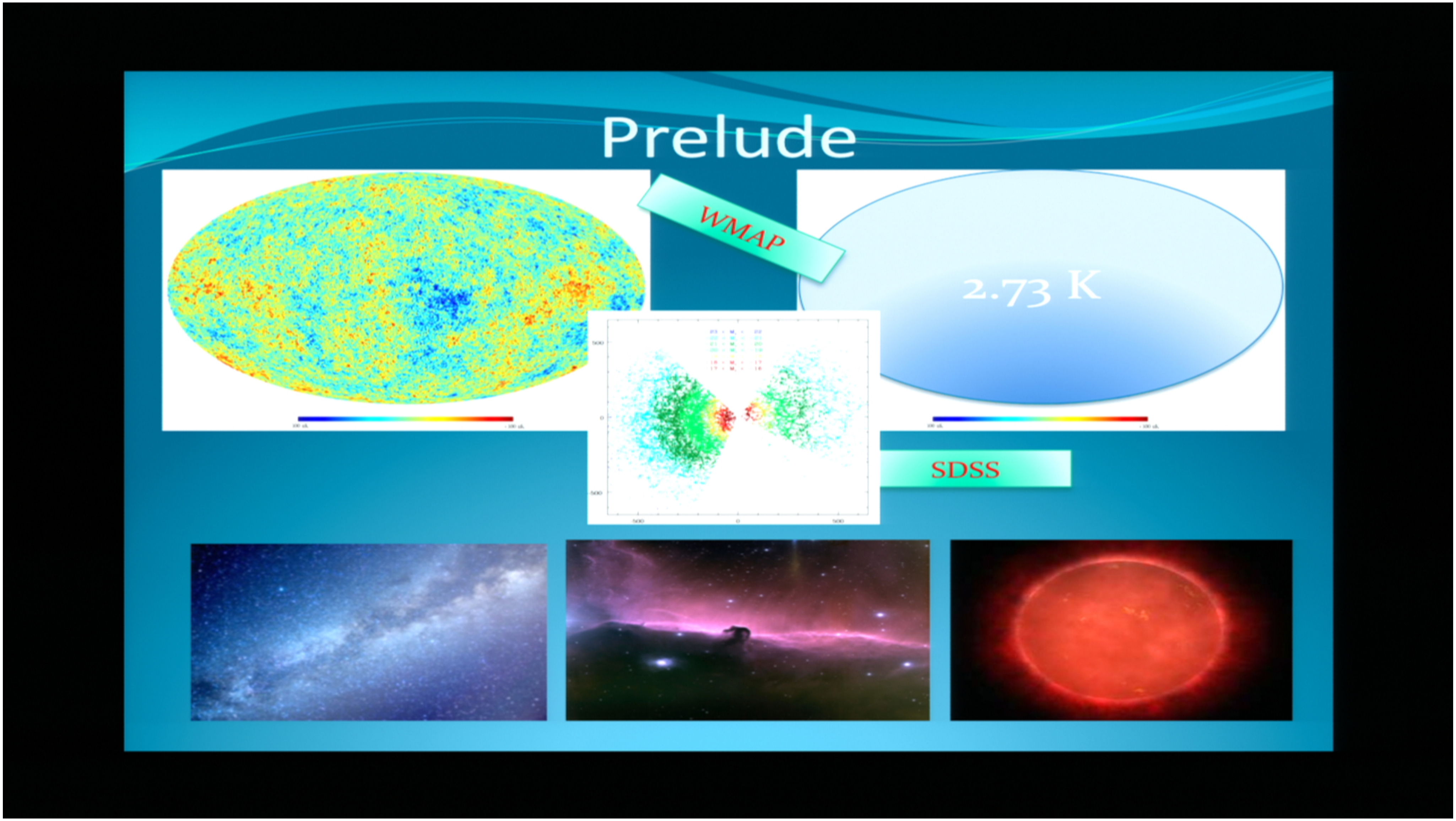
Title: The Effects of the Cosmological Backreaction on the Concordance Model of Cosmology

Date: Feb 21, 2012 11:00 AM

URL: http://pirsa.org/12020161

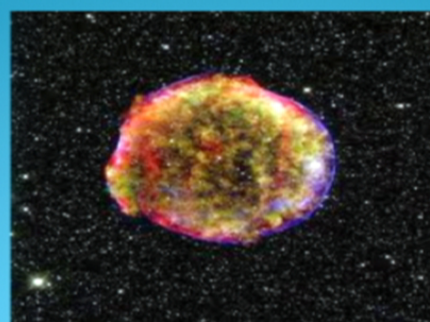
Abstract:

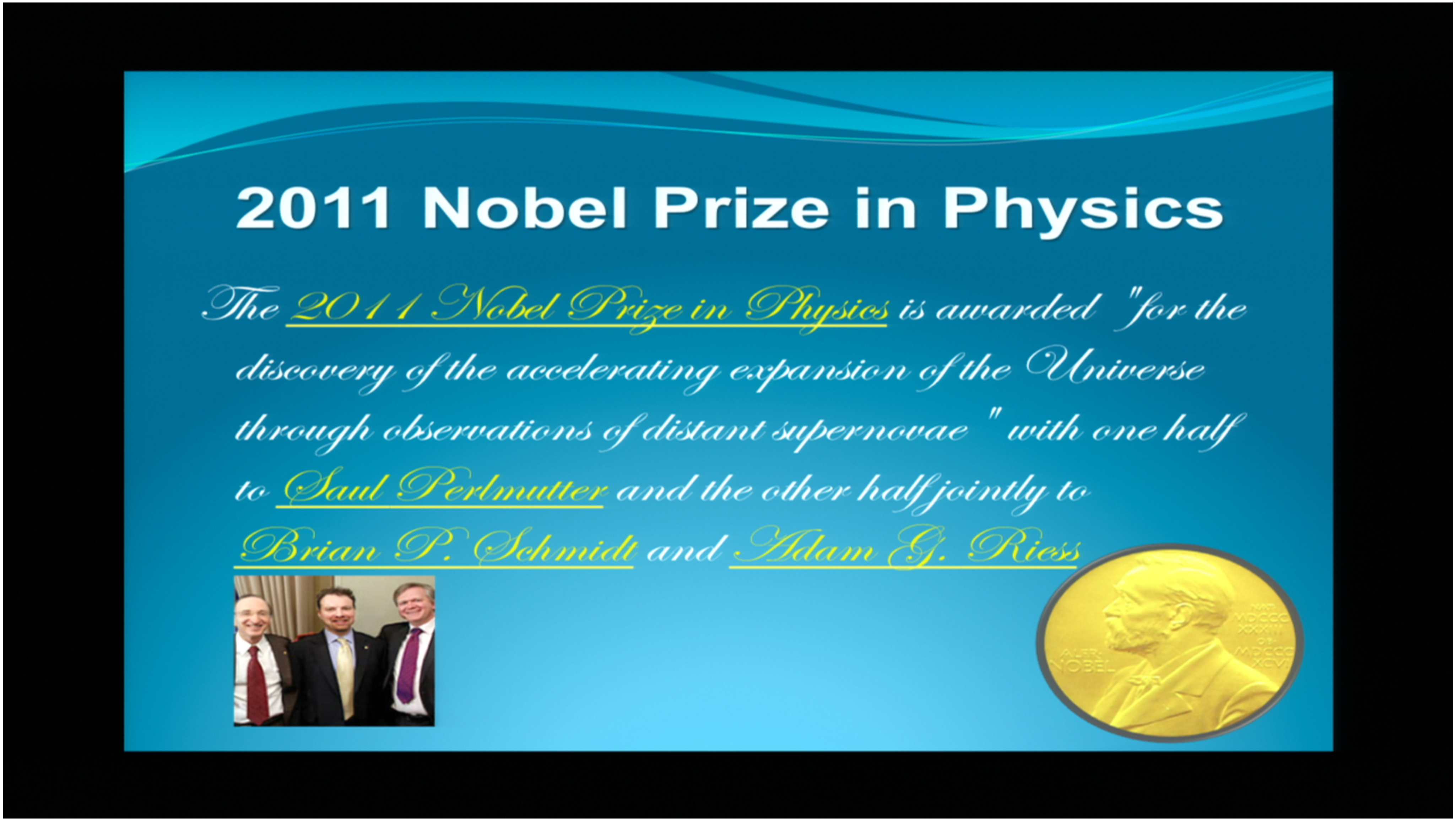

Prelude
WMAP
2.73 K
SDSS

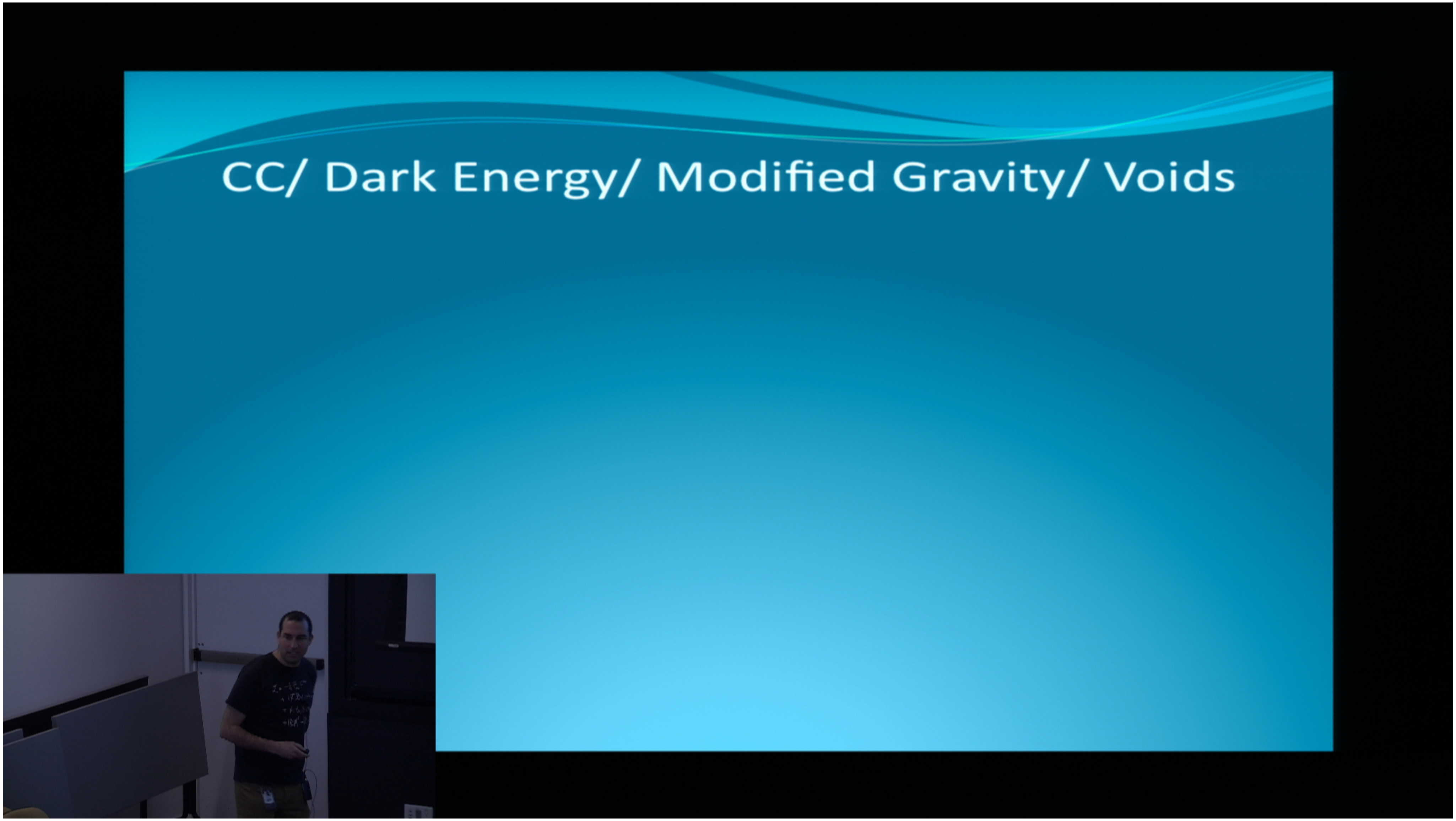
Perturbations at Low Redshift

- Measurements of SNIa ➔ Mostly neglected, naively argued as irrelevant $\sim 10^{-10}$
- The concordance model of cosmology:

~73% of the critical energy density is not accounted for by known matter, dark matter or curvature.




2011 Nobel Prize in Physics
The 2011 Nobel Prize in Physics is awarded "for the discovery of the accelerating expansion of the Universe through observations of distant supernovae " with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess


CC/ Dark Energy/ Modified Gravity/ Voids

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- **(Averaging of) perturbations in a consistent way. Does it have any effects?**
- **In this work, we are NOT:**

1. **Assuming voids**
2. **Modifying GR**
3. **Adding scalar fields**
4. **Having a CC....Doh!**

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Main stream Cosmology

- Assume FLRW = homogeneous and isotropic metric.

=>Implicit averaging

- Modeling the energy momentum tensor as a perfect fluid.
- Pert. give rise to structure, highly non-linear at some scale. Background unchanged.

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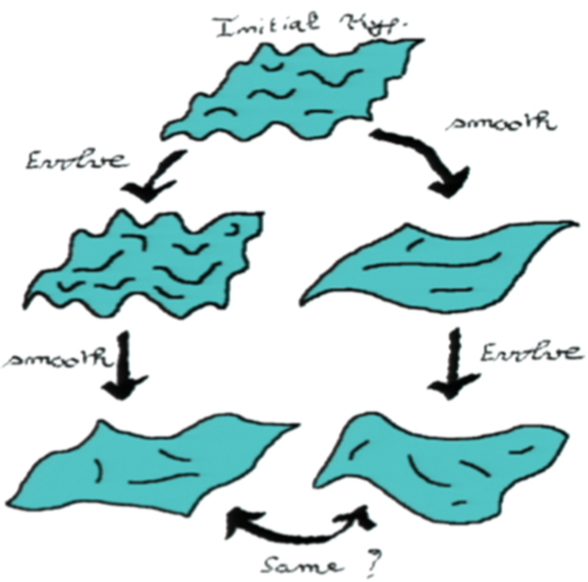
Importance of averaging



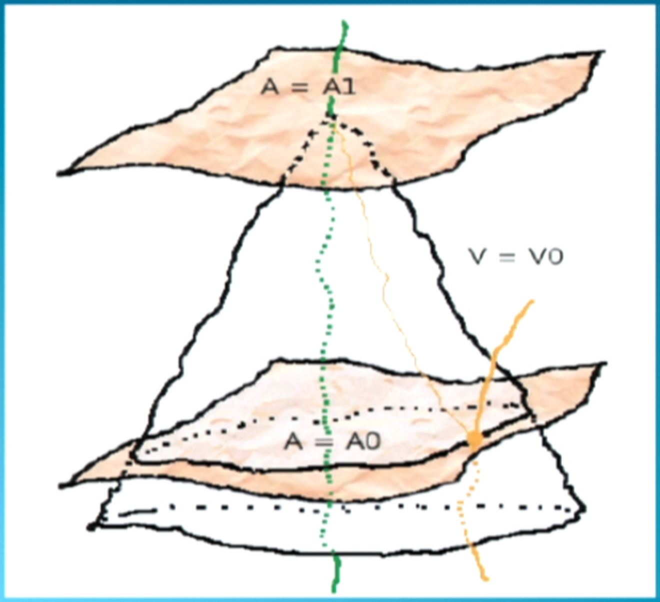
The evolution of an inhomogeneous spacetime after averaging differs from the evolution of its averaged spacetime.

