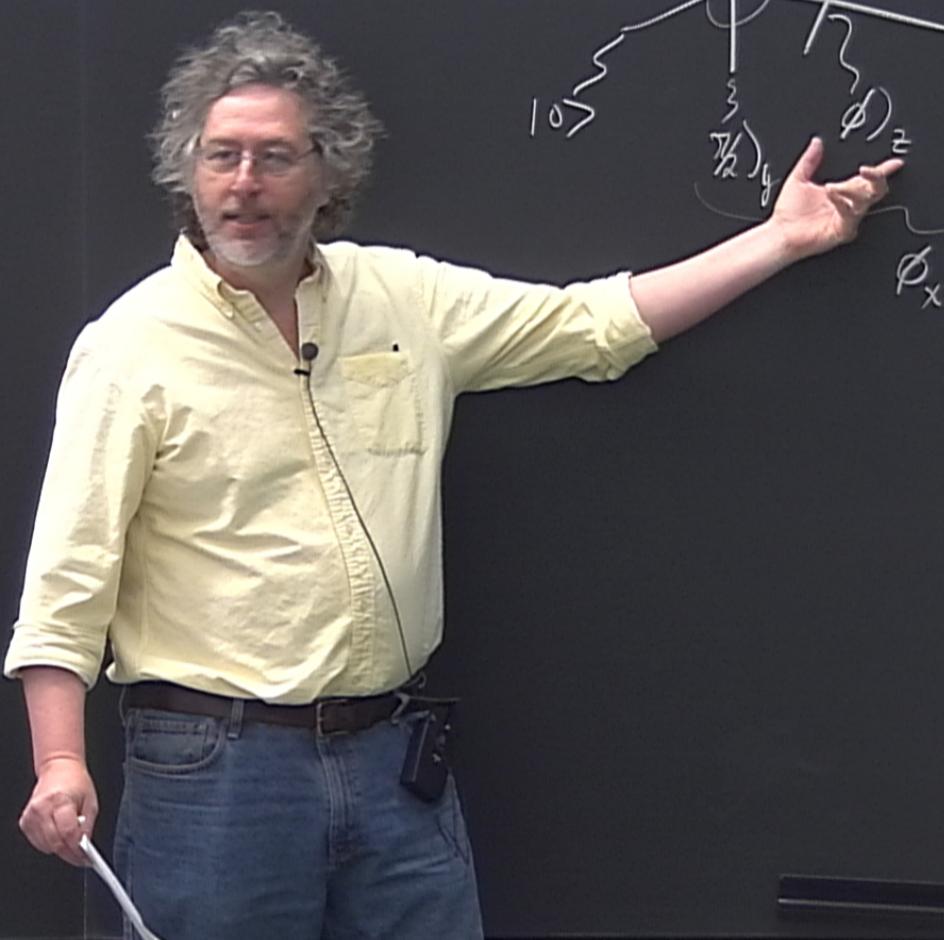


Title: Explorations in Quantum Information - Lecture 4

Date: Mar 15, 2012 09:00 AM

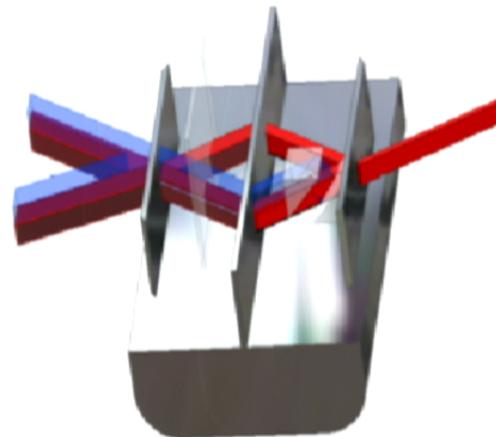
URL: <http://pirsa.org/12030007>

Abstract:

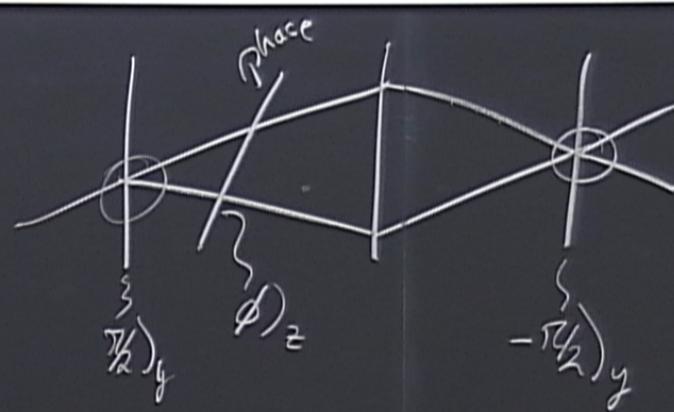


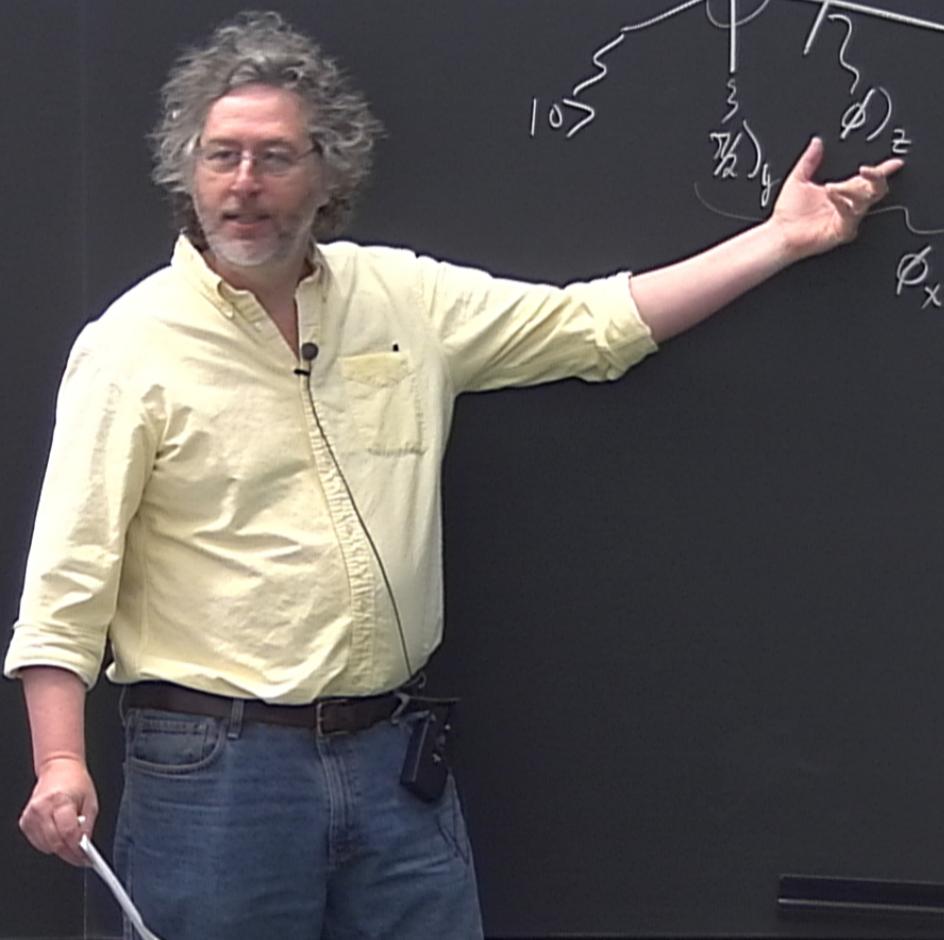
like we saw with the magnetic moment and a magnetic sample in the last lecture. I want to keep this distinct from decoherence. Since incoherence has an underlying time independent Hamiltonian structure with an associated classical probability distribution, it can always be recovered by post selection. So no information need be lost if we are willing to make a more complex measurement.

