Title: Quantum Integrability for Three-Point Functions

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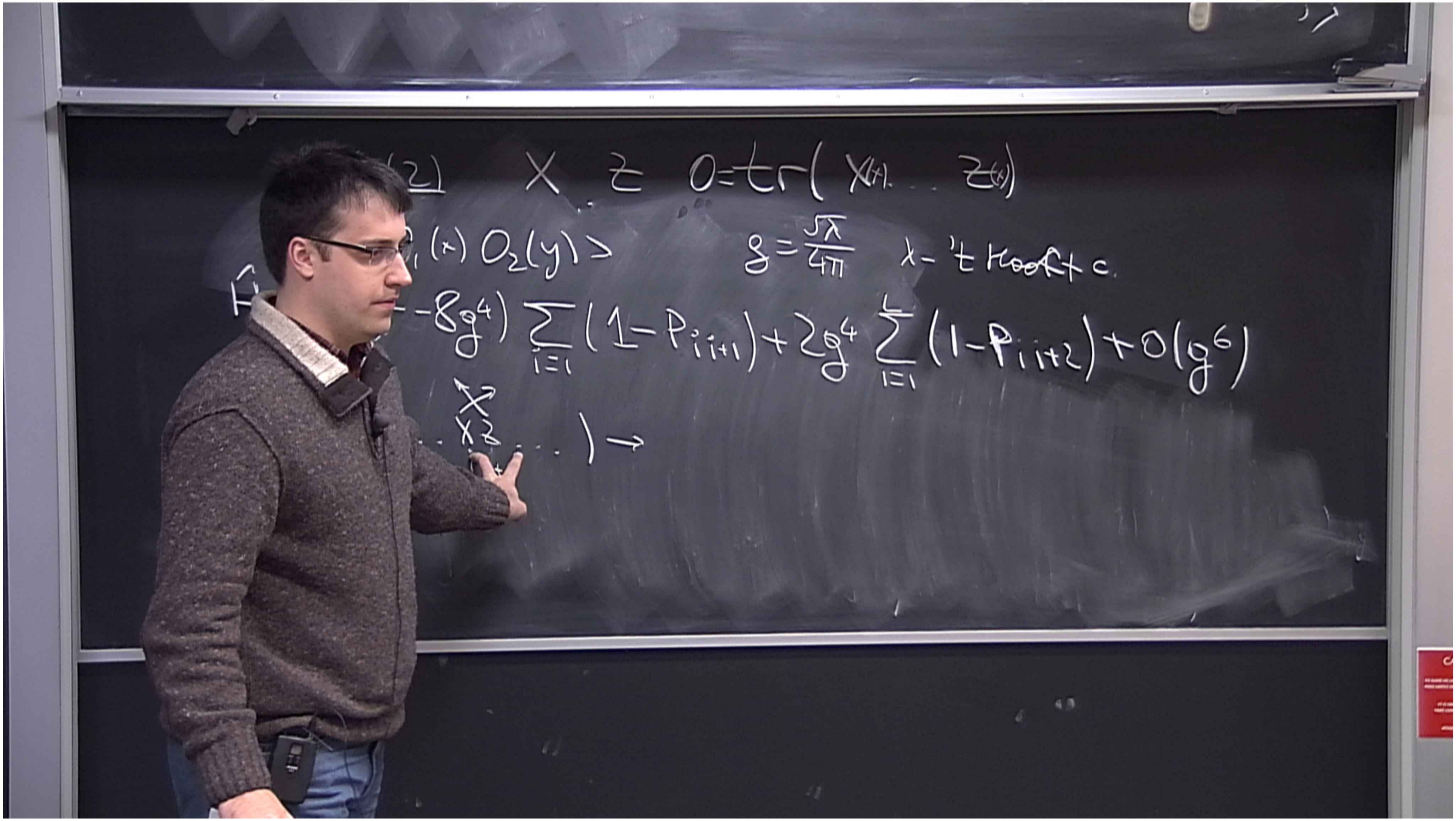
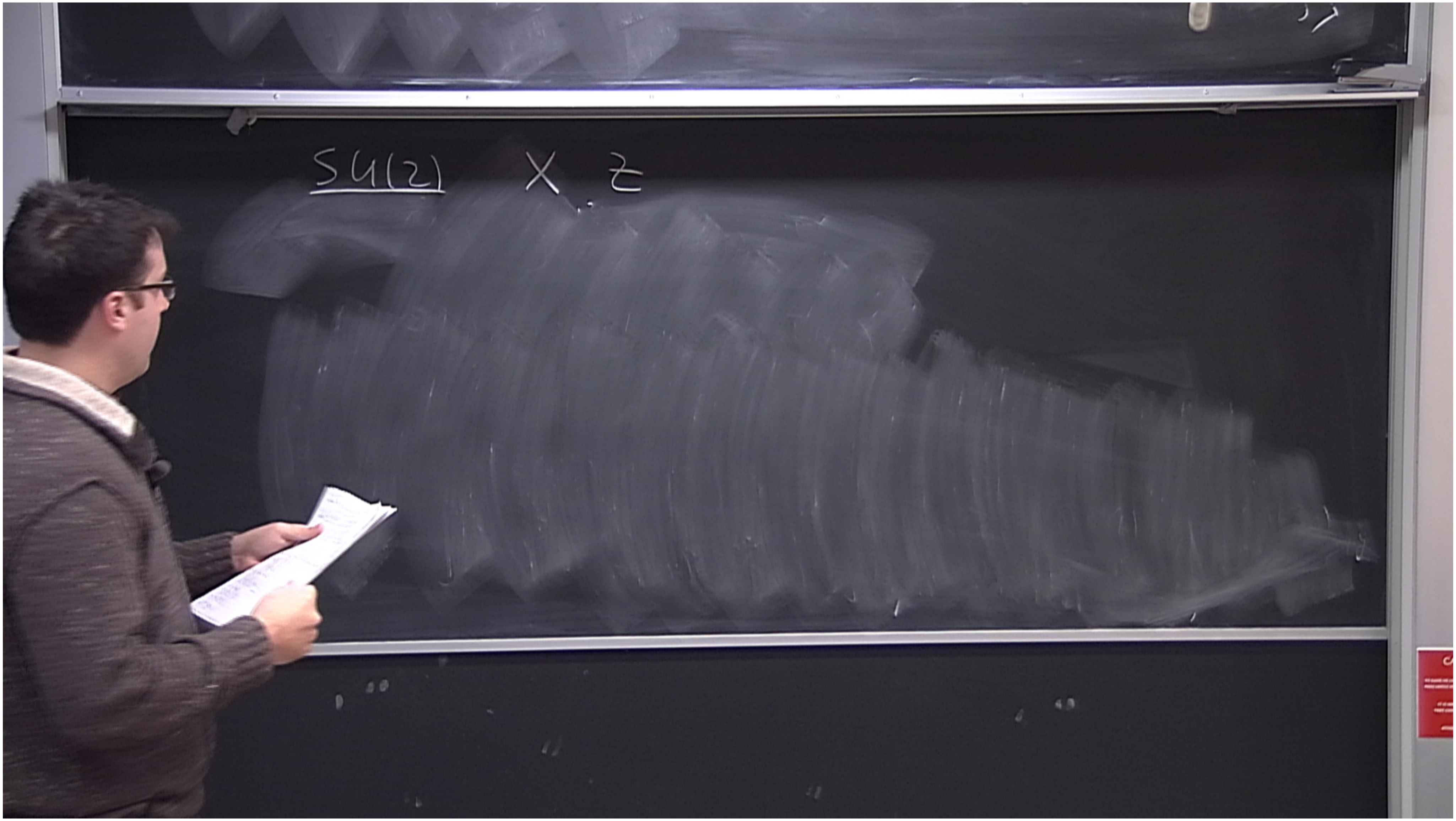
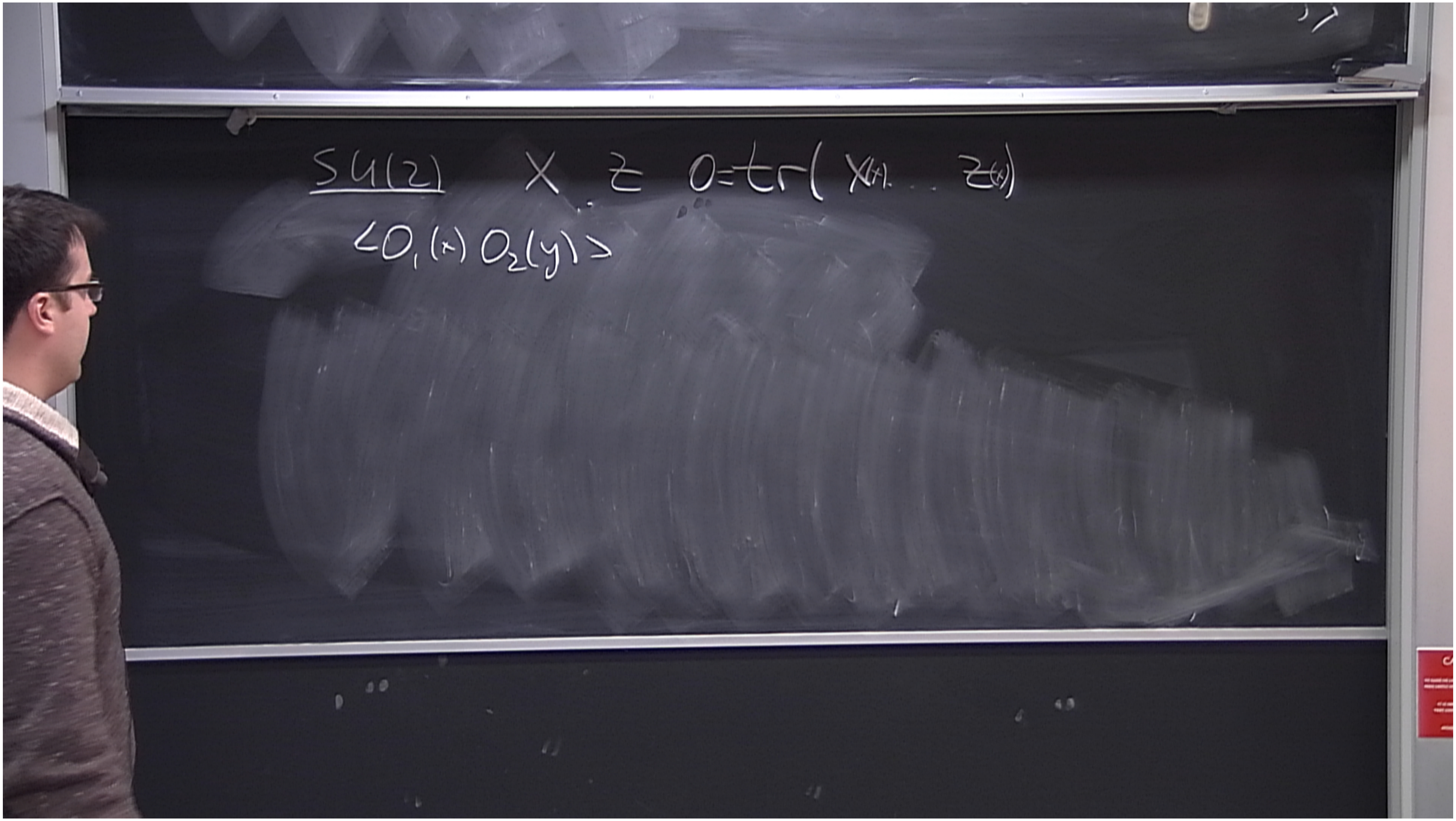
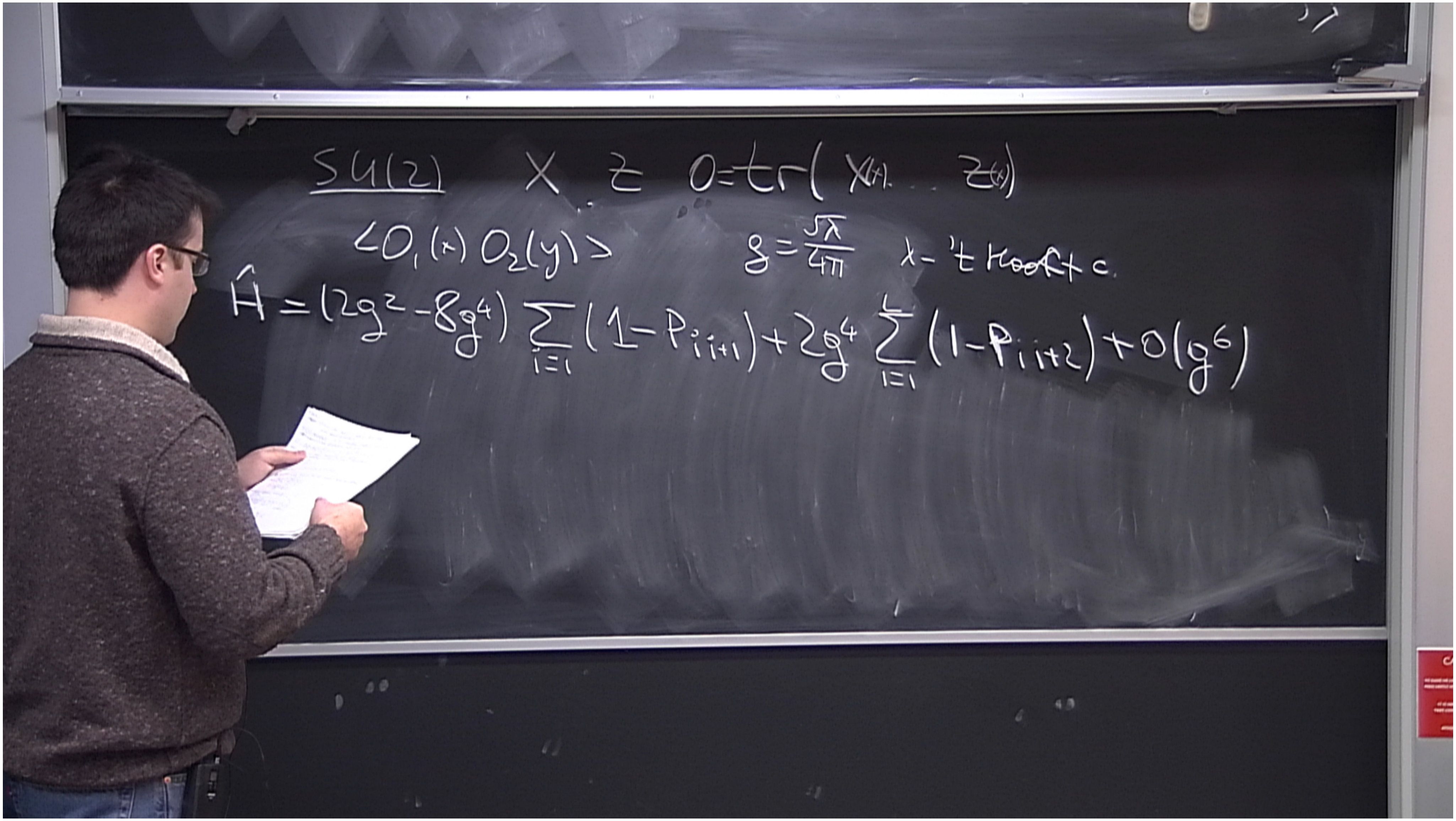
URL: http://pirsa.org/12020162

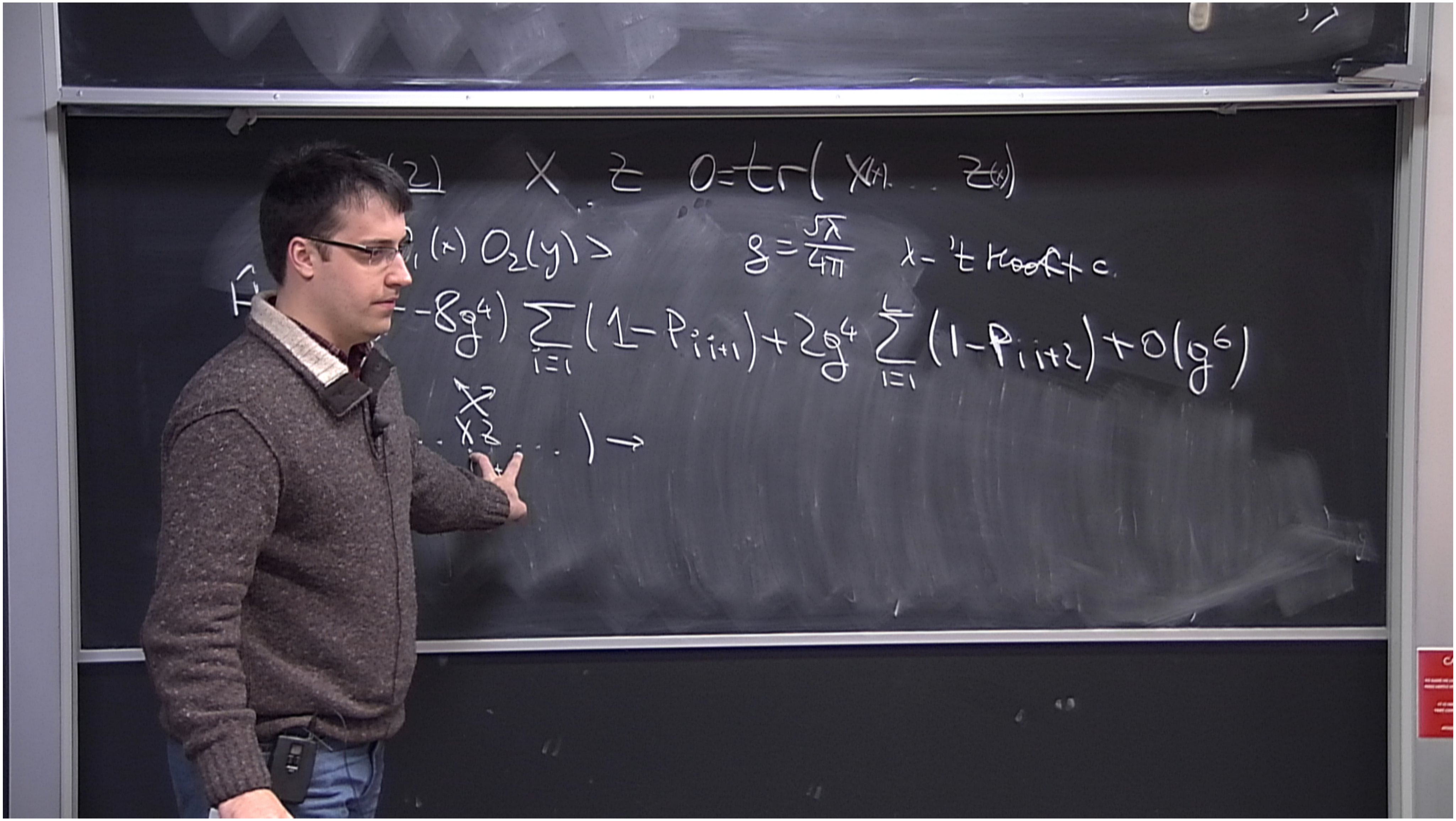
Abstract: Quantum corrections to three-point functions of scalar single trace operators in planar ${\cal N}=4$ Super-Yang-Mills theory are studied using integrability. At one loop, we find new algebraic structures that not only govern all two loop corrections to the mixing of the operators but also automatically incorporate all one loop diagrams correcting the tree level Wick contractions. Speculations about possible extensions of our construction to all loop orders are given. We also match our results with the strong coupling predictions in the classical (Frolov-Tseytlin) limit.

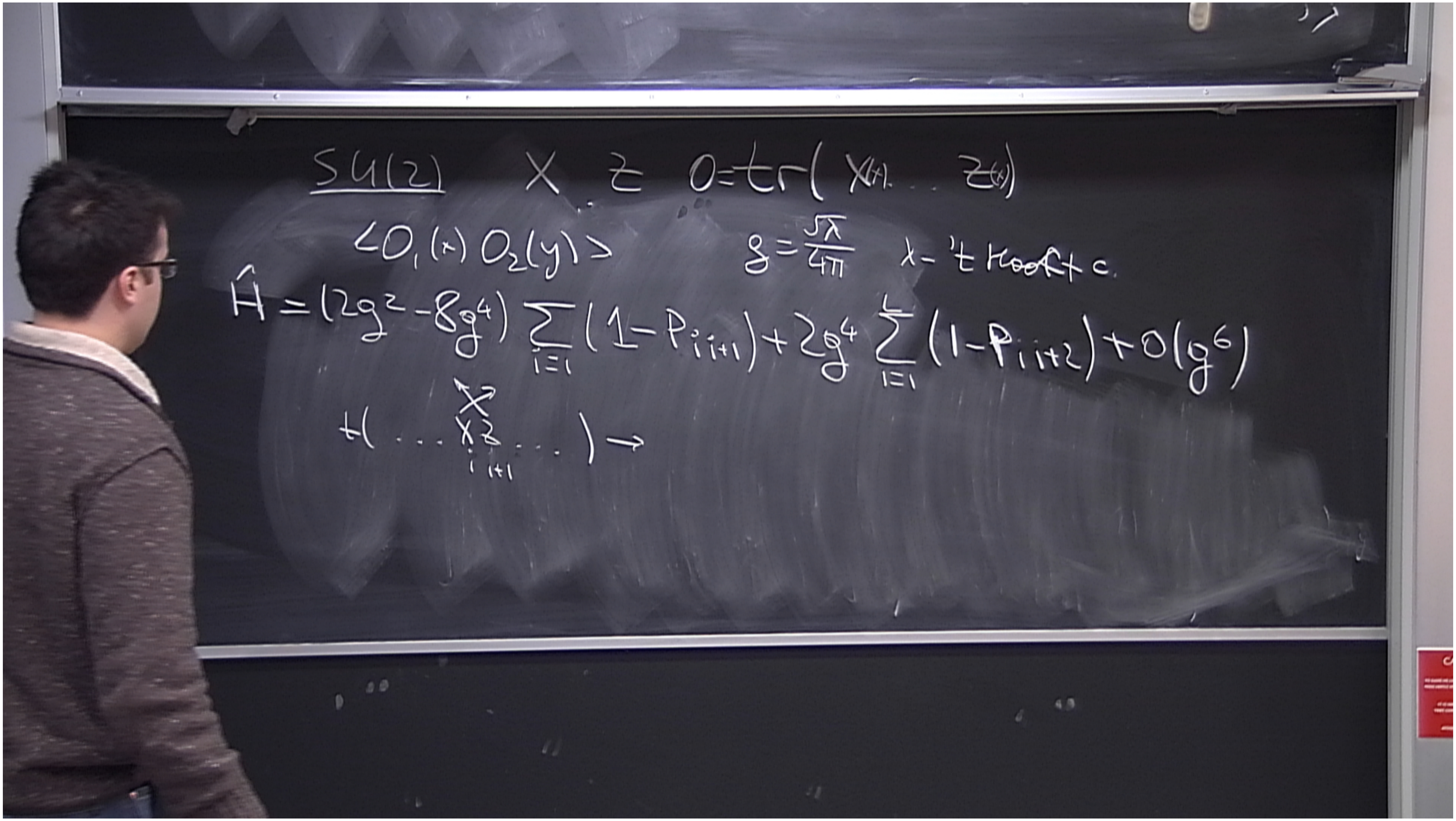

X Z O=tr(X(x) ... Z(x))
O₂(y)>
g = √λ/4π λ - 't Hooft c.
-8g⁴) Σ (1-P i i+1) + 2g⁴ Σ (1-P i i+2) + O(g⁶)
i=1 i=1

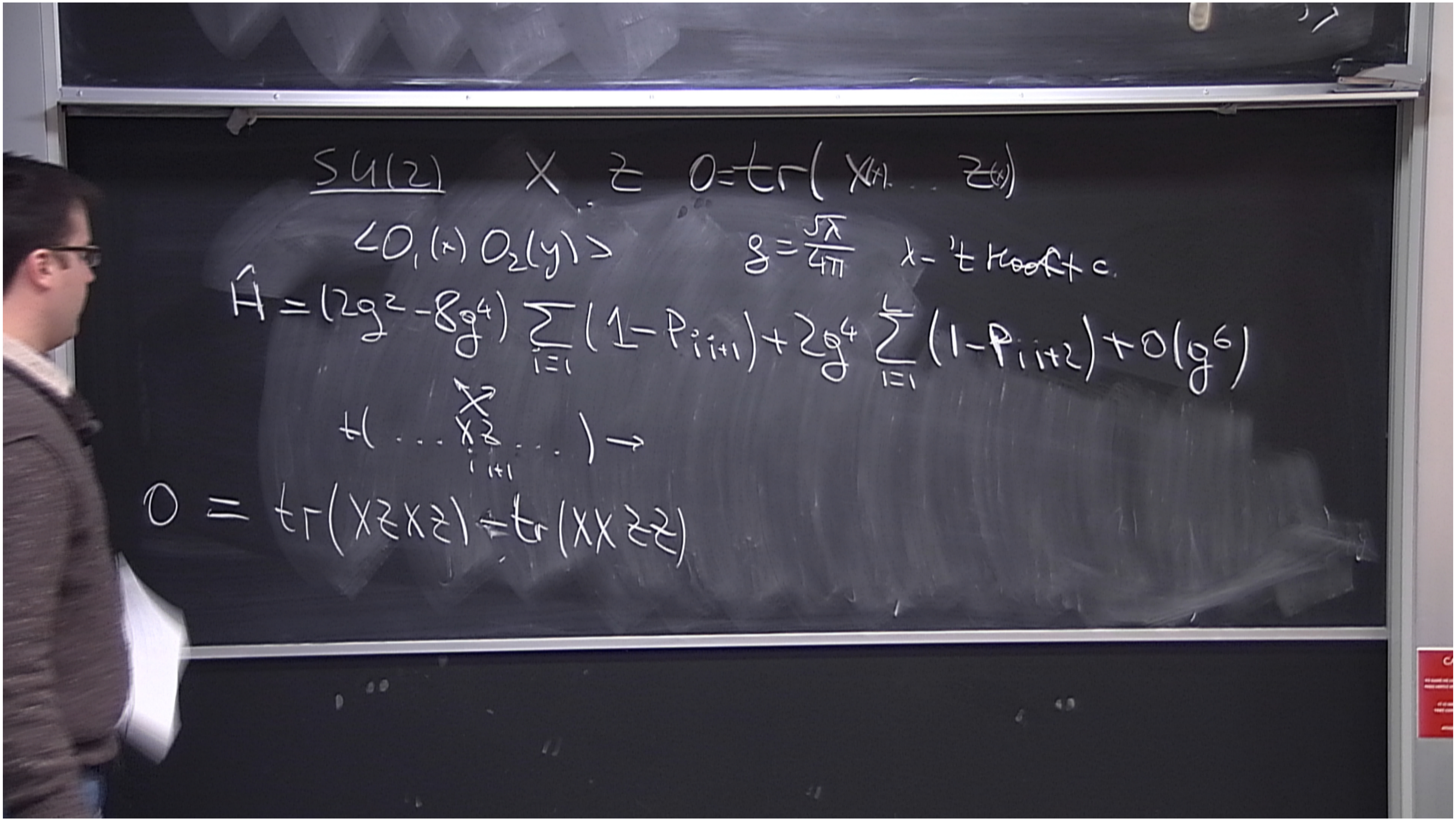

SU(2) X Z

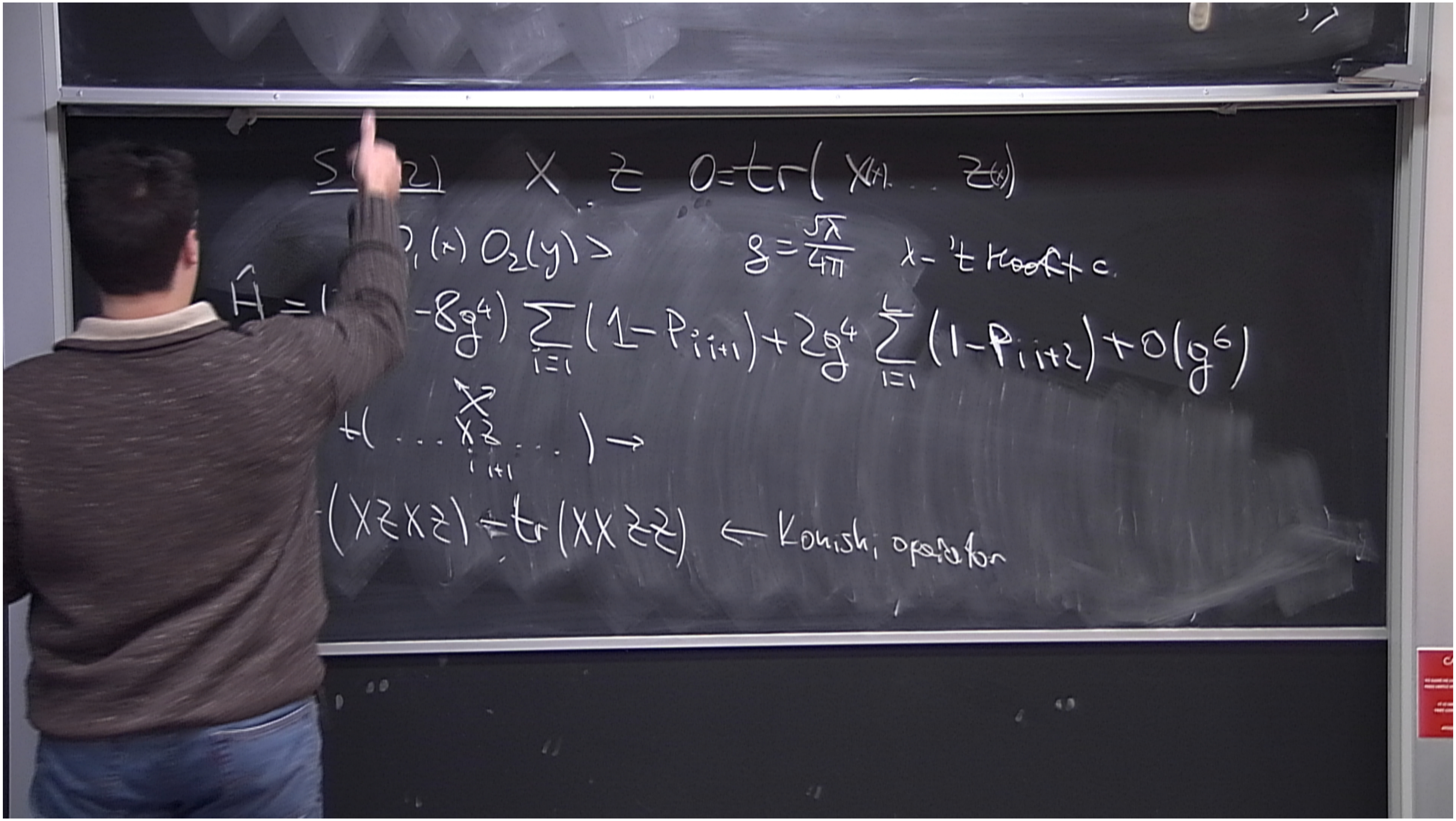

SU(2)   X   Z $O = \mathrm{tr}(X(x) \ldots Z(x))$
$\langle O_1(x) O_2(y) \rangle$


SU(2) X Z O=tr(X(x) ... Z(x))
⟨O1(x) O2(y)⟩
g = √λ/4π λ - 't Hooft c.
Ĥ = (2g² - 8g⁴) Σ i=1 (1 - P i i+1) + 2g⁴ Σ i=1 L (1 - P i i+2) + O(g⁶)

