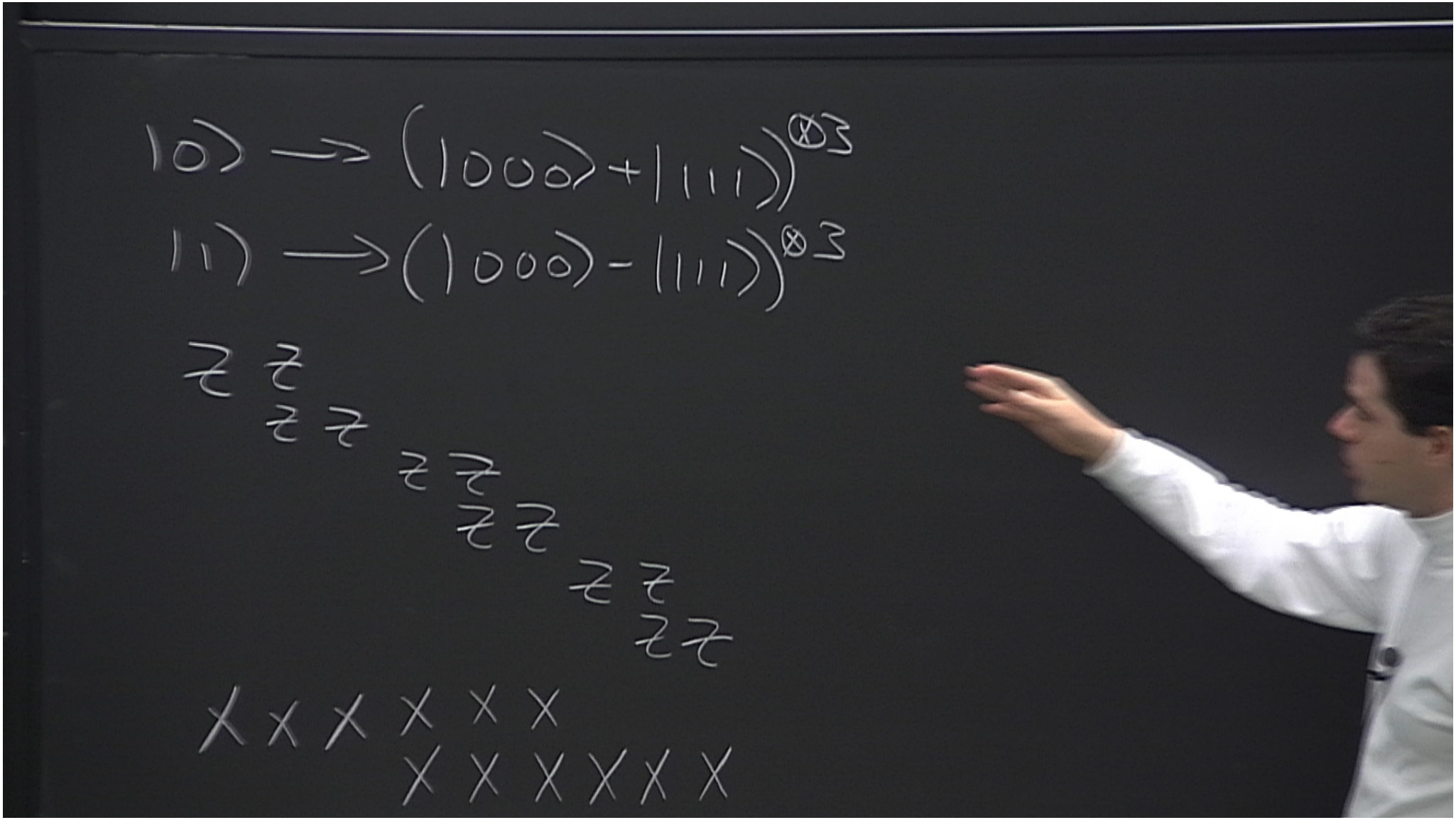
Title: Quantum Information (Review) - Lecture 13

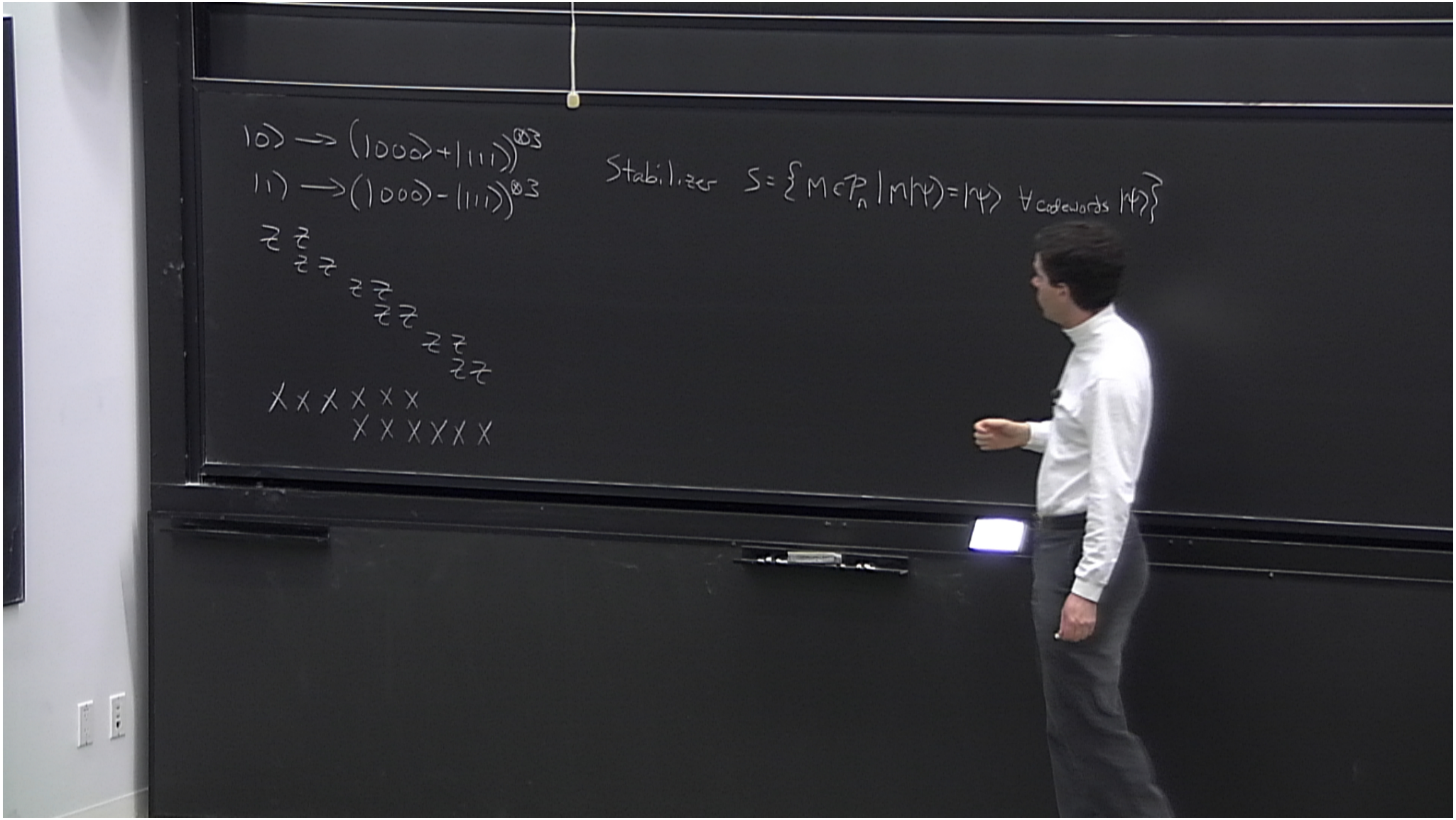
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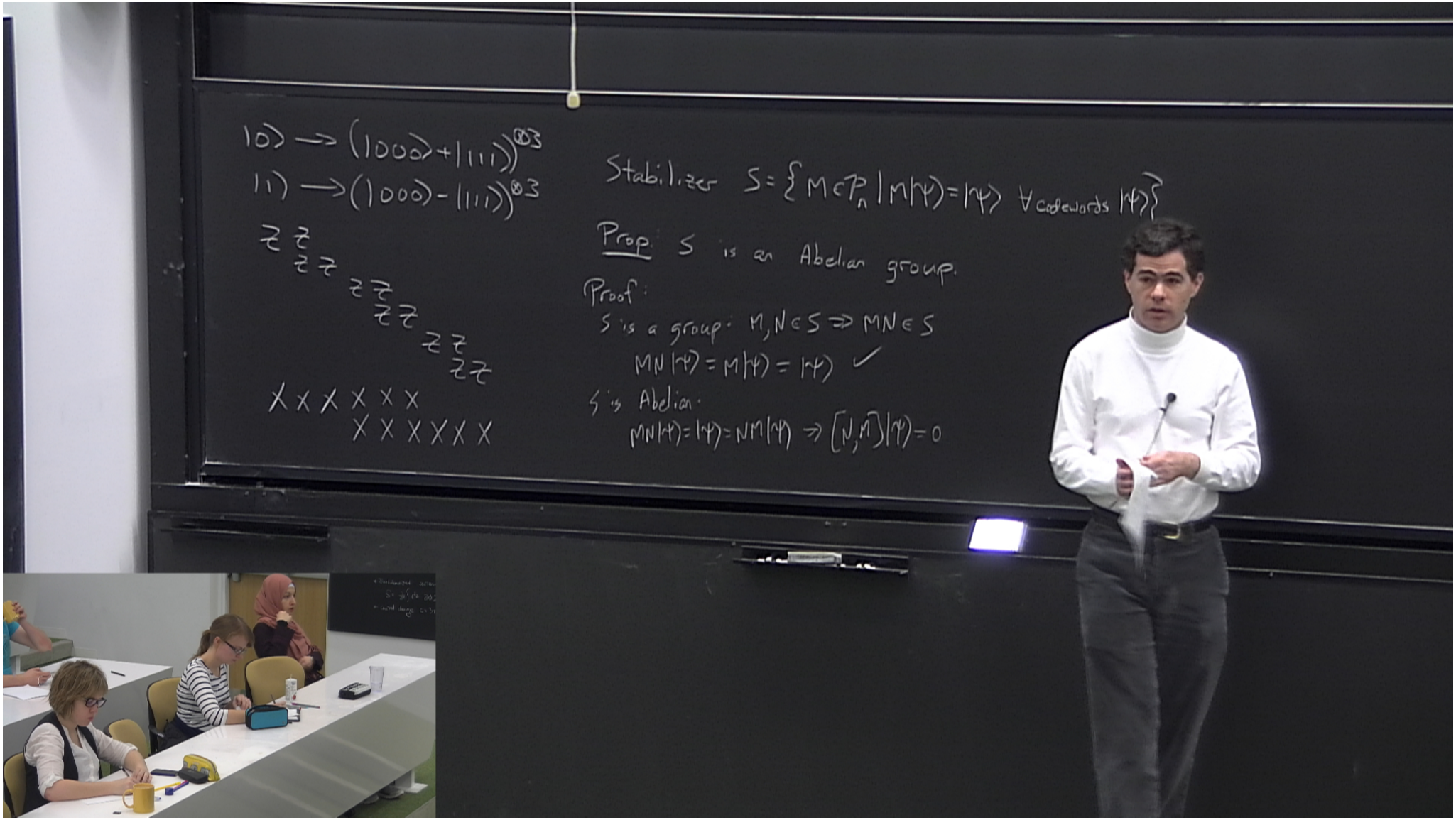
Abstract:


|0⟩ → (|000⟩+|111⟩)^⊗3


|0⟩ → (|000⟩ + |111⟩)^⊗3
|1⟩ → (|000⟩ − |111⟩)^⊗3


|0⟩ → (|000⟩+|111⟩)^⊗3
|1⟩ → (|000⟩-|111⟩)^⊗3
Stabilizer S = {M ∈ P_n | M|ψ⟩ = |ψ⟩ ∀ codewords |ψ⟩}
Z Z
Z Z
Z Z
Z Z
Z Z
Z Z
X X X X X X
X X X X X X


|0⟩ → (|000⟩+|111⟩)^⊗3
|1⟩ → (|000⟩-|111⟩)^⊗3
Stabilizer S = {M ∈ P_n | M|ψ⟩ = |ψ⟩ ∀ codewords |ψ⟩}
Prop S is an Abelian group.


