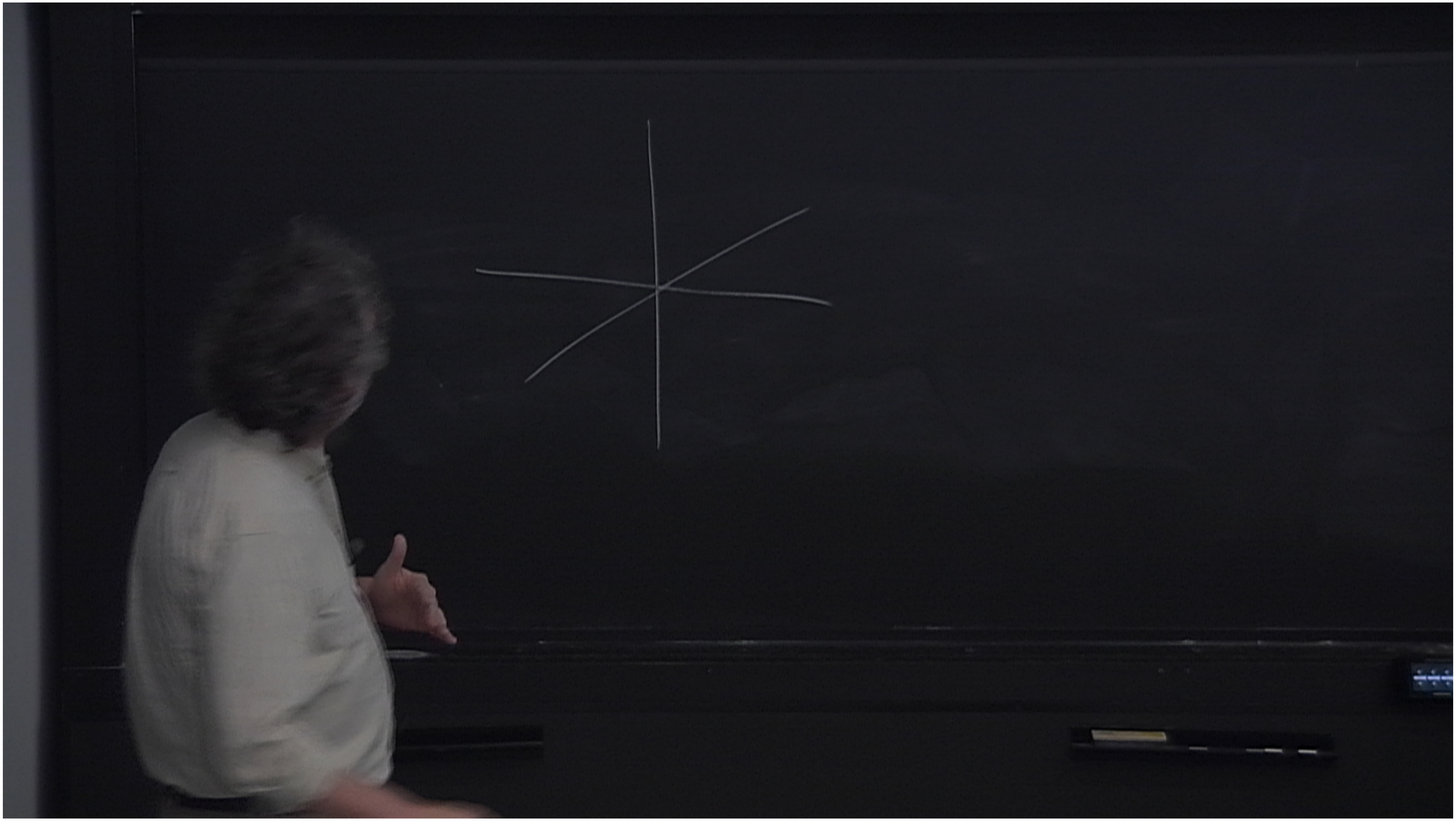
150%

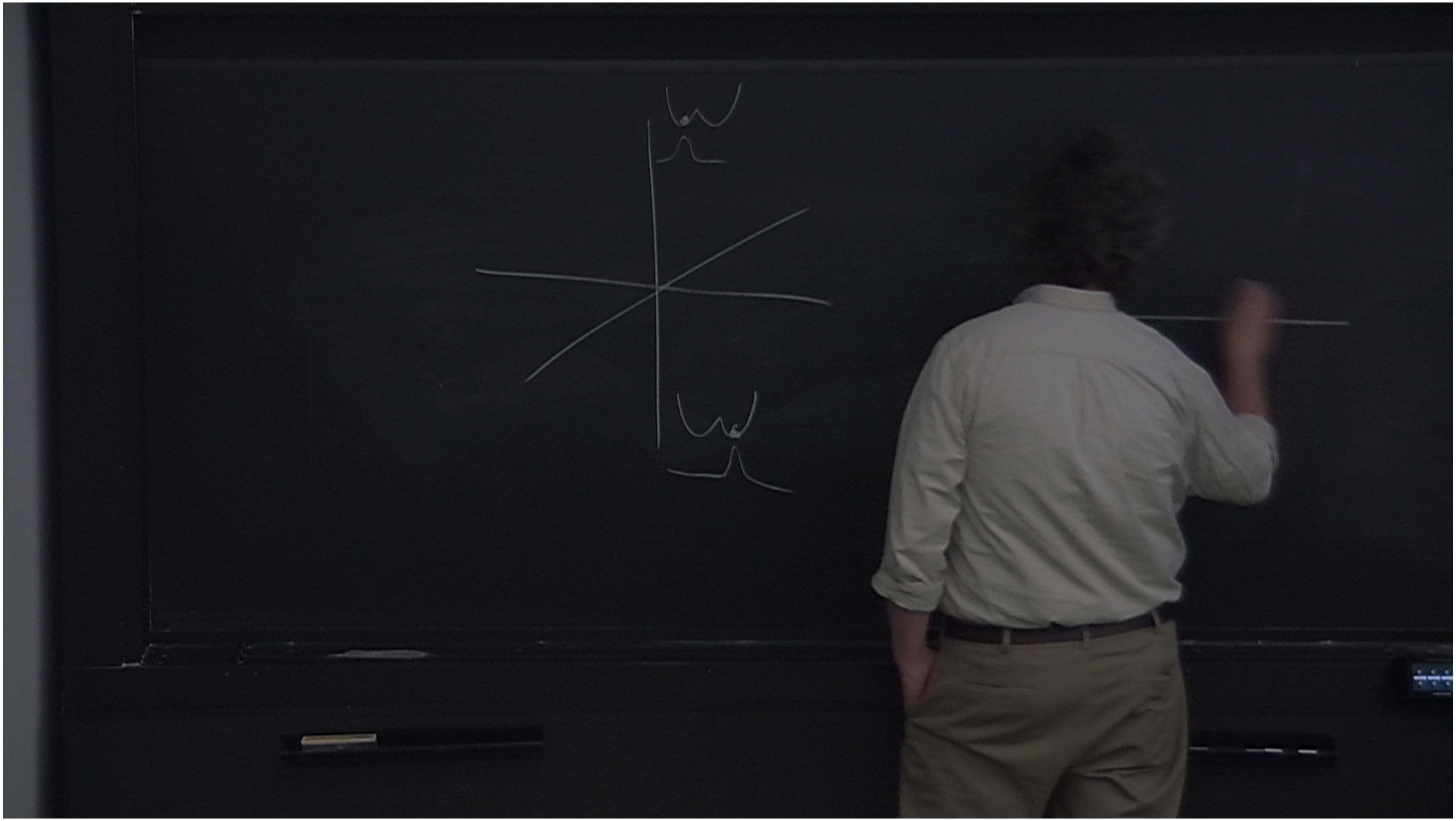

control
Idc
Icap
Idc = IR + IJJ + Icap
V=IR
I=C dV/dt

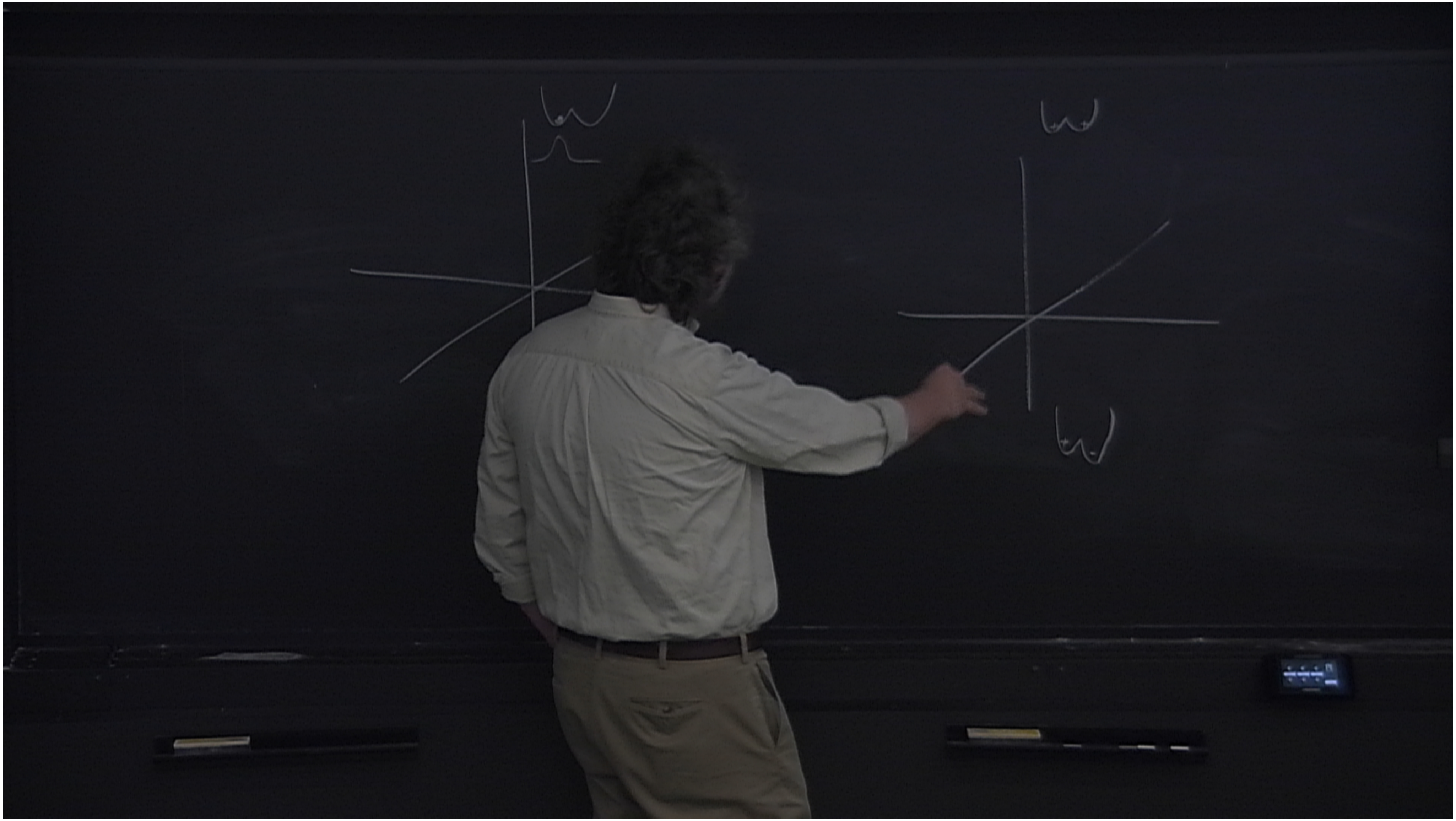

local bound states.
