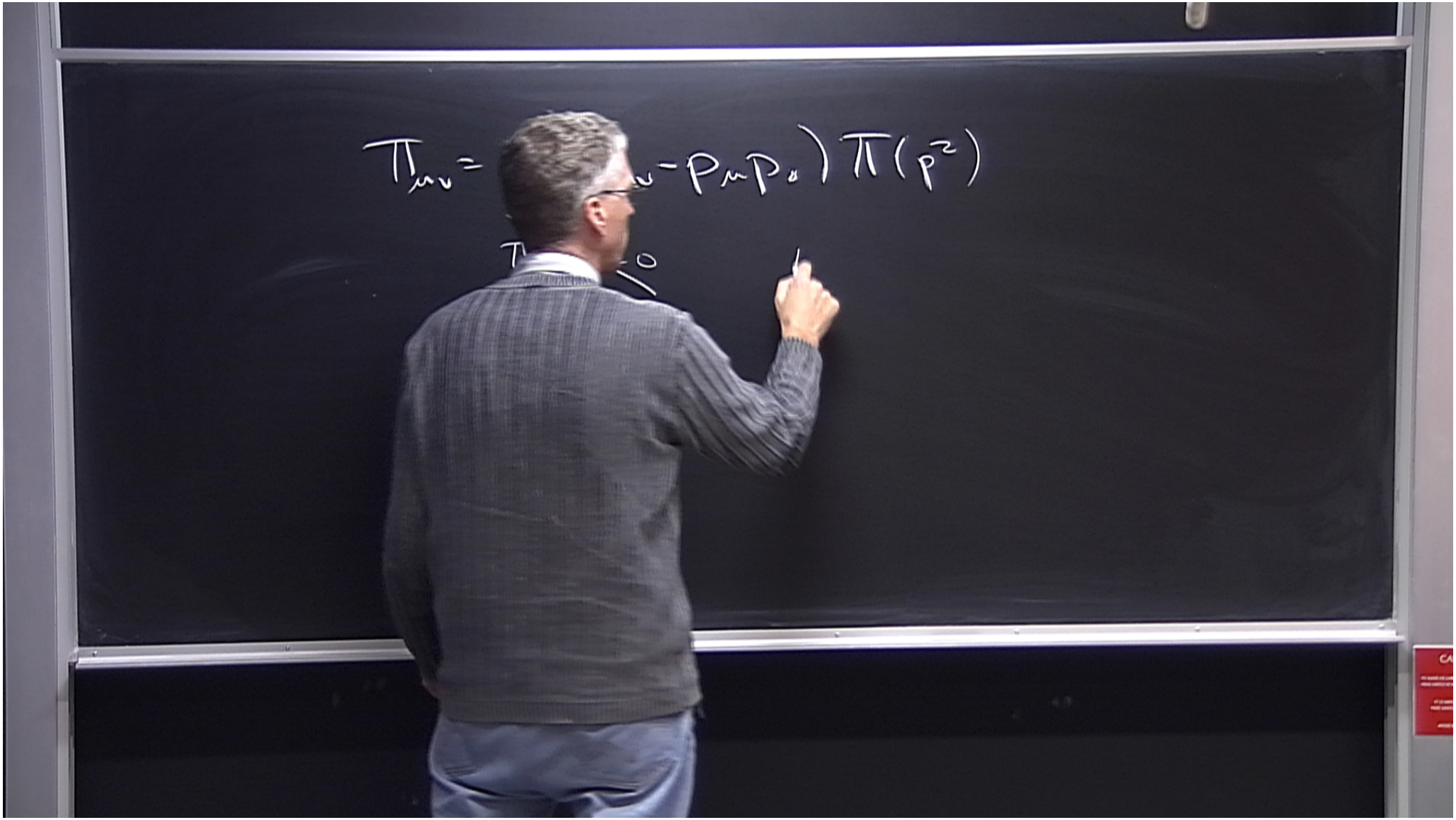
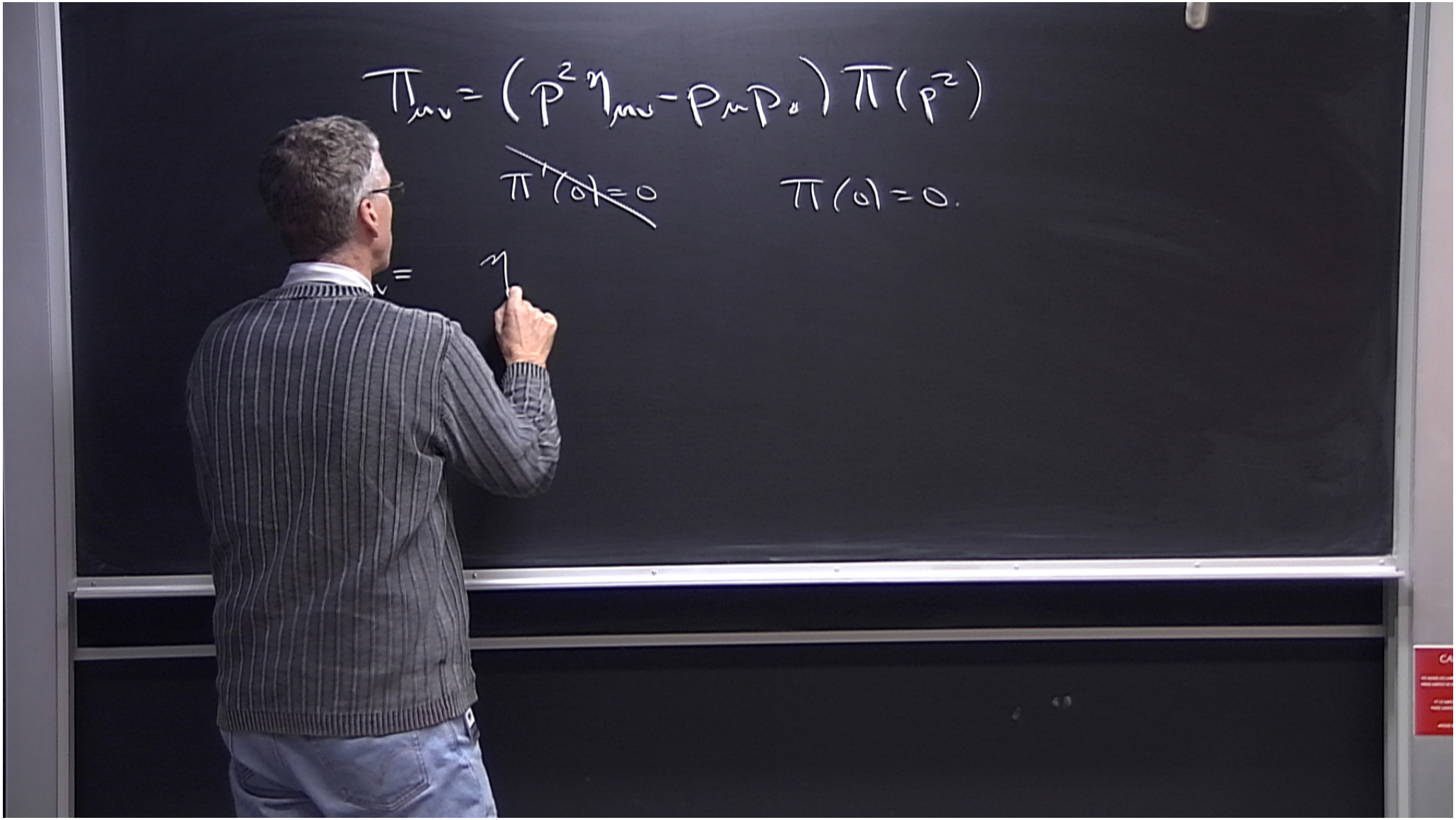
Title: Advanced Quantum Field Theory - Lecture 8