|0⟩ → (|000⟩+|111⟩)^⊗3
|1⟩ → (|000⟩-|111⟩)^⊗3
Stabilizer S = {M ∈ P_n | M|ψ⟩ = |ψ⟩ ∀ codewords |ψ⟩}
Prop: S is an Abelian group.
Proof:
S is a group: M,N ∈ S ⇒ MN ∈ S
MN|ψ⟩ = M|ψ⟩ = |ψ⟩ ✓
S is Abelian:
MN|ψ⟩ = |ψ⟩ = NM|ψ⟩ ⇒ [N,M]|ψ⟩ = 0

Stabilizer $S = \{ M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle \}$

Prop: S is an Abelian group.

Proof:

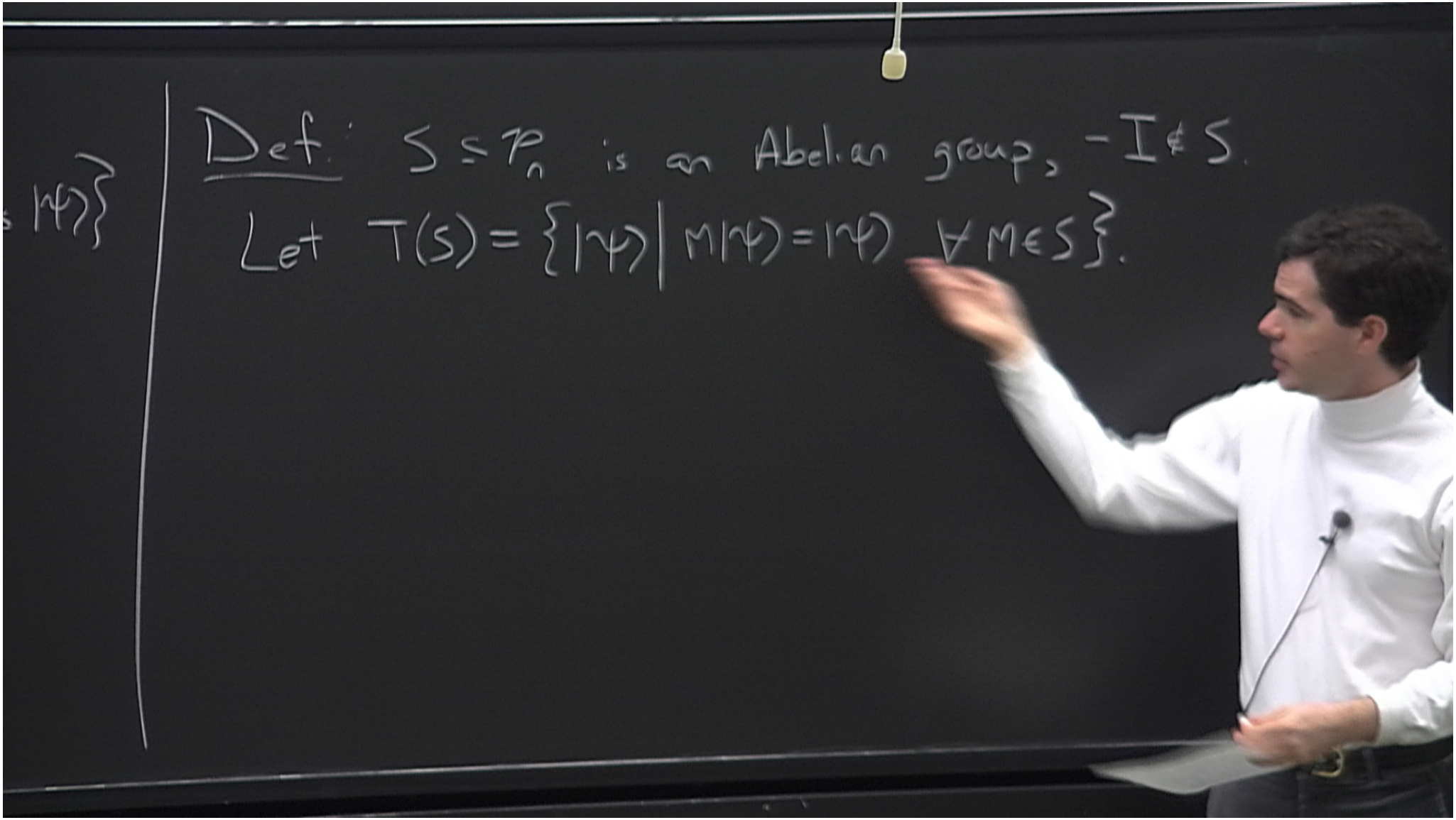
S is a group: $M, N \in S \Rightarrow MN \in S$

$MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$ ✓

S is Abelian:

$MN|\psi\rangle = |\psi\rangle = NM|\psi\rangle \Rightarrow [N,M]|\psi\rangle = 0 \Rightarrow [N,M] = 0$ (for Paulis)



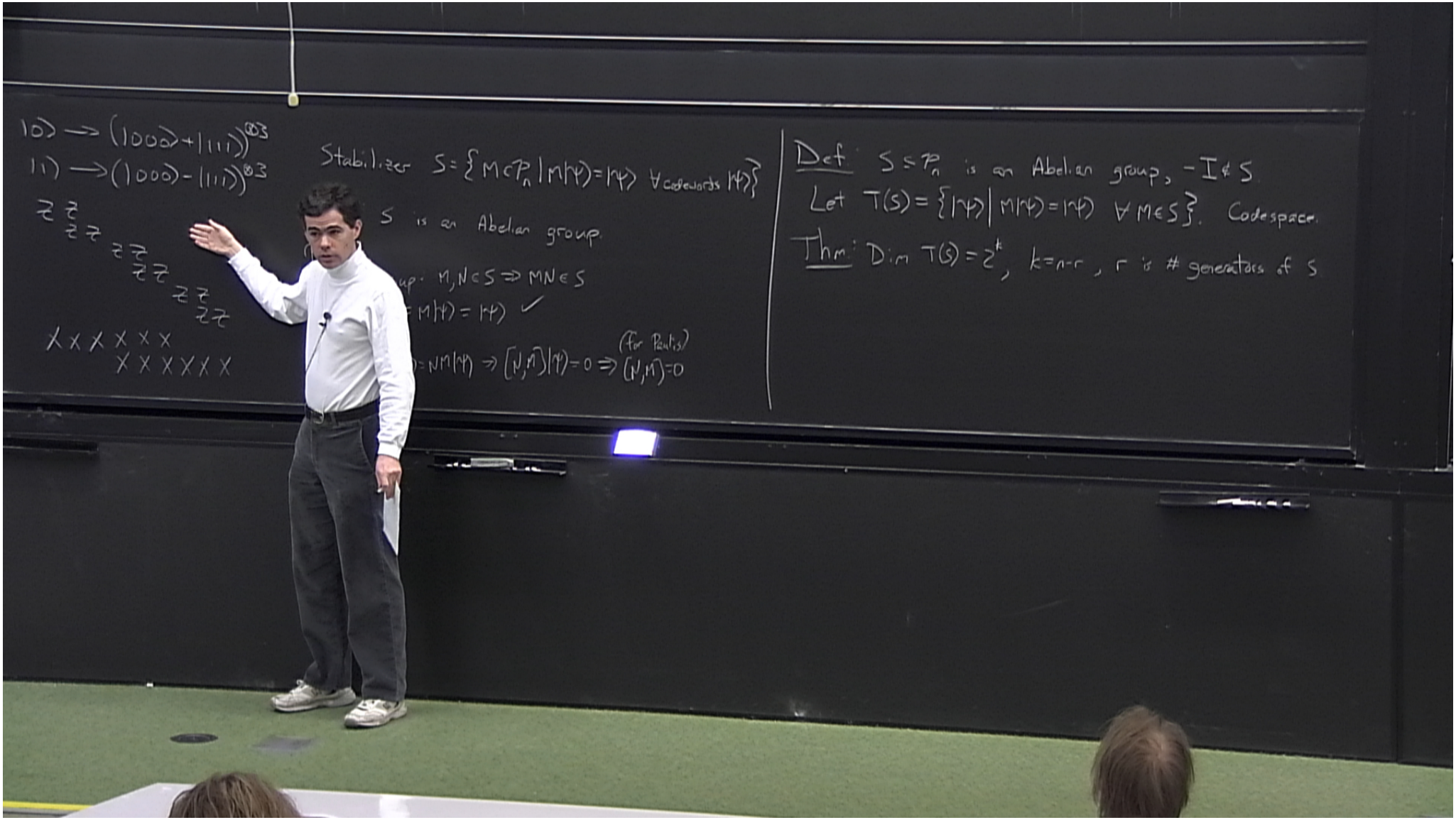

Def: $S \leq \mathcal{P}_n$ is an Abelian group, $-I \notin S$.
Let $T(S) = \{|\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S\}$.

