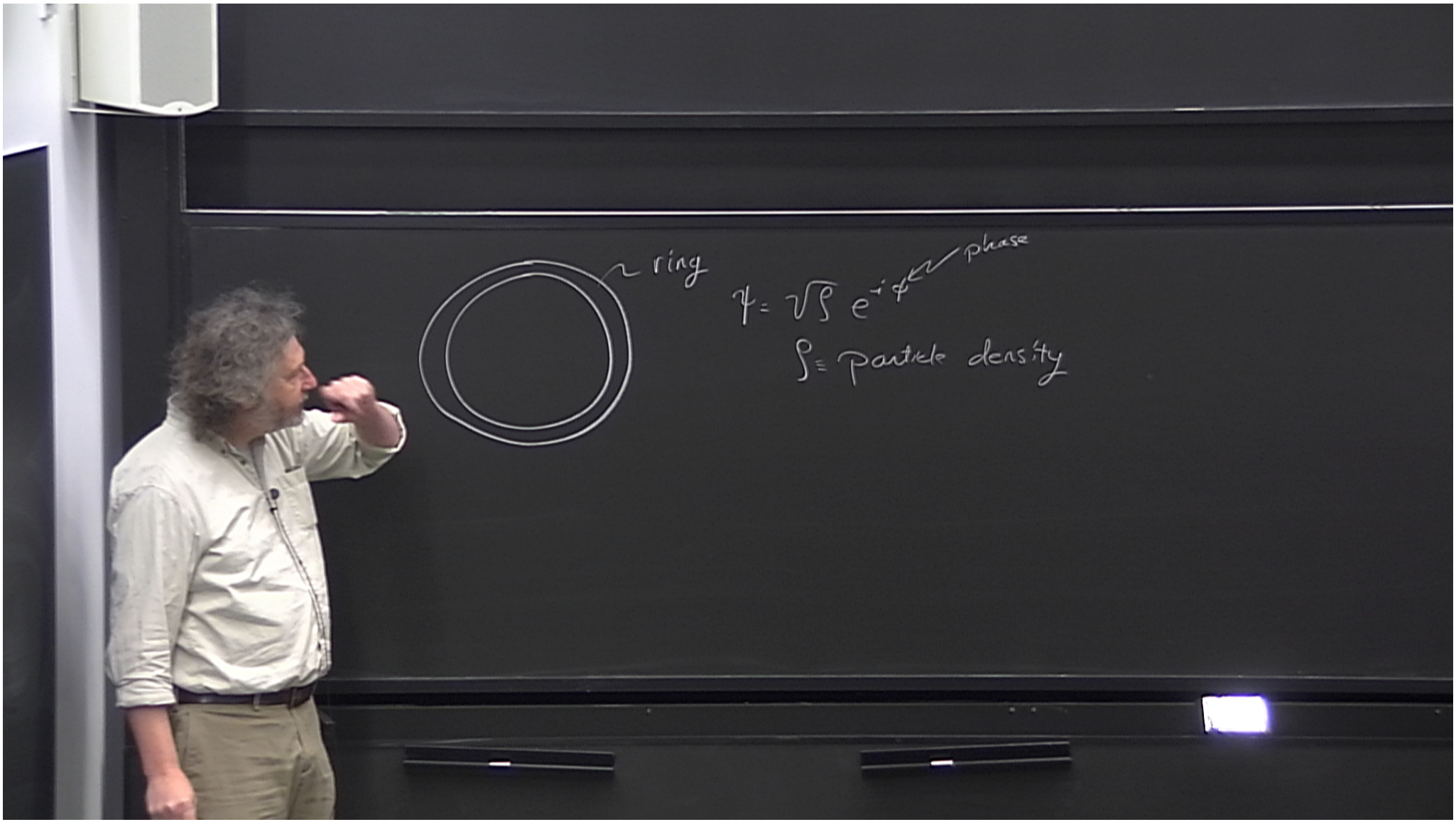
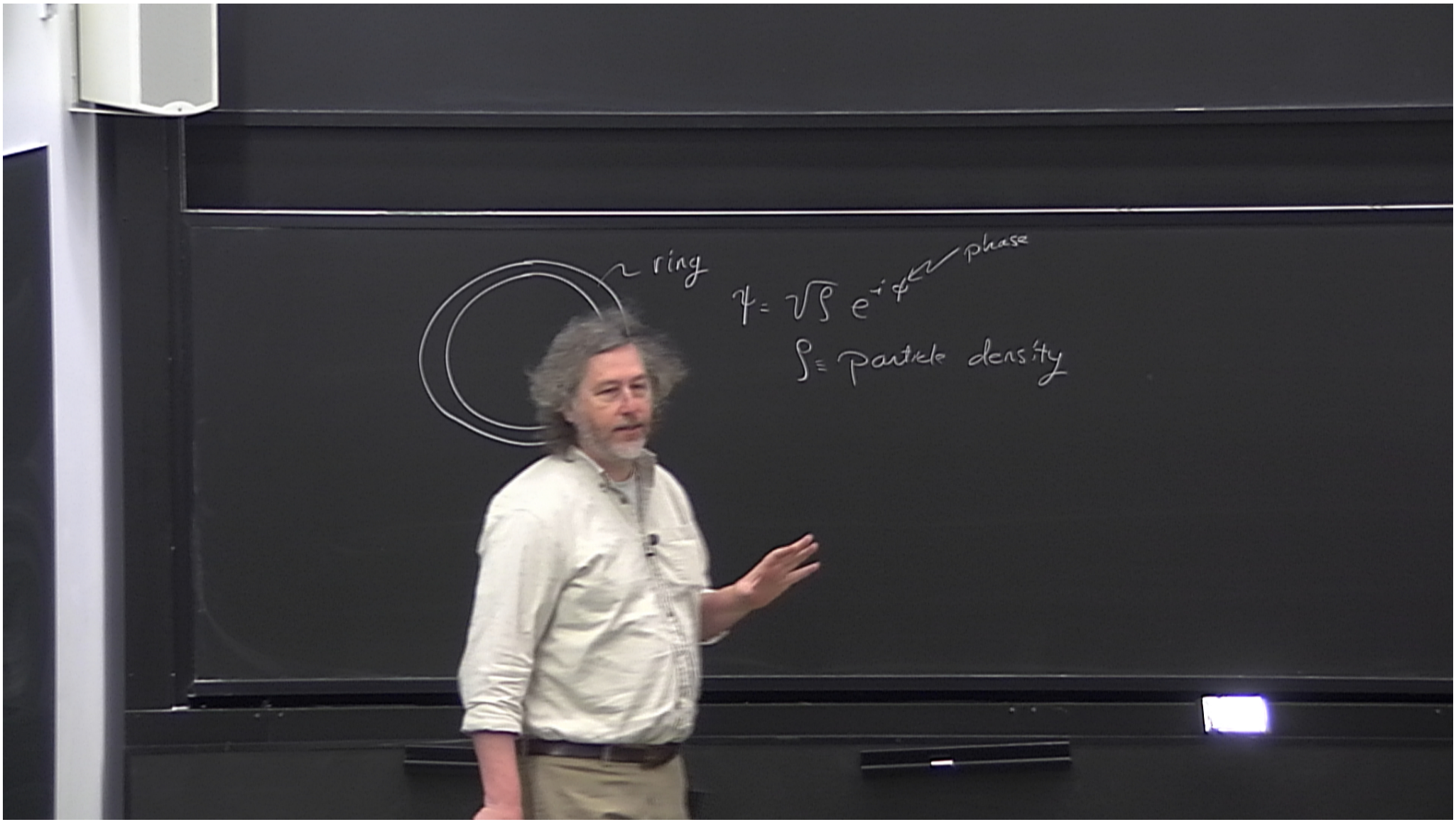
Title: Explorations in Quantum Information - Lecture 12

