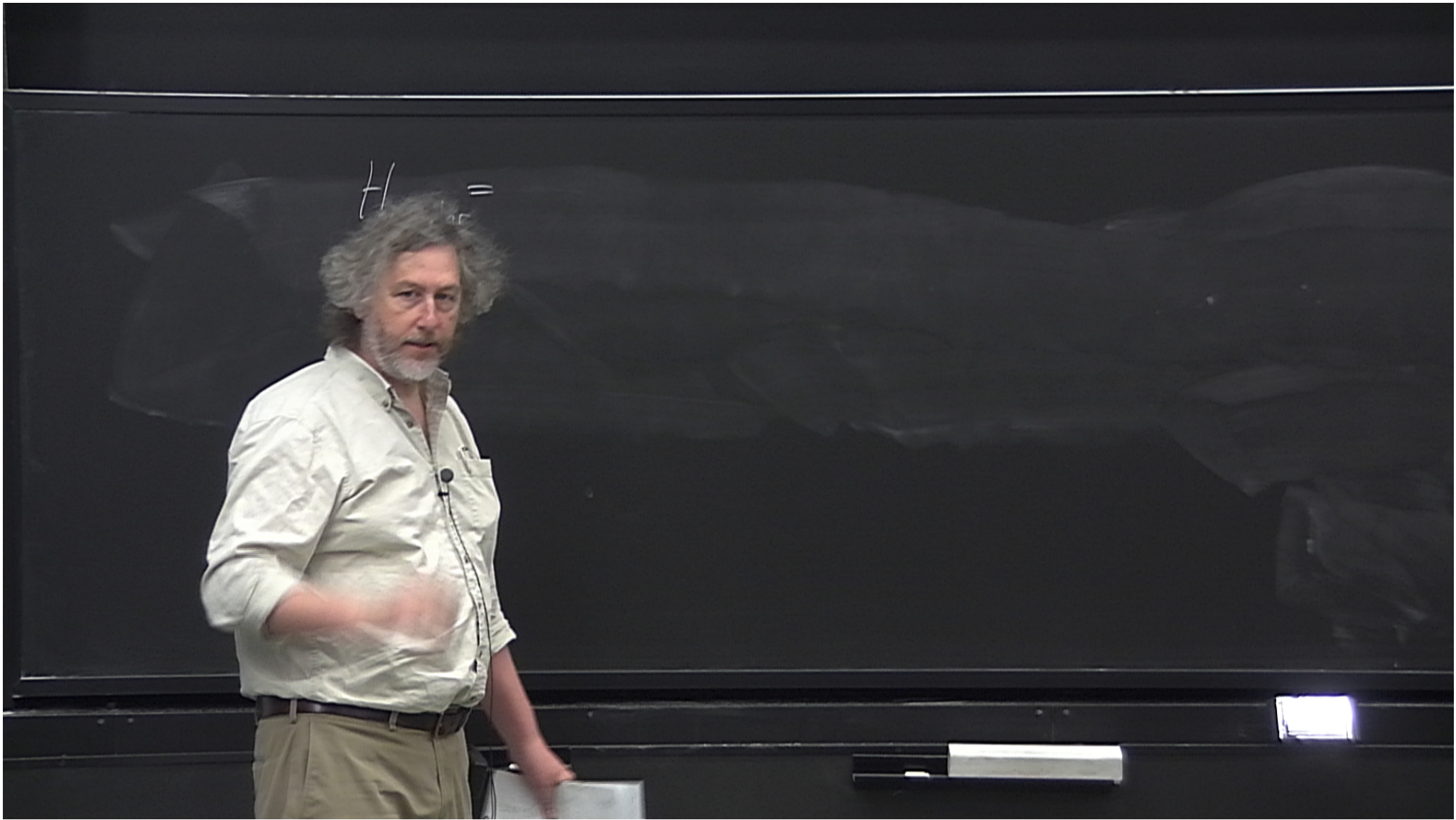


Title: Explorations in Quantum Information - Lecture 11

Date: Mar 26, 2012 02:00 PM

URL: <http://pirsa.org/12030018>

Abstract:



$$H_{\text{secular}} =$$

$$H_{\text{secular}} = A S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k$$

$$H_{\text{secular}} = A S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma}$$

$$H_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{\text{site}} \sum_{\alpha\beta} D_{\alpha\beta} \sigma_k^\alpha \sigma_k^\beta$$

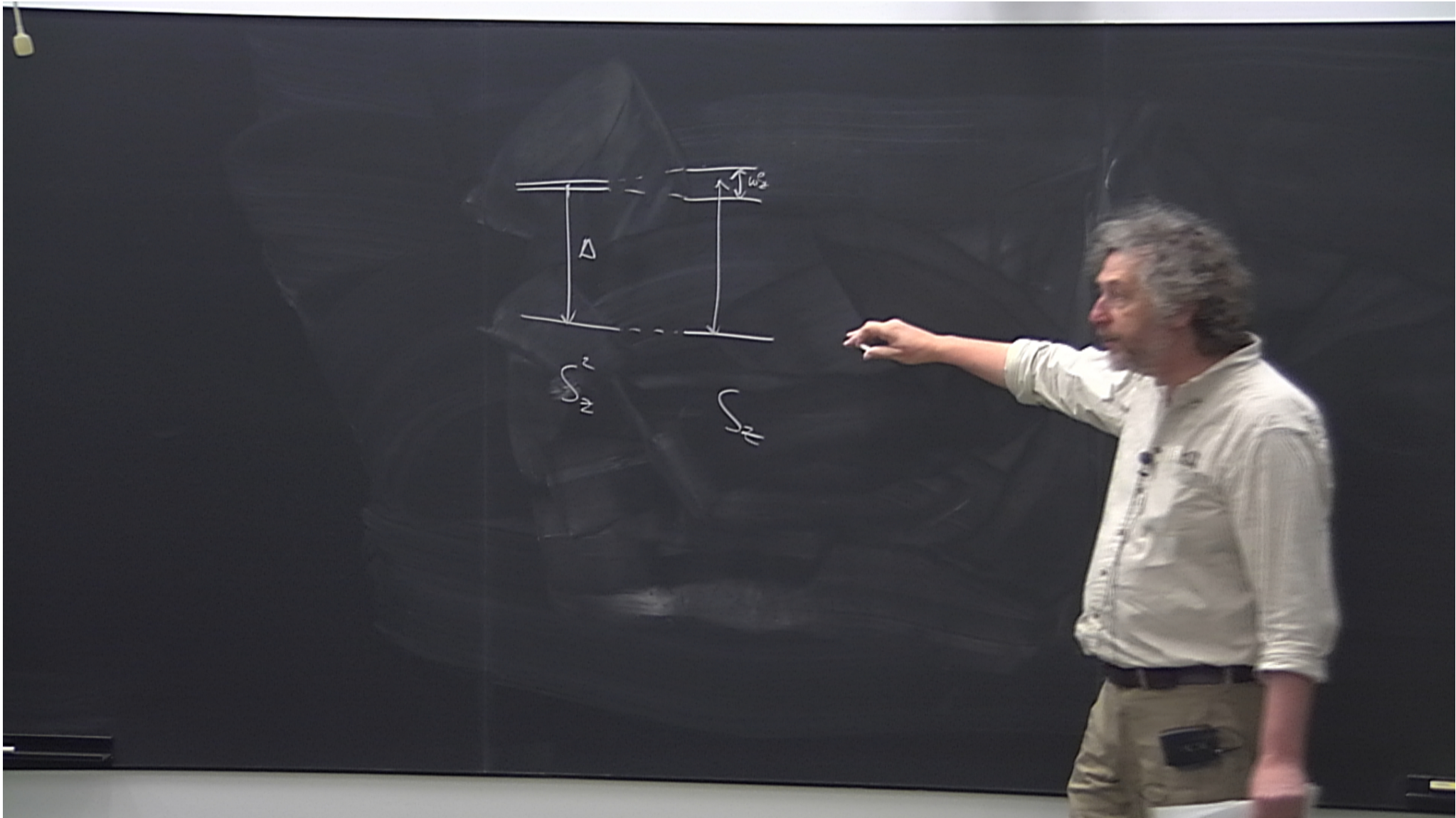
$$H_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{\text{site}} \sum_{\alpha\beta} D \sigma_k^\alpha \sigma_k^\beta$$

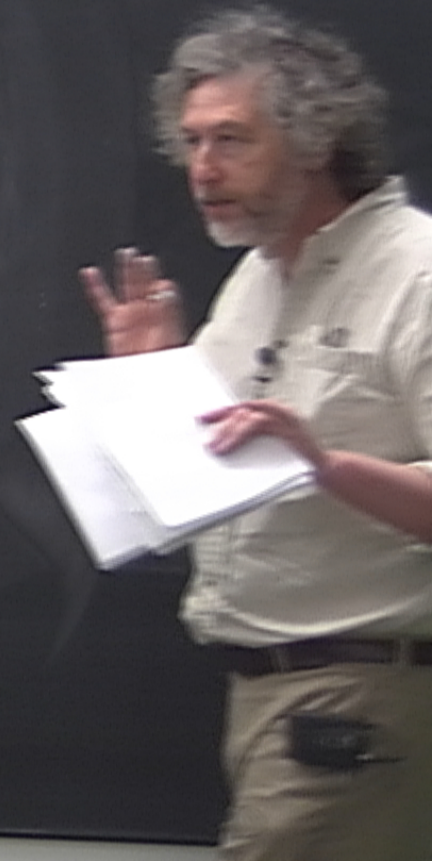
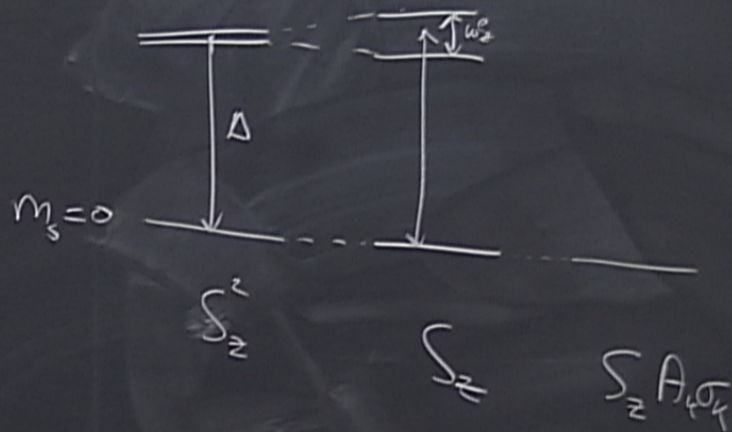
$$H_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z - \sum_k S_e A_{2k} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{\text{orb}} \vec{\sigma} \cdot \vec{D} \sigma^{\text{orb}}$$

