Title: Feature-finding in the cosmic microwave background

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Abstract:

Feature finding in the CMB

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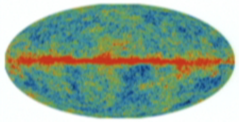
Perimeter Institute

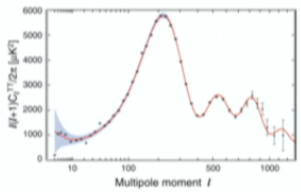
In collaboration with:

Stephen Feeney
Daniel Mortlock
Hiranya Peiris

The data

- Lambda-CDM: very successful at describing the CMB power spectrum.





WMAP 7-year data

Is there room for anything else?

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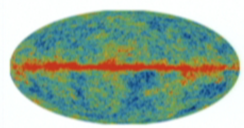

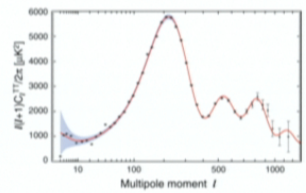

WMAP 7-year data

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Theories with features

- A number of theories predict specific features in the CMB on top of those predicted by LCDM:

$$\frac{\Delta T(\hat{\mathbf{n}})}{T} = (1 + f(\hat{\mathbf{n}}))\,(1 + \delta_{\rm LCDM}(\hat{\mathbf{n}})) - 1$$
$$\simeq f(\hat{\mathbf{n}}) + \delta_{\rm LCDM}(\hat{\mathbf{n}})$$

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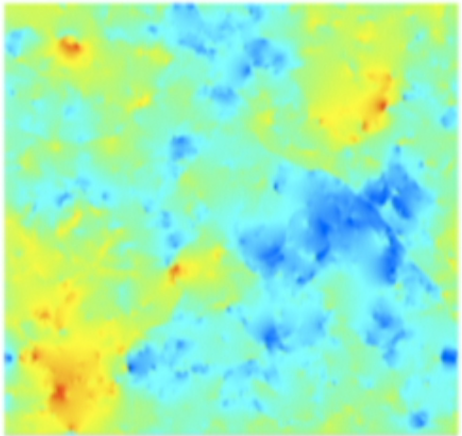
- Topological defects: cosmic strings



