Title: Mathematica and Symbolic Computations

Date: Feb 09, 2012 04:10 PM

URL: http://pirsa.org/12020150

Abstract:

# Daily use of Mathematica

**An example based on the study of three point functions in large N gauge theories**

[w/ J.Escobedo, N.Gromov and A.Sever]

## Checking a Conjecture

## Generating and Visualizing Data

## Using the Data

## Experiment vs Theory

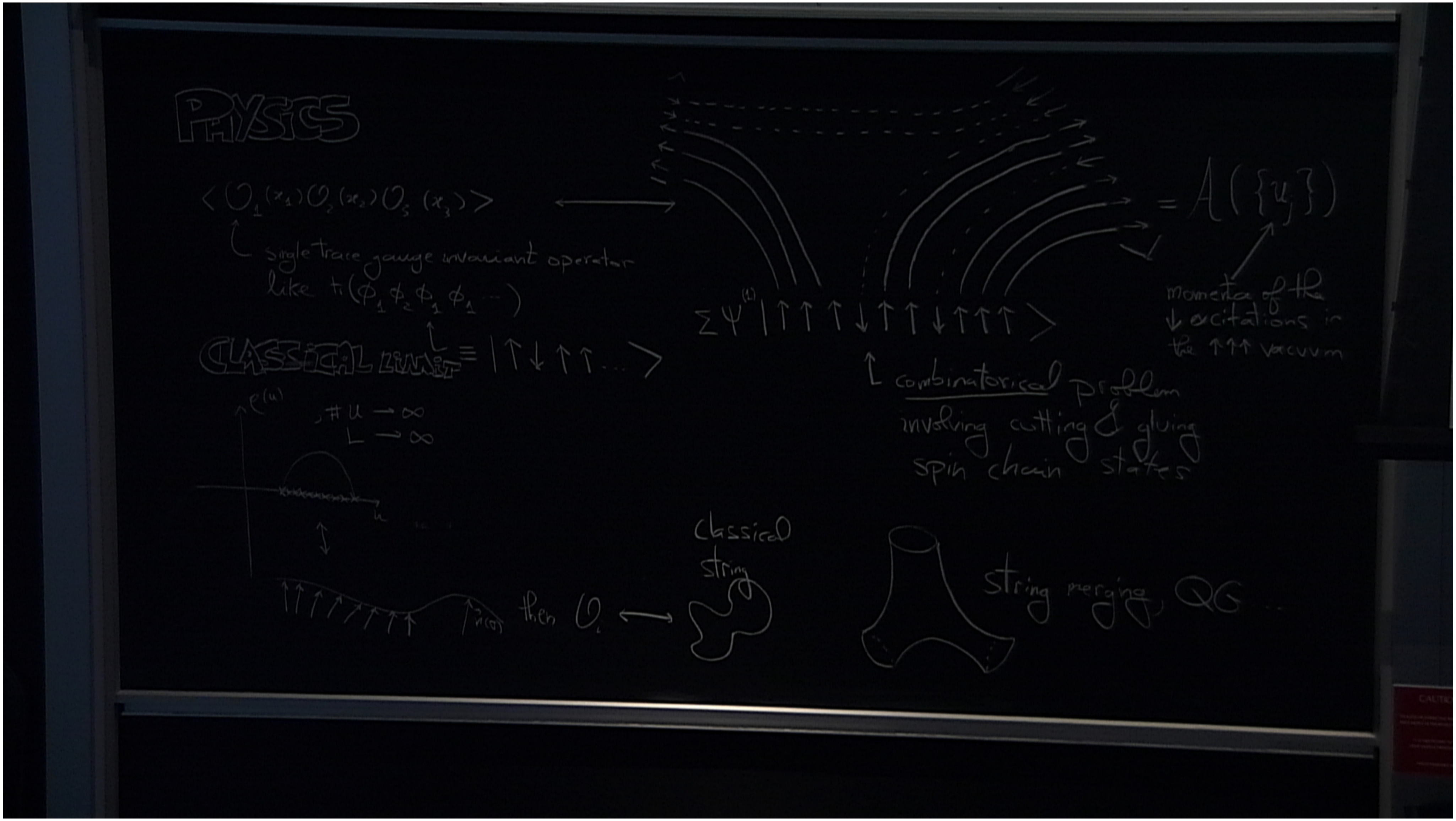