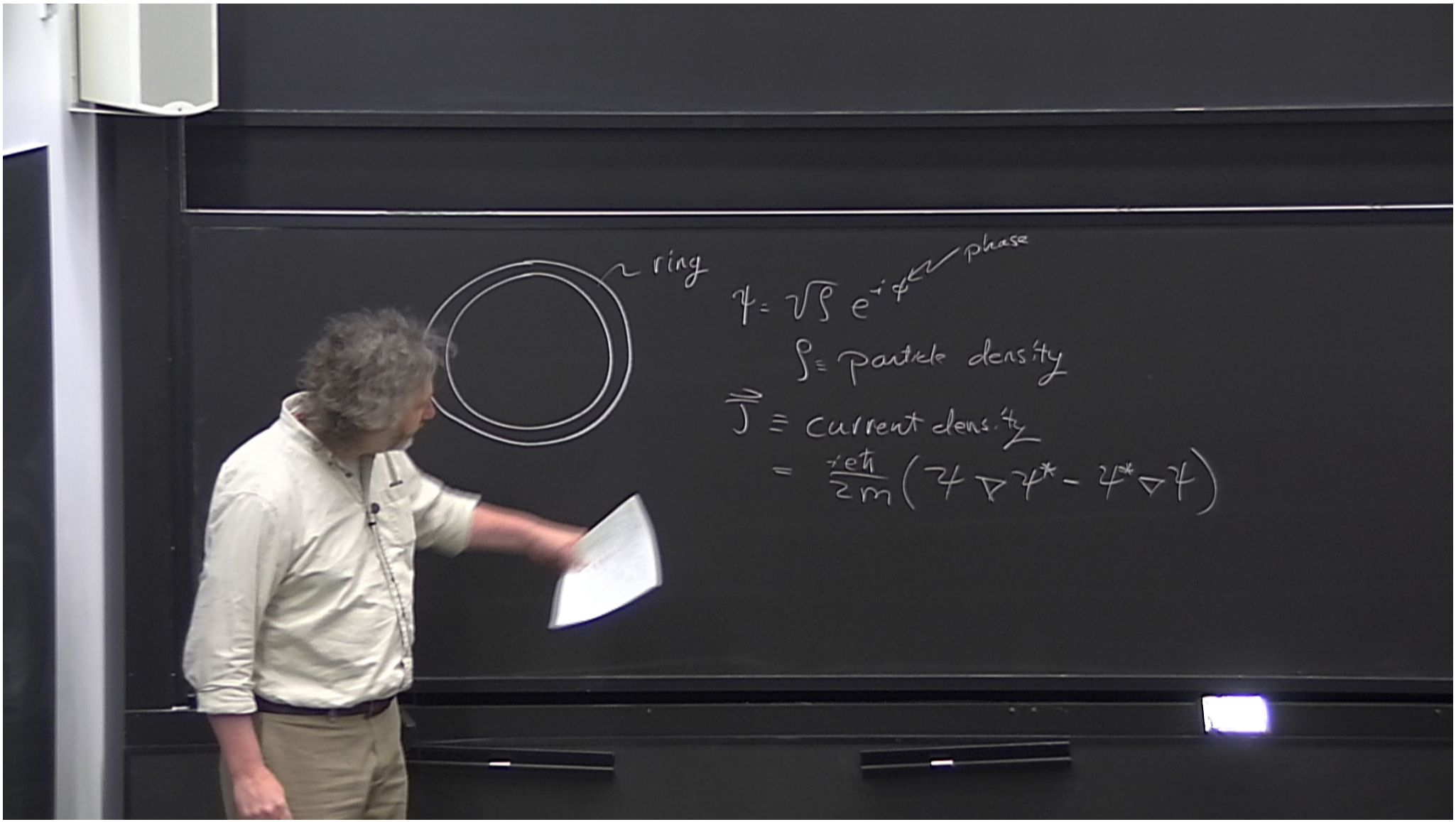
Date: Mar 27, 2012 09:00 AM

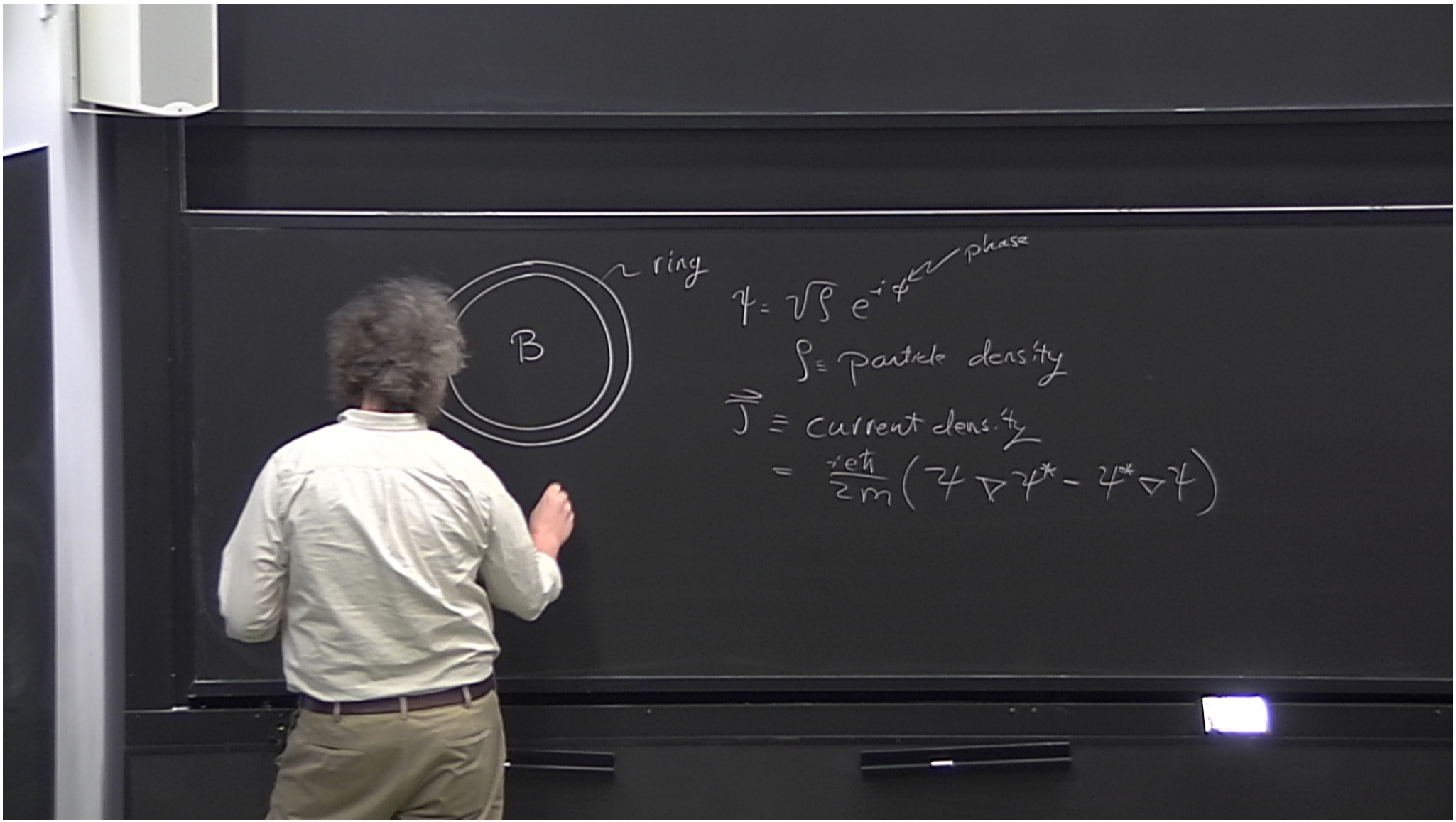
URL: http://pirsa.org/12030019

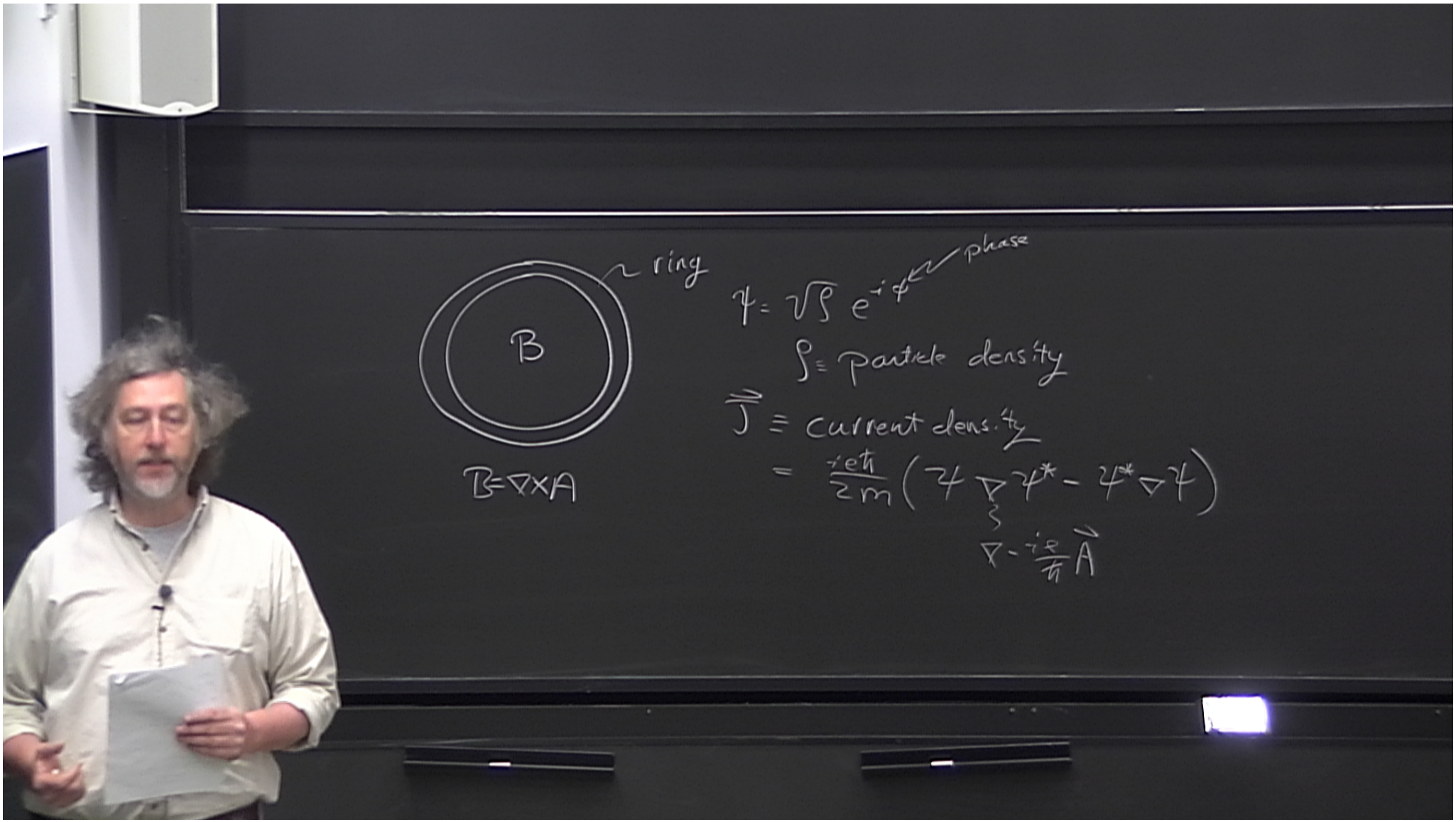
Abstract:

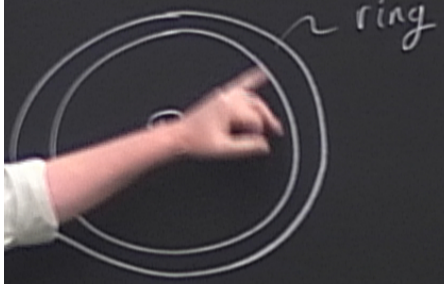

ring
$\psi = \sqrt{\rho}\, e^{-i\phi}$
phase
$\rho \equiv$ particle density


ring
$\psi = \sqrt{\rho}\, e^{-i\phi}$
phase
$\rho \equiv$ particle density


ring
phase
$\psi = \sqrt{\rho}\, e^{i\phi}$
$\rho \equiv$ particle density
$\vec{J} \equiv$ current density
$= \frac{ie\hbar}{2m}\left(\psi \nabla \psi^* - \psi^* \nabla \psi\right)$


ring
B
phase
$\psi = \sqrt{\rho}\, e^{-i\phi}$
$\rho \equiv$ particle density
$\vec{J} \equiv$ current density
$= \frac{-ie\hbar}{2m}\left(\psi \nabla \psi^* - \psi^* \nabla \psi\right)$


ring
B
B=∇×A
ψ = √ρ e^{-iφ}
phase
ρ ≡ particle density
J ≡ current density
= iℏ/2m (ψ ∇ψ* − ψ* ∇ψ)
∇ − ie/ℏ A



