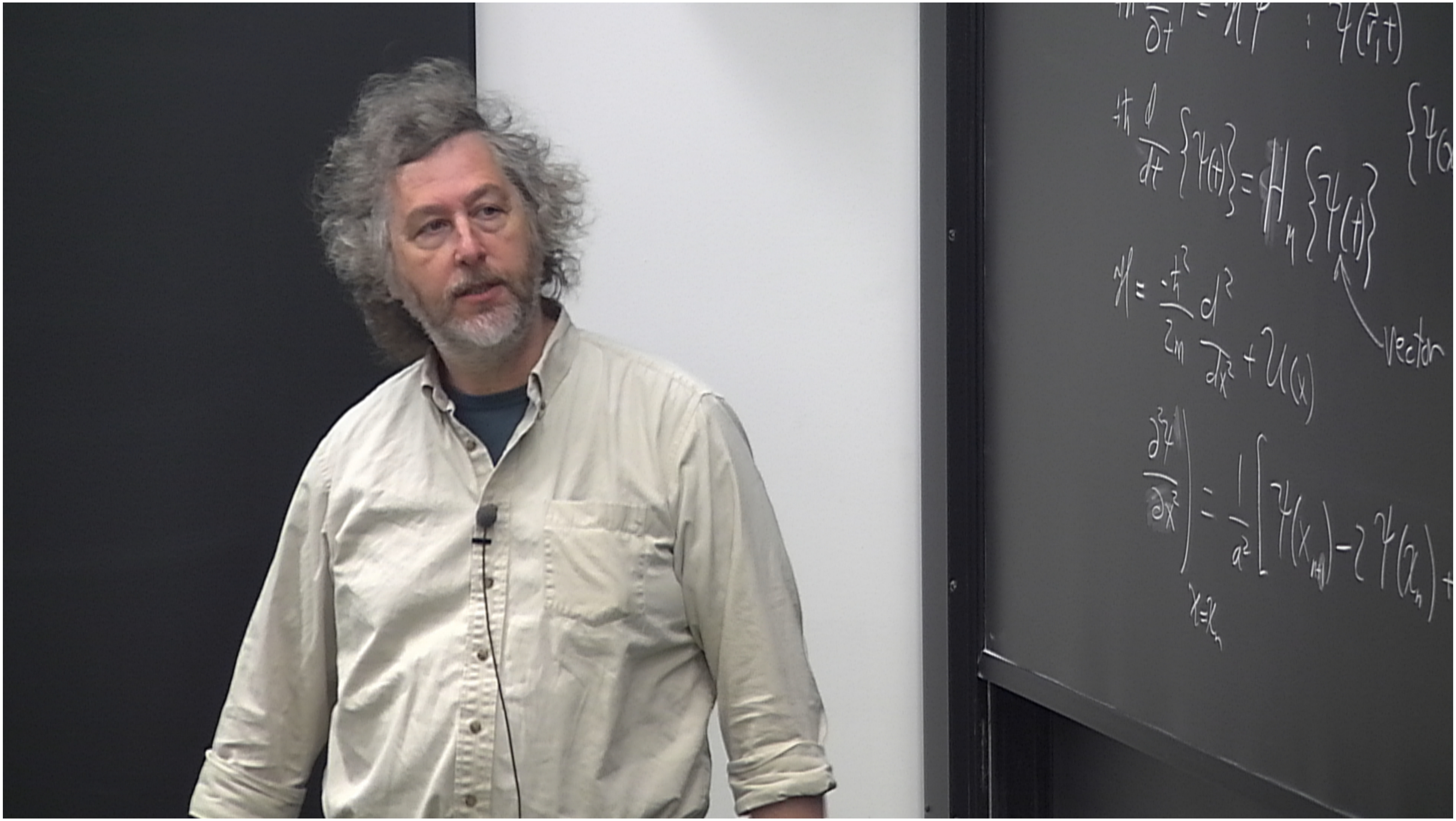
Title: Explorations in Quantum Information - Lecture 13

Date: Mar 29, 2012 09:00 AM

URL: http://pirsa.org/12030020

Abstract:


vector

Finite difference Method

$$i\hbar \frac{\partial}{\partial t}\psi = \mathcal{H}\psi \quad : \psi(\vec{r}, t)$$

$$\{\psi(x_0), \psi(x_1) \cdots\}$$

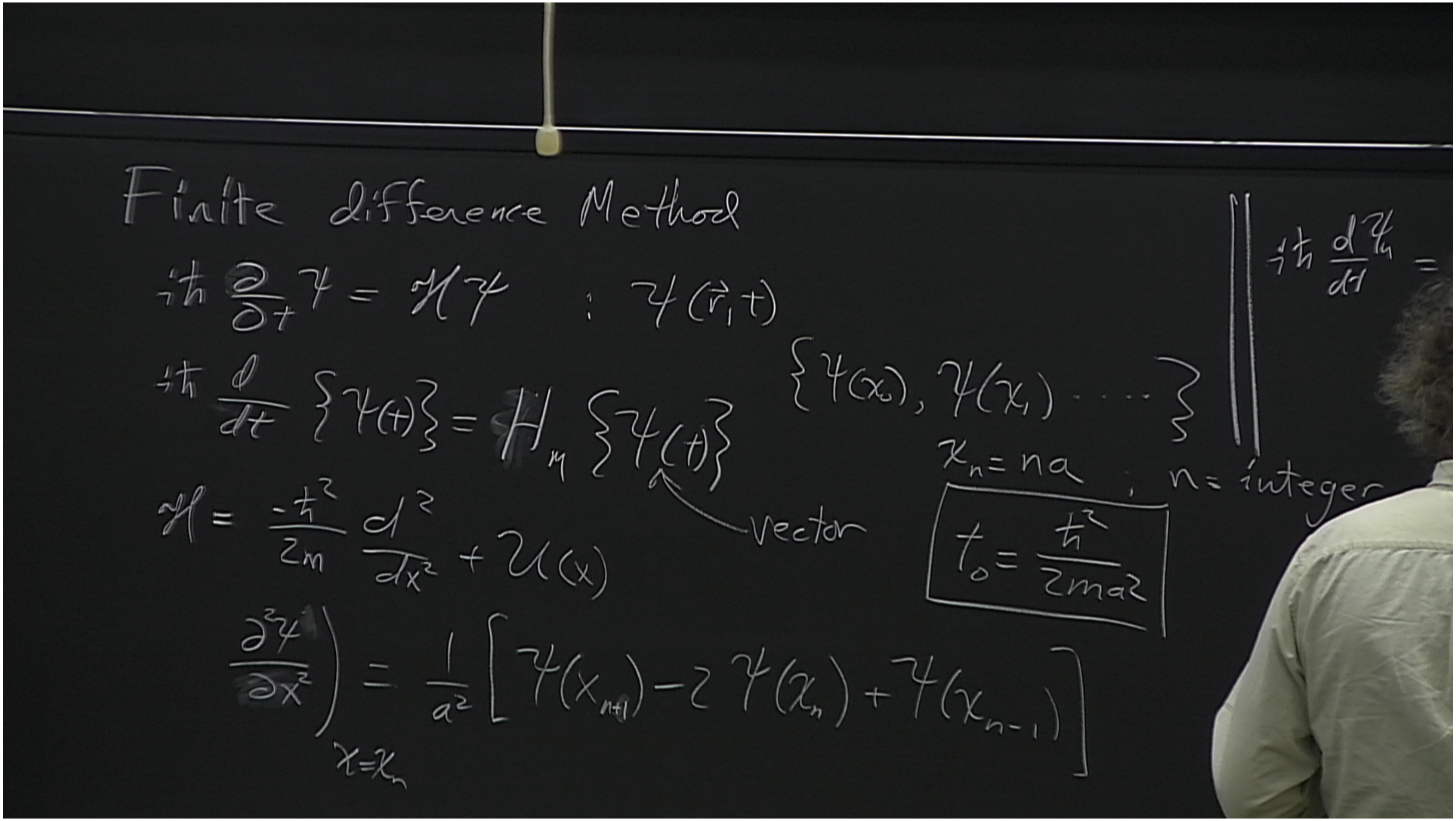
$$i\hbar \frac{d}{dt}\{\psi(t)\} = H_M \{\psi(t)\}$$

$x_n = na \quad ; \quad n = \text{integer}$

$\{\psi(t)\}$ ← vector

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

$$f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$


Finite difference Method
$i\hbar \frac{\partial}{\partial t} \psi = \mathcal{H} \psi$: $\psi(\vec{r}, t)$
$i\hbar \frac{d}{dt} \{\psi(t)\} = H_M \{\psi(t)\}$
vector
$\{\psi(x_0), \psi(x_1) \cdots\}$
$x_n = na$; n = integer
$t_0 = \frac{\hbar^2}{2ma^2}$
$\mathcal{H} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \mathcal{U}(x)$
$\left.\frac{\partial^2 \psi}{\partial x^2}\right)_{x=x_n} = \frac{1}{a^2}\left[\psi(x_{n+1}) - 2\psi(x_n) + \psi(x_{n-1})\right]$
$i\hbar \frac{d\psi_n}{dt} =$

$\psi(x_1) \cdots$

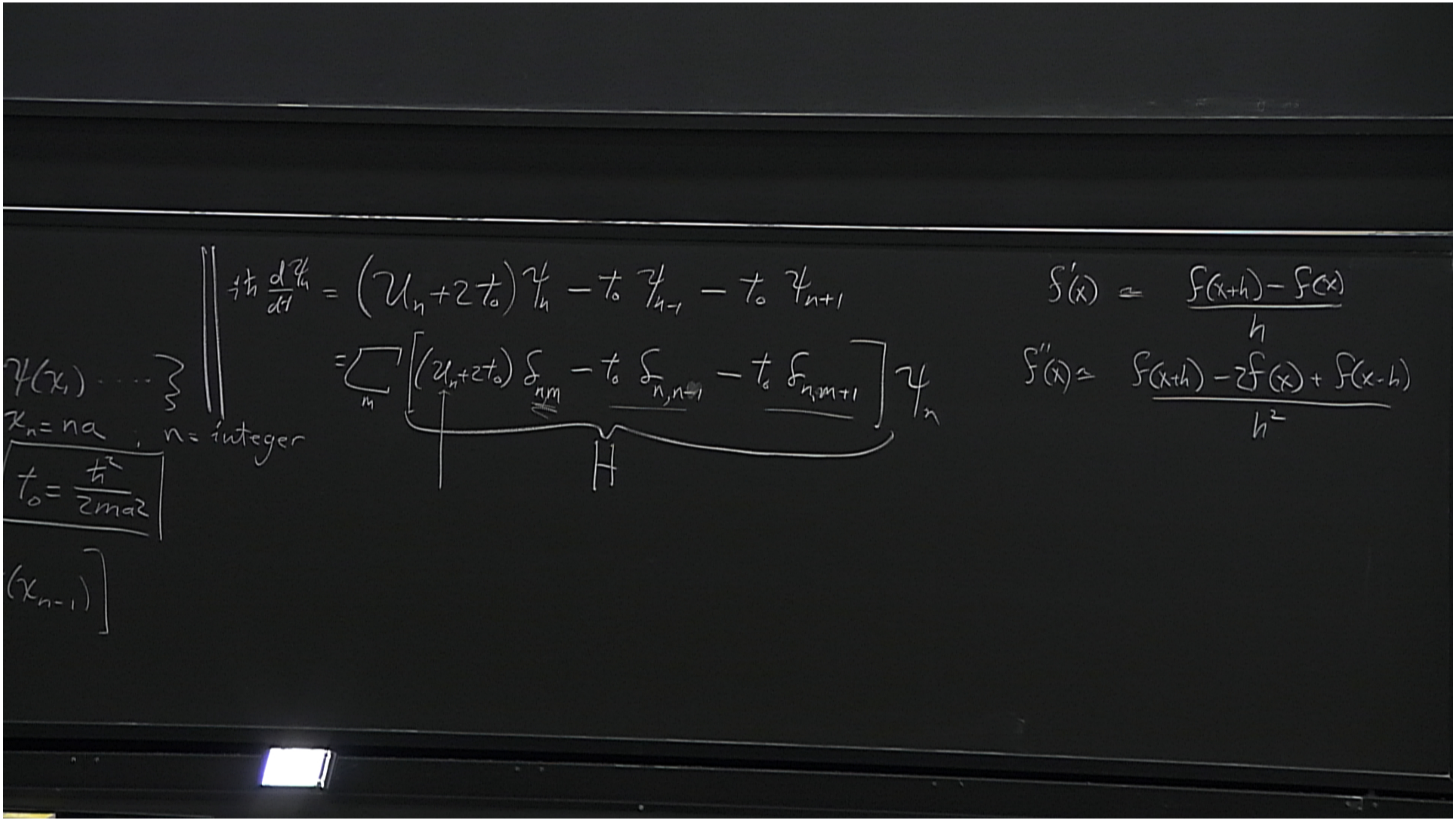
$x_n = na \; ; \; n = \text{integer}$

$$\boxed{t_0 = \frac{\hbar^2}{2ma^2}}$$

$$i\hbar \frac{d\psi_n}{dt} = (U_n + 2t_0)\psi_n - t_0\,\psi_{n-1} - t_0\,\psi_{n+1}$$

$$= \sum_m \left[(U_n + 2t_0)\,\delta_{n,m} - t_0\,\delta_{n,n-1} - t_0\,\delta_{n,m+1}\right]\psi_n$$

$\psi(x_{n-1})\Big]$


iħ dψn/dt = (Un+2t0) ψn − t0 ψn−1 − t0 ψn+1
= Σm [(Un+2t0) δn,m − t0 δn,n−1 − t0 δn,m+1] ψn
H
ψ(x1) ....}
xn = na , n = integer
t0 = ħ²/2ma²
(xn−1)]
f′(x) ≃ (f(x+h) − f(x))/h
f″(x) ≃ (f(x+h) − 2f(x) + f(x−h))/h²

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dcory@iqc.ca

Here we consider a physical system consisting of an electron bound in a 1-D well composed of an arbitrary potential.

- **constants**

```
h = 6.626 × 10^-34; (* Plank's constant in m^2 kg/s *)

a = 10^-9; (* length of quantum wire in m *)

m = 9.11 × 10^-31; (* mass of an electron in kg *)

eV = 1.602 × 10^-19; (* J *)
```

- **wave function and energy**

For comparison we use the eigenfunctions of the infinite square well.

```
ψ[n_, x_] := Sqrt[2 / a] Sin[n π x / a];

Energy[n_] := h^2 n^2 / (8 m a^2) (*in Joules*)
```

- **Numerical estimation of the wave function**

For most potentials we do not know an analytic function for the eigenfunctions. However we can numerically approximate them. We do so by writing down the Hamiltonian in a convenient basis and then numerically solving the eigenvalue problem. Of course the Hamiltonian will not be diagonal, but usually we can find a representation where most of the Hamiltonian is zero. Here we use a basis of position.

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```
DiagonalMatrix[Table[-1, {99}], -1]);
```

Since we are only looking to 2nd order the Hamiltonian has only three non-zero diagonal components.

```
MatrixForm[swH / t0]
```

2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.	-1.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	2.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.

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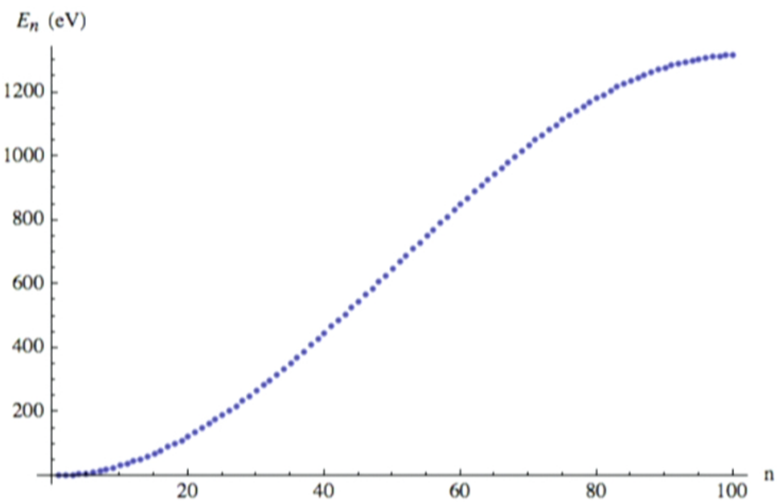
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

```
val = Reverse[Eigenvalues[swH] // N];
```

Reverse just swaps the order of the eigenvalues in the vector. *Mathematica* orders from largest to smallest, but we want the lowest energies first.

```
ListPlot[val 2 π m / h / eV, {AxesLabel → {"n", "En (eV)"}}]
```




```
Energy[1] / (val[[1]] 2 π m / h ) // N
```

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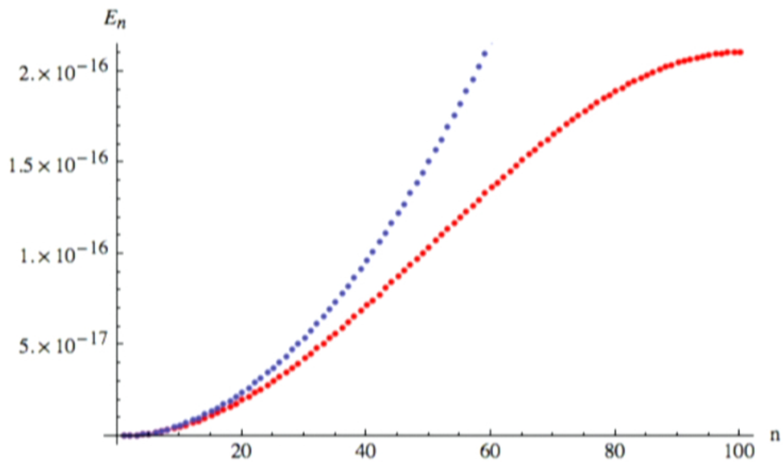
19	2.17471×10^{-17}	1.78866×10^{-17}
20	2.40965×10^{-17}	1.97564×10^{-17}

```
Show[{ListPlot[val 2 π m / h, {PlotStyle → {RGBColor[1, 0, 0]}, AxesLabel → {"n", "En"}}],
  ListPlot[Table[Energy[n], {n, 1, 100}]]}]
```



Red is numeric and blue is analytic. So for the first 20 eigenstates the approximation is good.

```
Ʊ = Table[If[Eigenvectors[swH][[101 - n]][[1]] < 0, - Eigenvectors[swH][[101 - n]] / Sqrt[a / 100] // N,
   Eigenvectors[swH][[101 - n]] / Sqrt[a / 100] // N], {n, 1, 100}];
```

Note that for each eigenfunction there are two choices, the first lobe can be either positive or negative. Since we selected for the analytic solution the positive functions, the above code makes the same choice for the numeric solutions.

```
x = Table[a n / 101. {n. 1. 100}];
```

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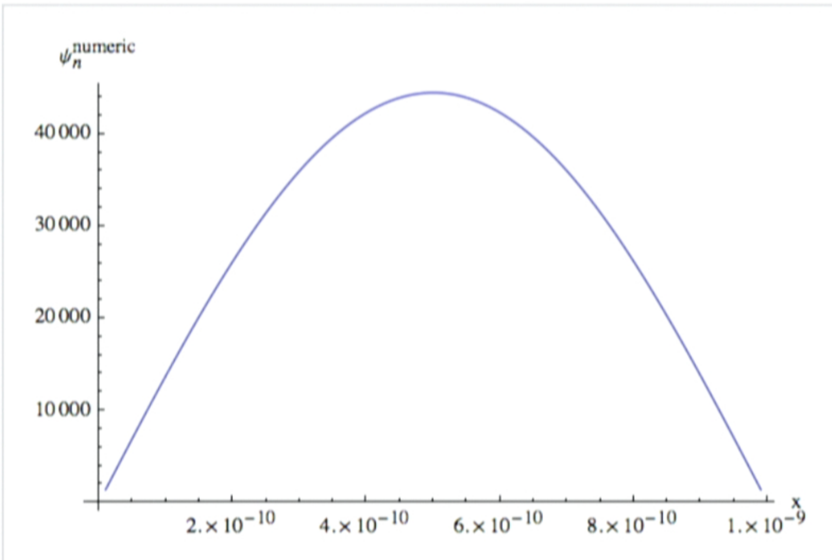
```
Manipulate[ListPlot[Transpose[{x, ʊ[[n]]}], {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True}],
 {n, 1, 100, 1}, SaveDefinitions → True]
```

n




```
Manipulate[
 Show[{Plot[ψ[n, x], {x, 0, a}, {AxesLabel → {"x", "ψn"}, PlotRange → All}],
   ListPlot[Transpose[{x, ʊ[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
 {n, 1, 100, 1}, SaveDefinitions → True]
```

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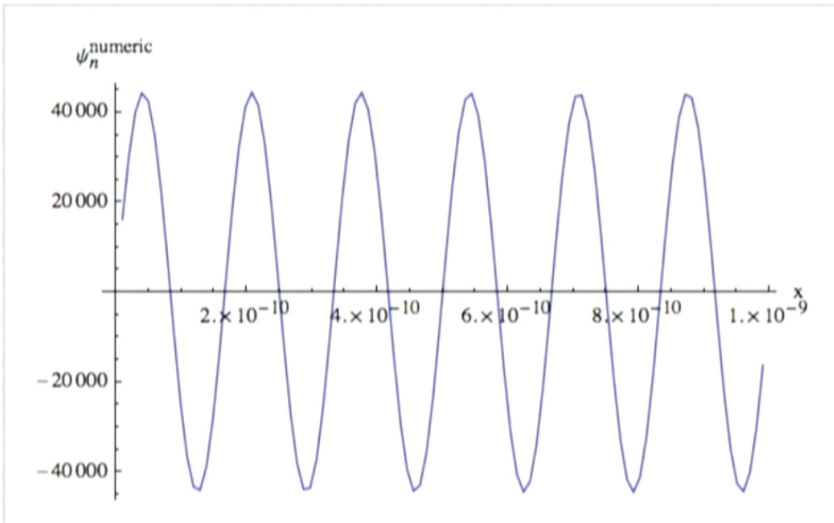
```
Manipulate[ListPlot[Transpose[{x, ℧[[n]]}], {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True}],
 {n, 1, 100, 1}, SaveDefinitions → True]
```

n




