Title: Bell's Theorem and Bayesian Networks

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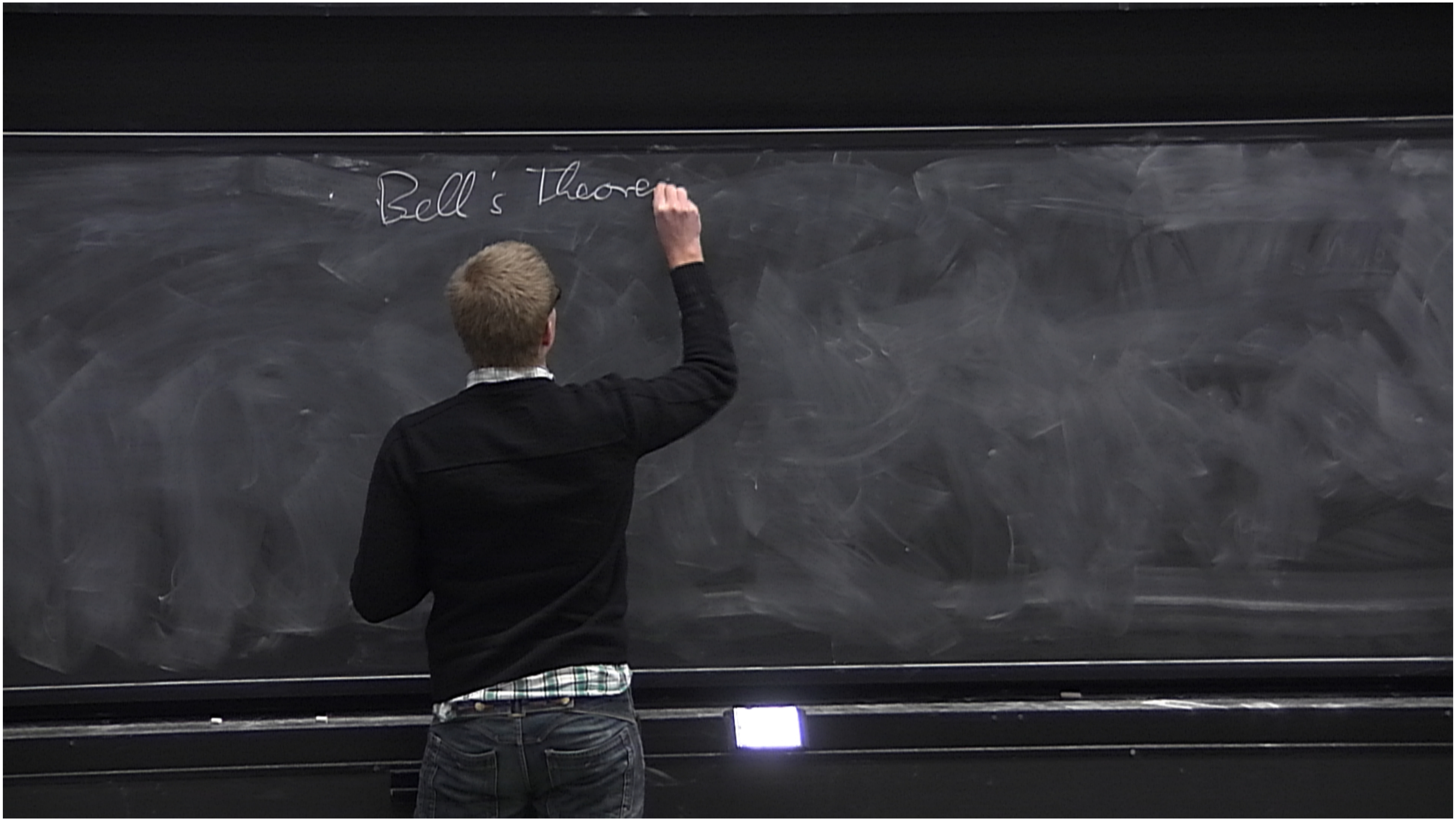
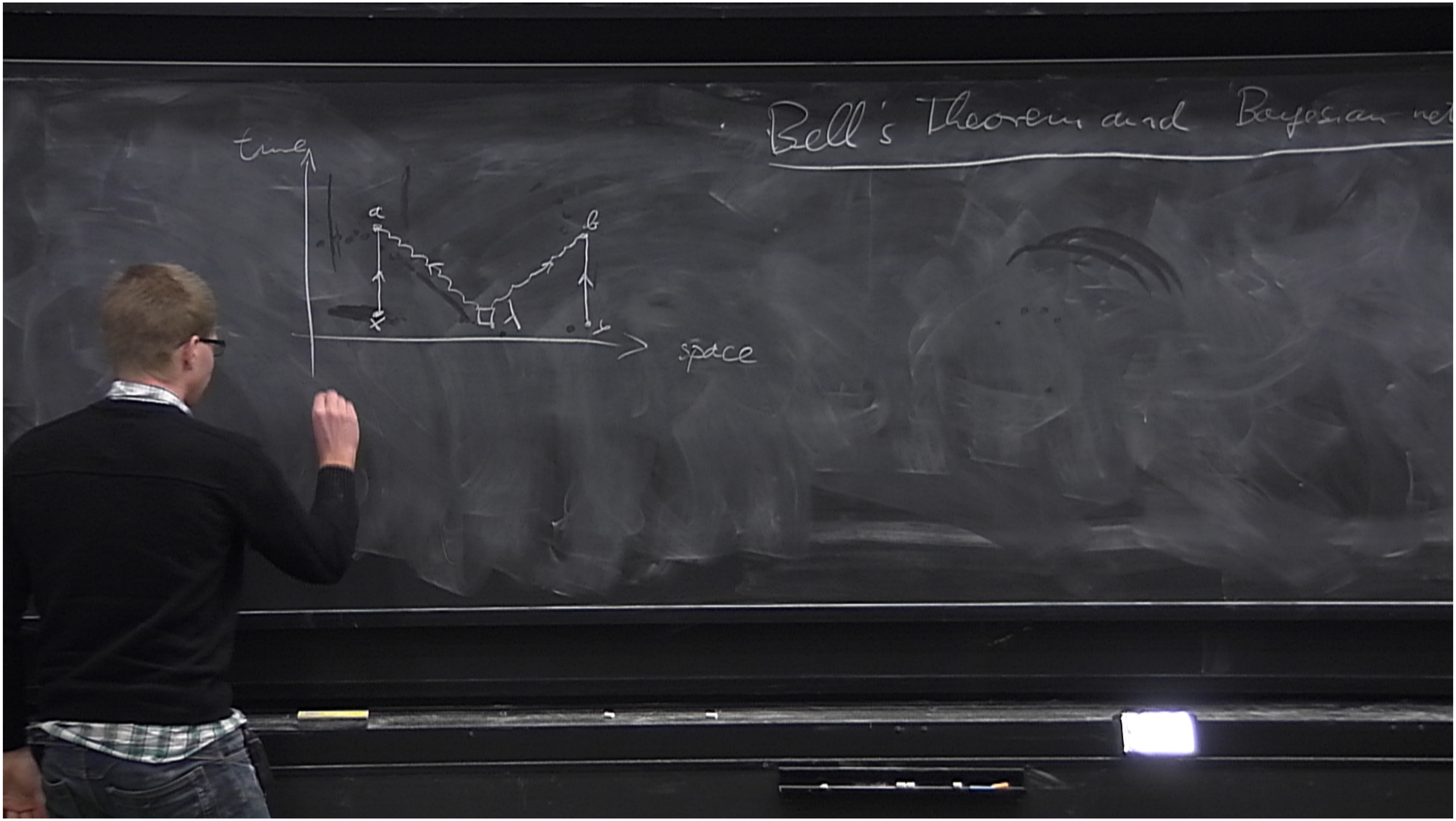
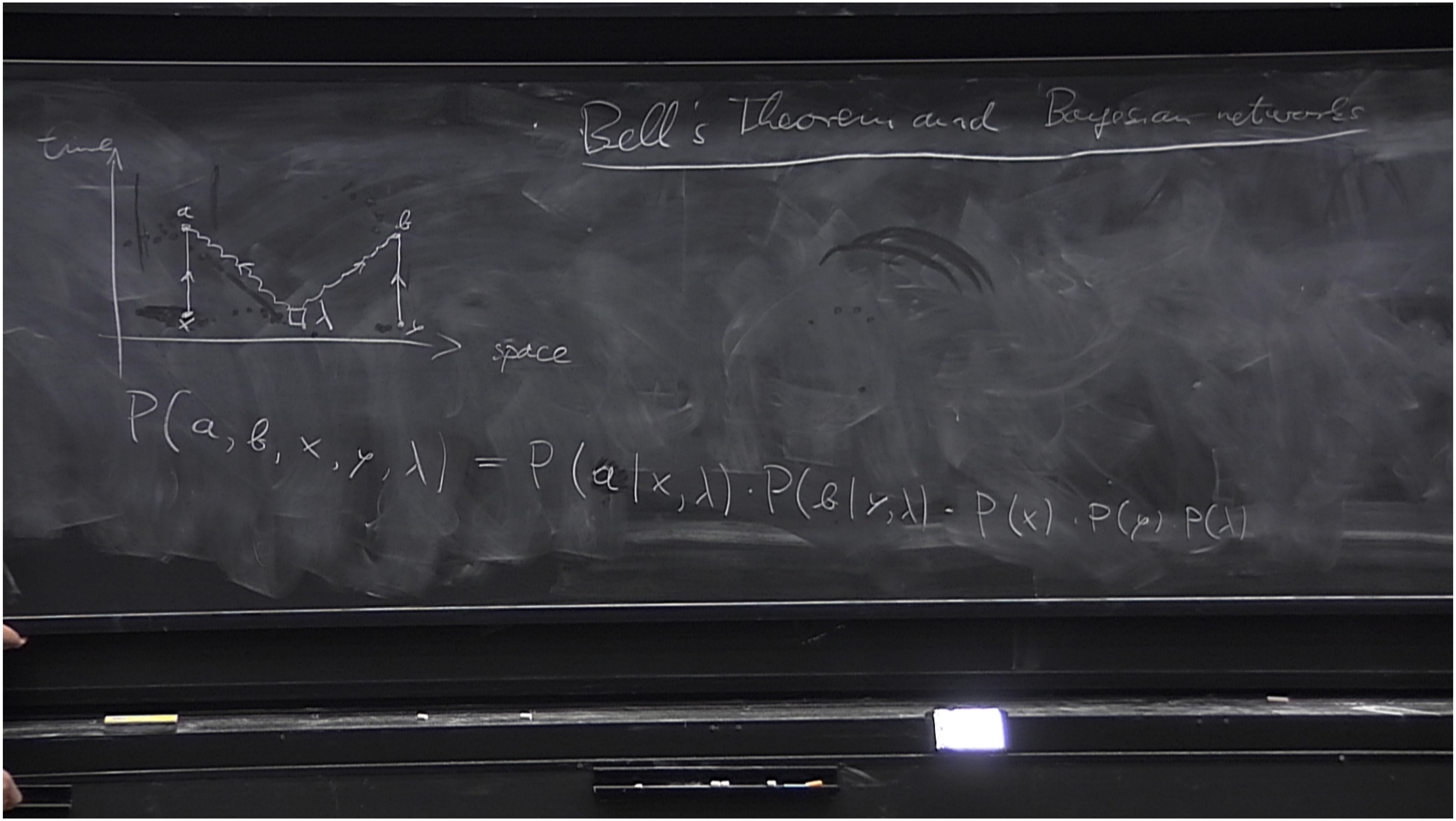
URL: http://pirsa.org/12020134

