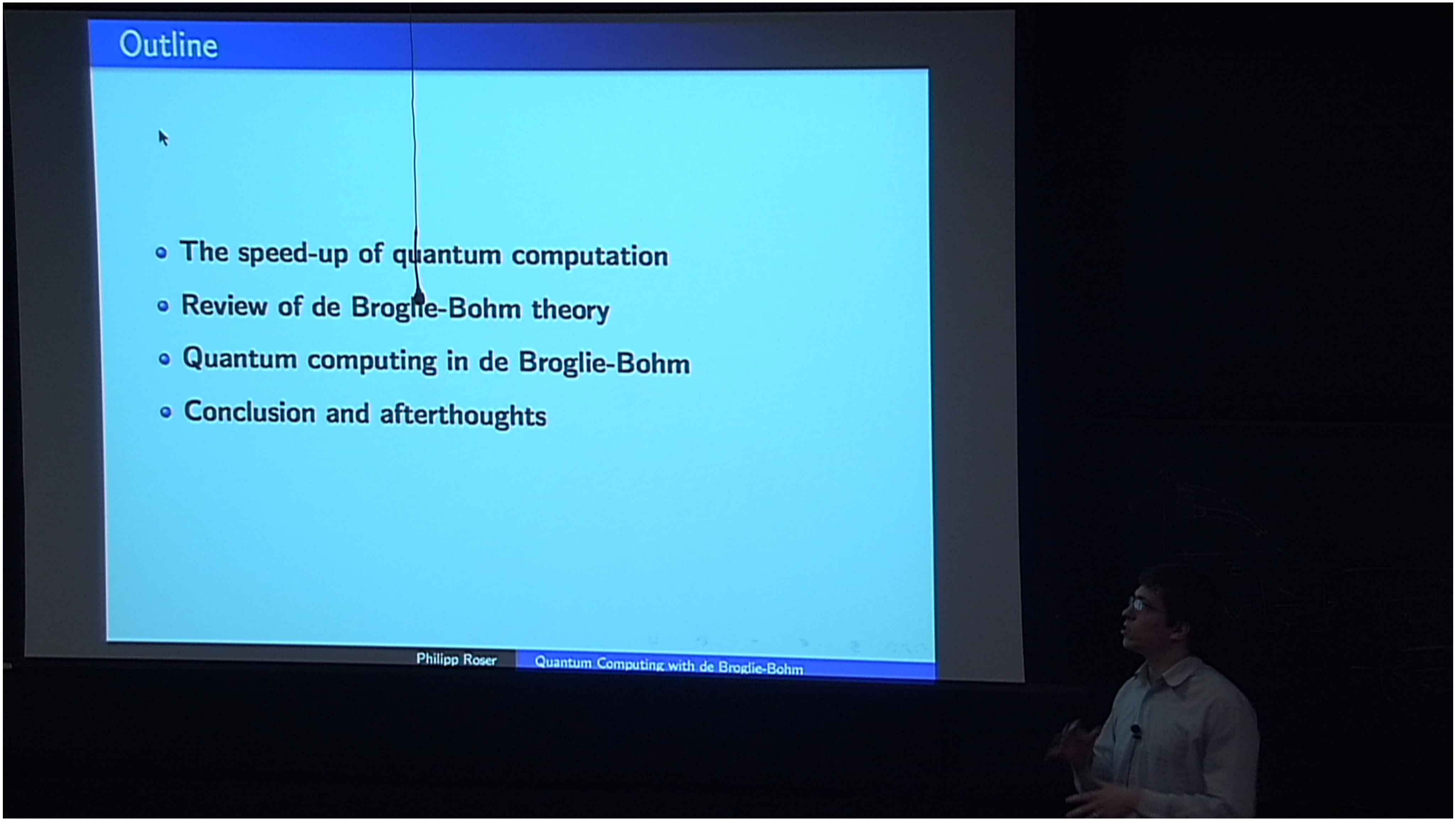
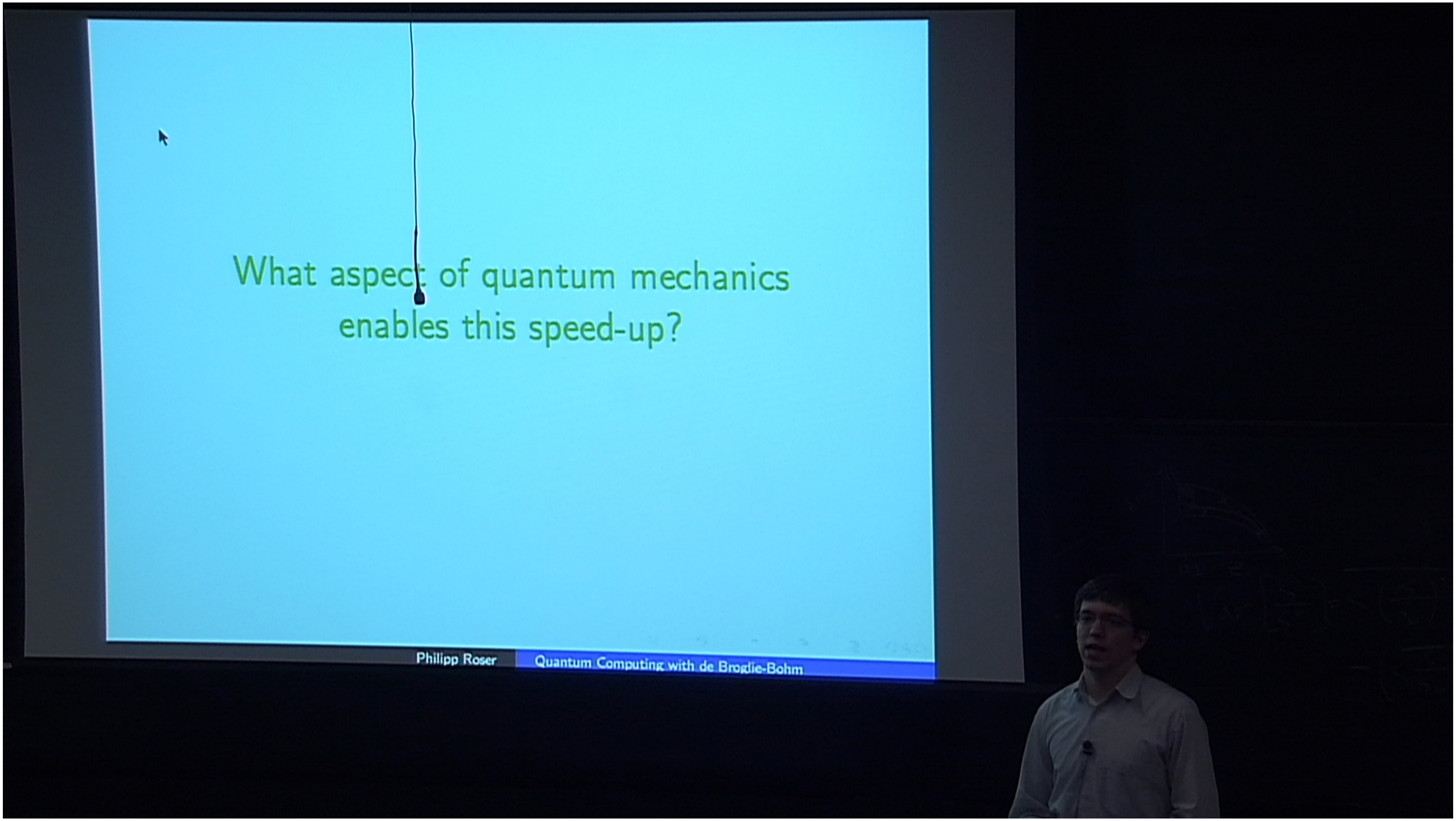
Title: Quantum Computing from the Perspective of the de Broglie-Bohm Formulation of Quantum Mechanics

Date: Feb 21, 2012 03:30 PM

URL: http://pirsa.org/12020135

Abstract:


Outline
The speed-up of quantum computation
Review of de Broglie-Bohm theory
Quantum computing in de Broglie-Bohm
Conclusion and afterthoughts
Philipp Roser
Quantum Computing with de Broglie-Bohm


What aspect of quantum mechanics
enables this speed-up?
Philipp Roser
Quantum Computing with de Broglie-Bohm

Deutsch's algorithm I

Scenario

We're given a black box whose dynamics can be abstractly described by one of the four possible functions $f : \{0,1\} \to \{0,1\}$.

$$
\begin{aligned}
f_0(0) &= 0 \qquad f_0(1) = 0 \\
f_1(0) &= 0 \qquad f_1(1) = 1 \\
f_2(0) &= 1 \qquad f_2(1) = 0 \\
f_3(0) &= 1 \qquad f_3(1) = 1
\end{aligned}
$$

We are not told *which* of these describes the box.