Map to qubit.
(1-α)L
αL
Flux, Φ
I
R
JJ
C
I_dc - I

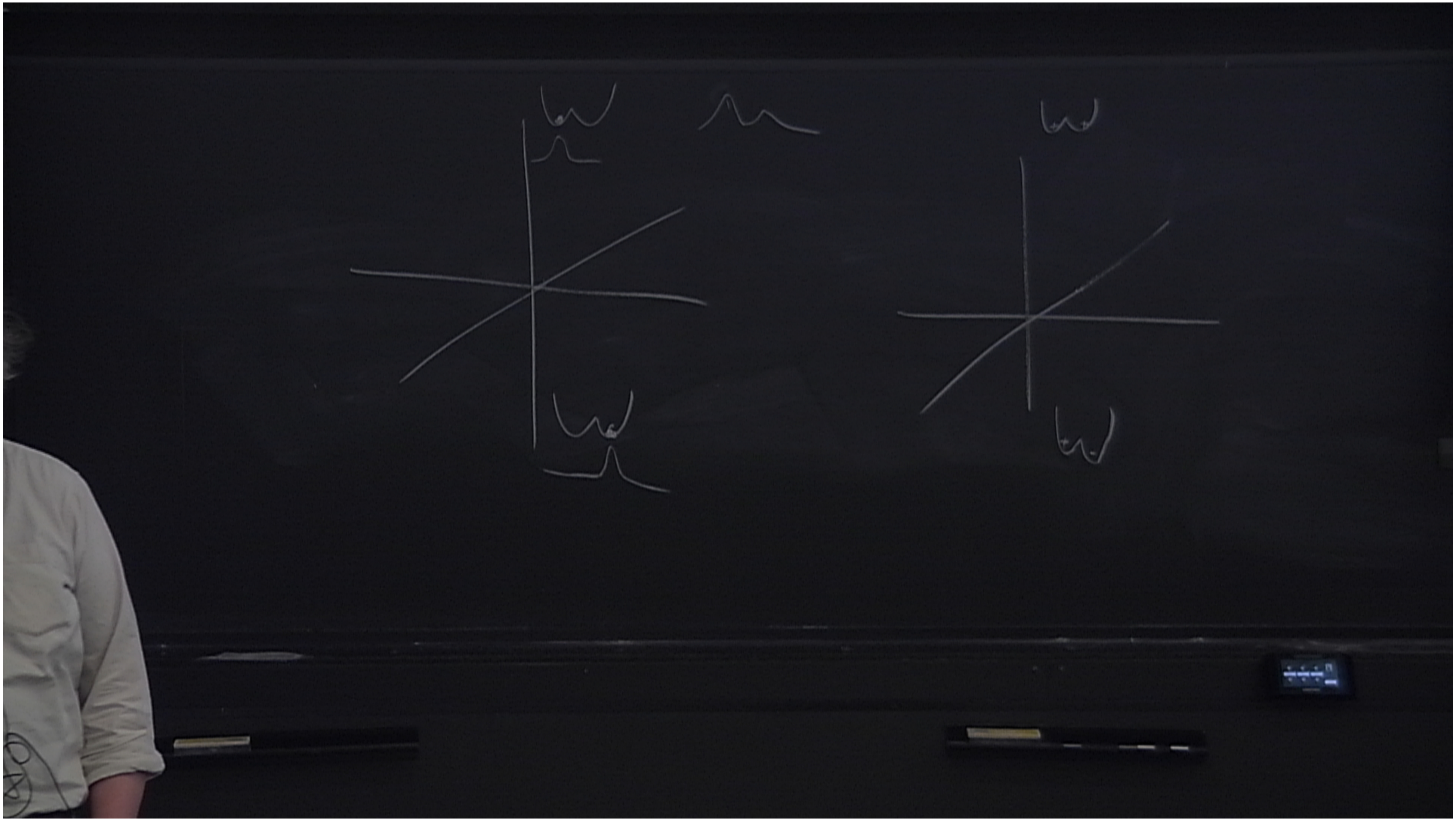

local bound states.
Map to qubit.
(1-α)L
αL
flux, Φ
I
I_dc
R
JJ
C
I_dc - I
∇φ = (2π/Φ₀) A
φ = 2π(n + Φ/Φ₀)

