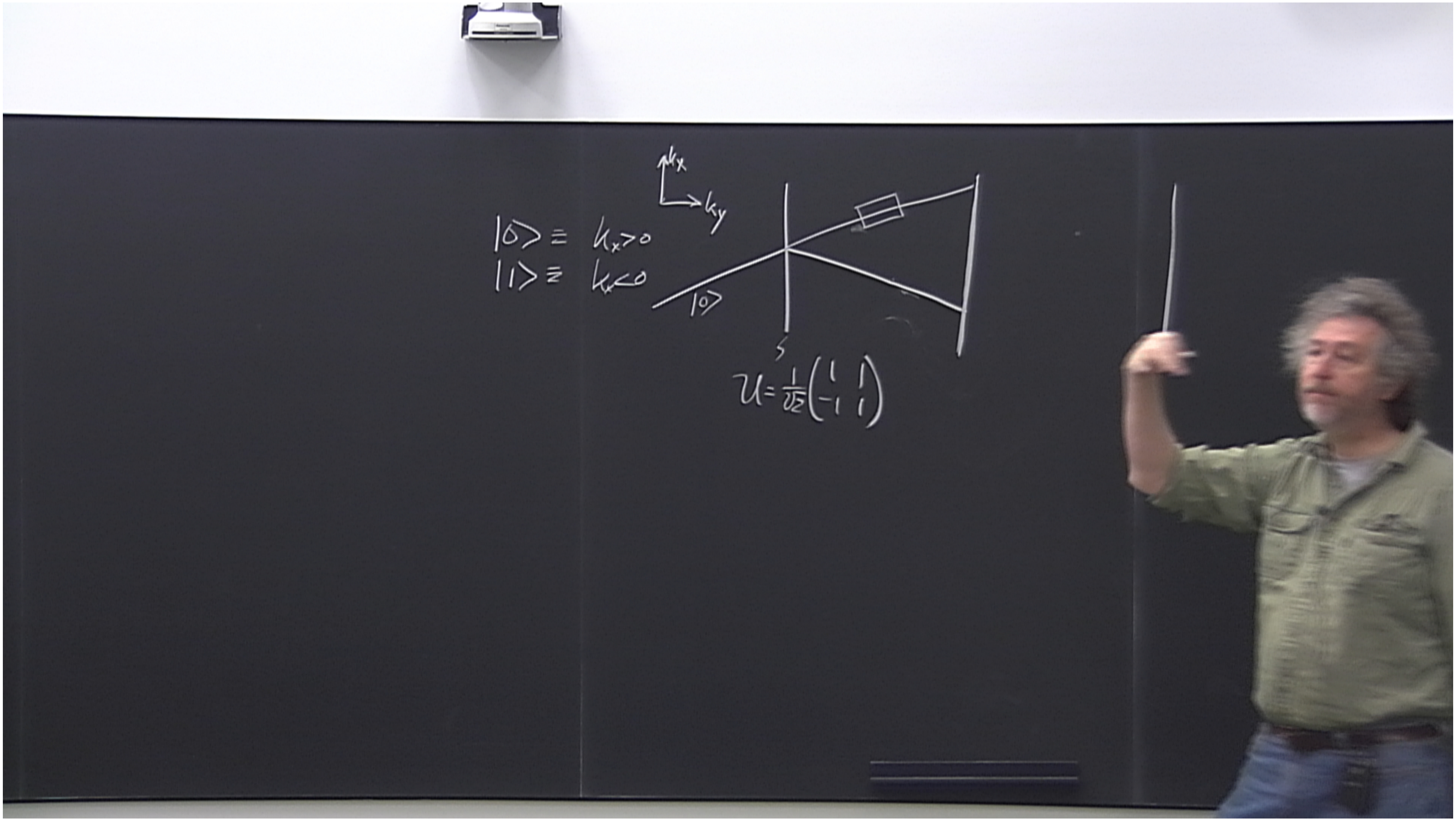
Title: Explorations in Quantum Information - Lecture 2

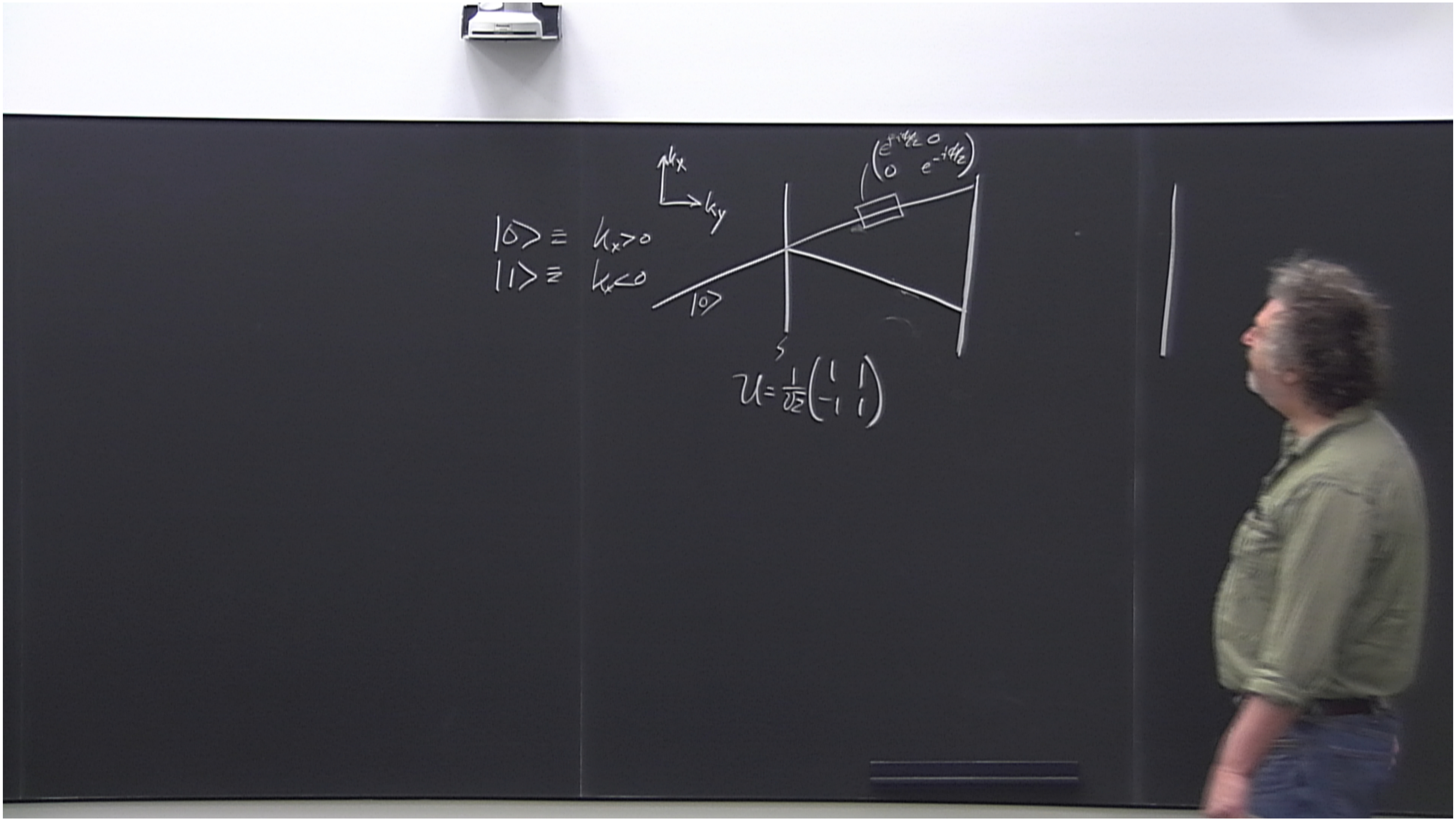
Date: Mar 13, 2012 09:00 AM

URL: http://pirsa.org/12030005

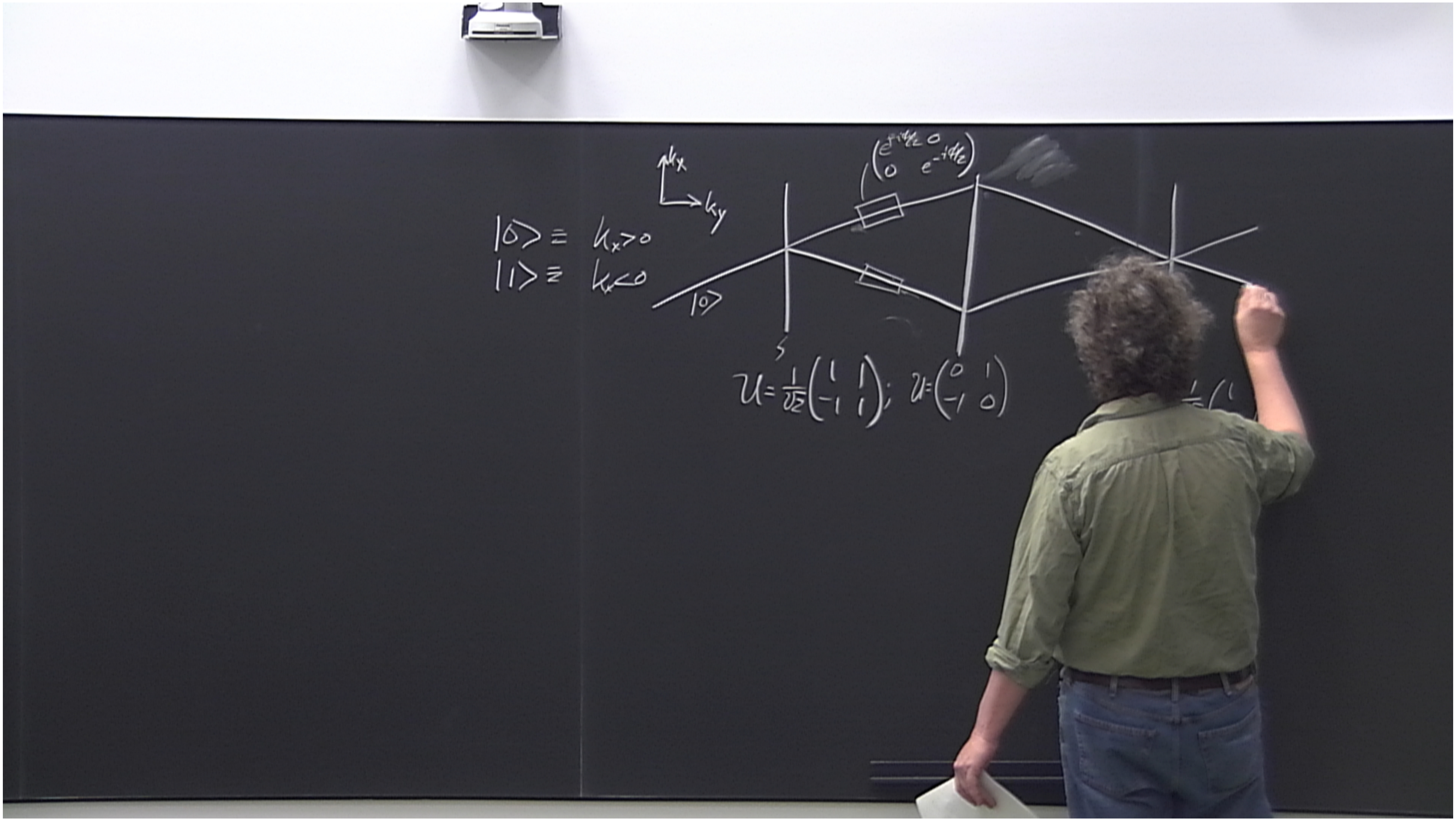
Abstract:

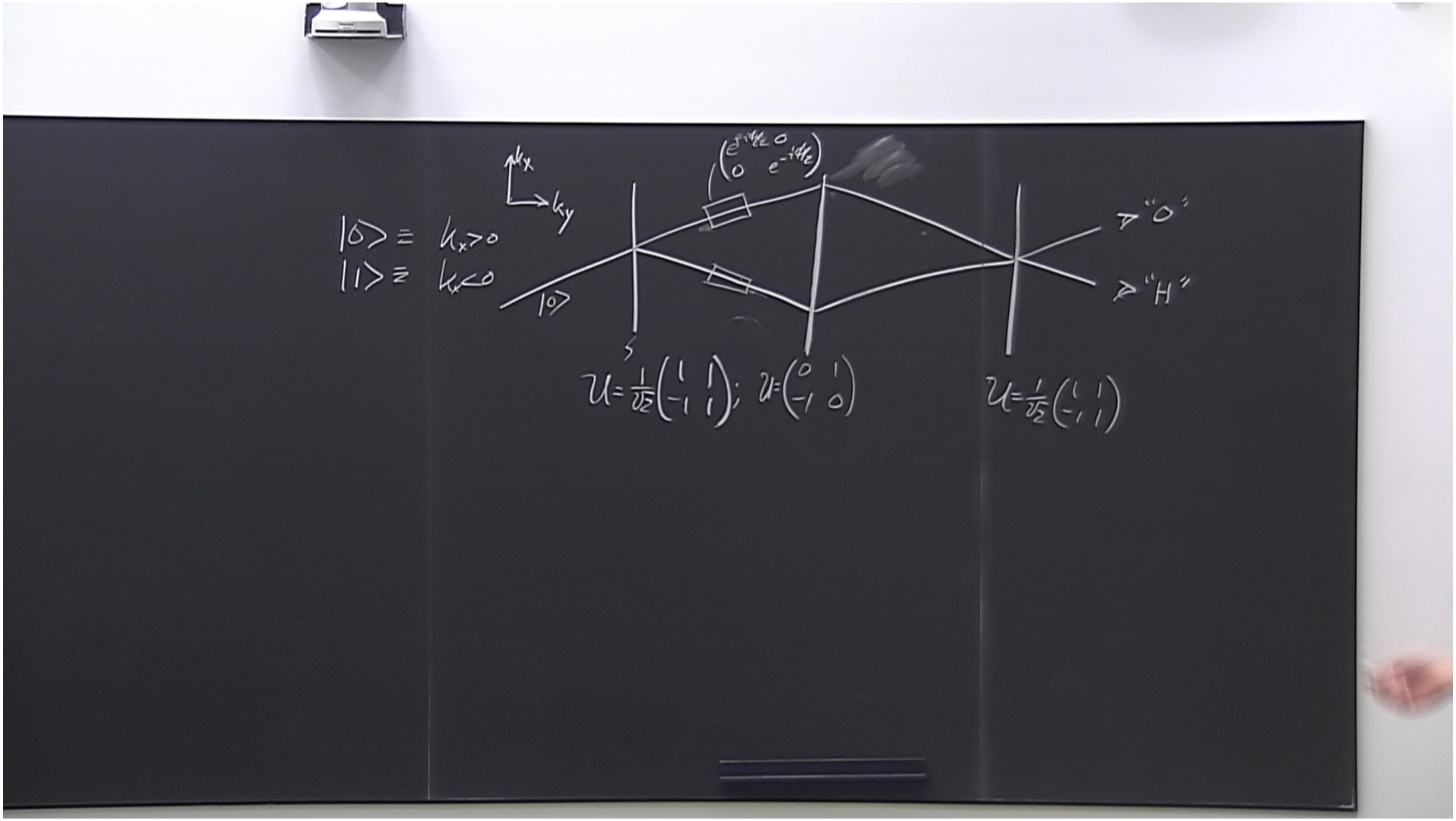

kx
ky

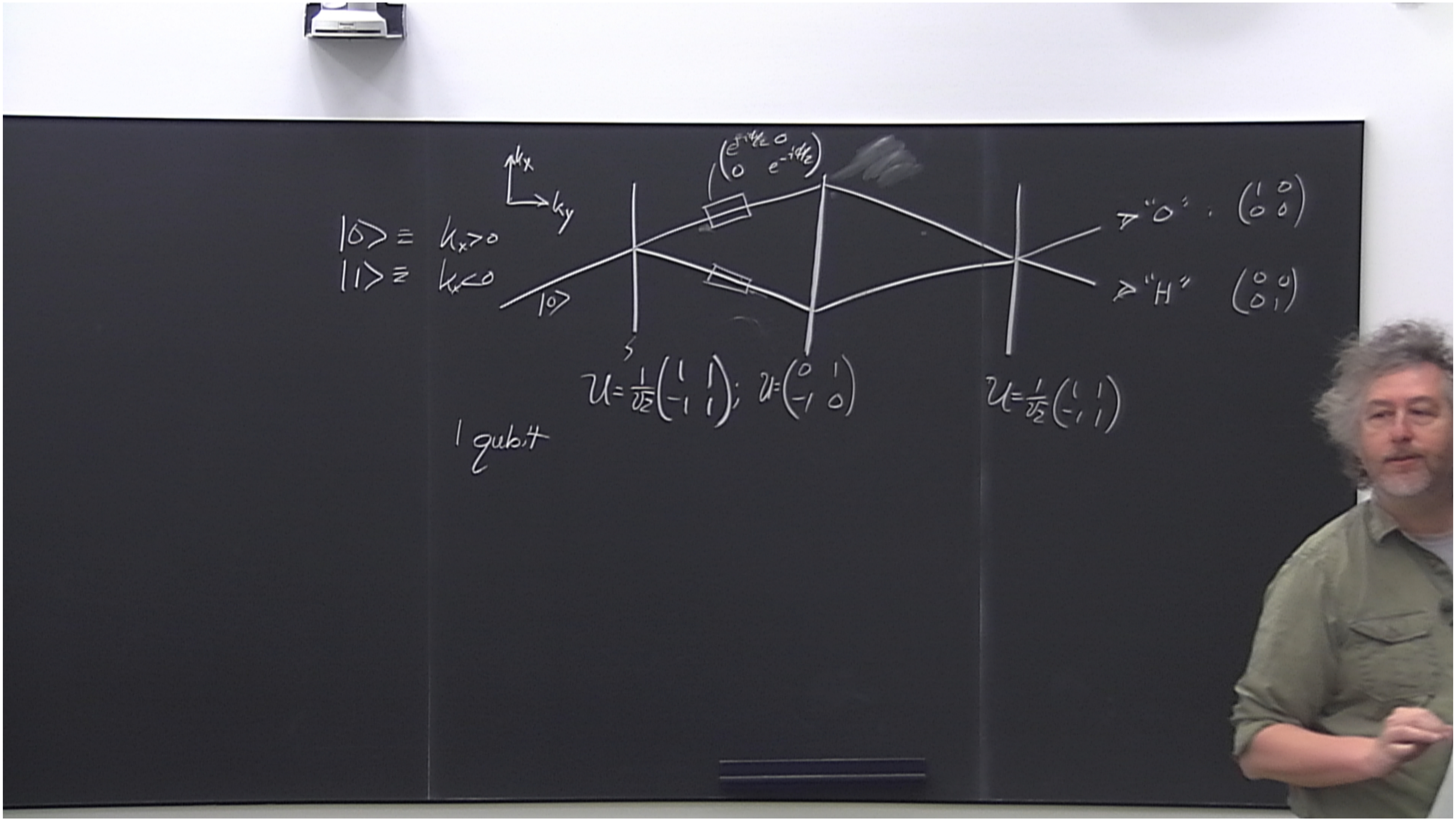

|0> ≡ kx>0
|1> ≡ kx<0
kx
ky
|0>
U = 1/√2 (1 1 ; -1 1)