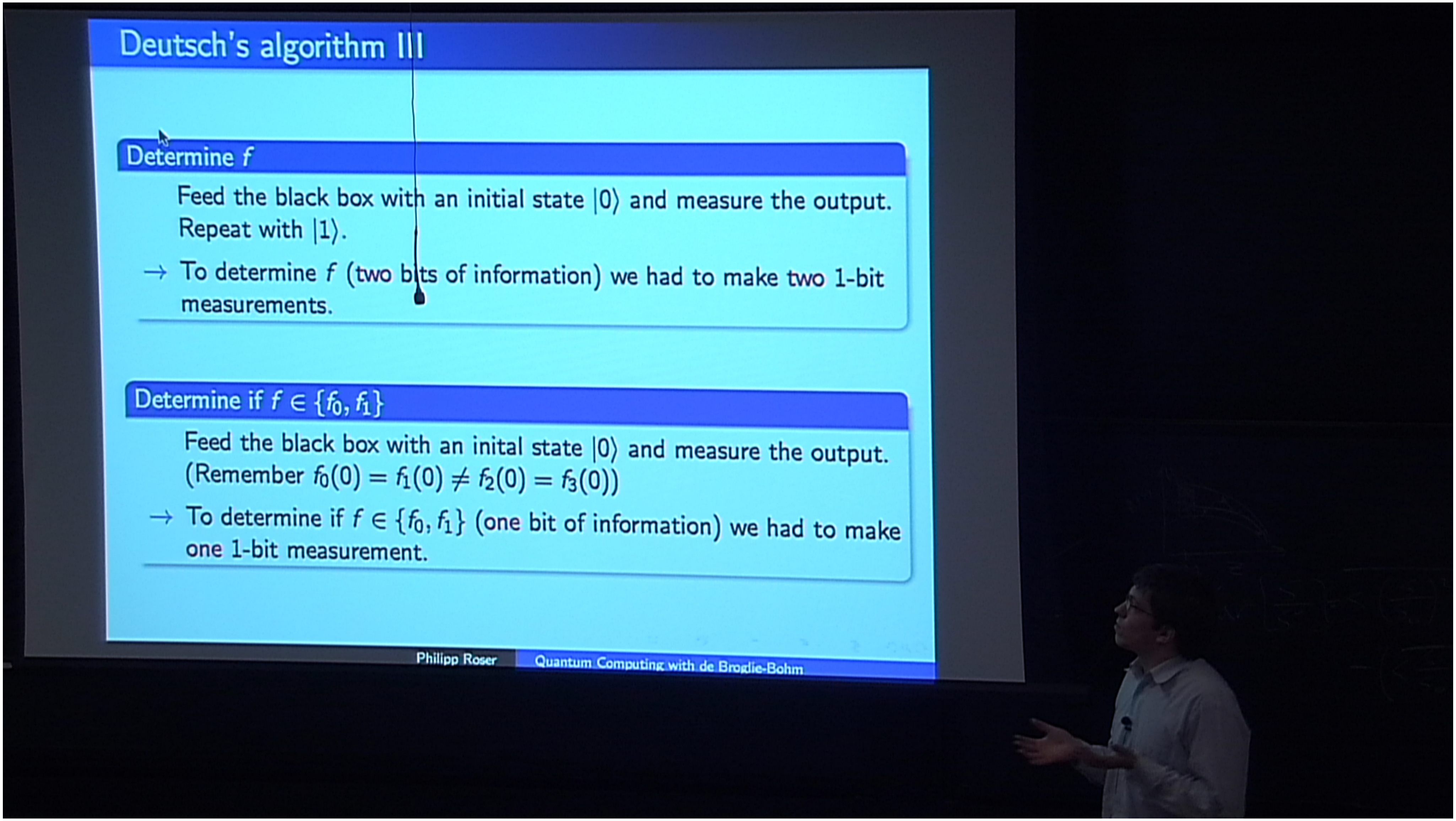
Deutsch's algorithm I

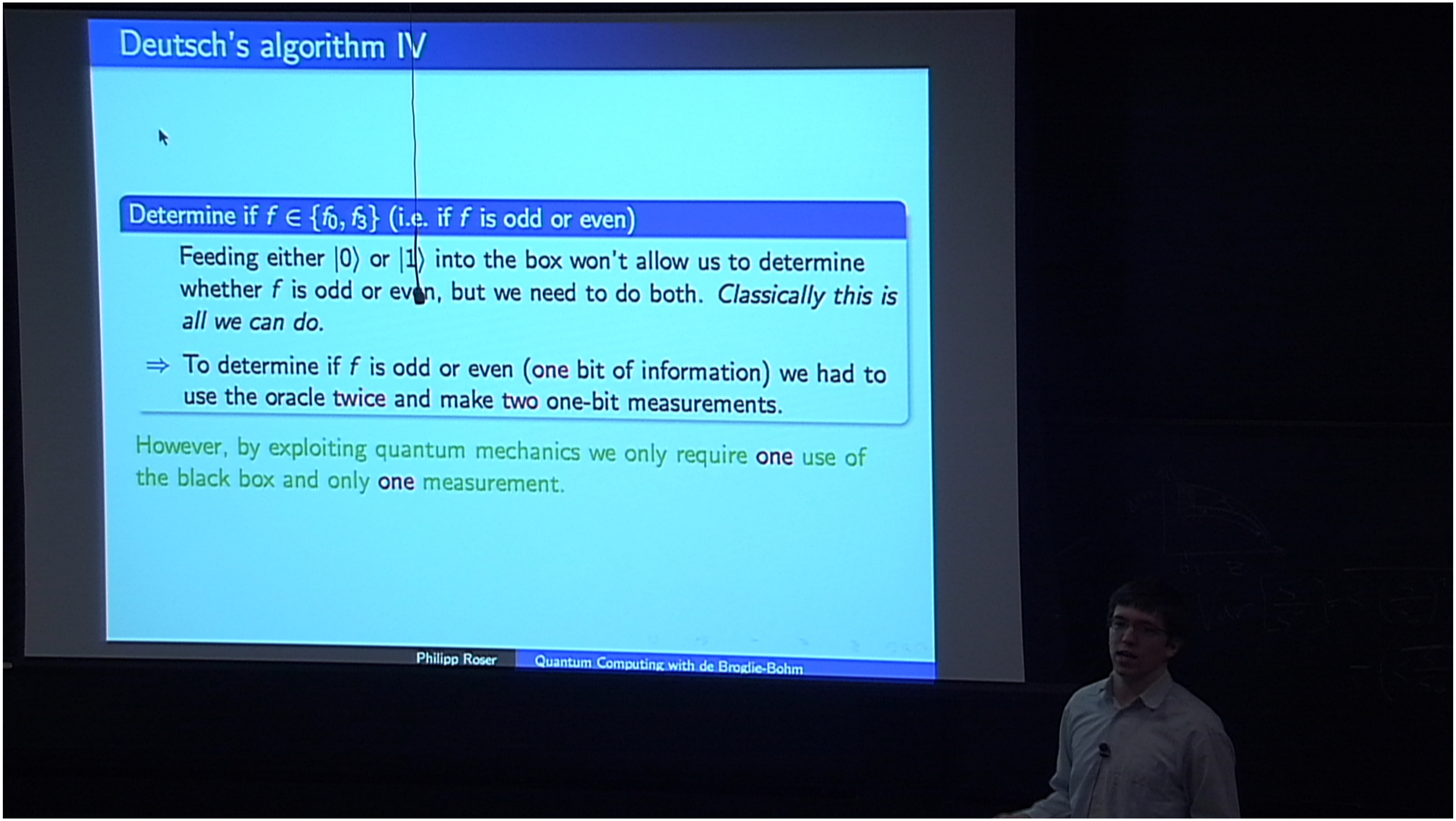
Scenario

We're given a black box whose dynamics can be abstractly described by one of the four possible functions $f : \{0,1\} \to \{0,1\}$.

$$\begin{aligned} f_0(0) &= 0 \qquad f_0(1) = 0 \\ f_1(0) &= 0 \qquad f_1(1) = 1 \\ f_2(0) &= 1 \qquad f_2(1) = 0 \\ f_3(0) &= 1 \qquad f_3(1) = 1 \end{aligned}$$

We are not told *which* of these describes the box.


Deutsch's algorithm III
Determine f
Feed the black box with an initial state $|0\rangle$ and measure the output. Repeat with $|1\rangle$.
→ To determine f (two bits of information) we had to make two 1-bit measurements.
Determine if $f \in \{f_0, f_1\}$
Feed the black box with an inital state $|0\rangle$ and measure the output. (Remember $f_0(0) = f_1(0) \neq f_2(0) = f_3(0)$)
→ To determine if $f \in \{f_0, f_1\}$ (one bit of information) we had to make one 1-bit measurement.
Philipp Roser
Quantum Computing with de Broglie-Bohm


Deutsch's algorithm IV
Determine if $f \in \{f_0, f_3\}$ (i.e. if f is odd or even)
Feeding either $|0\rangle$ or $|1\rangle$ into the box won't allow us to determine whether f is odd or even, but we need to do both. *Classically this is all we can do.*
⇒ To determine if f is odd or even (one bit of information) we had to use the oracle twice and make two one-bit measurements.
However, by exploiting quantum mechanics we only require one use of the black box and only one measurement.
Philipp Roser
Quantum Computing with de Broglie-Bohm

Deutsch's algorithm V

"Hadamard" Gate

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(\,|0\rangle + |1\rangle\,)$

$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(\,|0\rangle - |1\rangle\,)$

Measurement in $\{|0\rangle, |1\rangle\}$ basis

$$\text{State prior to measurement} = \begin{cases} \pm\frac{1}{\sqrt{2}}|0\rangle_D(|0\rangle_A - |1\rangle_A) & \text{if } f(0) = f(1) \\ \pm\frac{1}{\sqrt{2}}|1\rangle_D(|0\rangle_A - |1\rangle_A) & \text{if } f(0) \neq f(1) \end{cases}$$

