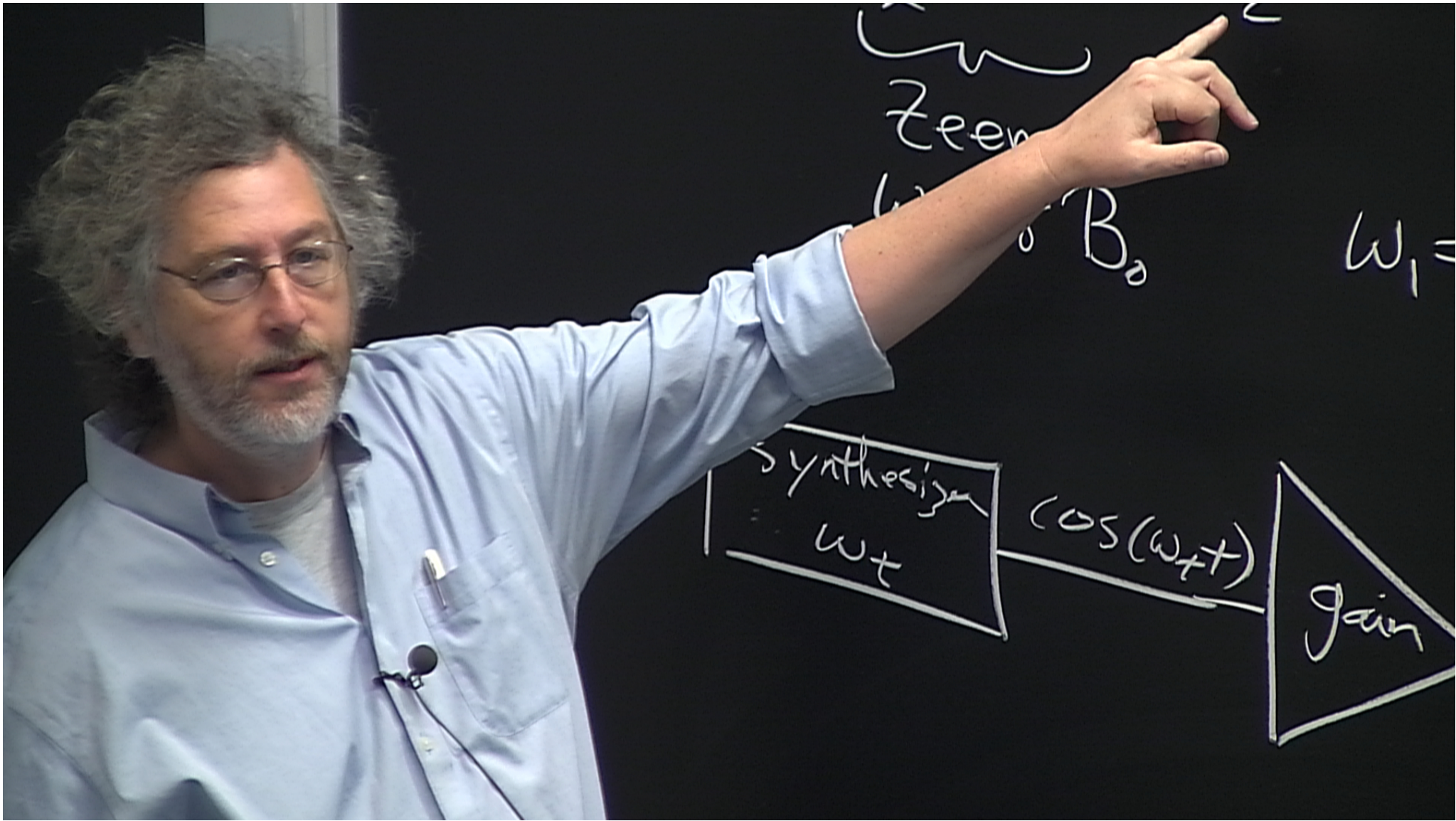


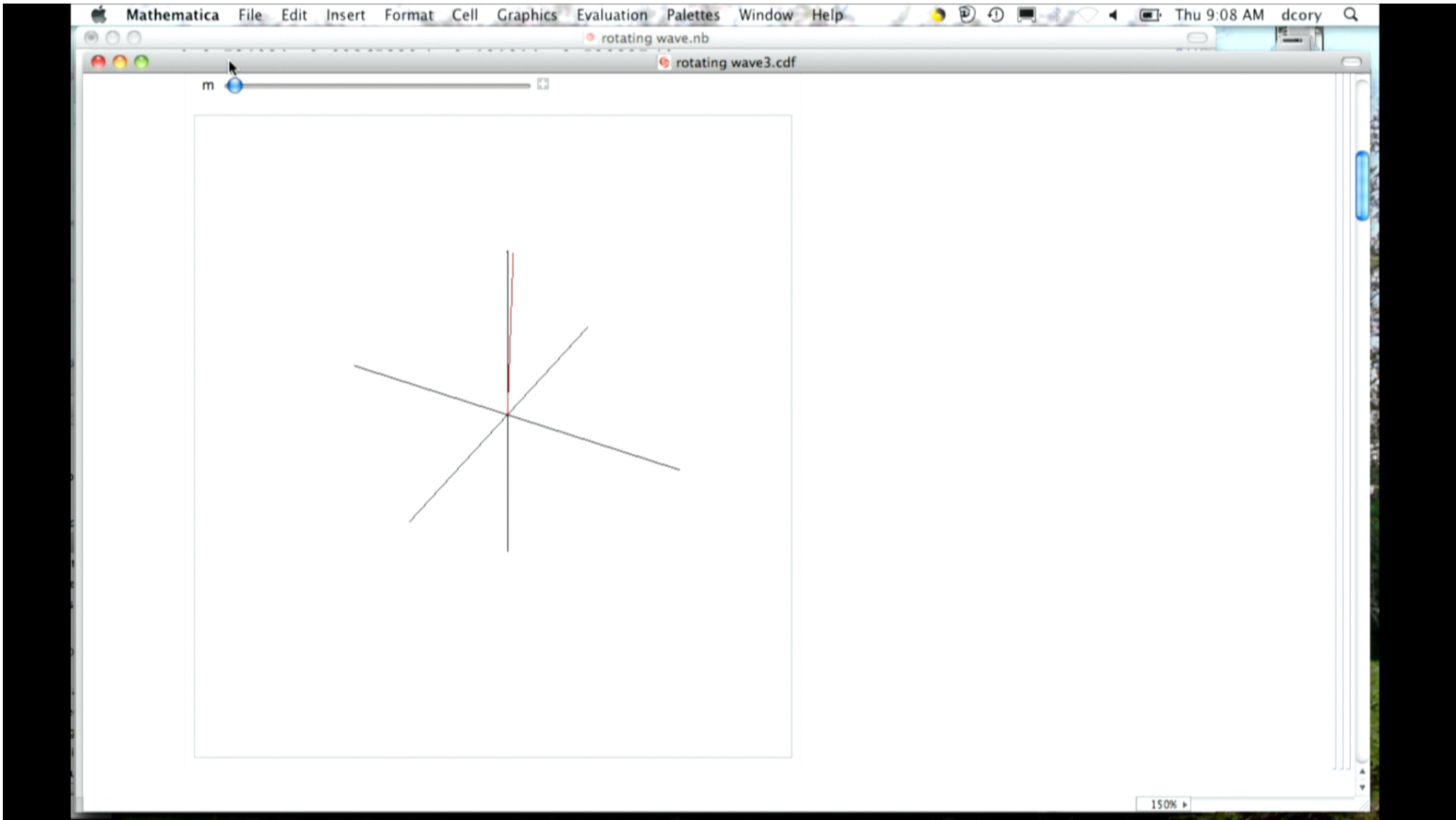
Title: Explorations in Quantum Information - Lecture 9