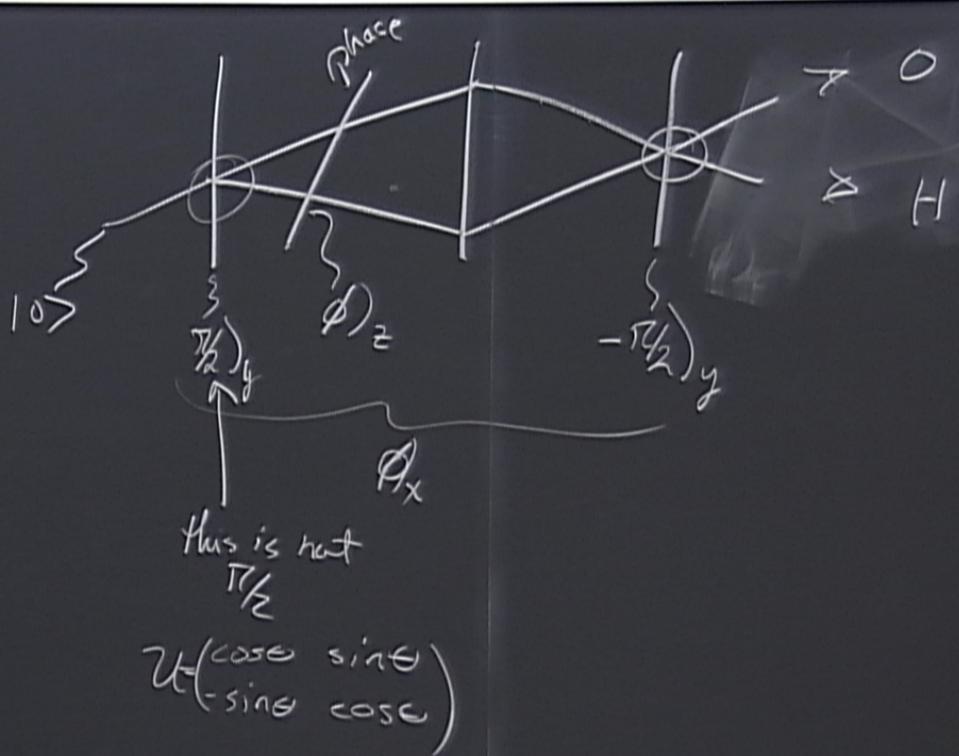
■ 3-blade single crystal interferometer

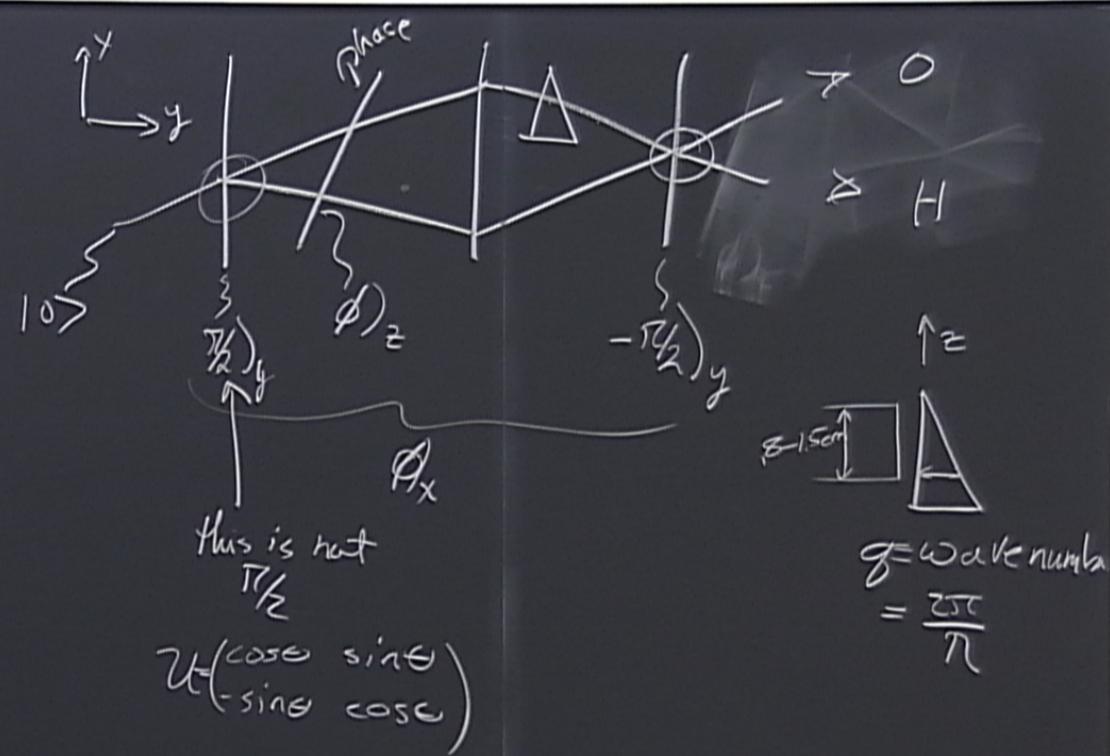


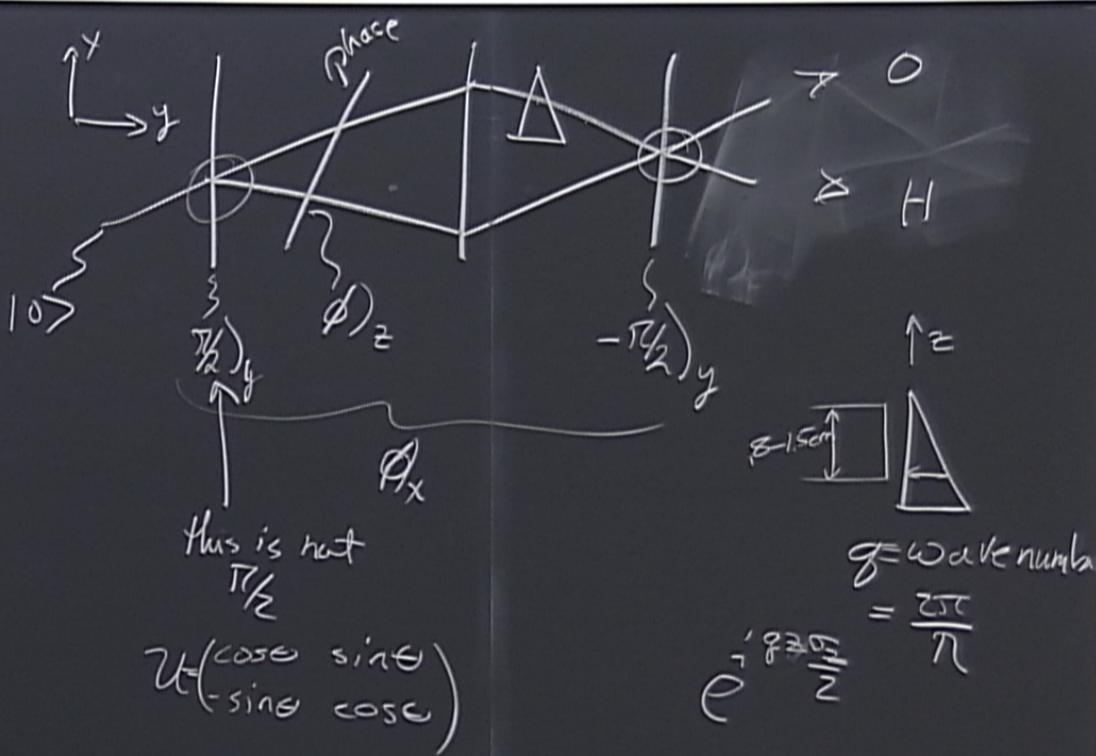
150%











$$\mathbb{I} = E_+ + E_-$$

$$\sigma_z = E_+ - E_-$$

$$E_{\pm} = \frac{1}{2}(\mathbb{I} \pm \sigma_z)$$

$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

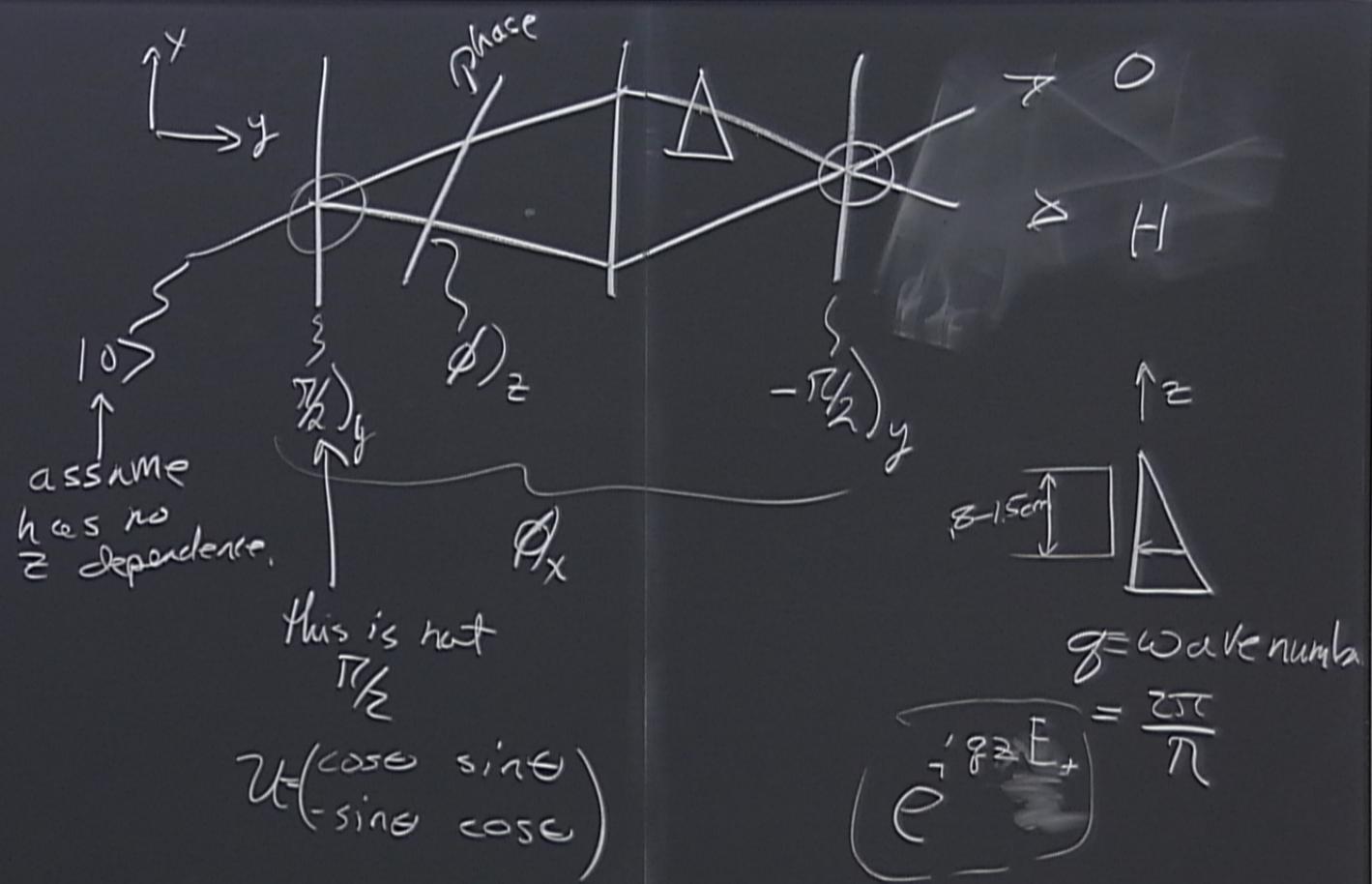
$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

