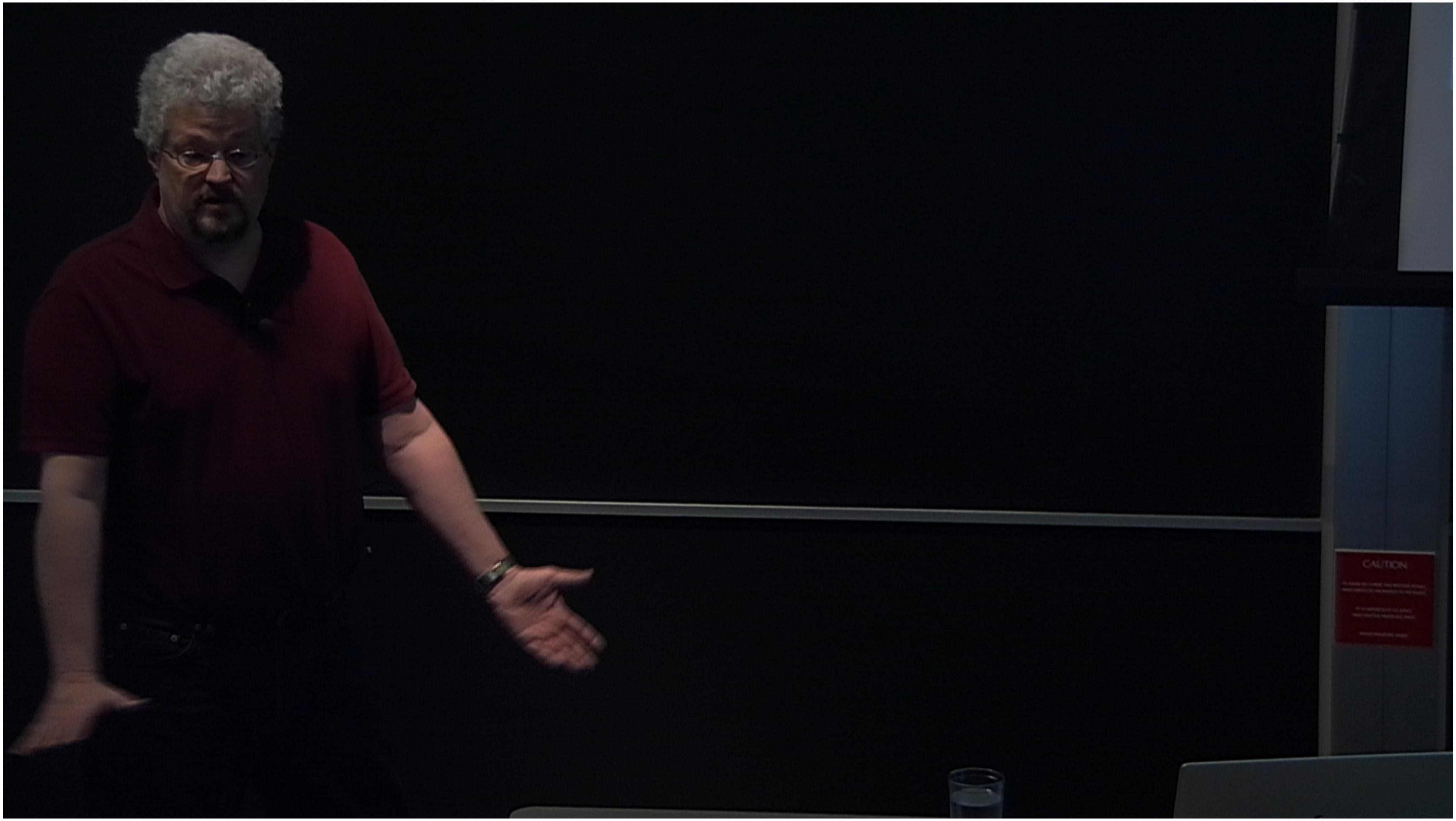
Title: Vorticity and Holography

Date: Feb 10, 2012 11:00 AM

URL: http://pirsa.org/12020126

Abstract: Three-dimensional fluids with nontrivial vorticity can be described holographically. It is well-known that the Kerr-AdS geometry gives rise to a 'cyclonic' flow. Lorentzian Taub--NUT--AdS_4 geometries give rise to a rotating fluid with vortex flow.



Aim

- focus of this talk:
  - consider AdS/CFT to describe rotating fluids
  - we'll consider several fixed bulk geometries
  - one of these geometries has classical 'problems' (CTC)
  - suggest that holographic rotating fluids may be viewed either
    - as genuine rotating near-perfect Bose or Fermi gases
    - or as analogue-gravity set ups for acoustics/optics in rotating media

      [see also Schäfer *et al.* '09, Das *et al.* '10]
  - will assemble a collection of pieces of evidence for the latter
- so perhaps bulk problematic geometries can have sensible holographic interpretations

Rotating Holographic Fluids

- Starting from a $3+1$-dim asymptotically AdS background, a $2+1$-dim holographic dual is endowed with a set of boundary data
  - boundary frame
  - boundary stress current (energy-momentum tensor)
- Within hydrodynamics, data interpreted as a $2+1$-dim fluid moving in a background – generically *with vorticity*
- we'll consider fixed backgrounds in this talk
  - AdS–Schwarzchild (S)
  - AdS–Kerr (K)
  - (Lorentzian) AdS–Taub-NUT (TN)

  — exact bulk solutions that will serve to illustrate various properties (all are stationary, axially symmetric)