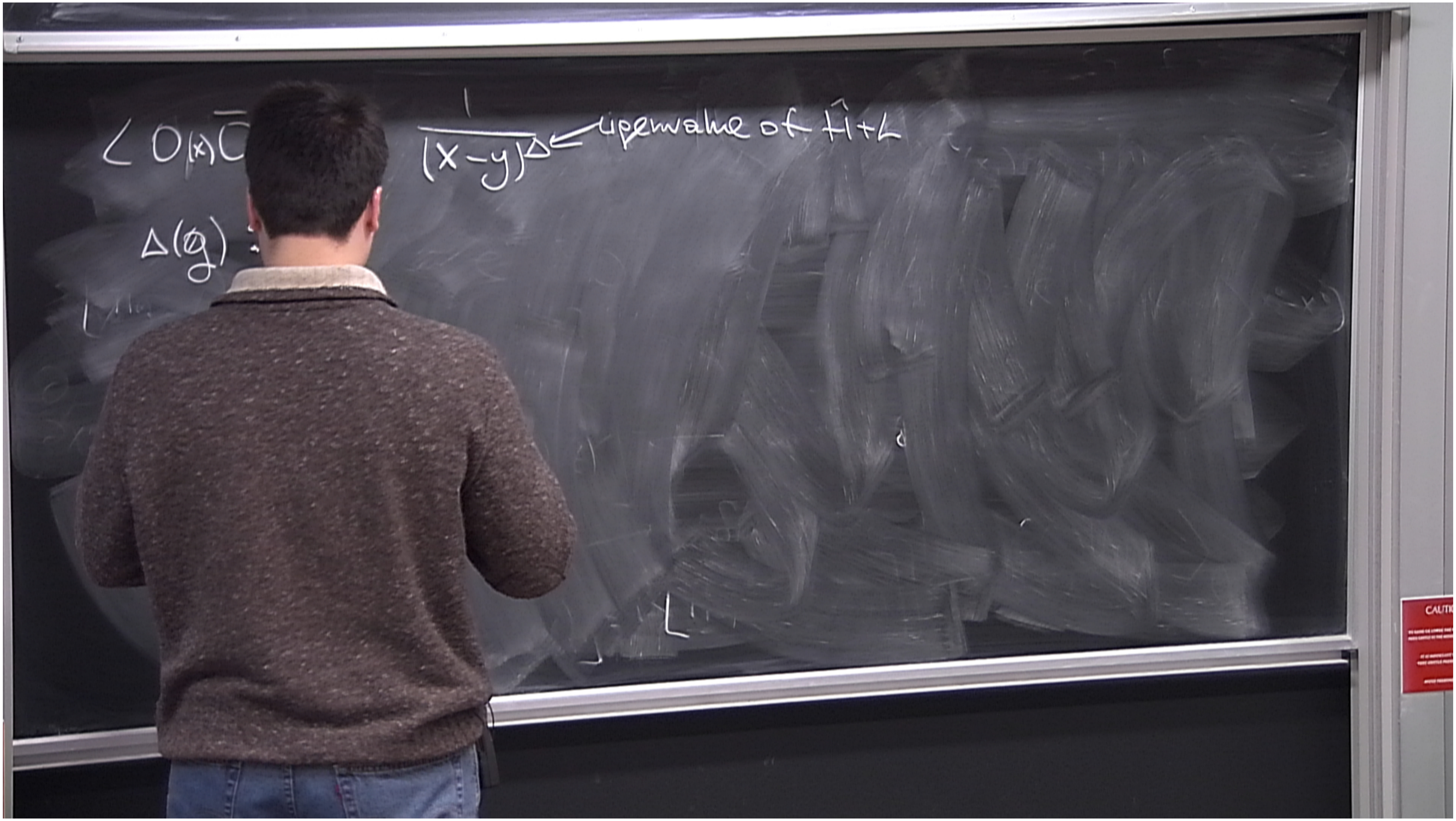

X Z O=tr(X(x) ... Z(x))
O2(y)>
g = √λ/4π λ - 't Hooft c.
-8g4) Σ (1-P i i+1) + 2g4 Σ (1-P i i+2) + O(g6)
i=1

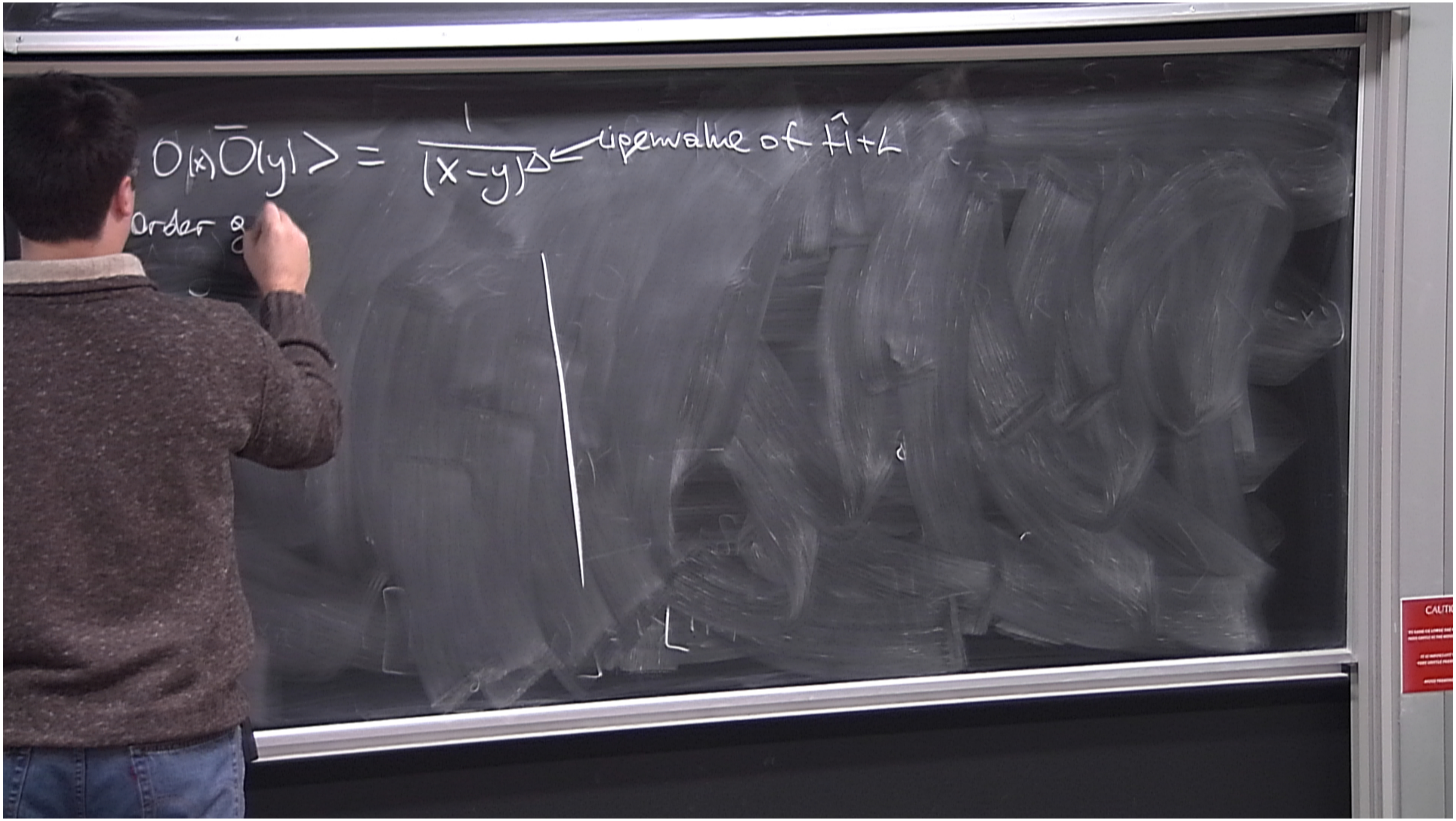

$SU(2)$ X Z $O = tr(X(x) \ldots Z(x))$
$\langle O_1(x) O_2(y) \rangle$
$g = \frac{\sqrt{\lambda}}{4\pi}$ λ - 't Hooft c.
$\hat{H} = (2g^2 - 8g^4) \sum_{i=1}^{L} (1 - P_{i\,i+1}) + 2g^4 \sum_{i=1}^{L} (1 - P_{i\,i+2}) + O(g^6)$
$tr(\ldots X_i Z_{i+1} \ldots) \rightarrow$


SU(2)   X   Z   O = tr(X(x) . . . Z(x))
⟨O₁(x) O₂(y)⟩
g = √λ / 4π   λ – 't Hooft c.
Ĥ = (2g² − 8g⁴) Σ_{i=1} (1 − P_{i i+1}) + 2g⁴ Σ_{i=1}^{L} (1 − P_{i i+2}) + O(g⁶)
tr(. . . X Z . . .) →
i i+1
O = tr(XZXZ) − tr(XXZZ)


X Z O=tr(X(x) ... Z(x))
O2(y)>
g = √λ/4π
λ - 't Hooft c.
-8g^4) Σ (1 - P i i+1) + 2g^4 Σ (1 - P i i+2) + O(g^6)
i=1
(XZXZ) = tr (XXZZ) ← Konishi operator

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}} \leftarrow \text{eigenvalue of } \hat{H}+L$$




O(x)Ō(y)> = 1/(x-y)^Δ

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}} \leftarrow \text{eigenvalue of } \hat{H}+L$$

Order g^2 (1-loop)

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}} \leftarrow \text{eigenvalue of } \hat{H}+L$$

Order g^2 (1-loop)

$$\left(\frac{u_k + i/2}{u_k - i/2}\right)^{L \leftarrow \#x+\#t} = \prod_{j\neq k} \frac{u_k - u_j + i}{u_k - u_j - i}$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

order g^2 (1-loop)

#X+#Z

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = \prod_{j\neq k}\frac{u_k-u_j+i}{u_k-u_j-i}$$

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

All-loops

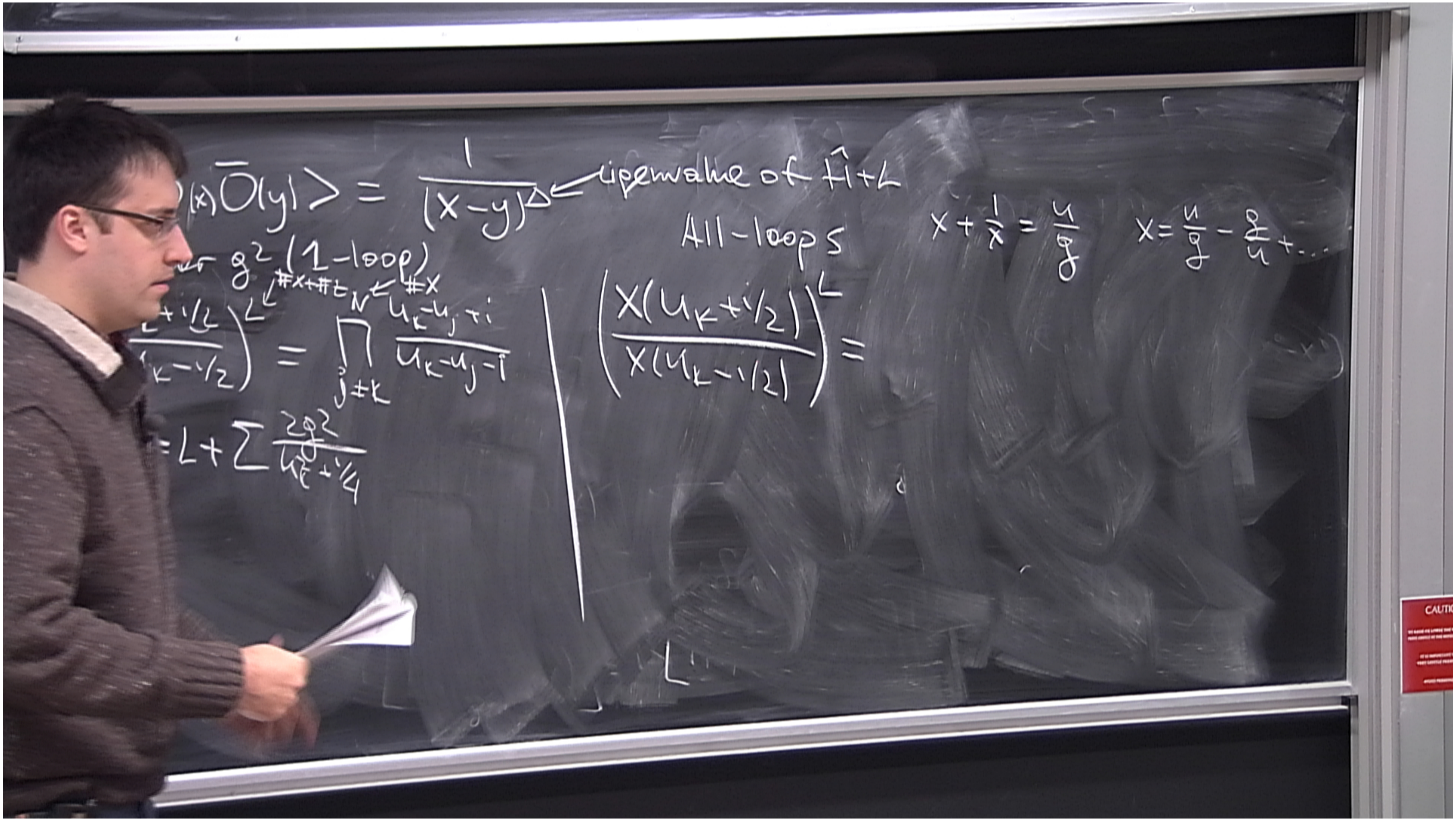
$$x+\frac{1}{x}=\frac{u}{g}$$

Order g^2 (1-loop)

$L = \#X + \#Z$

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = \prod_{j\neq k}\frac{u_k-u_j+i}{u_k-u_j-i}$$

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$


All-loops
x + 1/x = u/g
x = u/g - g/u + ...

$$\langle \mathcal{O}(x)\bar{\mathcal{O}}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

Order g^2 (1-loop)

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{\overset{\#X+\#Z}{L}} = \prod_{j\neq k}^{\overset{\#X}{N}} \frac{u_k-u_j+i}{u_k-u_j-i}$$

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

All-loops

$$x+\frac{1}{x}=\frac{u}{g} \qquad x = \frac{u}{g}-\frac{g}{u}+\dots$$

$$\left(\frac{x(u_k+i/2)}{x(u_k-i/2)}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}\,\sigma^2(u_k,u_j)$$

dressing

$$\sigma^2 = 1 + O(g^6)$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}} \leftarrow \text{eigenvalue of } \hat{H}+L$$

Order g^2 (1-loop)

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^L = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}$$

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

All-loops

$$x+\frac{1}{x}=\frac{u}{g} \qquad x=\frac{u}{g}-\frac{g}{u}+\dots$$

$$\left(\frac{x(u_k+i/2)}{x(u_k-i/2)}\right)^L = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}\,\underset{\text{dressing}}{\sigma^2(u_k,u_j)}$$

$$\sigma^2 = 1+O(g^6)$$

$$E = L + \sum_k \left(\frac{2ig}{x(u_k+i/2)} - \frac{2ig}{x(u_k-i/2)}\right)$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

Order g^2 (1-loop)

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}$$

L: #x+#L, N: #x

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

All-loops

$$x+\frac{1}{x} = \frac{u}{g} \qquad x = \frac{u}{g} - \frac{g}{u} + \dots$$

$$\left(\frac{x(u_k+i/2)}{x(u_k-i/2)}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}\,\sigma^2(u_k,u_j)$$

σ^2: dressing

$$\sigma^2 = 1 + O(g^6)$$

$$E = L + \sum_k \left(\frac{2ig}{x(u_k+i/2)} - \frac{2ig}{x(u_k-i/2)}\right)$$

$$p = \frac{1}{i}\ln\frac{x(u+i/2)}{x(u-i/2)}$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

Order g^2 (1-loop)

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}$$

L ← #x+#t, N ← #x

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

All-loops

$$x + \frac{1}{x} = \frac{u}{g} \qquad x = \frac{u}{g} - \frac{g}{u} + \ldots$$

$$\left(\frac{x(u_k+i/2)}{x(u_k-i/2)}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}\,\sigma^2(u_k,u_j)$$

σ^2 ← dressing

$$\sigma^2 = 1 + O(g^6)$$

$$\Delta = L + \sum_k \left(\frac{2ig}{x(u_k+i/2)} - \frac{2ig}{x(u_k-i/2)}\right)$$

$$p = \frac{1}{i}\ln\frac{x(u+i/2)}{x(u-i/2)}$$

$$\langle O(x)\bar{O}(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$ ← eigenvalue of $\hat{H}+L$

Order g^2 (1-loop)

$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}$$

L ← #x+#t, N ← #x

$$\Delta = L + \sum \frac{2g^2}{u_k^2+1/4}$$

$$e^{ip_k L} = \prod S(p_j,p_k)$$

All-loops

$$x+\frac{1}{x} = \frac{u}{g} \qquad x = \frac{u}{g} - \frac{g}{u} + \dots$$

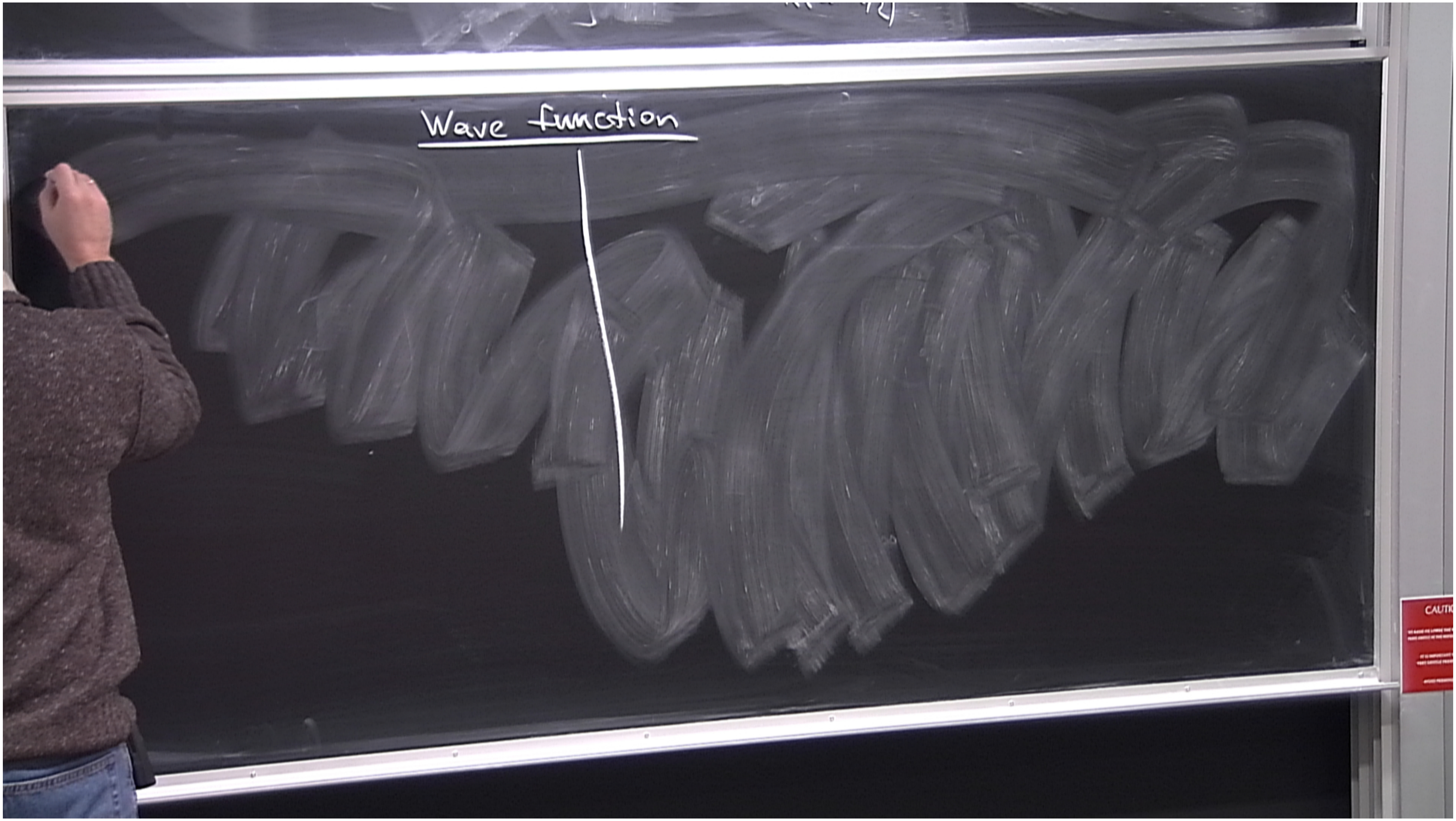
$$\left(\frac{x(u_k+i/2)}{x(u_k-i/2)}\right)^{L} = \prod_{j\neq k}^{N} \frac{u_k-u_j+i}{u_k-u_j-i}\,\sigma^2(u_k,u_j)$$

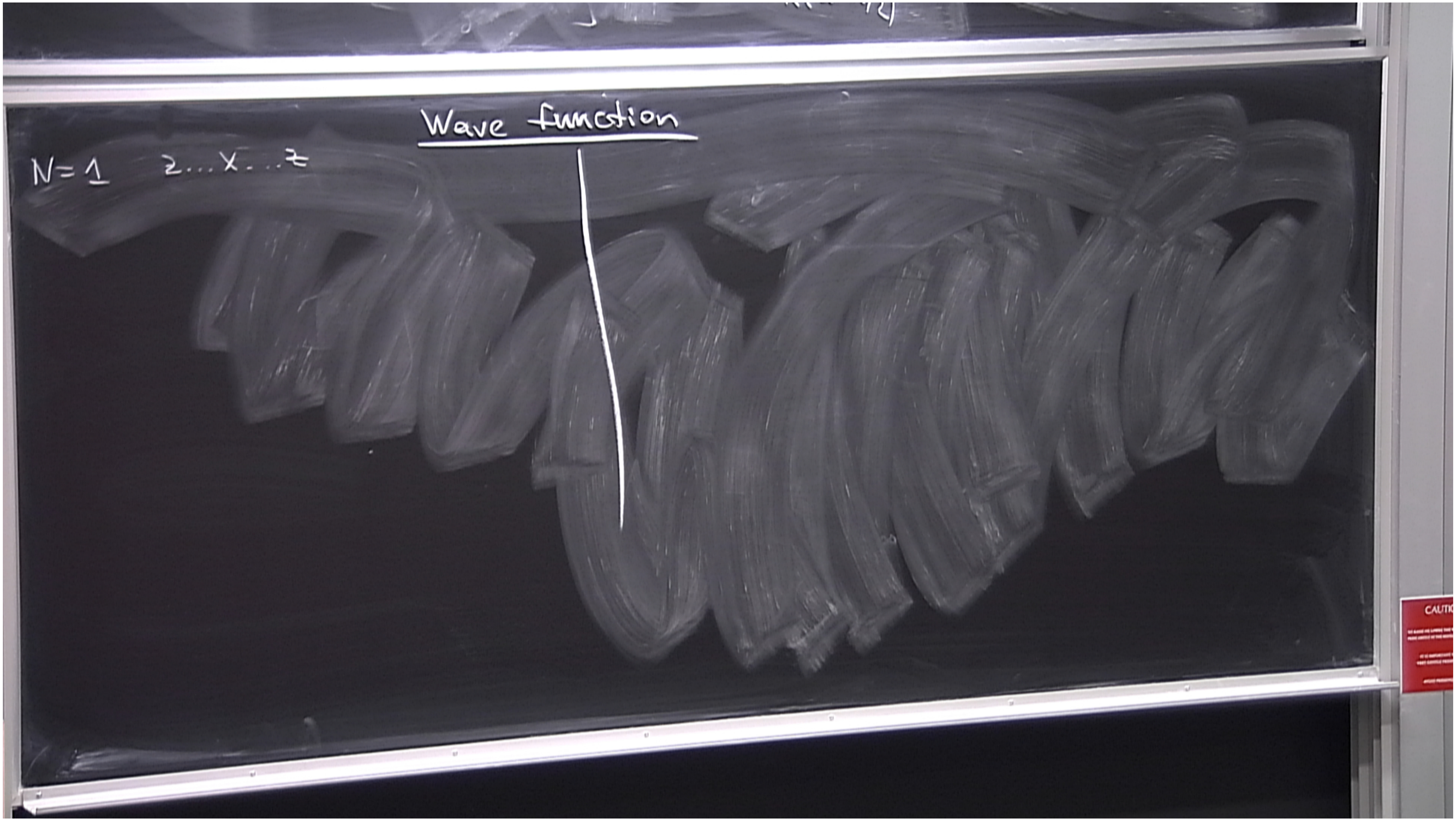
σ^2 ← dressing

$$\sigma^2 = 1 + O(g^6)$$

$$\Delta = L + \sum_k \left(\frac{2ig}{x(u_k+i/2)} - \frac{2ig}{x(u_k-i/2)}\right)$$

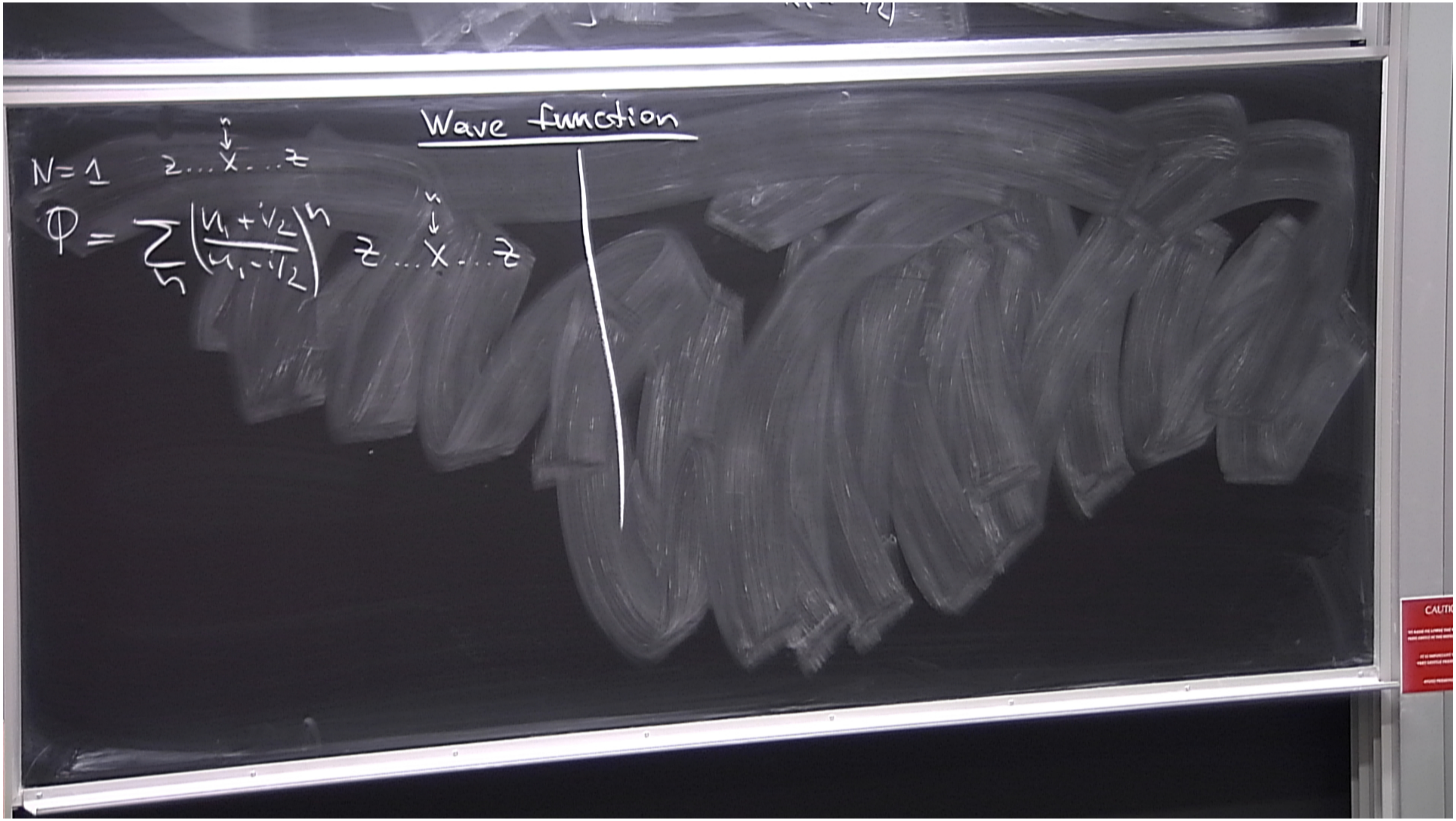
$$p = \frac{1}{i}\ln\frac{x(u+i/2)}{x(u-i/2)}$$

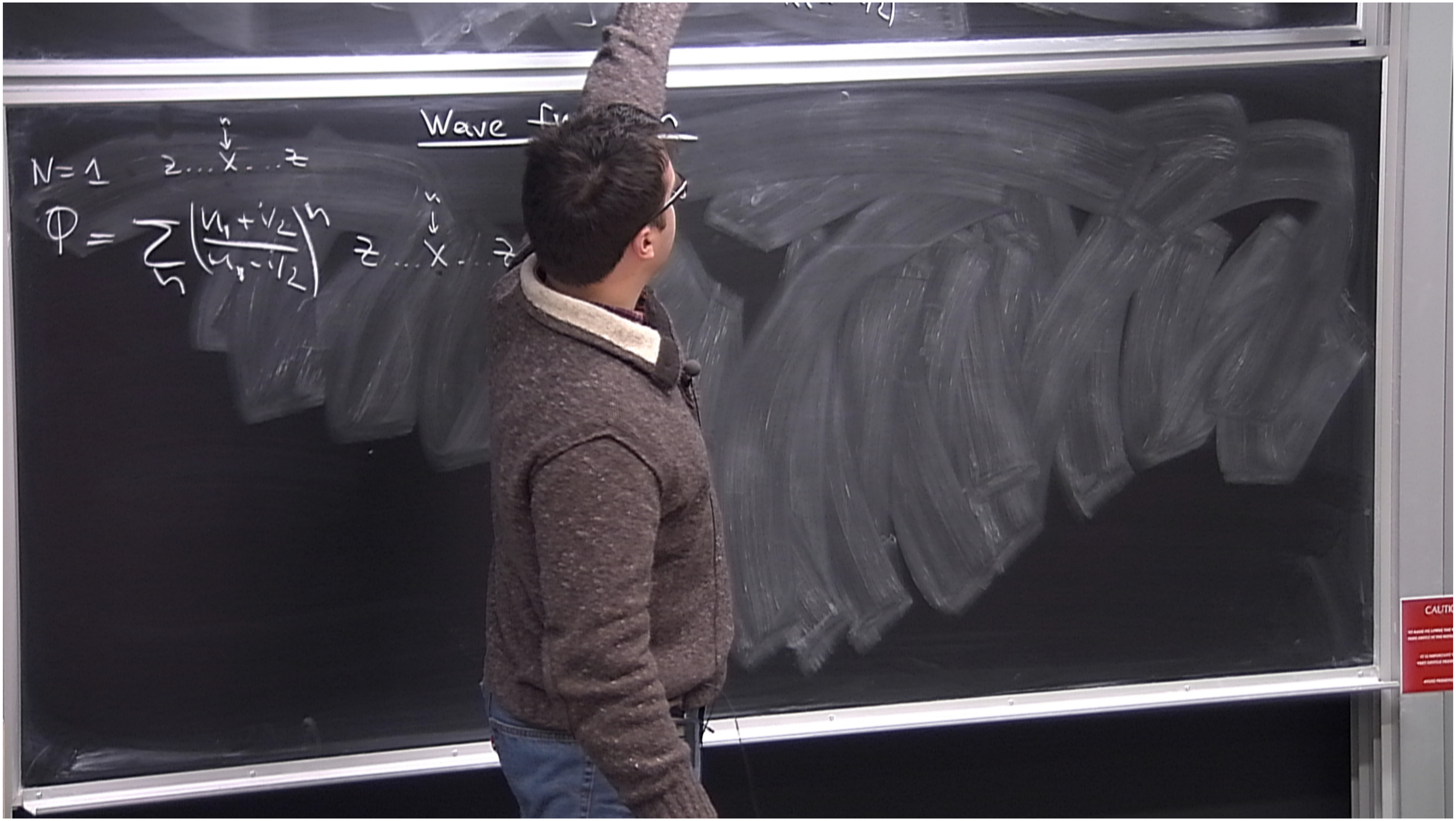

Wave function

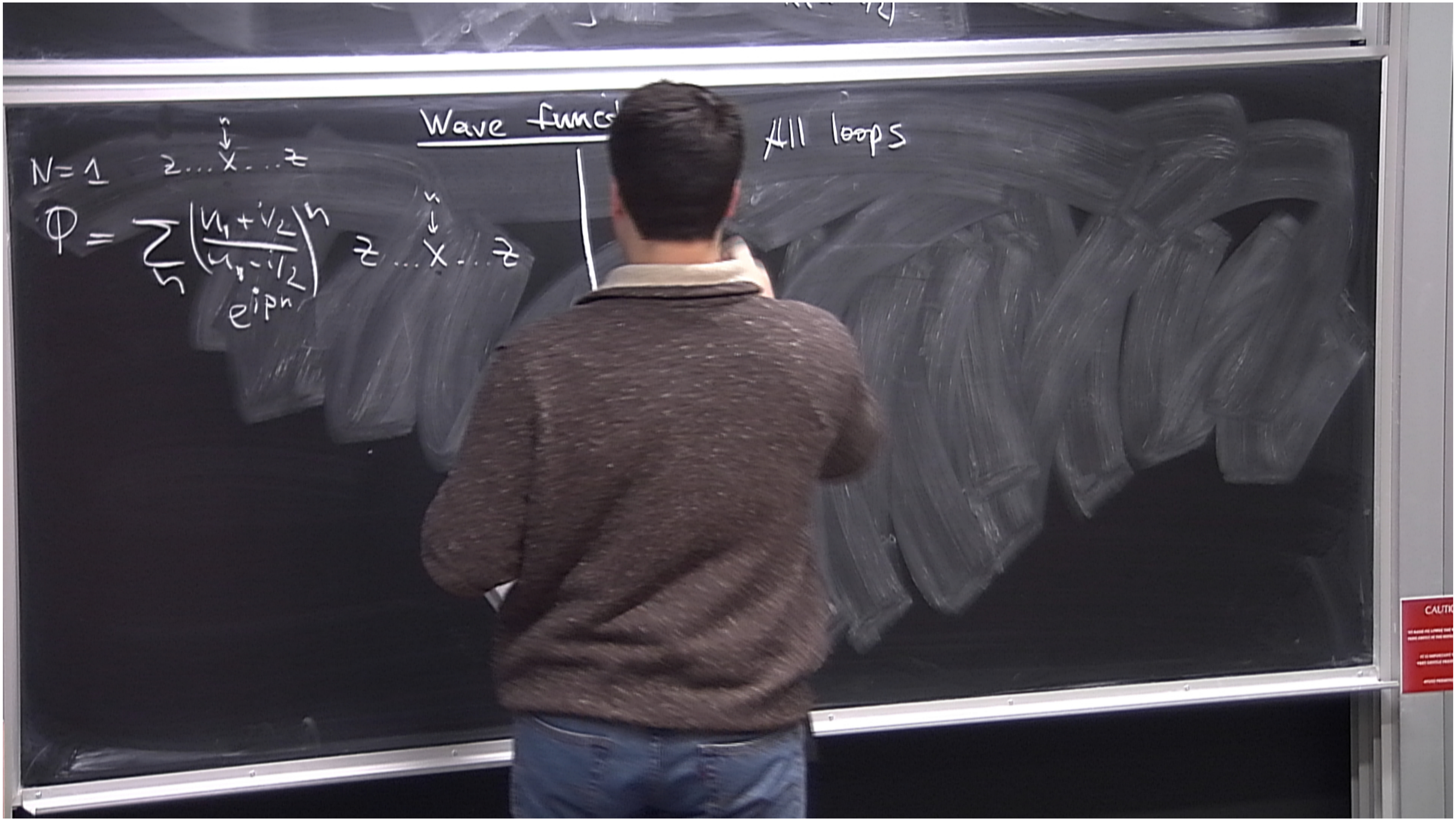

Wave function
N=1 z...x...z

Wave function

$N=1 \quad 2 \ldots \overset{n}{\downarrow}{x} \ldots 7$


Wave function
$N=1 \quad z \dots \overset{n}{\downarrow} x \dots z$
$\Phi = \sum_n \left(\frac{u_1 + i u_2}{u_1 - i u_2}\right)^n z \dots \overset{n}{\downarrow} x \dots z$