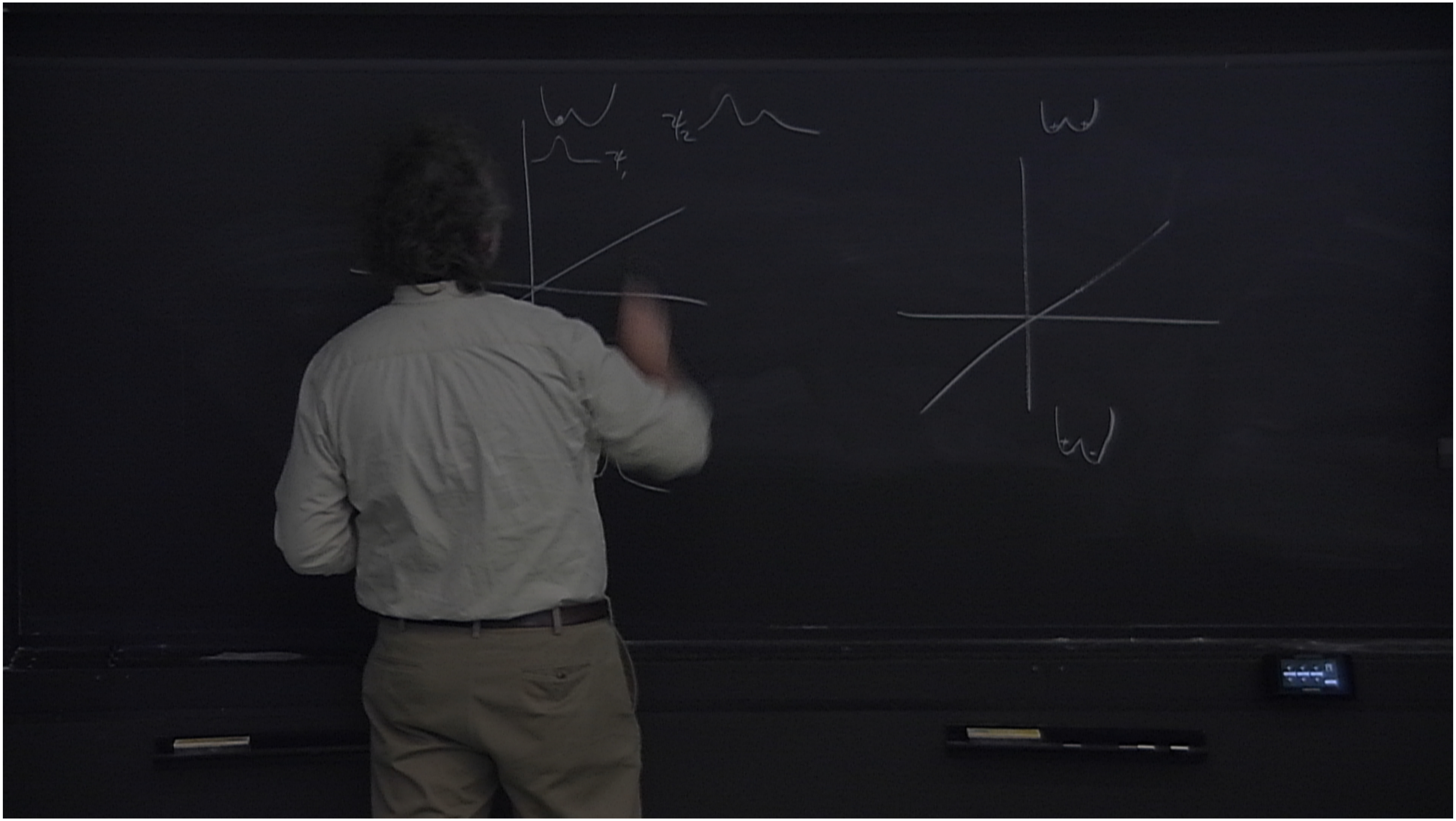


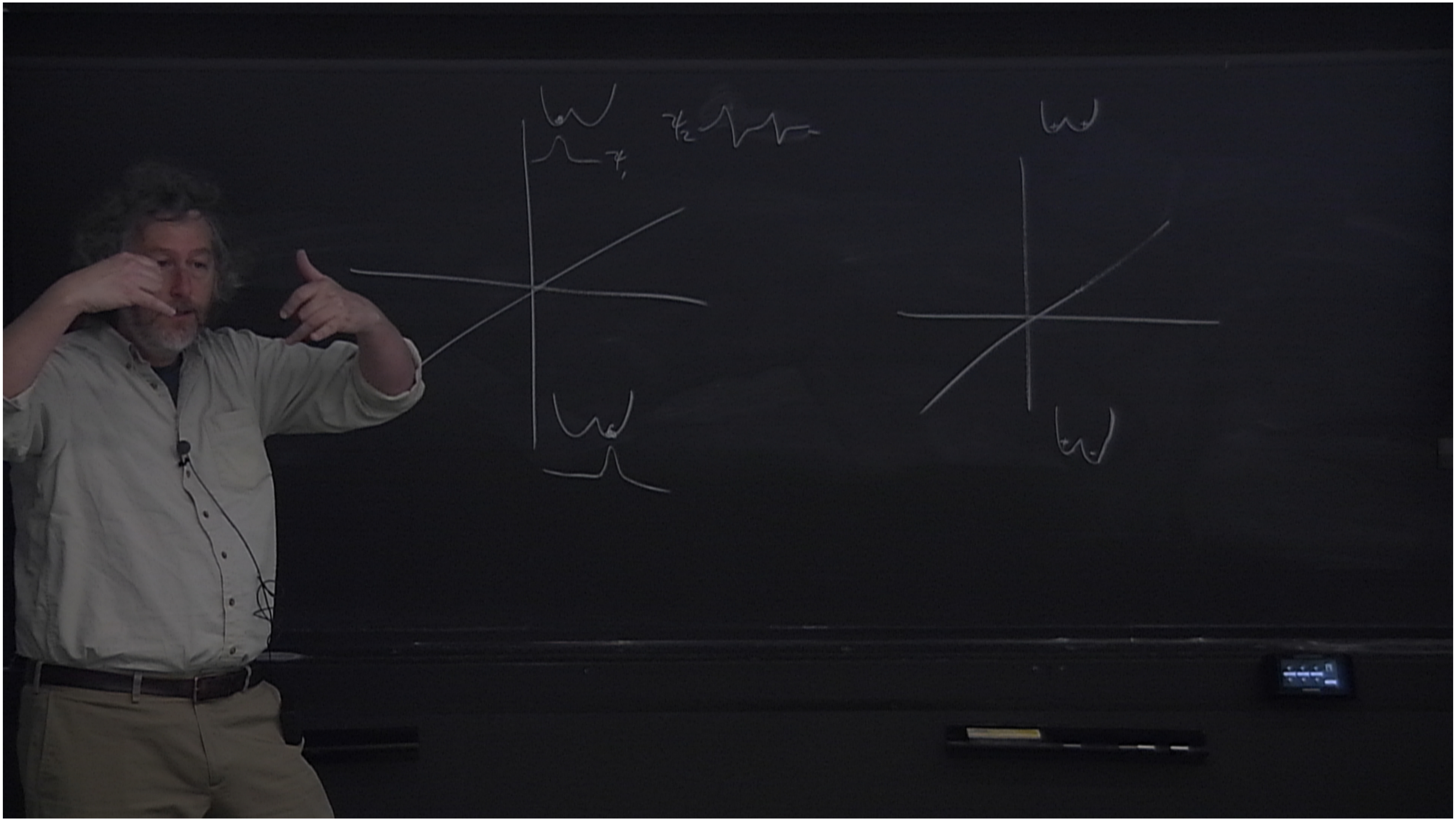


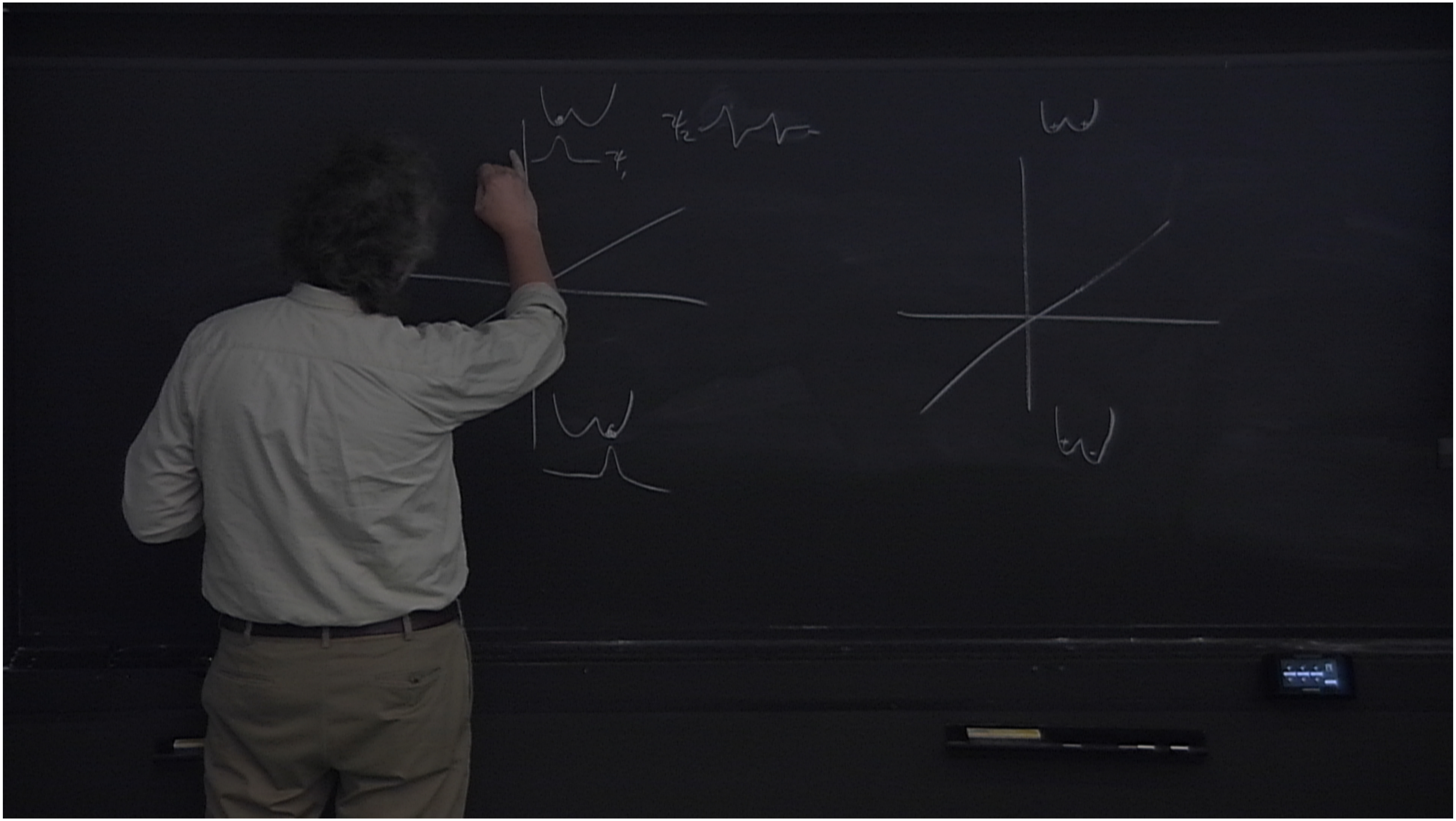












Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 2:55 PM dcory

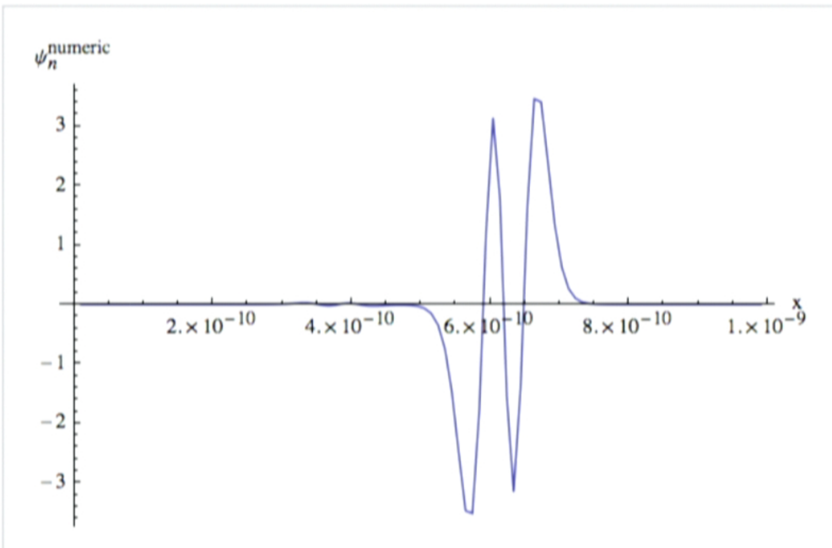
Phase qubit.nb

In[39]:=
```
Manipulate[ListPlot[ Transpose[{x, ψrndeg[[n]]}],
  {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
 SaveDefinitions → True]
```

n

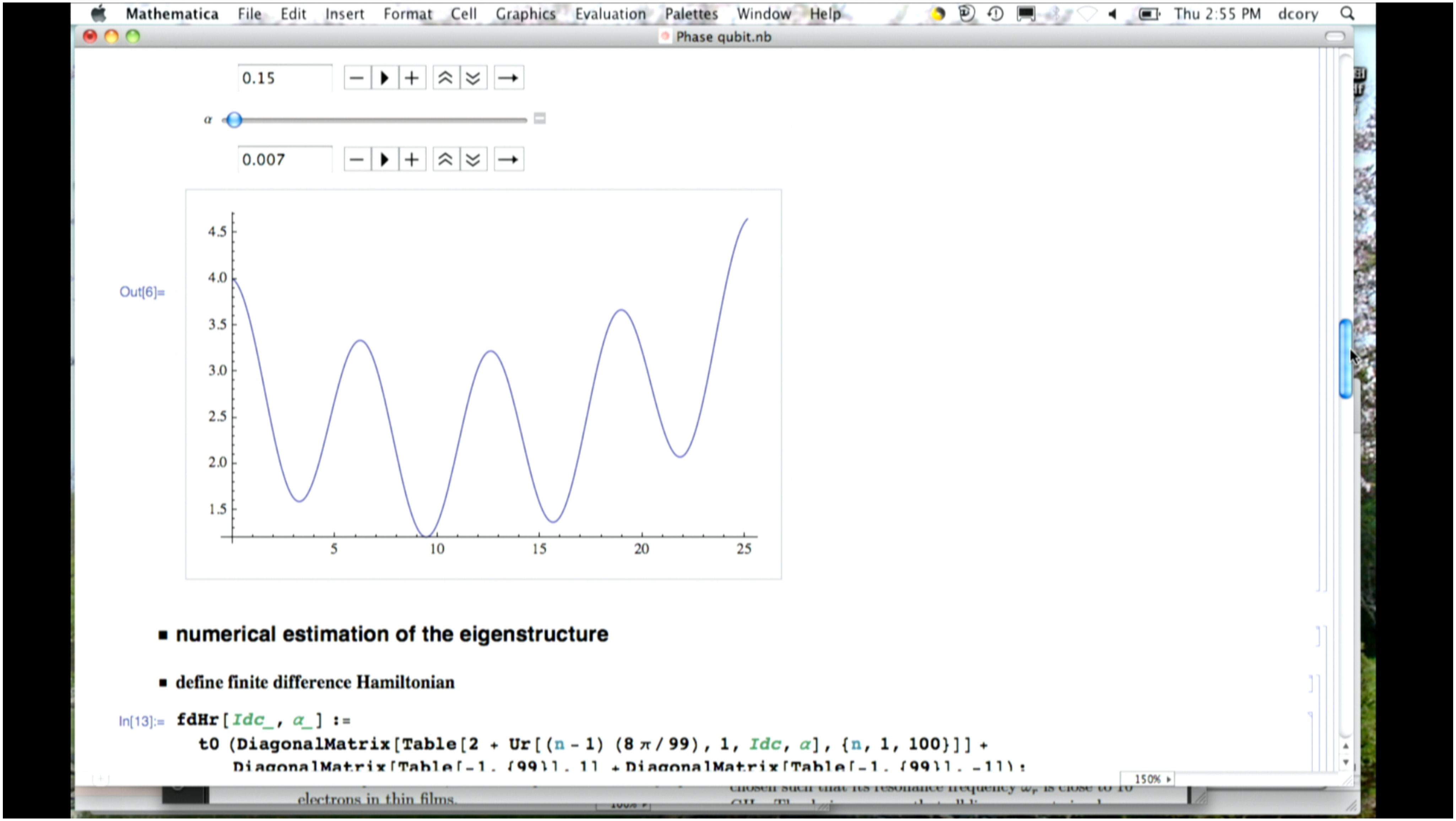
9

Out[39]=



- **overlap**

150%

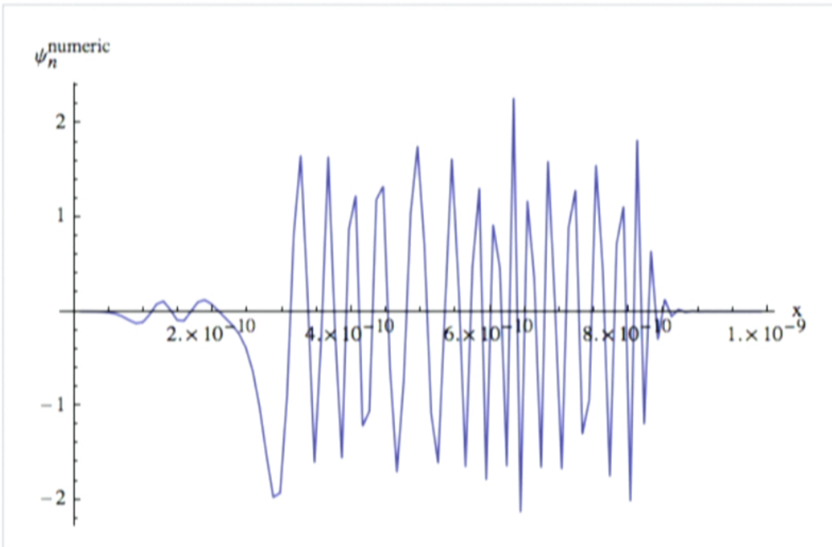

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 2:55 PM dcory
Phase qubit.nb
0.15
α
0.007
Out[6]=
4.5
4.0
3.5
3.0
2.5
2.0
1.5
5
10
15
20
25
numerical estimation of the eigenstructure