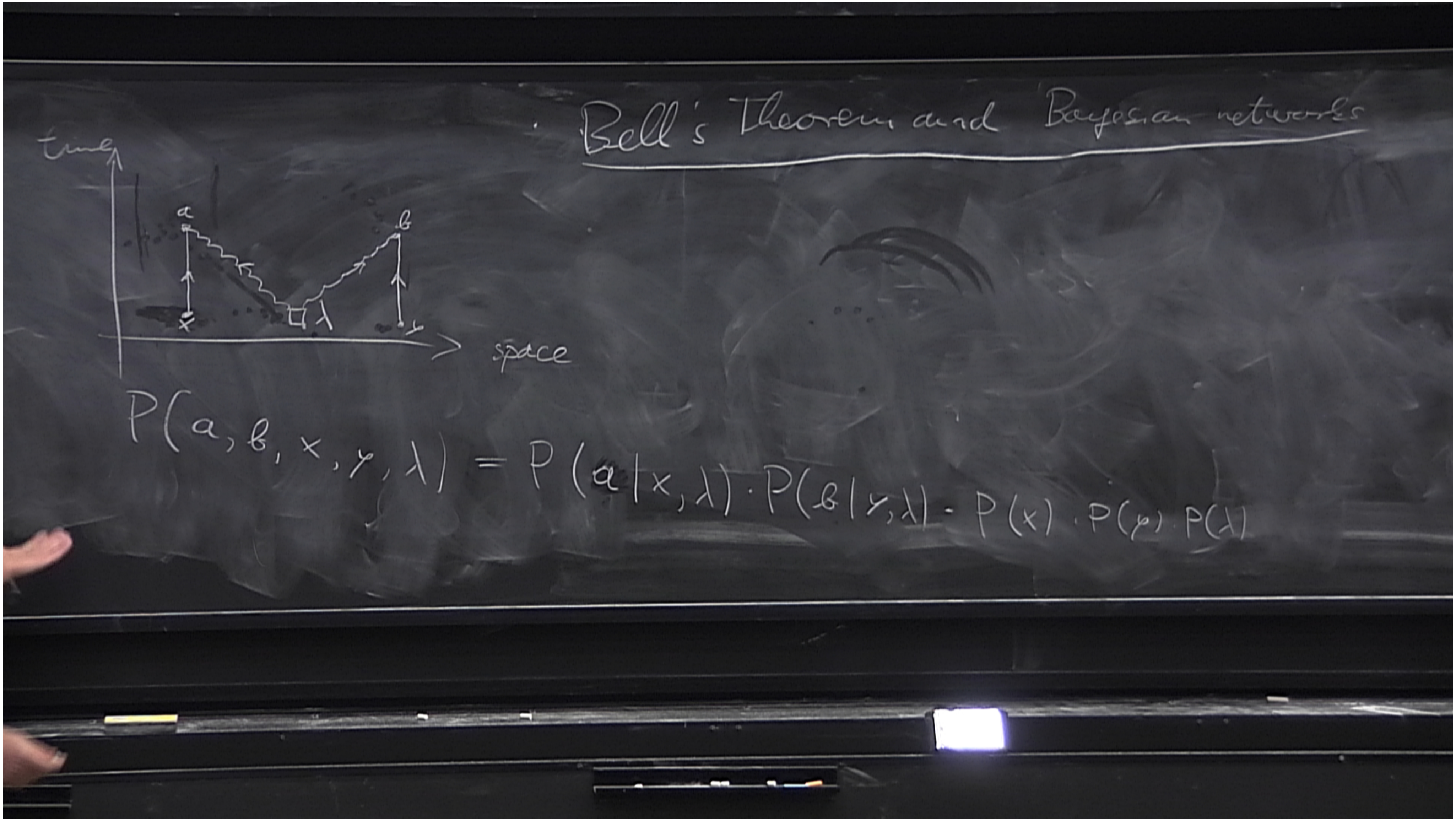
Abstract: The standard approach to quantum nonlocality (Bell's Theorem) relies on the assumption of the existence of "free will". I will explain how to get rid of this mysterious assumption in favor of the independence of sources. From this new point of view, Bell's Theorem becomes a statement about Bayesian networks. Besides allowing a more intuitive formulation of the standard result, our formalism also provides new network topologies giving rise to new kinds of nonlocality. Some of these relate to results by Steudel and Ay on the statistical inference of causal relations. Witnessing quantum nonlocality in new network topologies is a challenge which I will pose as an open problem.


Bell's Theore


Bell's Theorem and Bayesian net
time
space


Bell's Theorem and Bayesian net
time
a
b
x
λ
y
space


Bell's Theorem and Bayesian networks
time
space
a
b
x
y
λ
$P(a, b, x, y, \lambda) = P(a|x,\lambda) \cdot P(b|y,\lambda) \cdot P(x) \cdot P(y) \cdot P(\lambda)$