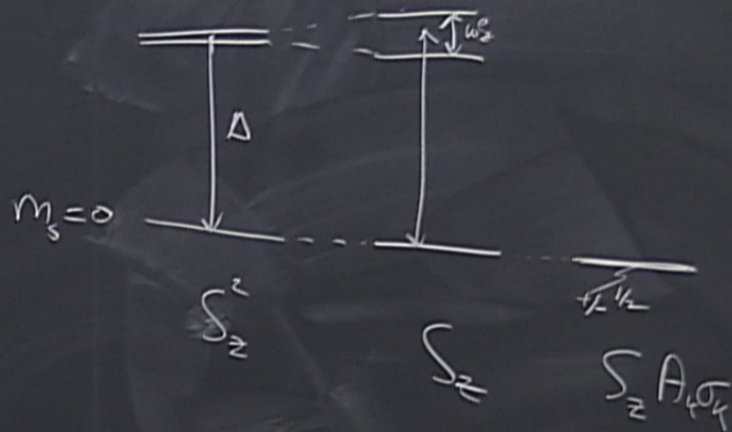
$$H_{\text{ns}} = \gamma (B_x S_x + B_y S_y)$$

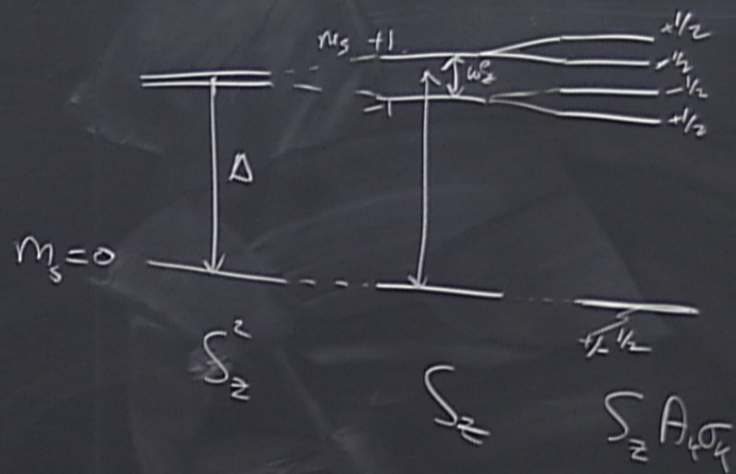
$$\mathcal{H}_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{\text{spin}} \vec{\sigma}^i \cdot \vec{D} \sigma^k$$

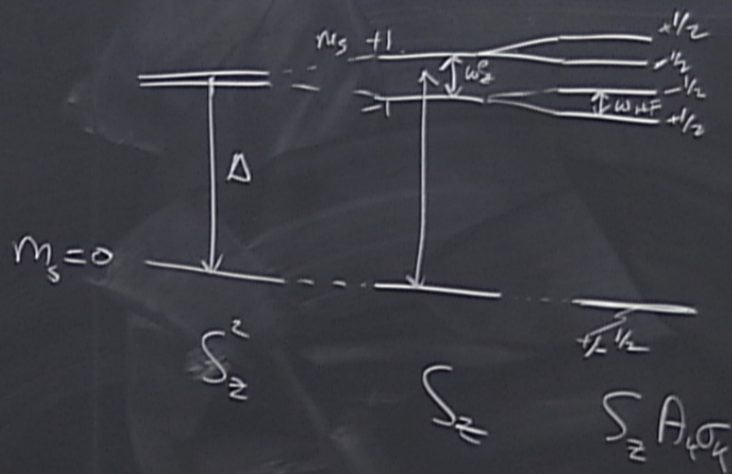
$$\mathcal{H}_{\text{ns}} = \gamma_e (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$











$$\delta_n \vec{B} \cdot \vec{\sigma} + \sum_{s \neq k} \vec{\sigma}^s \cdot \vec{D} \sigma^k$$

1st order $\frac{\langle n | \delta H_{ns} | n \rangle}{\Delta E} = 0$

$$\delta_n \vec{B} \cdot \vec{\sigma} + \sum_{s \neq k} \vec{\sigma}^s \cdot \vec{D} \sigma^k$$

$$\text{1st order } \frac{\langle n | \mathcal{H}_{ns} | n \rangle}{\Delta E} = 0$$

$$\text{2nd order } \frac{\langle n | \mathcal{H}_{ns} | m \rangle \langle m | \mathcal{H}_{ns} | n \rangle}{\Delta E}$$

$$E_{m_s=1} =$$

$$E_{m_s=1} = \frac{1}{2} \frac{\gamma}{\Delta} (B_x^2 + B_y^2)$$

$$E_{m_s=1} = \frac{1}{2} \frac{\gamma}{\Delta} (B_x^2 + B_y^2)$$

$$E_{m_s=0} =$$

$$E_{m_s=1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\overline{B}_x^2 + \overline{B}_y^2)$$

$$E_{m_s=0} = - \frac{\gamma^2}{\Delta} (\overline{B}_x^2 + \overline{B}_y^2)$$

$$E_{m_s=-1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\overline{B}_x^2 + \overline{B}_y^2)$$

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad E_{m_s=1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad E_{m_s=0} = -\frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \quad E_{m_s=-1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad E_{m_s=1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\overline{B}_x^2 + \overline{B}_y^2)$$

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$$P_{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \quad E_{m_s=-1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\overline{B}_x^2 + \overline{B}_y^2)$$

$$S_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; E_{m_s=1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; E_{m_s=0} = -\frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; E_{m_s=-1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$S_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; E_{m_s=1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

$$P_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; E_{m_s=0} = -\frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

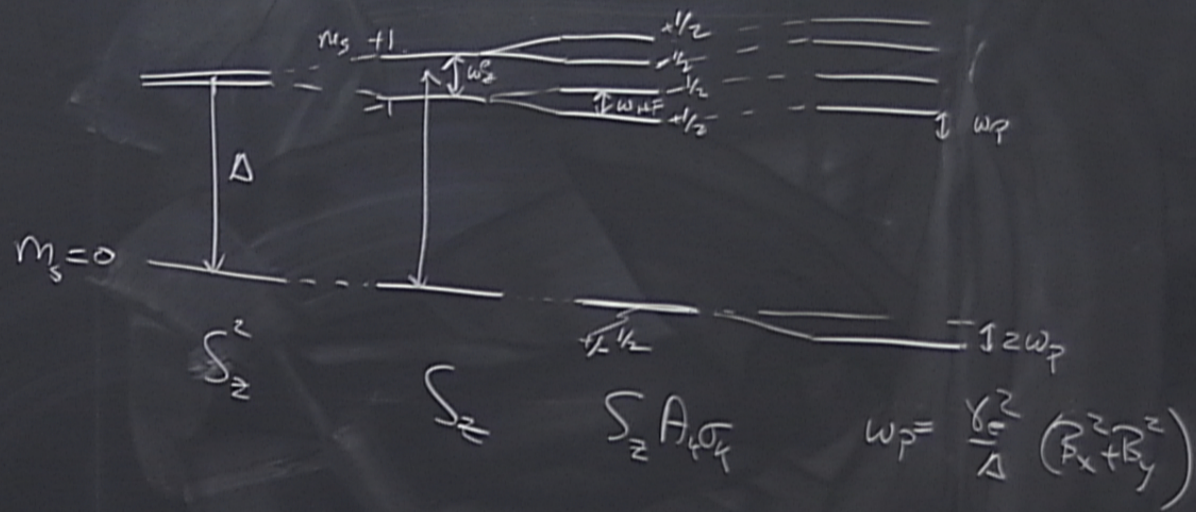
$$P_{-1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; E_{m_s=-1} = \frac{1}{2} \frac{\gamma^2}{\Delta} (\mathcal{B}_x^2 + \mathcal{B}_y^2)$$

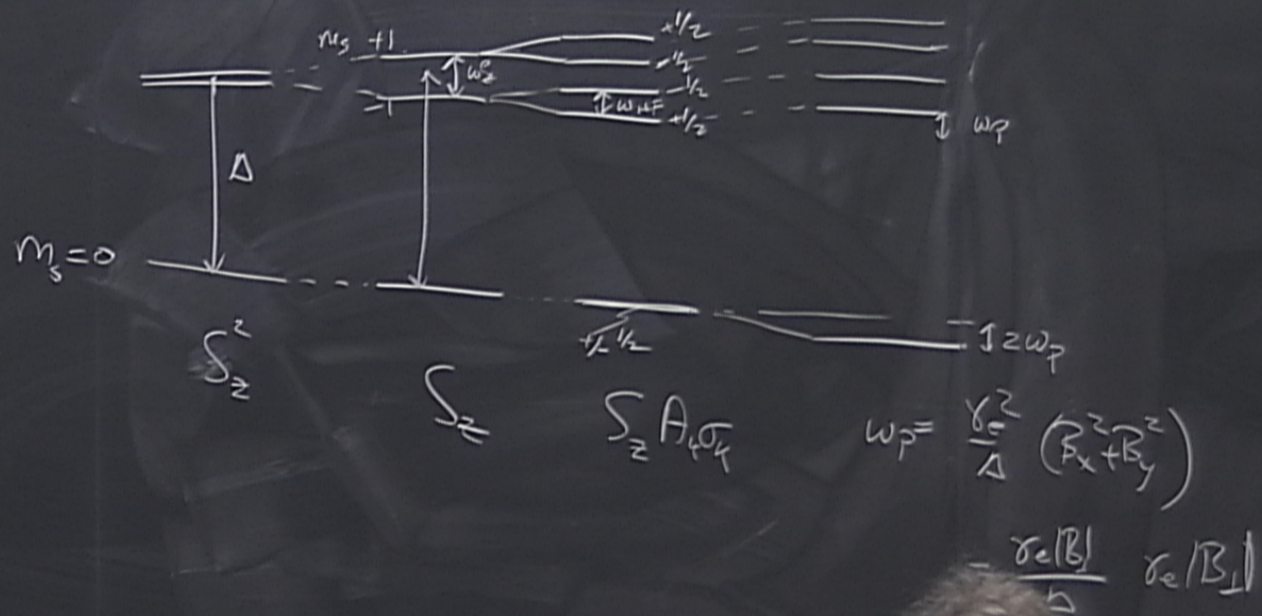
$$= \begin{pmatrix} 1 & \\ & -2 \end{pmatrix} = (3S_z^2 - 2\mathbb{1})$$

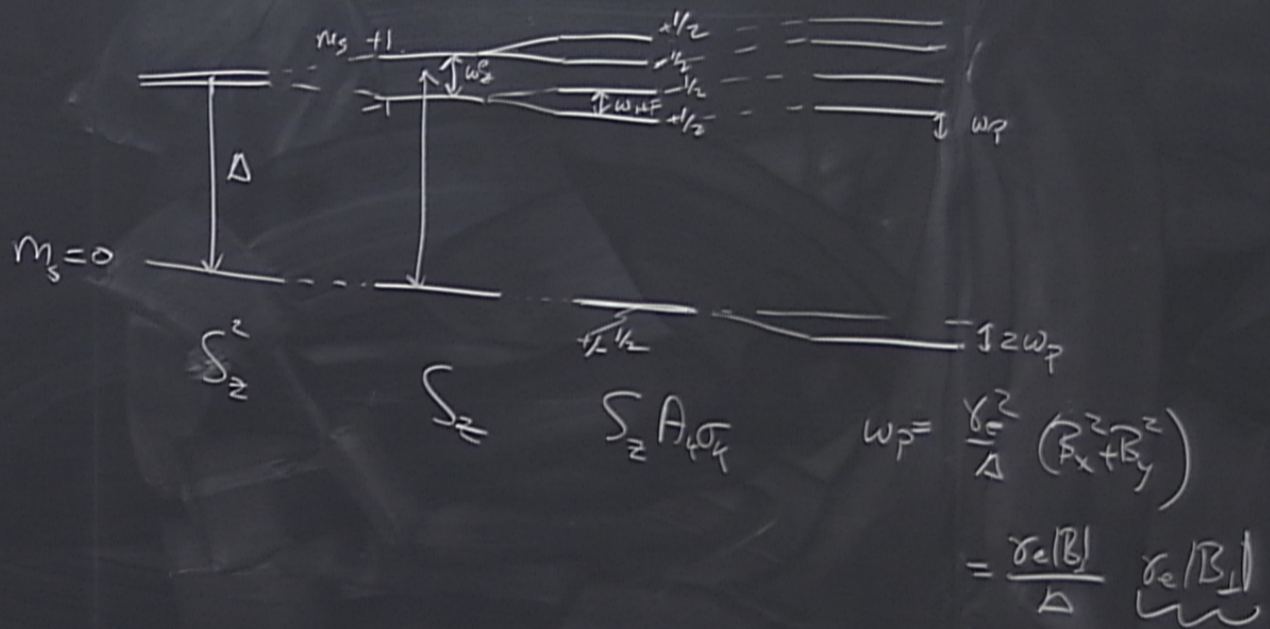
$$S_z^2 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$$

$$\mathbb{1} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$S_z = \begin{pmatrix} 1 & \\ & 0 \\ & & -1 \end{pmatrix}$$