define finite difference Hamiltonian
In[13]:=
fdHr[Idc_, α_] :=
t0 (DiagonalMatrix[Table[2 + Ur[(n - 1) (8 π / 99), 1, Idc, α], {n, 1, 100}]] +
150%

In[11]:=
```
Manipulate[ListPlot[ Transpose[{x, ψ[Idc][[n]]}],
  {AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
 {Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
```

n

Idc 0.2

Out[11]=



Potential for current biased JJ ring

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 2:56 PM dcory

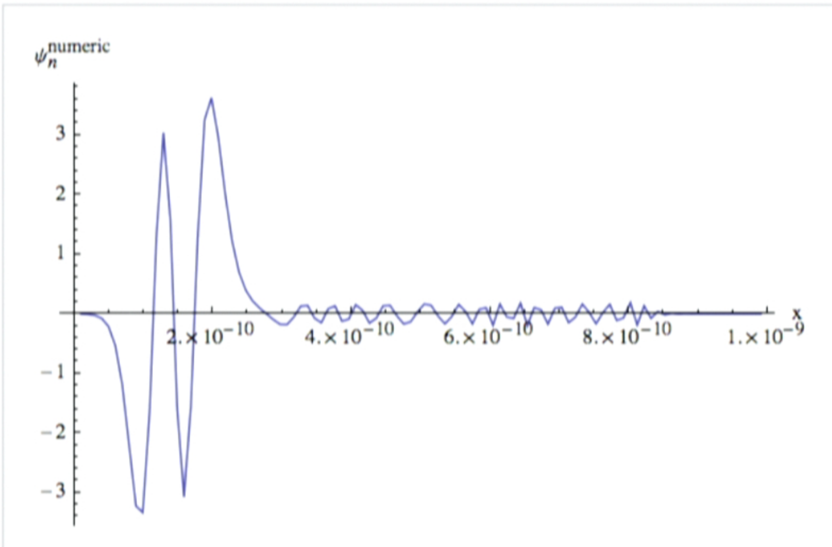
Phase qubit.nb

In[11]:=
```
Manipulate[ListPlot[ Transpose[{x, ψ[Idc][[n]]}],
  {AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
 {Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
```

