Title: Coarse graining spin nets with tensor networks

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Abstract:

Coarse graining with tensor networks

Bianca Dittrich
with
Frank Eckert, Mercedes Martin-Benito

BD, F.C. Eckert, M. Martin-Benito, to appear in NJP, **arXiv:1109.4927 [gr-qc]**

BD, F.C. Eckert, (short proceeding version), **arXiv:1111.0967 [gr-qc]**

Motivation

•spin foam models: candidates for quantum gravity, describe (very) small scale physics

•most important question: what do they describe at large scales?

•Spin foams can be understood as lattice systems:

Use coarse graining to construct effective models for larger scales.

Problem: Spin foam models for gravity have amazingly complicated amplitudes. No coarse graining methods available.

Here: Simplify models drastically, keeping 'spin foam construction principle', develop and test coarse graining methods.

Interest for quantum information/ condensed matter:
-models related to topological phases, string nets, symmetry breaking
-same techniques for coarse graining

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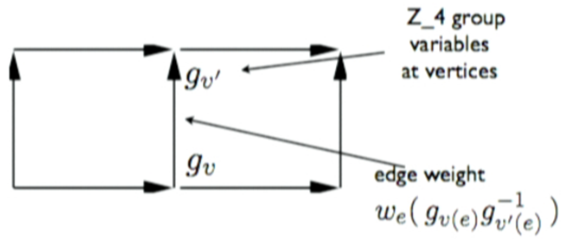
Simple example

generalized Ising model

$$Z \sim \sum_{g_v=0,1,2,3} \prod_e w_e(g_{v(e)} g^{-1}_{v'(e)})$$

↑ edge weight

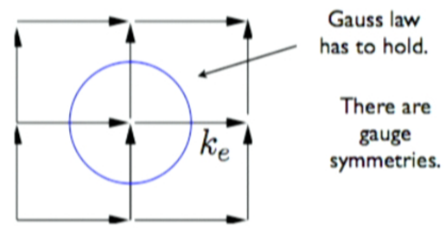


freeze:

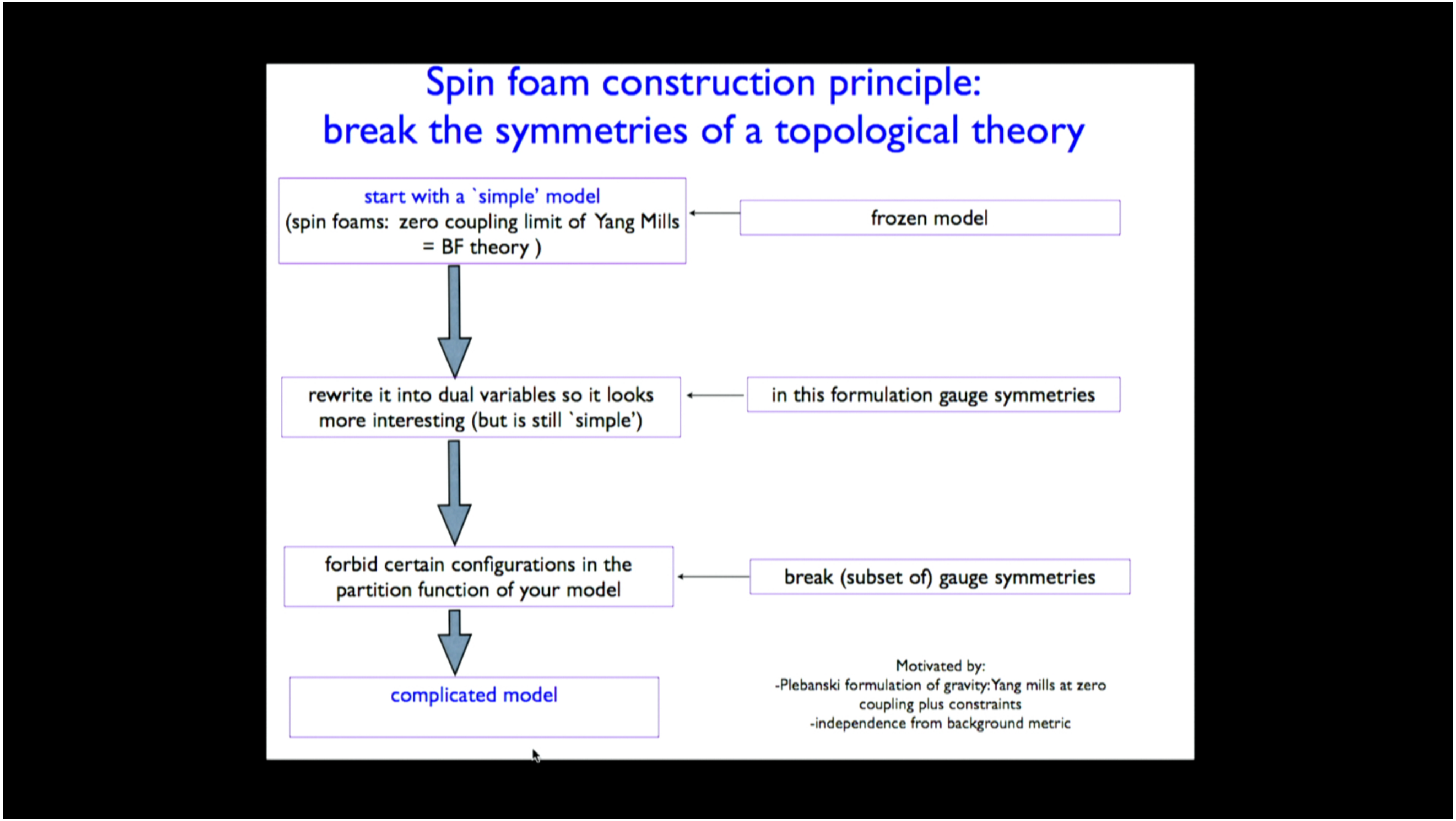
$$w_e(\, g_{v(e)} g^{-1}_{v'(e)}\,) = \delta(\, g_{v(e)} g^{-1}_{v'(e)}\,)$$