```
Manipulate[
 Show[{Plot[ψ[n, x], {x, 0, a}, {AxesLabel → {"x", "ψn"}, PlotRange → All}],
   ListPlot[Transpose[{x, ℧[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
 {n, 1, 100, 1}, SaveDefinitions → True]
```

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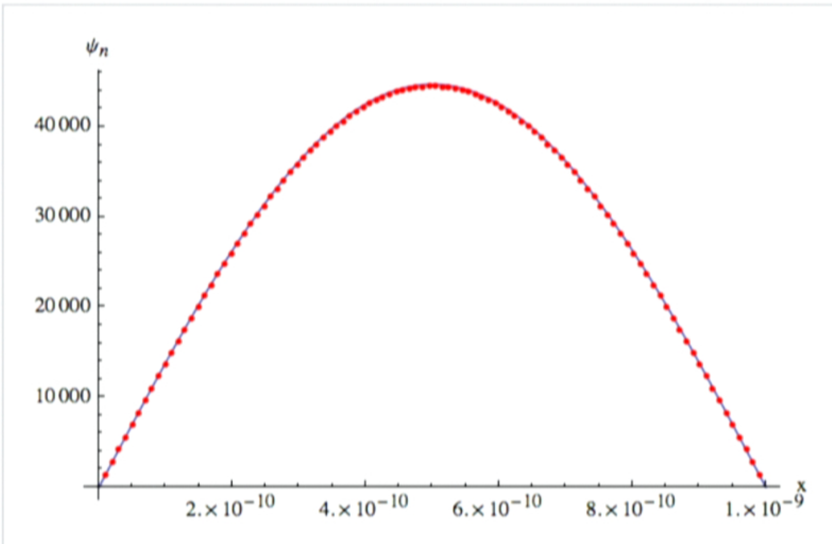
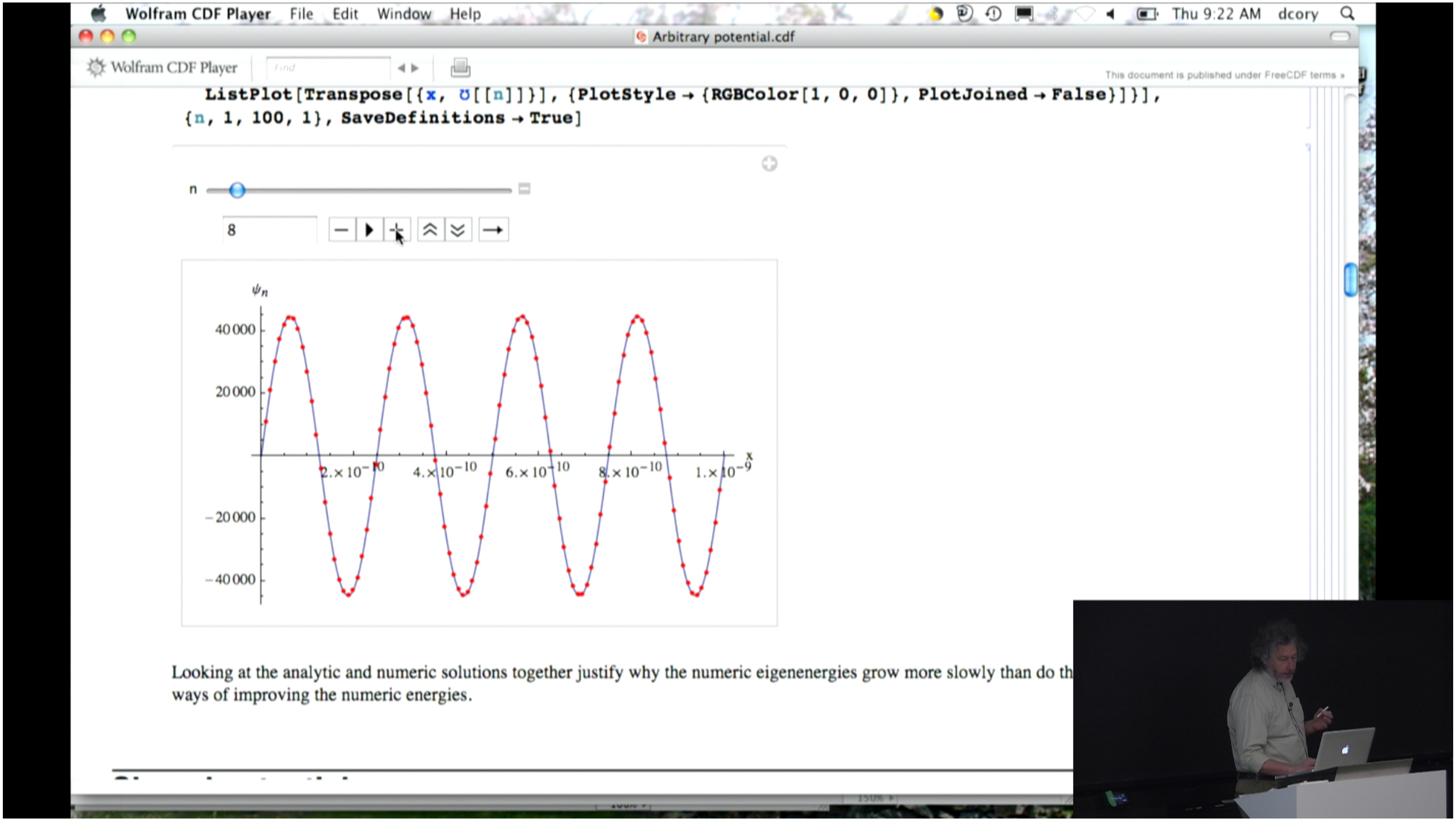
```
ListPlot[Transpose[{x, ʊ[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
{n, 1, 100, 1}, SaveDefinitions → True]
```



Looking at the analytic and numeric solutions together justify why the numeric eigenenergies grow more slowly than do the analytic ones. Suggest two ways of improving the numeric energies.

Sloped potential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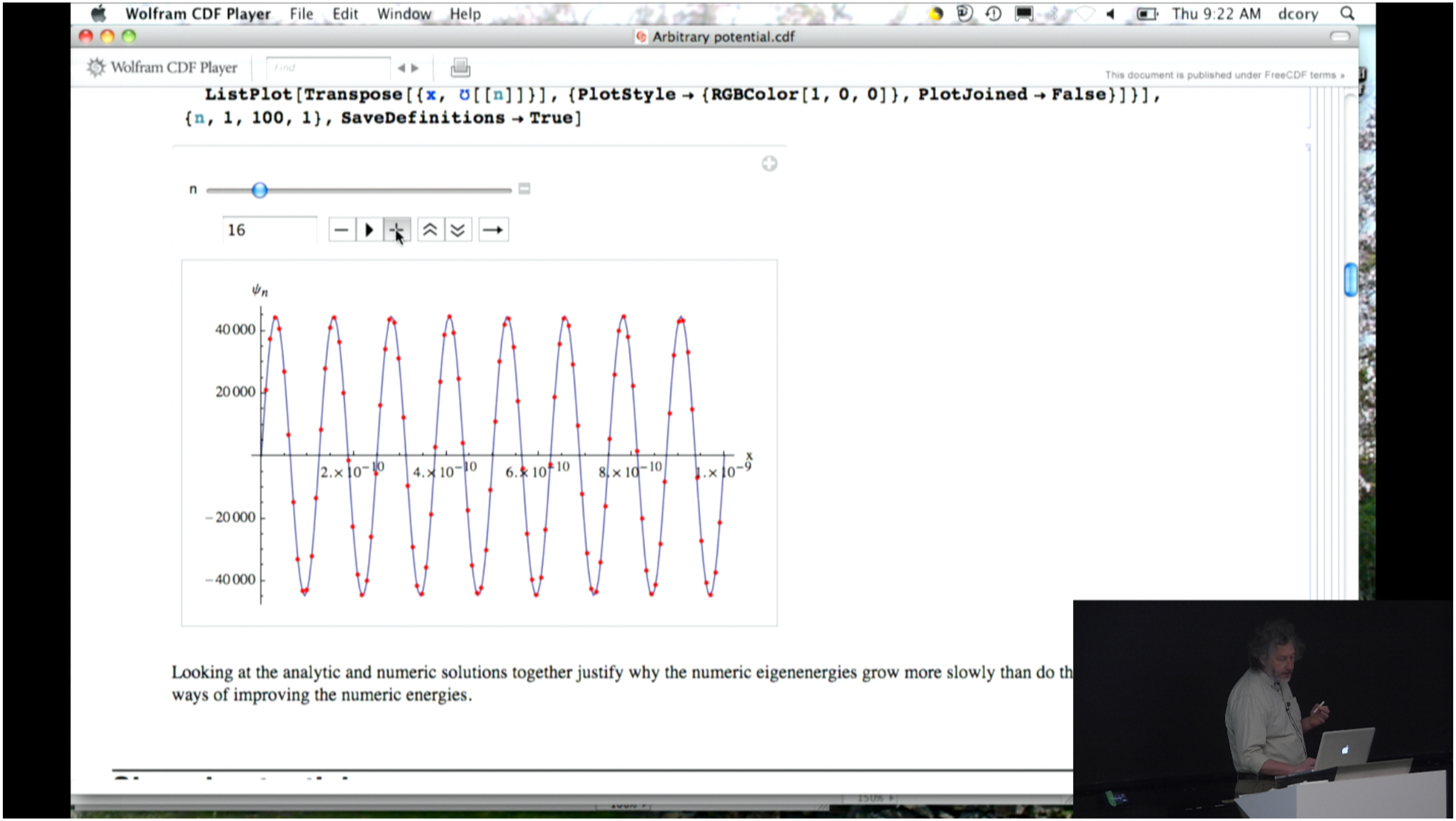
ListPlot[Transpose[{x, ʊ[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
{n, 1, 100, 1}, SaveDefinitions → True]
n
8
ψn
40 000
20 000
−20 000
−40 000
2.× 10−10
4.× 10−10
6.× 10−10
8.× 10−10
1.× 10−9
x
Looking at the analytic and numeric solutions together justify why the numeric eigenenergies grow more slowly than do th
ways of improving the numeric energies.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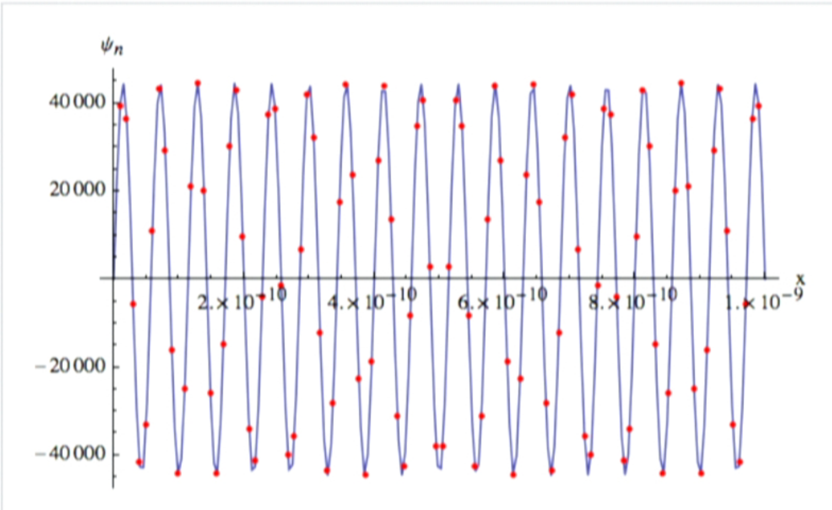
ListPlot[Transpose[{x, ʊ[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
{n, 1, 100, 1}, SaveDefinitions → True]
n
16
ψn
40 000
20 000
−20 000
−40 000
2.× 10−10
4.× 10−10
6.× 10−10
8.× 10−10
1.× 10−9
x
Looking at the analytic and numeric solutions together justify why the numeric eigenenergies grow more slowly than do th
ways of improving the numeric energies.

```
ListPlot[Transpose[{x, ℧[[n]]}], {PlotStyle → {RGBColor[1, 0, 0]}, PlotJoined → False}]}],
{n, 1, 100, 1}, SaveDefinitions → True]
```

n

35



Looking at the analytic and numeric solutions together justify why the numeric eigenenergies grow more slowly than do th
ways of improving the numeric energies.

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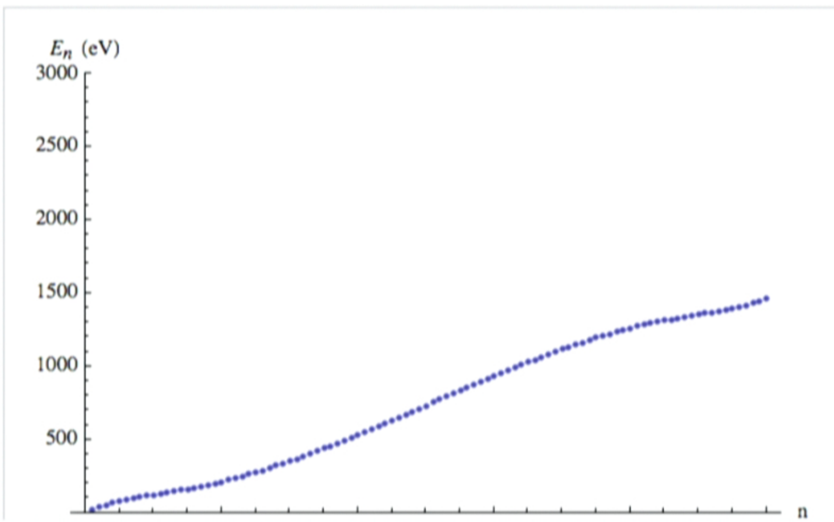
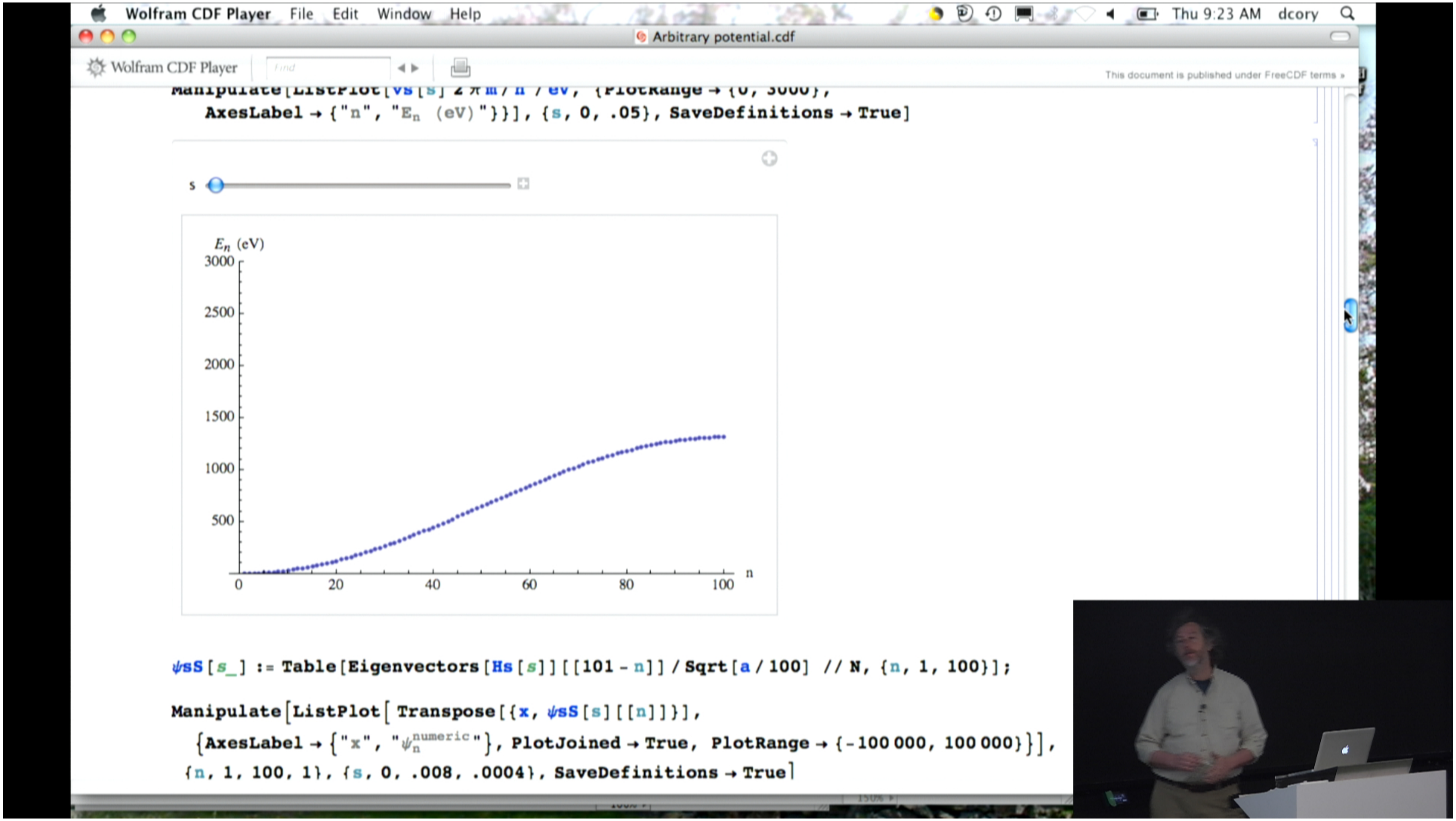
```
t0 = h^2 / (8 π^2 m a^2);

Hs[s_] :=
  t0 (DiagonalMatrix[Table[2 + n s, {n, 1, 100}]] + DiagonalMatrix[Table[-1, {99}], 1] +
     DiagonalMatrix[Table[-1, {99}], -1]);

vs[s_] := Reverse[Eigenvalues[Hs[s]]];

Manipulate[ListPlot[vs[s] 2 π m / h / eV, {PlotRange → {0, 3000},
   AxesLabel → {"n", "E_n (eV)"}}], {s, 0, .05}, SaveDefinitions → True]
```

s





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AxesLabel → {"n", "En (eV)"}}], {s, 0, .05}, SaveDefinitions → True]
s
En (eV)
3000
2500
2000
1500
1000
500
0
20
40
60
80
100
n
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
Manipulate[ListPlot[Transpose[{x, ψsS[s][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]

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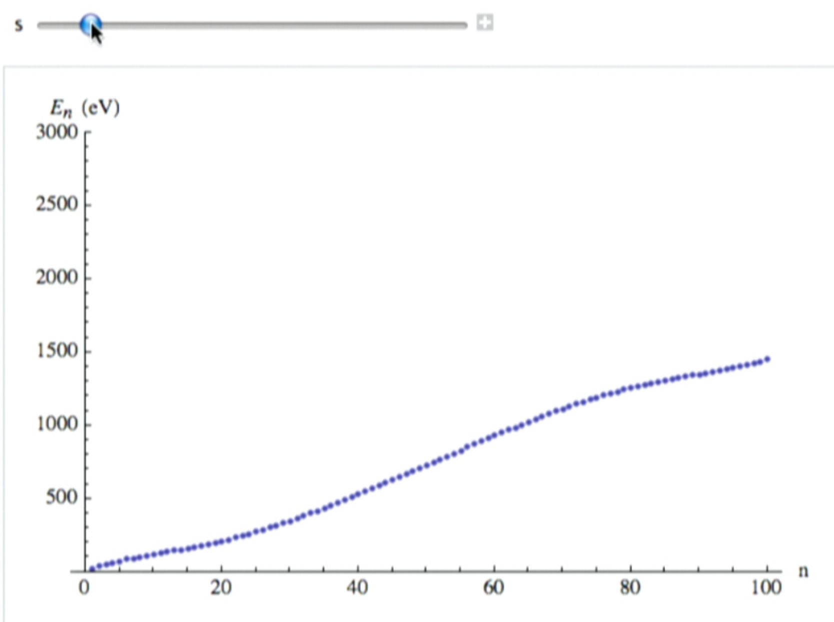
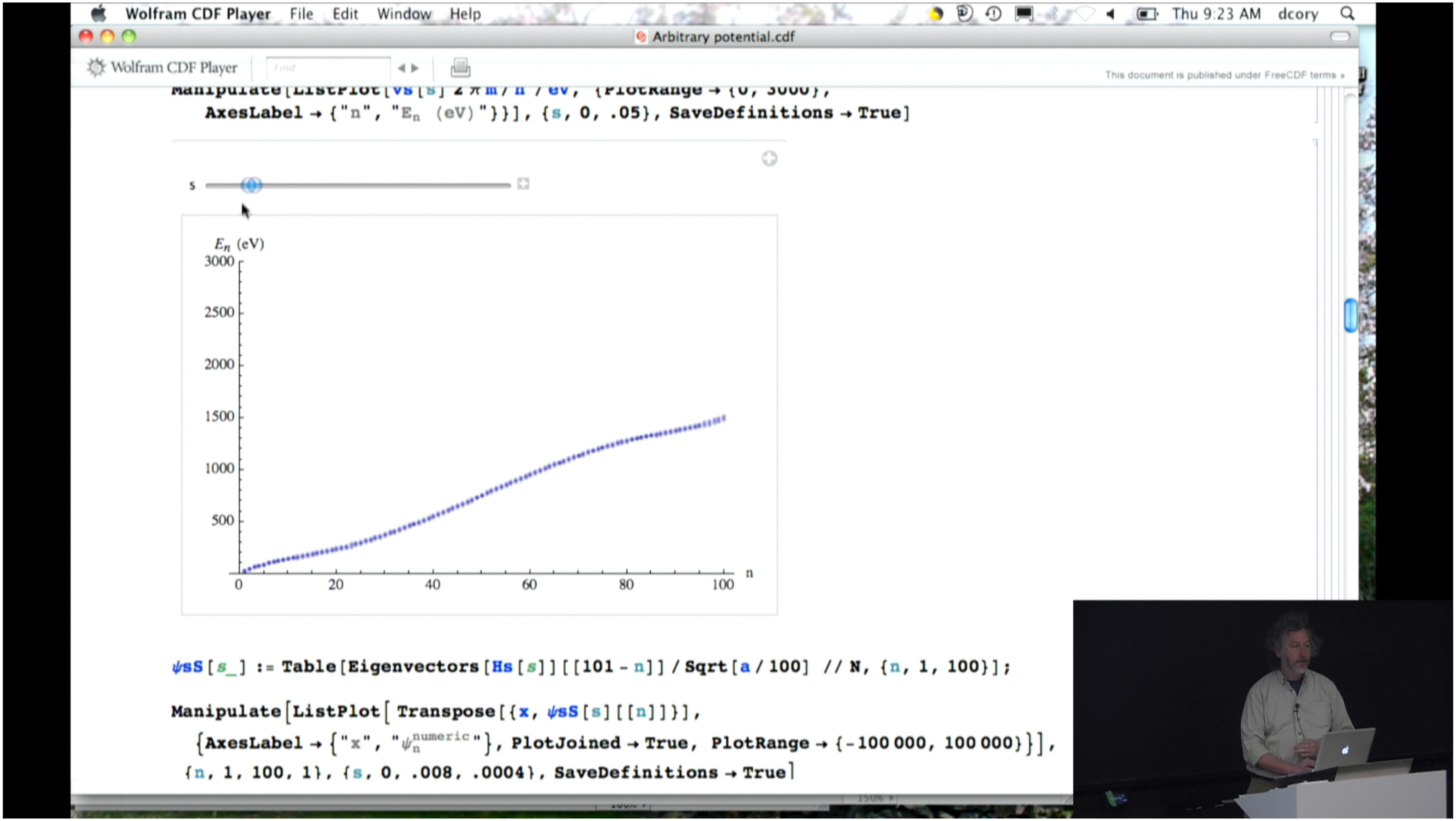
```
Manipulate[ListPlot[vs[s] 2 π m / h / ev, {PlotRange → {0, 3000},
   AxesLabel → {"n", "En (eV)"}}], {s, 0, .05}, SaveDefinitions → True]
```




