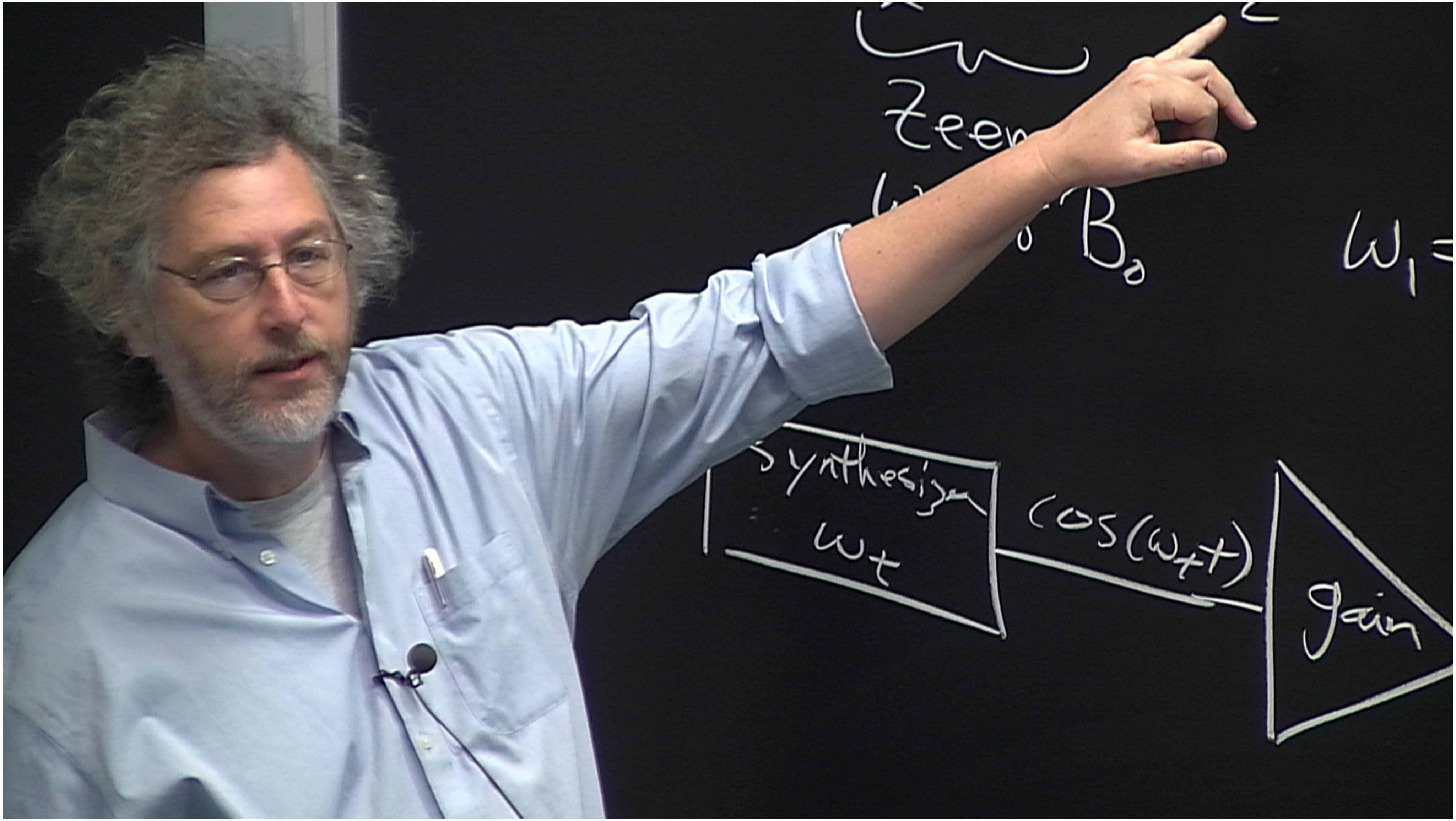
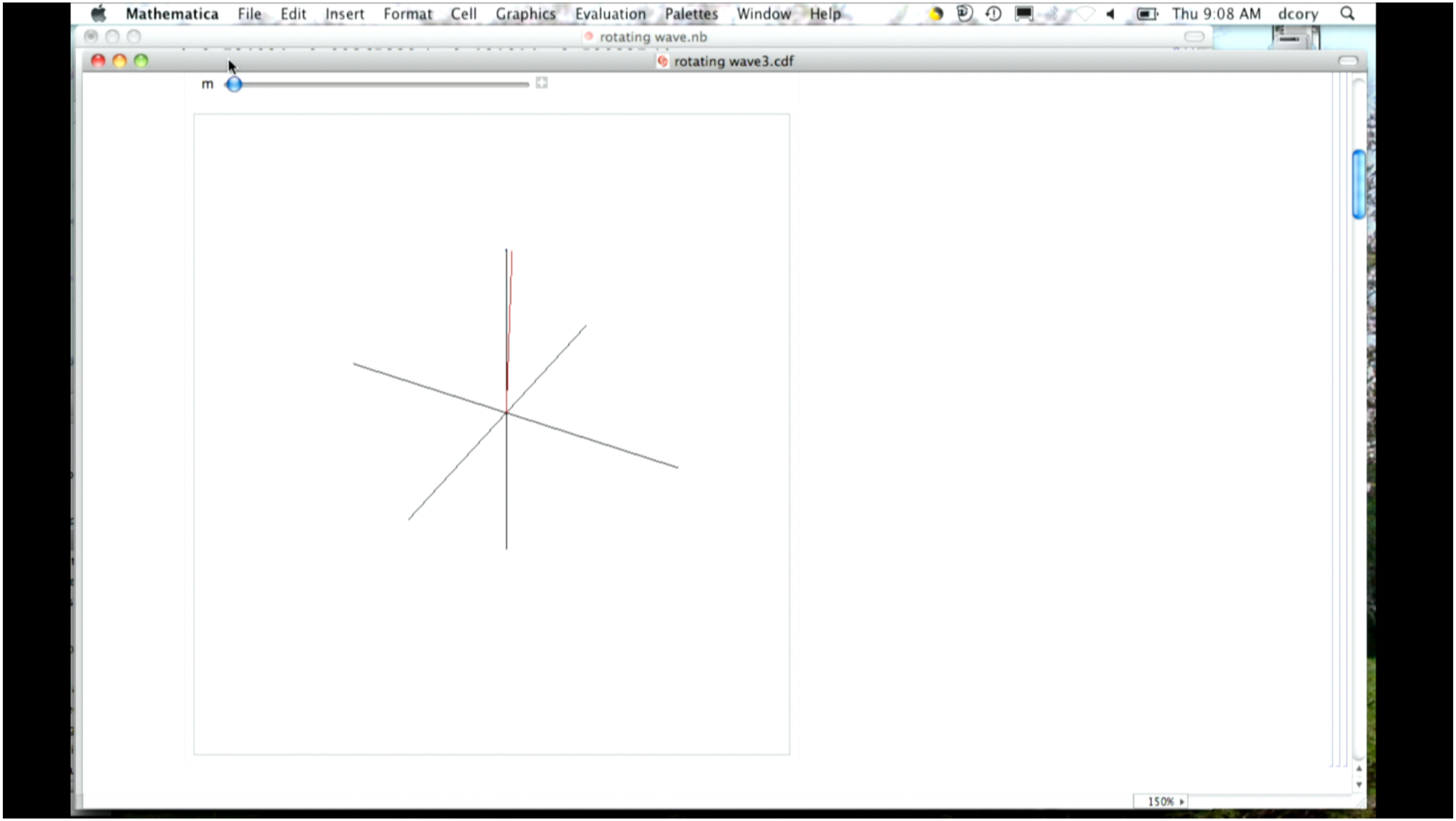
Title: Explorations in Quantum Information - Lecture 9