Wave fu
N = 1   z ... x ... z
Φ = Σ_n ((u_1 + i/2)/(u_1 − i/2))^n   z ... x ... z


Wave func
All loops
N=1

Wave function

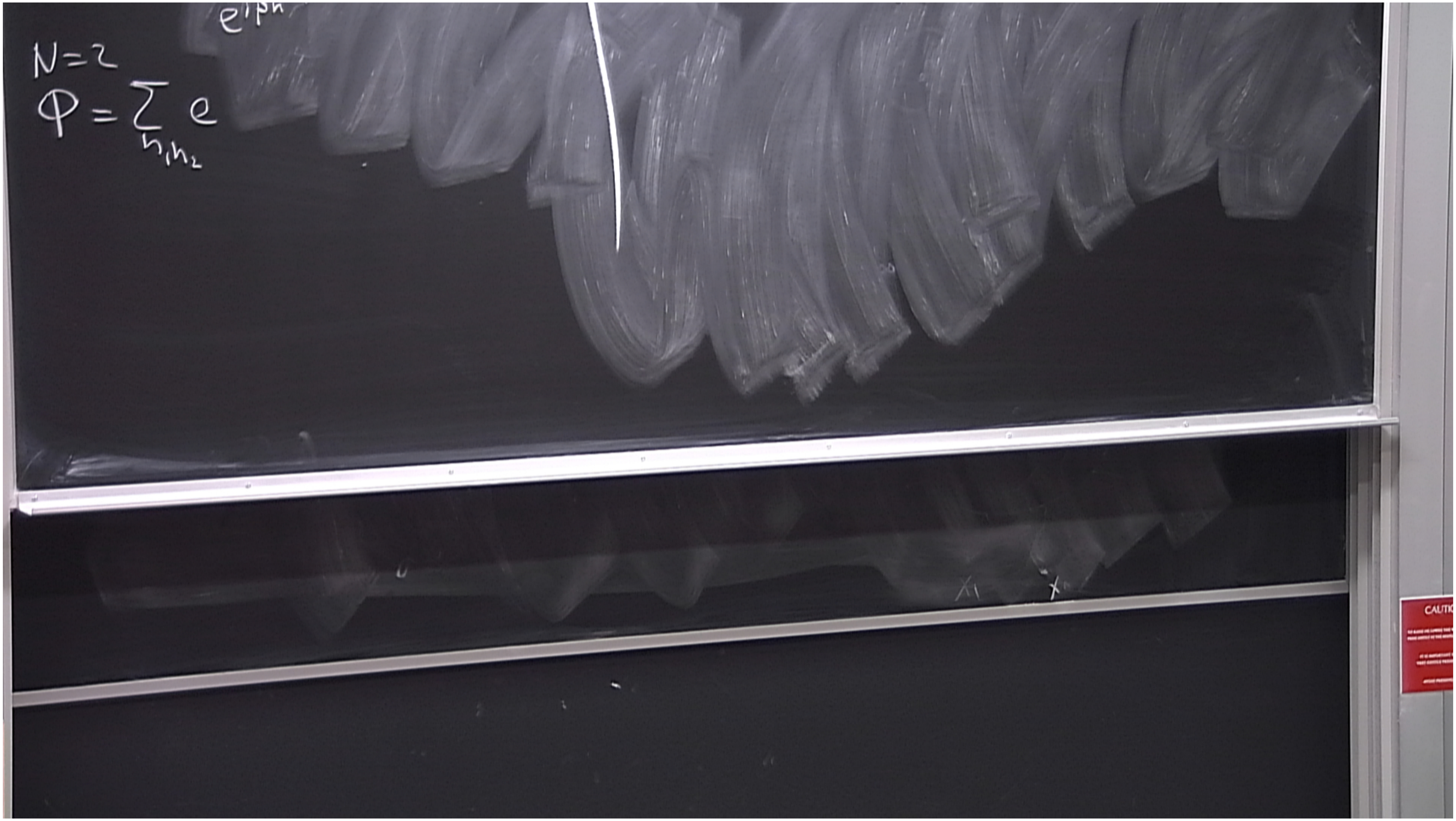
$N=1 \qquad z \ldots \overset{n}{\overset{\downarrow}{x}} \ldots z$

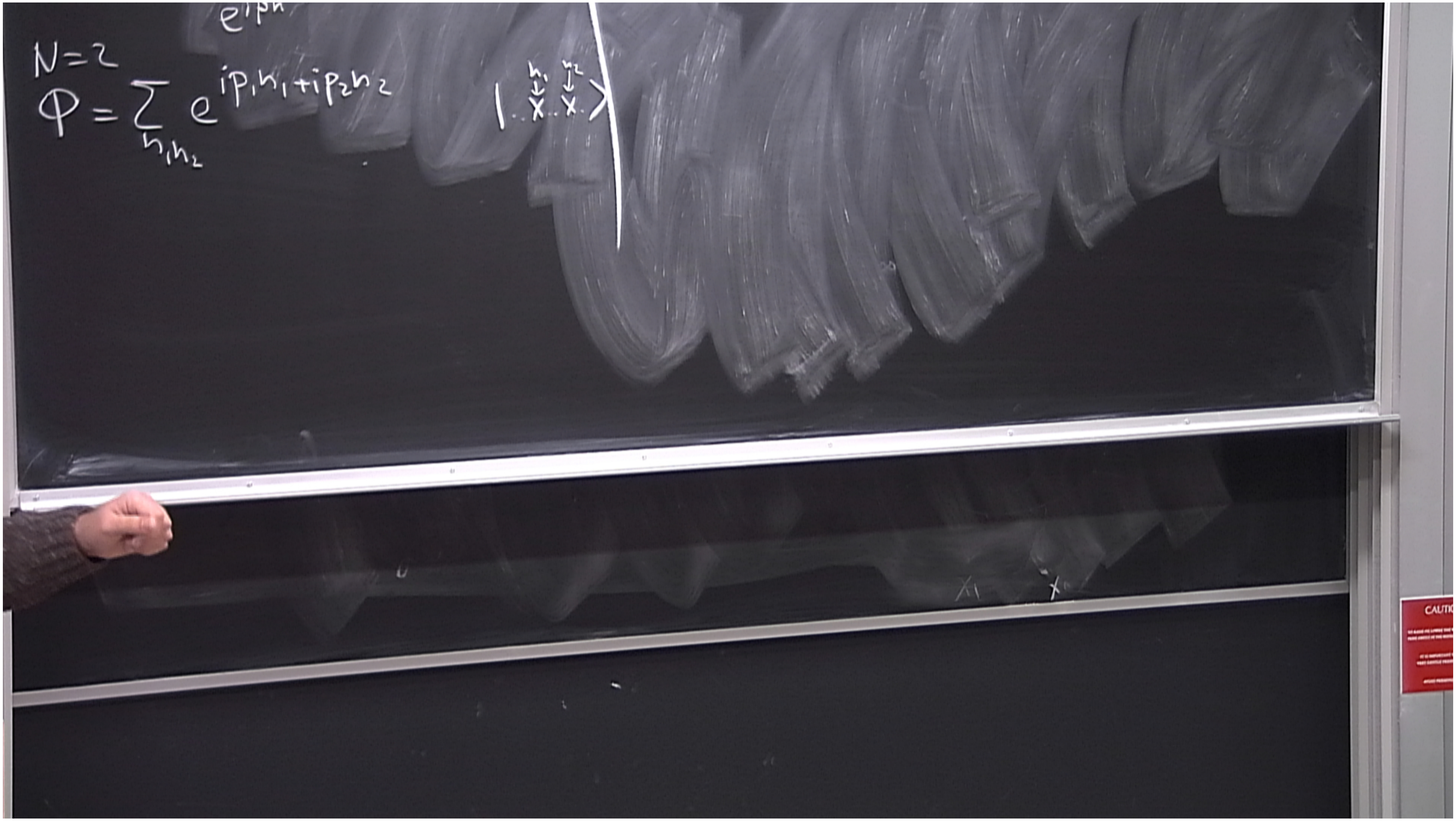
$$\Phi = \sum_n \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^n z \ldots \overset{n}{\overset{\downarrow}{x}} \ldots z$$

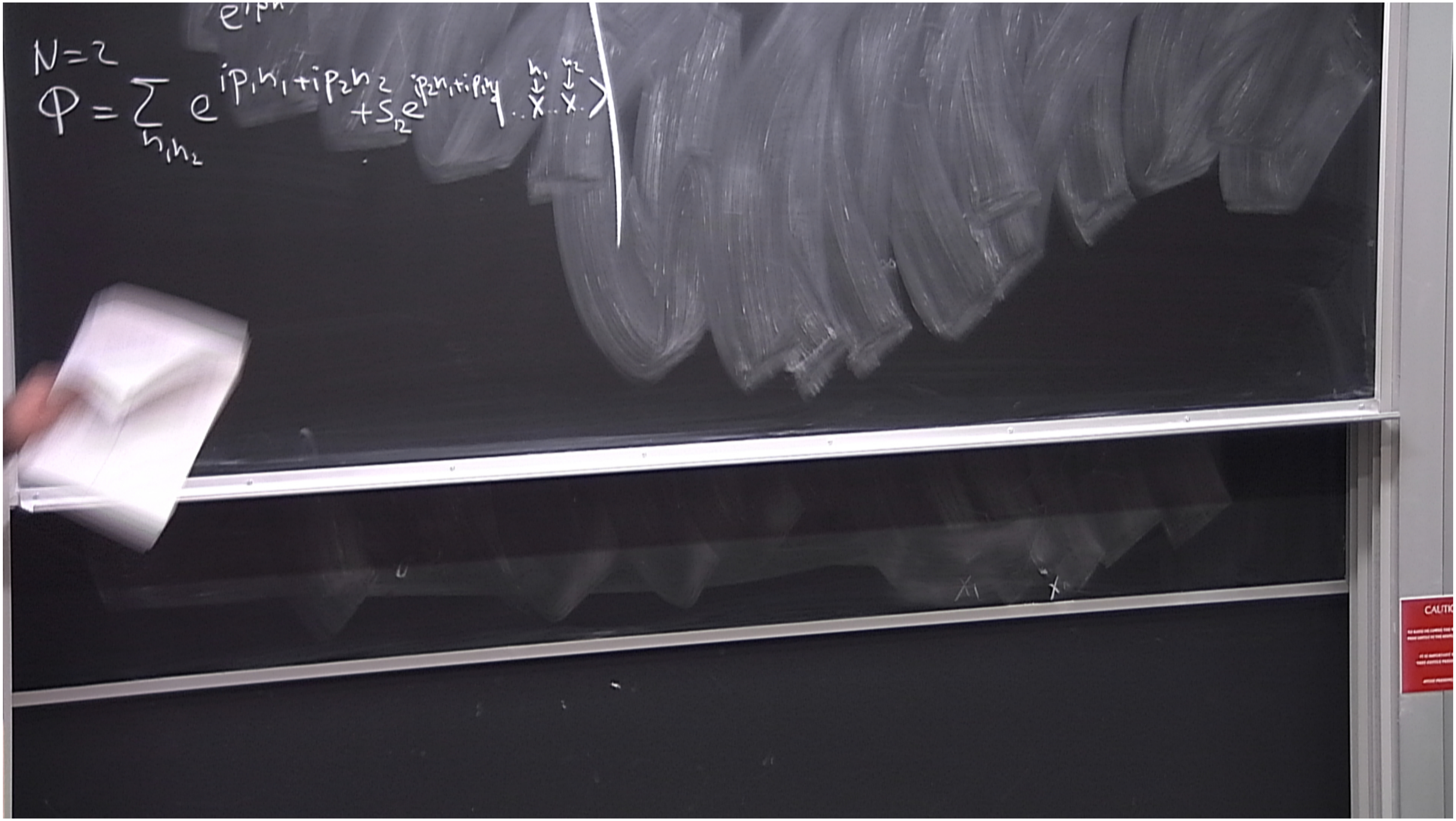
$$e^{ipn}$$

All loops

$$\varphi = \sum_n \left(\frac{X(u_1 + i/2)}{X(u_1 - i/2)}\right)^n | \ldots \overset{n}{\overset{\downarrow}{x}} \ldots z \rangle$$


N=2
Φ = Σ e
n1,n2


N=2
$\Phi = \sum_{n_1 n_2} e^{ip_1 n_1 + ip_2 n_2}$
$| \ldots \overset{n_1}{x} \ldots \overset{n_2}{x} \ldots \rangle$


N=2
$\Phi = \sum_{n_1,n_2} e^{ip_1n_1+ip_2n_2} + S_{12}\, e^{ip_2n_1+ip_1n_2} |\ldots \overset{n_1}{x} \ldots \overset{n_2}{x} \ldots\rangle$

Wave function

$N=1$ $\qquad z \ldots \overset{n}{x} \ldots z$

$$\Phi = \sum_n \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^n \; z \ldots \overset{n}{x} \ldots z$$

$$e^{ipn}$$

$N=2$

$$\Phi = \sum_{n_1, n_2} e^{ip_1 n_1 + ip_2 n_2} + S_{12} e^{ip_2 n_1 + ip_1 n_2} \; | \ldots \overset{n_1}{x} \ldots \overset{n_2}{x} \ldots \rangle$$

All loops

$$\Phi = \sum_n \left(\frac{x(u_1 + i/2)}{x(u_1 - i/2)}\right)^n \; | \ldots \overset{n}{x} \ldots$$

$$\Phi = \sum_n \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^n z \ldots \overset{n}{\underset{\downarrow}{}} x \ldots z$$

e^{ipn}

$N=2$

$$\Phi = \sum_{n_1 n_2} e^{ip_1 n_1 + ip_2 n_2} + S_{12} e^{ip_2 n_1 + ip_1 n_2} | \ldots \overset{n_1}{x} \ldots \overset{n_2}{x} \ldots \rangle$$

$$\varphi = \sum_n \left(\frac{x(u_1 + i/2)}{x(u_1 - i/2)}\right) | \ldots x \ldots z \rangle$$

$\varphi \overset{?}{=} \sum$ the same $\quad p_1 = \frac{1}{i} \ln \frac{x(u + i/2)}{x(u - i/2)}$

$$\varphi = \sum_n \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^n \quad z \ldots \overset{n}{\underset{\downarrow}{x}} \ldots z$$

e^{ipn}

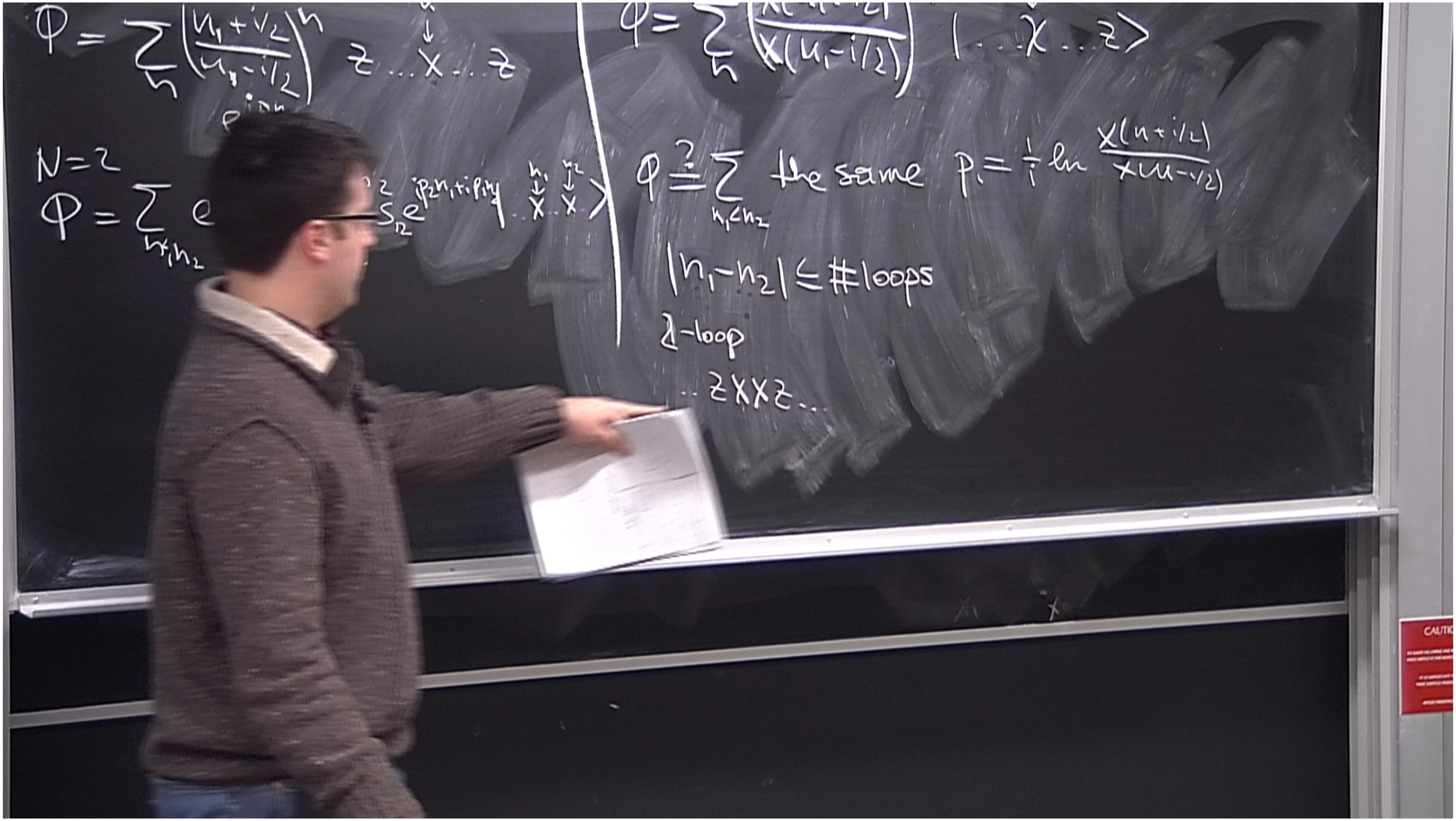
$N=2$

$$\varphi = \sum_{n_1 < n_2} e^{ip_1 n_1 + ip_2 n_2} + S_{12} e^{ip_2 n_1 + ip_1 n_2} \ldots \overset{n_1}{\underset{\downarrow}{x}} \ldots \overset{n_2}{\underset{\downarrow}{x}} \ldots \rangle$$

$$\varphi = \sum_n \left(\frac{x(u_1 + i/2)}{x(u_1 - i/2)}\right) \quad | \ldots x \ldots z \rangle$$

$$\varphi \overset{?}{=} \sum_{n_1 < n_2} \text{ the same } \quad p_1 = \frac{1}{i} \ln \frac{x(u + i/2)}{x(u - i/2)}$$

$$|n_1 - n_2| \leq \#\text{loops}$$


N=2
the same
|n1-n2| ≤ #loops
2-loop
... Z X X Z ...

$$\Phi = \sum_n \left(\frac{u_1+i/2}{u_1-i/2}\right)^n \quad z \ldots \overset{n}{\overset{\downarrow}{x}} \ldots z$$

e^{ipn}

$N=2$

$$\Phi = \sum_{n_1, n_2} e^{ip_1 n_1 + ip_2 n_2} + S_{12} e^{ip_2 n_1 + ip_1 n_2} \, | \ldots \overset{n_1}{\overset{\downarrow}{X}} \ldots \overset{n_2}{\overset{\downarrow}{X}} \ldots \rangle$$

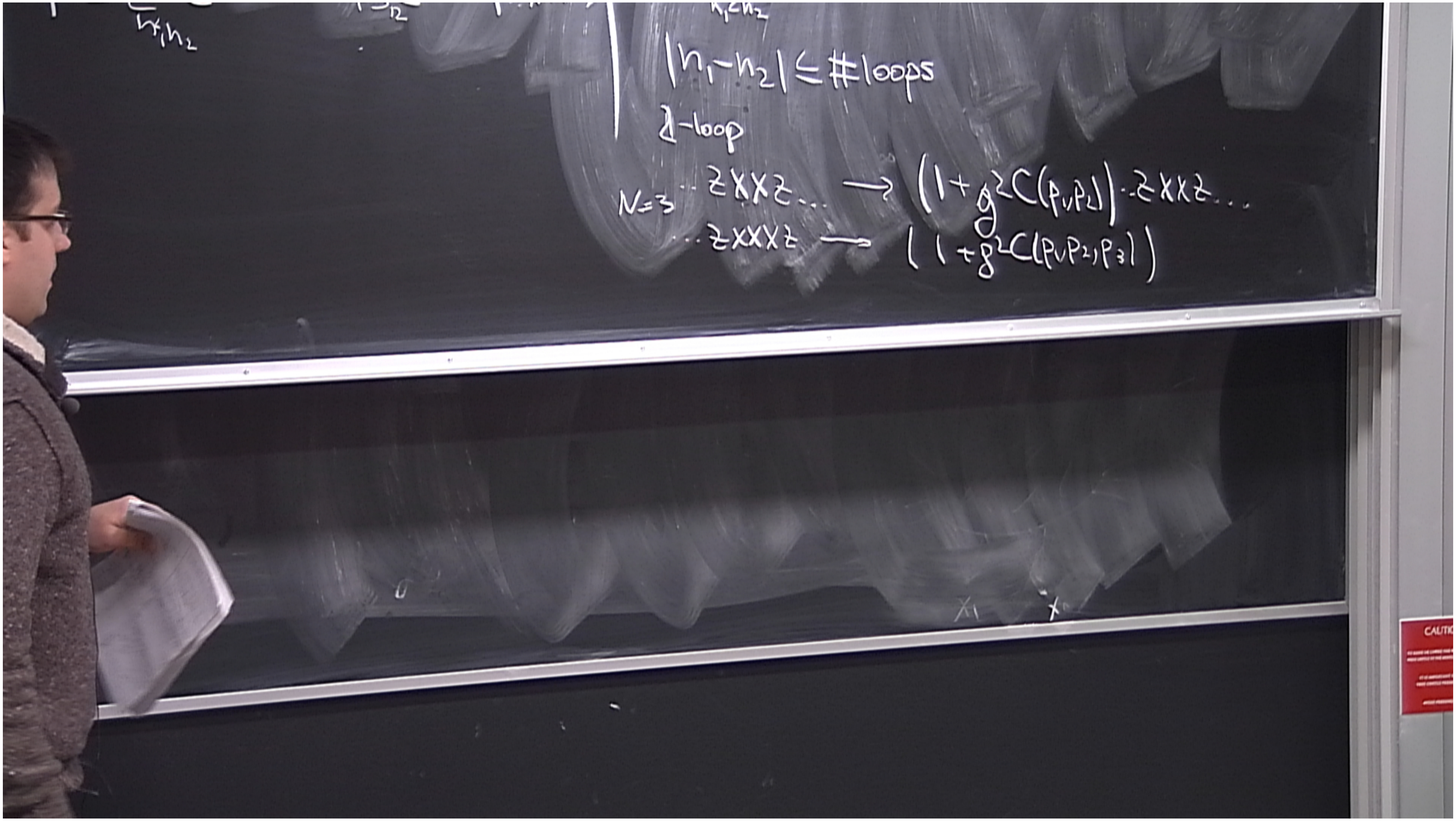
$$\phi = \sum_n \left(\frac{x(u_1+i/2)}{x(u_1-i/2)}\right)^n \, | \ldots X \ldots z \rangle$$

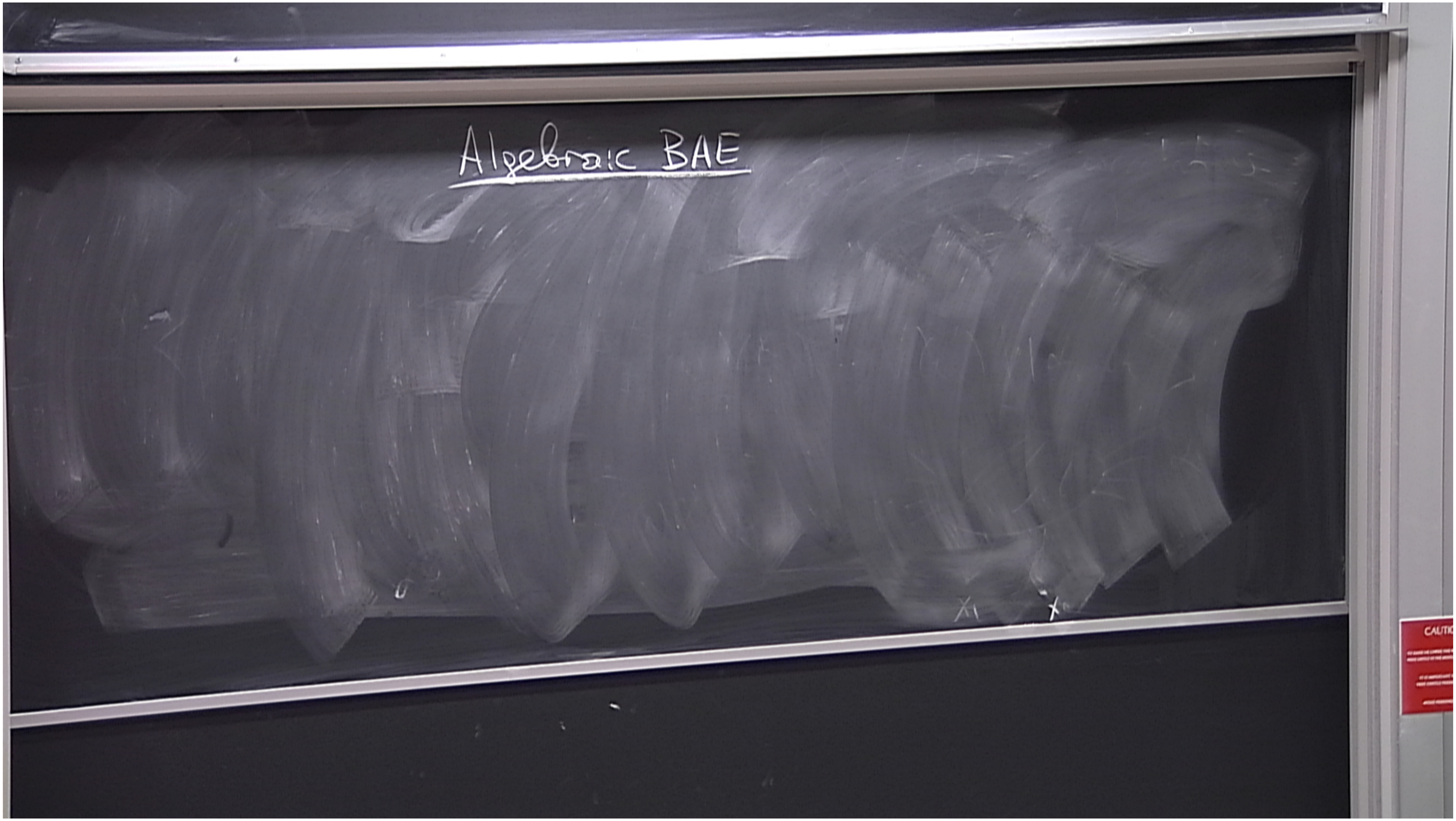
$$\phi \overset{?}{=} \sum_{n_1<n_2} \text{the same} \quad p_1 = \frac{1}{i}\ln\frac{x(u+i/2)}{x(u-i/2)}$$