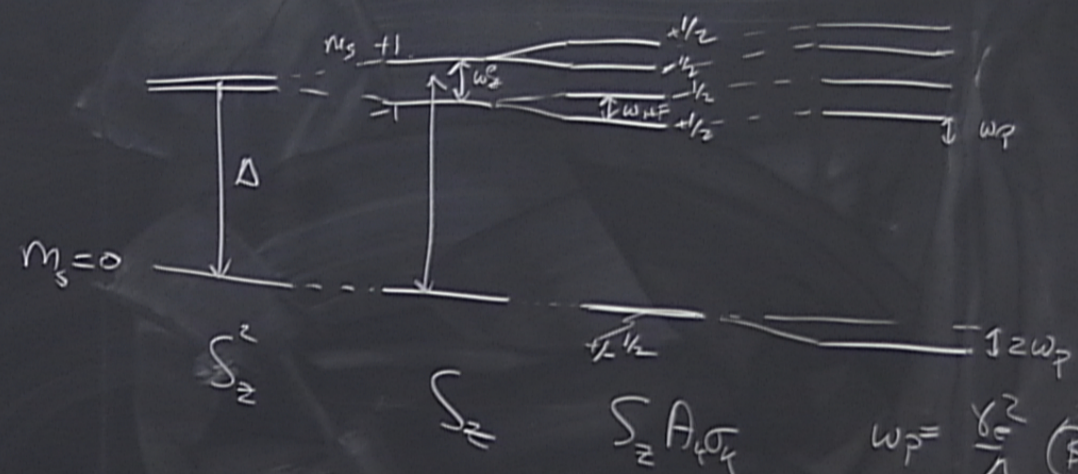


$$\mathcal{H}_{\text{secular}} = A S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{s \neq k} \dots$$

$$\mathcal{H}_{\text{ns}} = \gamma_p (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

Z/HF

Zeeman
 z z z



$$\omega_p = \frac{\gamma_e^2}{\Delta} (\overline{B_x^2} + \overline{B_y^2})$$

$$= \frac{\gamma_e |B|^2}{\Delta}$$

$$\mathcal{H}_{\text{secular}} = A S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \gamma_n \vec{B} \cdot \vec{\sigma} + \sum_{s \neq k} \tilde{D}^s \sigma^k$$

$$\mathcal{H}_{\text{ns}} = \gamma_e (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

Z/HF

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}_{HS} = \gamma_p (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

z/HF

$$E_{m_s=1} = \frac{\gamma_p}{\Delta} (B_x A_x + B_y A_y) \cdot \vec{\sigma}$$

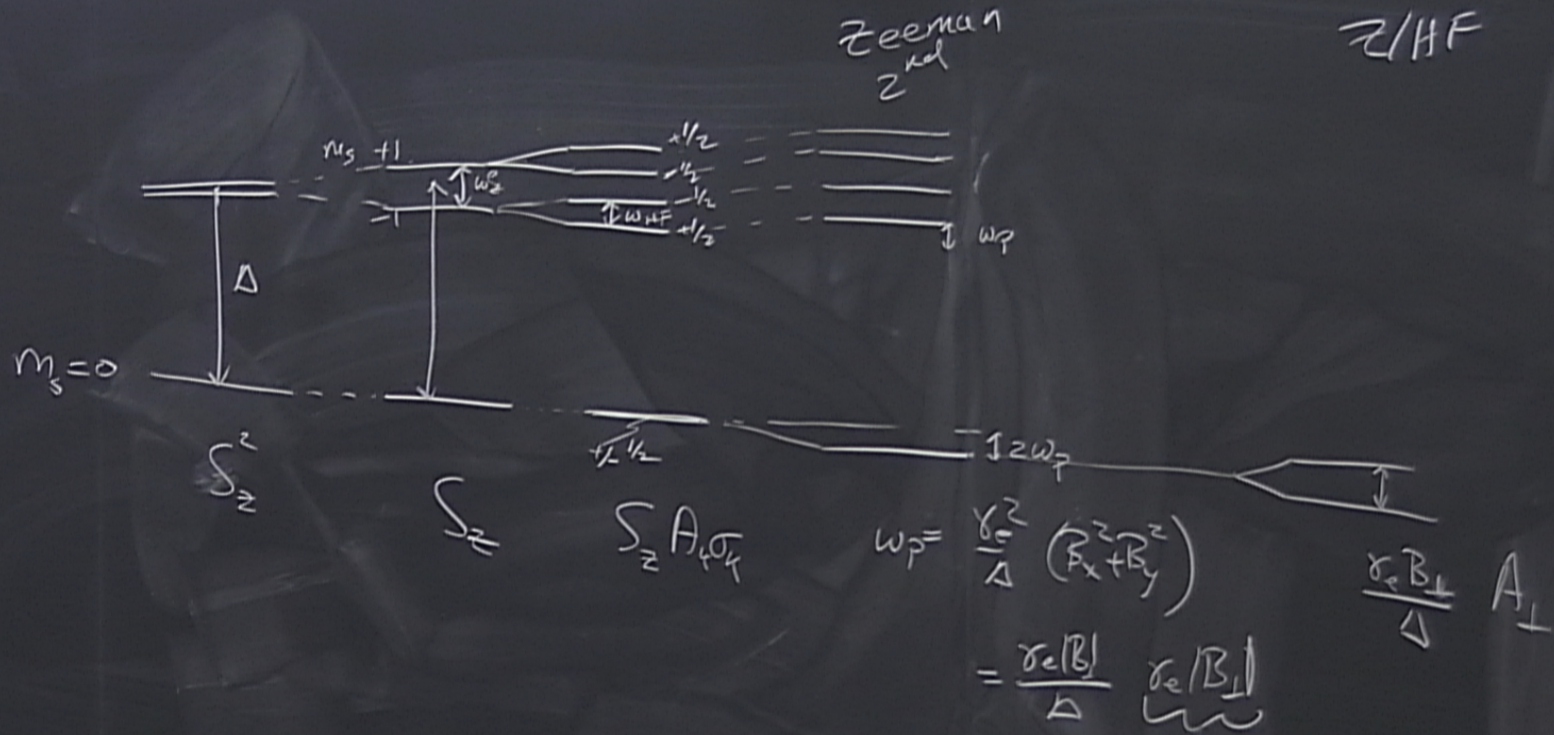
$$\mathcal{H}_{z/HF}^{(z)} = (3S_z^2 - 2I) \frac{\gamma_p}{\Delta} (B_x A_x + B_y A_y) \cdot \vec{\sigma}$$

$$\mathcal{H}_{NS} = \gamma_e (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

z/HF

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y)$$

$$\mathcal{H}_{z/HF}^{(z)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \quad \begin{matrix} \uparrow \\ \text{to} \\ \text{MM} \end{matrix}$$



$$\mathcal{H}_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \frac{\gamma_n \vec{B} \cdot \vec{\sigma}}{\sim 10 \text{ kHz}} + \sum_{\text{SAK}} \vec{\sigma}^i \cdot \vec{D} \sigma^k$$

$$\mathcal{H}_{\text{ns}} = \gamma_p (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

\mathbb{Z}/HF

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \cdot \vec{\sigma}$$

$$\mathcal{H}_{\mathbb{Z}/\text{HF}}^{(\mathbb{Z})} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \frac{\mu_B}{M}$$

$$\mathcal{H}_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \frac{\gamma_n \vec{B} \cdot \vec{\sigma}}{\sim 10 \text{ kHz}} + \sum_{\text{SAK}} \vec{\sigma}^i \cdot \vec{D} \sigma^k$$

$$\mathcal{H}_{\text{ns}} = \gamma_e (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

Z/HF

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \cdot \vec{\sigma}$$

$$\mathcal{H}_{\text{Z/HF}}^{(2)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \cdot \vec{\sigma}$$

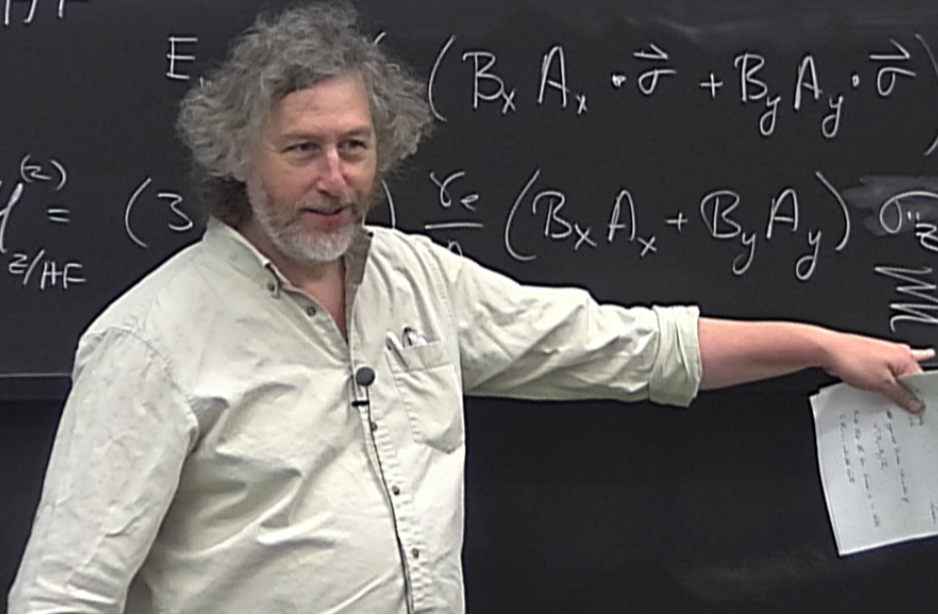
$$\mathcal{H}_{\text{secular}} = \Delta S_z^2 + \gamma_e B_z S_z + \sum_k S_z A_{zk} \sigma_k + \frac{\gamma_n \vec{B} \cdot \vec{\sigma}}{\sim 10 \text{ kHz}} + \sum_{\text{SAK}} \vec{\sigma} \cdot \mathcal{D} \sigma^k$$