Holographic Fluids

- here we're using a *bottom-up* approach to holography
- bulk 4-dim asymptotically AdS geometry, with $\Lambda = -3/L^2$
- typically use coordinates in which the conformal boundary is a constant-$r \to \infty$ slice
- the bulk metric encodes a boundary metric as well as the expectation value of the energy-momentum tensor of the dual theory
- for reasons that will become clear, we will use the Palatini formalism and focus on an asymptotic co-frame E^a
- written in the Fefferman-Graham form, we have a metric

$$ds^2 = L^2 \frac{dr^2}{r^2} + \frac{r^2}{L^2} \eta_{ab} E^a(r,x) E^b(r,x)$$

Holographic Fluids

- can do radial slicing of bulk co-frame and connection
- in absence of torsion, determined by the two coefficients e^a and f^a in the asymptotic expansion of the co-frame $E^a(r, x)$

$$E^a(r,x) = \left[e^a(x) + \frac{L^2}{r^2} F^a(x) + \cdots \right] + \frac{L^3}{r^3} \left[f^a(x) + \cdots \right]$$

- Other coefficients are determined by e^a and f^a and have interesting geometrical interpretation (e.g., $F^a \sim$ Schouten tensor)

e^a and f^a are boundary 1-forms and play the role of canonically-conjugate variables with respect to the radial evolution

Holographic Fluids

- the co-frame e^a determines the metric and f^a is interpreted as the expectation value of the stress current of the dual boundary theory
- here I'm using the term *stress current* to refer to the diffeomorphism current, and it determines the usual notion of $T_{\mu\nu}$ via

$$\kappa f^a = T(e^a, \cdot) = T^a{}_b e^b$$

- in all of the backgrounds that we will consider, the boundary metric is conformal to

$$ds^2 = -(dt - b_i(x)dx^i)^2 + a_{ij}(x)dx^i dx^j$$

- thus, a natural co-frame for the boundary is

$$e^0 = dt - b_i(x)dx^i, \qquad e^\alpha = \varepsilon_i^\alpha dx^i, \qquad \text{with } a_{ij} = \delta_{\alpha\beta}\varepsilon_i^\alpha \varepsilon_j^\beta$$

The Randers Frame

- we refer to this as the *Randers co-frame* and $b = b_i dx^i$ is the *Randers 1-form* [Randers '41] (see also [Gibbons '08])
- the corresponding Randers frame is

Randers frame

$$\underline{e}_0 = \partial_t, \qquad \underline{e}_\alpha = \varepsilon^i_\alpha \left(\partial_i + b_i(x)\partial_t\right), \qquad \text{with } \varepsilon^i_\alpha \varepsilon^\beta_i = \delta^\alpha_\beta$$

- the Randers frame is co-moving, $\nabla_{\underline{e}_0} \underline{e}_0 = 0$
- so the boundary data, as we've described it, corresponds to a relativistic fluid, as seen by a co-moving observer
- the velocity vector field of the fluid is $\underline{u} = \underline{e}_0$.
- the kinematical properties of the fluid are encoded entirely in the *leading* term of the FG expansion

Holographic Fluids

- in the backgrounds we'll consider, the stress current takes the perfect fluid form, and is conformal ($E = 2p$)

$$f^0 = -2e^0, \qquad f^\alpha = e^\alpha$$

- this corresponds to

$$T(e^a, \underline{e}_b) = \kappa f^a(\underline{e}_b)$$

or

$$T_{00} = 2\kappa, \qquad T_{\alpha\beta} = \kappa\delta_{\alpha\beta}$$

or

$$T^{ab} = \kappa\left(\eta^{ab} + 3u^a u^b\right) \equiv p\eta^{ab} + (E + p)u^a u^b$$

Holographic Fluids

- in the coordinate basis, we have

$$u^t = 1, \quad u^j = 0$$
$$u_t = -1, \quad u_j = b_j$$

$$\begin{aligned} T_{tt} &= 2\kappa \\ T_{ij} &= \kappa\left(a_{ij} + 2b_i b_j\right) \\ T_{tj} &= -2\kappa b_j \end{aligned} \tag{1}$$

General Fluid Properties

- given a generic normalized vector field $\underline{u}$, in a coordinate basis we decompose [Ehlers '61]

$$\nabla_\mu u_\nu = -u_\mu a_\nu + \sigma_{\mu\nu} + \frac{1}{2}\Theta h_{\mu\nu} + \omega_{\mu\nu}$$

where

- $h_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu}$: projector/metric on the transverse space
- $a_\mu = u^\nu \nabla_\nu u_\mu$: *acceleration* – transverse
- $\sigma_{\mu\nu}$: symmetric traceless part – *shear*
- $\Theta = \nabla_\mu u^\mu$: trace – *expansion*
- $\omega_{\mu\nu}$: antisymmetric part – *vorticity*

$$\omega = \frac{1}{2}\omega_{\mu\nu} dx^\mu \wedge dx^\nu = \frac{1}{2}(du + u \wedge a)$$

- The fluid may be perfect or not

$$T_{\text{visc}} = -\left(2\eta\sigma^{ab} + \zeta h^{ab}\Theta\right) \underline{e}_a \otimes \underline{e}_b$$

Holographic Fluids

- in all of the cases that we'll consider, the fluid is inertial ($a = 0$), shearless ($\sigma = 0$) and expansionless ($\Theta = 0$). It is also geodesic

[Caldarelli *et al.* '08]

$$\nabla_{\mathbf{u}} E = 0$$

$$\nabla p + u \nabla_{\mathbf{u}} p = 0$$

(here E, p are constants)