## Power of Determinants

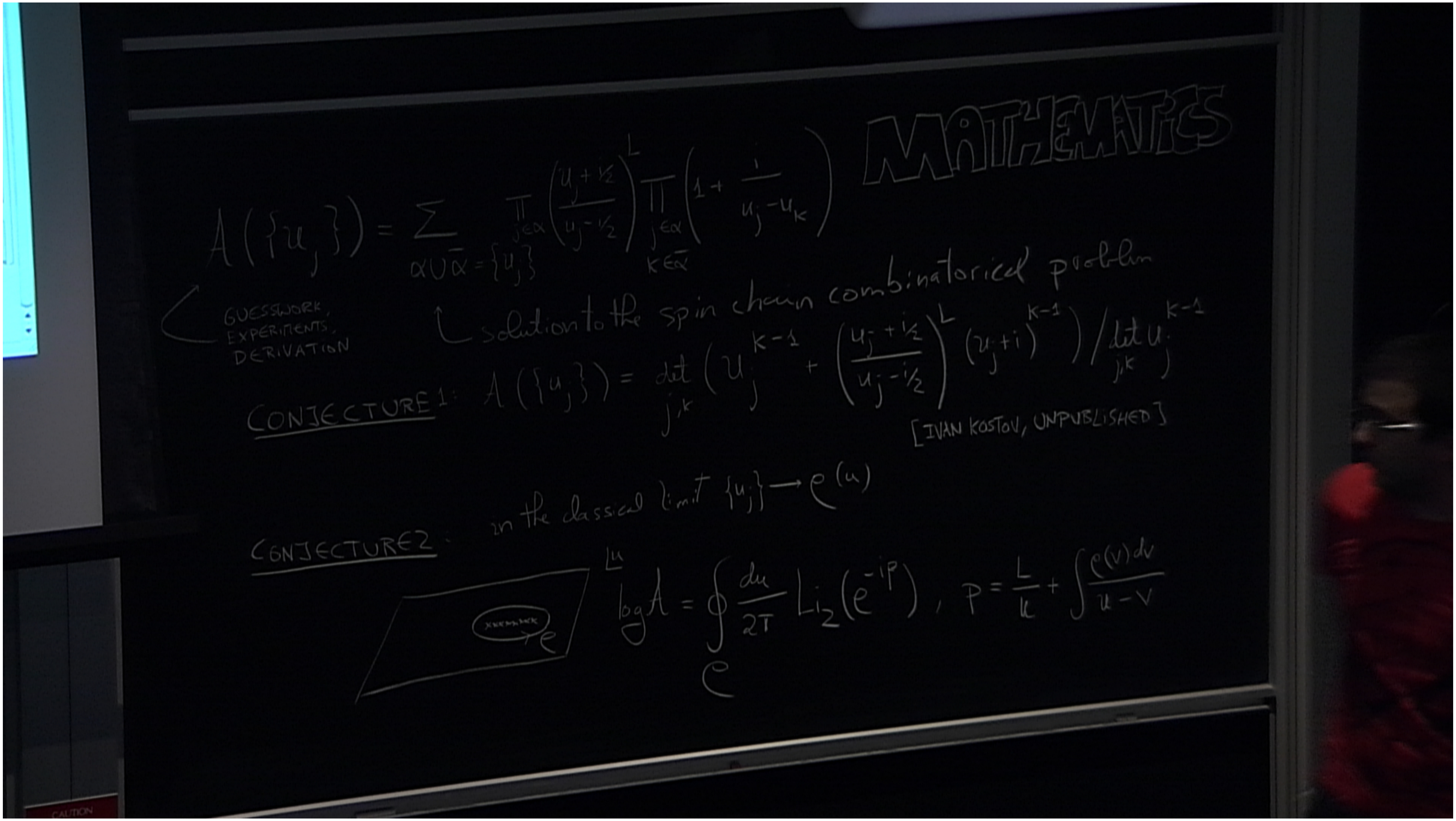
## What we could be doing better

## In the paper

## Yesterday

## What else we typically do


PHYSICS
single trace gauge invariant operator
like tr(φ1 φ2 φ1 φ1 ...)
CLASSICAL LIMIT
#u → ∞
L → ∞
combinatorical problem
involving cutting & gluing
spin chain states
momenta of the
↓ excitations in
the ↑↑↑ vacuum
then
classical
string
string merging, QG ...