```
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];

Manipulate[ListPlot[ Transpose[{x, ψsS[s][[n]]}],
  {AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
 {n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]
```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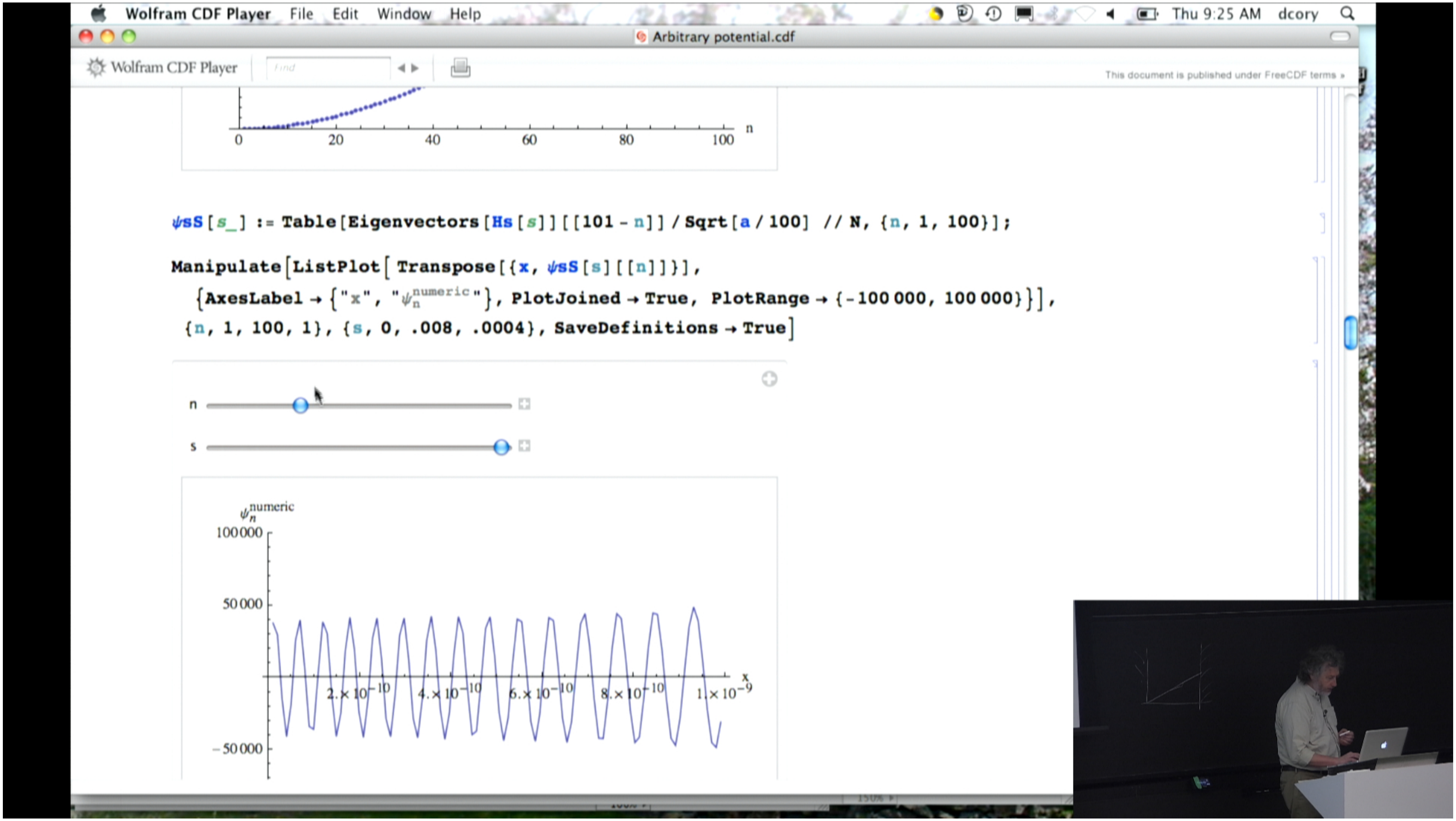
Manipulate[ListPlot[vs[s] 2 π m / h / ev, {PlotRange → {0, 3000},
AxesLabel → {"n", "En (eV)"}}], {s, 0, .05}, SaveDefinitions → True]
s
En (eV)
3000
2500
2000
1500
1000
500
0
20
40
60
80
100
n
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
Manipulate[ListPlot[Transpose[{x, ψsS[s][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]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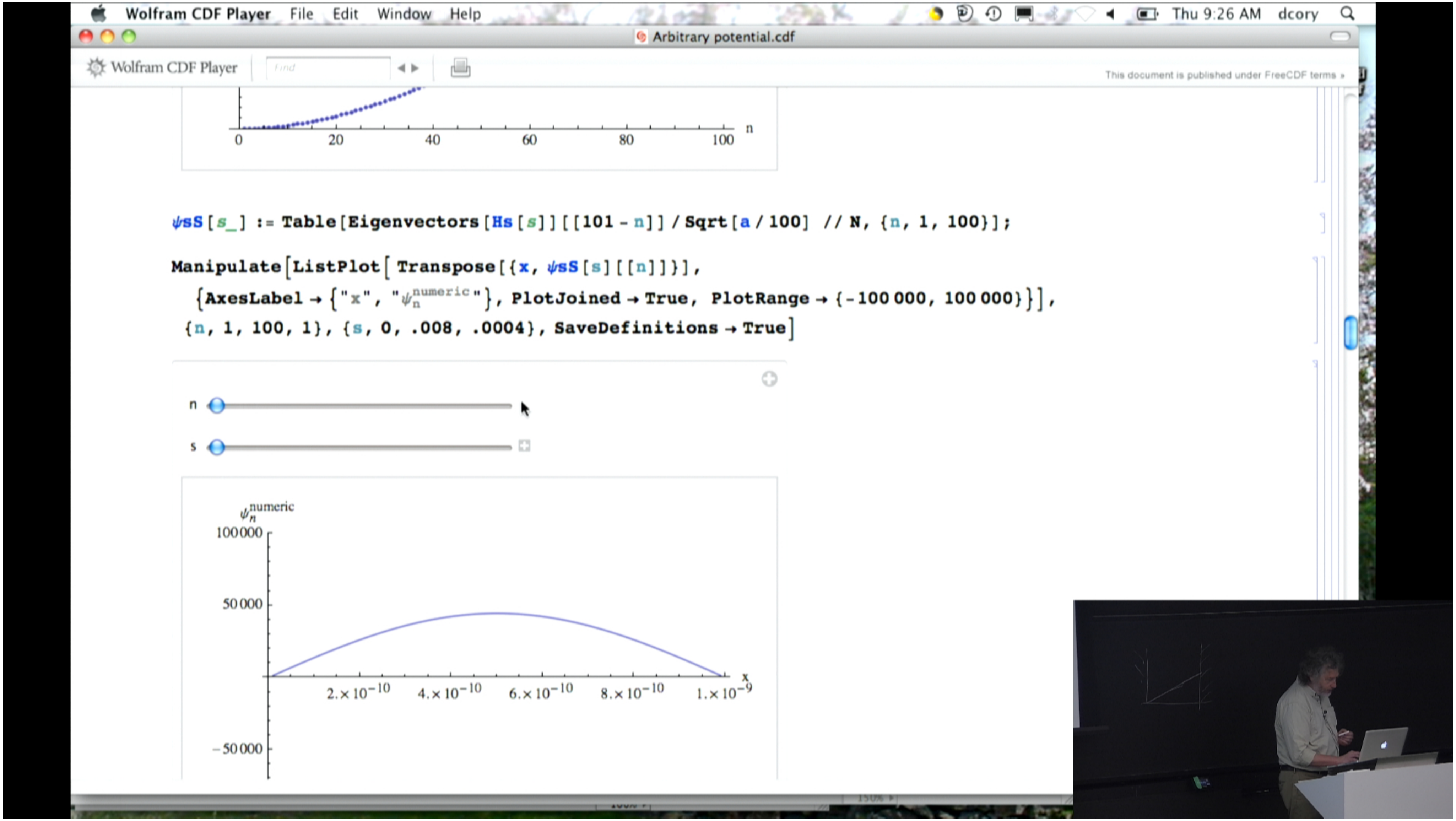
0 20 40 60 80 100 n
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
Manipulate[ListPlot[Transpose[{x, ψsS[s][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]
n
s
ψn numeric
100 000
50 000
−50 000
2.×10^−10 4.×10^−10 6.×10^−10 8.×10^−10 1.×10^−9 x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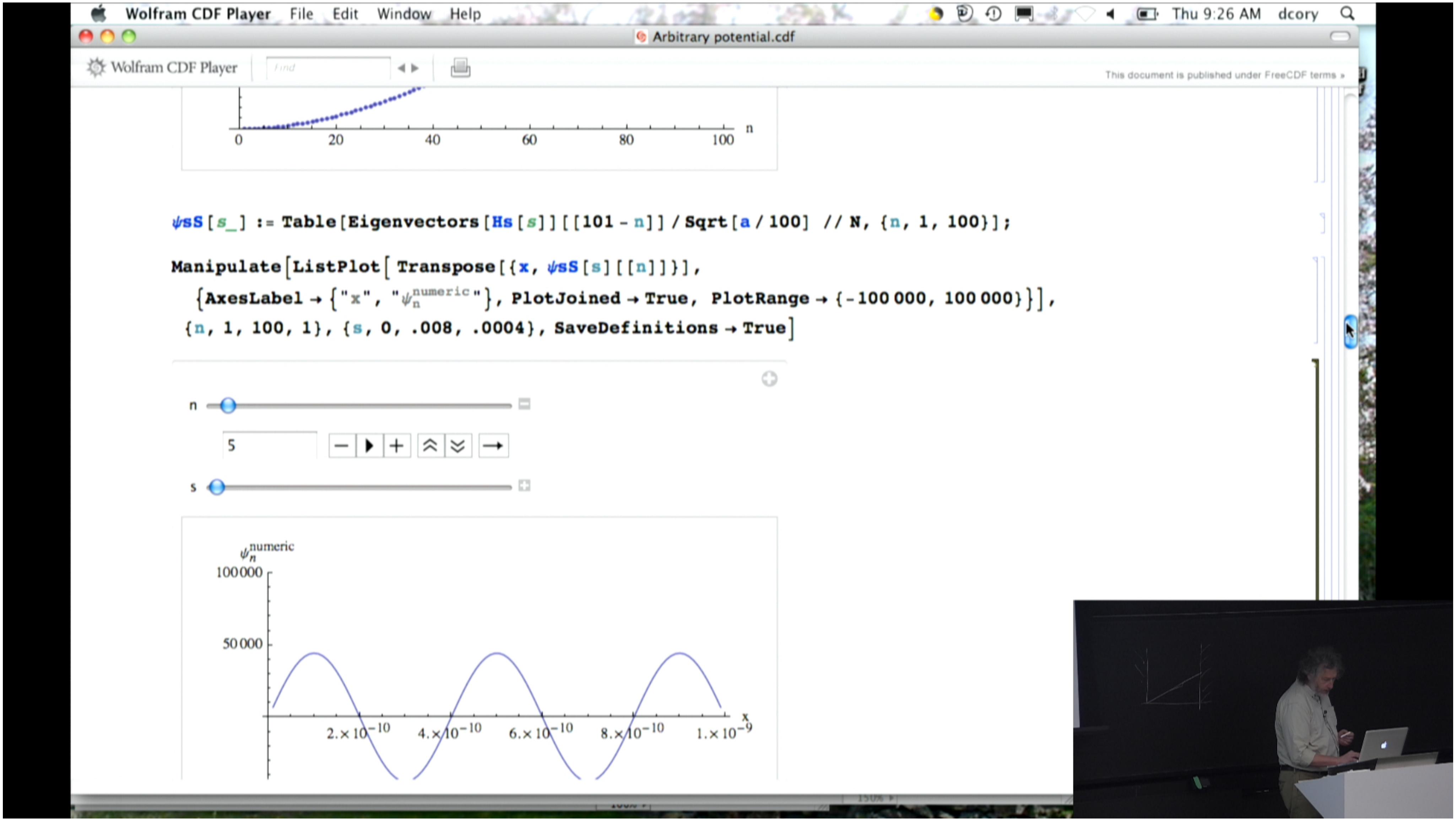
n
0 20 40 60 80 100
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
Manipulate[ListPlot[Transpose[{x, ψsS[s][[n]]}],
{AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]
n
s
ψ_n^numeric
100 000
50 000
-50 000
2.×10^-10 4.×10^-10 6.×10^-10 8.×10^-10 1.×10^-9
x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Arbitrary potential.cdf
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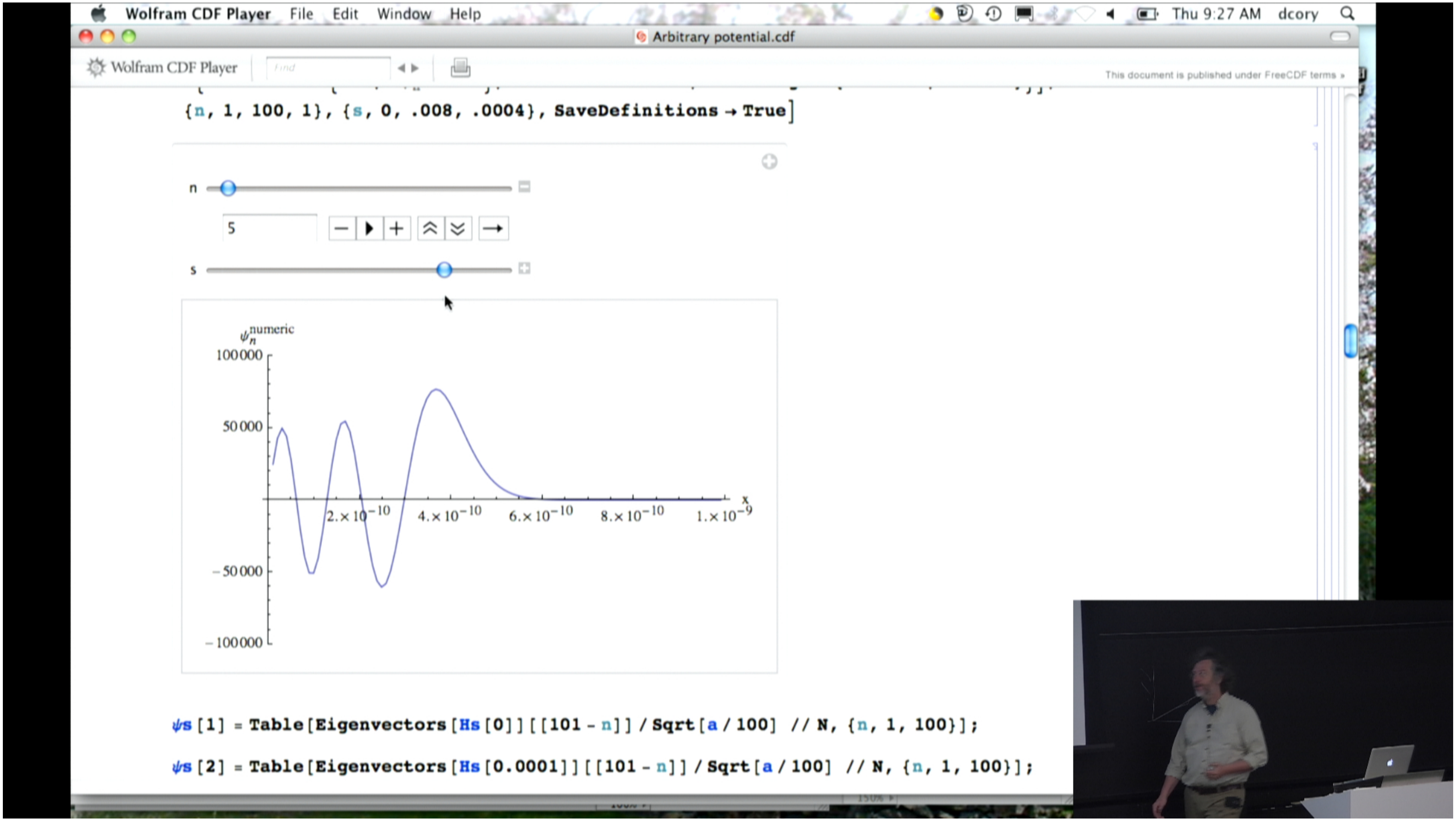
ψsS[s_] := Table[Eigenvectors[Hs[s]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
Manipulate[ListPlot[Transpose[{x, ψsS[s][[n]]}],
{AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → {-100 000, 100 000}}],
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]
n
5
s
ψ_n^numeric
100 000
50 000
x
2.×10^-10
4.×10^-10
6.×10^-10
8.×10^-10
1.×10^-9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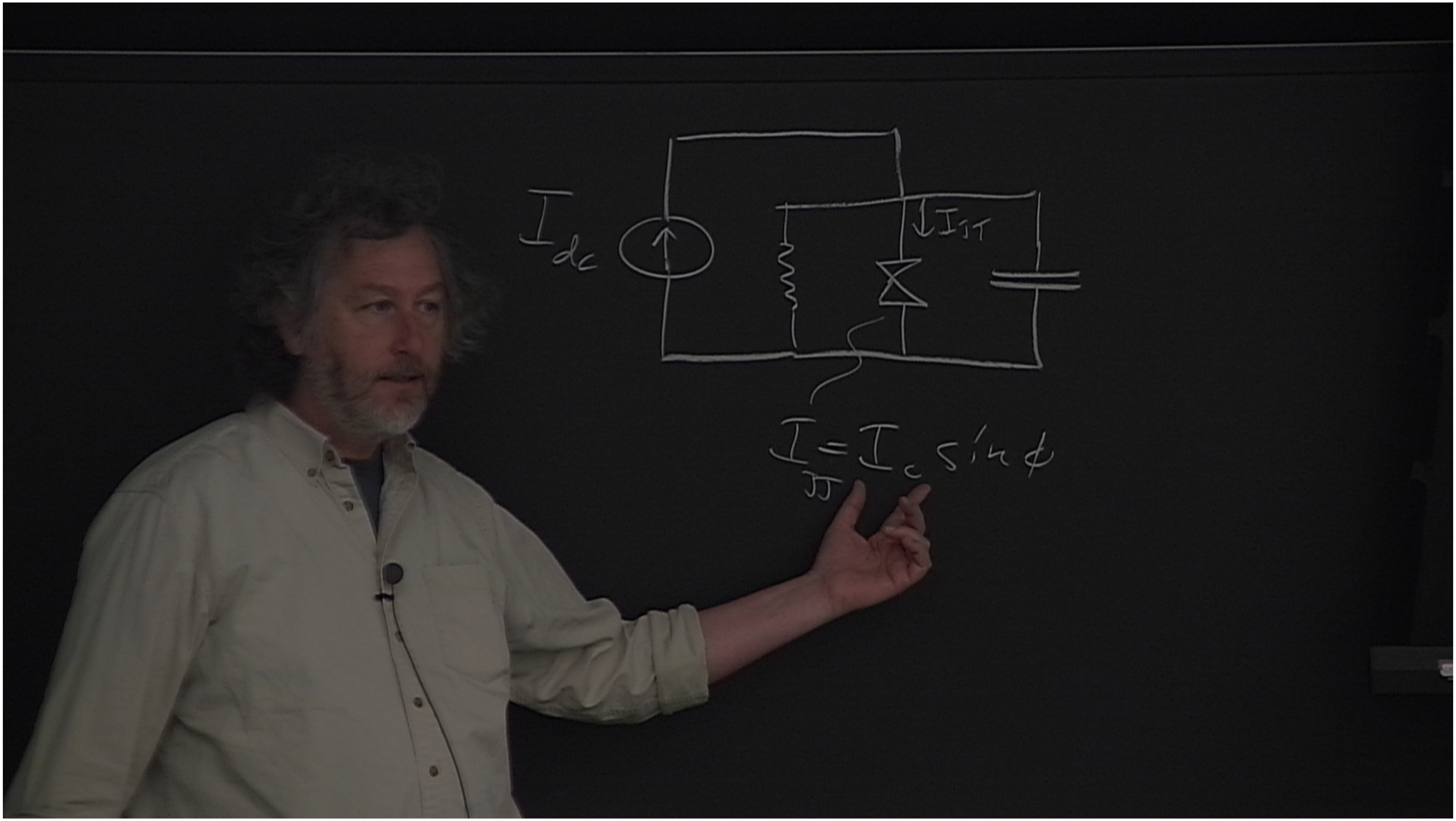
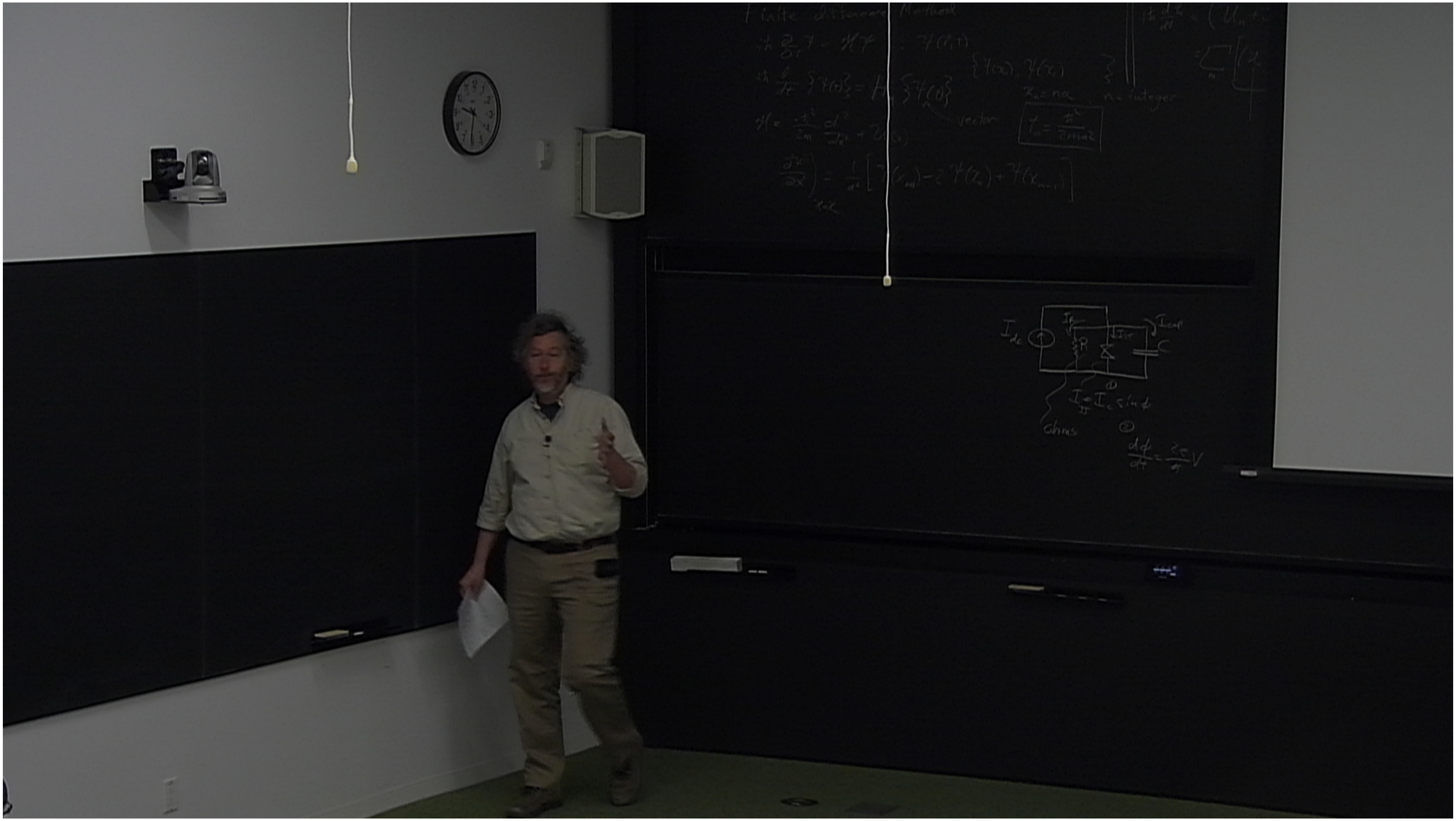
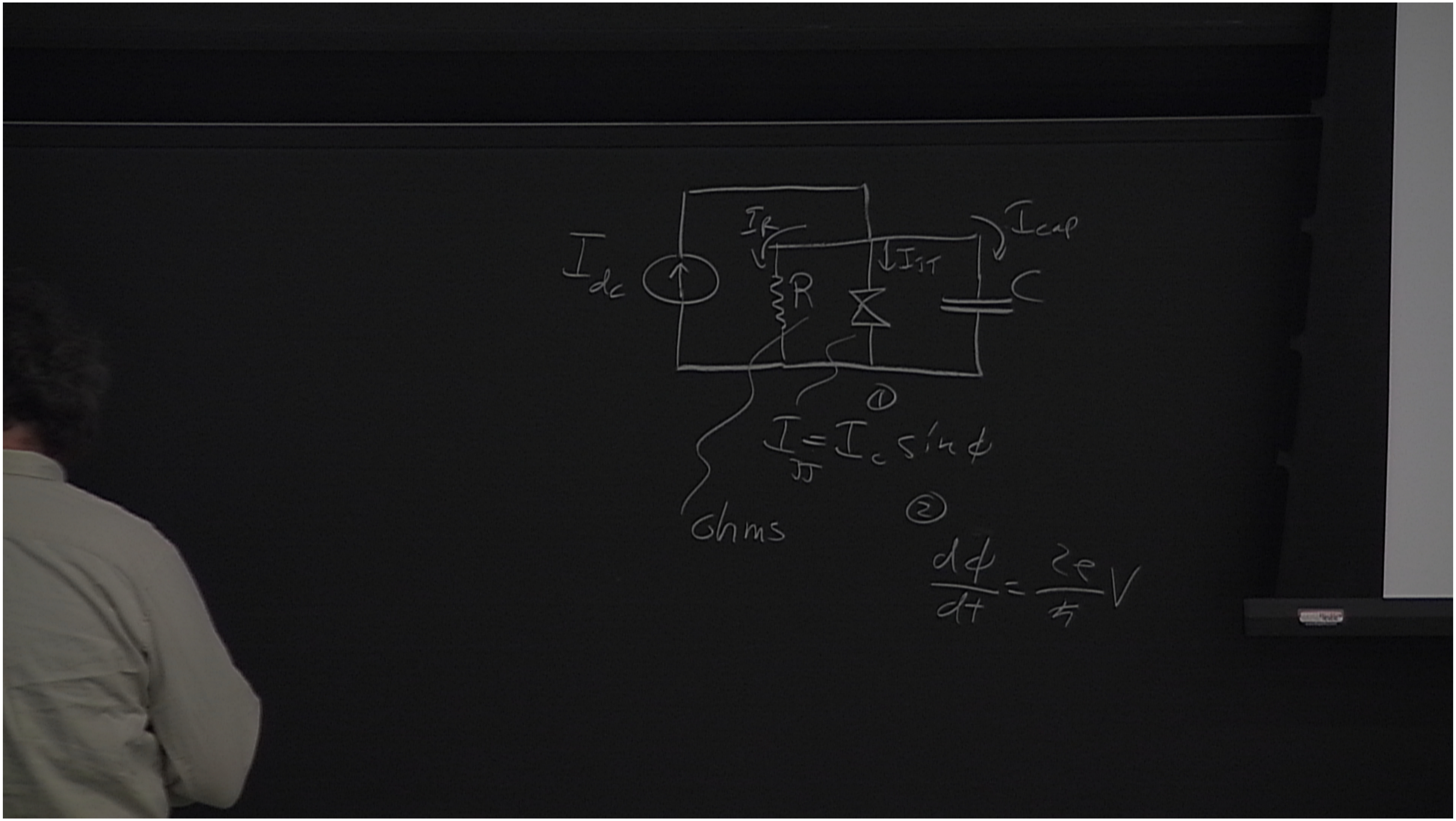
{n, 1, 100, 1}, {s, 0, .008, .0004}, SaveDefinitions → True]
n
5
s
ψ_n^numeric
100 000
50 000
− 50 000
− 100 000
2.× 10^−10
4.× 10^−10
6.× 10^−10
8.× 10^−10
1.× 10^−9
x
ψs[1] = Table[Eigenvectors[Hs[0]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];
ψs[2] = Table[Eigenvectors[Hs[0.0001]][[101 - n]] / Sqrt[a / 100] // N, {n, 1, 100}];



Idc
IJJ
I = Ic sin ϕ
JJ

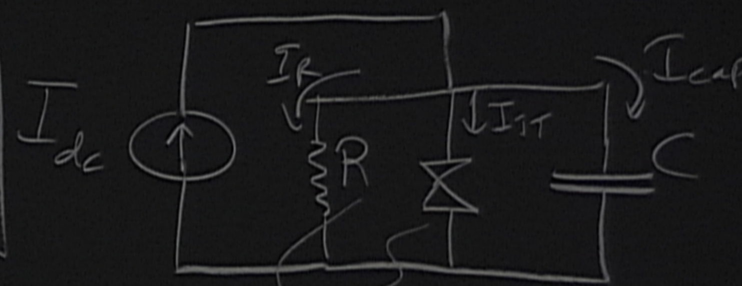

vector
integer
Ohms


I_dc
I_R
I_JJ
I_cap
R
C
①
I_JJ = I_c sin φ
②
dφ/dt = 2e/ħ V
ohms

$$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} = -I_c \sin\phi - I_{dc}$$

(force)

$$U(\phi) = -\cos\phi - \frac{I_{dc}}{I_c}\phi$$

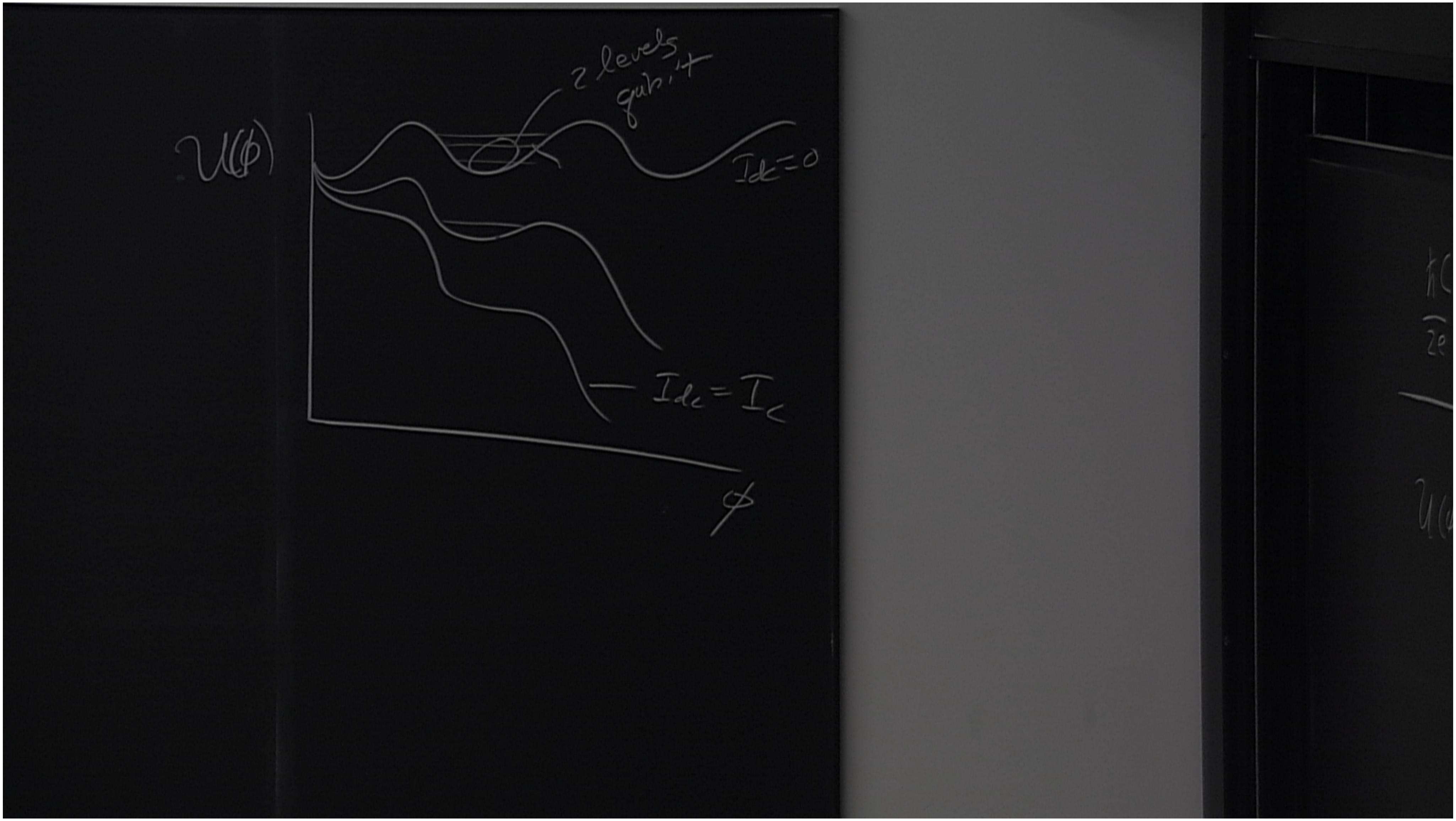


① $I_{JJ} = I_c \sin\phi$

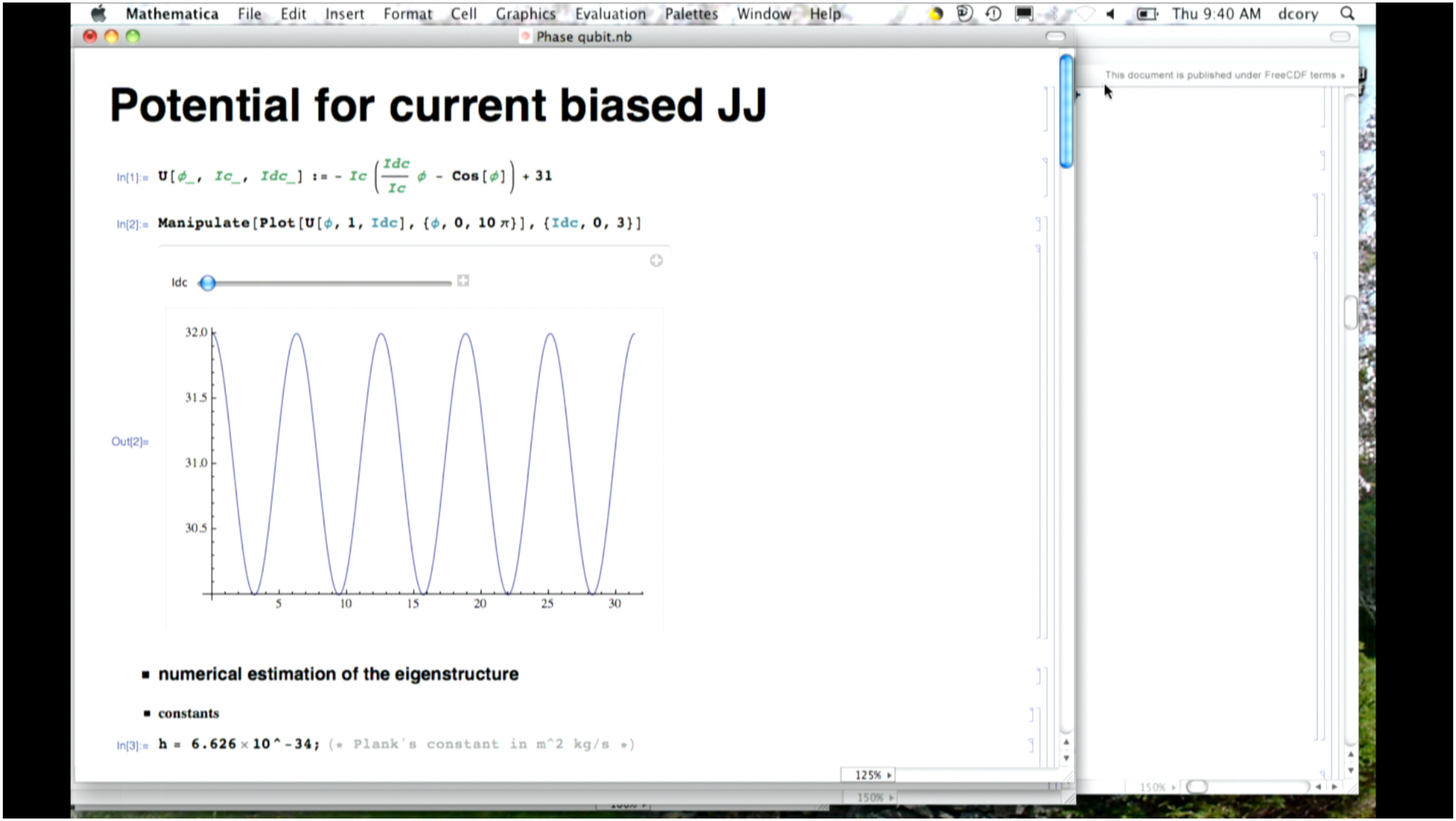
Ohms

② $\frac{d\phi}{dt} = \frac{2e}{\hbar}$

$$I_{dc} = I_R + I_{JJ} + I_{cap}$$



U(ϕ)
2 levels
qubit
Idc=0
Idc=Ic
ϕ



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Phase qubit.nb
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Potential for current biased JJ
In[1]:= U[ϕ_, Ic_, Idc_] := - Ic (Idc/Ic ϕ - Cos[ϕ]) + 31
In[2]:= Manipulate[Plot[U[ϕ, 1, Idc], {ϕ, 0, 10 π}], {Idc, 0, 3}]
Idc
Out[2]=
32.0
31.5
31.0
30.5
5
10
15
20
25
30
numerical estimation of the eigenstructure
constants
In[3]:= h = 6.626 × 10^-34; (* Plank's constant in m^2 kg/s *)
125%

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Phase qubit.nb

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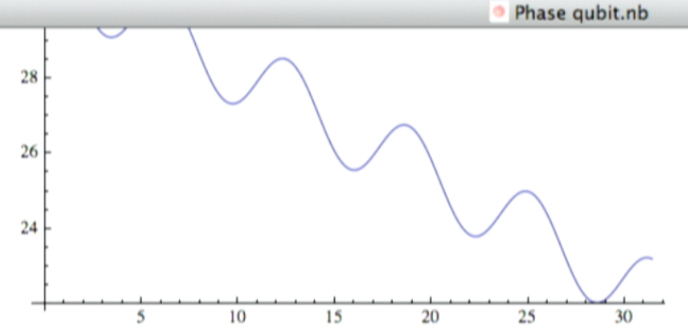
Out[2]=



- **numerical estimation of the eigenstructure**

  - **constants**