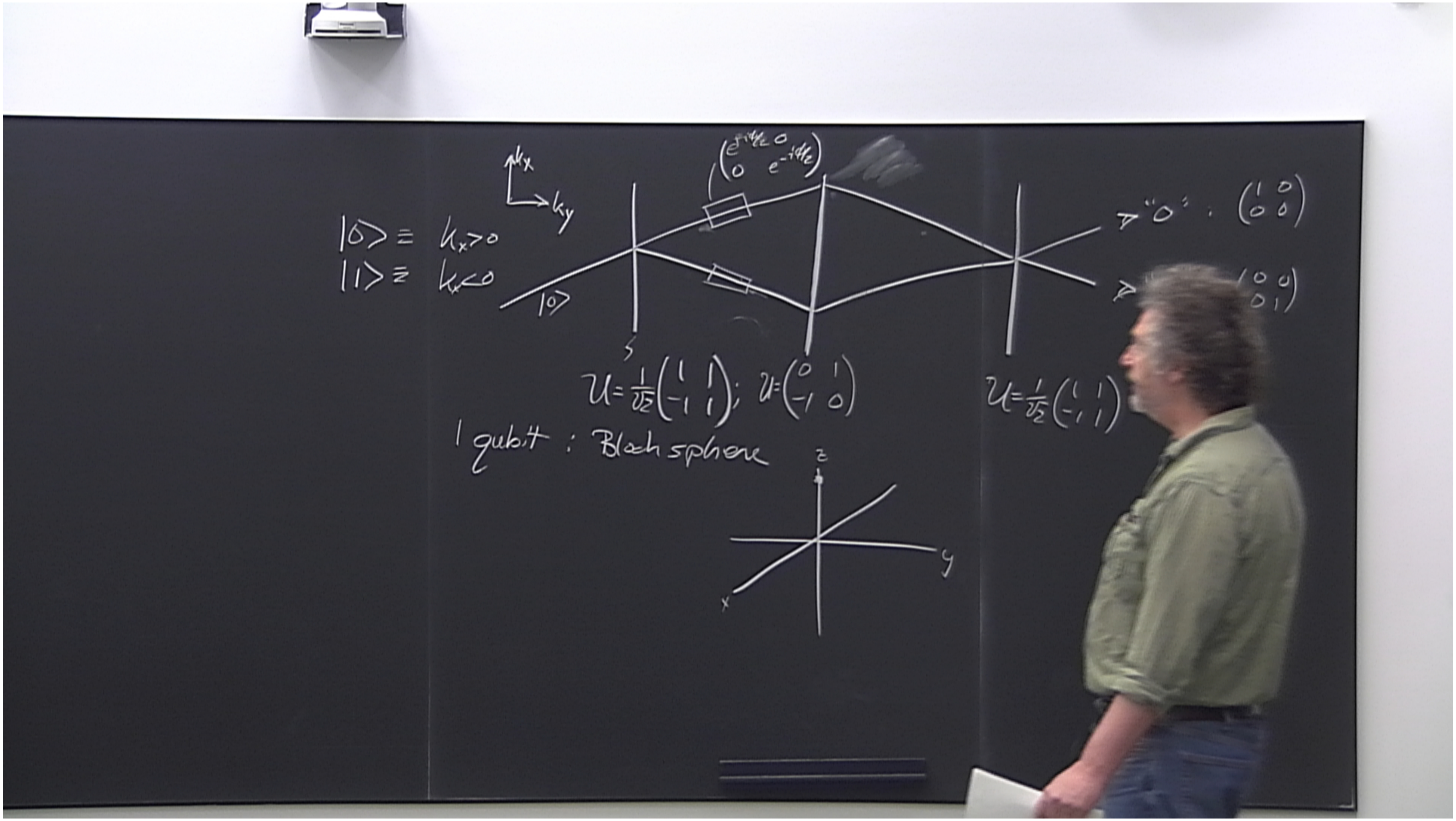
|0> ≡ k_x>0
|1> ≡ k_x<0
k_x
k_y
|0>
U = 1/√2 (1 1 / -1 1)
(e^{iφ/2} 0 / 0 e^{-iφ/2})

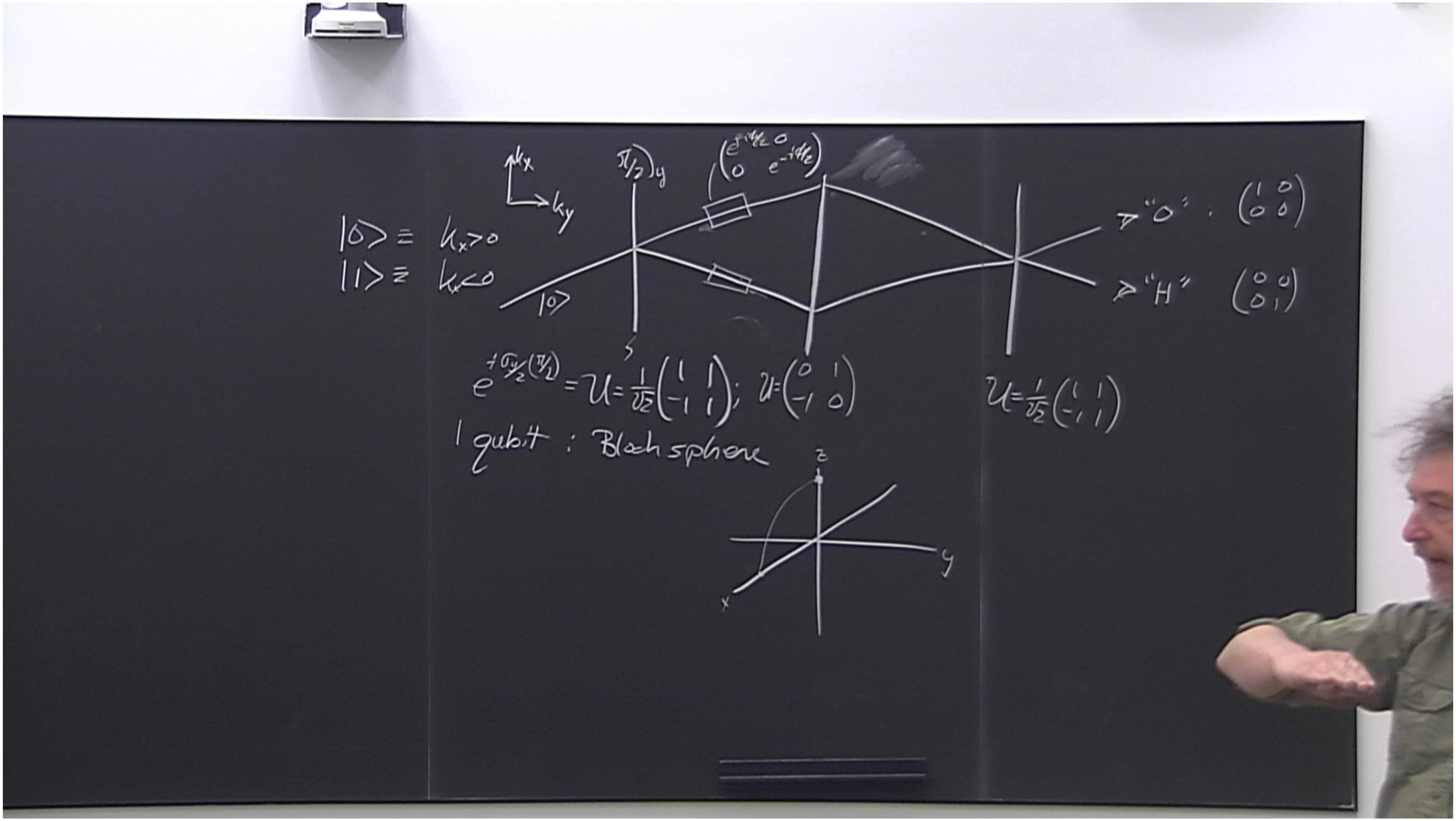

|0> ≡ kx>0
|1> ≡ kx<0
kx
ky
|0>
U = 1/√2 (1 1; -1 1); U = (0 1; -1 0)


|0> ≡ kx>0
|1> ≡ kx<0
kx
ky
|0>
"0"
"H"
U = 1/√2 (1 1; -1 1);
U = (0 1; -1 0)
U = 1/√2 (1 1; -1 1)

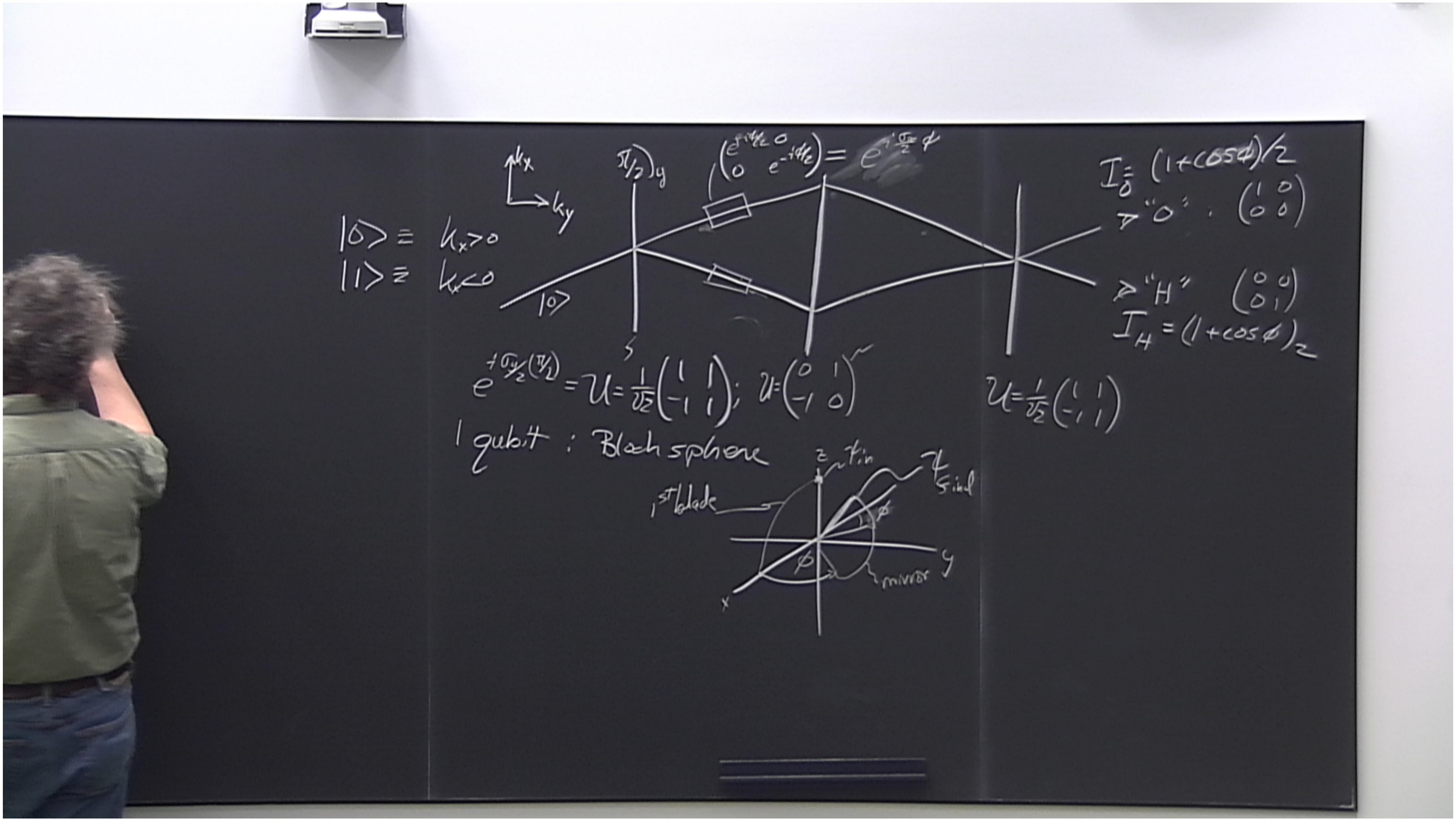


|0> ≡ k_x>0
|1> ≡ k_x<0
k_x
k_y
|0>
"0" (1 0 / 0 0)
"H" (0 0 / 0 1)
U = 1/√2 (1 1 / -1 1); U = (0 1 / -1 0)
U = 1/√2 (1 1 / -1 1)
1 qubit