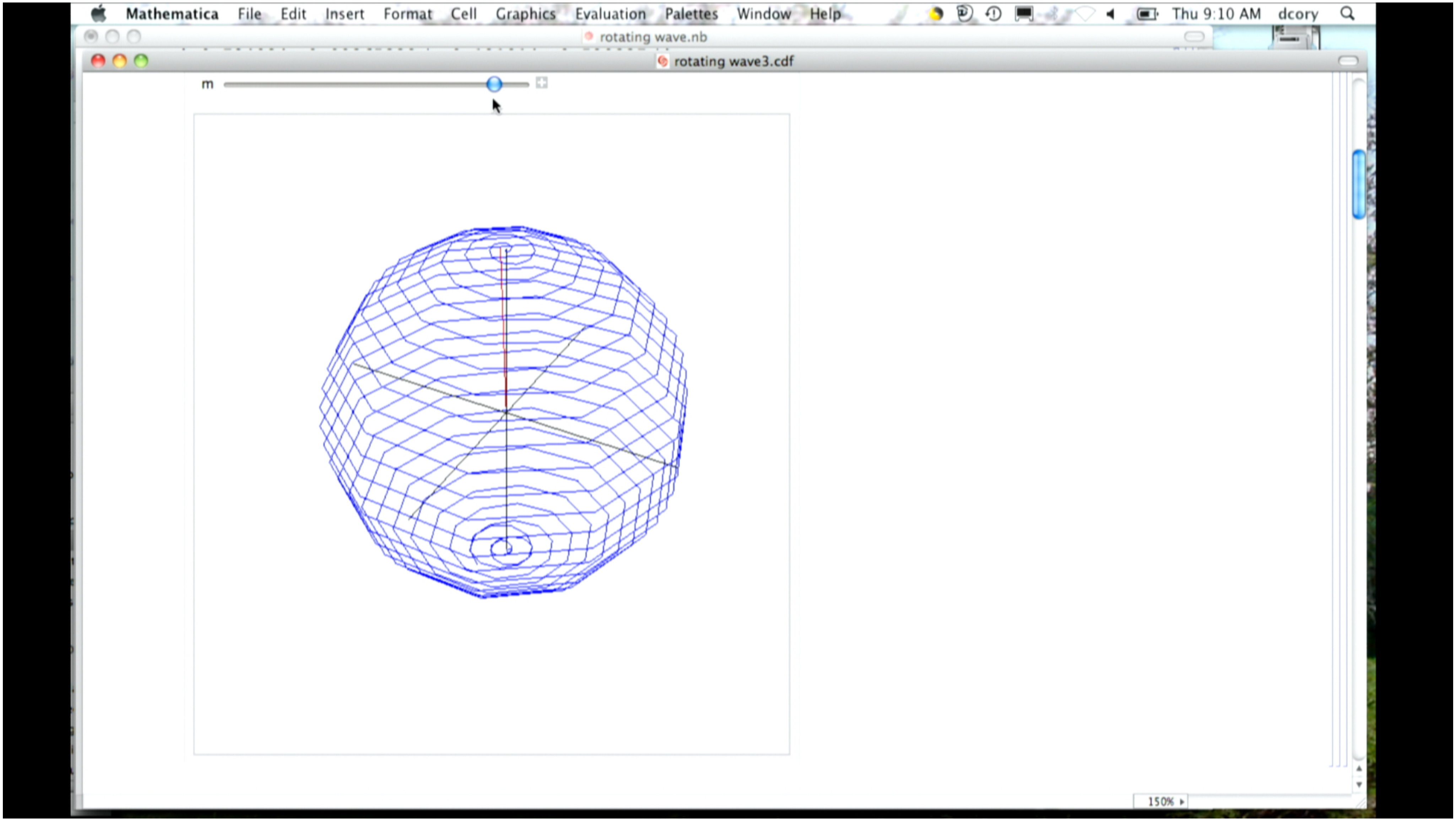
Date: Mar 22, 2012 09:00 AM

URL: http://pirsa.org/12030014

Abstract:


Zeem
B_0
synthesizer
ω_t
$\cos(\omega_t t)$
gain


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Thu 9:08 AM dcory
rotating wave.nb
rotating wave3.cdf
m
150%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m
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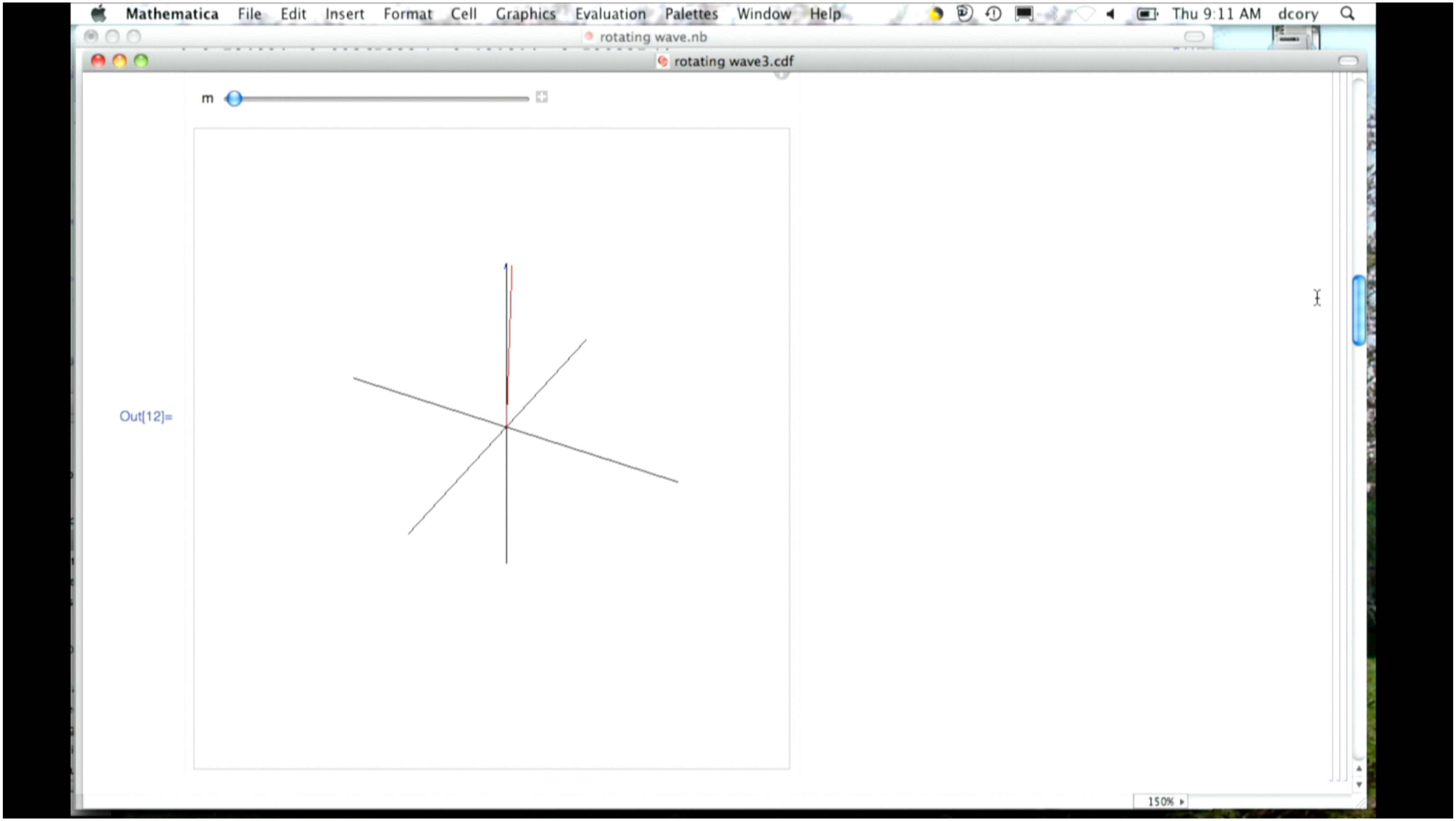
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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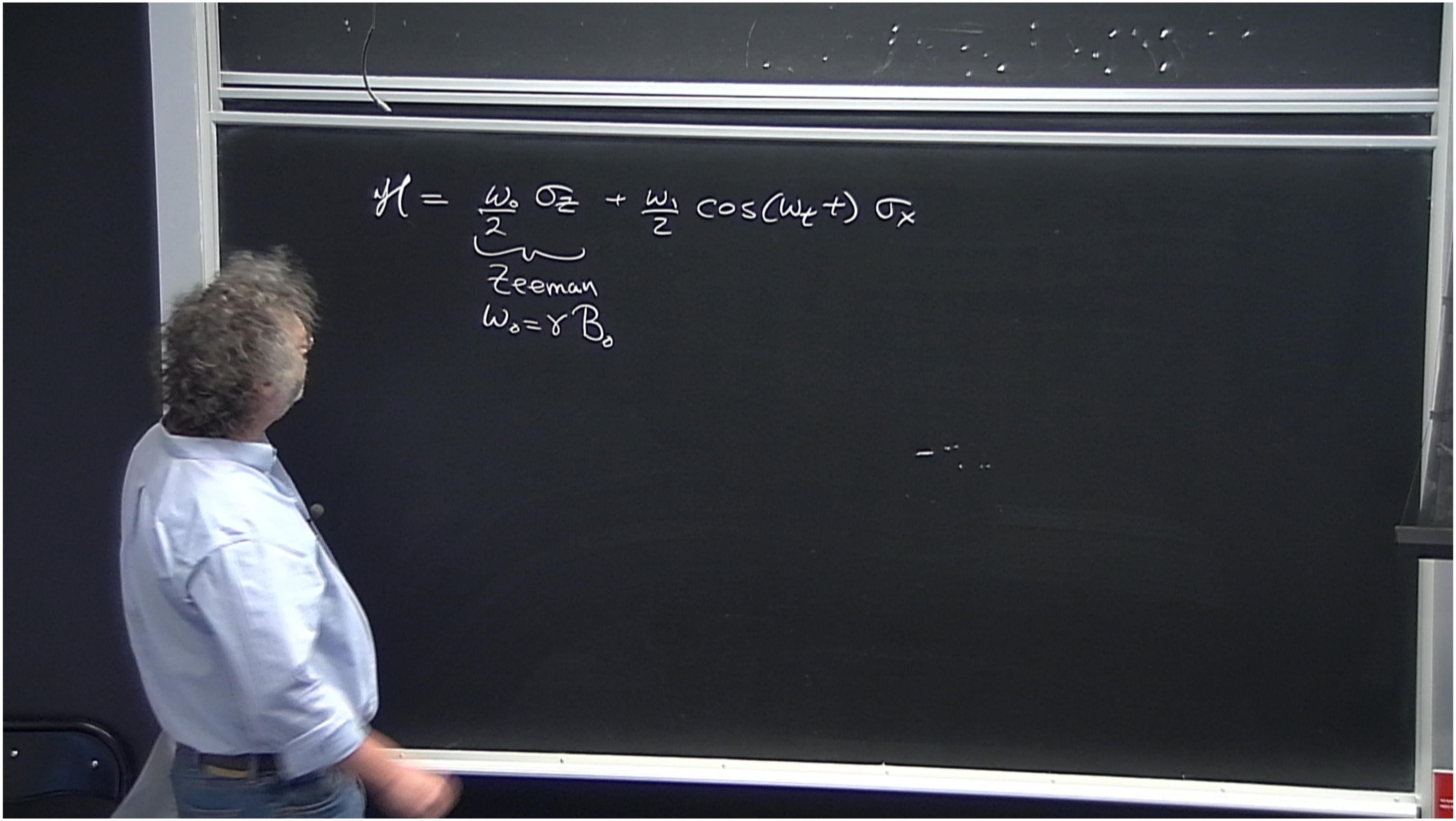
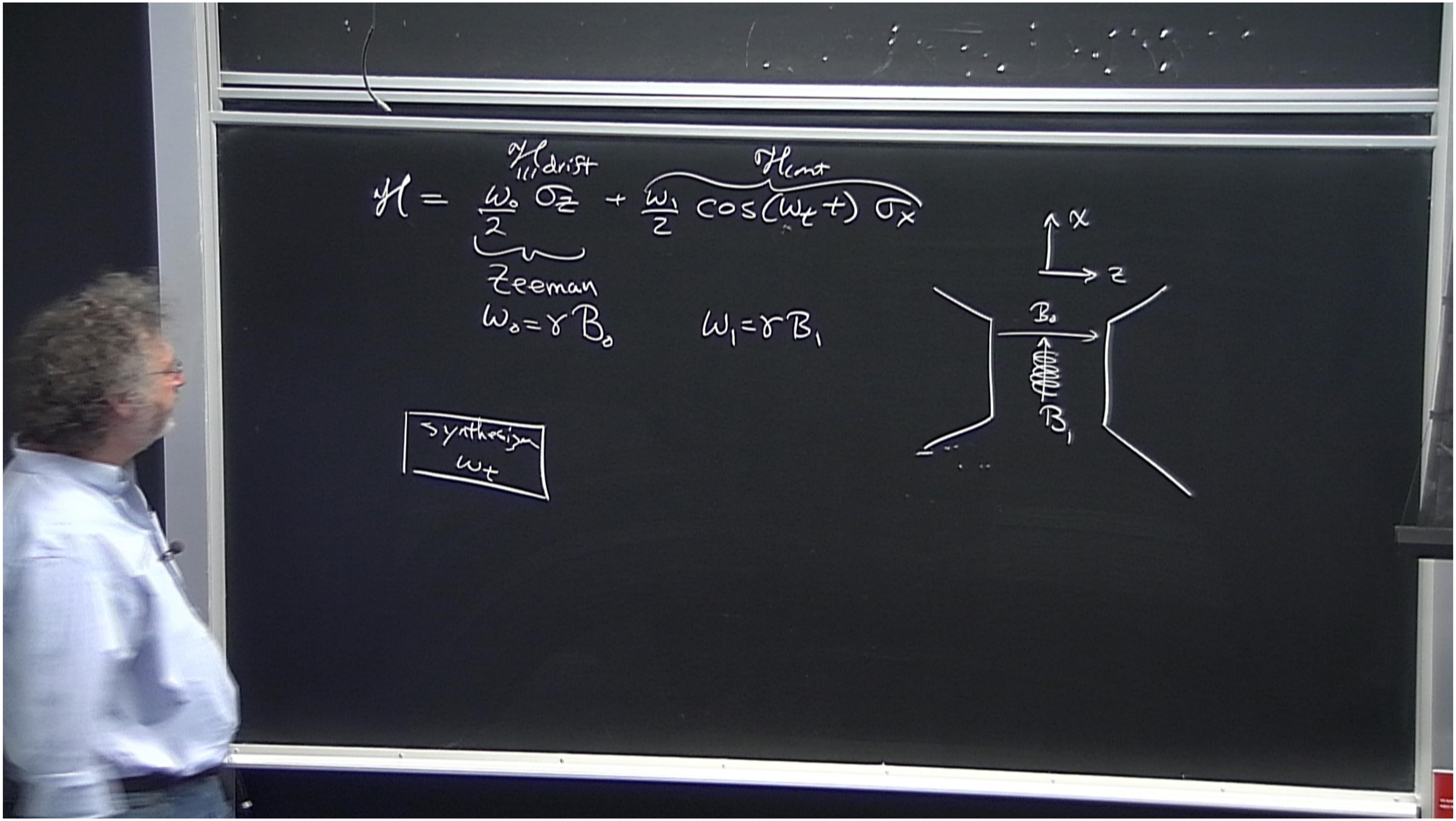
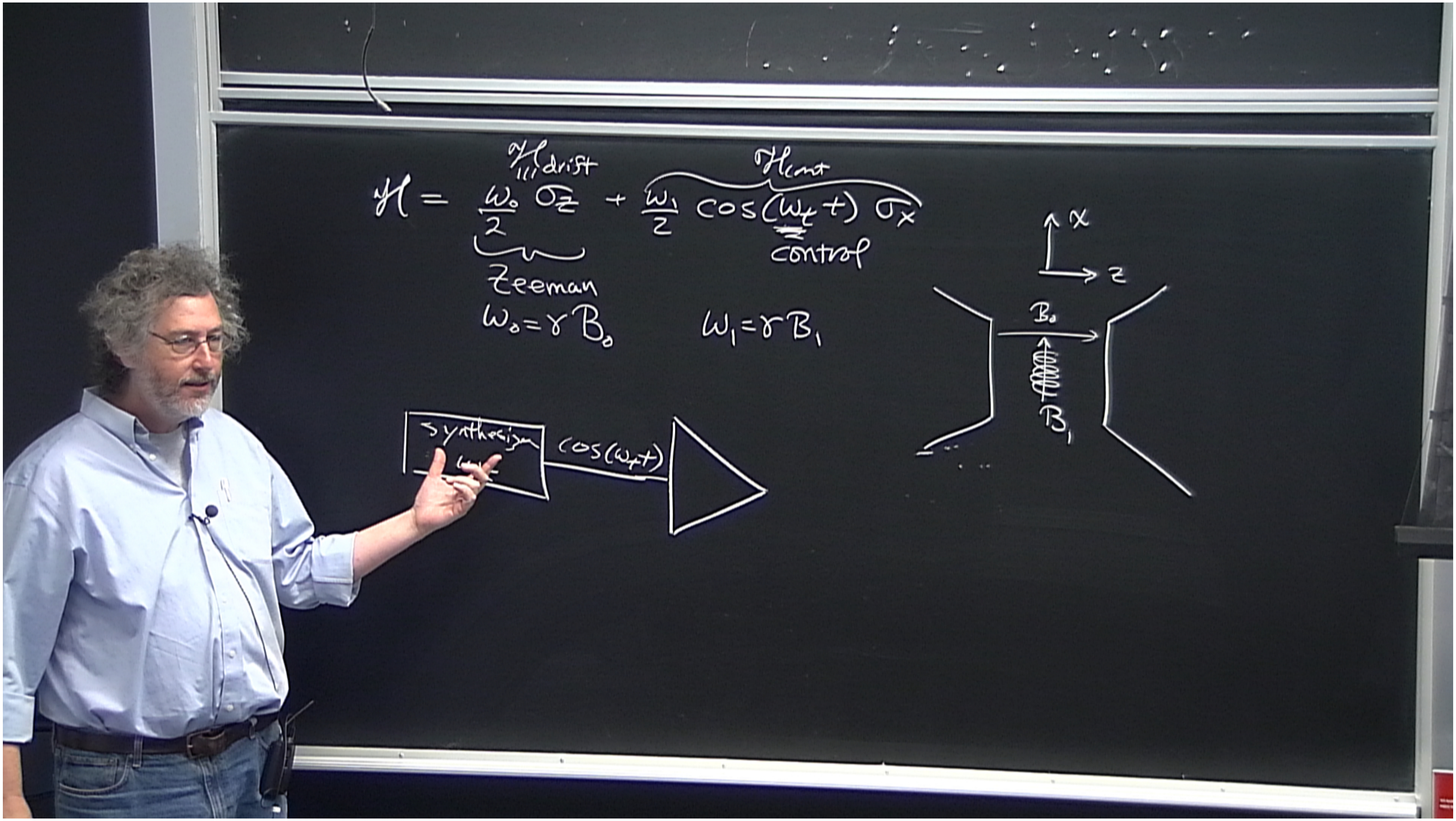
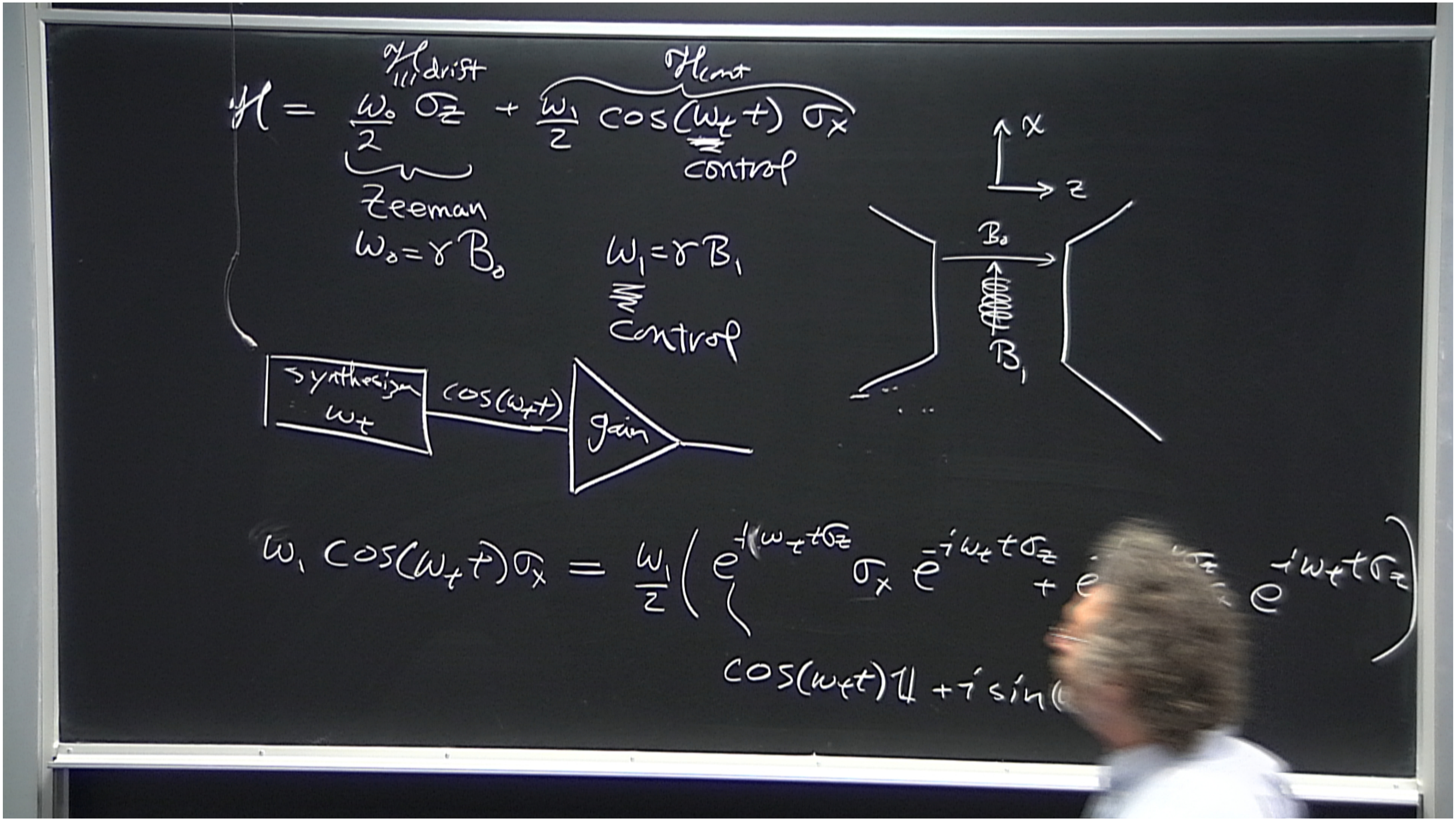
rotating wave.nb
rotating wave3.cdf
m
Out[12]=
150%