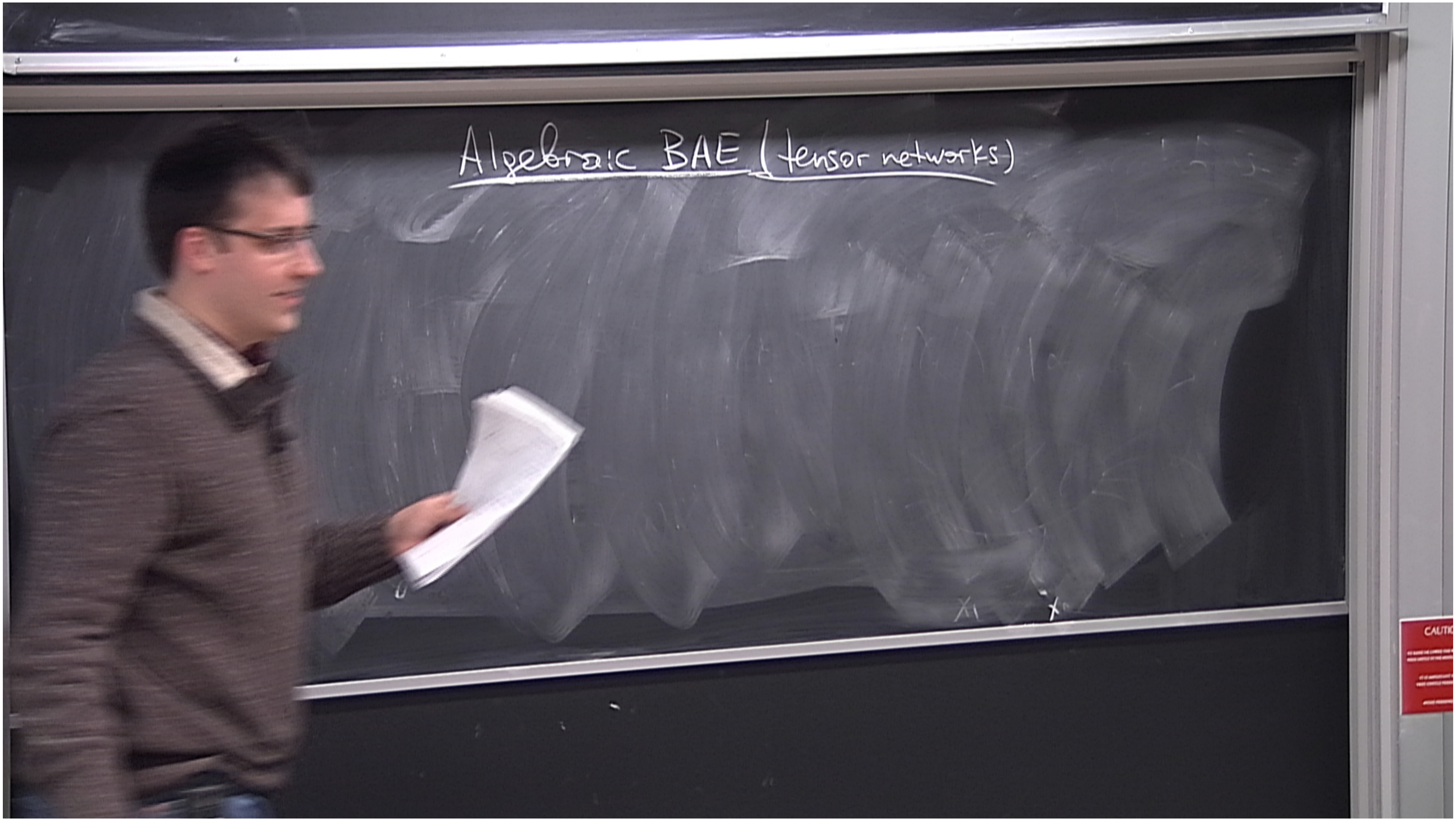
$|n_1-n_2| \leq$ #loops

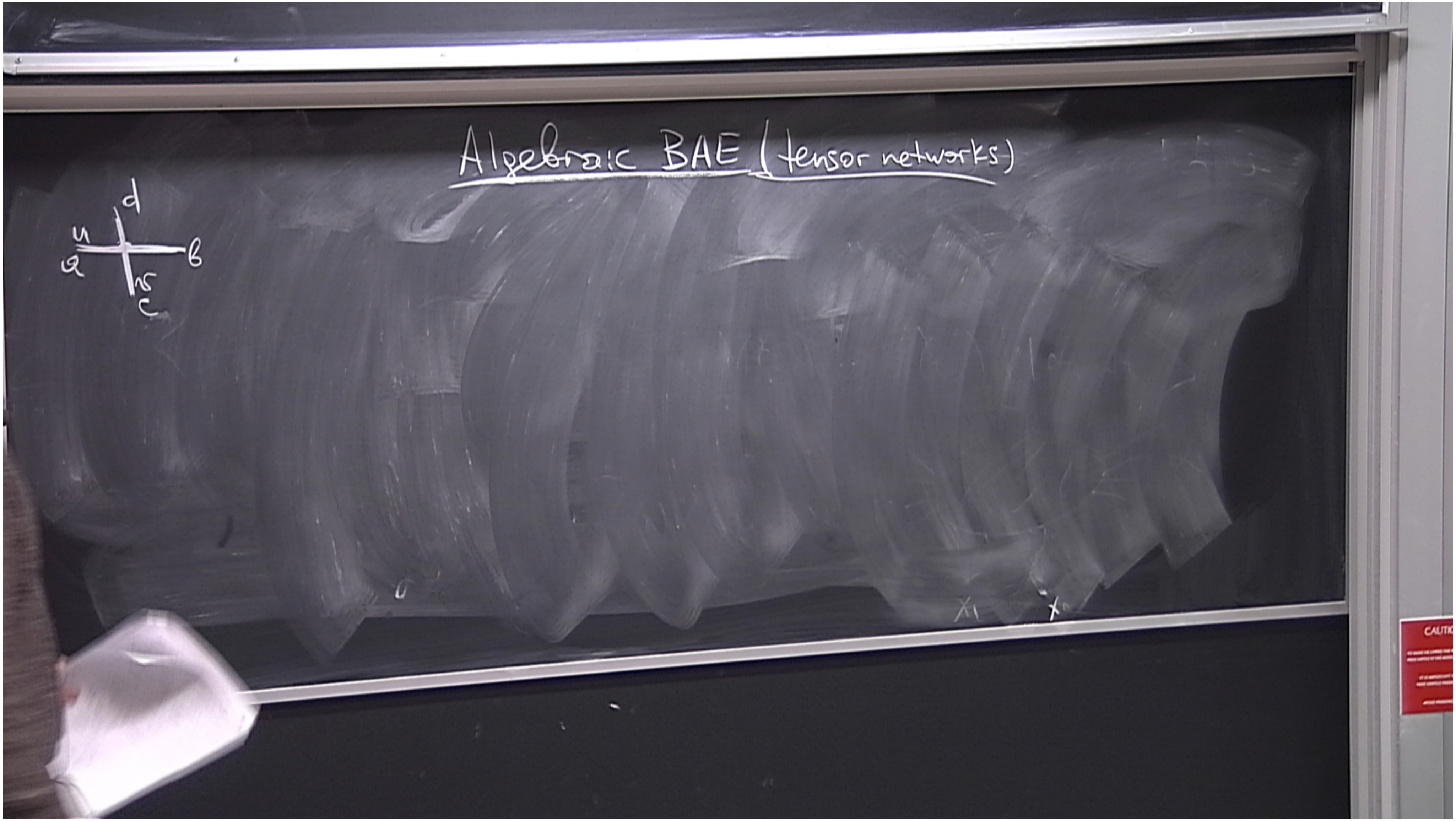
2-loop

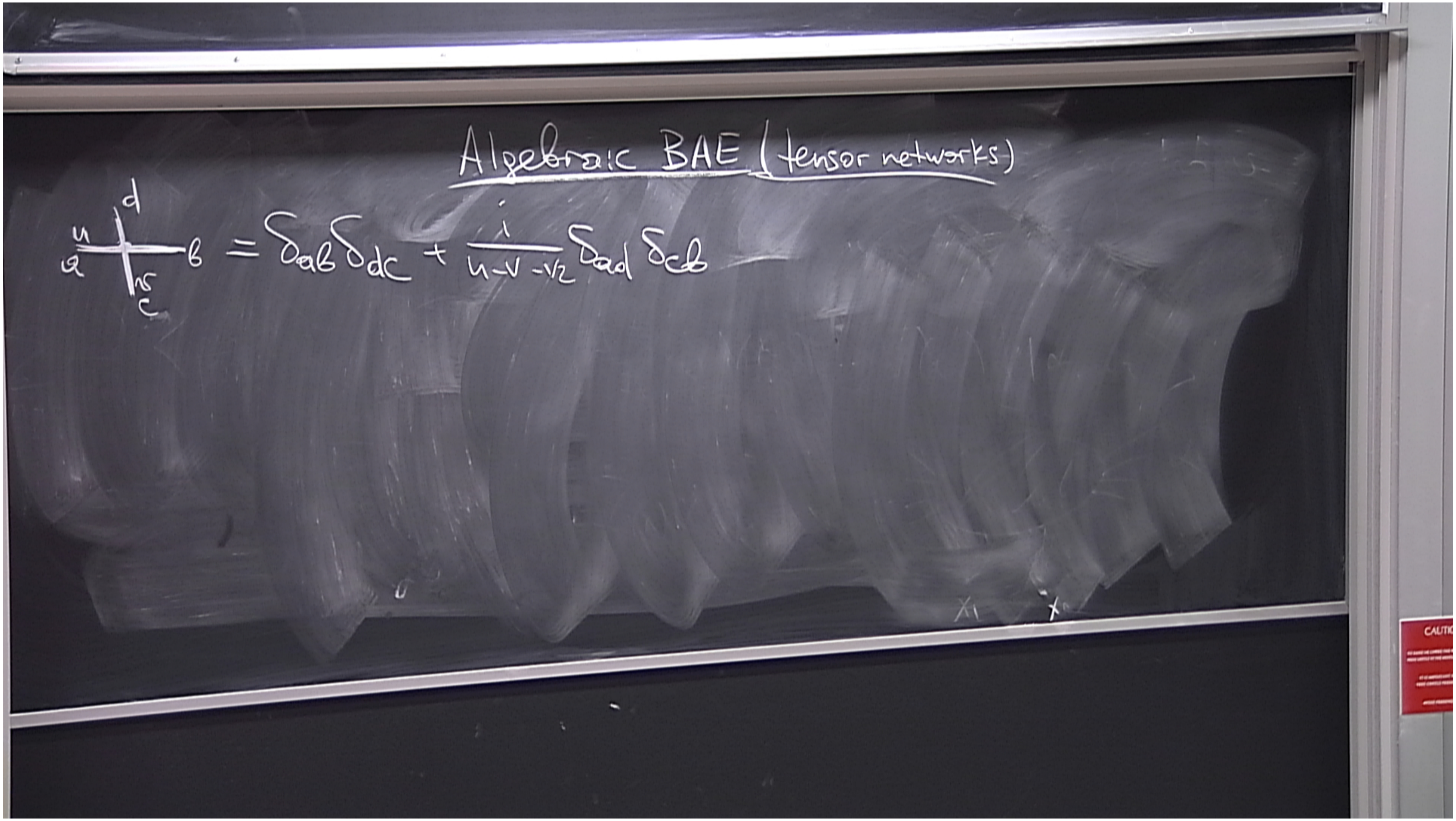
$$\ldots ZXXZ \ldots \rightarrow \left(1+g^2 C(p_1,p_2)\right) \ldots ZXXZ \ldots$$

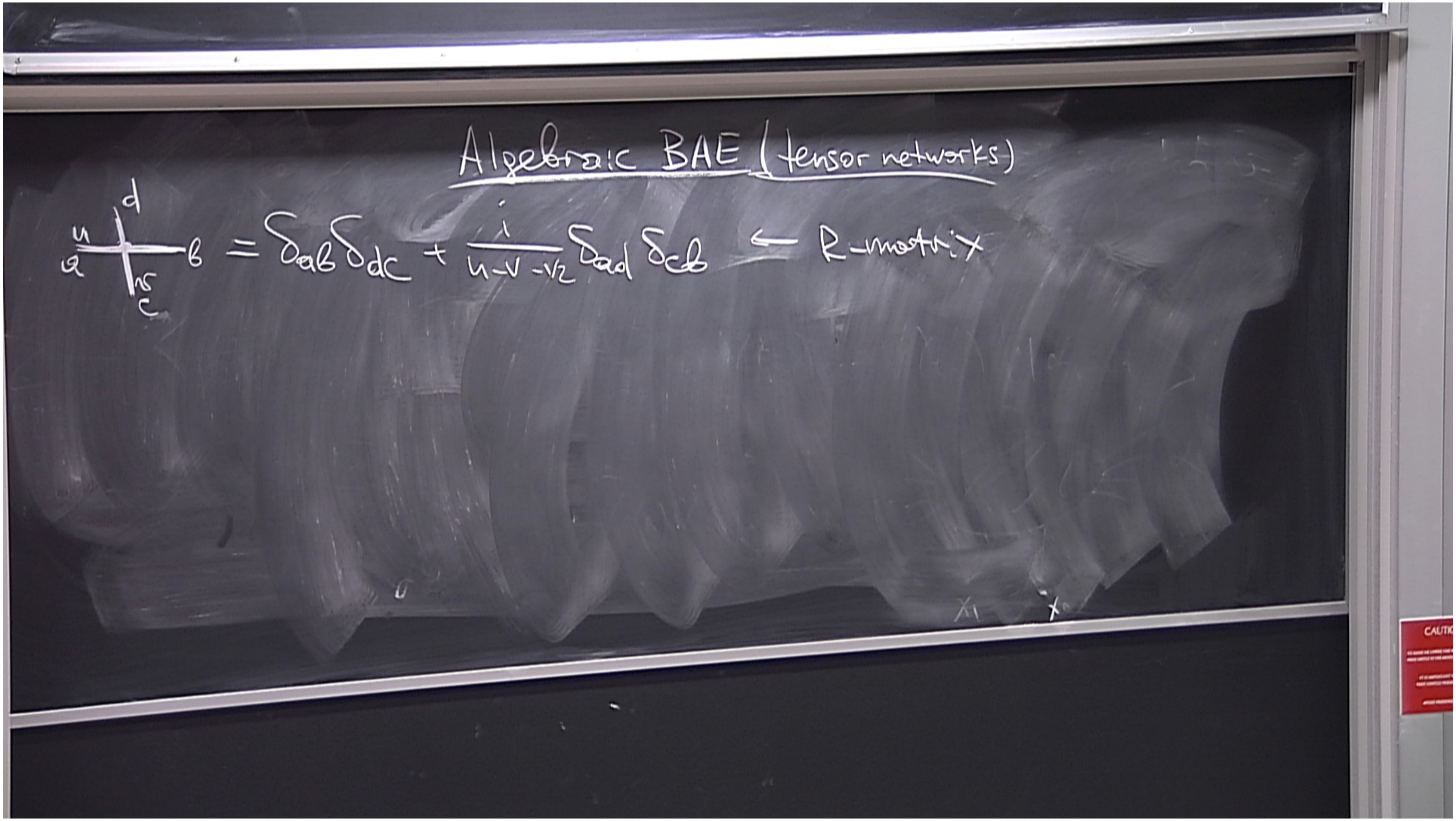

|n1-n2| ≤ #loops
2-loop
N=3 ··ZXXZ... → (1+g²C(p1,p2))··ZXXZ...
··ZXXXZ → (1+g²C(p1,p2,p3))

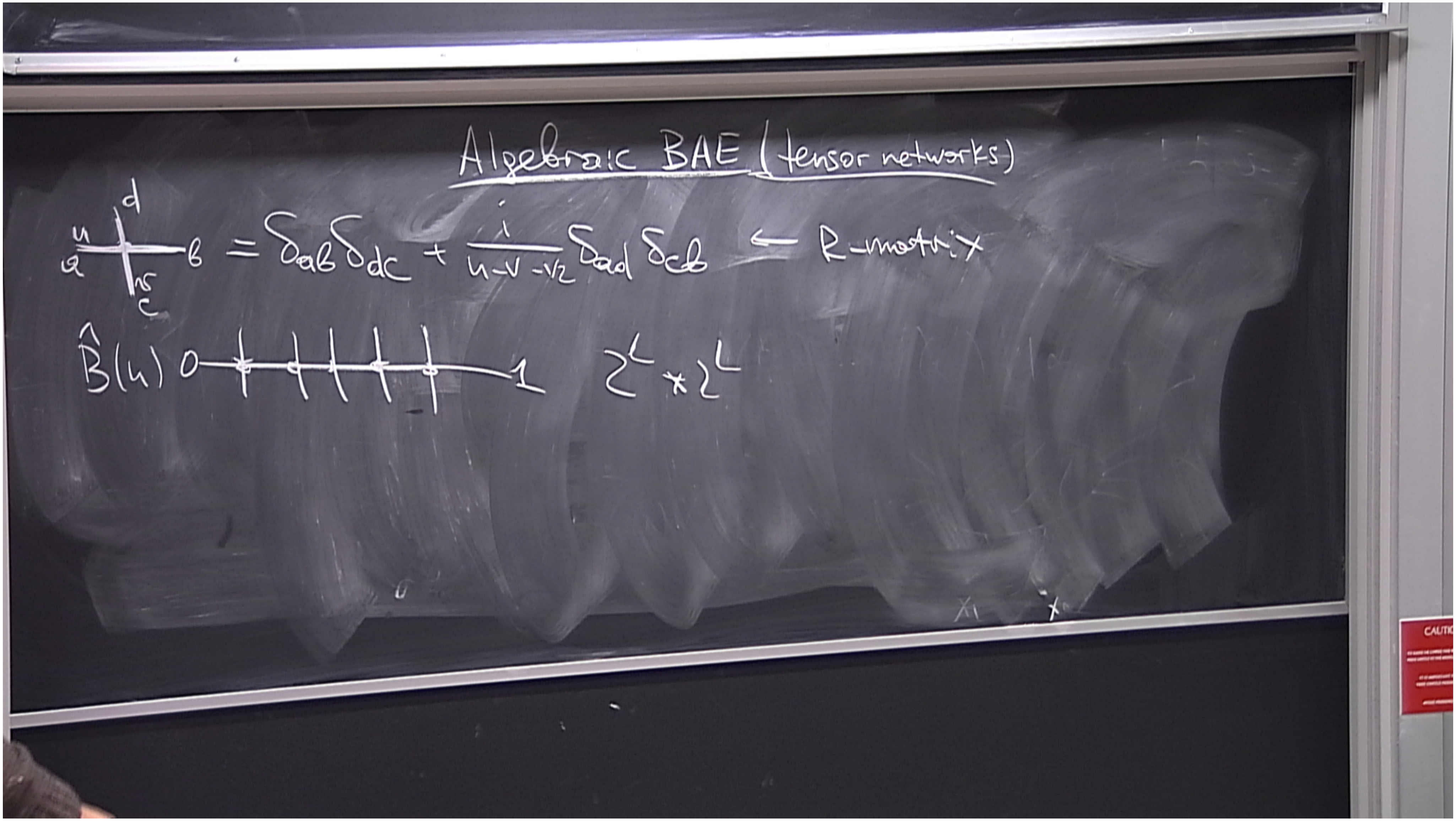

Algebraic BAE

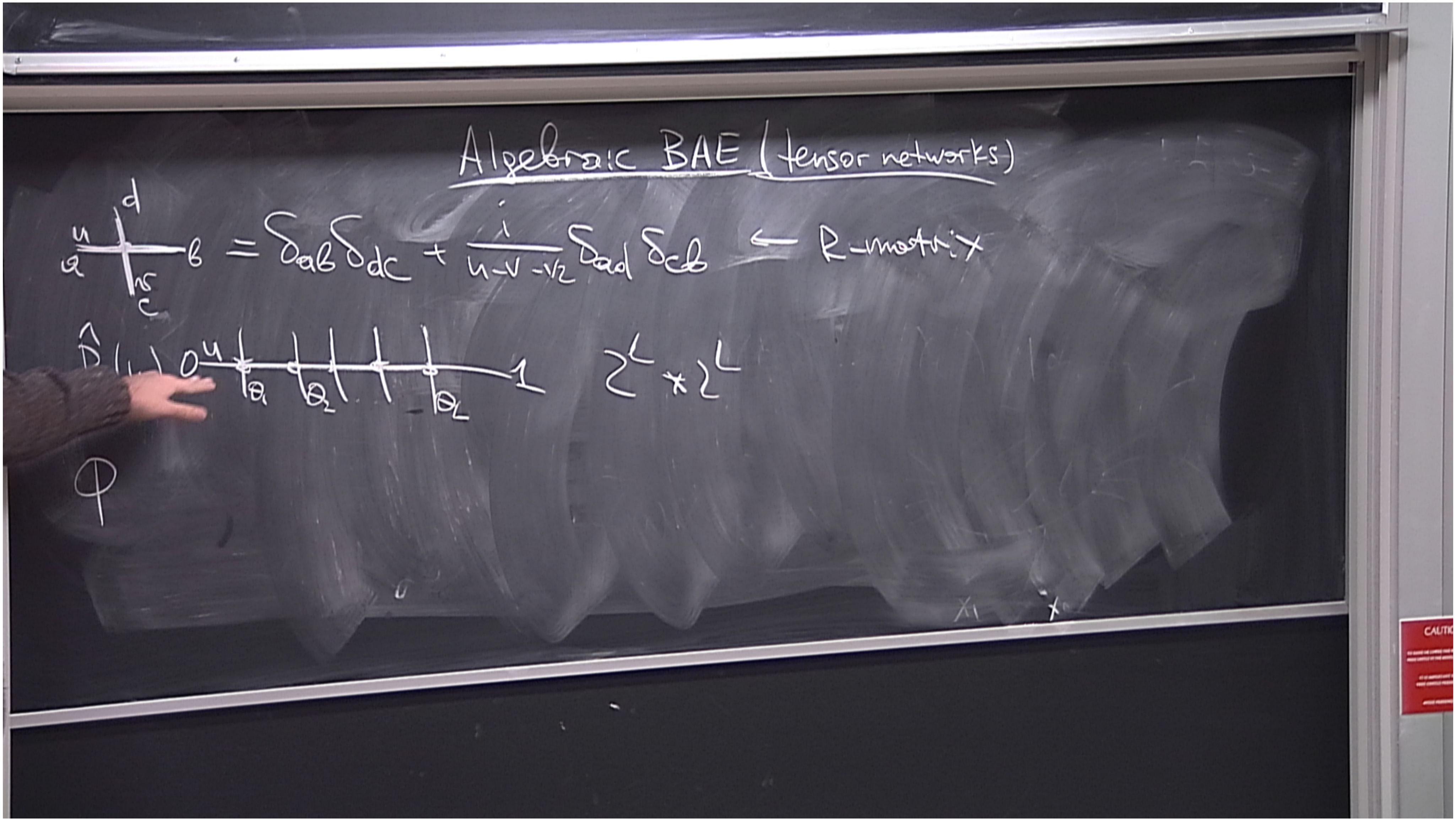

Algebraic BAE (tensor networks)


Algebraic BAE (tensor networks)
d
u
a
b
v
c


Algebraic BAE (tensor networks)
$= \delta_{ab}\delta_{dc} + \frac{i}{u-v-1/2}\delta_{ad}\delta_{cb}$


Algebraic BAE (tensor networks)
d
u
a
b
v
c
$= \delta_{ab}\delta_{dc} + \frac{i}{u-v-1/2}\delta_{ad}\delta_{cb}$ ← R-matrix


Algebraic BAE (tensor networks)
= δab δdc + i/(u−v−1/2) δad δcb ← R-matrix
B̂(u) 0 1
2^L × 2^L


Algebraic BAE (tensor networks)
= δab δdc + i/(u−v−1/2) δad δcb ← R-matrix
2^L × 2^L

Algebraic BAE (tensor networks)

$$\text{vertex}(a,b,c,d;\,u,v) = \delta_{ab}\delta_{dc} + \frac{i}{u-v-1/2}\delta_{ad}\delta_{cb} \leftarrow \text{R-matrix}$$

$$\hat{B}(u) \quad 0 \overset{u}{—} \theta_1 \; \theta_2 \; \cdots \; \theta_L — 1 \qquad 2^L \times 2^L$$

$$\Phi = \hat{B}(u_1) \cdots \hat{B}(u_N) |00000\rangle$$

$$e^{ip n}$$

$N=2$

$$\Phi = \sum_{n_1, n_2} \underbrace{e^{ip_1 n_1 + ip_2 n_2} + S_{12} e^{ip_2 n_1 + ip_1 n_2}}_{N!} \ldots \overset{n_1}{\underset{X}{\downarrow}} \ldots \overset{n_2}{\underset{X}{\downarrow}} \ldots \rangle$$

$$\varphi \overset{?}{=} \sum_{n_1 < n_2} \text{the same} \quad p_1 = \frac{1}{i} \ln \frac{x(u+i/2)}{x(u-i/2)}$$

$$|n_1 - n_2| \leq \# \text{loops}$$

λ-loop

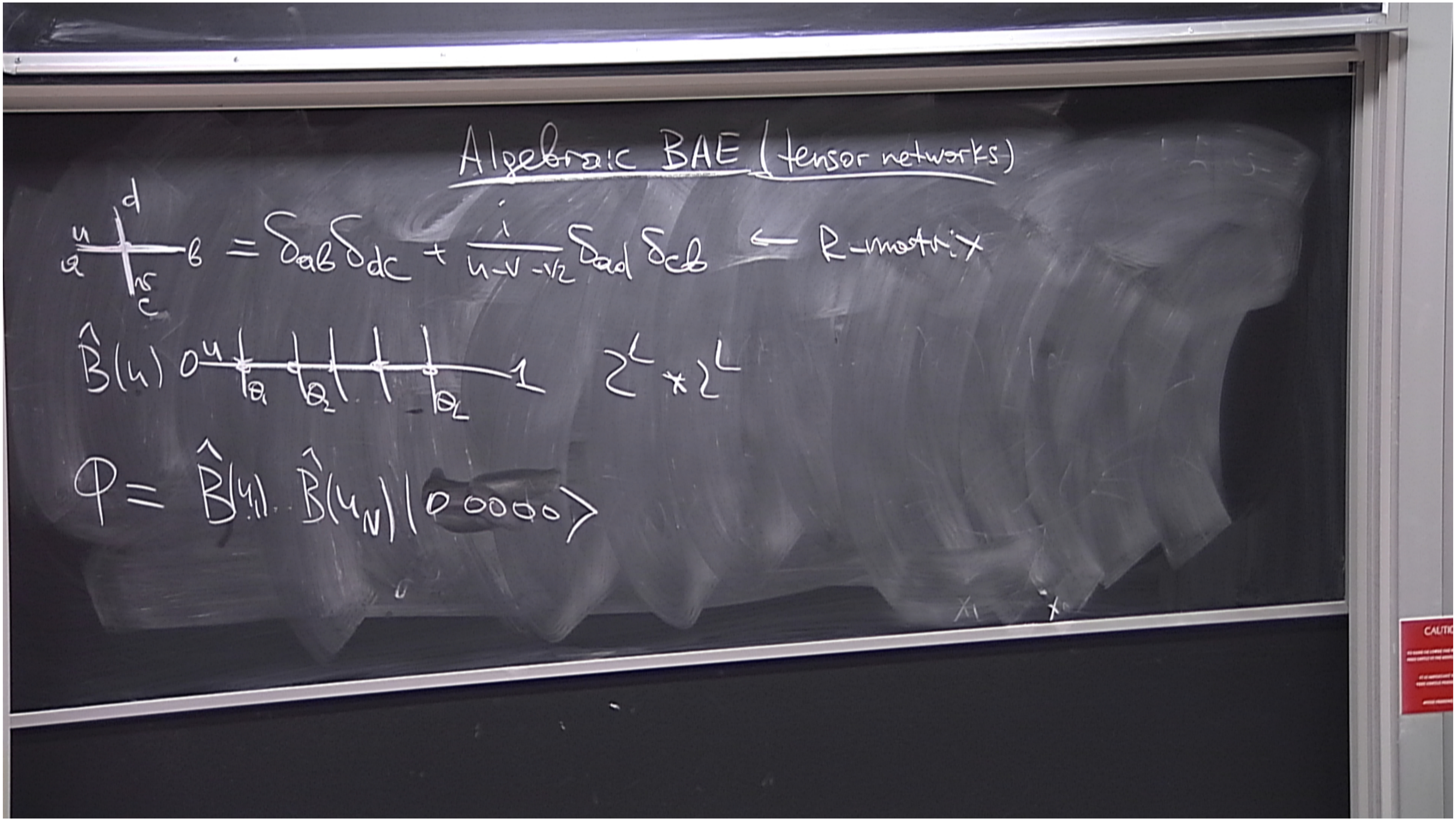
$$N=3 \quad \ldots ZXXZ \ldots \longrightarrow \left(1 + g^2 C(p_1, p_2)\right) \ldots ZXXZ \ldots$$

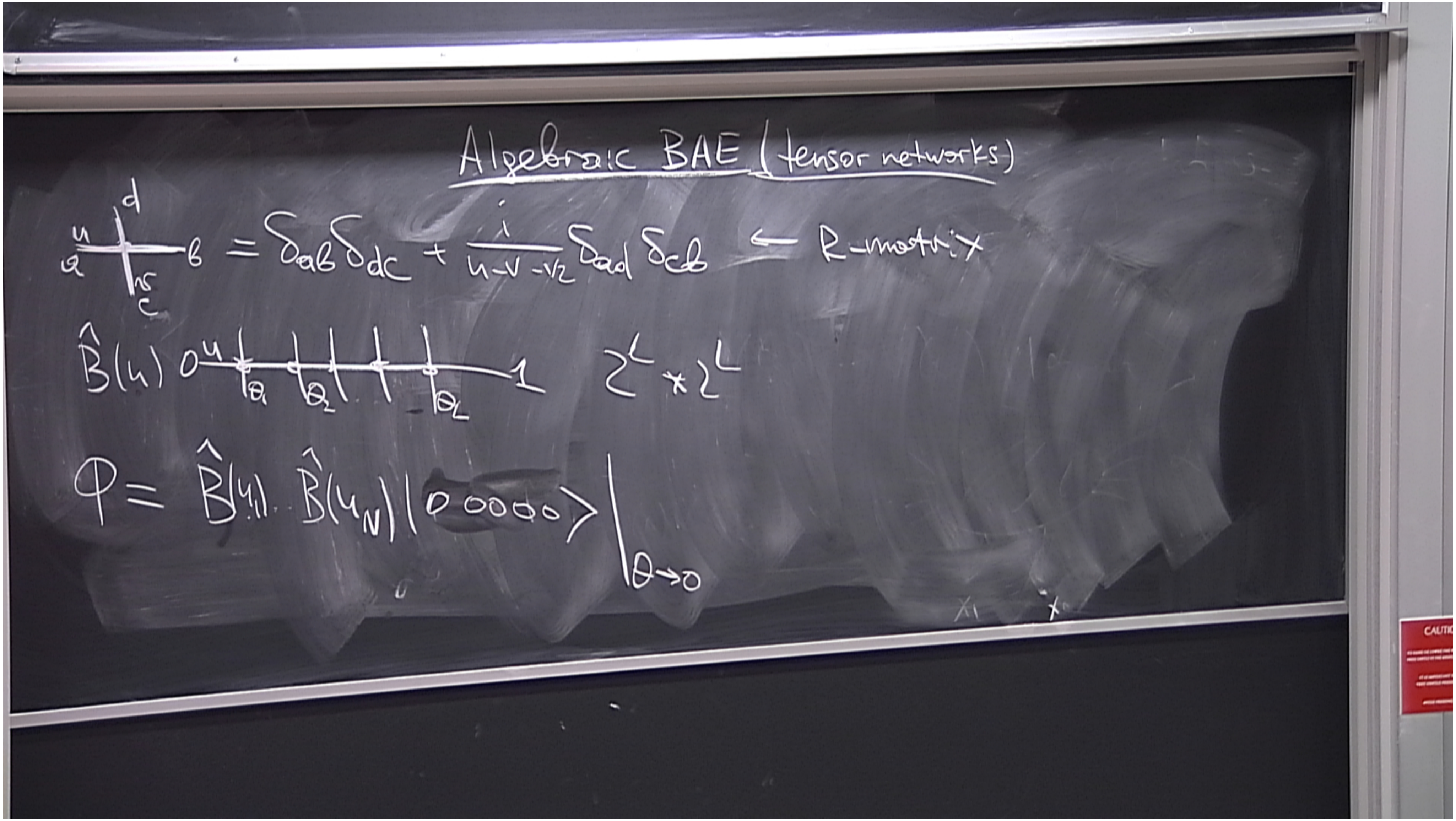
$$\ldots ZXXXZ \longrightarrow \left(1 + g^2 C(p_1, p_2, p_3)\right)$$

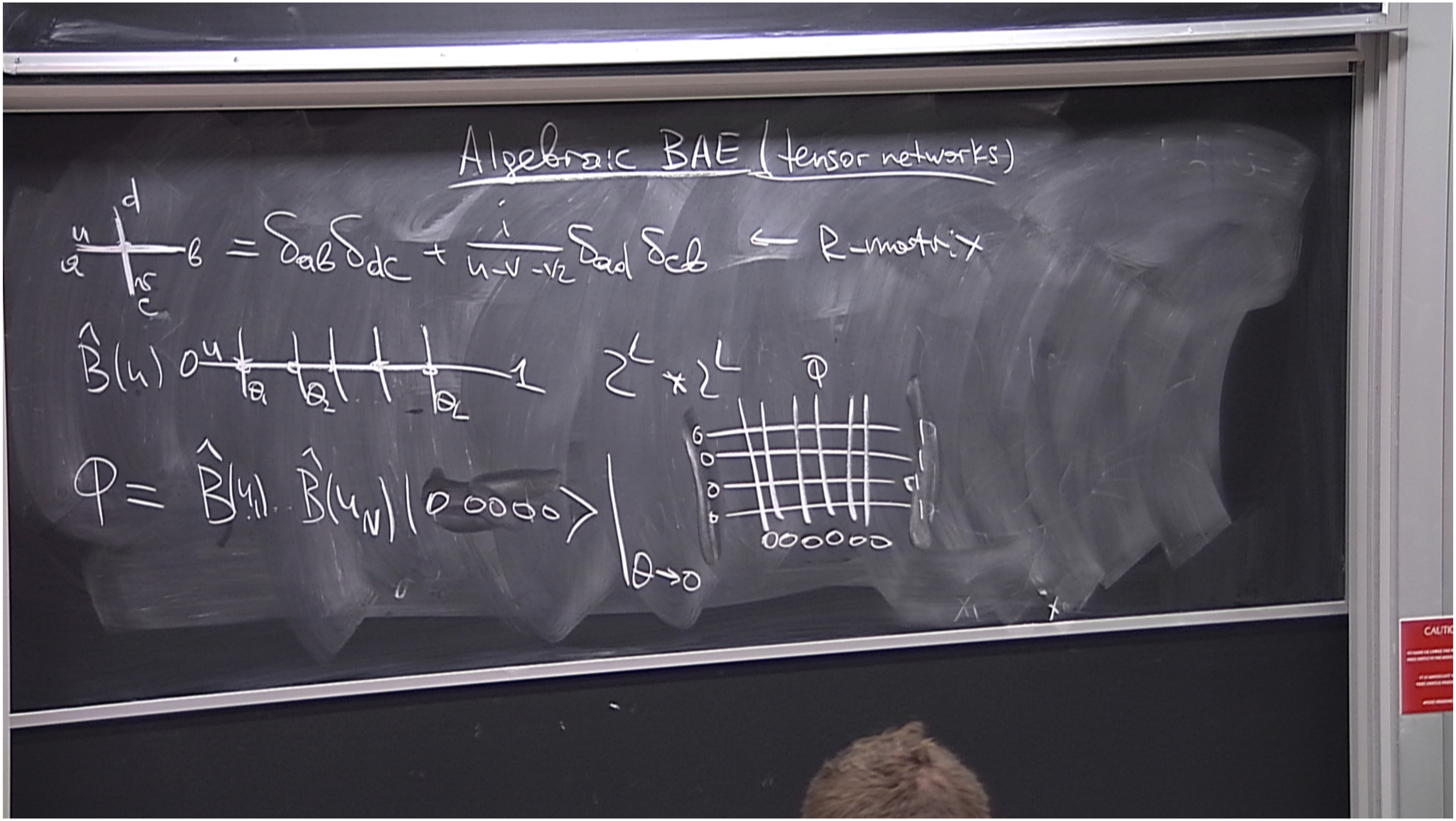
$$b = \delta_{ab}\delta_{dc} + \frac{1}{u - v - i/2} \delta_{ad} \delta_{cb} \Leftarrow \text{R-matrix}$$

$$\hat{B}(u) \quad 0 \; \theta_1 \; \theta_2 \; \ldots \; \theta_L \; 1 \qquad 2^L \times 2^L$$