duality (Fourier) transform

$$Z \sim \sum_{k_e=0,1,2,3} \left(\prod_v \delta\big(\sum_{e \supset v} (\pm) k_e\big) \right)$$



Remark: Clebsch-Gordon coefficients instead of delta for non-Abelian groups hide `simple' character of the model.


Spin foam construction principle:
break the symmetries of a topological theory
start with a `simple' model
(spin foams: zero coupling limit of Yang Mills = BF theory)
frozen model
rewrite it into dual variables so it looks more interesting (but is still `simple')
in this formulation gauge symmetries
forbid certain configurations in the partition function of your model
break (subset of) gauge symmetries
complicated model
Motivated by:
-Plebanski formulation of gravity: Yang mills at zero coupling plus constraints
-independence from background metric

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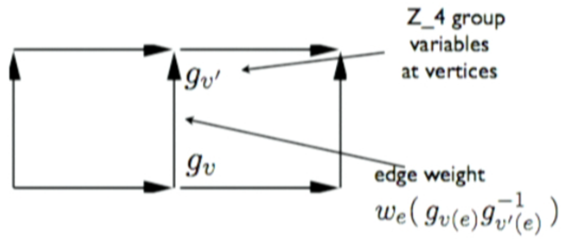

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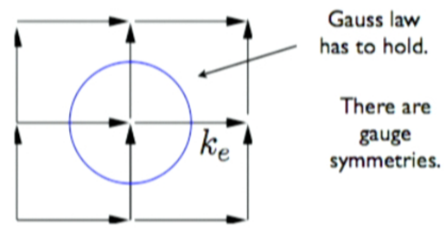

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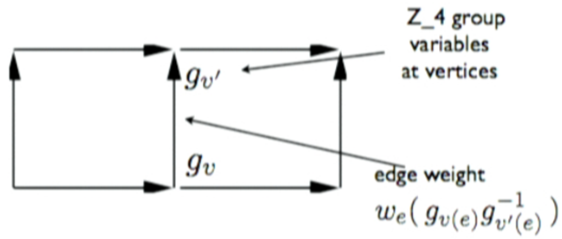

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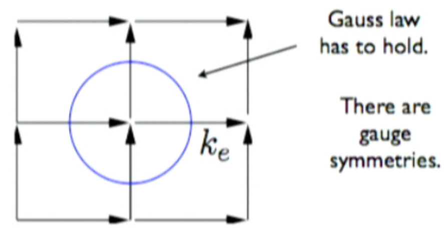

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Simple example

exclude some configurations

$$Z \sim \sum_{k_e=0,1,\not{2},3} \left(\prod_v \delta\big(\sum_{e \supset v} (\pm) k_e\big) \right)$$

Space of models we will consider here:

q= size of (Abelian group),

K= `cutoff' = 1/2 the number of variables you sum over

non-trivial model

Remark: crude approximation of heat kernel action by step function.