$B = \nabla \times A$

$\psi = \sqrt{\rho}\, e^{-i\phi}$ ← phase

$\rho \equiv$ particle density

$\vec{J} \equiv$ current density

$$= \frac{-e\hbar}{2m}\left(\psi \underbrace{\nabla}_{\nabla - \frac{ie}{\hbar}\vec{A}} \psi^* - \psi^* \nabla \psi\right)$$

$$\vec{J} = \frac{e\hbar\rho}{m}\nabla\phi - \frac{e^2\rho}{m}\vec{A}$$

$\sqrt{\rho}\, e^{-i\phi}$ phase

ρ = particle density

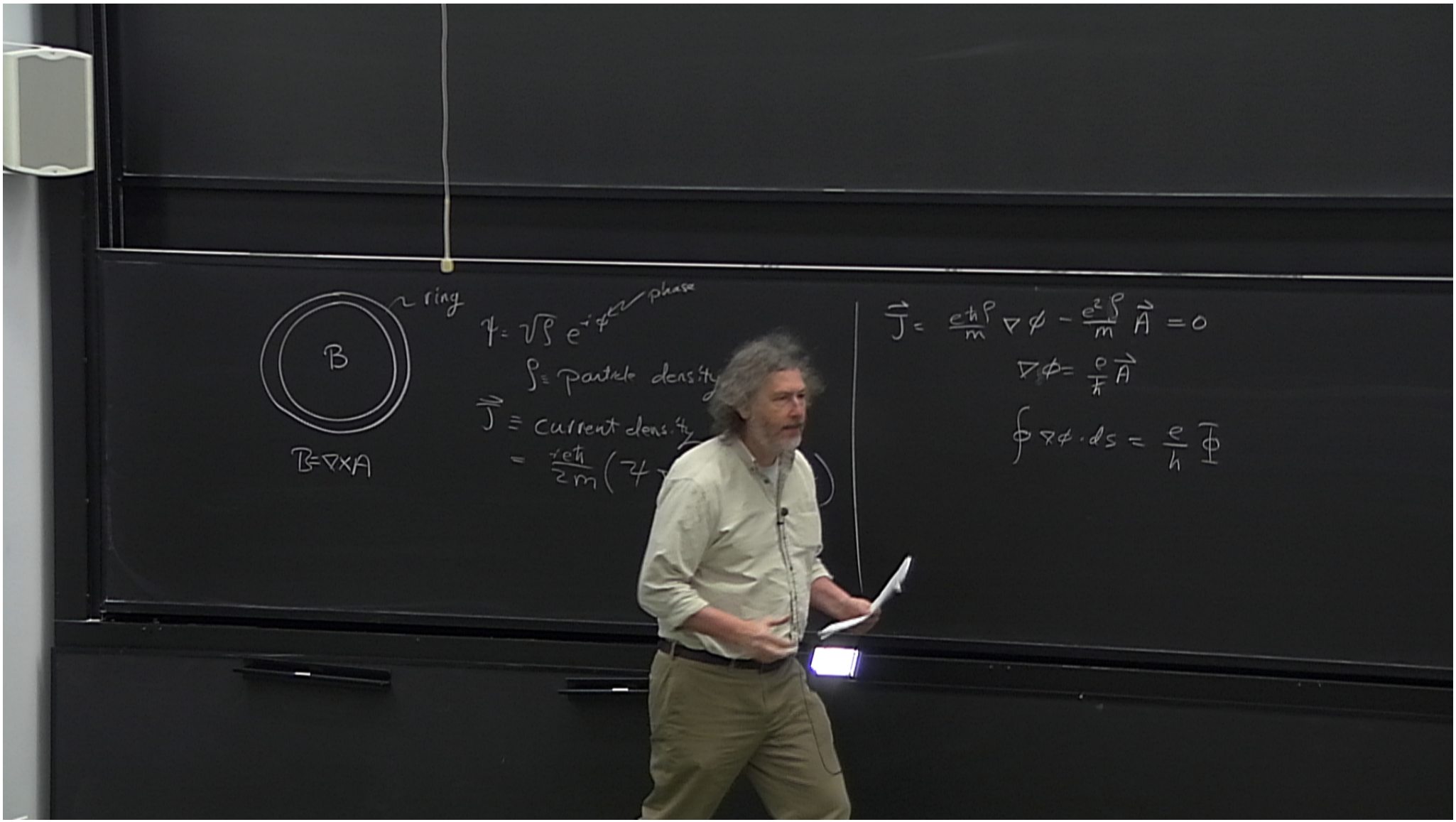
current density

$$\frac{-e\hbar}{2m}\left(\psi \nabla \psi^* - \psi^* \nabla \psi\right)$$

$$\nabla - \frac{ie}{\hbar}\vec{A}$$

$$\vec{J} = \frac{e\hbar\rho}{m}\nabla\phi - \frac{e^2\rho}{m}\vec{A} = 0$$

$$\nabla\phi = \frac{e}{\hbar}\vec{A}$$


ring
B
B=∇×A
ψ = √ρ e^{-iφ}
phase
ρ ≡ particle density
J ≡ current density
= ieħ/2m (ψ
J = eħρ/m ∇φ − e²ρ/m A = 0
∇φ = e/ħ A
∮ ∇φ · ds = e/h Φ

$\sqrt{\rho}\, e^{-i\phi}$ phase

ρ = particle density

current density

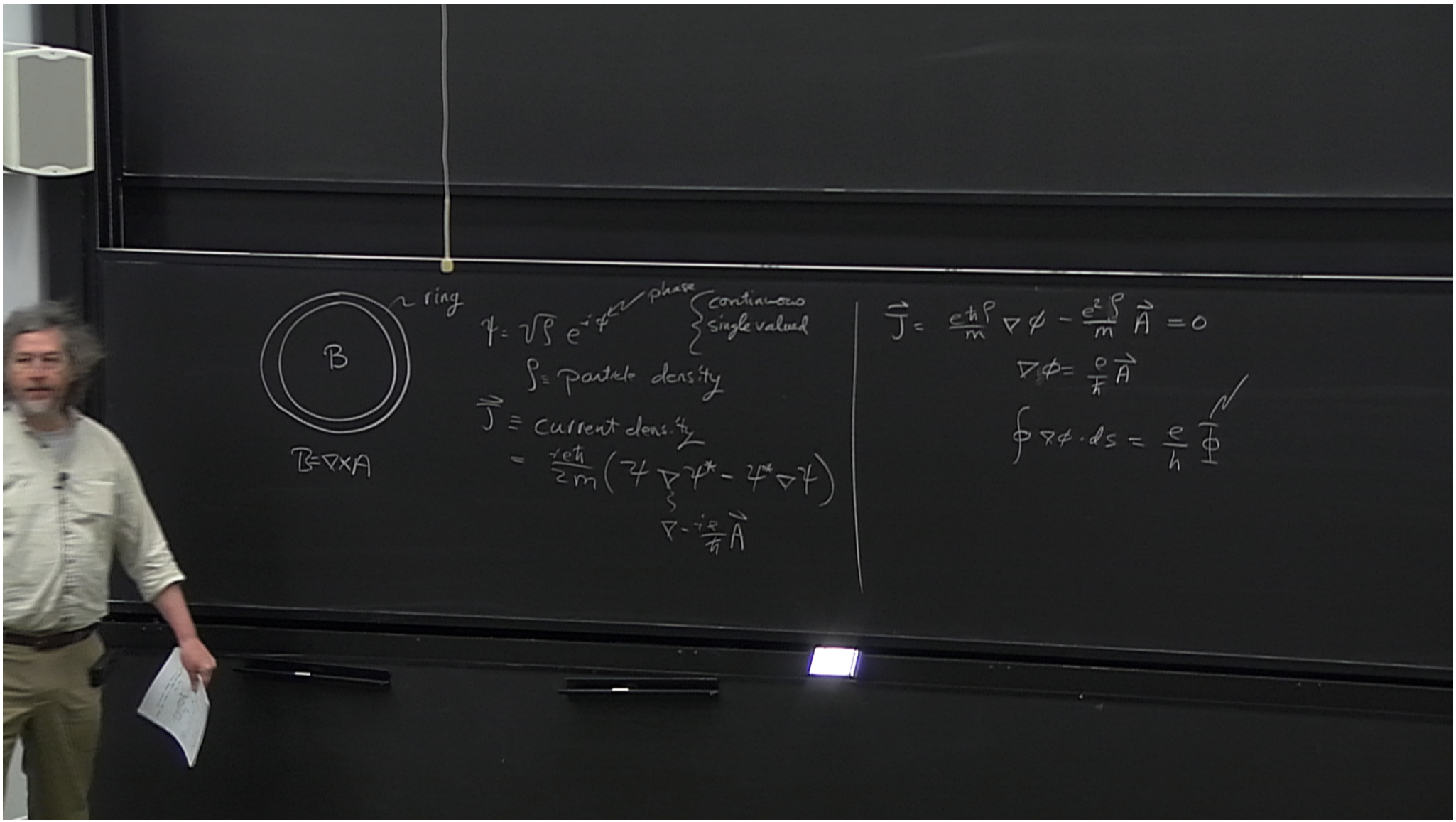
$$\frac{-e\hbar}{2m}\left(\psi \nabla \psi^* - \psi^* \nabla \psi\right)$$

$$\nabla - \frac{ie}{\hbar}\vec{A}$$

$$\vec{J} = \frac{e\hbar\rho}{m}\nabla\phi - \frac{e^2\rho}{m}\vec{A} = 0$$

$$\nabla\phi = \frac{e}{\hbar}\vec{A}$$

$$\oint \nabla\phi \cdot ds = \frac{e}{h}\Phi$$


ring
B
B=∇×A
ψ = √ρ e^{-iφ}
phase
continuous
single valued
ρ = particle density
J = current density
= iehm/2m (ψ ∇ψ* − ψ* ∇ψ)
∇ − ie/ħ A
J = eħρ/m ∇φ − e²ρ/m A = 0
∇φ = e/ħ A
∮ ∇φ · ds = e/h Φ

se

$\left\{ \begin{array}{l} \text{continuous} \\ \text{single valued} \end{array} \right.$

ns.ity

ity

$\nabla\psi^* - \psi^*\nabla\psi)$

$- \frac{ie}{\hbar}\vec{A}$

$$\vec{J} = \frac{e\hbar\rho}{m}\nabla\phi - \frac{e^2\rho}{m}\vec{A} = 0$$

$$\nabla\phi = \frac{e}{\hbar}\vec{A}$$

$$\oint \nabla\phi \cdot ds = \frac{e}{h}\Phi \quad ; \quad \boxed{\Phi = n\left(\frac{h}{e}\right)} \quad n = 1, 2, \cdots$$

se

$\left\{ \begin{array}{l} \text{continuous} \\ \text{single valued} \end{array} \right.$