- to deduce the coefficients η and ζ, we would need to look at transport – i.e. vary the metric
- (have not included here parity-violating effects)
- we do have *vorticity* in general though, determined by the Randers 1-form

$$\omega = \frac{1}{2} du = \frac{1}{2} db$$

Magnetic Analogue

- note that vorticity is the analogue of a magnetic field, in that it appears in the same place in the FG expansion
  - given a bulk gauge field

$$A_\mu(r,x) = [a_\mu(x) + \ldots] + \frac{L}{r}[\rho_\mu(x) + \ldots]$$

  charge density is given by $\rho_0 \equiv \langle j_0 \rangle$, a_0 has the interpretation of a chemical potential, and the (transverse) magnetic field is given by $B = \epsilon^{ij}\partial_i a_j$

- the analogue to magnetic fields runs deep – experiments on rotating superfluids, for example, show the quantum Hall effect in the absence of magnetic fields and charge

AdS–Schwarzchild

- let's begin with the simplest example, the AdS–Schwarzchild black hole
- the usual metric $ds^2 = \frac{dr^2}{V(r)} - V(r)dt^2 + r^2\delta_{ij}dx^i dx^j$ with

$$V(r) = 1 + \frac{r^2}{L^2} - \frac{2M}{r}$$

- the FG expansion gives the Randers co-frame as

$$e^0 = dt, \qquad e^\alpha = \delta_j^\alpha dx^j$$

and

$$\kappa = \frac{1}{3}ML^2$$

- here the fluid is irrotational ($b = 0$)

AdS-Kerr: the solid rotation

- now add rotation: the AdS–Kerr black hole

The bulk data

$$\begin{aligned} ds^2 &= \frac{d\tilde{r}^2}{V(\tilde{r},\vartheta)} - V(\tilde{r},\vartheta)\left[dt - \frac{a}{\Xi}\sin^2\vartheta\, d\varphi\right]^2 \\ &+ \frac{\rho^2}{\Delta_\vartheta} d\vartheta^2 + \frac{\sin^2\vartheta\,\Delta_\vartheta}{\rho^2}\left[a\,dt - \frac{r^2+a^2}{\Xi} d\varphi\right]^2 \end{aligned}$$

$V(\tilde{r},\vartheta) = {}^{\Delta}/_{\rho^2}$ with

$$\begin{aligned} \Delta &= (\tilde{r}^2 + a^2)\left(1 + \tilde{r}^2/L^2\right) - 2M\tilde{r} \\ \rho^2 &= \tilde{r}^2 + a^2\cos^2\vartheta \\ \Delta_\vartheta &= 1 - (a/L)^2\cos^2\vartheta \\ \Xi &= 1 - (a/L)^2 \end{aligned}$$

AdS–Kerr

The boundary metric – following FG expansion

$$ds^2_{\text{bry.}} = \eta_{ab} e^a e^b = g_{(0)\mu\nu} dx^\mu dx^\nu$$
$$= -\left(dt - \frac{a \sin^2\vartheta}{\Xi} d\varphi\right)^2 + \frac{L^2}{\Delta_\vartheta}\left(d\vartheta^2 + \left(\frac{\Delta_\vartheta \sin\vartheta}{\Xi}\right)^2 d\varphi^2\right)$$

- spatial section: squashed 2-sphere
- here the Randers 1-form is $b = \frac{a \sin^2\vartheta}{\Xi} d\varphi$ and the vorticity is

$$\omega = \frac{1}{2} db = \frac{a}{2\Xi} \sin 2\vartheta d\vartheta \wedge d\varphi = \frac{a}{L^2} \cos\vartheta e^1 \wedge e^2$$

- the dual fluid is perfect, conformal and *rotating*, with smooth vorticity [see also Caldarelli, Dias, Klemm '08]

AdS–Taub-NUT

- the Lorentzian AdS–TN black hole gives also the Randers form
- the bulk metric can be written

The bulk data [Taub '51, Newman, Tamburino, Unti '63]

$$ds^2 = \frac{d\tilde{r}^2}{V(\tilde{r})} - V(\tilde{r})\,[dt - 2n\cos\vartheta\, d\varphi]^2 + \rho^2\left[d\vartheta^2 + \sin^2\vartheta\, d\varphi\right]^2$$

$V(\tilde{r}) = {}^{\Delta}/_{\rho^2}$ with

$$\begin{aligned}\Delta &= \left(\tilde{r}^2 - n^2\right)\left(1 + \left(\tilde{r}^2 + 3n^2\right)/L^2\right) + 4n^2\tilde{r}^2/L^2 - 2M\tilde{r}\\ \rho^2 &= \tilde{r}^2 + n^2\end{aligned}$$

- No rotation parameter a, but NUT charge n
- a solution to Einstein eqs. with peculiar properties [Misner '63]

AdS–Taub-NUT

- Taub–NUT: rich geometry – foliation over squashed 3-spheres with $SU(2) \times U(1)$ isometry (homogeneous and axisymmetric)
  - horizon at $r = r_+ \neq n$: 2-dim fixed locus of $-2n\partial_t \to$ *bolt* (Killing becoming light-like)
  - extra fixed point of $\partial_\varphi - 4n\partial_t$ on the horizon at $\vartheta = \pi$

  NUT at $r = r_+, \vartheta = \pi$ from which departs a *Misner string* (coordinate singularity if $t \not\cong t + 8\pi n$) [Misner '63]
- compare to Kerr: stationary (rotating) black hole
  - horizon at $r = r_+$: fixed locus of $\partial_t + \Omega_{\mathrm{H}}\partial_\varphi \to$ bolt
  - pair of NUT–anti-NUT at $r = r_+, \vartheta = 0, \pi$ (fixed points of ∂_φ) connected by a Misner string

AdS–Taub-NUT

Pictorially: NUTs and Misner strings

Figure: Kerr vs. Taub–NUT

How is Taub–NUT related to rotation?