The speed-up I

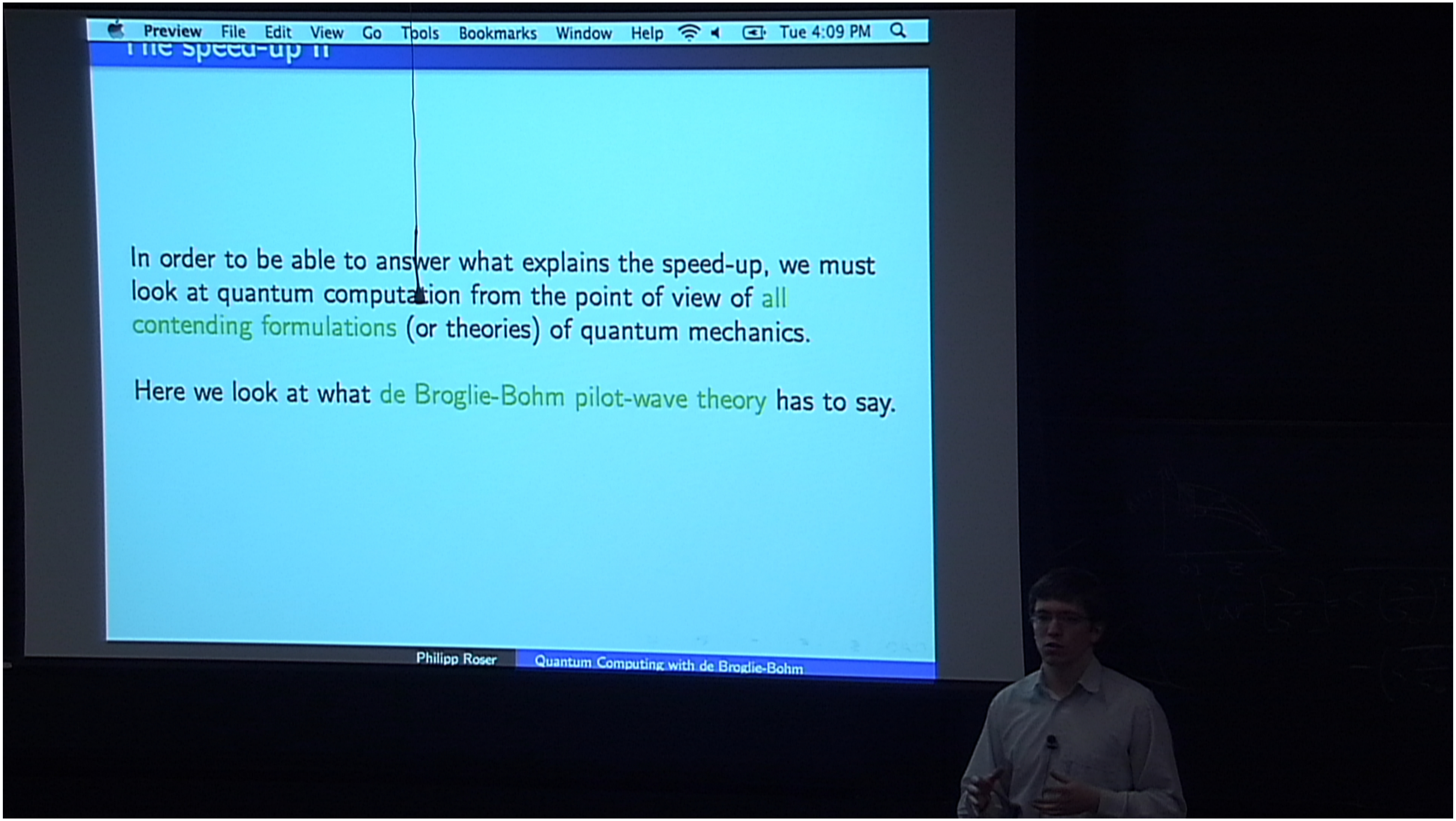
So what explains the speed-up?

- Entanglement? (Josza)
- "Wavelike" dynamics? (Mermin)
- Many worlds? (e.g. Deutsch)

The speed-up I

So what explains the speed-up?

- Entanglement? (Josza)
- "Wavelike" dynamics? (Mermin)
- Many worlds? (e.g. Deutsch)


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:09 PM
In order to be able to answer what explains the speed-up, we must look at quantum computation from the point of view of all contending formulations (or theories) of quantum mechanics.
Here we look at what de Broglie-Bohm pilot-wave theory has to say.
Philipp Roser
Quantum Computing with de Broglie-Bohm

dBB: Ontology and Dynamics

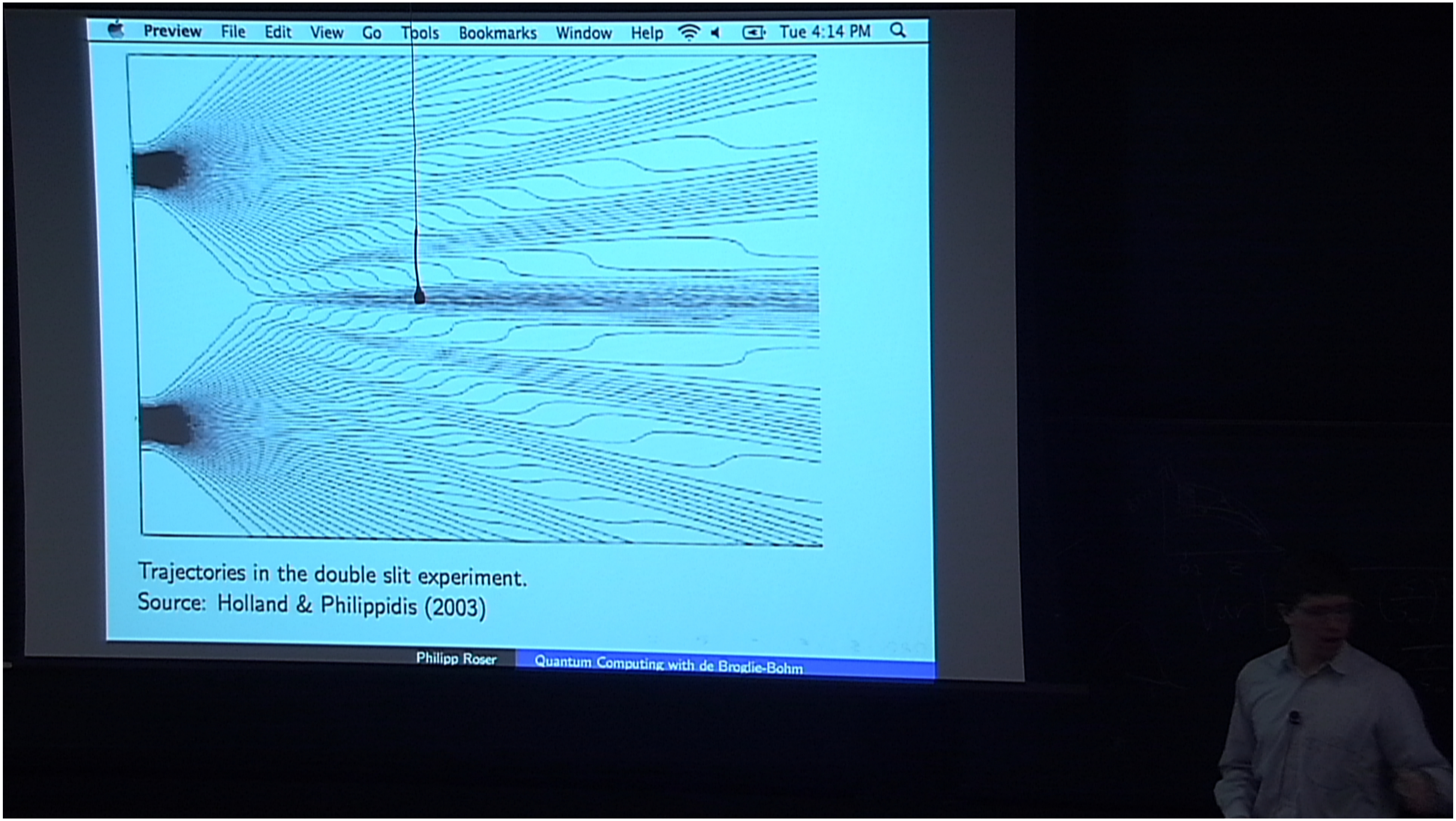
1 The *pilot wave*: a complex field $\psi(\vec{x}, t)$ in the configuration space of the chosen system $\mathfrak{S}$, obeying the equation

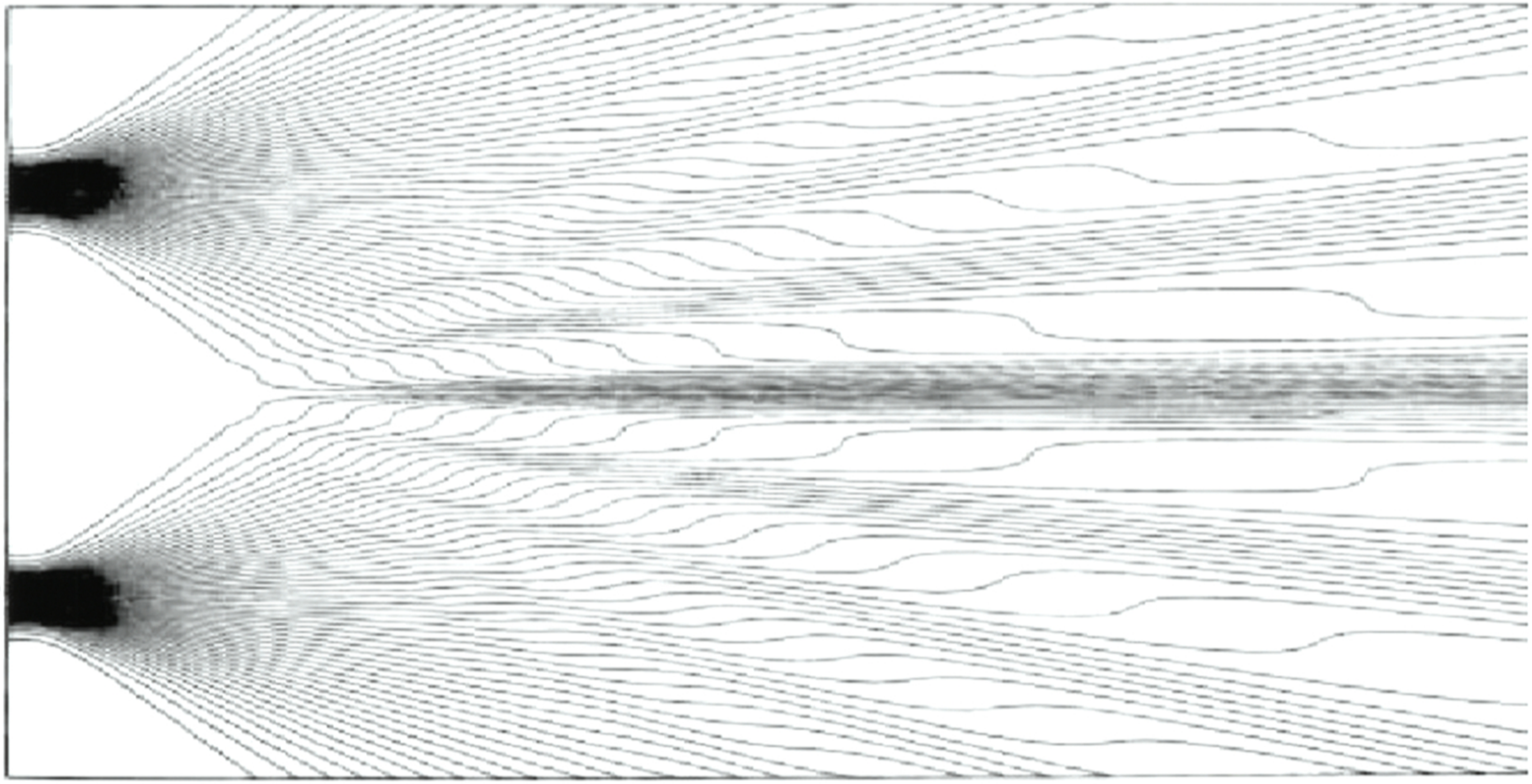
$$i\frac{\partial}{\partial t}\psi(\vec{x}, t) = \widehat{H}_{\mathfrak{S}}\psi(\vec{x}, t). \qquad (*)$$

2 The *actual configuration* as a function of time, tracing out a path $\vec{x}_c(t)$ in configuration space according to the "guidance equation"

$$\dot{\vec{x}}_c(t) = \frac{\vec{j}(\vec{x}_c, t)}{|\psi(\vec{x}_c, t)|^2},$$

where $\vec{j}(\vec{x}, t)$ is the current implicitly defined in the continuity equation $\frac{\partial}{\partial t}|\psi(\vec{x}, t)|^2 + \nabla \cdot \vec{j}(\vec{x}, t) = 0$ as derived from (*).