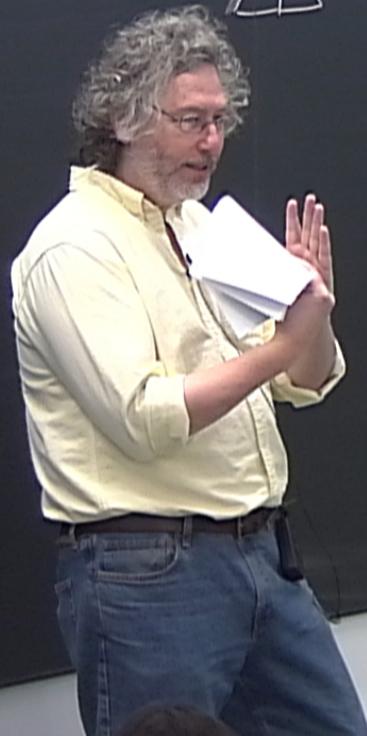
\uparrow^x
 \rightarrow^y

$|0\rangle$

Ph

$U(-)$





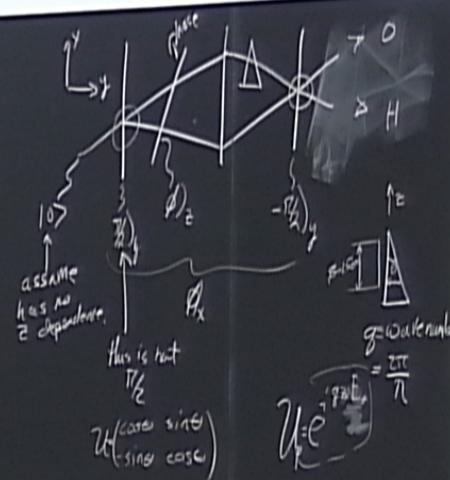
$$\mathbb{I} = E_+ + E_-$$

$$\sigma_z = E_+ - E_-$$

$$E_{\pm} = \frac{1}{2}(\mathbb{I} \pm i\sigma_z)$$

$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

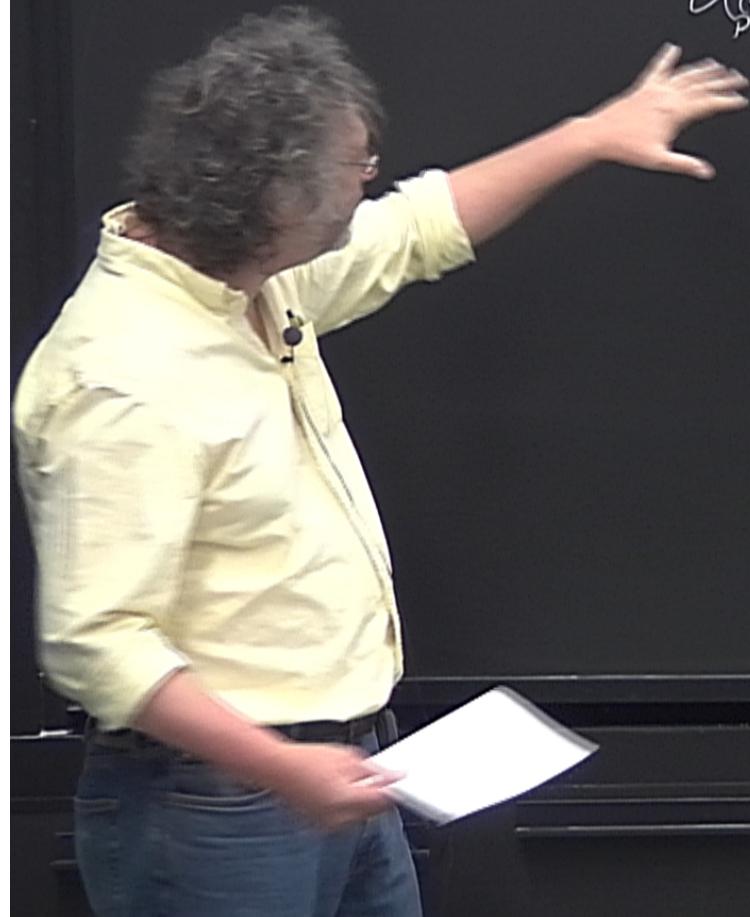
$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$





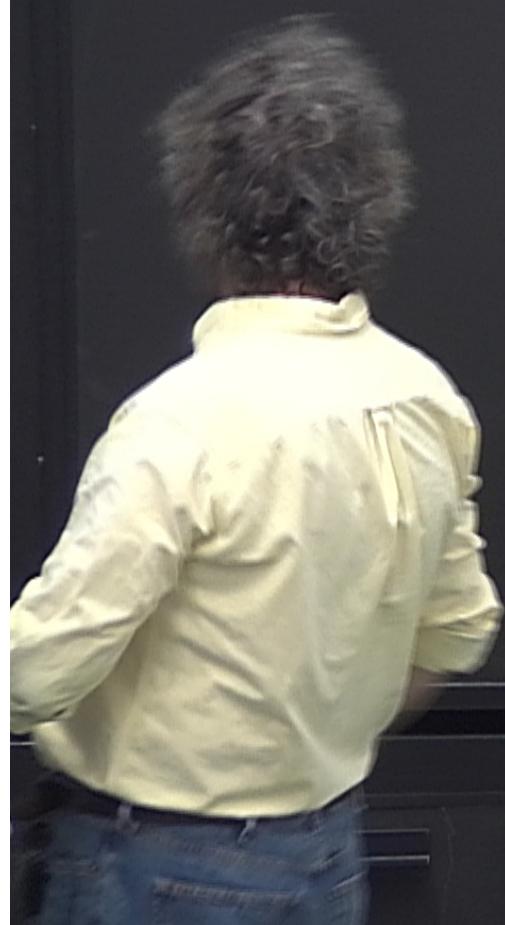
$$U_{\text{eff}} U_m U_{\text{eff}} U, \stackrel{\text{in}}{\circ} U^{-1} U' U_{\text{eff}}^{-1} U_m^{-1} U_{\text{eff}}^{-1}$$

\downarrow
 E_+



$$U_3 U_{p\bar{q}} U_m U_{\bar{q}\bar{p}} U_i S_{in} U^{-1} U_{\bar{q}\bar{p}} U_m^{-1} U_{\bar{q}\bar{p}}^{-1} U_{p\bar{q}}^{-1} U_3^{-1}$$

\downarrow
 E_r



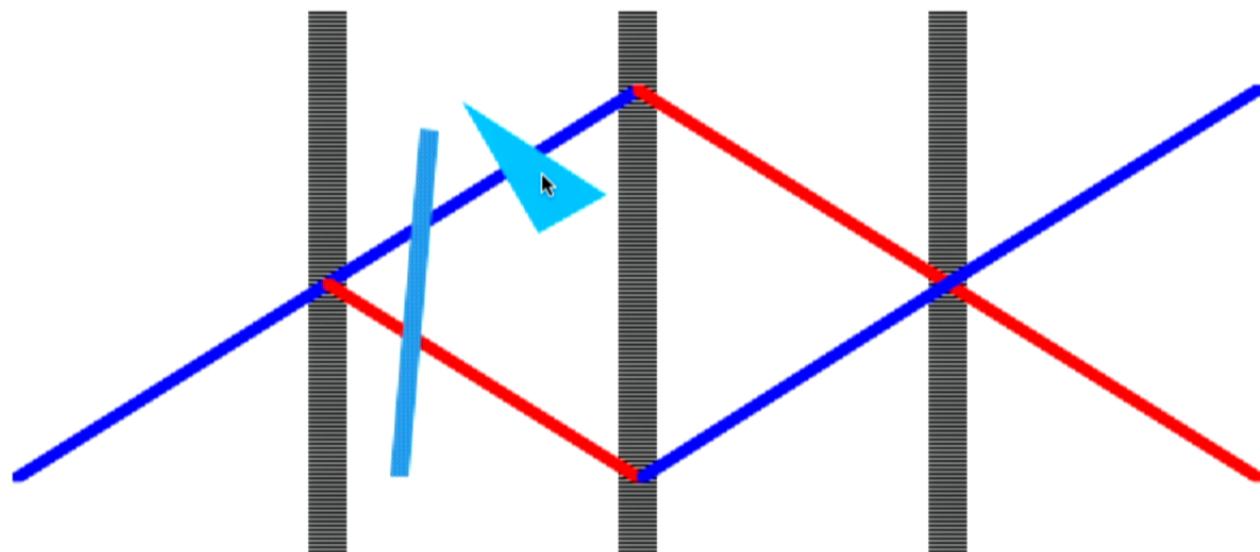
$$\int P(z) U_3 U_{\frac{m}{z}} U_m U(\phi) U_i \underbrace{S_{in} U^*}_{E_r} U(\bar{\phi}) U_m^{-1} U_{\frac{m}{z}}^{-1} U_{\frac{m}{z}} U_3^{-1} dz$$

$$\int_{\text{out}} \mathcal{P}(z) U_3 U_{\frac{p}{\lambda}} U_m U(\phi) U_i S_{\text{in}} U' U(\bar{\phi}) U_m^{-1} U_{\frac{c}{\lambda}} U_3^{-1} dz$$

$\underbrace{E_r}_{z}$

Experiments

- Sixth Experiment: incoherent distribution with a wedge in the path.