AdS–TN

The boundary metric – following FG expansion

$$\begin{aligned} ds^2_{\text{bry.}} &= \eta_{ab} e^a e^b = g_{(0)\mu\nu} dx^\mu dx^\nu \\ &= -\left(dt - 2n(\cos\vartheta - 1)d\varphi\right)^2 + L^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \end{aligned}$$

- here the Randers 1-form is $b = -4n\sin^2(\vartheta/2)d\varphi$
- in this case, there is a subtlety in the vorticity: $u = -e^0$ is ill-defined at $\vartheta = \pi$

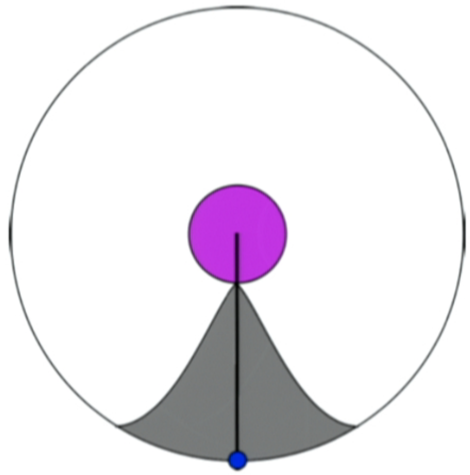
$$\omega = \frac{1}{2}db = -\frac{n}{L^2}e^1 \wedge e^2 - \frac{n}{L^2}\delta_2(\theta - \pi)$$

- this looks like rotation plus a point vortex

for the same reason the bulk LC connection has a δ-function, or a smooth connection has torsion with δ-function support

– this is the Misner string

AdS–TN



- the Misner string is physical (if time is Lorentzian and non-compact), and cloaked by a region of CTC
  - φ is compact coordinate, but $g_{\varphi\varphi}$ goes through zero at $\vartheta = \vartheta_c = 2arctan(L/2n)$

AdS–TN

- the stress current takes the same form as before
  - thus this is the same fluid, but with a different Randers frame
  - the bulk Misner string extends radially out to a "Misner vortex" that we've located at $\vartheta = \pi$, still cloaked by a region of CTCs
- the total vorticity of the fluid reproduces the NUT charge

$$n = -\frac{1}{4\pi}\int_{S^2}\omega$$

- this is a Dirac monopole-like vortex
- AdS–Kerr gives a dipole, without net NUT charge, $\int_{S^2}\omega = 0$
- there is presumably a multipole generalization, e.g., $\omega \sim P_{\ell-1}(\cos\vartheta)e^1 \wedge e^2$:— (S: $\ell = 0$, TN: $\ell = 1$, K: $\ell = 2$)

AdS–TN

- near $\theta = 0$, the boundary metric becomes (for both K and TN) the Som-Raychaudri metric, corresponding to ($\rho \sim \vartheta << 1$, $\Omega \sim a, n$)

$$e^0 \simeq dt + \Omega\rho^2 d\phi, \quad e^1 \simeq d\rho, \quad e^2 \simeq \rho d\phi$$

  this is in fact a solution to Einstein eqs. with rotating charged dust; it is of Gödel type, and has CTCs for $\rho > 1/\Omega$

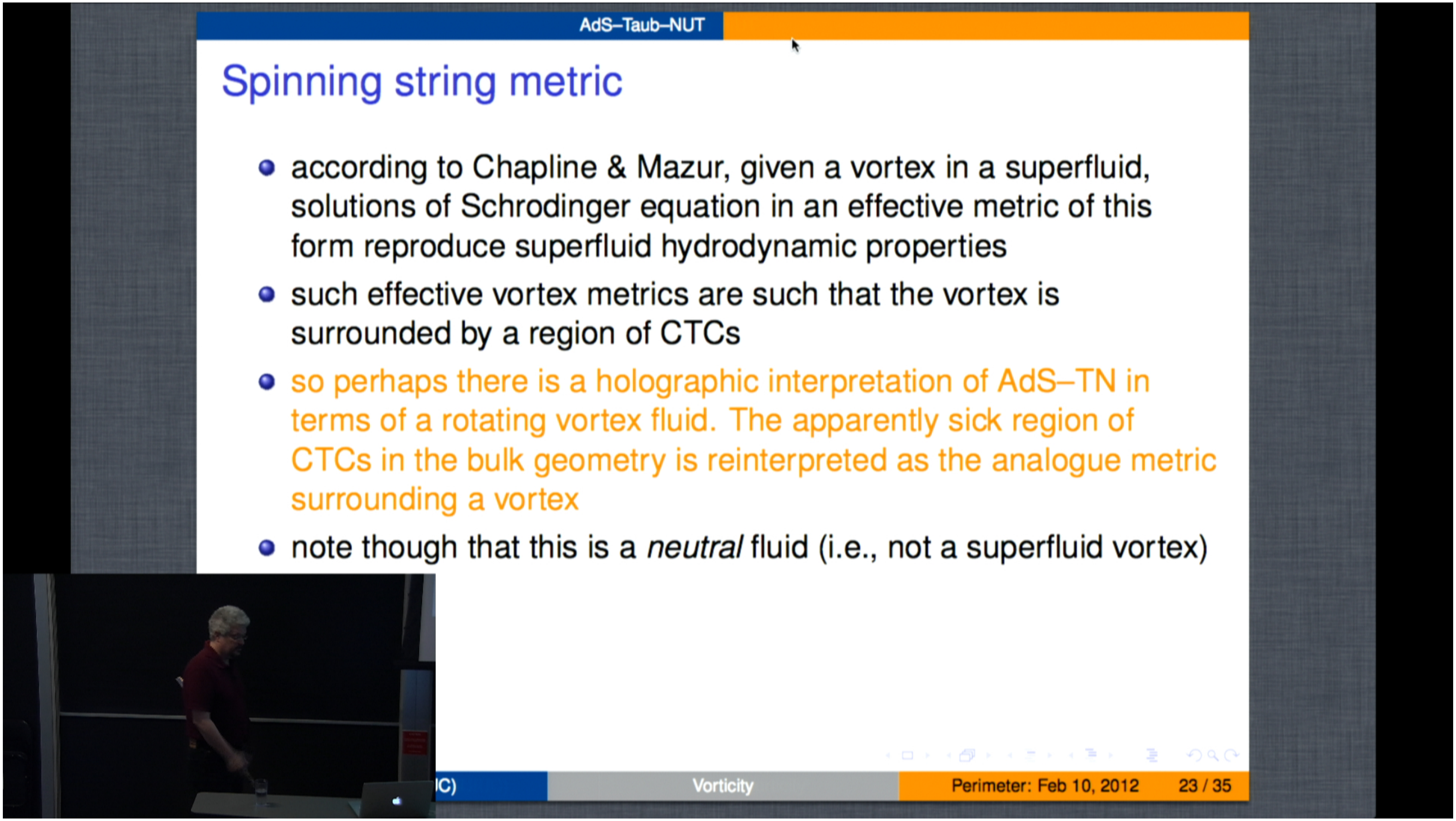
- the full Kerr boundary metric has no CTCs, but TN does.
- near $\vartheta \sim \pi$, the Kerr bdy. metric is again Som-Raychaudri, but the TN bdy. metric