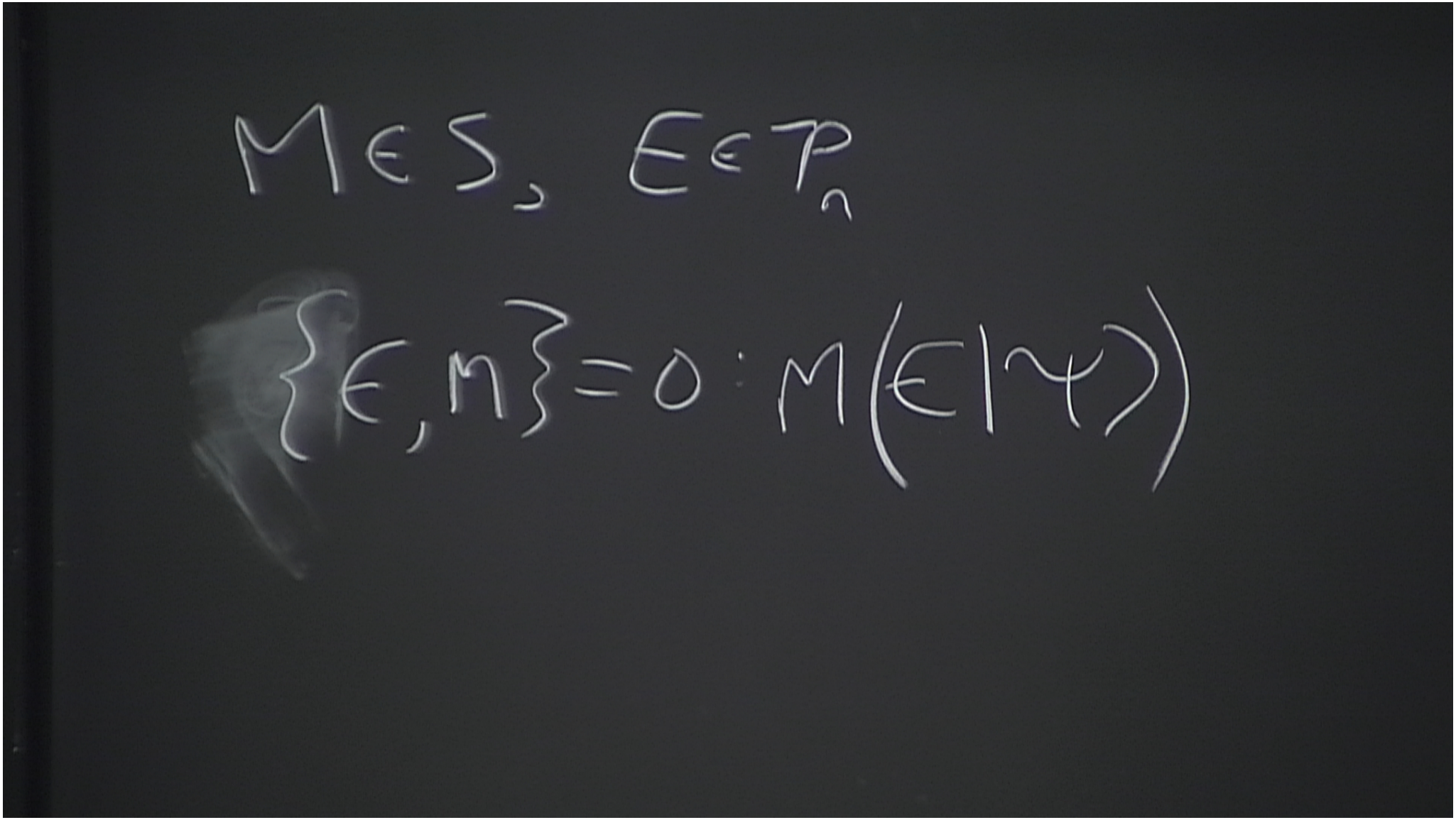
$|\psi\rangle\}$

Def: $S \leq \mathcal{P}_n$ is an Abelian group, $-I \notin S$.

Let $T(S) = \{|\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S\}$. Codespace.

Thm: $\dim T(S) = 2^k$, $k = n - r$, r is # generators of S.


Stabilizer
Def
Let
Thm:
Dim T(S) = 2^k, k=n-r, r is # generators of S


M ∈ S, E ∈ P_n
{E, M} = 0 : M(E|ψ)

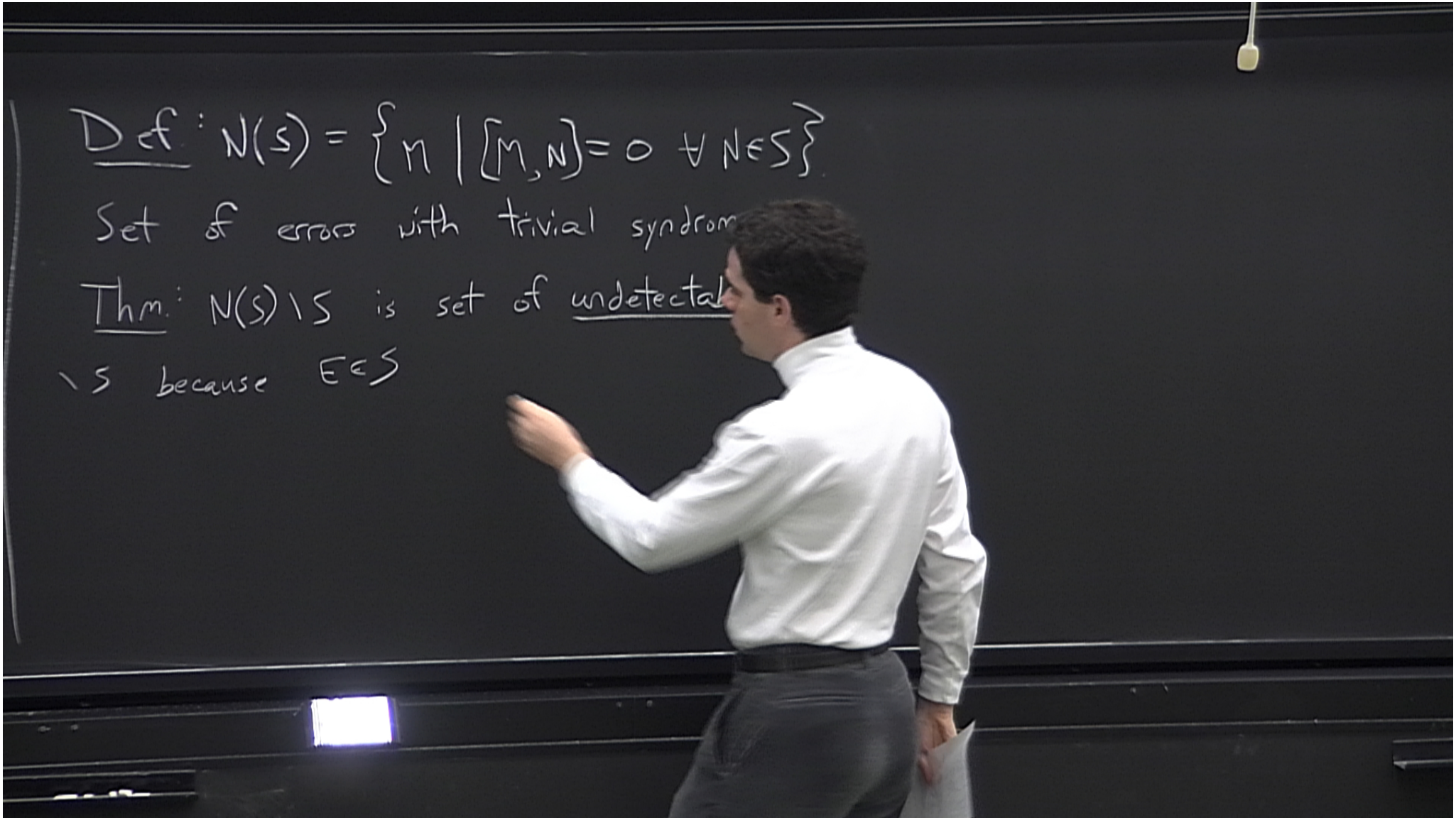
$M \in S$, $E \in P_n$

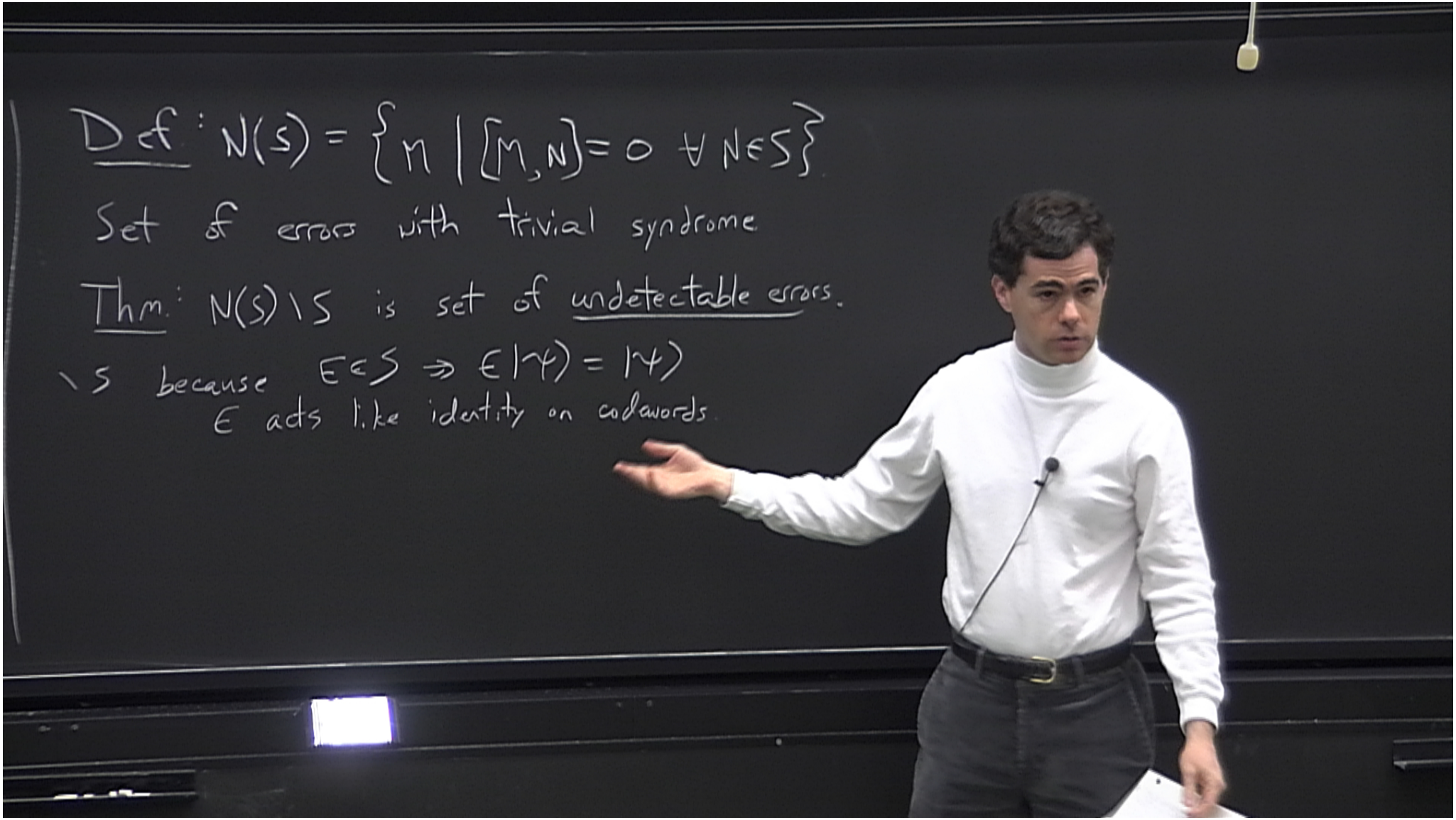
$\{E, M\} = 0 : M(E|\psi\rangle) = -EM|\psi\rangle = -E|\psi\rangle$