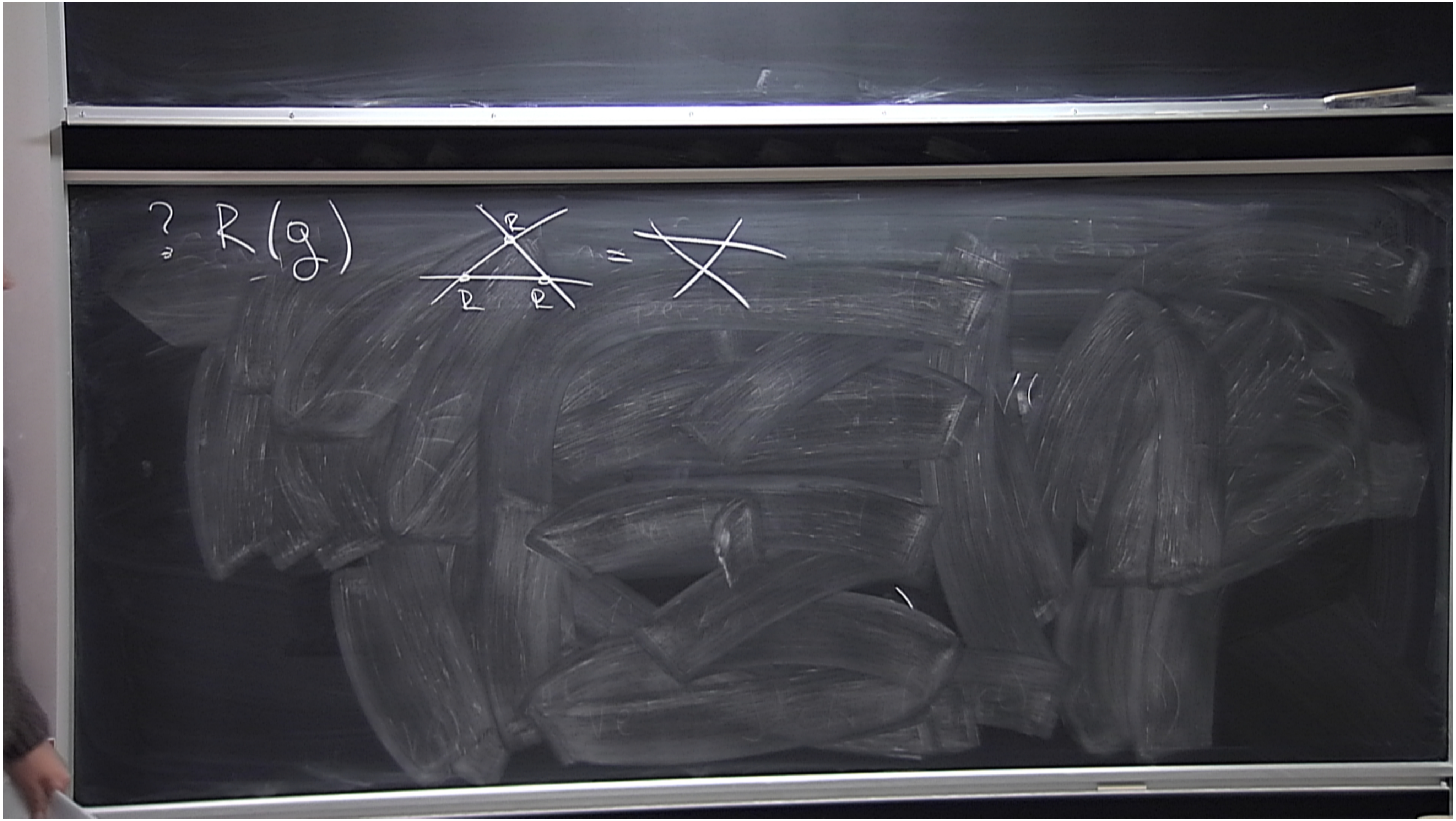
$$\Phi = \hat{B}(u_1) \hat{B}(u_2) \ldots$$

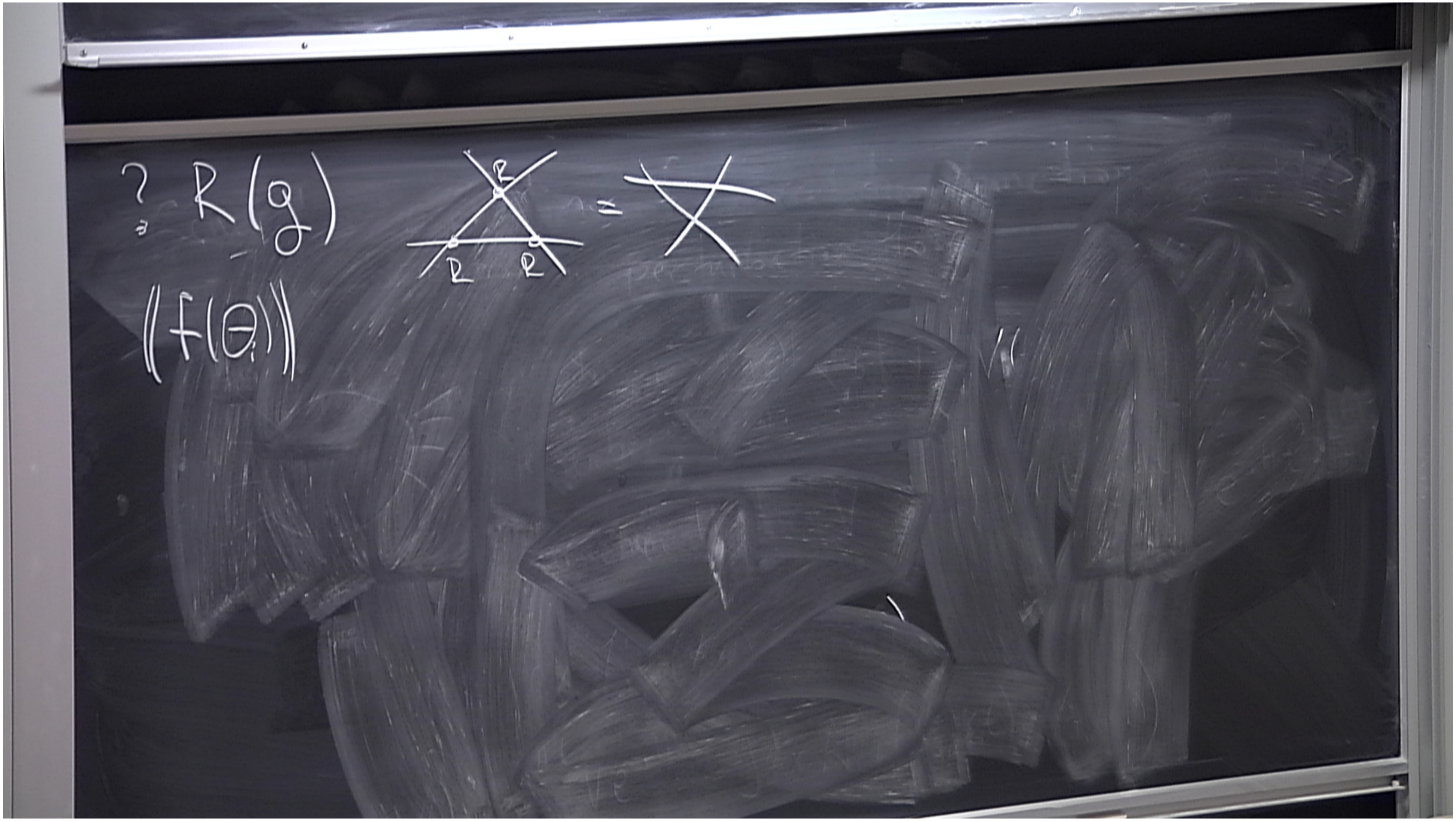

Algebraic BAE (tensor networks)
$= \delta_{ab}\delta_{dc} + \frac{i}{u-v-1/2}\delta_{ad}\delta_{cb}$ ← R-matrix
$\hat{B}(u)$
θ_1 θ_2 θ_L
$2^L \times 2^L$
$\Phi = \hat{B}(u_1) \ldots \hat{B}(u_N) |00000\rangle$


Algebraic BAE (tensor networks)
= δab δdc + i/(u−v−1/2) δad δcb ← R-matrix
B̂(u)
2^L × 2^L
Φ = B̂(u1) .. B̂(uN) |0 0000 ⟩ |θ→0

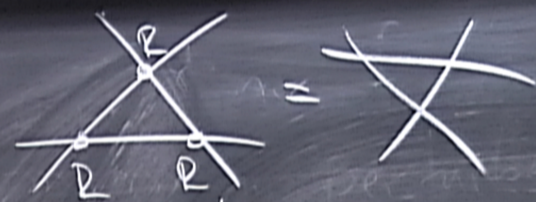

Algebraic BAE (tensor networks)
← R-matrix
$2^L \times 2^L$
Φ
$\theta \to 0$


? R(g)


? R(g)
R
R R


? R(g)
||f(θ)||
R
R R
=

$\overset{?}{\exists}\ R(g)$



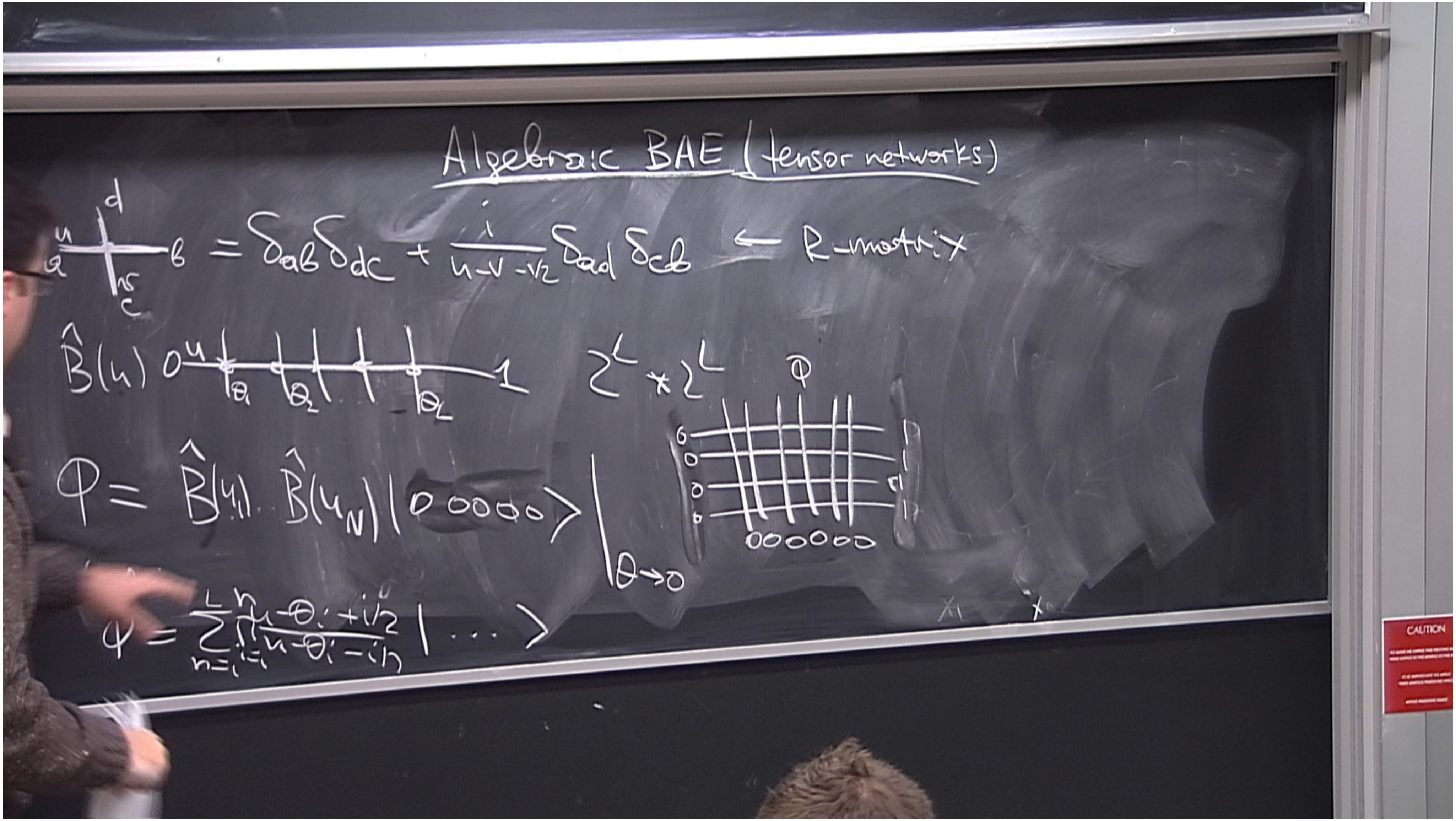
$$\langle\!\langle f(\Theta_i)\rangle\!\rangle = f + \frac{g^2}{2}\sum_{i=1}^{L}\left(\partial_{\theta_i}-\partial_{\theta_{i+1}}\right)^2 f + \dots \Big|_{\theta\to 0}$$

$$\left\| f(\Theta_i) \right\| = f + \frac{g^2}{2} \sum_{i=1}^{L} \left(\partial_{\Theta_i} - \partial_{\Theta_{i+1}}\right)^2 f + \dots \Big|_{\Theta \to 0}$$

Θ-derivative

Properties

- $$\left\| \prod_{i=1}^{L} \frac{u-\Theta_i + i/2}{u-\Theta_i - i/2} \right\|$$


Algebraic BAE (tensor networks)
$= \delta_{ab}\delta_{dc} + \frac{i}{u-v-i/2}\delta_{ad}\delta_{cb}$ ← R-matrix
$\hat{B}(u)$ 0 1
θ_1 θ_2 θ_L
$2^L \times 2^L$
Φ
$\Phi = \hat{B}(u_1) \ldots \hat{B}(u_N)|0\,0000\rangle \Big|_{\theta\to 0}$
000000
$\Phi = \sum_{n=1}^{L}\prod_{i=1}^{n}\frac{u-\theta_i+i/2}{u-\theta_i-i/2}|\ldots\rangle$

$$\langle\langle f(\theta_i)\rangle\rangle = f + \frac{g^2}{2}\sum_{i=1}^{L}\left(\partial_{\theta_i}-\partial_{\theta_{i+1}}\right)^2 f + \dots$$

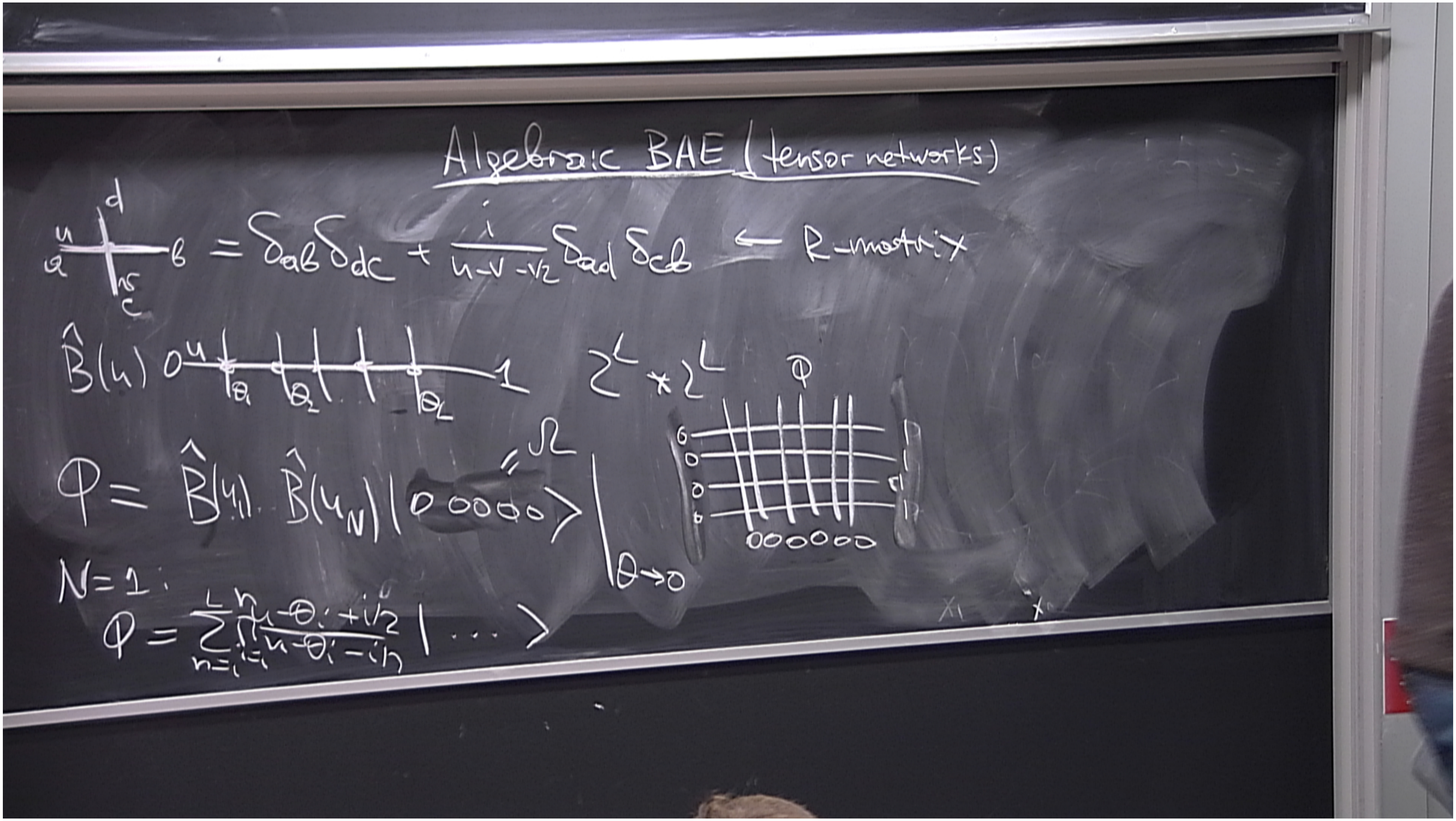
$\theta \to 0$

Θ-derivative

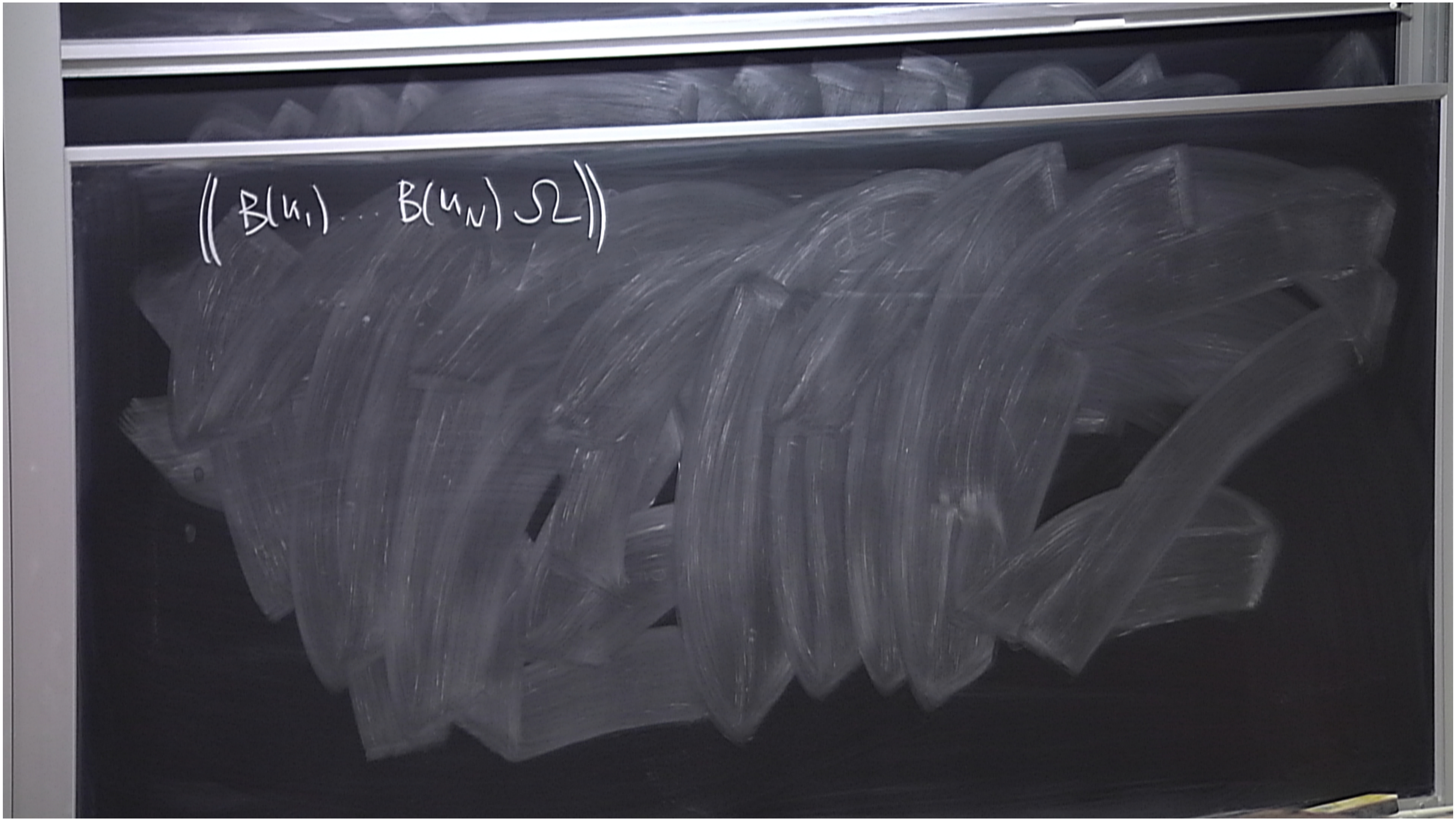
Properties

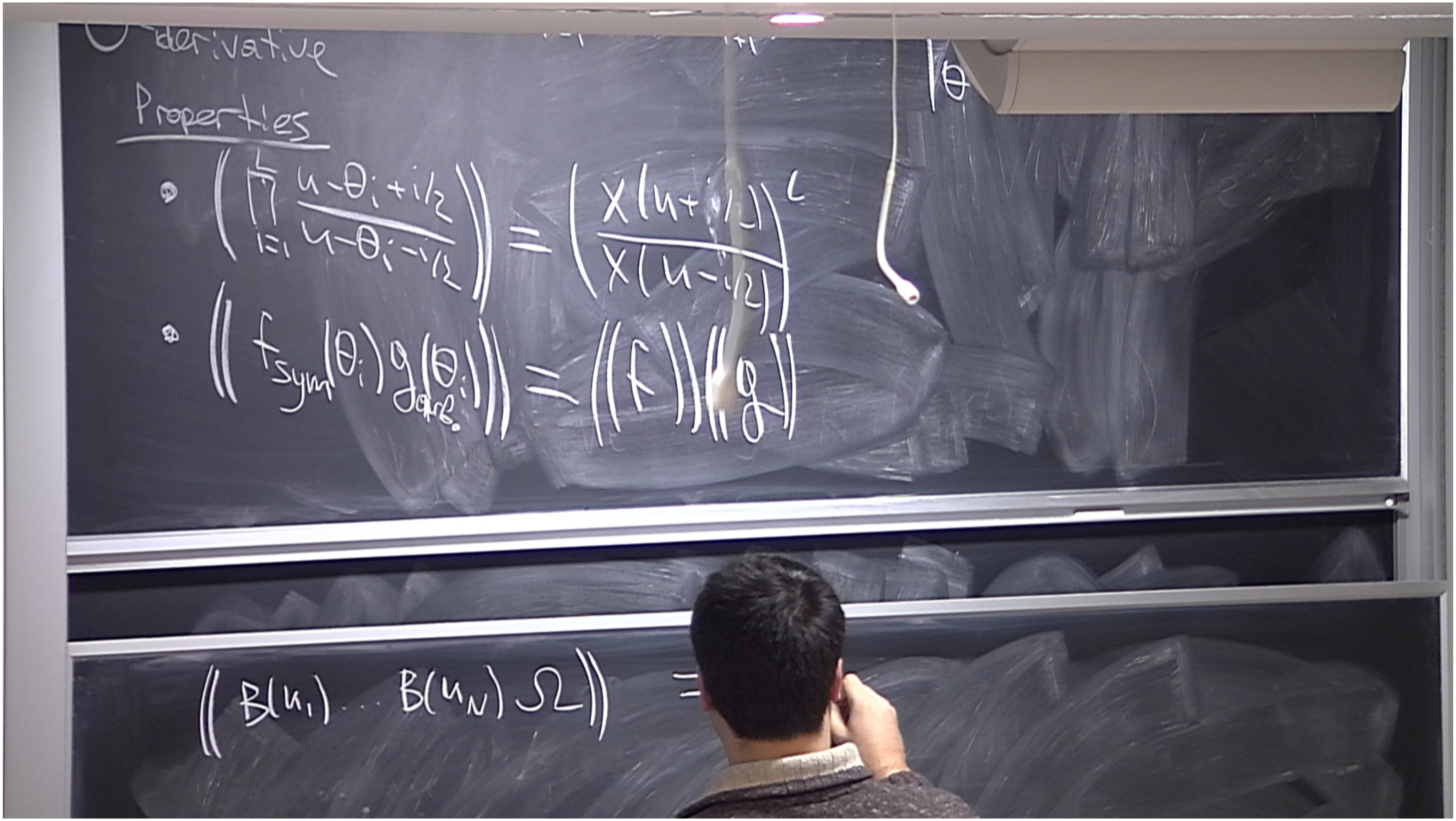
- $$\left\langle\left\langle \prod_{i=1}^{L} \frac{u-\theta_i+i/2}{u-\theta_i-i/2}\right\rangle\right\rangle = \left(\frac{x(u+i/2)}{x(u-i/2)}\right)^L$$
- $$\langle\langle f_{sym}(\theta_i)\, g_{arb}(\theta_i)\rangle\rangle = \langle\langle f\rangle\rangle\langle\langle g\rangle\rangle$$

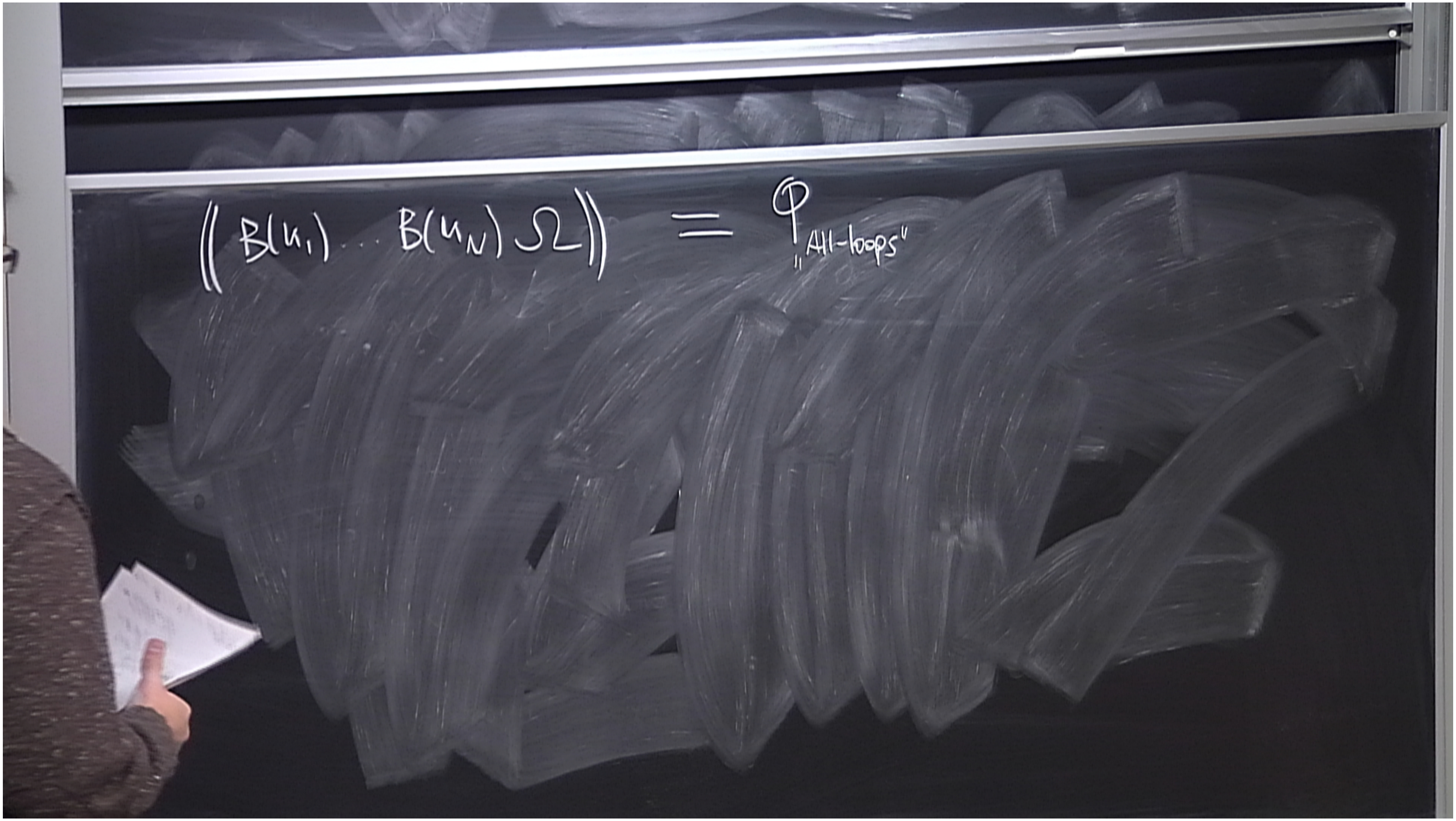
$$B(u_1)\ldots B(u_N)\,\Omega$$



Algebraic BAE (tensor networks)
= δab δdc + i/(u−v−i/2) δad δcb ← R-matrix
B̂(u) 0 u θ1 θ2 θL 1
2^L × 2^L
Φ
Φ = B̂(u1) … B̂(uN) |0 0000⟩
= Ω
θ→0
N = 1 :
Φ = Σ_{n=1}^{L} Π_{i=1}^{n} (u−θi+i/2)/(u−θi−i/2) | … ⟩