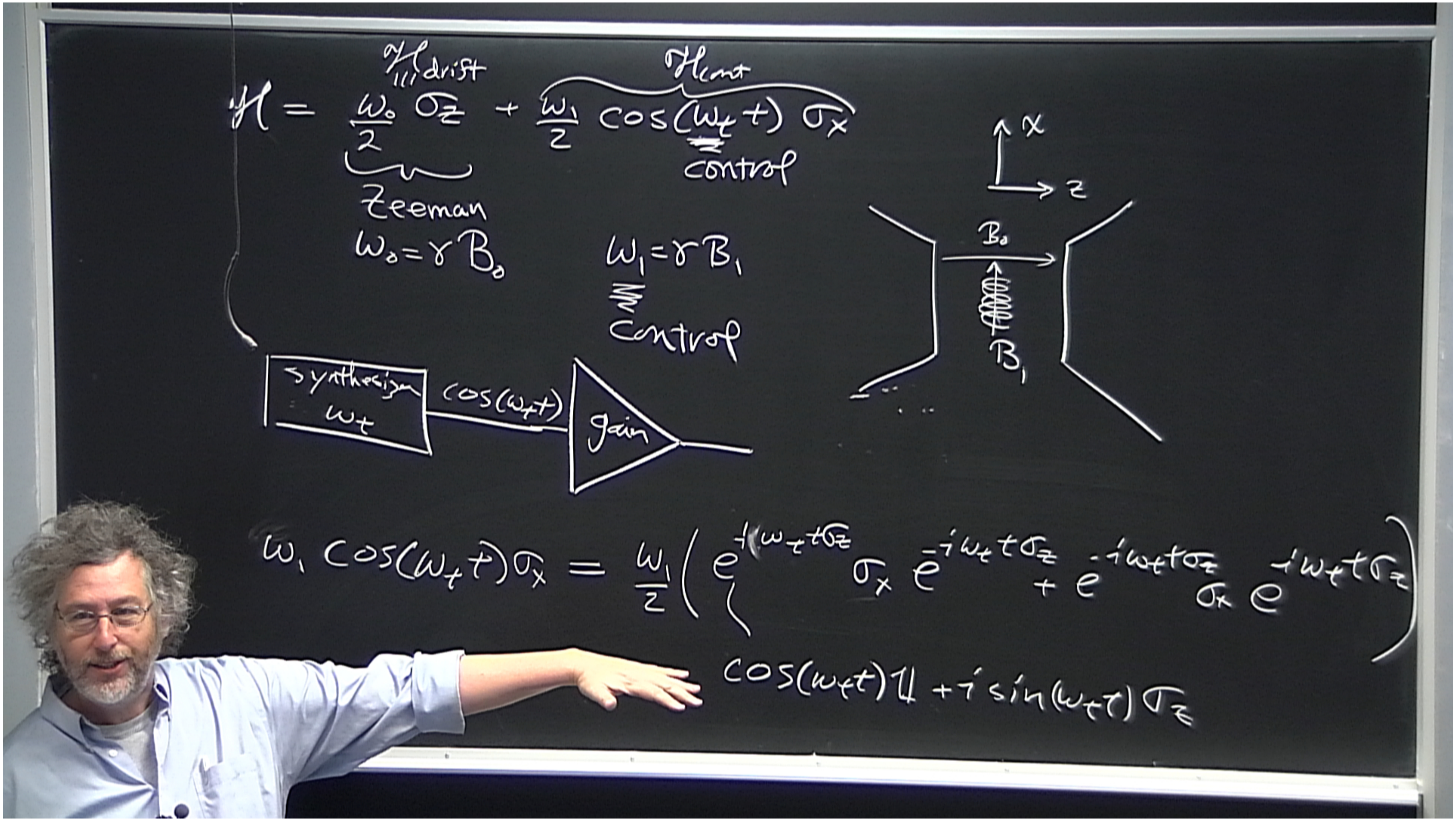

$\mathcal{H} = \underbrace{\frac{\omega_0}{2}\sigma_z}_{\text{Zeeman}} + \frac{\omega_1}{2}\cos(\omega_t t)\,\sigma_x$
$\omega_0 = \gamma B_0$


$\mathcal{H}_{drift}$
$\mathcal{H}_{cont}$
$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\cos(\omega_t t)\,\sigma_x$
Zeeman
$\omega_0 = \gamma B_0$
$\omega_1 = \gamma B_1$
synthesizer
ω_t
x
z
B_0
B_1