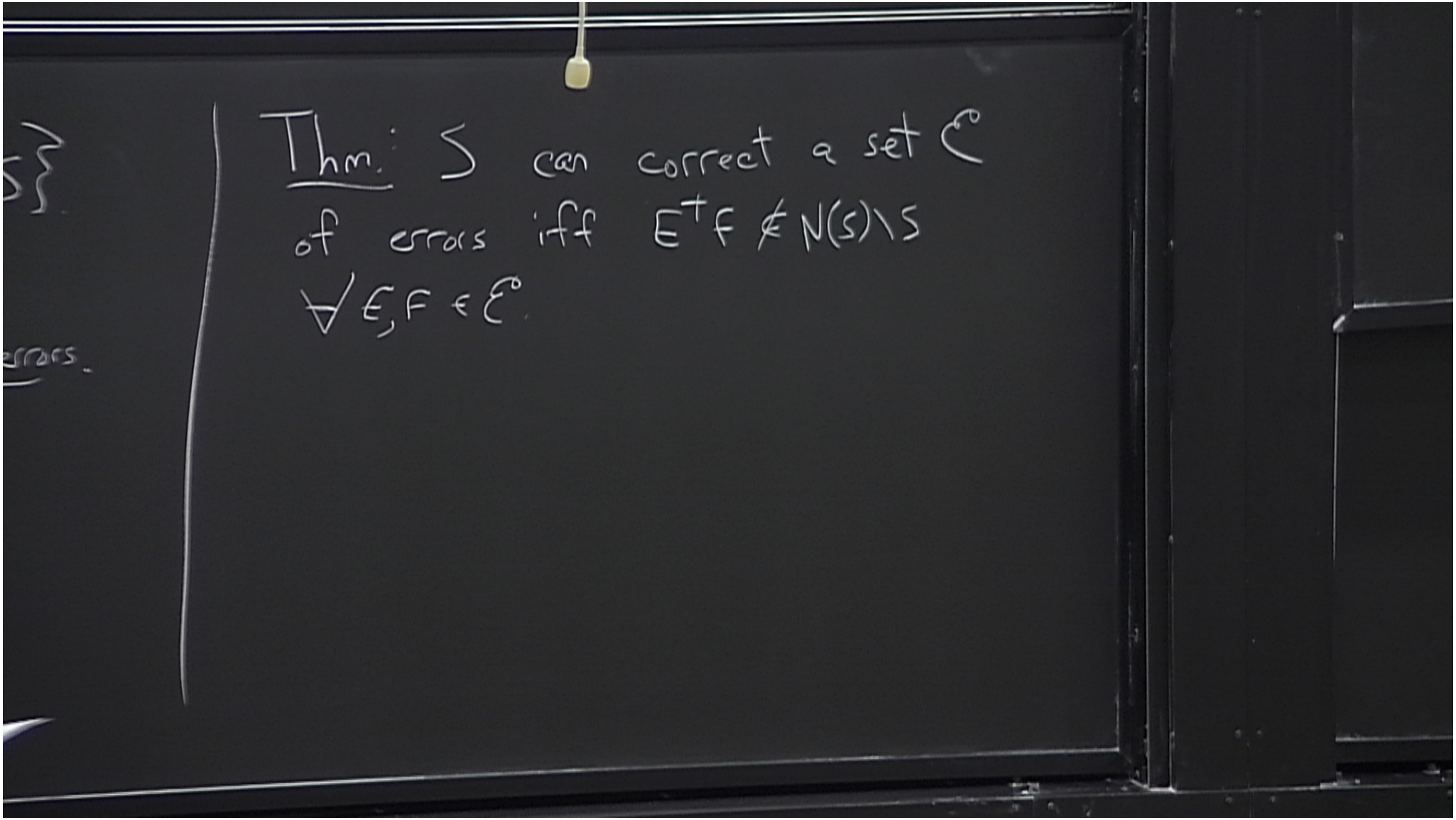
$E|\psi\rangle$ is a -1 eigenstate of M.

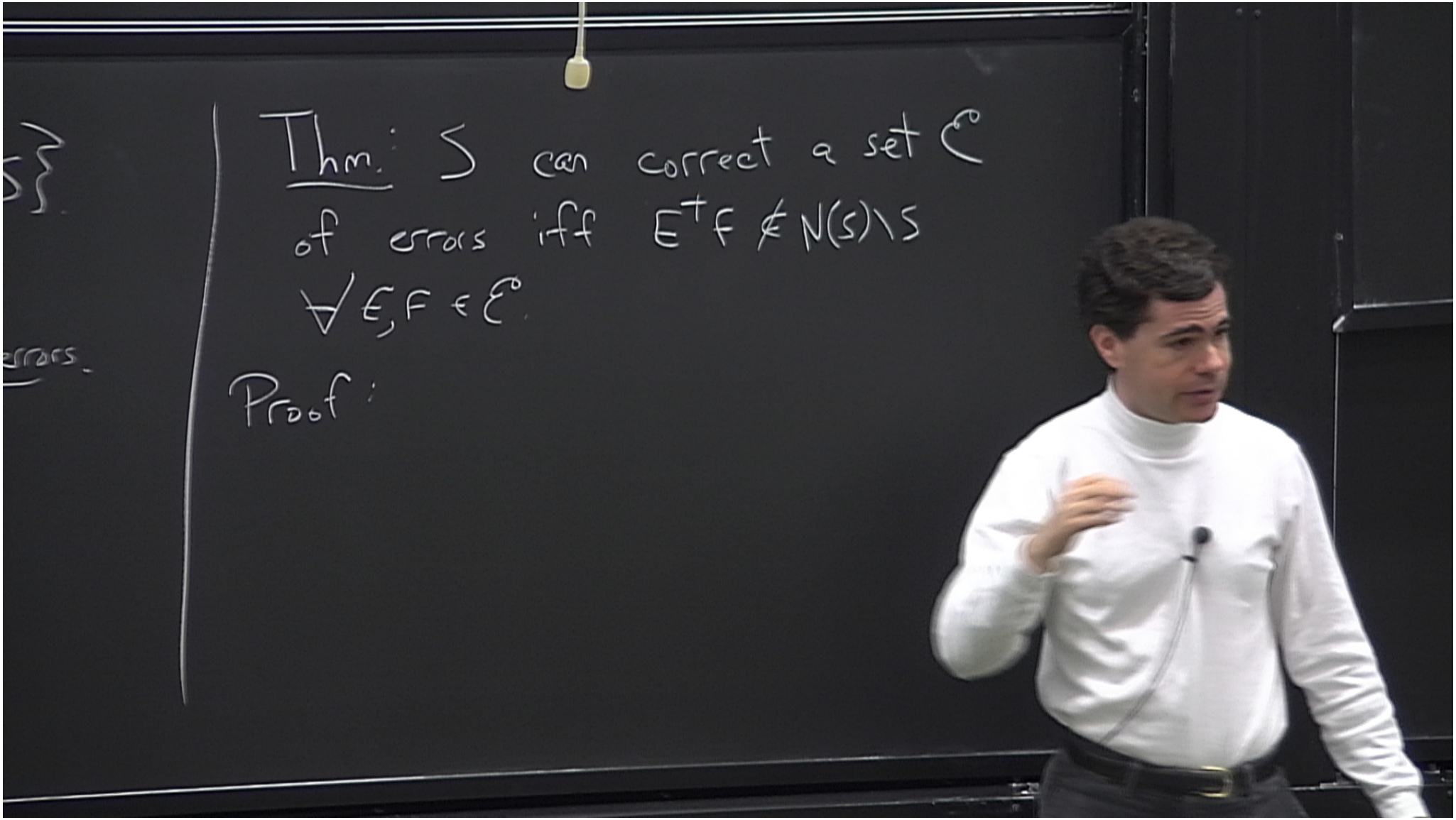
$[E, M] = 0 : M(E|\psi\rangle) = EM|\psi\rangle = E|\psi\rangle$

$E|\psi\rangle$ is a $+1$ eigenstate


Def: $N(S) = \{n \mid [M,N] = 0 \ \forall N \in S\}$
Set of errors with trivial syndrom
Thm: $N(S) \setminus S$ is set of undetectab
$\setminus S$ because $E \in S$


Def: $N(S) = \{M \mid [M,N] = 0 \ \forall N \in S\}$
Set of errors with trivial syndrome
Thm: $N(S) \setminus S$ is set of undetectable errors.
$\setminus S$ because $E \in S \Rightarrow E|\psi\rangle = |\psi\rangle$
E acts like identity on codewords


errors.
Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.


errors.
Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.
Proof:

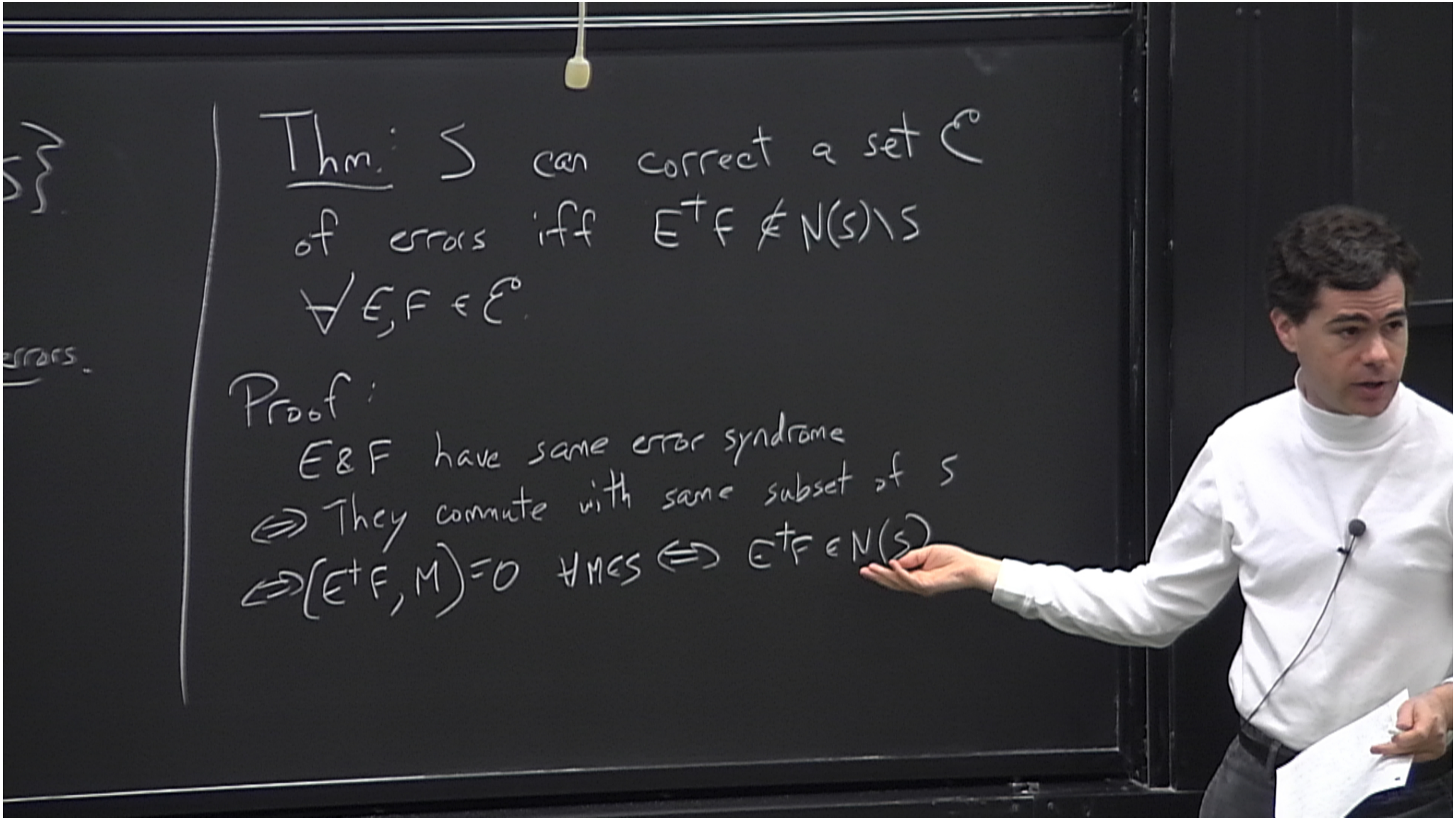
errors.