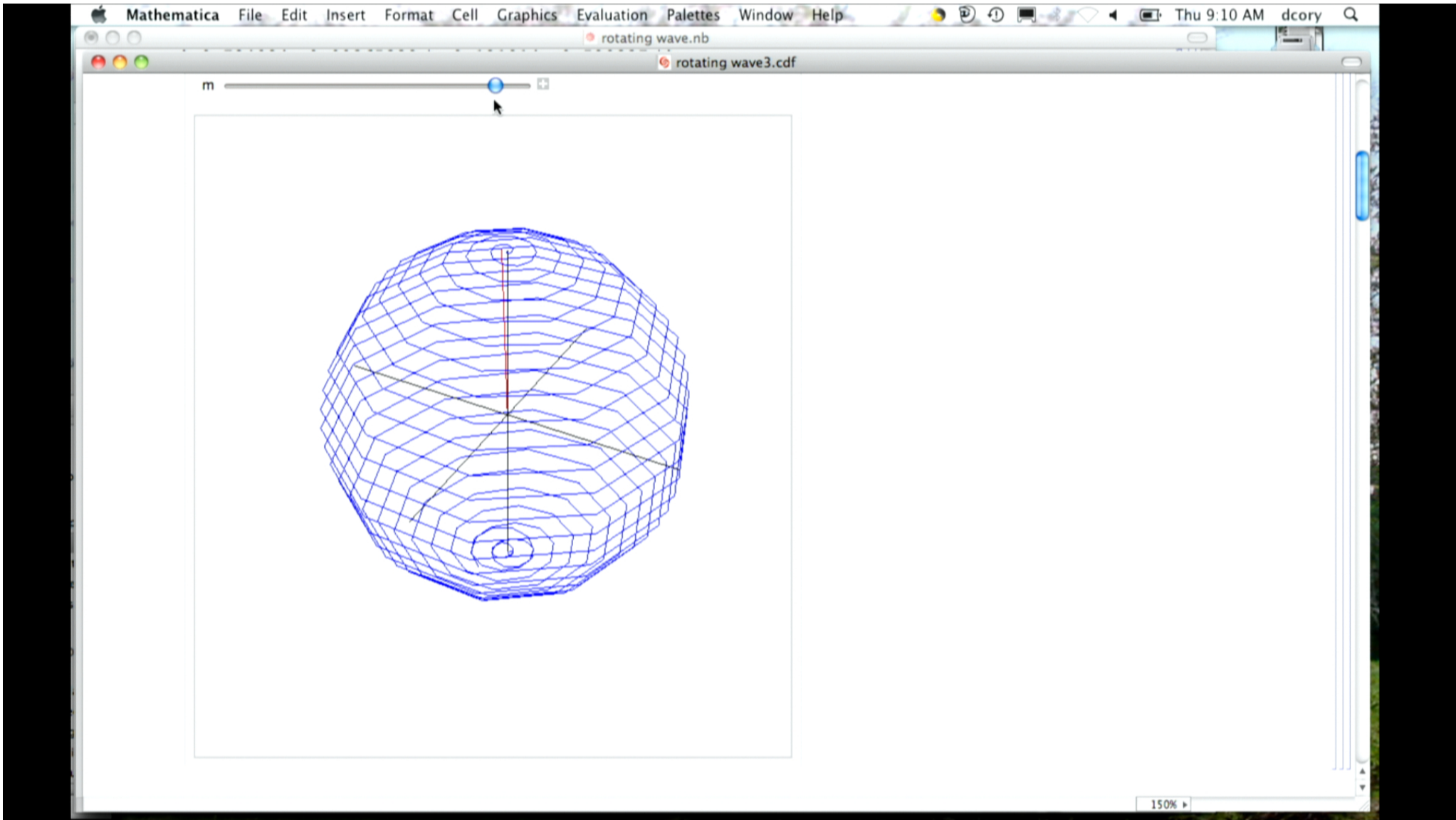
Date: Mar 22, 2012 09:00 AM

URL: <http://pirsa.org/12030014>

Abstract:





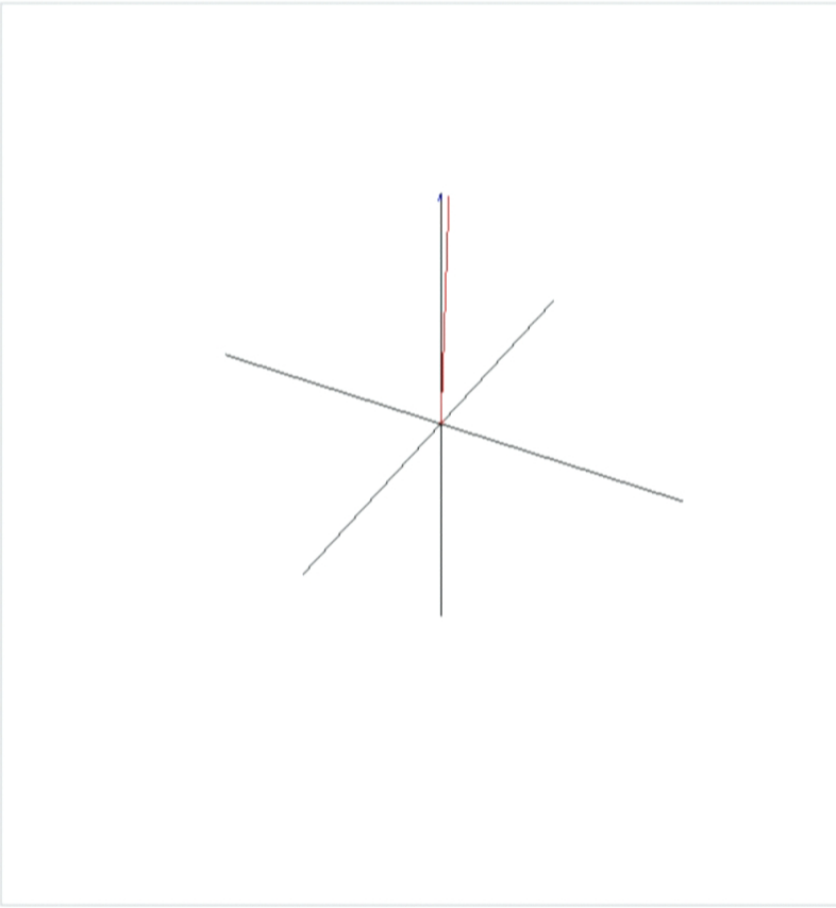


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Thu 9:11 AM dcory

rotating wave.nb rotating wave3.cdf

m

Out[12]=



150%

The image shows a Mathematica notebook window titled "rotating wave3.cdf". At the top, there is a menu bar with options: Mathematica, File, Edit, Insert, Format, Cell, Graphics, Evaluation, Palettes, Window, Help. The system tray on the right shows the date and time as "Thu 9:11 AM" and the user name "dcory". Below the menu bar, the notebook title bar shows "rotating wave.nb" and "rotating wave3.cdf". The main content area contains a plot labeled "Out[12]=". The plot shows a central point from which several lines radiate outwards. One vertical line is highlighted with a red and blue gradient, suggesting it is a rotating wave. Above the plot, there is a slider control labeled "m". The plot area is enclosed in a white box with a gray border. The bottom right corner of the plot area shows a zoom level of "150%".