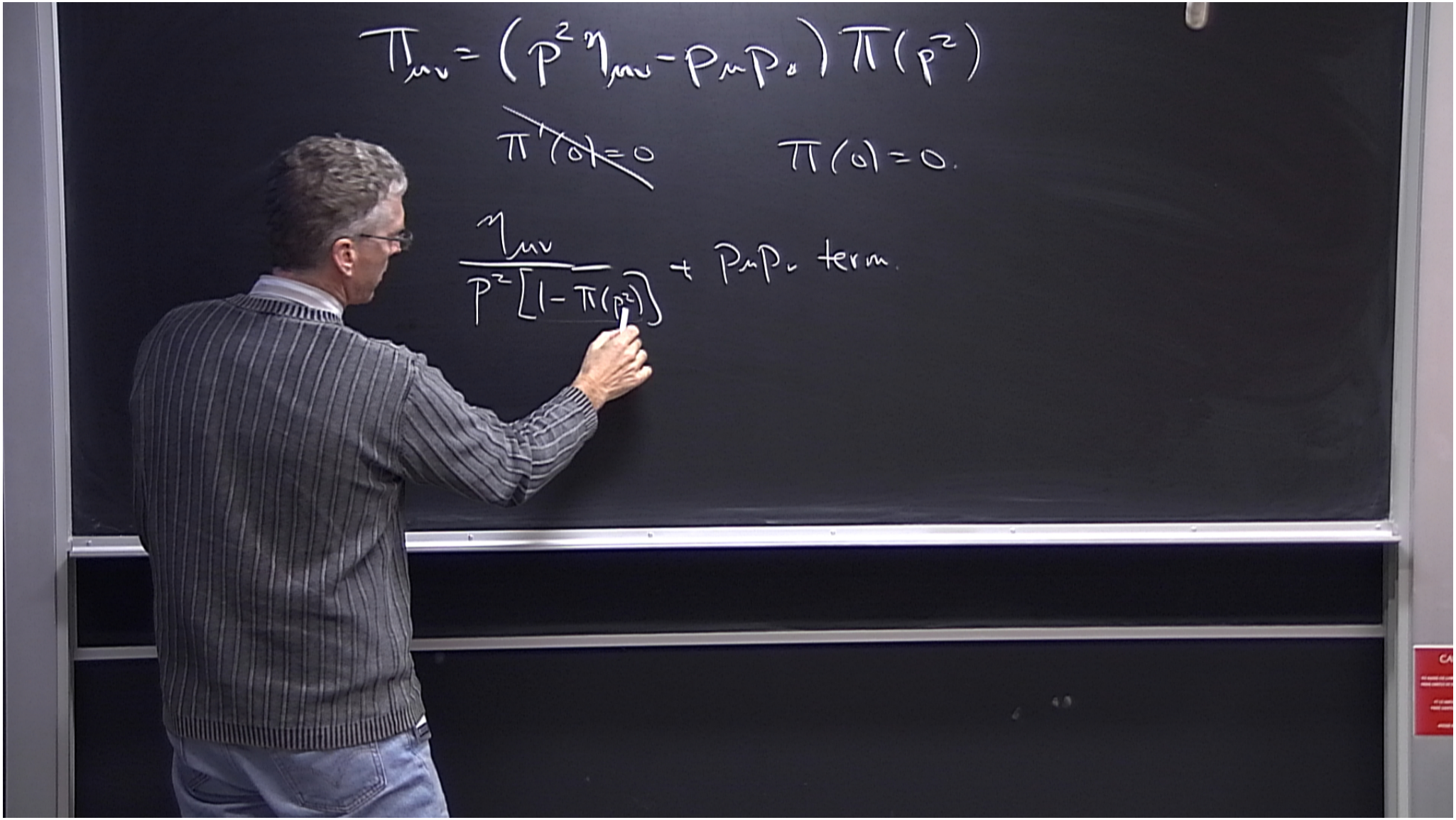
Date: Feb 03, 2012 09:30 AM

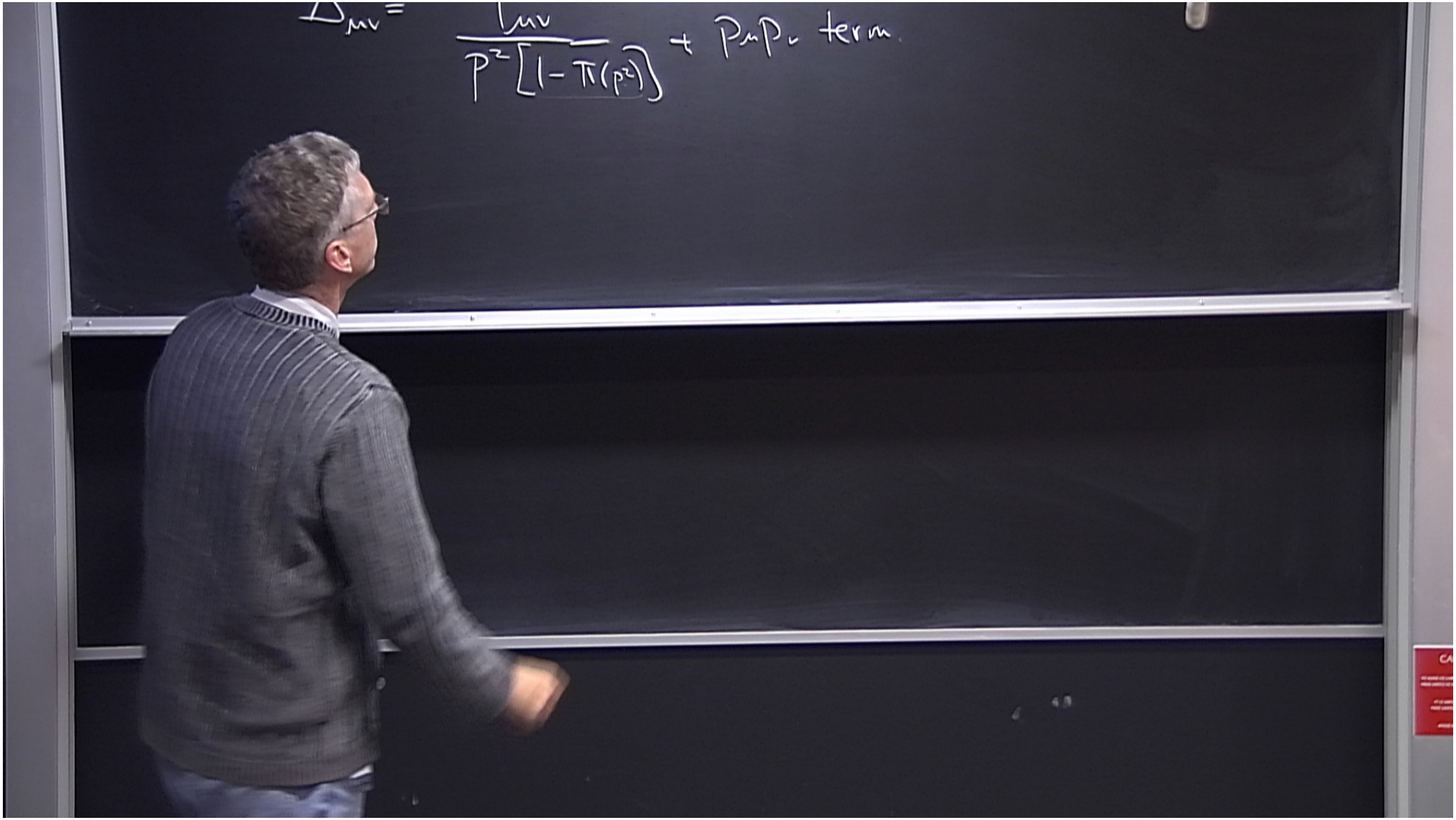
URL: http://pirsa.org/12020117

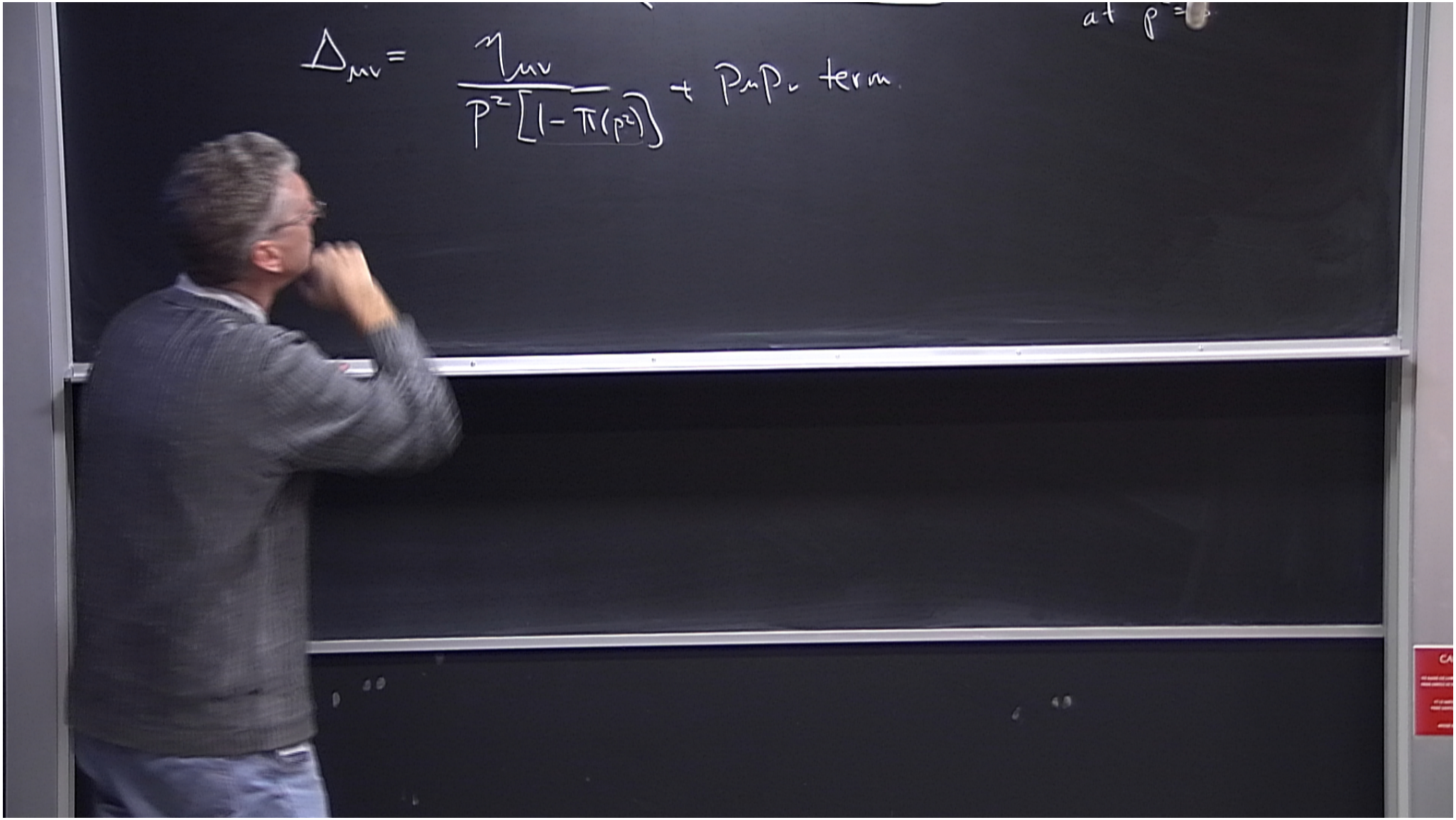
Abstract:


$\Pi_{\mu\nu} =$
$- p_\mu p_\nu) \Pi(p^2)$

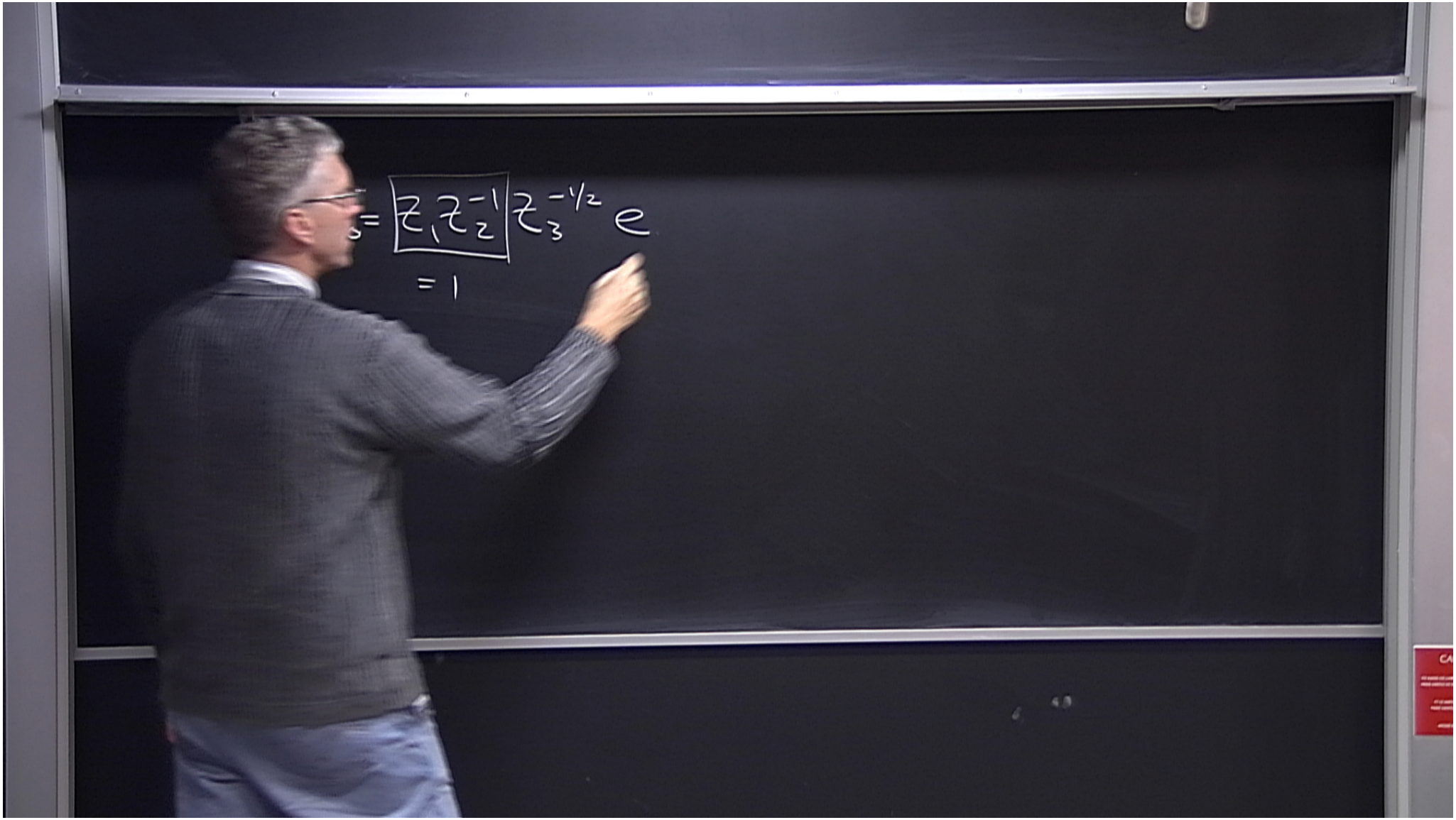

$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\,\Pi(p^2)$
$\Pi'(0) = 0$
$\Pi(0) = 0.$