$\| B(u_1) \ldots B(u_N)\Omega \|$

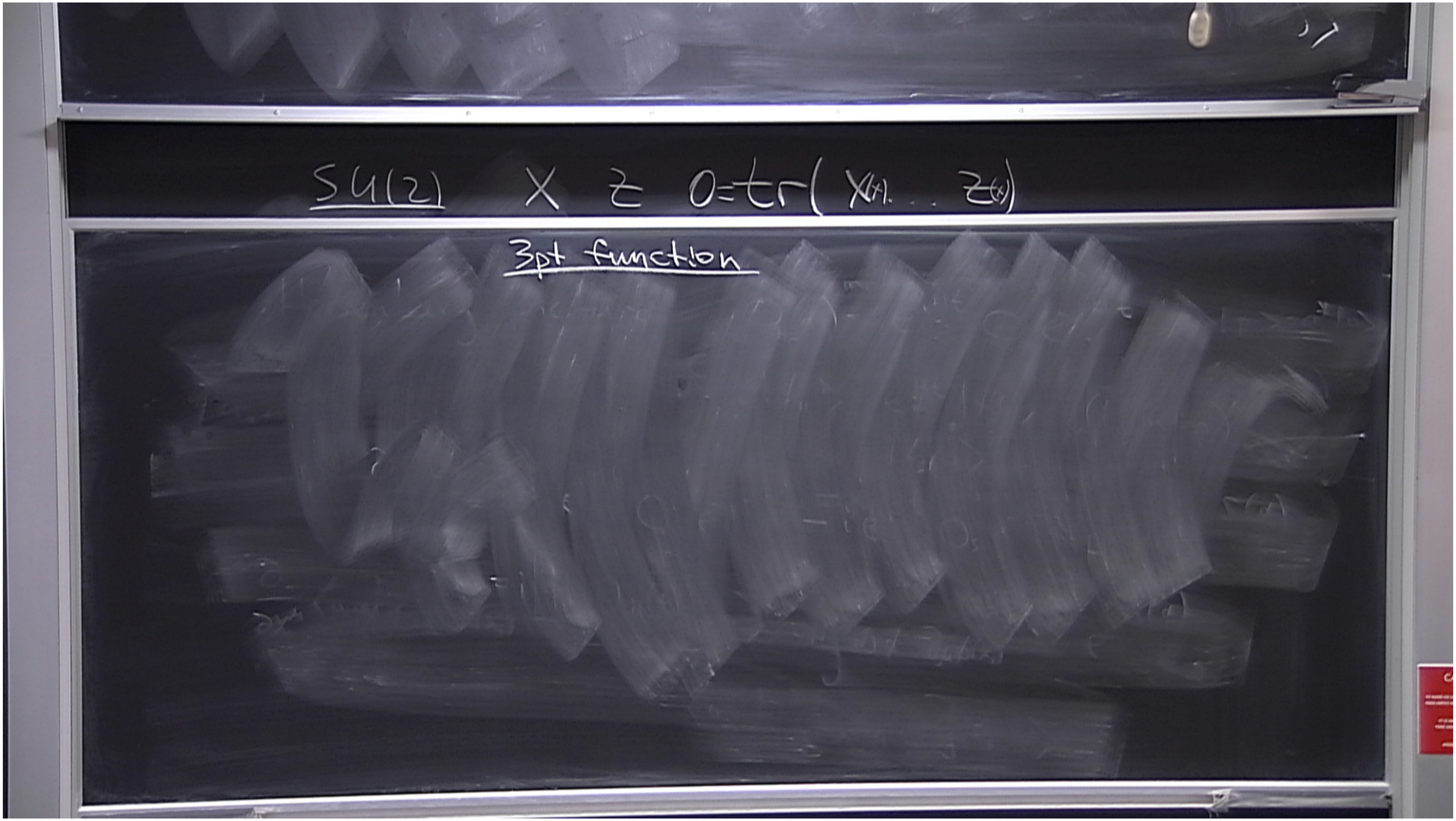

derivative
Properties
$\left(\!\left(\prod_{i=1}^{L}\frac{u-\theta_i+i/2}{u-\theta_i-i/2}\right)\!\right)=\left(\frac{x(u+i/2)}{x(u-i/2)}\right)^{L}$
$\left(\!\left(f_{sym}(\theta_i)\,g_{arb.}(\theta_i)\right)\!\right)=\left(\!\left(f\right)\!\right)\left(\!\left(g\right)\!\right)$
$\left(\!\left(B(u_1)\dots B(u_N)\,\Omega\right)\!\right)=$

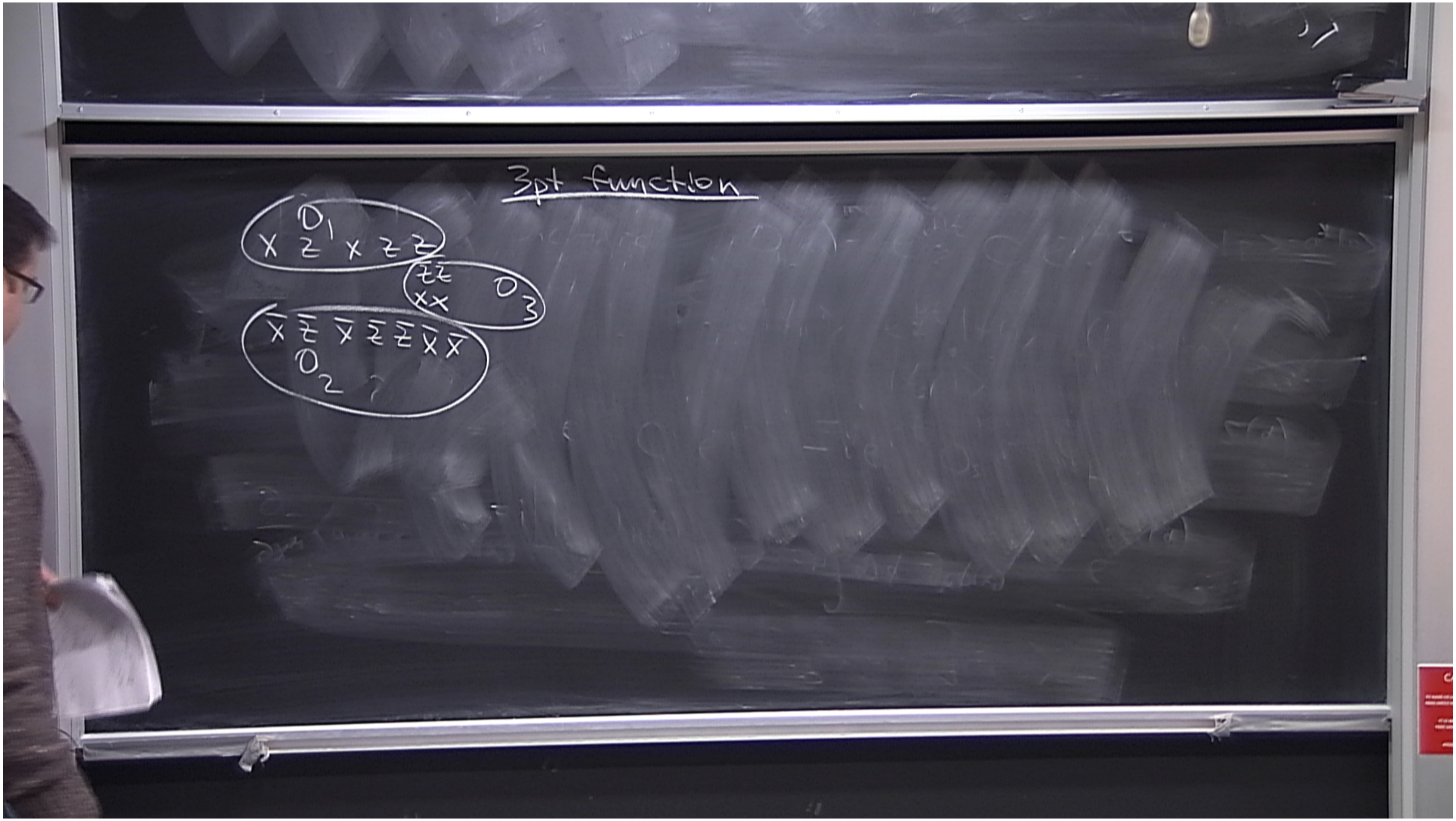

$\left(\!\left(B(u_1) \ldots B(u_N)\, \Omega \right)\!\right) = \Phi_{\text{"All-loops"}}$

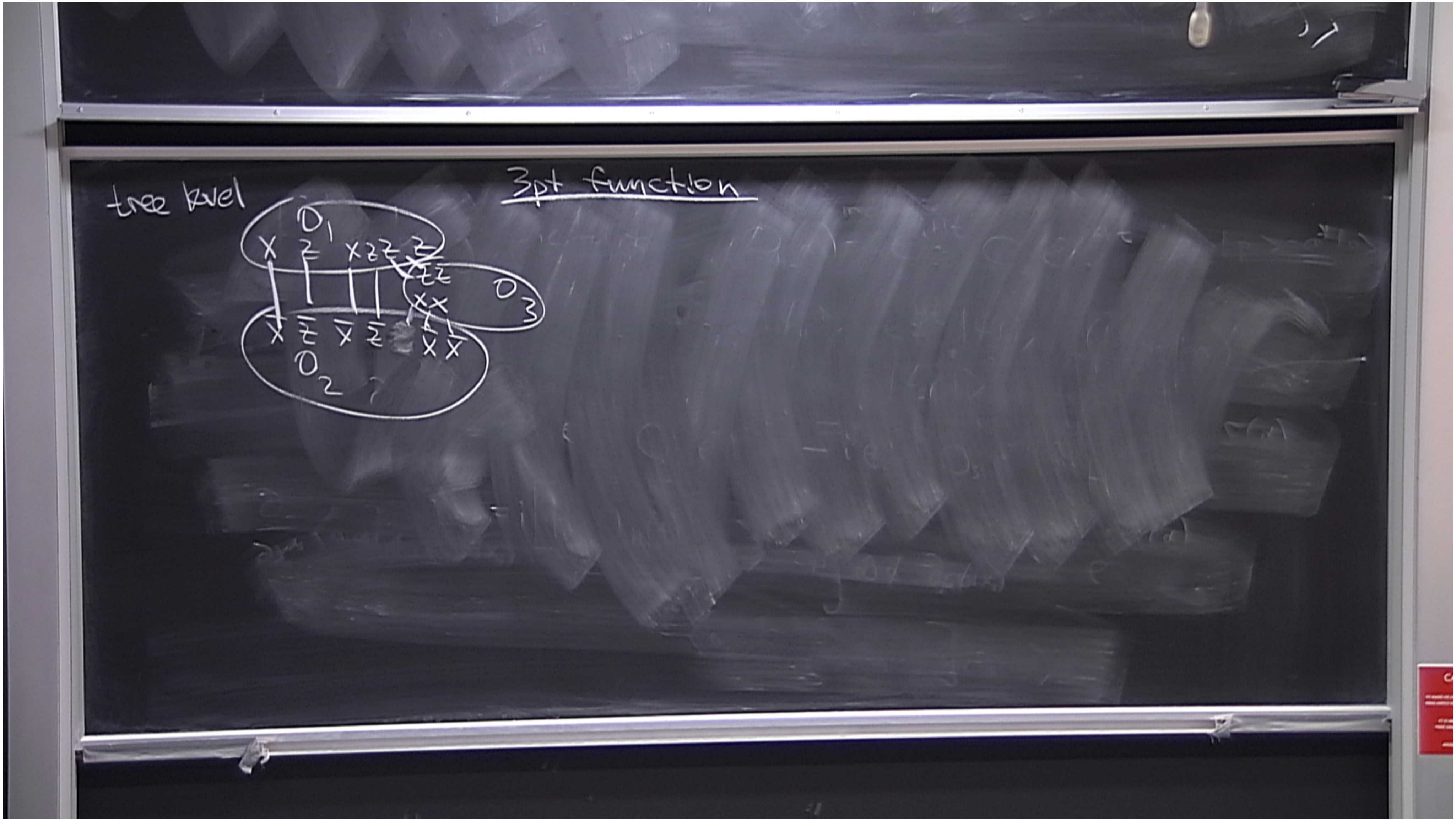
$$\left(1-g^2 P_{1L}\right)\left(\left(B(u_1) \cdots B(u_N)\,\Omega \right)\right) = \Phi_{\text{"All-loops"}}$$

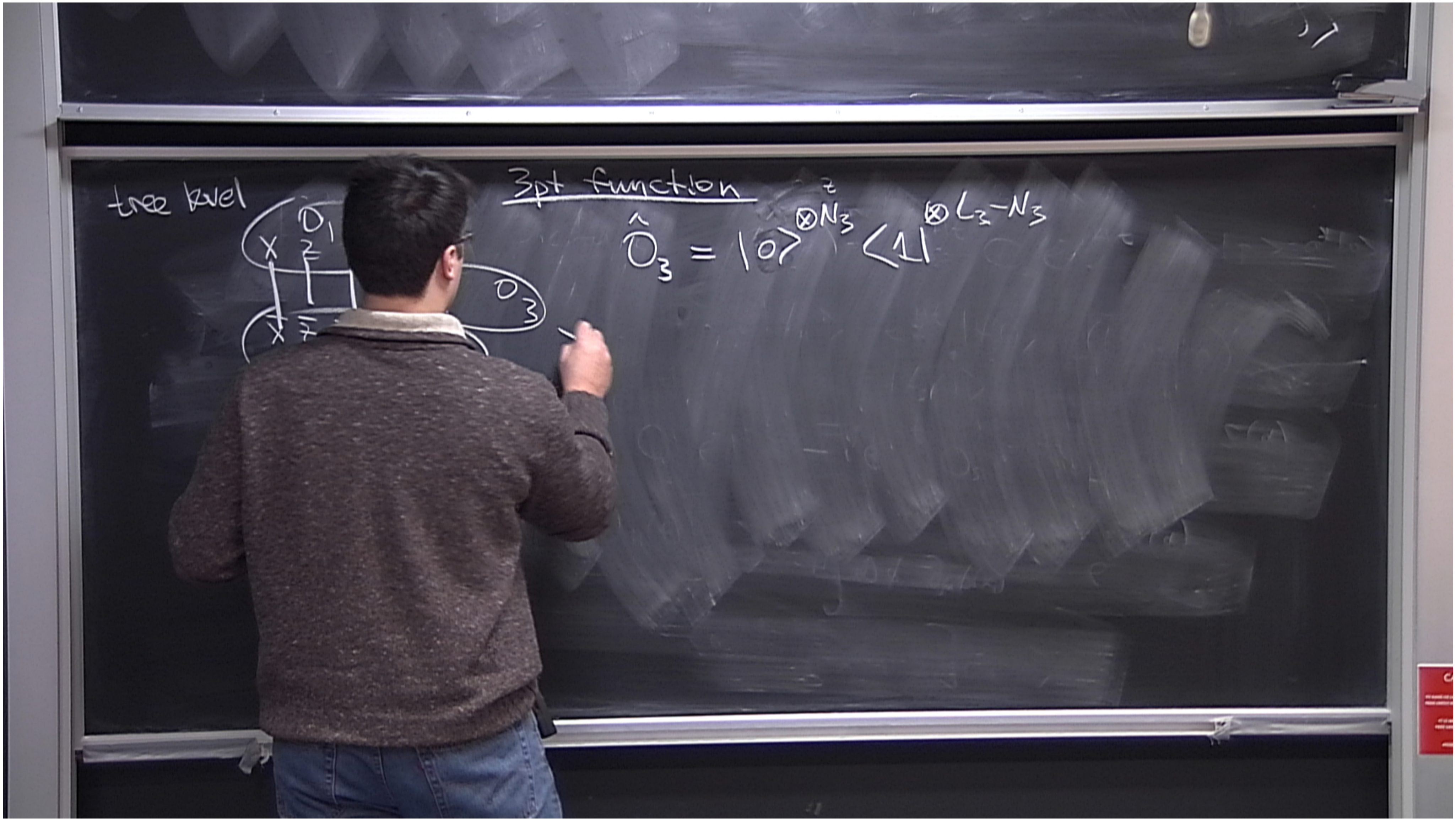
$$Z \ldots\ldots Z$$

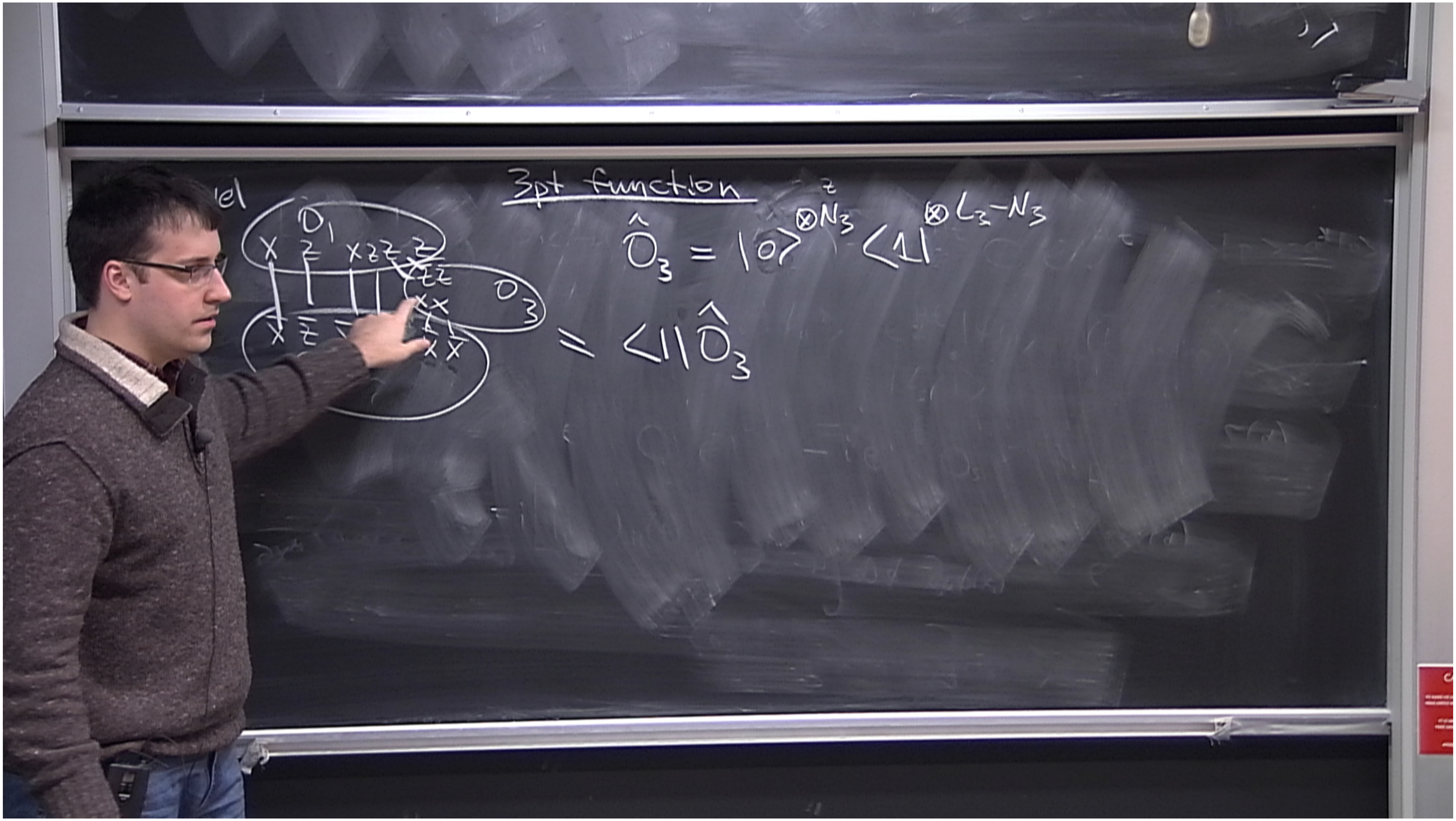
$$\left(1-g^2 P^{+\dots}_{1L}\right)\left(\left(\underset{Z}{B(u_1)} \;\cdots\cdots\; \underset{Z}{B(u_N)}\, \Omega \right)\right) = \varphi_{\text{"All-loops"}}$$

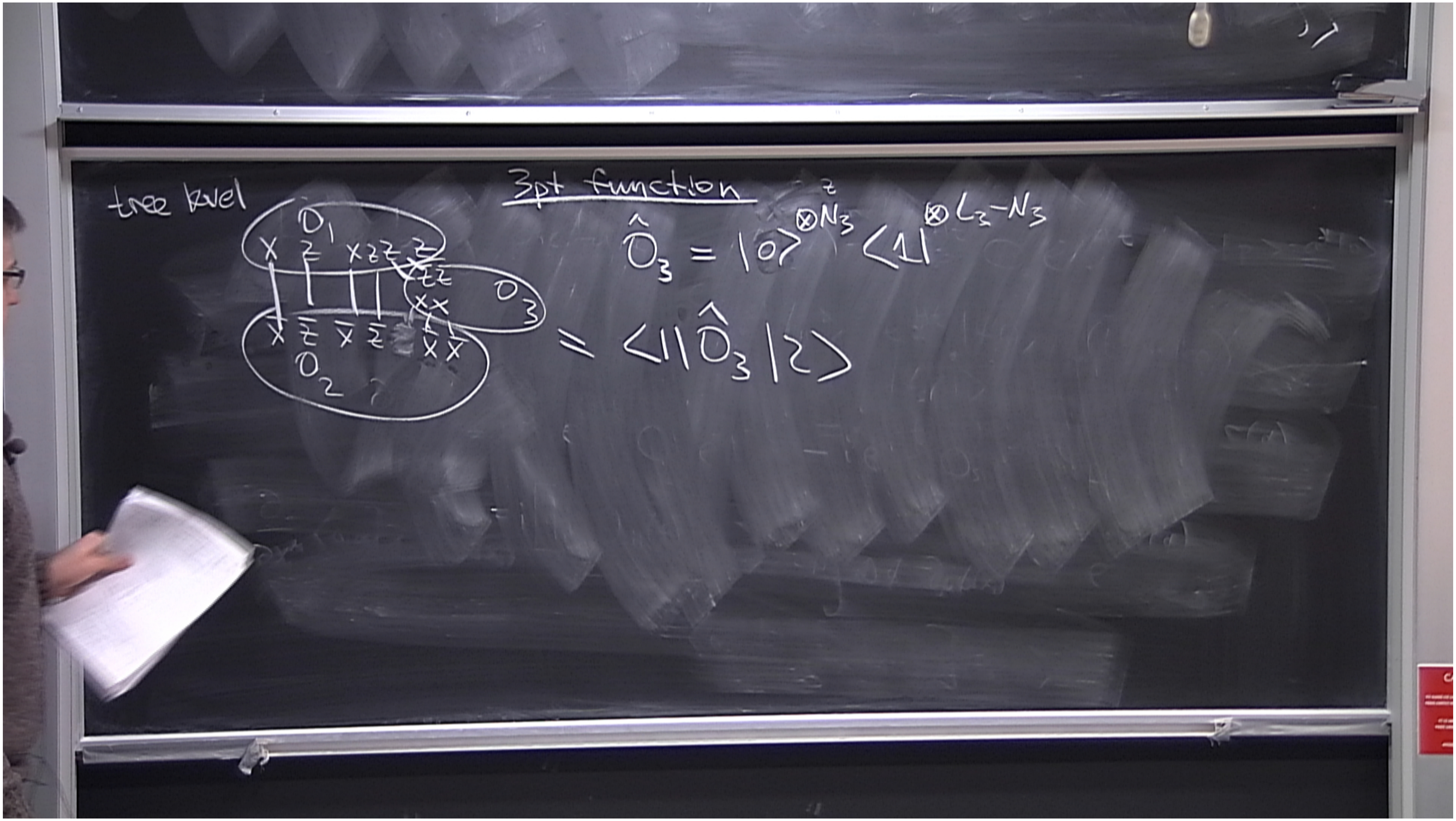

SU(2) X Z O=tr(X(x)... Z(x))
3pt function

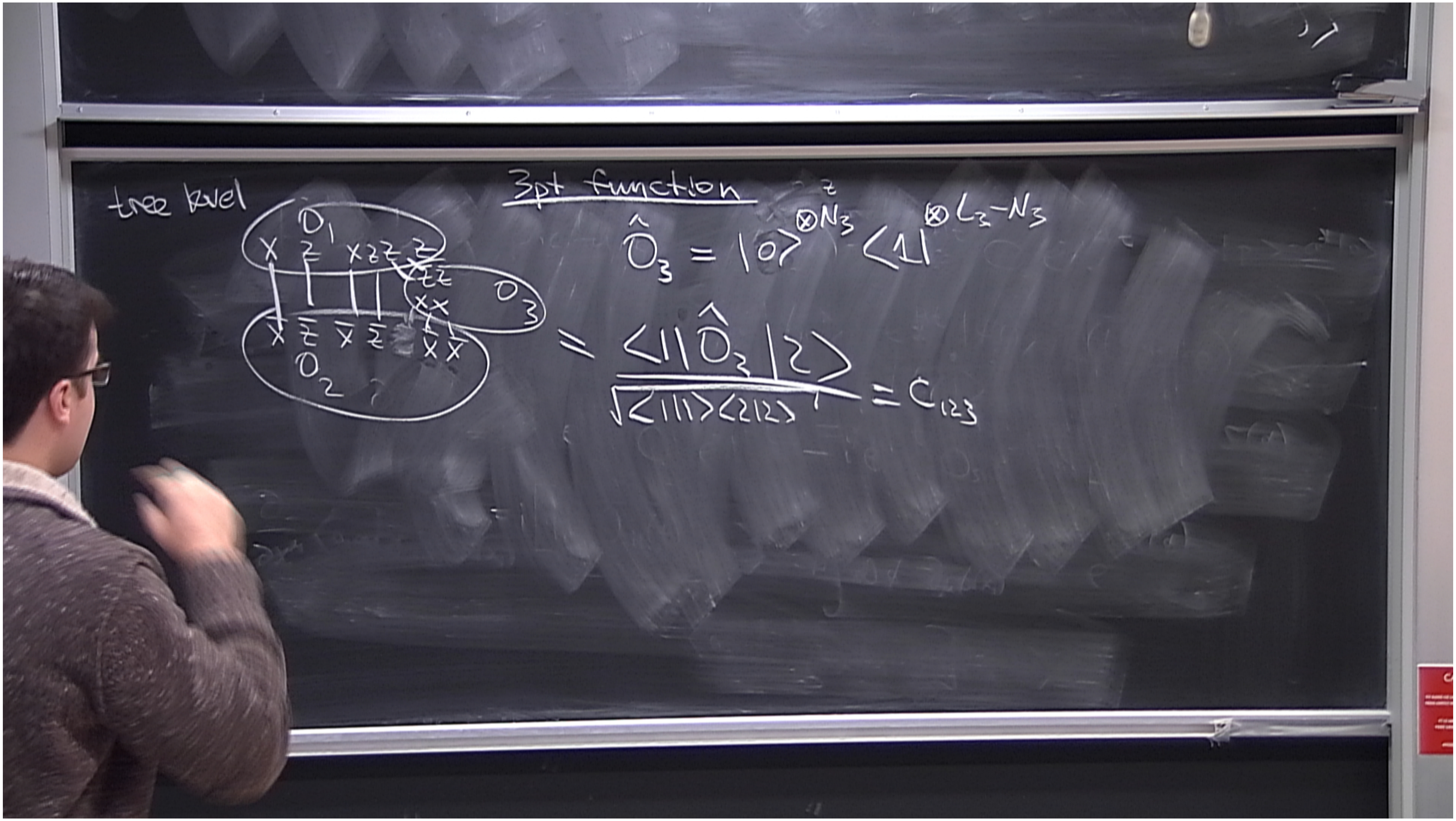

3pt function
O_1
X Z X Z Z
ZZ
XX
O_3
$\bar{X}\ \bar{Z}\ \bar{X}\ \bar{Z}\ \bar{Z}\ \bar{X}\ \bar{X}$
O_2

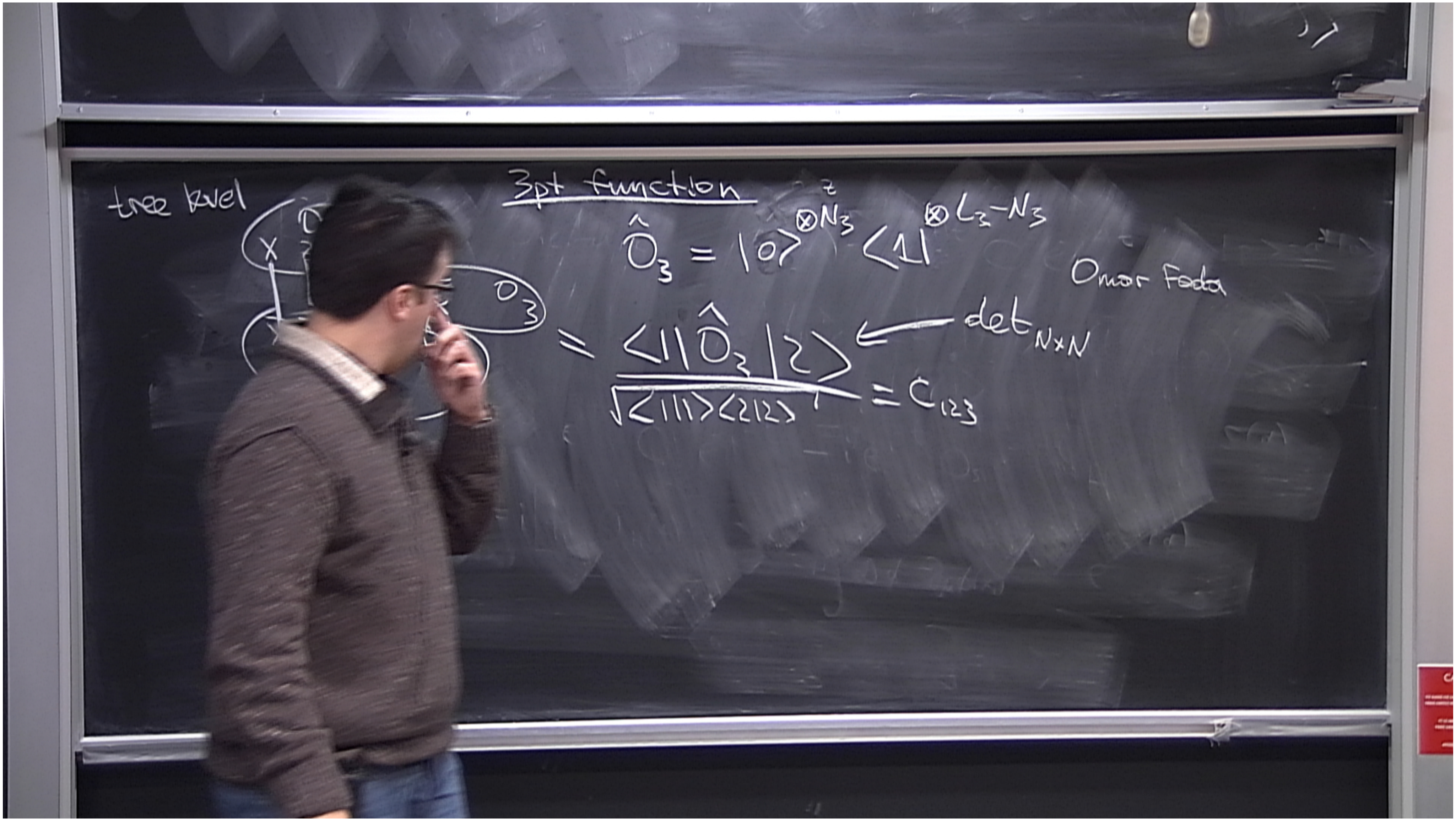

tree level
3pt function
O1
O2
O3

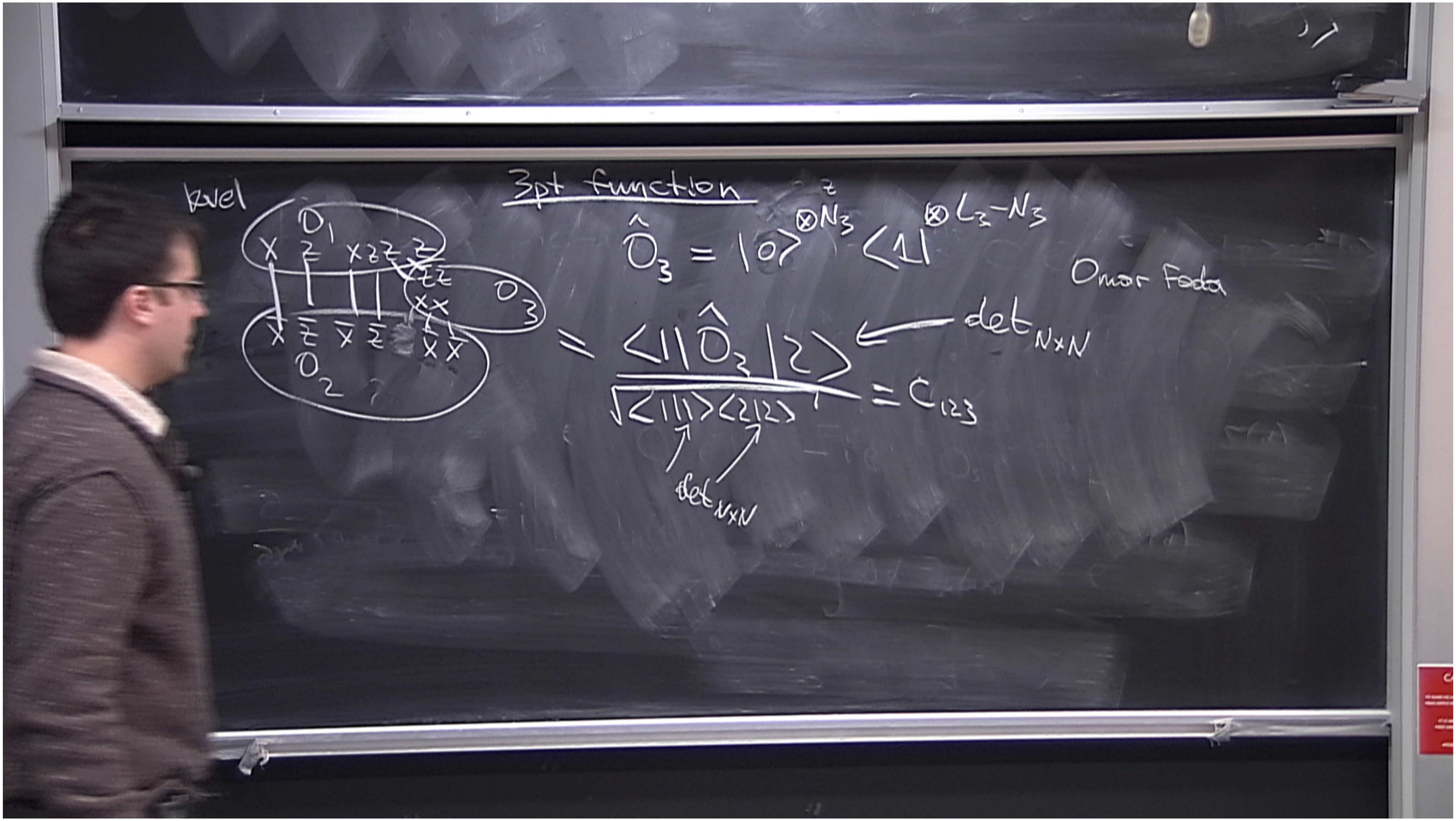

tree level
3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
O_1
O_3

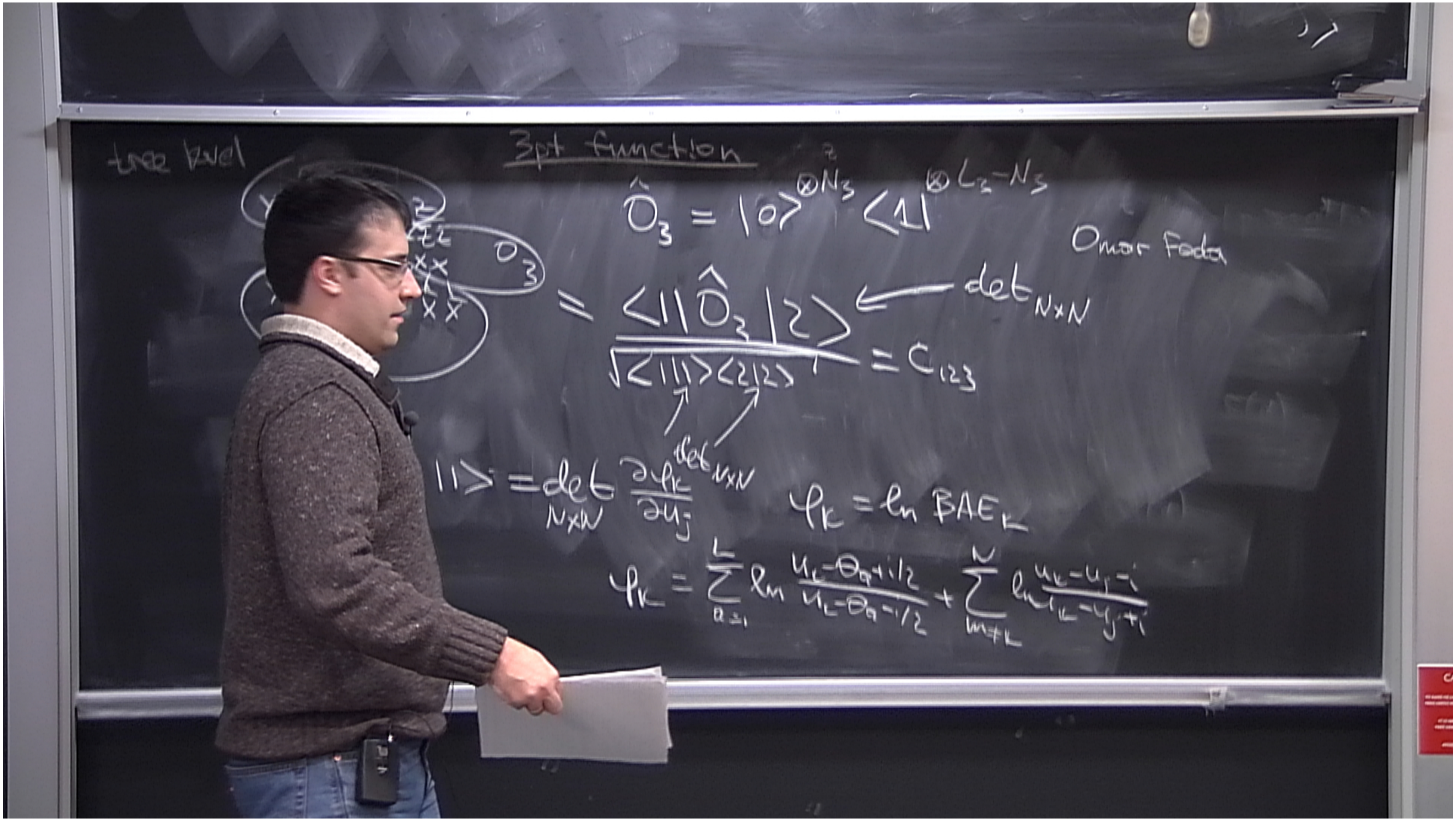

3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
$= \langle 1| \hat{O}_3$
O_1
O_3