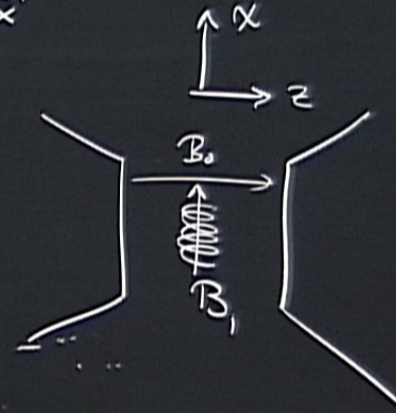
$$\mathcal{H} = \underbrace{\frac{\omega_0}{2} \sigma_z}_{\text{Zeeman}} + \frac{\omega_1}{2} \cos(\omega_L t) \sigma_x$$

$\omega_0 = \gamma B_0$

$$H = \underbrace{\frac{\omega_0}{2} \sigma_z}_{\text{Zeeman}} + \underbrace{\frac{\omega_1}{2} \cos(\omega_1 t)}_{\text{Hint}} \sigma_x$$

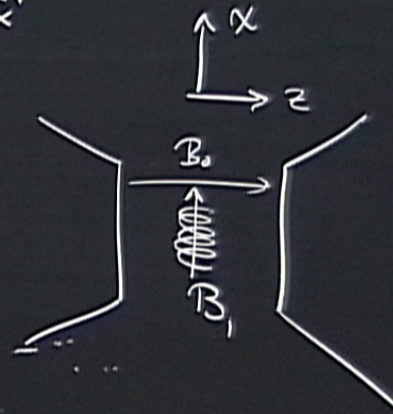
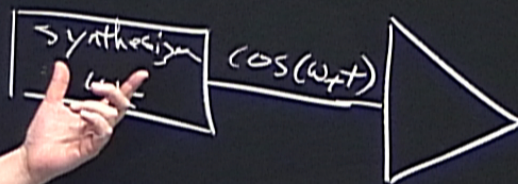
$\omega_0 = \gamma B_0$ $\omega_1 = \gamma B_1$

Synthesizer
 ω_1



$$H = \underbrace{\frac{\omega_0}{2} \sigma_z}_{\text{Zeeman}} + \underbrace{\frac{\omega_1}{2} \cos(\omega_1 t)}_{\text{control}} \sigma_x$$

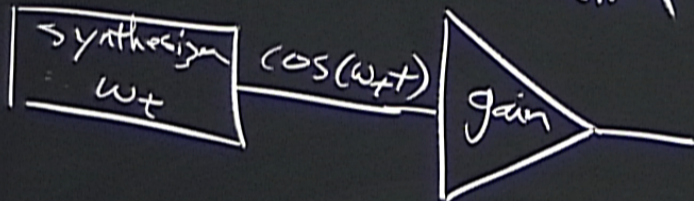
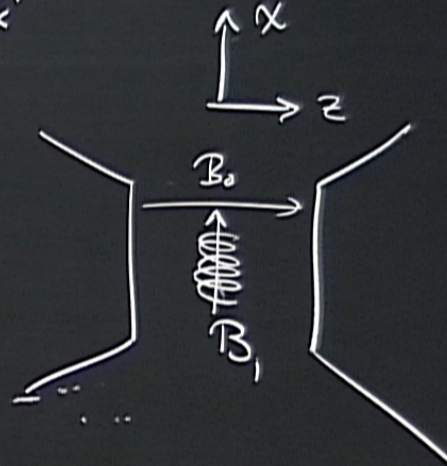
$\omega_0 = \gamma B_0$ $\omega_1 = \gamma B_1$



$$H = \underbrace{\frac{\omega_0}{2} \sigma_z}_{\text{Zeeman}} + \underbrace{\frac{\omega_1}{2} \cos(\omega_1 t)}_{\text{control}} \sigma_x$$

Zeeman
 $\omega_0 = \gamma B_0$

$\omega_1 = \gamma B_1$
 MW
 Control



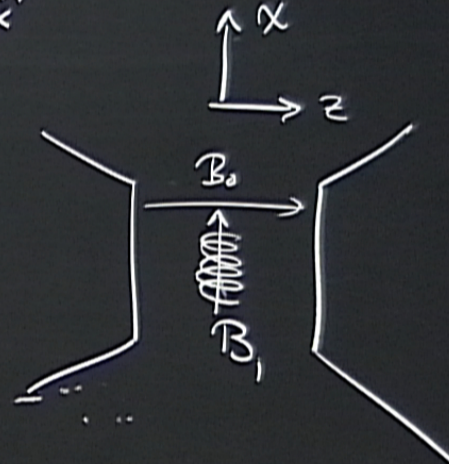
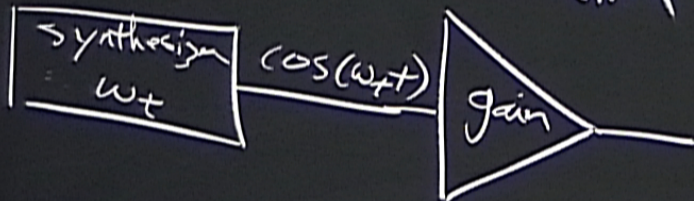
$$\omega_1 \cos(\omega_1 t) \sigma_x = \frac{\omega_1}{2} \left(e^{+i(\omega_1 t + \sigma_z)} \sigma_x e^{-i\omega_1 t \sigma_z} + e^{-i(\omega_1 t + \sigma_z)} \sigma_x e^{+i\omega_1 t \sigma_z} \right)$$

$$\cos(\omega_1 t) \mathbb{1} + i \sin(\omega_1 t) \sigma_x$$

$$\mathcal{H} = \underbrace{\frac{\omega_0}{2} \sigma_z}_{\mathcal{H}_{\text{drift}}} + \underbrace{\frac{\omega_1}{2} \cos(\omega_1 t) \sigma_x}_{\mathcal{H}_{\text{int}}}$$