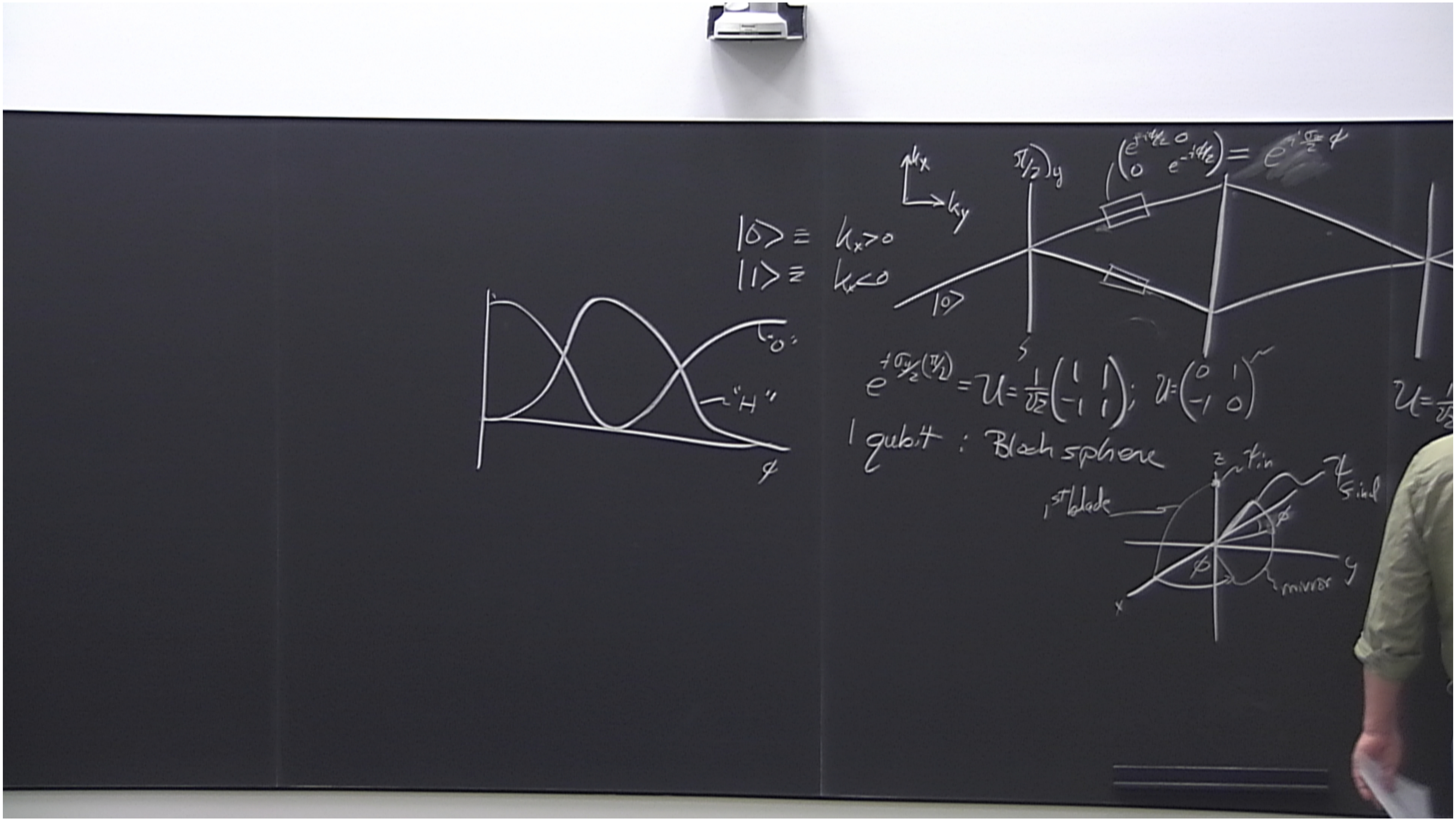

|0> ≡ kx>0
|1> ≡ kx<0
|0>
kx
ky
1 qubit : Bloch sphere
z
y
x

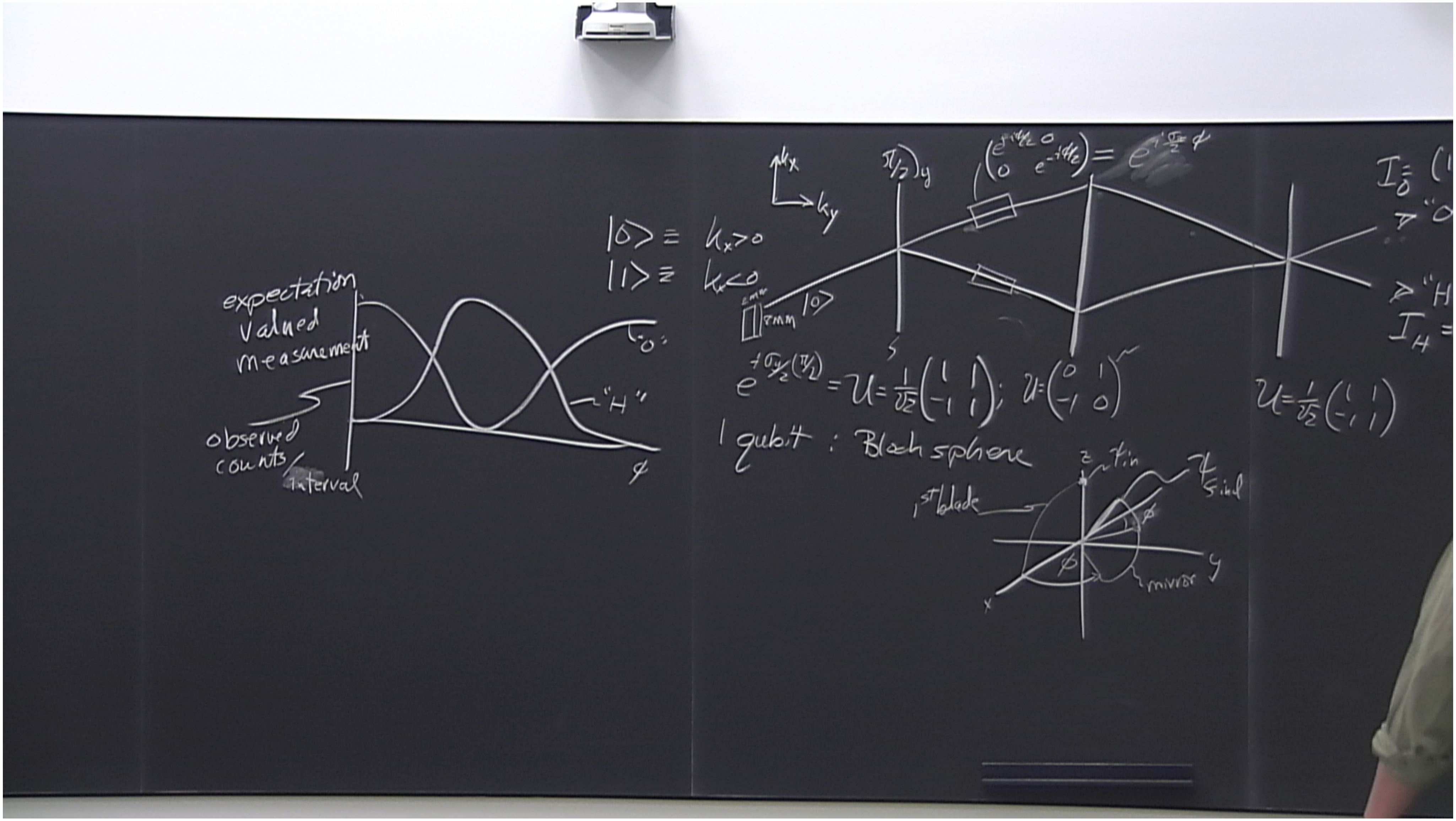


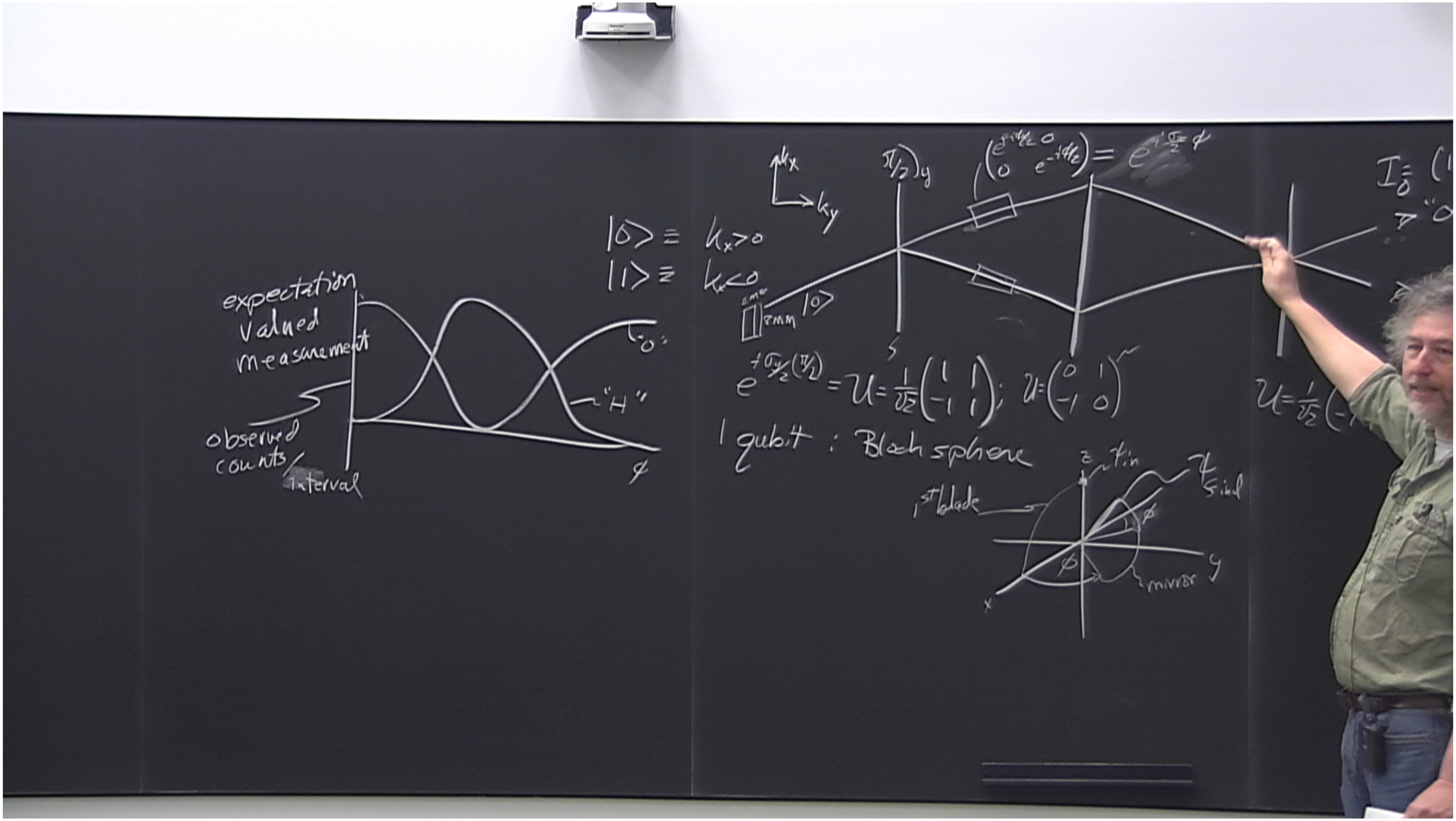
|0> ≡ kx>0
|1> ≡ kx<0
|0>
π/2)y
"0"
"H"
U = 1/√2 (1 1; -1 1)
1 qubit : Bloch sphere
z
y
x

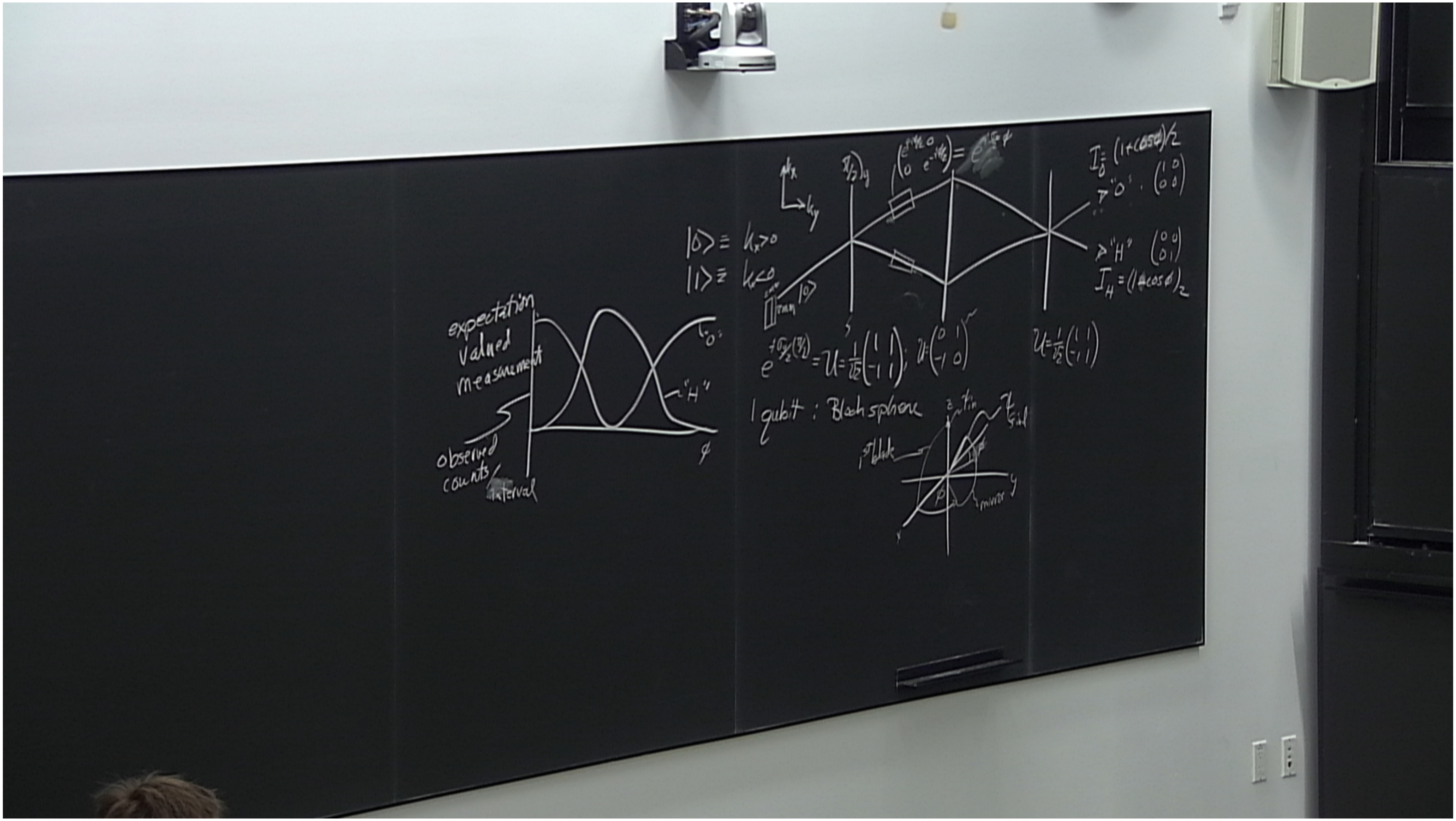


|0> ≡ k_x>0
|1> ≡ k_x<0
|0>
π/2)_y
I_0 = (1+cosφ)/2
"0"
"H"
I_H = (1+cosφ)/2
1 qubit : Bloch sphere
1st blade
mirror

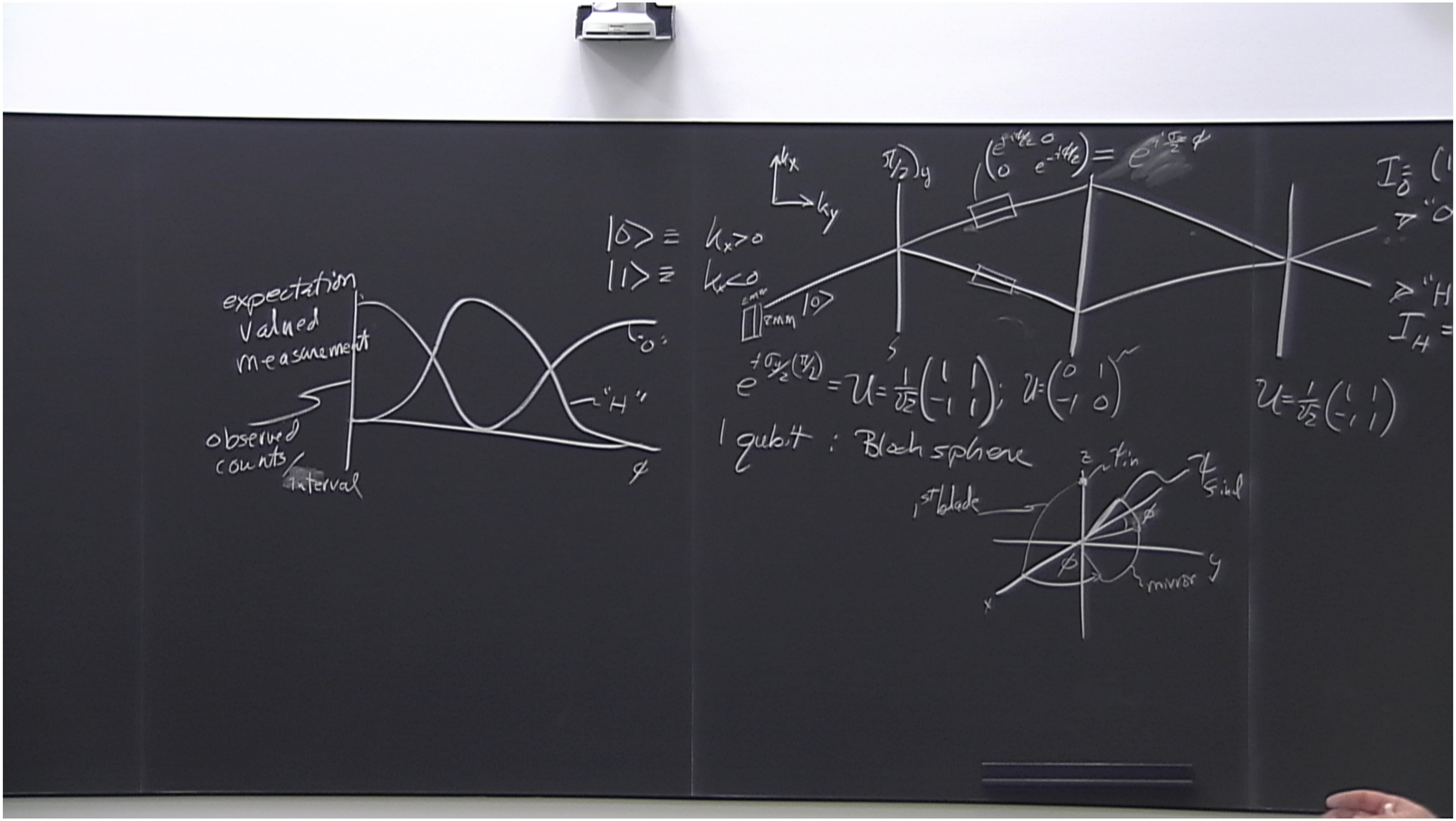

|0> ≡ kx>0
|1> ≡ kx<0
1 qubit : Bloch sphere
1st blade
mirror
φ
"H"