Bell's Theorem and Bayesian networks
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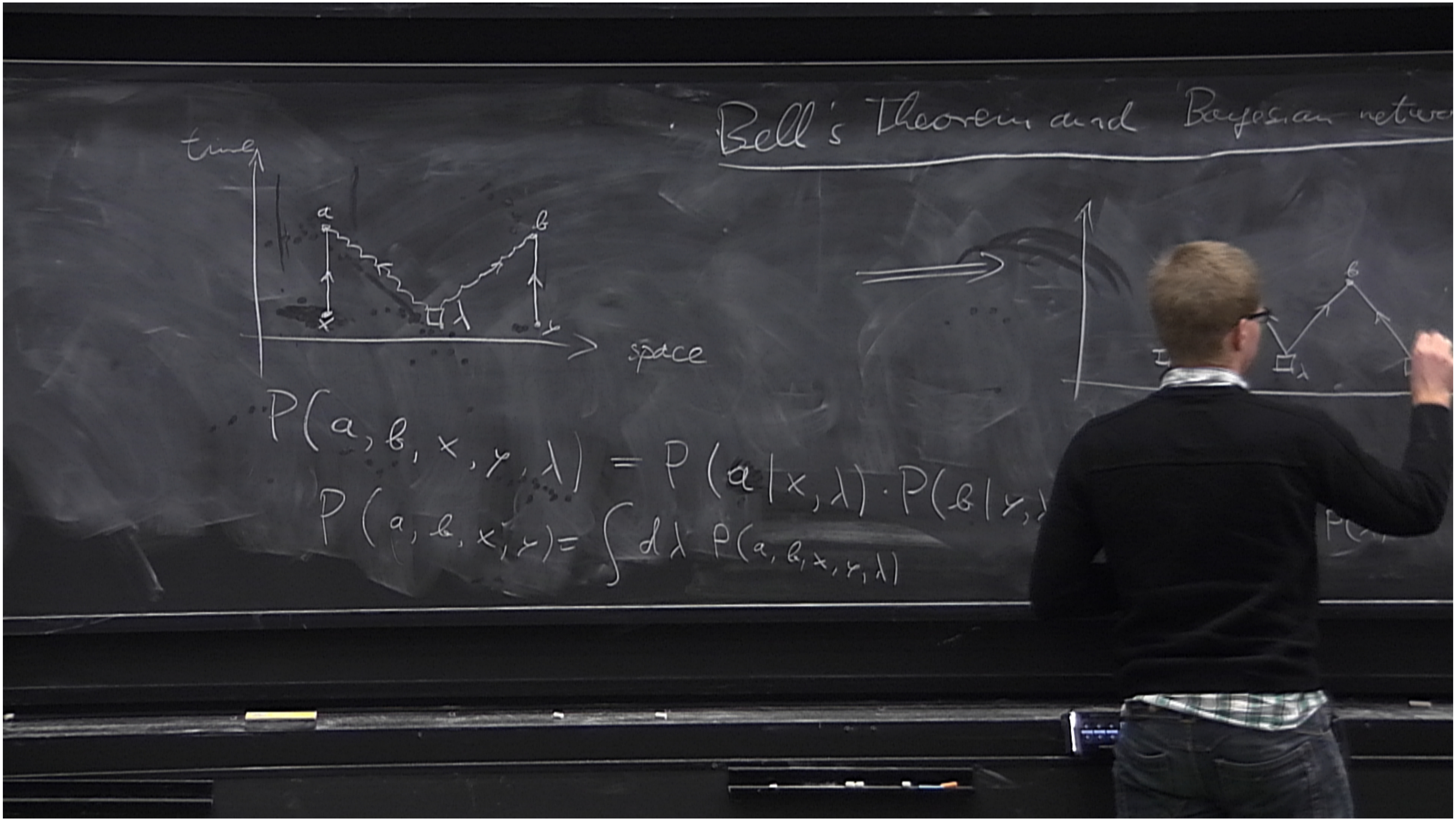