n

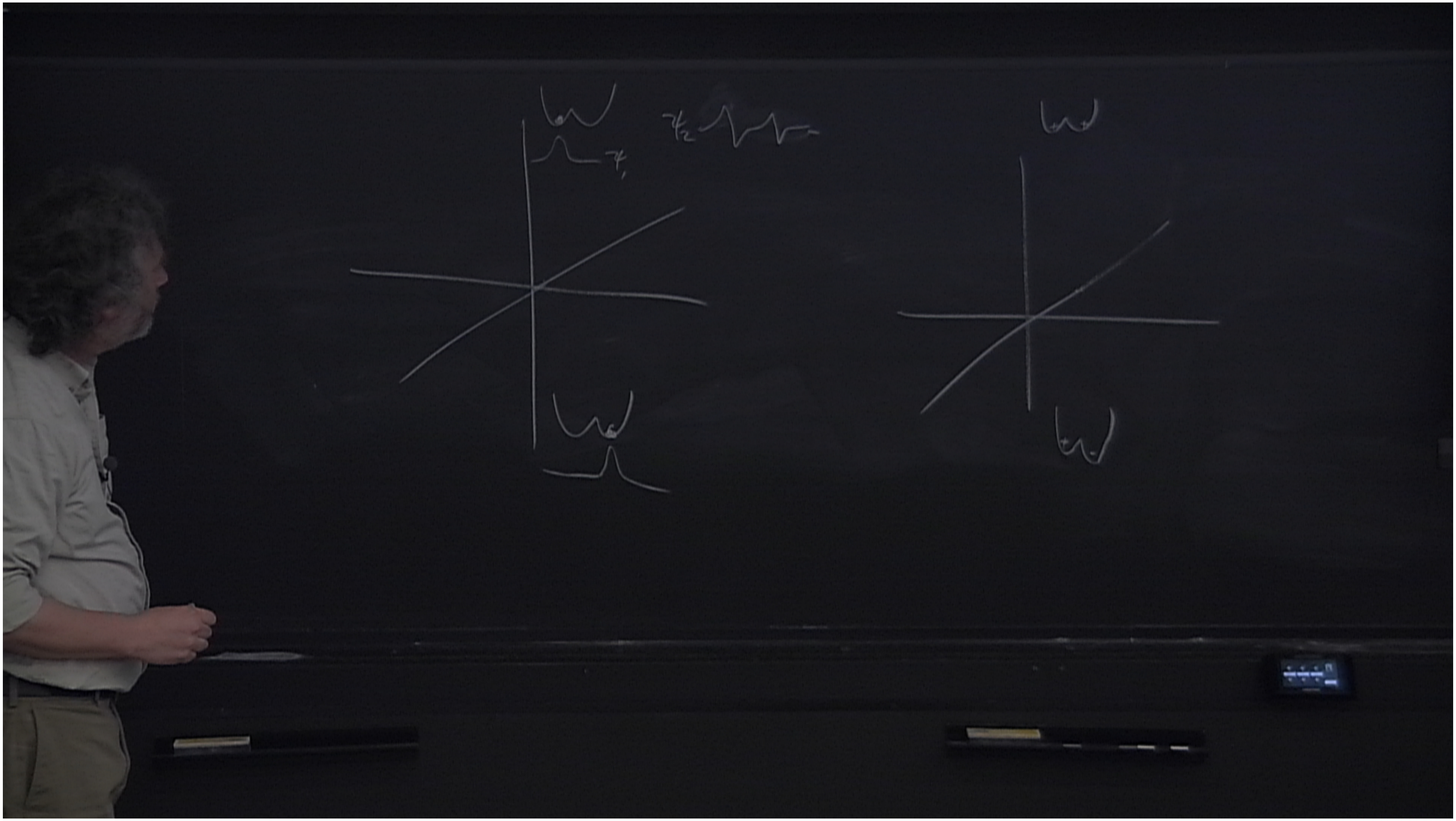
Idc 0.2

Out[11]=

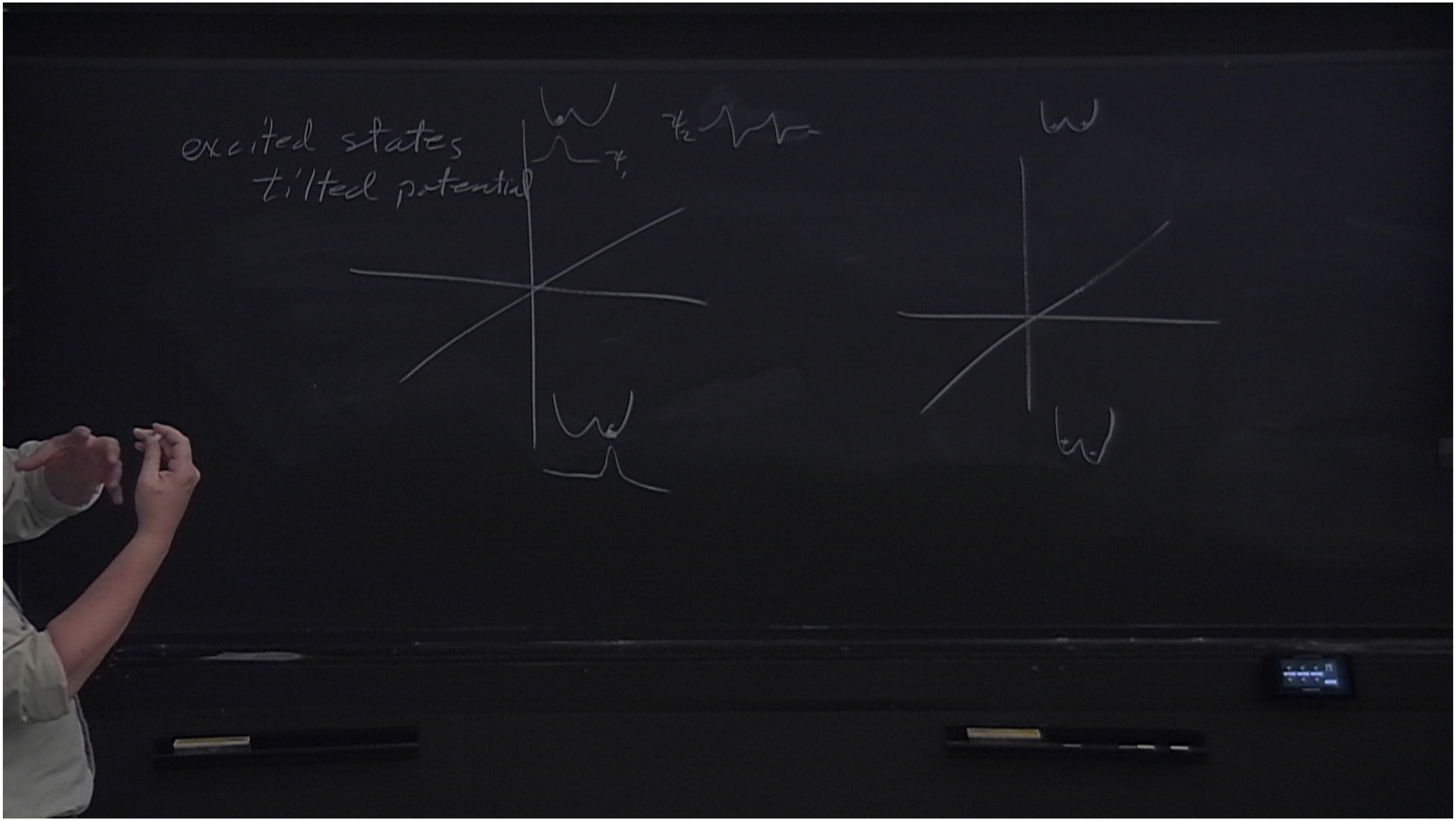


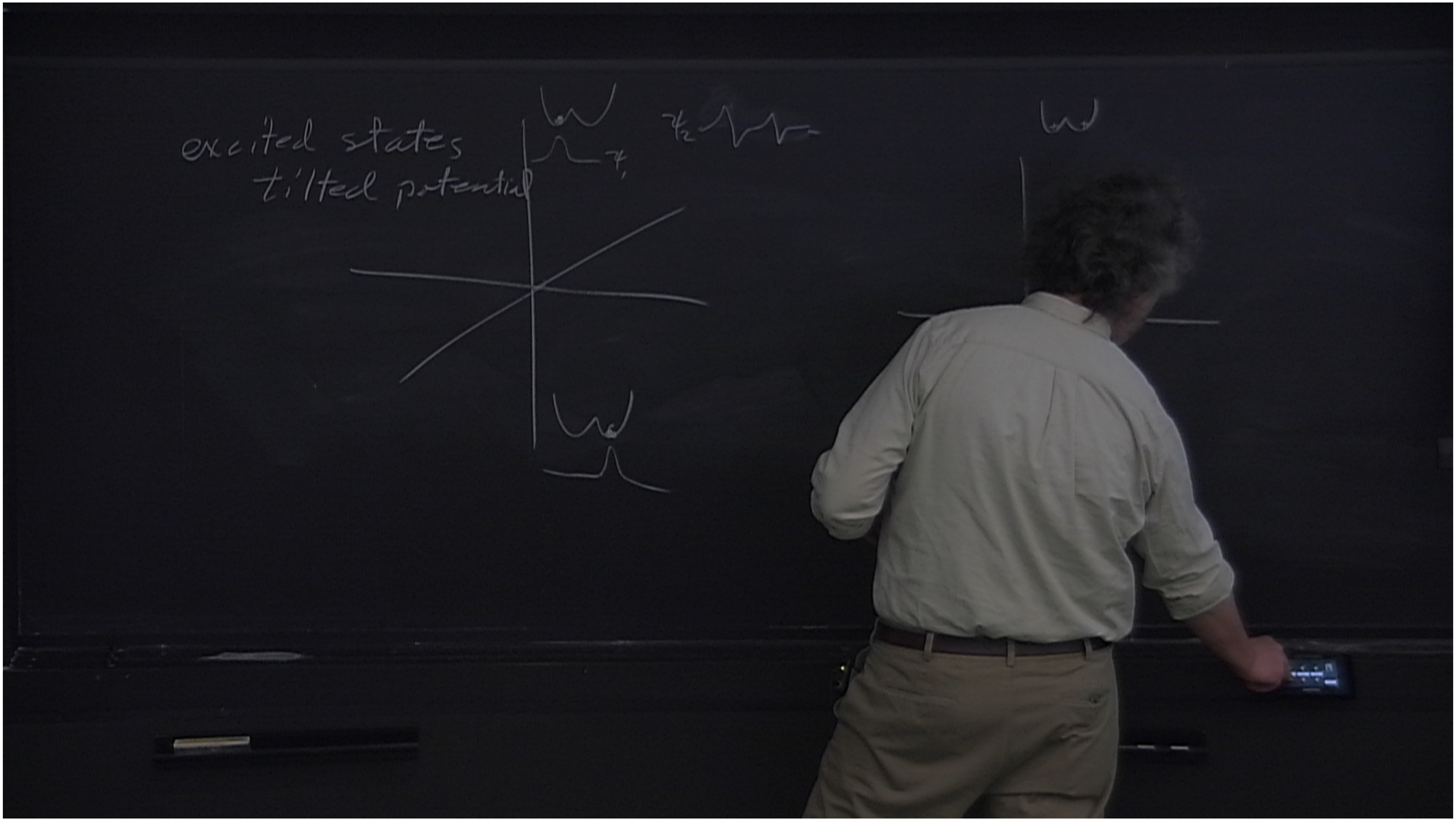
Potential for current biased JJ ring

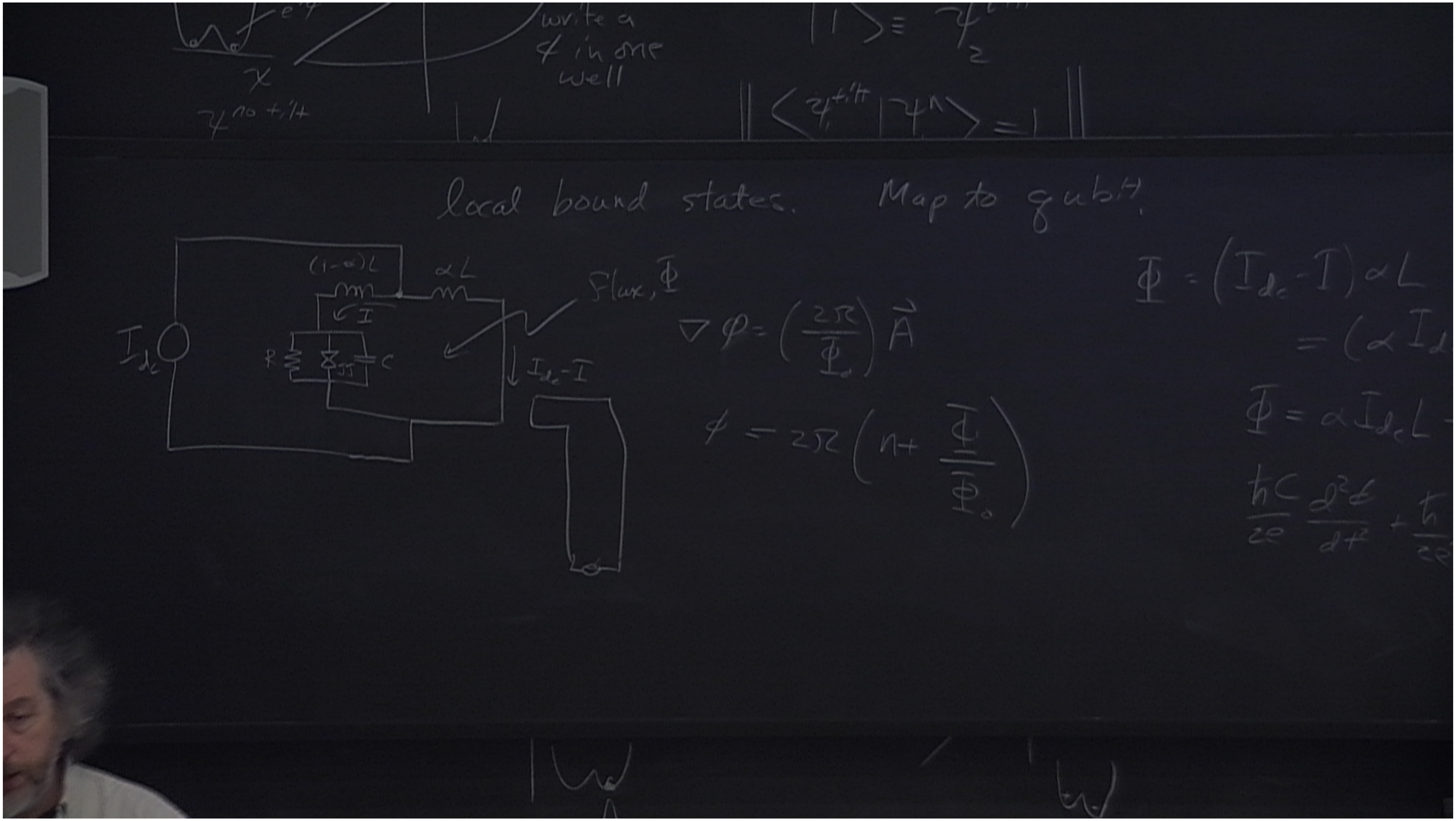
150%

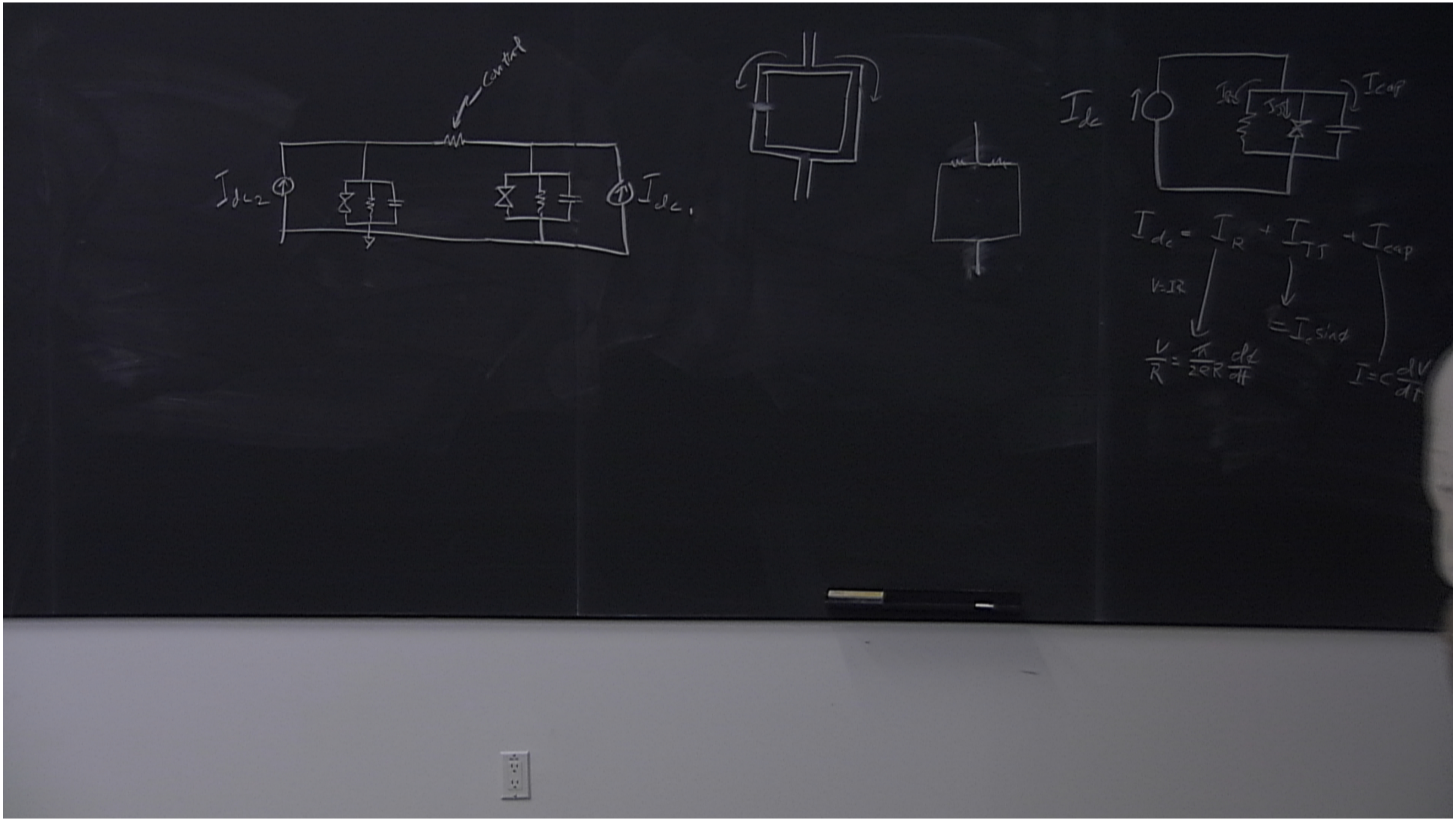


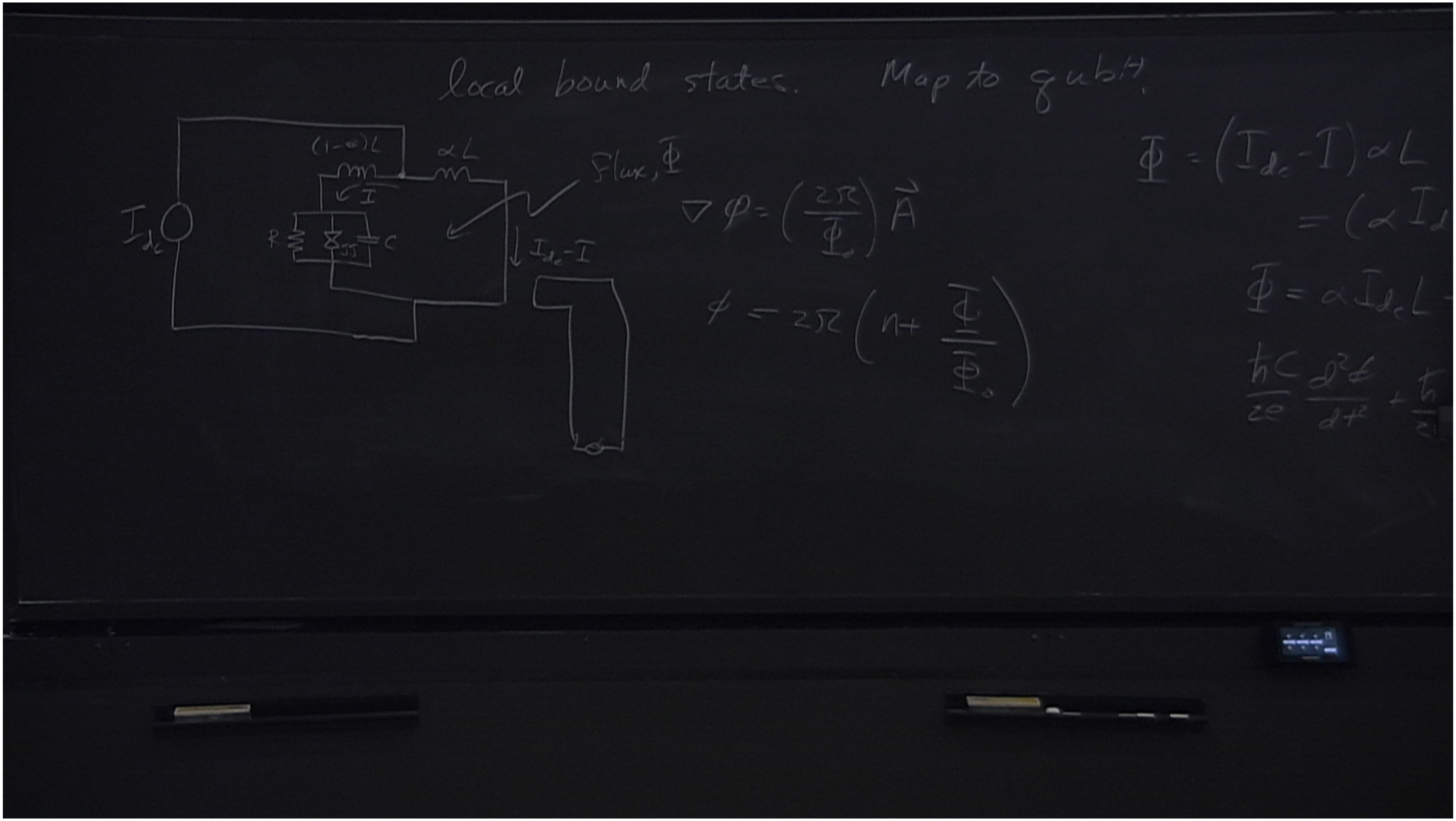
ψ_1