Few questions:

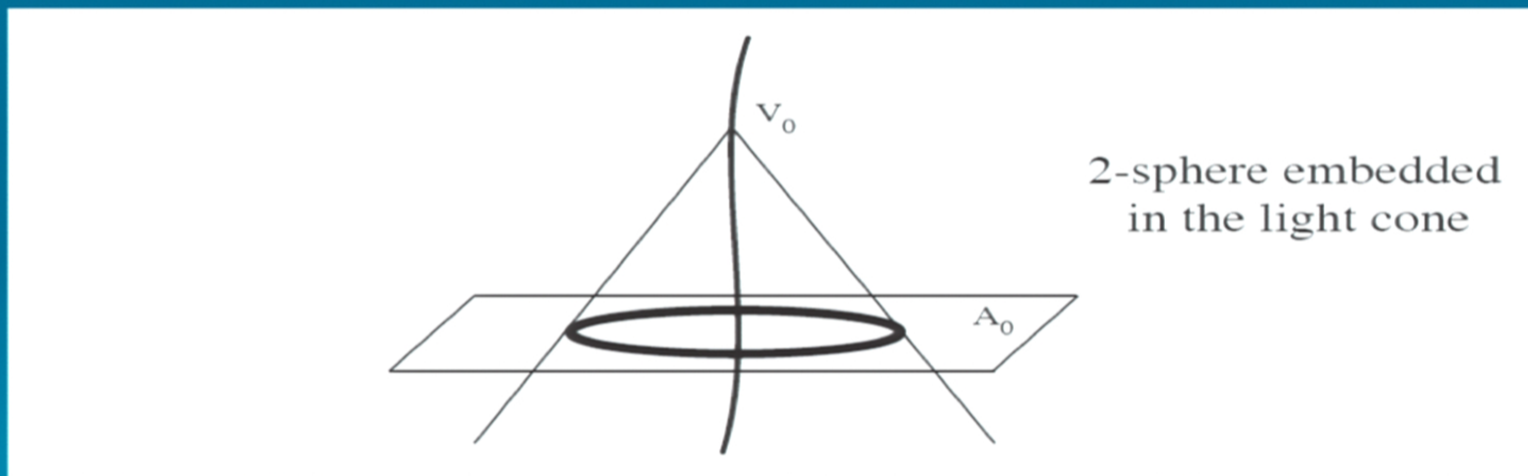
- Can smoothing of structure contribute to an acceleration term (DE)?
  Is there an effect from small scales to large scales ?
  ⇒ Nice way out of the coincidence problem? (T. Buchert)
- Consequences on cosmological parameters? (C. Clarkson, J. Larena)
  Negligible for a physical reason?
- What is the scale of homogeneity in the Universe? $100Mpc$?
- Fitting Problem : What is the best-fit FLRW model to a given lumpy Universe?
- How Einstein field equations transform after a coarse-graining procedure? How do we average vectors and tensors?

LC Averaging

- Hyper-surfaces using meaningful physical quantities: Redshift, temperature etc.
- Observations are made on the light-cone.
- Past attempts: Coley 0905.2442; Rasanen 1107.1176, 0912.3370



Average dL on the past LC at constant red-shift



A= redshift, V= light-cone coordinate

Light Cone Averaging 1104.1167

- A-priori - the averaging is a geometric procedure, does not assume a specific energy momentum tensor.
- Assume light travels on geodesics.

$$I(S; V_0, A_0; -) = \int d^4x \sqrt{-g}\, \delta(V_0 - V)\delta(A - A_0)|\partial_\mu V \partial^\mu A| S(x)$$

$$k_\mu \equiv \partial_\mu V \qquad \langle S \rangle_{V_0, A_0} = \frac{I(S; V_0, A_0; -)}{I(1; V_0, A_0; -)}$$

The prescription is gauge inv., {field reparam. A->A'(A), V->V'(V)} and invariant under general coordinate transformation. A(x) is a time-like scalar, V(x) is null.

This gives a procedure for general space-times.

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Prescription Properties

- Dynamical properties: Generalization of Buchert-Ehlers commutation rule:

$$\frac{\partial}{\partial A_0}\langle S\rangle_{V_0,A_0} = \left\langle \frac{k\cdot\partial S}{k\cdot\partial A}\right\rangle_{V_0,A_0} + \left\langle \frac{\nabla\cdot k}{k\cdot\partial A} S\right\rangle_{V_0,A_0} - \left\langle \frac{\nabla\cdot k}{k\cdot\partial A}\right\rangle_{V_0,A_0} \langle S\rangle_{V_0,A_0}$$

- For actual physical calculations, use EFE/ energy momentum tensor for evaluation. Example: which gravitational potential to use in evaluating the d_L-z relation.
- In this talk, we take a geodetic observer, and the gravitational potential of pure CDM.

GLC Metric and Averages

$$ds^2_{GLC} = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\widetilde{\theta}^a - U^a dw)(d\widetilde{\theta}^b - U^b dw).$$

- Ideal Observational Cosmology – Ellis et al.
- Evaluating scalars at a constant redshift for a geodetic observer.

$$I(S, w_0, z) = \int d^2\widetilde{\theta}\sqrt{\gamma(w_0, z, \widetilde{\theta}^a)}S(w_0, z, \widetilde{\theta}^a);$$

$$\langle S \rangle = \frac{I(S, w_0, z)}{I(1, w_0, z)};$$

$$1 + z = \frac{\Upsilon_O}{\Upsilon_S}$$

GLC Metric

- FLRW

$$w = r + \eta, \qquad \tau = t, \qquad \Upsilon = a(t), \qquad U^a = 0,$$
$$\gamma_{ab} d\theta^a d\theta^b = a^2(t) r^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

- τ can be identified as the time coordinate in the synchronous gauge of arbitrary space-time.

$$= -\left[\partial_\tau + \Upsilon^{-1}(\partial_w + U^a \partial_a)\right] X^\mu = -u^\nu \partial_\nu X^\mu = -\frac{dX^\mu}{d\lambda},$$

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Averaged d_L at Constant Redshift

- Ethrington's Reciprocity Law, for any spacetime:

$$d_L(z) = (1+z)^2 d_S$$

$$\langle d_L \rangle_{w_0,z} = (1+z)^2 \frac{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}\, d_s(w_0,\tau(z,w_0,\theta^a),\theta^a)}{\int d^2\theta \sqrt{|\gamma(w_0,\tau(z,w_0,\theta^a),\theta^a)|}}\,.$$

Measure of Integration

Fluc. in scalar

$$d_s^2(w = w_0,\tau,\theta^a) = \lim_{r_p \to 0} r_p^2 \frac{\sqrt{|\gamma(w_0,\tau,\tilde{\theta}^a)|}}{\sqrt{|\gamma(w_0,\tau_p,\tilde{\theta}^a)|}}\,.$$

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Measure of Integration

Fluc. in scalar

$$d_s^2(w = w_0, \tau, \theta^a) = \lim_{r_p \to 0} r_p^2 \frac{\sqrt{|\gamma(w_0, \tau, \tilde{\theta}^a)|}}{\sqrt{|\gamma(w_0, \tau_p, \tilde{\theta}^a)|}} .$$

Averaged d_L at Constant Redshift

- Novelty: In principle, exact treatment of the geodesic equations and the averaging hyper-surface for any spacetime.
- Previous attempts are limited to perturbations about FLRW/LTB/Swiss Cheese and had to solve order by order: Vanderveld et al. – post Newtonian, Barausse et al., Kolb et al. – SG superhorizon, Pyne at al…

Statistical Properties

- In principle, we can now calculate <d_L>(z) to first order in the gravitational potential ~ void model.
- In order not to resort to a specific realization we need LC+statistical/ensemble average. If perturbations come from primordial Gaussian fluc. (inflation) $\overline{\Psi} = 0, \ \overline{\Psi^2} \neq 0$

$$\langle S \rangle_\Sigma = \frac{\int_\Sigma d^2\mu \, S}{\int_\Sigma d^2\mu}$$

$$d^2\mu = (d^2\mu)^{(0)}(1+\mu), \qquad S = S^{(0)}(1+\sigma),$$

$$\mu = \sum_i \mu_i, \qquad \sigma = \sum_i \sigma_i.$$

BR of Statistical and LC Averaging

- The mean of a scalar:

$$\overline{\langle S/S^{(0)}\rangle} = 1 + \overline{\langle\sigma_2\rangle} + \mathrm{IBR}_2 + \overline{\langle\sigma_3\rangle} + \mathrm{IBR}_3 + \ldots$$

$$\begin{aligned}\mathrm{IBR}_2 &= \overline{\langle\mu_1\sigma_1\rangle} - \overline{\langle\mu_1\rangle\langle\sigma_1\rangle}, \\ \mathrm{IBR}_3 &= \overline{\langle\mu_2\sigma_1\rangle} - \overline{\langle\mu_2\rangle\langle\sigma_1\rangle} + \overline{\langle\mu_1\sigma_2\rangle} - \overline{\langle\mu_1\rangle\langle\sigma_2\rangle} - \overline{\langle\mu_1\rangle\langle\mu_1\sigma_1\rangle} + \overline{\langle\mu_1\rangle\langle\mu_1\rangle\langle\sigma_1\rangle},\end{aligned}$$

=> Effects are second order, but we have the full backreaction of the inhomogeneities of the metric at this order!