```
In[3]:= h = 6.626 × 10^-34; (* Plank's constant in m^2 kg/s *)

In[4]:= m = 9.11 × 10^-31; (* mass of an electron in kg *)

In[5]:= eV = 1.602 × 10^-19; (* J *)

In[6]:= t0 = h^2 / (8 π^2 m);
```

  - **define finite difference Hamiltonian**

```
In[7]:= fdH[Idc_] :=
          t0 (DiagonalMatrix[Table[2 + U[(n - 1) (10 π / 99), 1, Idc], {n, 1, 100}]] +
             DiagonalMatrix[Table[-1, {99}], 1] + DiagonalMatrix[Table[-1, {99}], -1]);

In[8]:= vs[Idc_] := Reverse[Eigenvalues[fdH[Idc]]];

In[9]:= Manipulate[ListPlot[vs[Idc] 2 π m / h / eV, {PlotRange → All,
          AxesLabel → {"n", "En (eV)"}}], {Idc, 0, 2}, SaveDefinitions → True]
```

Idc

125%

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Phase qubit.nb

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```
In[6]:= t0 = h^2 / (8 π^2 m);
```

- **define finite difference Hamiltonian**

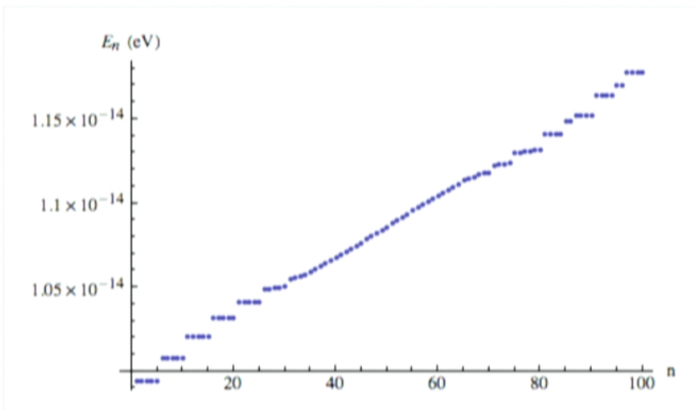
```
In[7]:= fdH[Idc_] :=
          t0 (DiagonalMatrix[Table[2 + U[(n - 1) (10 π / 99), 1, Idc], {n, 1, 100}]] +
             DiagonalMatrix[Table[-1, {99}], 1] + DiagonalMatrix[Table[-1, {99}], -1]);

In[8]:= vs[Idc_] := Reverse[Eigenvalues[fdH[Idc]]];

In[9]:= Manipulate[ListPlot[vs[Idc] 2 π m / h / eV, {PlotRange → All,
           AxesLabel → {"n", "En (eV)"}}], {Idc, 0, 2}, SaveDefinitions → True]
```

