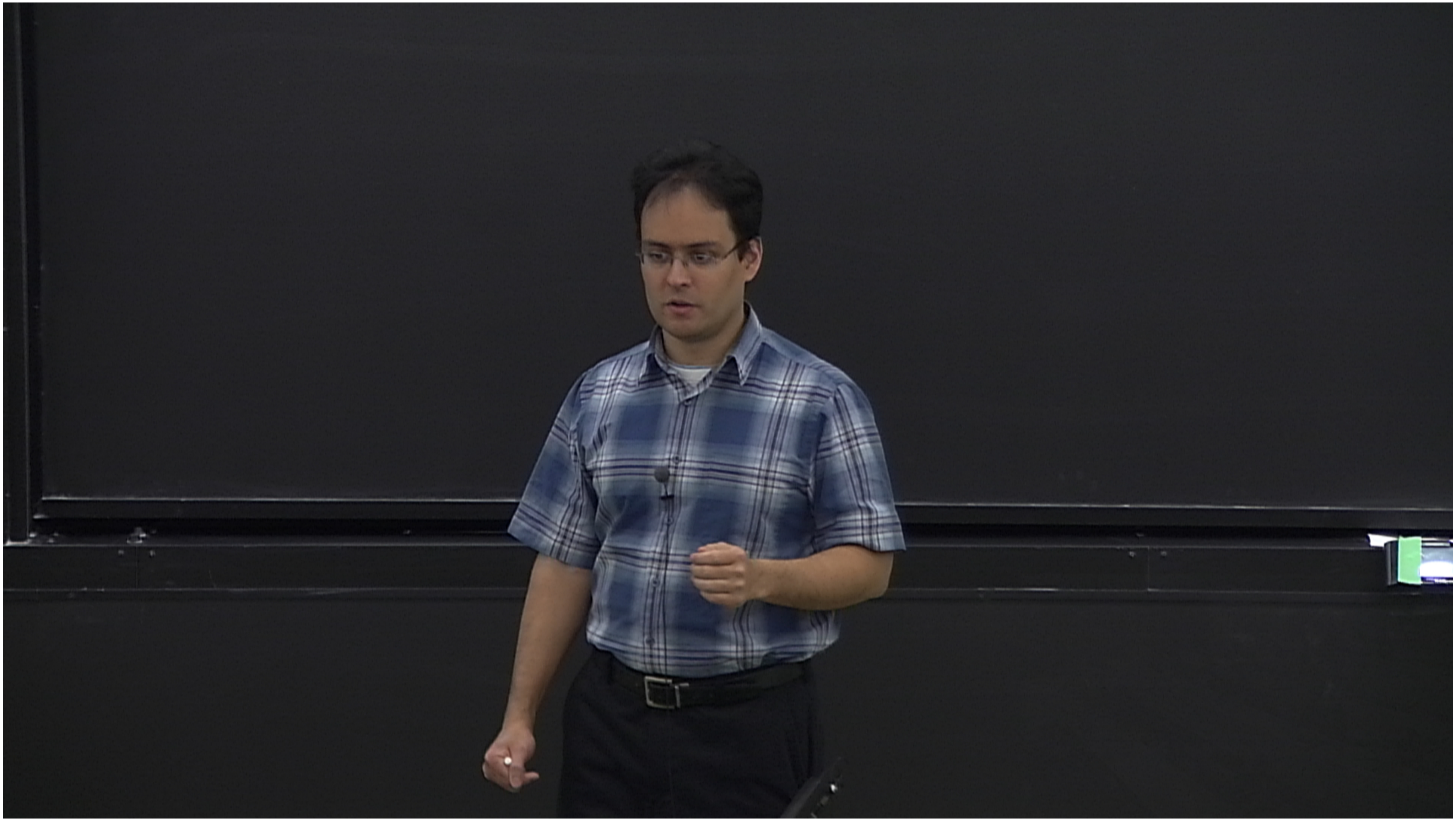
Title: String Theory (Review) - Lecture 14

Date: Mar 02, 2012 11:30 AM

URL: http://www.pirsa.org/12030003

Abstract:



$\mathcal{N}=1$ 11 Dim. SUGRA

or 32 supercharges.

Low ener

11 Dim SUGRA on $\mathbb{R}^{1,9} \times S^1_R$.

$$e^{4\Phi/3} = R^2$$

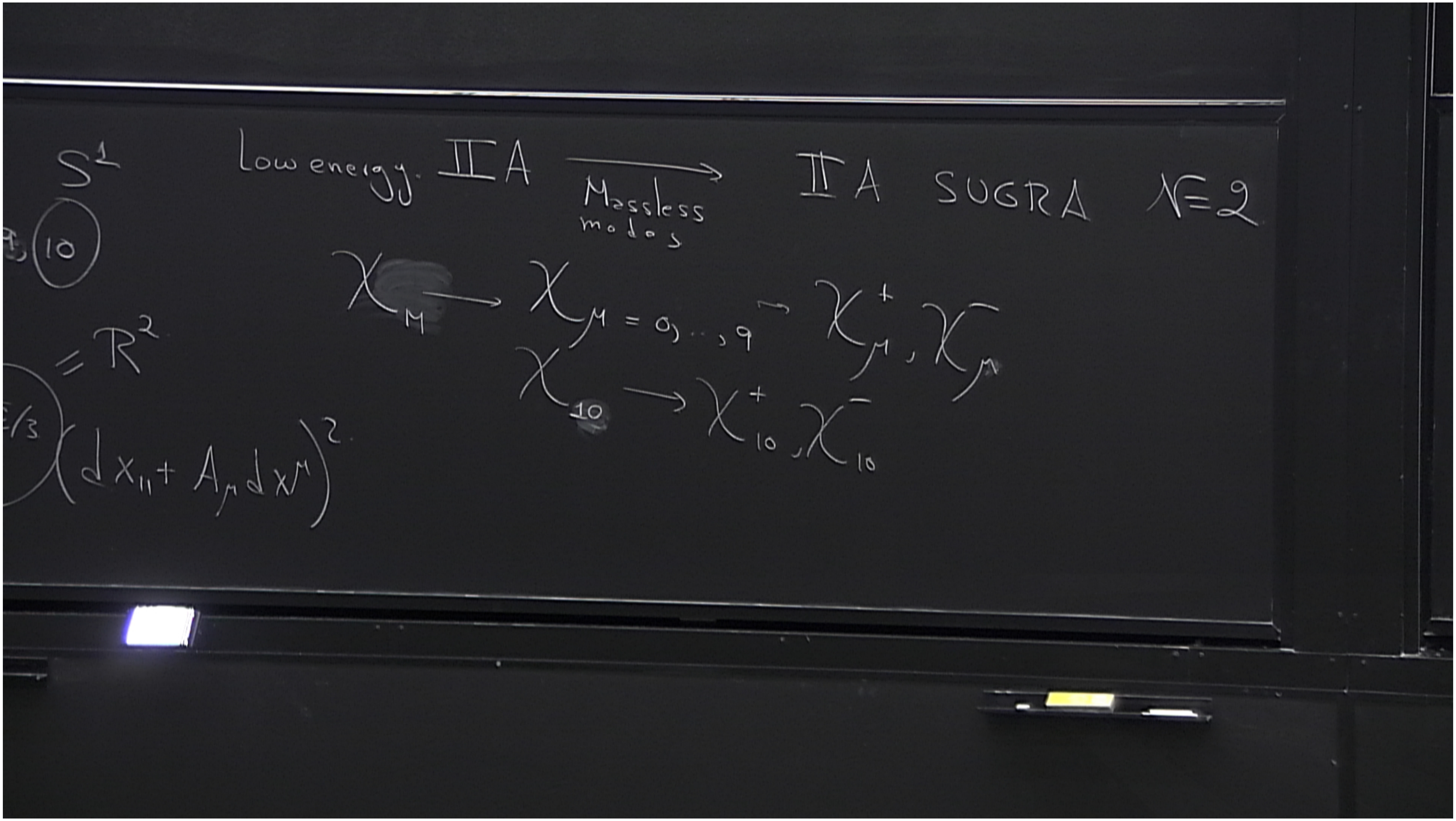
$$ds^2 = e^{-2\Phi/3}\left(g_{\mu\nu}\, dx^\mu dx^\nu\right) + e^{4\Phi/3}\left(dx_{11} + A_\mu dx^\mu\right)^2$$

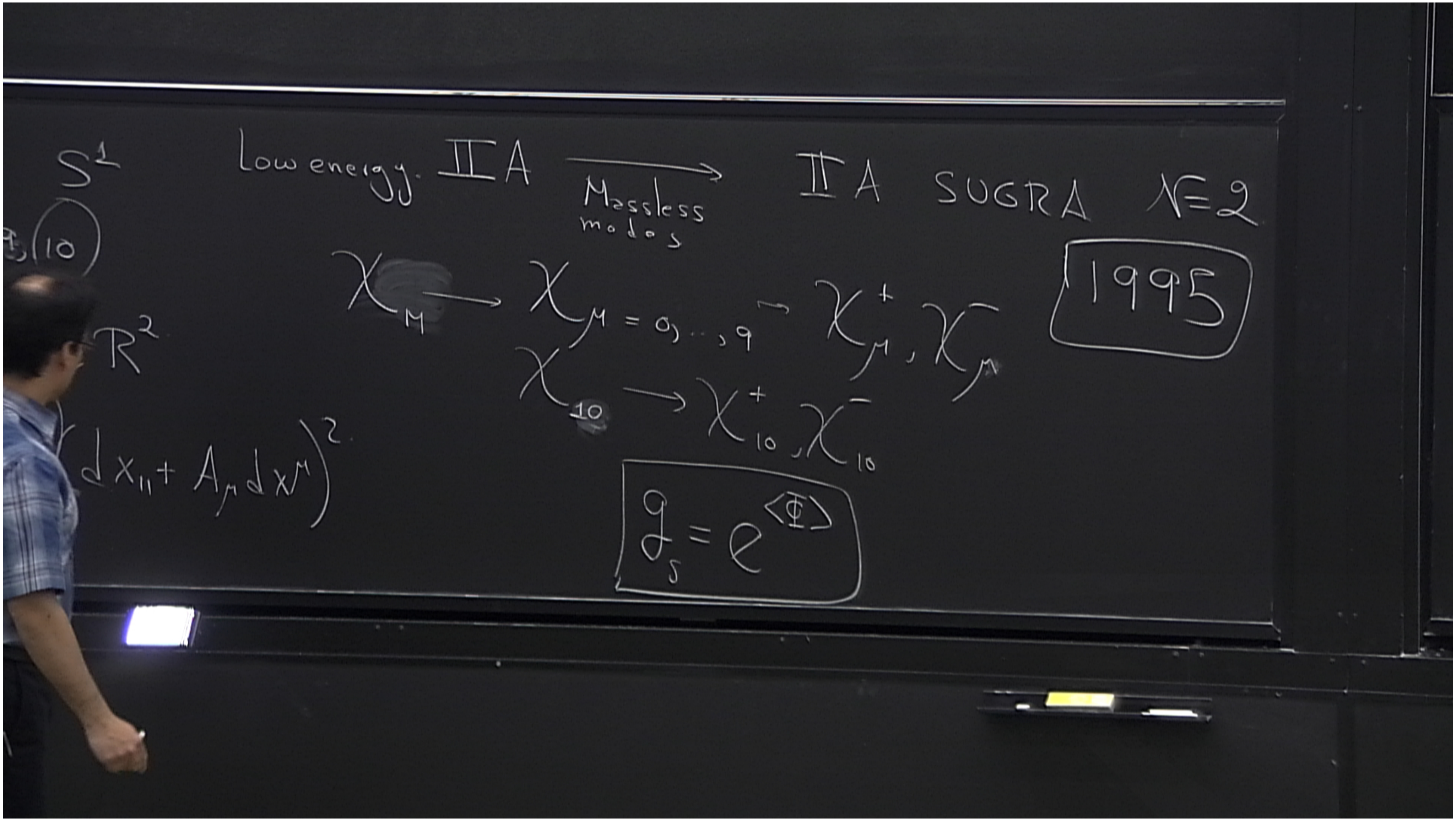
Low energy IIA $\xrightarrow[\text{modes}]{\text{Massless}}$ IIA SUGRA $N=2$

$$X_M \longrightarrow X_\mu,\; X_{11}$$

$$= R^2$$

$$\ldots/3 \, (dx_{11} + A_\mu dx^\mu)^2$$


S^1
Low energy IIA
Massless modes
IIA SUGRA $N=2$
10
$\chi_M \to \chi_{\mu=0,\dots,9} \to \chi^+_\mu, \chi^-_\mu$
$\chi_{10} \to \chi^+_{10}, \chi^-_{10}$
$= R^2$
$(dx_{11} + A_\mu dx^\mu)^2$


S1
Low energy IIA
Massless
modos
IIA SUGRA N=2
1995
R2
(dx11 + Aμ dxμ)2
gs = e⟨Φ⟩

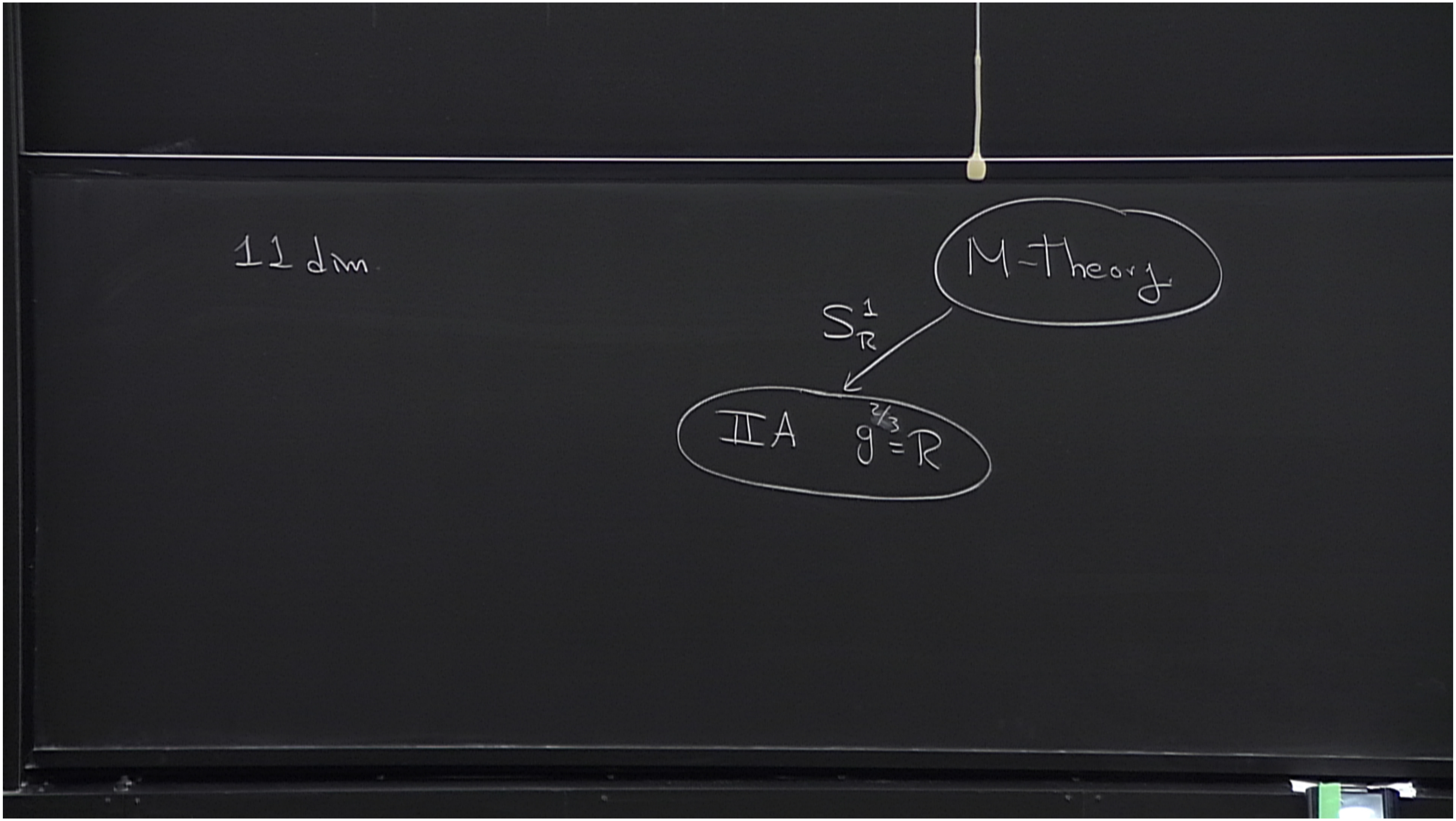
$$ds^2 = e^{-2\Phi/3}\left(g_{\mu\nu}\,dx^\mu dx^\nu\right) + e^{4\Phi/3}\left(dx_{11} + A_\mu dx^\mu\right)^2$$

$$\boxed{g_s = e^{\langle\Phi\rangle}}$$

$$g_s^{2/3} = R_{10}$$

Crazy idea: $\exists$ a theory in 11 dimensions which has a weakly couple description around flat space-time with Lowest modes those of 11 D SUGRA.

M-Theory


11 dim
M-Theory
S^1_R
IIA $g^{2/3}=R$

$$ds^2 = e^{-2\Phi/3}\left(g_{\mu\nu}dx^\mu dx^\nu\right) + e^{4\Phi/3}\left(dx_{11} + A_\mu dx^\mu\right)^2$$

$\chi_{10} \to \chi_{10}, \chi_{10}$

$$\boxed{g_s = e^{\langle\Phi\rangle}}$$

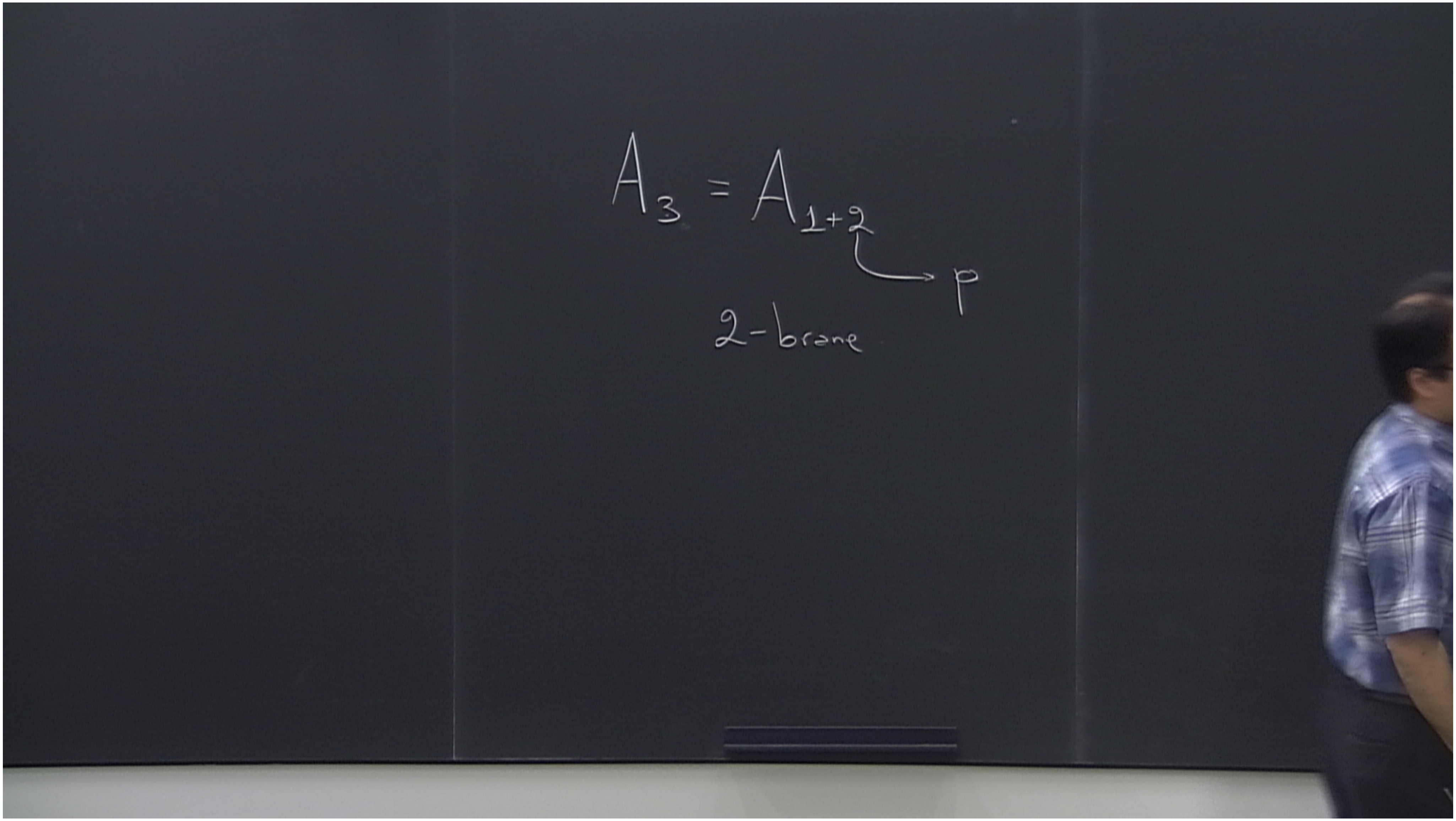
$$g_s^{2/3} = R_{10}$$

G_{MN}, χ_M, A_{MNP}

↳ 3-form gauge potential

Crazy idea: ∃ a theory in 11 dimensions which has a weakly couple description around flat space-time with Lowest modes those of 11D SUGRA.