$$e^0 \simeq dt + (4n - n\rho^2/L^2)d\phi, \quad e^1 \simeq d\rho, \quad e^2 \simeq \rho d\phi$$

  "spinning string" or vortex metric [Chapline & Mazur 09]

Spinning string metric

- according to Chapline & Mazur, given a vortex in a superfluid, solutions of Schrodinger equation in an effective metric of this form reproduce superfluid hydrodynamic properties
- such effective vortex metrics are such that the vortex is surrounded by a region of CTCs
- so perhaps there is a holographic interpretation of AdS–TN in terms of a rotating vortex fluid. The apparently sick region of CTCs in the bulk geometry is reinterpreted as the analogue metric surrounding a vortex
- note though that this is a *neutral* fluid (i.e., not a superfluid vortex)


AdS–Taub–NUT
Spinning string metric
according to Chapline & Mazur, given a vortex in a superfluid, solutions of Schrodinger equation in an effective metric of this form reproduce superfluid hydrodynamic properties
such effective vortex metrics are such that the vortex is surrounded by a region of CTCs
so perhaps there is a holographic interpretation of AdS–TN in terms of a rotating vortex fluid. The apparently sick region of CTCs in the bulk geometry is reinterpreted as the analogue metric surrounding a vortex
note though that this is a neutral fluid (i.e., not a superfluid vortex)
IC)
Vorticity
Perimeter: Feb 10, 2012
23 / 35

The Zermelo Frame

- the Randers frame is inertial: the integral lines of $\underline{e}_0 = \partial_t$ are geodesics ($\underline{e}_0$ generates geodesic congruence)
- vorticity shows up as the rotation of the spatial frame along the congruence

$$\nabla_{\underline{e}_0}\underline{e}_\alpha = \omega_{\alpha\beta}\eta^{\beta\gamma}\underline{e}_\gamma$$

- the fluid's *physical surfaces* consist of points synchronous in this frame. Since $dt(\partial_j) = 0$, we can define a physical orthonormal spatial frame $\underline{z}_\alpha = L^j_\alpha\partial_j$. These do not coincide with $\underline{e}_\alpha$.
- an orthonormal Lorentz frame is then given by

$$\underline{z}_0 = \frac{1}{\gamma}(\partial_t + W^i\partial_i), \quad \underline{z}_\alpha = L^j_\alpha\partial_j$$

$$z^0 = \gamma dt, \quad z^\alpha = L^\alpha_i(dx^i - W^i dt)$$

with

$$\gamma^{-2} = 1 - a^{ij}b_ib_j, \quad W^i = -\gamma^2 a^{ij}b_j$$

The Zermelo Frame

- this frame of reference is known as the Zermelo frame
- it has an old interpretation in terms of minimum time problems for a ship sailing in a wind W^i
- in this frame, the boundary metric reads

$$\hat{g} = \tfrac{1}{\lambda}\left[-dt^2 + h_{ij}(dx^i - W^i dt)\,(dx^j - W^j dt)\right]$$
$$h_{ij} = \lambda(a_{ij} - b_i b_j) = \lambda L^{\alpha}{}_i L^{\beta}{}_j \delta_{\alpha\beta}\,, \quad \lambda \equiv 1/\gamma^2\,.$$