tree level
3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
$= \langle 1|\hat{O}_3|2\rangle$
O_1
O_2
O_3


tree level
3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
$= \frac{\langle 1|\hat{O}_3|2\rangle}{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}} = C_{123}$


tree level
3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
Omar Foda
$= \frac{\langle 1|\hat{O}_3|2\rangle}{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}} = C_{123}$
$\det_{N\times N}$


level
3pt function
O1
O2
O3
Ô3 = |0>⊗N3 <1|⊗L3−N3
Omar Foda
det N×N
= <1|Ô3|2> / √(<1|1><2|2>) = C123
det N×N


tree level
3pt function
$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$
Omar Foda
$= \frac{\langle 1|\hat{O}_3|2\rangle}{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}} = C_{123}$
$\det_{N\times N}$
$\det_{N\times N}$
$|1\rangle = \det_{N\times N} \frac{\partial \varphi_k}{\partial u_j}$
$\varphi_k = \ln \mathrm{BAE}_k$
$\varphi_k = \sum_{a=1}^{L} \ln \frac{u_k - \theta_a + i/2}{u_k - \theta_a - i/2} + \sum_{m\neq k}^{N} \ln \frac{u_k - u_j - i}{u_k - u_j + i}$

$$\underbrace{\overset{\uparrow}{X}\ \overset{\uparrow}{Z}\ \overset{\uparrow}{X}\ \overset{\uparrow}{Z}\ \cdots\ \overset{\uparrow}{X}\ \overset{\uparrow}{X}}_{3}$$

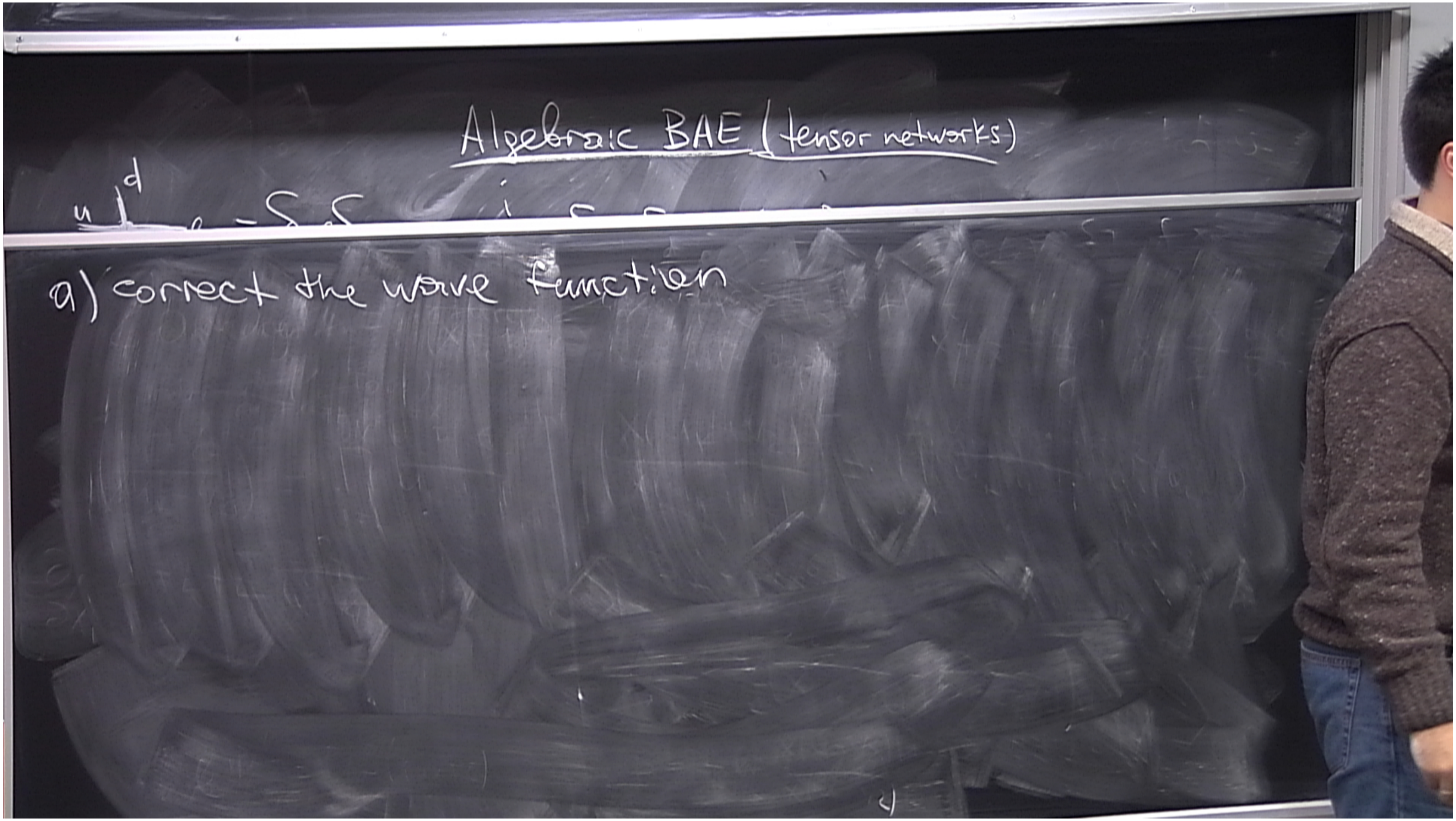
O_2 ?

$$= \frac{\langle 1|\hat{O}_3|2\rangle}{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}} = C_{123} \qquad \leftarrow \det_{N\times N}$$

$\det_{N\times N}$

$$\langle 1|1\rangle = \det_{N\times N} \frac{\partial \varphi_k}{\partial u_j} \qquad \varphi_k = \ln \mathrm{BAE}_k$$

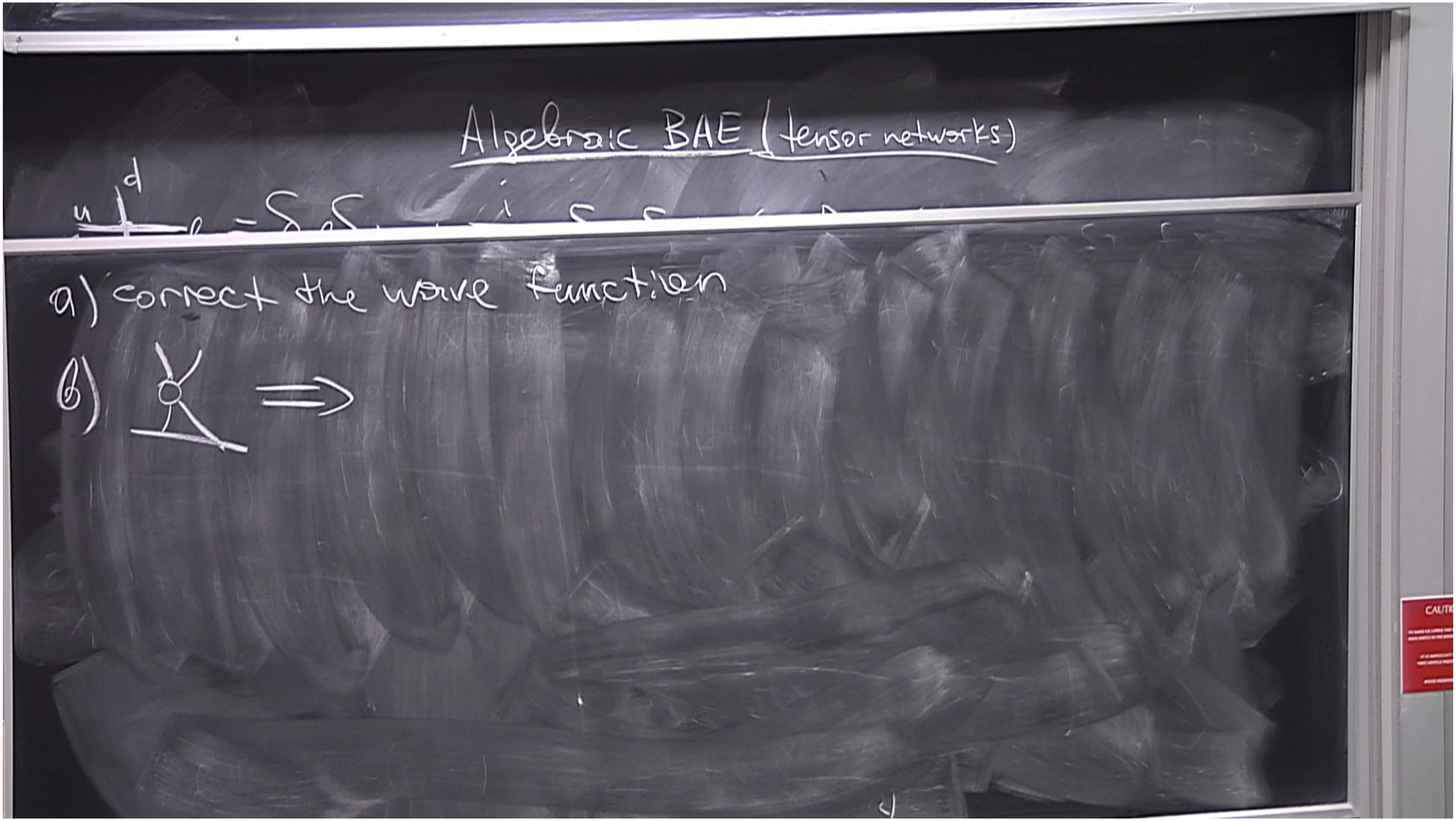
$$\varphi_k = \sum_{a=1}^{L} \ln \frac{u_k-\theta_a+i/2}{u_k-\theta_a-i/2} + \sum_{m\neq k}^{N} \ln \frac{u_k-u_j-i}{u_k-u_j+i}$$

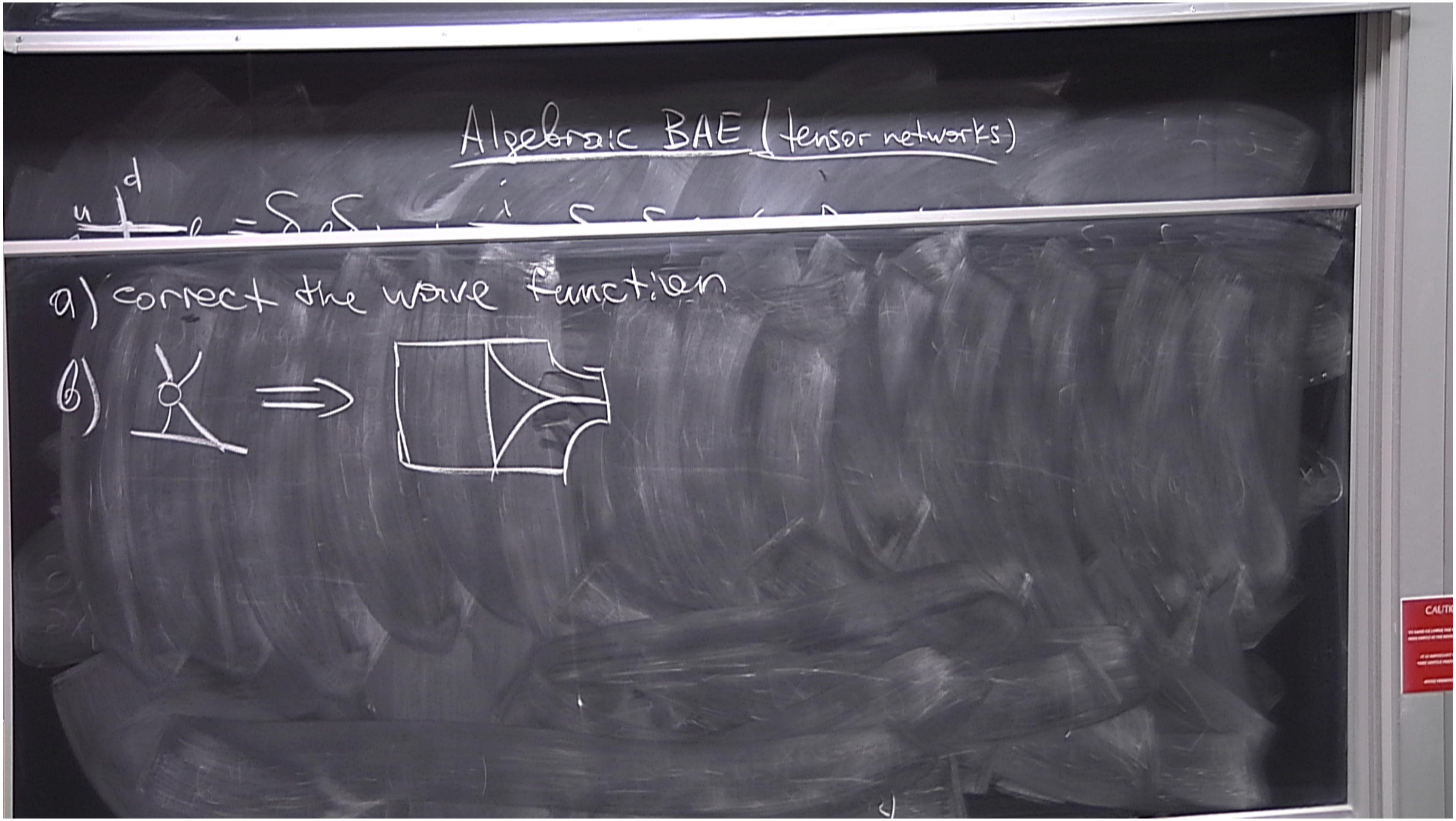

Algebraic BAE (tensor networks)
a) correct the wave function

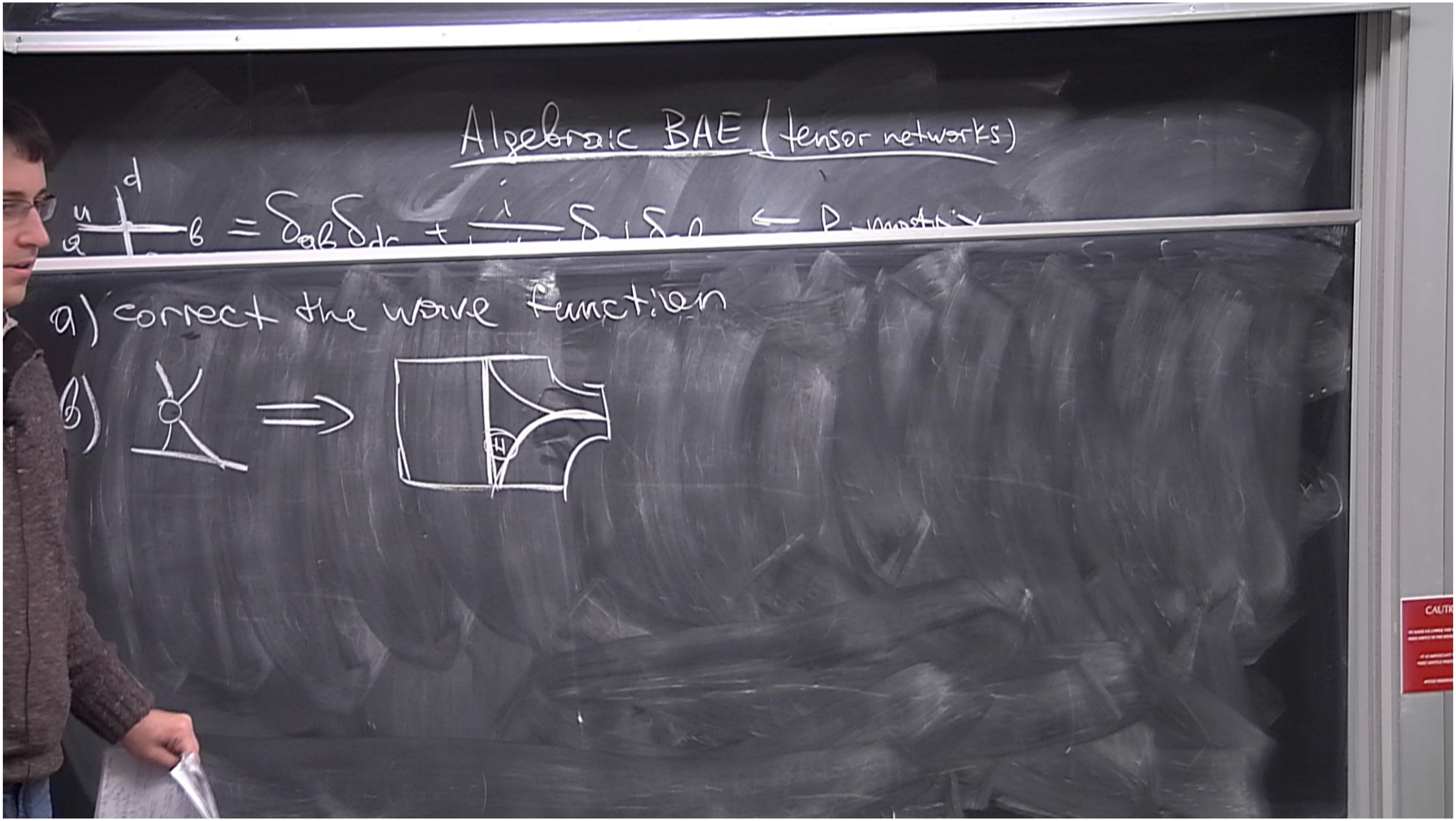
Algebraic BAE (tensor networks)

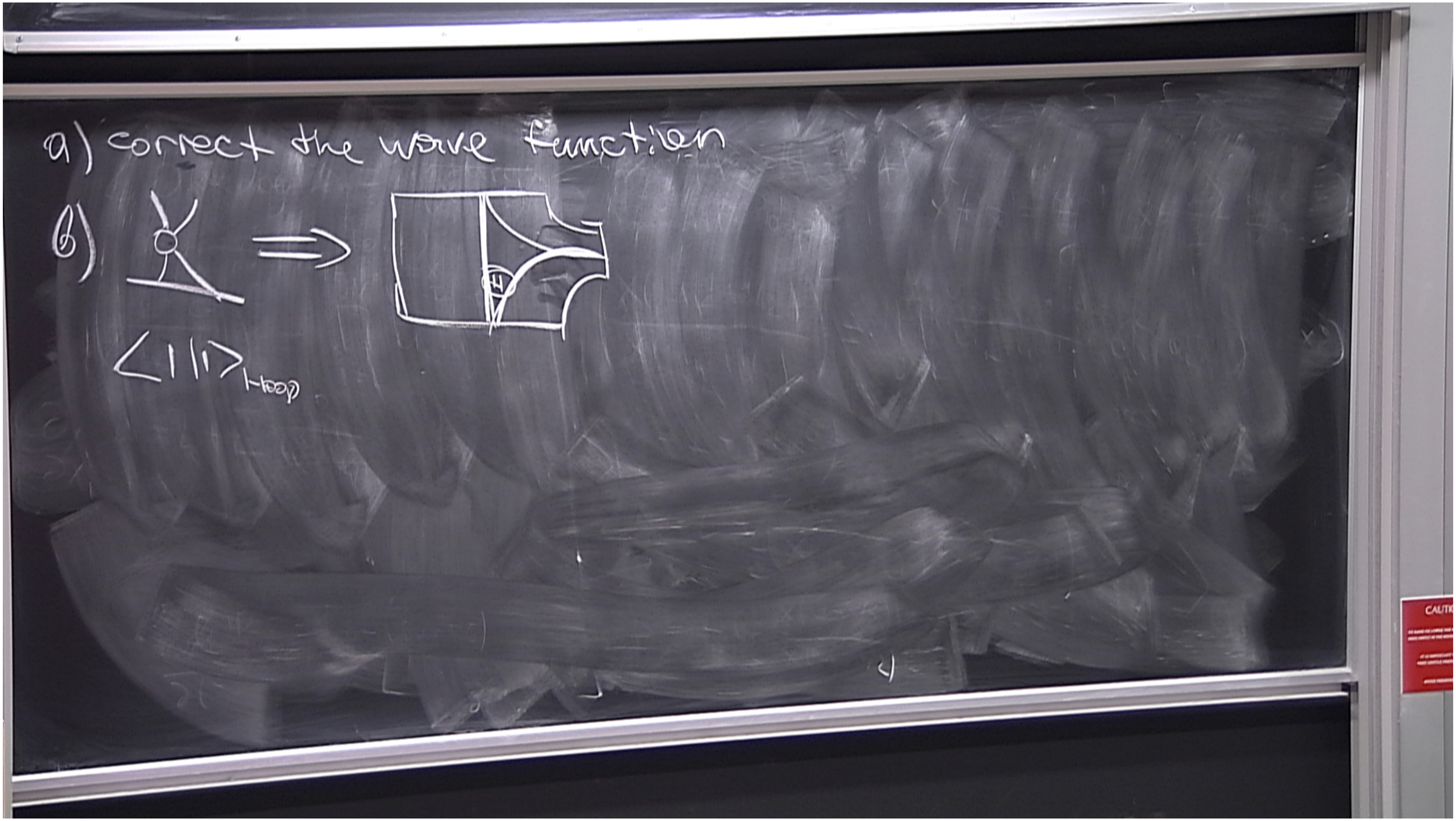
a) correct the wave function

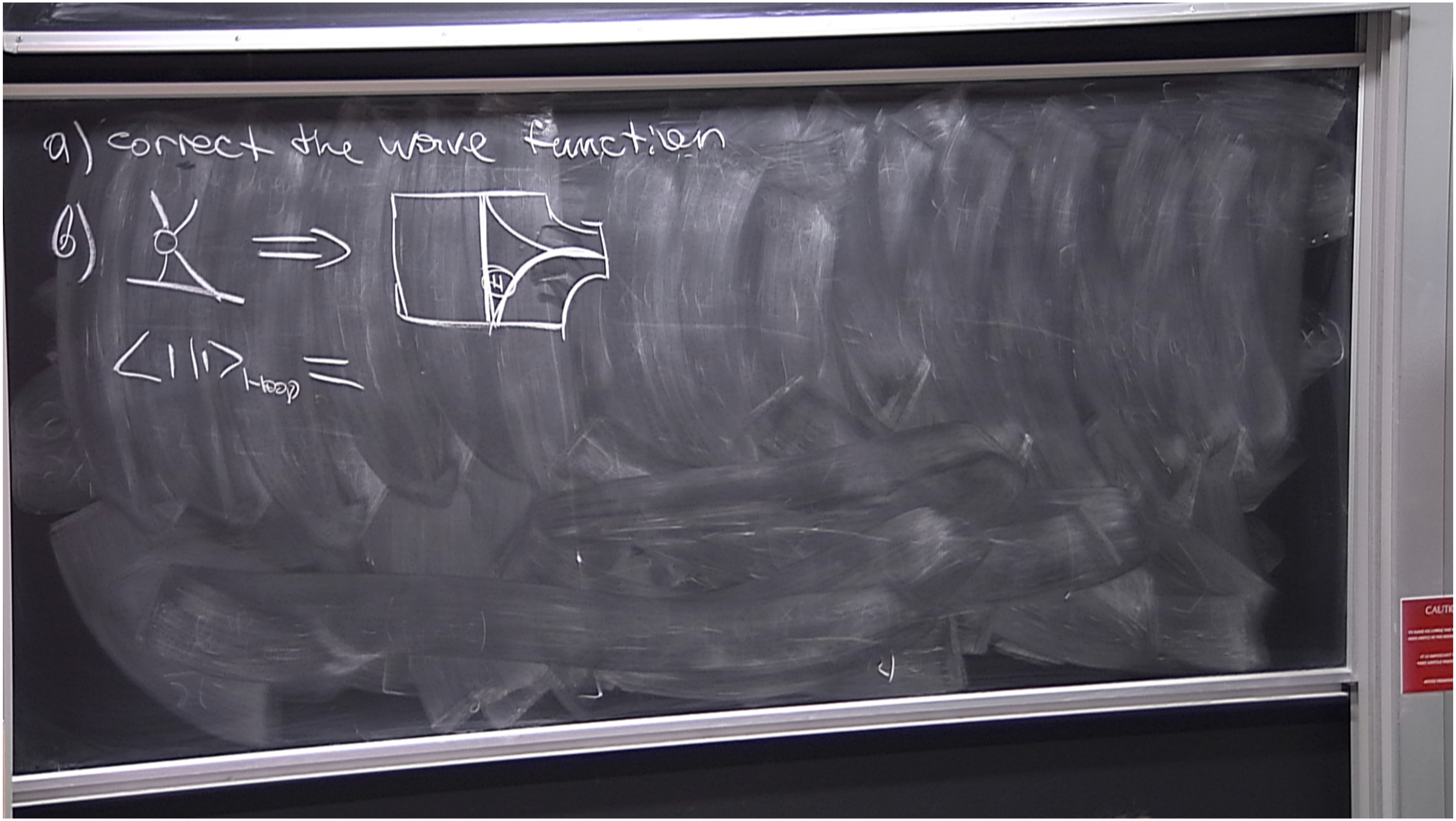
b) 