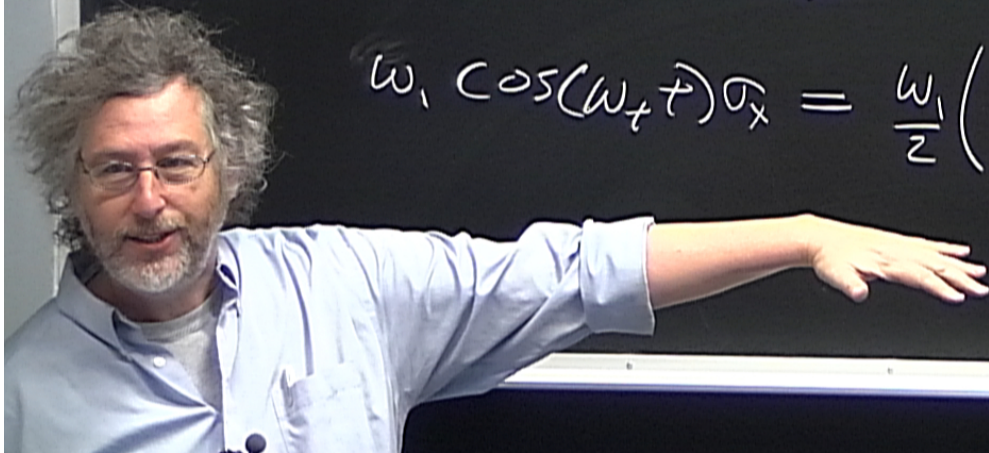
Zeeman
 $\omega_0 = \gamma B_0$

$\omega_1 = \gamma B_1$
 MW
 Control



$$\omega_1 \cos(\omega_1 t) \sigma_x = \frac{\omega_1}{2} \left(e^{+i\omega_1 t \sigma_z} \sigma_x e^{-i\omega_1 t \sigma_z} + e^{-i\omega_1 t \sigma_z} \sigma_x e^{+i\omega_1 t \sigma_z} \right)$$

$$\cos(\omega_1 t) \mathbb{1} + i \sin(\omega_1 t) \sigma_z$$



$$\cos(\omega_1 t) \mathbb{1} + i \sin(\omega_1 t) \sigma_z$$

$$\tilde{H} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \left(e^{+i\omega_1 t \sigma_z} \sigma_x e^{-i\omega_1 t \sigma_z} + e^{-i\omega_1 t \sigma_z} \sigma_x e^{+i\omega_1 t \sigma_z} \right)$$

$$\cos(\omega t) \mathbb{1} + i \sin(\omega t) \sigma_z$$

$$\mathcal{H}_{\text{lab}} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \left(e^{+i\omega t + \sigma_z} \sigma_x e^{-i\omega t + \sigma_z} + e^{-i\omega t + \sigma_z} \sigma_x e^{+i\omega t + \sigma_z} \right)$$

$$U_{\text{rot}} = e^{-i\frac{\omega_1}{2} t \sigma_z} ; \mathcal{H}_{\text{rot}} = -\frac{\omega_1}{2} \sigma_z$$

lab $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] ; \rho(t) = U(t) \rho(0) U^\dagger(t) ; U(t) = e^{-i\mathcal{H}t}$

rotating $\frac{d\tilde{\rho}}{dt}$

$$\tilde{\rho} = U_{\text{rot}} \rho U_{\text{rot}}^{-1}$$

$$\cos(\omega t) \mathbb{1} + i \sin(\omega t) \sigma_z$$

$$\mathcal{H}_{\text{lab}} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \left(e^{+i\omega t + \sigma_z} \sigma_x e^{-i\omega t + \sigma_z} + e^{-i\omega t + \sigma_z} \sigma_x e^{+i\omega t + \sigma_z} \right)$$

$$U_{\text{rot}} = e^{-i\frac{\omega_1}{2} t \sigma_z} ; \mathcal{H}_{\text{rot}} = -\frac{\omega_1}{2} \sigma_z$$

lab $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho] ; \rho(t) = U(t) \rho(0) U^\dagger(t) ; U(t) = e^{-i\mathcal{H}t}$

rotating $\frac{d\tilde{\rho}}{dt}$

$$\tilde{\rho} = U_{\text{rot}} \rho U_{\text{rot}}^{-1}$$

$$\begin{aligned}
 \frac{d\vec{S}}{dt} = -i \left[\mathcal{H}_{\text{eff}} \vec{S} \right] &= \underbrace{\frac{dU_{\text{rot}}}{dt}}_{i \frac{\omega_L}{2} \sigma_z U_{\text{rot}}} U_{\text{rot}}^{-1} + \underbrace{U_{\text{rot}} \frac{d\vec{S}}{dt} U_{\text{rot}}^{-1}}_{-i [\mathcal{H}, \vec{S}]} + U_{\text{rot}} \underbrace{\left[\mathcal{H}, \vec{S} \right]}_{-i \frac{\omega_L}{2} \sigma_z U_{\text{rot}}^{-1}} \\
 &= \underbrace{U_{\text{rot}} \mathcal{H}_{\text{eff}} U_{\text{rot}}^{-1}}_{U_{\text{rot}} \mathcal{H}_{\text{eff}} U_{\text{rot}}^{-1}} - U_{\text{rot}} \mathcal{H}_{\text{eff}} U_{\text{rot}}^{-1}
 \end{aligned}$$