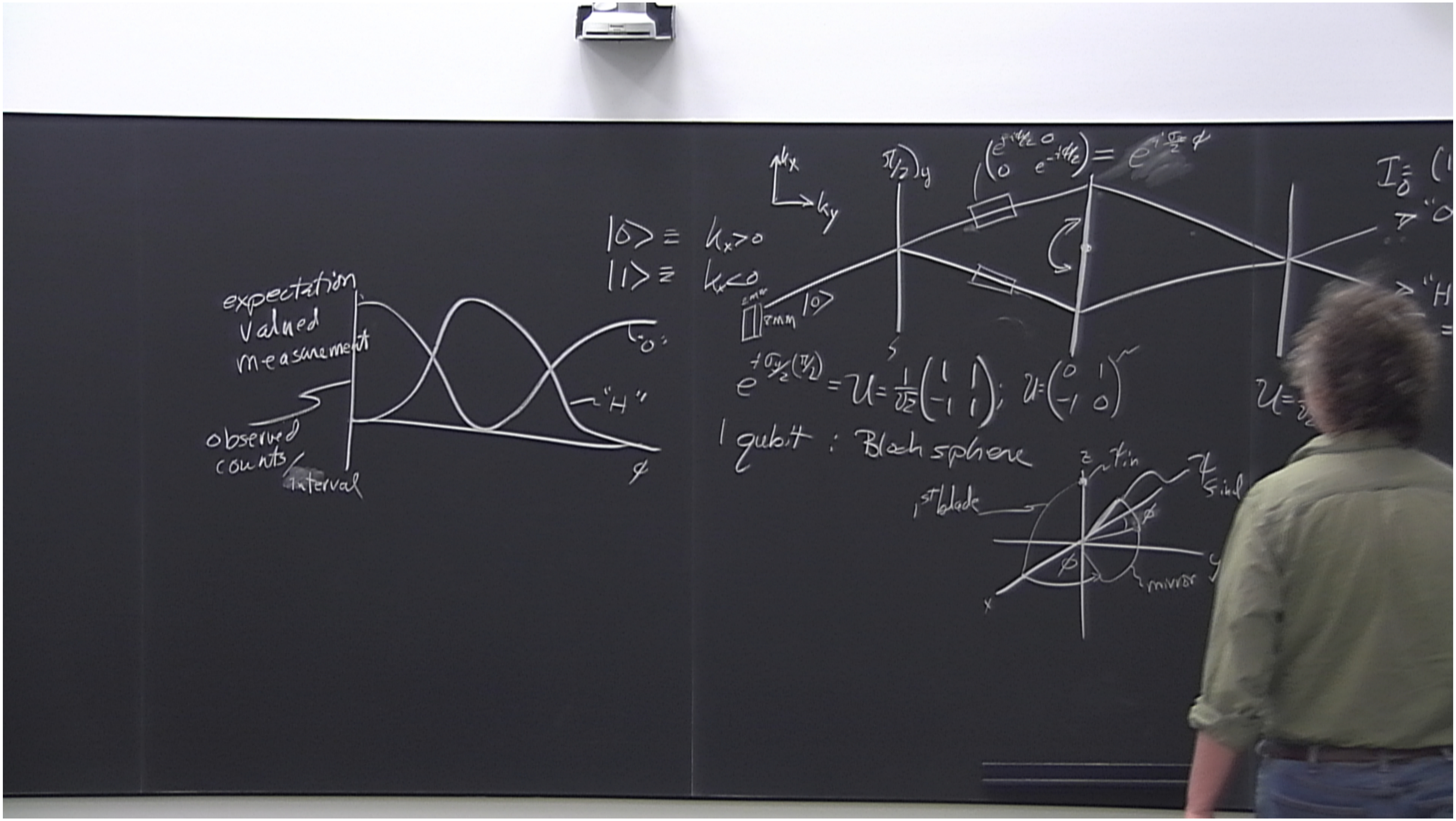

expectation valued measurement
observed counts / interval
"H"
|0> ≡ kx>0
|1> ≡ kx<0
kx
ky
1 qubit : Bloch sphere
1st blade
mirror
z
y
x


expectation valued measurement
observed counts/interval
"0"
"H"
|0> ≡ kx>0
|1> ≡ kx<0
1 qubit : Bloch sphere
1st blade
mirror

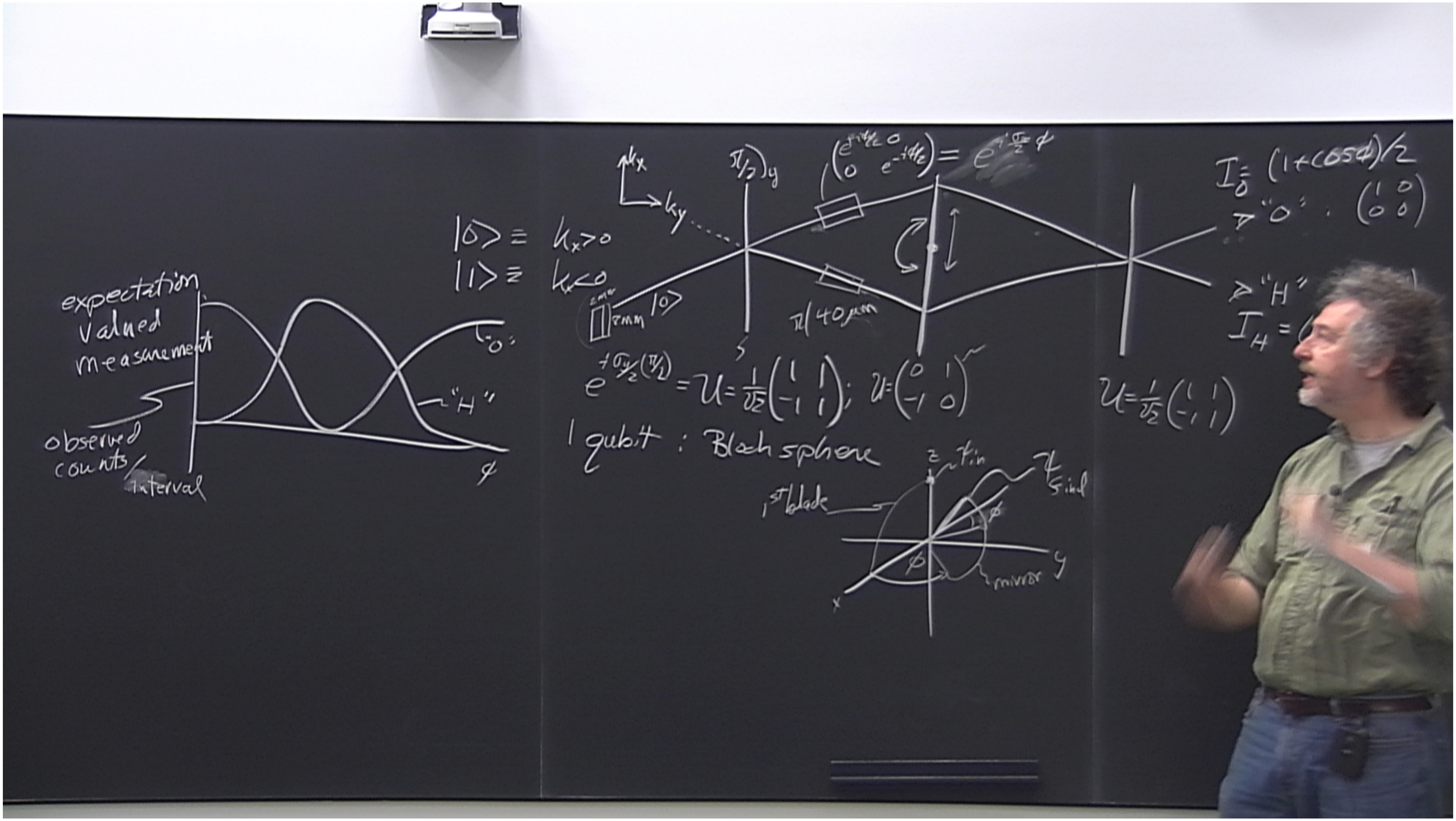


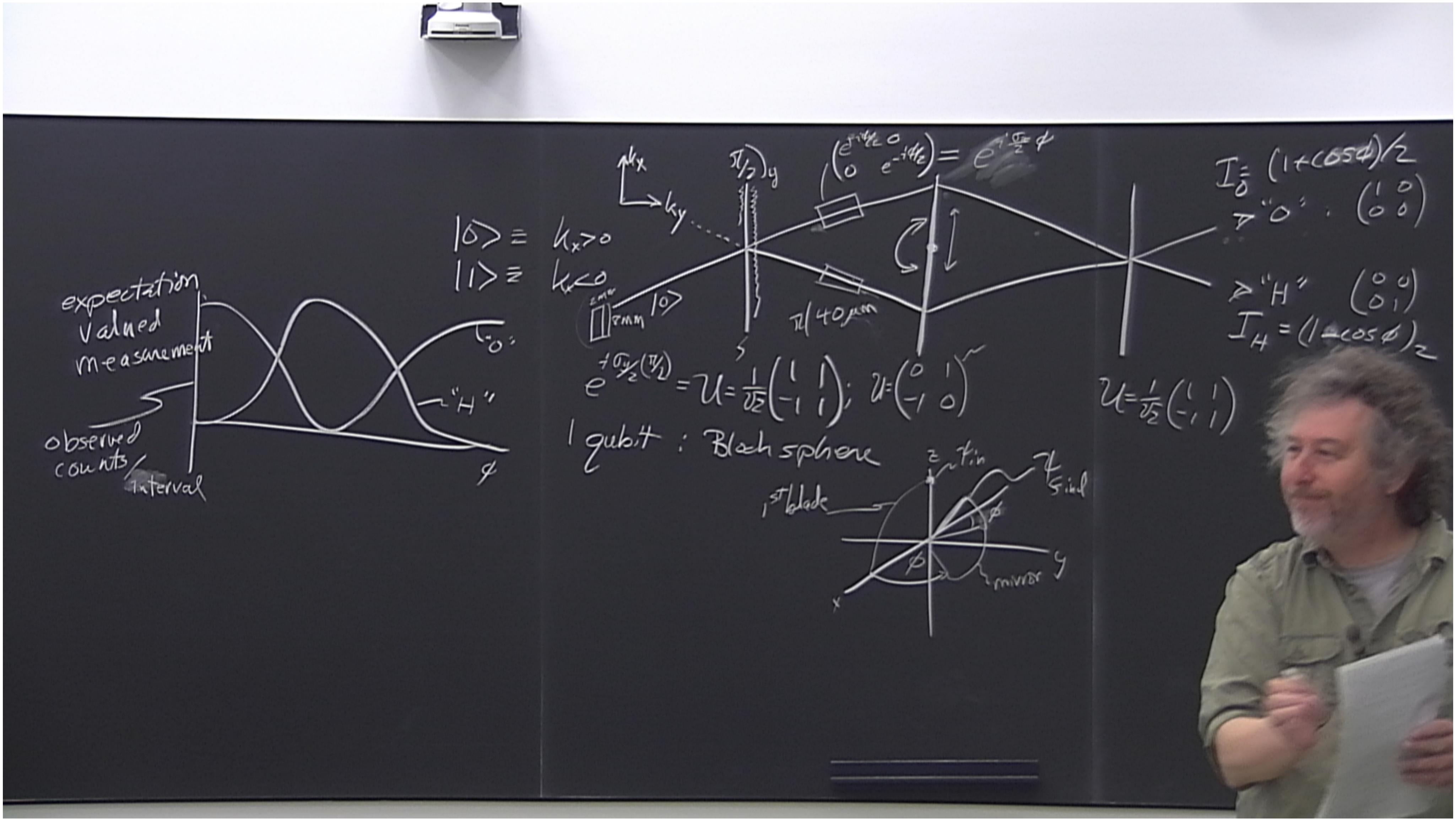
expectation valued measurement
observed counts/interval
1 qubit : Bloch sphere

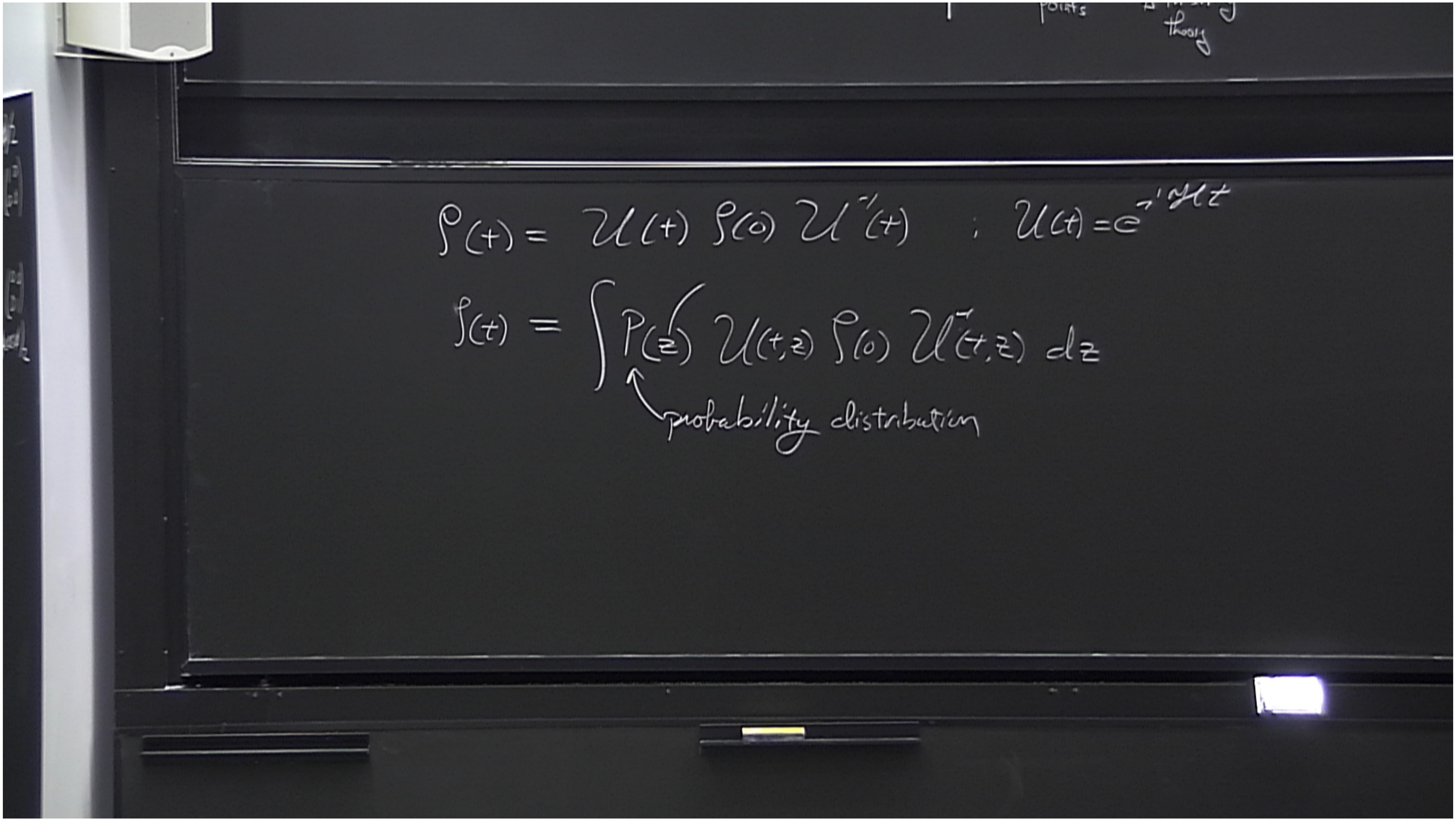

expectation valued measurement
observed counts
interval
"0"
"H"
|0> ≡ kx>0
|1> ≡ kx<0
1 qubit : Bloch sphere
1st blade
mirror