This is the basic idea for constructing spin foam models, starting with a topological field theory.

For non-Abelian groups, we end up with genuine vertex models (basically tensor networks).

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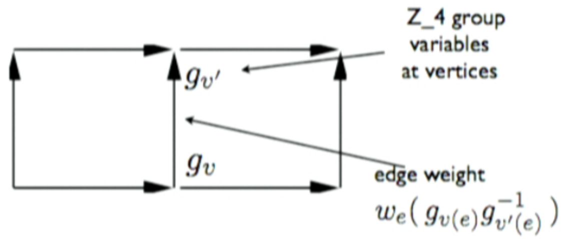

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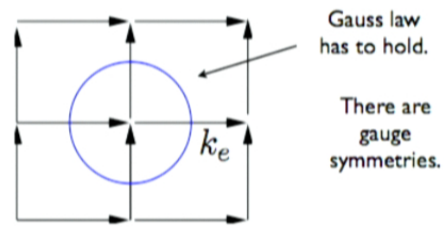

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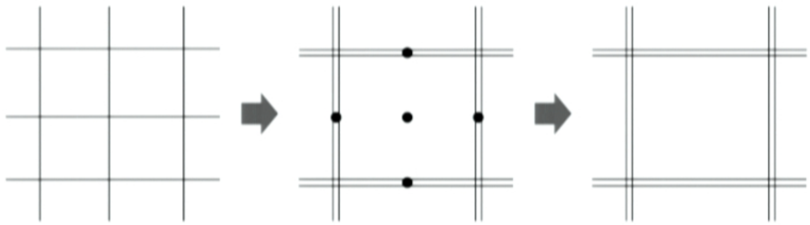
Coarse graining

•standard method: Migdal-Kadanoff ('75/76)

-approximation to local couplings
-leads to simple recursion relation on weights

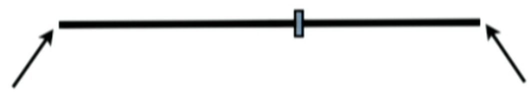


$$Z = \sum_{g_v} \prod_e w_e(g_{v(e)} g^{-1}_{v'(e)}) \longrightarrow Z \sim \sum_{g_V} \prod_E w'_E(g_{V(E)} g^{-1}_{V'(E)})$$

sum over subset of variables

approximate with local couplings and effective weight

To which fixed points do the models flow under coarse graining (symmetry restoration)?



zero temperature with gauge symmetries

$$\tilde{w}_e(k_e) \equiv 1 \quad \Rightarrow \quad w_e(\,g_{v(e)}g^{-1}_{v'(e)}\,) \sim \delta^{(q)}(\,g_{v(e)}g^{-1}_{v'(e)}\,)$$

infinite temperature

$$\tilde{w}_e(k_e) = \delta_{k_e,0} \quad \Rightarrow \quad w_e(\,g_{v(e)}g^{-1}_{v'(e)}\,) \sim 1$$

Results [BD, Eckert, Martin-Benito 11]

3d lattice gauge (spin foam)

10										LTF
9									LTF	
8								LTF		
7							LTF			
6						LTF				
5					LTF					
4				LTF						
3			LTF							
2		LTF								
1	LTF									
0	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF
K/q	2	4	6	8	10	12	14	16	18	20

flow to LTF

flow to HTF

- convergence after small number of iteration steps either to HTF or LTF
- mostly to HTF

- for U(1), SU(N): Migdal-Kadanoff approximation flows all configurations to HTF: there is only one phase [Ito 84; Mueller, Schiemann 85]
- as 3d is to near the critical dimension of 2

Results [BD, Eckert, Martin-Benito, 11]

4d lattice gauge (spin foam)/ 2d edge model (spin net)