- The variance to leading order: $\mathrm{Var}[S/S^{(0)}] = \overline{\langle\sigma_1^2\rangle}.$

GLC to FLRW NG 1st Order

$$g^{\mu\nu}_{NG} = a^{-2}(\eta)\,\mathrm{diag}\left(-1+2\Phi, 1+2\Psi, (1+2\Psi)\gamma_0^{ab}\right) .$$

$$\begin{aligned}
\tau &= \int_{\eta_{in}}^{\eta} d\eta' a(\eta')\left[1+\Psi(\eta', r, \theta^a)\right] \ , \\
w &= \eta_+ + \int_{\eta_+}^{\eta_-} dx\, \hat{\Psi}(\eta_+, x, \theta^a) \ , \\
\widetilde{\theta}^a &= \theta^a + \frac{1}{2}\int_{\eta_+}^{\eta_-} dx\, \hat{\gamma}_0^{ab}(\eta_+, x, \theta^a) \int_{\eta_+}^{x} dy\, \partial_b \hat{\Psi}(\eta_+, y, \theta^a) \ ,
\end{aligned}$$

$$\hat{\Psi}(\eta_+, \eta_-, \theta^a) \equiv \Psi(\eta, r, \theta^a)$$
$$\hat{\gamma}_0^{ab}(\eta_+, \eta_-, \theta^a) \equiv \gamma_0^{ab}(\eta, r, \theta^a) = diag(r^{-2}, r^{-2}\sin^{-2}\theta)$$

$$\eta_\pm = \eta \pm r$$

Luminosity Distance Terms

$$J = I_+ - I_r.$$

$$I_+ = \int_{\eta_+^s}^{\eta_-^s} dx\, \partial_+ \Psi(\eta_s^+, x, \theta^a) = \Psi_s - \Psi_o - 2\int_{\eta_s}^{\eta_0} d\eta\, \partial_r \Psi(\eta, r, \theta^a),$$

$$I_r = \int_{\eta_{in}}^{\eta_s} d\eta \frac{a(\eta)}{a(\eta_s)} \partial_r \Psi(\eta, r_s, \theta^a) - \int_{\eta_{in}}^{\eta_0} d\eta \frac{a(\eta)}{a(\eta_0)} \partial_r \Psi(\eta, 0, \theta^a).$$

$$I_r = (\vec{v}_S - \vec{v}_0) \cdot \hat{n}$$

$$\vec{v}_{s,o} = -\int_{\eta_{in}}^{\eta_{s,o}} d\eta' \frac{a(\eta')}{a(\eta_{s,0})} \vec{\nabla} \Psi(\eta', r, \theta^a)$$

- Can be associated with "peculiar velocities" due to the perturbation of the geodesic, NOT due to intrinsic motion.
- The Lensing Term:

$$J_2 = \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \left[\partial_\theta^2 + \cot\theta\, \partial_\theta + (\sin\theta)^{-2} \partial_\phi^2\right] \Psi(\eta', \eta 0 - \eta', \theta^a) \equiv \frac{1}{\eta_0 - \eta_s} \int_{\eta_s}^{\eta_0} d\eta \frac{\eta - \eta_s}{\eta_0 - \eta} \Delta_2 \Psi$$

Measure of Integration

$$\int d^2\tilde{\theta}\sqrt{\gamma} = \int d\phi \sin\theta \, d\theta a^2 r^2 (1 - 2\Psi)$$

- The deviation from the unperturbed measure:

$$\mu_1 = -2\Psi_s + 4\Psi_{\rm av} + 2\left(1 - \frac{1}{\mathcal{H}_s \Delta\eta}\right) J(z_s, \theta^a)$$

$$\mathrm{IBR}_2 = \int_0^\infty \frac{dk}{k}\, P_\Psi(k) \sum_{i=1}^{4}\sum_{j=1}^{5} 2\Big[\mathcal{C}_{ij}(k,\eta_0,\eta_s) - \mathcal{C}_i(k,\eta_0,\eta_s)\, \mathcal{C}_j(k,\eta_0,\eta_s)\Big]$$

$$\left(\mathrm{Var}\left[\frac{d_L}{d_L^{FLRW}}\right]\right)^{1/2} = \sqrt{\langle\sigma_1^2\rangle} = \left[\int_0^\infty \frac{dk}{k}\, P_\Psi(k) \sum_{i=1}^{5}\sum_{j=1}^{5} \mathcal{C}_{ij}(k,\eta_0,\eta_s)\right]^{1/2}.$$

$\langle A_i A_j\rangle$	$\mathcal{C}_{ij}(k,\eta_0,\eta_s)$	$\mathcal{C}_{ij}$ for $k\Delta\eta \ll 1$	$\mathcal{C}_i\,\mathcal{C}_j$ for $k\Delta\eta \ll 1$
$\langle A_1 A_1\rangle$	1	1	$1-\frac{l^2}{3}$
$\langle A_1 A_2\rangle$	$-\frac{2}{l}\mathrm{SinInt}(l)$	$-2+\frac{l^2}{9}$	$-2+\frac{4}{9}l^2$
$\langle A_1 A_3\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$
$\langle A_1 A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$-\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\langle A_1 A_5\rangle$	$-2\left[1-\frac{\sin(l)}{l}\right]$	$-\frac{l^2}{3}$	0
$\langle A_2 A_2\rangle$	$\frac{8}{l^2}\left[-1+\cos l+l\,\mathrm{SinInt}(l)\right]$	$4-\frac{l^2}{9}$	$4-\frac{4}{9}l^2$
$\langle A_2 A_3\rangle$	0	0	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\langle A_2 A_4\rangle$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0+f_s}{\Delta\eta}\left[1-\frac{\sin(l)}{l}\right]$	$\frac{f_0+f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{2}{3}l^2$
$\langle A_2 A_5\rangle$	$\frac{2}{3l^2}\left[-4+(4+l^2)\cos l+l\sin l+l^3\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0
$\langle A_3 A_3\rangle$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{36}$
$\langle A_3 A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{f_0-f_s}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{18}$
$\langle A_3 A_5\rangle$	$-2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\langle A_4 A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[\frac{f_0^2+f_s^2}{\Delta\eta^2}\frac{l^2}{3}-\frac{2f_0f_s}{\Delta\eta^2}\left(2\cos l+(-2+l^2)\frac{\sin l}{l}\right)\right]$	$\left(\frac{f_0-f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(\frac{f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{9}$
$\langle A_4 A_5\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[\frac{f_0+3f_s}{\Delta\eta}\cos l+\frac{f_0-f_s}{\Delta\eta}\frac{\sin l}{l}+\frac{(f_0+f_s)(-2+l\,\mathrm{SinInt}(l))}{\Delta\eta}\right]$	$\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\langle A_5 A_5\rangle$	$\frac{1}{15l^2}\left[-24+20l^2+(24-2l^2+l^4)\cos(l)+l(-6+l^2)\sin(l)+l^5\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0

A_i	$\mathcal{C}_i(k,\eta_0,\eta_s)$
A_1	$\frac{\sin l}{l}$
A_2	$-\frac{2}{l}\mathrm{SinInt}(l)$
A_3	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left(1-\frac{\sin l}{l}\right)$
A_4	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_s}{\Delta\eta}\left(\cos l-\frac{\sin l}{l}\right)$
A_5	0

$$l = k\Delta\eta$$

$$f_{0,s} \approx \frac{\eta_{0,s}}{3}$$

Power Spectrum

- We use the WMAP7 best fit value and the transfer function of Eisenstein & Hu 1997 for CDM.
- We are interested in the overall magnitude so we neglect the baryonic oscillations.

$$P_\Psi(k) \equiv \frac{k^3}{2\pi^2}|\Psi_k|^2 = \left(\frac{3}{5}\right)^2 \Delta_R^2 T(k)^2, \quad \Delta_R^2 = A\left(\frac{k}{k_0}\right)^{n_s-1}$$

- Only subhorizon fluc. $H_0<k$. Superhorizon fluc. are subdominant.
- No UV ($k \rightarrow \infty$) or IR ($z \rightarrow 0$, $k \rightarrow 0$) divergences.

$$\mathrm{IBR}_2 = \int_0^\infty \frac{dk}{k}\, P_\Psi(k) \sum_{i=1}^{4}\sum_{j=1}^{5} 2\left[\mathcal{C}_{ij}(k,\eta_0,\eta_s) - \mathcal{C}_i(k,\eta_0,\eta_s)\ \mathcal{C}_j(k,\eta_0,\eta_s)\right]$$