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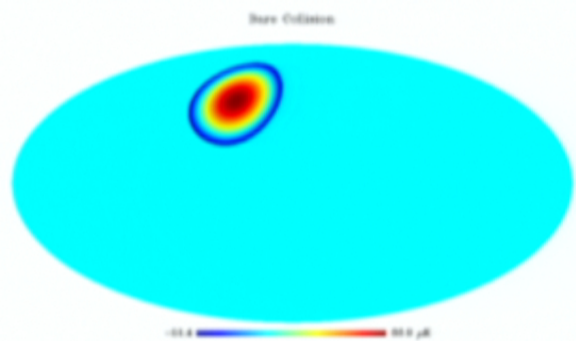
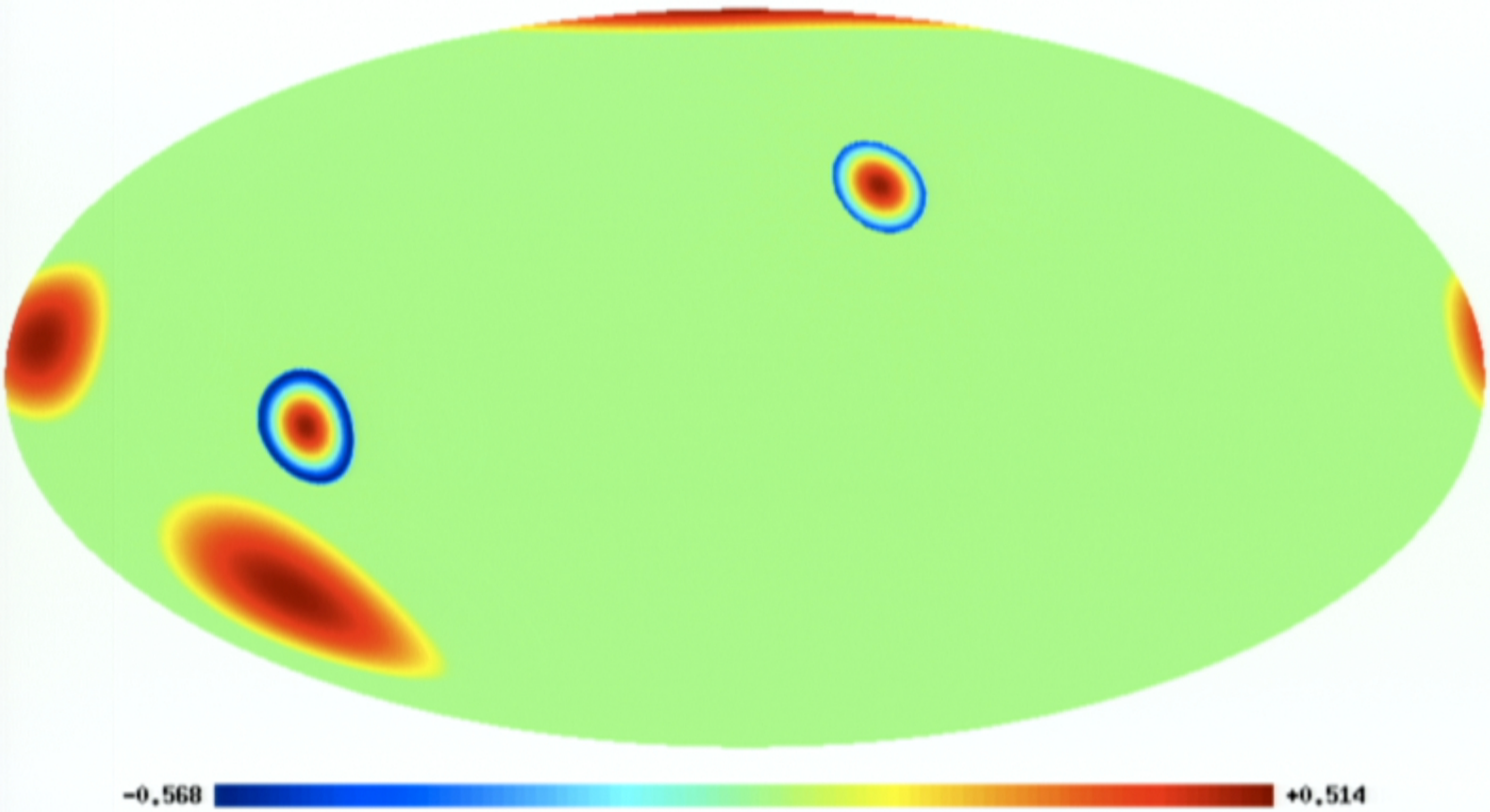
- Topological defects: cosmic strings and cosmic textures.
- Cosmic bubble collisions: observational signature of eternal inflation.



Features

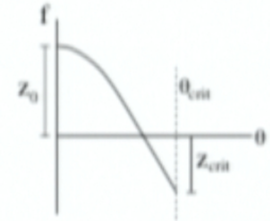


A framework for searching for features

- **A theory that predicts features:**

Template with free parameters $\mathbf{m}$

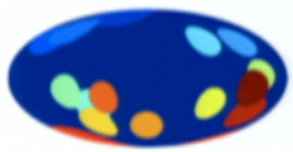
size, location, amplitude....



PDF for the number of features and template parameters

$$\Pr(\bar{\mathrm{N}}_s, \mathbf{m})$$

How many features of each type do I expect to see?



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$$\Pr(\boldsymbol{d}|N_s) = \int \mathrm{d}\boldsymbol{m}_1\, \mathrm{d}\boldsymbol{m}_2 \ldots \mathrm{d}\boldsymbol{m}_{N_s} \Pr(\boldsymbol{m}_1, \boldsymbol{m}_2, \ldots \boldsymbol{m}_{N_s})\Pr(\boldsymbol{d}|N_s, \boldsymbol{m}_1, \boldsymbol{m}_2, \ldots \boldsymbol{m}_{N_s})$$