MATHEMATICS
$A(\{u_j\}) = \sum_{\alpha \cup \bar{\alpha} = \{u_j\}} \prod_{j \in \alpha} \left(\frac{u_j + i/2}{u_j - i/2}\right)^L \prod_{j \in \alpha, k \in \bar{\alpha}} \left(1 + \frac{i}{u_j - u_k}\right)$
GUESSWORK, EXPERIMENTS, DERIVATION
solution to the spin chain combinatorical problem
CONJECTURE 1: $A(\{u_j\}) = \det_{j,k}\left(u_j^{k-1} + \left(\frac{u_j + i/2}{u_j - i/2}\right)^L (u_j + i)^{k-1}\right) / \det_{j,k} u_j^{k-1}$
[IVAN KOSTOV, UNPUBLISHED]
CONJECTURE 2: in the classical limit $\{u_j\} \to \rho(u)$
$\log A = \oint_{\mathcal{C}} \frac{du}{2\pi} \mathrm{Li}_2(e^{-ip})$, $p = \frac{L}{u} + \int \frac{\rho(v)\,dv}{u - v}$

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example.nb

# Daily use of Mathematica

**An example based on the study of three point functions in large N gauge theories**

[w/ J.Escobedo, N.Gromov and A.Sever]

## Checking a Conjecture

In[1]:=

$$\texttt{A}[L\_, M\_] := \sum_{\alpha}^{\texttt{\{Complement[Range[}M\texttt{],\#],\#\}\&/@Subsets[Range[}M\texttt{],\{0,}M\texttt{\}]}} \left(\prod_{j}^{\alpha[[1]]} \left(\frac{u[j]+\frac{i}{2}}{u[j]-\frac{i}{2}}\right)^{L}\right) \prod_{j}^{\alpha[[1]]} \prod_{k}^{\alpha[[2]]} \left(1+\frac{i}{u[j]-u[k]}\right);$$

In[2]:=

$$\texttt{det}[L\_, M\_] := \frac{\texttt{Det@Table}\left[u[j]^{k-1} + \left(\frac{u[j]+i/2}{u[j]-i/2}\right)^{L} (u[j]+i)^{k-1}, \{j, M\}, \{k, M\}\right]}{\texttt{Det@Table}\left[u[j]^{k-1}, \{j, M\}, \{k, M\}\right]};$$

In[3]:=

```
det[L, 4] / A[L, 4] // Simplify
```

## Generating and Visualizing Data

## Using the Data

## Experiment vs Theory

## Power of Determinants

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example.nb

## An example based on the study of three point functions in large N gauge theories

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# Checking a Conjecture

In[1]:=

$$\texttt{A[L\_, M\_] :=} \sum_{\alpha}^{\texttt{\{Complement[Range[M],\#],\#\}\&/@Subsets[Range[M],\{0,M\}]}} \left(\prod_{j}^{\alpha[[1]]}\left(\frac{u[j]+\frac{i}{2}}{u[j]-\frac{i}{2}}\right)^{L}\right)\prod_{j}^{\alpha[[1]]}\prod_{k}^{\alpha[[2]]}\left(1+\frac{i}{u[j]-u[k]}\right);$$

In[2]:=

$$\texttt{det[L\_, M\_] :=} \frac{\texttt{Det@Table}\left[u[j]^{k-1}+\left(\frac{u[j]+i/2}{u[j]-i/2}\right)^{L}(u[j]+i)^{k-1},\ \{j, M\},\ \{k, M\}\right]}{\texttt{Det@Table}\left[u[j]^{k-1},\ \{j, M\},\ \{k, M\}\right]};$$

In[3]:=

```
det[L, 4] / A[L, 4] // Simplify
```

# Generating and Visualizing Data

# Using the Data

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example.nb

In[1]:=

$$A[L\_,\ M\_] := \sum_{\alpha}^{\{\text{Complement}[\text{Range}[M],\#],\#\}\&/@\text{Subsets}[\text{Range}[M],\{0,M\}]} \left(\prod_{j}^{\alpha[[1]]}\left(\frac{u[j]+\frac{i}{2}}{u[j]-\frac{i}{2}}\right)^{L}\right)\prod_{j}^{\alpha[[1]]}\prod_{k}^{\alpha[[2]]}\left(1+\frac{i}{u[j]-u[k]}\right);$$