- this is a non-inertial frame, $\nabla_{\underline{z}_0}\underline{z}_0 \neq 0$ — Zermelo observers see a rotating fluid
- the Fermi derivative, which disentangles the intrinsic fluid rotation from that of the frame motion is

$$\mathrm{D}_{\underline{z}_0}\underline{u} = \frac{a^{ij}}{\sqrt{\lambda}}\left(\left(\frac{\partial_i\lambda}{2\lambda} + \omega_{ik}b^k\right)L^{\alpha}{}_j\underline{z}_{\alpha} + \frac{\partial_i\lambda}{2\lambda}b_j\underline{e}_0\right)\,.$$

The Zermelo Frame

- at each point, W^i are the components of the relative velocity of the two observers
- for AdS–Kerr, the Fermi derivative vanishes and $W = -a/L^2 \underline{e}_2$: the Zermelo metric can be made conformal to a static metric by a global Lorentz boost [Bhattacharyya et al, 08]
- for AdS–TN, this is not possible
  - the fluid velocity in the Zermelo frame has norm $||V|| = 2n/L \tan(\theta/2)$, which exceeds unity at $\theta > \theta_*$, which coincides with the region of CTC
  - indeed, the (local) Lorentz transformation between Randers and Zermelo goes singular there
- so a fluid observer can't boost himself into the synchronous frame

Acoustic Metrics

- note also that the Zermelo form of the metric belongs to the class of acoustic metrics

$$ds^2 = \frac{\rho}{c_s}\left(-c_s^2 dt^2 + h_{ij}(dx^i - W^i dt)(dx^j - W^j dt)\right)$$

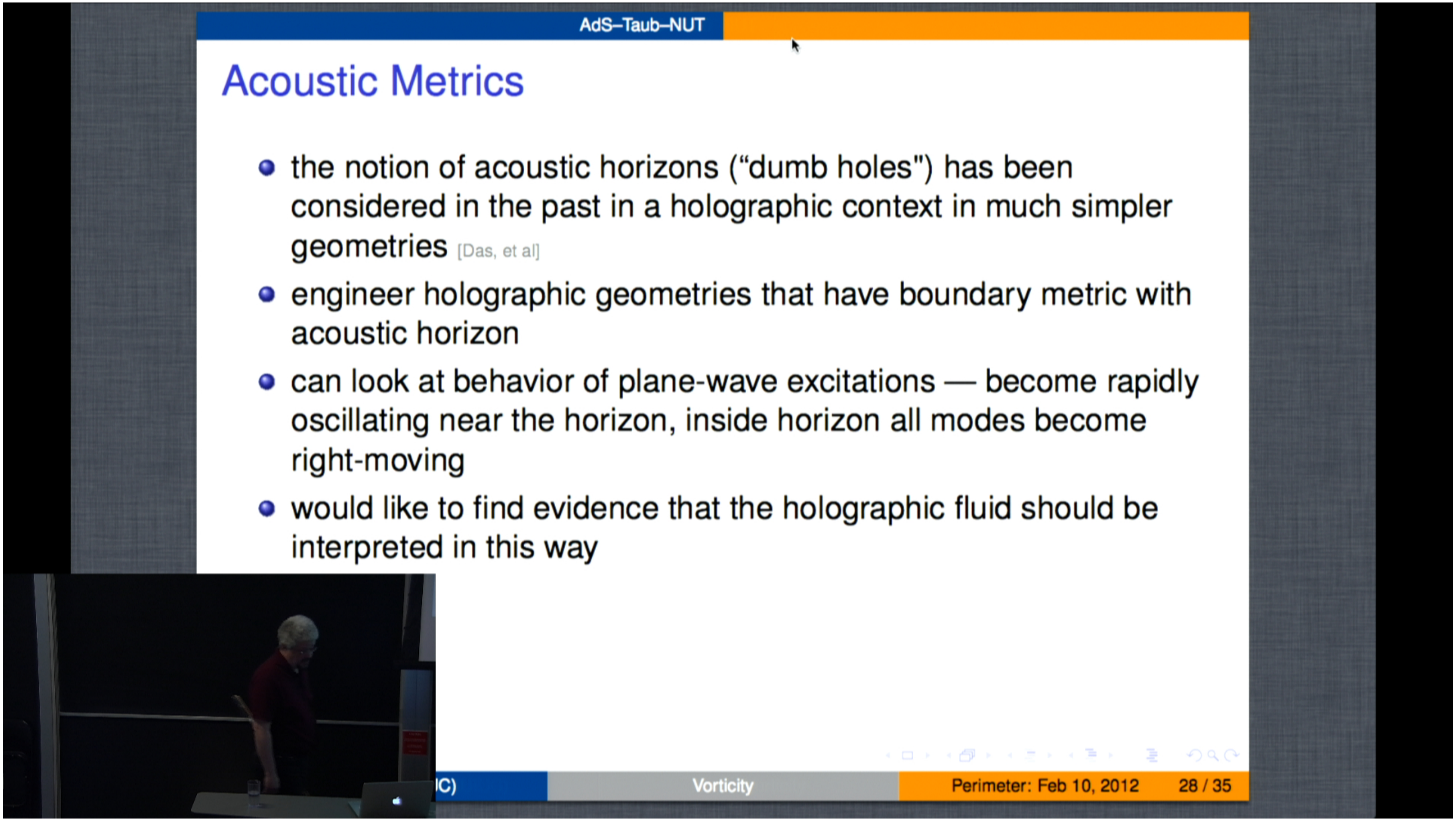
  that describes sound propagation in a (perfect) classical fluid, where

  $c_s = 1/\sqrt{\frac{\partial \rho}{\partial p}}$ ($\rho, p =$ local mass density and pressure

  h_{ij} is local geometry supporting fluid

  W^i is the local fluid velocity field

- sound propagates on the acoustic cone (null surfaces)
- should interpret the boundary metric of AdS–TN in terms of such an analogue geometry?
  - the CTC boundary would then be interpreted as an *acoustic horizon*