M-Theory


$A_3 = A_{1+2}$
p
2-brane

$= 1$ 11 Dim. SUGRA

32 supercharges. $0, 1, 2, \dots, 9, 10$

S^1

Low energy IIA $\xrightarrow{\text{Massless modes}}$ IIA SUGRA $N=2$

$\boxed{1995}$

11 Dim SUGRA on $\mathbb{R}^{1,9} \times S^1_R$

$\chi_M \to \chi_{\mu = 0, \dots, 9} \to \chi^+_\mu, \chi^-_\mu$

$\chi_{10} \to \chi^+_{10}, \chi^-_{10}$

$e^{4\Phi/3} = R^2$

$$ds^2 = e^{-2\Phi/3}\left(g_{\mu\nu} dx^\mu dx^\nu\right) + e^{4\Phi/3}\left(dx_{11} + A_\mu dx^\mu\right)^2$$

$$\boxed{g_s = e^{\langle \Phi \rangle}}$$

$g_s^{2/3} = R_{10}$

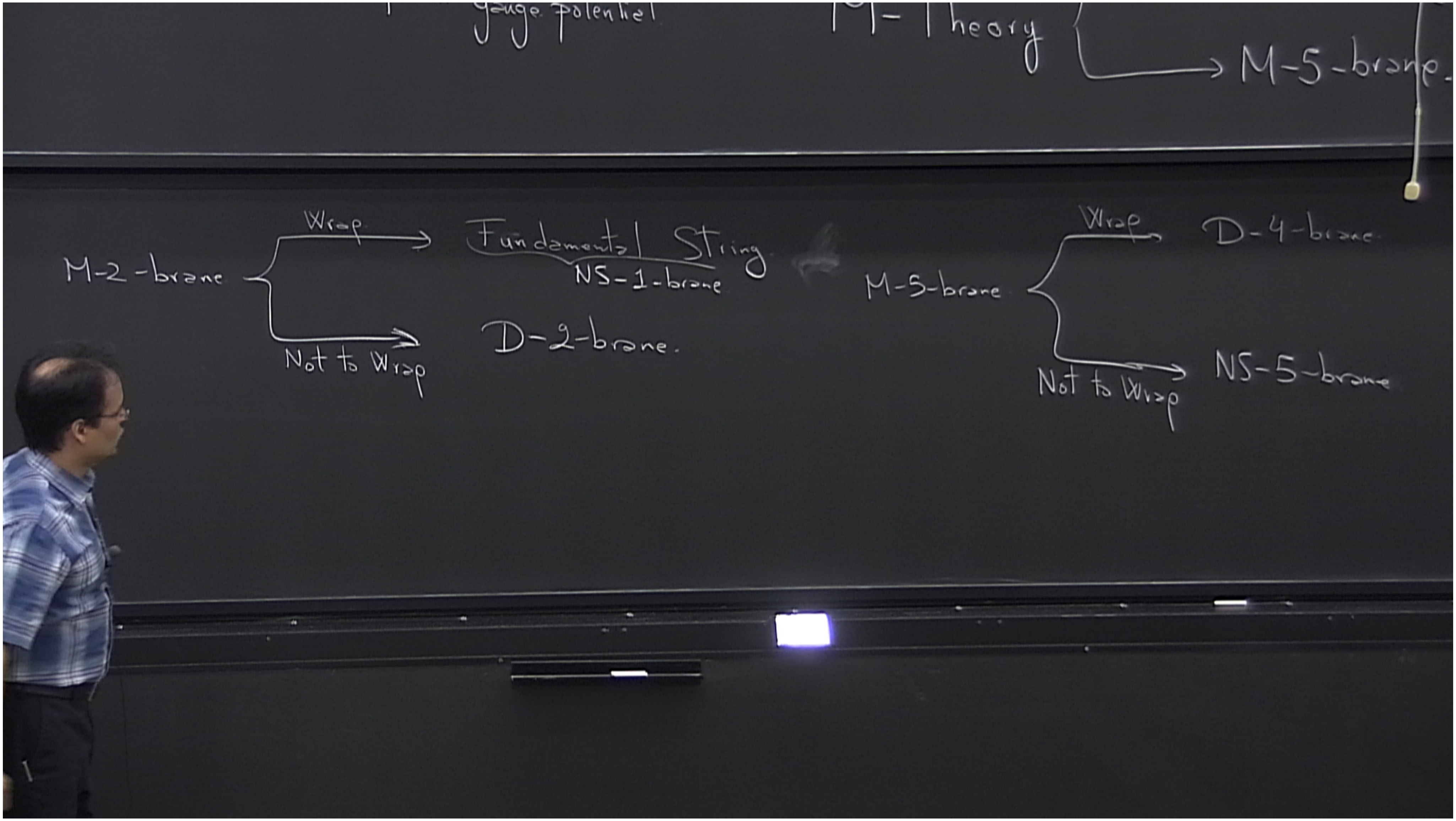
G_{MN}, χ_M, A_{MNP}

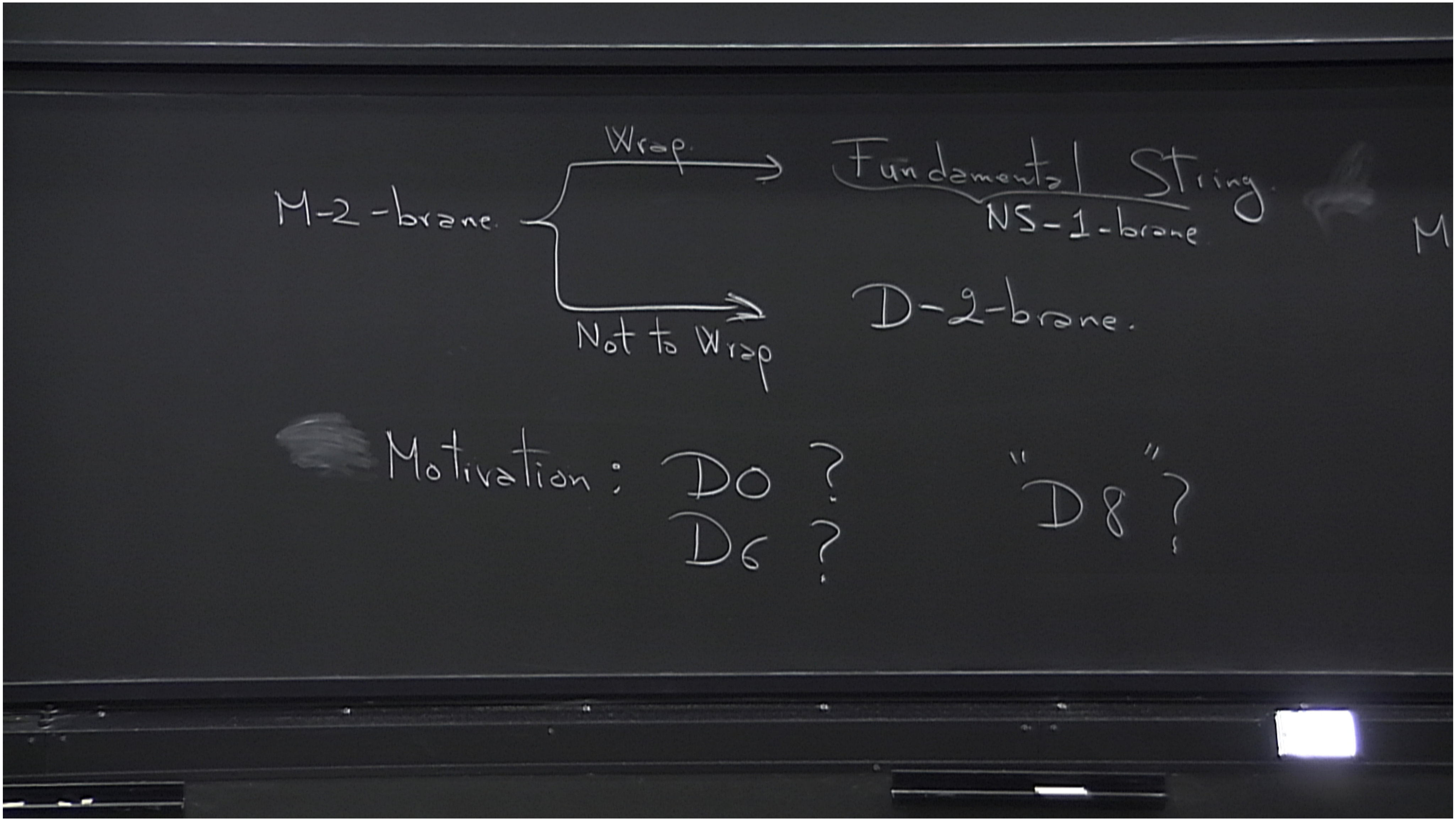
↳ 3-form gauge potential

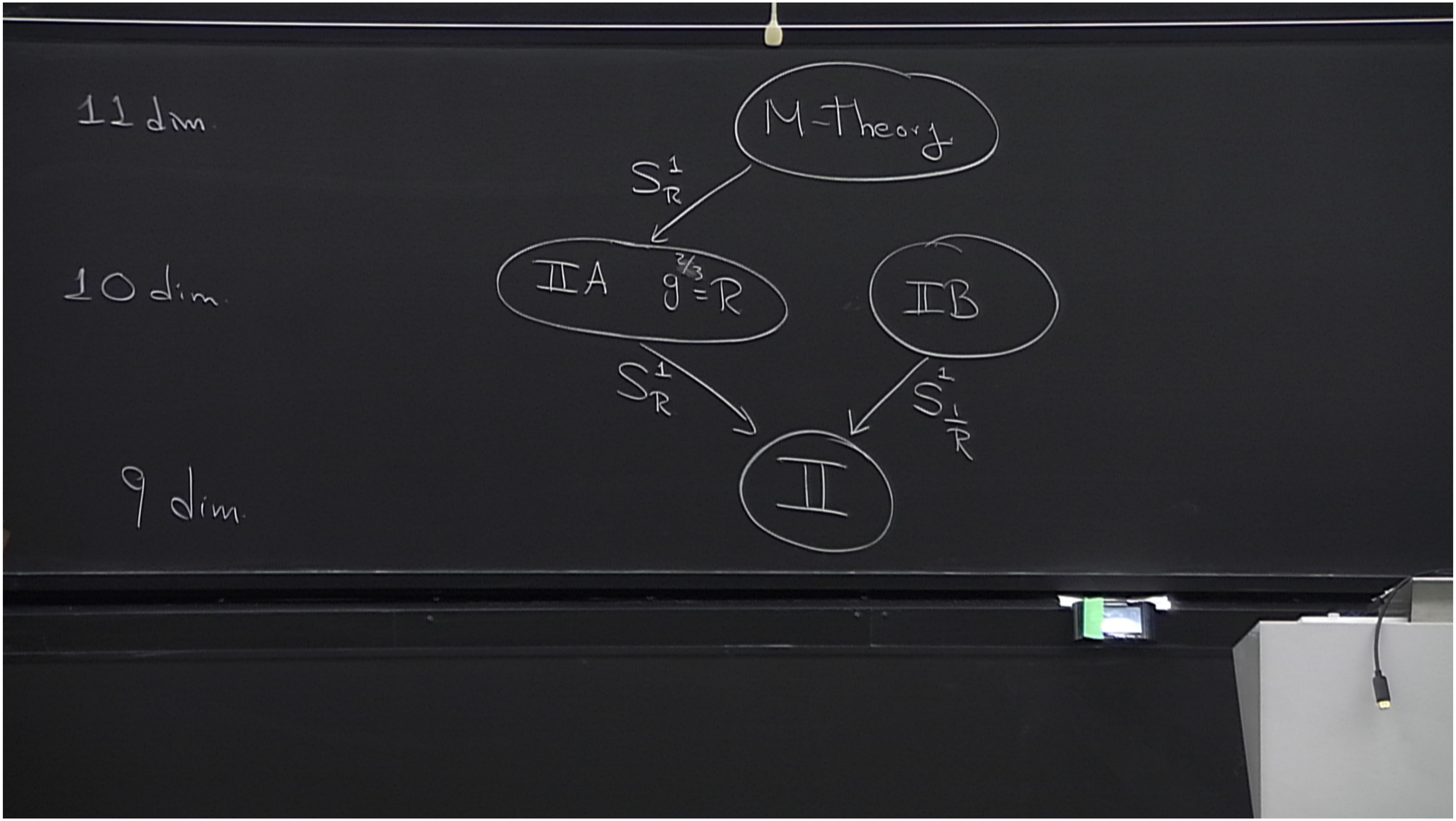
Crazy idea: $\exists$ a theory in 11 dimensions which has a weakly couple description around flat space-time with lowest modes those of 11D SUGRA.

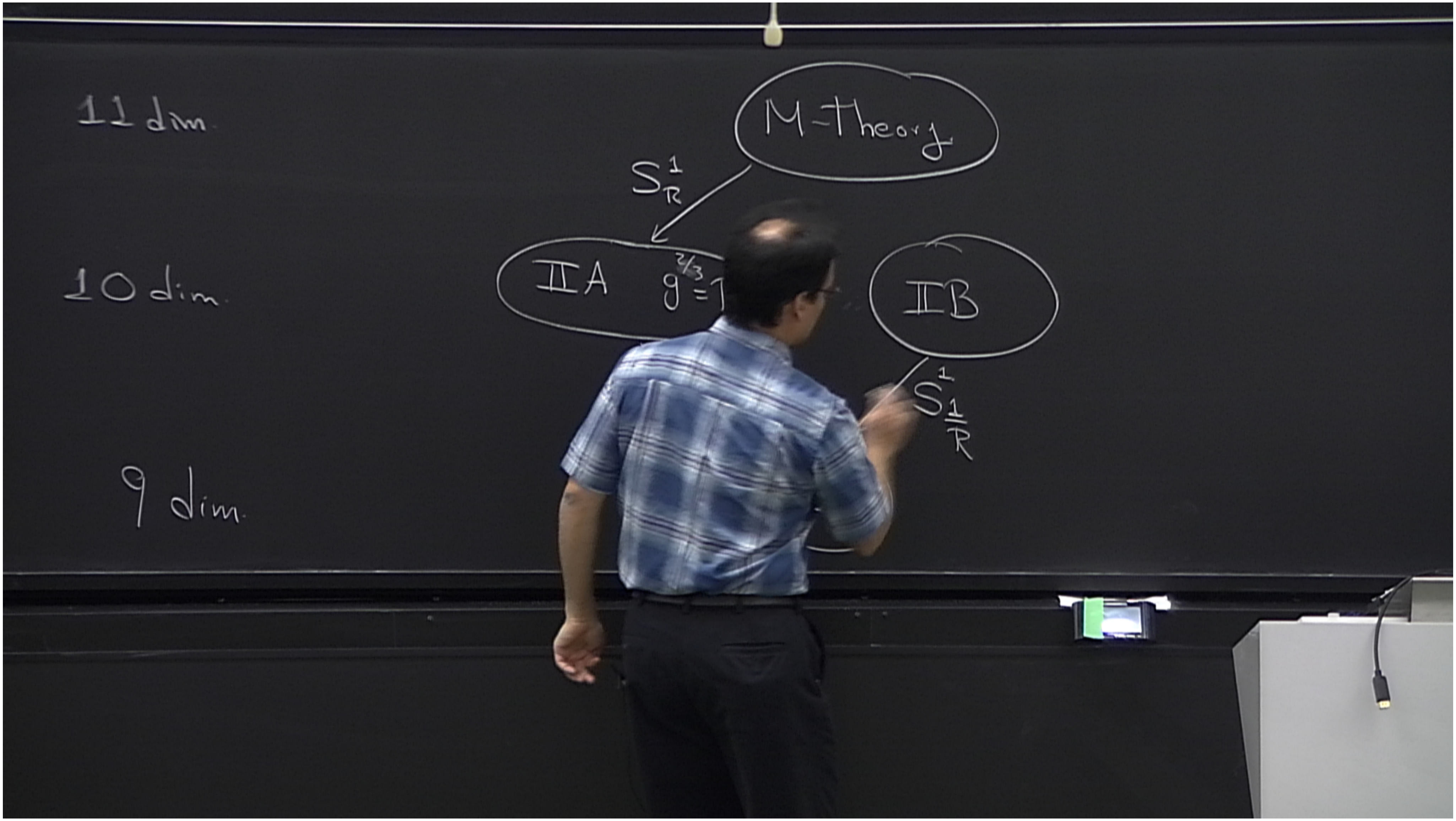
M-Theory → M-2-brane.

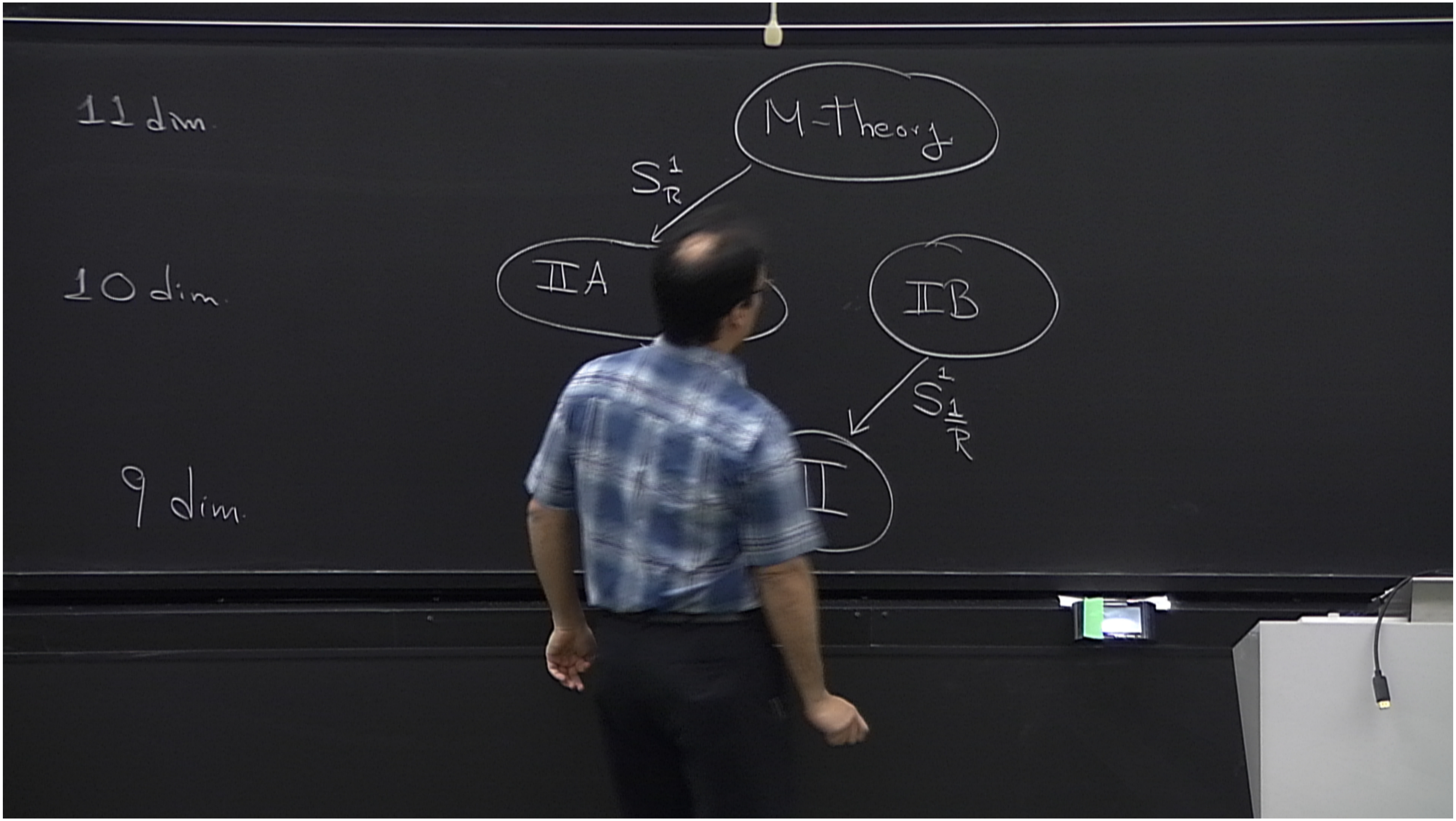
M-Theory → M-5-brane.

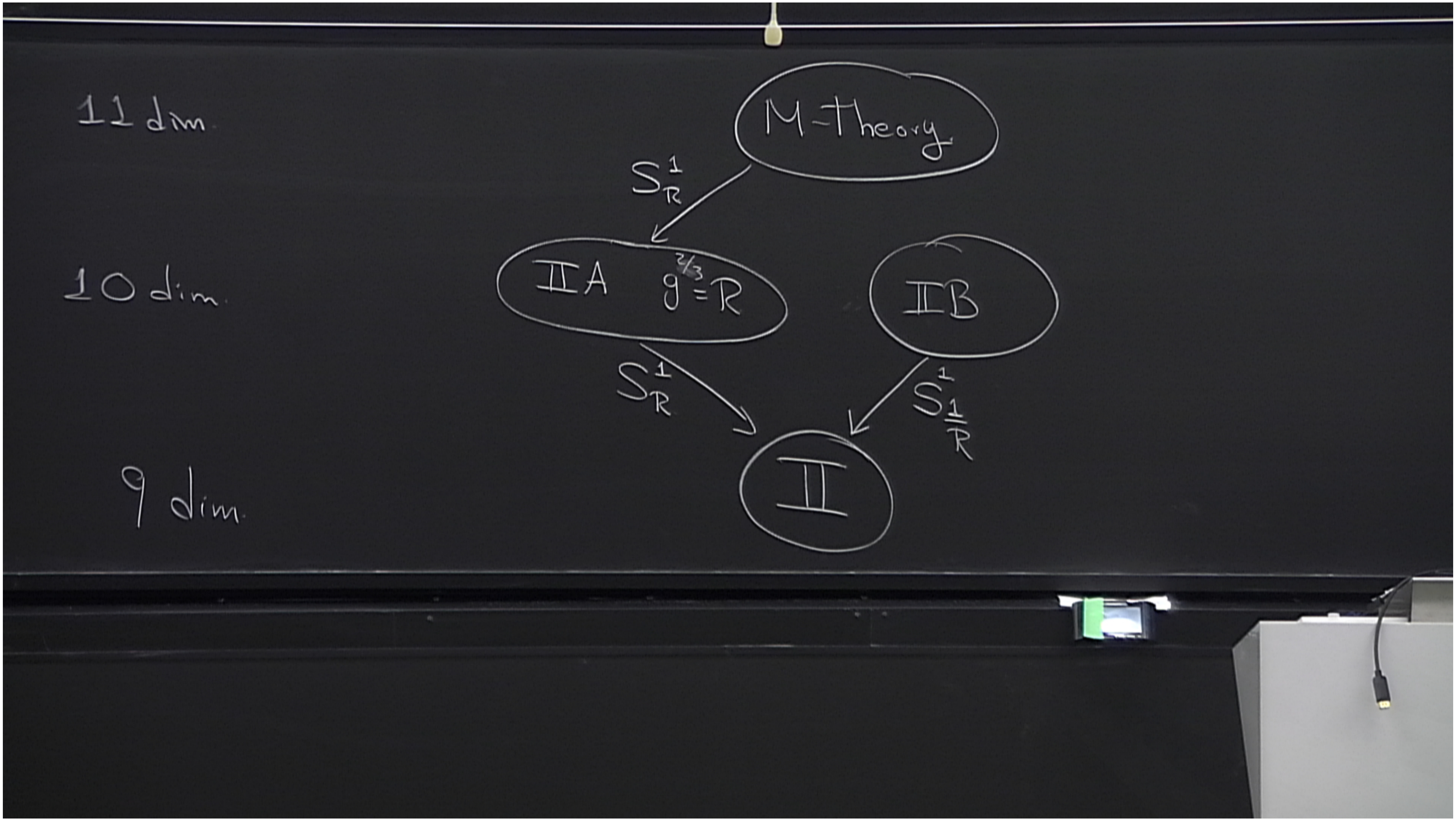

gauge potential
M-Theory
M-5-brane.
M-2-brane
Wrap
Fundamental String
NS-1-brane
Not to Wrap
D-2-brane.
M-5-brane
Wrap
D-4-brane
Not to Wrap
NS-5-brane

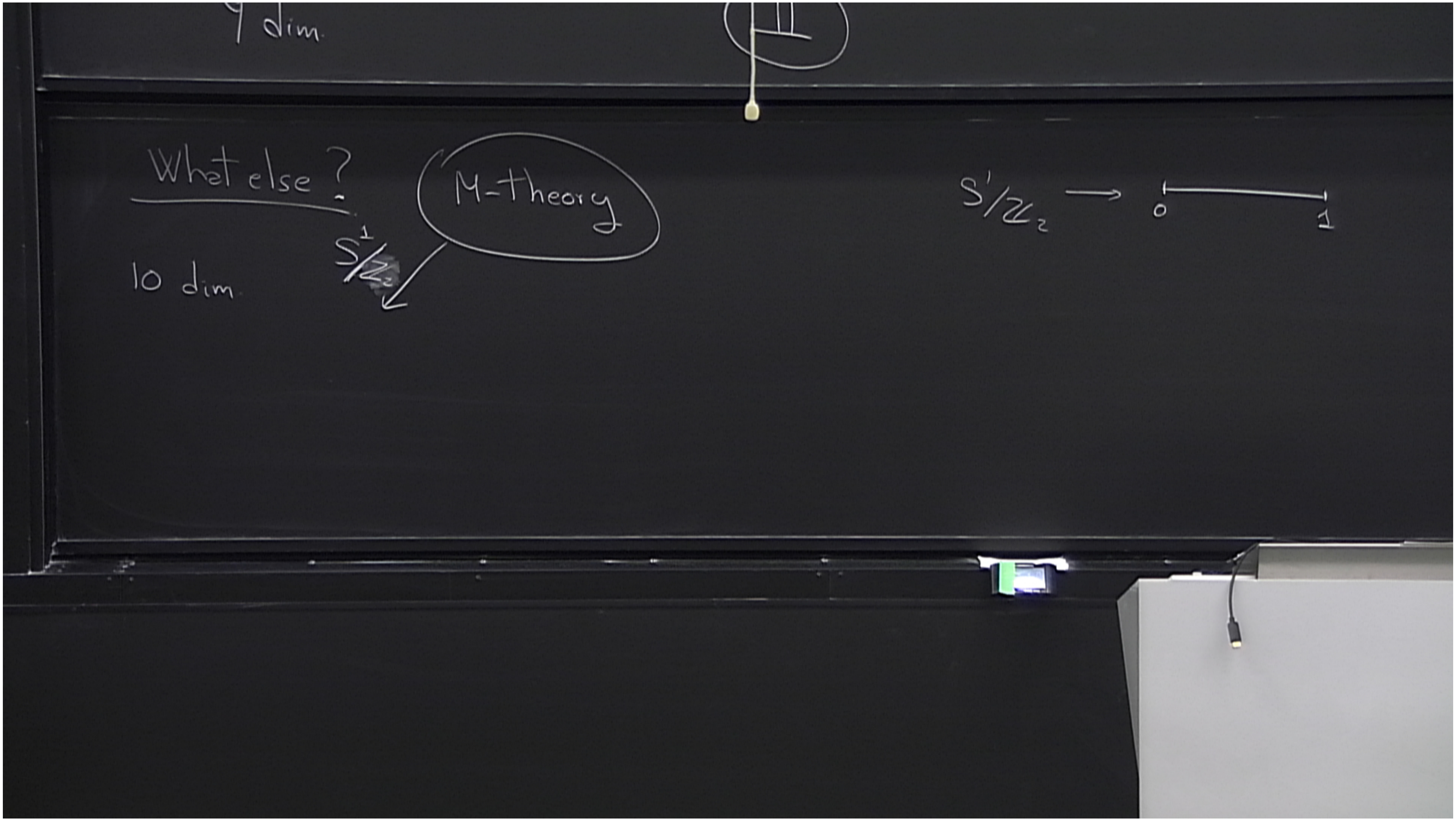

M-2-brane
Wrap.
Fundamental String.
NS-1-brane
Not to Wrap
D-2-brane.
Motivation: D0 ?
D6 ?
"D8" ?