In[2]:=

A[4, 4]

Out[2]=

$$1+\frac{\left(\frac{i}{2}+u[1]\right)^4\left(1+\frac{i}{u[1]-u[2]}\right)\left(1+\frac{i}{u[1]-u[3]}\right)\left(1+\frac{i}{u[1]-u[4]}\right)}{\left(-\frac{i}{2}+u[1]\right)^4}+$$

$$\frac{\left(\frac{i}{2}+u[2]\right)^4\left(1+\frac{i}{-u[1]+u[2]}\right)\left(1+\frac{i}{u[2]-u[3]}\right)\left(1+\frac{i}{u[2]-u[4]}\right)}{\left(-\frac{i}{2}+u[2]\right)^4}+$$

$$\frac{\left(\frac{i}{2}+u[1]\right)^4\left(\frac{i}{2}+u[2]\right)^4\left(1+\frac{i}{u[1]-u[3]}\right)\left(1+\frac{i}{u[2]-u[3]}\right)\left(1+\frac{i}{u[1]-u[4]}\right)\left(1+\frac{i}{u[2]-u[4]}\right)}{\left(-\frac{i}{2}+u[1]\right)^4\left(-\frac{i}{2}+u[2]\right)^4}+$$

$$\frac{\left(\frac{i}{2}+u[3]\right)^4\left(1+\frac{i}{-u[1]+u[3]}\right)\left(1+\frac{i}{-u[2]+u[3]}\right)\left(1+\frac{i}{u[3]-u[4]}\right)}{\left(-\frac{i}{2}+u[3]\right)^4}+$$

$$\frac{\left(\frac{i}{2}+u[1]\right)^4\left(1+\frac{i}{u[1]-u[2]}\right)\left(\frac{i}{2}+u[3]\right)^4\left(1+\frac{i}{-u[2]+u[3]}\right)\left(1+\frac{i}{u[1]-u[4]}\right)\left(1+\frac{i}{u[3]-u[4]}\right)}{\left(-\frac{i}{2}+u[1]\right)^4\left(-\frac{i}{2}+u[3]\right)^4}+$$