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:14 PM
Trajectories in the double slit experiment.
Source: Holland & Philippidis (2003)
Philipp Roser
Quantum Computing with de Broglie-Bohm



Trajectories in the double slit experiment.
Source: Holland & Philippidis (2003)

Preview File Edit View Go Tools Bookmarks Window Help Tue 4:15 PM

dBB: The ensemble density ρ

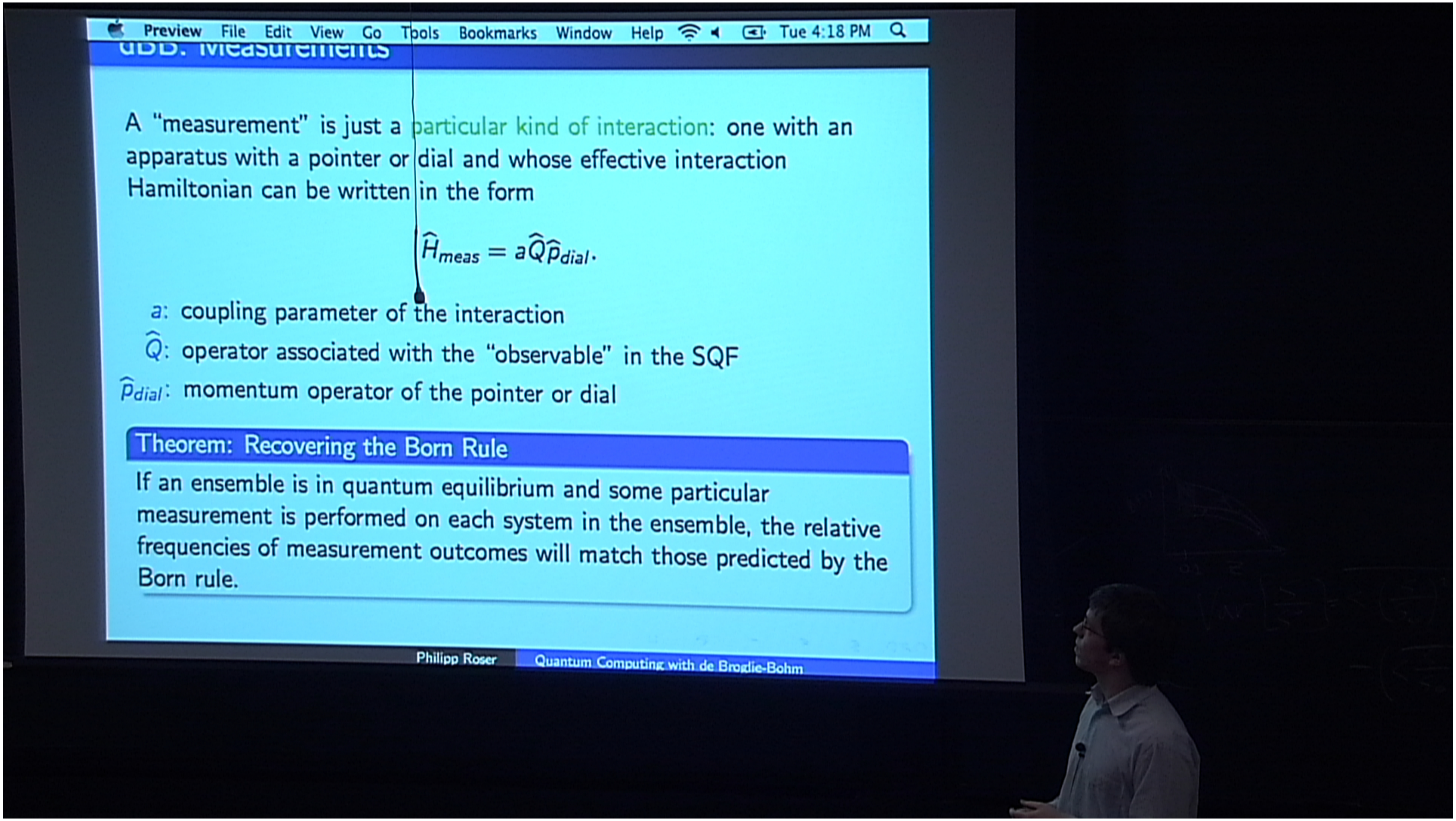
Consider a large ensemble of identical systems with normalised ensemble density $\rho(\vec{x}, t)$ of obtaining configurations.

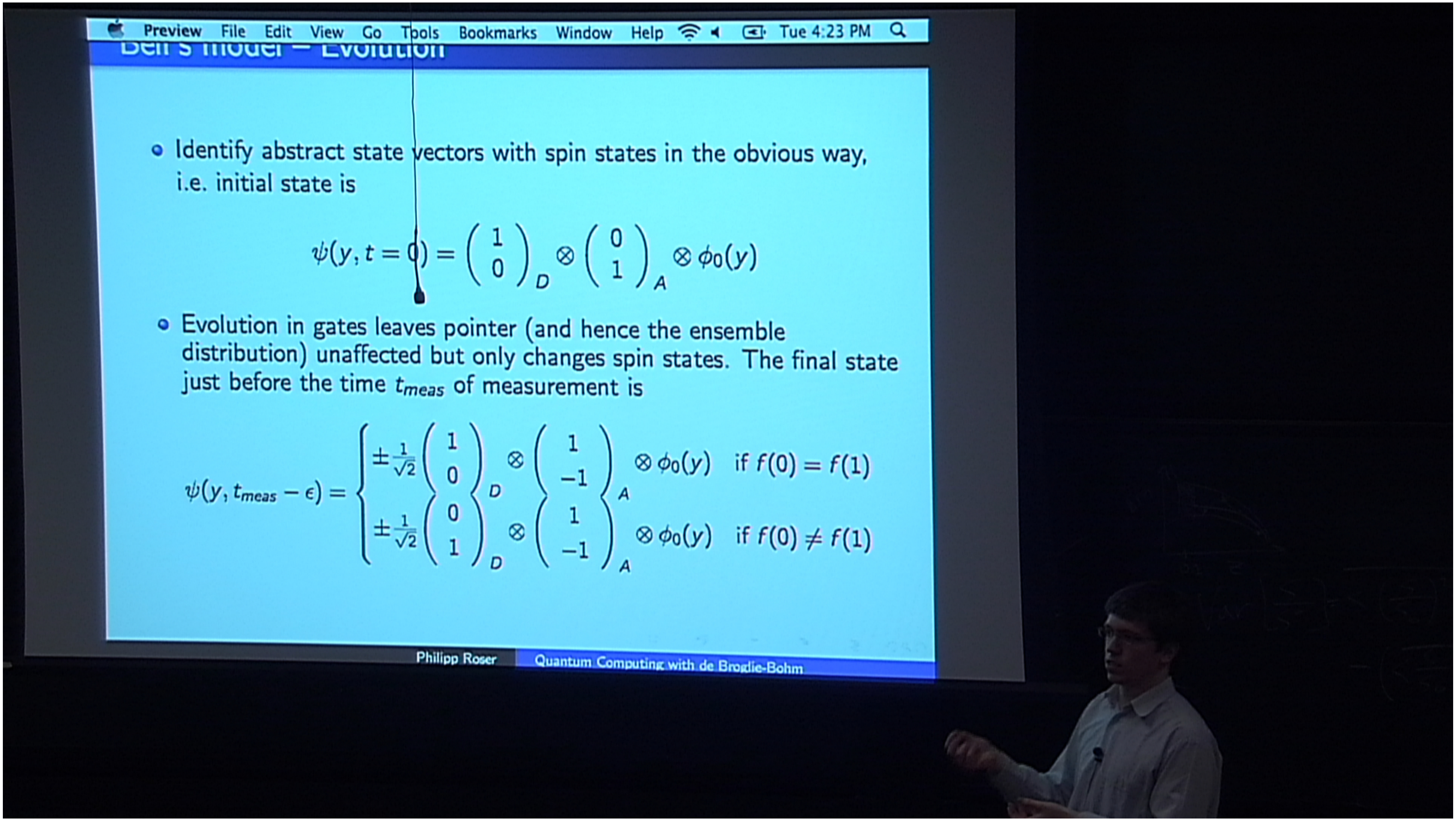
Theorem: The existence of quantum equilibrium

If $\rho(\vec{x}, t_0) = |\psi(\vec{x}, t_0)|^2$ for some time t_0,
then $\rho(\vec{x}, t) = |\psi(\vec{x}, t)|^2$ for all times t.

Theorem: Approach of equilibrium

An ensemble density $\rho(\vec{x}, t)$ not in quantum equilibrium monotonically approaches quantum equilibrium (given certain assumptions about the initial conditions).