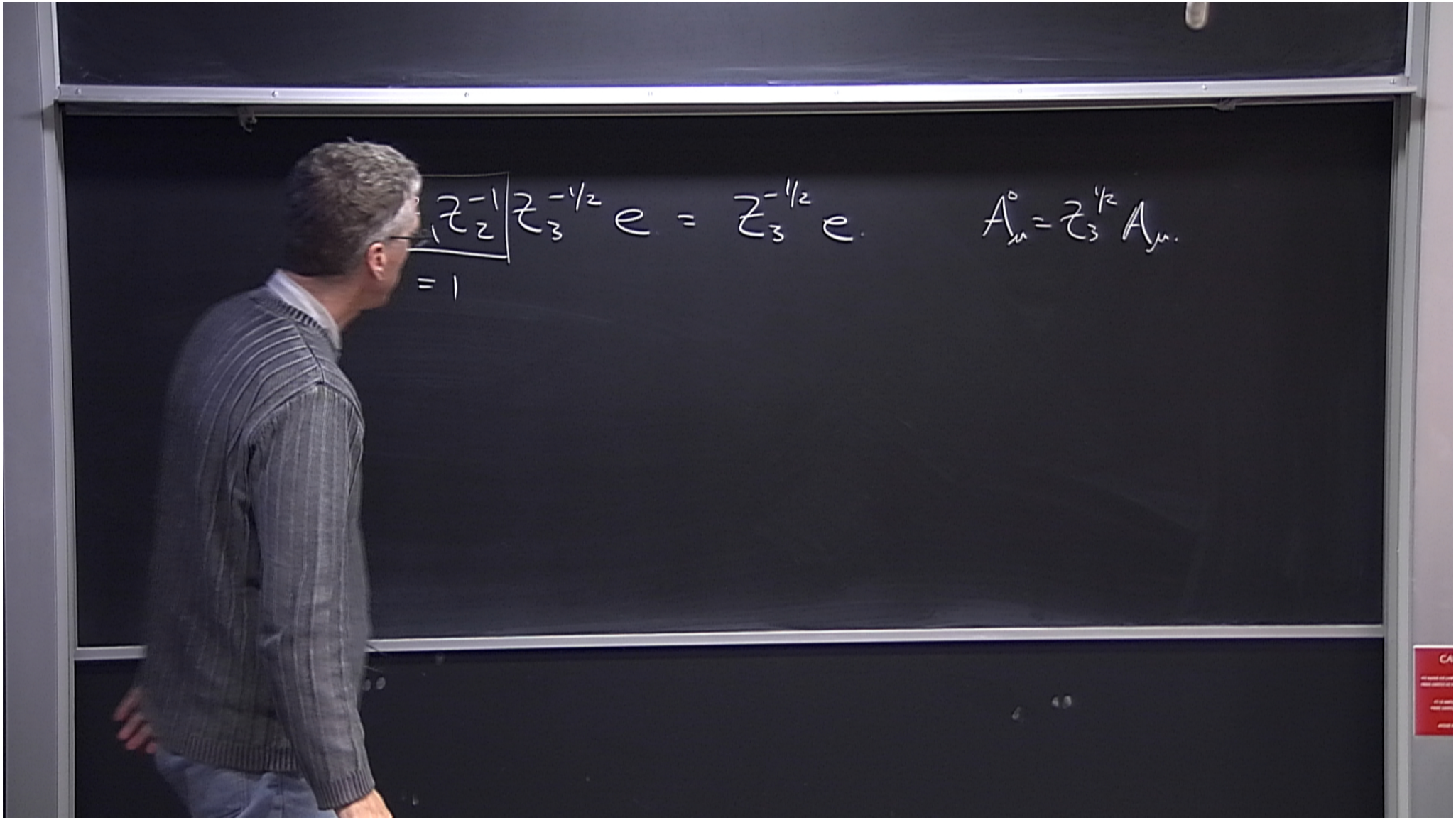

$\Pi_{\mu\nu} = (p^2 \eta_{\mu\nu} - p_\mu p_\nu)\,\Pi(p^2)$
$\Pi'(0) = 0$
$\Pi(0) = 0.$
$\frac{\eta_{\mu\nu}}{p^2[1-\Pi(p^2)]}$ + $p_\mu p_\nu$ term.

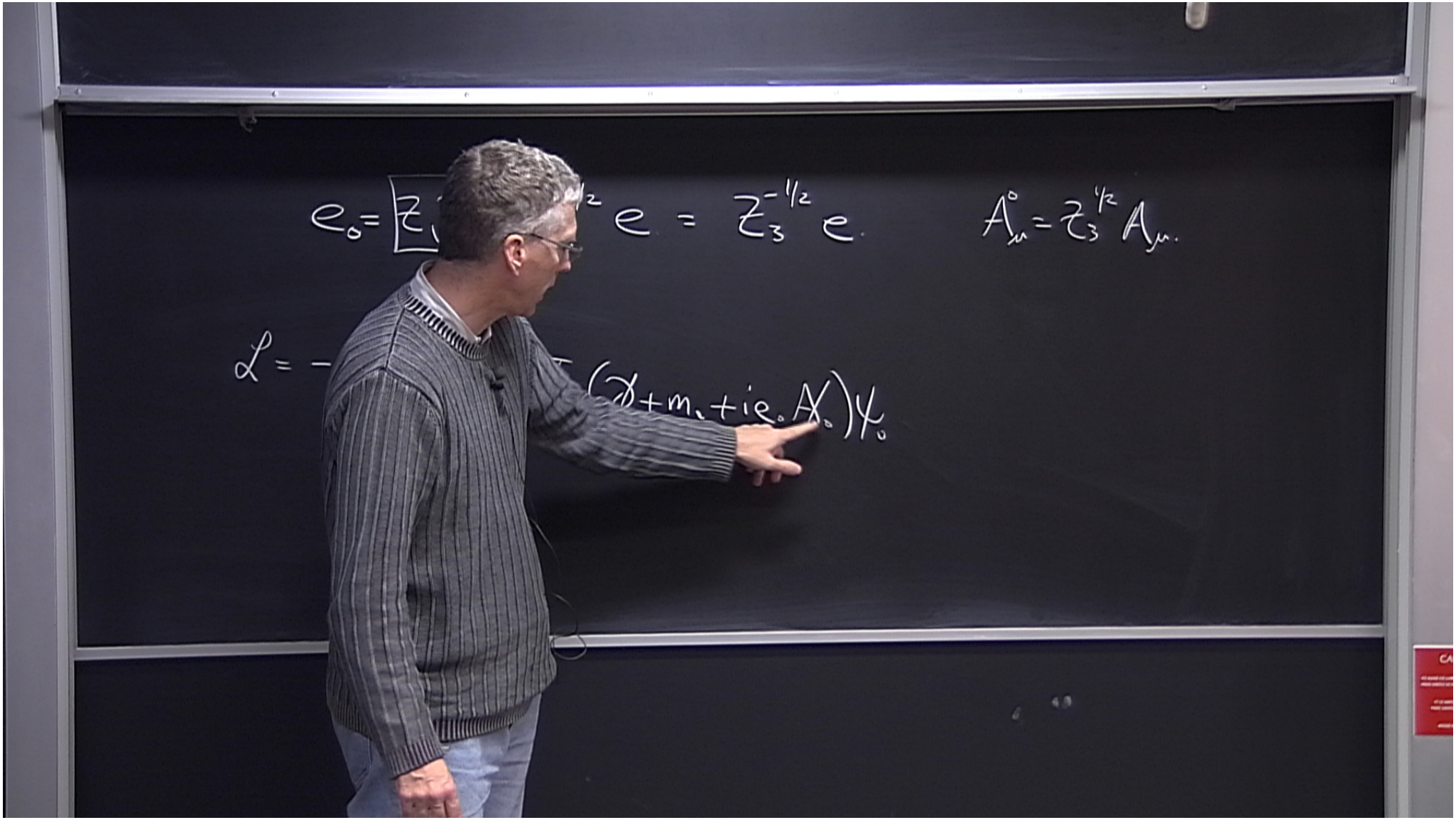

$D_{\mu\nu} = \frac{\eta_{\mu\nu}}{p^2[1-\pi(p^2)]} + p_\mu p_\nu$ term

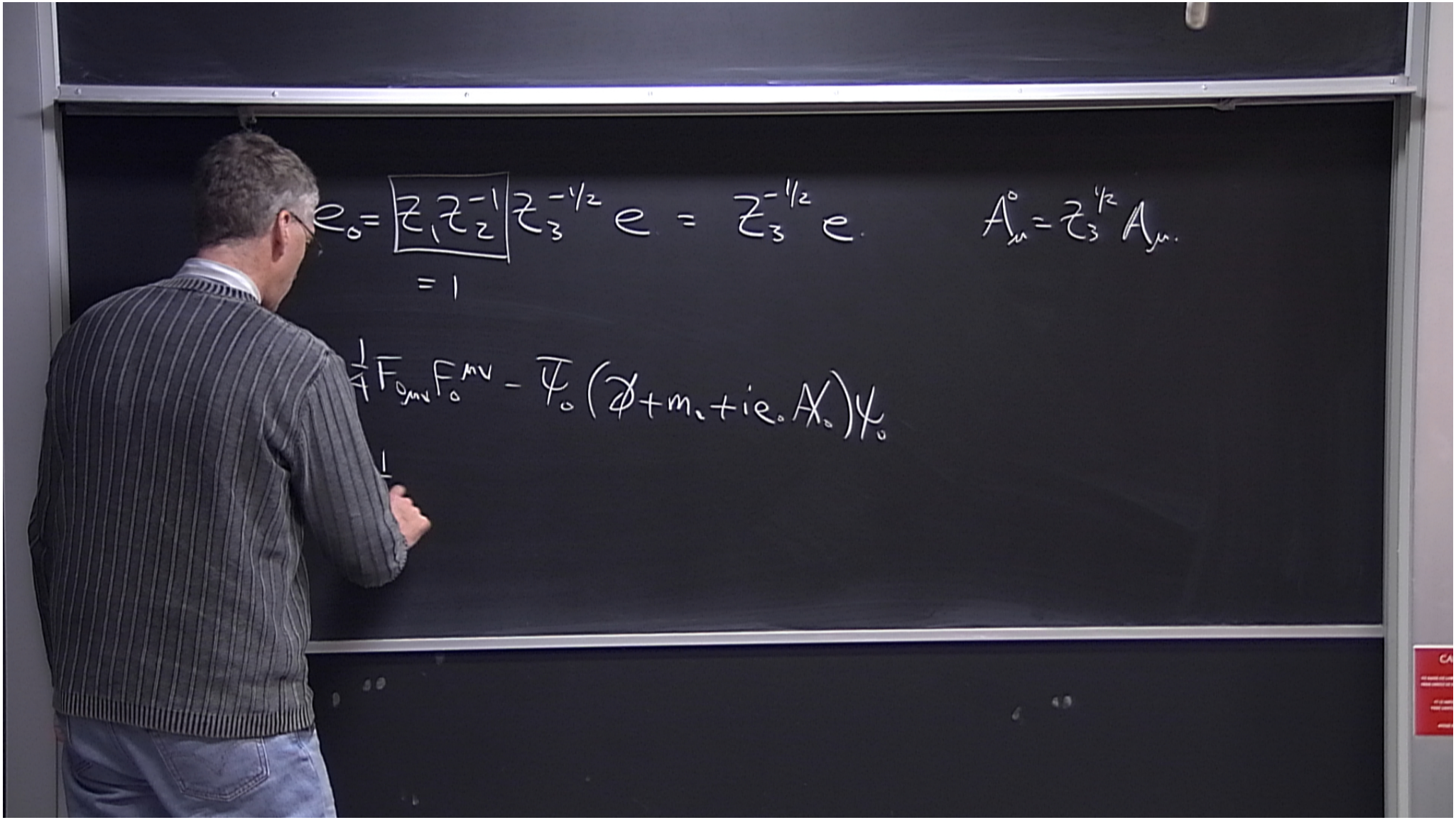

$\Delta_{\mu\nu} = \frac{\eta_{\mu\nu}}{p^2[1-\Pi(p^2)]} + p_\mu p_\nu$ term
at $p^2 =$