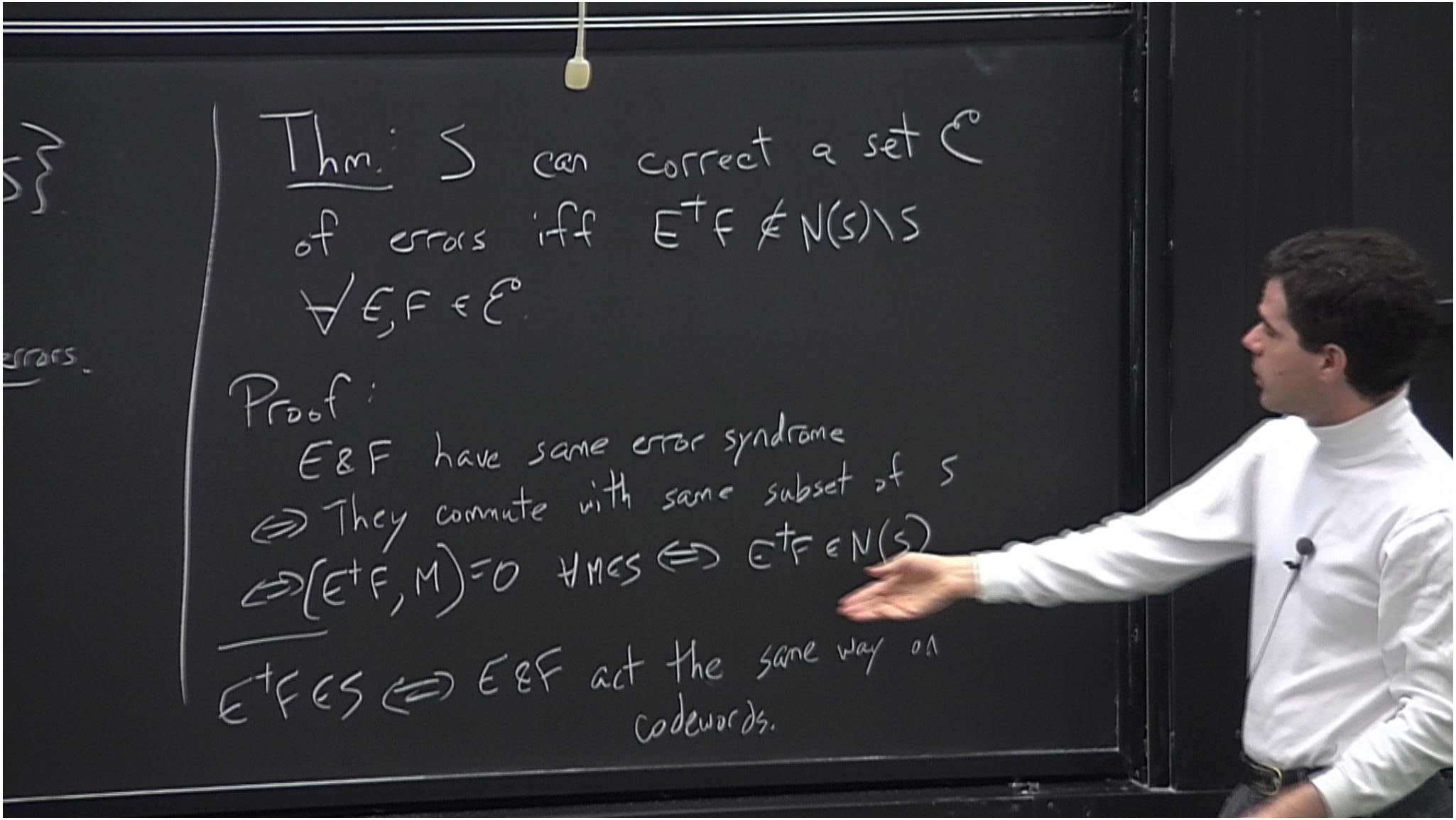
Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.

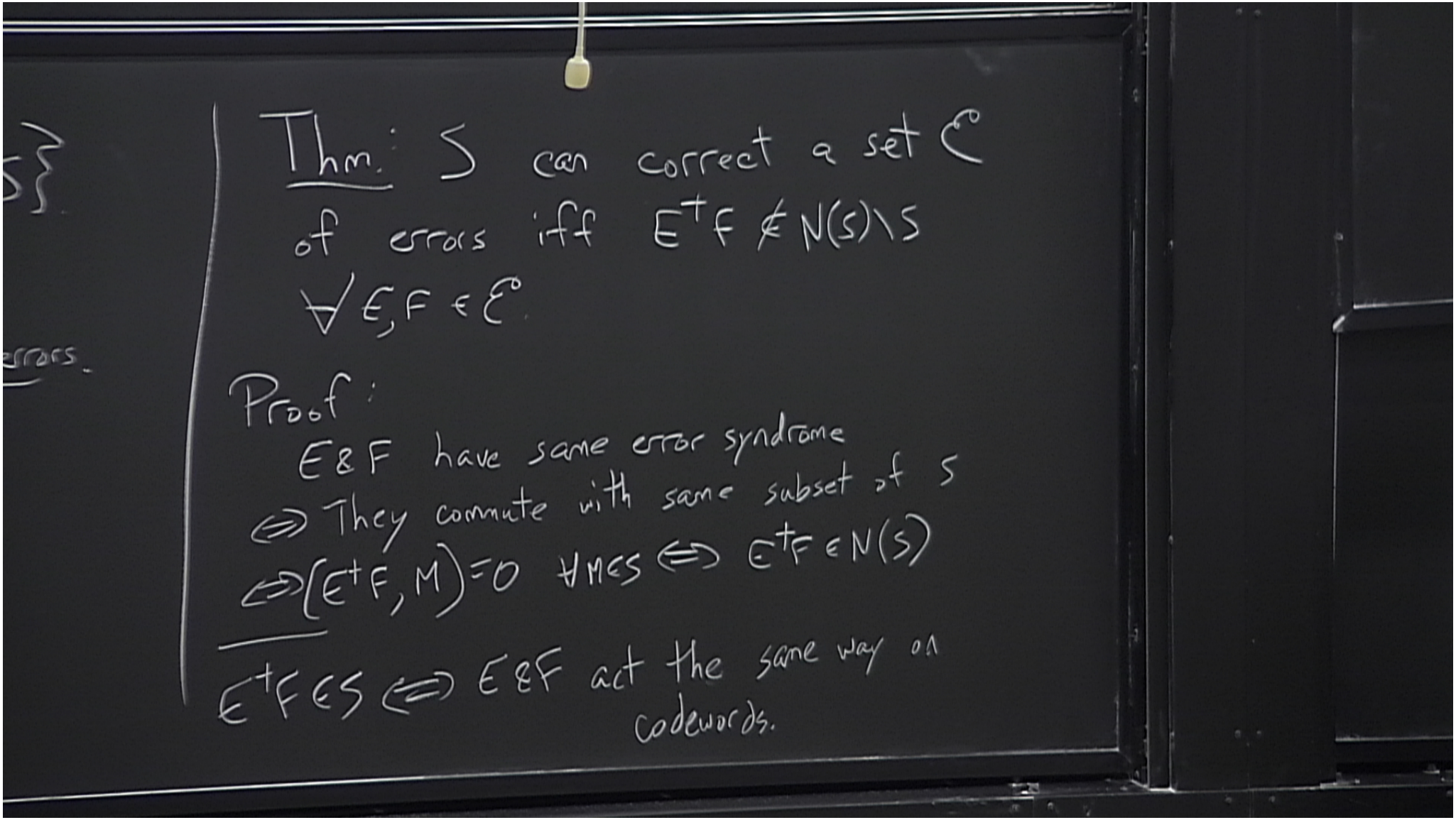
Proof:

E & F have same error syndrome

$\Leftrightarrow$ They commute with same subset of S


Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.
Proof:
E & F have same error syndrome
⇔ They commute with same subset of S
⇔ $(E^\dagger F, M) = 0$ $\forall M \in S$ ⇔ $E^\dagger F \in N(S)$


Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.
errors.
Proof:
E&F have same error syndrome
⇔ They commute with same subset of S
⇔ $(E^\dagger F, M) = 0$ $\forall M \in S$ ⇔ $E^\dagger F \in N(S)$
$E^\dagger F \in S$ ⇔ E&F act the same way on codewords.


Thm: S can correct a set $\mathcal{E}$ of errors iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.
Proof:
E&F have same error syndrome
⇔ They commute with same subset of S
⇔ $(E^\dagger F, M) = 0$ $\forall M \in S$ ⇔ $E^\dagger F \in N(S)$
$E^\dagger F \in S$ ⇔ E&F act the same way on codewords.

If 1S condition is needed,
the code is degenerate.

Def: The distance of S
is $\min\{wt\, E \mid E \in N(S) \setminus S\}$



wt E = weight = # of qubits
on which E acts non-trivially.

ed,

wt E = weight = # of qubits on which E acts non-trivially.

Summarize code as

$[[n, k, d]]$

n: # physical qubits

k: # logical qubits

d: distance

ed,

wt $\in$ = weight = # of qubits on which $\in$ acts non-trivially.