We've marginalized over template parameters

$$\boldsymbol{m}_i = \{\theta_0^{(i)}, \phi_0^{(i)}, \theta_{\mathrm{crit}}^{(i)}, z_0^{(i)}, z_{\mathrm{crit}}^{(i)}\}$$

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C	covariance matrix

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$$\mathsf{C}_{ij} = \sum_{\ell} \frac{2\ell+1}{4\pi} C_\ell \, P_\ell(\cos\theta_{ij})$$

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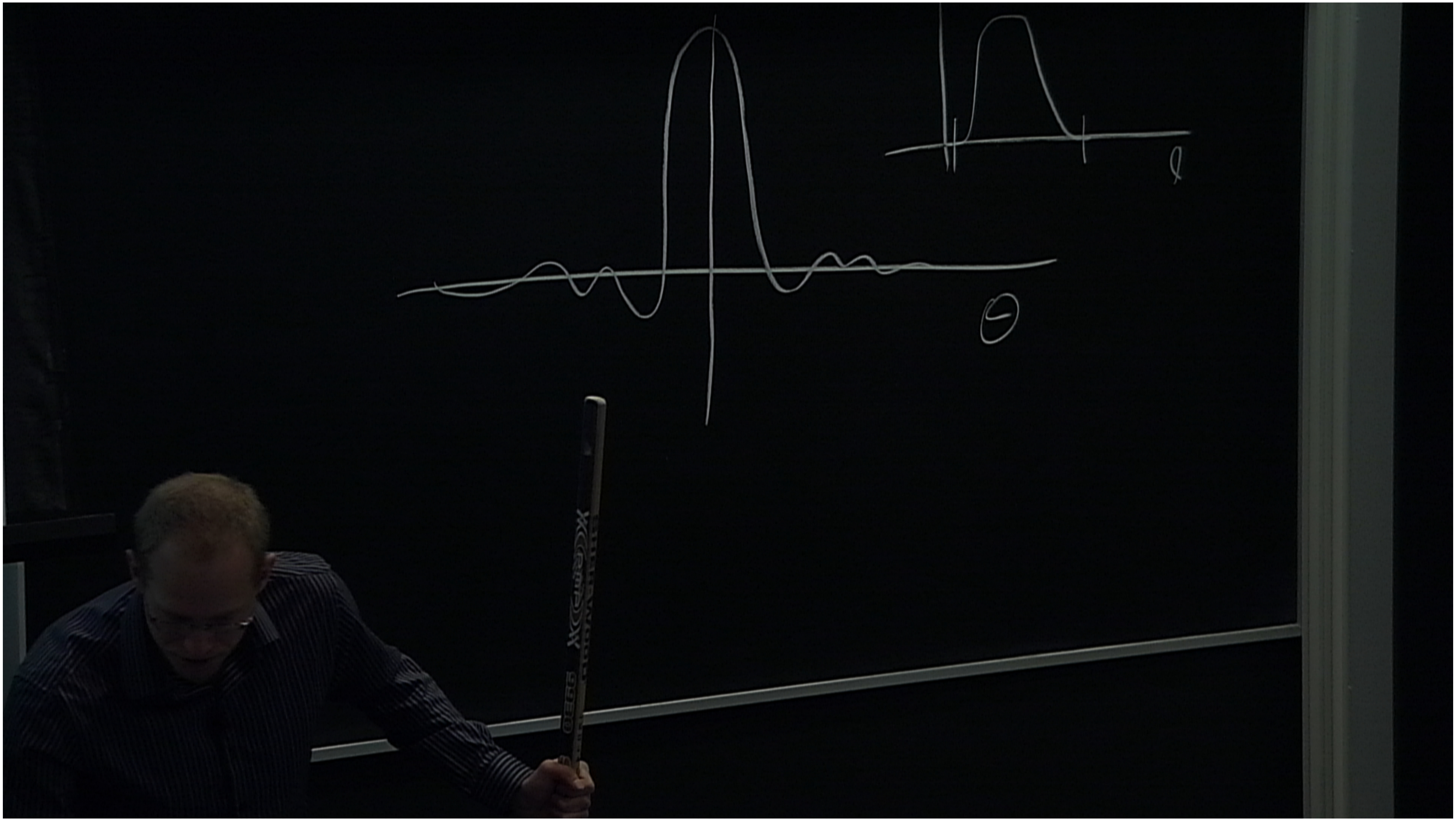
A framework for searching for features

- The current pipeline uses needlets (a type of wavelet transform) to identify candidates: sensitive to scale (j) and location (k).

$$T(\hat{\gamma}) = \sum_{j,k} \beta_{jk}\psi_{jk}(\hat{\gamma}) \qquad \beta_{jk} = \int T(\hat{\gamma})\psi_{jk}(\hat{\gamma})d\Omega.$$

- The coefficients are computed in harmonic space.