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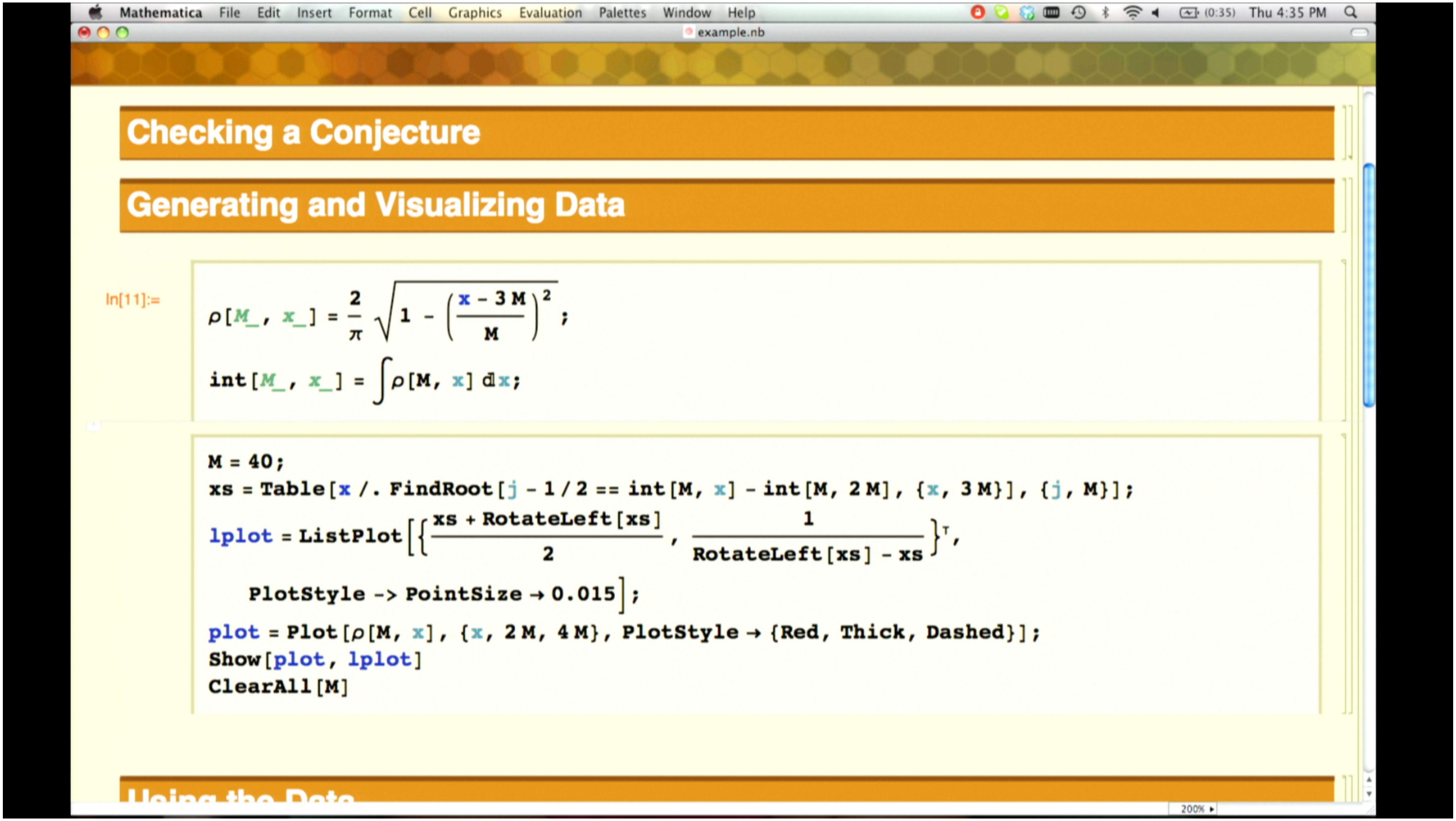
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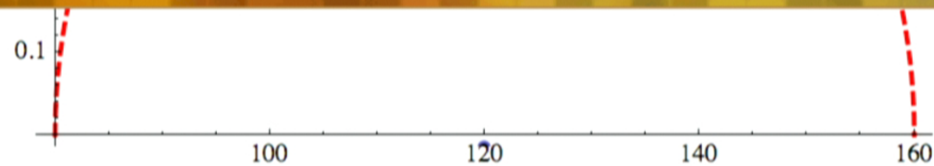
Thu 4:35 PM
example.nb
Checking a Conjecture
Generating and Visualizing Data
In[11]:=
ρ[M_, x_] = 2/π Sqrt[1 - ((x - 3 M)/M)^2];
int[M_, x_] = ∫ρ[M, x] dx;
M = 40;
xs = Table[x /. FindRoot[j - 1 / 2 == int[M, x] - int[M, 2 M], {x, 3 M}], {j, M}];
lplot = ListPlot[{(xs + RotateLeft[xs])/2, 1/(RotateLeft[xs] - xs)}^T,
PlotStyle -> PointSize → 0.015];
plot = Plot[ρ[M, x], {x, 2 M, 4 M}, PlotStyle → {Red, Thick, Dashed}];
Show[plot, lplot]
ClearAll[M]
Using the Data
200%



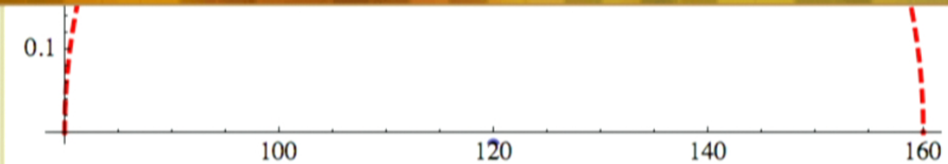

## Using the Data

In[60]:=
```
do[L_, M_] :=
 Block[
  {us =
    Table[x /. FindRoot[j - 1/2 == int[M, x] - int[M, 2 M], {x, 3 M},
       WorkingPrecision → 30], {j, M}], u}, u[k_] := us[[k]]; Re@Log@A[L, M]]
```

In[61]:=
```
tb = Monitor[Table[{M, do[3 M/2, M]/M}, {M, 4, 14}], M];
```

In[62]:=
```
num = Fit[tb, {1, 1/M, 1/M^2, 1/M^3, 1/M^4}, M] /. M → ∞
```

## Experiment vs Theory




## Using the Data

In[21]:=

```
do[L_, M_] :=
 Block[
  {us =
    Table[x /. FindRoot[j - 1 / 2 == int[M, x] - int[M, 2 M], {x, 3 M},
       WorkingPrecision → 30], {j, M}], u}, u[k_] := us[[k]]; Re@Log@A[L, M]]
```