The wedge is a precise prism that introduces a phase shift that is proportional to the height of the neutron in the beam.

Note the picture is misleading, the wedge is arranged so that it becomes narrower as it moves out of the paper towards you.

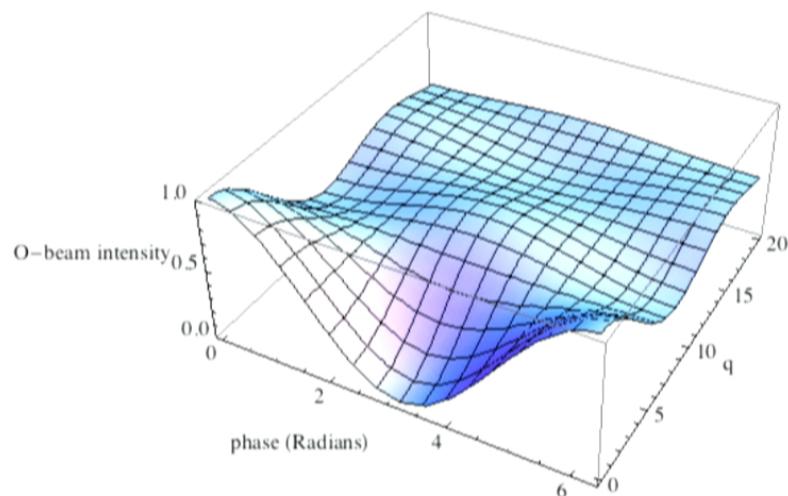
Here we need to note that the neutron beam has some height, so we must introduce a new variable (z) as well as a classical probability distribution over z .

We will define the spatially dependent phase shift in terms of the position (z) and a wave-number q . Note that the product qz is an angle.

- Wedge Propagators

We define a propagator for a wedge in both the $+k_x$ and in the $-k_x$ paths.

150%

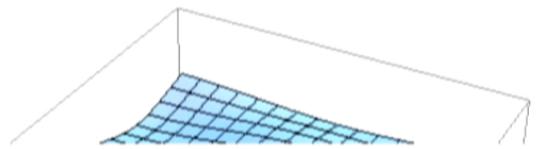


```
M6H[q_, a_] := Integrate[Tr[Ezm . res6[q, z, a]], {z, -0.5, 0.5}]
```

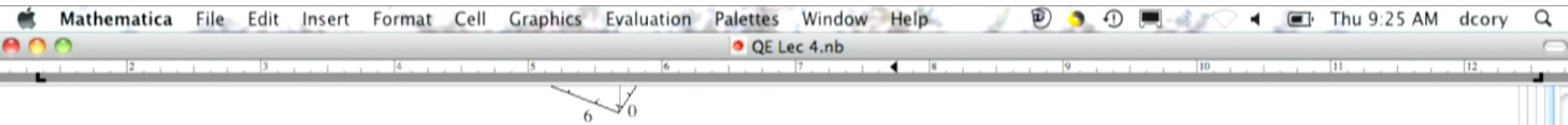
```
M6H[q, a]
```

$$\frac{0.5 q + 0.5 \sin[a - 0.5 q] - 0.5 \sin[a + 0.5 q]}{q}$$

```
Plot3D[M6H[q, a], {a, 0, 2 \pi}, {q, 0, 20},  
{AxesLabel \rightarrow {"phase (Radians)", "q", "O-beam intensity"}, PlotRange \rightarrow {0, 1}}]
```



150%

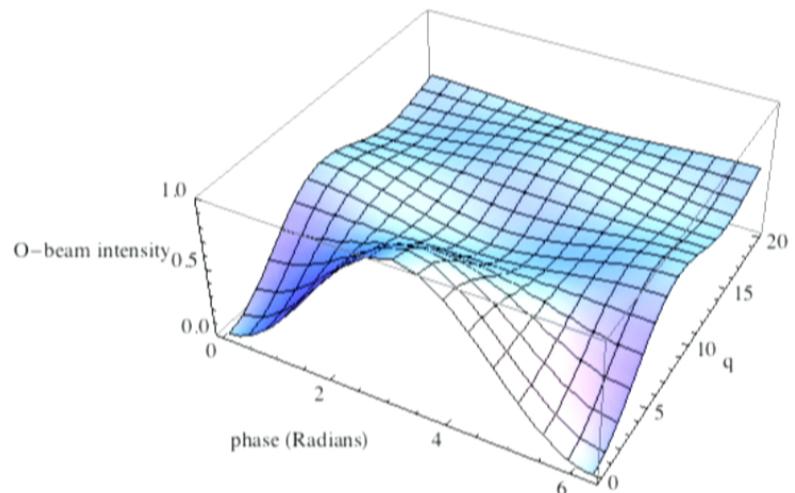


```
M6H[q_, a_] := Integrate[Tr[Ezm . res6[q, z, a]], {z, -0.5, 0.5}]
```

```
M6H[q, a]
```

$$\frac{0.5q + 0.5 \sin[a - 0.5q] - 0.5 \sin[a + 0.5q]}{q}$$

```
Plot3D[M6H[q, a], {a, 0, 2π}, {q, 0, 20},
AxesLabel -> {"phase (Radians)", "q", "O-beam intensity"}, PlotRange -> {0, 1}]
```



We see that as expected all neutrons are detected at one beam or the other, but that as the wave-number of the prism is in-



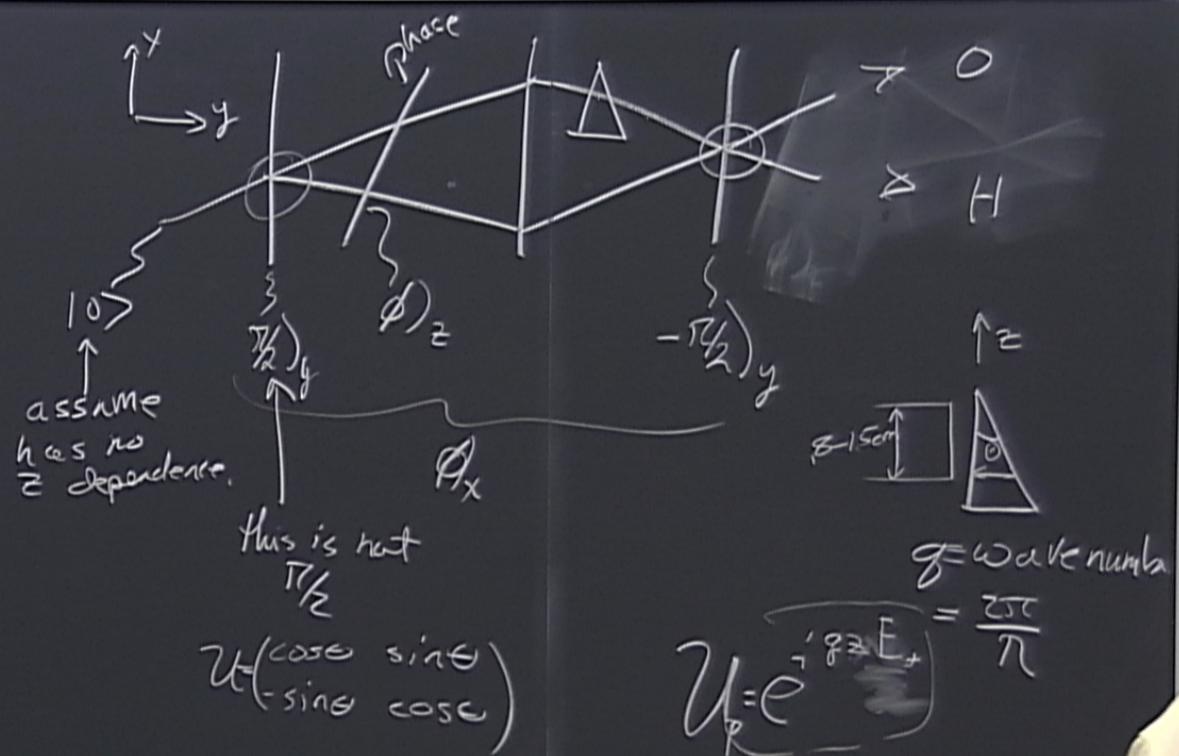
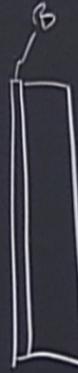
$$\mathbb{I} = E_+ + E_-$$