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$\langle A_1A_1\rangle$	1	1	$1-\frac{l^2}{3}$
$\langle A_1A_2\rangle$	$-\frac{2}{l}\mathrm{SinInt}(l)$	$-2+\frac{l^2}{9}$	$-2+\frac{4}{9}l^2$
$\langle A_1A_3\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{6}$
$\langle A_1A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$-\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\langle A_1A_5\rangle$	$-2\left[1-\frac{\sin(l)}{l}\right]$	$-\frac{l^2}{3}$	0
$\langle A_2A_2\rangle$	$\frac{8}{l^2}\left[-1+\cos l+l\,\mathrm{SinInt}(l)\right]$	$4-\frac{l^2}{9}$	$4-\frac{4}{9}l^2$
$\langle A_2A_3\rangle$	0	0	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$
$\langle A_2A_4\rangle$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_0+f_s}{\Delta\eta}\left[1-\frac{\sin(l)}{l}\right]$	$\frac{f_0+f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{2}{3}l^2$
$\langle A_2A_5\rangle$	$\frac{2}{3l^2}\left[-4+(4+l^2)\cos l+l\sin l+l^3\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0
$\langle A_3A_3\rangle$	$2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[1-\frac{\sin(l)}{l}\right]$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{36}$
$\langle A_3A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{f_0-f_s}{\Delta\eta}\left[\cos l-\frac{\sin(l)}{l}\right]$	$-\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\frac{f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{18}$
$\langle A_3A_5\rangle$	$-2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[1-\frac{\sin(l)}{l}\right]$	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\langle A_4A_4\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\left[\frac{f_0^2+f_s^2}{\Delta\eta^2}\frac{l^2}{3}-\frac{2f_0f_s}{\Delta\eta^2}\left(2\cos l+(-2+l^2)\frac{\sin l}{l}\right)\right]$	$\left(\frac{f_0-f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^2}{3}$	$\left(\frac{f_s}{\Delta\eta}\right)^2\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)^2\frac{l^4}{9}$
$\langle A_4A_5\rangle$	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left[\frac{f_0+3f_s}{\Delta\eta}\cos l+\frac{f_0-f_s}{\Delta\eta}\frac{\sin l}{l}+\frac{(f_0+f_s)(-2+l\,\mathrm{SinInt}(l))}{\Delta\eta}\right]$	$\frac{f_0-f_s}{\Delta\eta}\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{l^2}{3}$	0
$\langle A_5A_5\rangle$	$\frac{1}{15l^2}\left[-24+20l^2+(24-2l^2+l^4)\cos(l)+l(-6+l^2)\sin(l)+l^5\mathrm{SinInt}(l)\right]$	$\frac{l^2}{3}$	0

A_i	$\mathcal{C}_i(k,\eta_0,\eta_s)$
A_1	$\frac{\sin l}{l}$
A_2	$-\frac{2}{l}\mathrm{SinInt}(l)$
A_3	$-\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\left(1-\frac{\sin l}{l}\right)$
A_4	$\left(1-\frac{1}{\mathcal{H}_s\Delta\eta}\right)\frac{f_s}{\Delta\eta}\left(\cos l-\frac{\sin l}{l}\right)$
A_5	0

$$l = k\Delta\eta$$

$$f_{0,s} \approx \frac{\eta_{0,s}}{3}$$

Interpretation & Analysis

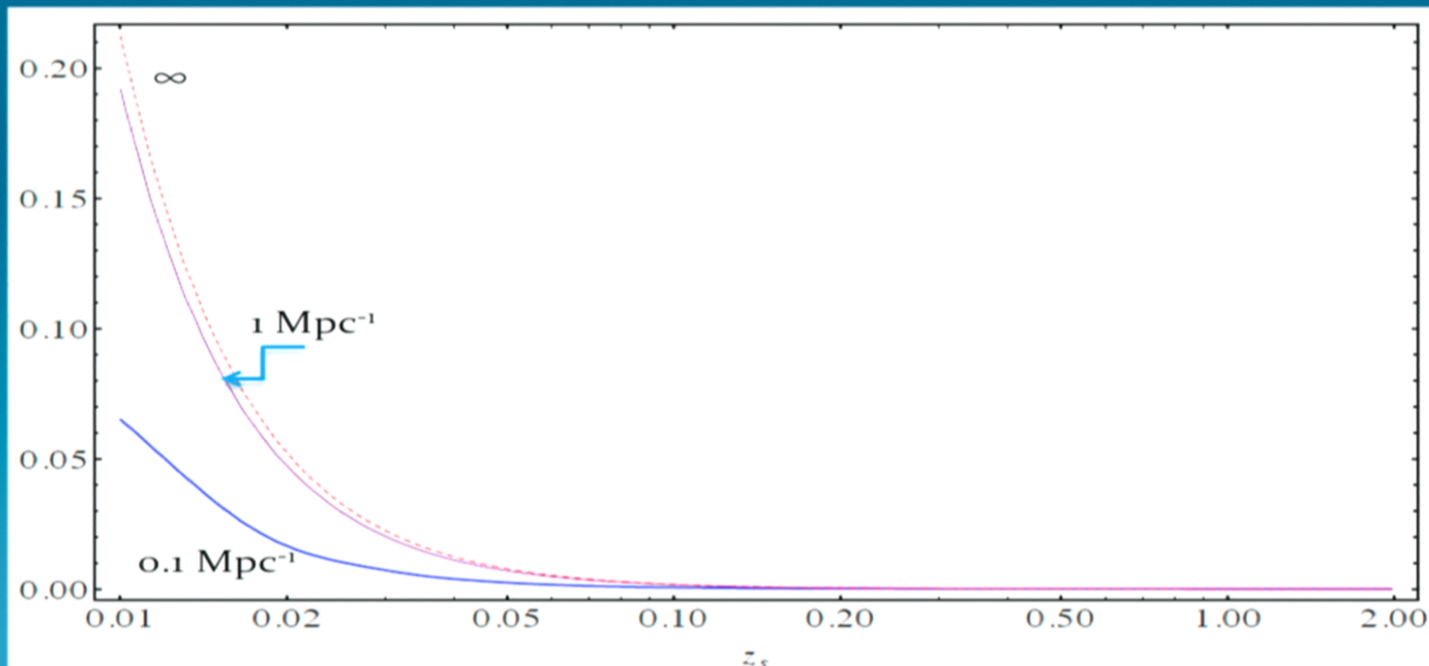
- d_L is a stochastic observable – mean, dispersion, skewness…
- z<<1, autocorrelation of the "velocity" term, z~1 autocorrelation lensing term, which at current analysis exists only in the dispersion.
- At small enough scales, the transfer function wins.
- At intermediate scales, the phase space factor competes successfully with the initial small amplitude