$$\frac{d\vec{P}}{dt} = -i \left[\underbrace{\mathcal{H}}_{\text{PSE}} \vec{P} \right] = \underbrace{\frac{dU_{\text{rot}}}{dt}}_{i \frac{\omega}{2} \sigma_z U_{\text{rot}}} U_{\text{rot}}^{-1} + \underbrace{U_{\text{rot}} \frac{d\vec{P}}{dt} U_{\text{rot}}^{-1}}_{-i [\mathcal{H}, \vec{P}]} + U_{\text{rot}} \underbrace{\left[\frac{dU_{\text{rot}}}{dt} \right]}_{-i \frac{\omega}{2} \sigma_z U_{\text{rot}}^{-1}}$$

$$\boxed{\frac{d\vec{P}}{dt} = -i [\vec{H}, \vec{P}] \rightarrow -i [-\mathcal{H}_{\text{rot}}, \vec{P}]}$$

$$U_{\text{rot}} \mathcal{H} U_{\text{rot}}^{-1} - U_{\text{rot}} \mathcal{H} U_{\text{rot}}$$

$$\mathcal{H}_{\text{lab}} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \left(e^{+i\omega_1 t/2} \sigma_x e^{-i\omega_1 t/2} + e^{-i\omega_1 t/2} \sigma_x e^{+i\omega_1 t/2} \right)$$

$$U_{\text{rot}} = e^{-i\omega_1 t/2 \sigma_z}$$

$$\mathcal{H}_{\text{rot}} = -\frac{\omega_1}{2} \sigma_z$$

lab $\frac{d\rho}{dt} = -i[\mathcal{H}, \rho]$; $\rho(t) = U(t)\rho(0)U^\dagger(t)$; $U(t) = e^{-i\mathcal{H}t}$

rotating $\frac{d\tilde{\rho}}{dt}$

$$\tilde{\rho} = U_{\text{rot}} \rho U_{\text{rot}}^{-1}$$

$$\frac{d\vec{P}}{dt} = -i \left[\underbrace{\mathcal{H}}_{\text{PSE}} \vec{P} \right] = \underbrace{\frac{dU_{\text{rot}}}{dt}}_{i\frac{\omega_+}{2} \sigma_z U_{\text{rot}}} U_{\text{rot}}^{-1} + \underbrace{U_{\text{rot}} \frac{d\vec{P}}{dt} U_{\text{rot}}^{-1}}_{-i[\mathcal{H}, \vec{P}]} + U_{\text{rot}} \underbrace{\frac{dU_{\text{rot}}^{-1}}{dt}}_{-i\frac{\omega_+}{2} \sigma_z U_{\text{rot}}^{-1}}$$

$$\frac{d\vec{P}}{dt} = -i \left[\vec{\mathcal{H}}, \vec{P} \right] - i \left[-\mathcal{H}_{\text{rot}}, \vec{P} \right]$$

$$U_{\text{rot}} \mathcal{H} U_{\text{rot}}^{-1} - U_{\text{rot}}^{-1} \mathcal{H} U_{\text{rot}}$$

$$\frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \sigma_x + e^{-i\frac{2\omega_+ t}{2} \sigma_z} \sigma_x e^{i\frac{2\omega_+ t}{2} \sigma_z} - i\frac{\omega_+}{2} \sigma_z$$

$$\frac{d\vec{P}}{dt} = -i \left[\mathcal{H}_{\text{eff}}, \vec{P} \right] = \underbrace{\frac{dU_{\text{rot}}}{dt}}_{i\frac{\omega_r}{2}\sigma_z U_{\text{rot}}} U_{\text{rot}}^{-1} + \underbrace{U_{\text{rot}} \frac{d\vec{P}}{dt} U_{\text{rot}}^{-1}}_{-i[\mathcal{H}, \vec{P}]} + U_{\text{rot}} \underbrace{\left[\frac{dU_{\text{rot}}^{-1}}{dt} \right]}_{-i\frac{\omega_r}{2}\sigma_z U_{\text{rot}}^{-1}}$$

$$\frac{d\vec{P}}{dt} = -i[\mathcal{H}, \vec{P}] - i[-\mathcal{H}_{\text{rot}}, \vec{P}]$$

$$U_{\text{rot}} \mathcal{H} U_{\text{rot}}^{-1} - U_{\text{rot}}^{-1} \mathcal{H}_{\text{rot}}$$

$$\frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \sigma_x + e^{-i\frac{2\omega_r t}{2}\sigma_z} \sigma_x e^{i\frac{2\omega_r t}{2}\sigma_z} - i\frac{\omega_r}{2} \sigma_z$$

$$\mathcal{H}_{\text{eff}} = \mathcal{H} - \mathcal{H}_{\text{rot}} =$$

$$\cos(\omega_1 t) \mathbb{1} + i \sin(\omega_1 t) \sigma_z$$

$$\mathcal{H}_{\text{lab}} = \frac{\omega_0}{2} \sigma_z + \frac{\omega_1}{4} \left(\overset{\text{rotating field}}{e^{+i\omega_1 t} \sigma_x} e^{-i\omega_1 t} \sigma_z + \overset{\text{countercorotating}}{e^{-i\omega_1 t} \sigma_x} e^{+i\omega_1 t} \sigma_z \right)$$