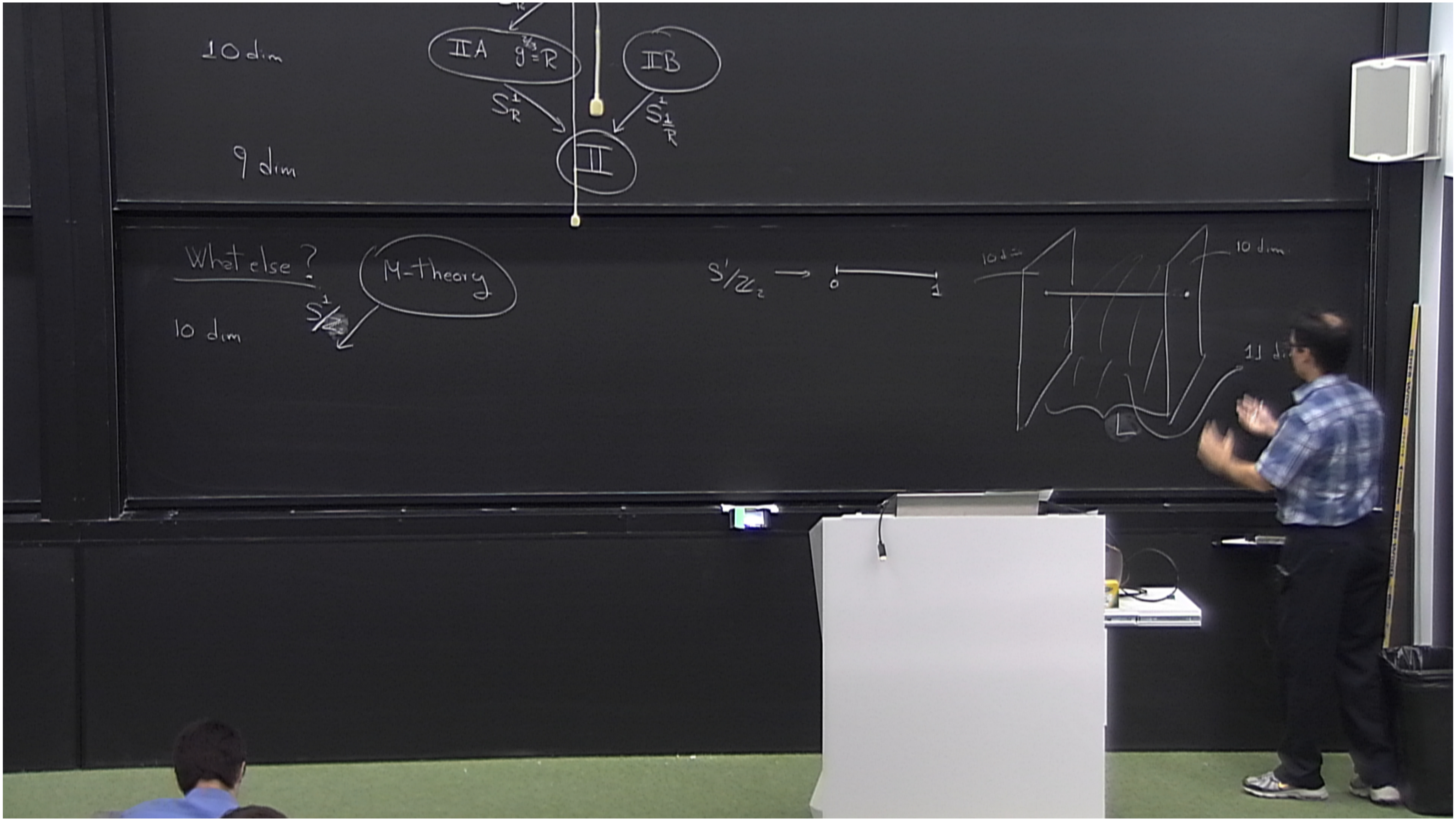

11 dim.
10 dim.
9 dim.
M-Theory
S^1_R
IIA
$g^{2/3} = R$
IIB
S^1_R
$S^1_{\frac{1}{R}}$
II

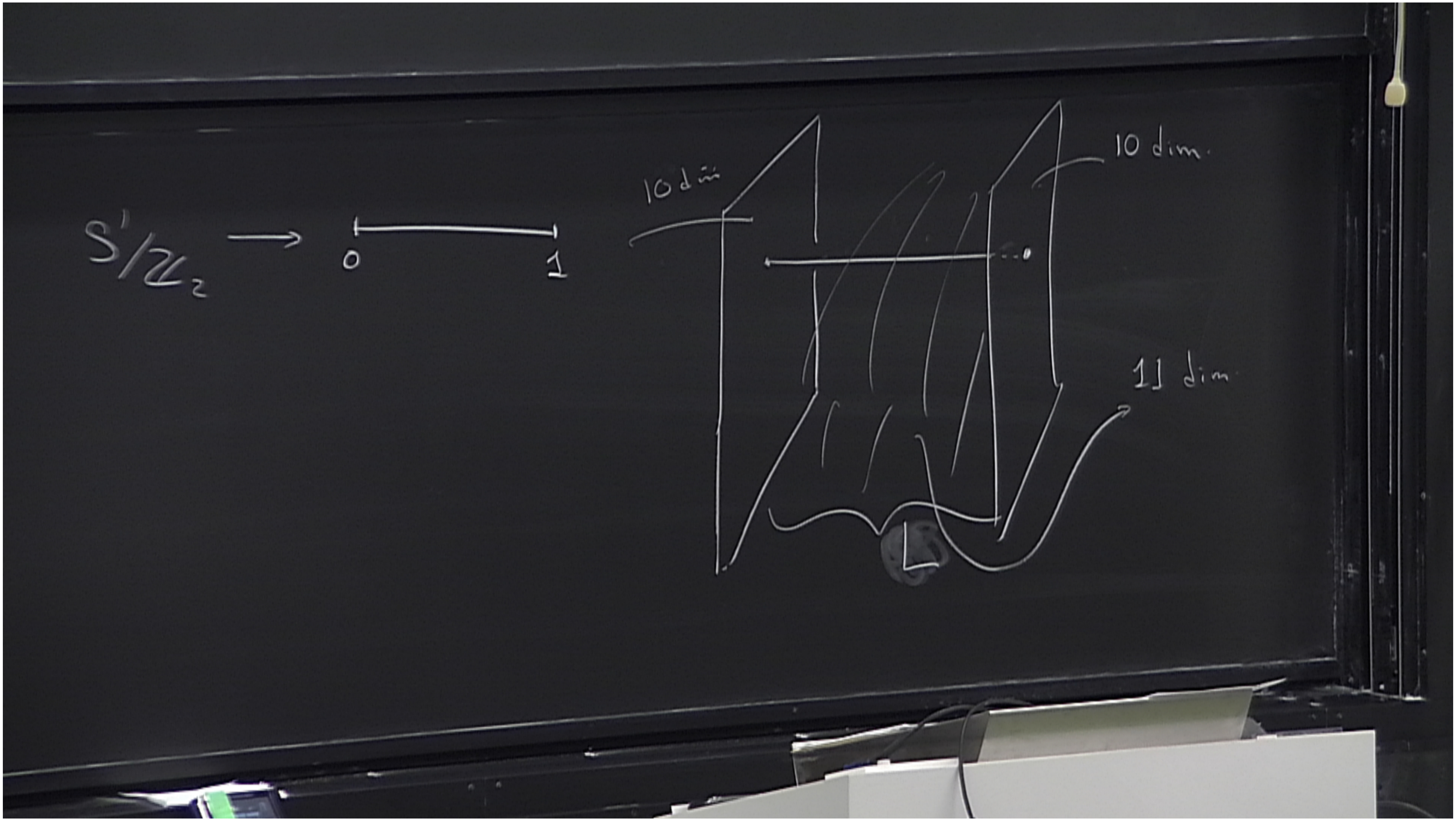

11 dim.
M-Theory
S^1_R
10 dim.
IIA
IIB
$S^1_{\frac{1}{R}}$
9 dim.

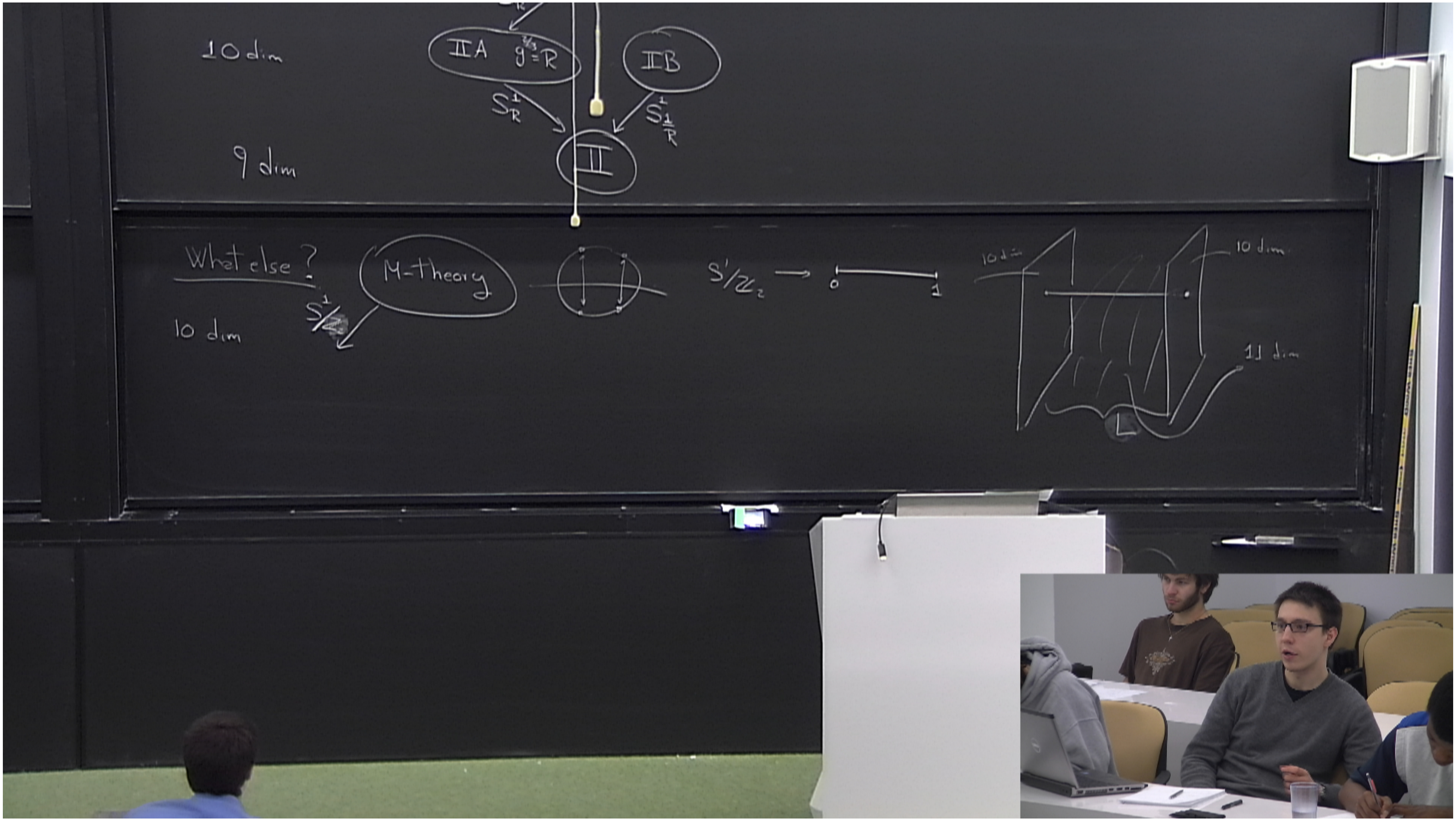

11 dim.
M-Theory
S^1_R
10 dim.
IIA
IIB
$S^1_{\frac{1}{R}}$
9 dim.

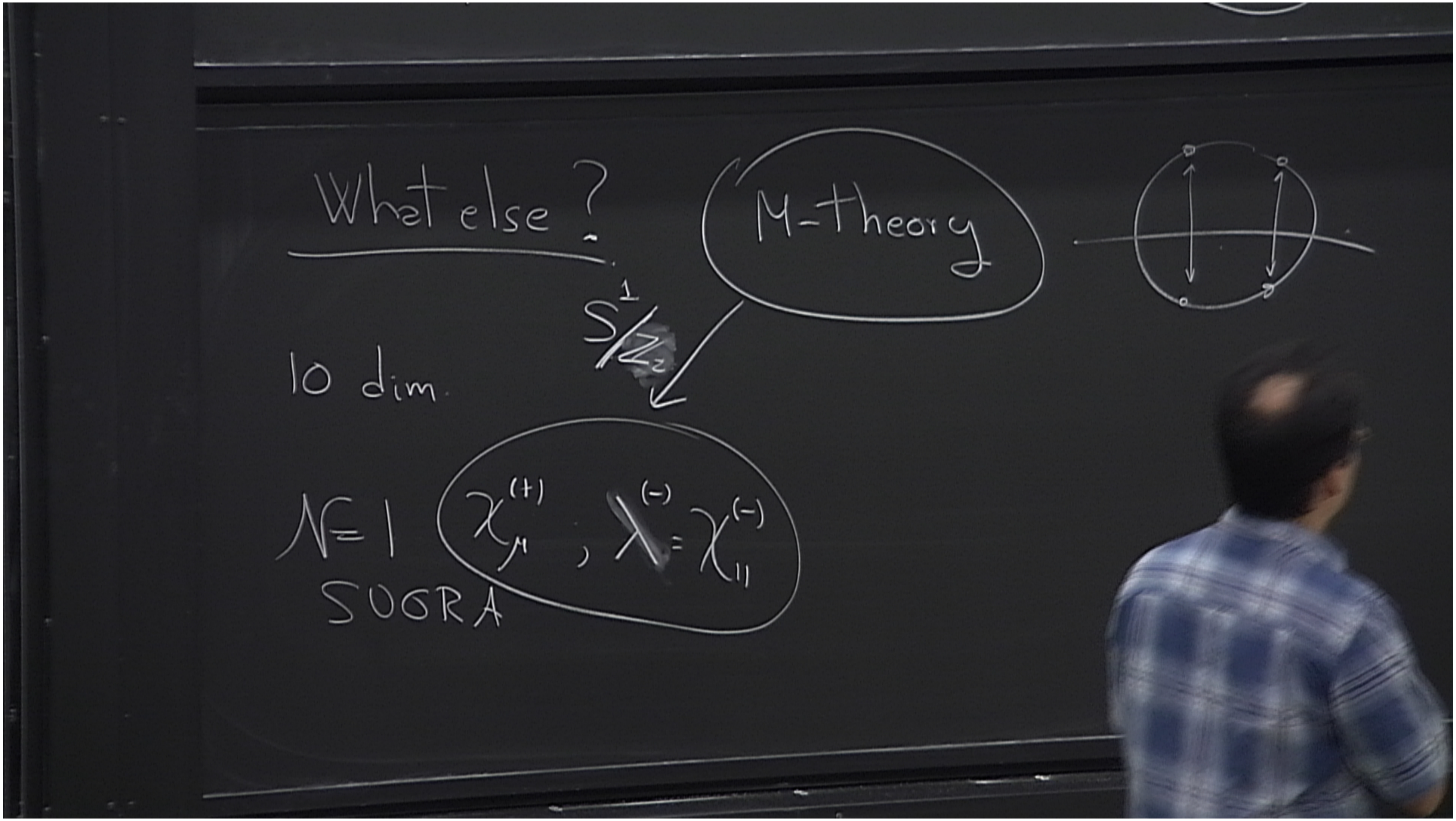

11 dim.
10 dim.
9 dim.
M-Theory
S^1_R
IIA $g^{2/3} = R$
IIB
S^1_R
$S^1_{\frac{1}{R}}$
II

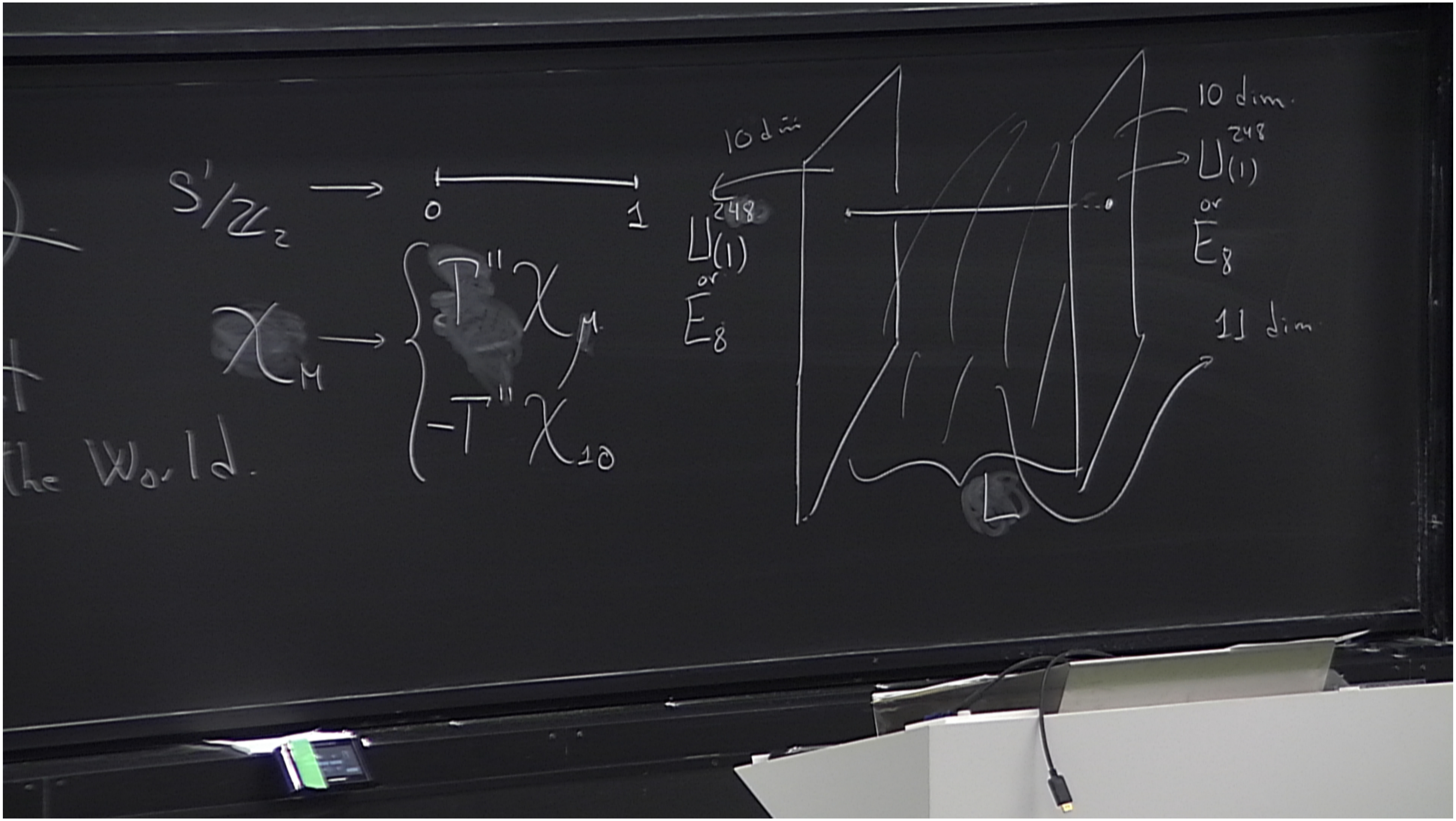

What else ?
M-theory
10 dim.
$S^1/\mathbb{Z}_2$
$S^1/\mathbb{Z}_2 \longrightarrow$
0
1


10 dim
IIA
IIB
9 dim
II
What else?
M-theory
10 dim
S'/Z2
0
1
10 dim
11 dim
L

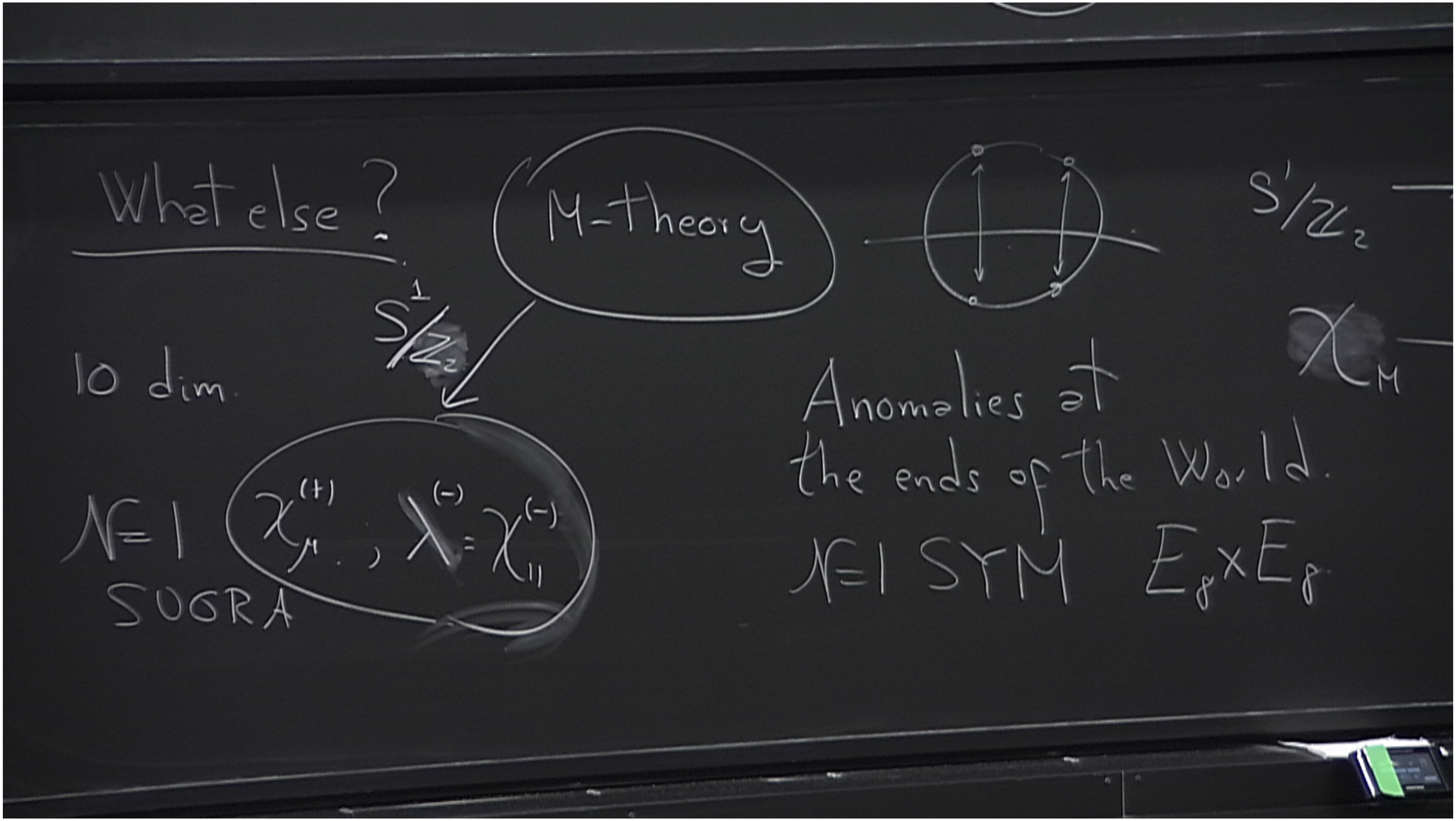

S1/Z2 →
0
1
10 dim
10 dim.
11 dim.