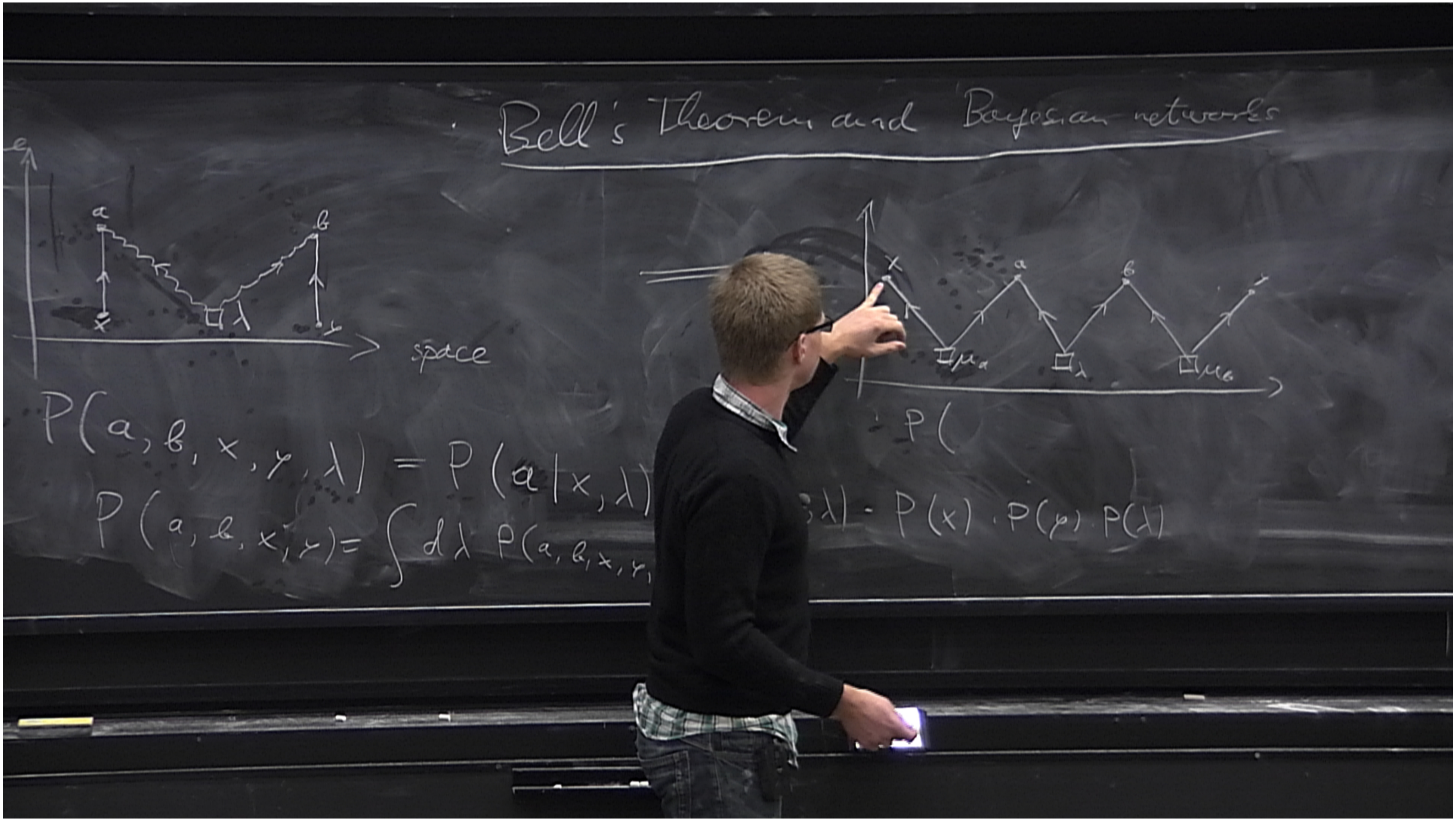
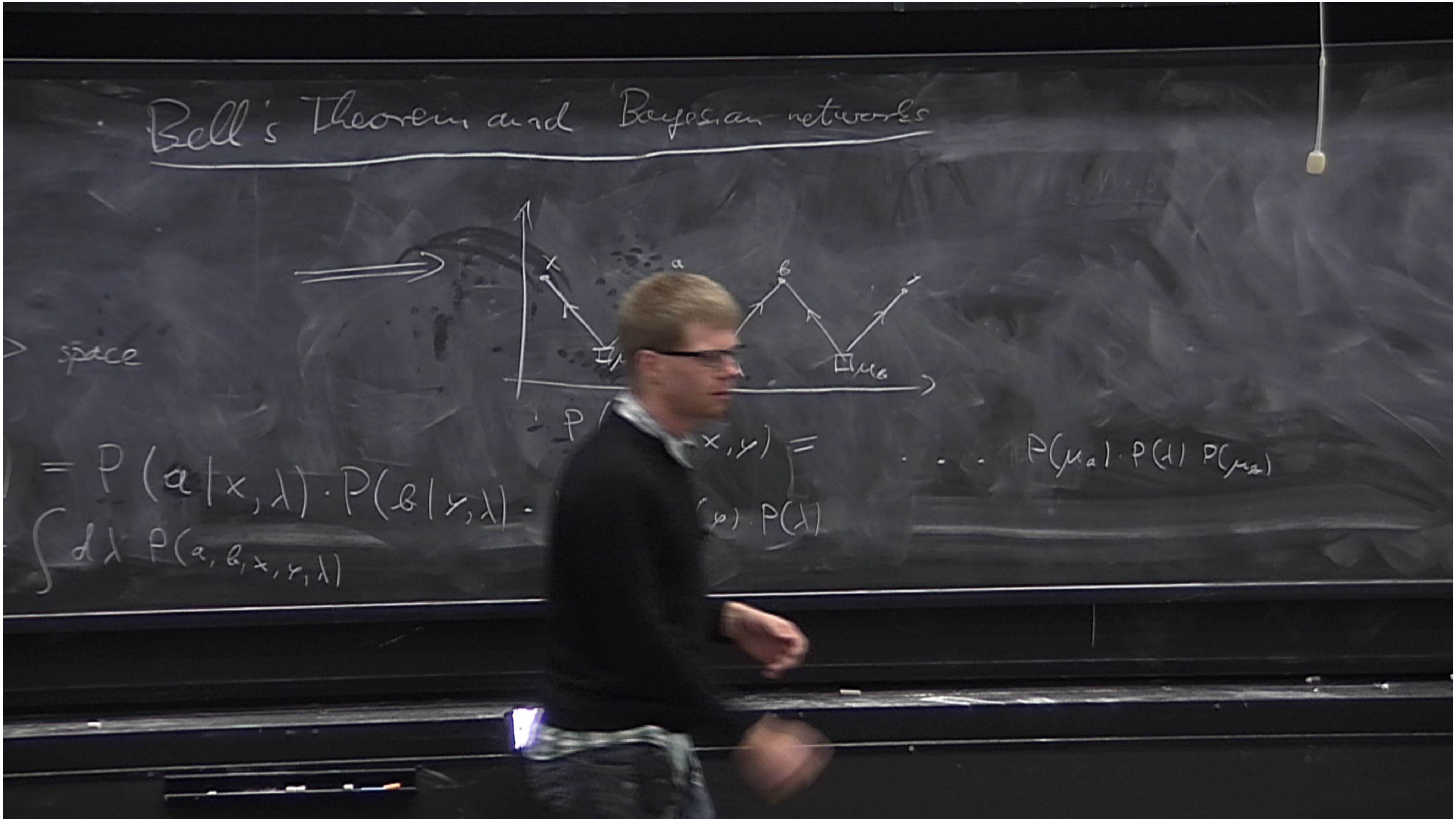
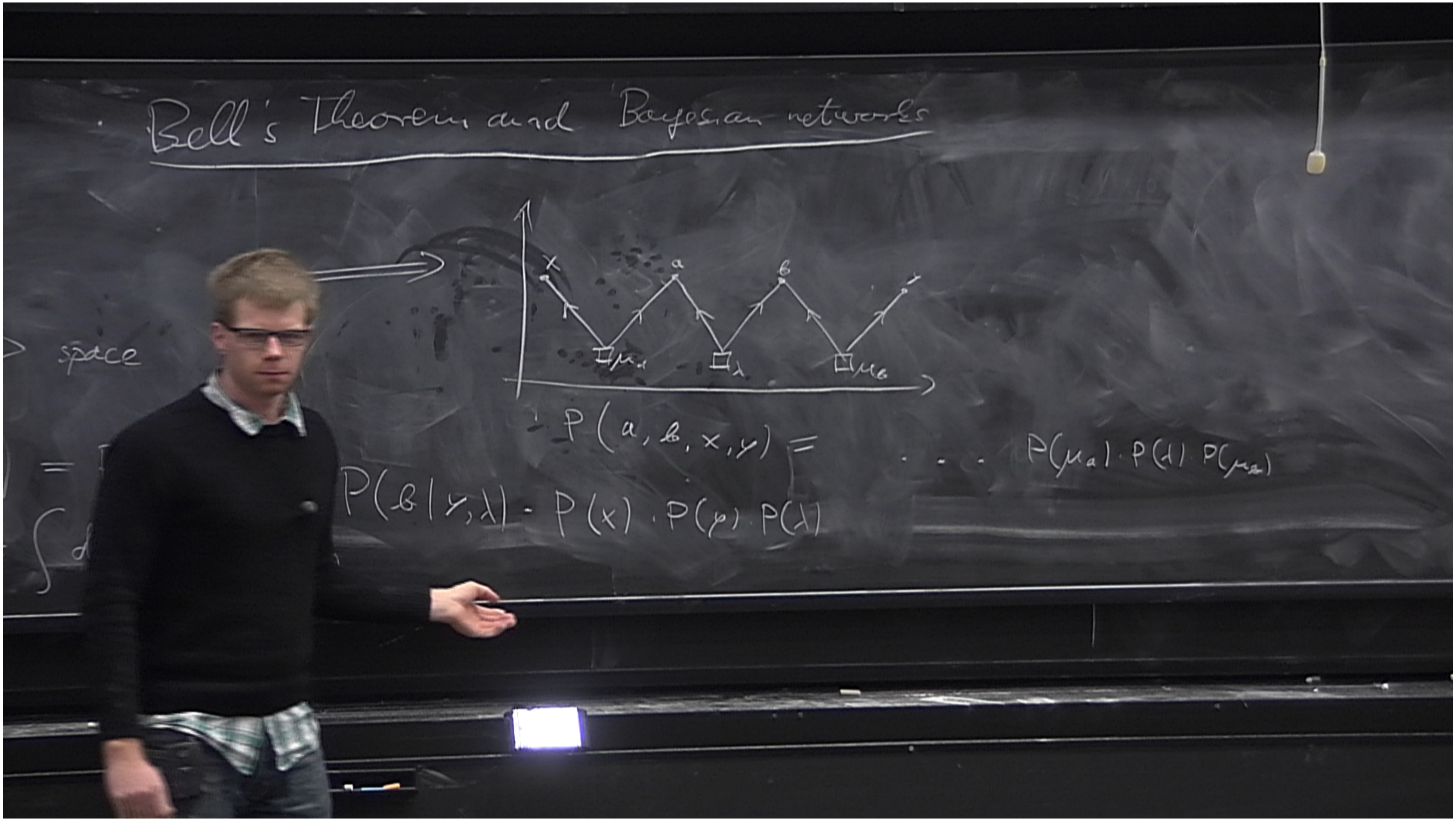
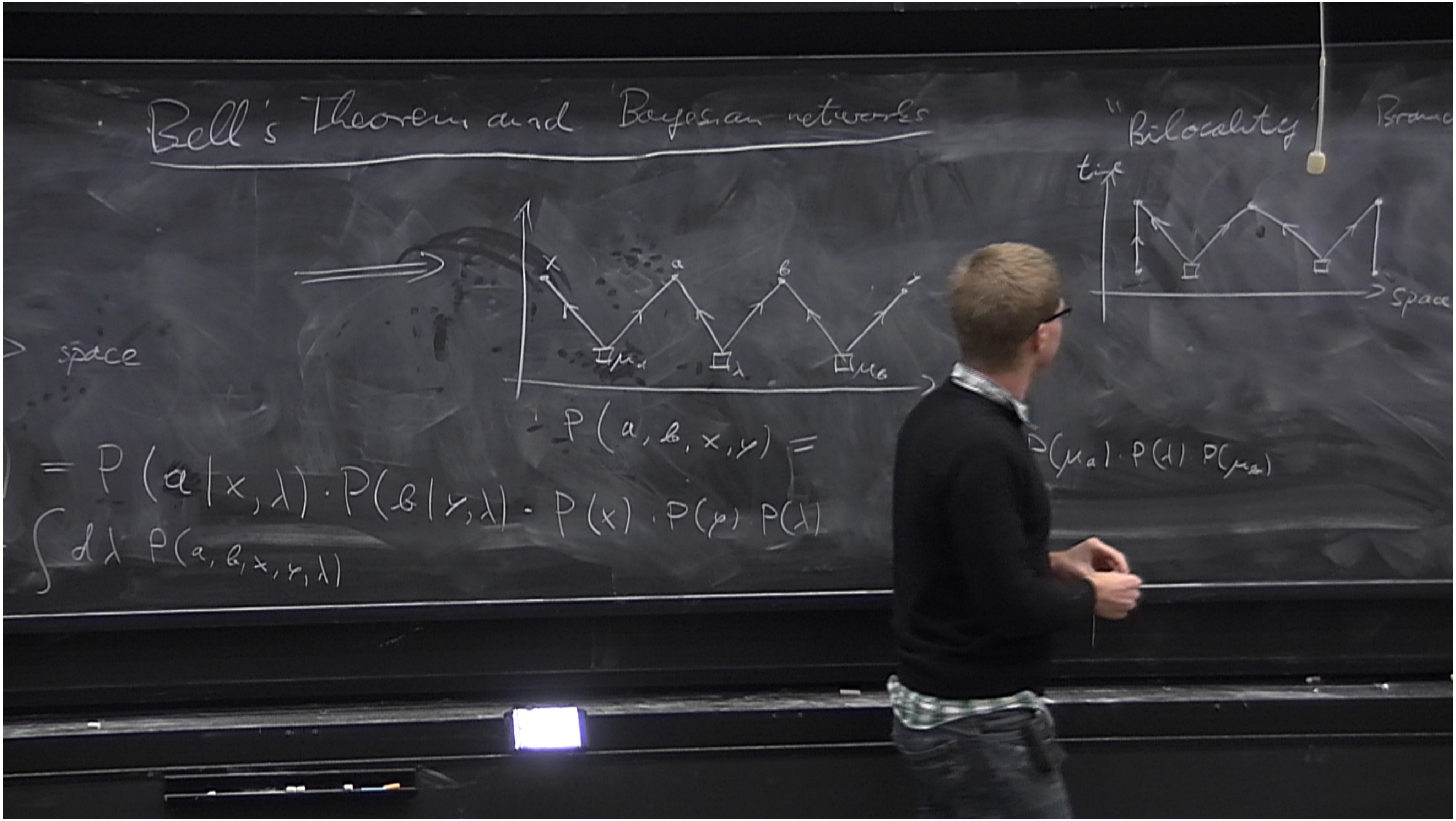