10										LTF
9									LTF	
8								LTF		
7							LTF			
6						LTF				
5					LTF					
4				LTF						
3			LTF							
2		LTF								
1	LTF									
0	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF
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flow to LTF

flow to nontrivial unstable (quasi) fixed point

flow to HTF

- more configurations flow to LTF
- but also to unstable fixed point!

very weak phase transition

- for U(1): in 4d there are two phases [Polyakov 75; Banks, Myerson, Kogut 77]
- but: MK relations miss Kosterlitz Thouless transitions (flow to HTF) [Ito '85]

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0	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF	HTF
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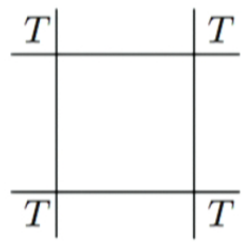
Tensor network representation

•partition function can be written in (different) tensor network form

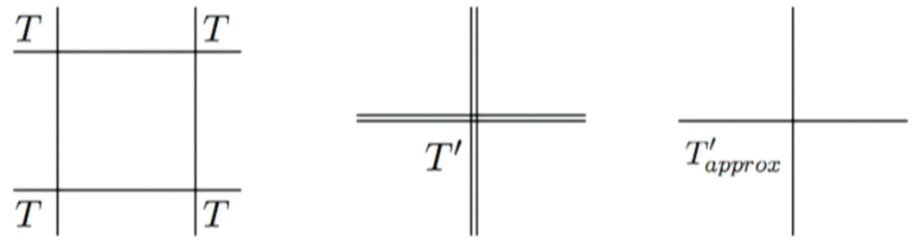


$$Z \sim \text{Tensor-Tr}(T_v T_{v'} T_{v''} \dots)$$

•graphical calculus: associate tensors to vertices, contract along edges

•[Wen et al] tensor networks can describe topological phases

Tensor network renormalization

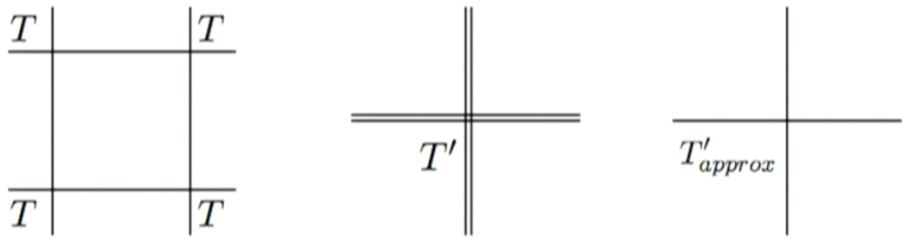


- contract four tensors to effective tensors
- approximate by cutting off the index range to some fixed value (for instance by singular value decomposition)
- [Levin, Nave '07, Gu, Wen '09] specific proposals for algorithms in 2d

gist of the method:

- cut-off in index range determines number of fields (implicit non-local couplings)
- in each step apply field redefinitions, so to keep number of fields minimal

Tensor network renormalization



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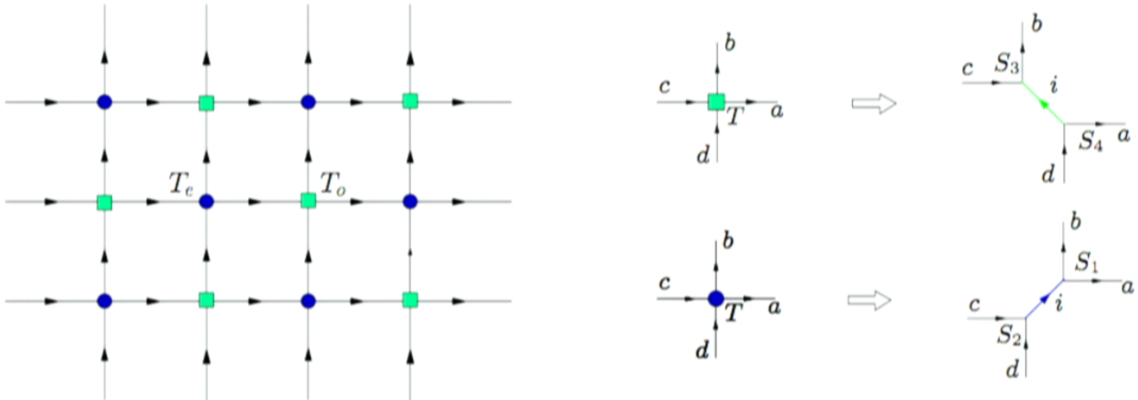
The renormalization algorithm