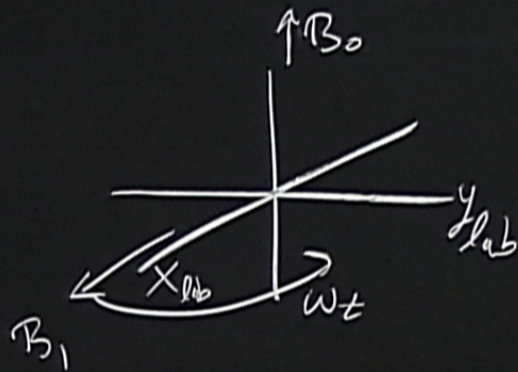
$$U_{\text{rot}} = e^{-i\frac{\omega_1}{2} t \sigma_z}; \quad \mathcal{H}_{\text{rot}} = -\frac{\omega_1}{2} \sigma_z$$

$$\text{lab} \quad \frac{d\rho}{dt} = -i [\mathcal{H}, \rho]; \quad \rho(t) = U(t) \rho(0) U^\dagger(t); \quad U(t) = e^{-i\mathcal{H}t}$$

$$\text{rotating} \quad \frac{d\tilde{\rho}}{dt}$$

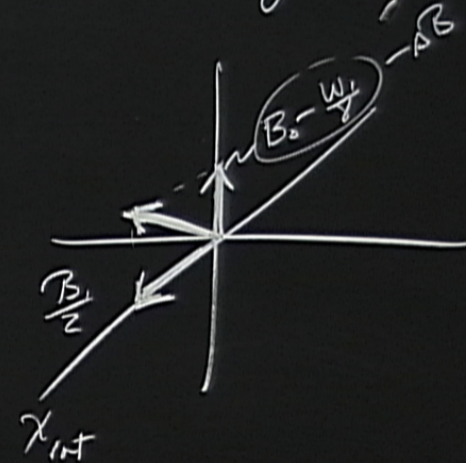
$$\tilde{\rho} = U_{\text{rot}} \rho U_{\text{rot}}^\dagger$$