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:18 PM
dBB: Measurements
A "measurement" is just a particular kind of interaction: one with an apparatus with a pointer or dial and whose effective interaction Hamiltonian can be written in the form
$$\widehat{H}_{meas} = a\widehat{Q}\widehat{p}_{dial}.$$
a: coupling parameter of the interaction
$\widehat{Q}$: operator associated with the "observable" in the SQF
$\widehat{p}_{dial}$: momentum operator of the pointer or dial
Theorem: Recovering the Born Rule
If an ensemble is in quantum equilibrium and some particular measurement is performed on each system in the ensemble, the relative frequencies of measurement outcomes will match those predicted by the Born rule.
Philipp Roser
Quantum Computing with de Broglie-Bohm


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:23 PM
Bell's model – Evolution
Identify abstract state vectors with spin states in the obvious way, i.e. initial state is
$$\psi(y, t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_A \otimes \phi_0(y)$$
Evolution in gates leaves pointer (and hence the ensemble distribution) unaffected but only changes spin states. The final state just before the time t_{meas} of measurement is
$$\psi(y, t_{meas} - \epsilon) = \begin{cases} \pm\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) = f(1) \\ \pm\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) \neq f(1) \end{cases}$$
Philipp Roser
Quantum Computing with de Broglie-Bohm

Preview File Edit View Go Tools Bookmarks Window Help Tue 4:23 PM

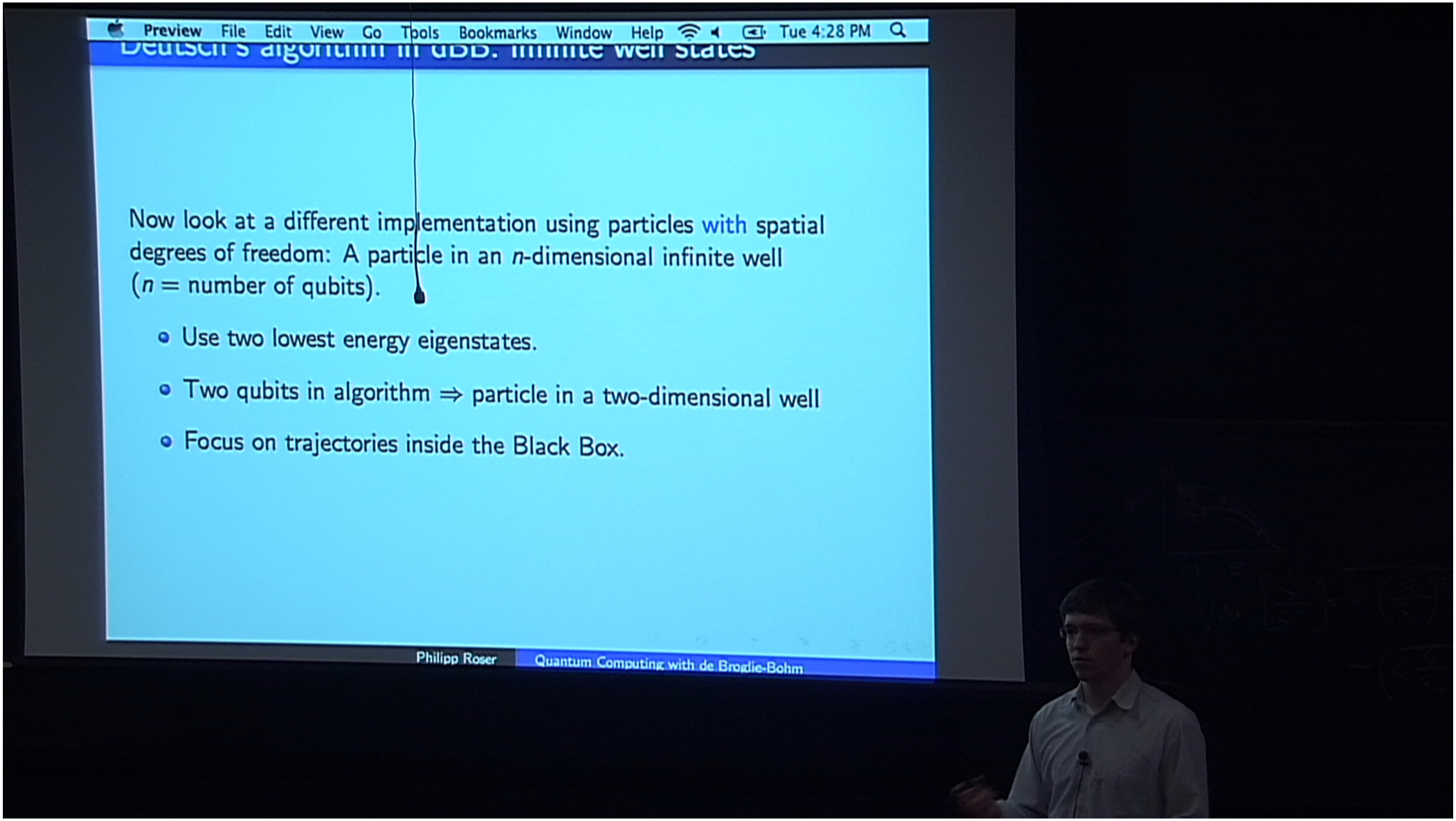
Bell's model – Evolution

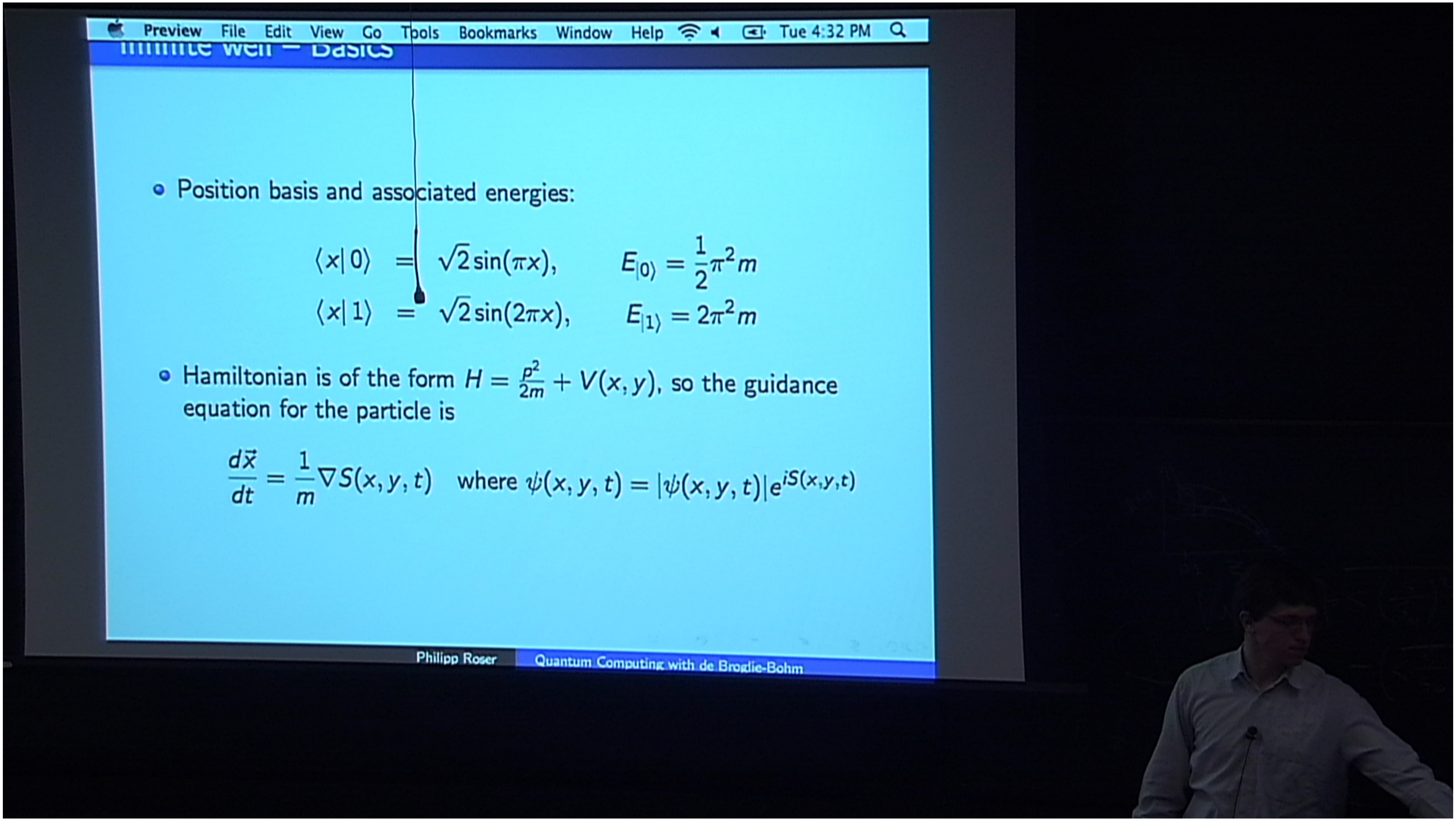
- Identify abstract state vectors with spin states in the obvious way, i.e. initial state is

$$\psi(y, t=0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_A \otimes \phi_0(y)$$