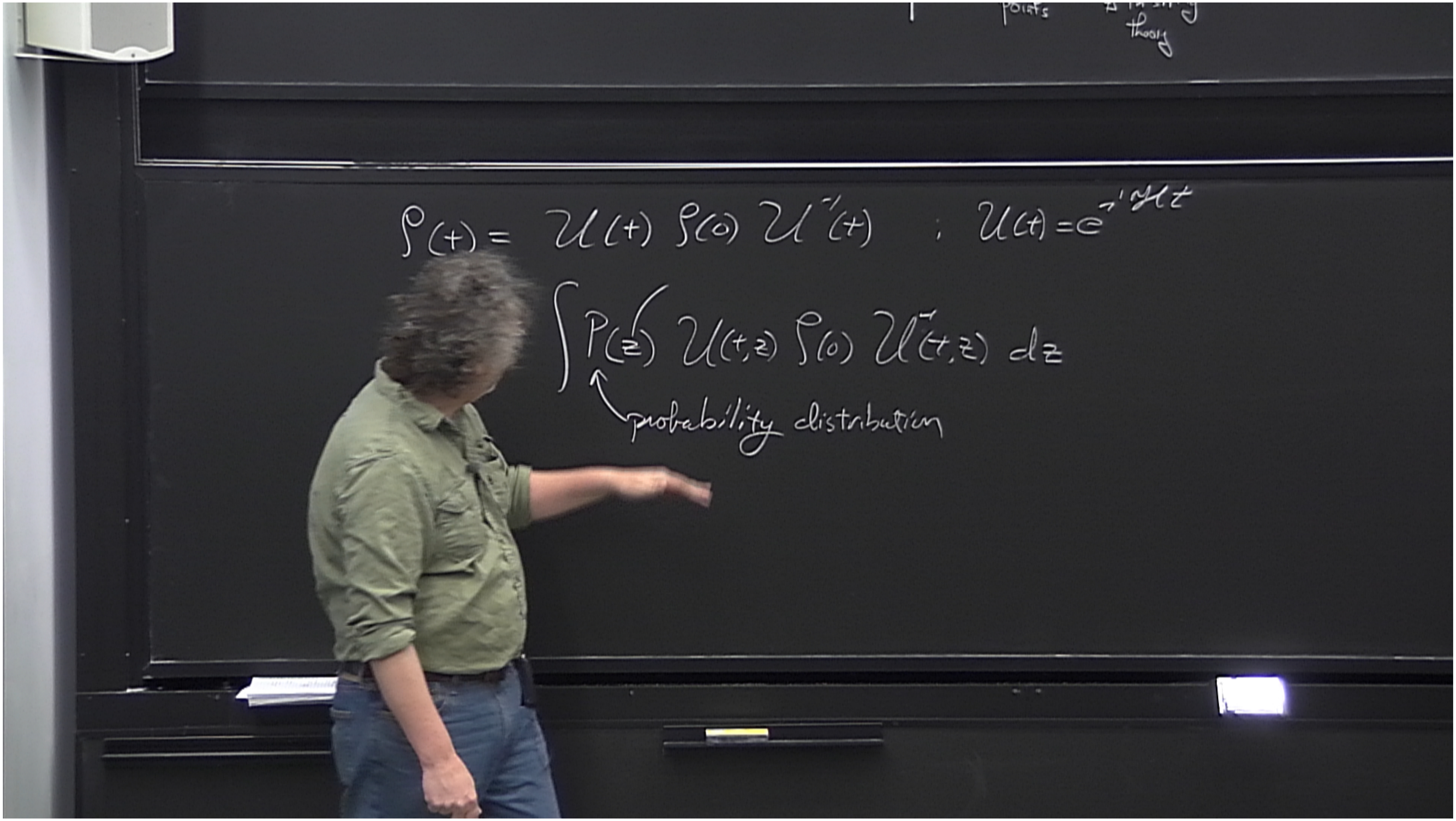


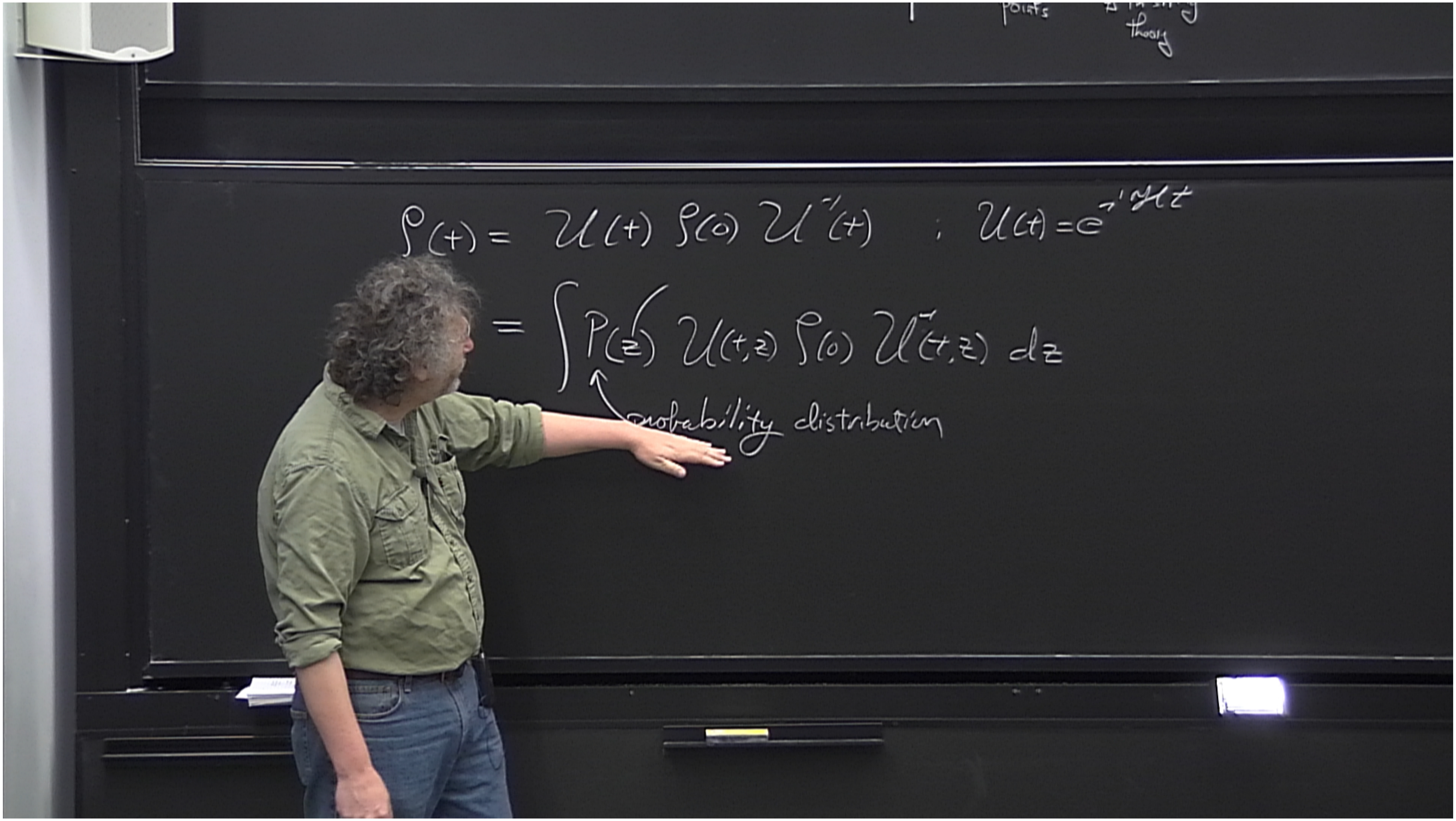
expectation valued measurement
observed counts/interval
"O"
"H"
|0> ≡ kx>0
|1> ≡ kx<0
|0>
1 qubit : Bloch sphere
1st blade
mirror

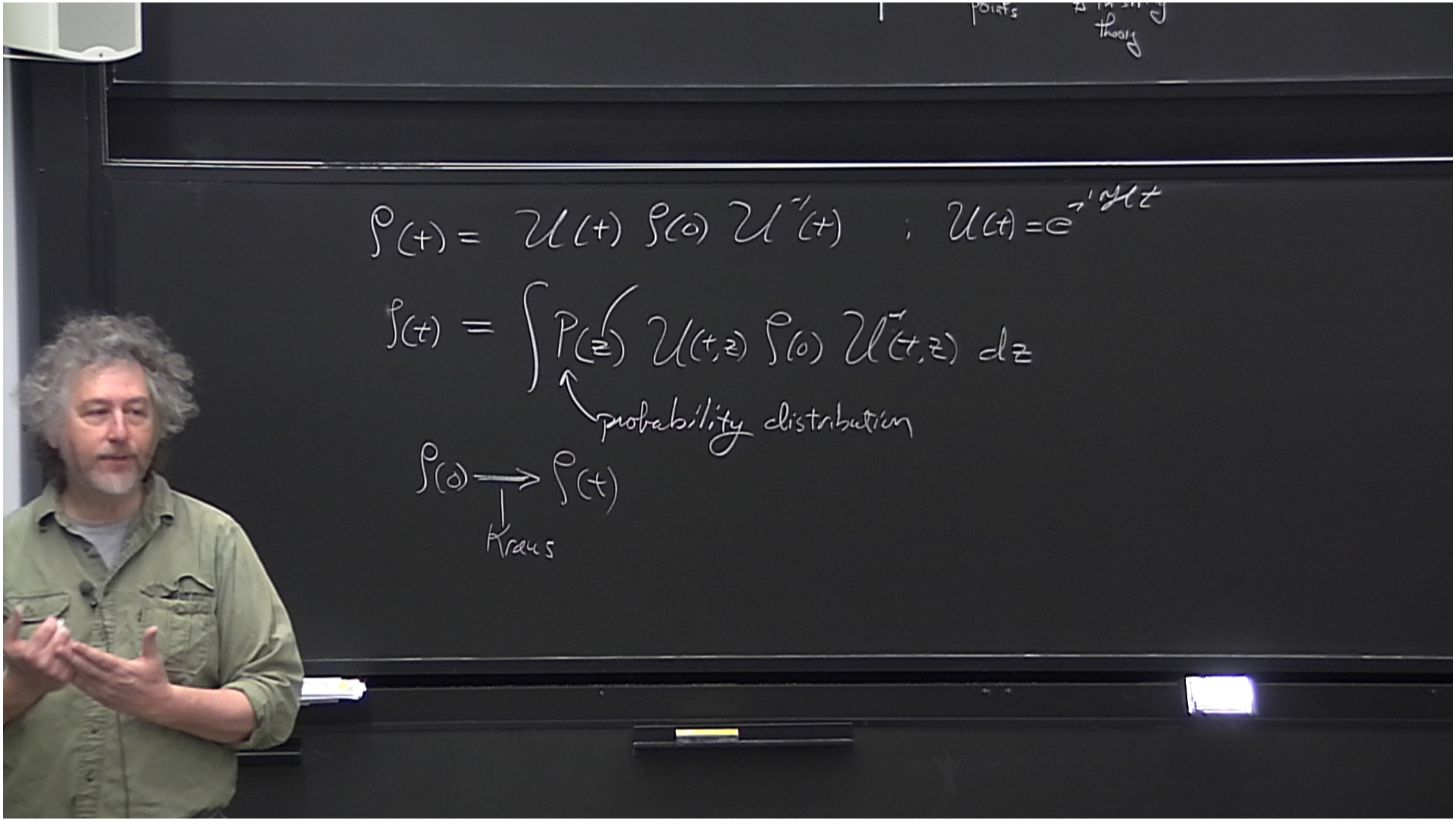

expectation valued measurement
observed counts/interval
"0"
"H"
ϕ
$|0\rangle \equiv k_x > 0$
$|1\rangle \equiv k_x < 0$
2mm
8mm
$|0\rangle$
$\pi/2)_y$
$\pi/40\mu m$
k_x
k_y
$\begin{pmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{pmatrix} = e^{i\frac{\sigma_z}{2}\phi}$
$e^{i\frac{\sigma_y}{2}(\frac{\pi}{2})} = U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$; $U = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$
$I_0 = (1 + \cos\phi)/2$
"0" . $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
"H"
I_H =
1 qubit : Bloch sphere
z
ψ_{in}
ψ_{final}
1st blade
mirror
x
y
ϕ

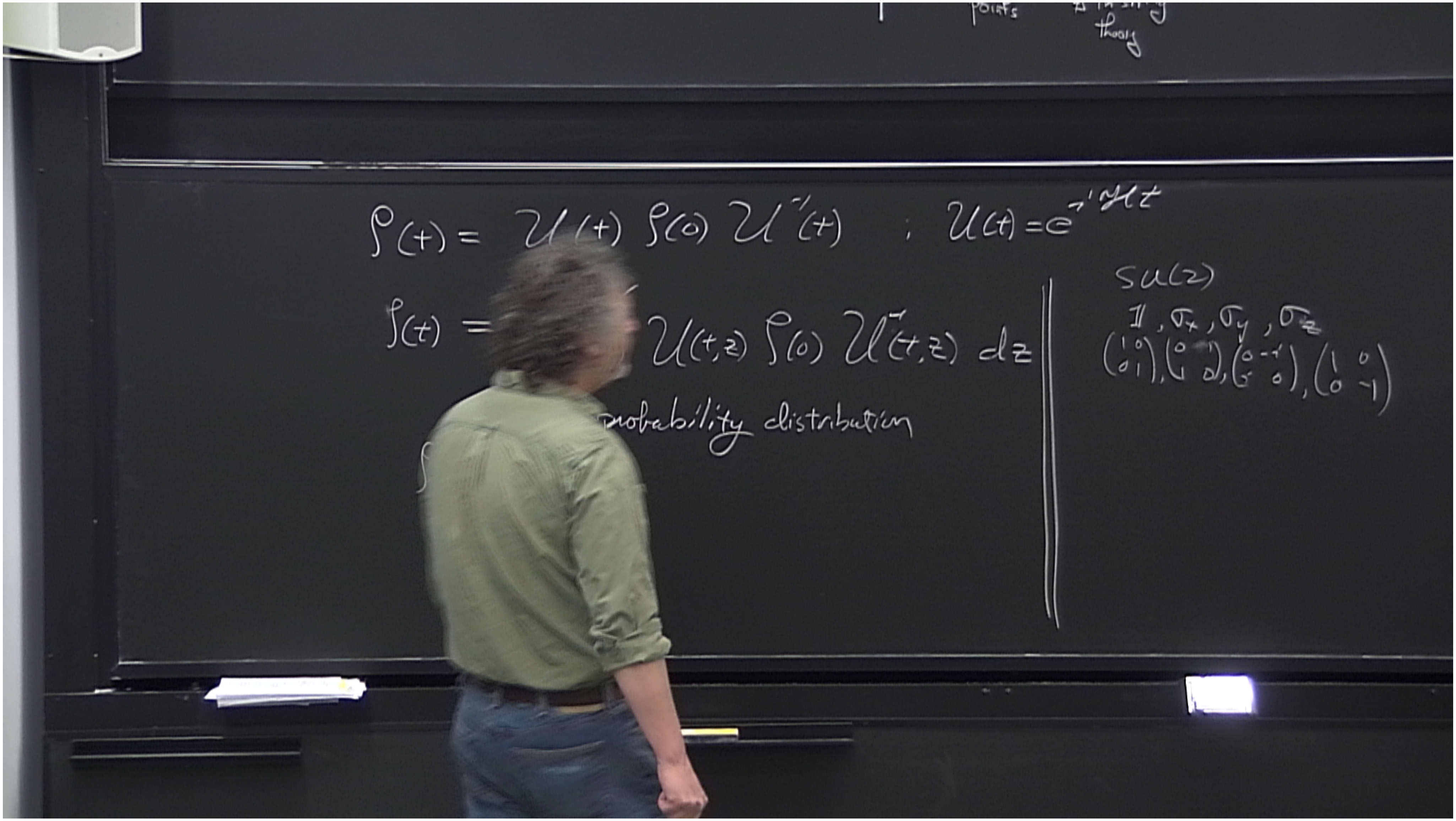

expectation valued measurement
observed counts/interval
|0> ≡ kx>0
|1> ≡ kx<0
1 qubit : Bloch sphere
1st blade
mirror
U = 1/√2 (1 1; -1 1)
I0 = (1+cosφ)/2
IH = (1-cosφ)/2
π/40μm
8mm


$\rho(t) = \mathcal{U}(t)\,\rho(0)\,\mathcal{U}^{-1}(t)$; $\mathcal{U}(t) = e^{-i\mathcal{H}t}$
$\rho(t) = \int P(z)\,\mathcal{U}(t,z)\,\rho(0)\,\mathcal{U}^{-1}(t,z)\,dz$
probability distribution