Bell's Theorem and Bayesian
time
space
a
b
x
y
λ
P(a, b, x, y, λ) = P(a | x, λ) · P(b | y, λ)
P(a, b, x, y) = ∫ dλ P(a, b, x, y, λ)

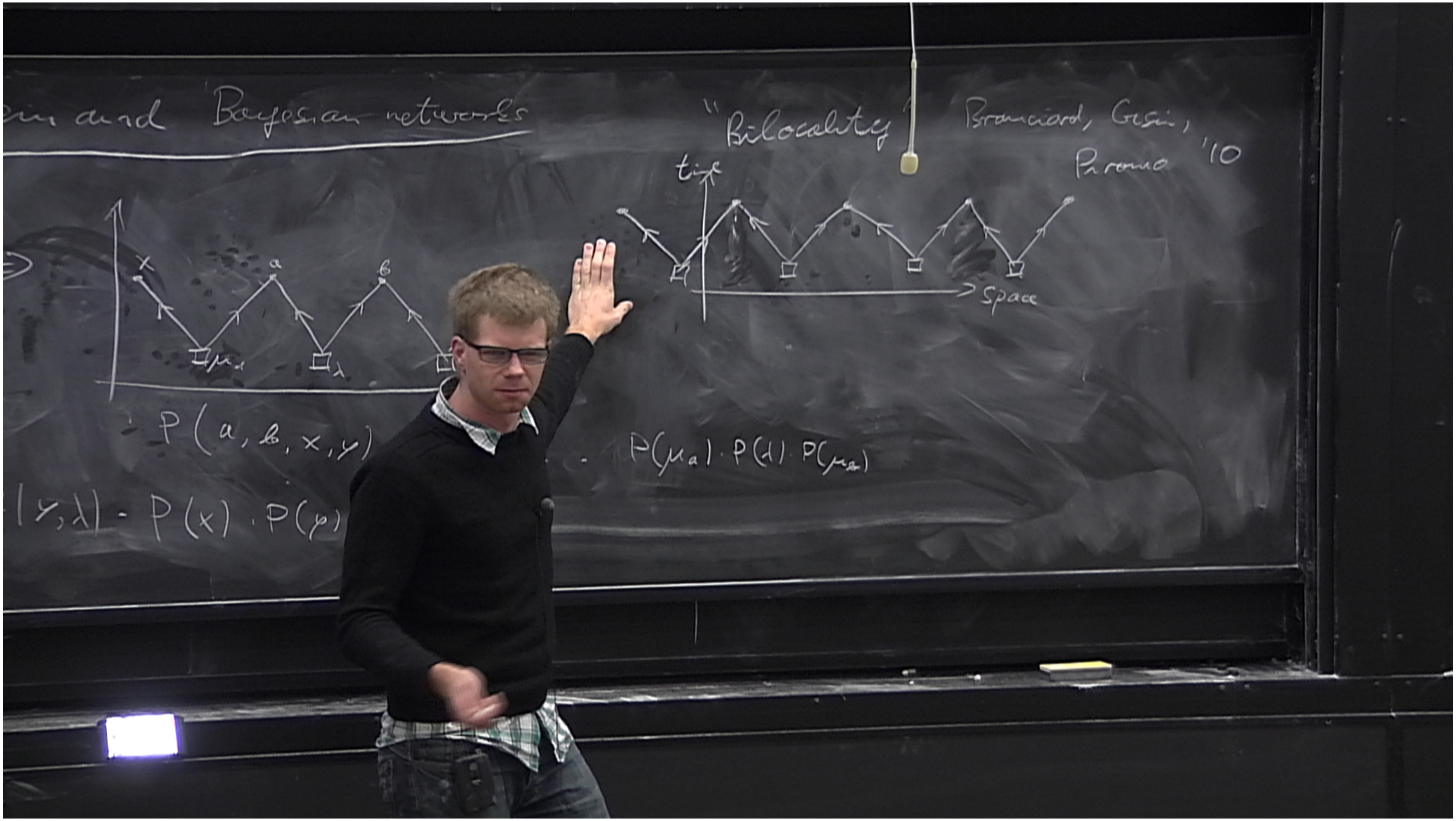

Bell's Theorem and Bayesian networks
space
P(a, b, x, y, λ) = P(a | x, λ)
P(a, b, x, y) = ∫ dλ P(a, b, x, y,
λ) · P(x) · P(y) · P(λ)
P(

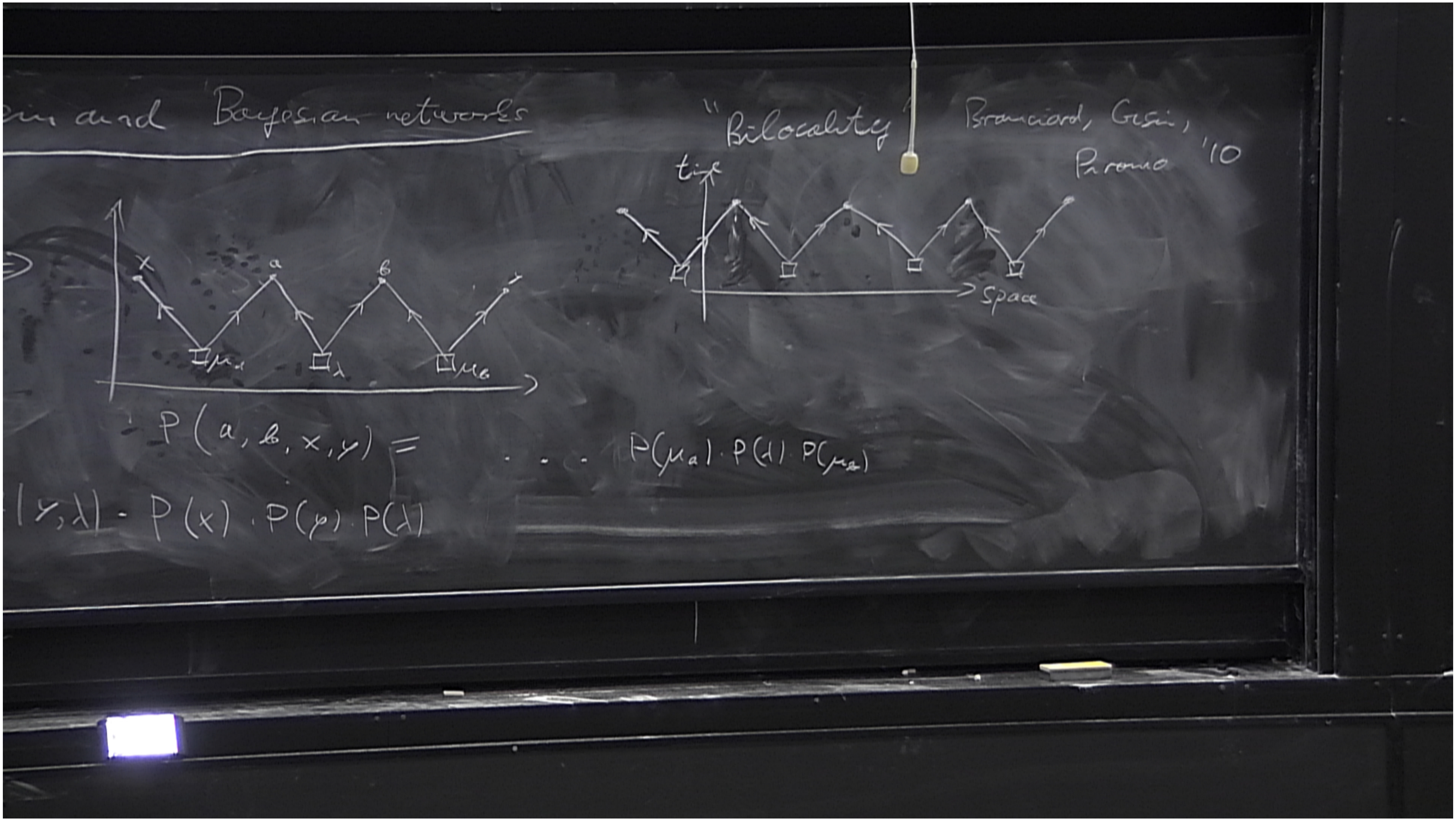

Bell's Theorem and Bayesian networks
space
P(a | x, λ) · P(b | y, λ)
∫ dλ P(a, b, x, y, λ)
P(μa) · P(λ) · P(μb)