ns.ity

ty

$\psi^* - \psi^* \nabla \psi)$

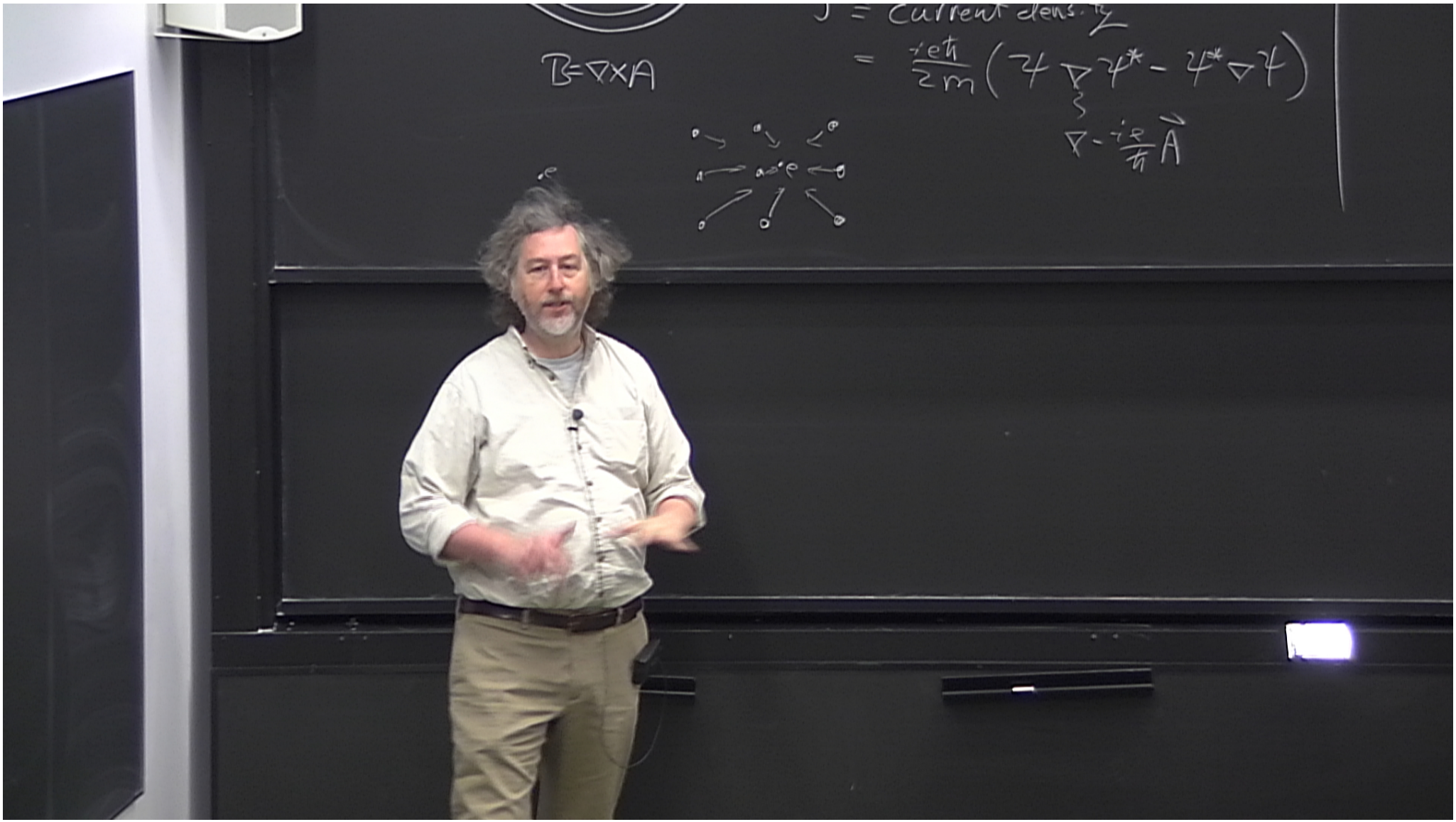
$- \frac{ie}{\hbar} \vec{A}$

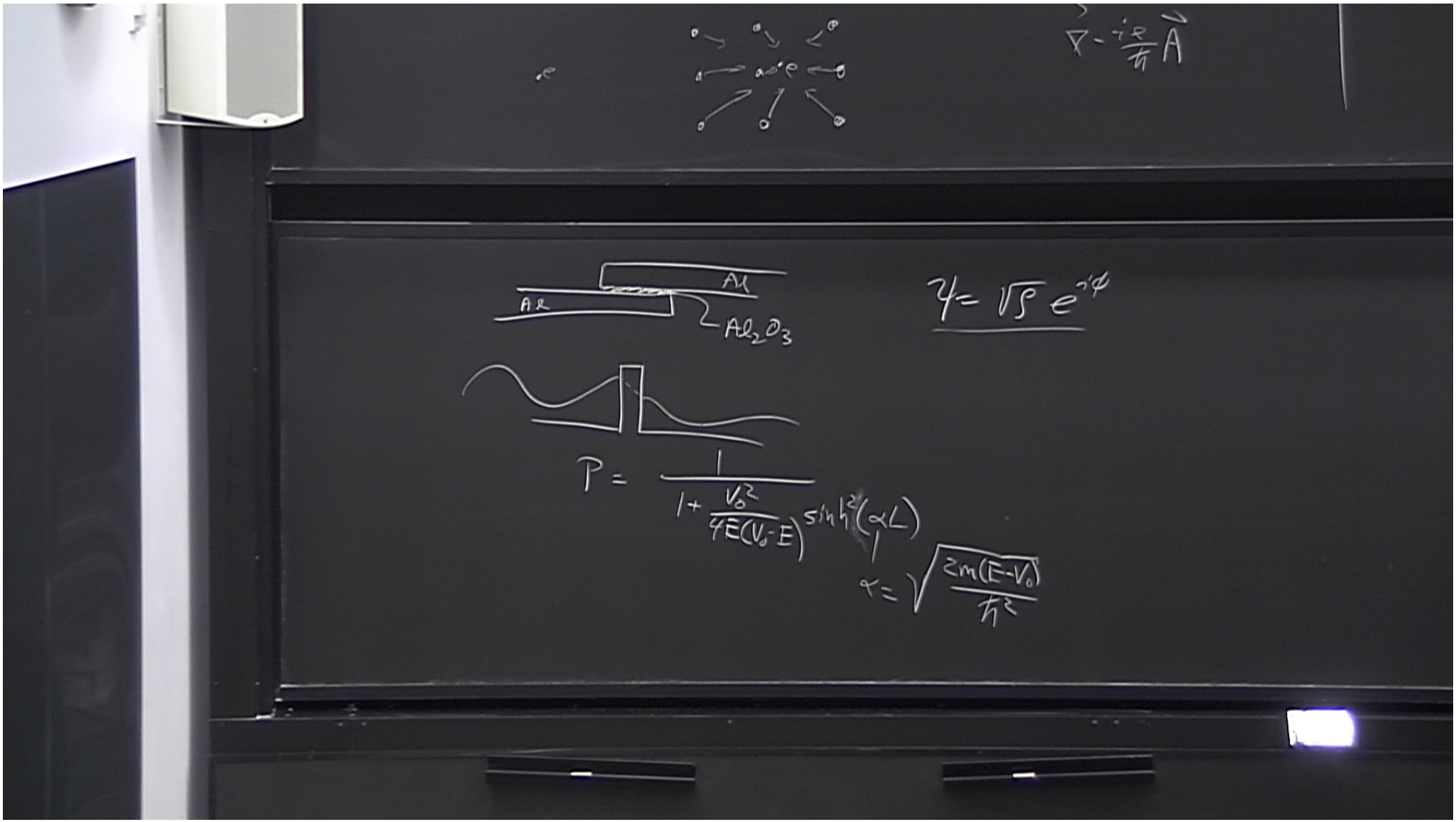
$$\vec{J} = \frac{e\hbar\rho}{m} \nabla \phi - \frac{e^2 \rho}{m} \vec{A} = 0$$

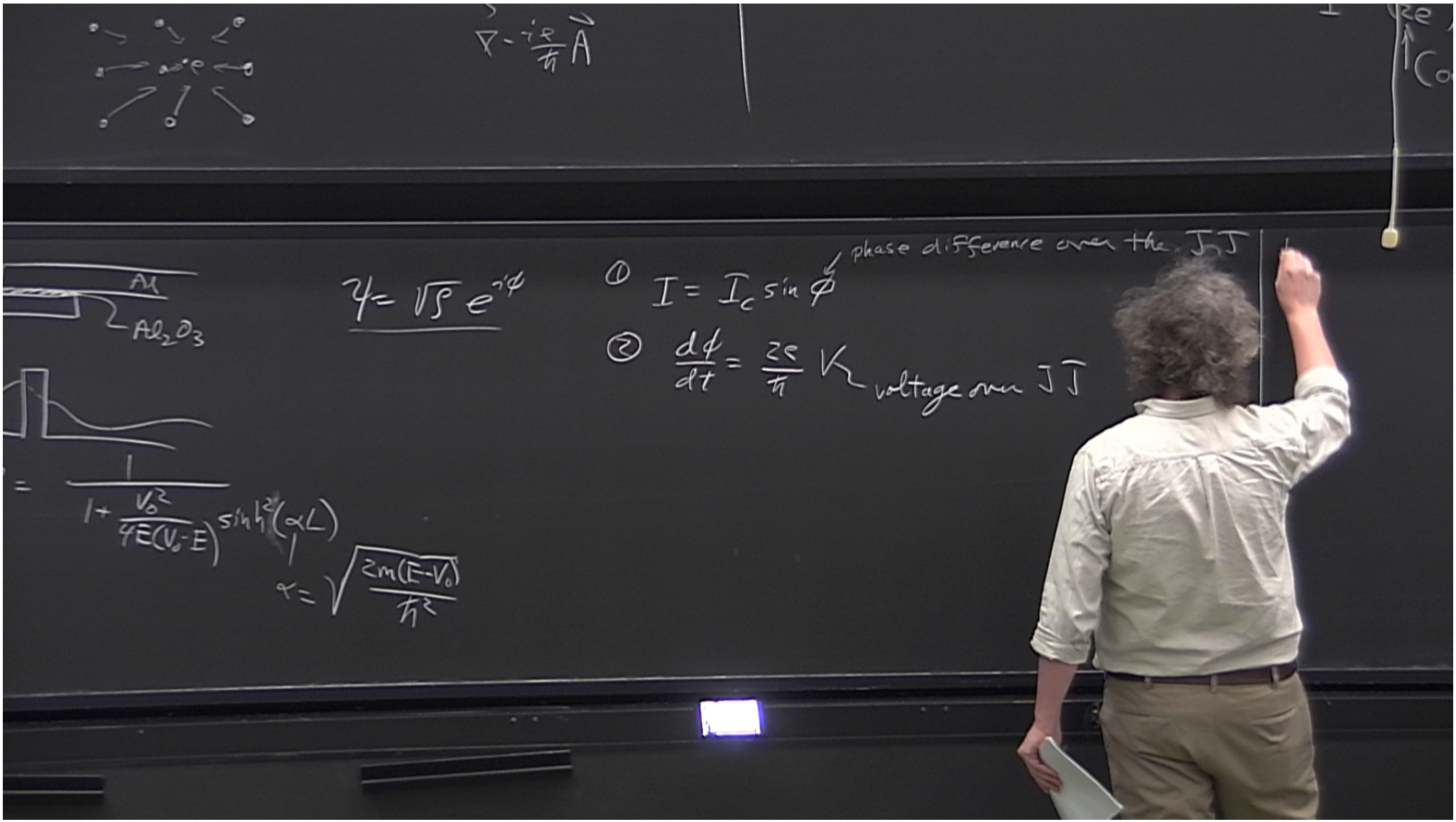
$$\nabla \phi = \frac{e}{\hbar} \vec{A}$$