AdS–Taub–NUT
Acoustic Metrics
the notion of acoustic horizons ("dumb holes") has been considered in the past in a holographic context in much simpler geometries [Das, et al]
engineer holographic geometries that have boundary metric with acoustic horizon
can look at behavior of plane-wave excitations — become rapidly oscillating near the horizon, inside horizon all modes become right-moving
would like to find evidence that the holographic fluid should be interpreted in this way
IC)
Vorticity
Perimeter: Feb 10, 2012
28 / 35

Holography of AdS–TN

- in the rest of the talk, I will describe simple attempts to understand the holographic physics
- we are far from a complete picture
- so consider a massive bulk scalar field propagating on the AdS–TN geometry
- AdS–TN has $SU(2) \times U(1)$ isometry and scalar solutions will organize in representations.
- the generators are (introduce $x \equiv \sin^2 \theta/2$, $x \in [0, 1]$)

$$\begin{aligned} H &= -i\partial_t = -i\hat{e}_0 \\ L_3 &= -i(\partial_\varphi - 2n\partial_t) \\ L_\pm &= ie^{\pm i\varphi}\left(2n\sqrt{\frac{x}{1-x}}\partial_t \mp i\sqrt{x(1-x)}\partial_x + \frac{1-2x}{2\sqrt{x(1-x)}}\partial_\varphi\right) \end{aligned} \tag{2}$$

Holography of AdS–TN

- the $SU(2)$ generator is twisted with ∂_t

$$L_3 = -i(\partial_\varphi - 2n\partial_t)$$

and the orbits of the $SU(2)$ will not be closed curves, but helices

$$e^{i\chi L_3}\Phi(t,\phi) = \Phi(t - 2n\chi, \phi + \chi)$$

- consequently, scalar modes of $L_3 = m$, $H = \omega$ will have the form

$$\Phi(r,t,x,\varphi) = \sum_{m,\omega} \Phi_{m,\omega}(r,x) e^{i(m-2n\omega)\varphi} e^{-i\omega t}$$

- for brevity, we will write $\Omega \equiv 2n\omega$

Holography of AdS–TN

- the resulting Klein-Gordon operator for mass μ is

Klein-Gordon

$$\left[\partial_r\left[(r^2+n^2)V(r)\partial_r\right]+(r^2+n^2)\left(\frac{\omega^2}{V(r)}-\mu^2\right)-C+\Omega^2\right]\Phi_{m,\omega}(r,x)=0$$

- where C is the $SU(2)$ Casimir

$$C=-\partial_x\left[x(1-x)\partial_x\right]+m^2+\frac{(\Omega+(2x-1)m)^2}{4x(1-x)}$$

- the KG equation is fully separable in the form (for any mass μ)

$$\Phi_{m,\omega}(r,x)=\sum_\lambda R_{\lambda,\omega}(r)Y_{\lambda,m,\Omega}(x)$$

Holography of AdS–TN

$$\Phi_{m,\omega}(r,x) = \sum_{\lambda} R_{\lambda,\omega}(r) Y_{\lambda,m,\Omega}(x)$$

where

$$C[Y_{\lambda,m,\Omega}] = (\lambda + \Omega^2) Y_{\lambda,m,\Omega}$$

$$\partial_r \left[(r^2+n^2)V(r)R'_{\lambda,\omega}(r)\right] + \left[(r^2+n^2)\left(\frac{\omega^2}{V(r)} - \mu^2\right) - \lambda\right] R_{\lambda,\omega}(r) = 0$$

- the functions $Y_{\lambda,m,\Omega}[x]e^{i(m-2n\omega)}$ are deformed spherical harmonics, and form representations of $su(2)$
- they are hypergeometrics ($C = q(q+1)$)

$$Y_{\lambda,m,\Omega}[x] \sim x^{\pm(m-\Omega)/2}(1-x)^{\pm(m+\Omega)/2}{}_2F_1(1+q\pm m, -q\pm m, 1\pm(m-\Omega); x)$$

Holography of AdS–TN

- as we've seen, the boundary space-time is smooth near $x = 0$, so we should require that the solutions are well-defined there. Depending on (m, ω) this removes one solution.
- for $\omega = 0$, there is a complete basis of unitary representations (spherical harmonics), and these are finite at $x = 1$ as well
- generally, we have