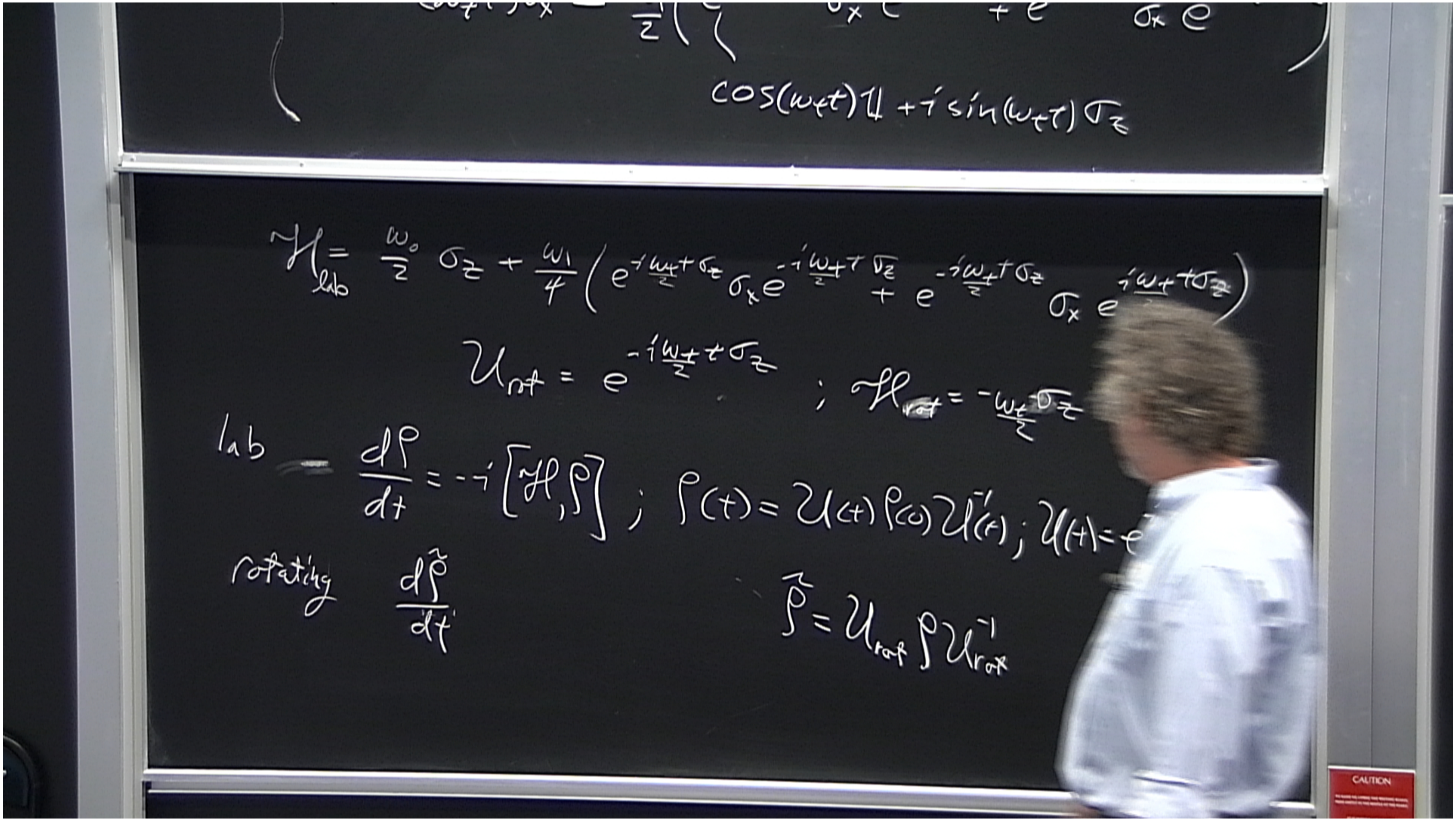
$\mathcal{H}_{drift}$
$\mathcal{H}_{cont}$
$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\cos(\omega_t t)\,\sigma_x$
Zeeman
control
$\omega_0 = \gamma B_0$
$\omega_1 = \gamma B_1$
x
z
B_0
B_1
synthesizer
$\cos(\omega_t t)$


H drift
H cont
H = ω0/2 σz + ω1/2 cos(ωt t) σx
control
Zeeman
ω0 = γ B0
ω1 = γ B1
control
x
z
B0
B1
synthesizer
ωt
cos(ωt t)
gain
ω1 cos(ωt t) σx = ω1/2 (e^{iωt t σz} σx e^{-iωt t σz}
+
cos(ωt t) 1 + i sin(


$\mathcal{H}_{drift}$
$\mathcal{H}_{cont}$
$\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{2}\cos(\omega_t t)\,\sigma_x$
control
Zeeman
$\omega_0 = \gamma B_0$
$\omega_1 = \gamma B_1$
control
x
z
B_0
B_1
synthesizer
ω_t
$\cos(\omega_t t)$
gain
$\omega_1\cos(\omega_t t)\sigma_x = \frac{\omega_1}{2}\left(e^{i\omega_t t\sigma_z}\sigma_x e^{-i\omega_t t\sigma_z} + e^{-i\omega_t t\sigma_z}\sigma_x e^{i\omega_t t\sigma_z}\right)$
$\cos(\omega_t t)\mathbb{1} + i\sin(\omega_t t)\sigma_z$

$$\cos(\omega_t t)\mathbb{1} + i\sin(\omega_t t)\sigma_z$$

$$\tilde{H} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\left(e^{i\omega_t t\sigma_z}\sigma_x e^{-i\omega_t t\sigma_z} + e^{-i\omega_t t\sigma_z}\sigma_x e^{i\omega_t t\sigma_z}\right)$$


$\cos(\omega_t t)\mathbb{1} + i\sin(\omega_t t)\sigma_z$
$\mathcal{H}_{lab} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\left(e^{i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{-i\frac{\omega_t}{2}t\sigma_z} + e^{-i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{i\frac{\omega_t}{2}t\sigma_z}\right)$
$U_{rot} = e^{-i\frac{\omega_t}{2}t\sigma_z}$; $\mathcal{H}_{rot} = -\frac{\omega_t}{2}\sigma_z$
lab $\frac{d\rho}{dt} = -i[\mathcal{H},\rho]$; $\rho(t) = U(t)\rho(0)U^{-1}(t)$; $U(t) = e$
rotating $\frac{d\tilde{\rho}}{dt}$
$\tilde{\rho} = U_{rot}\,\rho\,U_{rot}^{-1}$

$$\cos(\omega_t t)\,\sigma_x = \frac{1}{2}\Big(\big(\ \ \sigma_x e \ \ + e \ \ \sigma_x e \Big)$$

$$\cos(\omega_t t)\mathbb{1} + i\sin(\omega_t t)\,\sigma_z$$

$$\mathcal{H}_{lab} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\left(e^{-i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{-i\frac{\omega_t}{2}t\sigma_z} + e^{-i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{i\frac{\omega_t}{2}t\sigma_z} \right)$$

$$\mathcal{U}_{rot} = e^{-i\frac{\omega_t}{2}t\sigma_z} \quad ; \quad \mathcal{H}_{rot} = -\frac{\omega_t}{2}\sigma_z$$

lab $\quad \dfrac{d\rho}{dt} = -i[\mathcal{H},\rho] \; ; \; \rho(t) = \mathcal{U}(t)\rho(0)\mathcal{U}^{-1}(t) \; ; \; \mathcal{U}(t) = e^{-i\mathcal{H}t}$

rotating $\quad \dfrac{d\tilde{\rho}}{dt}$

$$\tilde{\rho} = \mathcal{U}_{rot}\,\rho\,\mathcal{U}_{rot}^{-1}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{H}_{\text{eff}}, \tilde{\rho}\right] = \frac{dU_{rot}}{dt}\,\rho\, U_{rot}^{-1} + \overbrace{U_{rot}\frac{d\rho}{dt}U_{rot}^{-1}}^{[\tilde{H},\,\tilde{\rho}]} + \underbrace{U_{rot}\,\rho\,\frac{dU_{rot}^{-1}}{dt}}$$

$$\frac{dU_{rot}}{dt} = i\frac{\omega_t}{2}\sigma_z U_{rot}$$

$$\frac{d\rho}{dt} = -i[H, \rho]$$

$$\frac{dU_{rot}^{-1}}{dt} = -i\frac{\omega_t}{2}\sigma_z U_{rot}^{-1}$$

$$U_{rot} H_{\cdot} \rho\, U_{rot}^{-1} -$$

$$U_{rot}^{-1} U_{rot}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{\mathcal{H}}_{\text{eff}}, \tilde{\rho}\right] = \underbrace{\frac{d\,\mathcal{U}_{rot}}{dt}}_{i\frac{\omega_t}{2}\sigma_z\mathcal{U}_{rot}}\rho\,\mathcal{U}_{rot}^{-1} + \overbrace{\mathcal{U}_{rot}\underbrace{\frac{d\rho}{dt}}_{-i[\mathcal{H},\rho]}\mathcal{U}_{rot}^{-1}}^{[\tilde{\mathcal{H}},\tilde{\rho}]} + \mathcal{U}_{rot}\,\rho\,\underbrace{\frac{d\,\mathcal{U}_{rot}^{-1}}{dt}}_{-i\frac{\omega_t}{2}\sigma_z\mathcal{U}_{rot}^{-1}}$$

$$\mathcal{U}_{rot}\,\mathcal{H}\,\rho\,\mathcal{U}_{rot}^{-1} - $$

$$\mathcal{U}_{rot}^{-1}\,\mathcal{U}_{rot}$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{\mathcal{H}}, \tilde{\rho}\right] - i\left[-\mathcal{H}_{rot}, \tilde{\rho}\right]}$$

$$\mathcal{H}_{lab} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\left(e^{i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{-i\frac{\omega_t}{2}t\sigma_z} + e^{-i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{i\frac{\omega_t}{2}t\sigma_z}\right)$$

$$\mathcal{U}_{rot} = e^{-i\frac{\omega_t}{2}t\sigma_z} \quad ; \quad \mathcal{H}_{rot} = -\frac{\omega_t}{2}\sigma_z$$

lab $\quad \dfrac{d\rho}{dt} = -i\left[\mathcal{H}, \rho\right] \;;\; \rho(t) = \mathcal{U}(t)\,\rho(0)\,\mathcal{U}^{-1}(t) \;;\; \mathcal{U}(t) = e^{-i\mathcal{H}t}$

rotating $\quad \dfrac{d\tilde{\rho}}{dt}$

$$\tilde{\rho} = \mathcal{U}_{rot}\,\rho\,\mathcal{U}_{rot}^{-1}$$

$$\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{\mathcal{H}}_{(eff)}, \tilde{\rho}\right] = \frac{dU_{rot}}{dt}\rho U_{rot}^{-1} + \overbrace{U_{rot}\frac{d\rho}{dt}U_{rot}^{-1}}^{[\tilde{\mathcal{H}},\tilde{\rho}]} + U_{rot}\,\rho\,\underbrace{\frac{dU_{rot}^{-1}}{dt}}$$