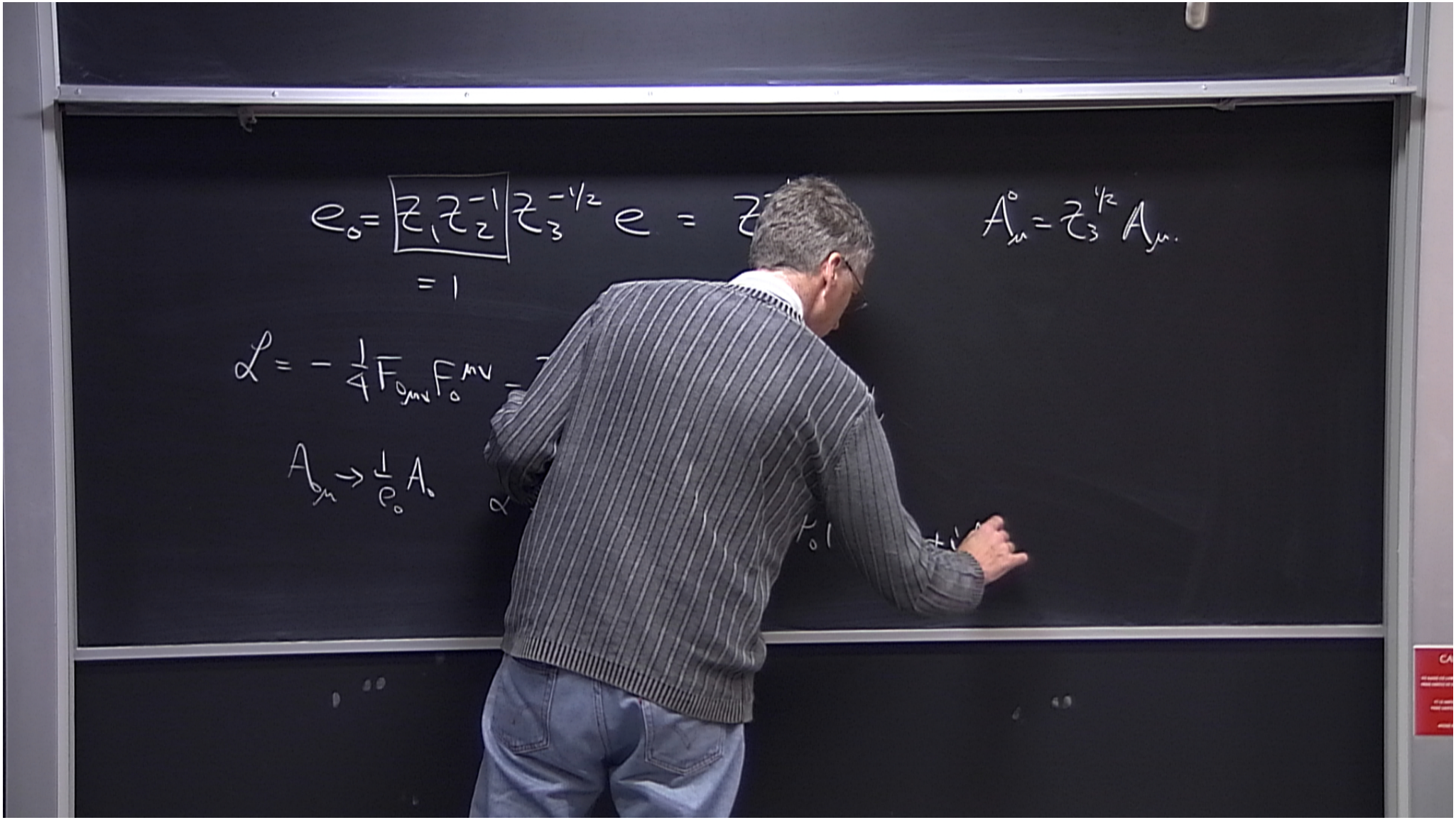



$= |z_1 z_2^{-1}| z_3^{-1/2} e$
$= 1$


$Z_1^{-1} Z_2 \, Z_3^{-1/2} e = Z_3^{-1/2} e$
$= 1$
$A^0_\mu = Z_3^{1/2} A_\mu$


$e_0 = \boxed{Z_1} e = Z_3^{-1/2} e$
$A_\mu^0 = Z_3^{1/2} A_\mu$
$\mathcal{L} = -$
$(\not\partial + m_0 + ie_0 \not A_0)\psi_0$


$e_0 = \boxed{Z_1 Z_2^{-1}} Z_3^{-1/2} e = Z_3^{-1/2} e$
$= 1$
$A_\mu^0 = Z_3^{1/2} A_\mu$
$\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + i e_0 \not{A}_0) \psi_0$

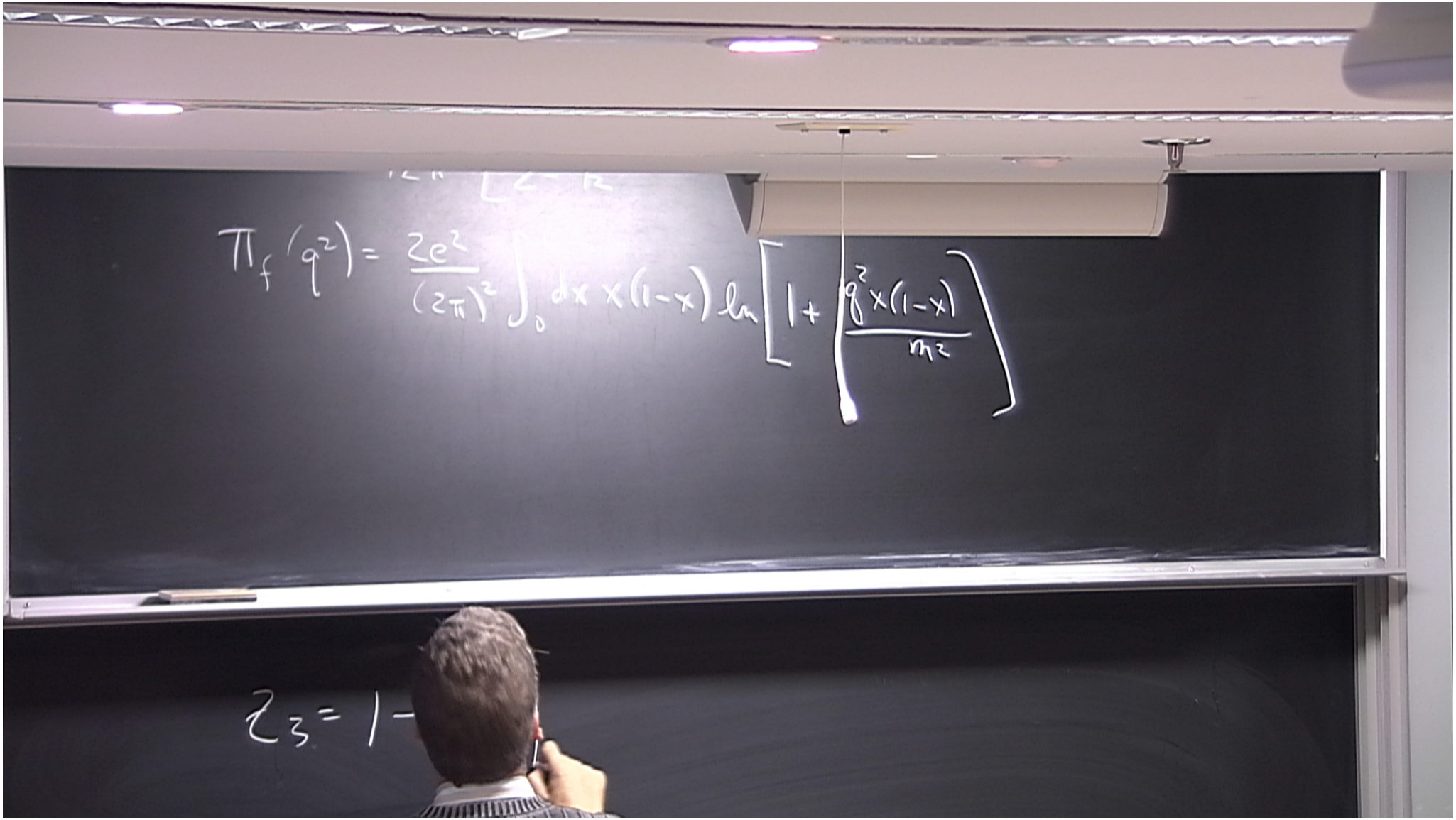

$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e = Z$
$= 1$
$A^0_\mu = Z_3^{1/2} A_\mu$
$\mathcal{L} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu}$
$A_{0\mu} \to \frac{1}{e_0} A_0$

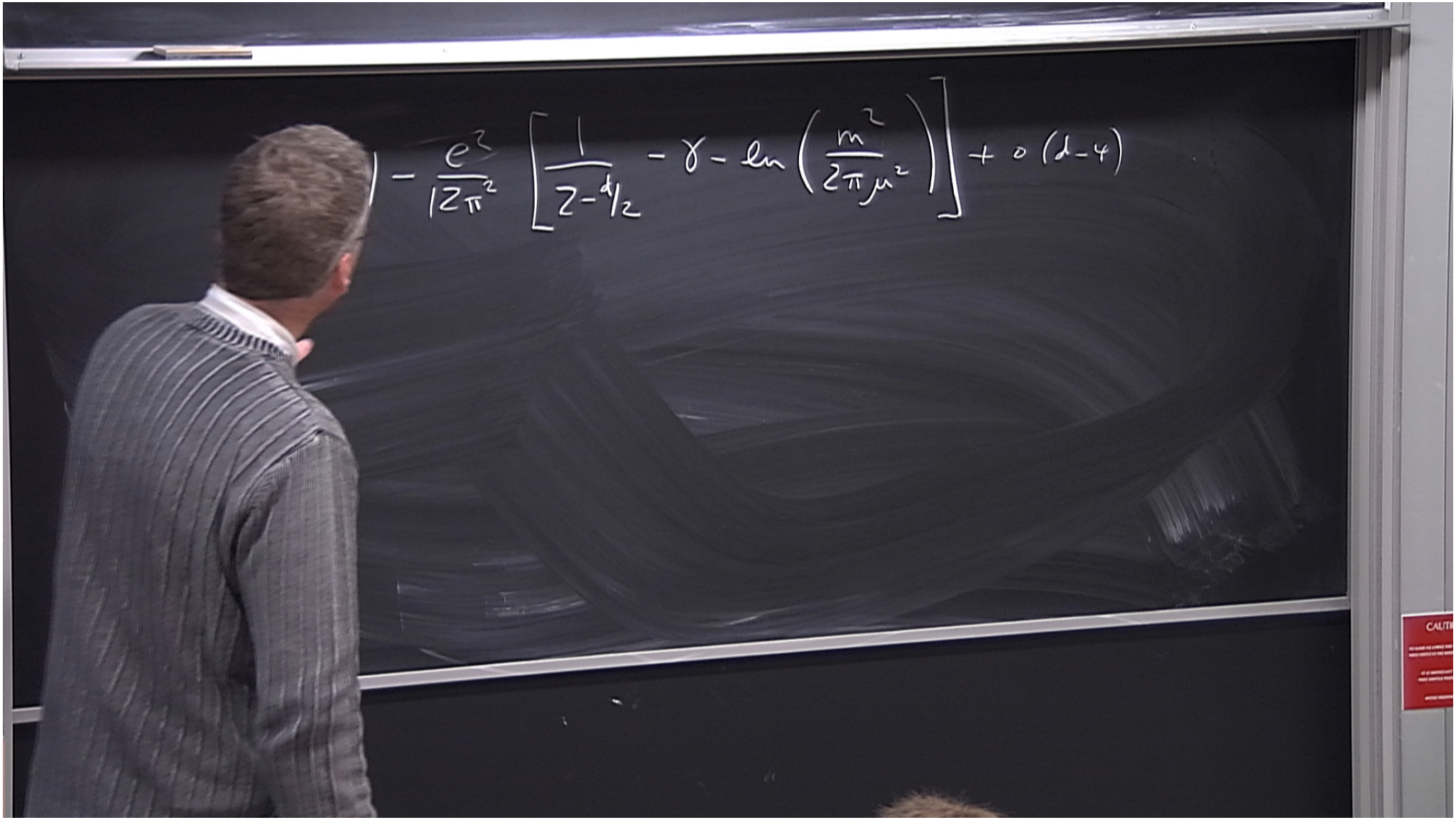
$$e_0 = \boxed{Z_1 Z_2^{-1}} Z_3^{-1/2} e = \underline{\underline{Z_3^{-1/2} e}} \qquad A_\mu^0 = Z_3^{1/2} A_\mu .$$