In[22]:=

```
tb = Monitor[Table[{M, do[3 M / 2, M] / M}, {M, 4, 14}], M];
```

13

In[62]:=

```
num = Fit[tb, {1, 1 / M, 1/M^2, 1/M^3, 1/M^4}, M] /. M → ∞
```

example.nb

```
In[21]:=
do[L_, M_] :=
 Block[
  {us =
    Table[x /. FindRoot[j - 1/2 == int[M, x] - int[M, 2 M], {x, 3 M},
       WorkingPrecision → 30], {j, M}], u}, u[k_] := us[[k]]; Re@Log@A[L, M]]

In[22]:=
tb = Monitor[Table[{M, do[3 M/2, M]/M}, {M, 4, 14}], M];

In[24]:=
tb // ListPlot

Out[24]=
```

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example.nb

```
an
an - num2
```

Out[41]=

0.68134639069071366264715356252053784692910210477200672136028428275

Out[42]=

0.681346393668330356081565129129

Out[43]=

$2.977616693434411566608 \times 10^{-9}$

## What we could be doing better

Several densities, M' s, L' s etc in different computers

## In the paper

## Yesterday

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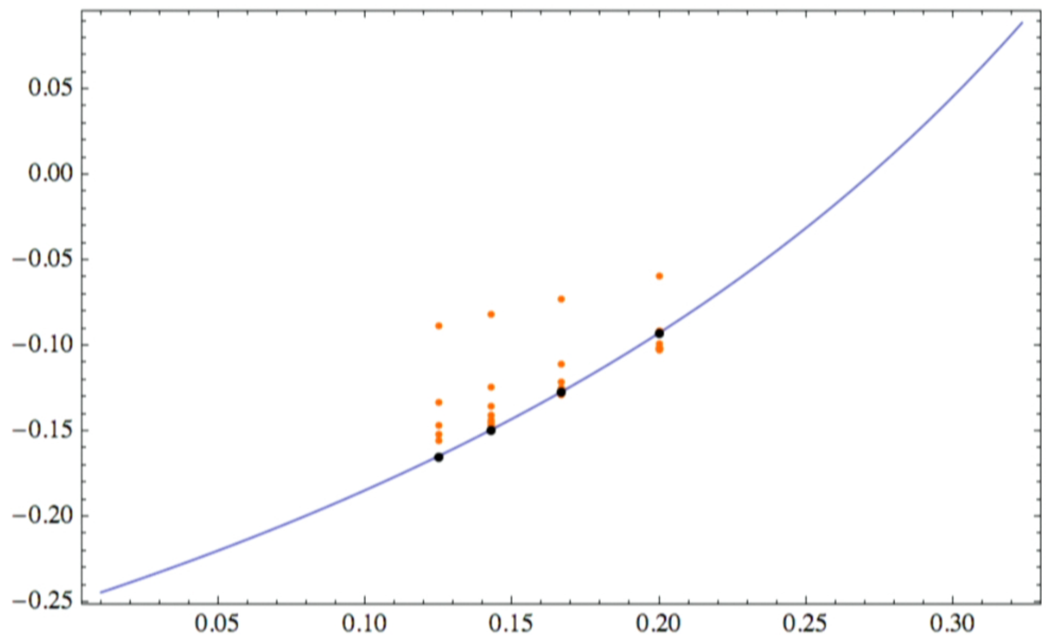
# Yesterday

Took a few hours on Kolya's laptop

Out[206]=




**Weak/strong coupling match in AdS/CFT**

**Weak/strong coupling match in AdS/CFT**

## What else we typically do

1. TBA stuff
2. PDE's
3. //Simplify , //Series , //Integrate
4. Wick Contractions and other mechanical QFT stuff
5. Recursion Relations and Component Extraction for Amplitudes
6. Pattern recognition
7. Algebra manipulation (Transfer matrices stuff e.g.)

Typical times involved: 5 seconds - 40 hours.

**Factor of 10 improvement would make a huge difference, specially for the $\tau \sim$ coffee time computations.**

**Weak/strong coupling match in AdS/CFT**

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