Summarize code as

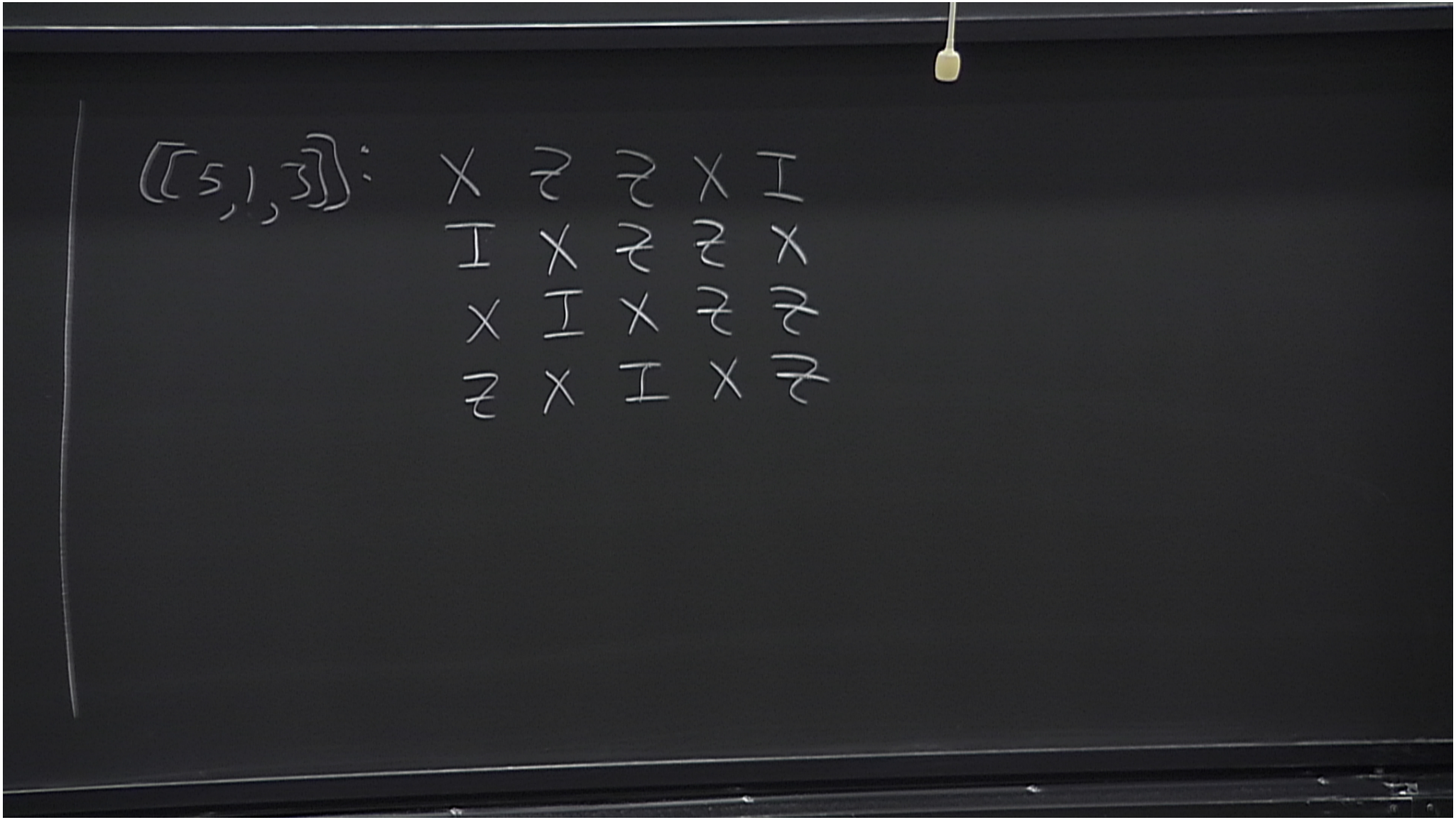
$[[n, k, d]]$

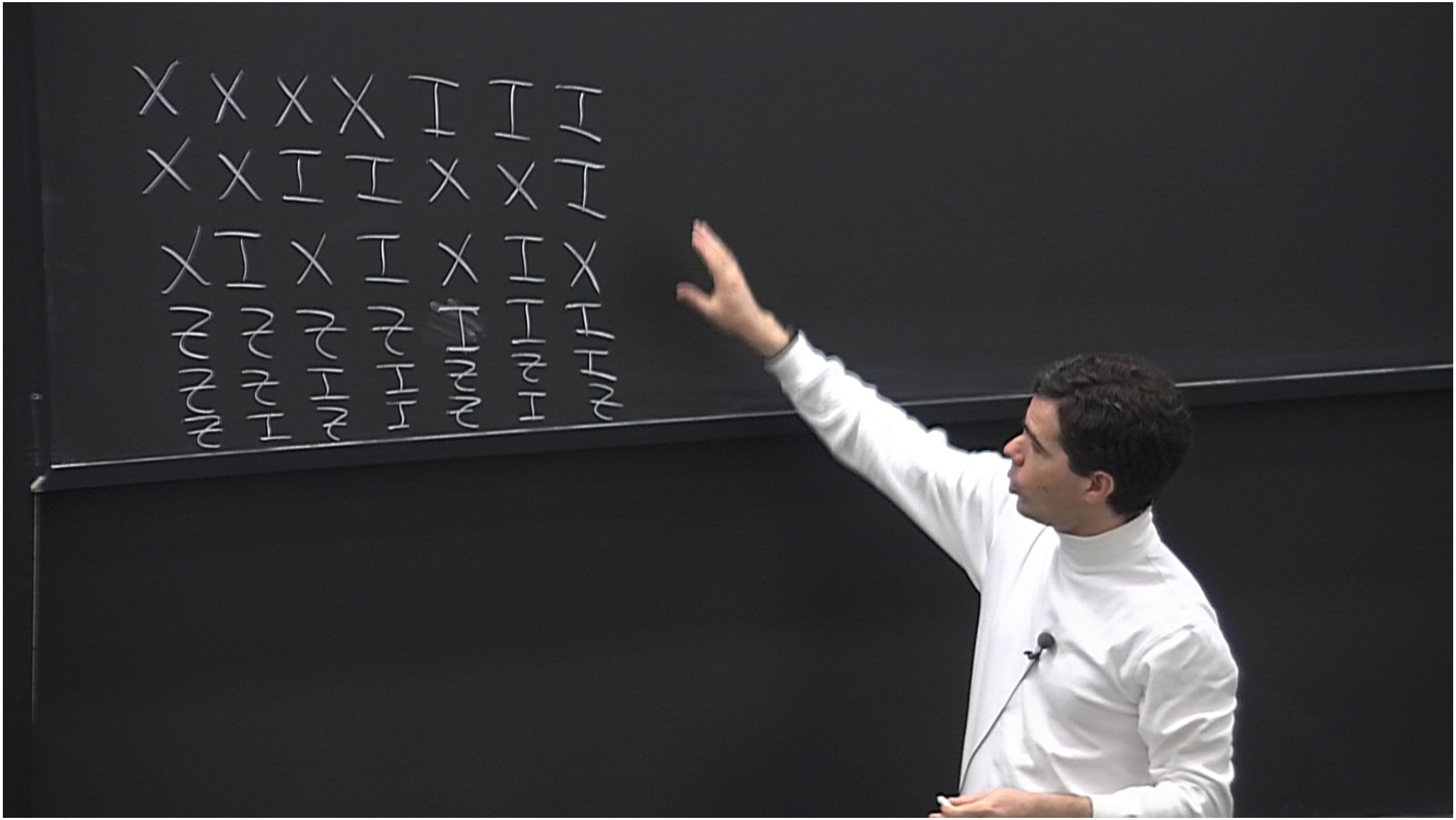
physical qubits

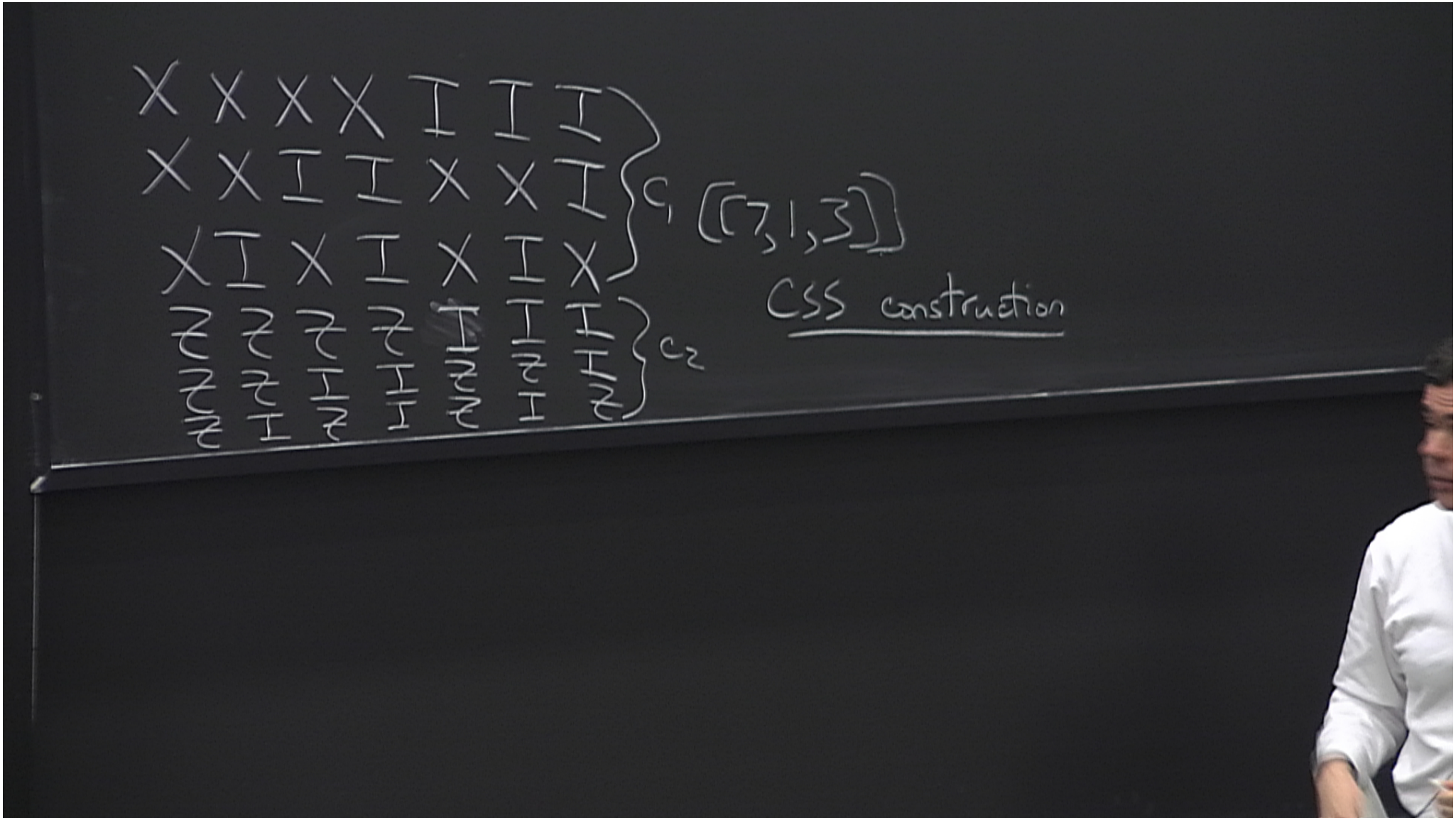
logical qubits

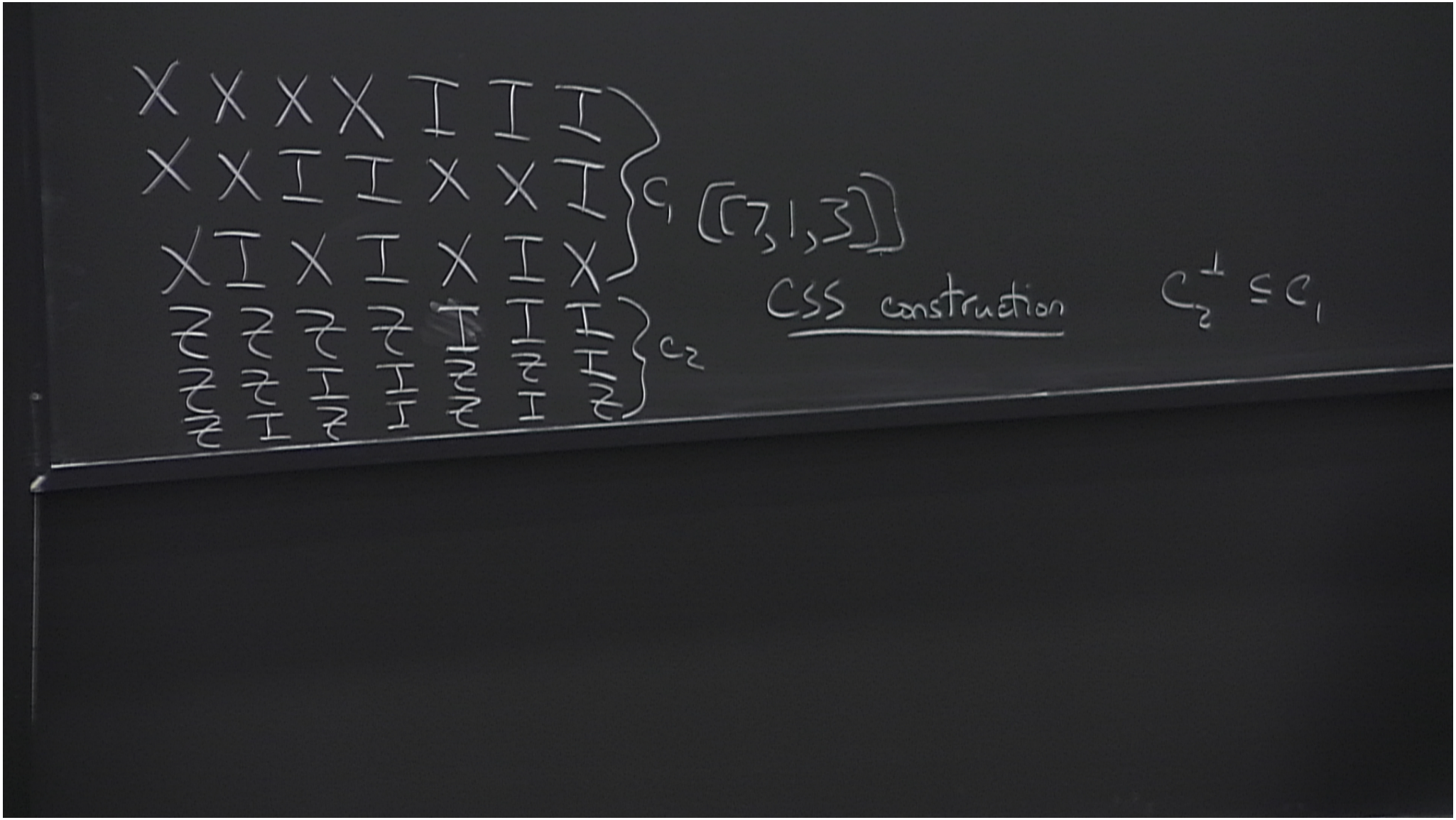
distance

$[[5, 1, 3]]$:

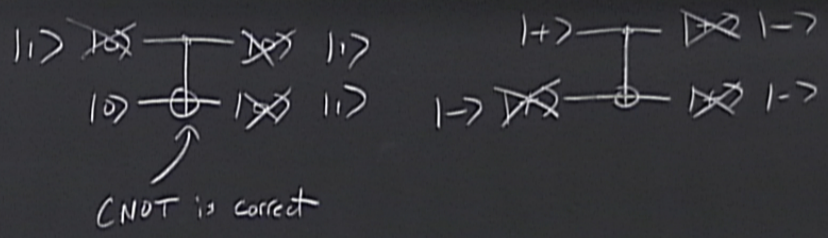

[[5,1,3]]:
X Z Z X I
I X Z Z X
X I X Z Z
Z X I X Z




C1 ([7,1,3])
CSS construction
C2


X X X X I I I
X X I I X X I
X I X I X I X
Z Z Z Z I I I
Z Z I I Z Z I
Z I Z I Z I Z