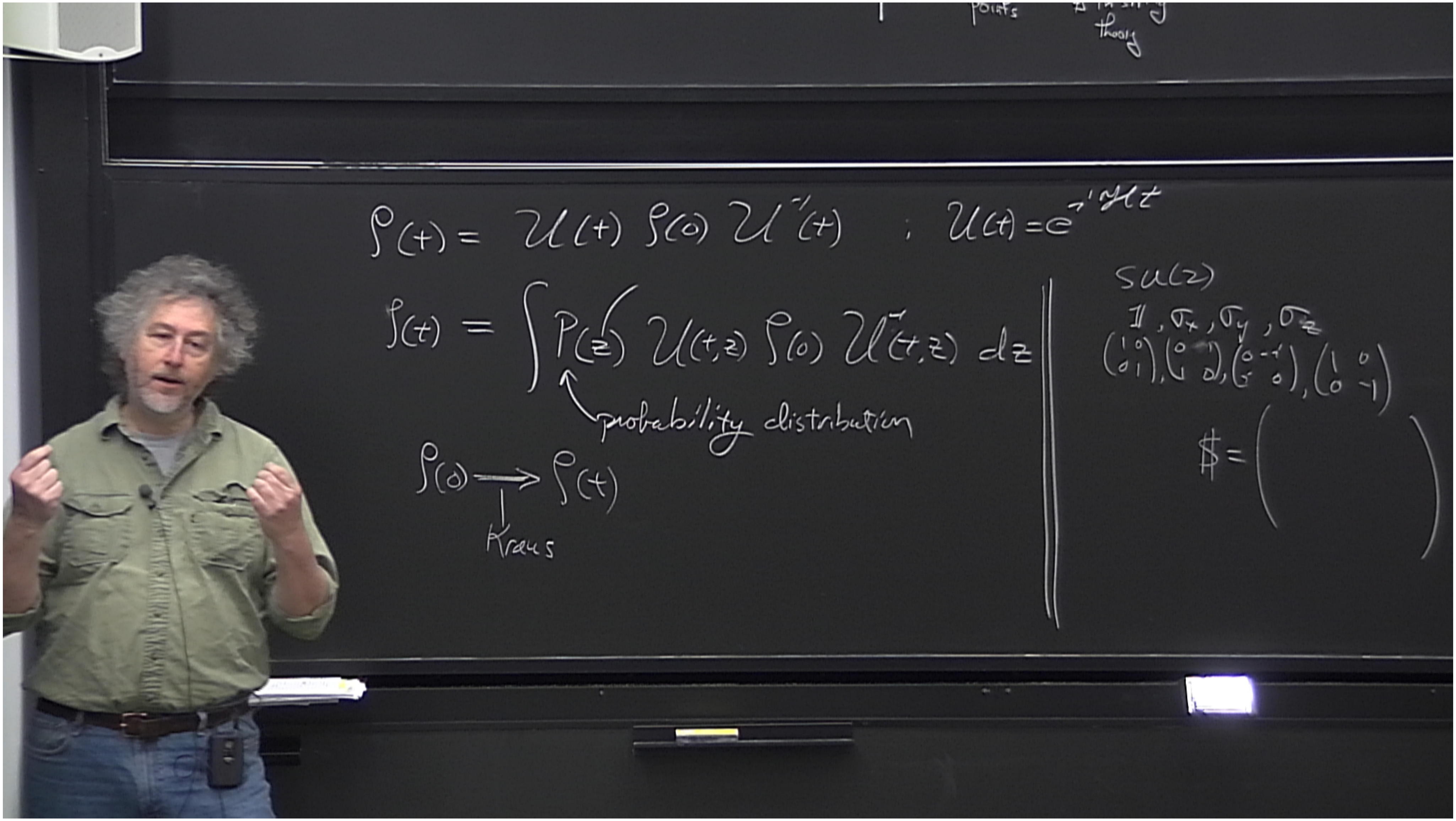

points
theory
$\rho(t) = \mathcal{U}(t)\,\rho(0)\,\mathcal{U}^{-1}(t)$; $\mathcal{U}(t) = e^{-i\mathcal{H}t}$
$\int P(z)\,\mathcal{U}(t,z)\,\rho(0)\,\mathcal{U}^{-1}(t,z)\,dz$
probability distribution

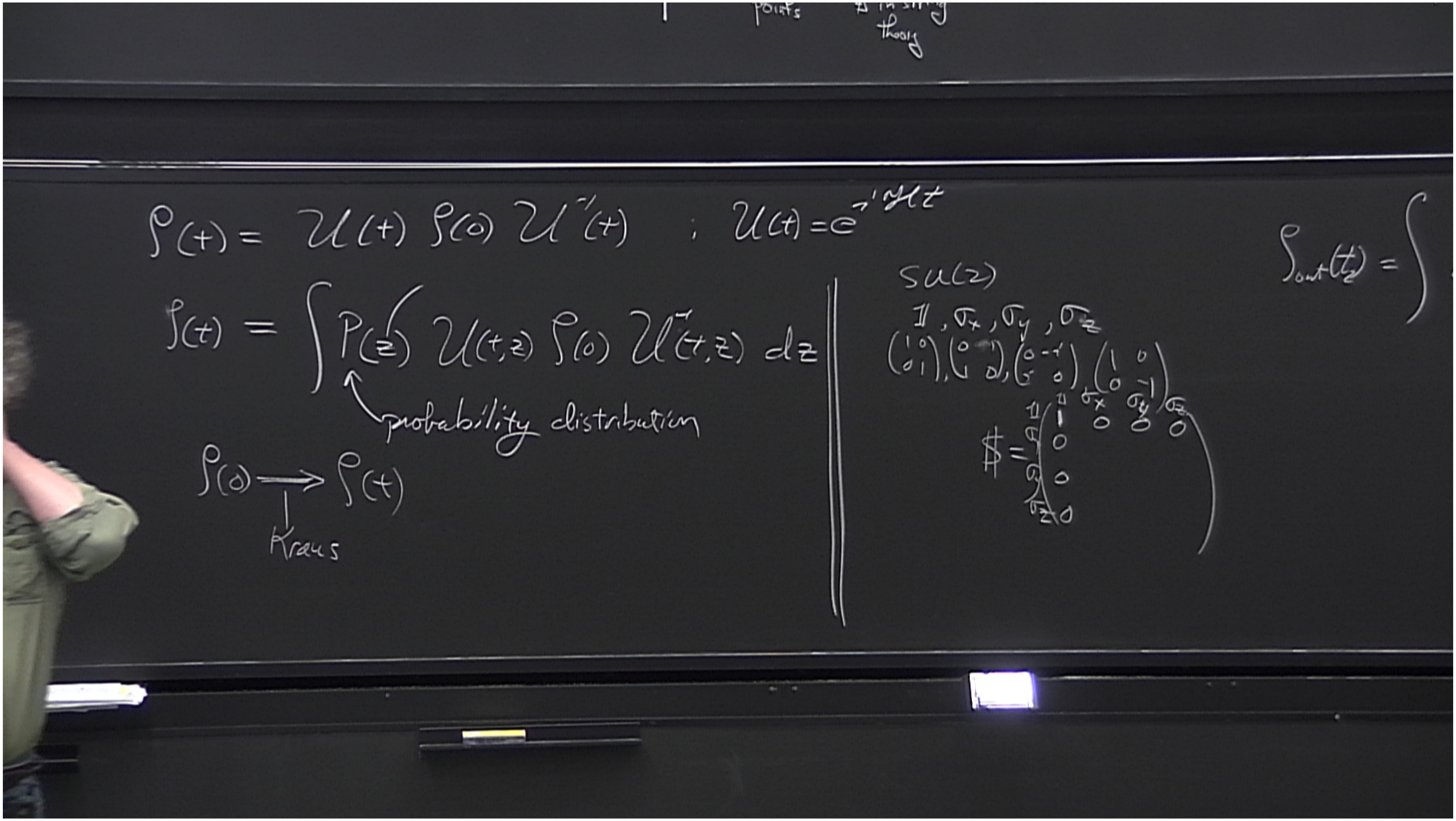

ρ(t) = U(t) ρ(0) U⁻¹(t) ; U(t) = e^{-iHt}
= ∫ P(z) U(t,z) ρ(0) U⁻¹(t,z) dz
probability distribution

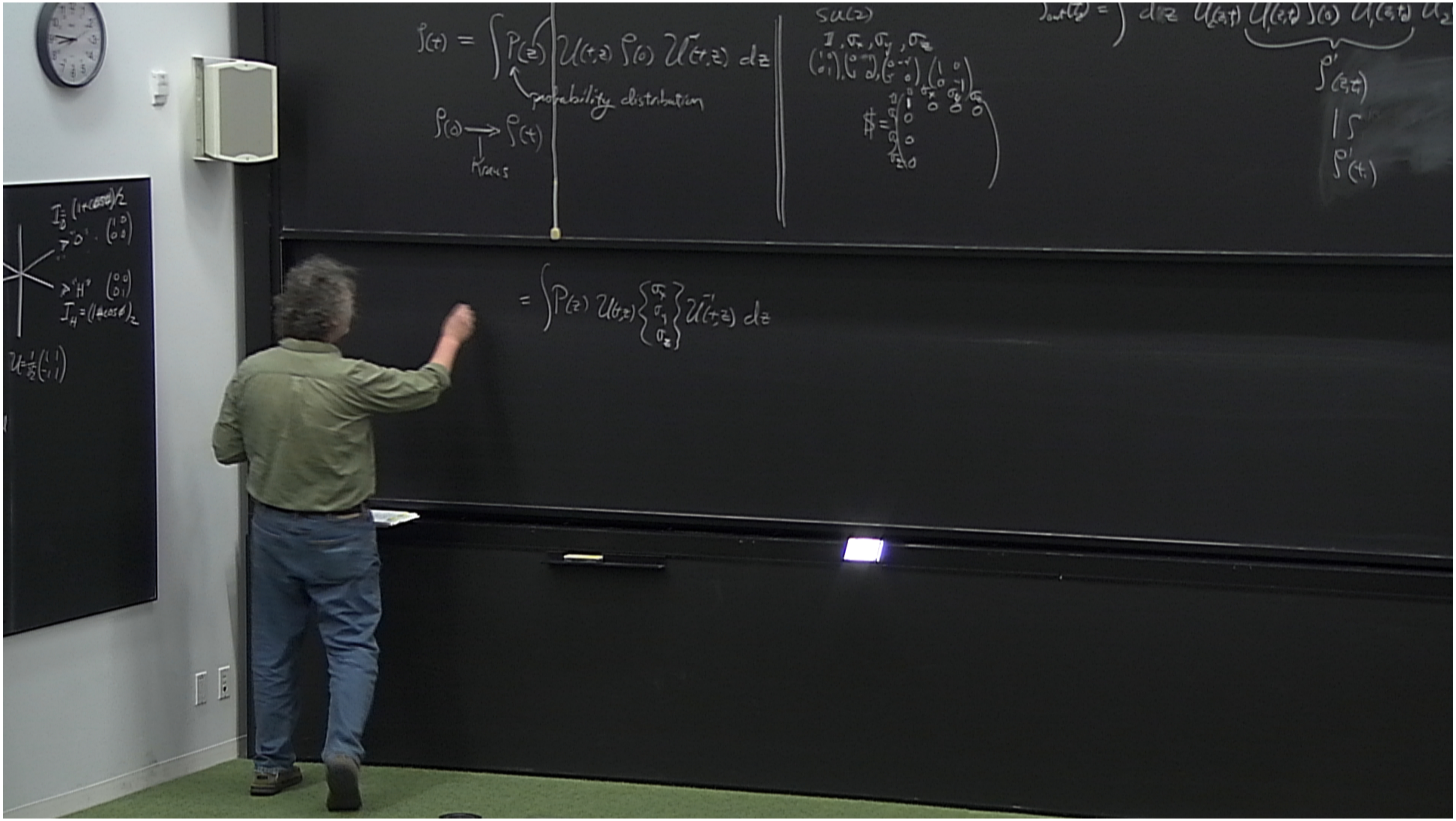

points
theory
$\rho(t) = \mathcal{U}(t)\,\rho(0)\,\mathcal{U}^{-1}(t) \quad ; \quad \mathcal{U}(t) = e^{-i\mathcal{H}t}$
$\rho(t) = \int P(z)\,\mathcal{U}(t,z)\,\rho(0)\,\mathcal{U}^{-1}(t,z)\,dz$
probability distribution
$\rho(0) \longrightarrow \rho(t)$
Kraus