$$= 1$$

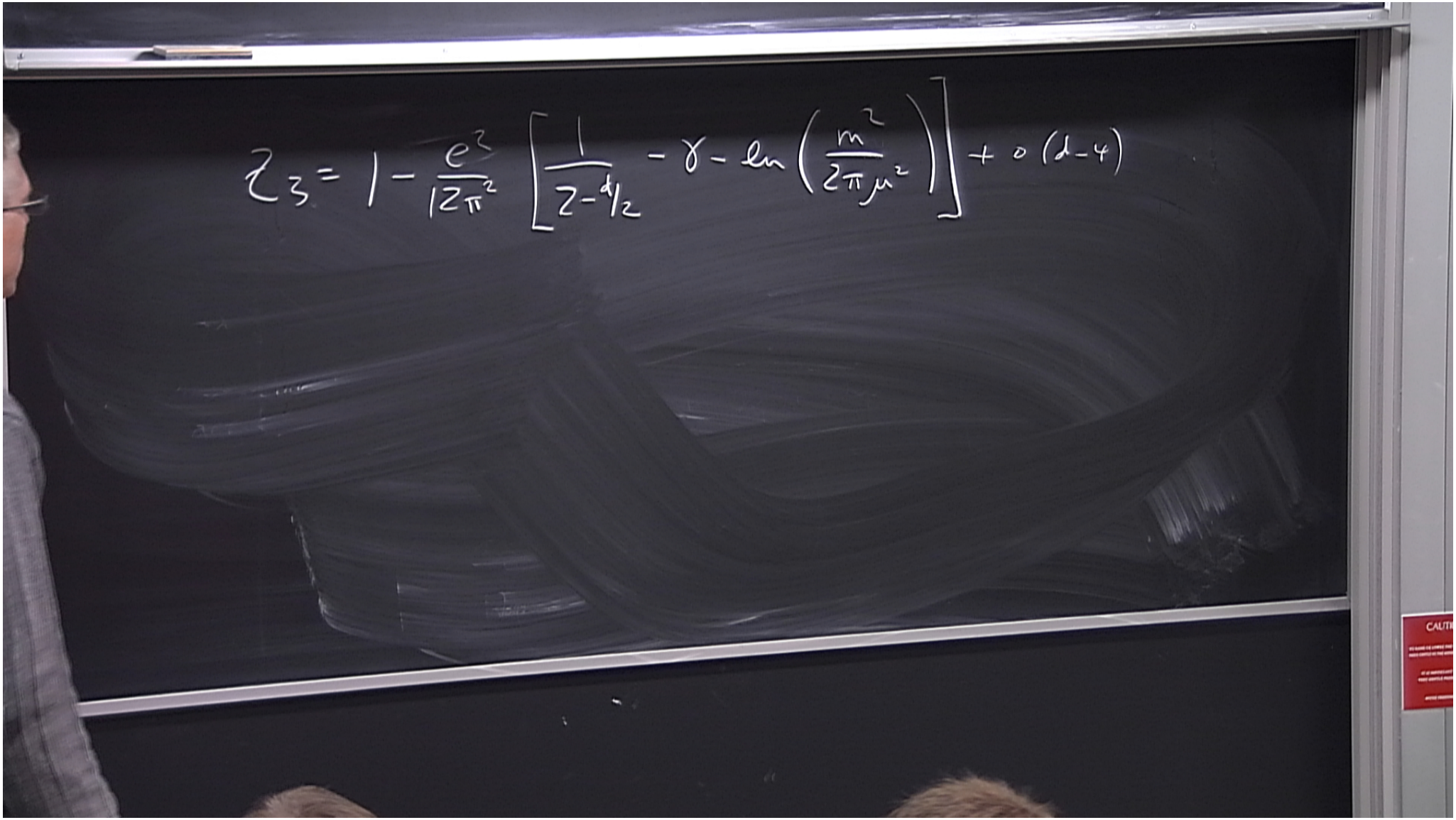
$$\mathcal{L} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + i e_0 \not{A}_0) \psi_0$$

$$A_\mu \to \frac{1}{e_0} A_0 \qquad \mathcal{L} = -\frac{1}{4 e_0^2} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}_0 (\not{\partial} + m_0 + i \not{A}_0) \psi_0$$


$\Pi_f(q^2) = \frac{2e^2}{(2\pi)^2} \int_0 dx\, x(1-x) \ln\left[1 + \frac{q^2 x(1-x)}{m^2}\right]$
$Z_3 = 1 -$


$) - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$

$$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{4\pi\mu^2}\right)\right] + O(d-4)$$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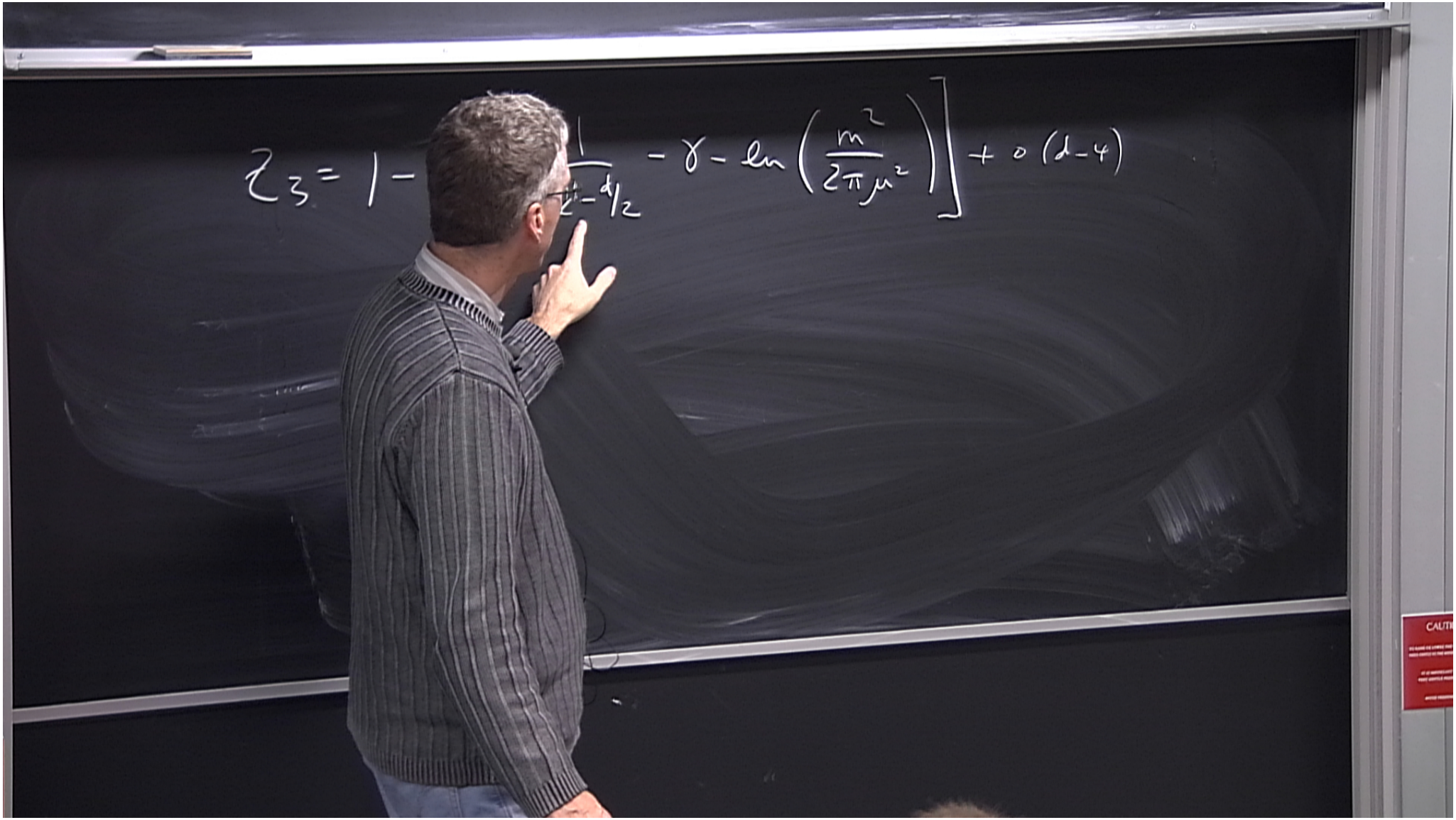
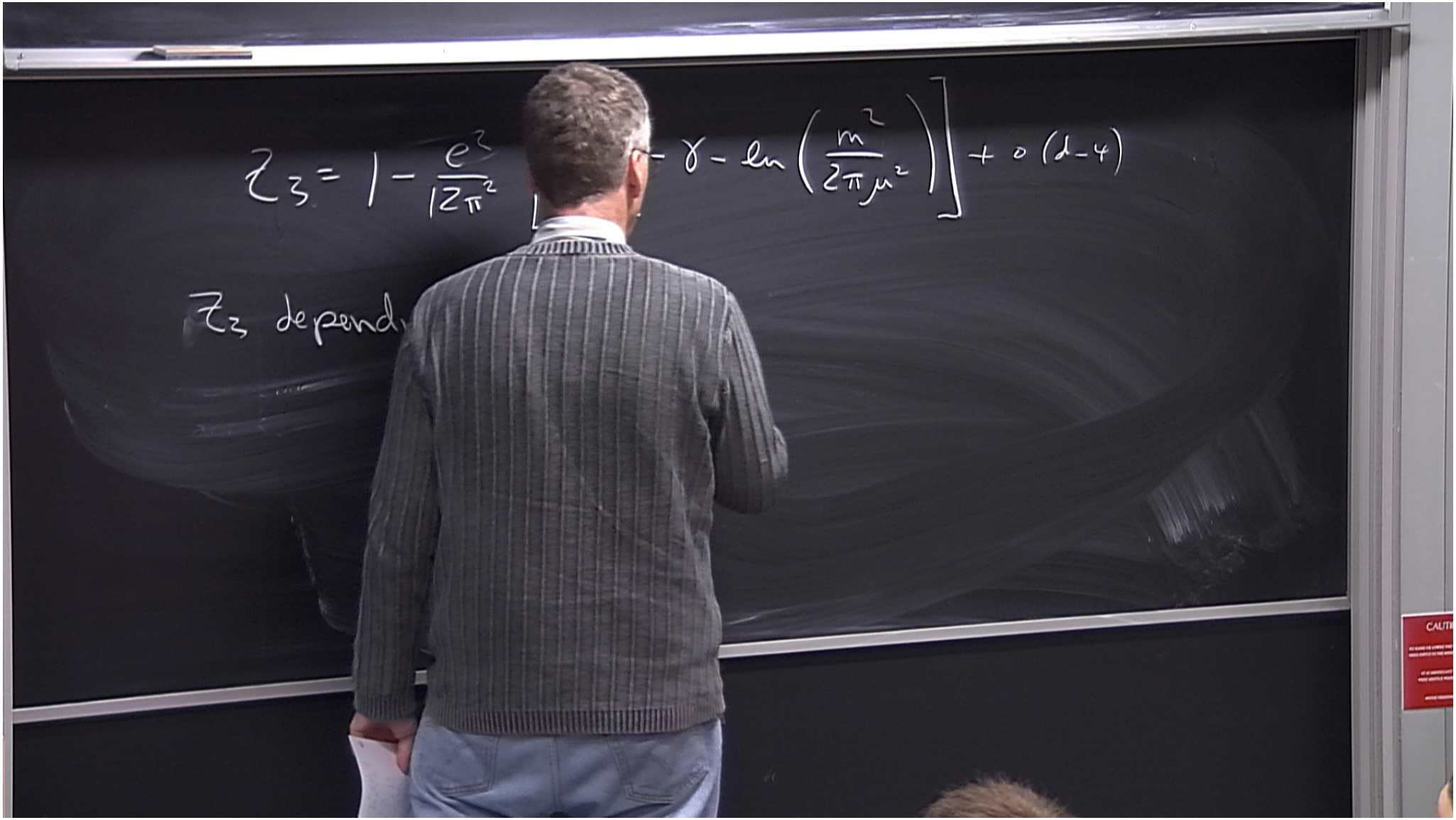
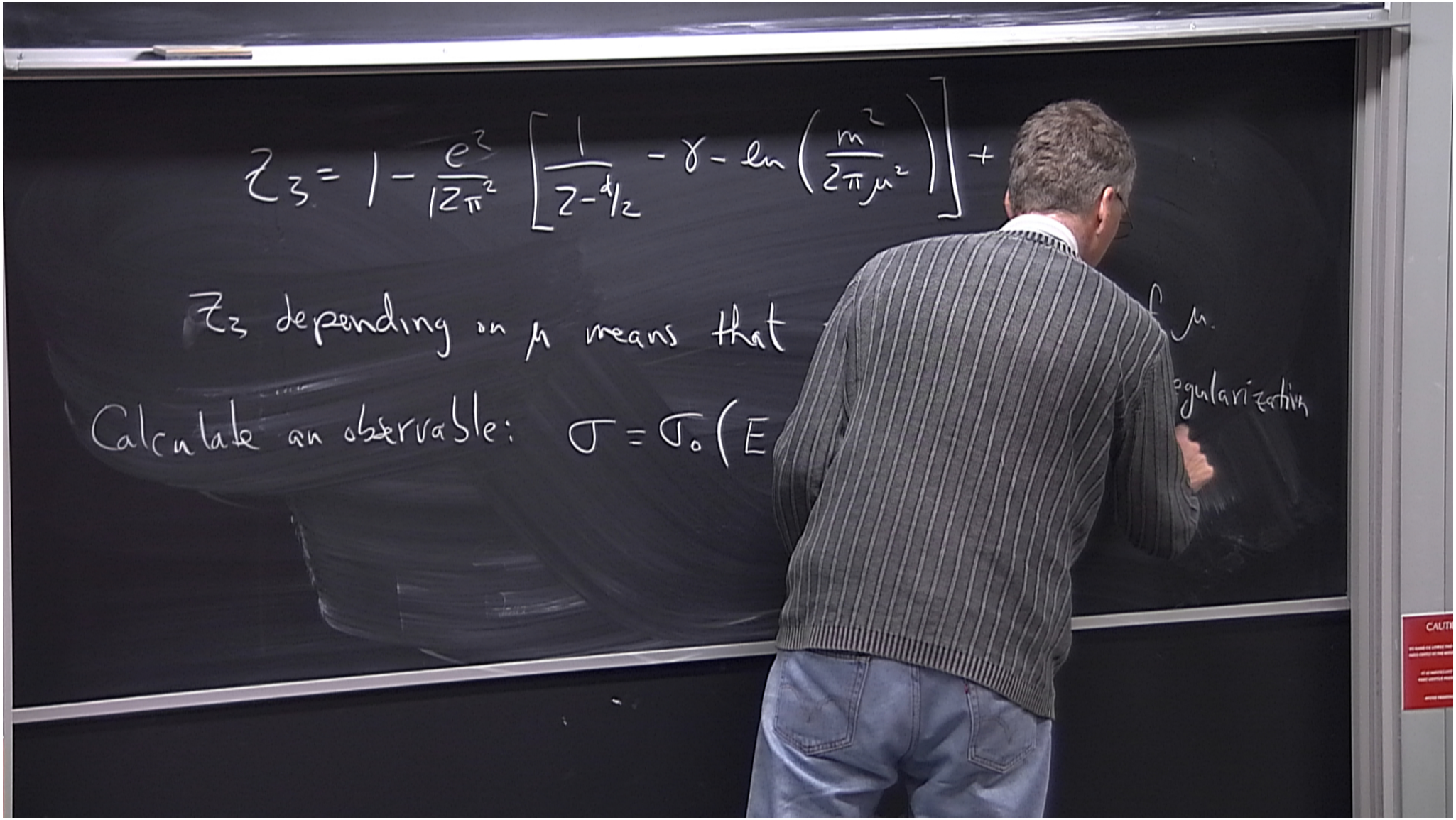
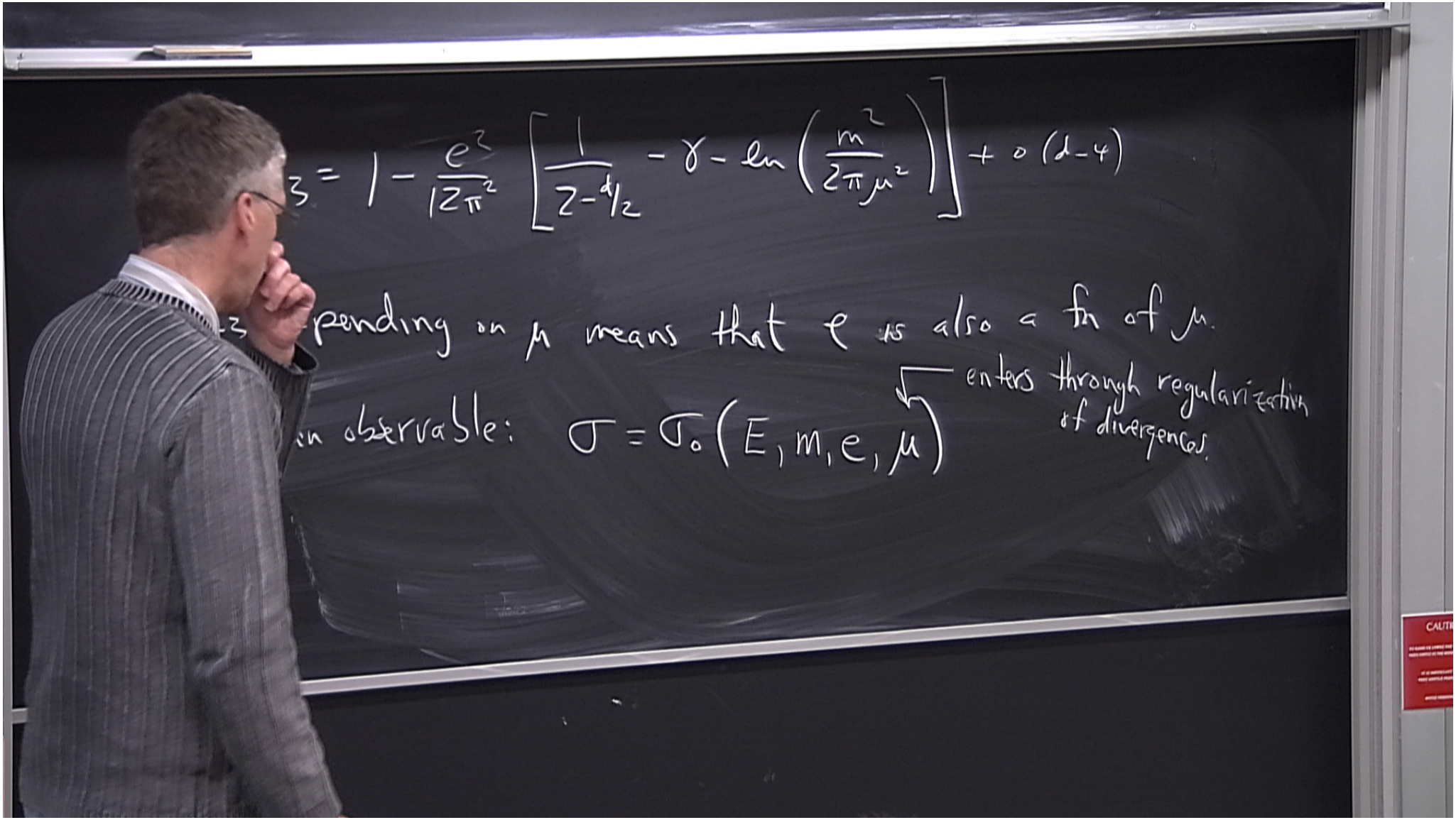
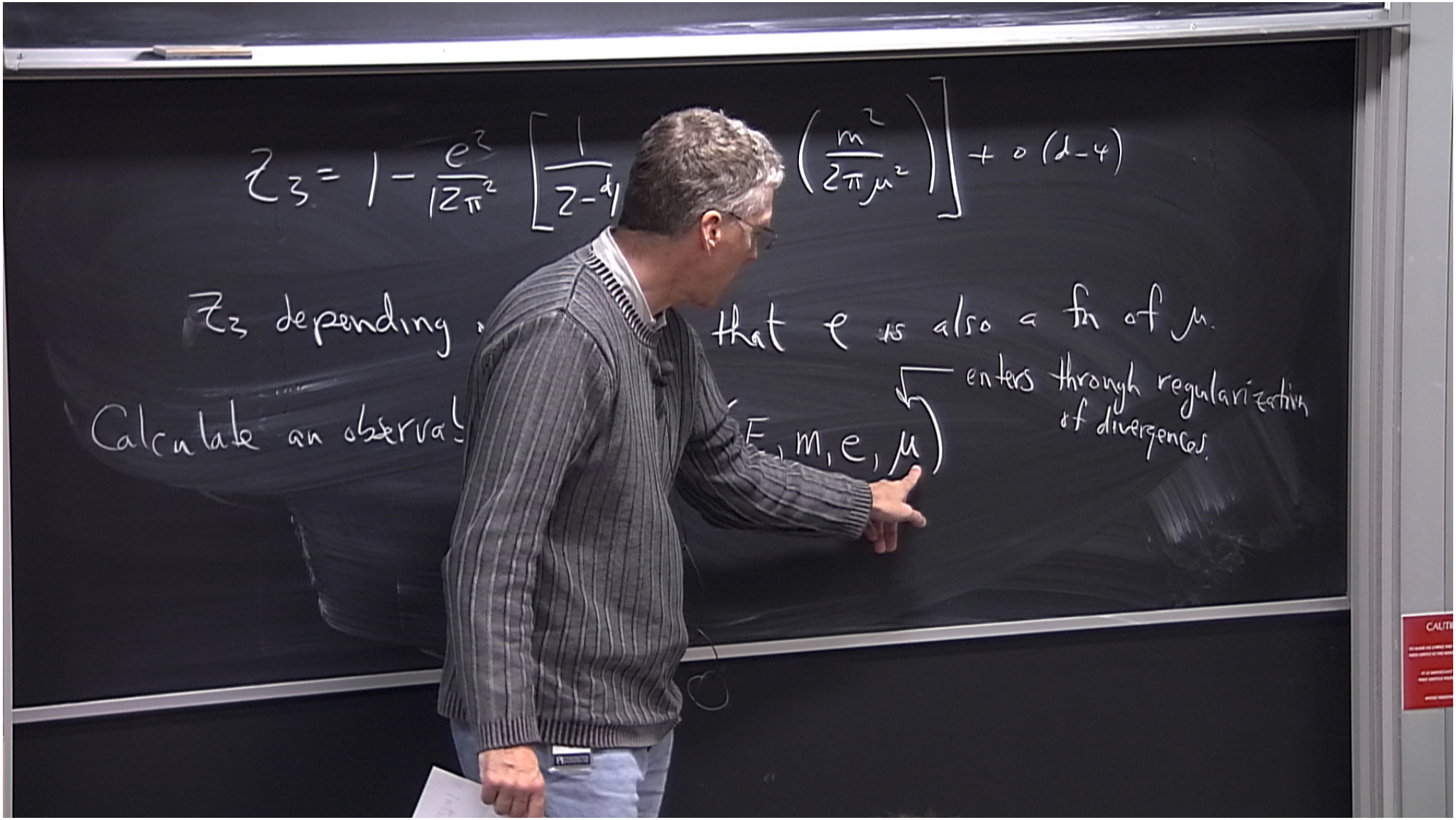
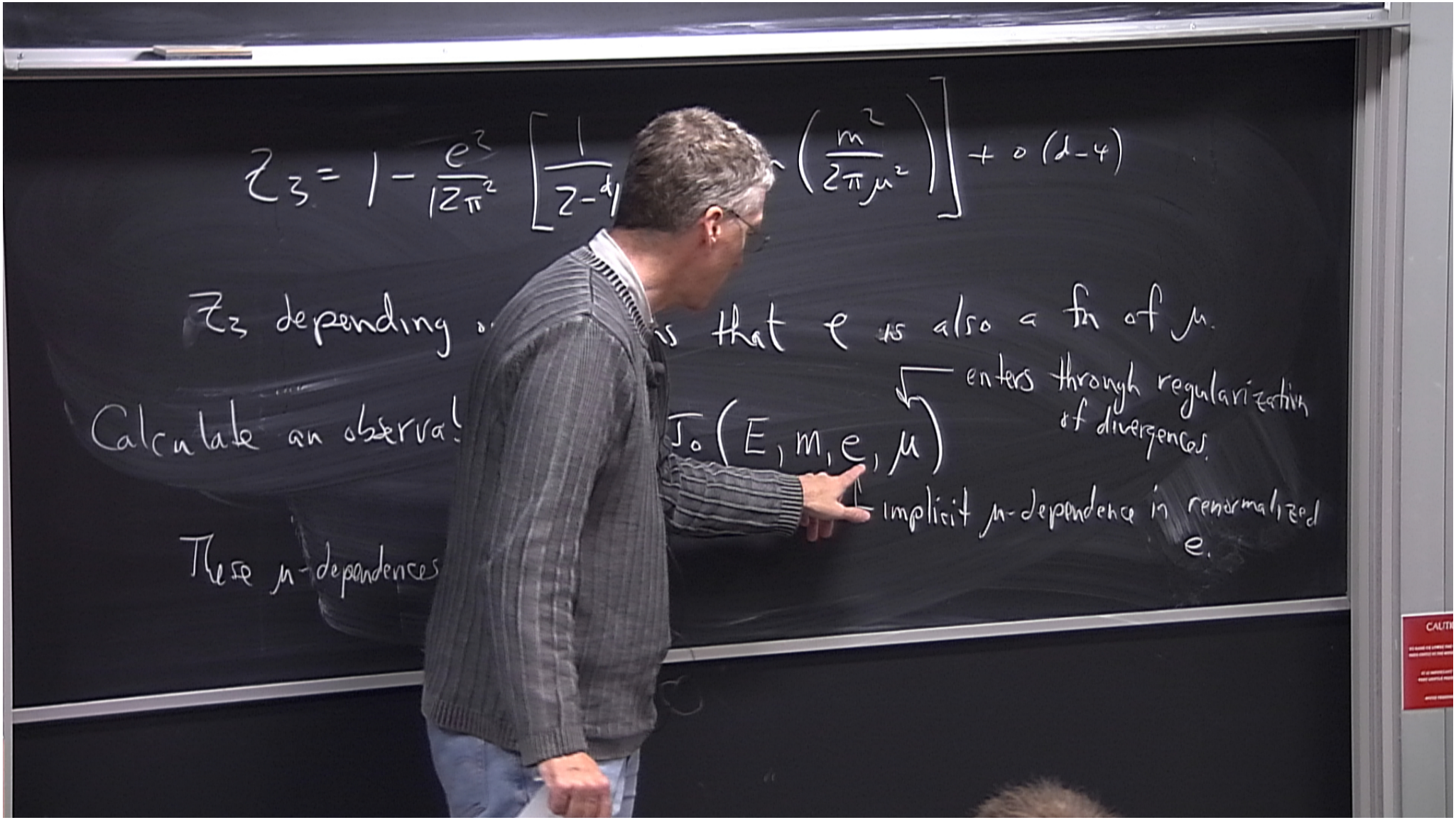