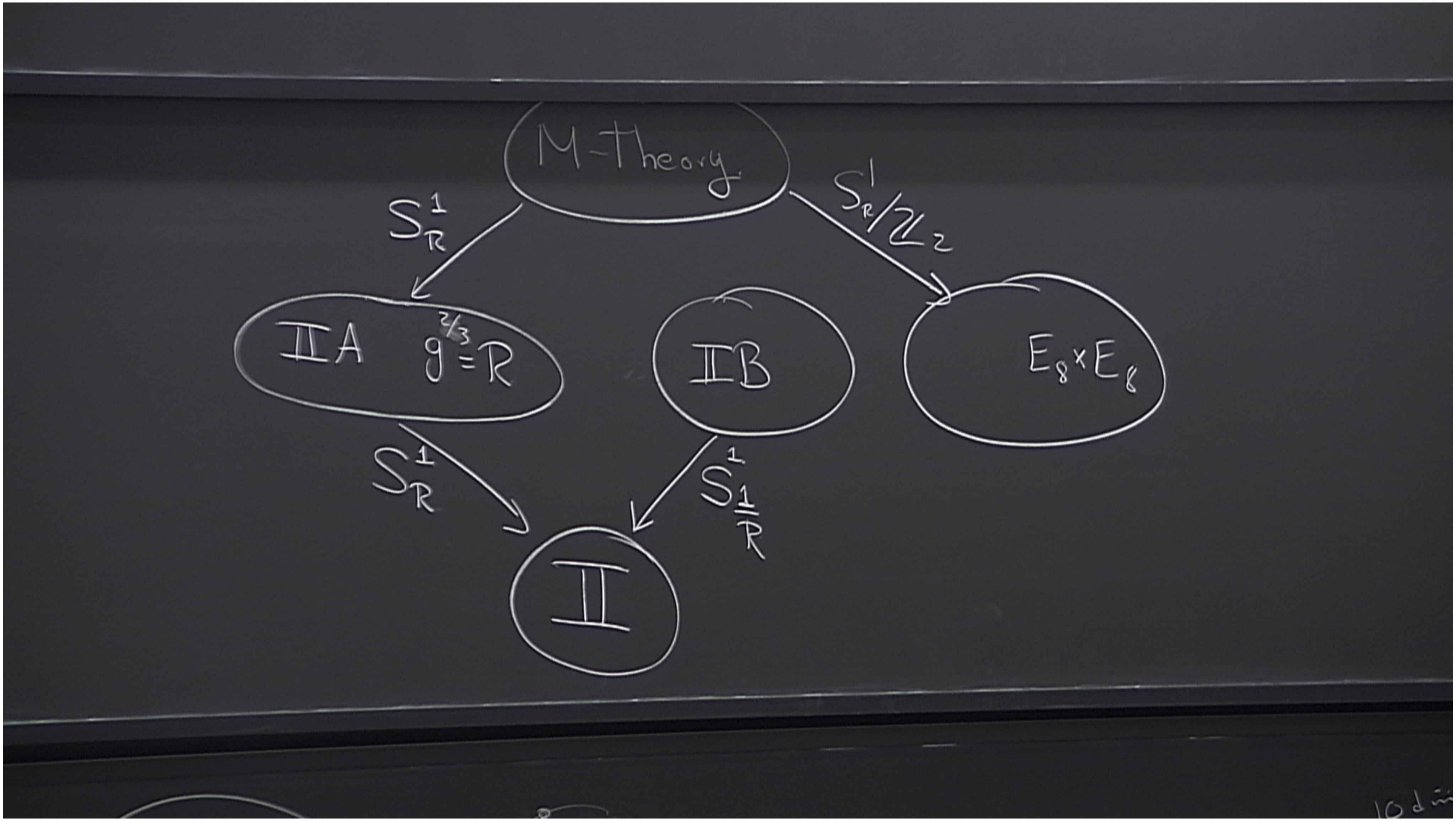

10 dim
IIA
IIB
9 dim
II
What else?
10 dim
M-theory
10 dim
11 dim
L


What else ?
M-theory
$S^1/\mathbb{Z}_2$
10 dim.
$\mathcal{N}=1$
SUGRA
$\chi^{(+)}_{\mu}$, $\lambda^{(-)} = \chi^{(-)}_{11}$



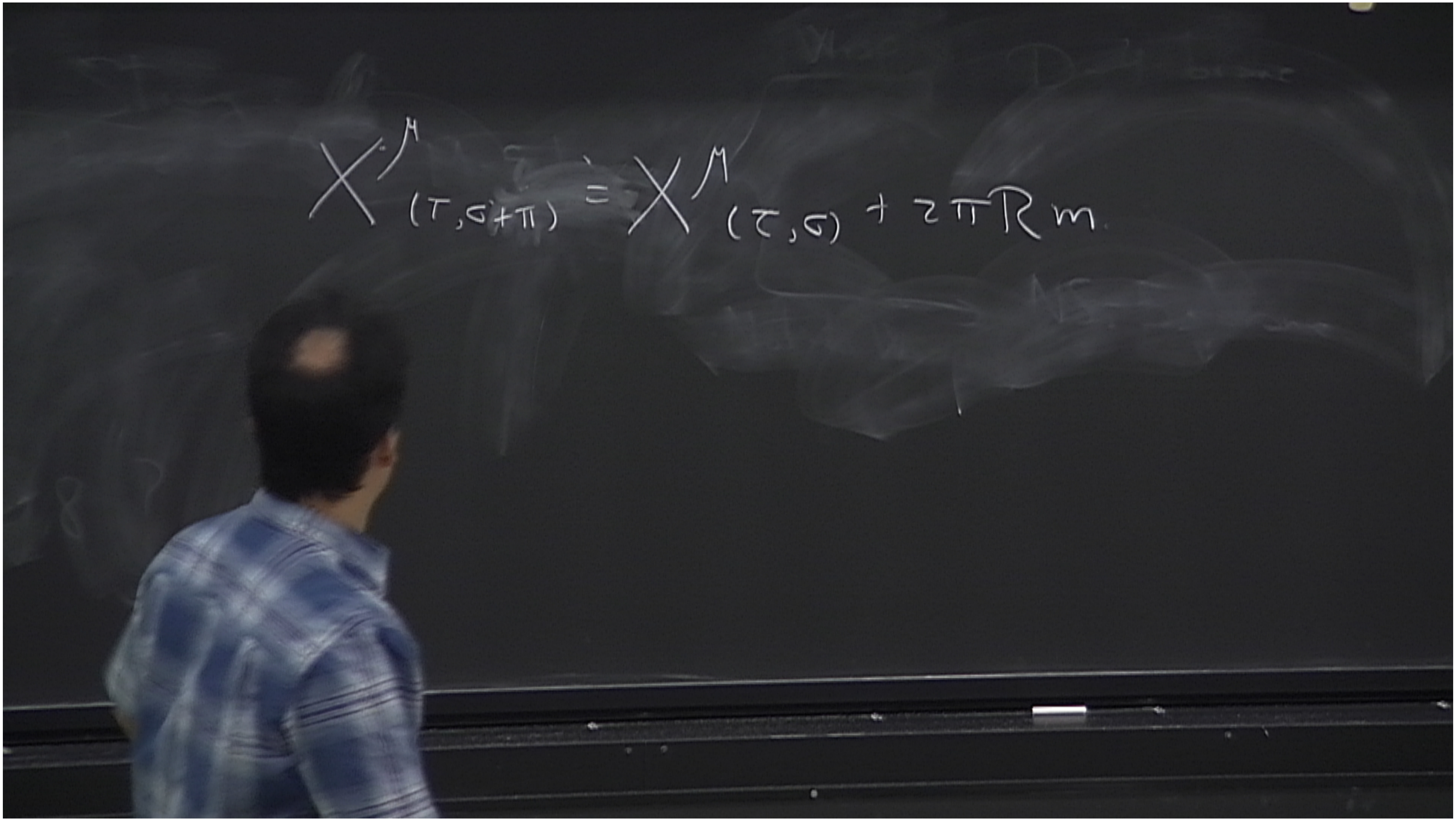
$S^1/\mathbb{Z}_2 \longrightarrow$ 0 1
$\chi_M \longrightarrow \begin{cases} T'' \chi_M \\ -T'' \chi_{10} \end{cases}$
the World.
10 dim
$U(1)^{248}$ or E_8
10 dim.
$U(1)^{248}$ or E_8
11 dim
L


What else?
M-theory
$S^1/\mathbb{Z}_2$
10 dim.
$\mathcal{N}=1$
SUGRA
$\chi^{(+)}_{\mu}$, $\lambda^{(-)} = \chi^{(-)}_{11}$
Anomalies at
the ends of the World.
$\mathcal{N}=1$ SYM $E_8 \times E_8$
$S^1/\mathbb{Z}_2$
χ_M

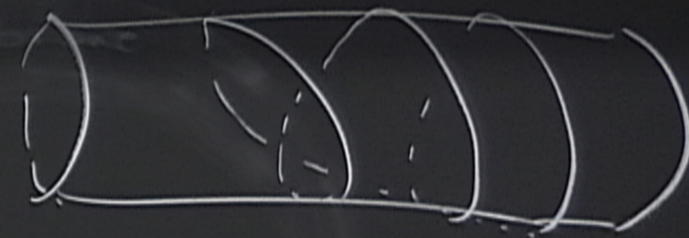

M-Theory
S^1_R
$S^1_R/\mathbb{Z}_2$
IIA
$g^{2/3} = R$
IIB
$E_8 \times E_8$
S^1_R
$S^1_{\frac{1}{R}}$
II

Torus Compactifications

Closed Bosonic String on $\mathbb{R}^{1,24} \times S^1_R$


$X^\mu(\tau, \sigma+\pi) = X^\mu(\tau, \sigma) + 2\pi R m$

$$X^{\mu}(\tau, \sigma + \pi) = X^{\mu}(\tau, \sigma) + 2\pi R m$$



$$\frac{1}{2}\left(P_L^2 - P_R^2\right) = n m$$

Momentum

$$X^{25}(\tau,\sigma+\pi) = X^{25}(\tau,\sigma) + 2\pi R m$$

$$\frac{1}{2}\left(P_L^2 - P_R^2\right) = n m$$

Momentum

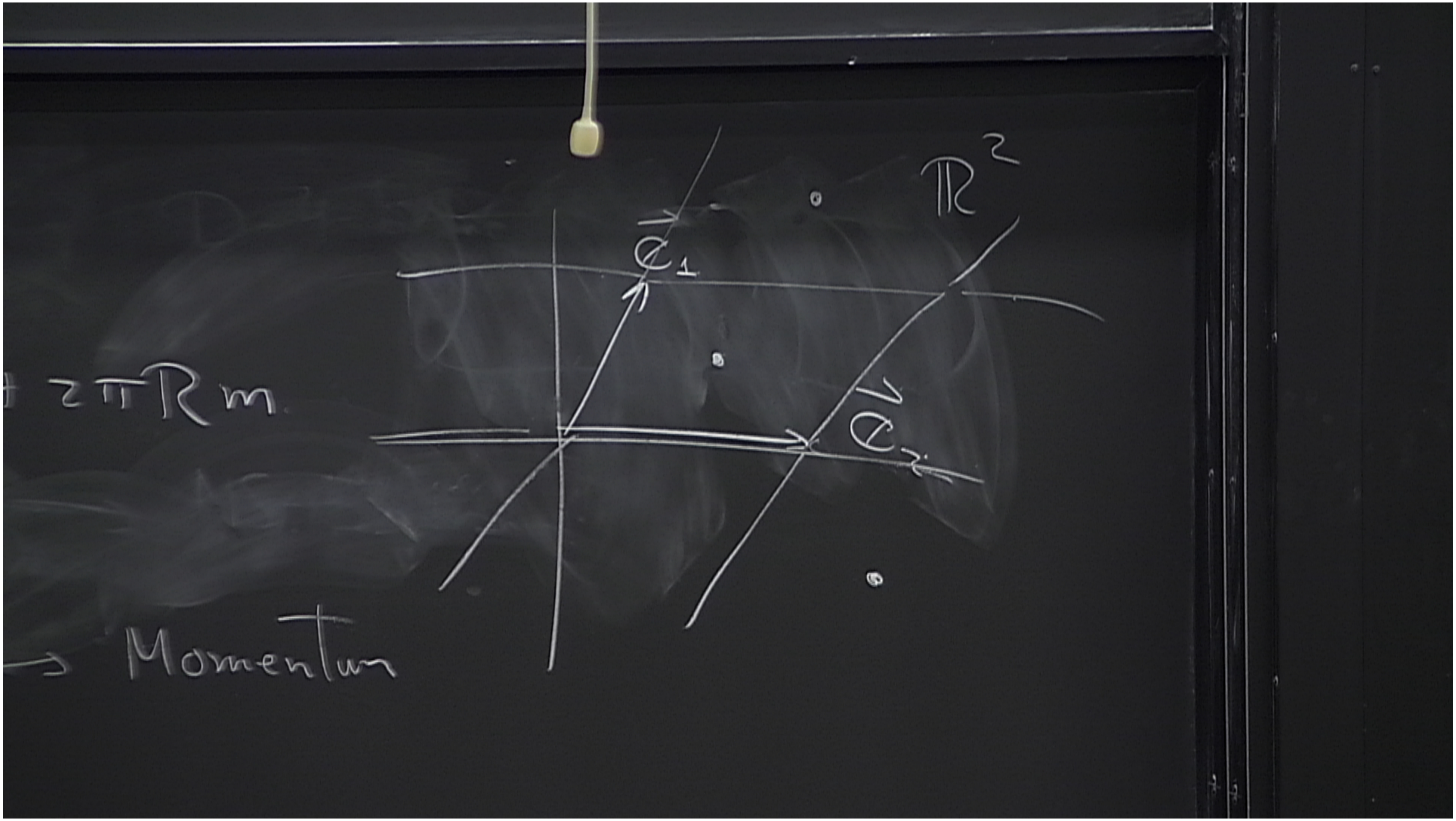
Torus Compactifications

Closed Bosonic String on $\mathbb{R}^{1,24} \times S^1_R$

More general: $X^I_{(\tau,\sigma+\pi)} = X^I_{(\tau,\sigma)} + 2\pi R\, m\, e^I$

$I = 1, \dots, d$

$T^d = (S^1)^d$


$2\pi R m.$
$\mathbb{R}^2$
$\vec{e}_1$
$\vec{e}_2$
Momentum

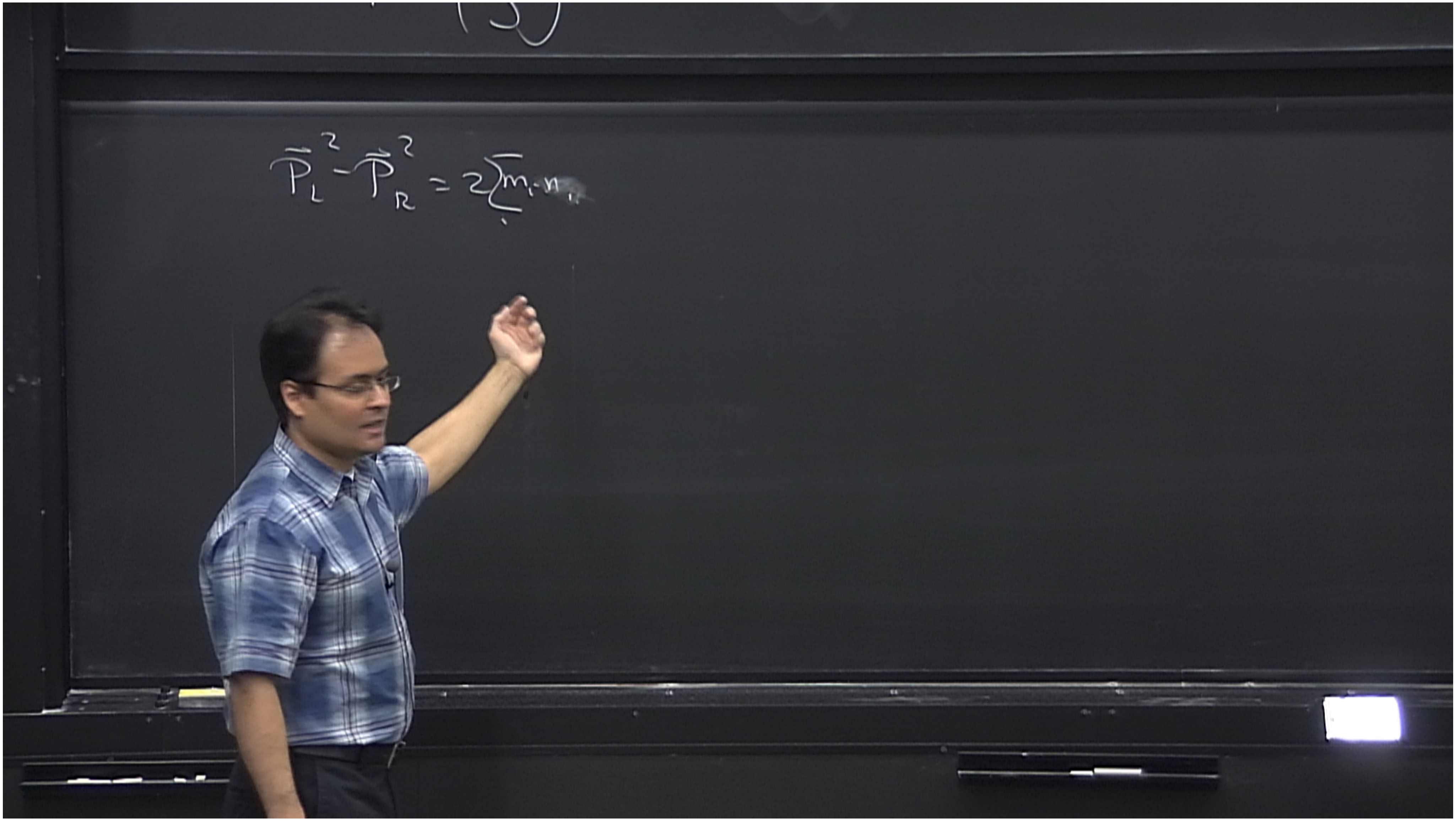
Torus Compactifications

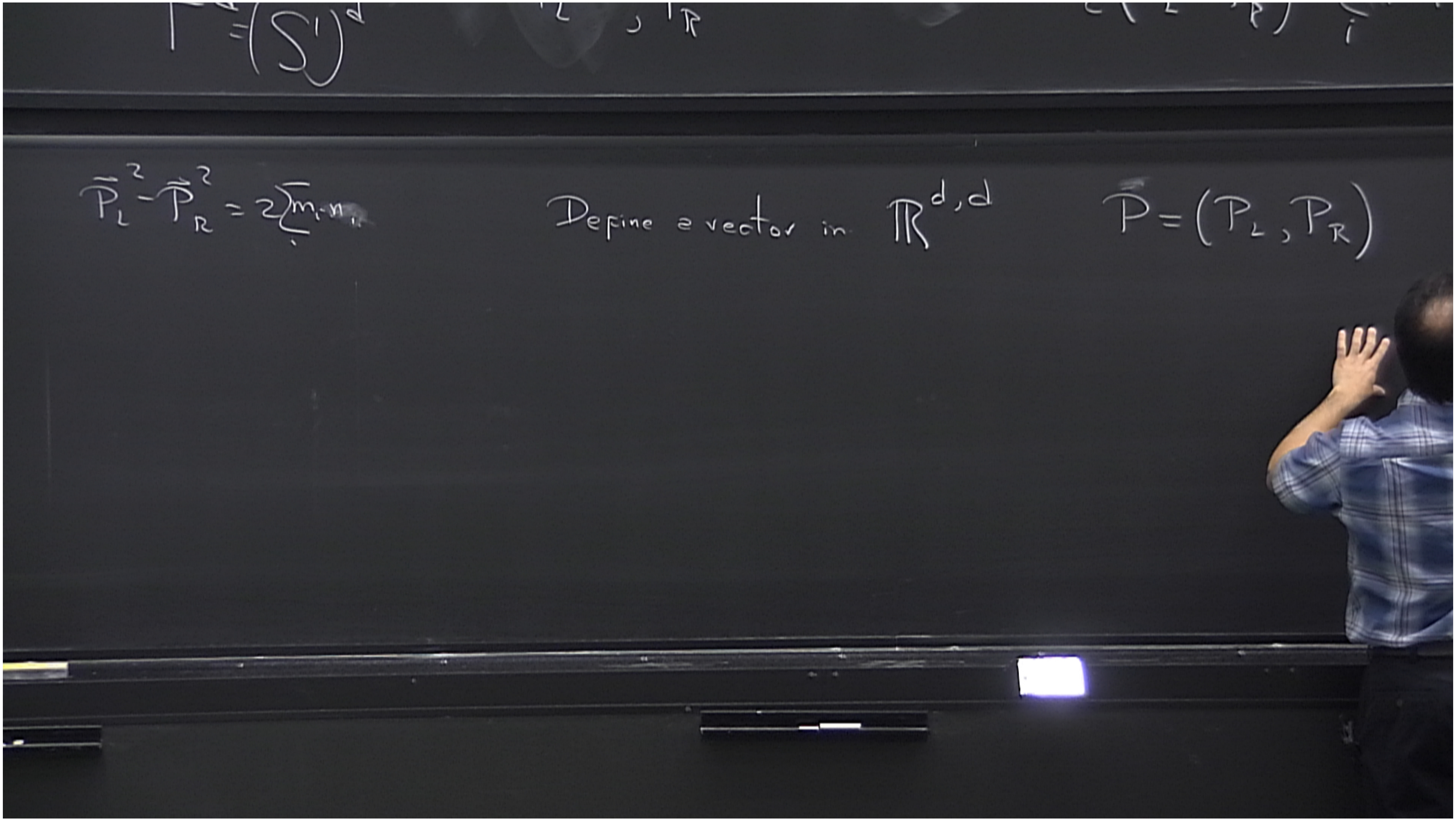
Closed Bosonic String on $\mathbb{R}^{1,24} \times S^1_R$

More general: $\vec{X}(\tau, \sigma+\pi) = \vec{X}(\tau,\sigma) + 2\pi R \sum_{i=1}^{d} m_i \vec{e}_i$

$I = 1, \ldots, d$

$T^d = (S^1)^d$


$\vec{P}_L^2 - \vec{P}_R^2 = 2\sum_i m_i n_i$

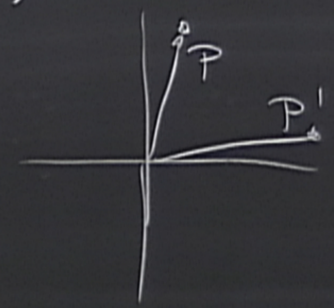

$=(S^1)^d$
$\vec{P}_L^2 - \vec{P}_R^2 = 2\sum_i m_i n_i$
Define a vector in $\mathbb{R}^{d,d}$
$\vec{P} = (P_L, P_R)$

$$\text{metric} = (\underbrace{1, 1, \ldots, 1}_{d}, \underbrace{-1, -1, \ldots, -1}_{d})$$

P_L, P_R $\sum (P_L - P_R) = \sum_i n_i m_i$

Define a vector in $\mathbb{R}^{d,d}$ $\vec{P} = (P_L, P_R)$ metric $= (\underbrace{1,1,\dots,1}_{d}, \underbrace{-1,-1,\dots,-1}_{d})$

Lattices in $\mathbb{R}^{d,d}$ that are "even"



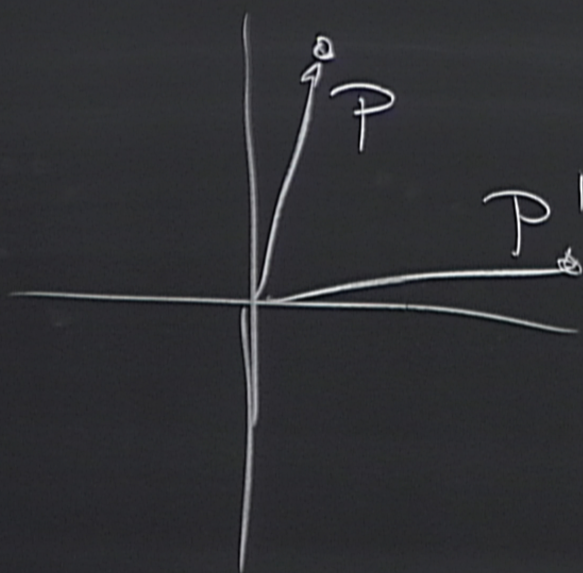
$P \cdot P' \in 2\mathbb{Z}$

$P_L, P_R)$

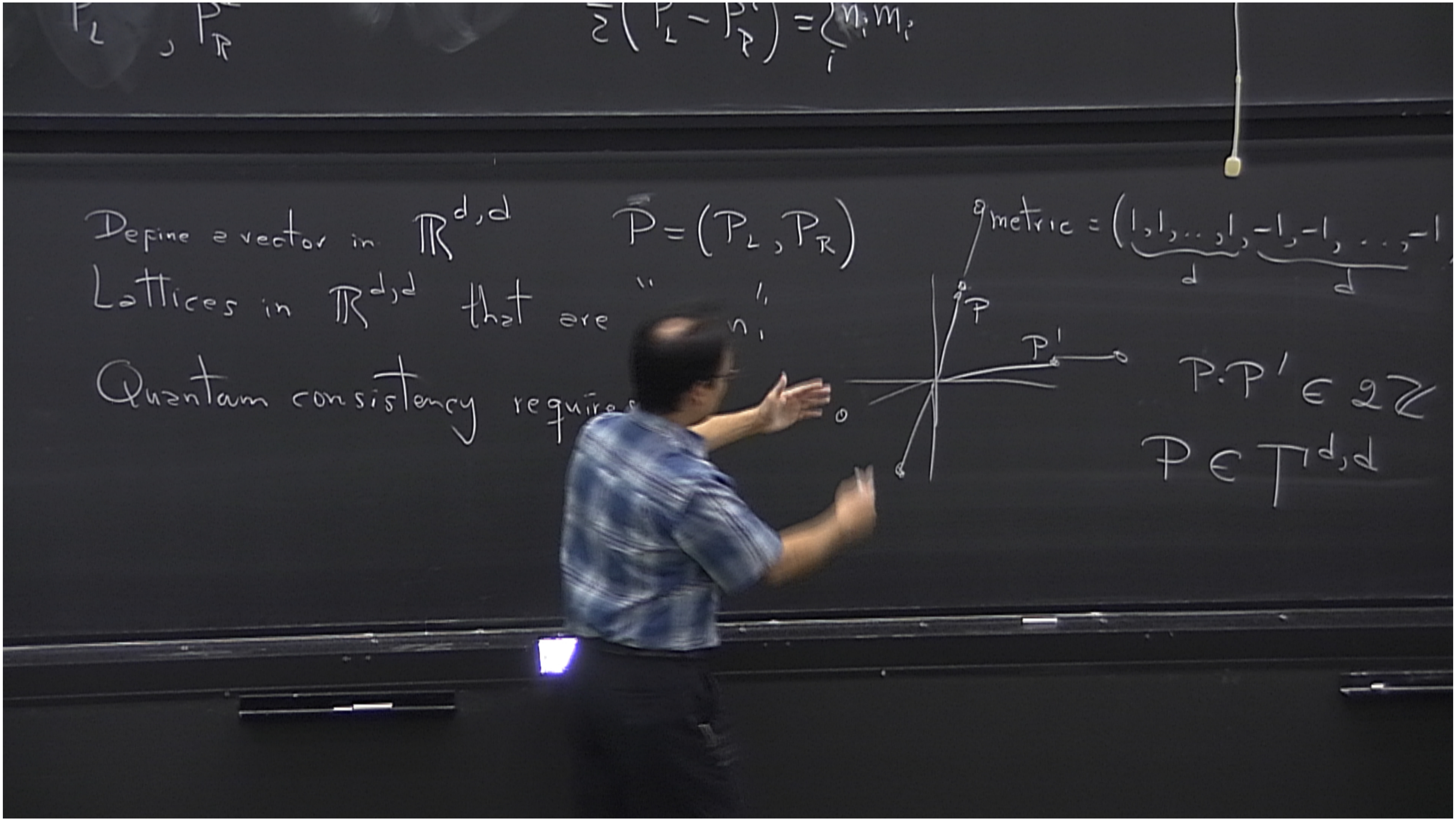
$l,$

$n,$

$$\text{metric} = (\underbrace{1, 1, \ldots, 1}_{d}, \underbrace{-1, -1, \ldots, -1}_{d})$$