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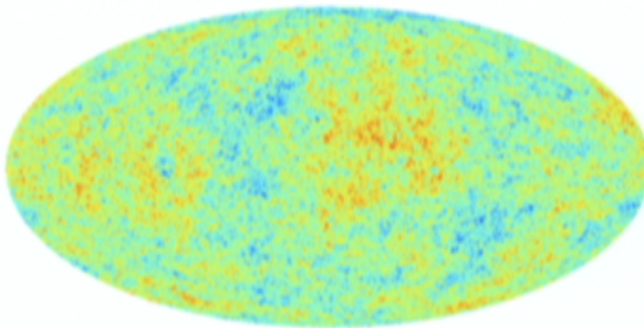

A framework for searching for features

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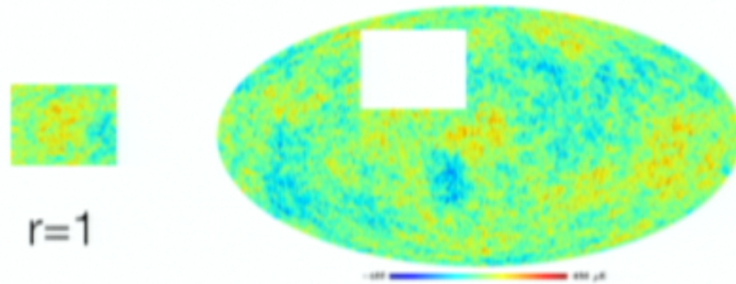

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r=2

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Evidence ratio in the patch: how much better does one describe the data by adding a template

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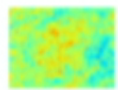

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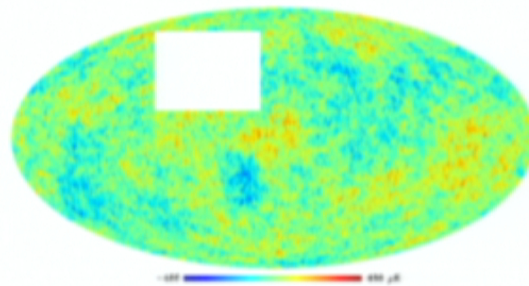

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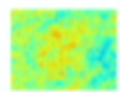

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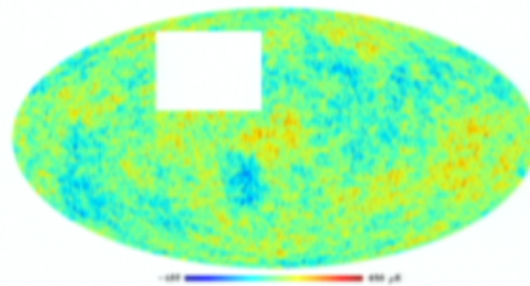

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- The general expression for N_b candidates:

Theory prior

Cosmic variance

All combos of templates and blobs

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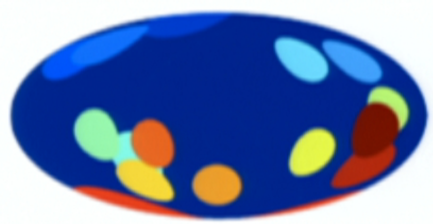
Expected number of features

Poisson process

Evidence ratio in each blob



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How do we evaluate this expression?