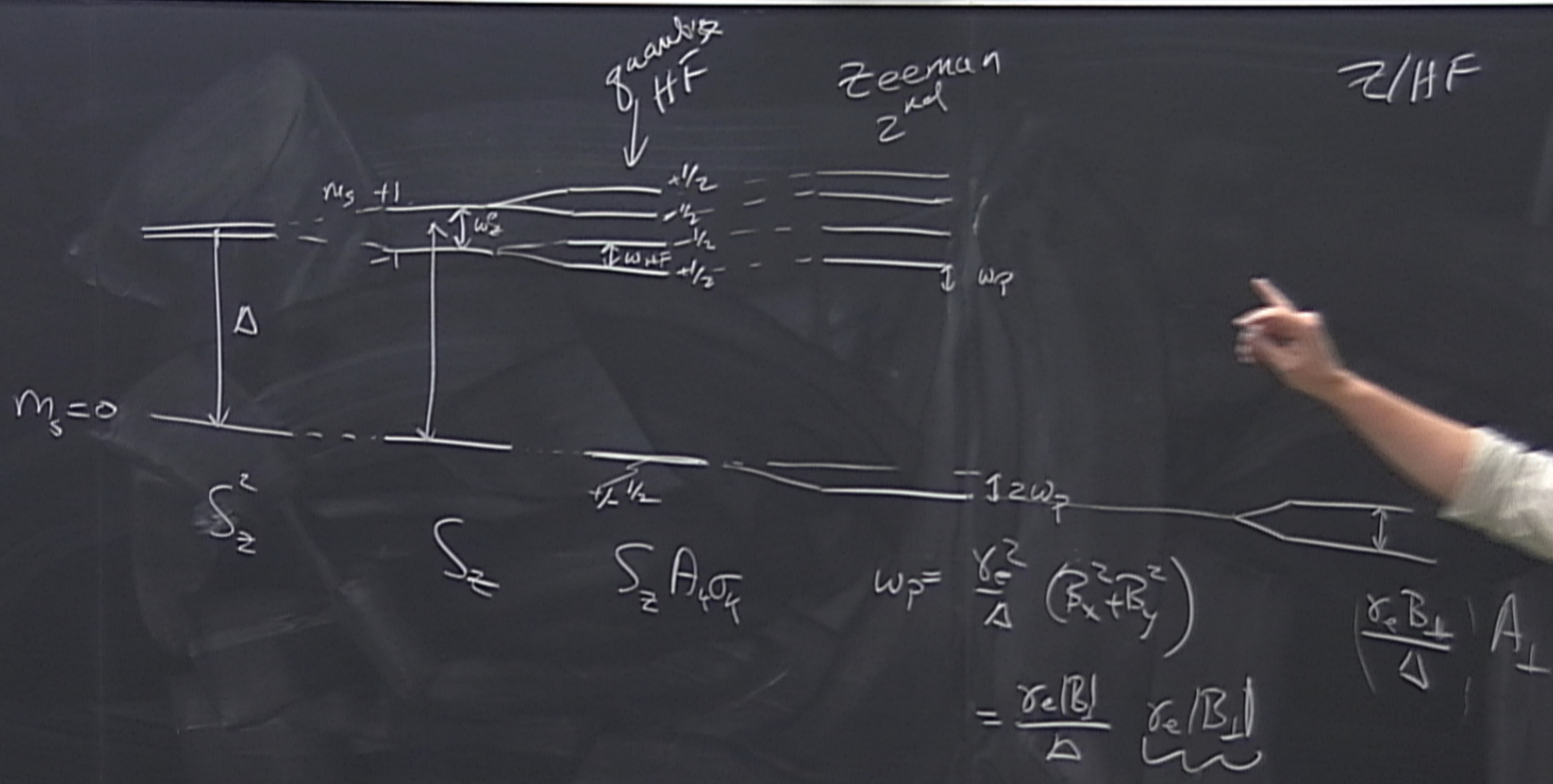
$$\mathcal{H}_{\text{ns}} = \gamma_e (B_x S_x + B_y S_y) + \sum_k (S_x A_{xk} \sigma_k + S_y A_{yk} \sigma_k)$$

Z/HF

$$E_{\text{ns}} (\vec{B}_x A_x \cdot \vec{\sigma} + \vec{B}_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}_{\text{Z/HF}}^{(z)} = (3) \frac{\gamma_e}{n} (\vec{B}_x A_x + \vec{B}_y A_y) \cdot \vec{\sigma} \quad \sigma_{\text{PAS}}$$





Zeeman
Z_{red}

Z/HF

H F' / H F

ω_p

$\downarrow 2\omega_p$

$$\omega_p = \frac{\gamma_e^2}{\Delta} (\vec{F}_x + \vec{B}_y)$$
$$= \frac{\gamma_e |B|}{\Delta} \underbrace{\gamma_e |B_{\perp}|}$$

$$\left(\frac{\gamma_e B_{\perp}}{\Delta} \right) A_{\perp}$$

$$E_{ms=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}^{(z)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \underbrace{\sigma_x \sigma_x + \sigma_y \sigma_y}_{\sigma_{PAS}}$$

$$E_{ms=1}^{(z)} = \langle 1 | S_x A_x \cdot \sigma + S_y A_y \cdot \sigma | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle$$

$$E_{ms=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$H_{\frac{z}{\hbar c}}^{(z)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \underbrace{\sum_i \vec{\sigma}_i}_{\sigma_{PAS}}$$

$$E_{ms=1}^{(z)} = \langle 1 | S_x A_x \cdot \sigma + S_y A_y \cdot \sigma | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle + hc$$

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}_{\text{Z/HF}}^{(2)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \underbrace{\sigma_{\text{PAS}}^{(z)}}_{\sigma_{\text{PAS}}}$$

$$\begin{aligned} E_{m_s=1}^{(2)} &= \langle 1 | \frac{S_x A_x \cdot \sigma + S_y A_y \cdot \sigma}{\Delta} | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle + hc \\ &= \frac{1}{\Delta} \left[(A_x \cdot \sigma)(A'_x \cdot \sigma) + (A_y \cdot \sigma)(A'_y \cdot \sigma') \right] \end{aligned}$$

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}^{(2)}_{\frac{e}{\hbar c}} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \underbrace{\sigma_{\alpha\beta}}_{\sigma_{PAS}}$$

$$E_{m_s=1}^{(2)} = \langle 1 | \frac{S_x A_x \cdot \sigma + S_y A_y \cdot \sigma}{\Delta} | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle + hc$$

$$= \frac{1}{\Delta} \left[(A_x \cdot \sigma)(A'_x \cdot \sigma') + (A_y \cdot \sigma)(A'_y \cdot \sigma') \right]$$

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}_{\text{eff}}^{(2)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \sum_{\sigma, \sigma'} \sigma_{\text{PAS}}$$

$$E_{m_s=1}^{(2)} = \langle 1 | \frac{S_x A_x \cdot \sigma + S_y A_y \cdot \sigma}{\Delta} | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle + hc$$

$$= \frac{1}{\Delta} \left[(A_x \cdot \sigma)(A'_x \cdot \sigma') + (A_y \cdot \sigma)(A'_y \cdot \sigma') \right]$$

effective n/n interaction

$$E_{m_s=1} = \frac{\gamma_e}{\Delta} (B_x A_x \cdot \vec{\sigma} + B_y A_y \cdot \vec{\sigma})$$

$$\mathcal{H}_{\text{eff}}^{(2)} = (3S_z^2 - 2I) \frac{\gamma_e}{\Delta} (B_x A_x + B_y A_y) \underbrace{\sum_{\sigma, \sigma'} \sigma_{\text{PAS}}}_{\sigma_{\text{PAS}}}$$

$$E_{m_s=1}^{(2)} = \langle 1 | \frac{S_x A_x \cdot \sigma + S_y A_y \cdot \sigma}{\Delta} | 0 \rangle \langle 0 | S_x A'_x \cdot \sigma' + S_y A'_y \cdot \sigma' | 1 \rangle + hc$$

$$= \frac{1}{\Delta} \left[(A_x \cdot \sigma)(A'_x \cdot \sigma') + (A_y \cdot \sigma)(A'_y \cdot \sigma') \right]$$

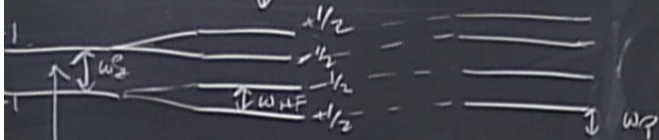
effective n/n interaction, fast

quadrup
HF

Zeeman
 z_{rel}

z/HF

$H F' / H F'$



$\pm 1/2$

$\pm 2w_p$

$\left(\frac{\gamma_e B_{\perp}}{\Delta} \right) A_{\perp}$

S_z

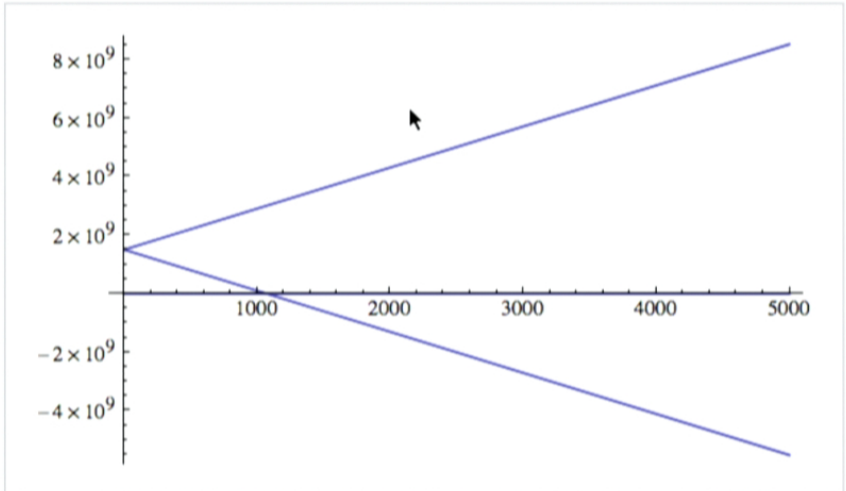
$S_z A_{\perp} \sigma_K$

$$w_p = \frac{\gamma_e^2}{\Delta} (\vec{F}_x + \vec{B}_y) = \frac{\gamma_e |B_{\perp}|}{\Delta} \underbrace{\gamma_e |B_{\perp}|}$$

```
In[25]:= Manipulate[Plot[Energy[3 × 10^9, γe B, γn B, wHF, θ], {B, 0, 5000}], {wHF, 0, 2.5 × 10^5}, {θ, 0, π/2}]
```

wHF

θ



Out[25]=

```
In[26]:= Manipulate[Plot[Energy[3 × 10^9, γe B, γn B, wHF, θ], {B, 1060, 1080}, {PlotRange → {-0.1 × 10^8, 0.1 × 10^8}}, {wHF, 0, 2.5 × 10^7}, {θ, 0, π/2}]
```