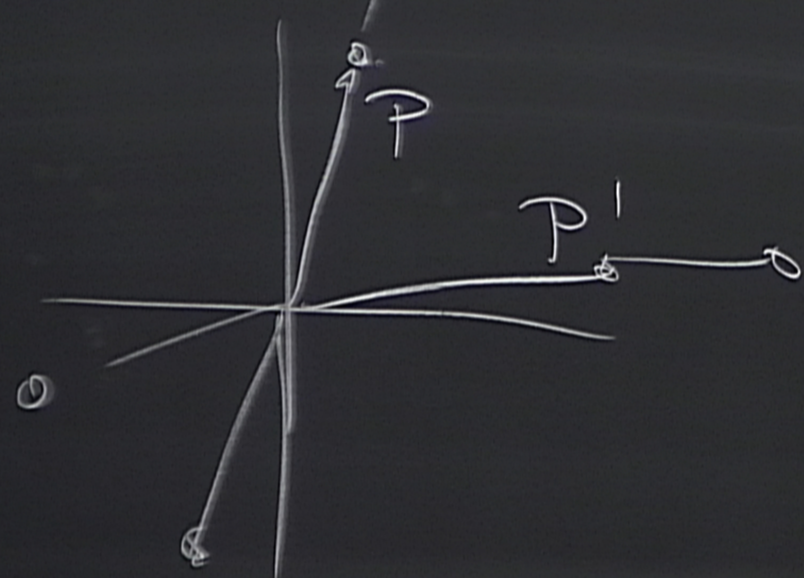


$$P \cdot P' \in 2\mathbb{Z}$$


Define a vector in ℝ^{d,d} P = (P_L , P_R)
metric = (1, 1, . . , 1, −1, −1, . . . , −1)
d d
Lattices in ℝ^{d,d} that are
Quantum consistency requires
P′
P·P′ ∈ 2ℤ
P ∈ Γ^{d,d}

$, P_R)$

$$g_{\text{metric}} = (\underbrace{1,1,\dots,1}_{d},\underbrace{-1,-1,\dots,-1}_{d})$$



$$P \cdot P' \in 2\mathbb{Z}$$

$$P \in \Gamma^{d,d}$$

$$\vec{P}_L^{\,2} - \vec{P}_R^{\,2} = 2\sum_i m_i n_i$$

Γ = even & self dual

Define a vector

Lattices in $\mathbb{R}$

Quantum cons

Γ

$$\vec{P}_L^{\,2} - \vec{P}_R^{\,2} = 2\sum_i m_i n_i$$

Γ = even & self dual

Γ^{d_1, d_2}

even & self dual

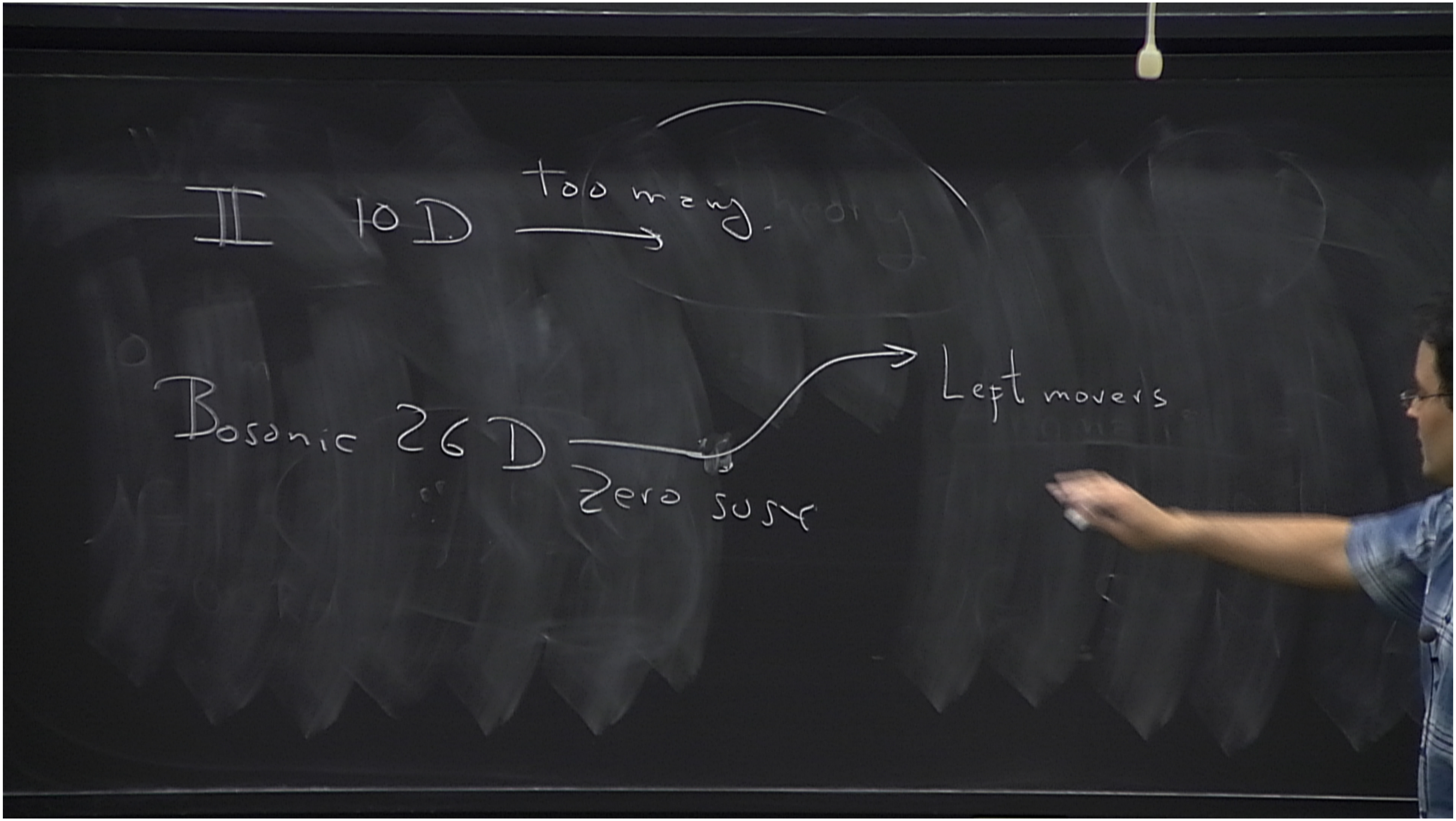
$$d_1 - d_2 \equiv 0 \mod 8.$$

Define a vector

Lattices in $\mathbb{R}$

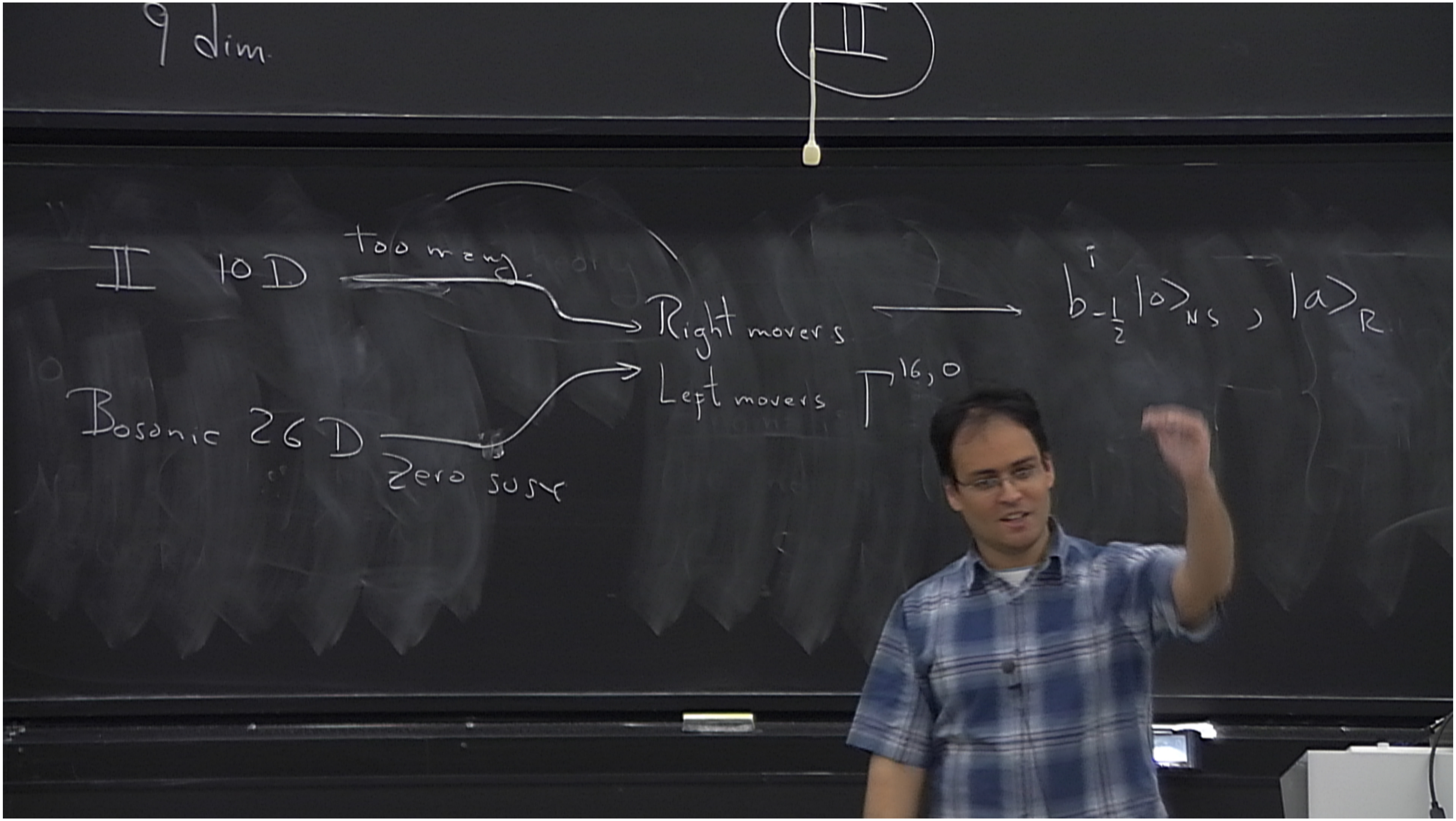
Quantum cons

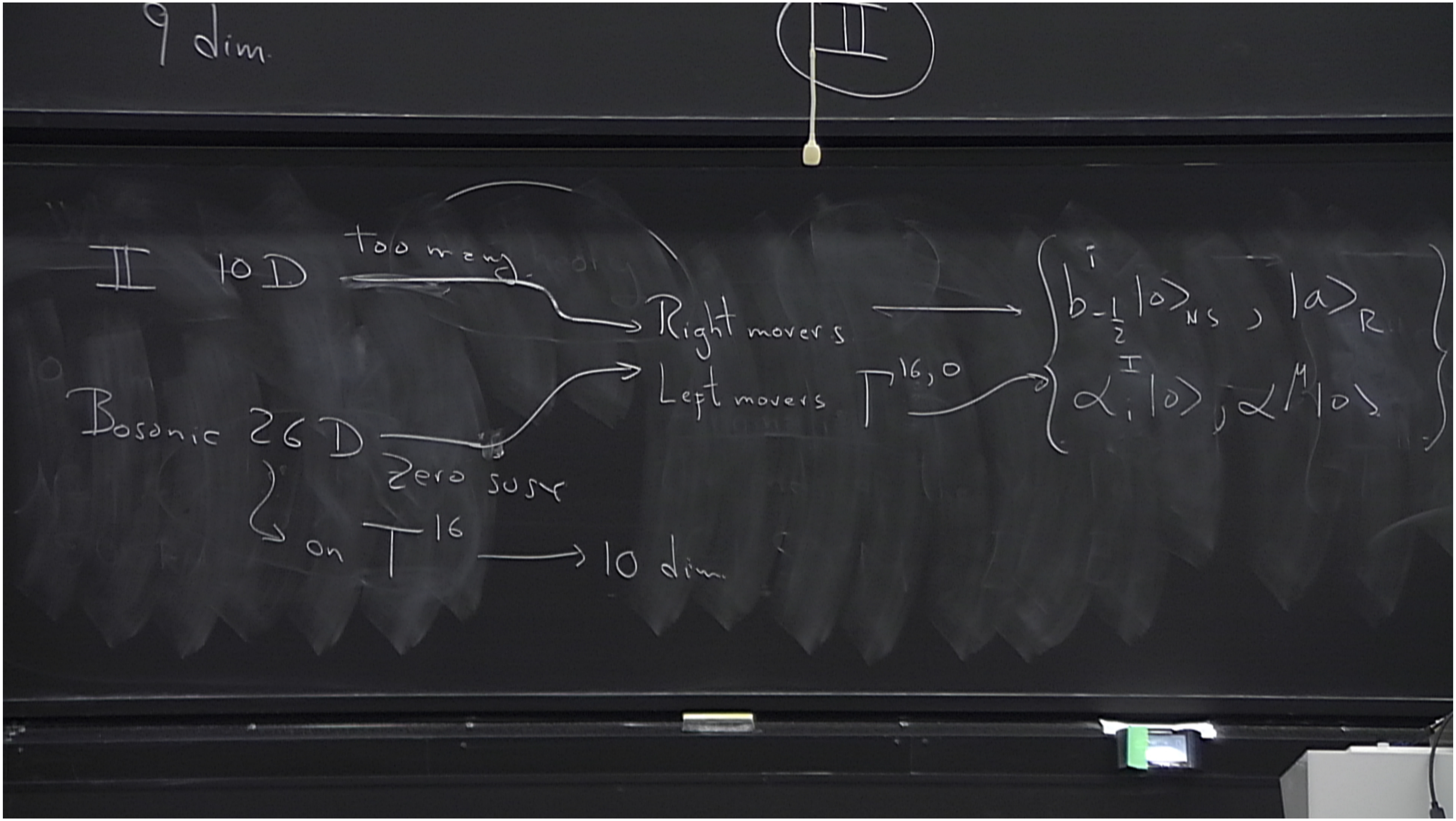
Γ

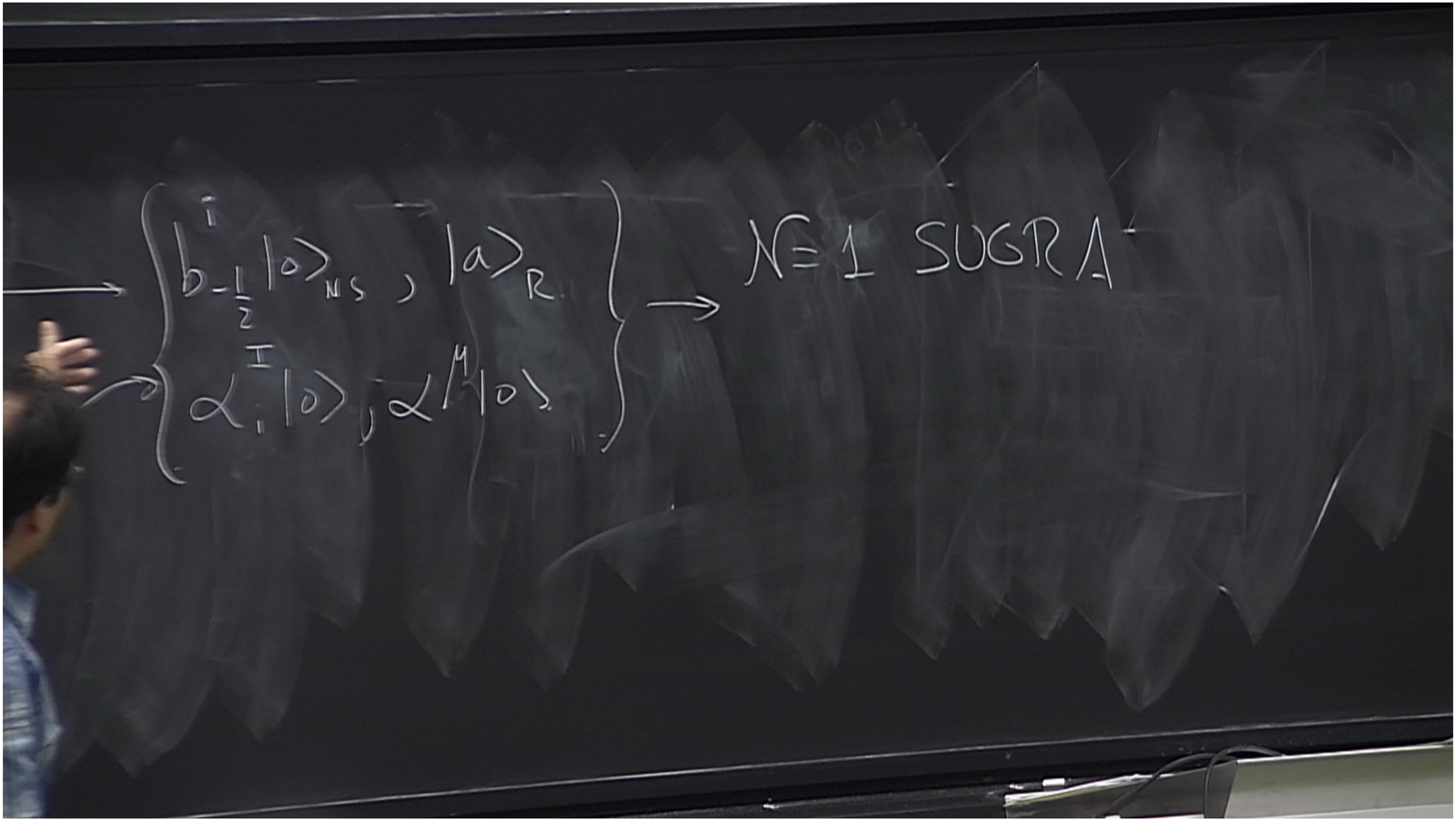

II 10D
too many.
Bosonic 26D
Zero susy
Left movers

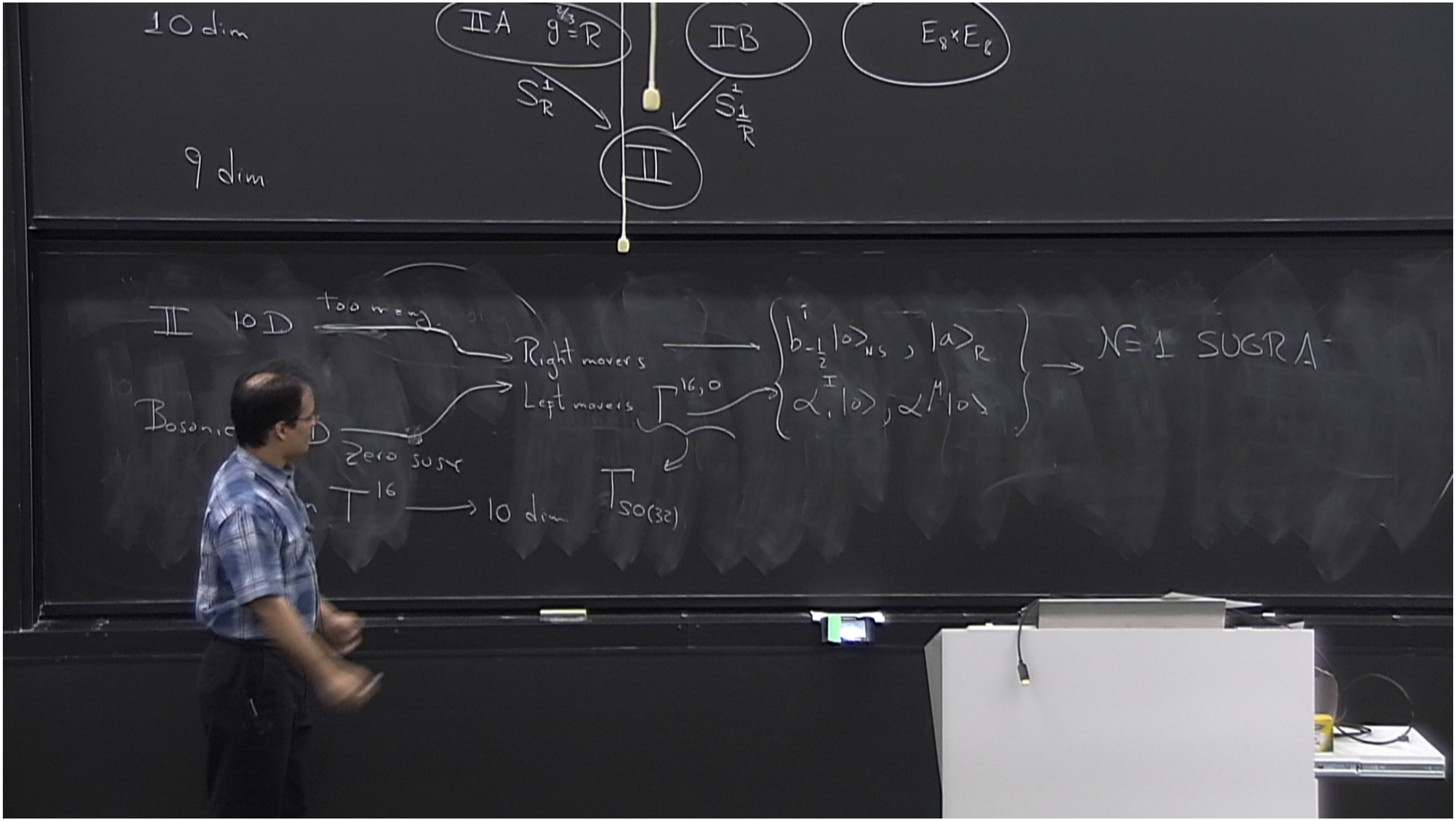
II 10D $\xrightarrow{\text{too many}}$

Bosonic 26D $\xrightarrow[\text{Zero susy}]{}$ Left movers $\Gamma^{16,0}$

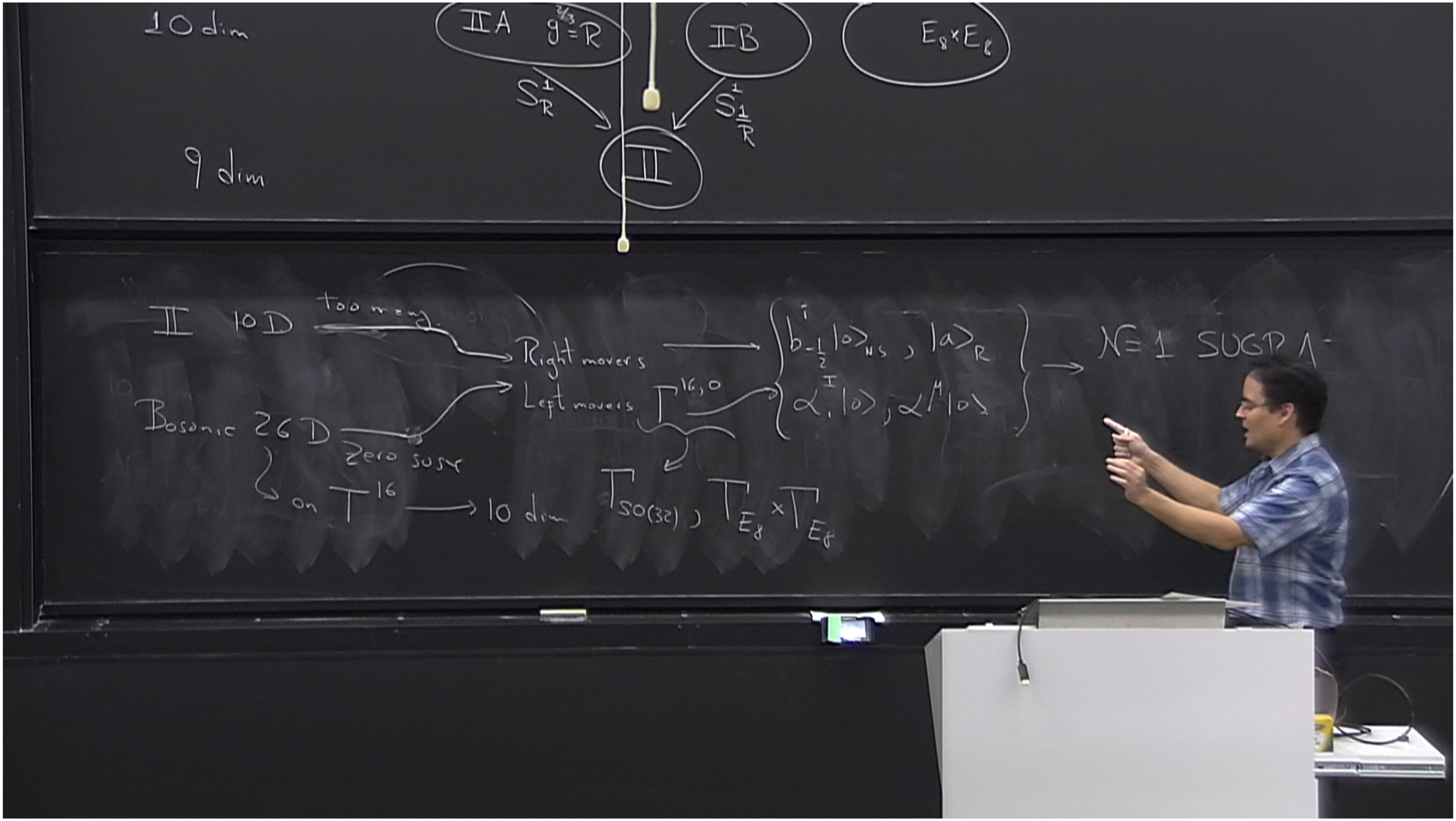

9 dim
II
II 10D
too many
Right movers
$b^i_{-\frac{1}{2}}|0\rangle_{NS}$, $|a\rangle_R$
Left movers $\Gamma^{16,0}$
Bosonic 26D
Zero susy