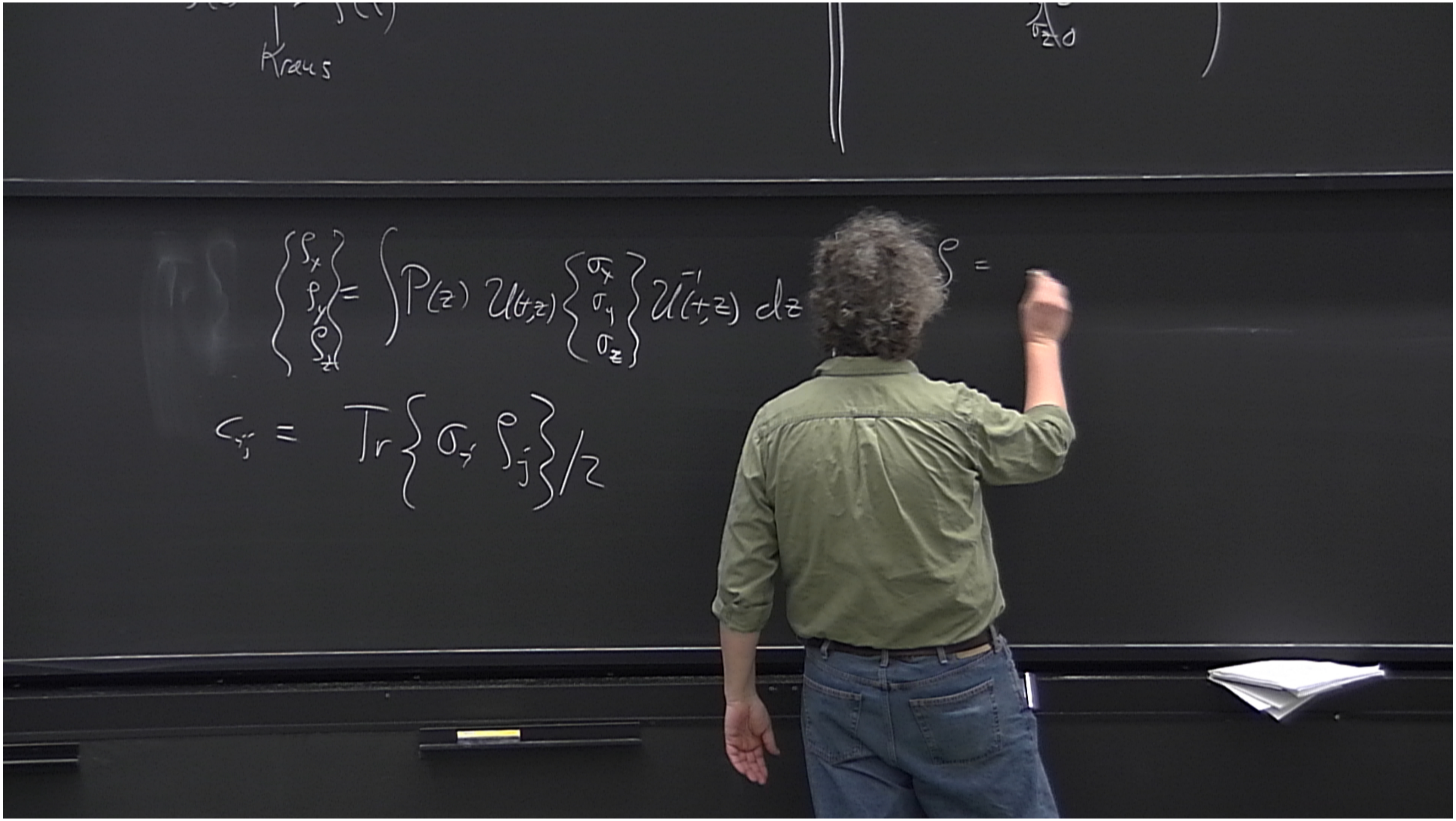


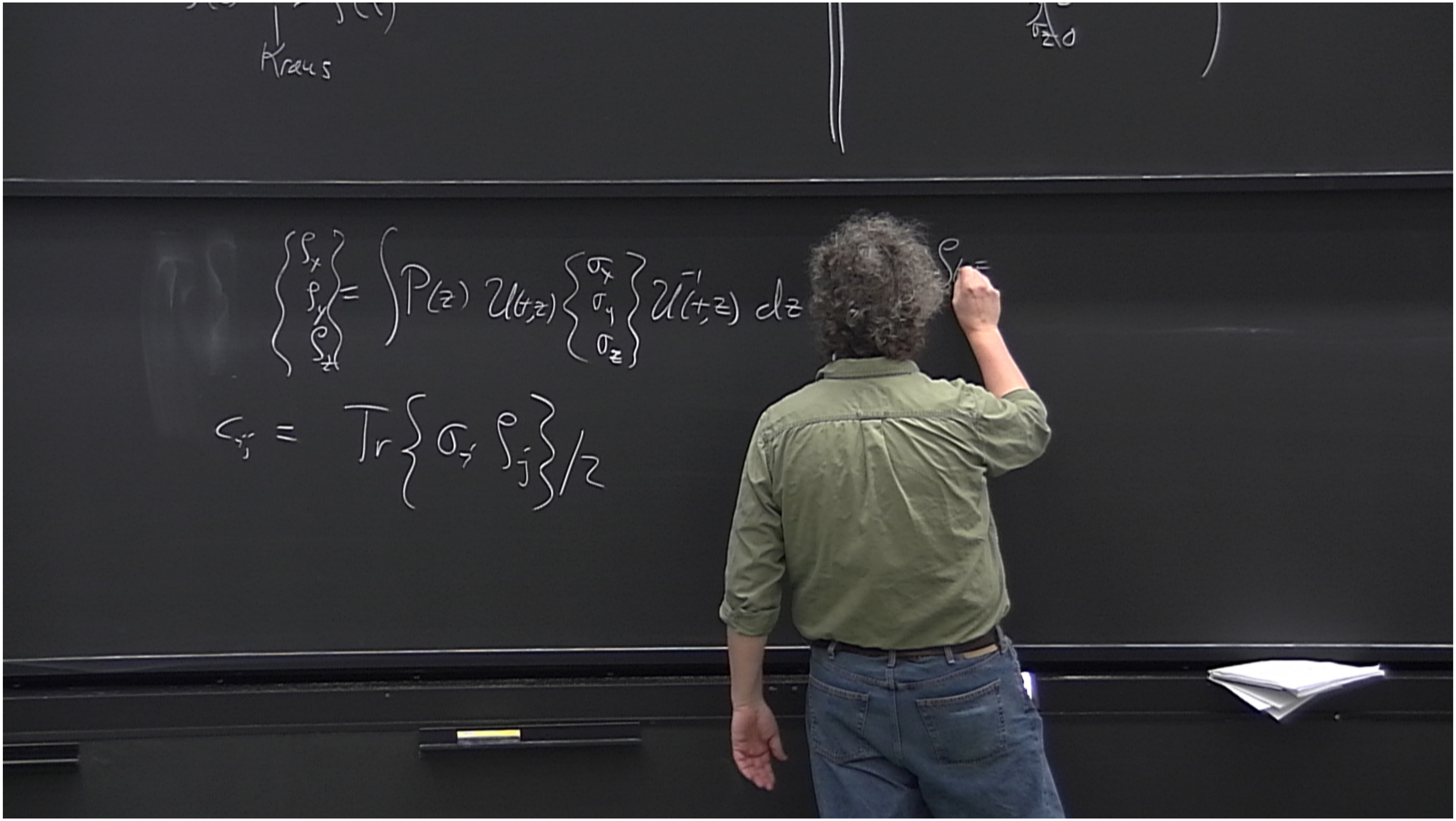
ρ(t) = U(t) ρ(0) U⁻¹(t) ; U(t) = e^{-iHt}
ρ(t) = U(t,z) ρ(0) U⁻¹(t,z) dz
probability distribution
SU(2)
𝟙, σx, σy, σz

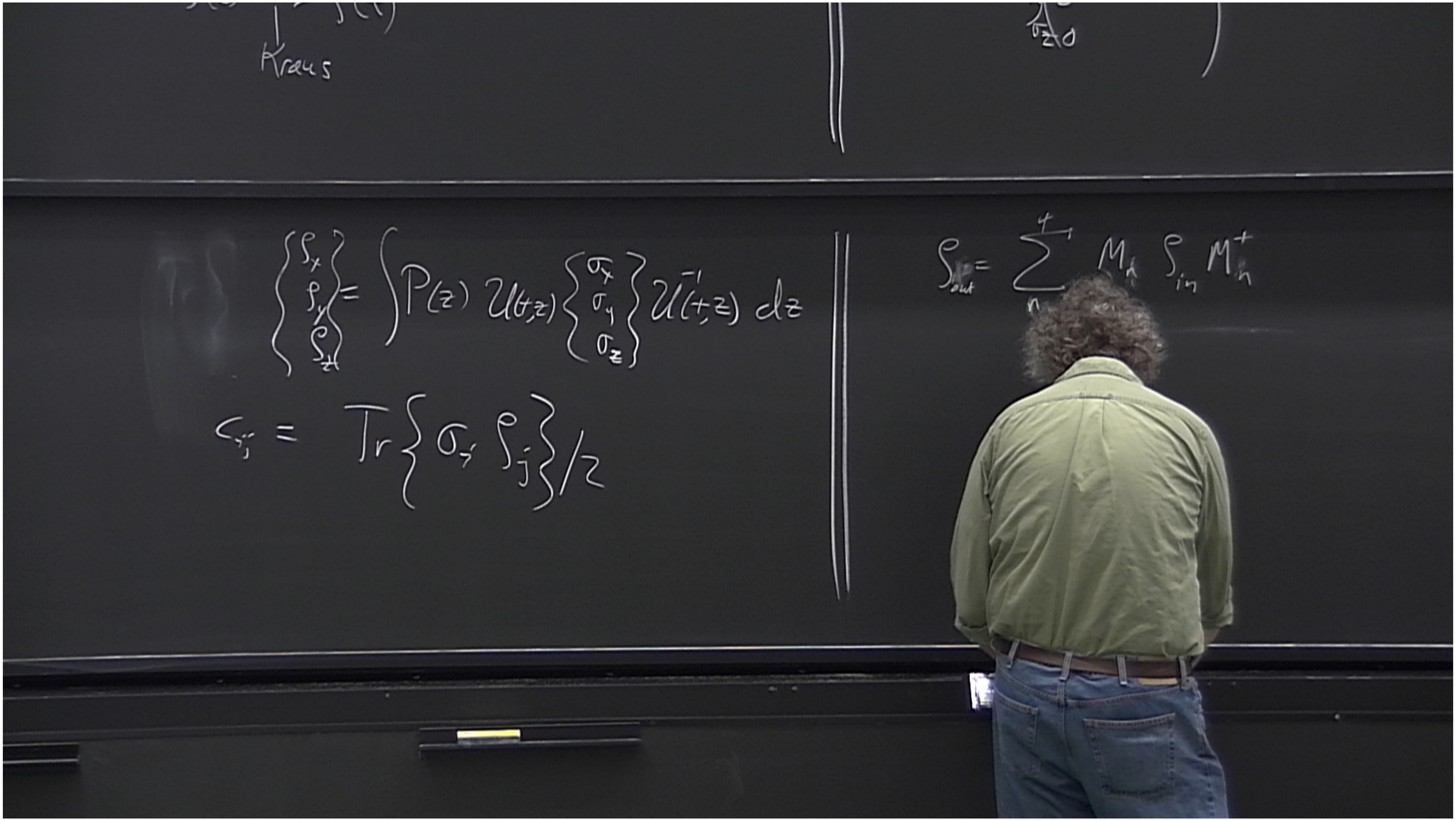

$\rho(t) = \mathcal{U}(t)\,\rho(0)\,\mathcal{U}^{-1}(t)$; $\mathcal{U}(t) = e^{-i\mathcal{H}t}$
$\rho(t) = \int P(z)\,\mathcal{U}(t,z)\,\rho(0)\,\mathcal{U}^{-1}(t,z)\,dz$
probability distribution
$\rho(0) \longrightarrow \rho(t)$
Kraus
SU(2)
$\mathbb{1}, \sigma_x, \sigma_y, \sigma_z$
$\begin{pmatrix}1&0\\0&1\end{pmatrix}, \begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}$
$ = ( )

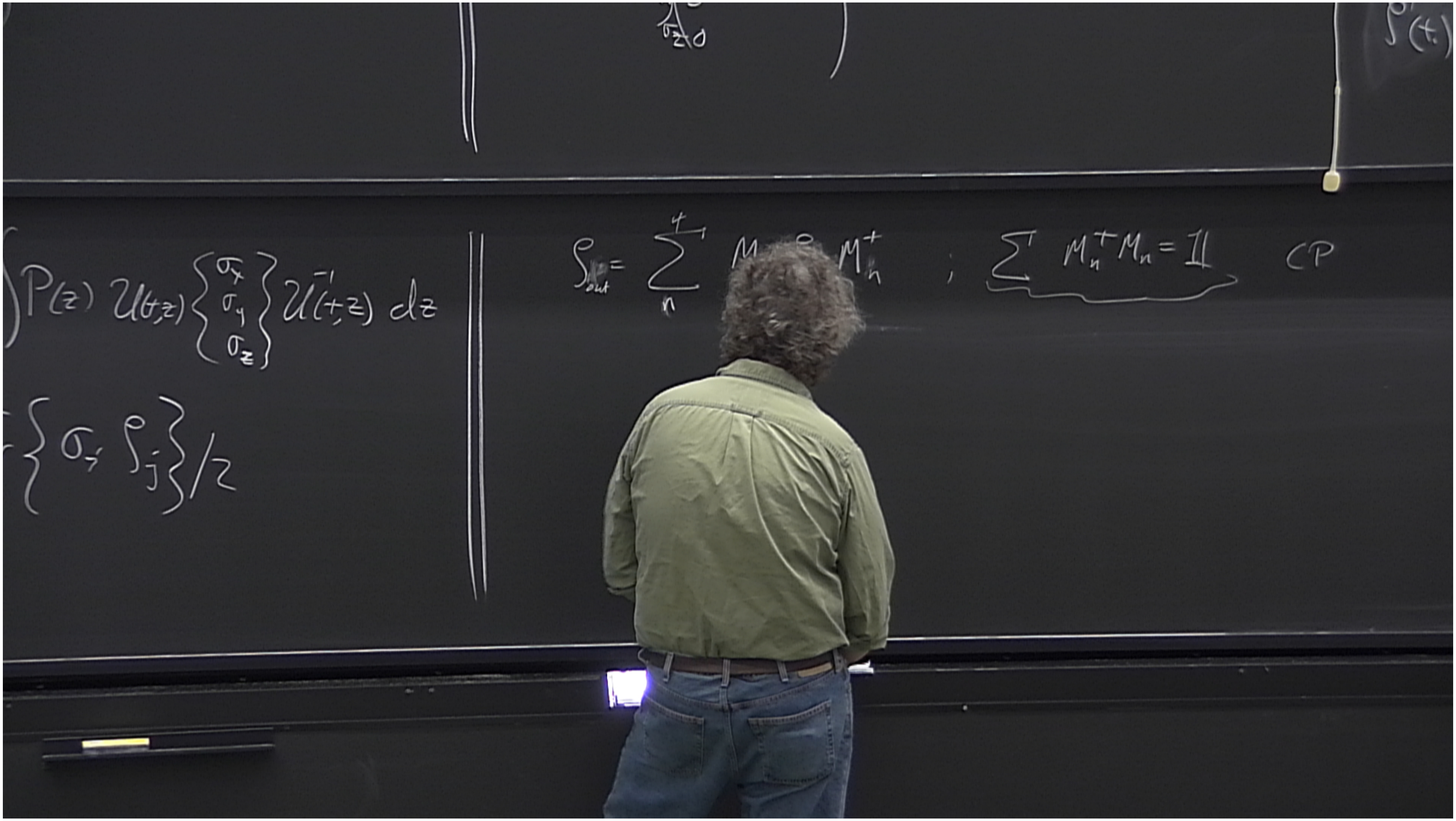

points
theory
ρ(t) = U(t) ρ(0) U⁻¹(t) ; U(t) = e^{-iHt}
ρ(t) = ∫ P(z) U(t,z) ρ(0) U⁻¹(t,z) dz
probability distribution
ρ(0) → ρ(t)
Kraus
SU(2)
𝟙, σx, σy, σz
ρout(t2) = ∫



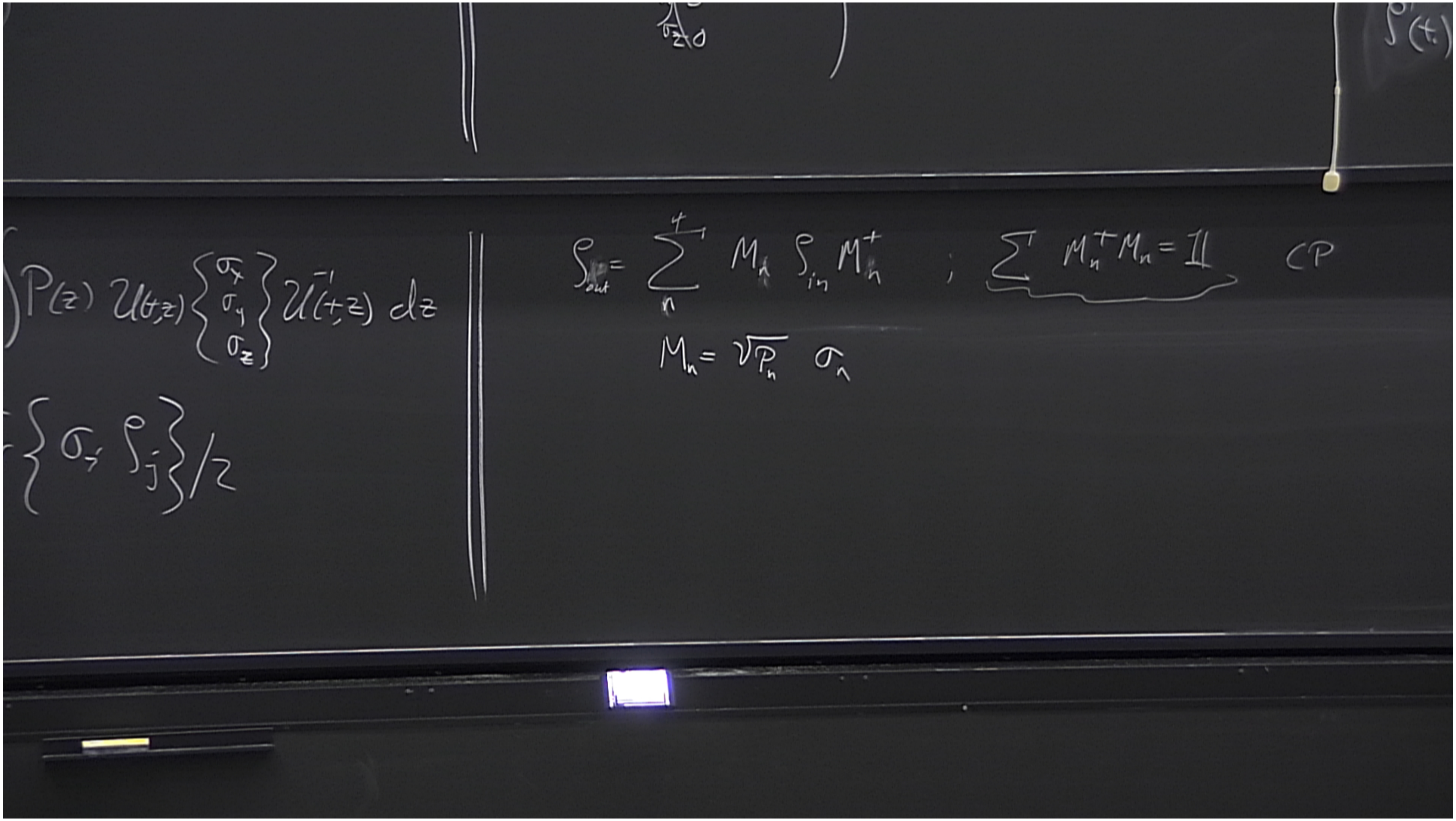

Kraus
σz≠0
{ρx, ρy, ρz} = ∫ P(z) U(t,z) {σx, σy, σz} U⁻¹(t,z) dz
ρ =
cij = Tr{σi ρj}/2