wHF

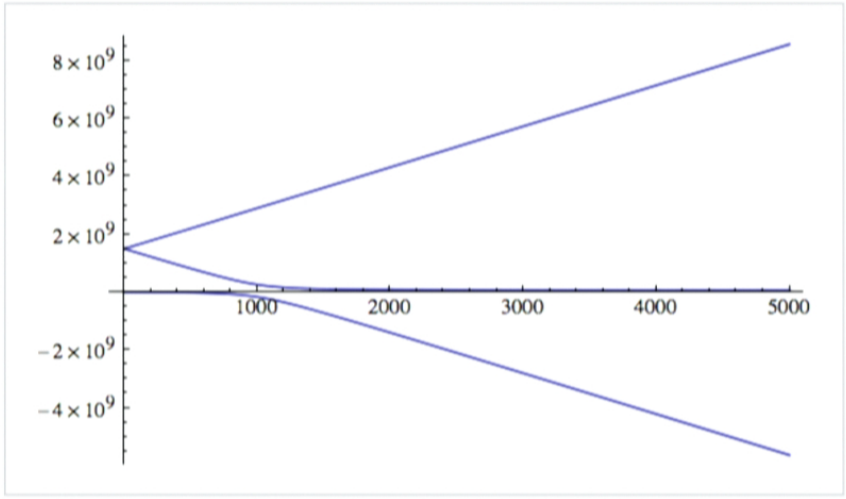
θ

```
In[25]:= Manipulate[Plot[Energy[3 × 10^9, γe B, γn B, wHF, θ], {B, 0, 5000}], {wHF, 0, 2.5 × 10^5}, {θ, 0, π/2}]
```

wHF

θ

Out[25]=



```
In[26]:= Manipulate[Plot[Energy[3 × 10^9, γe B, γn B, wHF, θ], {B, 1060, 1080}, {PlotRange → {-0.1 × 10^8, 0.1 × 10^8}}, {wHF, 0, 2.5 × 10^7}, {θ, 0, π/2}]
```

wHF

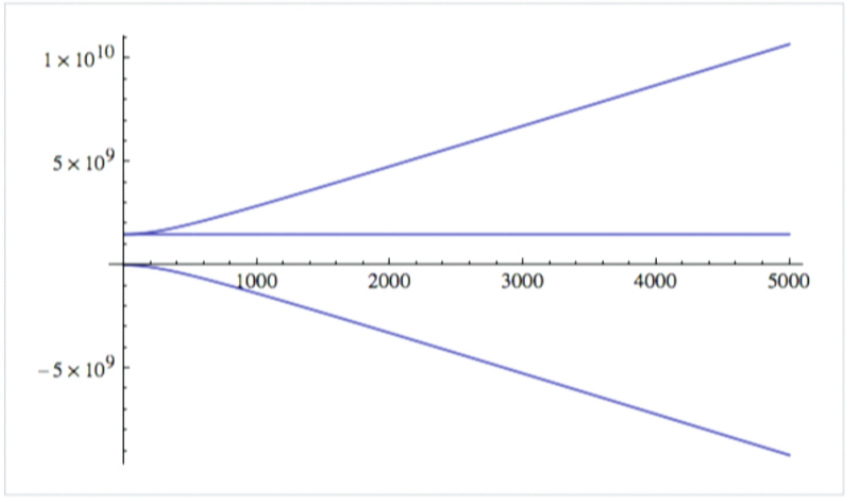
θ

```
In[25]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, 5000}], {wHF, 0, 2.5 × 10^5}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

θ

Out[25]=



```
In[26]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 1060, 1080}, {PlotRange → {-0.1 × 10^8, 0.1 × 10^8}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

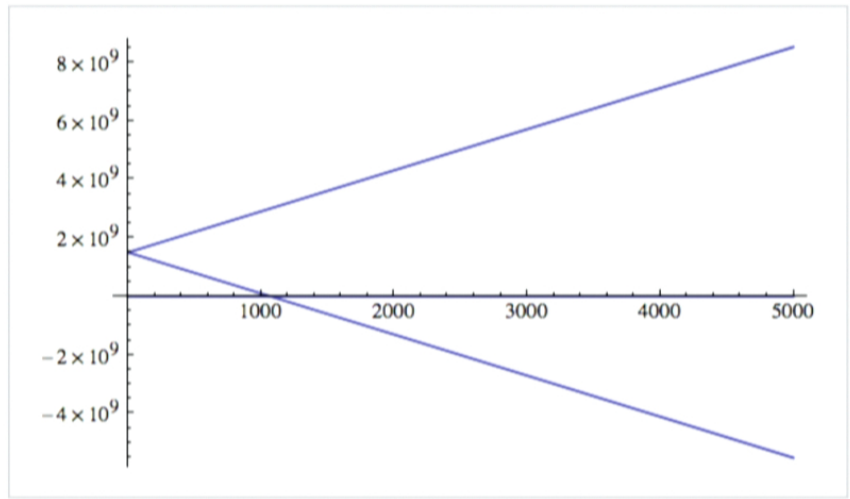
θ


```
In[24]:= Energy[wZF_,we_,wn_,wHF_,  $\theta$ _]:= Sort[Eigenvalues[H[wZF,we,wn,wHF,  $\theta$ ]]]
```

```
In[25]:= Manipulate[Plot[Energy[ $3 \times 10^9$ ,  $\gamma_e B$ ,  $\gamma_n B$ , wHF,  $\theta$ ], {B, 0, 5000}], {wHF, 0,  $2.5 \times 10^5$ }, { $\theta$ , 0,  $\pi/2$ }]
```

wHF θ

Out[25]=



```
In[26]:= Manipulate[Plot[Energy[ $3 \times 10^9$ ,  $\gamma_e B$ ,  $\gamma_n B$ , wHF,  $\theta$ ], {B, 1060, 1080}, {PlotRange -> {- $.1 \times 10^8$ ,  $.1 \times 10^8$ }}, {wHF, 0,  $2.5 \times 10^7$ }, { $\theta$ , 0,  $\pi/2$ }]
```

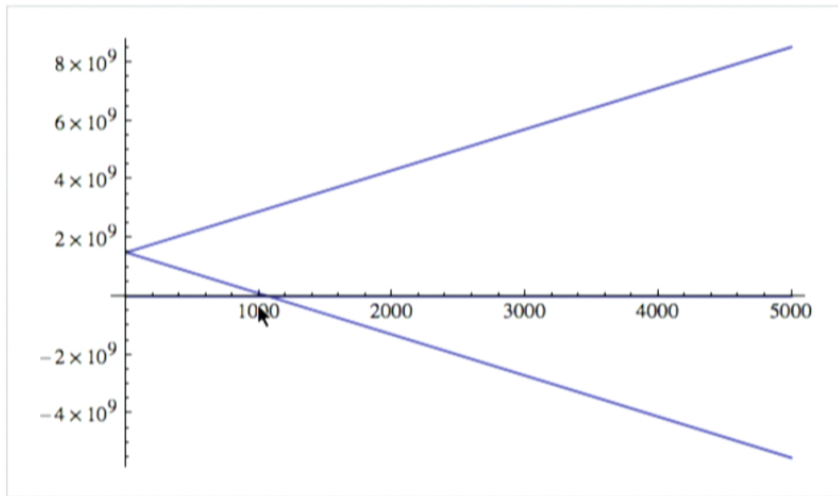
wHF θ

```
In[24]:= Energy[wZF_, we_, wn_, wHF_,  $\theta$ _]:= Sort[Eigenvalues[H[wZF, we, wn, wHF,  $\theta$ ]]]
```

```
In[25]:= Manipulate[Plot[Energy[ $3 \times 10^9$ ,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, 5000}], {wHF, 0,  $2.5 \times 10^5$ }, { $\theta$ , 0,  $\pi/2$ }]
```



Out[25]=



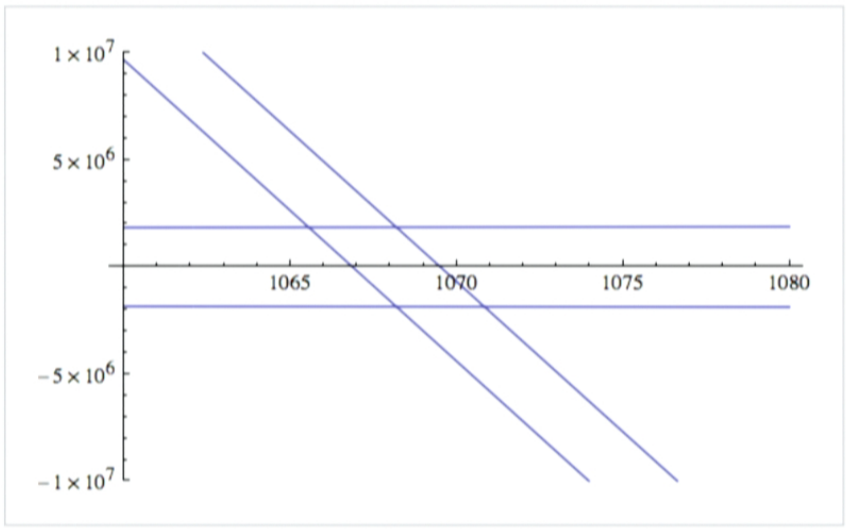
```
In[26]:= Manipulate[Plot[Energy[ $3 \times 10^9$ ,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 1060, 1080}, {PlotRange  $\rightarrow$   $\{-.1 \times 10^8, .1 \times 10^8\}$ }}, {wHF, 0,  $2.5 \times 10^7$ }, { $\theta$ , 0,  $\pi/2$ }]
```



wHF

θ

Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

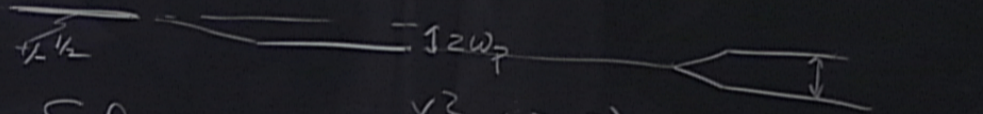
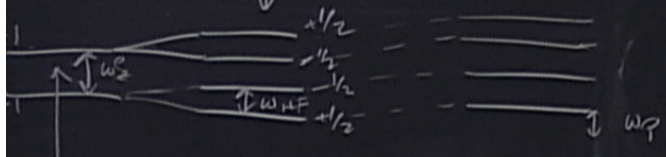
θ

quadrup
HF

Zeeman
 z rel

z/HF

HF/HF'