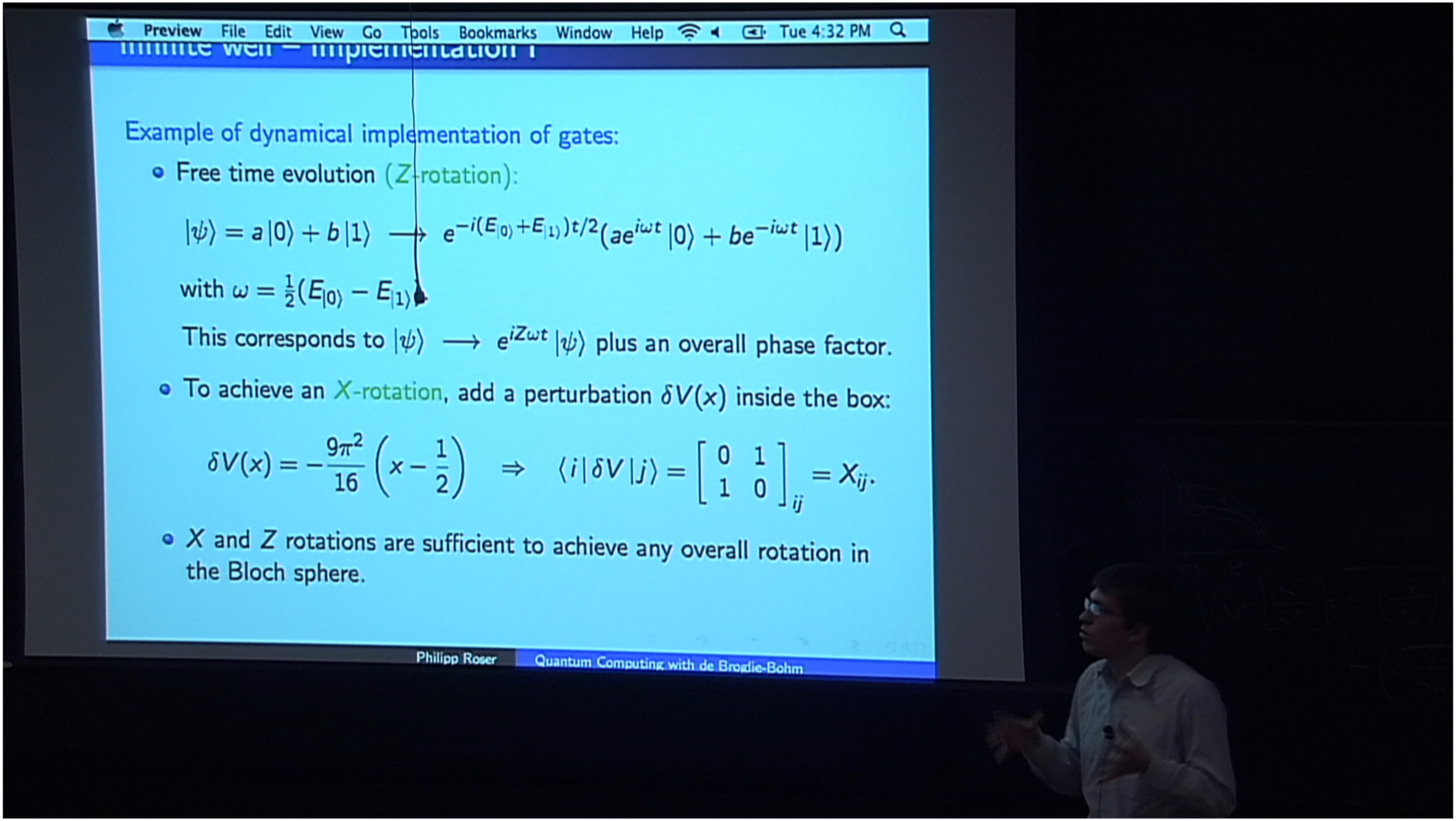
- Evolution in gates leaves pointer (and hence the ensemble distribution) unaffected but only changes spin states. The final state just before the time t_{meas} of measurement is

$$\psi(y, t_{meas} - \epsilon) = \begin{cases} \pm\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) = f(1) \\ \pm\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) \neq f(1) \end{cases}$$


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:28 PM
Now look at a different implementation using particles with spatial degrees of freedom: A particle in an n-dimensional infinite well (n = number of qubits).
Use two lowest energy eigenstates.
Two qubits in algorithm ⇒ particle in a two-dimensional well
Focus on trajectories inside the Black Box.
Philipp Roser
Quantum Computing with de Broglie-Bohm



Preview File Edit View Go Tools Bookmarks Window Help Tue 4:32 PM
Position basis and associated energies:
$\langle x|0\rangle = \sqrt{2}\sin(\pi x), \quad E_{|0\rangle} = \frac{1}{2}\pi^2 m$
$\langle x|1\rangle = \sqrt{2}\sin(2\pi x), \quad E_{|1\rangle} = 2\pi^2 m$
Hamiltonian is of the form $H = \frac{p^2}{2m} + V(x,y)$, so the guidance equation for the particle is
$\frac{d\vec{x}}{dt} = \frac{1}{m}\nabla S(x,y,t)$ where $\psi(x,y,t) = |\psi(x,y,t)|e^{iS(x,y,t)}$
Philipp Roser
Quantum Computing with de Broglie-Bohm


Example of dynamical implementation of gates:
Free time evolution (Z-rotation):
$|\psi\rangle = a|0\rangle + b|1\rangle \longrightarrow e^{-i(E_{|0\rangle}+E_{|1\rangle})t/2}(ae^{i\omega t}|0\rangle + be^{-i\omega t}|1\rangle)$
with $\omega = \frac{1}{2}(E_{|0\rangle} - E_{|1\rangle})$.
This corresponds to $|\psi\rangle \longrightarrow e^{iZ\omega t}|\psi\rangle$ plus an overall phase factor.
To achieve an X-rotation, add a perturbation $\delta V(x)$ inside the box:
$\delta V(x) = -\frac{9\pi^2}{16}\left(x - \frac{1}{2}\right) \Rightarrow \langle i|\delta V|j\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{ij} = X_{ij}.$
X and Z rotations are sufficient to achieve any overall rotation in the Bloch sphere.
Philipp Roser
Quantum Computing with de Broglie-Bohm

Infinite well — implementation II

One possibility to implement the two-qubit evolution inside the black box:

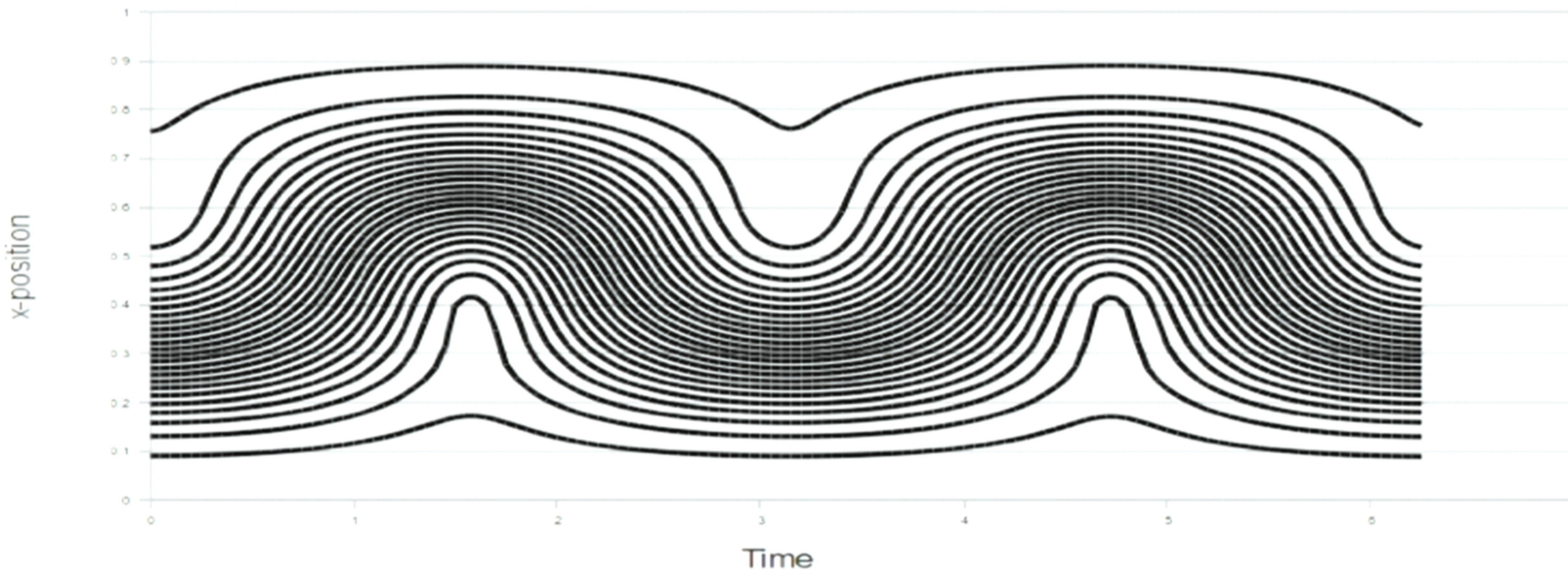
$$U(x,y) = (A + B\cos(x) + Cx\cos(x))\left[-\frac{9\pi^2}{16}\left(y - \frac{1}{2}\right) - 1\right]$$