(a) square lattice

(b) splitting of vertices

$$T^{abcd} = M_1^{ab,cd} = \sum_{i=0}^{q^2-1} U_1^{ab,i} \lambda_i (V_1^\dagger)^{i,cd} \approx \sum_{i=0}^{D_c-1} (U_1^{ab,i} \sqrt{\lambda_i})(\sqrt{\lambda_i} (V_1^\dagger)^{i,cd}) = \sum_i S_1^{ab,i} S_2^{cd,i}$$

singular value decomposition

approximation

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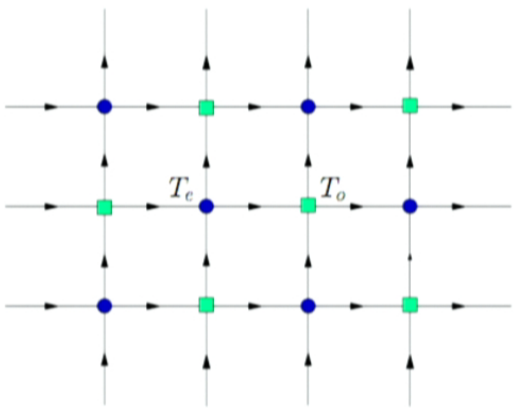

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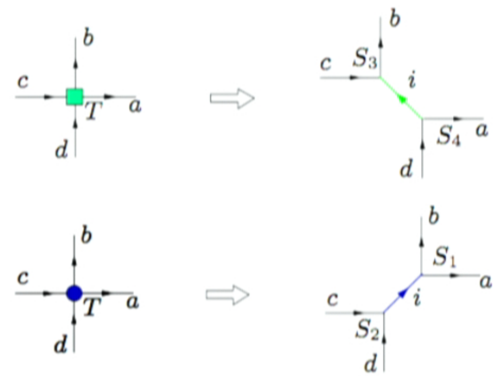

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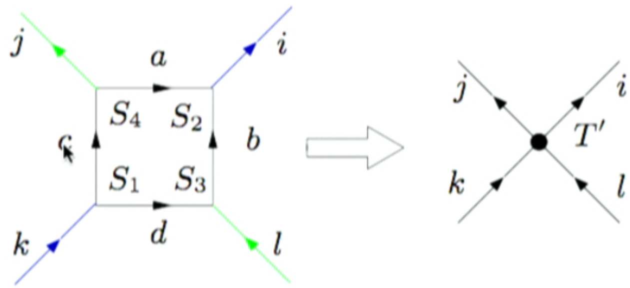
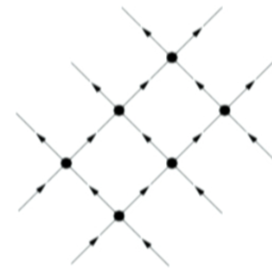


(a) contraction



(b) coarse grained lattice

$$T'^{ijkl} = \sum_{a,b,c,d} S_2^{ab,i} S_4^{ac,j} S_1^{dc,k} S_3^{db,l}$$

New `effective' tensor describing coarse grained model.

New feature and difficulty

- we look at 'large' groups, to which the TNW algorithm has not been applied before
- with increasing size of group weaker phase transition
- equivalence of models with the same K starting from sufficiently high q