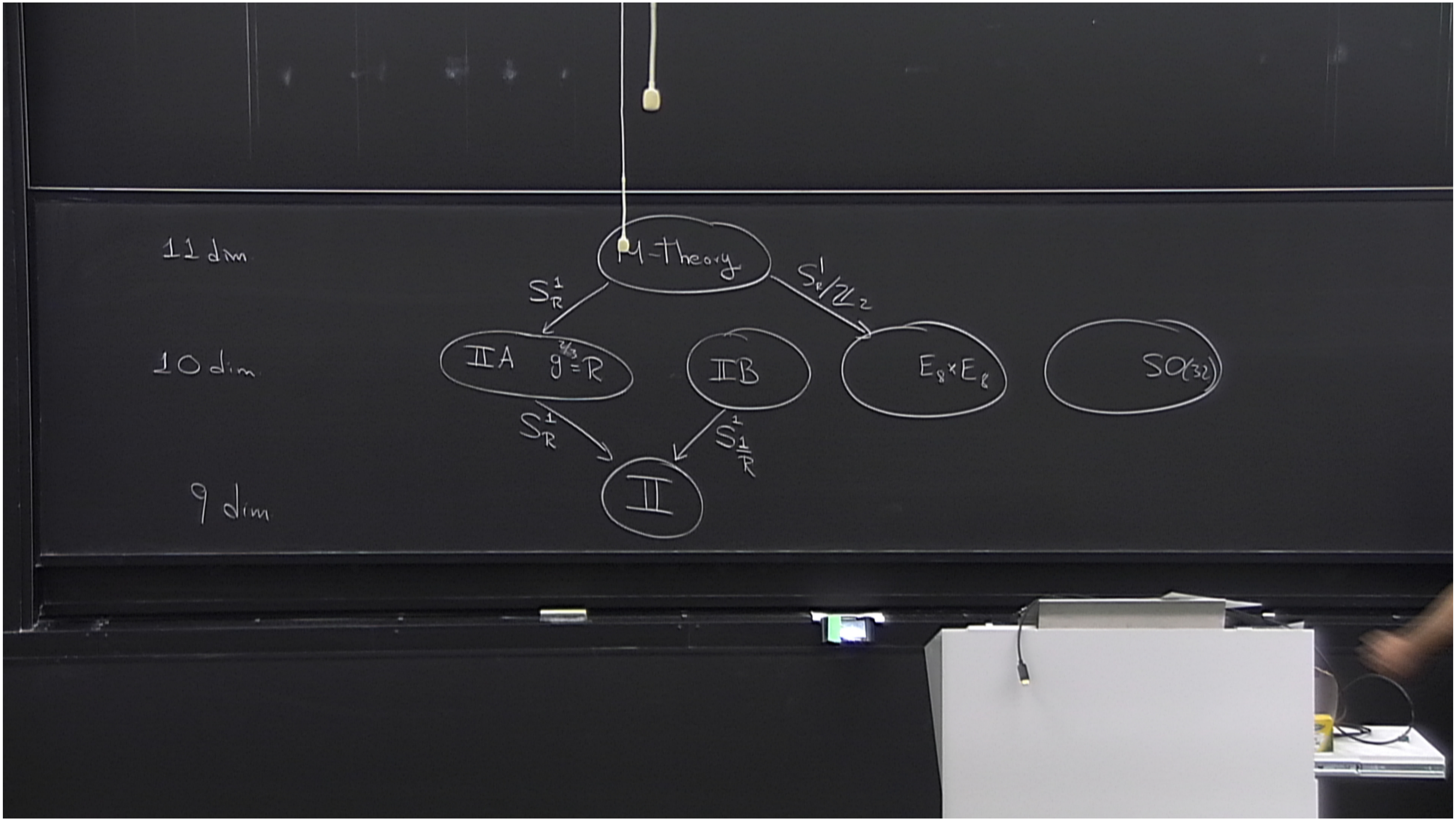

9 dim.
II
II 10D
too many
Right movers
Left movers $\Gamma^{16,0}$
$b^{i}_{-\frac{1}{2}}|0\rangle_{NS}$, $|a\rangle_{R}$
$\alpha^{I}_{-1}|0\rangle$, $\alpha^{\mu}|0\rangle$
Bosonic 26D
Zero susy
on T^{16}
10 dim.


$b^{i}_{-\frac{1}{2}}|0\rangle_{NS}$, $|a\rangle_{R}$
$\alpha^{I}_{-1}|0\rangle$, $\alpha^{\mu}|0\rangle$
N = 1 SUGRA

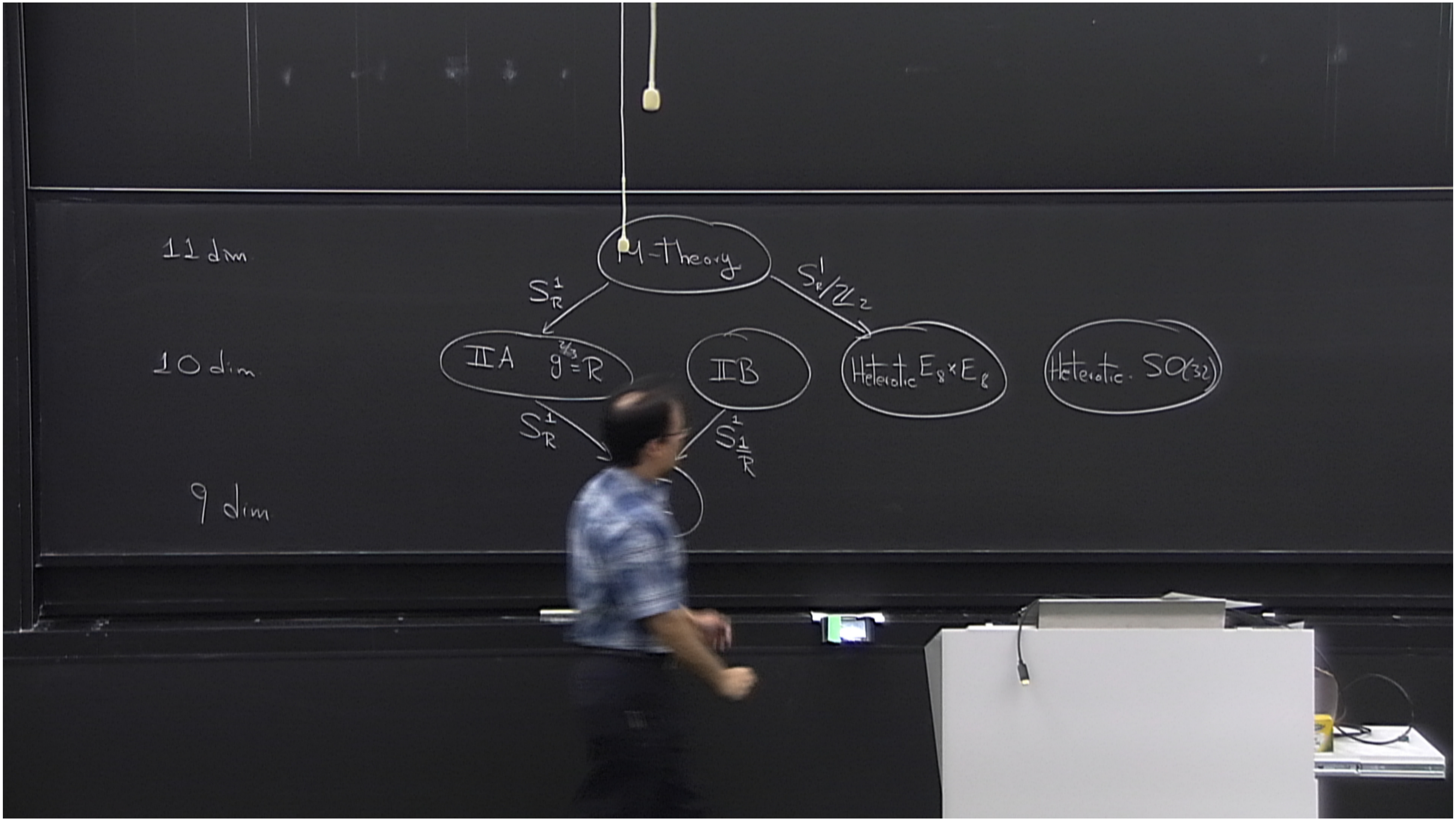


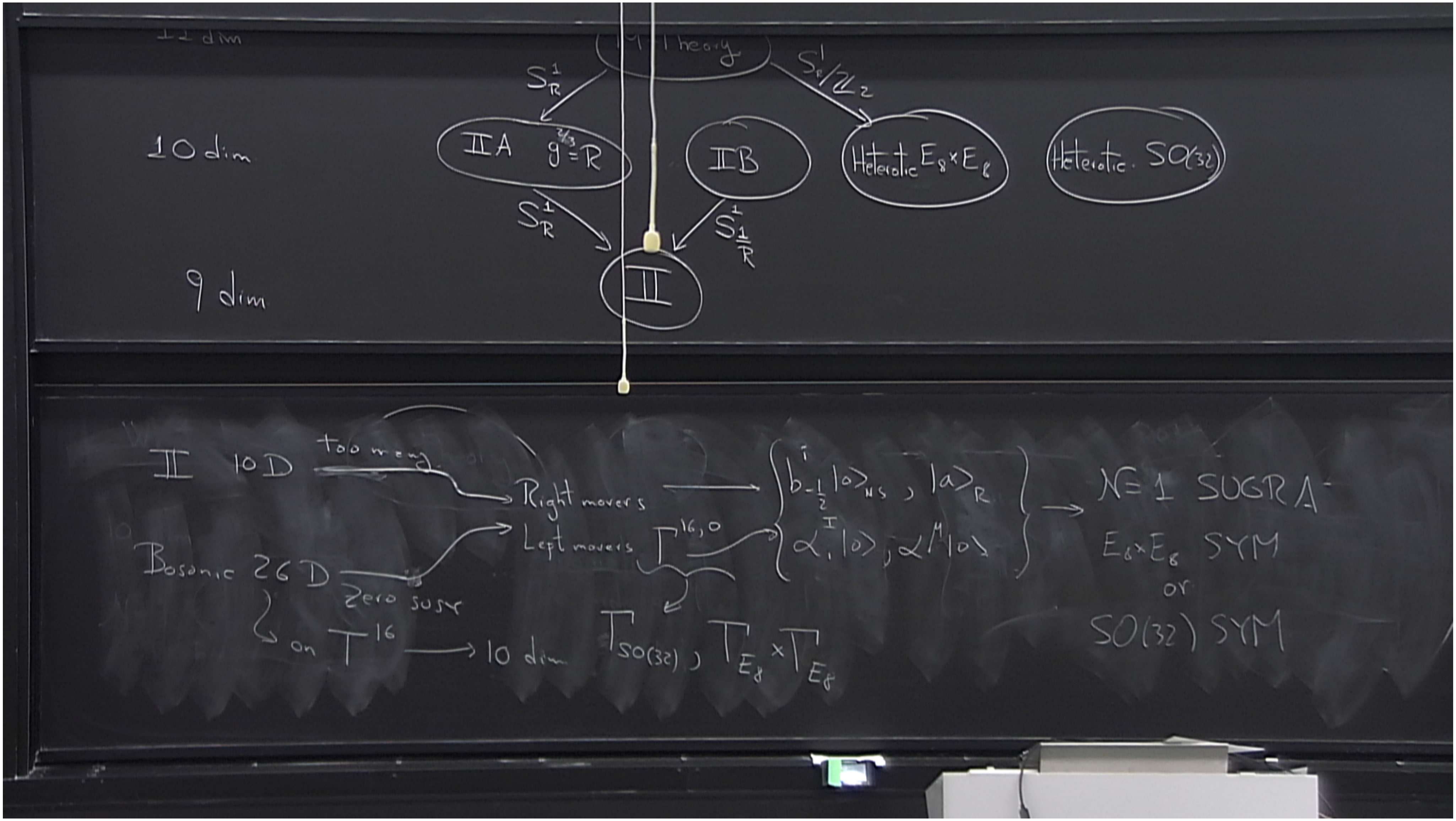
10 dim
IIA $g^{2/3} = R$
IIB
$E_8 \times E_8$
S^1_R
$S^1_{\frac{1}{R}}$
9 dim
II
II 10D
too many
Right movers
Left movers $\Gamma^{16,0}$
$b^i_{-\frac{1}{2}}|0\rangle_{NS}$, $|a\rangle_R$
$\alpha^I_{-1}|0\rangle$, $\alpha^\mu|0\rangle$
$N = 1$ SUGRA
Bosonic
Zero susy
T^{16}
10 dim
$\Gamma_{SO(32)}$


10 dim
IIA
IIB
E8×E8
9 dim
II
II 10D
too many
Right movers
Left movers
Bosonic 26D
zero susy
on T 16
10 dim
N=1 SUGRA



11 dim
10 dim
9 dim
M-Theory
S^1_R
$S^1_R/\mathbb{Z}_2$
IIA $g^{2/3}=R$
IIB
$E_8\times E_8$
SO(32)
S^1_R
$S^1_{\frac{1}{R}}$
II

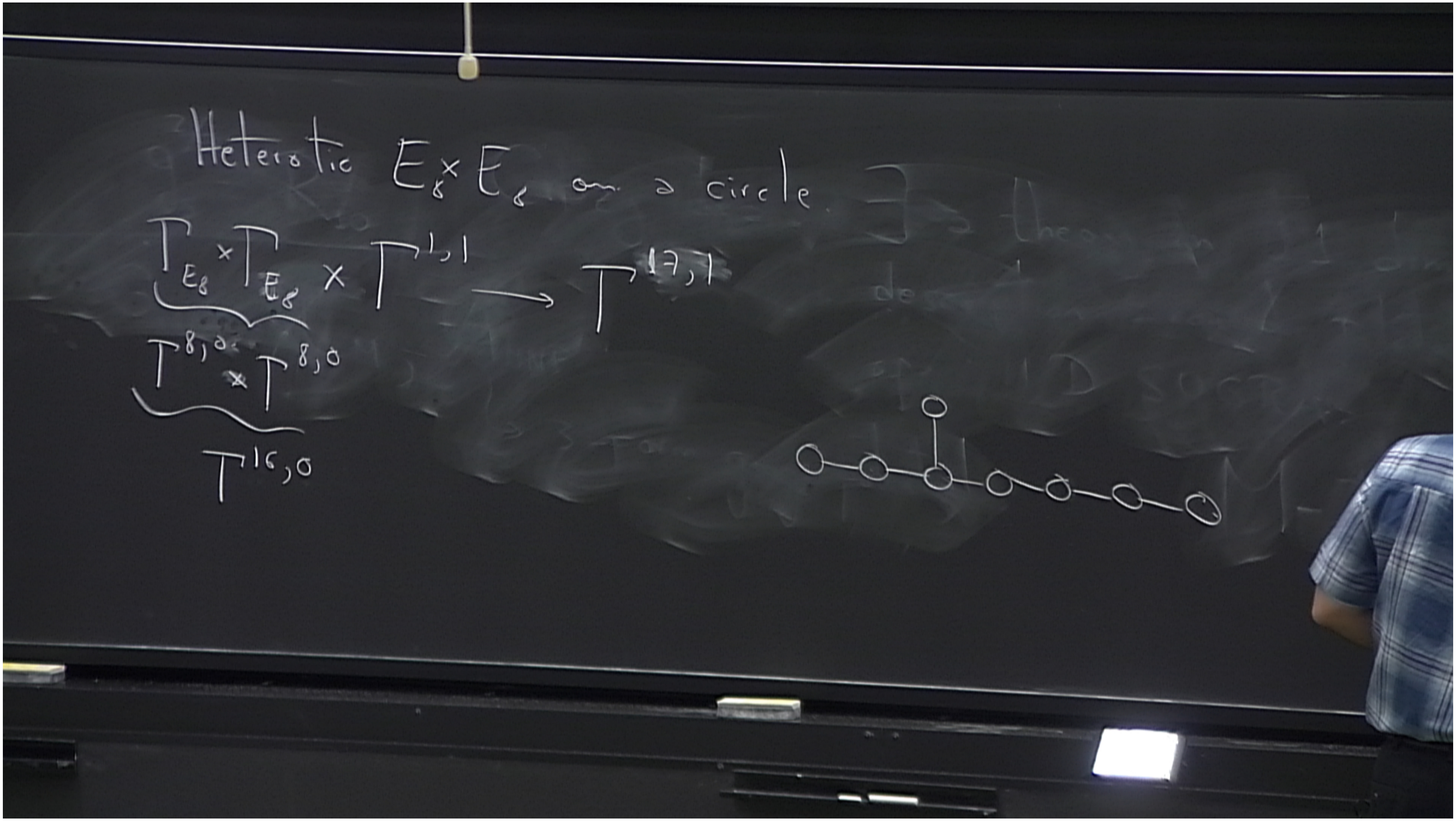

11 dim
M-Theory
10 dim
IIA
IIB
Heterotic E8 x E8
Heterotic SO(32)
9 dim

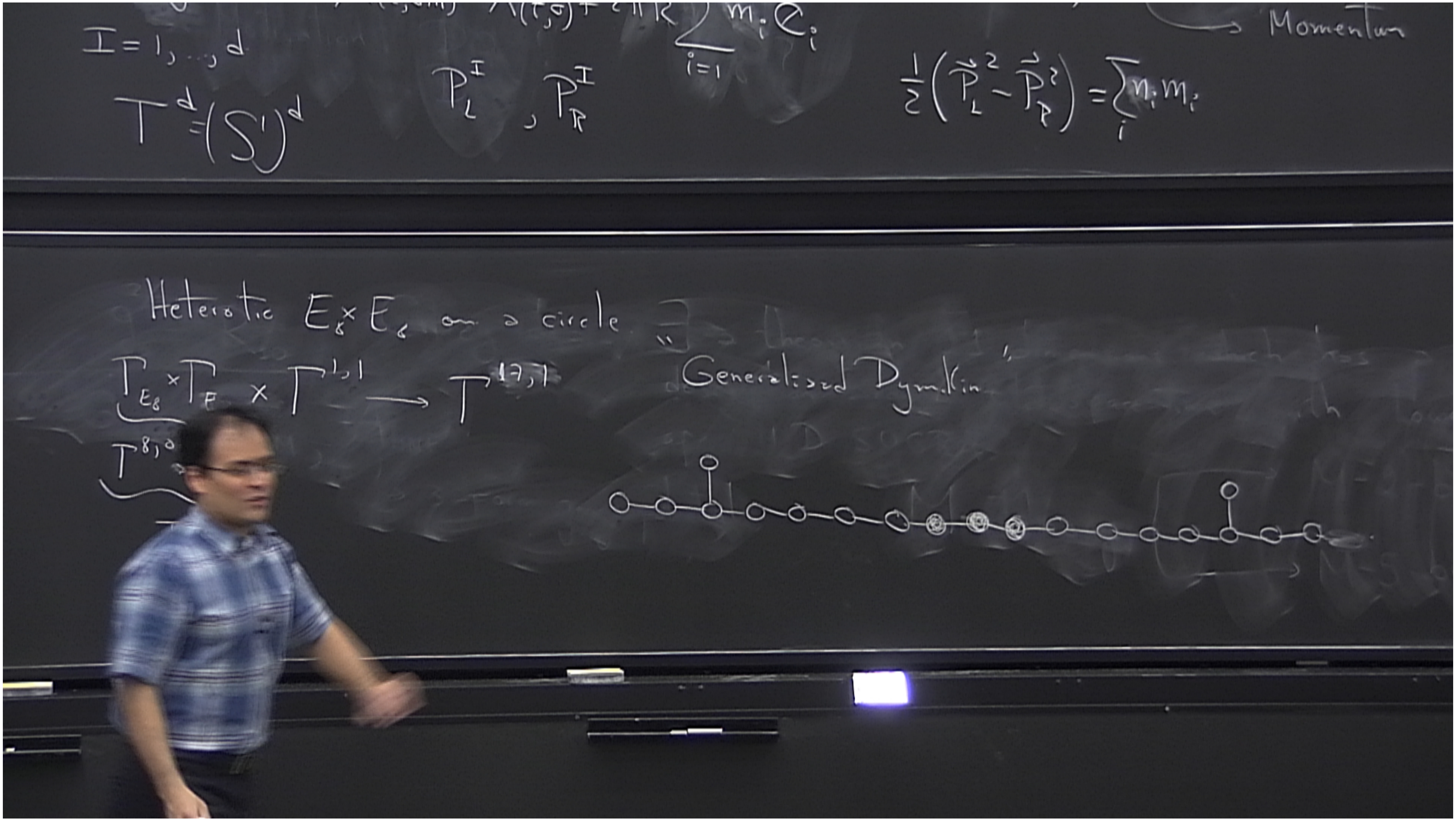

M-Theory
S^1_R
$S^1/\mathbb{Z}_2$
10 dim
IIA $g^{2/3}=R$
IIB
Heterotic $E_8\times E_8$
Heterotic $SO(32)$
S^1_R
$S^1_{1/R}$
9 dim
II
II 10D
too many
Right movers
Left movers $\Gamma^{16,0}$
Bosonic 26D
Zero susy
on T^{16}
10 dim
$\Gamma_{SO(32)}$, $\Gamma_{E_8}\times\Gamma_{E_8}$
$b^i_{-\frac{1}{2}}|0\rangle_{NS}$, $|a\rangle_R$
$\alpha^I_{-1}|0\rangle$, $\alpha^M_{-1}|0\rangle$
$\mathcal{N}=1$ SUGRA
$E_8\times E_8$ SYM
or
$SO(32)$ SYM

Heterotic $E_8 \times E_8$ on a circle

$$\underbrace{\Gamma_{E_8} \times \Gamma_{E_8}}_{} \times \Gamma^{1,1} \longrightarrow \Gamma^{17,1}$$

$$\underbrace{\Gamma^{8,0} \times \Gamma^{8,0}}_{\Gamma^{16,0}}$$


Heterotic $E_8 \times E_8$ on a circle
$\underbrace{\Gamma_{E_8} \times \Gamma_{E_8}} \times \Gamma^{1,1} \longrightarrow \Gamma^{17,1}$
$\underbrace{\Gamma^{8,0} \times \Gamma^{8,0}}$
$\Gamma^{16,0}$


I = 1, ..., d
Momentum
T^d = (S^1)^d
P_L^I , P_R^I
1/2 (P_L^2 − P_R^2) = Σ_i n_i m_i
Heterotic E_8 × E_8 on a circle
Γ_{E_8} × Γ_{E_8} × Γ^{1,1} → Γ^{17,1}
"Generalized Dynkin"
Γ^{8,0}

$\Lambda(\tau,\sigma) + 2\pi R \sum_{i=1} m_i e_i$ → Momentum

$I = 1, \ldots, d$

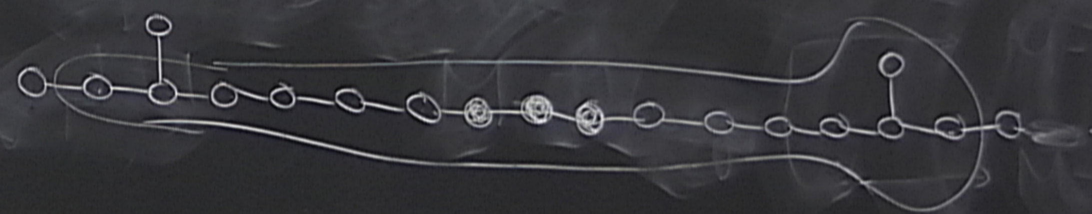
$T^d = (S^1)^d$ $\qquad P_L^I , P_R^I$ $\qquad \frac{1}{2}\left(\vec{P}_L^2 - \vec{P}_R^2\right) = \sum_i n_i m_i$