$$\sigma_z = E_+ - E_-$$

$$E_{\pm} = \frac{1}{2}(\mathbb{I} \pm \sigma_z)$$

$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



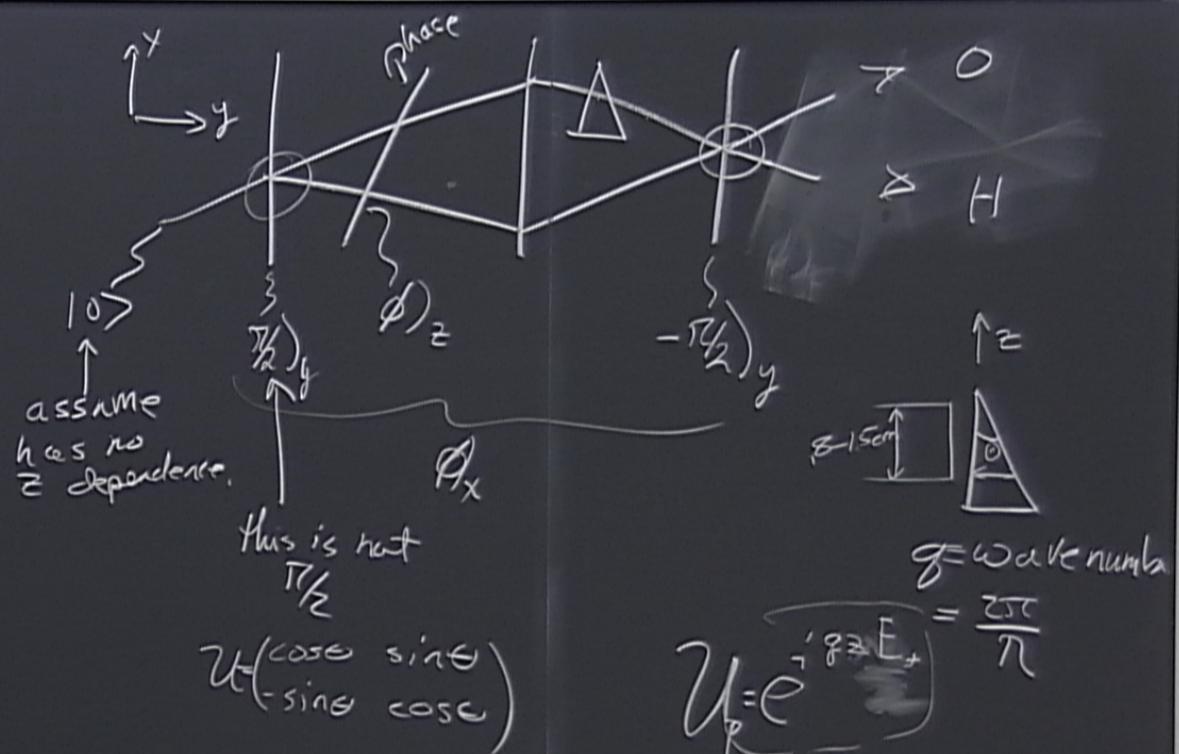
$$\Pi = E_+ + E_-$$

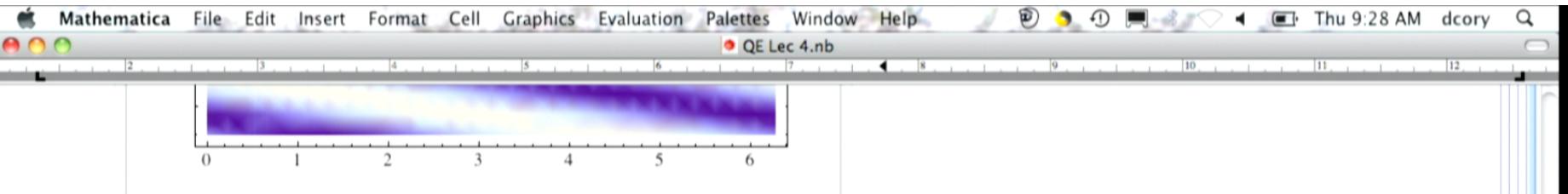
$$\sigma_z = E_+ - E_-$$

$$E_{\pm} = \frac{1}{2}(\Pi \pm \sigma_z)$$

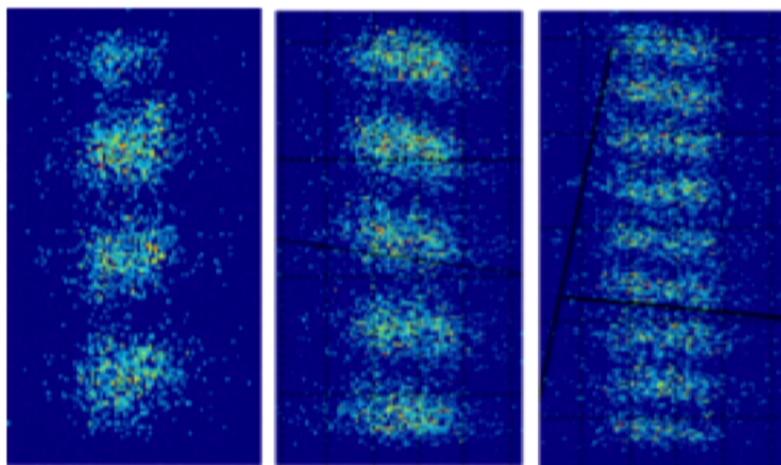
$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$





Clearly, the interferometer actually retains the desired contrast, it is just that the contrast curves from the various spatial locations are shifted in phase and add incoherently. For larger wave-numbers, this results in the integrated contrast vanishing even though at each spatial location the contrast is preserved.



Measurements from a position sensitive detector showing the fringes and the beam profile. Note that the position sensitivity is increased but at a cost of resolution. The scintillation event acts as a point source for photons.

Notice that as the position sensitive detector is moved from one path to the other that the fringes are complementary.



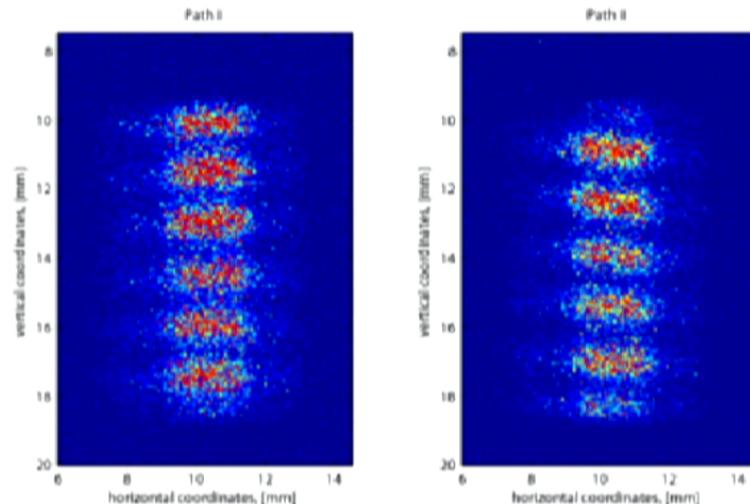
$$S_{out}^{(q)} = \int P(z) U_3 U_{\frac{p(q)}{z}} U_m U(\phi) U_i S_{in} U_i^{-1} U_{\bar{q}} U_m^{-1} U_{\frac{q}{z}} U_3^{-1} dz$$

\downarrow
 E_r

$$I = \frac{1 + \cos(qz + \phi)}{2}$$

Measurements from a position sensitive detector showing the fringes and the beam profile. Note that the position sensitive detector has low quantum efficiency. The neutron is converted to light in a scintillator which is then collected in a CCD. If the scintillator is thick, then the quantum efficiency is increased but at a cost of resolution. The scintillation event acts as a point source for photons.

Notice that as the position sensitive detector is moved from one path to the other that the fringes are complementary.



- **Problem 20:**

Why are the fringes tilted, and why are the O and H fringes complementary. How can you show from this that the state is an incoherent distribution of pure states?

$$\mathbb{I} = E_+ + E_-$$

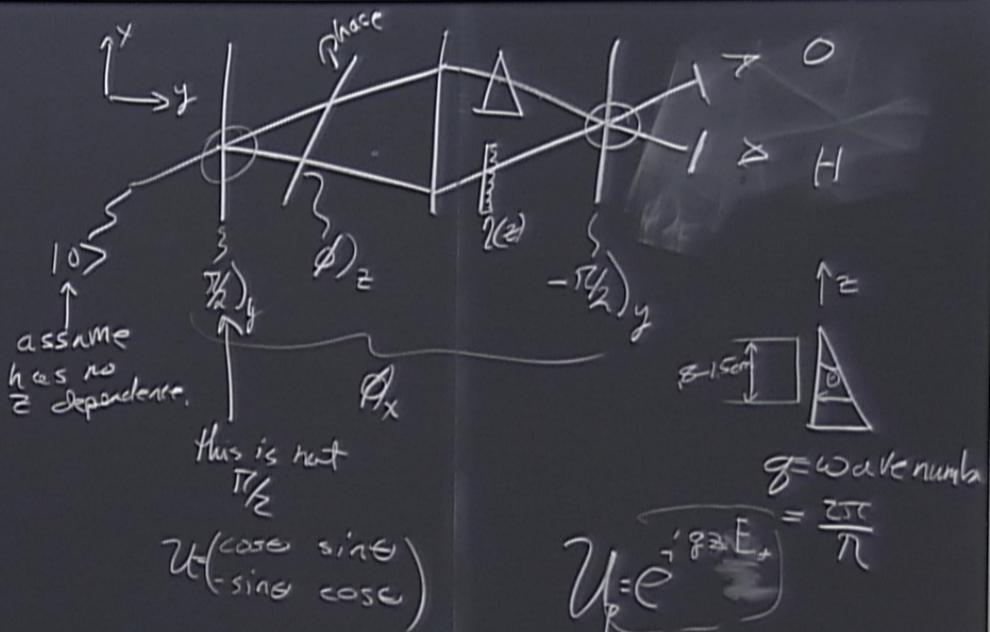
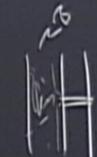
$$\sigma_z = E_+ - E_-$$