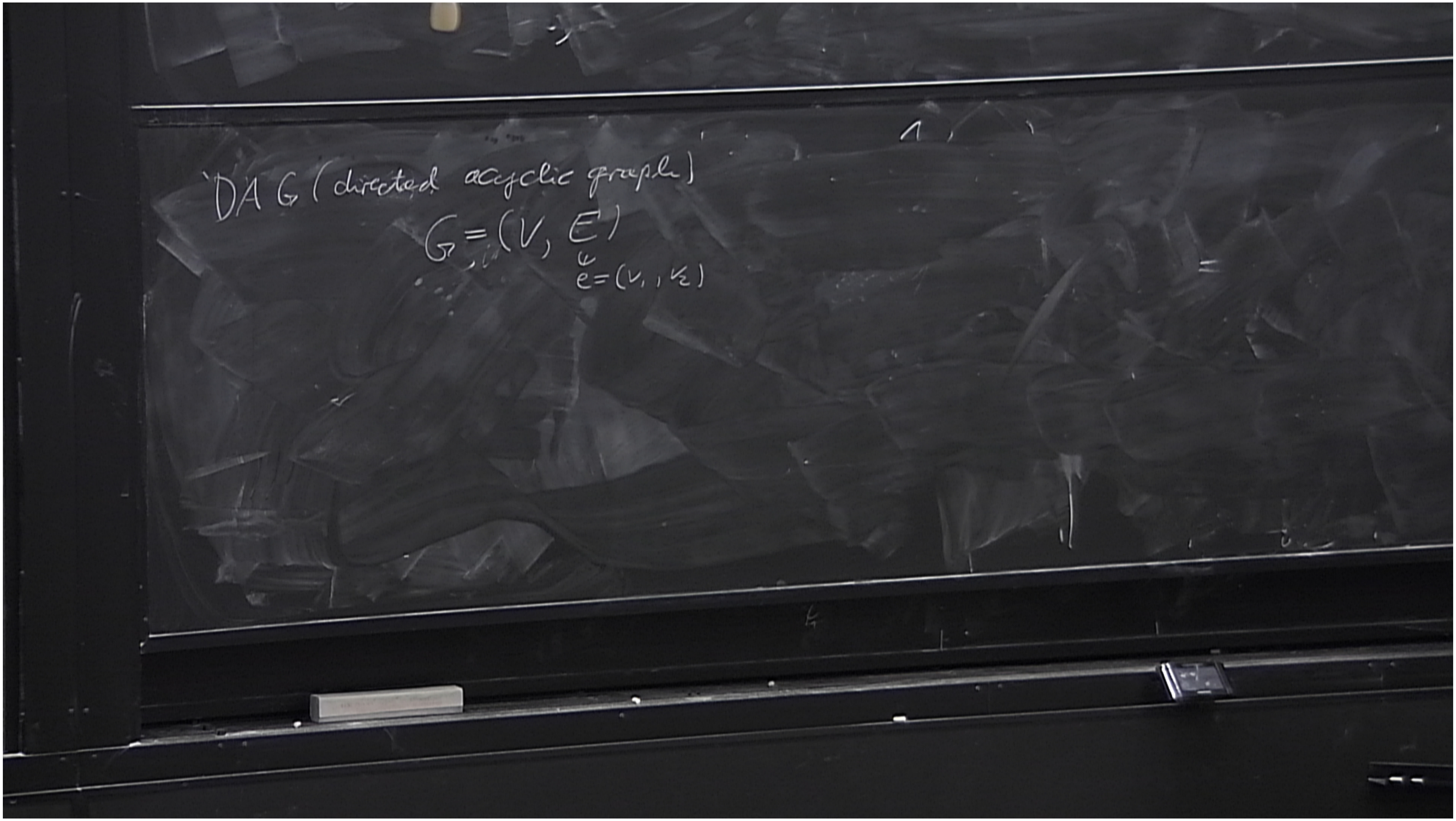

Bell's Theorem and Bayesian networks
space
P(a, b, x, y) =
P(b|y,λ) · P(x) · P(y) · P(λ)
P(μa) · P(λ) · P(μb)



Bell's Theorem and Bayesian networks
"Bilocality"
space
time
space
$P(a,b,x,y) =$
$= P(a|x,\lambda) \cdot P(b|y,\lambda) \cdot P(x) \cdot P(y) \cdot P(\lambda)$
$\int d\lambda\, P(a,b,x,y,\lambda)$


and Bayesian networks
"Bilocality"
Branciard, Gisin, Pironio '10
time
space
P(a, b, x, y)
P(μa) · P(λ) · P(μb)
P(x) · P(y)


and Bayesian networks
"Bilocality"
Branciard, Gisin, Pironio '10
time
space
P(a, b, x, y) =
P(μa)·P(λ)·P(μb)
P(x)·P(y)·P(λ)

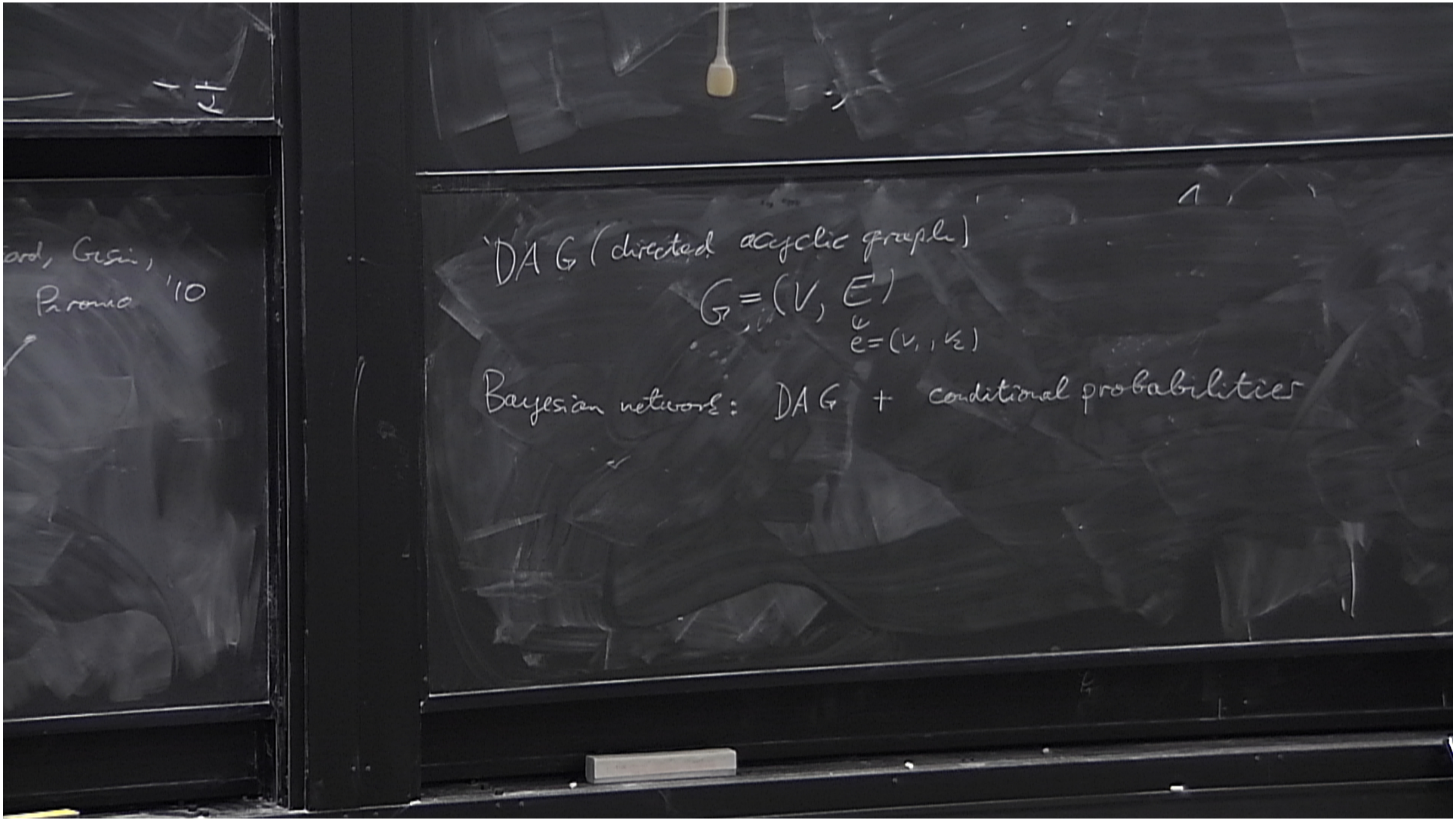

DAG (directed acyclic graph)
$G=(V, E)$
$e=(v_1, v_2)$

DAG G (directed acyclic graph)