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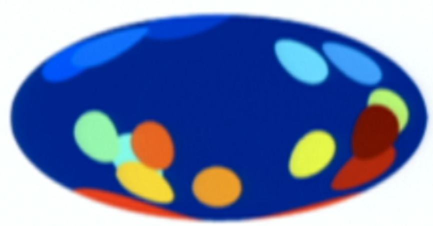

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- Full sky data is available at 5 frequencies: useful to remove foregrounds.
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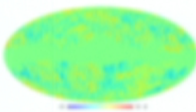

T map

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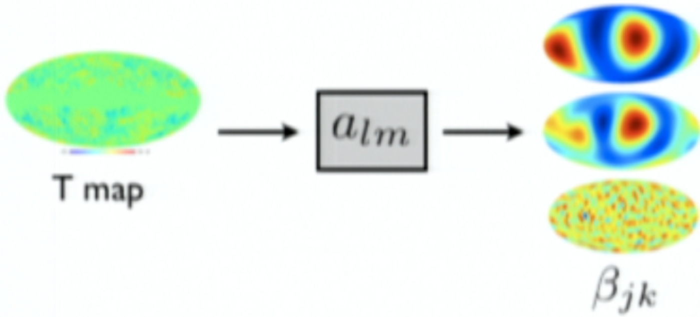

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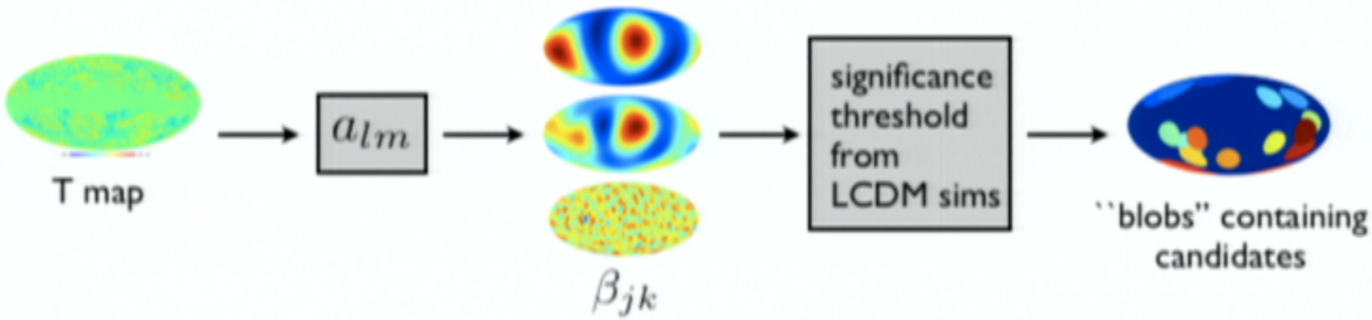
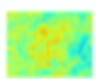


For each blob:



blob

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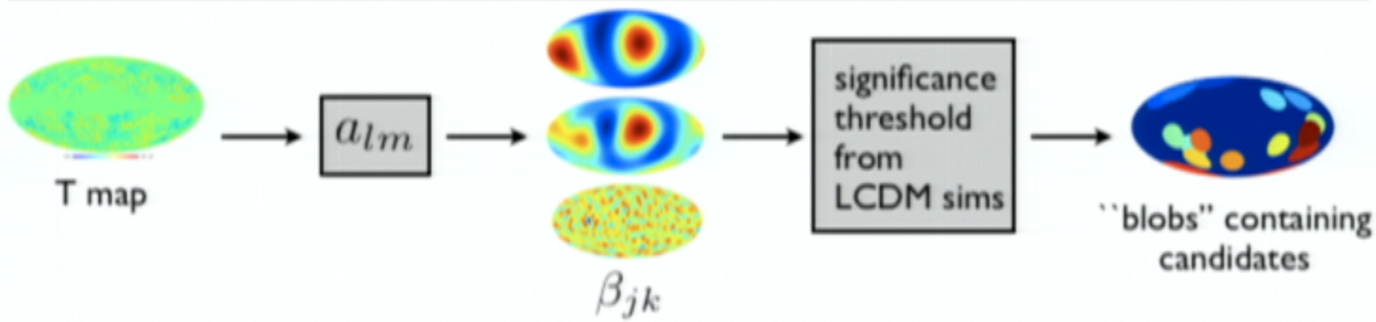