Idc

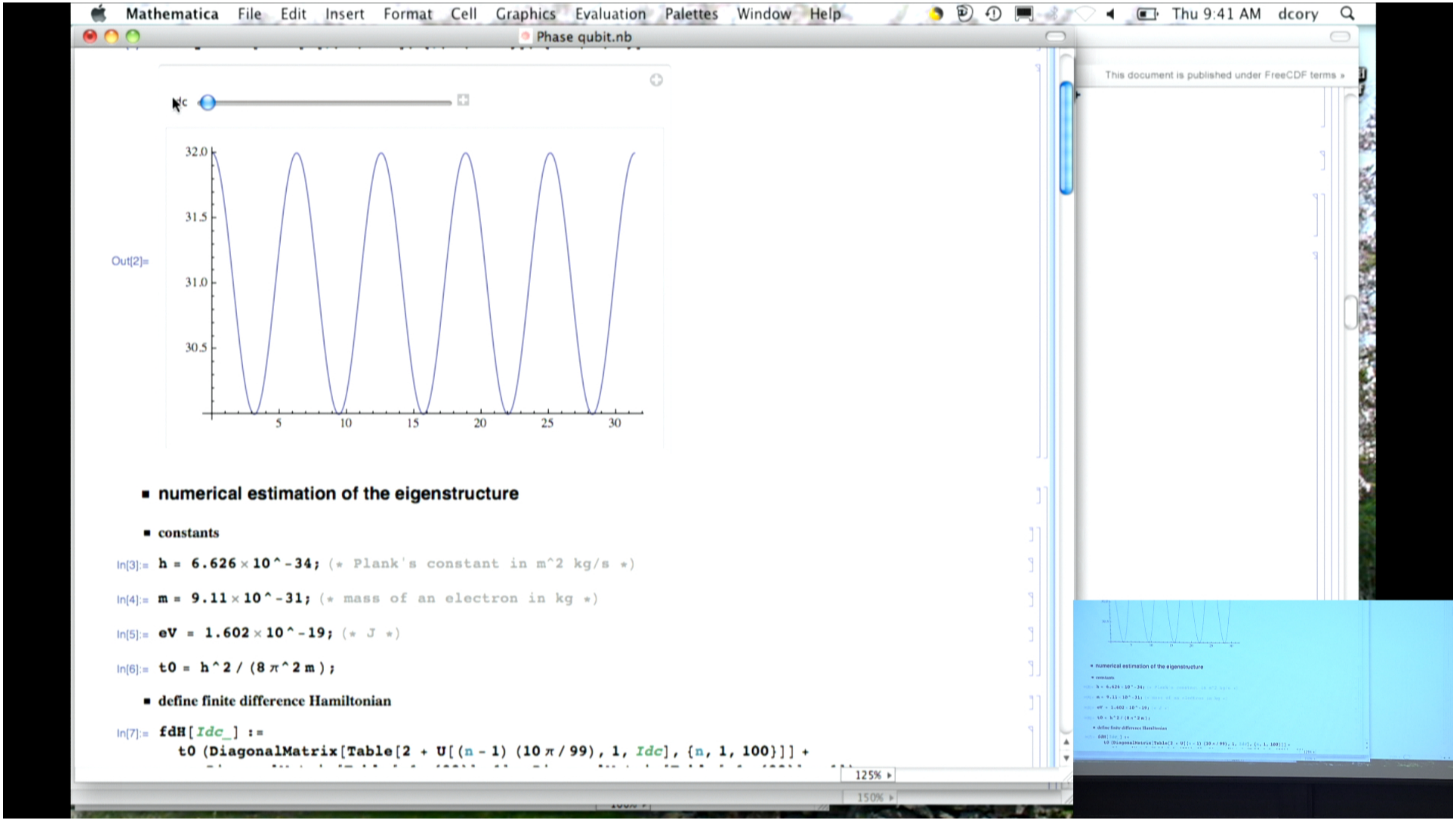
Out[9]=




```
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];

In[11]:= Manipulate[ListPlot[ Transpose[{x, ψ[Idc][[n]]}],
           {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
```

125%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Phase qubit.nb
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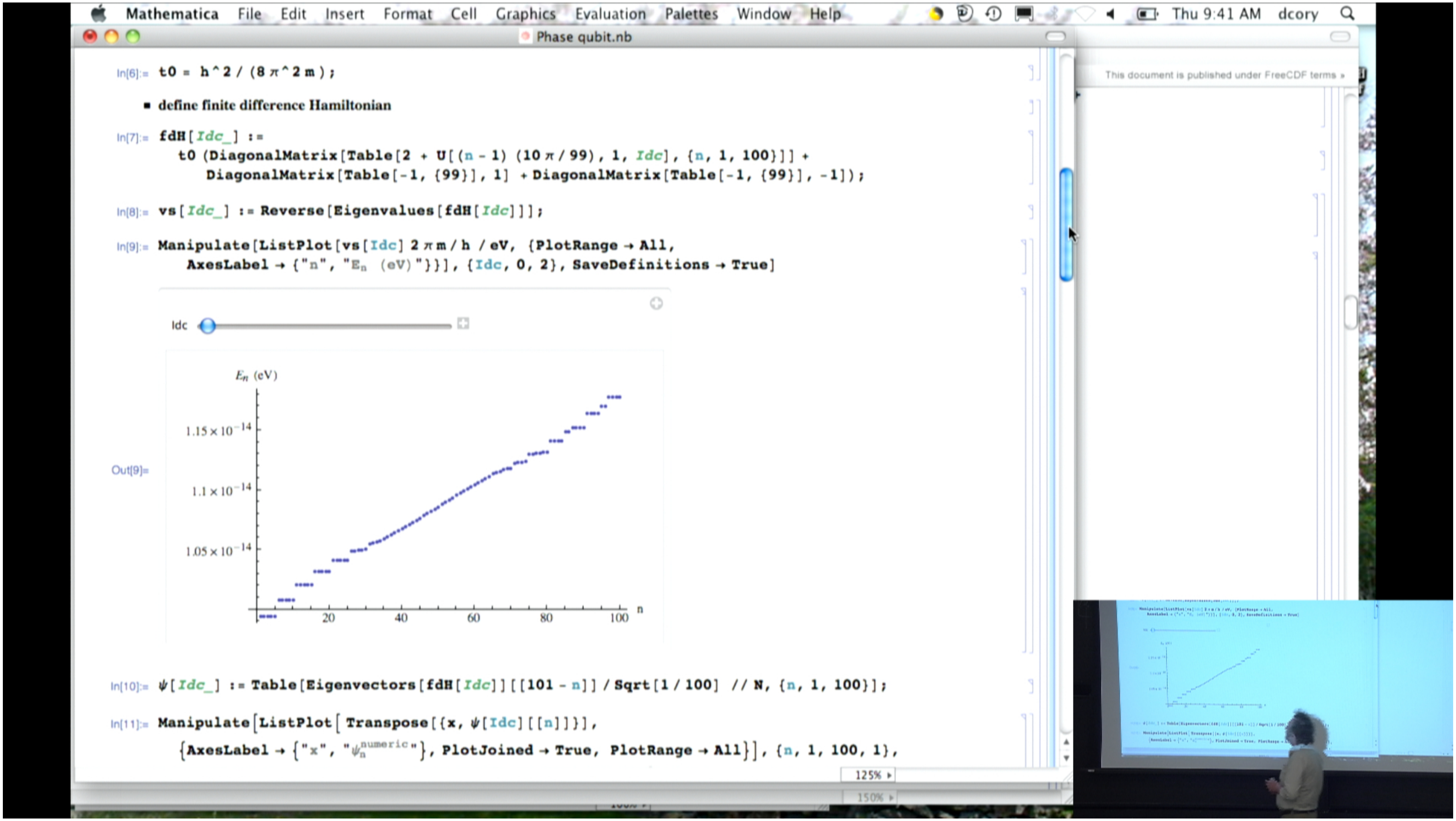
Out[2]=
numerical estimation of the eigenstructure
constants
In[3]:= h = 6.626 × 10^-34; (* Plank's constant in m^2 kg/s *)
In[4]:= m = 9.11 × 10^-31; (* mass of an electron in kg *)
In[5]:= eV = 1.602 × 10^-19; (* J *)
In[6]:= t0 = h^2 / (8 π^2 m);
define finite difference Hamiltonian
In[7]:= fdH[Idc_] :=
t0 (DiagonalMatrix[Table[2 + U[(n - 1) (10 π / 99), 1, Idc], {n, 1, 100}]] +
125%
150%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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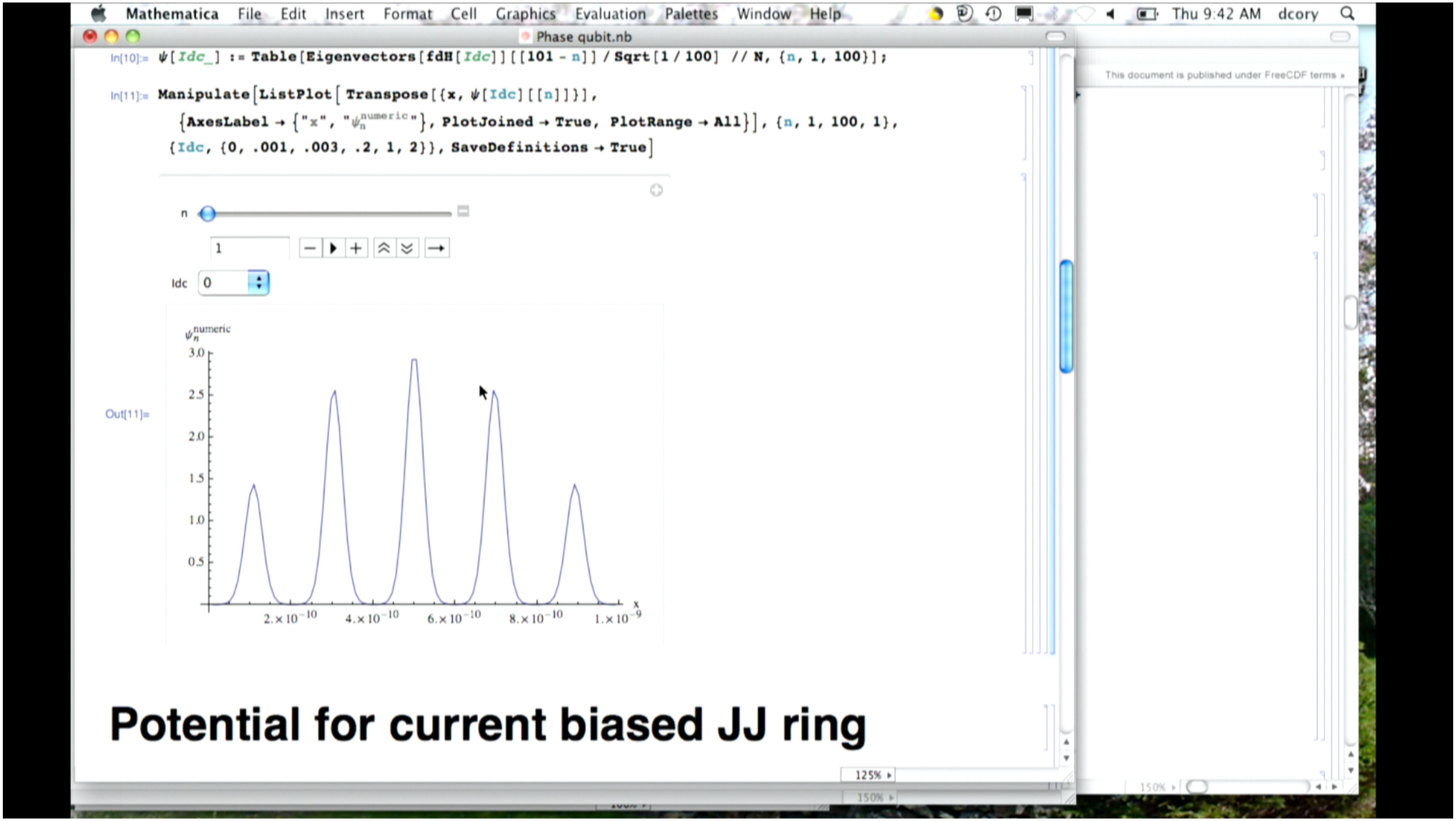
Phase qubit.nb
This document is published under FreeCDF terms »
In[6]:= t0 = h^2 / (8 π^2 m);
define finite difference Hamiltonian
In[7]:= fdH[Idc_] :=
t0 (DiagonalMatrix[Table[2 + U[(n - 1) (10 π / 99), 1, Idc], {n, 1, 100}]] +
DiagonalMatrix[Table[-1, {99}], 1] + DiagonalMatrix[Table[-1, {99}], -1]);
In[8]:= vs[Idc_] := Reverse[Eigenvalues[fdH[Idc]]];
In[9]:= Manipulate[ListPlot[vs[Idc] 2 π m / h / eV, {PlotRange → All,
AxesLabel → {"n", "E_n (eV)"}}], {Idc, 0, 2}, SaveDefinitions → True]
Idc
Out[9]=
E_n (eV)
1.15 × 10^-14
1.1 × 10^-14
1.05 × 10^-14
20
40
60
80
100
n
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
125%
150%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Phase qubit.nb
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
1
Idc 0
Out[11]=
ψn numeric
3.0
2.5
2.0
1.5
1.0
0.5
x
2. × 10^-10
4. × 10^-10
6. × 10^-10
8. × 10^-10
1. × 10^-9
Potential for current biased JJ ring
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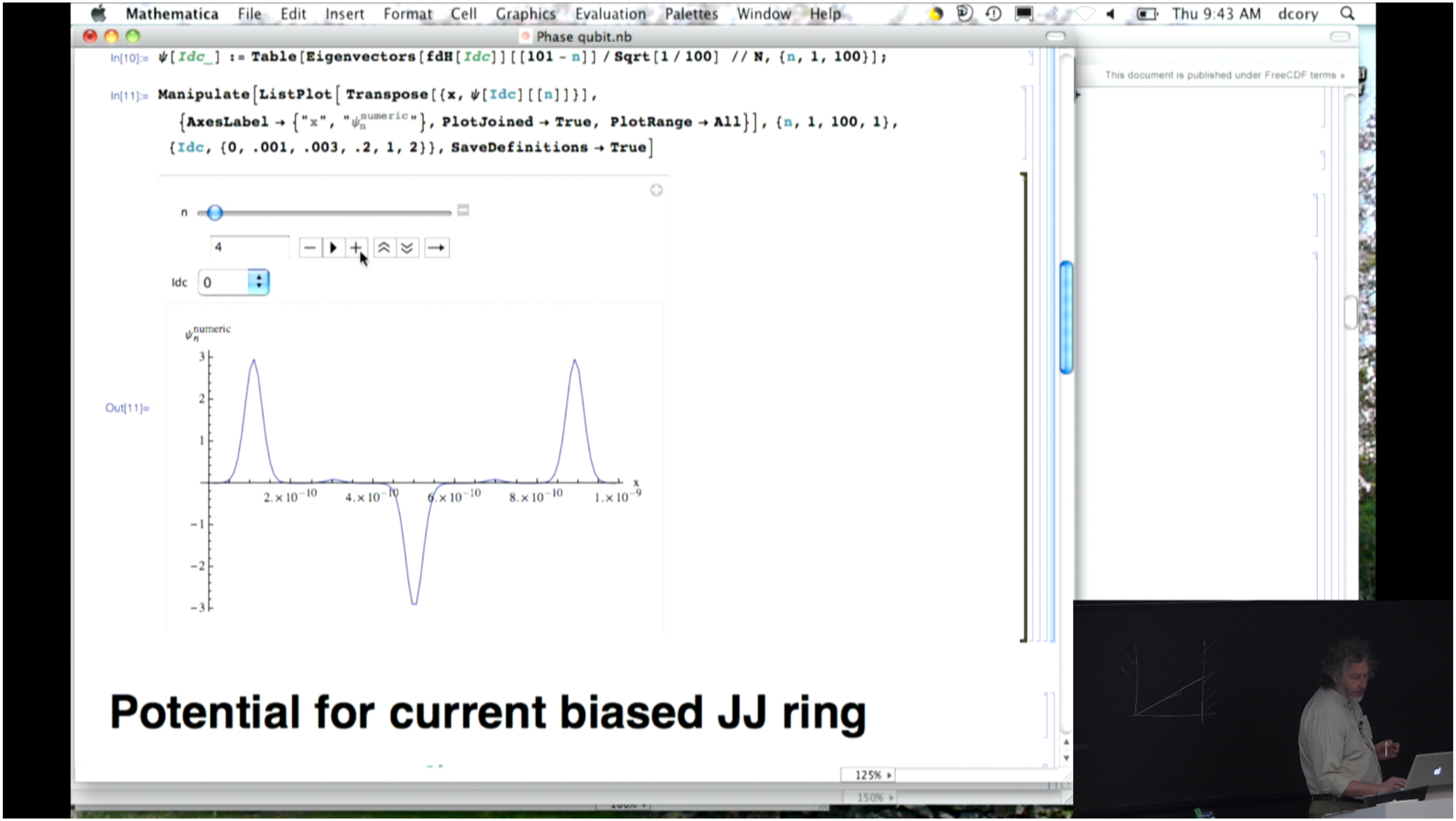
125%
150%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Phase qubit.nb
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
4
Idc
0
Out[11]=
ψn numeric
3
2
1
-1
-2
-3
2.×10^-10
4.×10^-10
6.×10^-10
8.×10^-10
1.×10^-9
x
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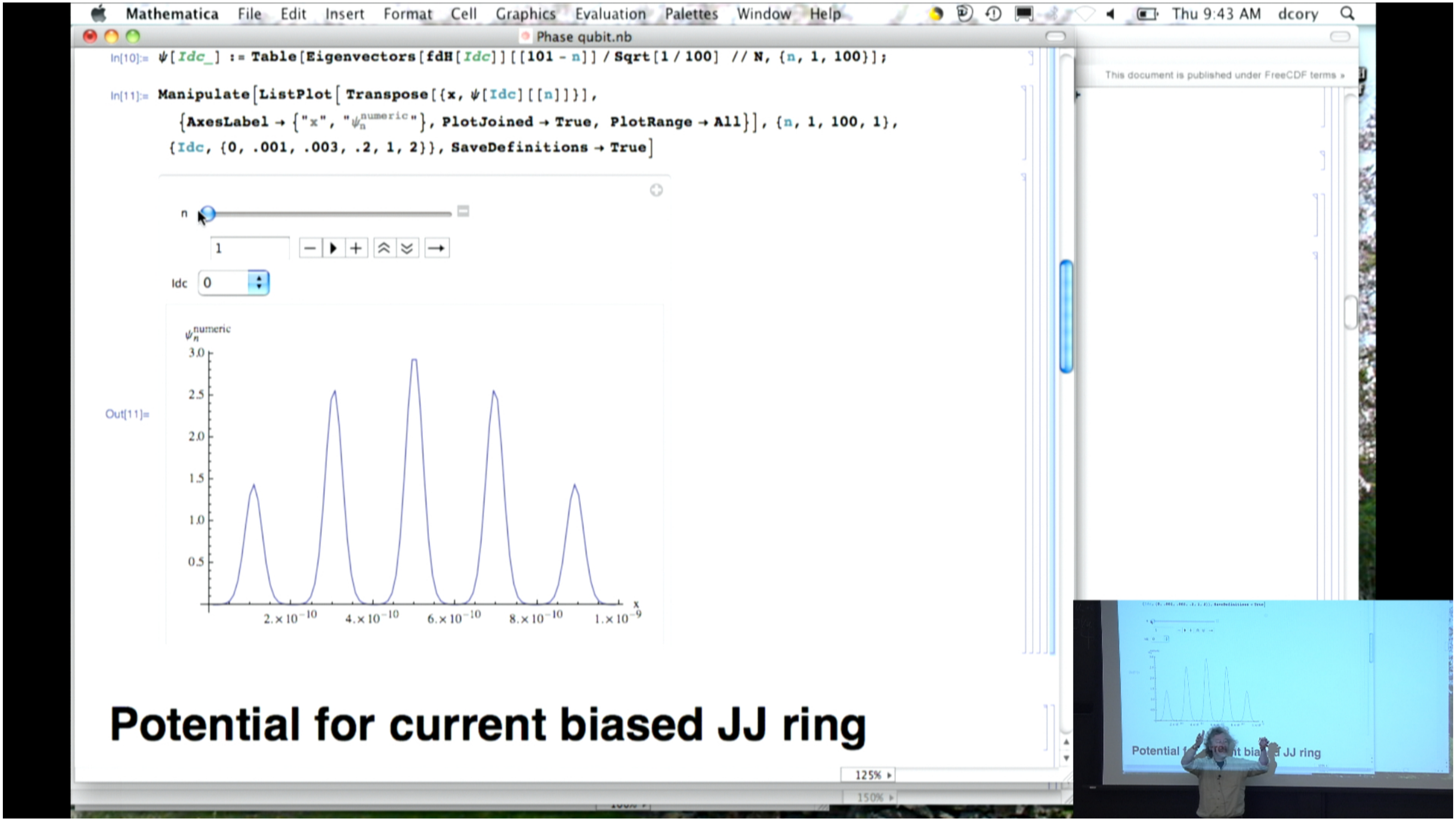
Potential for current biased JJ ring
125%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 9:43 AM dcory
Phase qubit.nb
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψ_n^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
1
Idc 0
Out[11]=
ψ_n^numeric
3.0
2.5
2.0
1.5
1.0
0.5
x
2.×10^-10
4.×10^-10
6.×10^-10
8.×10^-10
1.×10^-9
Potential for current biased JJ ring
125%
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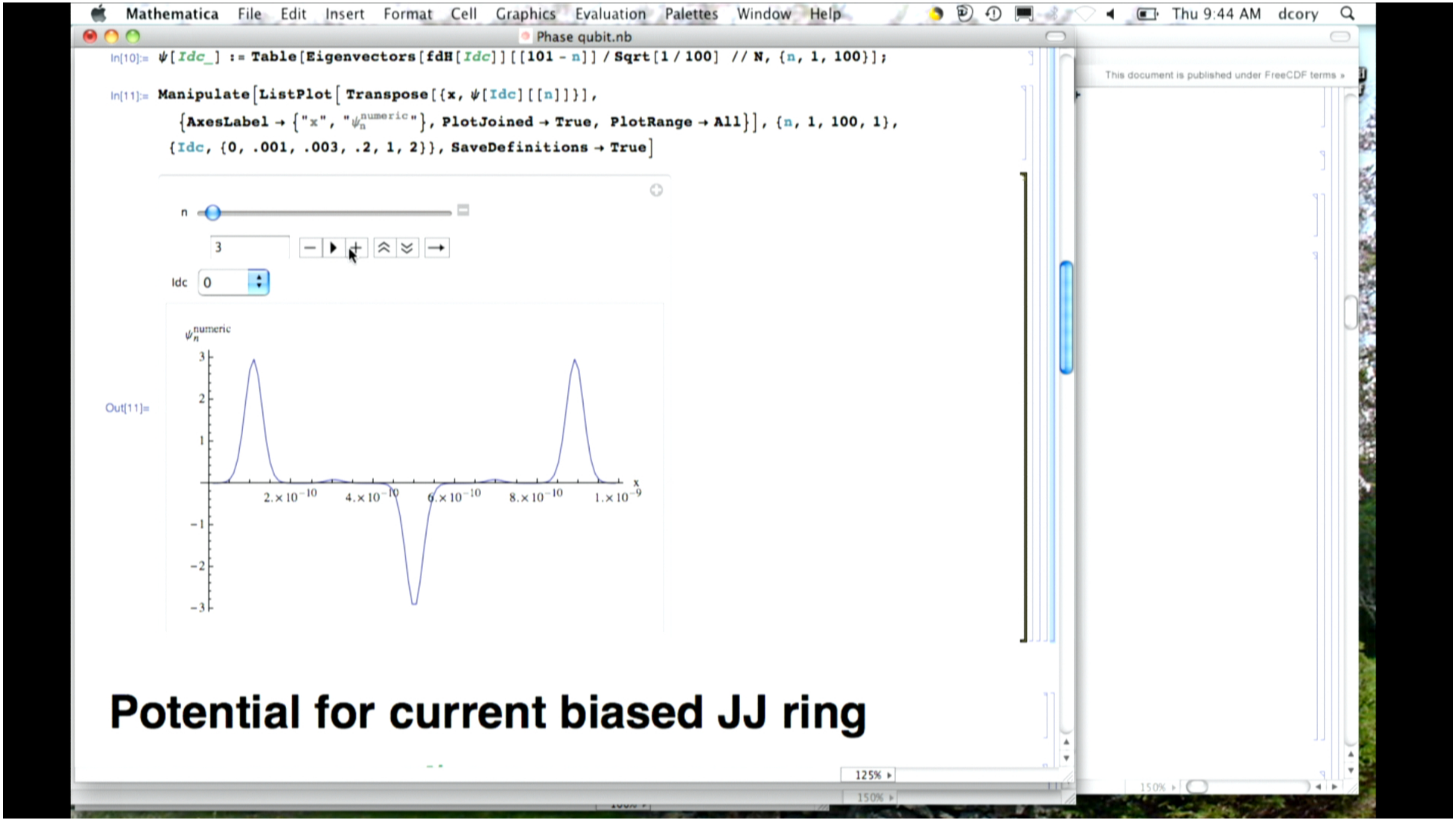
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Phase qubit.nb
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
3
Idc 0
Out[11]=
ψn numeric
x
2.×10^-10
4.×10^-10
6.×10^-10
8.×10^-10
1.×10^-9
Potential for current biased JJ ring
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125%