ψ_2

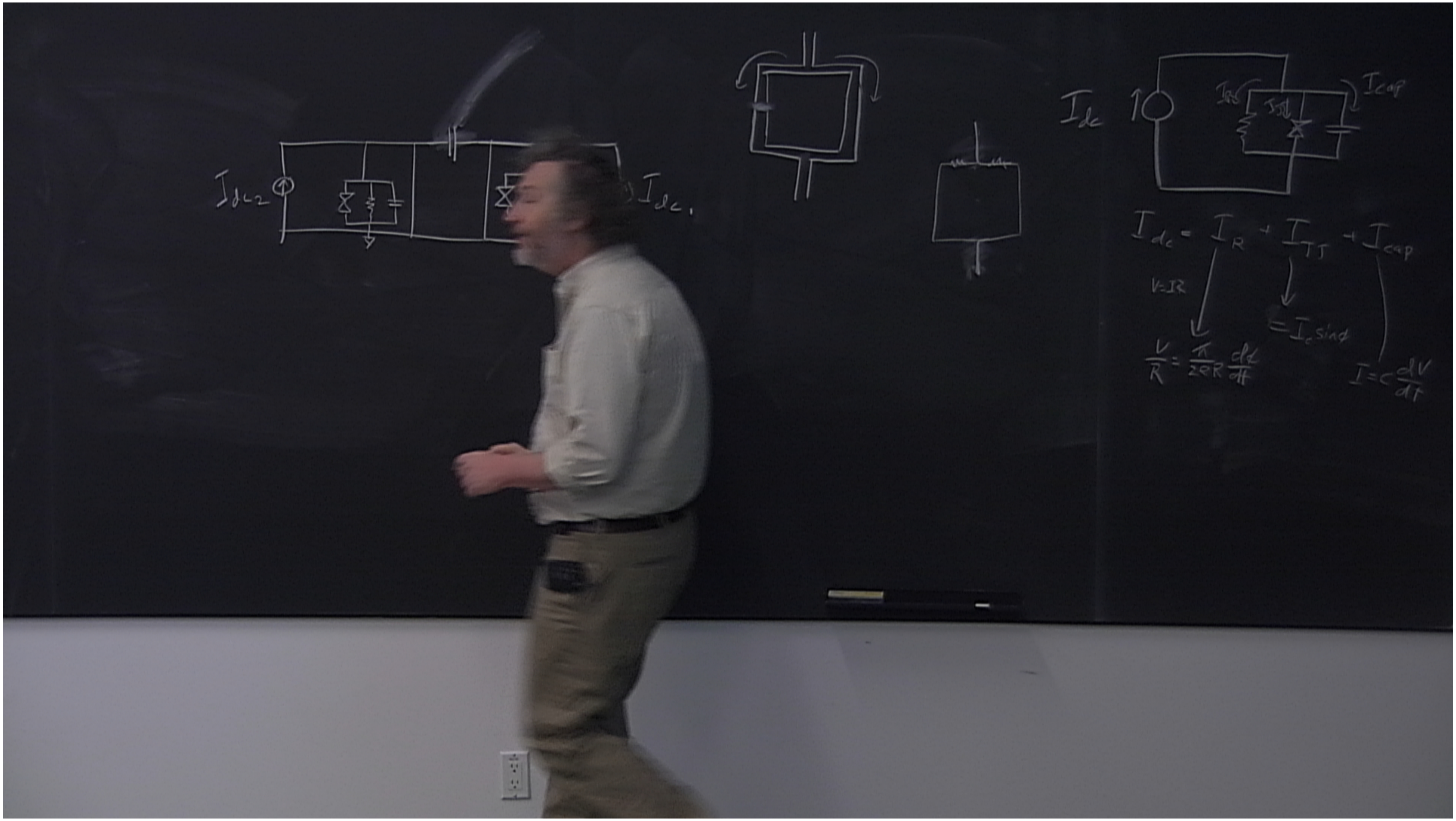

excited states
tilted potential

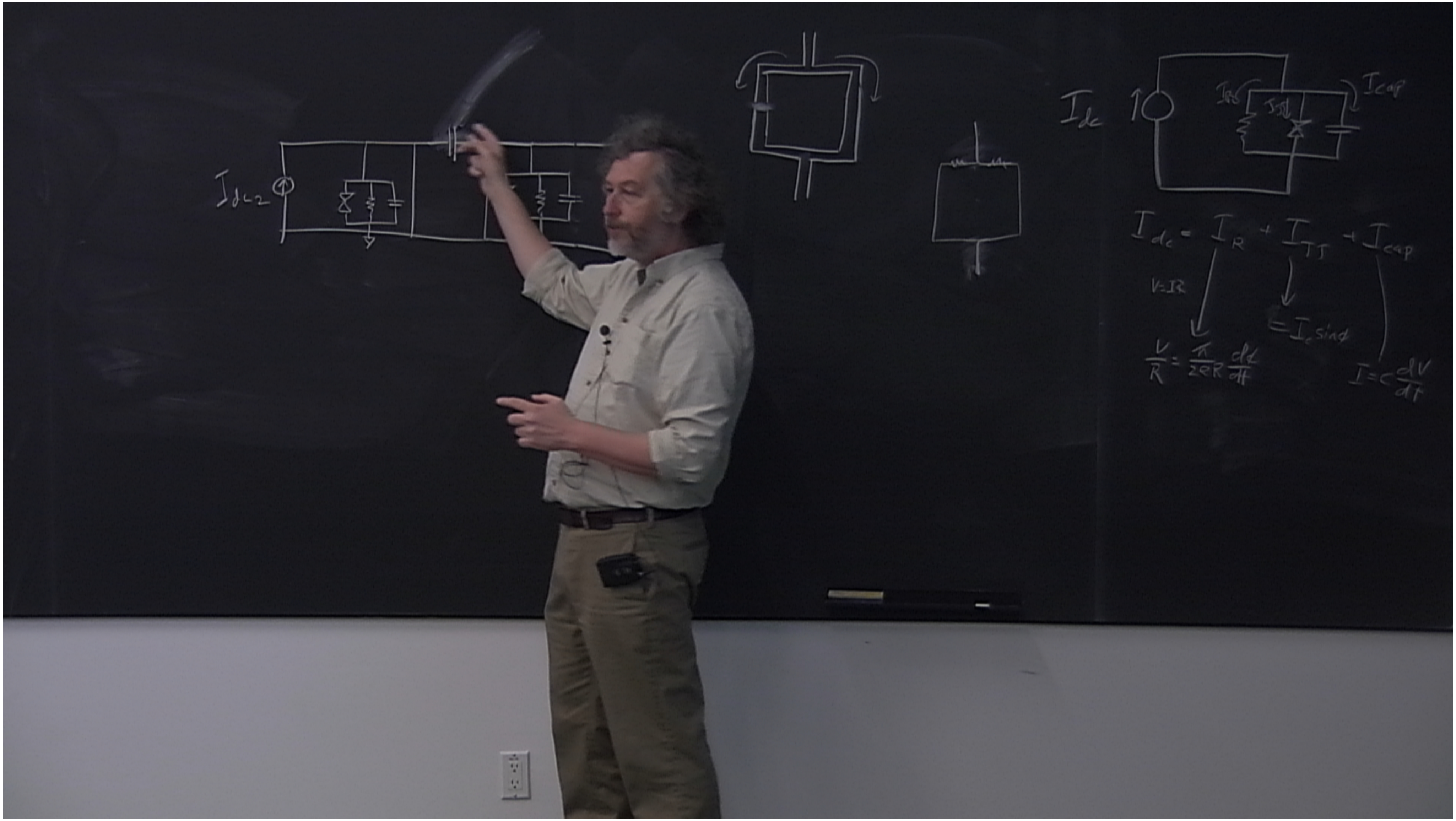

excited states
tilted potential

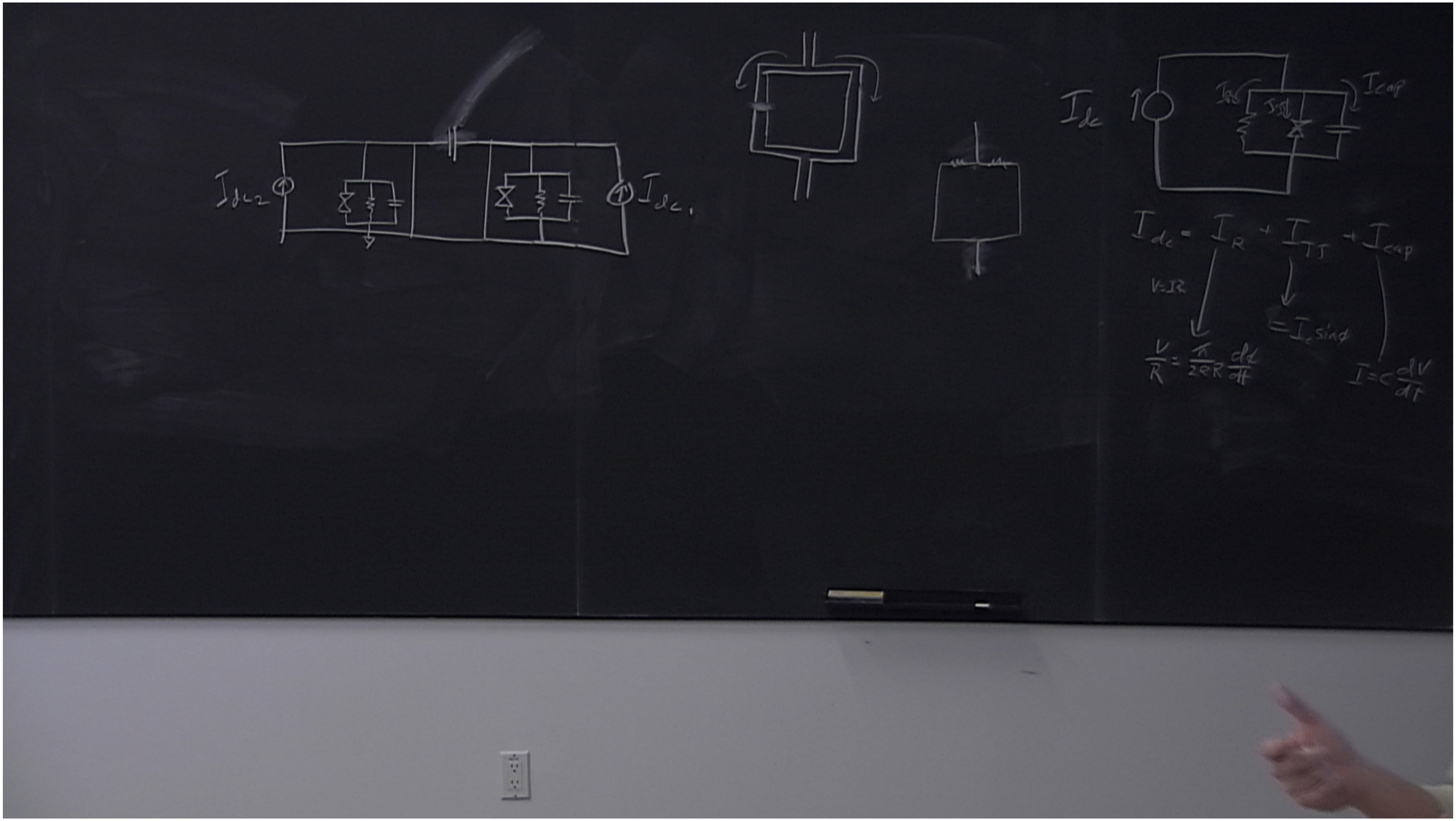

write a
ψ in one
well
local bound states. Map to qubit.
(1−α)L
αL
flux, Φ
I
R
JJ
C
I_dc − I
Φ = (I_dc − I) α L
Φ = α I_dc L


control
Idc2
Idc1
Idc
Icap
Idc = IR + IJJ + Icap
V=IR
= Ic sinφ
I = C dV/dt


local bound states. Map to qubit.
(1-α)L
αL
flux, Φ
I
I_{dc}
R
JJ
C
$I_{dc} - I$
$\nabla \varphi = \left(\frac{2\pi}{\Phi_0}\right) \vec{A}$
$\phi = 2\pi \left(n + \frac{\Phi}{\Phi_0}\right)$
$\Phi = (I_{dc} - I)\alpha L$
$= (\alpha I_{d}$
$\Phi = \alpha I_{dc} L$
$\frac{\hbar C}{2e} \frac{d^2\phi}{dt^2} +$


I_{dc2}
I_{dc1}
I_{dc}
I_{cap}
$I_{dc} = I_R + I_{JJ} + I_{cap}$
V=IR
$= I_c \sin\phi$
$\frac{V}{R} = \frac{\hbar}{2eR}\frac{d\phi}{dt}$
$I = C\frac{dV}{dt}$





Idc2
Idc1
Idc
Icap
Idc = IR + IJJ + Icap
V=IR
= Ic sinφ
V/R = ħ/2eR dφ/dt
I = C dV/dt