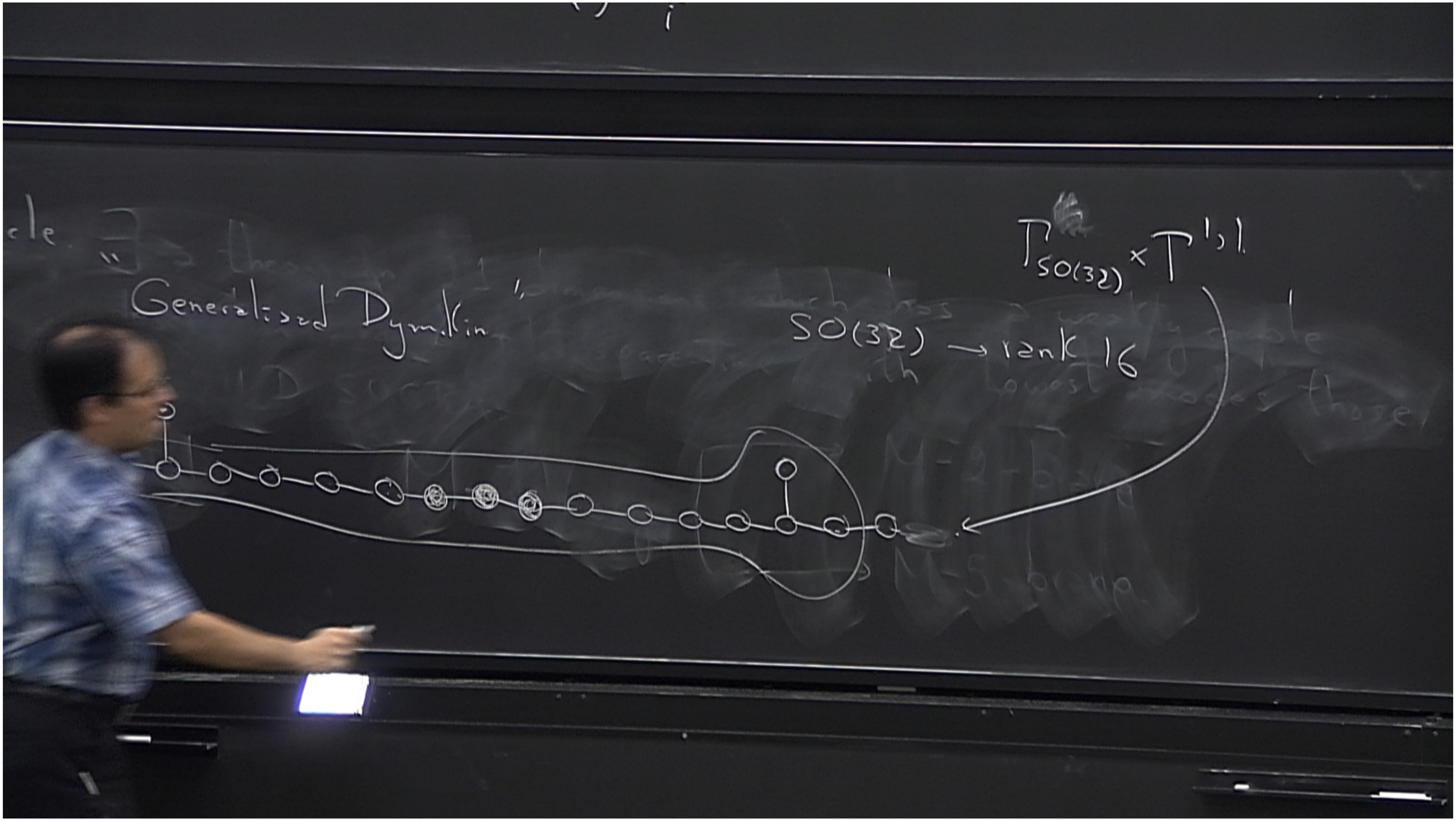
Heterotic $E_8 \times E_8$ on a circle

$\underbrace{\underbrace{\Gamma_{E_8} \times \Gamma_{E_8}}_{\Gamma^{8,0} \times \Gamma^{8,0}}}_{\Gamma^{16,0}} \times \Gamma^{1,1} \longrightarrow \Gamma^{17,1}$

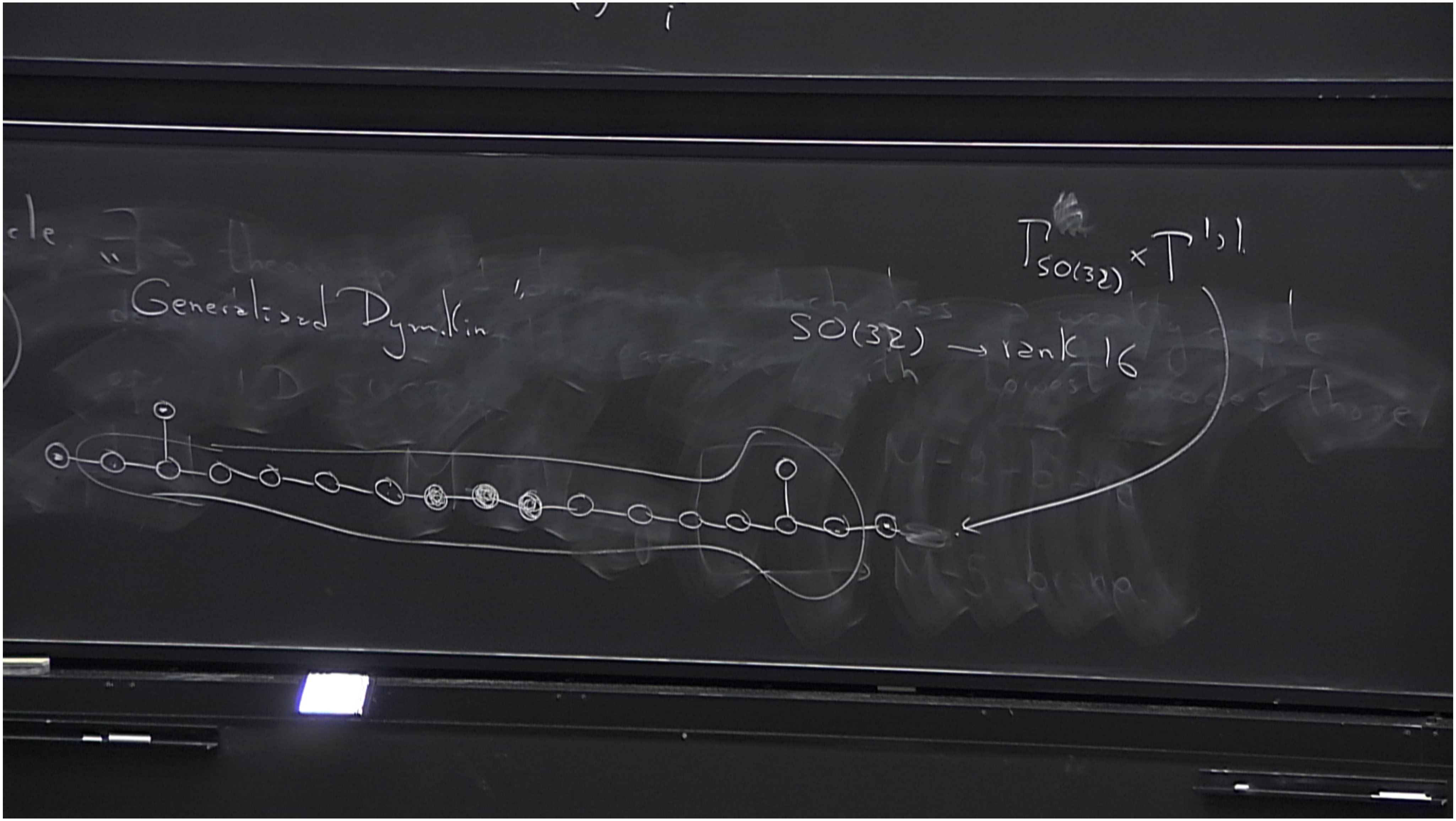
"Generalized Dynkin"

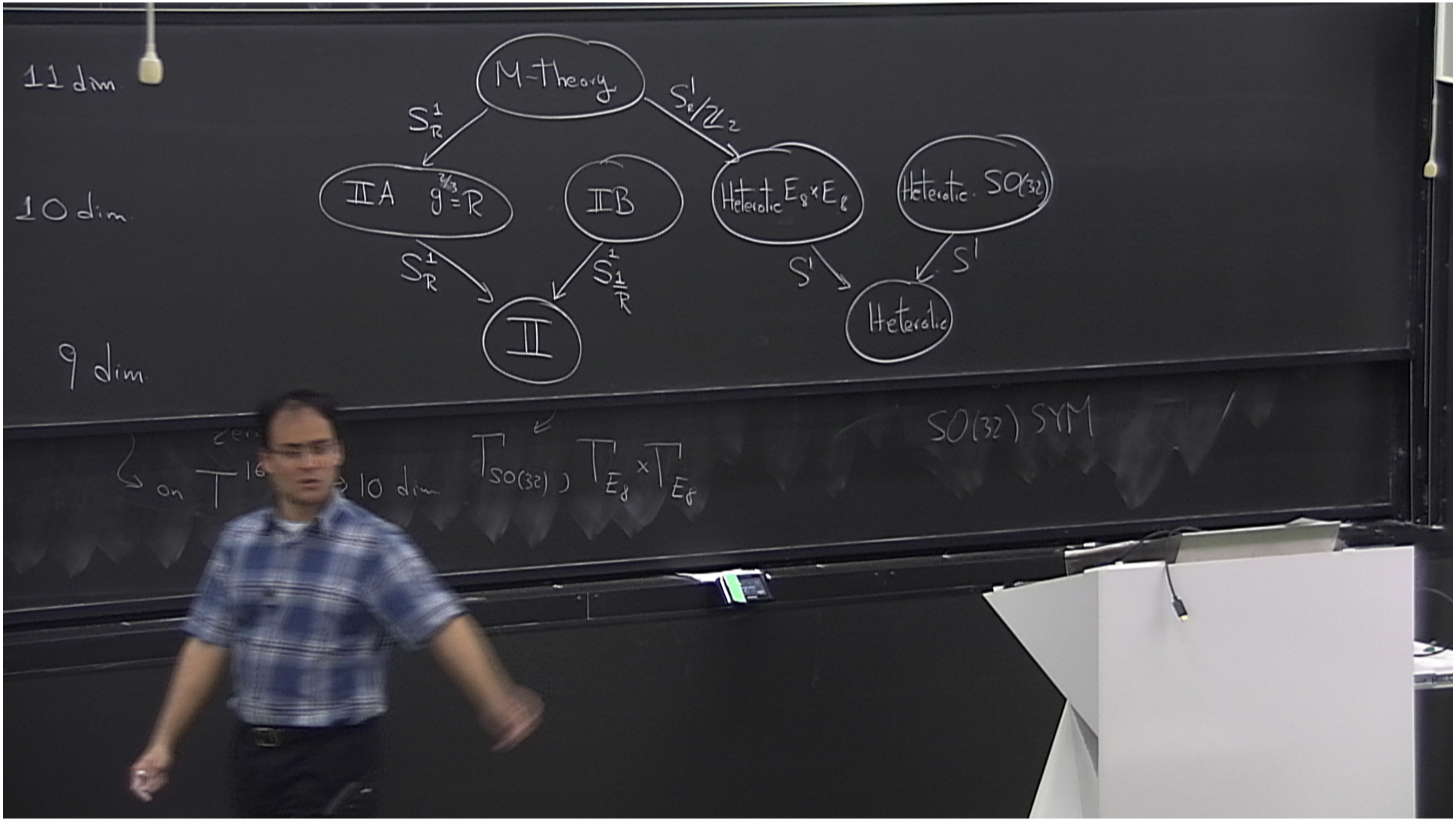
SO(32) → 12n



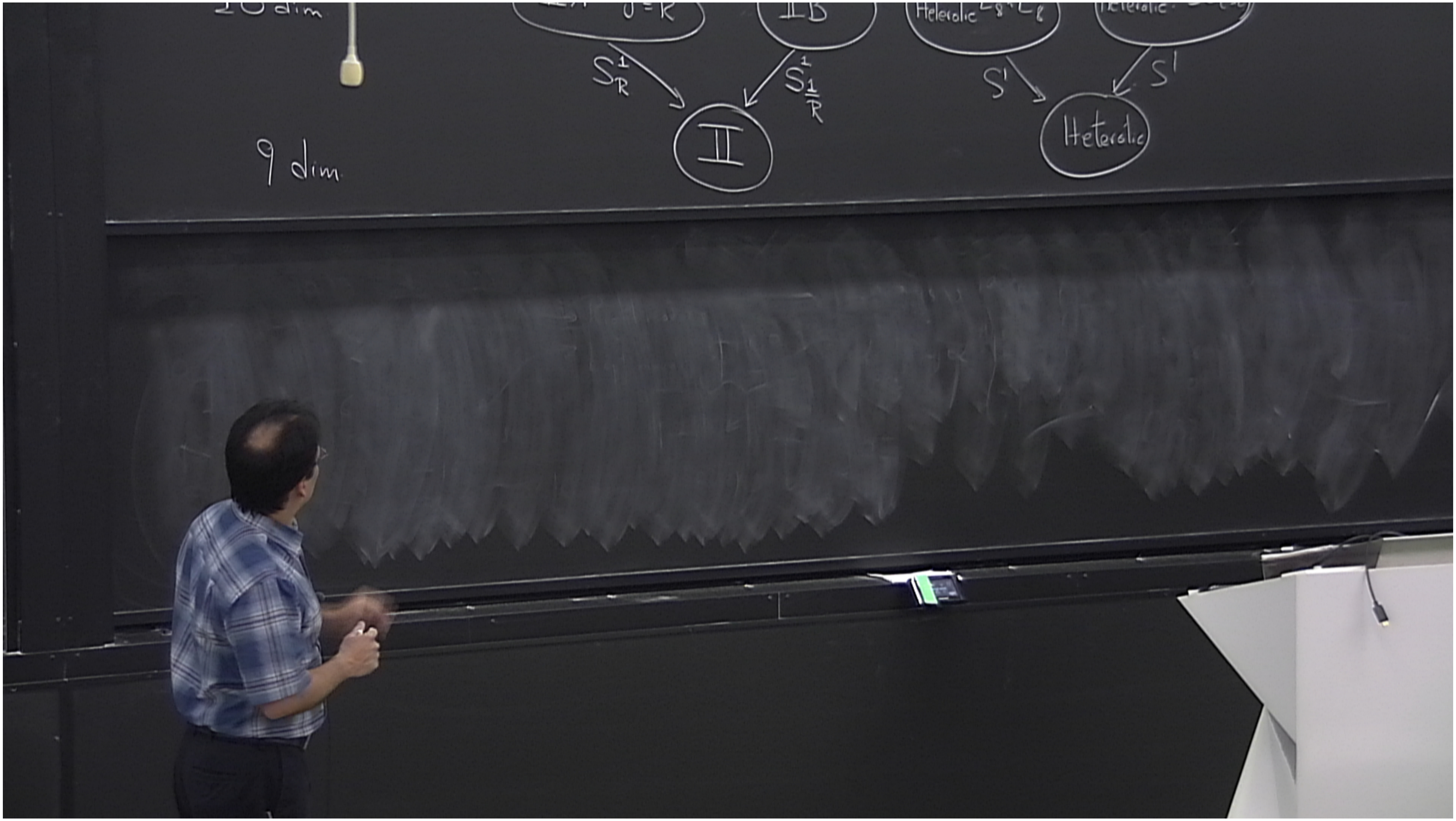


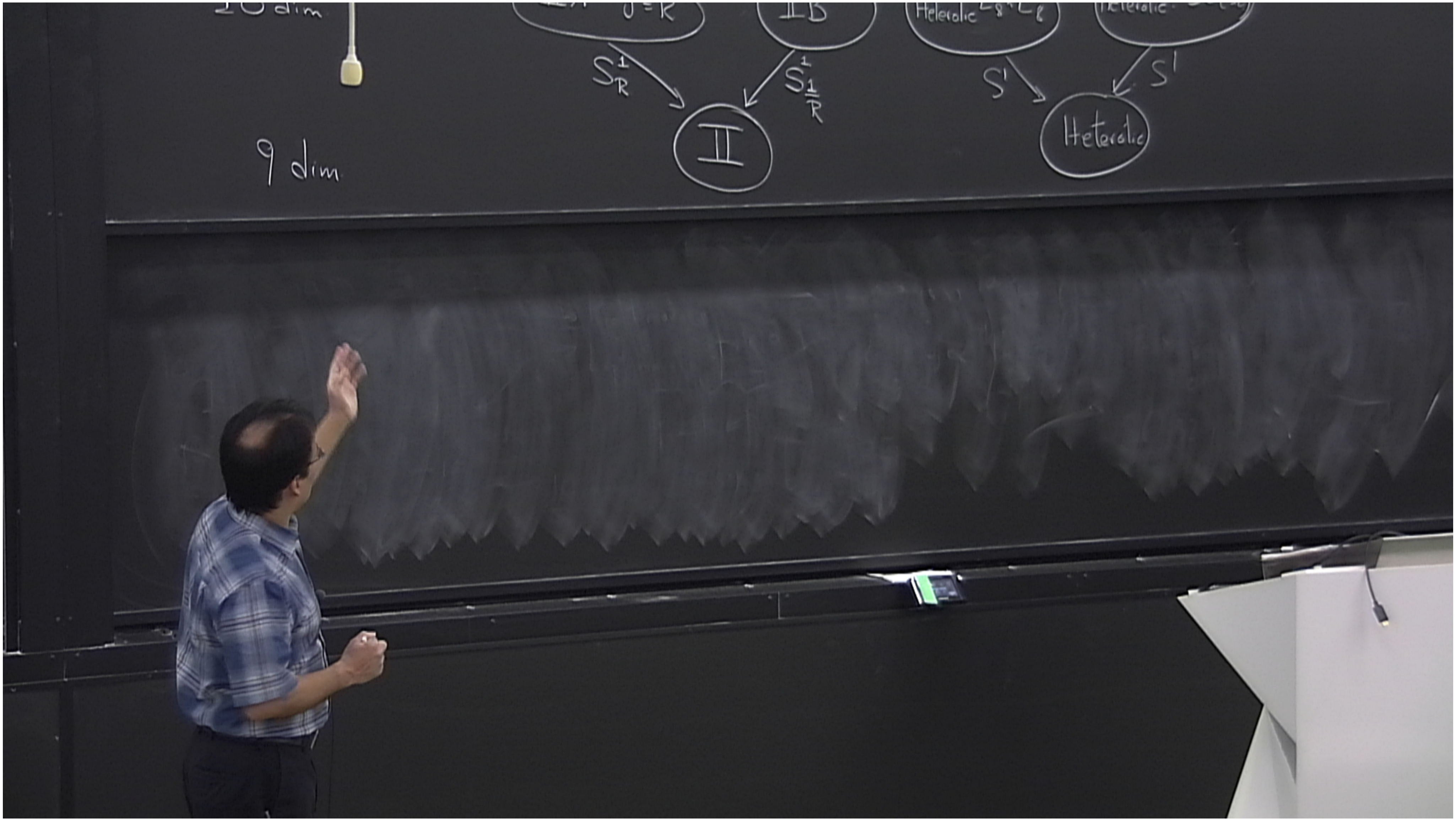
"Generalised Dynkin"
T SO(32) × T 1,1
SO(32) → rank 16

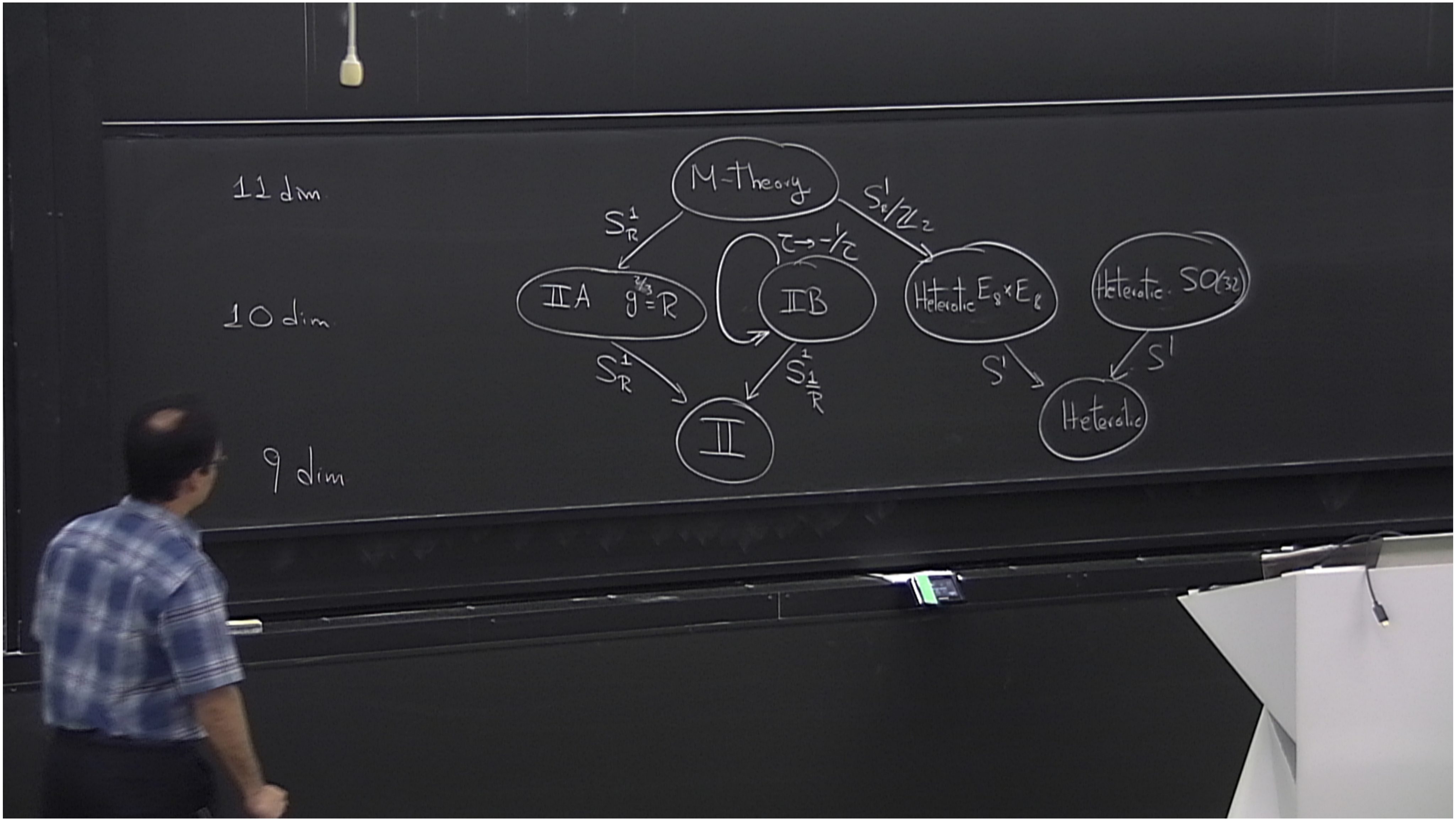

"Generalised Dynkin"
$T_{SO(32)} \times T^{1,1}$
SO(32) → rank 16



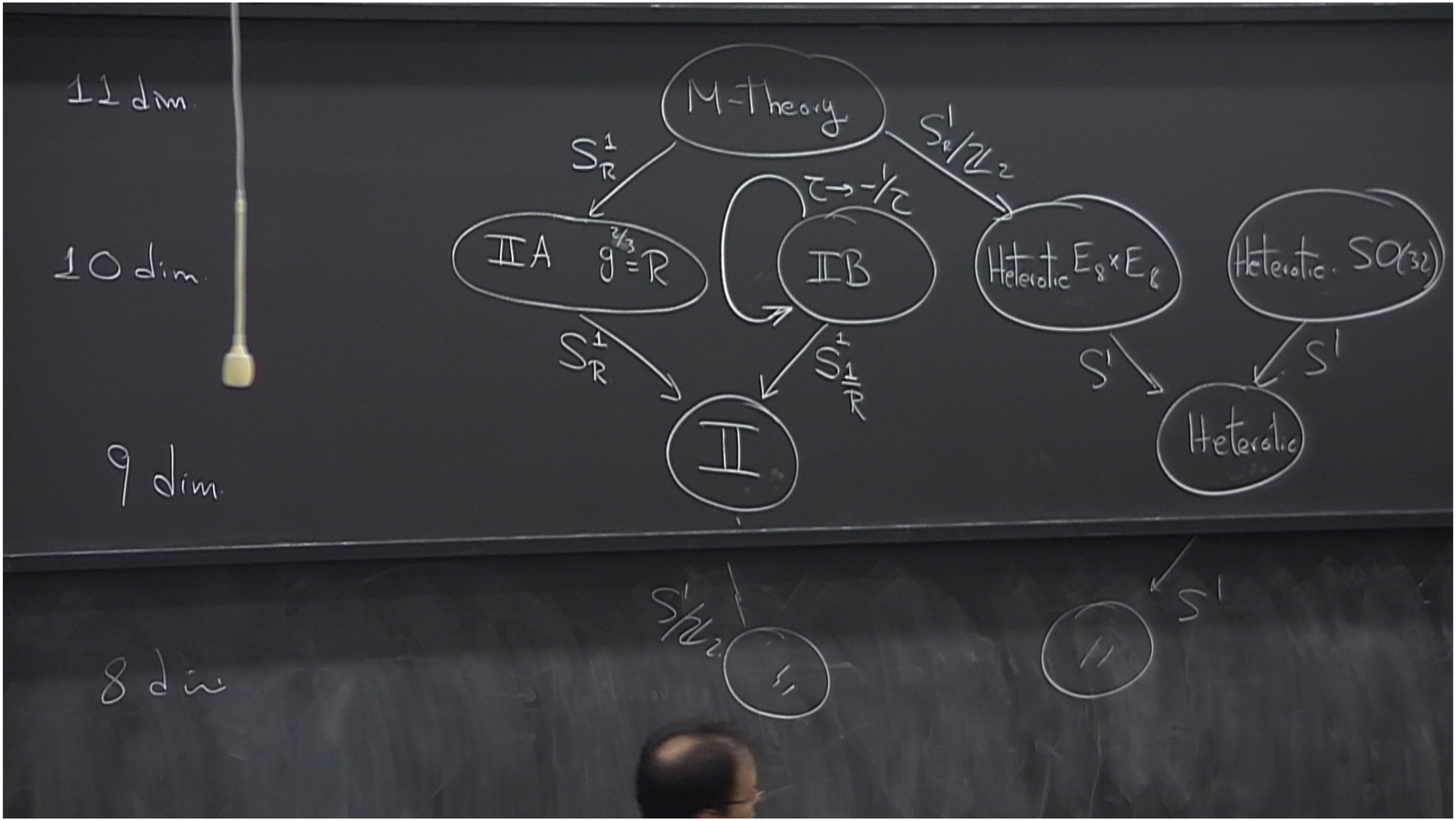
11 dim
10 dim
9 dim
M-Theory
S^1_R
$S^1_R/\mathbb{Z}_2$
IIA $g^{2/3}=R$
IIB
Heterotic $E_8\times E_8$
Heterotic SO(32)
S^1_R
$S^1_{\frac{1}{R}}$
S^1
S^1
II
Heterotic
SO(32) SYM
on T^{16} 10 dim
$\Gamma_{SO(32)}$, $\Gamma_{E_8}\times\Gamma_{E_8}$