C_1 $[[7,1,3]]$
C_2
CSS construction
$C_2^\perp \subseteq C_1$

Fault-tolerance: Careful about error propagation.


|+>
|0>
CNOT is correct
|+>
|->
|->
|->

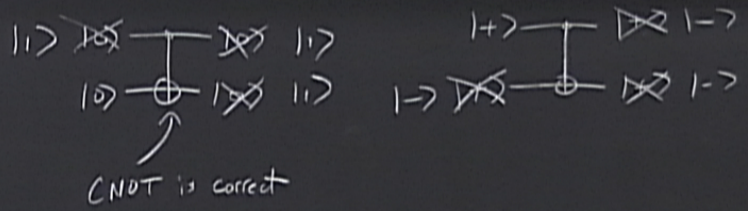
$E|\psi\rangle$ is a $+1$ eigenstate

$\Leftrightarrow$ They commute

$\Leftrightarrow (E^\dagger F, M) = 0$

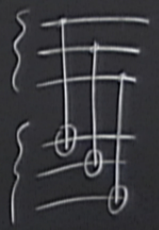
$E^\dagger F \in S \Leftrightarrow E \in F$

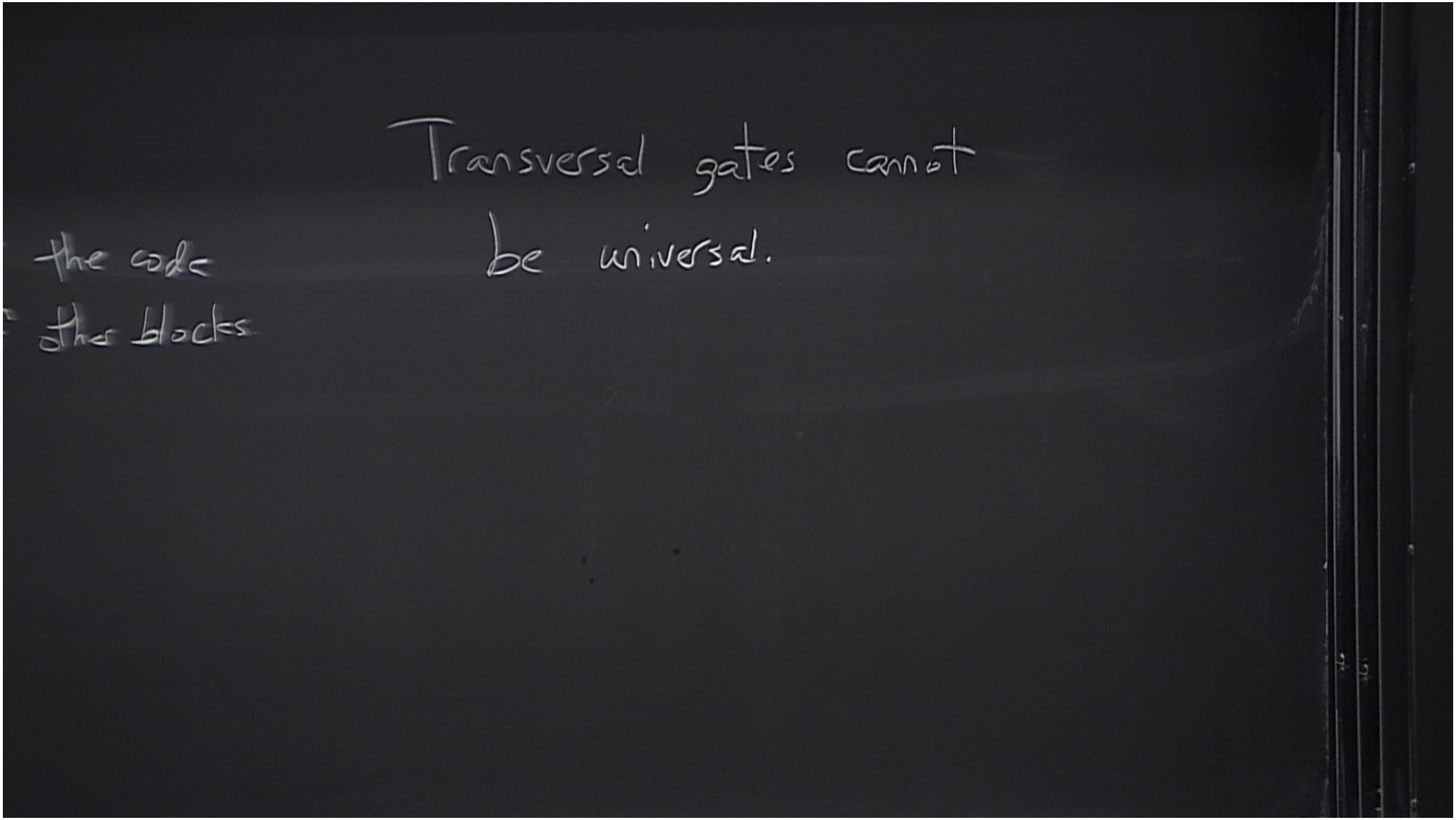
Fault-tolerance: Careful about error propagation.