$$G = (V, E)$$

$$\underset{e = (v_1, v_2)}{\overset{\cup}{}}$$

Bayesian network: DAG + conditional probabilities


DAG (directed acyclic graph)
G = (V, E)
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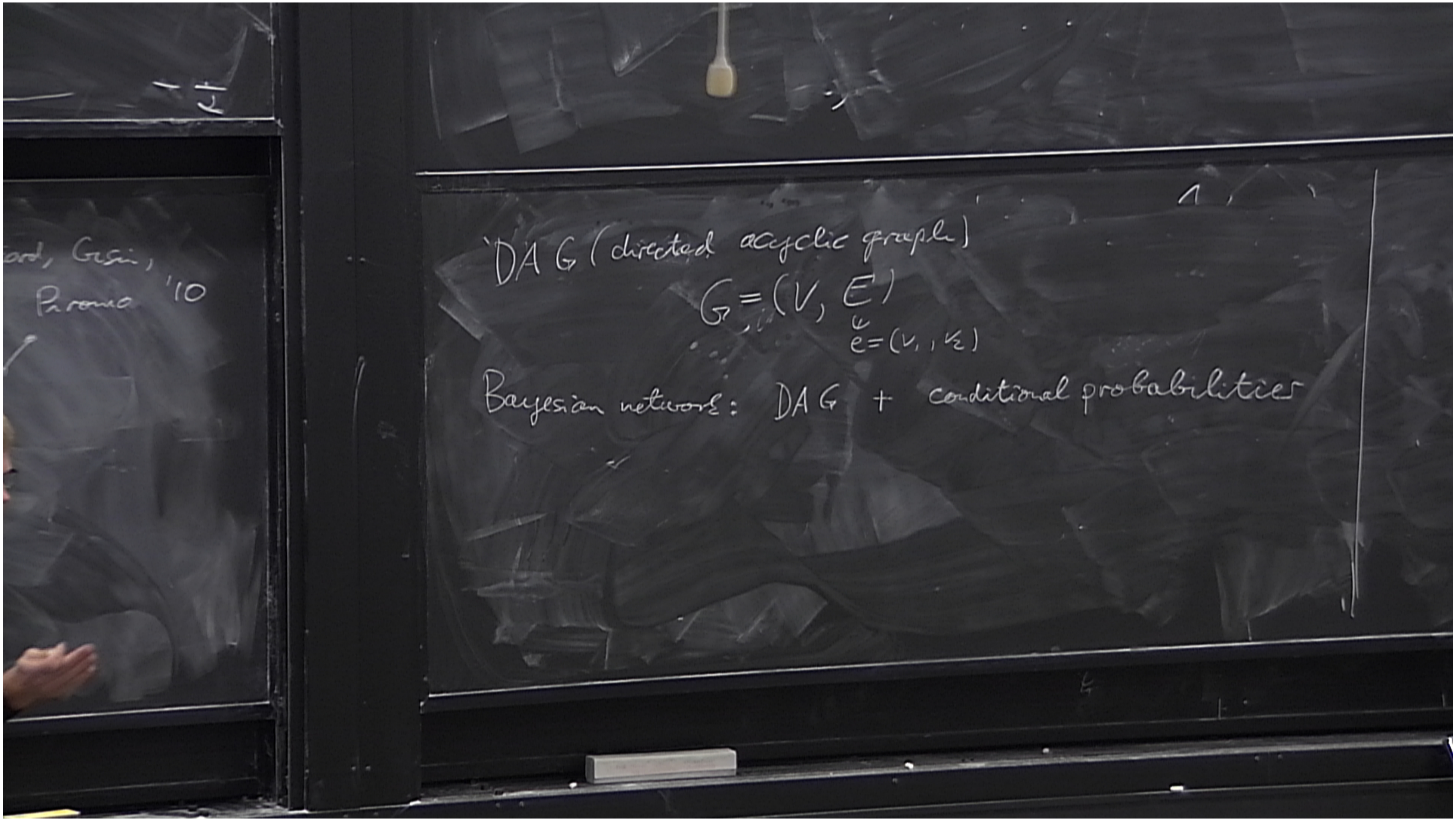
Bayesian network: DAG + conditional probabilities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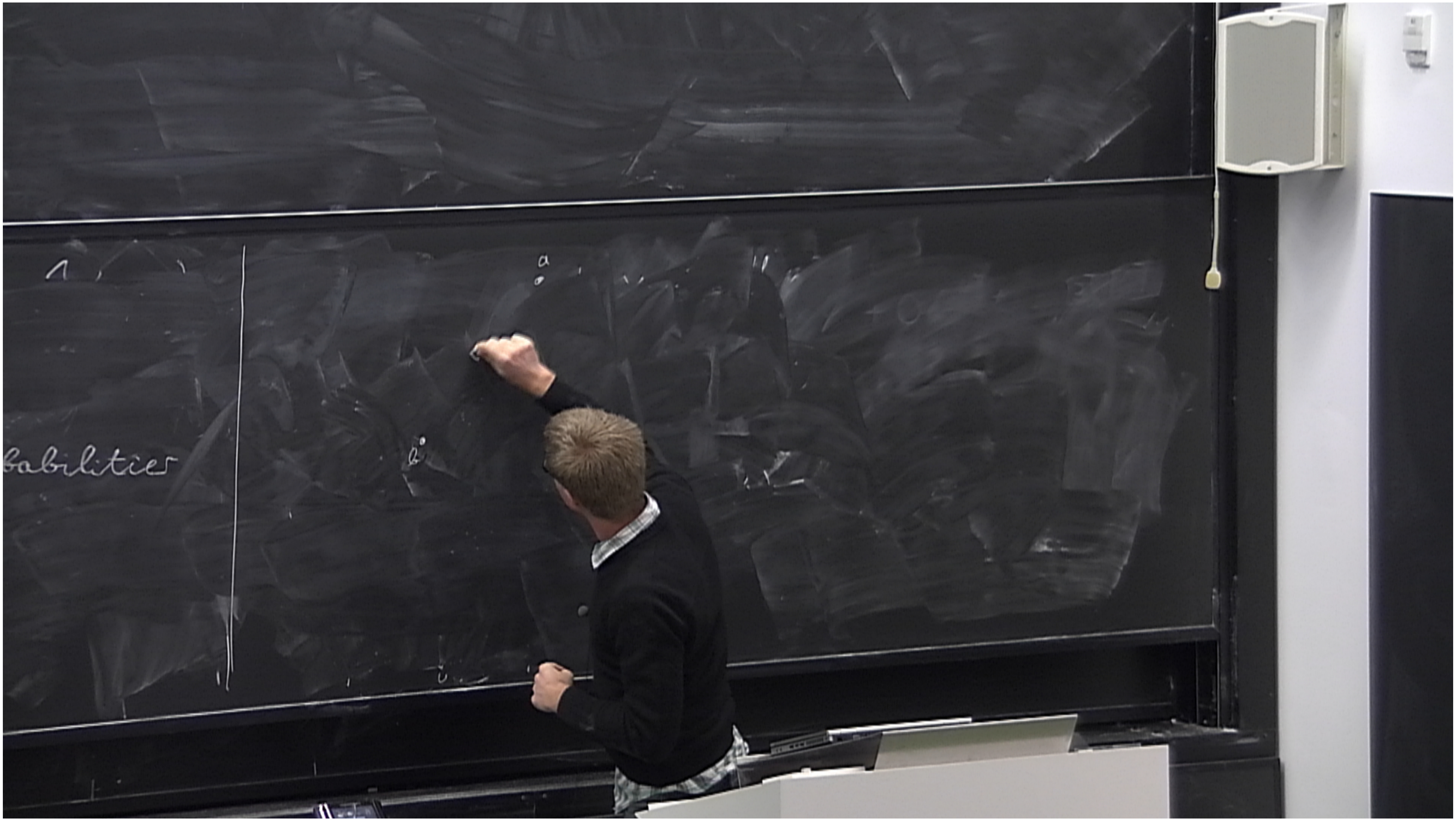
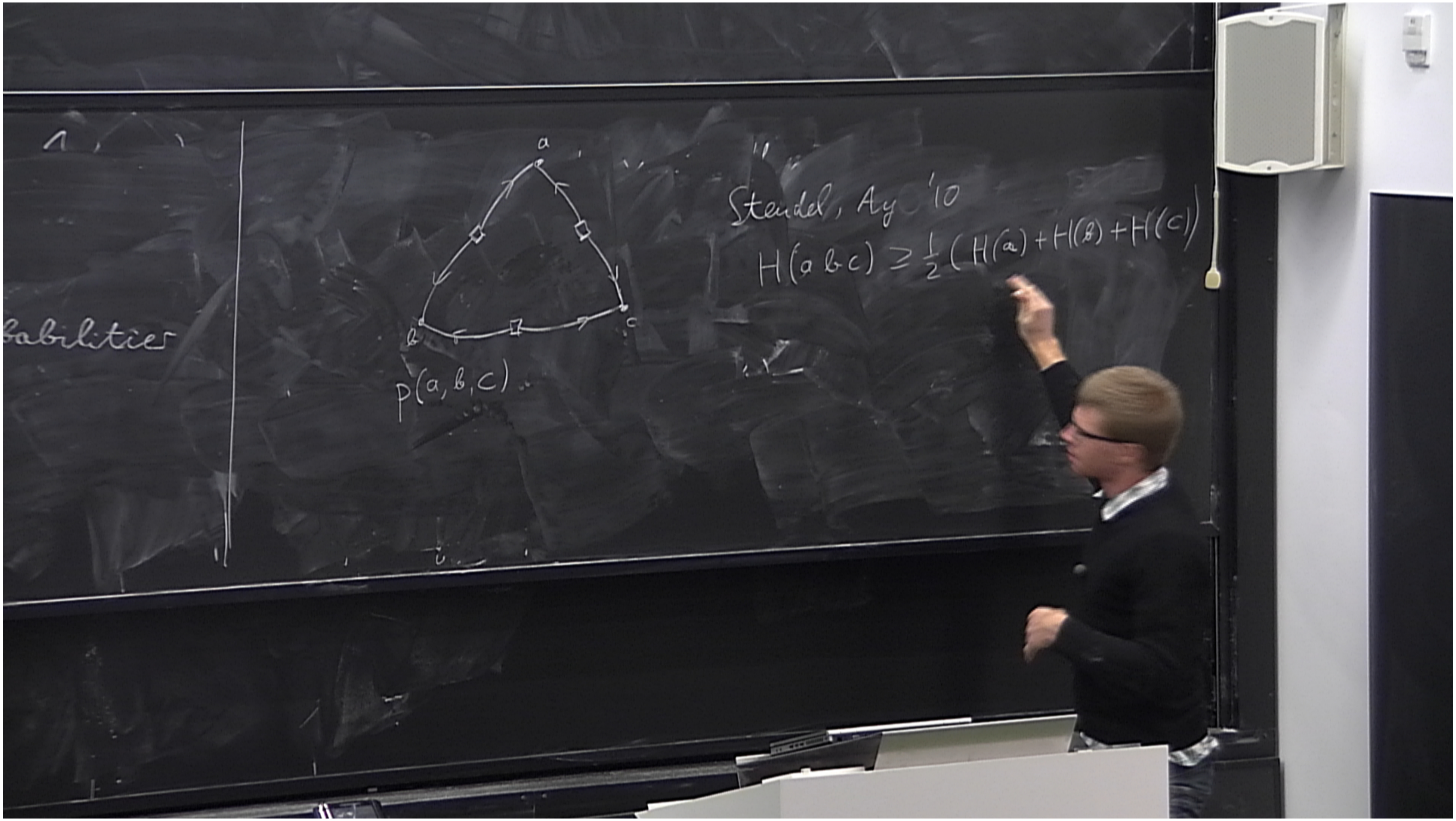