$$i\frac{\omega_t}{2}\sigma_z U_{rot} \qquad -i[\mathcal{H},\rho] \qquad -i\frac{\omega_t}{2}\sigma_z U_{rot}^{-1}$$

$$U_{rot}\mathcal{H}_1\rho U_{rot}^{-1} -$$

$$U_{rot}^{-1}U_{rot}$$

$$\boxed{\frac{d\tilde{\rho}}{dt} = -i\left[\tilde{\mathcal{H}},\tilde{\rho}\right] - i\left[-\mathcal{H}_{rot},\tilde{\rho}\right]}$$

$$\frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\sigma_x + e^{-i\frac{2\omega_t t}{2}\sigma_z}\sigma_x e^{i\frac{2\omega_t t}{2}\sigma_z} \; ; \qquad -i\frac{\omega_t}{2}\sigma_z$$

$$\frac{d\tilde\rho}{dt} = -i[\tilde{\mathcal{H}}_{eff}, \tilde\rho] = \underbrace{\frac{d\mathcal{U}_{rot}}{dt}}_{i\frac{\omega_t}{2}\sigma_z \mathcal{U}_{rot}} \rho\, \mathcal{U}_{rot}^{-1} + \overbrace{\mathcal{U}_{rot} \underbrace{\frac{d\rho}{dt}}_{-i[\mathcal{H},\rho]} \mathcal{U}_{rot}^{-1}}^{[\tilde{\mathcal{H}}, \tilde\rho]} + \mathcal{U}_{rot}\, \rho\, \underbrace{\frac{d\mathcal{U}_{rot}^{-1}}{dt}}_{-i\frac{\omega_t}{2}\sigma_z \mathcal{U}_{rot}^{-1}}$$

$$\mathcal{U}_{rot}\, \mathcal{H}\, \rho\, \mathcal{U}_{rot}^{-1} -$$

$$\mathcal{U}_{rot}^{-1} \mathcal{U}_{rot}$$

$$\boxed{\frac{d\tilde\rho}{dt} = -i[\tilde{\mathcal{H}}, \tilde\rho] - i[-\mathcal{H}_{rot}, \tilde\rho]}$$

$$\tilde{\mathcal{H}}: \quad \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\sigma_x + e^{-i\frac{2\omega_t t}{2}\sigma_z}\sigma_x e^{i\frac{2\omega_t t}{2}\sigma_z} ; \qquad \mathcal{H}_{rot}: \quad -i\frac{\omega_t}{2}\sigma_z$$

$$\mathcal{H}_{eff} = \tilde{\mathcal{H}} - \mathcal{H}_{rot} =$$

$$\cos(\omega_t t)\mathbb{1} + i\sin(\omega_t t)\,\sigma_z$$

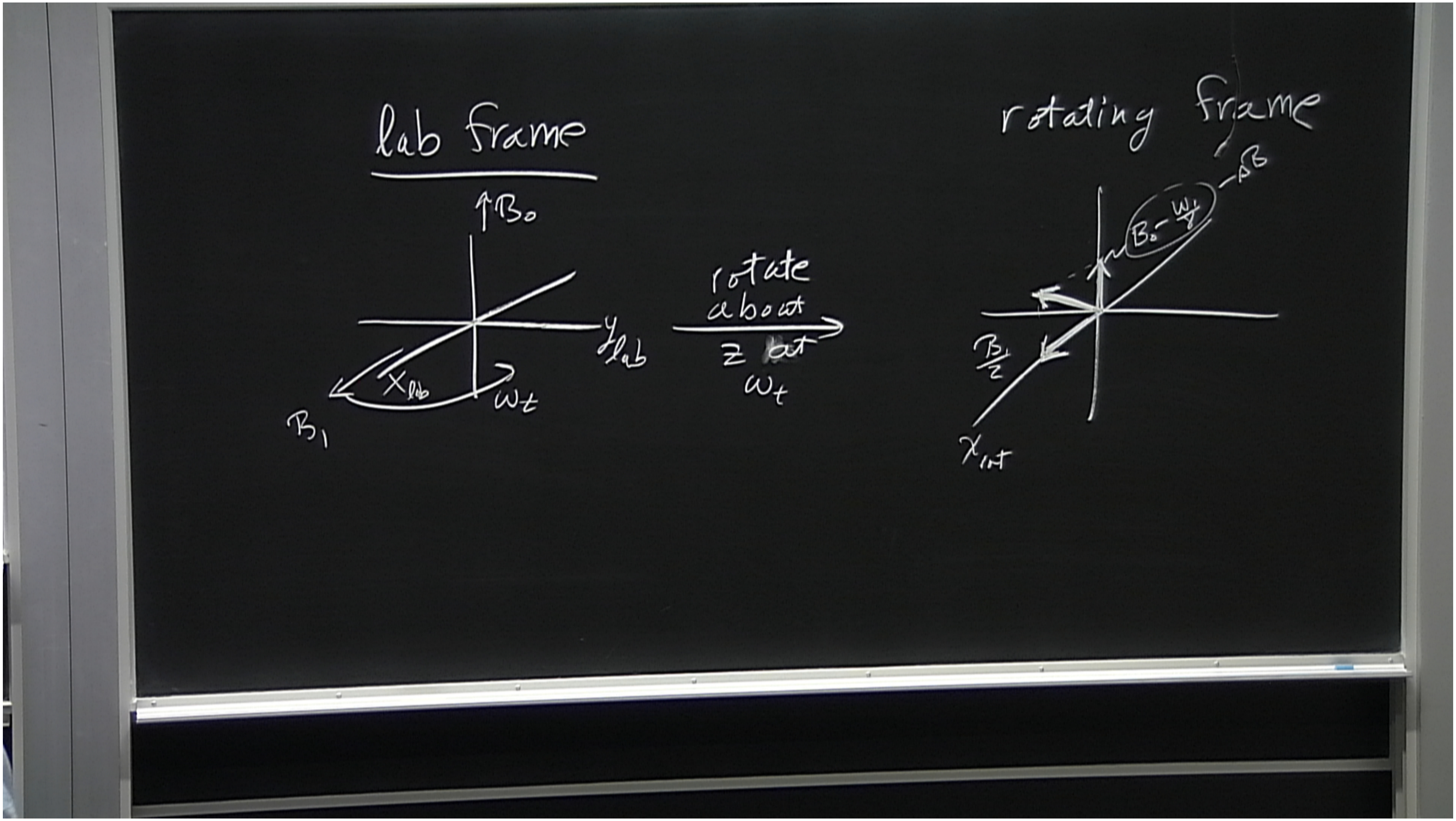
$$\mathcal{H}_{lab} = \frac{\omega_0}{2}\sigma_z + \frac{\omega_1}{4}\Big(\overbrace{e^{i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{-i\frac{\omega_t}{2}t\sigma_z}}^{\text{rotating field}} + \overbrace{e^{-i\frac{\omega_t}{2}t\sigma_z}\sigma_x e^{i\frac{\omega_t}{2}t\sigma_z}}^{\text{countarotating}} \Big)$$

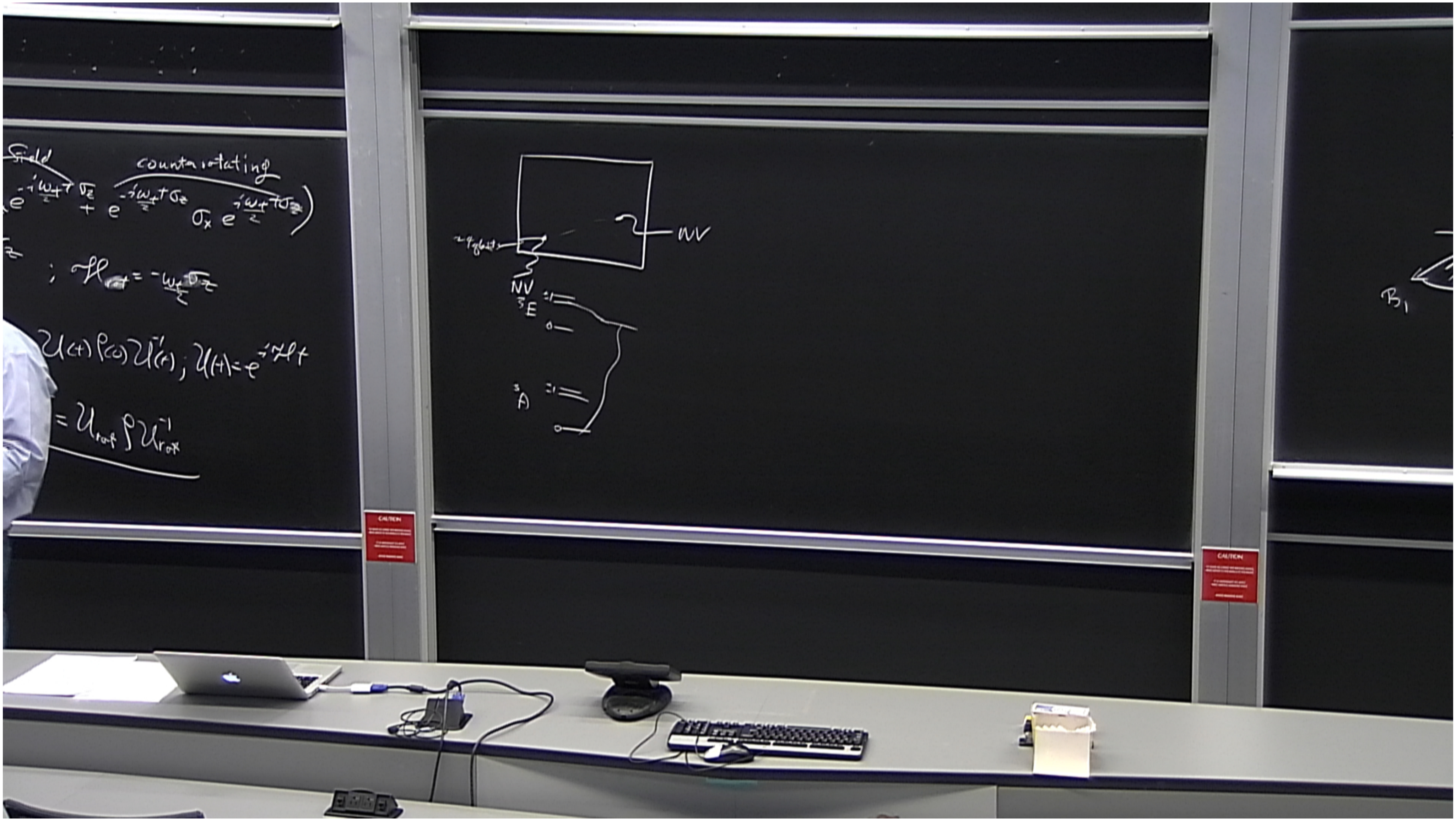
$$\mathcal{U}_{rot} = e^{-i\frac{\omega_t}{2}t\sigma_z} \quad ; \quad \mathcal{H}_{rot} = -\frac{\omega_t}{2}\sigma_z$$