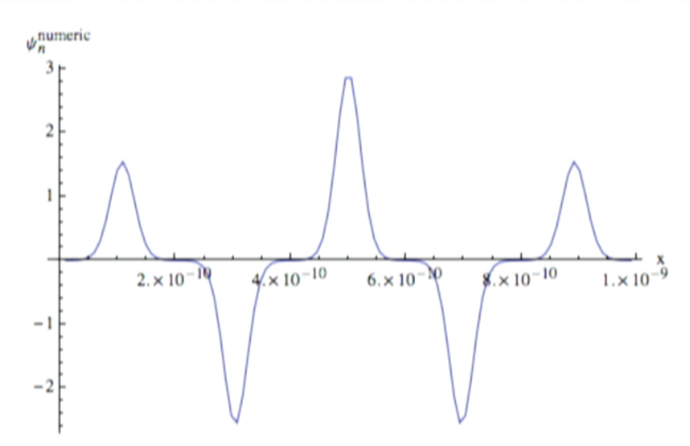
```
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];

In[11]:= Manipulate[ListPlot[ Transpose[{x, ψ[Idc][[n]]}],
    {AxesLabel → {"x", "ψn^numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
   {Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
```

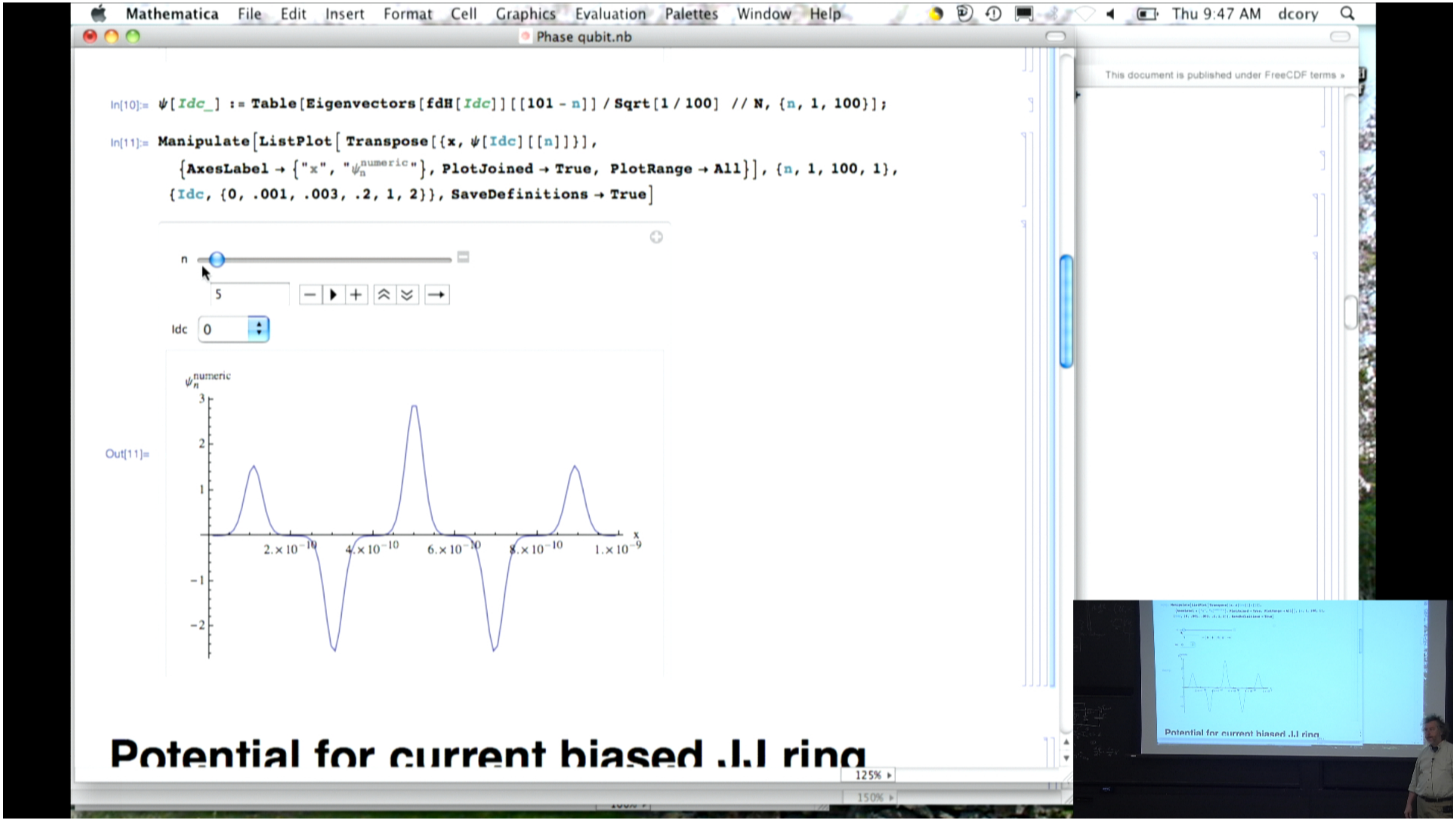
n 5

Idc 0

Out[11]=



Potential for current biased JJ ring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Phase qubit.nb
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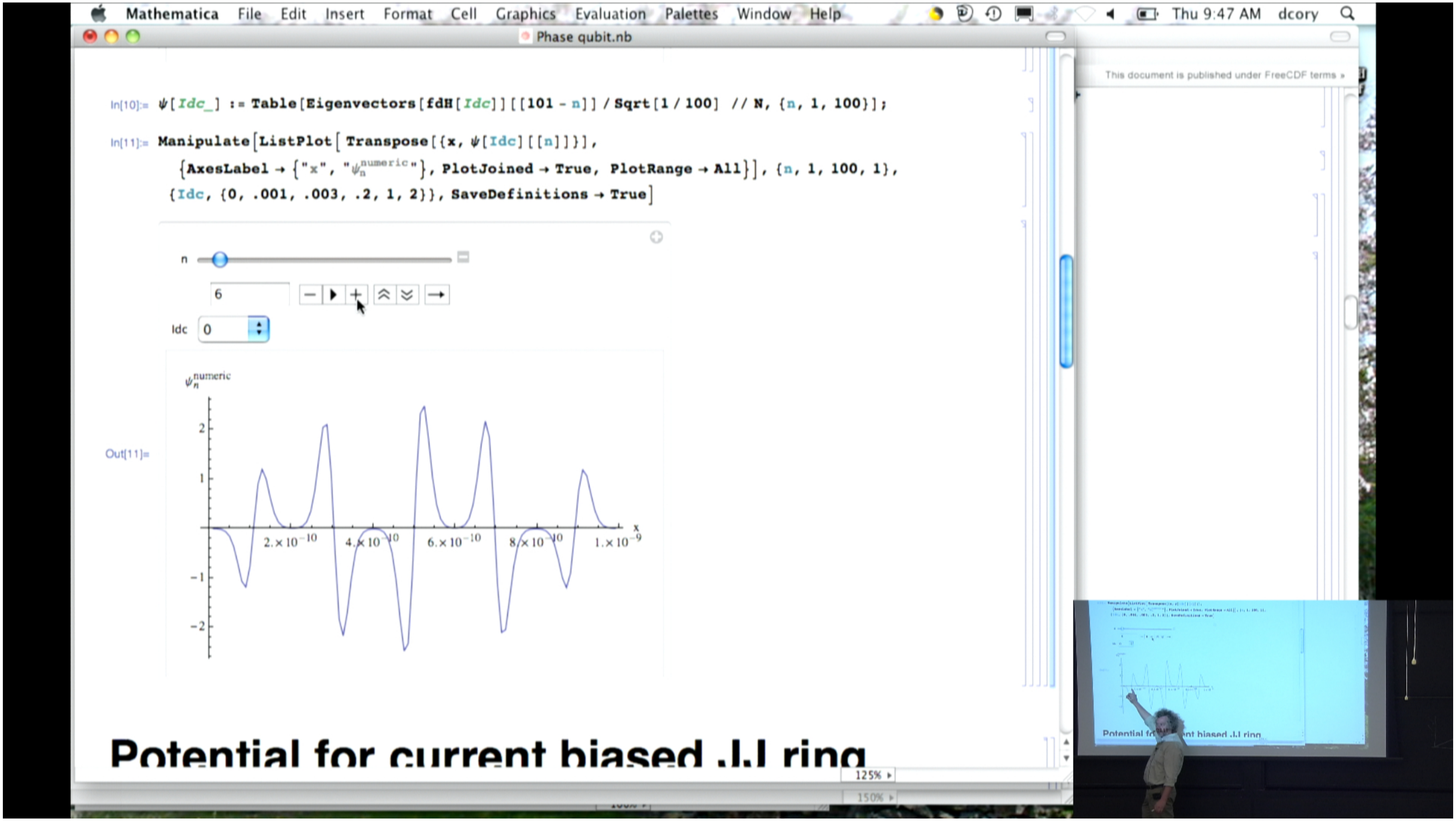
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
5
Idc 0
Out[11]=
ψn numeric
3
2
1
−1
−2
2.×10−10
4.×10−10
6.×10−10
8.×10−10
1.×10−9
x
Potential for current biased JJ ring
125%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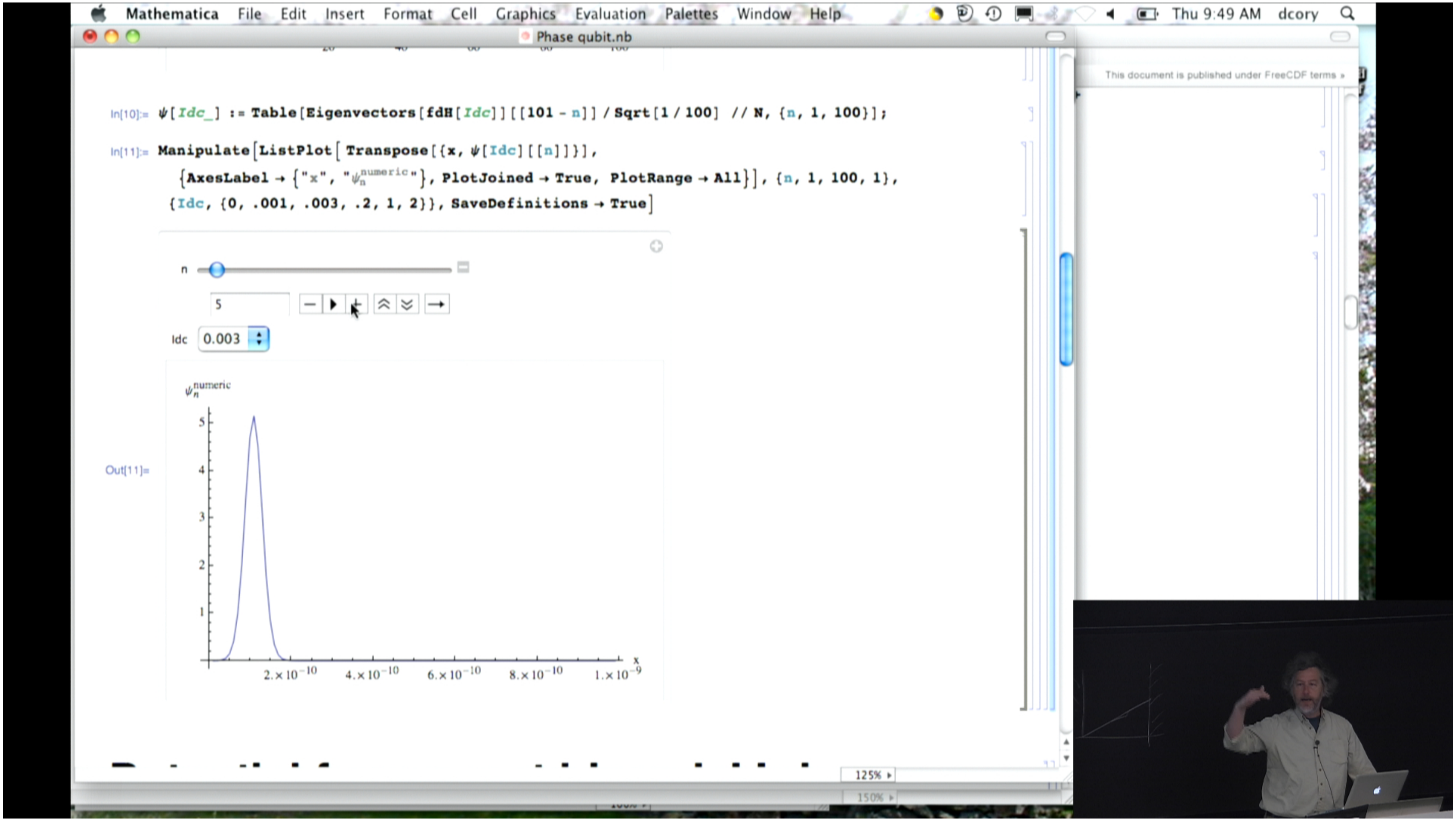
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
6
Idc 0
Out[11]=
ψn numeric
2
1
-1
-2
2. × 10^-10
4. × 10^-10
6. × 10^-10
8. × 10^-10
1. × 10^-9
x
Potential for current biased JJ ring
125%