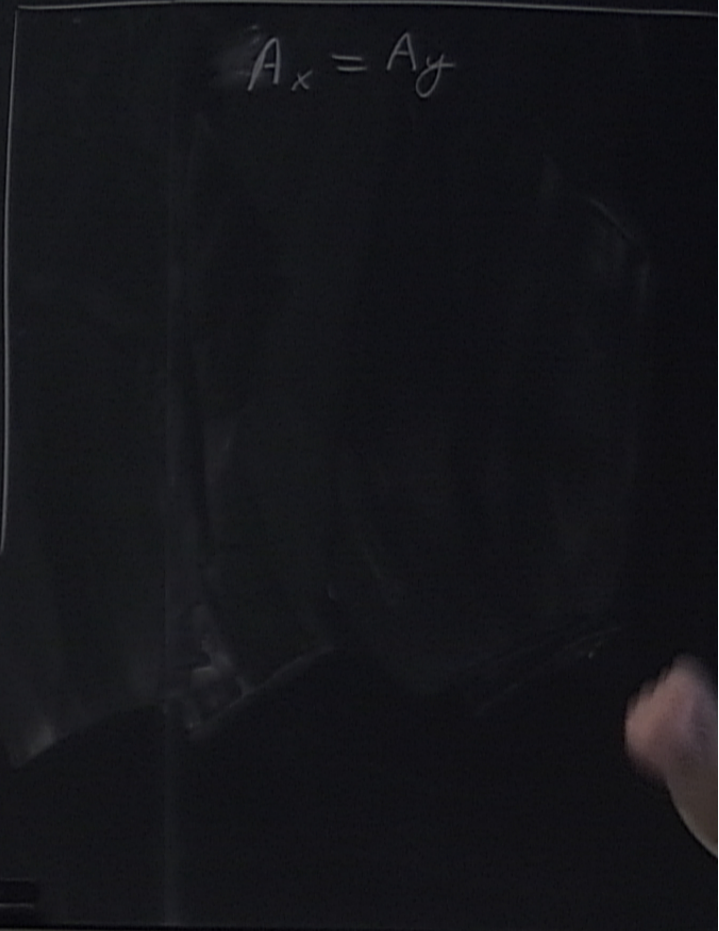


S_z
 $S_z A_{\sigma_K}$

$$w_p = \frac{\gamma_e^2}{\Delta} (\vec{F}_x + \vec{B}_y)$$

$$= \frac{\gamma_e |B_{\parallel}|}{\Delta} \underbrace{\gamma_e |B_{\perp}|}$$

$\left(\frac{\gamma_e B_{\perp}}{\Delta} \right) A_{\perp}$



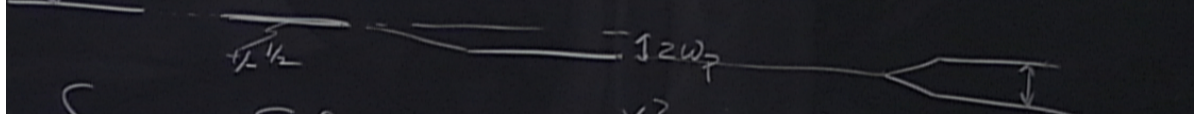
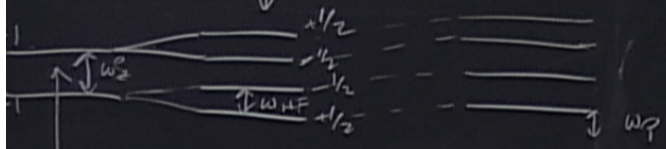
$A_x = A_y$

quadrup
HF

Zeeman
Z

Z/HF

HF/HF'



$$A_x = A_y$$

$$S_z$$

$$S_z A_{\uparrow\downarrow}$$

$$w_p = \frac{\gamma_e^2}{\Delta} (\vec{B}_x + \vec{B}_y)$$

$$= \frac{\gamma_e |B_x|}{\Delta} \gamma_e |B_y|$$

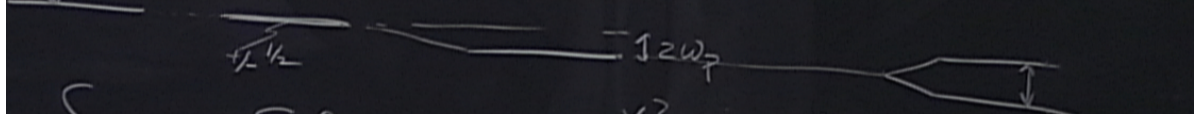
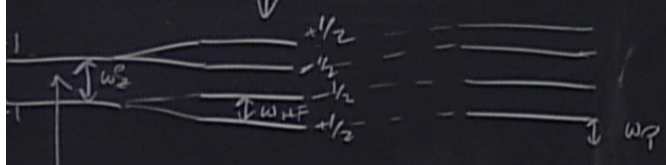
$$\left(\frac{\gamma_e B_{\perp}}{\Delta} \right) A_{\perp}$$

quadrup
HF

Zeeman
Z rel

Z/HF

HF/HF'



S_z
 $S_z A_4 \sigma_4$

$$w_p = \frac{\gamma_e^2}{\Delta} (\vec{F}_x + \vec{B}_y)$$

$$= \frac{\gamma_e |B_{\parallel}|}{\Delta} \underbrace{\gamma_e |B_{\perp}|}$$

$\frac{\gamma_e B_{\perp}}{\Delta} A_{\perp}$

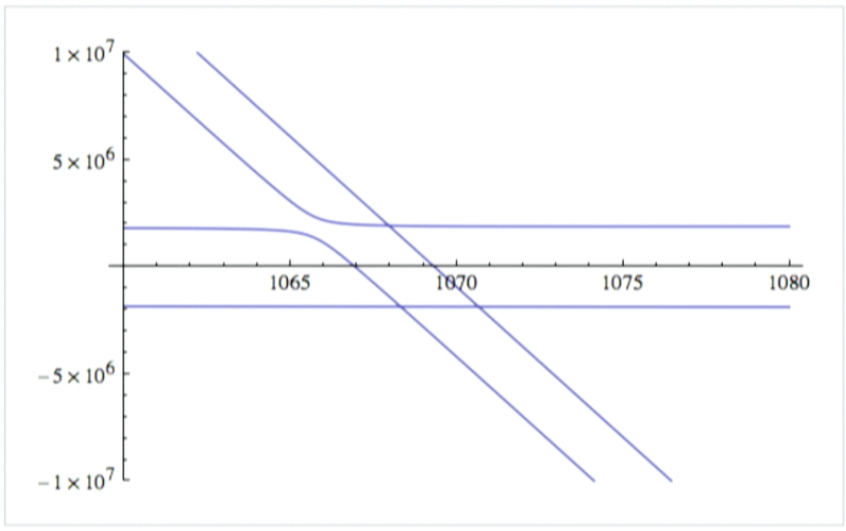
$A_x = A_y$

$S_x \sigma_x + S_y \sigma_y$

wHF

θ

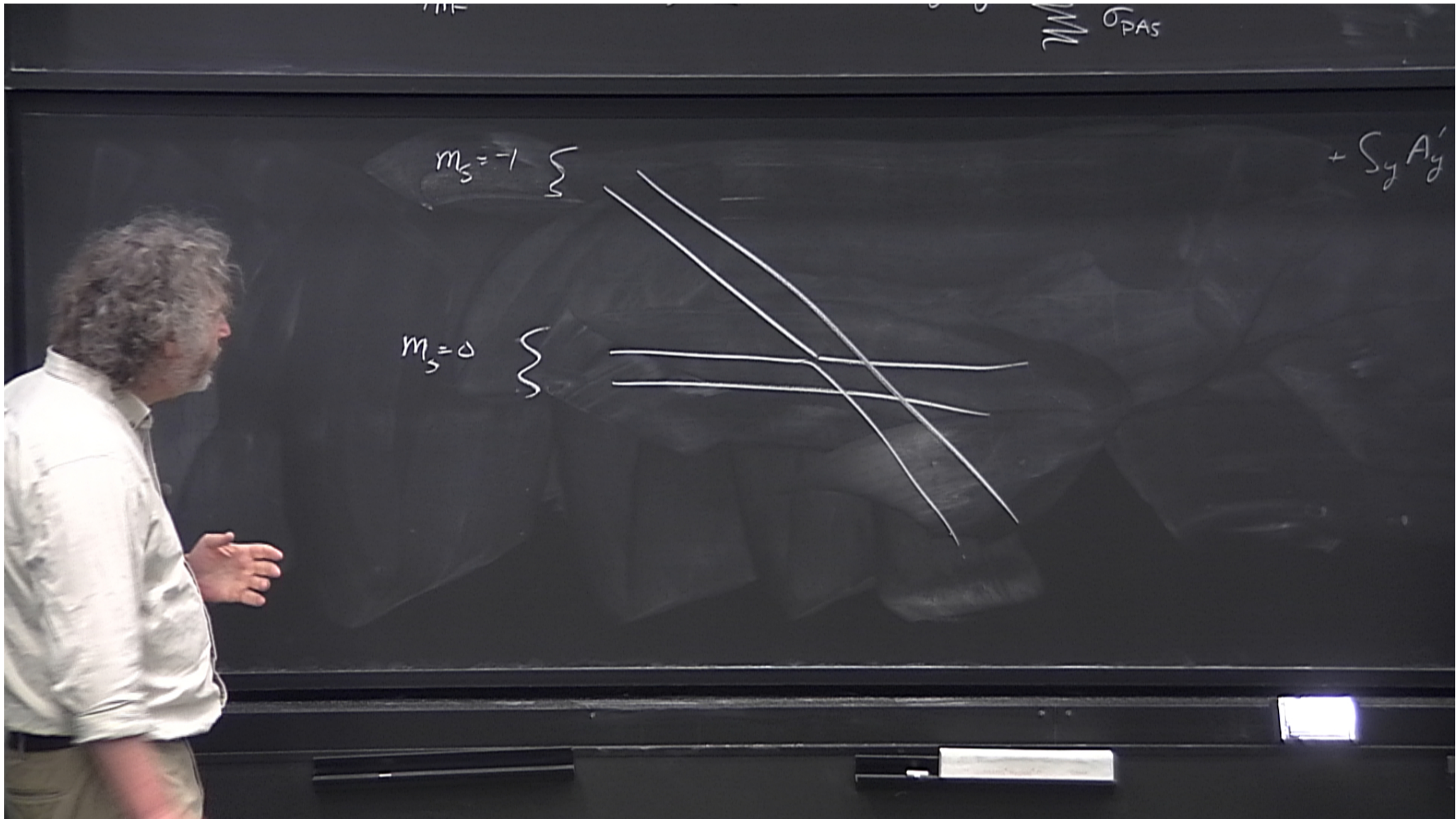
Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