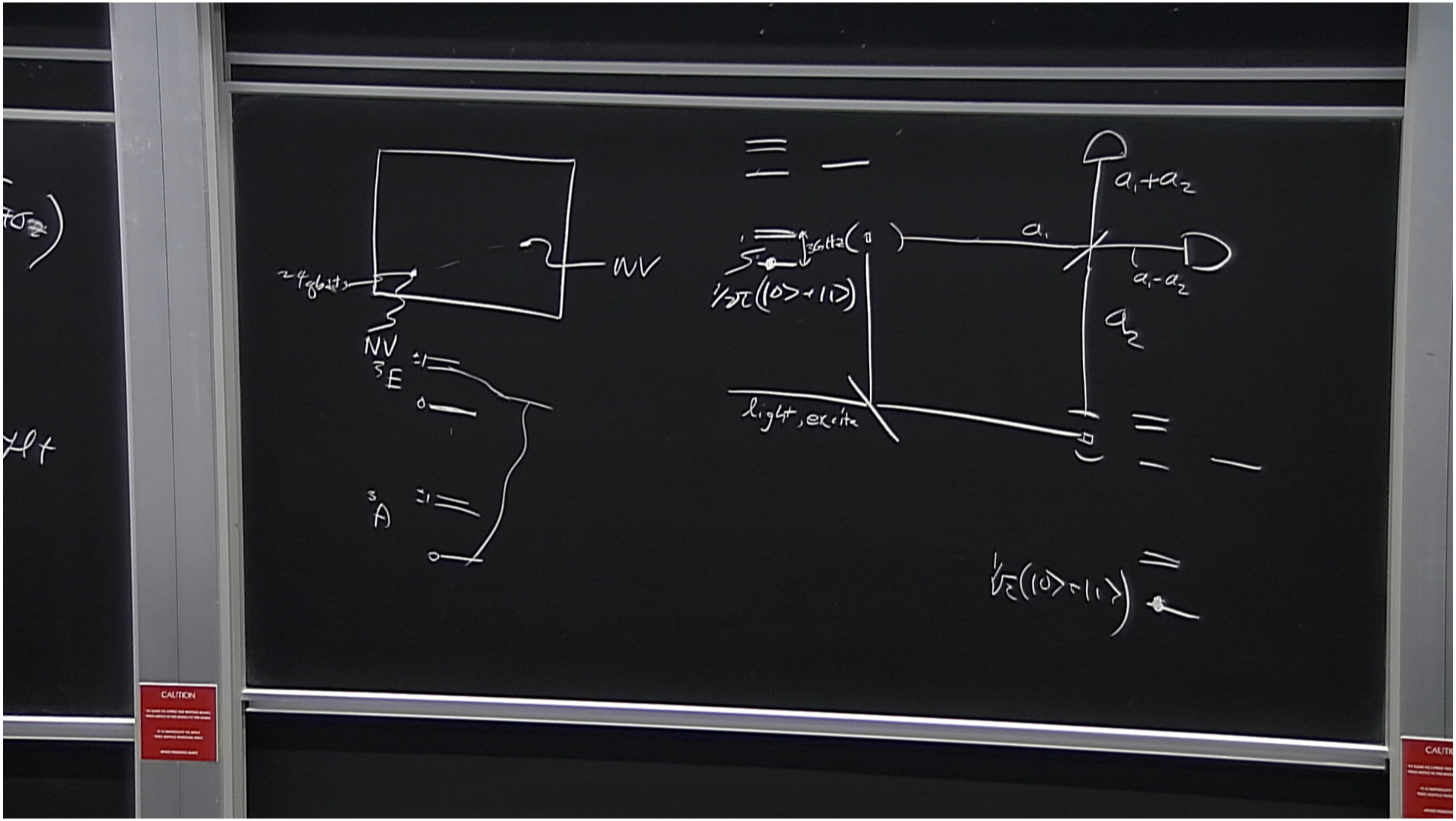
lab $\quad \dfrac{d\rho}{dt} = -i[\mathcal{H},\rho]\ ;\ \rho(t) = \mathcal{U}(t)\rho(0)\mathcal{U}^{-1}(t)\ ;\ \mathcal{U}(t) = e^{-i\mathcal{H}t}$

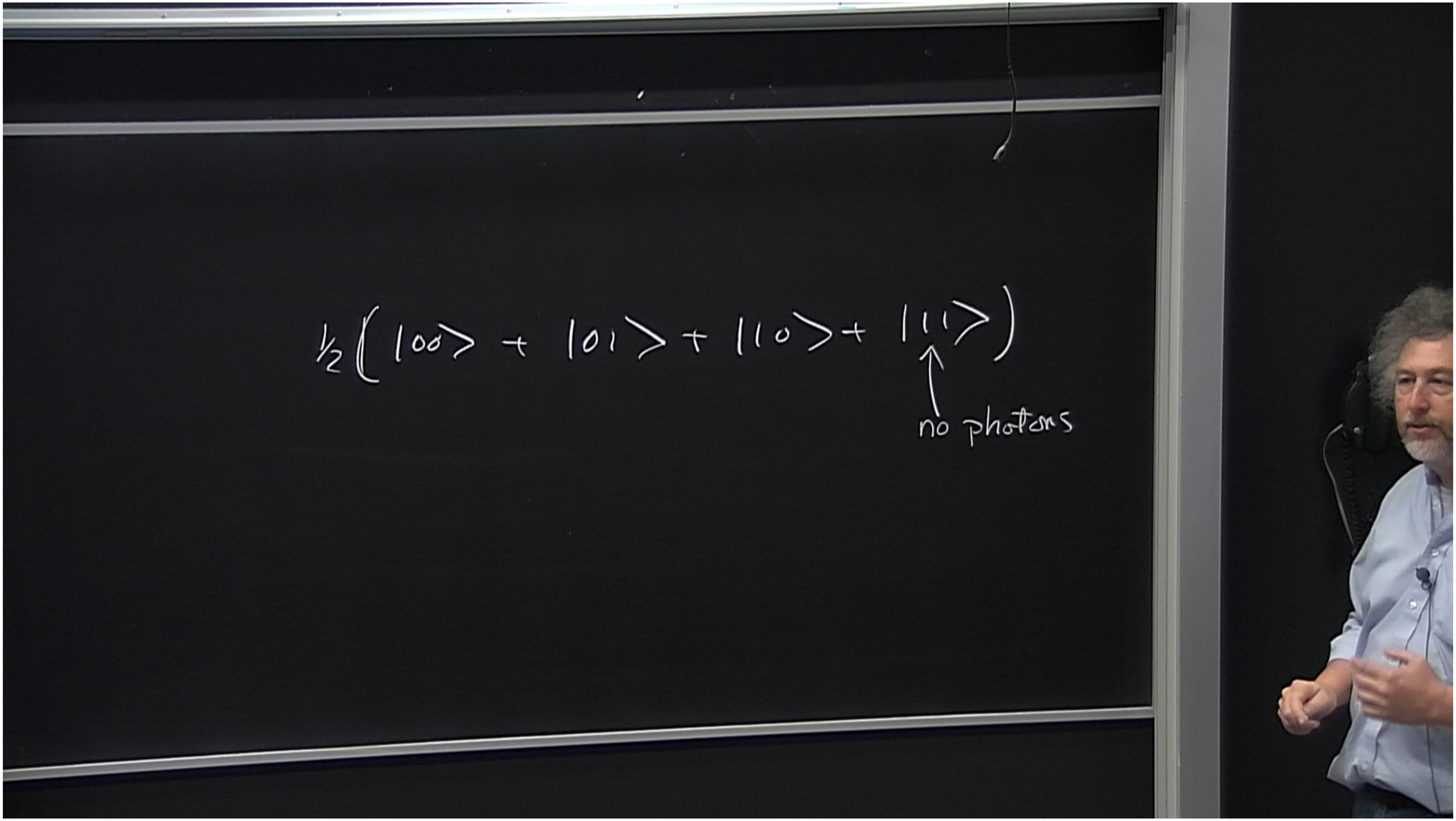
rotating $\quad \dfrac{d\tilde{\rho}}{dt}$