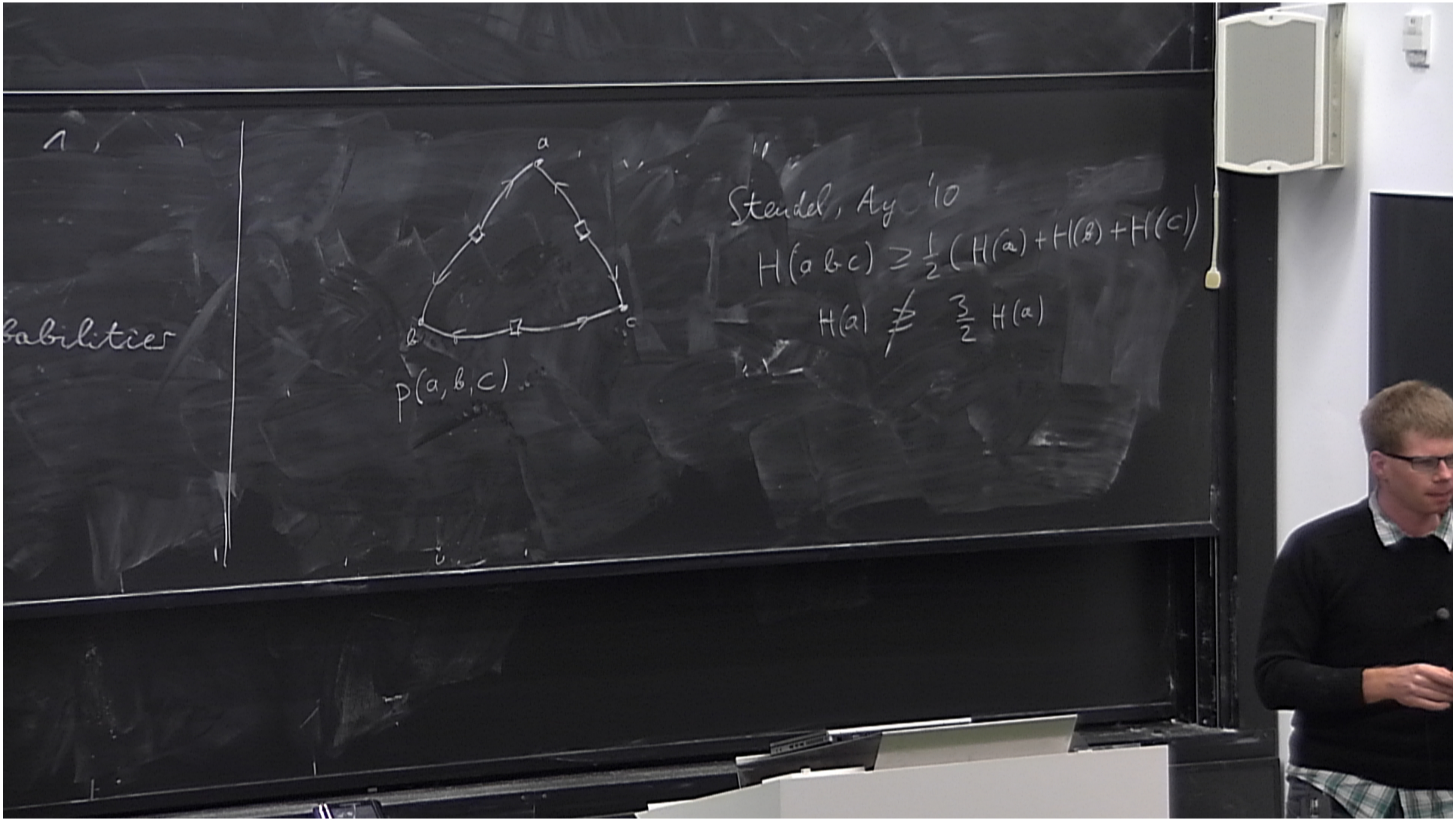
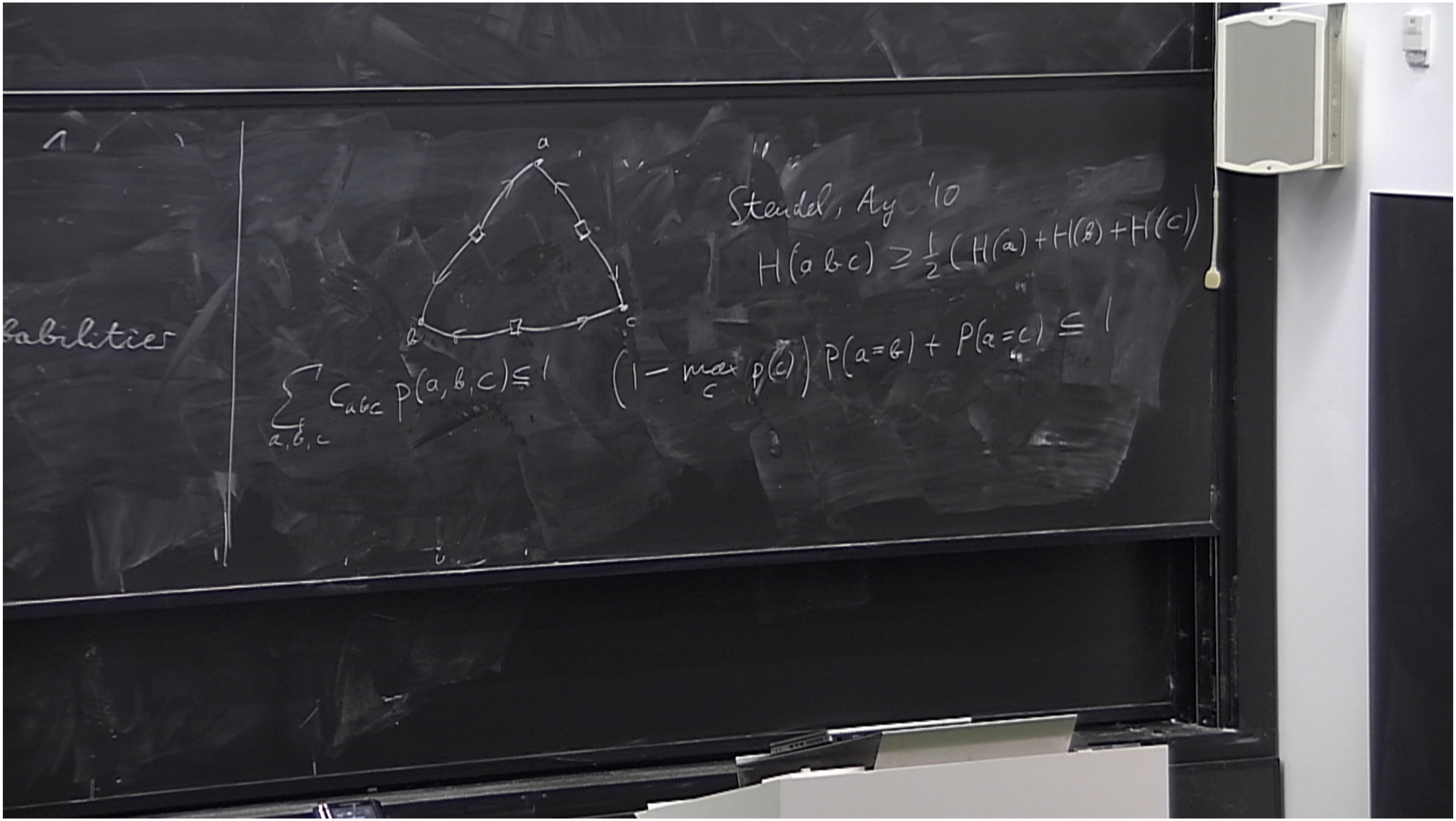
Bayesian network: DAG + conditional probabilities


babilities


abilities
a
b
c
p(a, b, c)


a
b
c
p(a,b,c)
Steudel, Ay '10
H(abc) ≥ ½ (H(a) + H(b) + H(c))


babilities
a
b
c
p(a, b, c)
Steudel, Ay '10
H(a b c) ≥ 1/2 ( H(a) + H(b) + H(c))
H(a) ≱ 3/2 H(a)


babilities
a
b
c
Steudel, Ay '10
H(abc) ≥ 1/2 (H(a) + H(b) + H(c))
(1 − max_c p(c)) P(a=b) + P(a=c) ≤ 1
Σ_{a,b,c} c_abc p(a,b,c) ≤ 1