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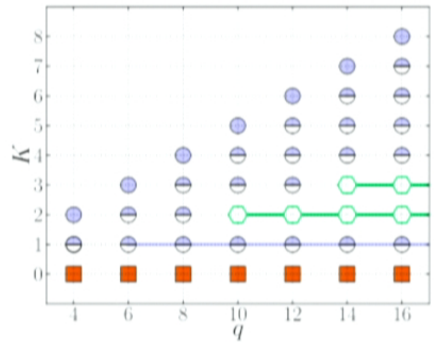
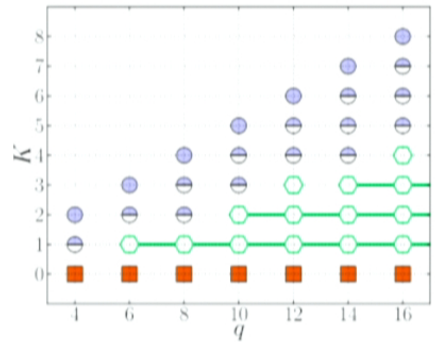
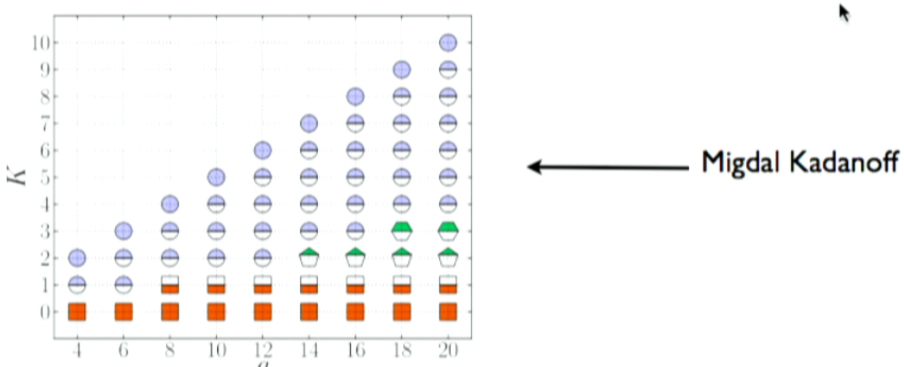
Results: TNW algortihm [BD, Eckert, Martin-Benito, 11]



(a) $D_c = 16$



(b) $D_c = 25$



Remarks

•we developed a Gauss constraint preserving algorithm: [see also Singh, Pfeifer, VIdal 2011]
- -enhanced speed and stability
- -physically important to preserve Gauss constraints
- -confirms (with few changes and additions) MK analysis so far

•algorithm leads to non-isolated fixed points
- -[Gu, Wen] have to implement entanglement filtering
- -needs to be much better understood

•our first algorithm suffered from instabilities for larger groups and cut-offs
- -due to `unsymmetric cut-offs' (in Gauss constraint preserving algorithm)
- -and very weak phase transitions

• improved stability by a `self-adaptive cut-off'
- -this helps a lot, but still some problems with stability
- -aim: can we see Kosterlitz Thouless transitions to appear for large groups?

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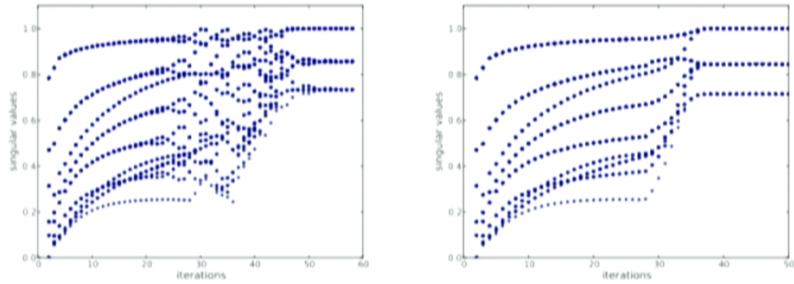
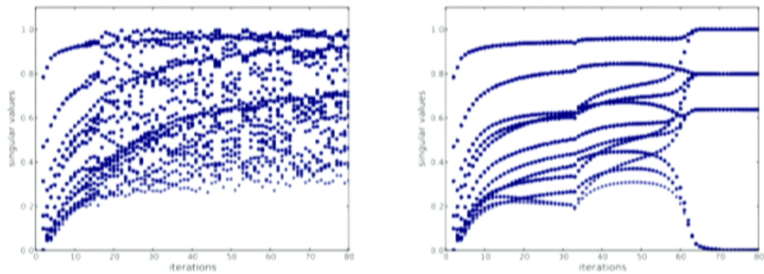
Improving: adaptive cutoff and symmetrization



(a) Adaptive cutoff (b) Adaptive cutoff and symmetrization

q=6,K=1, D_c=16

Improving: adaptive cutoff



(a) Original TNR (b) TNR with adaptive cutoff

q=6,K=1, D_c=21

wip

- relation Migdal Kadanoff and Tensor Network method
- understand the approximation in tensor network better:
  understand significance of negative eigenvalues (in singular value decomposition)
- detect phase transition temperature in Abelian models
- non-Abelian models: genuine vertex models, not clear which statistical properties to expect