$$e^{i\chi L_3} Y_{\lambda,m,\omega}(x) = e^{im\chi} Y_{\lambda,m,\omega}(x)$$

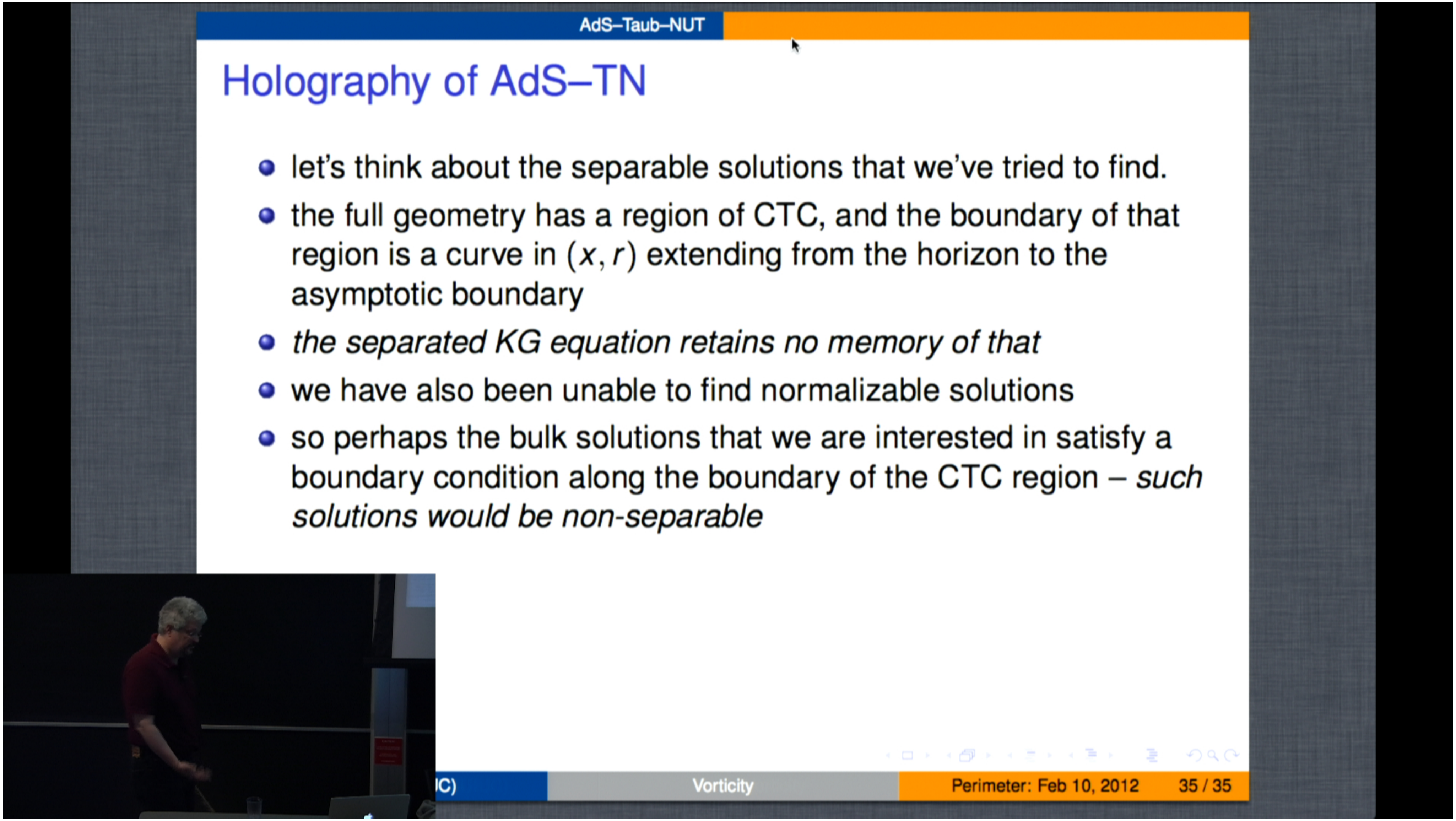
  and so if we require a faithful representation, we conclude $m \in \mathbb{Z}$
- one can show that $L_{\pm}$ shift m by ± 1, and thus we get either finite dimensional reps ($q \in \mathbb{Z}$) or non-unitary infinite dimensional reps
- careful study of the hypergeometrics show that solutions are singular at $x = 1$ unless $|\Omega|$ is bounded (essentially by $|m|$).

Holography of AdS–TN

- the Ω outside this range, solutions blow up near $x = 1$, are non-normalizable in the KG norm and are anyonic:
  - traversing a circle near $x = 1$ (at constant time), they pick up a phase
  $$\Phi(t, \varphi + 2\pi, x \sim 1) = e^{-2\pi i \Omega} \Phi(t, \varphi, x \sim 1)$$
- this phase (and the blowing up of solutions) is the manifestation of the Misner vortex at $x = 1$. This is avoided only for quantized ω – one would get such a quantization by insisting on the solutions being $L_2[0, 1]$.
- perhaps the conclusion that should be drawn is that a complete set of solutions can be constructed, but L_2 should not be the criterion
  - perhaps infalling at the CTC horizon??

Holography of AdS–TN

- the Ω outside this range, solutions blow up near $x = 1$, are non-normalizable in the KG norm and are anyonic:
  - traversing a circle near $x = 1$ (at constant time), they pick up a phase
$$\Phi(t, \varphi + 2\pi, x \sim 1) = e^{-2\pi i \Omega} \Phi(t, \varphi, x \sim 1)$$
- this phase (and the blowing up of solutions) is the manifestation of the Misner vortex at $x = 1$. This is avoided only for quantized ω – one would get such a quantization by insisting on the solutions being $L_2[0, 1]$.
- perhaps the conclusion that should be drawn is that a complete set of solutions can be constructed, but L_2 should not be the criterion
  - perhaps infalling at the CTC horizon??


AdS–Taub–NUT
Holography of AdS–TN
let's think about the separable solutions that we've tried to find.
the full geometry has a region of CTC, and the boundary of that region is a curve in (x, r) extending from the horizon to the asymptotic boundary
the separated KG equation retains no memory of that
we have also been unable to find normalizable solutions
so perhaps the bulk solutions that we are interested in satisfy a boundary condition along the boundary of the CTC region – such solutions would be non-separable
C)
Vorticity
Perimeter: Feb 10, 2012
35 / 35

Conclusions

- there has been a great deal of work in the literature engineering bulk geometries that reproduce specific hydrodynamics
- here we've considered specific bulk geometries with 'interesting' properties and attempt to give them a sensible boundary interpretation
- the scalar probe shows some interesting behaviors, but the details remain to be understood.