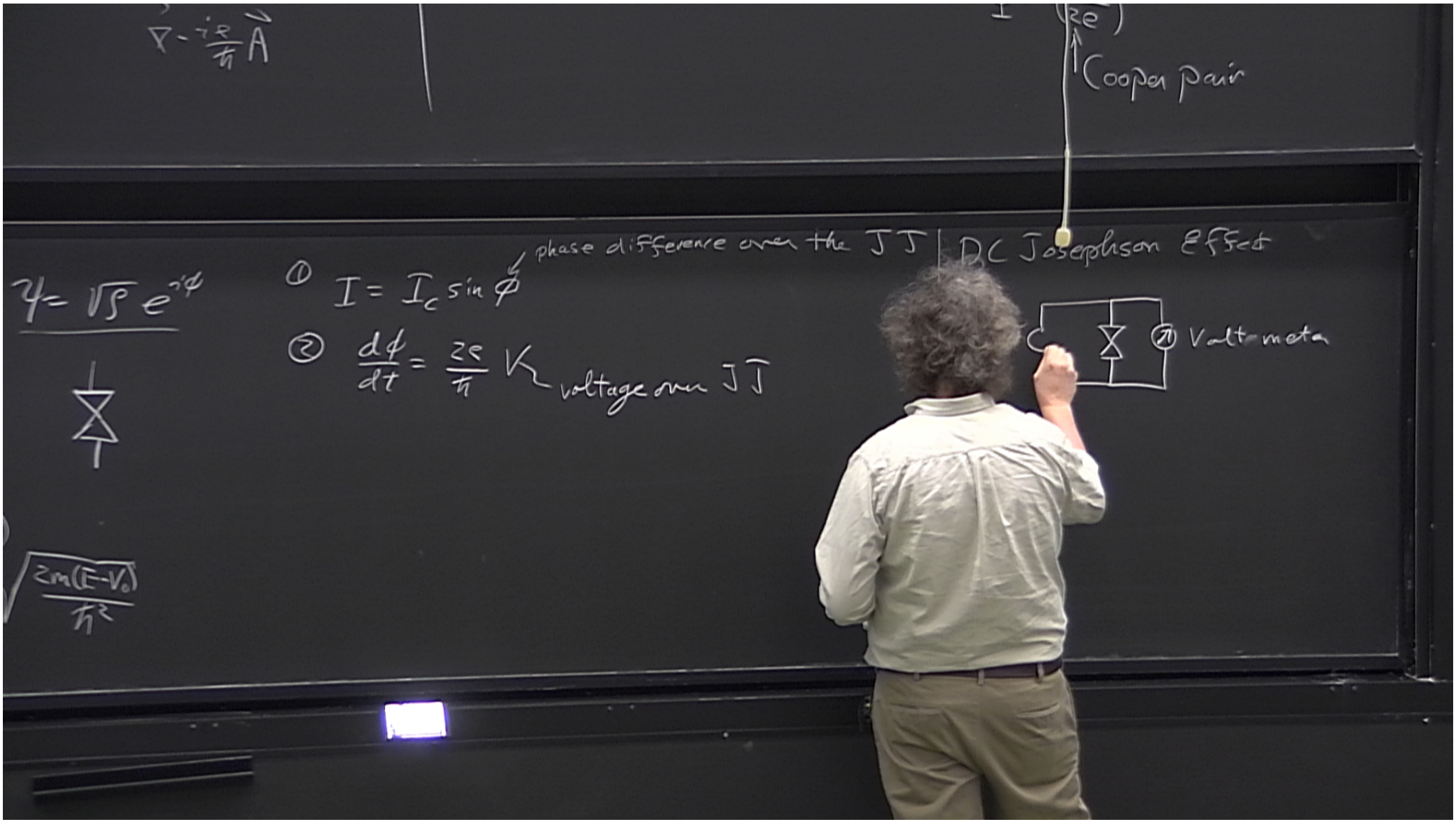
$$\oint \nabla \phi \cdot ds = \frac{e}{h} \Phi \quad ; \quad \boxed{\Phi = n\left(\frac{h}{e}\right)} \quad n = 1, 2, \cdots$$

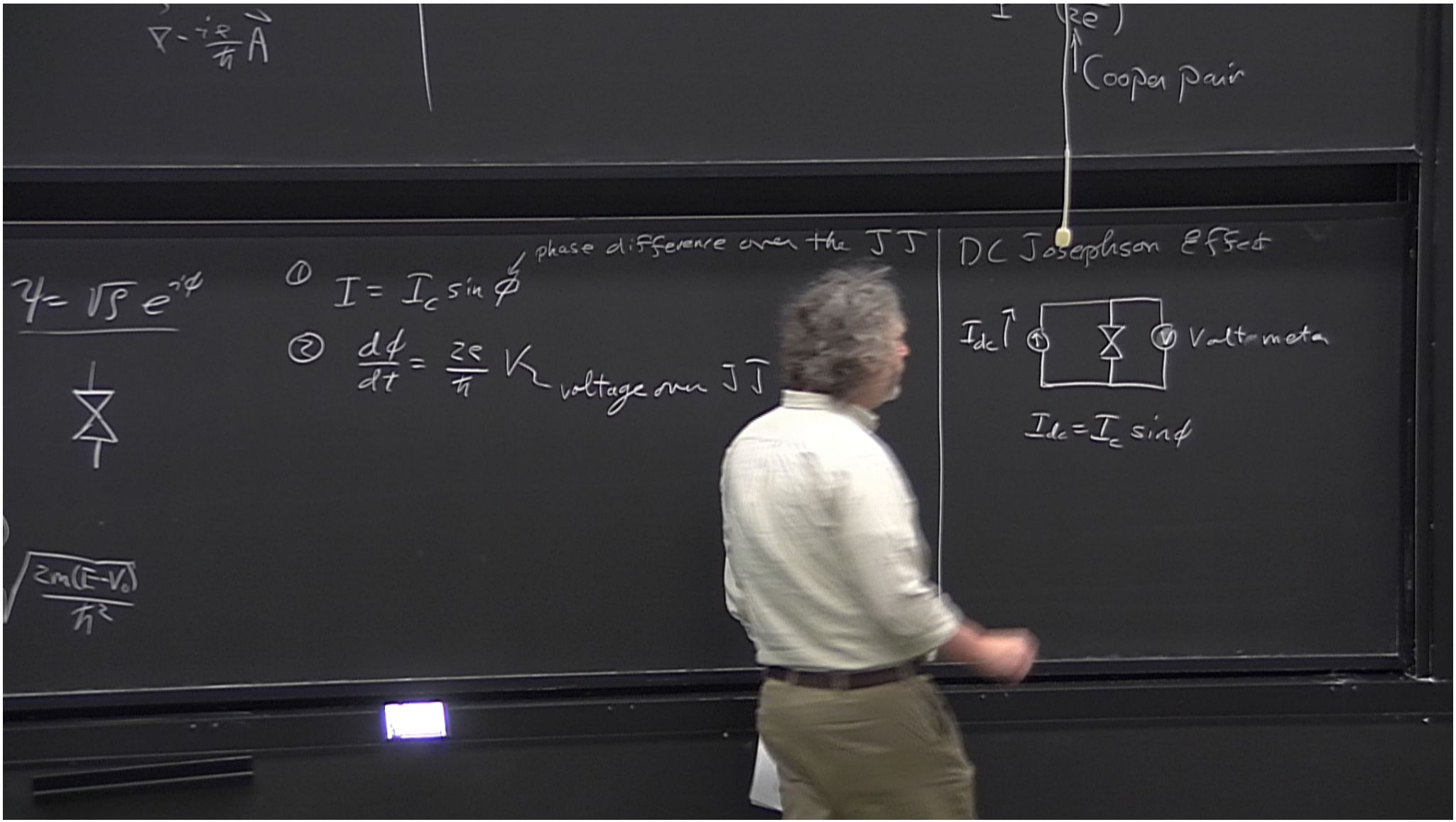
$$\Phi = n\left(\frac{h}{2e}\right)$$

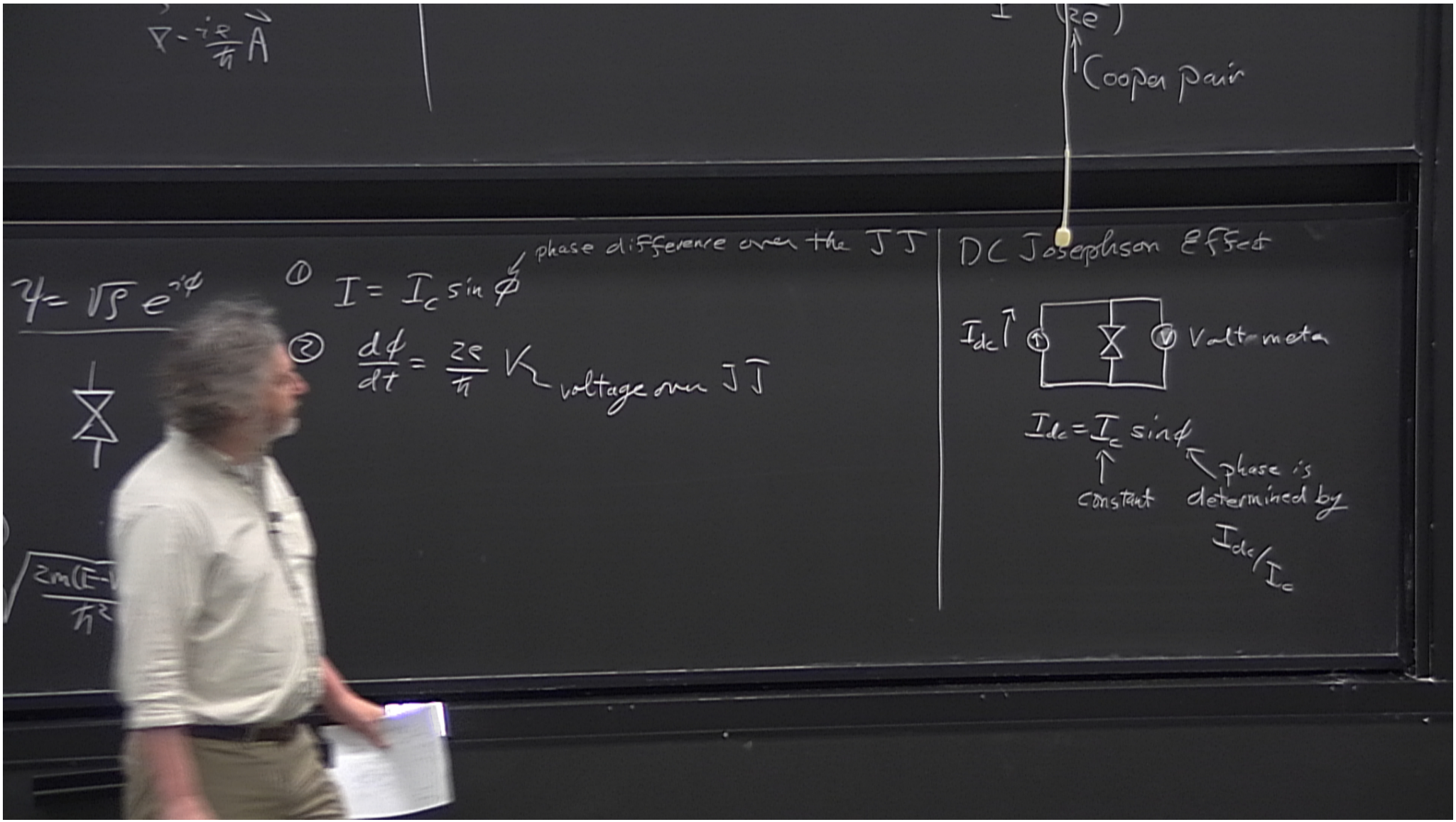

B=∇×A
J = current dens.
= ieħ/2m (ψ ∇ψ* − ψ* ∇ψ)
∇ − ie/ħ A


$\vec{\nabla} - \frac{ie}{\hbar}\vec{A}$
Al
Al
Al_2O_3
$\psi = \sqrt{\rho}\, e^{i\phi}$
$P = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\alpha L)}$
$\alpha = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$