$$T^2(k << k_{eq}) \sim 1$$

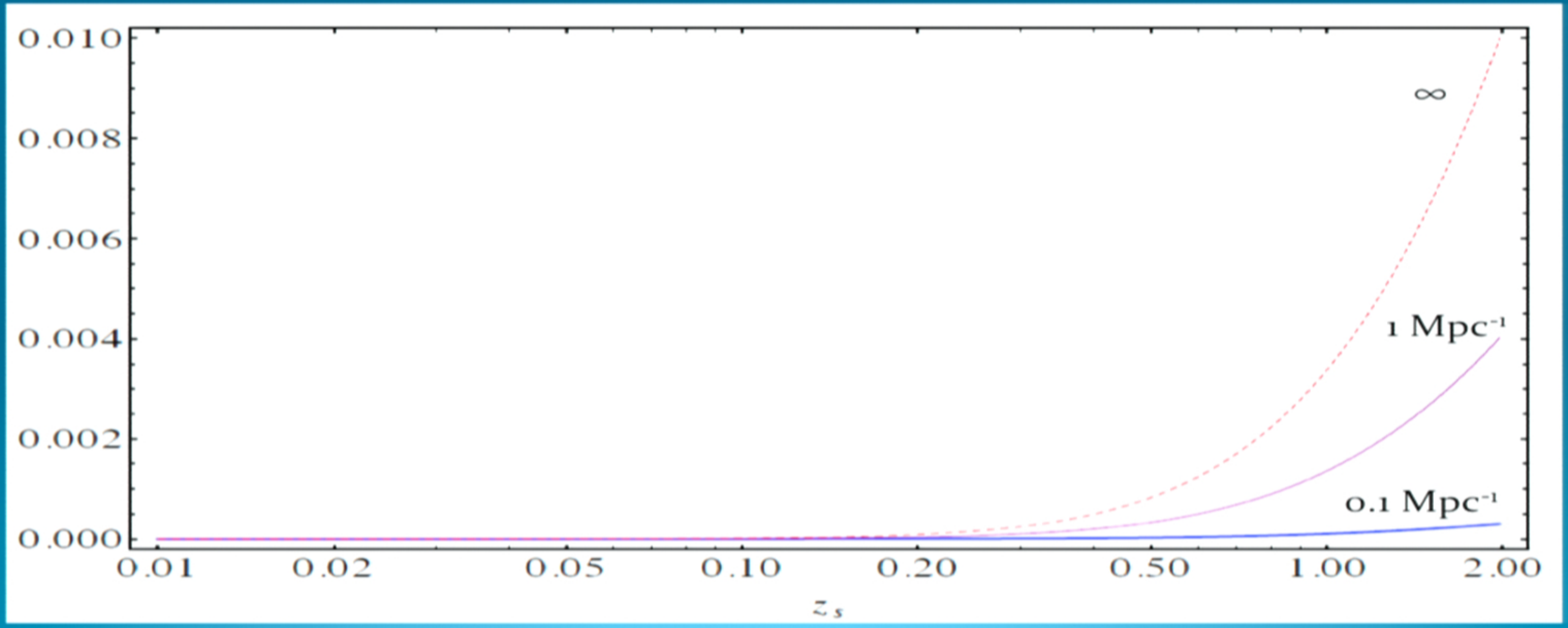
$$T^2(k >> k_{eq}) \sim \ln^2 k / k^4$$

$$Integrand \sim A \times T^2 \times \left(\frac{k}{H_0}\right)^p$$

"Velocity^2" term



Lensing² Term

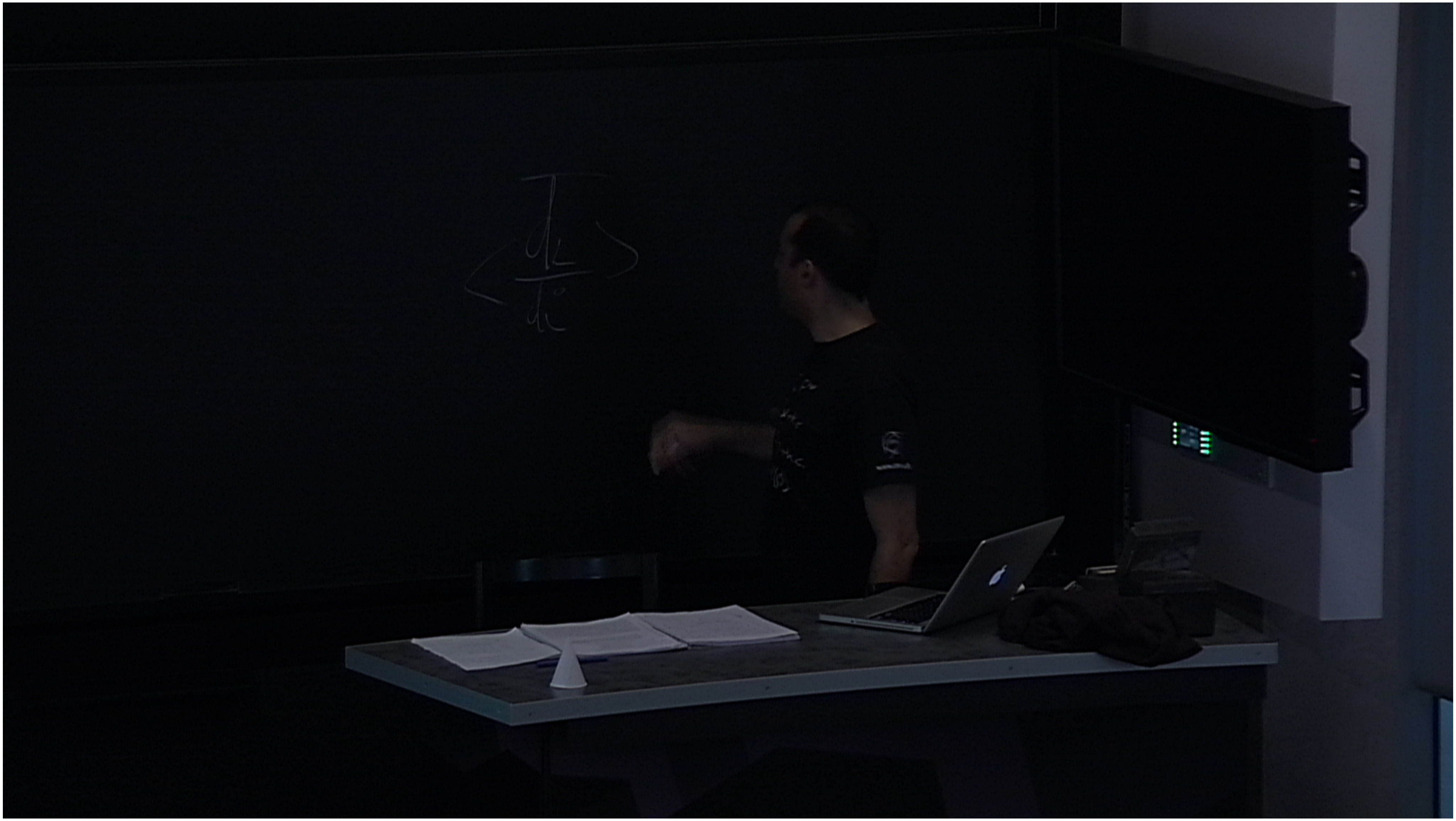


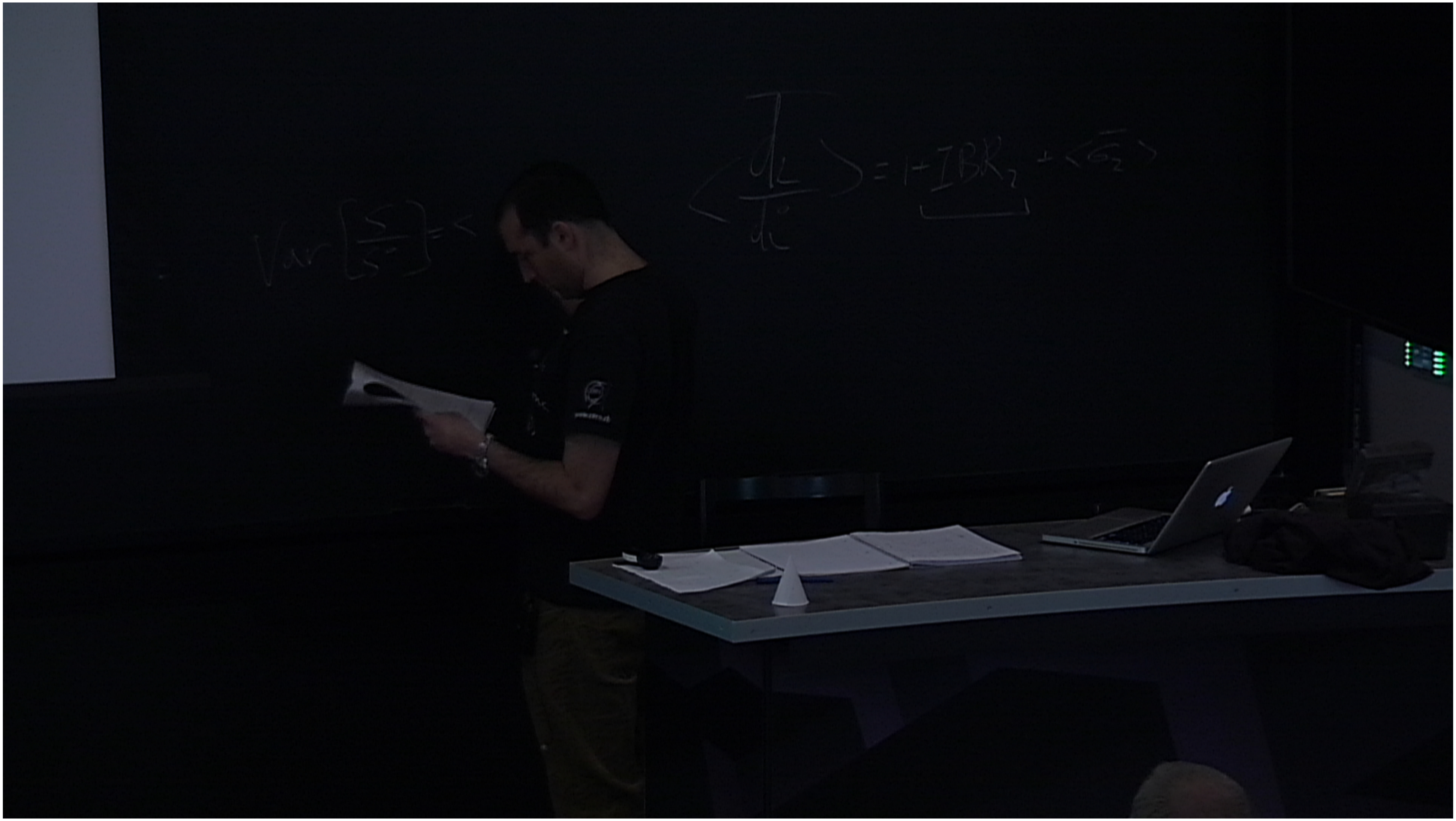
Results

- From z=0.03 IBR_2 term gives a contribution of 1 part in 10000 to the mean of d_L

<u>However</u> it is still 6 orders of magnitude bigger than the naive expectation.