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Phase qubit.nb
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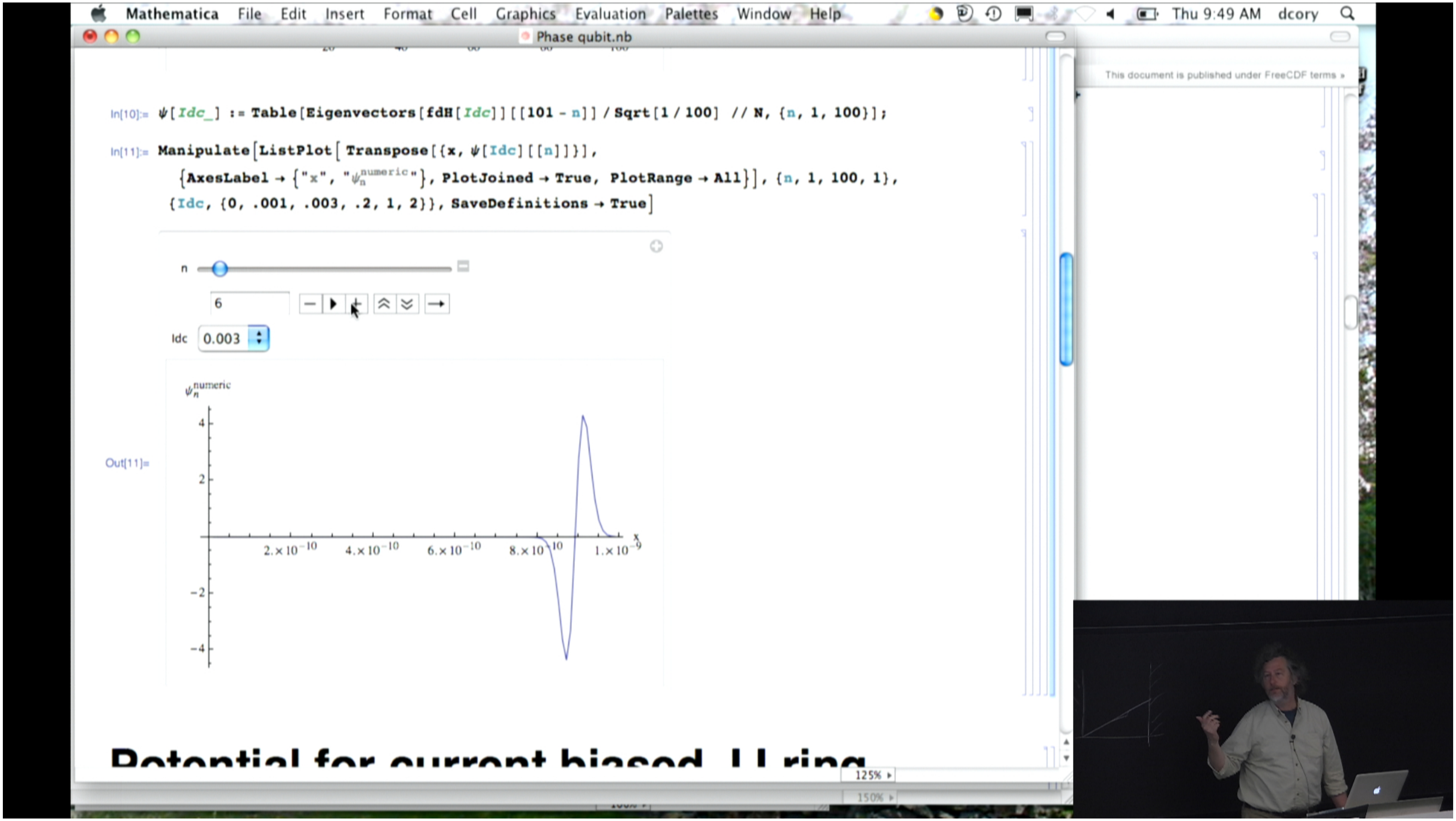
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
5
Idc 0.003
Out[11]=
ψn numeric
x
2.× 10^-10
4.× 10^-10
6.× 10^-10
8.× 10^-10
1.× 10^-9
125%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Phase qubit.nb
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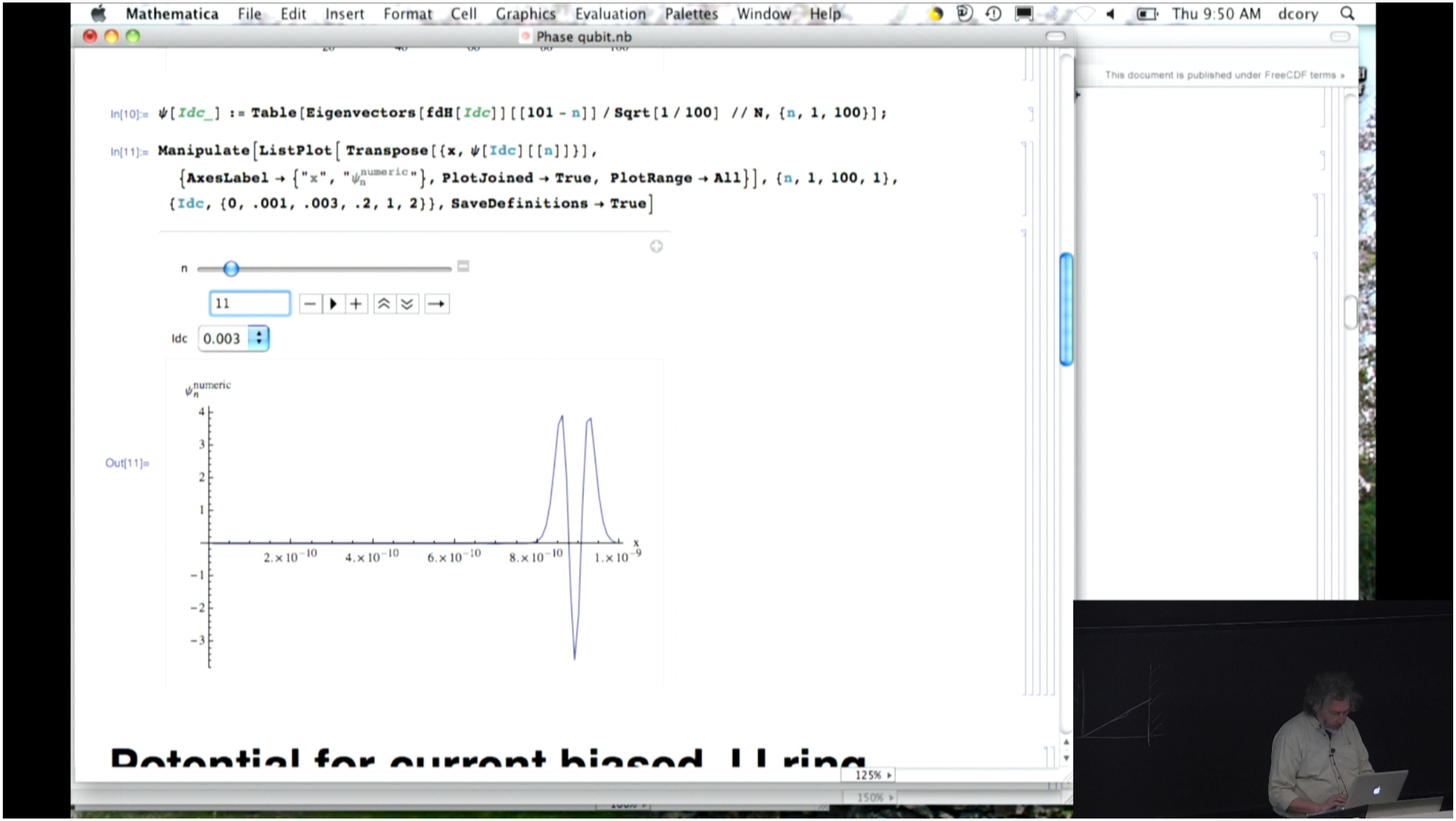
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψnumeric n"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
6
Idc 0.003
Out[11]=
ψnumeric n
4
2
-2
-4
2.× 10^-10
4.× 10^-10
6.× 10^-10
8.× 10^-10
1.× 10^-9
x
Potential for current biased JJ ring
125%
150%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Phase qubit.nb
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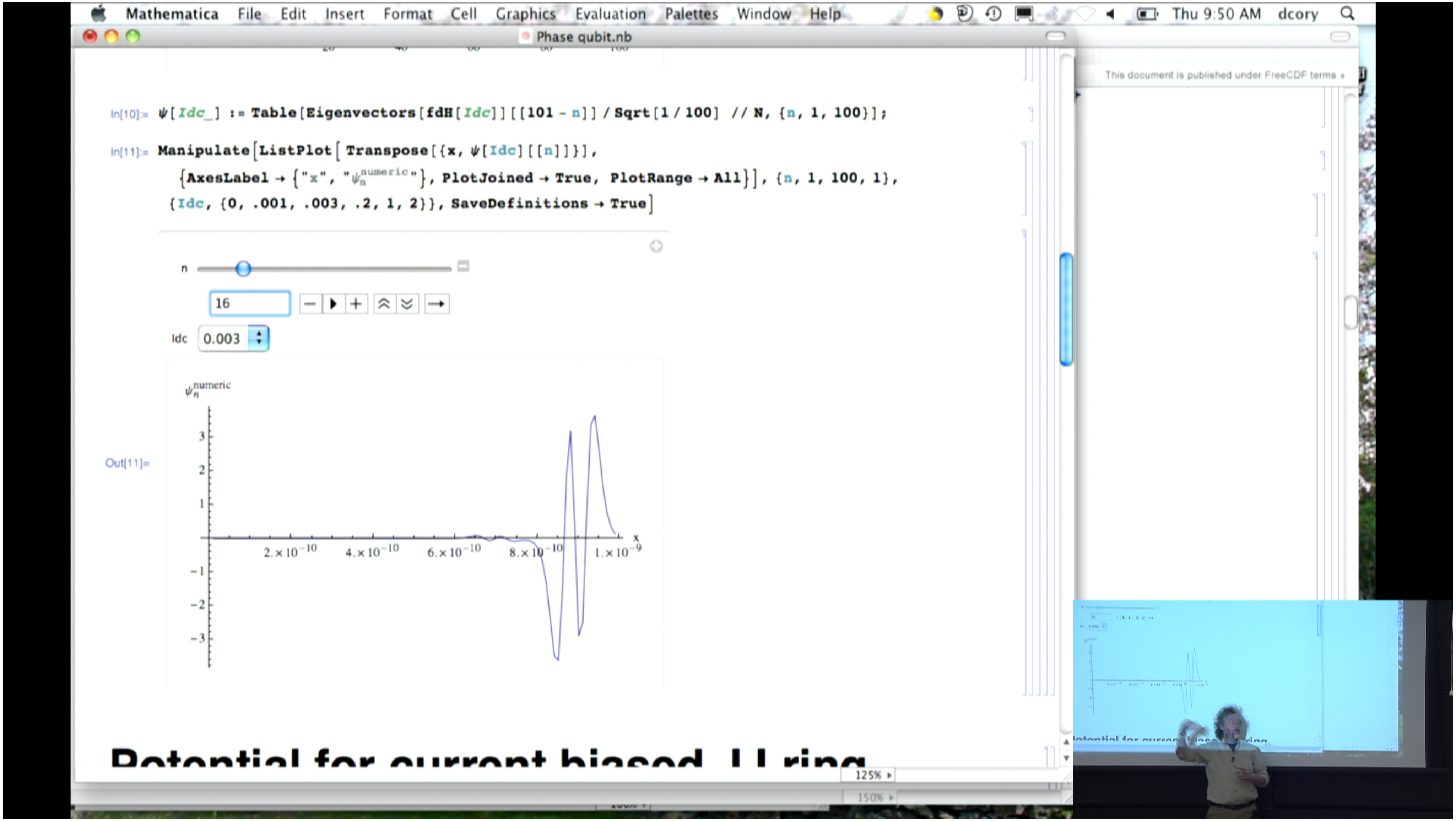
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψnumeric n"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
11
Idc 0.003
Out[11]=
ψnumeric n
x
2.× 10−10
4.× 10−10
6.× 10−10
8.× 10−10
1.× 10−9
125%
150%



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Phase qubit.nb
In[10]:= ψ[Idc_] := Table[Eigenvectors[fdH[Idc]][[101 - n]] / Sqrt[1 / 100] // N, {n, 1, 100}];
In[11]:= Manipulate[ListPlot[Transpose[{x, ψ[Idc][[n]]}],
{AxesLabel → {"x", "ψn numeric"}, PlotJoined → True, PlotRange → All}], {n, 1, 100, 1},
{Idc, {0, .001, .003, .2, 1, 2}}, SaveDefinitions → True]
n
16
Idc 0.003
Out[11]=
ψn numeric
x
Potential for current biased JJ ring
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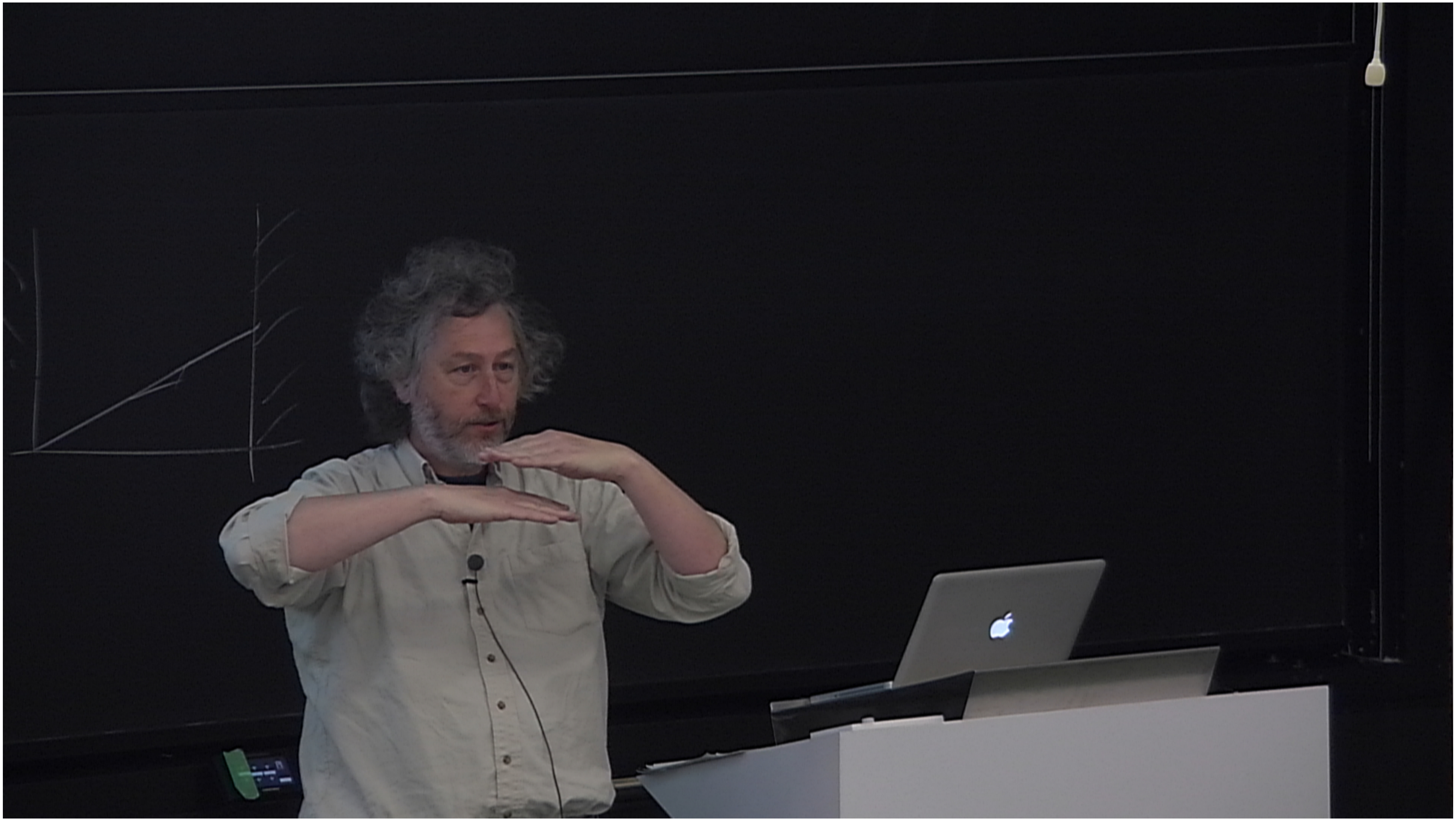

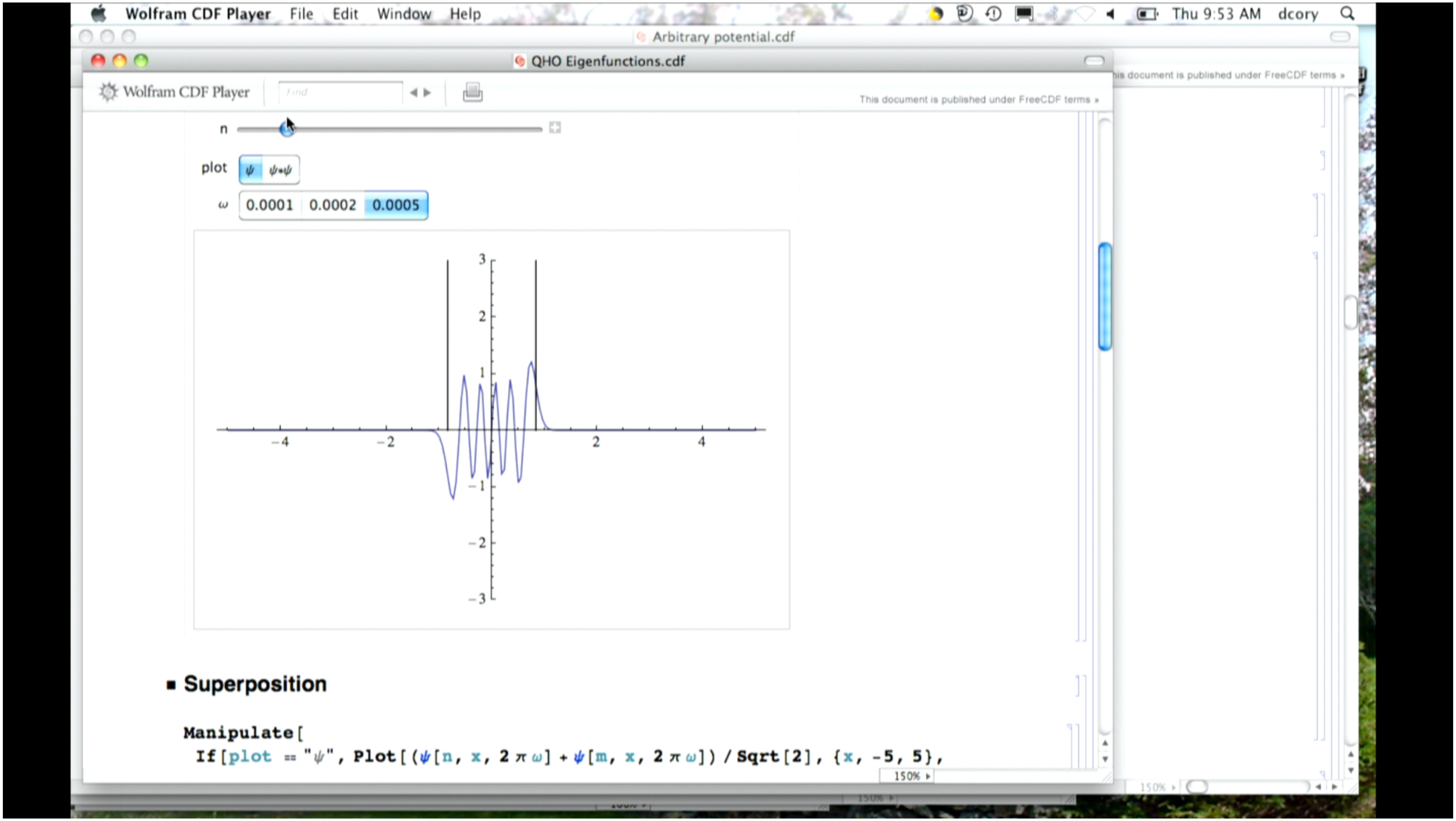
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Arbitrary potential.cdf
QHO Eigenfunctions.cdf
Wolfram CDF Player
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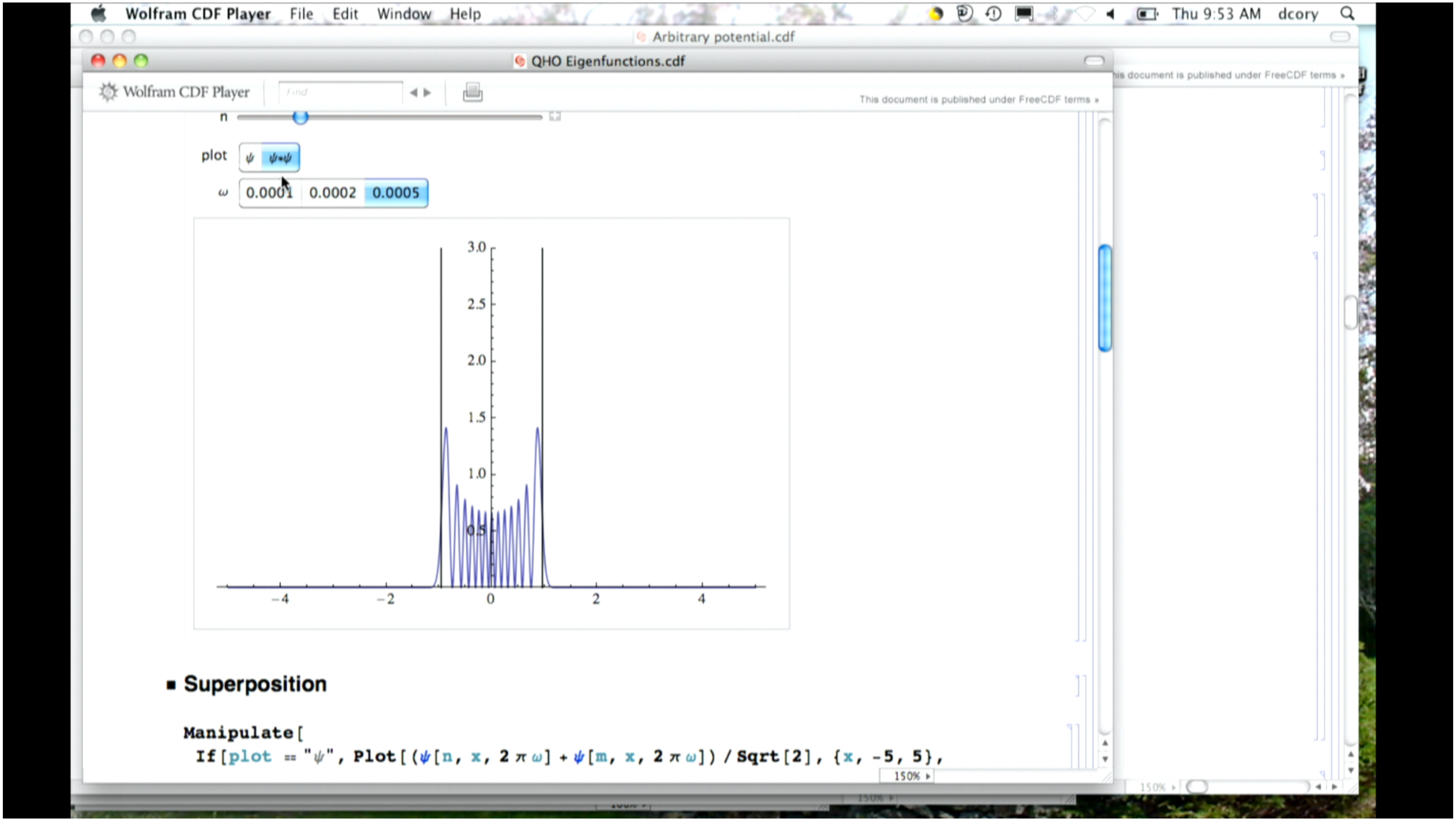
n
plot
ψ ψ*ψ
ω 0.0001 0.0002 0.0005
Superposition
Manipulate[
If[plot == "ψ", Plot[(ψ[n, x, 2 π ω] + ψ[m, x, 2 π ω]) / Sqrt[2], {x, -5, 5},
150%



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Thu 9:53 AM dcory
Arbitrary potential.cdf
QHO Eigenfunctions.cdf
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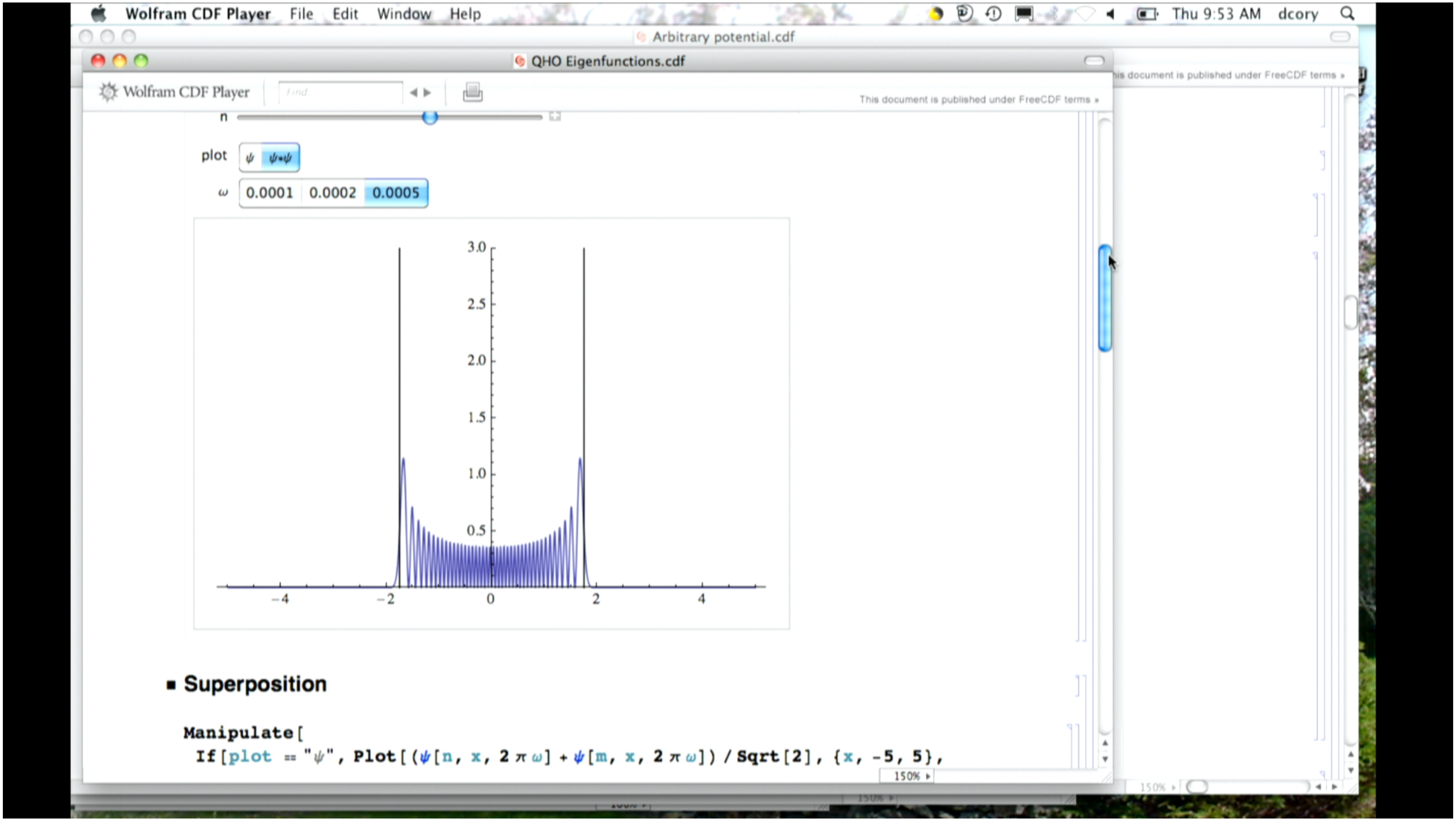
n
plot
ω 0.0001 0.0002 0.0005
3.0
2.5
2.0
1.5
1.0
0.5
−4 −2 0 2 4
Superposition
Manipulate[
If[plot == "ψ", Plot[(ψ[n, x, 2 π ω] + ψ[m, x, 2 π ω]) / Sqrt[2], {x, -5, 5},
150%