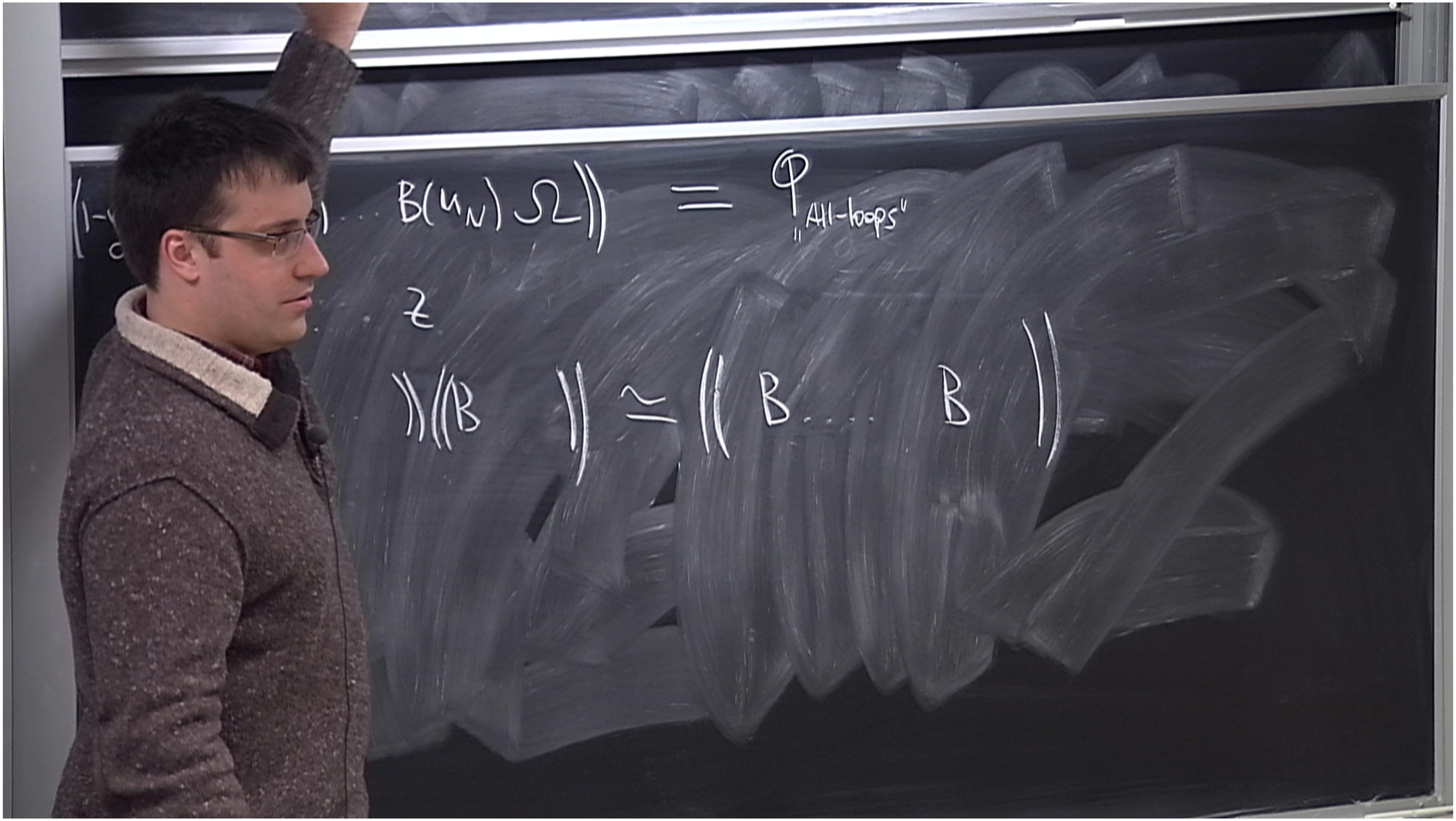

Algebraic BAE (tensor networks)
a) correct the wave function
b) ⟹

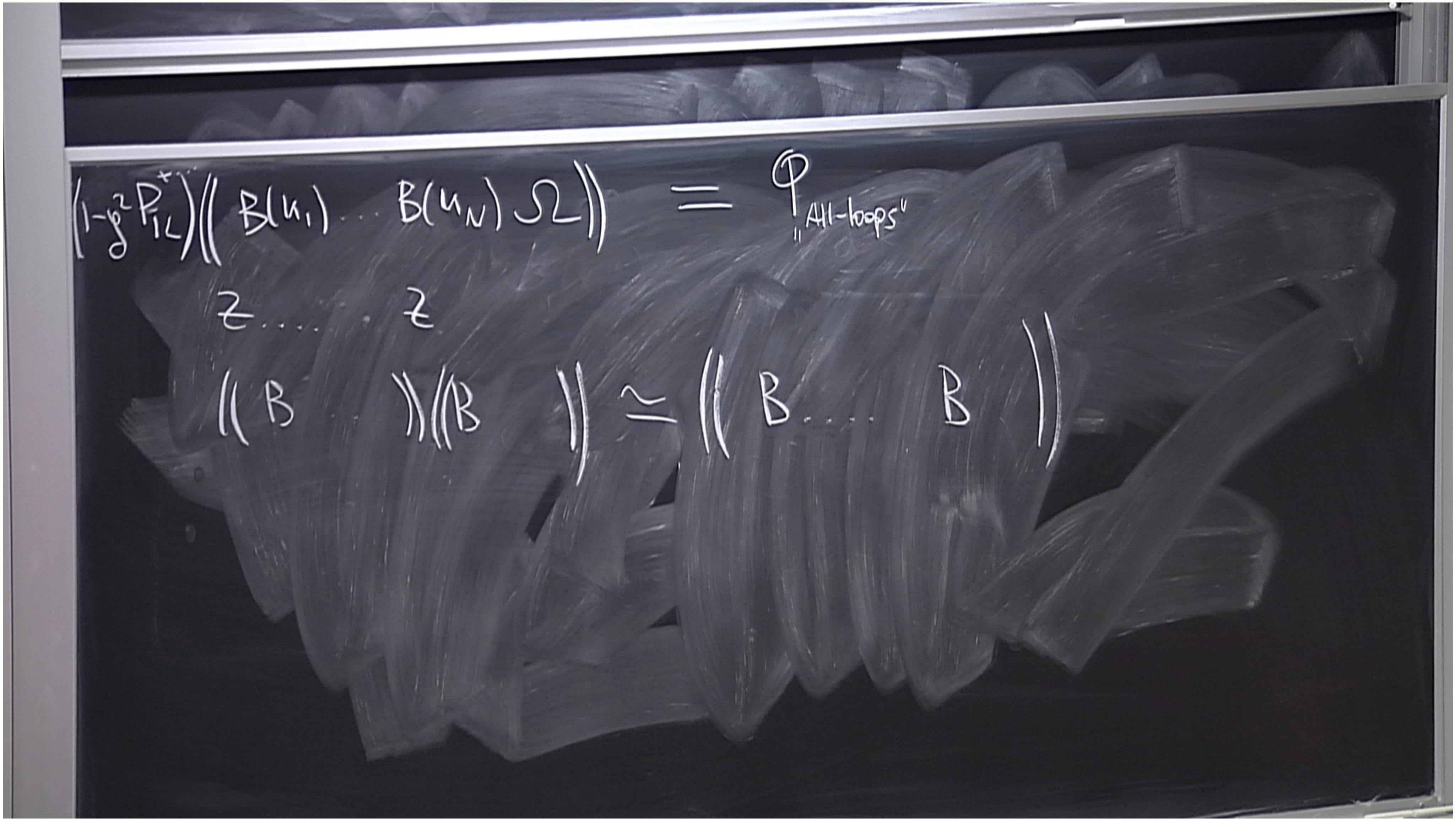

Algebraic BAE (tensor networks)
a) correct the wave function
b)


Algebraic BAE (tensor networks)
a) correct the wave function
b)


a) correct the wave function
b)
⟨1|1⟩ 1-loop


a) correct the wave function
b)
⟨1|1⟩1-loop =


B(u_N) Ω))
= Φ
"All-loops"
z
))((B
)) ≃ ((B ..... B))

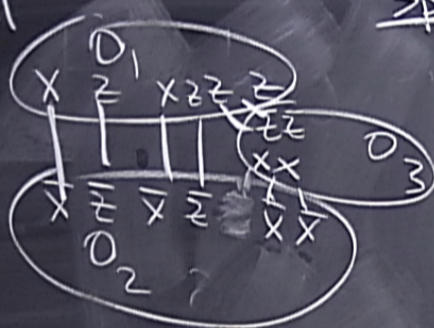

$(1-g^2 P^{+\cdots}_{1L})\,((\, B(u_1) \cdots B(u_N)\, \Omega\,)) = \Phi_{\text{"All-loops"}}$
$Z \cdots\cdots Z$
$((\,B \quad))((\,B \quad)) \simeq ((\,B \cdots\cdots B\,))$

a) correct the wave function

b) 

$$\langle 1|1\rangle_{\text{1-loop}} = \left(\left(\langle 1|1\rangle_{\theta} \right)\right)$$

tree level



3pt function

$$\hat{O}_3 = |0\rangle^{\otimes N_3} \langle 1|^{\otimes L_3 - N_3}$$

Omar Foda

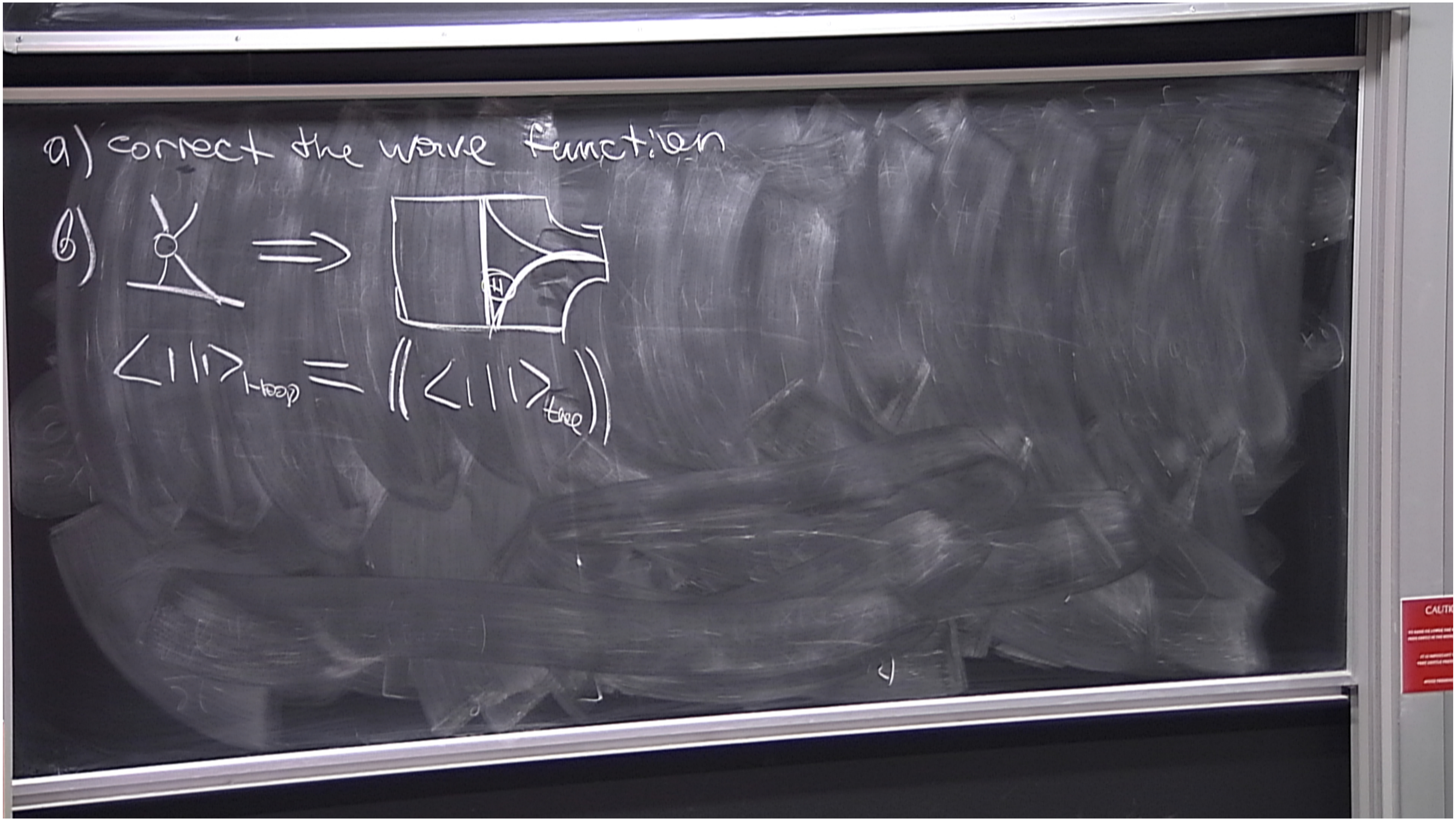
$$= \frac{\langle 1|\hat{O}_3|2\rangle}{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}} = C_{123} \qquad \leftarrow \det_{N\times N}$$

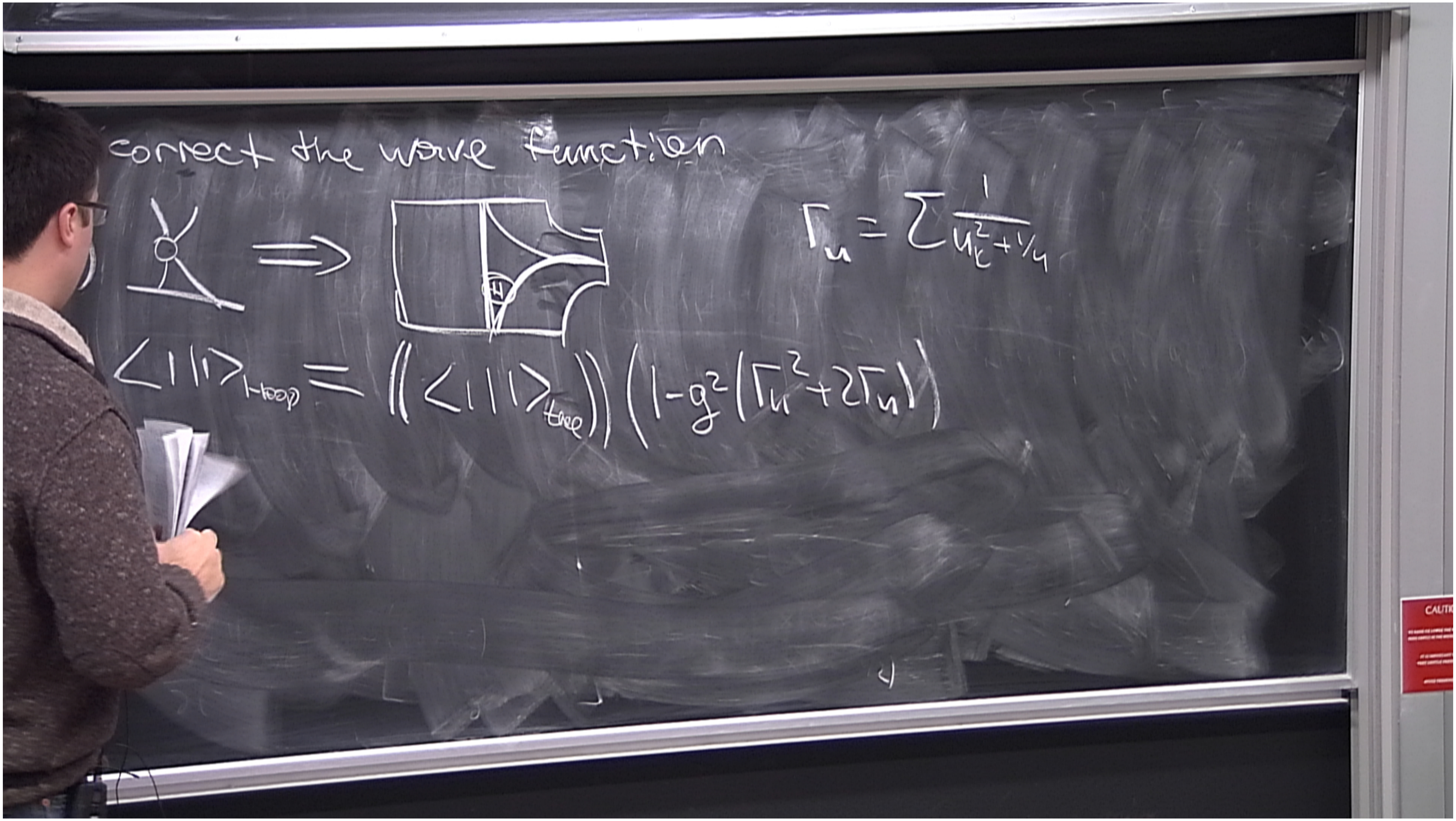
$\det_{N\times N}$

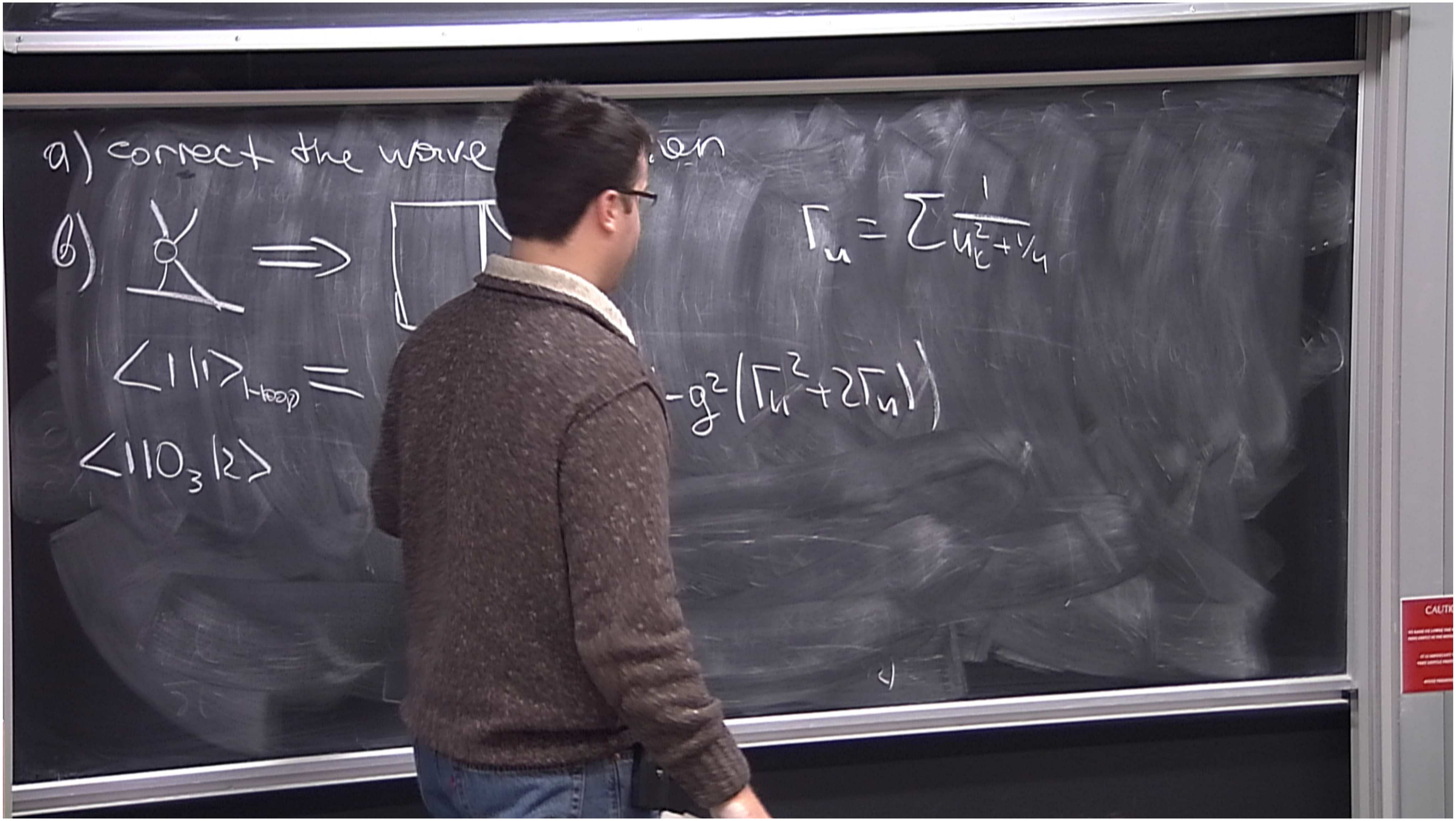
$$\langle 1|1\rangle_{\text{tree}} = \det_{N\times N} \frac{\partial \varphi_k}{\partial u_j}$$

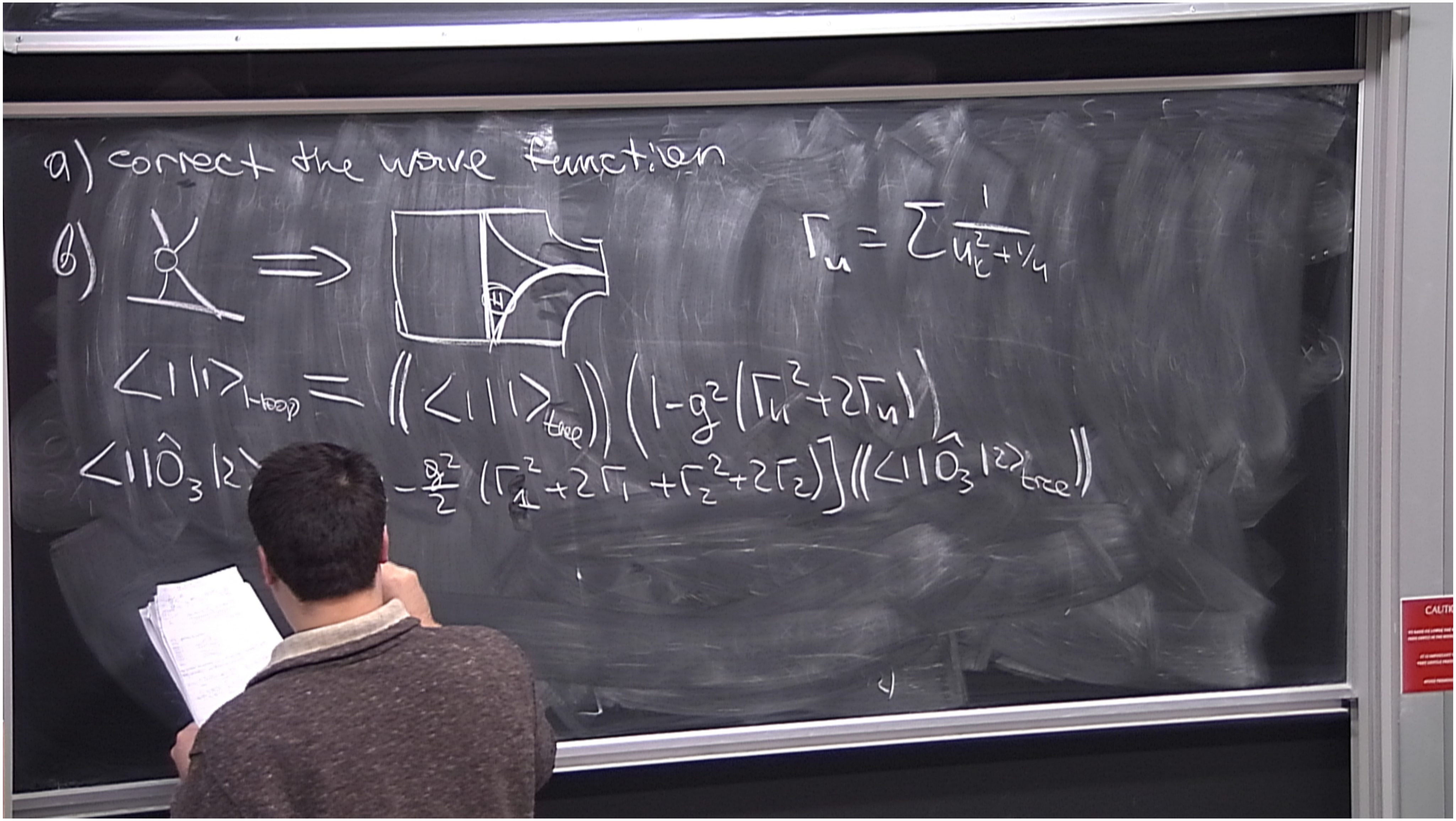
$$\varphi_k = \ln BAE_k$$

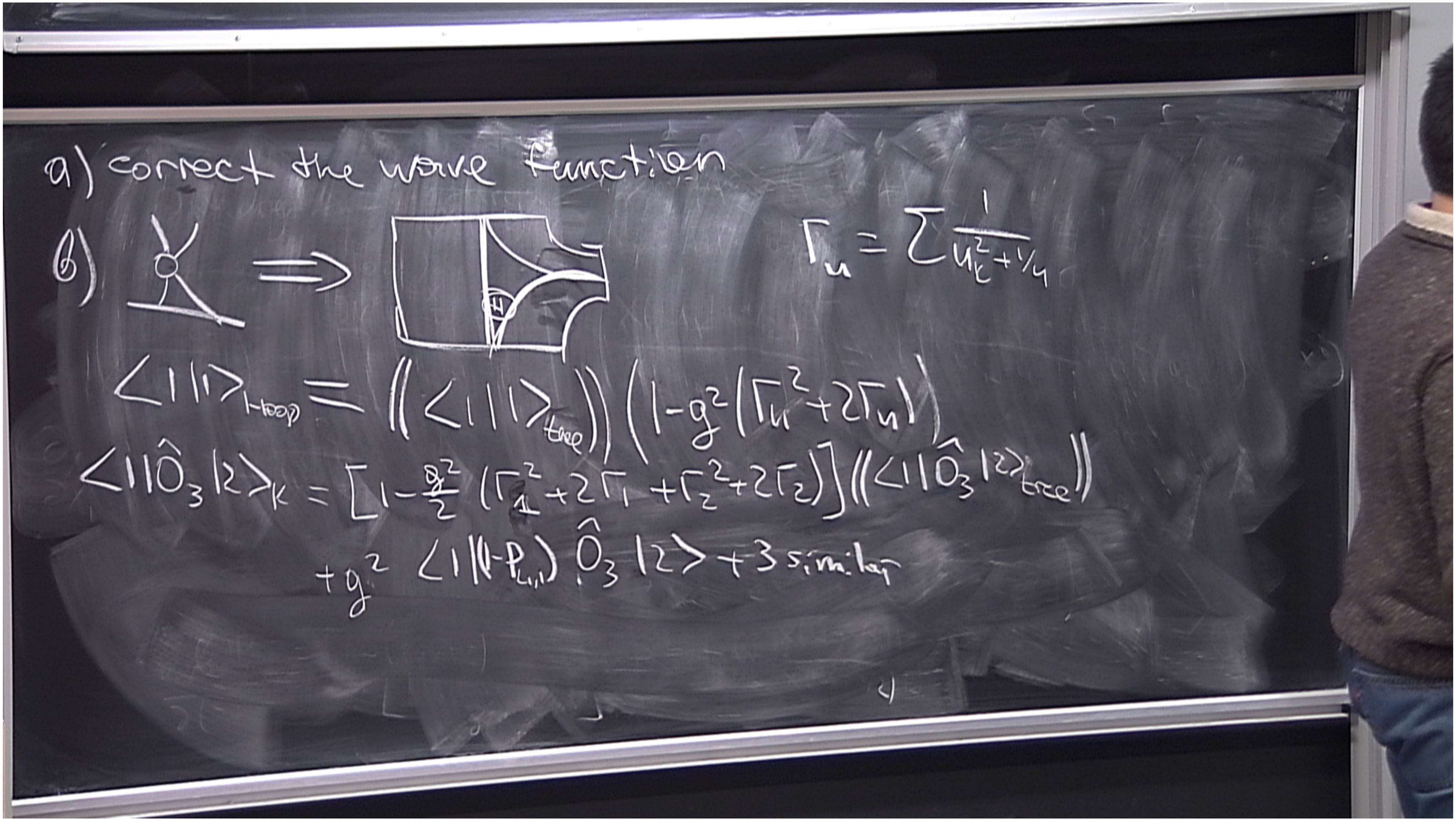
$$\varphi_k = \sum_{a=1}^{L} \ln \frac{u_k-\theta_a+i/2}{u_k-\theta_a-i/2} + \sum_{j\neq k}^{N} \ln \frac{u_k-u_j-i}{u_k-u_j+i}$$


a) correct the wave function
b)
⟨1|1⟩1-loop = ((⟨1|1⟩tree))

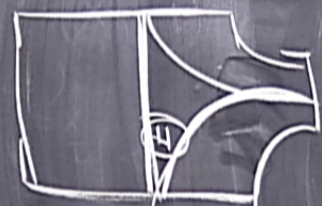

correct the wave function
$\Gamma_u = \sum \frac{1}{u_k^2 + 1/4}$
$\langle 1|1\rangle_{1\text{-loop}} = \left(\langle 1|1\rangle_{tree}\right)\left(1 - g^2\left(\Gamma_u^2 + 2\Gamma_u\right)\right)$


a) correct the wave
b)
$\langle 1|1\rangle_{1\text{-loop}} =$
$-g^2(\Gamma_u^2+2\Gamma_u)$
$\Gamma_u = \sum \frac{1}{u_k^2+1/4}$
$\langle 1|O_3|2\rangle$


a) correct the wave function
b)
$\Gamma_u = \sum \frac{1}{u_k^2 + 1/4}$
$\langle 1|1\rangle_{1\text{-loop}} = ((\langle 1|1\rangle_{tree}))(1 - g^2(\Gamma_u^2 + 2\Gamma_u))$
$\langle 1|\hat{O}_3|2\rangle$
$- \frac{g^2}{2}(\Gamma_1^2 + 2\Gamma_1 + \Gamma_2^2 + 2\Gamma_2)]((\langle 1|\hat{O}_3|2\rangle_{tree}))$


a) correct the wave function
b)
u
$\Gamma_u = \sum \frac{1}{u_k^2 + 1/4}$
$\langle 1|1\rangle_{1\text{-loop}} = \left(\langle 1|1\rangle_{tree}\right)\left(1 - g^2(\Gamma_u^2 + 2\Gamma_u)\right)$
$\langle 1|\hat{O}_3|2\rangle_K = \left[1 - \frac{g^2}{2}(\Gamma_1^2 + 2\Gamma_1 + \Gamma_2^2 + 2\Gamma_2)\right]\left(\langle 1|\hat{O}_3|2\rangle_{tree}\right)$
$+ g^2 \langle 1|(1-P_{i,i})\hat{O}_3|2\rangle + 3$ similar

a) correct the wave function

b)  $\Longrightarrow$ 

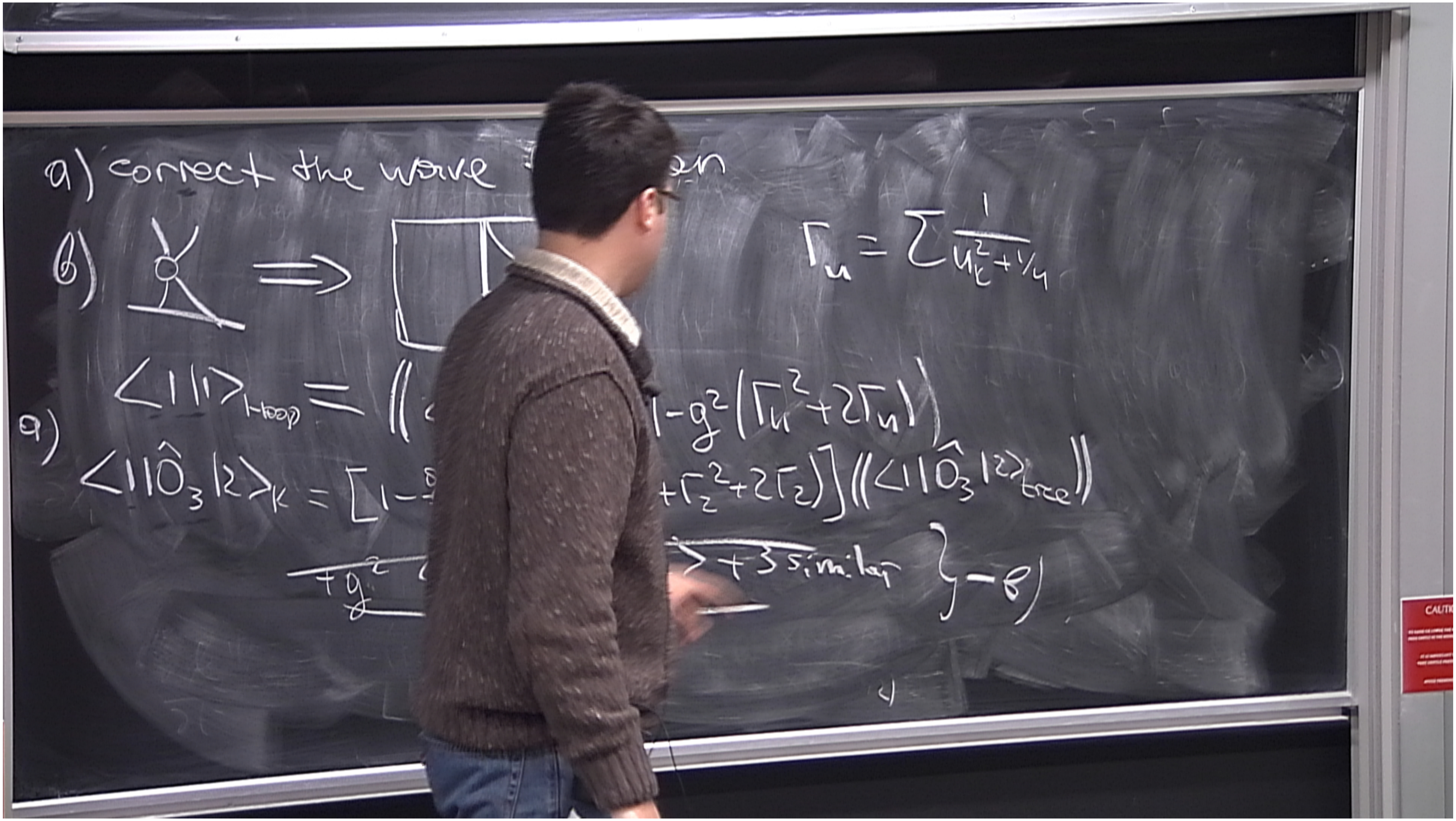
$$\Gamma_n = \sum \frac{1}{u_k^2 + 1/4}$$

$$\langle 1|1\rangle_{1\text{-loop}} = \left(\langle 1|1\rangle_{\text{tree}}\right)\left(1-g^2\left(\Gamma_n^2+2\Gamma_n\right)\right)$$

$$\langle 1|\hat{O}_3|2\rangle_K = \left[1-\frac{g^2}{2}\left(\Gamma_1^2+2\Gamma_1+\Gamma_2^2+2\Gamma_2\right)\right]\left(\langle 1|\hat{O}_3|2\rangle_{\text{tree}}\right)$$

$$\left.\cancel{+g^2\langle 1|(1-P_{i,i})\hat{O}_3|2\rangle + 3\text{ similar}}\right\} - b)$$

c)


a) correct the wave
b)
Γu = Σ 1/(uk²+1/4)
a)
= [1-

$$\left(1-g^2 P^{+\cdots}_{1L}\right)\left(\!\left(B(u_1)\cdots B(u_N)\,\Omega \right)\!\right) = \Phi_{\text{"All-loops"}}$$

$$z \;\ldots\ldots\; z$$

$$\left(\!\left(B \quad \right)\!\right)\left(\!\left(B \quad \right)\!\right) \simeq \left(\!\left(B \ldots\ldots B \right)\!\right)$$

$$C_{123} = \left(\!\left(C^{\text{tree}}_{123} \right)\!\right)$$