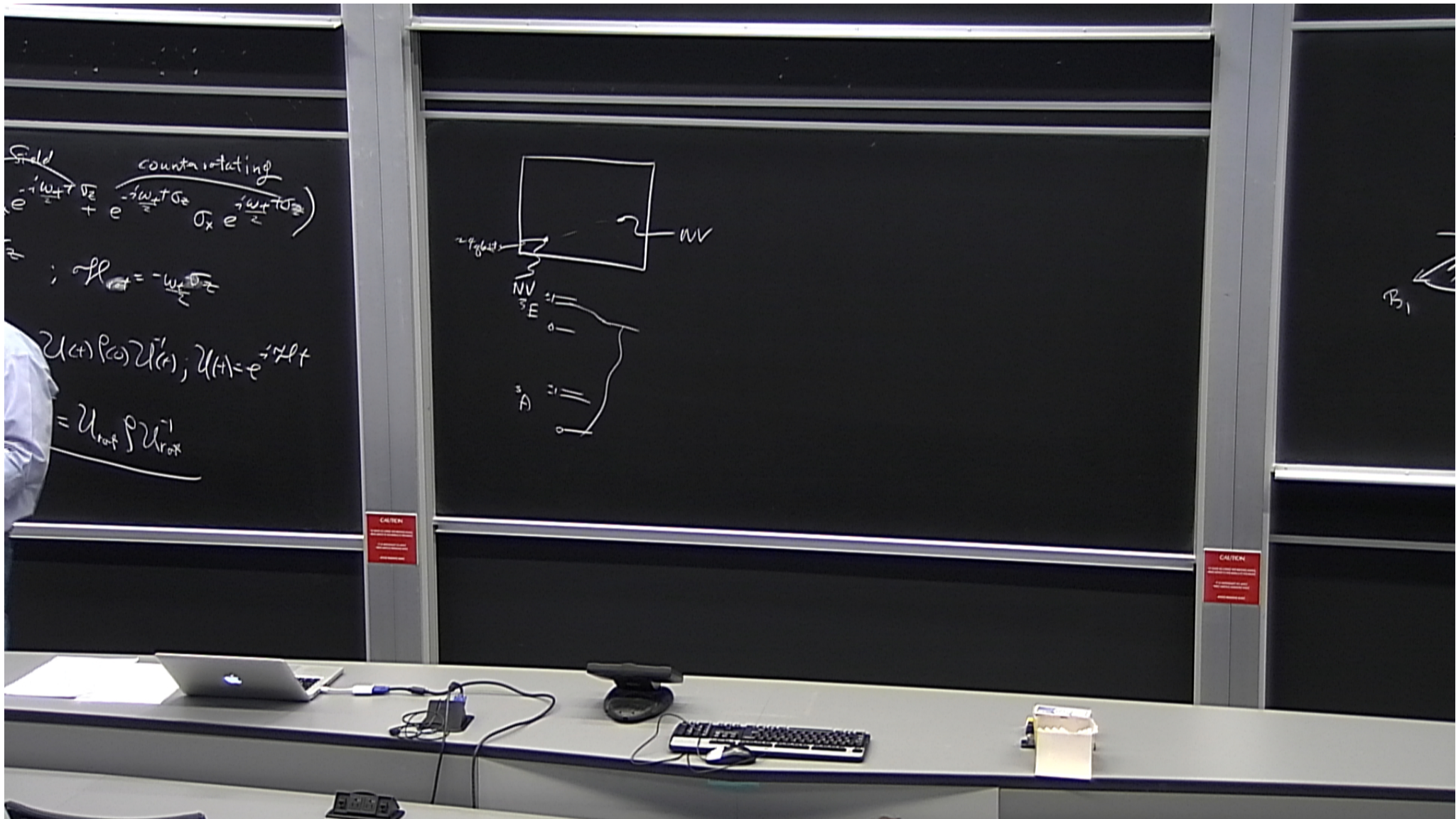
lab frame



rotate
about
z axis
 ω_t

rotating frame





Field

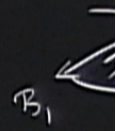
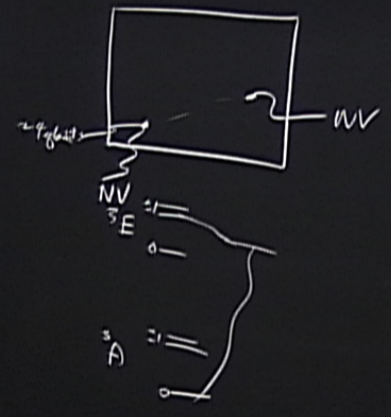
$$e^{-i\omega_+ t \sigma_z} + e^{-i\omega_+ t \sigma_z} \sigma_x e^{i\omega_+ t \sigma_z}$$

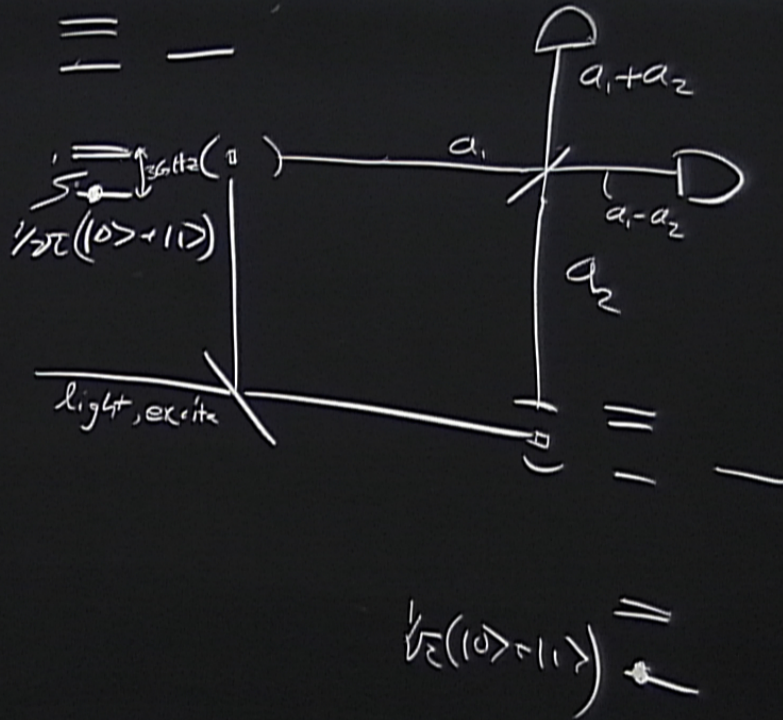
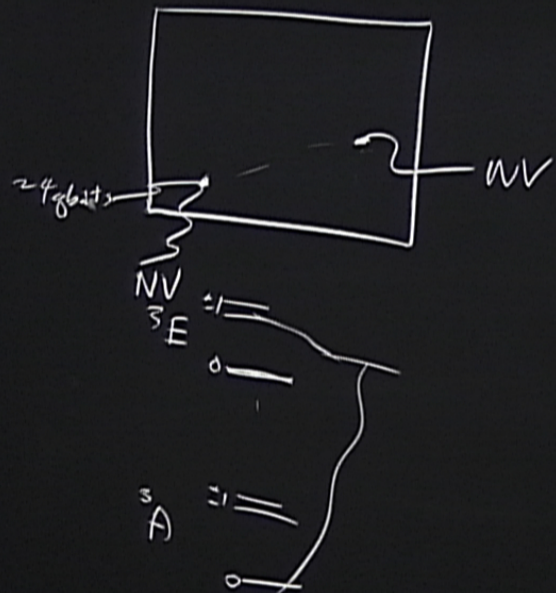
counterclockwise

$$; \mathcal{H}_{rot} = -\frac{\omega_+ \sigma_z}{2}$$

$$U(t) \rho(0) U^\dagger(t); U(t) = e^{-i\mathcal{H}t}$$

$$= U_{rot} \rho U_{rot}^\dagger$$





CAUTION
DO NOT TOUCH THE SURFACE OF THE MIRROR
OR THE MOUNTING OF THE MIRROR OR THE MOUNTING OF THE MIRROR

CAUTION
DO NOT TOUCH THE SURFACE OF THE MIRROR
OR THE MOUNTING OF THE MIRROR OR THE MOUNTING OF THE MIRROR

$$\frac{1}{2} (|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

↑
no photons



$$\frac{1}{2} (|100\rangle + \underbrace{|101\rangle + |110\rangle}_{\text{no photons}} + |111\rangle)$$

↑
photons
 $a_1 + a_2$
detector

↑
no photons