θ

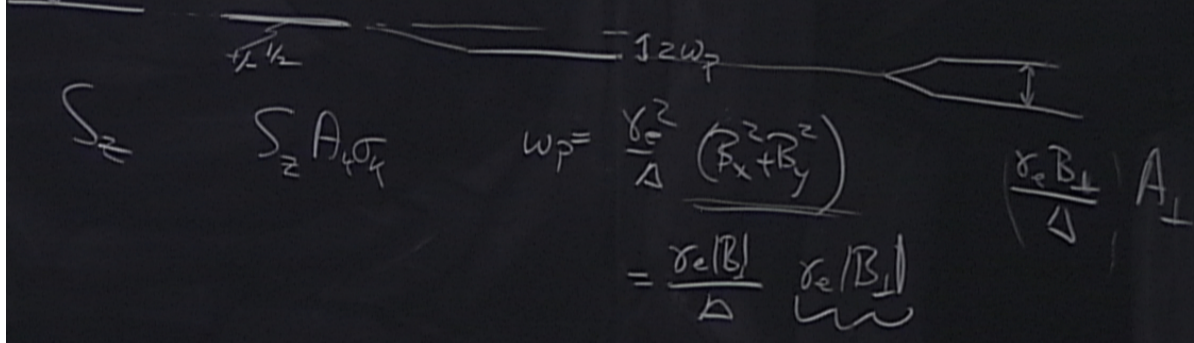
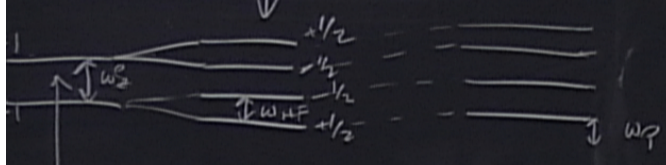


quadrup
HF

Zeeman
 z_{rel}

z/HF

HF/HF'



$$S_z$$

$$S_z A_{\pm} \sigma_{\mp}$$

$$w_p = \frac{\gamma_e^2}{\Delta} (\vec{r}_x + \vec{r}_y)^2$$

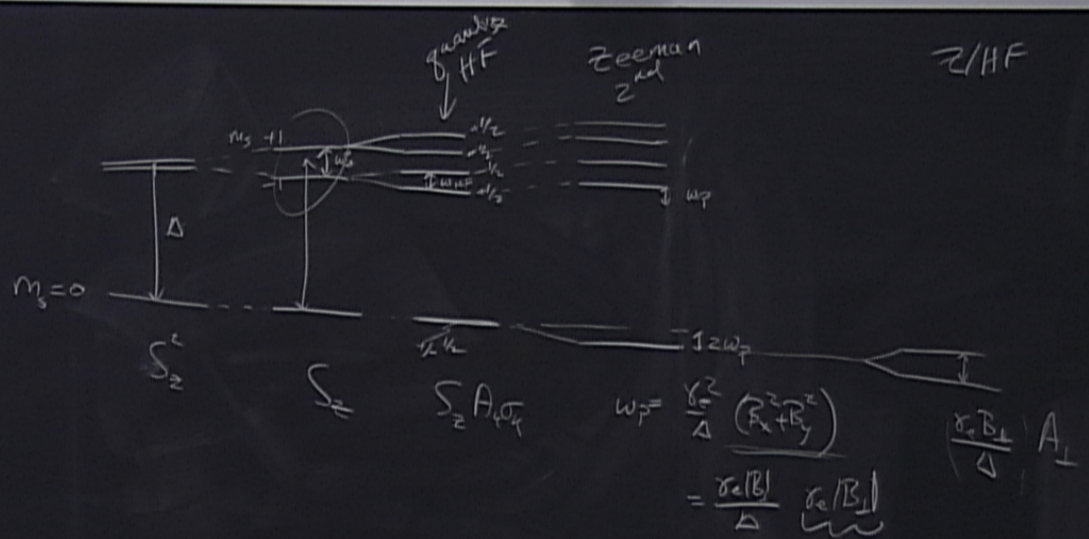
$$= \frac{\gamma_e |B_{\parallel}|}{\Delta} \underbrace{\gamma_e |B_{\perp}|}$$