$\nabla - \frac{ie}{\hbar}\vec{A}$
Al
Al_2O_3
$\psi = \sqrt{\rho}\, e^{i\phi}$
① $I = I_c \sin\phi$
phase difference over the JJ
② $\frac{d\phi}{dt} = \frac{2e}{\hbar} V$
voltage over JJ
$= \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\alpha L)}$
$\alpha = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$


Cooper pair
phase difference over the JJ
DC Josephson Effect
① I = I_c sin φ
② dφ/dt = 2e/ħ V
voltage over JJ
Voltmeter


Cooper pair
phase difference over the JJ
$\psi = \sqrt{\rho}\, e^{i\phi}$
① $I = I_c \sin\phi$
② $\frac{d\phi}{dt} = \frac{2e}{\hbar} V$
voltage over JJ
DC Josephson Effect
I_{dc}
Voltmeter
$I_{dc} = I_c \sin\phi$


$\nabla - \frac{ie}{\hbar}\vec{A}$
(2e)
Cooper pair
$\psi = \sqrt{\rho}\, e^{i\phi}$
① $I = I_c \sin\phi$
phase difference over the JJ
② $\frac{d\phi}{dt} = \frac{2e}{\hbar} V$
voltage over JJ
DC Josephson Effect
I_{dc}
V
Voltmeter
$I_{dc} = I_c \sin\phi$
constant
phase is determined by I_{dc}/I_c

$\nabla - \frac{ie}{\hbar}\vec{A}$

$(2e)$

Cooper pair

$\psi = \sqrt{\rho}\, e^{i\phi}$

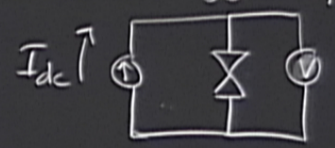
① $I = I_c \sin\phi$ phase difference over the JJ