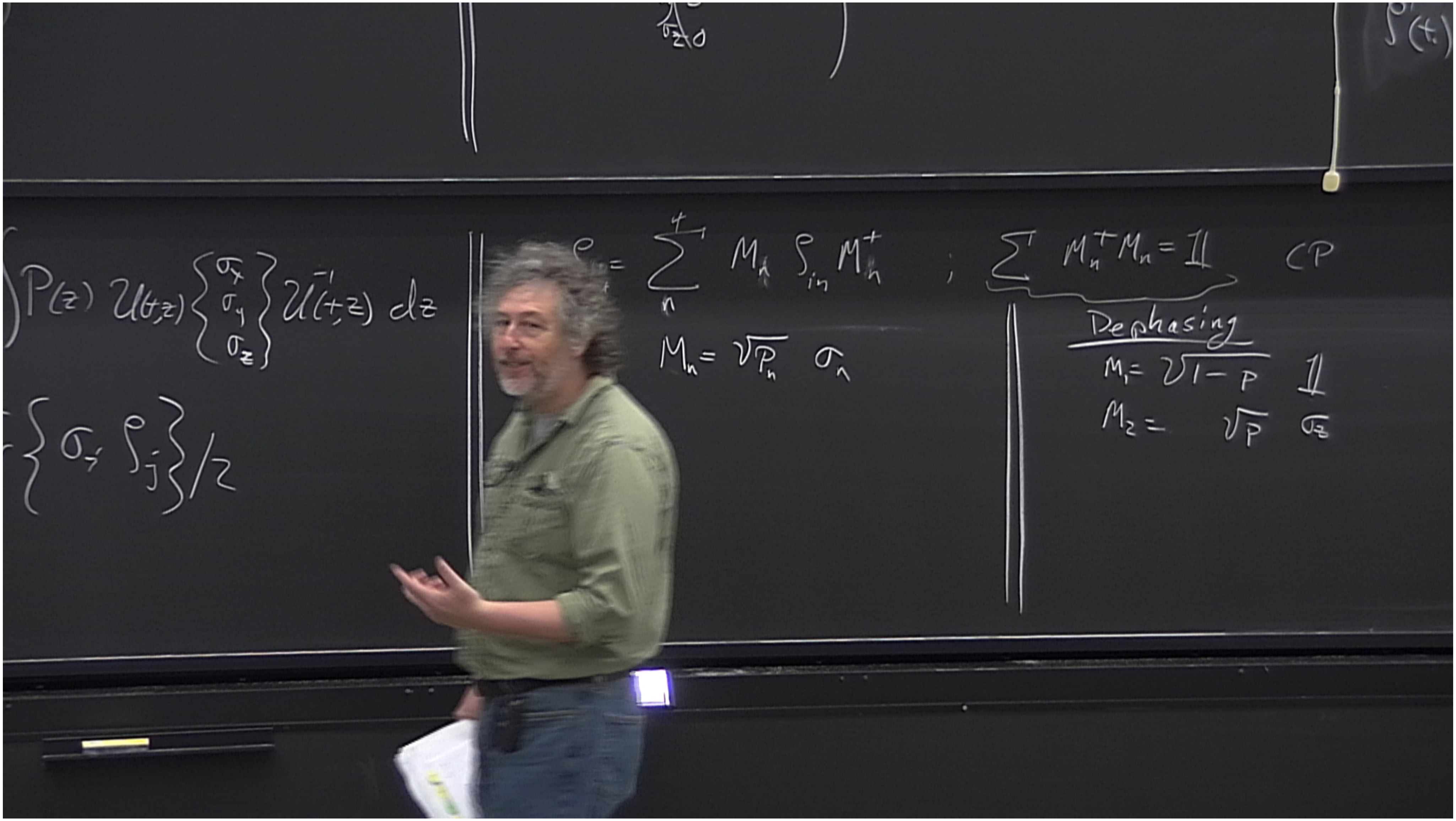

Kraus
{ρx, ρy, ρz} = ∫ P(z) U(t,z) {σx, σy, σz} U⁻¹(t,z) dz
c_ij = Tr{σ_i ρ_j}/2

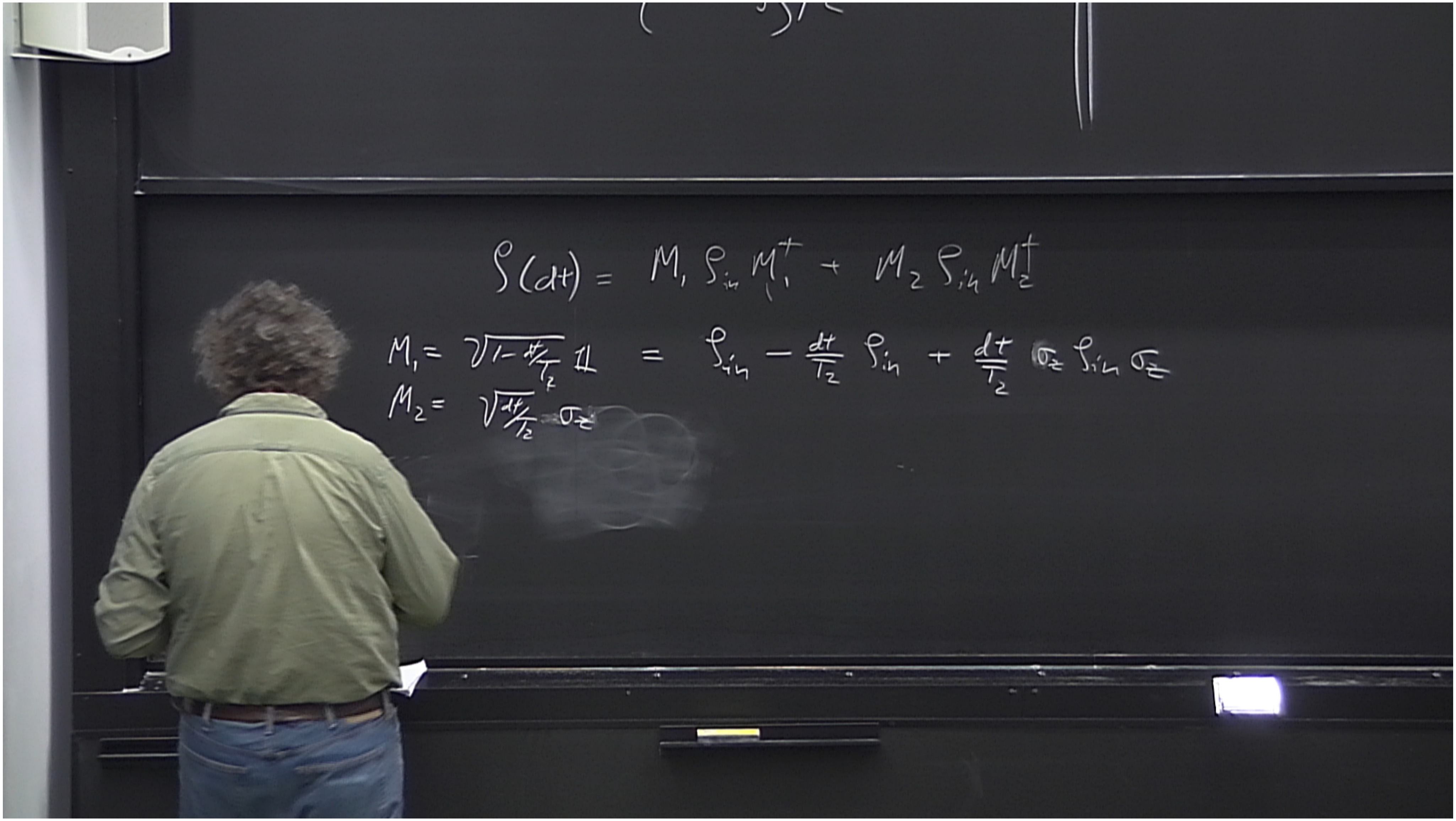

Kraus



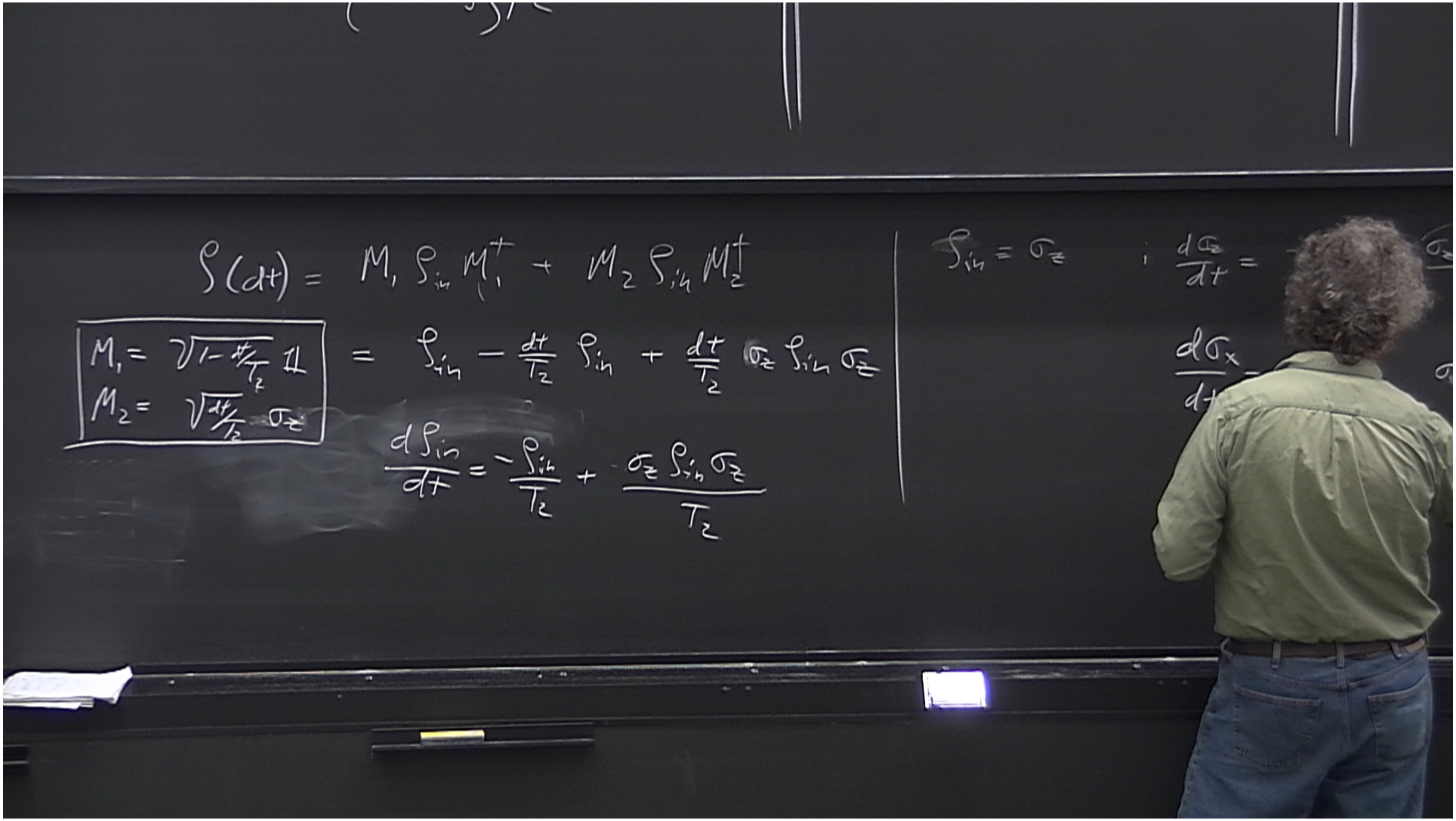
∫P(z) U(t,z) {σx σy σz} U⁻¹(t,z) dz
CP


$\int P(z)\, \mathcal{U}(t,z) \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix} \mathcal{U}^{-1}(t,z)\, dz$
$\{\sigma_i, \rho_j\}/2$
$\rho_{out} = \sum_n^4 M_n \rho_{in} M_n^+ \; ; \; \sum M_n^+ M_n = \mathbb{1}$
CP
$M_n = \sqrt{P_n}\, \sigma_n$


Dephasing
CP



$\rho(dt) = M_1 \rho_{in} M_1^\dagger + M_2 \rho_{in} M_2^\dagger$
$M_1 = \sqrt{1 - dt/T_2}\, \mathbb{1} = \rho_{in} - \frac{dt}{T_2} \rho_{in} + \frac{dt}{T_2} \sigma_z \rho_{in} \sigma_z$
$M_2 = \sqrt{dt/T_2}\, \sigma_z$


$\rho(dt) = M_1 \rho_{in} M_1^\dagger + M_2 \rho_{in} M_2^\dagger$
$M_1 = \sqrt{1 - dt/T_2}\, \mathbb{1}$
$M_2 = \sqrt{dt/T_2}\, \sigma_z$
$= \rho_{in} - \frac{dt}{T_2} \rho_{in} + \frac{dt}{T_2} \sigma_z \rho_{in} \sigma_z$
$\frac{d\rho_{in}}{dt} = \frac{-\rho_{in}}{T_2} + \frac{\sigma_z \rho_{in} \sigma_z}{T_2}$
$\rho_{in} = \sigma_z$

$$p(t) = t/T_2$$

$$M_1 \rho_{in} M_1^\dagger + M_2 \rho_{in} M_2^\dagger$$

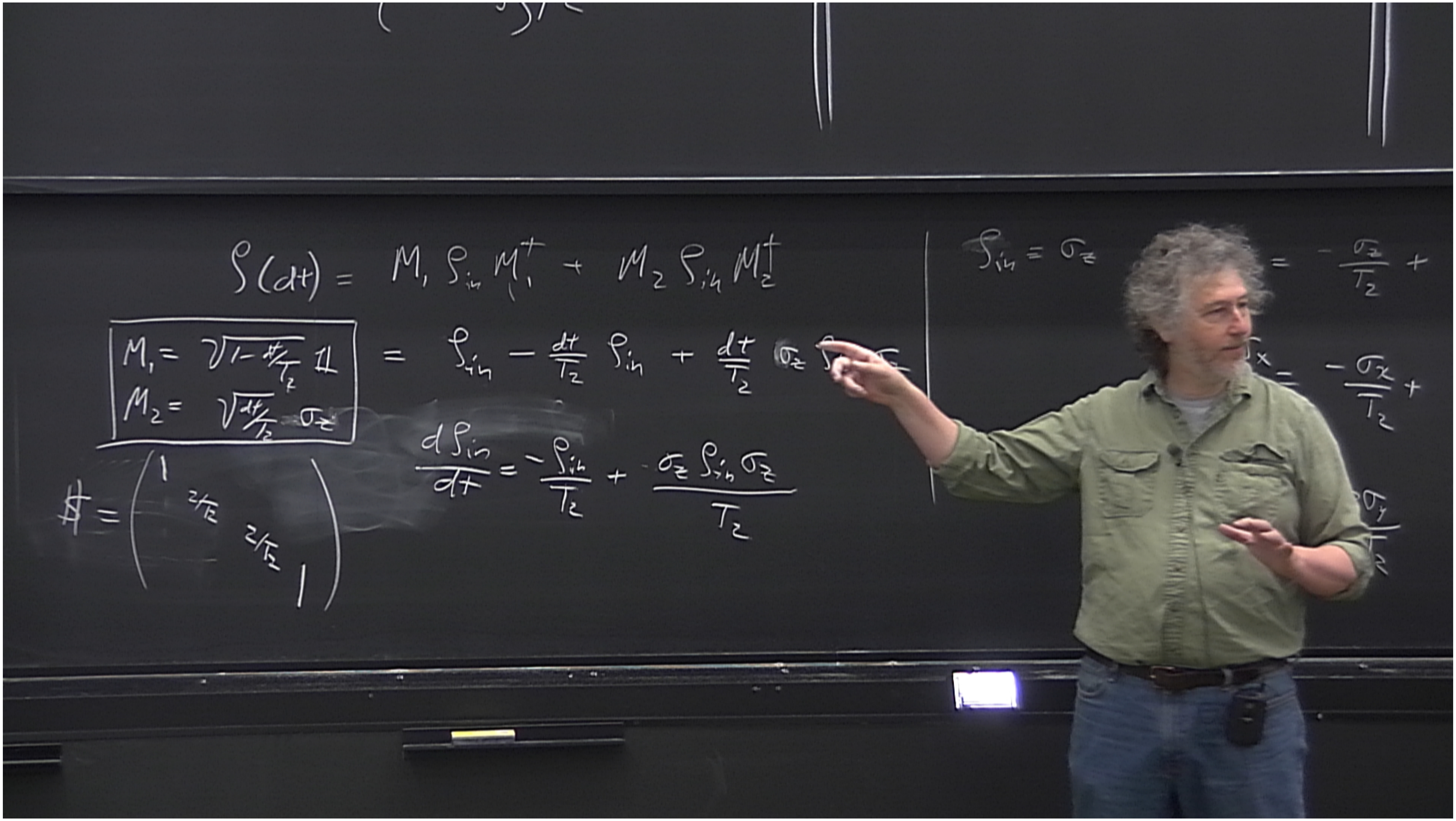
$$= \rho_{in} - \frac{dt}{T_2}\rho_{in} + \frac{dt}{T_2}\sigma_z \rho_{in} \sigma_z$$

$$\frac{d\rho_{in}}{dt} = -\frac{\rho_{in}}{T_2} + \frac{\sigma_z \rho_{in} \sigma_z}{T_2}$$

$$\rho_{in} = \sigma_z \quad ; \quad \frac{d\sigma_z}{dt} = -\frac{\sigma_z}{T_2} + \frac{\overbrace{\sigma_z\sigma_z\sigma_z}^{\sigma_z}}{T_2} = 0$$

$$\frac{d\sigma_x}{dt} = -\frac{\sigma_x}{T_2} + \frac{\sigma_z \sigma_x \sigma_z}{T_2} = \frac{2\sigma_x}{T_2}$$

$$\frac{d\sigma_y}{dt} = \frac{-2\sigma_y}{T_2}$$


ρ(dt) = M1 ρin M1† + M2 ρin M2†
M1 = √(1 − dt/T2) 𝟙
M2 = √(dt/T2) σz
= ρin − (dt/T2) ρin + (dt/T2) σz ρin σz
dρin/dt = −ρin/T2 + σz ρin σz/T2
ρin = σz