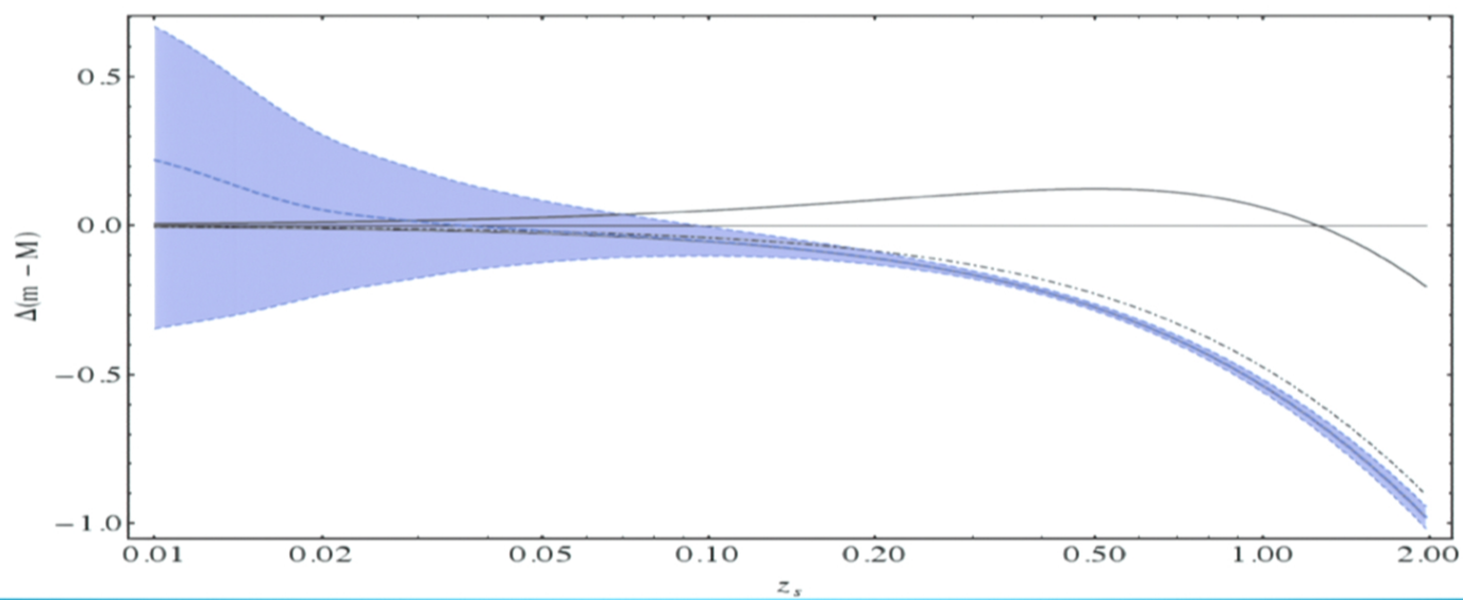
- The dispersion is very large~ 10% of the critical density.
- The lensing term squared also appears in the 2^{nd} order term of the mean, The term exists independently of the averaging prescription. Barring cancellations, this term will dominate over the IBR_2 contribution $\overline{\langle \sigma_2 \rangle}$?