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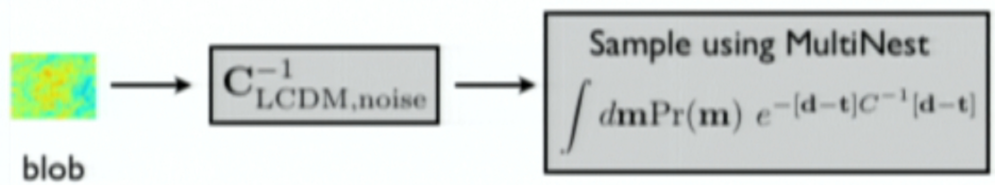

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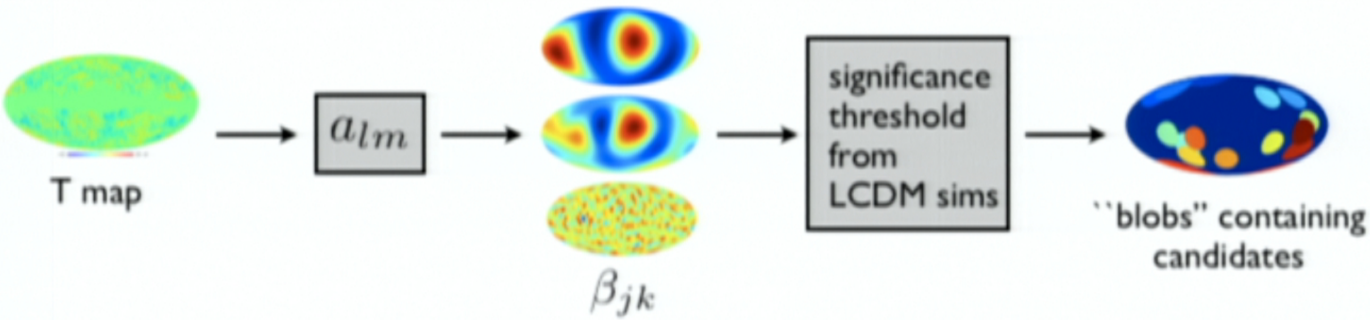

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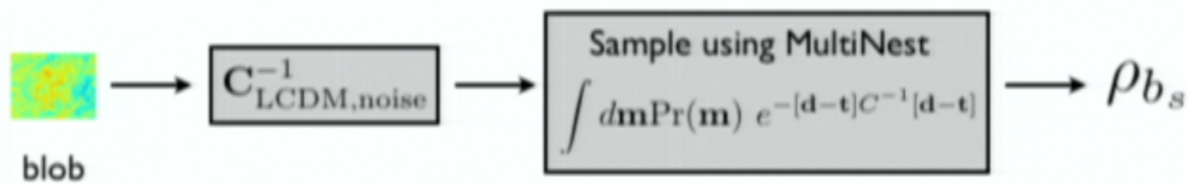

Combine according to:

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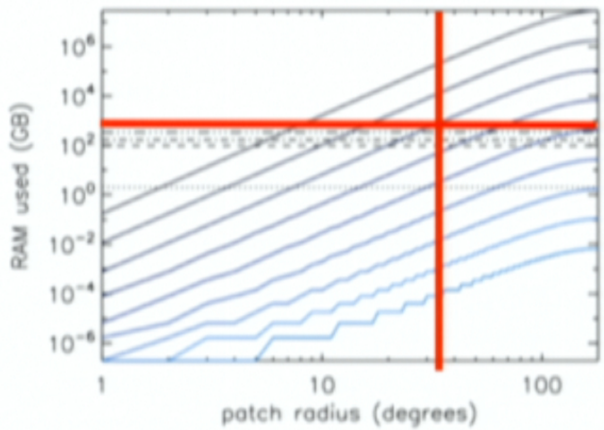
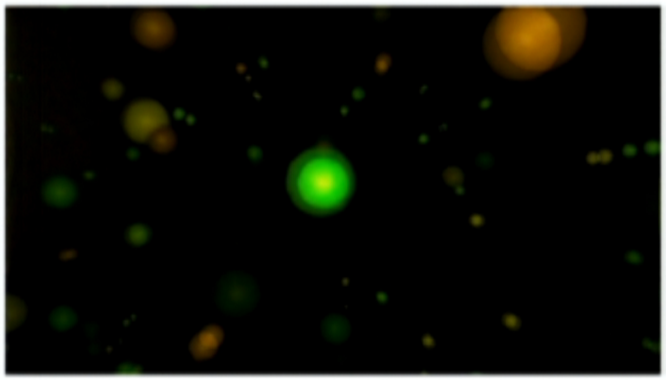
Limitations