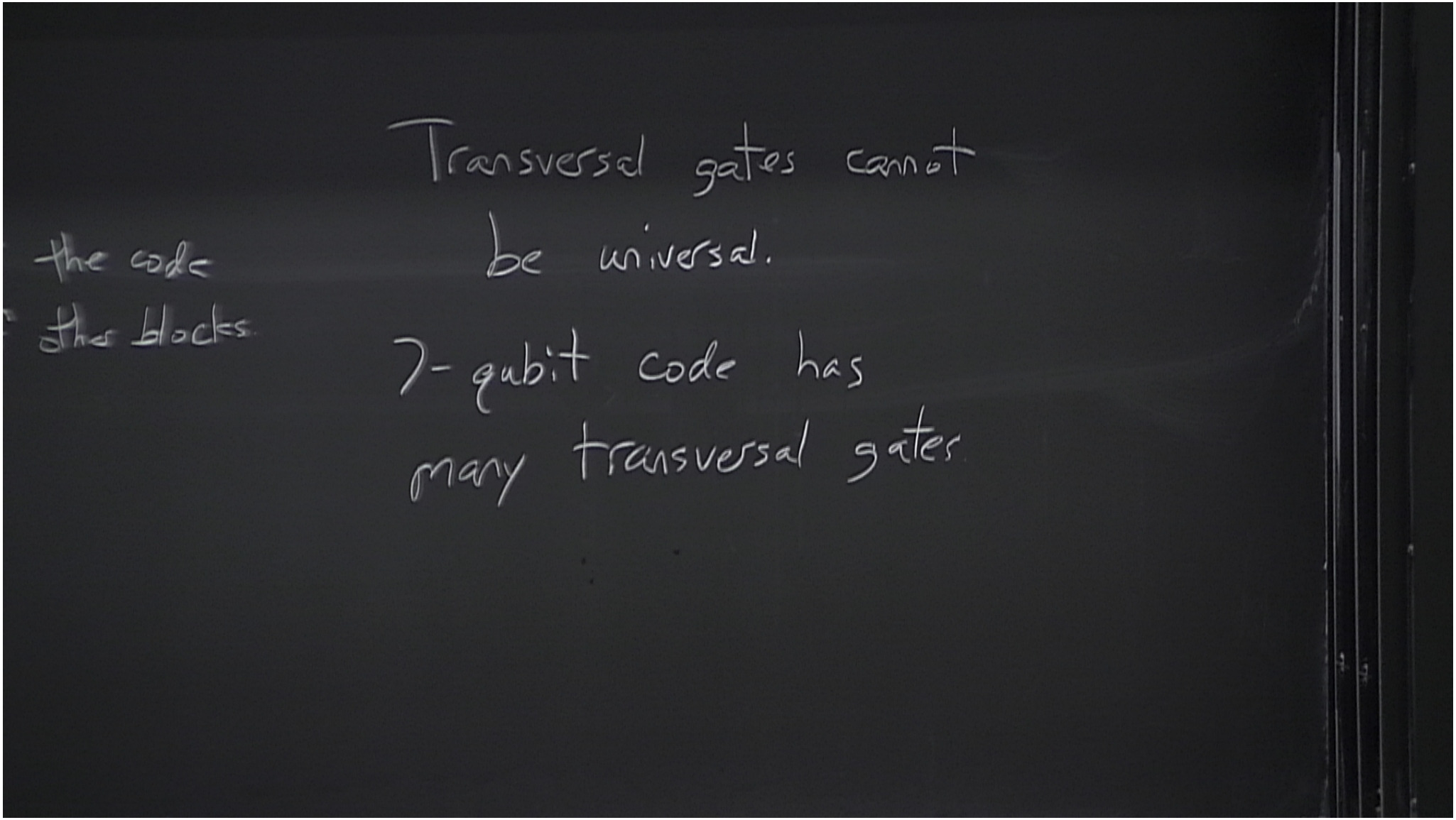


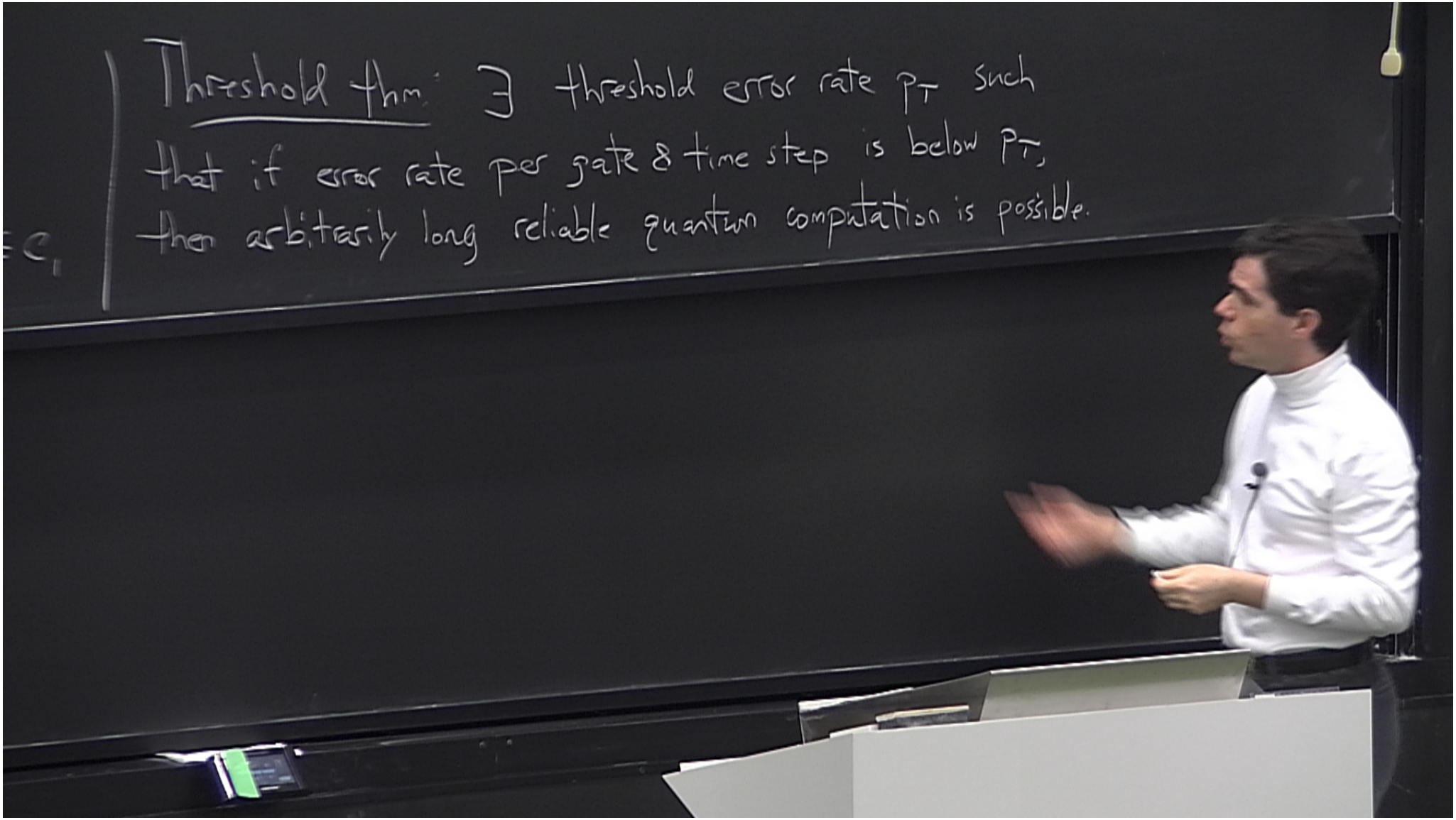
Transversal gates

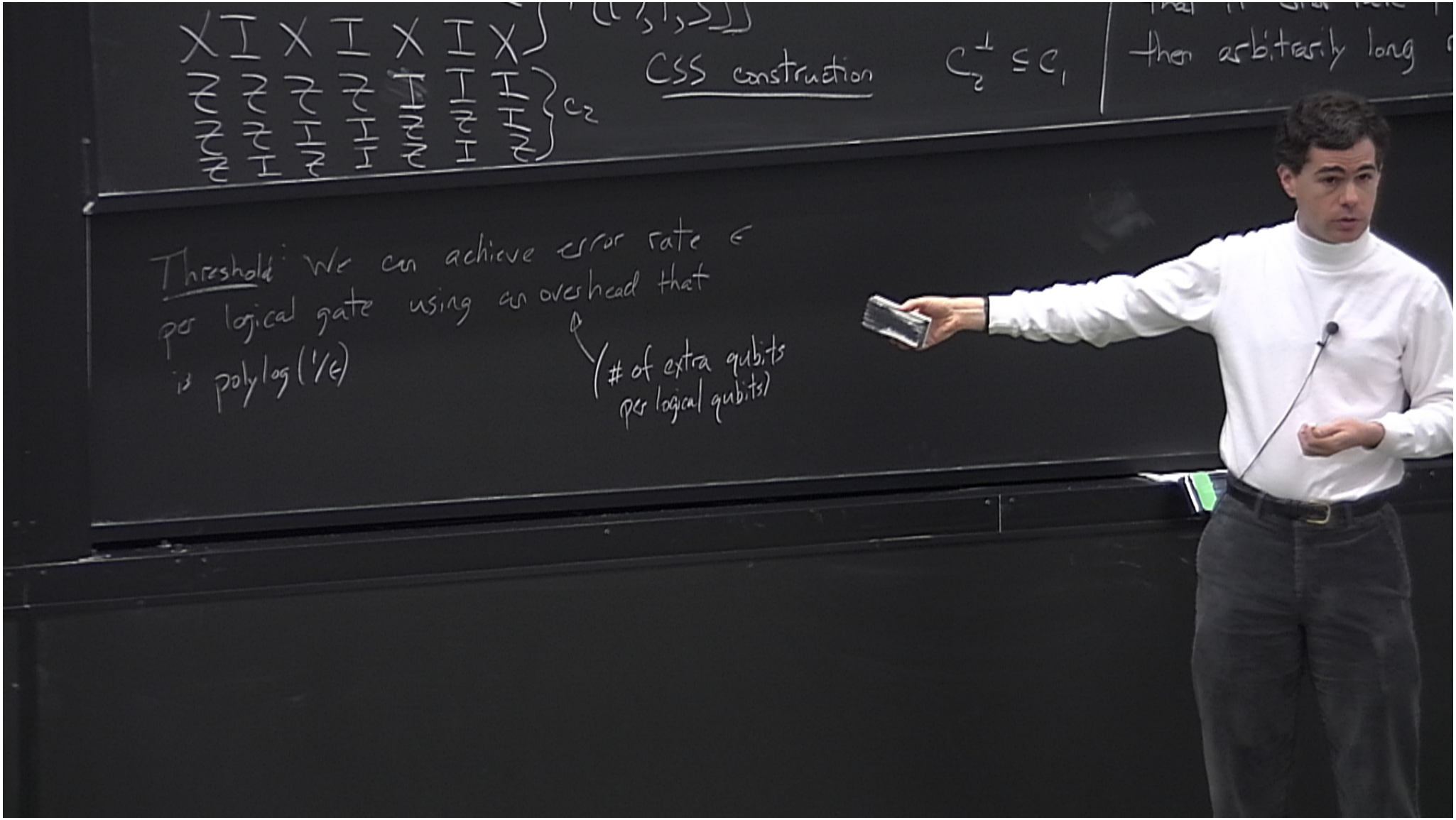
ith qubit of a block of the code only interact w/ ith qubit of other blocks




Transversal gates cannot
be universal.
the code
the blocks


Transversal gates cannot be universal.
7-qubit code has many transversal gates.
the code
the blocks


Threshold thm: ∃ threshold error rate p_T such
that if error rate per gate & time step is below p_T,
then arbitrarily long reliable quantum computation is possible.


CSS construction
$C_2^\perp \subseteq C_1$
then arbitrarily long
Threshold: We can achieve error rate ϵ per logical gate using an overhead that is $\text{poly}\log(1/\epsilon)$
(# of extra qubits per logical qubits)