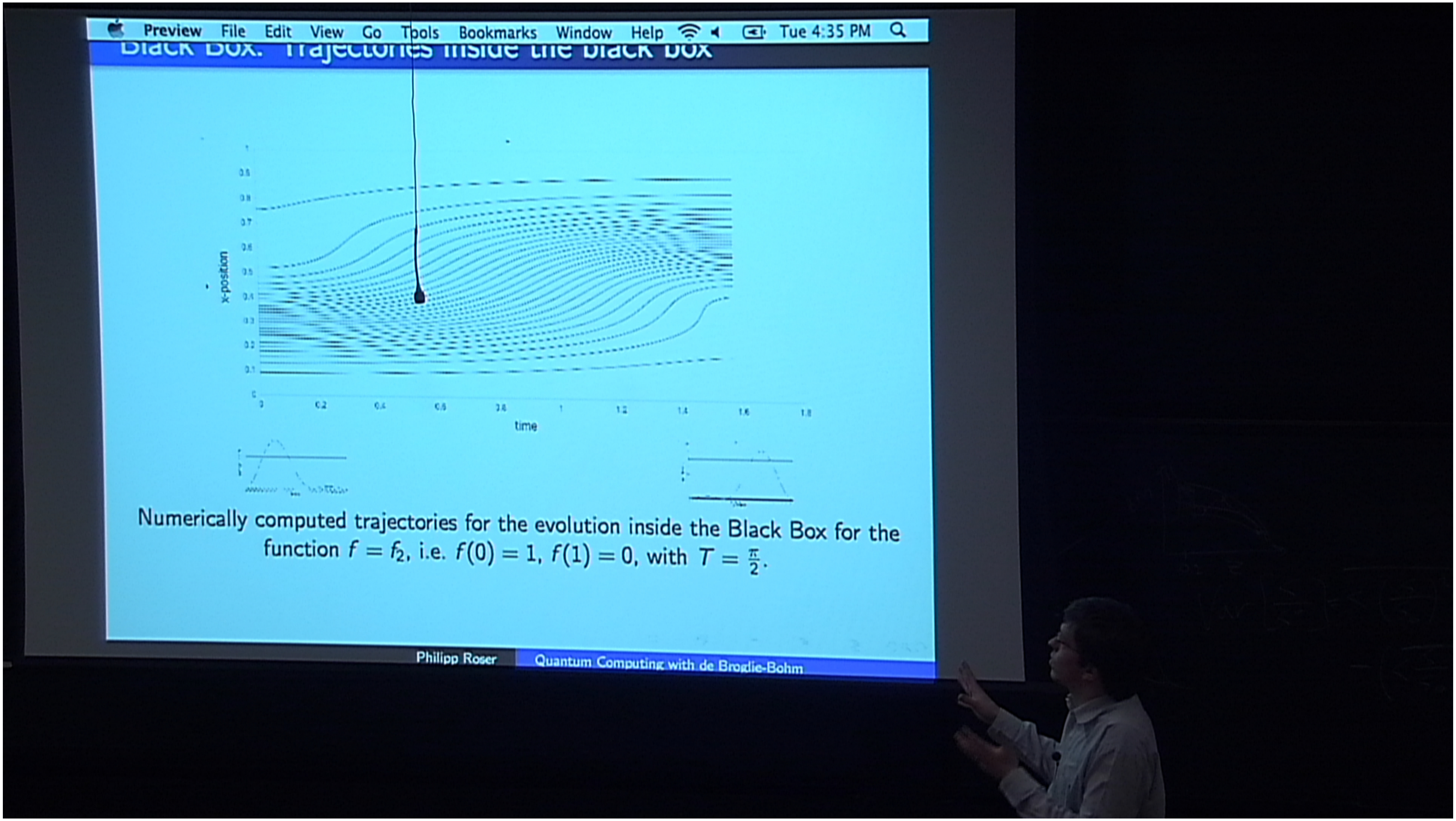
with

$$A = \frac{52}{27}, \qquad B = -\frac{225}{432}\pi^2, \qquad C = \frac{225}{216}\pi^2$$

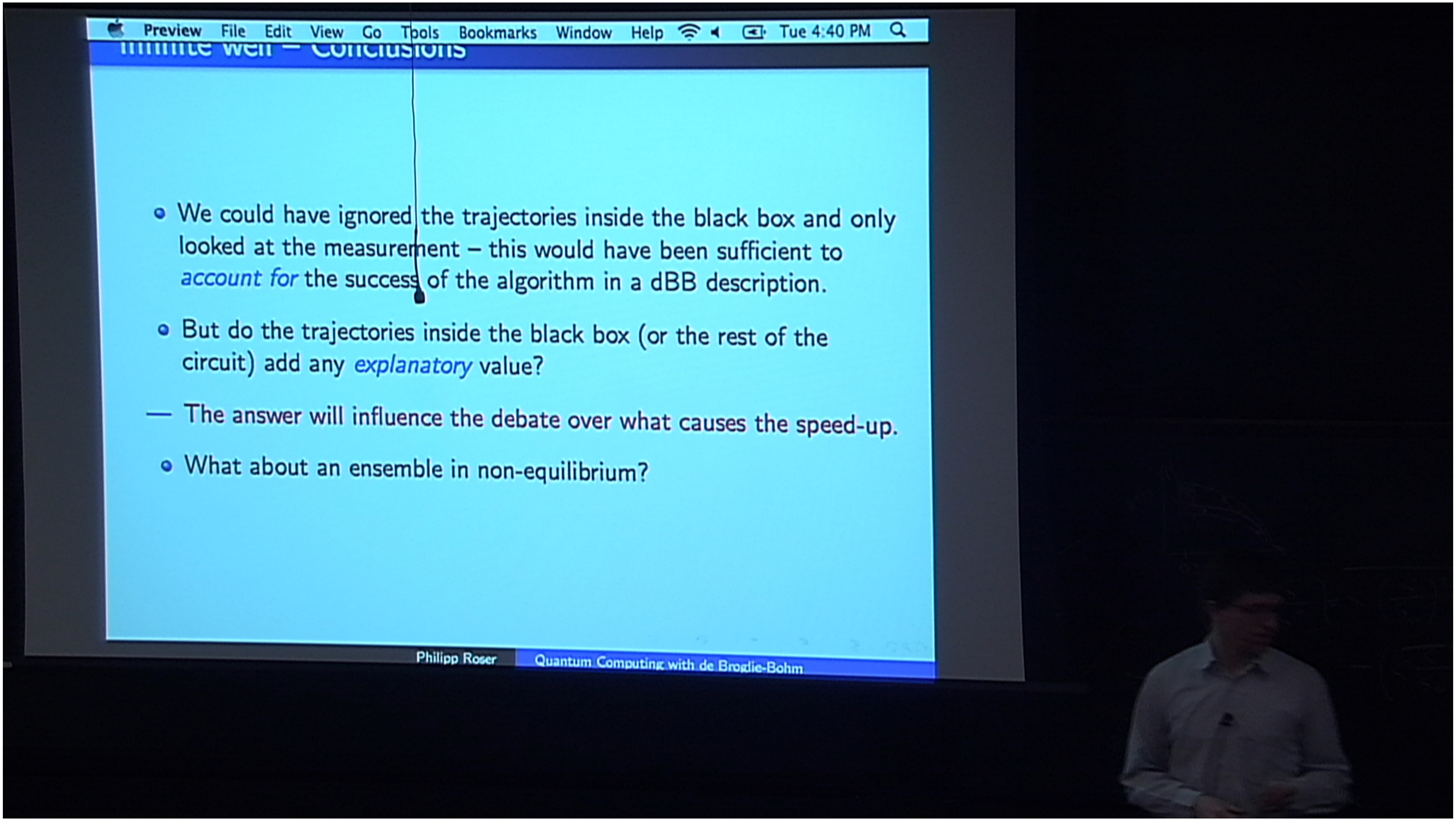
Preview File Edit View Go Tools Bookmarks Window Help Tue 4:34 PM