② $\frac{d\phi}{dt} = \frac{2e}{\hbar} V$ voltage over JJ

$\sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

DC Josephson Effect

JJ = superconducting wire

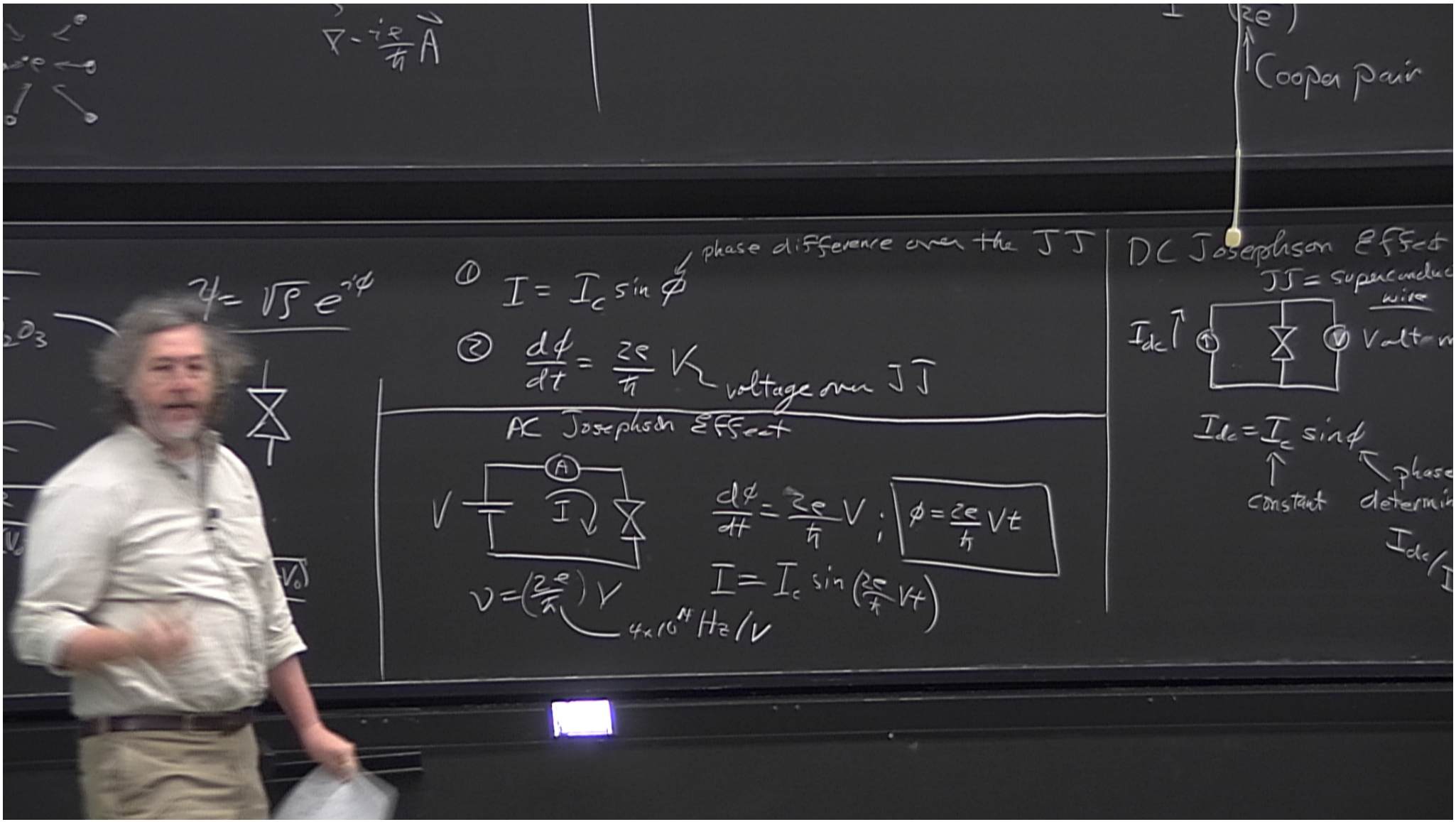


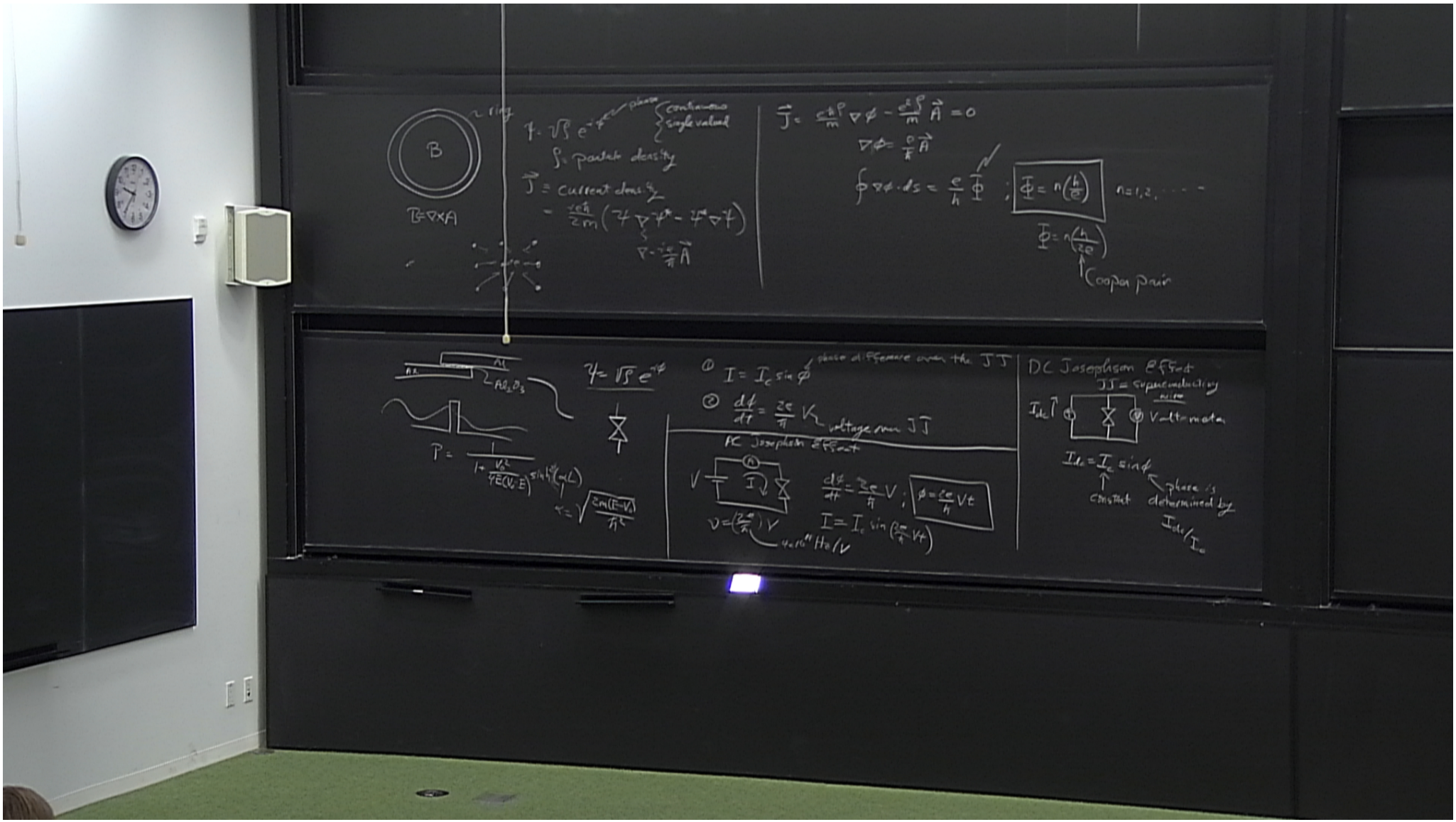
Voltmeter

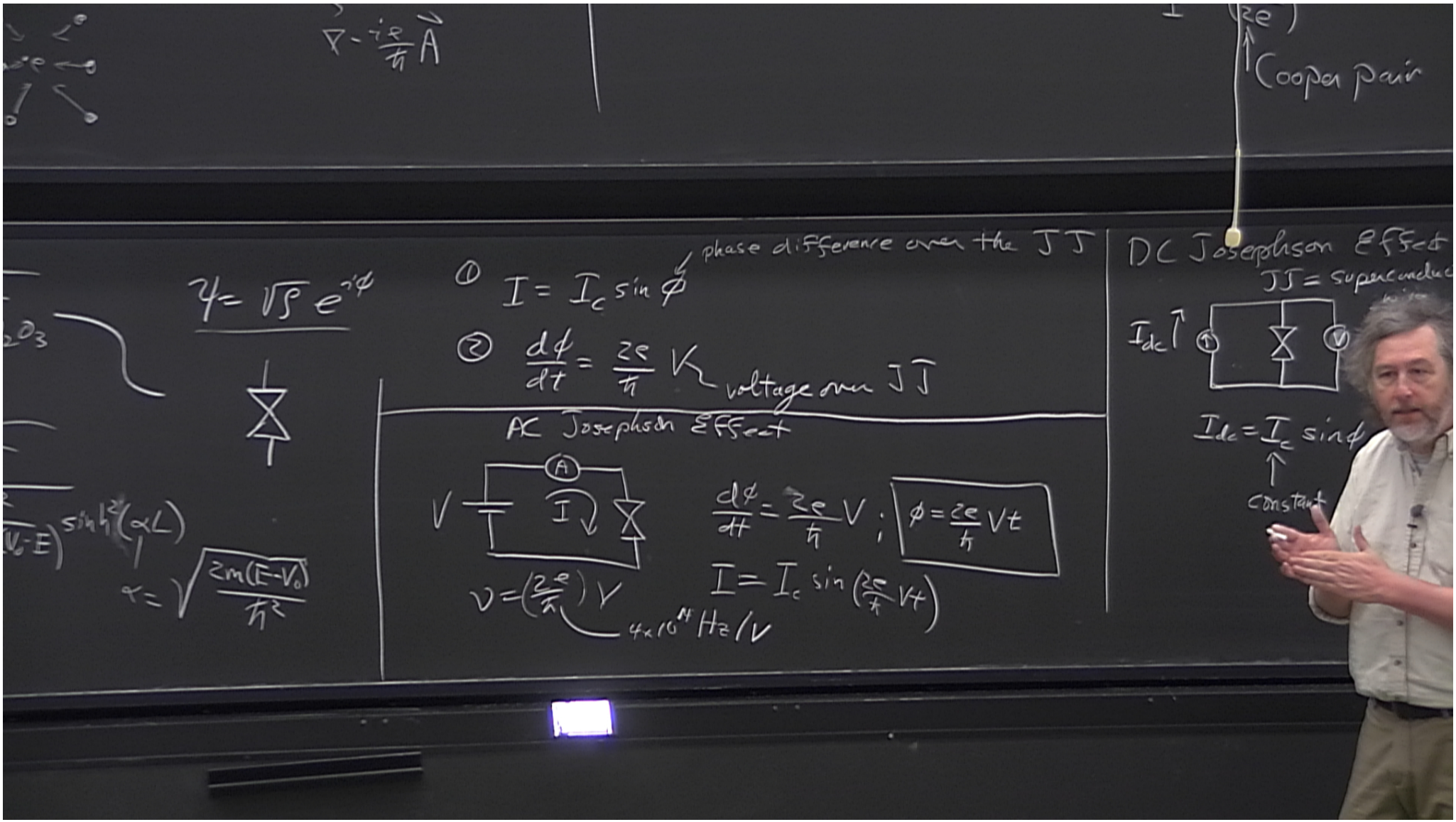
$I_{dc} = I_c \sin\phi$

constant

phase is determined by I_{dc}/I_c


Cooper pair
phase difference over the JJ
① I = Ic sin φ
② dφ/dt = 2e/ħ V
voltage over JJ
AC Josephson Effect
dφ/dt = 2e/ħ V ; φ = 2e/ħ Vt
I = Ic sin (2e/ħ Vt)
ν = (2e/ħ) V
4×10^14 Hz/V
DC Josephson Effect
JJ = superconduc
wire
Idc
Idc = Ic sin φ
constant
phase
determin


B
ring
B = ∇×A
ψ = √ρ e^{-iφ}
phase
continuous
single valued
ρ = particle density
J = current density
= eħ/2m (ψ ∇ψ* − ψ* ∇ψ)
∇ − ie/ħ A
J = eħρ/m ∇φ − e²ρ/m A = 0
∇φ = e/ħ A
∮ ∇φ · ds = e/h Φ ; Φ = n(h/e)
n = 1, 2, ...
Φ = n(h/2e)
Cooper pair
Al
Al₂O₃
ψ = √ρ e^{-iφ}
T = 1 / (1 + V₀²/(4E(V₀−E)) sinh²(κL))
κ = √(2m(E−V₀)/ħ²)
① I = I_c sin φ
phase difference over the JJ
② dφ/dt = 2e/ħ V
voltage over JJ
AC Josephson effect
V
I
dφ/dt = 2e/ħ V ; φ = 2e/ħ Vt
ν = (2e/ħ) V
4×10¹⁴ Hz/V
I = I_c sin (2e/ħ Vt)
DC Josephson effect
JJ = superconducting wire
I_dc
Voltmeter
I_dc = I_c sin φ
constant
phase is determined by I_dc/I_c


Cooper pair
phase difference over the JJ
① I = Ic sin φ
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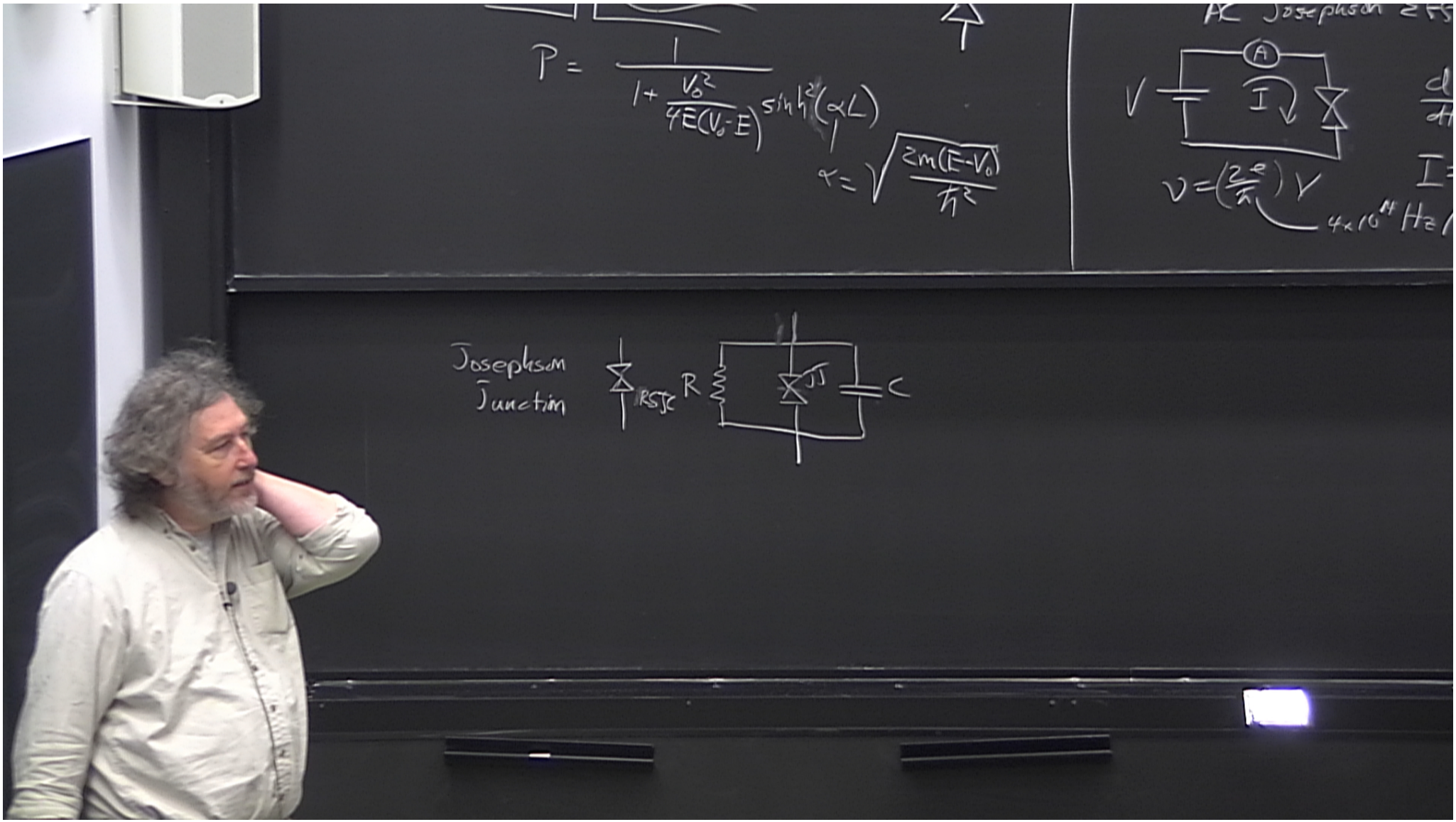
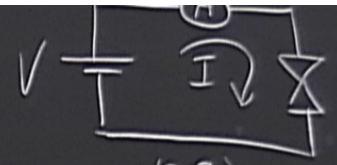
voltage over JJ
AC Josephson Effect
dφ/dt = 2e/ħ V ; φ = 2e/ħ Vt
I = Ic sin (2e/ħ Vt)
ν = (2e/ħ) V
4×10^14 Hz/V
DC Josephson Effect
JJ = superconduc
Idc = Ic sin φ
constant
ψ = √ρ e^{iφ}
α = √(2m(E−V0)/ħ²)


P = 1 / (1 + V0^2 / (4E(V0 - E)) sinh^2(αL))
α = √(2m(E - V0) / ħ^2)
AC Josephson Eff
A
V
I
ν = (2e/h) V
4×10^14 Hz
Josephson Junction
RSJC
R
JJ
C