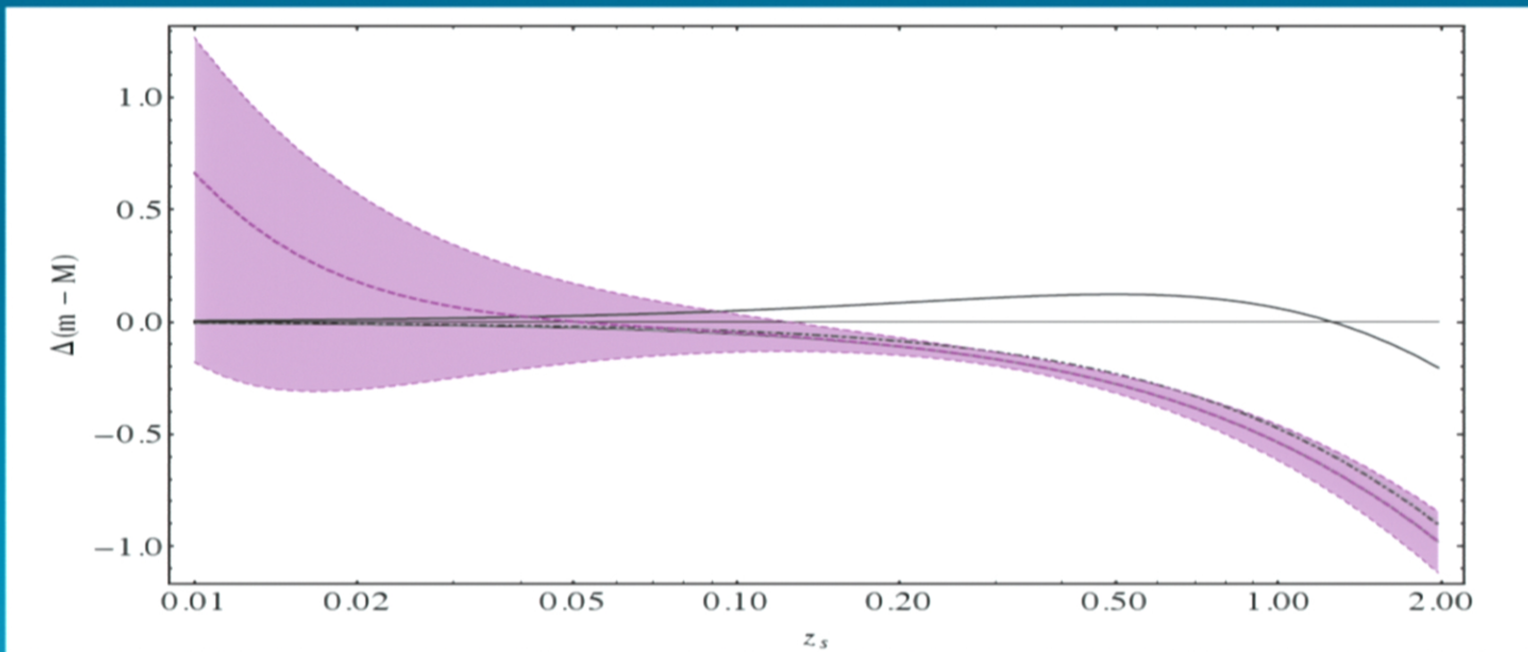


k_{max}=0.1 Mpc^{-1}

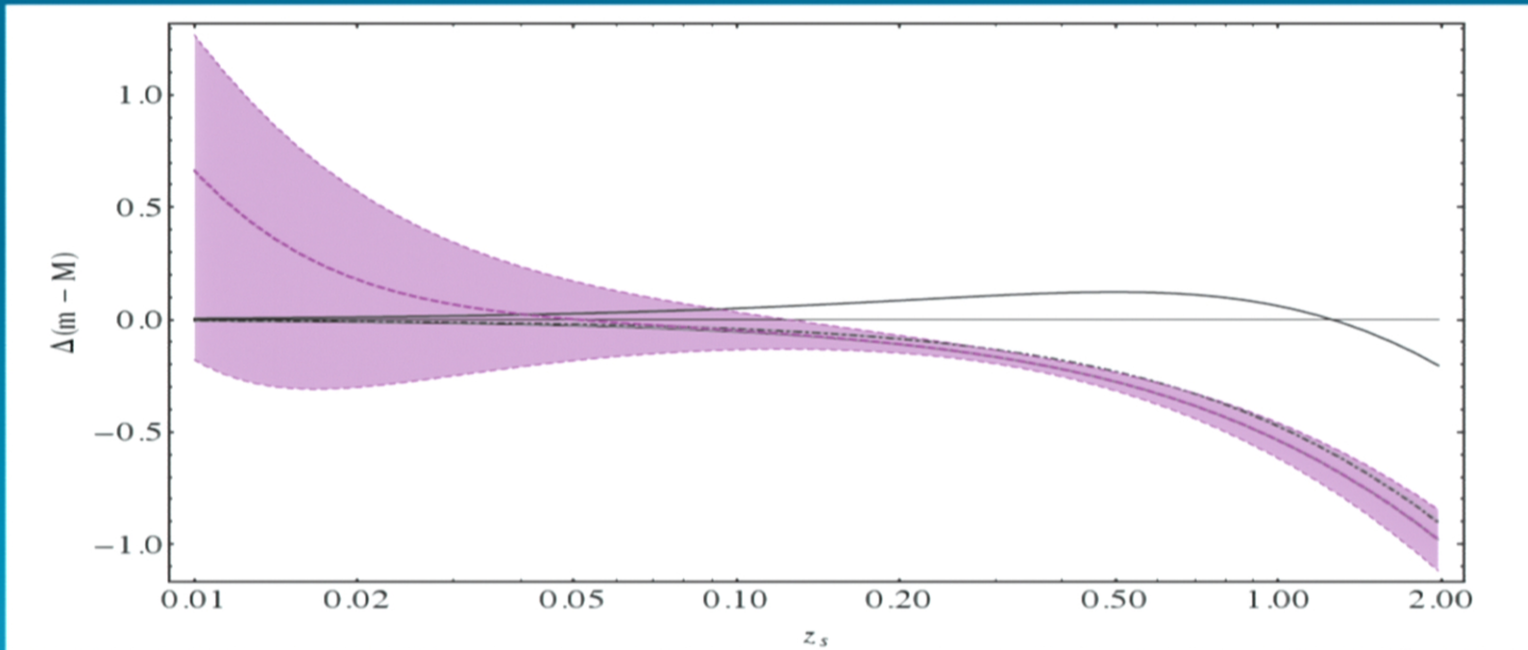
$$\Delta(m - M) = 5\log_{10}\left[\overline{ld_L\rangle}\right] - 5\log_{10}\left[\frac{(2+z_s)z_s}{2H_0}\right]$$

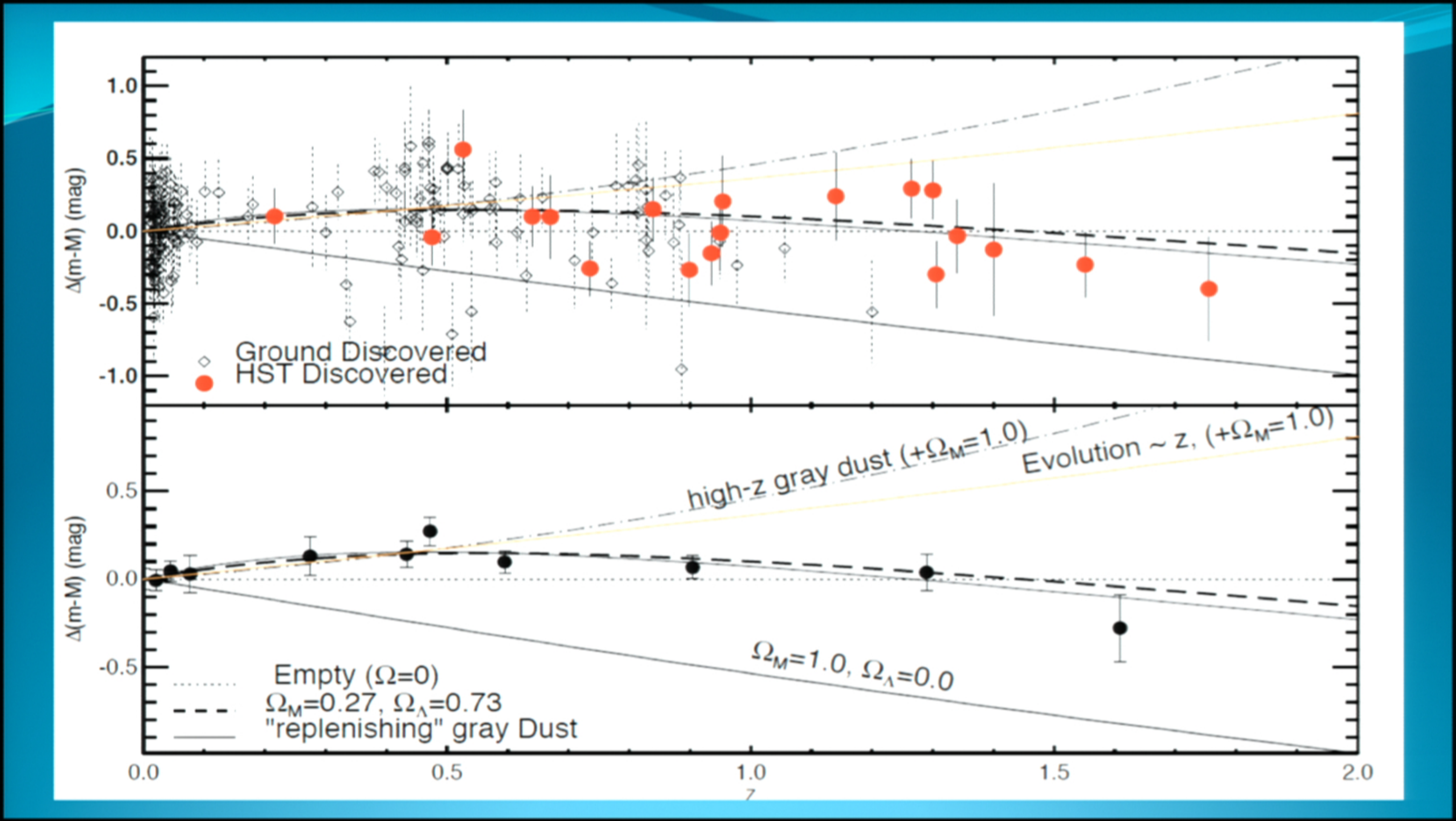


k_{max}=1 Mpc^{-1}



k_{max}=1 Mpc^{-1}





Ground Discovered
HST Discovered
Δ(m-M) (mag)
high-z gray dust (+Ω_M=1.0)
Evolution ~ z, (+Ω_M=1.0)
Ω_M=1.0, Ω_Λ=0.0
Empty (Ω=0)
Ω_M=0.27, Ω_Λ=0.73
"replenishing" gray Dust
z

Summary

- We applied the light cone averaging formalism to the d_L-z relation.
- We analyzed in full the contributions coming from correlations between the luminosity distance and the covariant integration measure.
- We derived to leading order the variance of the luminosity distance.
- We identified significant terms in 2^{nd} order pert. theory which deviate from pure FLRW more than the correlated terms.

Summary

- We applied the light cone averaging formalism to the d_L-z relation.
- We analyzed in full the contributions coming from correlations between the luminosity distance and the covariant integration measure.
- We derived to leading order the variance of the luminosity distance.
- We identified significant terms in 2^{nd} order pert. theory which deviate from pure FLRW more than the correlated terms.

Open Issues/Future Prospects

1. Repeating the calculation for LambdaCDM.
2. Magnitude of the full 2nd order calculation $\overline{\langle\sigma_2\rangle}$?

$$(\overline{\langle d_L^{-2}\rangle})^{-1/2} = d_L^{FLRW}\left(1+\overline{\langle\sigma_2\rangle}+IBR_2-\frac{3}{2}\overline{\langle\sigma_1^2\rangle}\right) \; ; \qquad \overline{\langle d_L\rangle} = d_L^{FLRW}\left(1+\overline{\langle\sigma_2\rangle}+IBR_2\right).$$

3. Better estimates of the non-linear regime, especially 0.1 $Mpc^{-1}<k<1\ Mpc^{-1}$.
4. Better understanding of the (re)normalization of H_o

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5. Cosmological parameter analysis.
6. Matching the effect to other probes: CMB, LSS
7. Other applications, averaging of EFE ....Many open theoretical and pheno. problems.

Conclusions

- GR + Standard Pert. Theory + Averaging challenge the concordance model.
- Perturbations cannot be discarded as negligible. Any explanation of the cosmic acceleration will have to take them into account.

BACKREACTION IS HERE TO STAY!