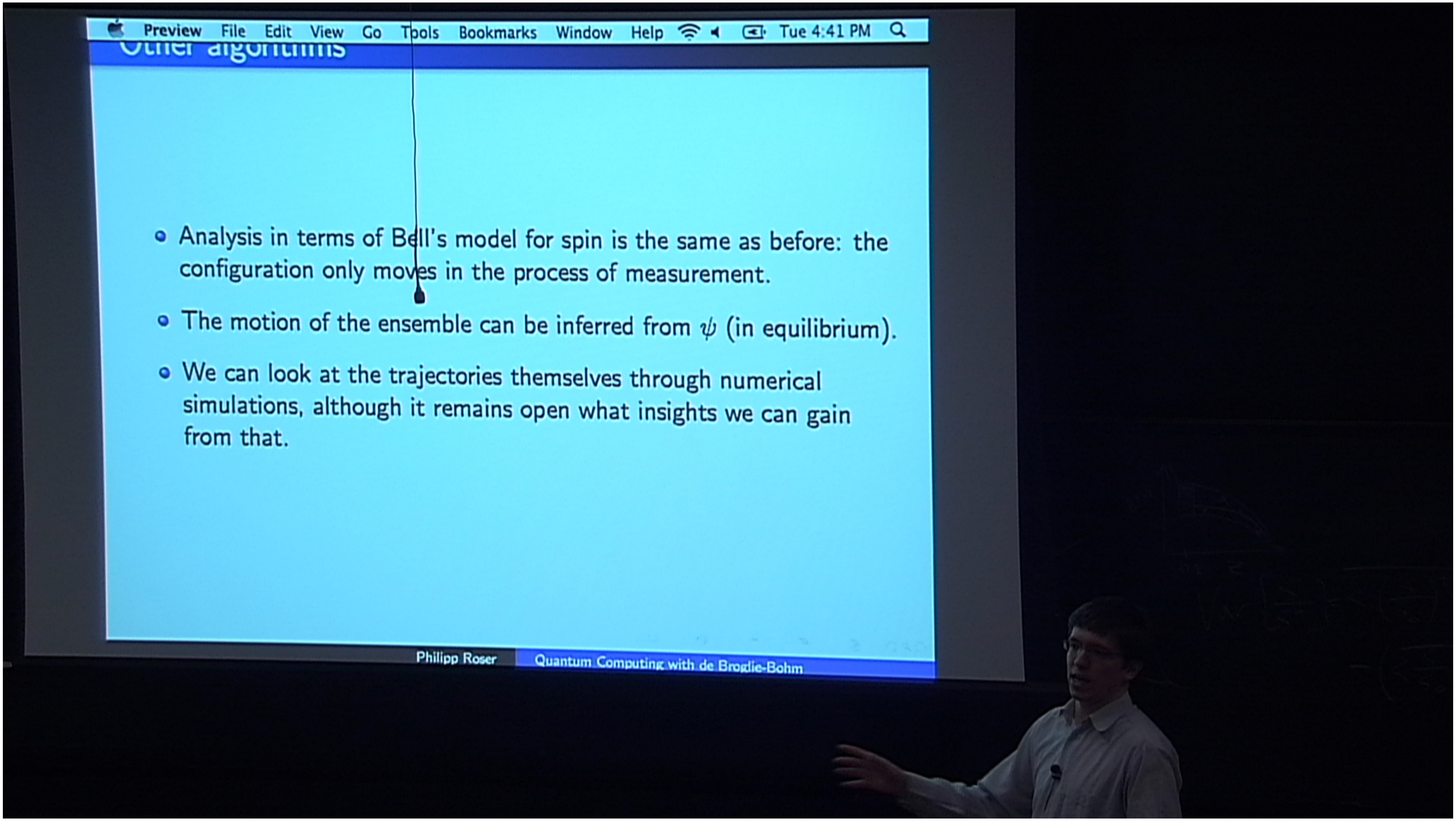
Infinite well – Free evolution

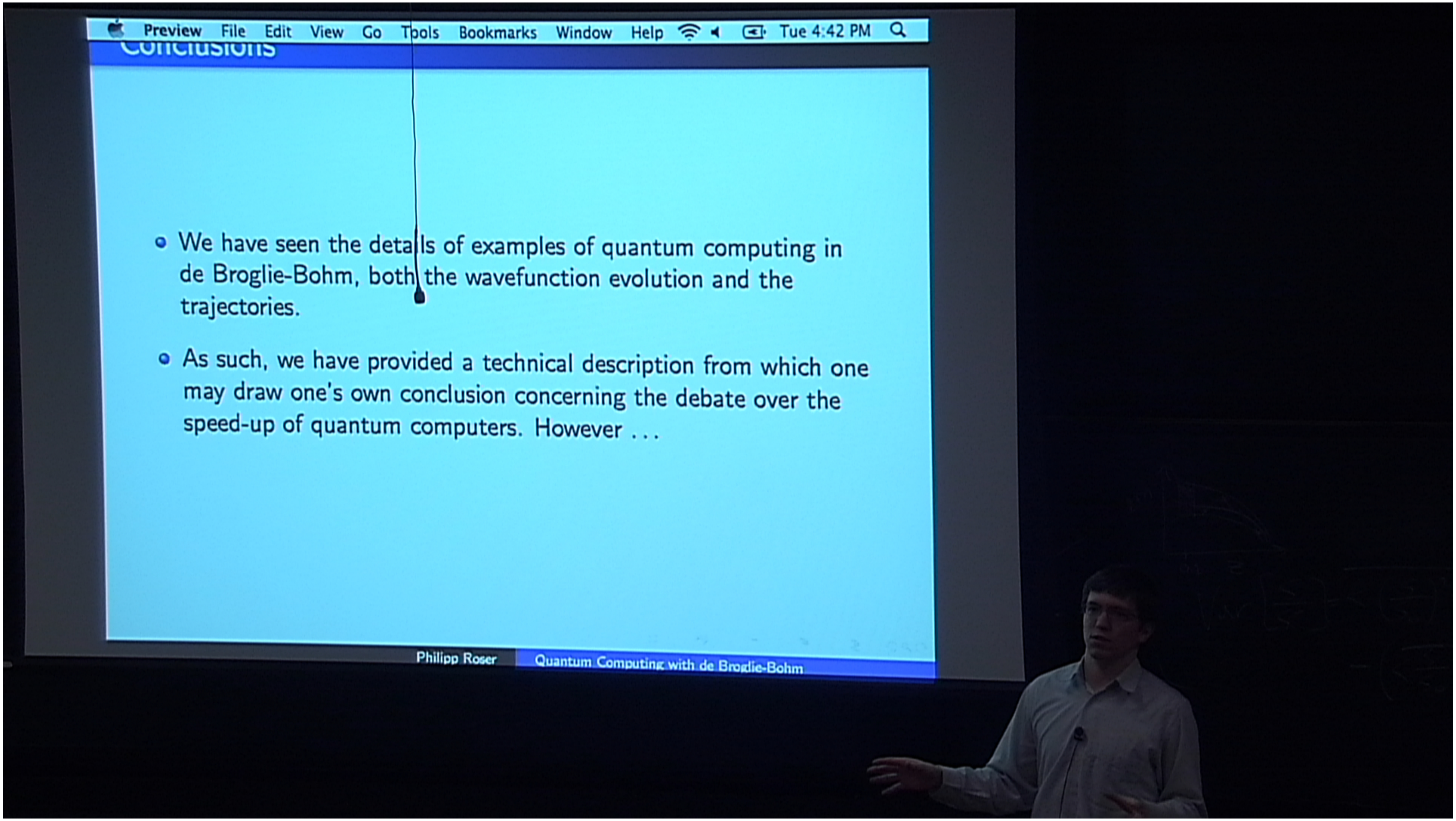


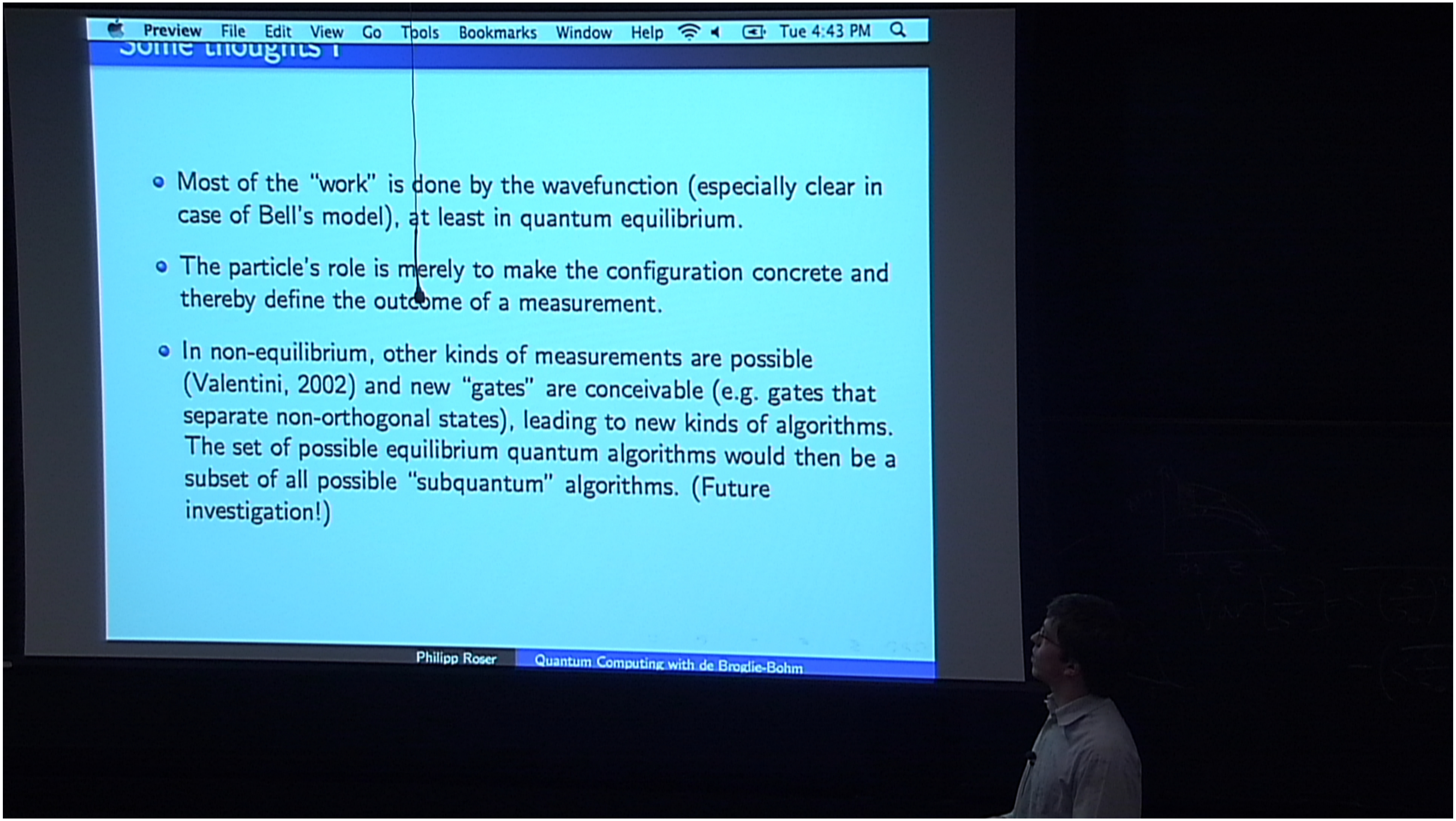


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:35 PM
Black Box: Trajectories inside the black box
x-position
time
Numerically computed trajectories for the evolution inside the Black Box for the function $f = f_2$, i.e. $f(0) = 1$, $f(1) = 0$, with $T = \frac{\pi}{2}$.
Philipp Roser
Quantum Computing with de Broglie-Bohm


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:40 PM
We could have ignored the trajectories inside the black box and only looked at the measurement – this would have been sufficient to account for the success of the algorithm in a dBB description.
But do the trajectories inside the black box (or the rest of the circuit) add any explanatory value?
— The answer will influence the debate over what causes the speed-up.
What about an ensemble in non-equilibrium?
Philipp Roser
Quantum Computing with de Broglie-Bohm


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:41 PM
Other algorithms
Analysis in terms of Bell's model for spin is the same as before: the configuration only moves in the process of measurement.
The motion of the ensemble can be inferred from ψ (in equilibrium).
We can look at the trajectories themselves through numerical simulations, although it remains open what insights we can gain from that.
Philipp Roser
Quantum Computing with de Broglie-Bohm


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:42 PM
Conclusions
We have seen the details of examples of quantum computing in de Broglie-Bohm, both the wavefunction evolution and the trajectories.
As such, we have provided a technical description from which one may draw one's own conclusion concerning the debate over the speed-up of quantum computers. However . . .
Philipp Roser
Quantum Computing with de Broglie-Bohm


Preview File Edit View Go Tools Bookmarks Window Help Tue 4:43 PM
Most of the "work" is done by the wavefunction (especially clear in case of Bell's model), at least in quantum equilibrium.
The particle's role is merely to make the configuration concrete and thereby define the outcome of a measurement.
In non-equilibrium, other kinds of measurements are possible (Valentini, 2002) and new "gates" are conceivable (e.g. gates that separate non-orthogonal states), leading to new kinds of algorithms. The set of possible equilibrium quantum algorithms would then be a subset of all possible "subquantum" algorithms. (Future investigation!)
Philipp Roser
Quantum Computing with de Broglie-Bohm


inds of algorithms.
ns would then be a
(Future
de Broglie-Bohm

Some thoughts I

- Most of the "work" is done by the wavefunction (especially clear in case of Bell's model), at least in quantum equilibrium.
- The particle's role is merely to make the configuration concrete and thereby define the outcome of a measurement.
- In non-equilibrium, other kinds of measurements are possible (Valentini, 2002) and new "gates" are conceivable (e.g. gates that separate non-orthogonal states), leading to new kinds of algorithms. The set of possible equilibrium quantum algorithms would then be a subset of all possible "subquantum" algorithms. (Future investigation!)