$$P = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\alpha L)}$$

$$\alpha = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

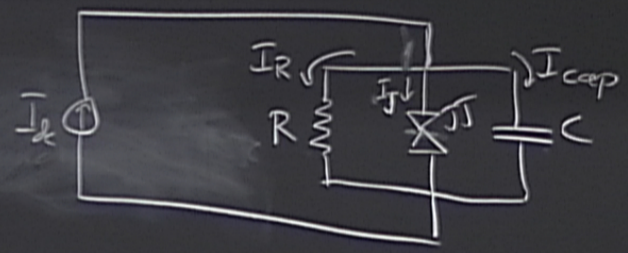


$$\frac{d\phi}{dt} = \frac{2e}{\hbar} V \; ; \; \phi =$$

$$I = I_c \sin\left(\frac{2e}{\hbar} V t\right)$$

$$\nu = \left(\frac{2e}{h}\right) V$$

4×10^{14} Hz/V

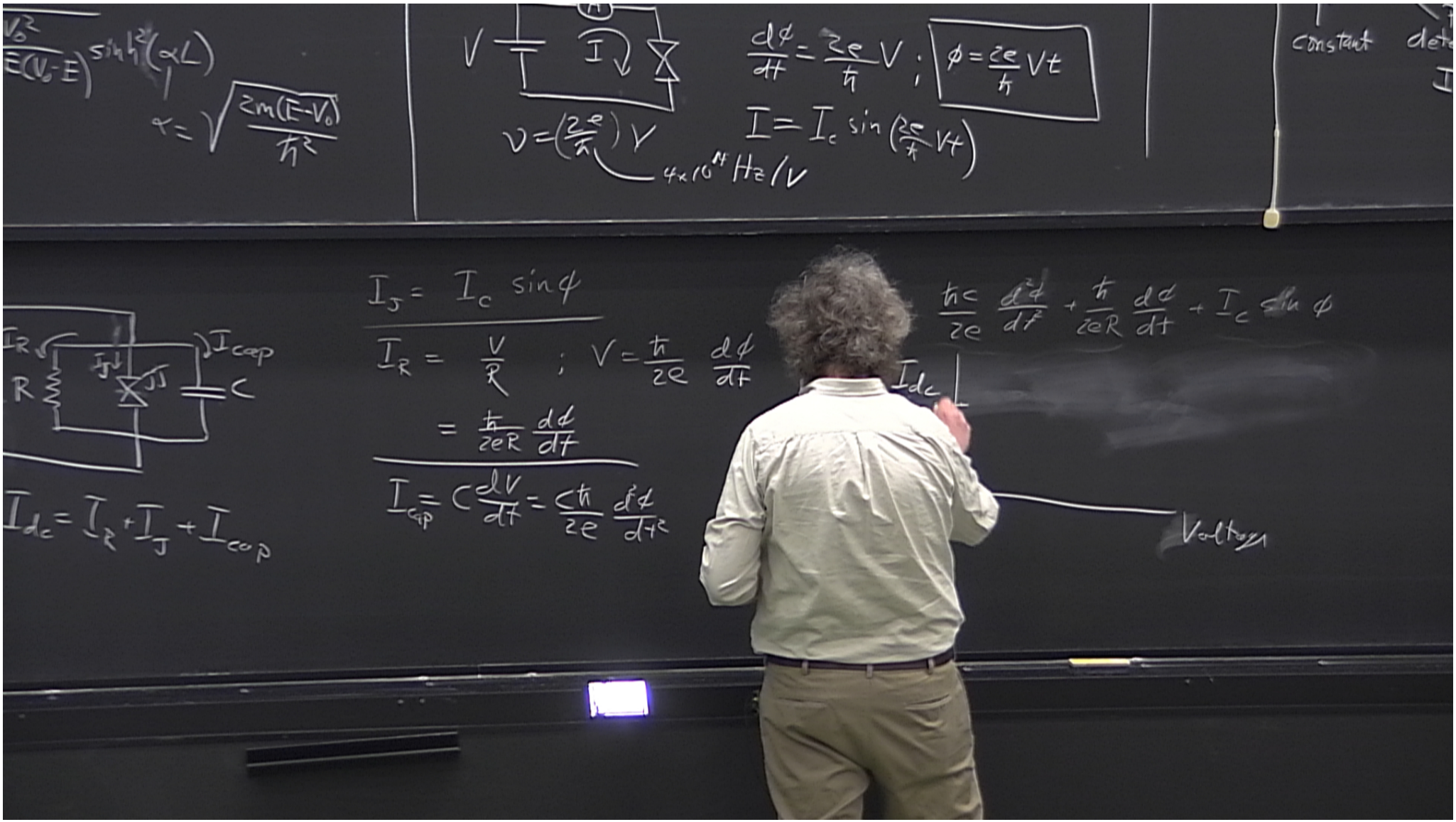


$$I_J = I_c \sin\phi$$

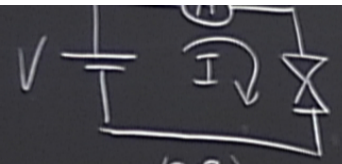
$$I_R = \frac{V}{R} \; ; \; V = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

$$= \frac{\hbar}{2eR}\frac{d\phi}{dt}$$

$$I_{cap} = C\frac{dV}{dt} = \frac{C\hbar}{2e}\frac{d^2\phi}{dt^2}$$


$\alpha = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$
$\nu = \left(\frac{2e}{h}\right) V$
4×10^{14} Hz/V
$\frac{d\phi}{dt} = \frac{2e}{\hbar} V$; $\phi = \frac{2e}{\hbar} Vt$
$I = I_c \sin\left(\frac{2e}{\hbar} Vt\right)$
constant
$I_J = I_c \sin\phi$
$I_R = \frac{V}{R}$; $V = \frac{\hbar}{2e}\frac{d\phi}{dt}$
$= \frac{\hbar}{2eR}\frac{d\phi}{dt}$
$I_{cap} = C\frac{dV}{dt} = \frac{C\hbar}{2e}\frac{d^2\phi}{dt^2}$
$I_{dc} = I_R + I_J + I_{cap}$
$\frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} + I_c \sin\phi$
I_{dc}
Voltage

$$\frac{m(E-V_0)}{\hbar^2}$$



$$\frac{d\phi}{dt} = \frac{2e}{\hbar} V \; ; \; \boxed{\phi = \frac{2e}{\hbar} V t}$$

$$I = I_c \sin\left(\frac{2e}{\hbar} V t\right)$$

$$\nu = \left(\frac{2e}{h}\right) V$$

4×10^{14} Hz/V

constant

phase is determined by I_{dc}/I_c

$$I_J = I_c \sin\phi$$

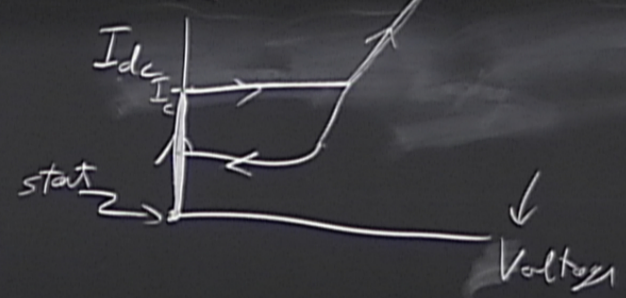
$$I_R = \frac{V}{R} \; ; \; V = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

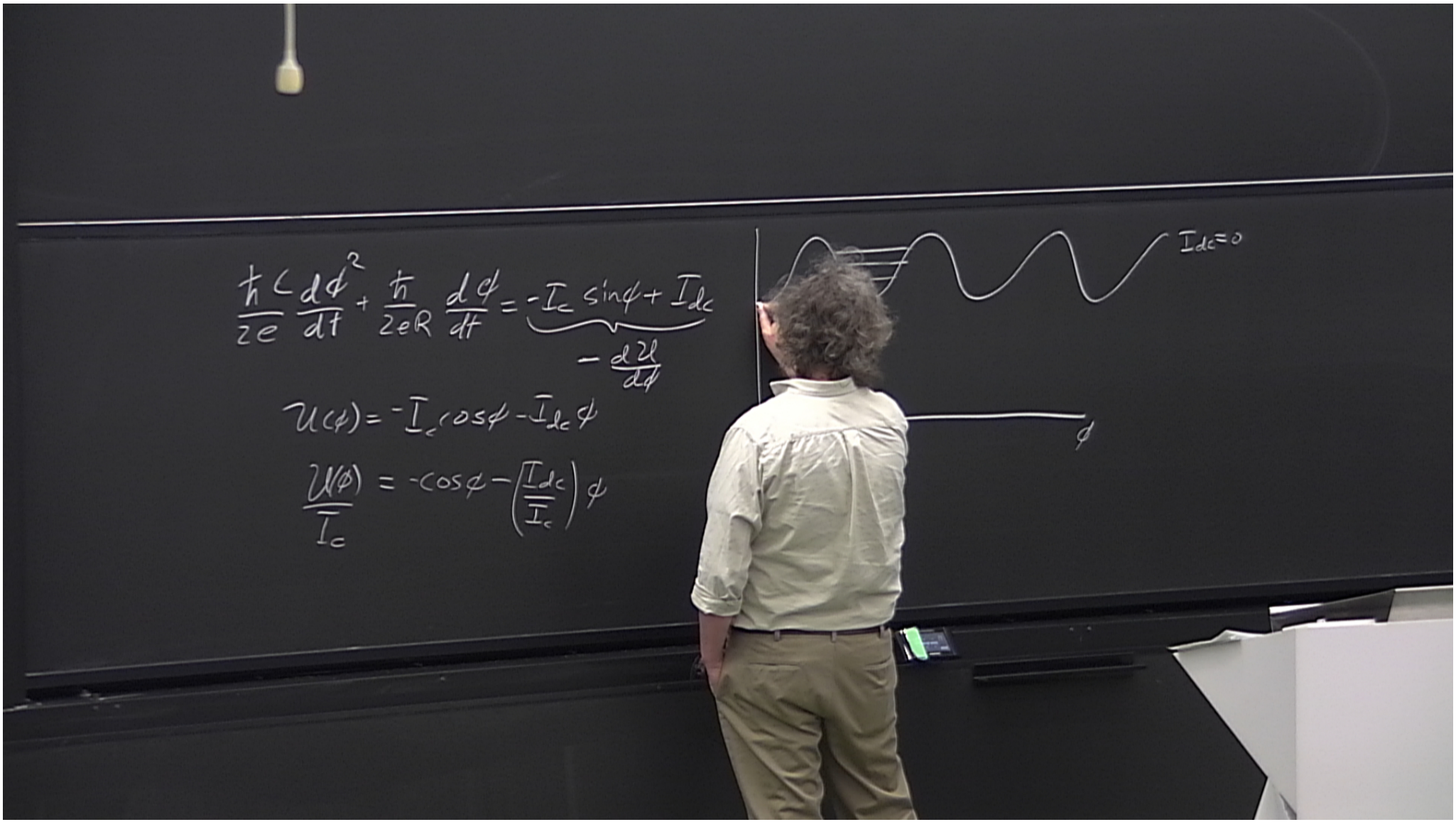
$$= \frac{\hbar}{2eR}\frac{d\phi}{dt}$$

$$I_{cap} = C\frac{dV}{dt} = \frac{C\hbar}{2e}\frac{d^2\phi}{dt^2}$$

$$I_{dc} = \frac{\hbar C}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} + I_c \sin\phi$$

HW




$\frac{\hbar C}{2e}\frac{d\phi^2}{dt} + \frac{\hbar}{2eR}\frac{d\phi}{dt} = \underbrace{-I_c \sin\phi + I_{dc}}_{-\frac{d\mathcal{U}}{d\phi}}$
$\mathcal{U}(\phi) = -I_c \cos\phi - I_{dc}\phi$
$\frac{\mathcal{U}(\phi)}{I_c} = -\cos\phi - \left(\frac{I_{dc}}{I_c}\right)\phi$
$I_{dc}=0$
ϕ

$$\frac{\hbar C}{2e}\frac{d\phi^2}{dt} + \frac{\hbar}{2eR}\frac{d\phi}{dt} = \underbrace{-I_c \sin\phi + I_{dc}}_{-\frac{d\mathcal{U}}{d\phi}}$$

$$\mathcal{U}(\phi) = -I_c \cos\phi - I_{dc}\,\phi = \mathcal{H}(\phi)$$

$$\frac{\mathcal{U}(\phi)}{I_c} = -\cos\phi - \left(\frac{I_{dc}}{I_c}\right)\phi$$