- Each blob contains a huge amount of data.
- Storing and inverting the covariance matrix requires lots of RAM:



- Currently running on a 672GB shared memory machine.
- The code is parallelized.
- The resolution inside a blob is degraded according to the resources available.

Results: Cosmic bubble collisions



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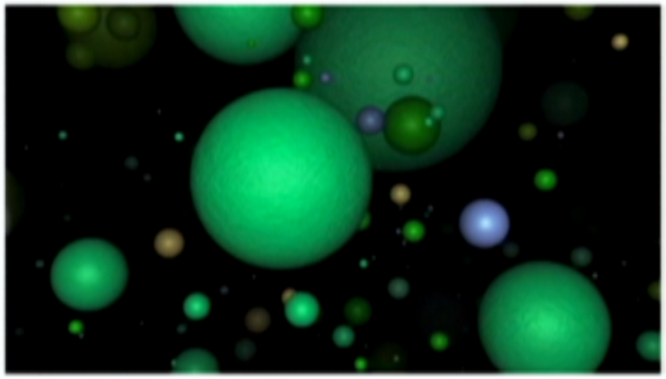
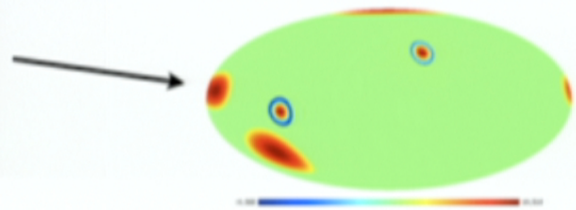


- The collision of our bubble with others provides an observational test of eternal inflation.



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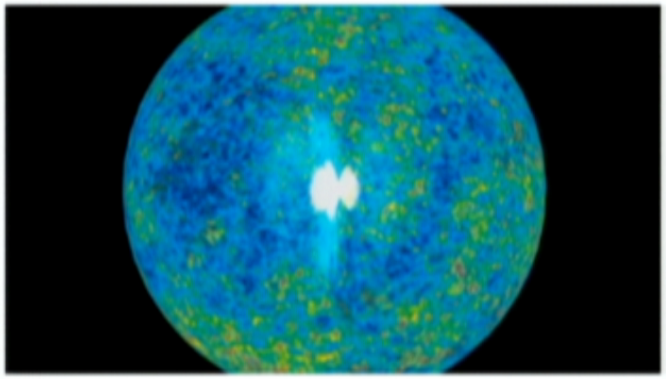