$$\left(\frac{\gamma_e B_{\perp}}{\Delta} \right) A_{\perp}$$

$$A_x = A_y$$

$$S_x \sigma_x + S_y \sigma_y$$

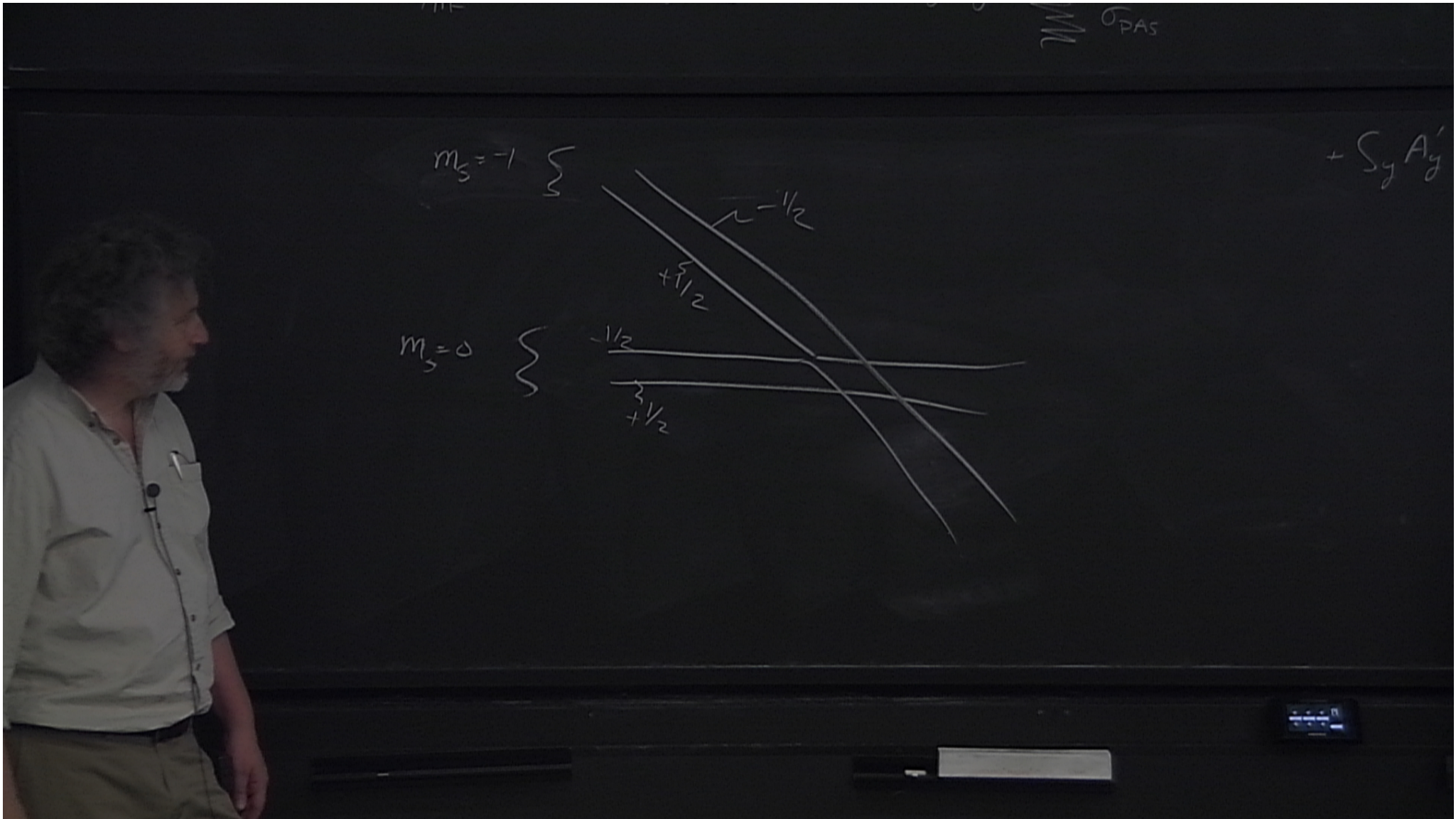
$$S_+ \sigma_- + S_- \sigma_+$$

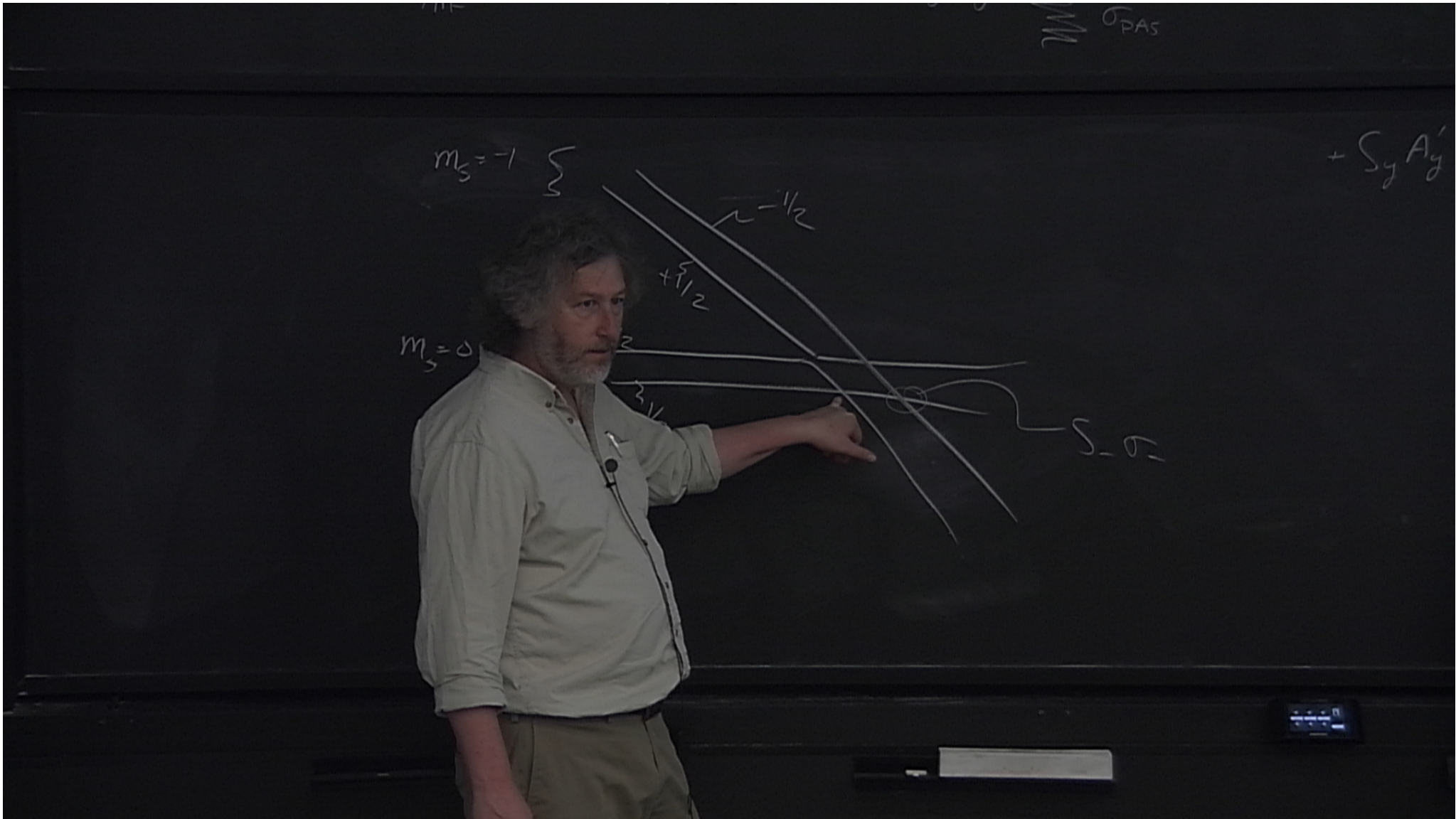


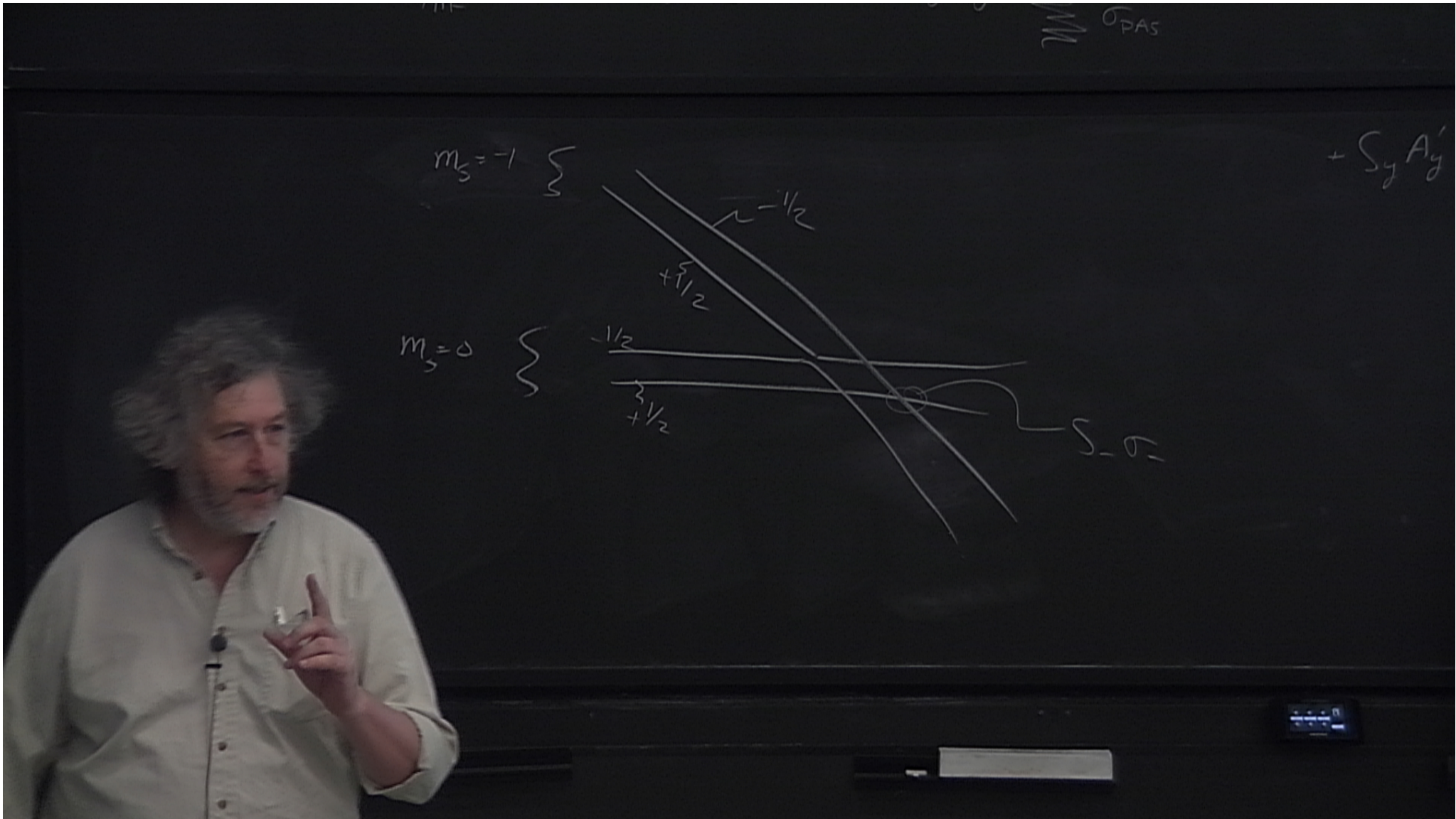
$$A_x = A_y$$

$$S_x \sigma_x + S_y \sigma_y$$

$$S_+ \sigma_- + S_- \sigma_+$$





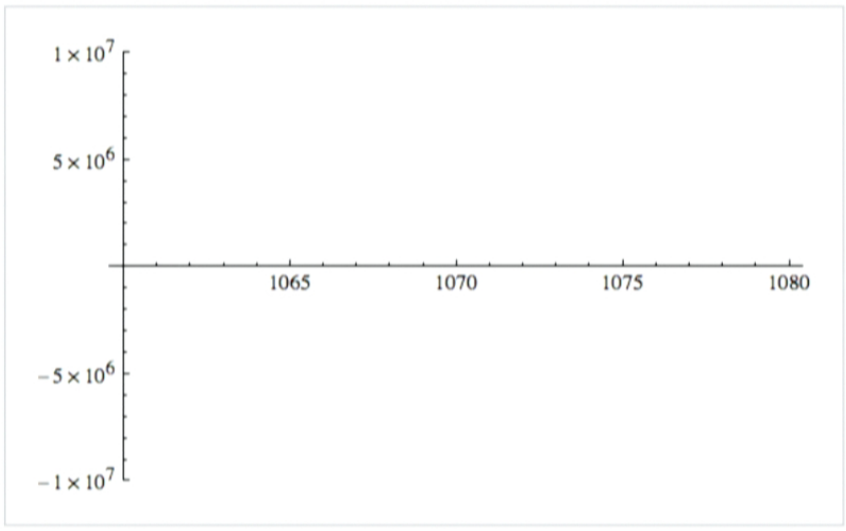


wHF

θ

0.0502655 | - ▶ + ⌵ ⌶ →

Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

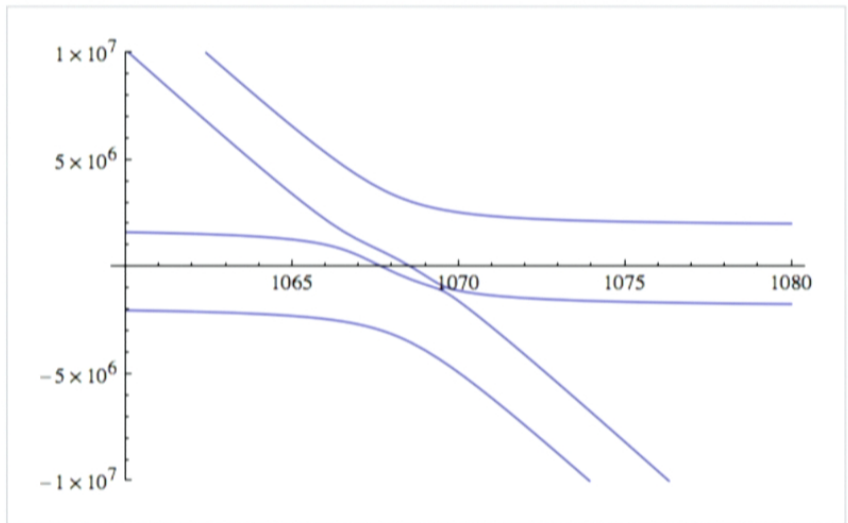
wHF

θ

wHF

θ

Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

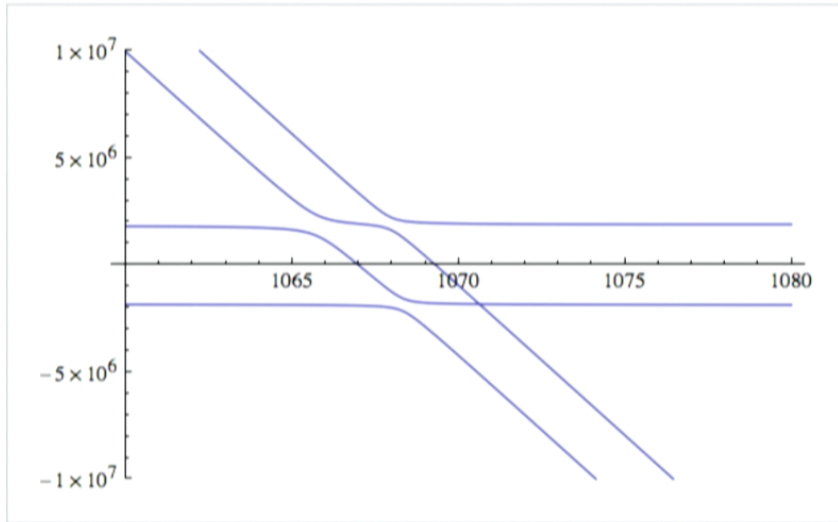
θ

wHF

θ

0.00021

Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

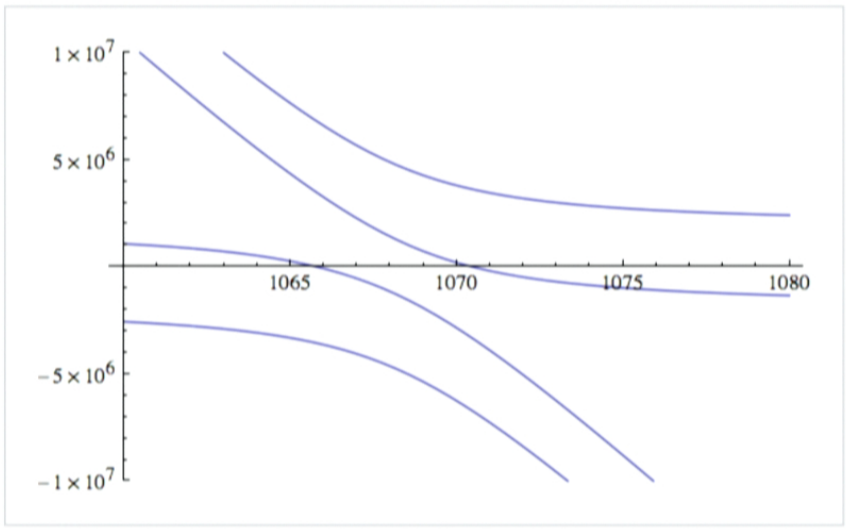
wHF

θ

wHF

θ

Out[26]=



```
In[27]:= Manipulate[Plot[Energy[3 × 10^9,  $\gamma_e$  B,  $\gamma_n$  B, wHF,  $\theta$ ], {B, 0, .1},
  {PlotRange → {1.5 × 10^9 - .4 × 10^6, 1.5 × 10^9 + .4 × 10^6}}, {wHF, 0, 2.5 × 10^7}, { $\theta$ , 0,  $\pi/2$ }]
```

wHF

θ

quantized HF
 Zeeman 2nd
 Z/HF
 HF/HF'

$m_s = 0$
 Δ
 S_z
 S_z
 $12\omega_p$
 ω_p
 $\frac{\mu_B^2}{\Delta} (\overline{B_x^2} + \overline{B_y^2})$
 $= \frac{\sigma_z |B_x|}{\Delta} \frac{\sigma_z |B_y|}{\Delta}$
 $\frac{\sigma_z B_{\perp}}{\Delta} A_{\perp}$

$A_x = A_y$
 $S_x \sigma_x + S_y \sigma_y$
 $S_+ \sigma_- + S_- \sigma_+$

Zeeman 2nd
 quantiz of HF
 $\mathcal{H}/\mathcal{H}F$
 $\mathcal{H}F'/\mathcal{H}F'$

Δ
 $m_s = 0$
 S_z
 S_z
 $S_z A_{\alpha} \sigma_{\alpha}$
 $\omega_p = \frac{\chi^2}{\Delta} (\beta_x^2 + \beta_y^2)$
 $= \frac{\sigma_{\alpha} |\beta_{\perp}|}{\Delta} \underbrace{\sigma_{\alpha} |\beta_{\perp}|}$
 $\frac{\sigma_{\alpha} |\beta_{\perp}|}{\Delta} A_{\perp}$
 $A_x = A_y$
 $\sigma_x + \sum_y \beta_y \sigma_y + \sum_z \beta_z \sigma_z$