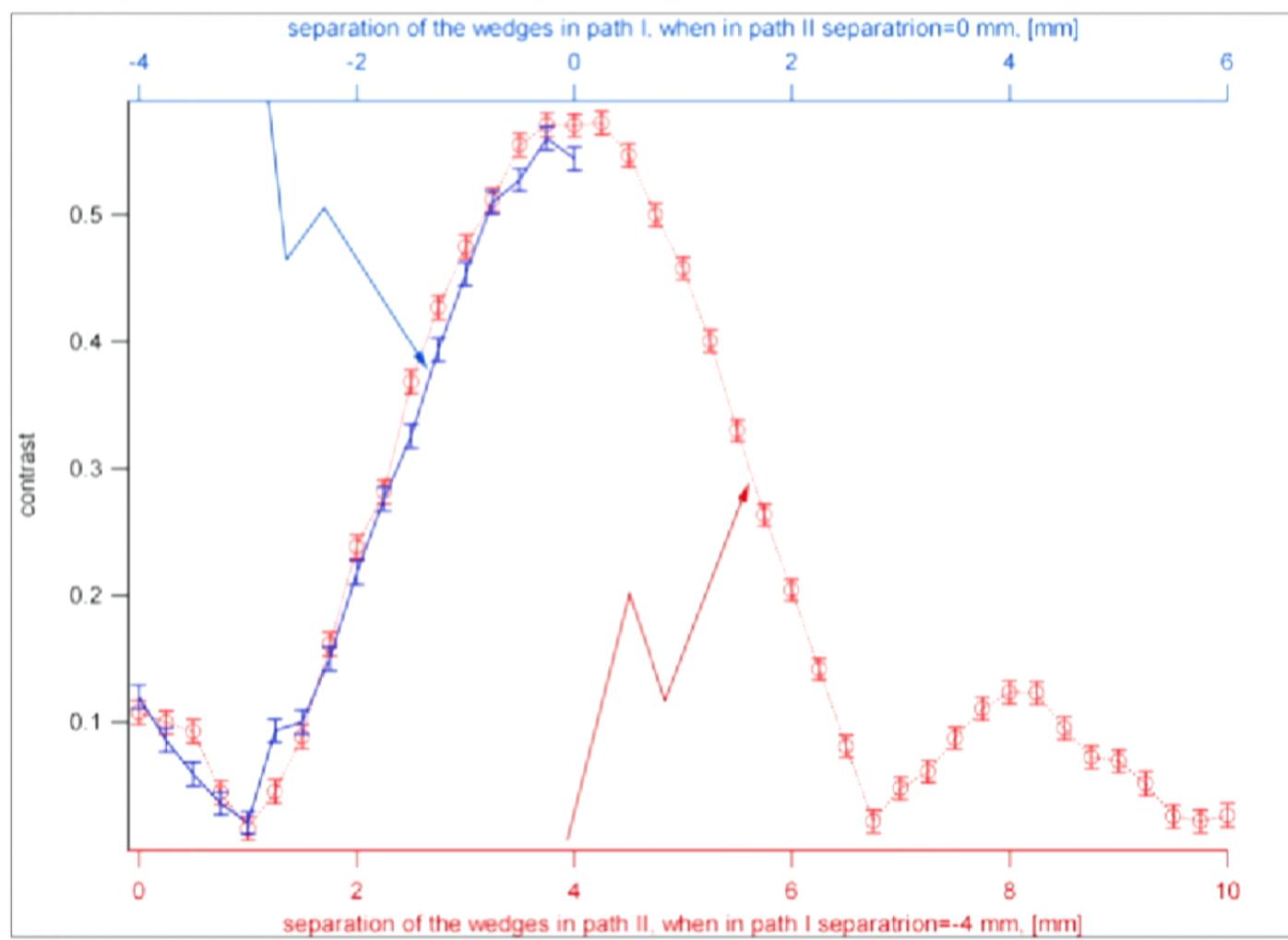
$$E_{\pm} = \frac{1}{2}(\mathbb{I} \pm \sigma_z)$$

$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

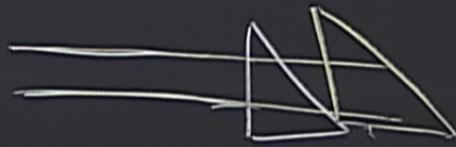
$$c(\phi) = \int \gamma(z) e^{iz\phi} dz$$





$$\int_{\text{out}} \rho(\phi) = \int P(z) U_3 U_{\text{out}}^{\dagger} U_m U(\phi) U_i S_{\text{in}} U_i^{\dagger} U_{\text{out}}^{\dagger} U_m^{\dagger} U(\phi)^{\dagger} U_3^{\dagger} dz$$

$$I = \frac{1 + \cos(\phi z + \phi)}{2}$$



$$\int_{\text{old}} \rho(\vec{q}) = \int P(z) U_3 U_n^\dagger U_m U_\phi U_i S_{in} U_i^\dagger U_n^\dagger U_m^\dagger U_\phi^\dagger U_3^\dagger dz$$