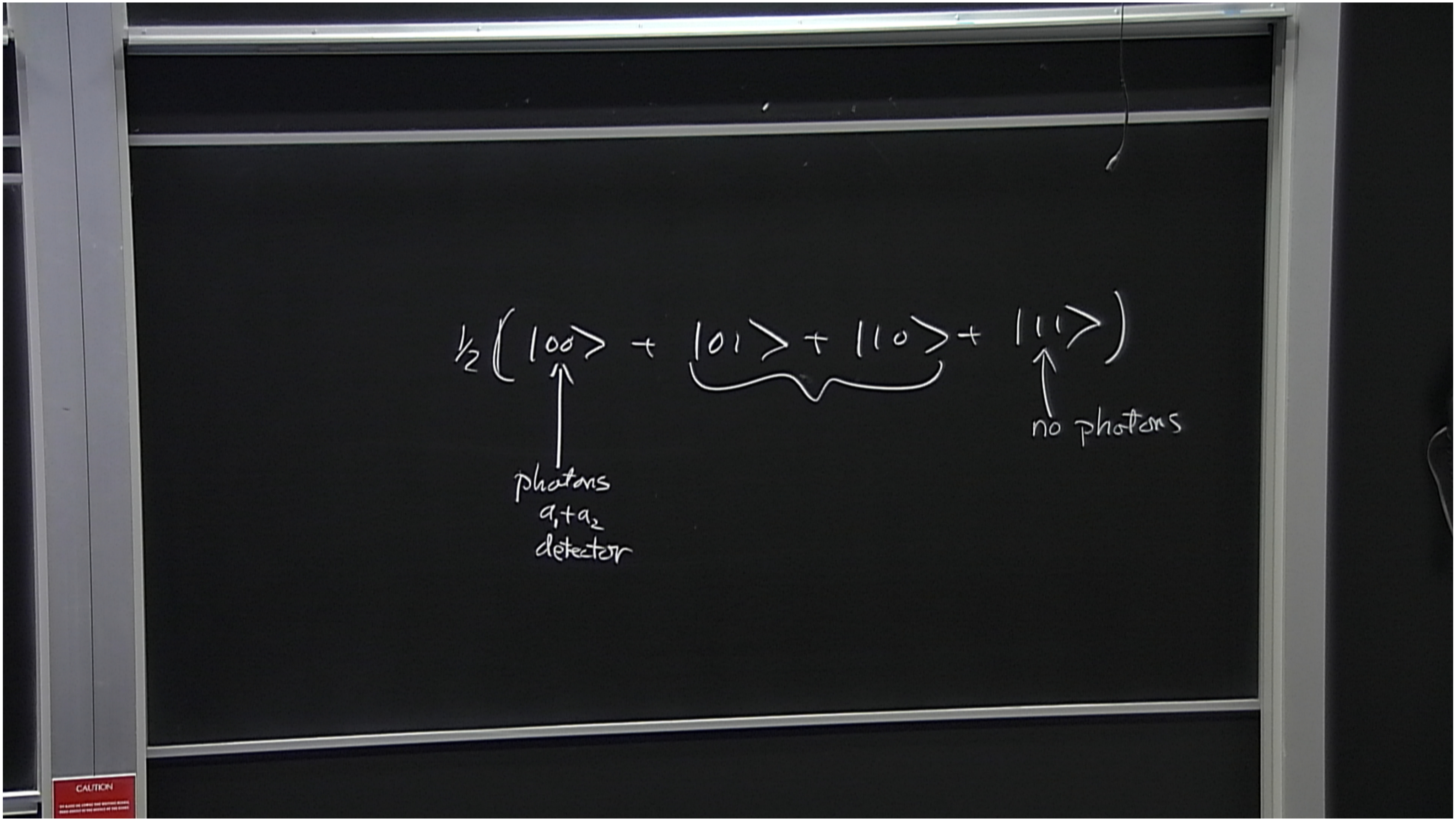
$$\tilde{\rho} = \mathcal{U}_{rot}\,\rho\,\mathcal{U}_{rot}^{-1}$$

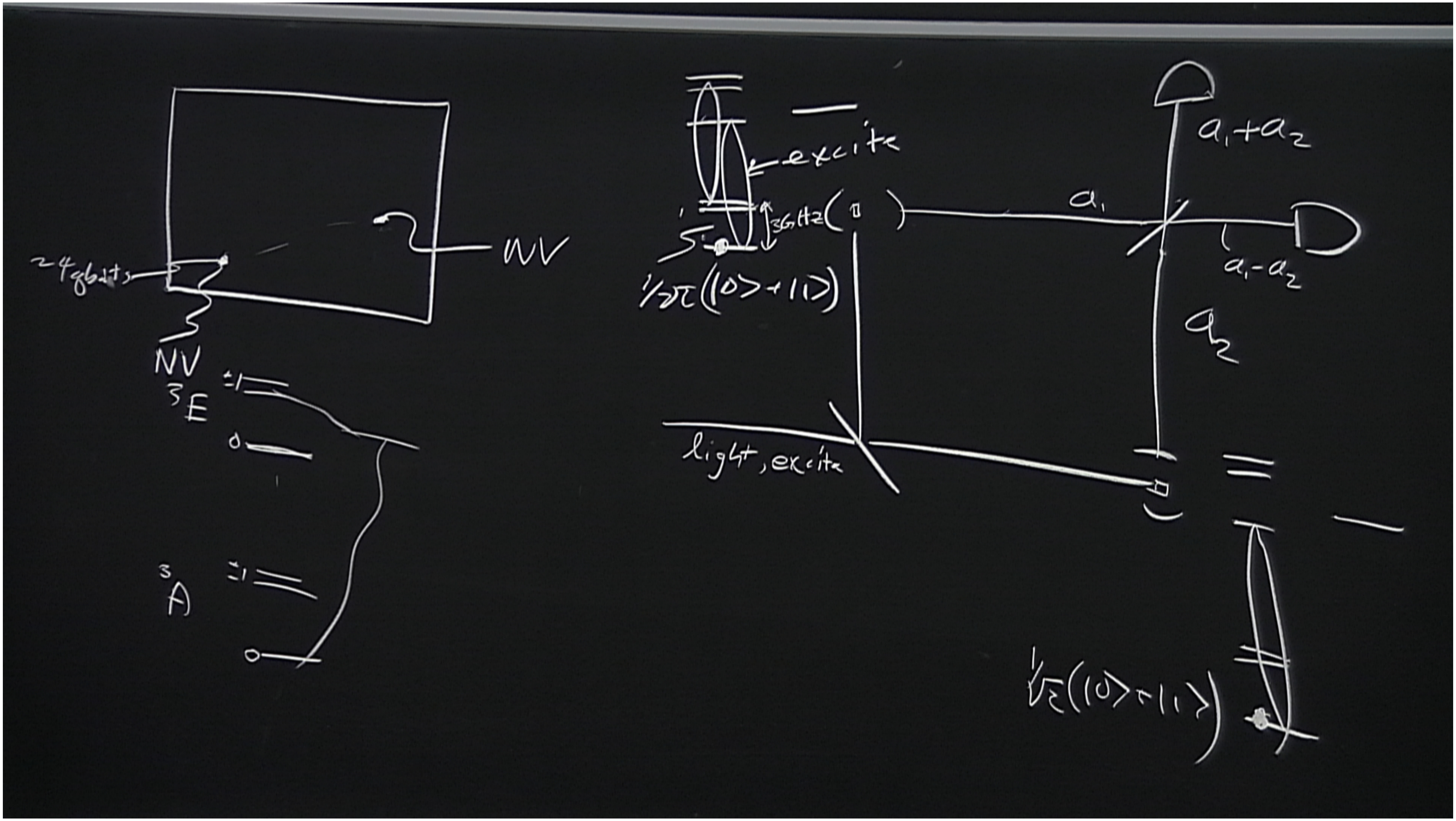

lab frame
B₀
y lab
X lab
B₁
ωt
rotate
about
z at
ωt
rotating frame
B₀ − ω/γ
−ΔB
B₁/2
x rot


counterrotating
NV
NV
^3E
^3A
B_1


24 qbits
NV
NV
^3E
^3A
$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$
light, excite
a_1
a_1+a_2
a_1-a_2
a_2
$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$


$\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$
no photons


$\frac{1}{2}\left(|00\rangle + \underbrace{|01\rangle + |10\rangle} + |11\rangle\right)$
photons
$a_1 + a_2$
detector
no photons
CAUTION



~4 qbits
NV
NV
^3E
±1
0
^3A
±1
0
excite
3GHz
$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$
light, excite
a_1
a_1+a_2
a_1-a_2
a_2
$\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$