$Z_3 = 1 -$
$\frac{1}{2^{1-d/2}}$
$- \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\Big] + o(d-4)$

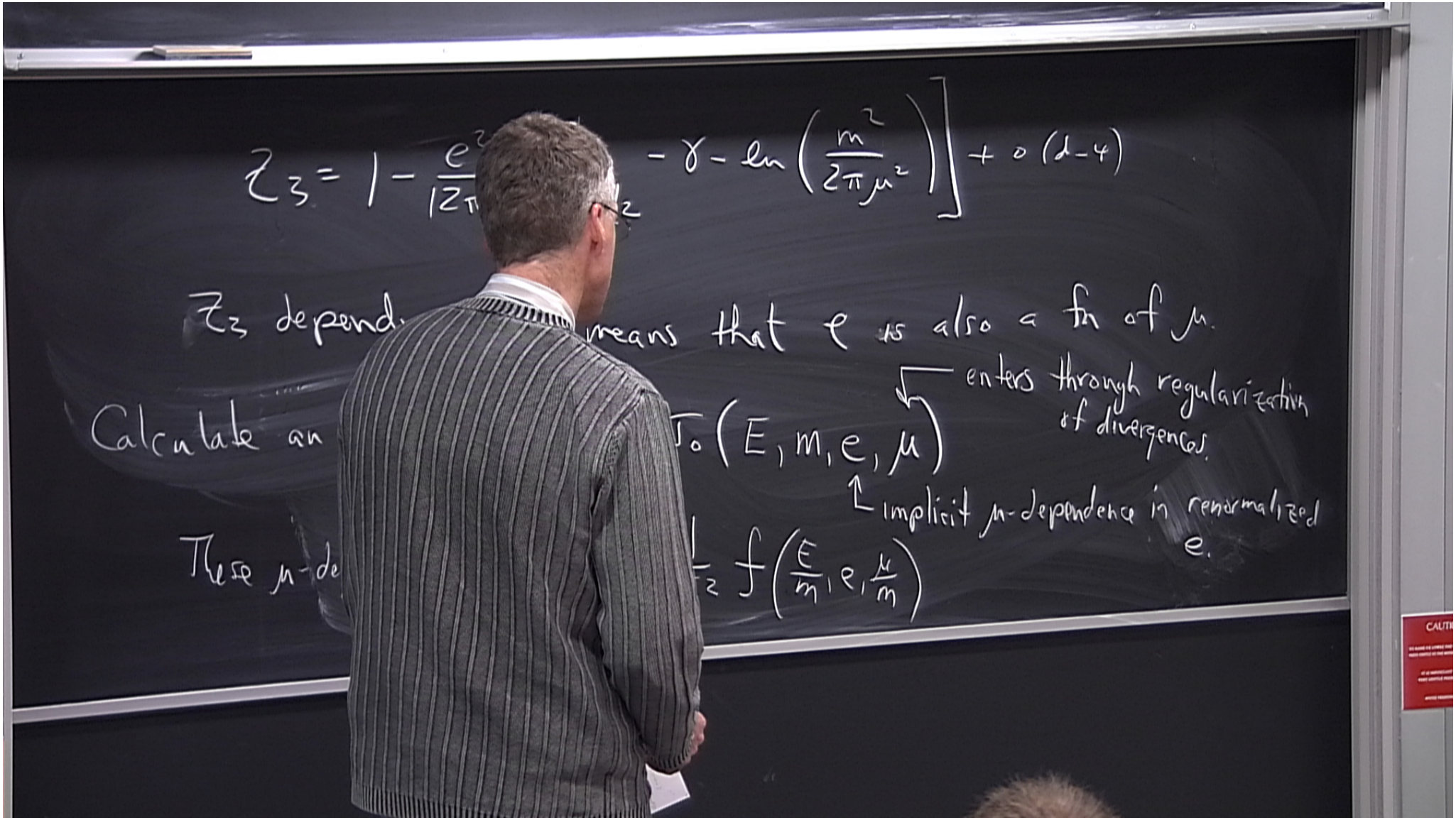

$Z_3 = 1 - \frac{e^2}{12\pi^2}$
$- \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\Big] + o(d-4)$
Z_3 depend