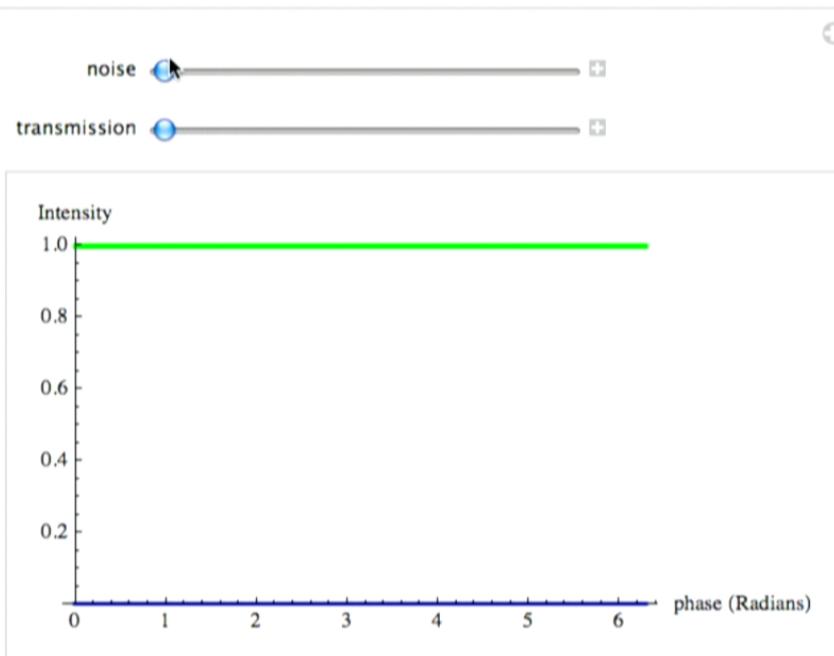
$\underbrace{\hspace{10em}}$
 E_r

$$I = \frac{1 + \cos(qz + \phi)}{2}$$

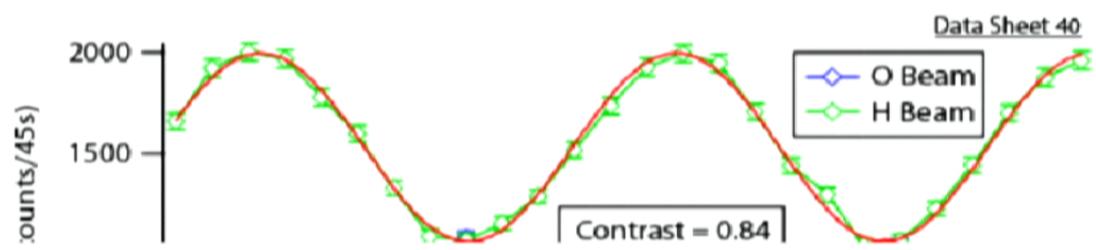
dummy
variable

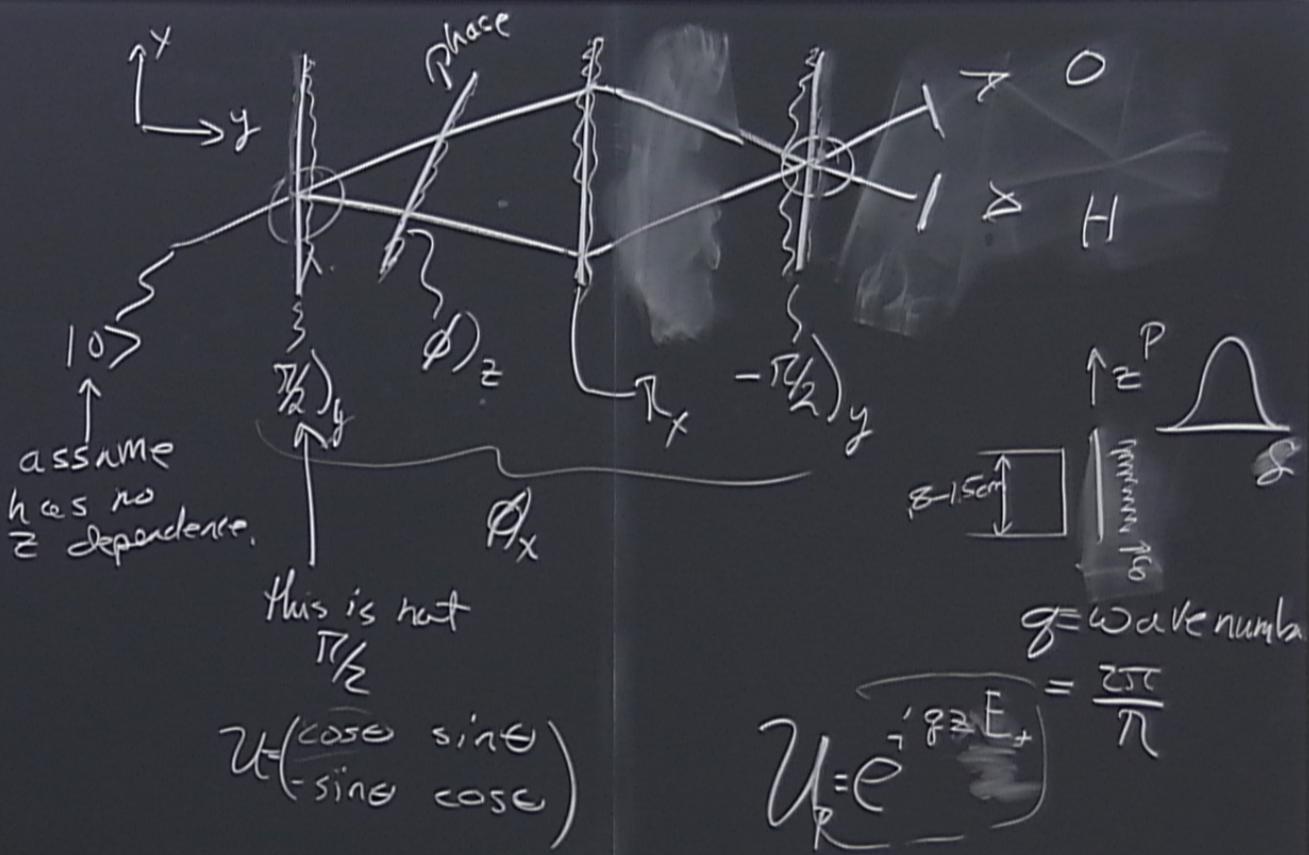
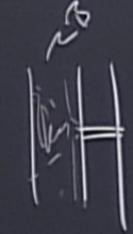
$$P(z) = \frac{e^{-\frac{z^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma}$$

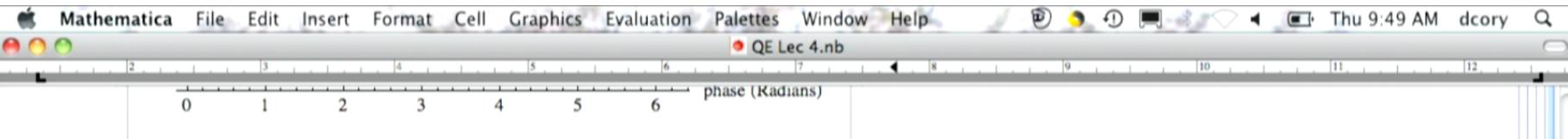
$$U_n(z) = e^{iz\tau_z/2}$$



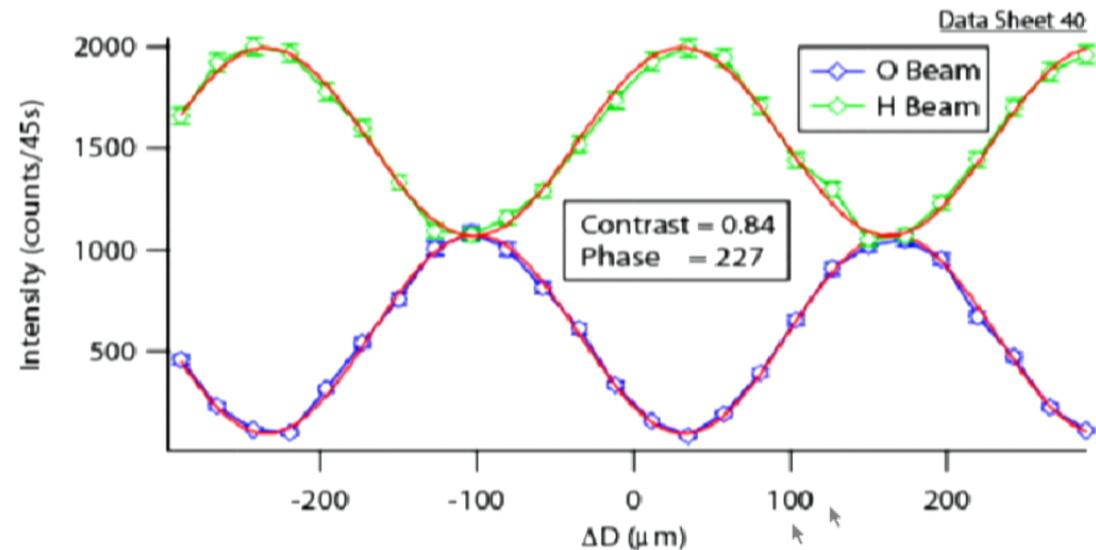
• Problem 24:







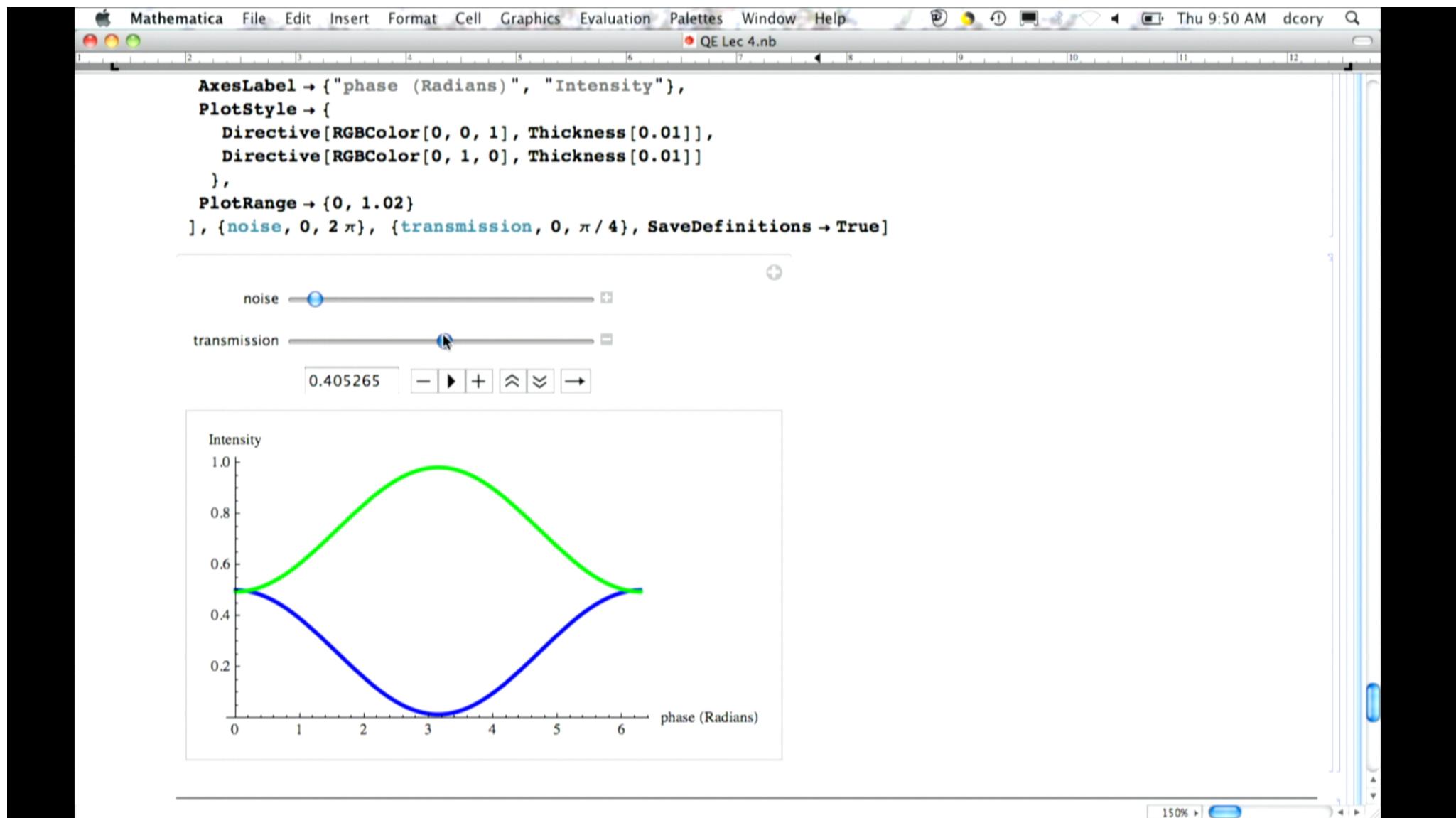
• Problem 24:

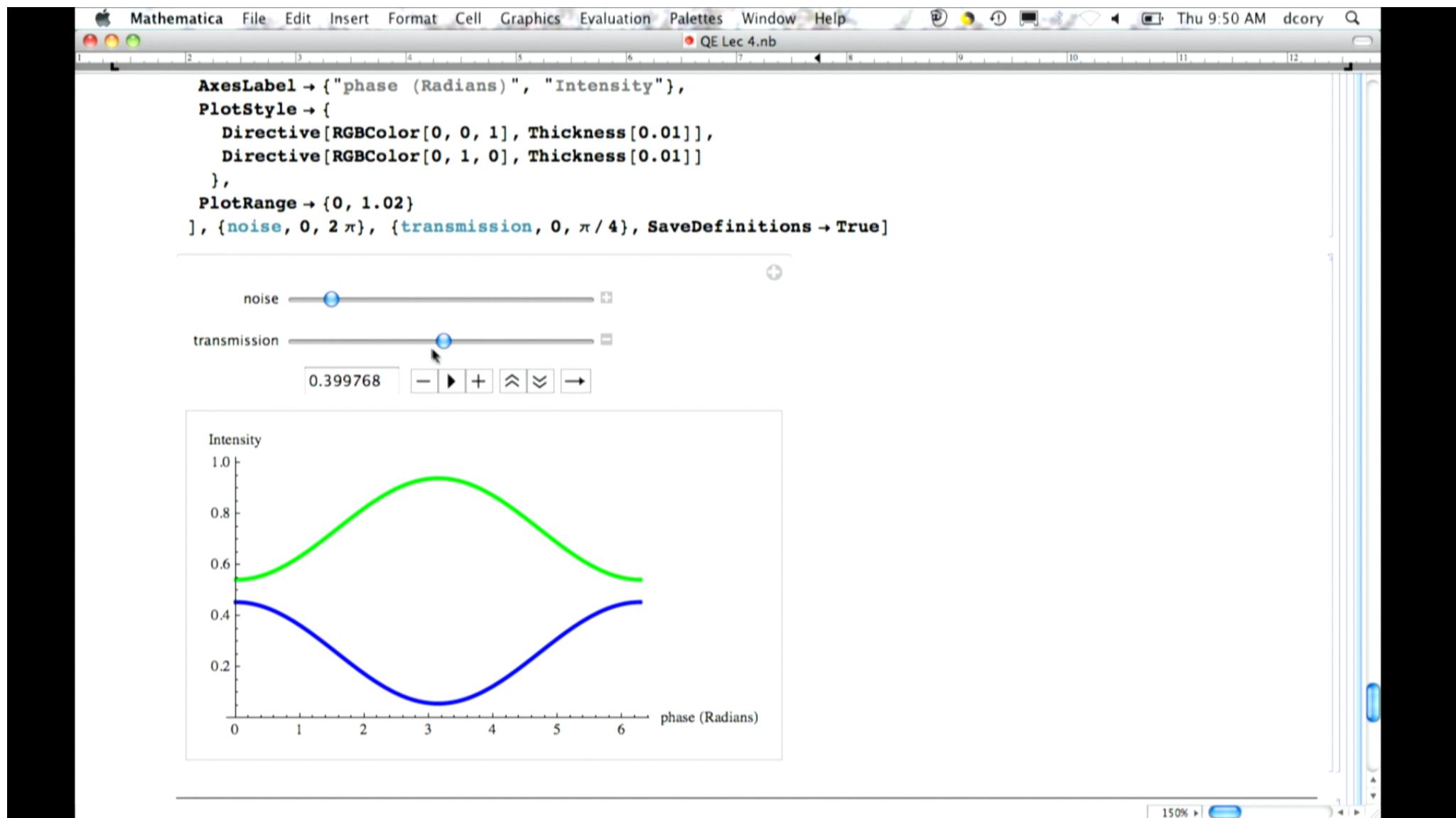


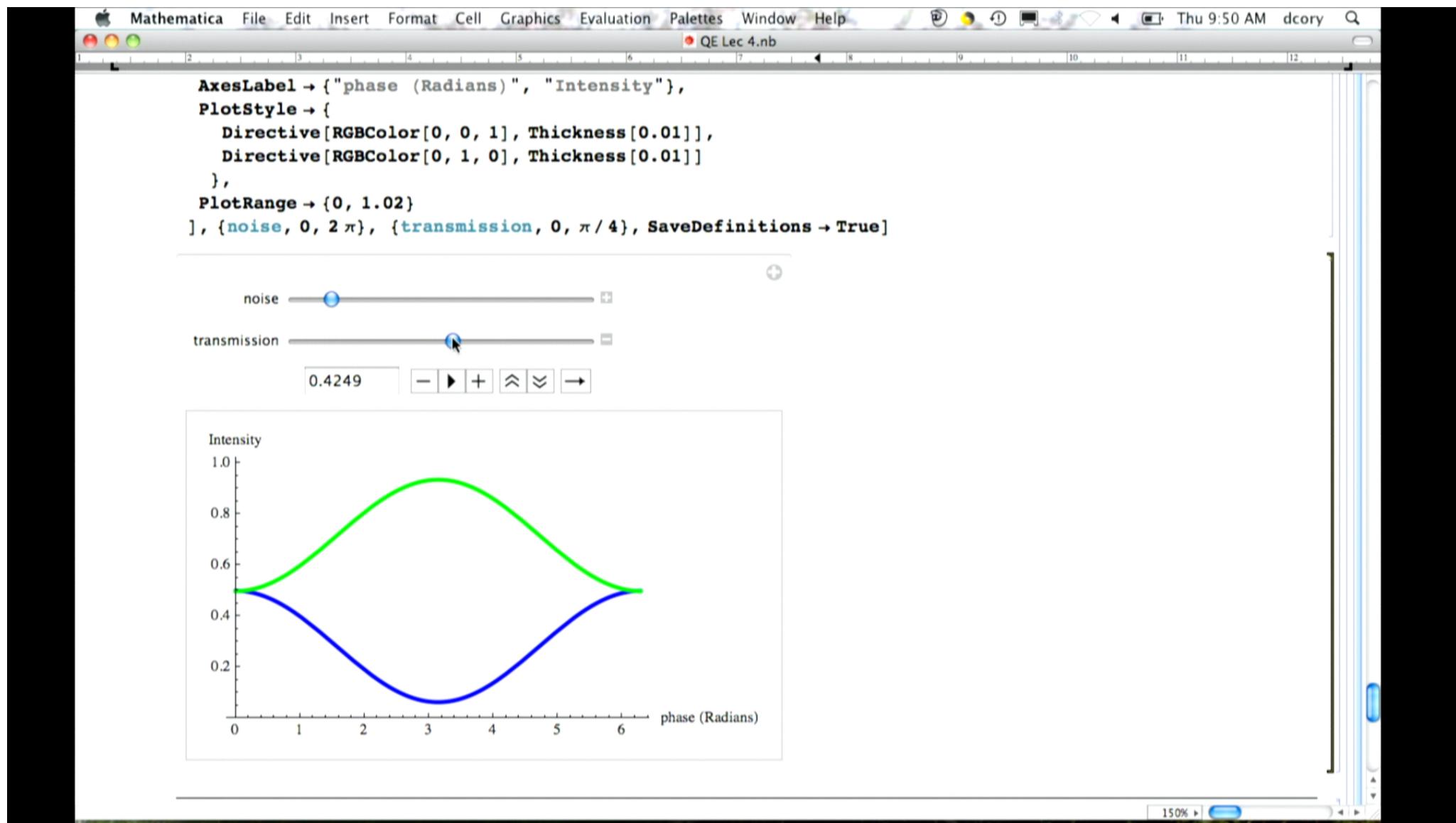
Estimate the transmission coefficient and the loss for the experimental NI setup shown above.

■ Discussion Question

If you are using NI to explore quantum foundations do the problems of noise and transmission enter the same way?







$$U_n(z) = e^{-iz}$$

ideal $U = \phi(x)$
 $= e^{i\phi(x)}$

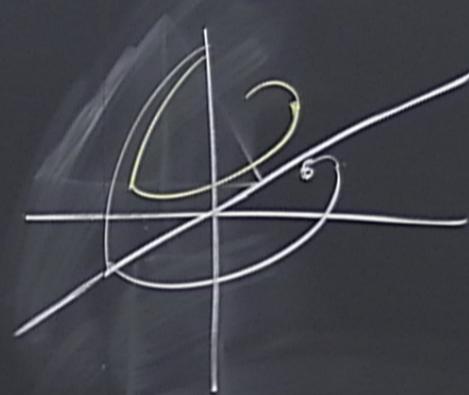
$$\mathbb{I} = E_+ +$$

$$S_z = E_+ - E_-$$

$$E_{\pm} = \frac{1}{2}(\mathbb{I} \pm$$

$$E_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$E_- = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$



$t \text{ passed}$

$$\sigma = 0$$

ideal

$$U = \phi)_{\alpha} \\ = e^{-i\phi \frac{\sigma_x}{2}}$$

$$P=1 \\ M_1 = e^{-i\phi \sigma_x/2}$$

Some

$$U = II$$

$$M_1 = (1-p)^{\frac{1}{2}} e^{-i\phi \sigma_x/2}$$

$$M_2 = p^{\frac{1}{2}} II$$

$$\sigma \neq 0$$

$$M_1 = (1-p)^{\frac{1}{2}} e^{-i\phi \sigma_x/2}$$

$$M_2 = p^{\frac{1}{2}} \sigma_x e^{-i\phi \sigma_x/2}$$



rotate ϕ so that all n are detected at 0
add Cd block