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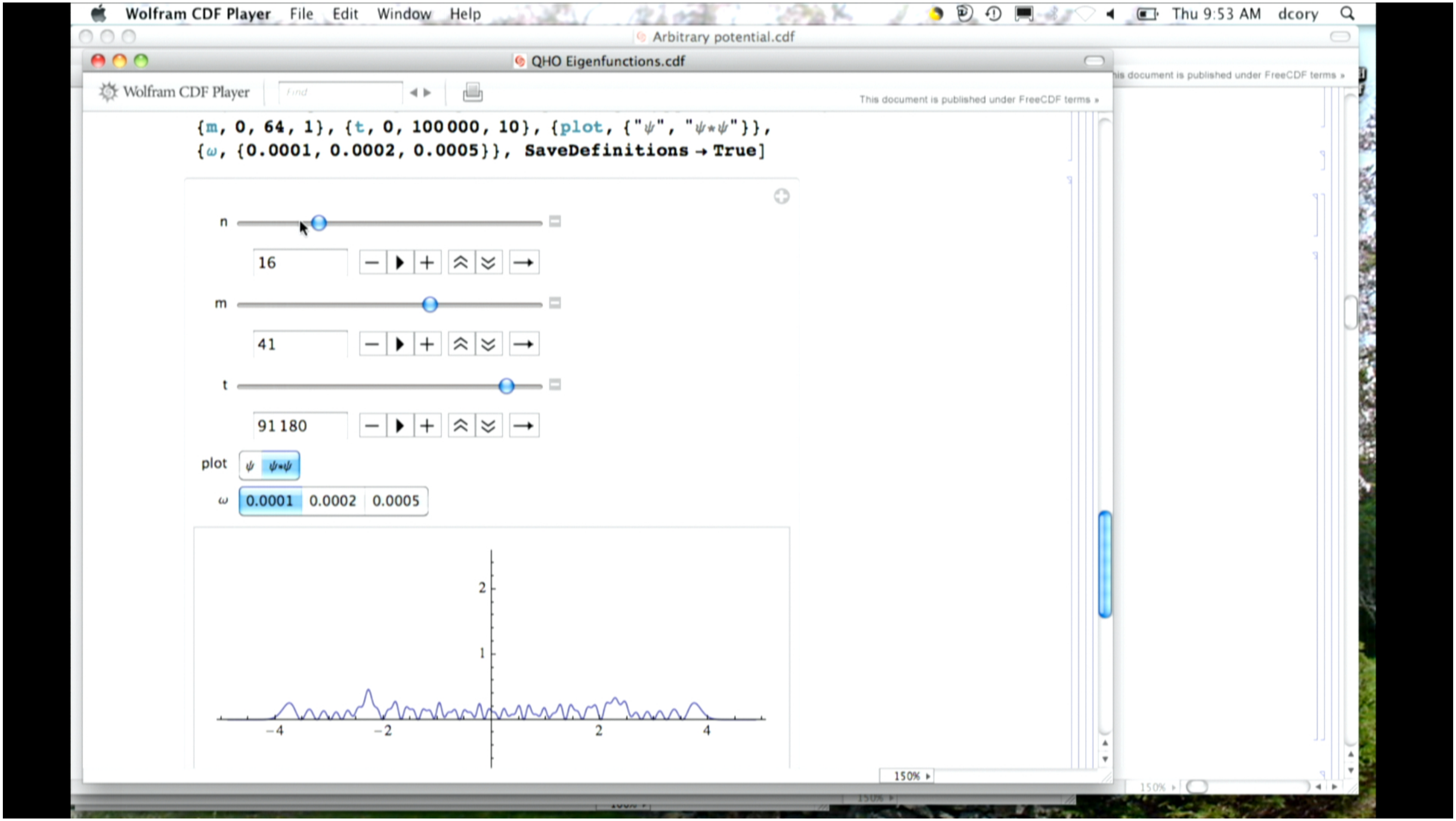
n
plot
ψ ψ*ψ
ω
0.0001 0.0002 0.0005
3.0
2.5
2.0
1.5
1.0
0.5
−4 −2 0 2 4
Superposition
Manipulate[
If[plot == "ψ", Plot[(ψ[n, x, 2 π ω] + ψ[m, x, 2 π ω]) / Sqrt[2], {x, -5, 5},
150%



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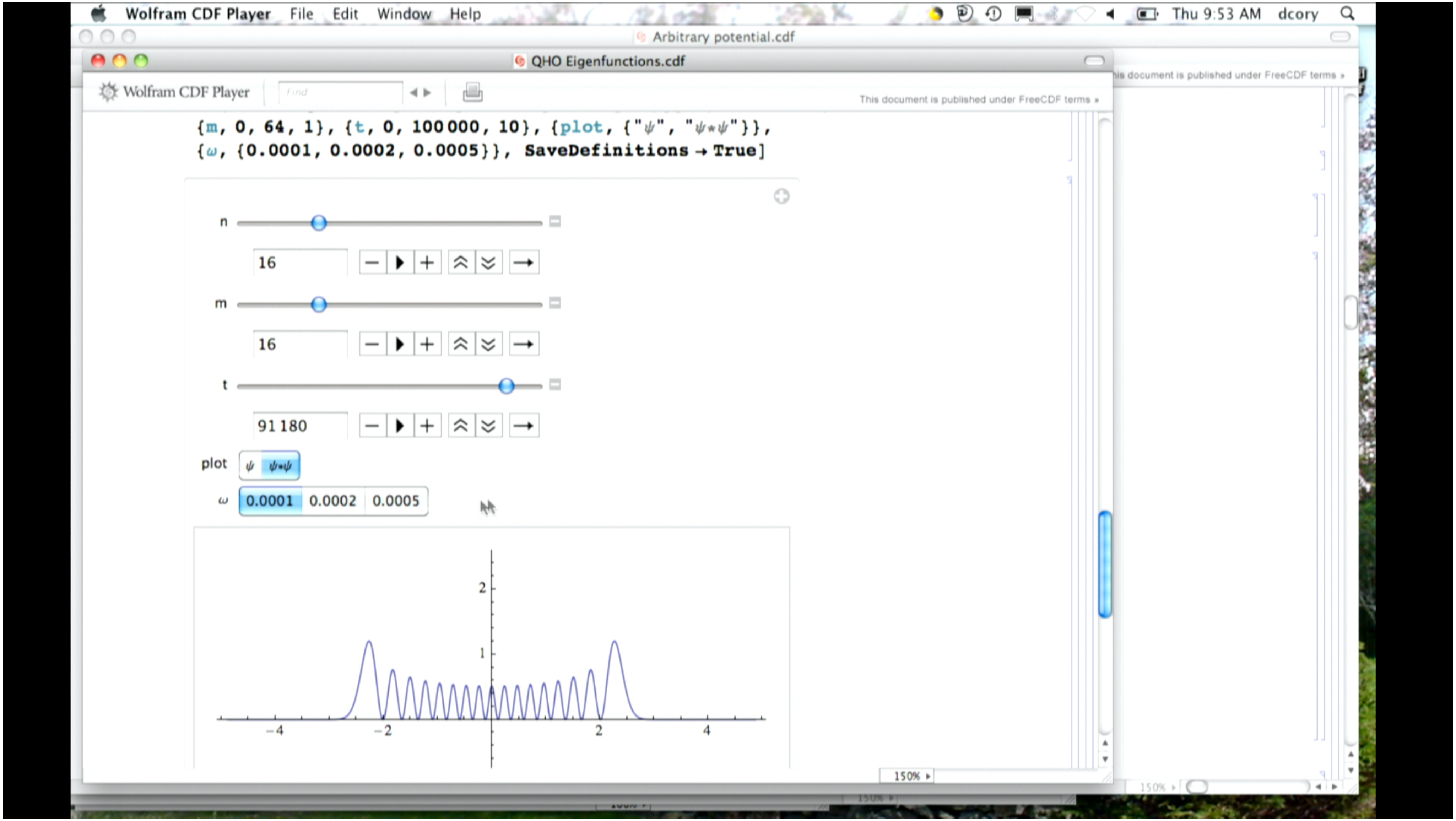
{m, 0, 64, 1}, {t, 0, 100 000, 10}, {plot, {"ψ", "ψ*ψ"}},
{ω, {0.0001, 0.0002, 0.0005}}, SaveDefinitions → True]
n
16
m
41
t
91 180
plot
ψ
ψ*ψ
ω
0.0001
0.0002
0.0005
2
1
−4
−2
2
4
150%



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QHO Eigenfunctions.cdf
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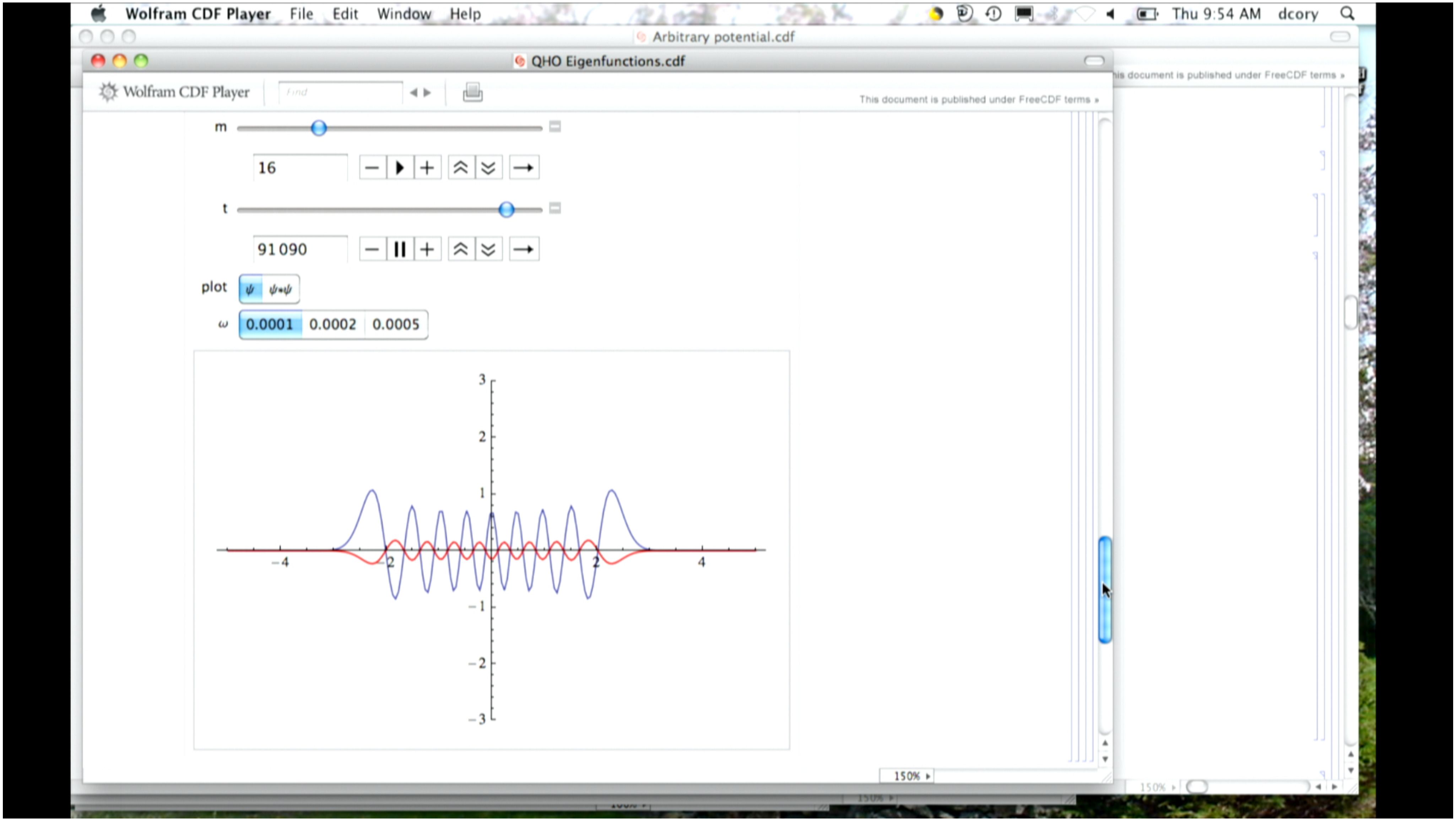
{m, 0, 64, 1}, {t, 0, 100000, 10}, {plot, {"ψ", "ψ*ψ"}},
{ω, {0.0001, 0.0002, 0.0005}}, SaveDefinitions → True]
n
16
m
16
t
91180
plot
ψ ψ*ψ
ω
0.0001 0.0002 0.0005
2
1
−4
−2
2
4
150%



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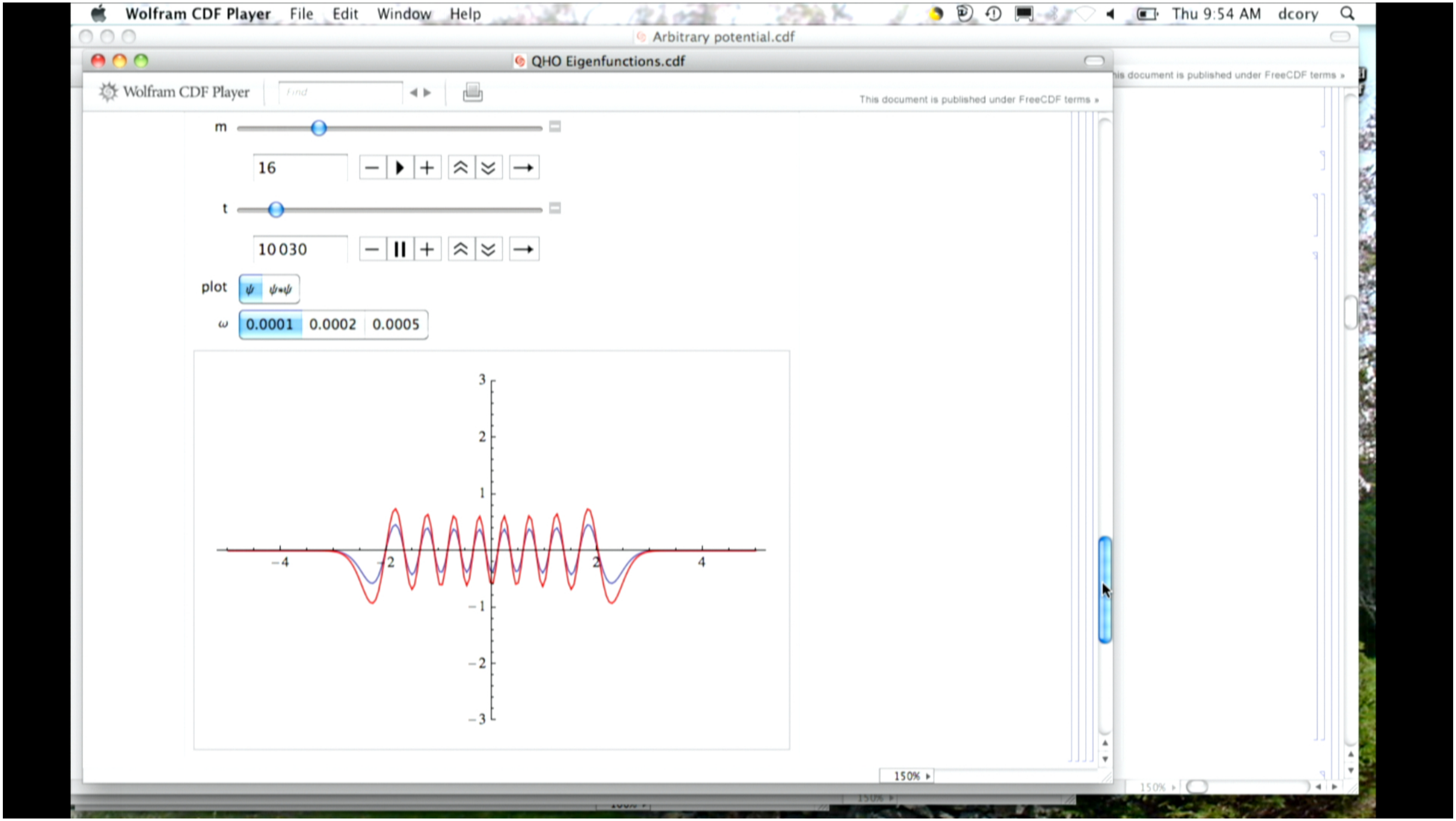
m
16
t
91 090
plot
ω
0.0001 0.0002 0.0005
150%



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QHO Eigenfunctions.cdf
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m
16
t
10030
plot
ω
0.0001 0.0002 0.0005
150%

"local"

start $\psi_n(\alpha)$ (tilt) $\equiv$ bound state

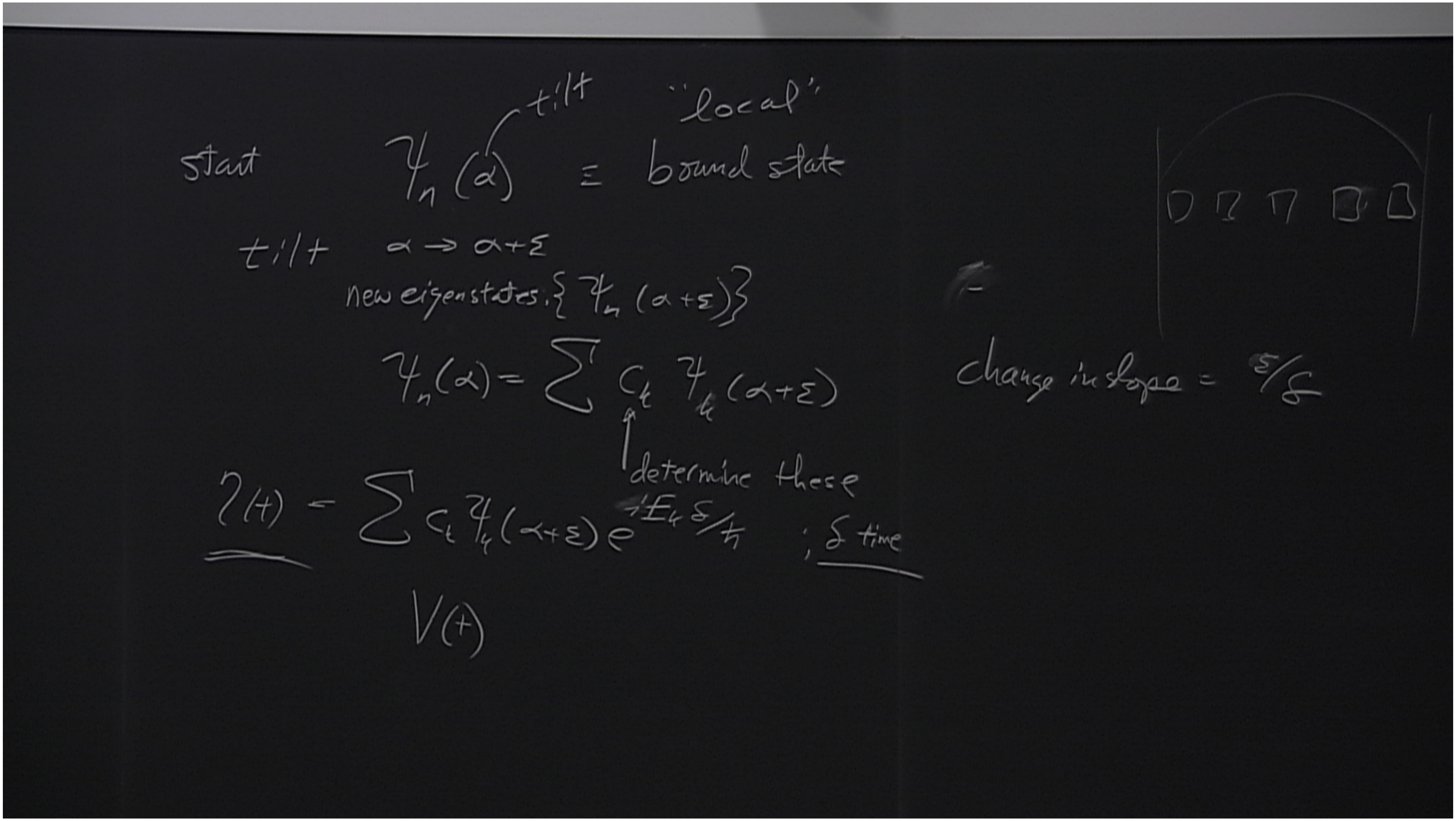
tilt $\alpha \to \alpha + \varepsilon$

new eigenstates: $\{\psi_n(\alpha+\varepsilon)\}$

$$\psi_n(\alpha) = \sum c_k \, \psi_k(\alpha+\varepsilon)$$

↑ determine these

$$\psi(t) = \sum c_k \, \psi_k(\alpha+\varepsilon)\, e^{-iE_k \delta/\hbar} \quad ; \ \delta \text{ time}$$


tilt
"local"
start ψn(α) ≡ bound state
tilt α → α+ε
new eigenstates: {ψn(α+ε)}
ψn(α) = Σ ck ψk(α+ε)
determine these
ψ(t) = Σ ck ψk(α+ε) e^(-iEk δ/ħ) ; δ time
V(t)
change in slope = ε/δ