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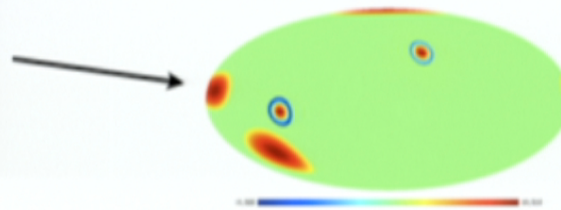
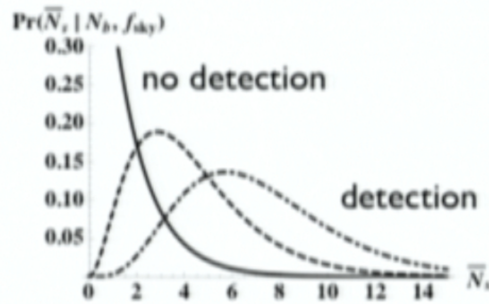
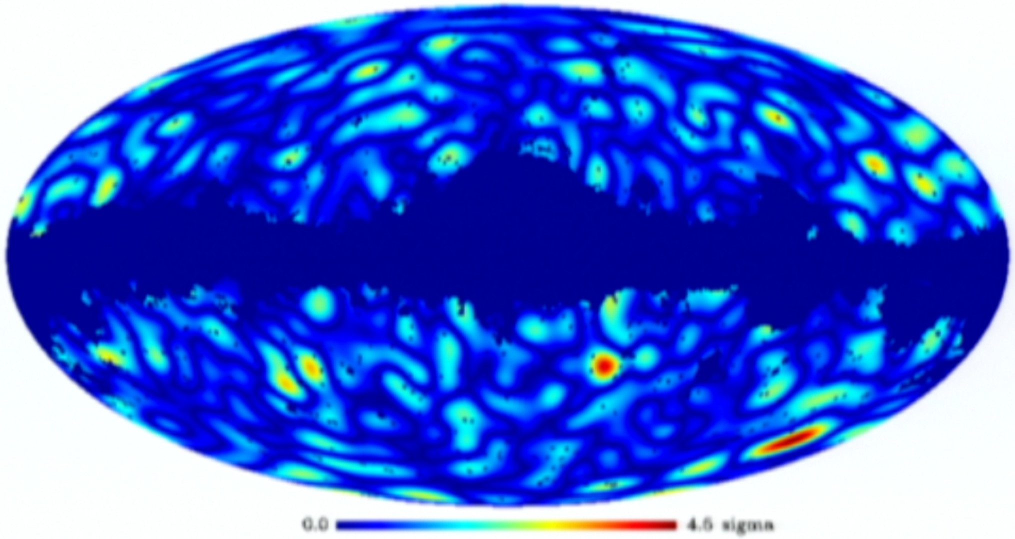


- Some possible outcomes:



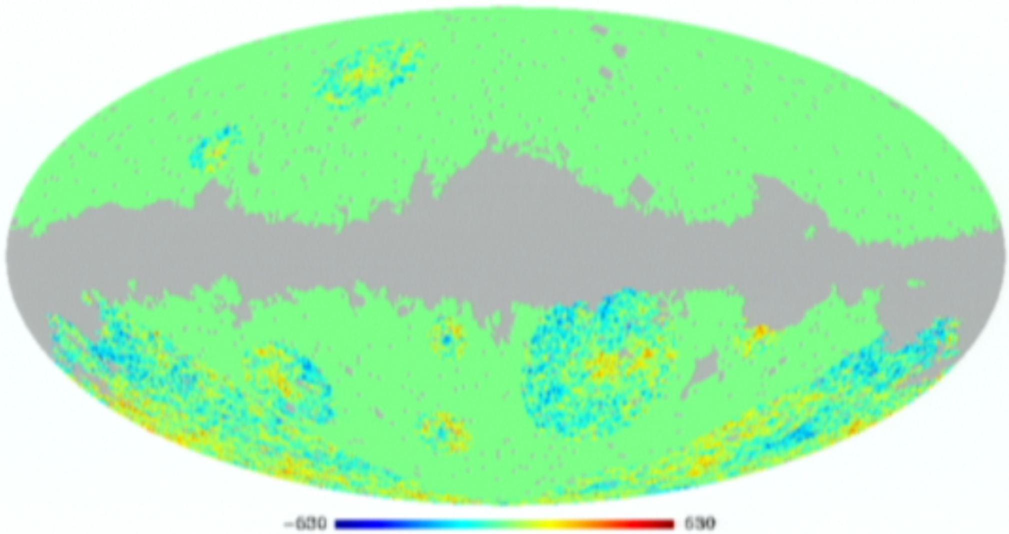
WMAP7 W-Band (94 GHz) : Needlets



(sensitive to 5-14 degree templates)

11 features pass thresholds, with detections in multiple needlet types/frequencies

WMAP7 W-Band (94 GHz) : Blobs



Divide the sky into “blobs” containing each candidate collision.

WMAP7 W-Band (94 GHz) : Posterior

- The posterior is peaked around $\bar{N}_s = 0$

  The data does not support the bubble collision hypothesis.

- From the shape of the posterior, we can rule out

$$\bar{N}_s < 1.6 \text{ at } 68\% \text{ CL}$$

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