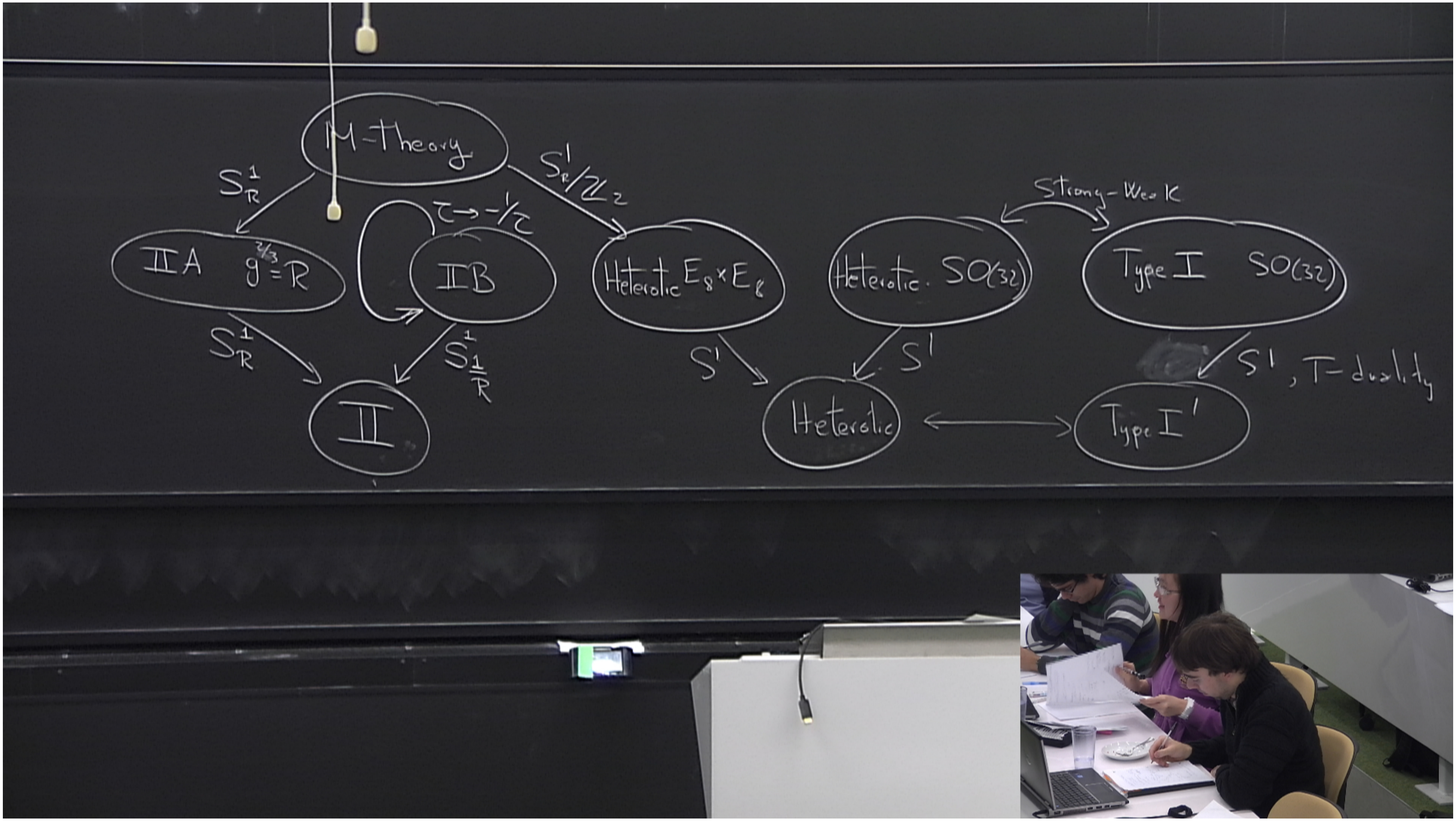

IIB
S^1_R
$S^1_{\frac{1}{R}}$
S^1
S^1
9 dim
II
Heterotic

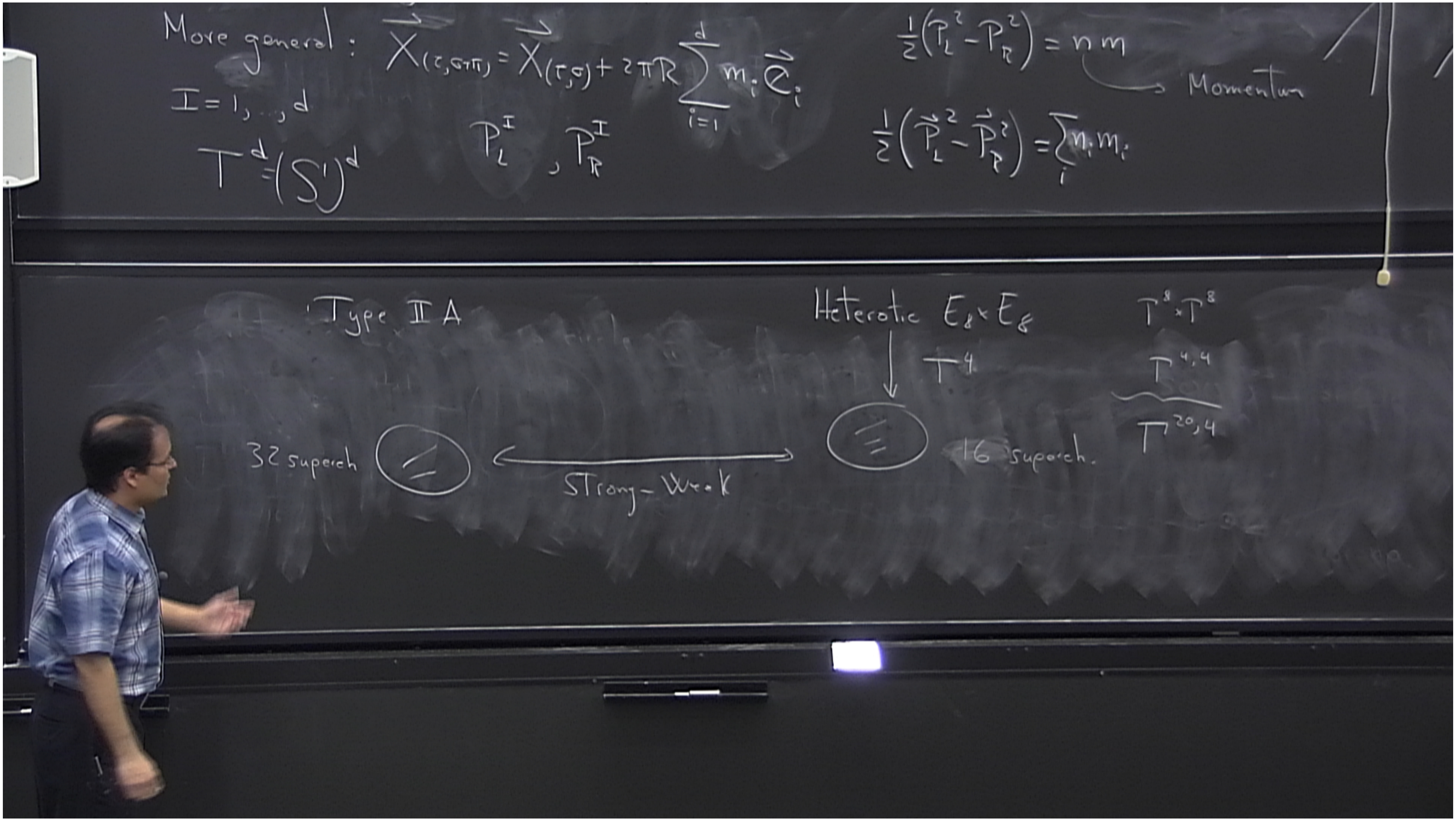

9 dim
IIB
II
Heterotic

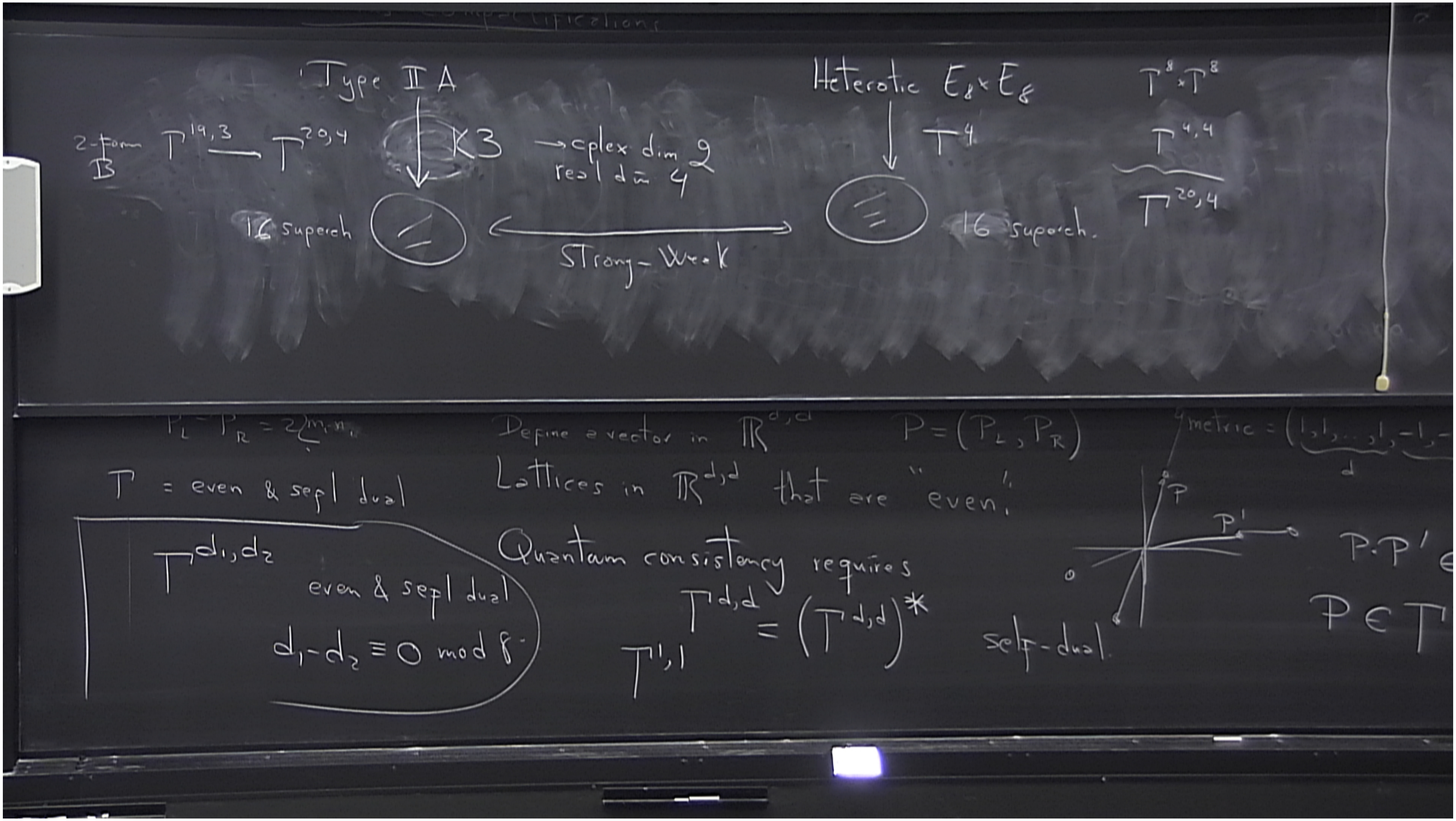


11 dim
10 dim
9 dim
M-Theory
S^1_R
$S^1/\mathbb{Z}_2$
$\tau \to -1/\tau$
IIA
$g^{2/3} = R$
IIB
Heterotic $E_8 \times E_8$
Heterotic SO(32)
S^1_R
$S^1_{1/R}$
S^1
S^1
II
Heterotic

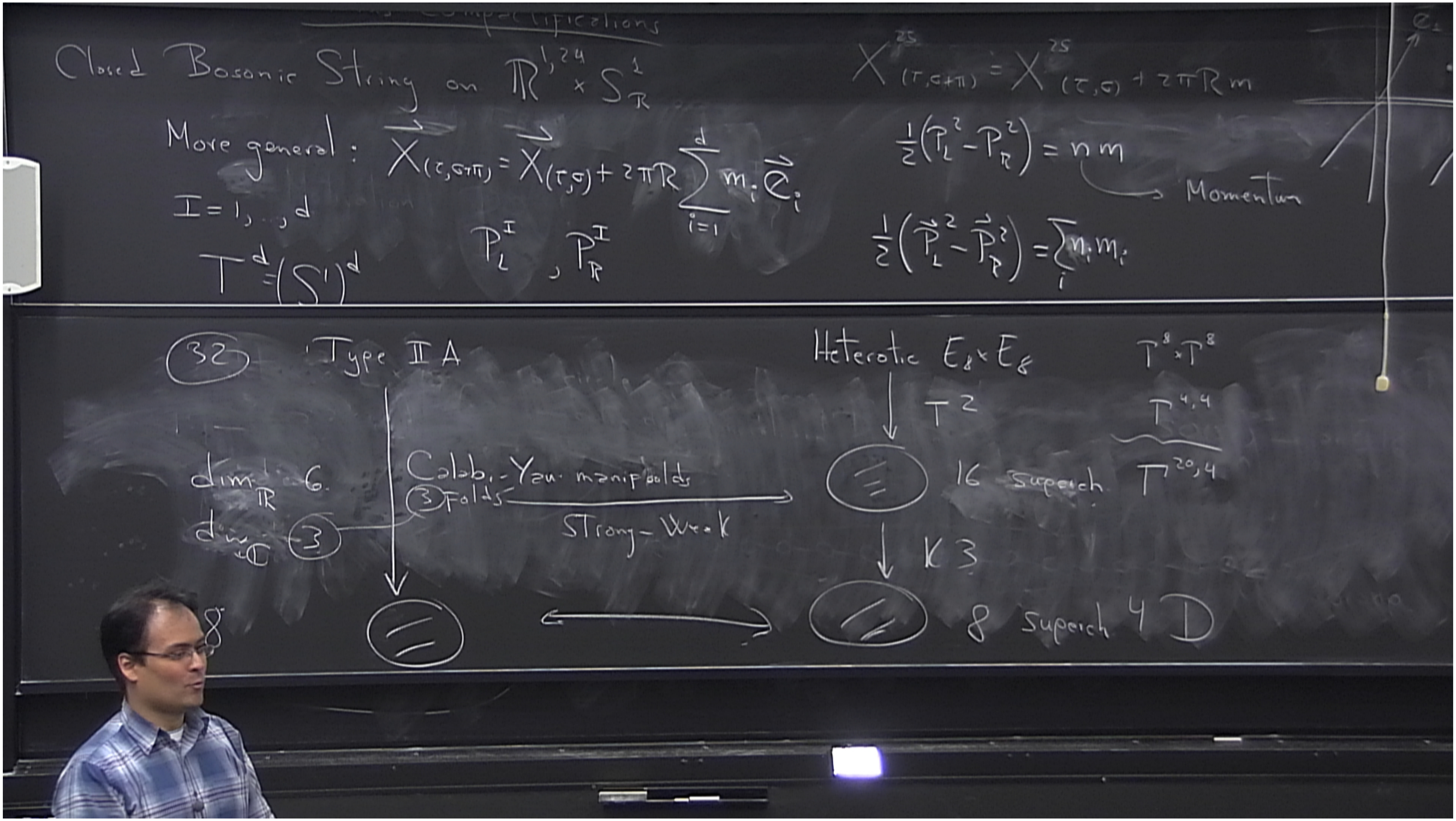

11 dim.
10 dim.
9 dim.
8 dim
M-Theory
S^1_R
$S^1_R/\mathbb{Z}_2$
IIA $g^{2/3}=R$
IIB
$\tau\to -1/\tau$
Heterotic $E_8\times E_8$
Heterotic. SO(32)
S^1_R
$S^1_{1/R}$
S^1
S^1
II
Heterotic
$S^1/\mathbb{Z}_2$
S^1

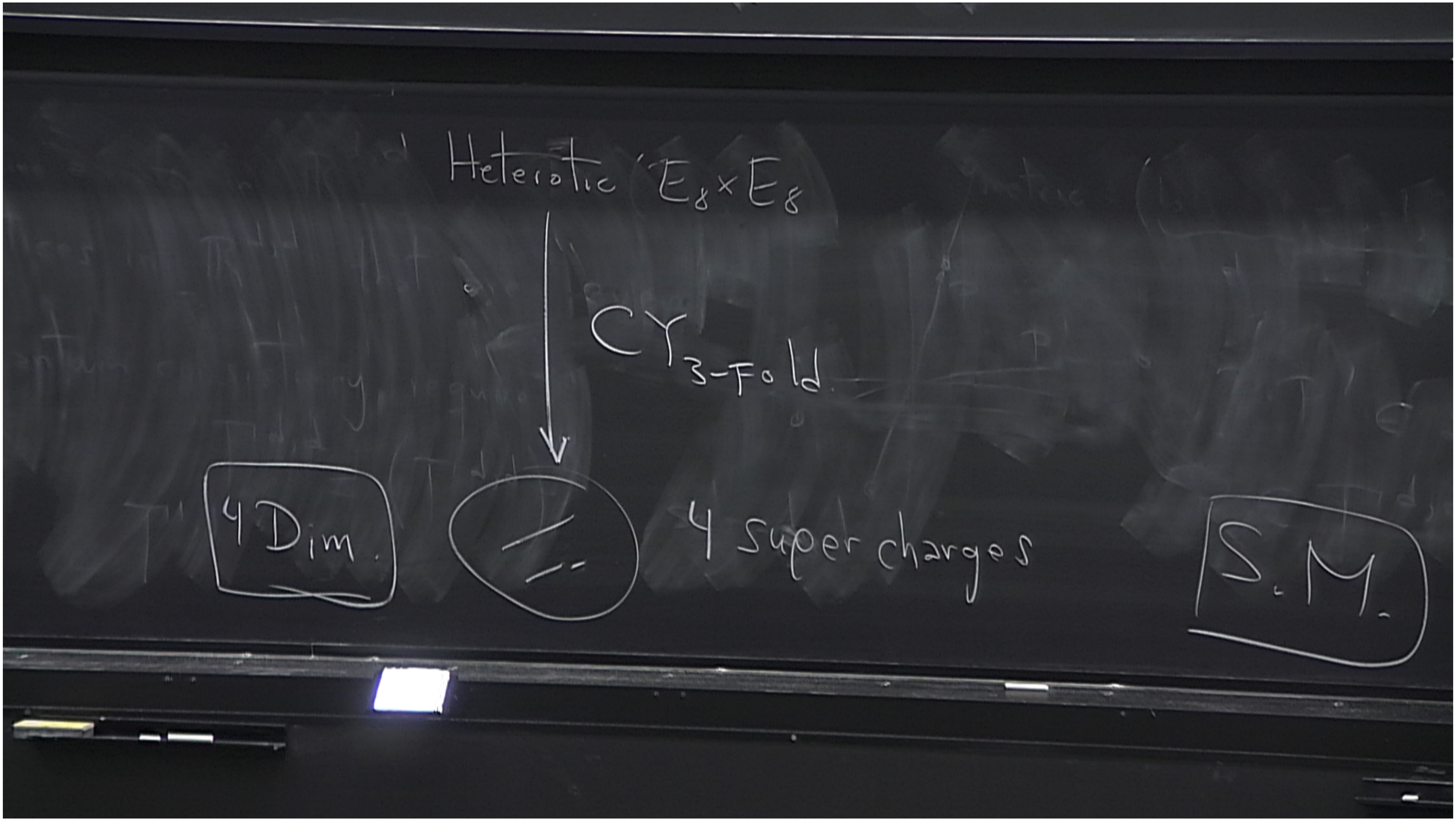

M-Theory
S^1_R
$S^1_R/\mathbb{Z}_2$
IIA $g^{2/3} = R$
$\tau \to -1/\tau$
IIB
Heterotic $E_8 \times E_8$
Heterotic $SO(32)$
Strong-Weak
Type I $SO(32)$
S^1_R
$S^1_{1/R}$
II
S^1
S^1
S^1, T-duality
Heterotic
Type I'


More general:
$\vec{X}(\tau,\sigma+\pi) = \vec{X}(\tau,\sigma) + 2\pi R \sum_{i=1}^{d} m_i \vec{e}_i$
$I = 1, \ldots, d$
$T^d = (S^1)^d$
$P_L^I \,,\, P_R^I$
$\frac{1}{2}(P_L^2 - P_R^2) = nm$
Momentum
$\frac{1}{2}(\vec{P}_L^2 - \vec{P}_R^2) = \sum_i n_i m_i$
Type IIA
Heterotic $E_8 \times E_8$
$T^8 \times T^8$
T^4
$T^{4,4}$
$T^{20,4}$
32 superch
Strong - Weak
16 superch.



Type II A
Heterotic E8 × E8
T8 × T8
2-form B
Γ19,3 — Γ20,4
K3
→ cplex dim 2
real dim 4
T4
Γ4,4
Γ20,4
16 superch
Strong – Weak
16 superch.
Define a vector in ℝd,d
P = (PL, PR)
metric = (1, 1, ..., 1, −1, −
d
Γ = even & self dual
Lattices in ℝd,d that are "even"
Γd1,d2
even & self dual
d1 − d2 ≡ 0 mod 8
Quantum consistency requires
Γd,d = (Γd,d)*
Γ1,1
self-dual
P
P′
P·P′ ∈
P ∈ Γ


Compactifications
Closed Bosonic String on R^{1,24} × S^1_R
X^25(τ,σ+π) = X^25(τ,σ) + 2πRm
More general: X(τ,σ+π) = X(τ,σ) + 2πR Σ_{i=1}^{d} m_i e_i
I = 1,..,d
T^d = (S^1)^d
P_L^I , P_R^I
1/2 (P_L^2 − P_R^2) = n m
Momentum
1/2 (P_L^2 − P_R^2) = Σ_i n_i m_i
32
Type IIA
Heterotic E8×E8
T^8×T^8
T^2
T^{4,4}
dim_R 6
dim_C 3
Calabi-Yau manifolds
3 Folds
Strong-Weak
16 superch
T^{20,4}
K3
8
8 superch 4D


Heterotic E8×E8
CY3-Fold
4 Dim.
4 super charges
S.M.