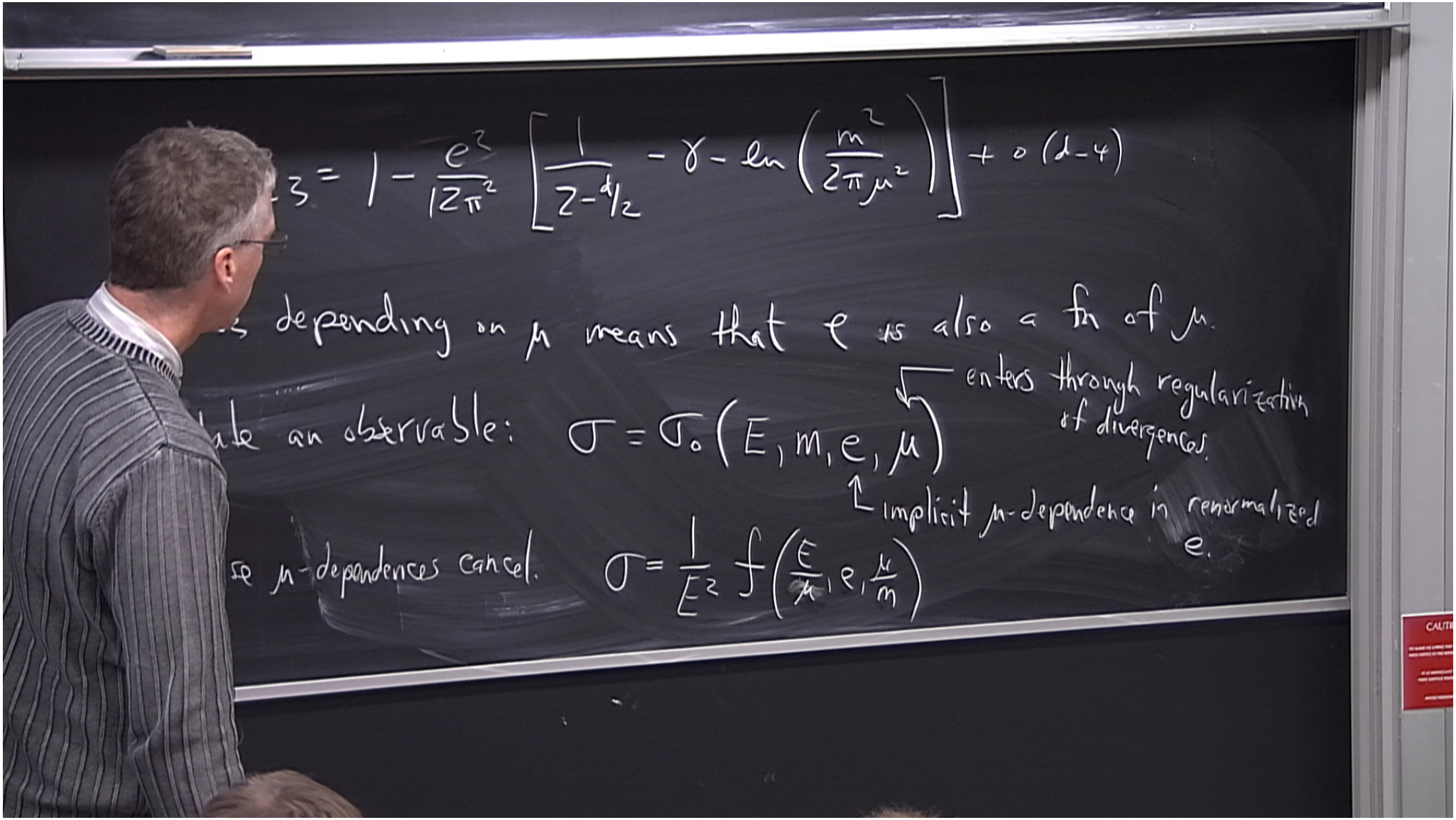

$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] +$
Z_3 depending on μ means that
Calculate an observable: $\sigma = \sigma_0($


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
pending on μ means that e is also a fn of μ.
in observable: $\sigma = \sigma_0(E, m, e, \mu)$
enters through regularization of divergences.


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-4}\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
Z_3 depending
that e is also a fn of μ.
Calculate an observa
$E, m, e, \mu)$
enters through regularization of divergences.


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-4}\ldots\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
Z_3 depending ... is that e is also a fn of μ.
Calculate an observa... $J_0(E, m, e, \mu)$
enters through regularization of divergences.
implicit μ-dependence in renormalized e.
These μ-dependences


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{\epsilon} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
Z_3 depend
means that e is also a fn of μ.
enters through regularization of divergences.
Calculate an
$\Gamma_0(E, m, e, \mu)$
implicit μ-dependence in renormalized e.
These μ-de
$\frac{1}{2} f\left(\frac{E}{m}, e, \frac{\mu}{m}\right)$


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
depending on μ means that e is also a fn of μ.
an observable: $\sigma = \sigma_0(E, m, e, \mu)$
enters through regularization of divergences.
implicit μ-dependence in renormalized e.
μ-dependences cancel. $\sigma = \frac{1}{E^2} f\left(\frac{E}{\mu}, e, \frac{\mu}{m}\right)$

$$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$$

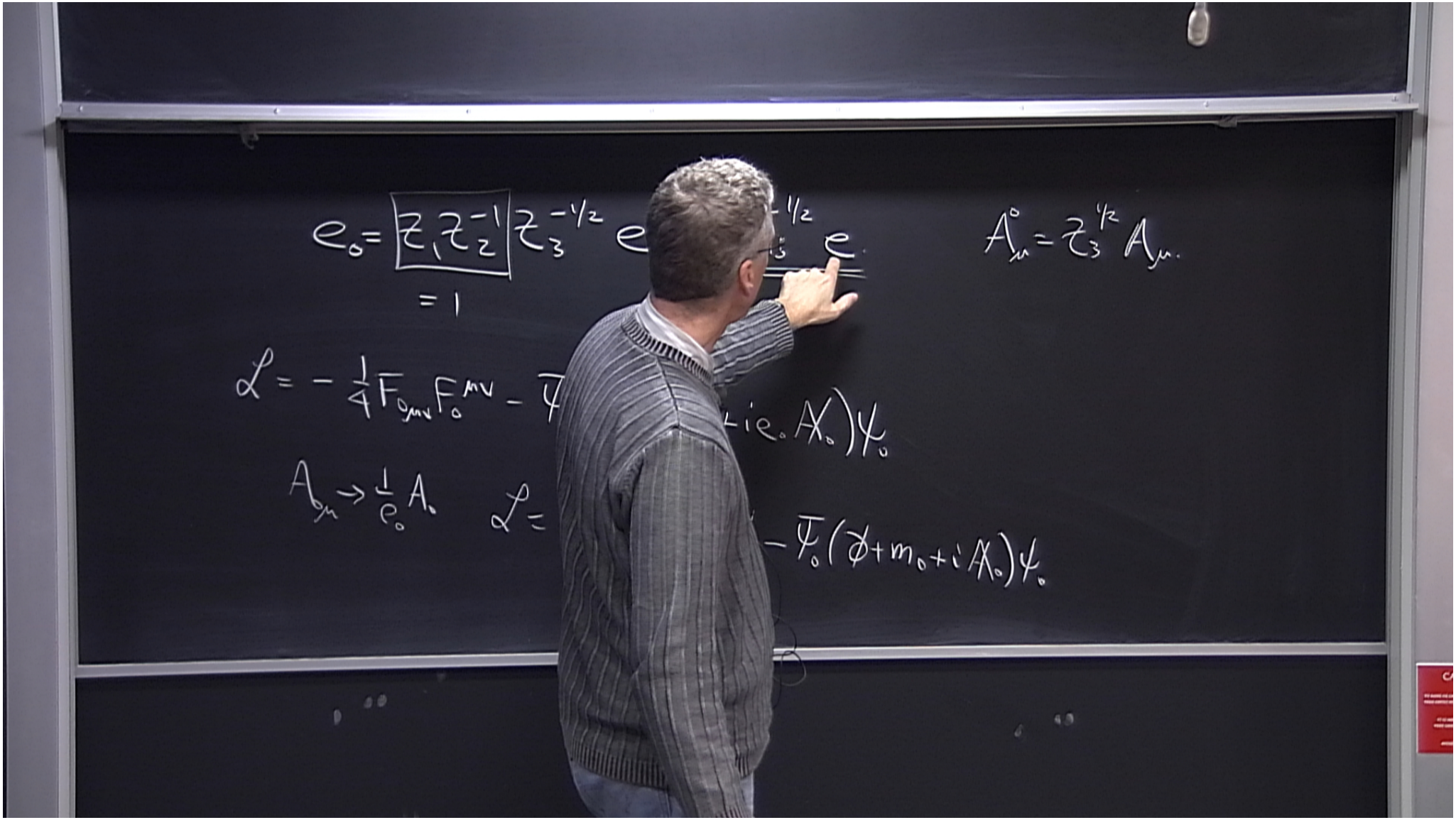
Z_3 depending on μ means that e is also a fn of μ.

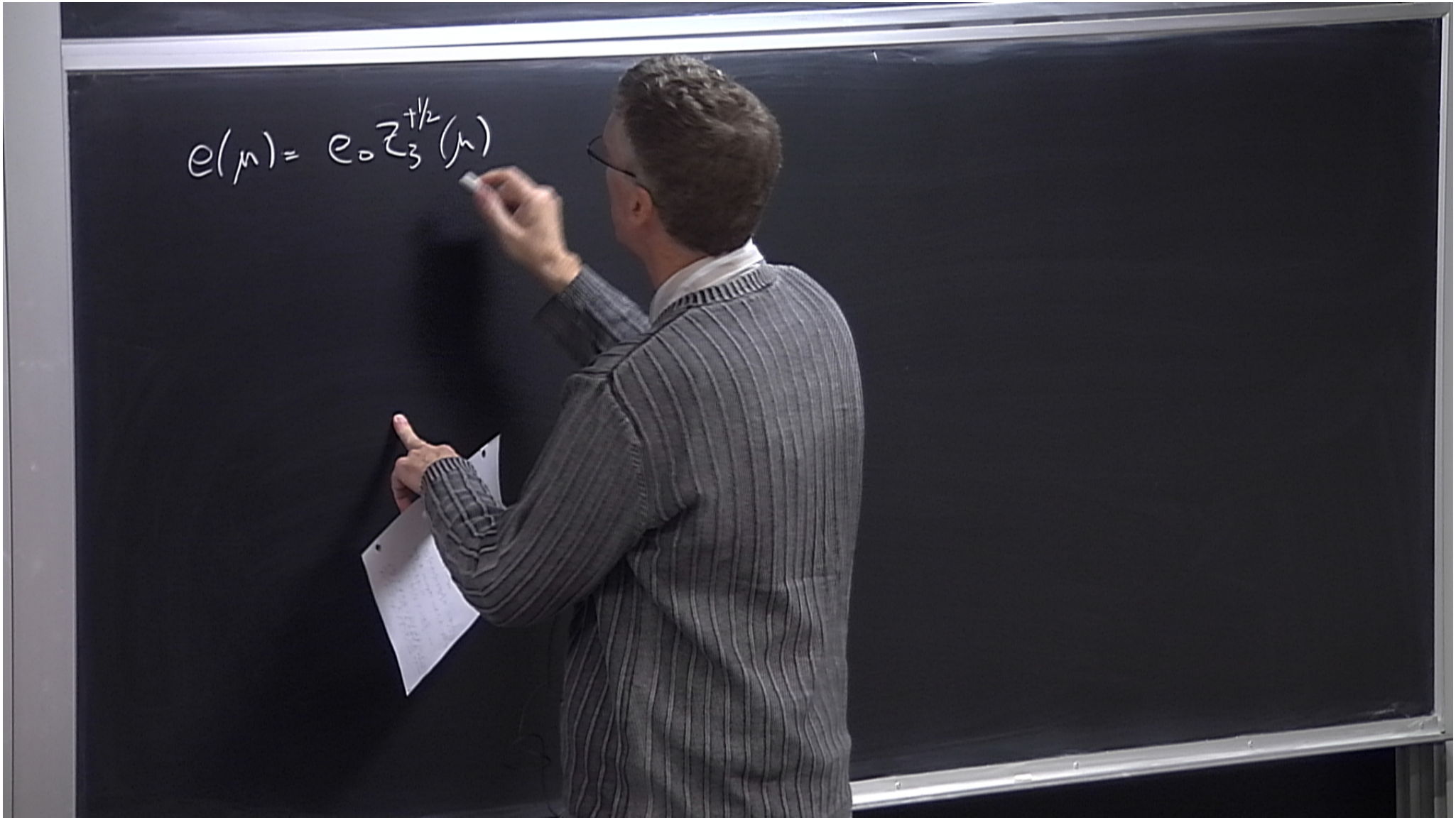
Calculate an observable: $\sigma = \sigma_0(E, m, e, \mu)$

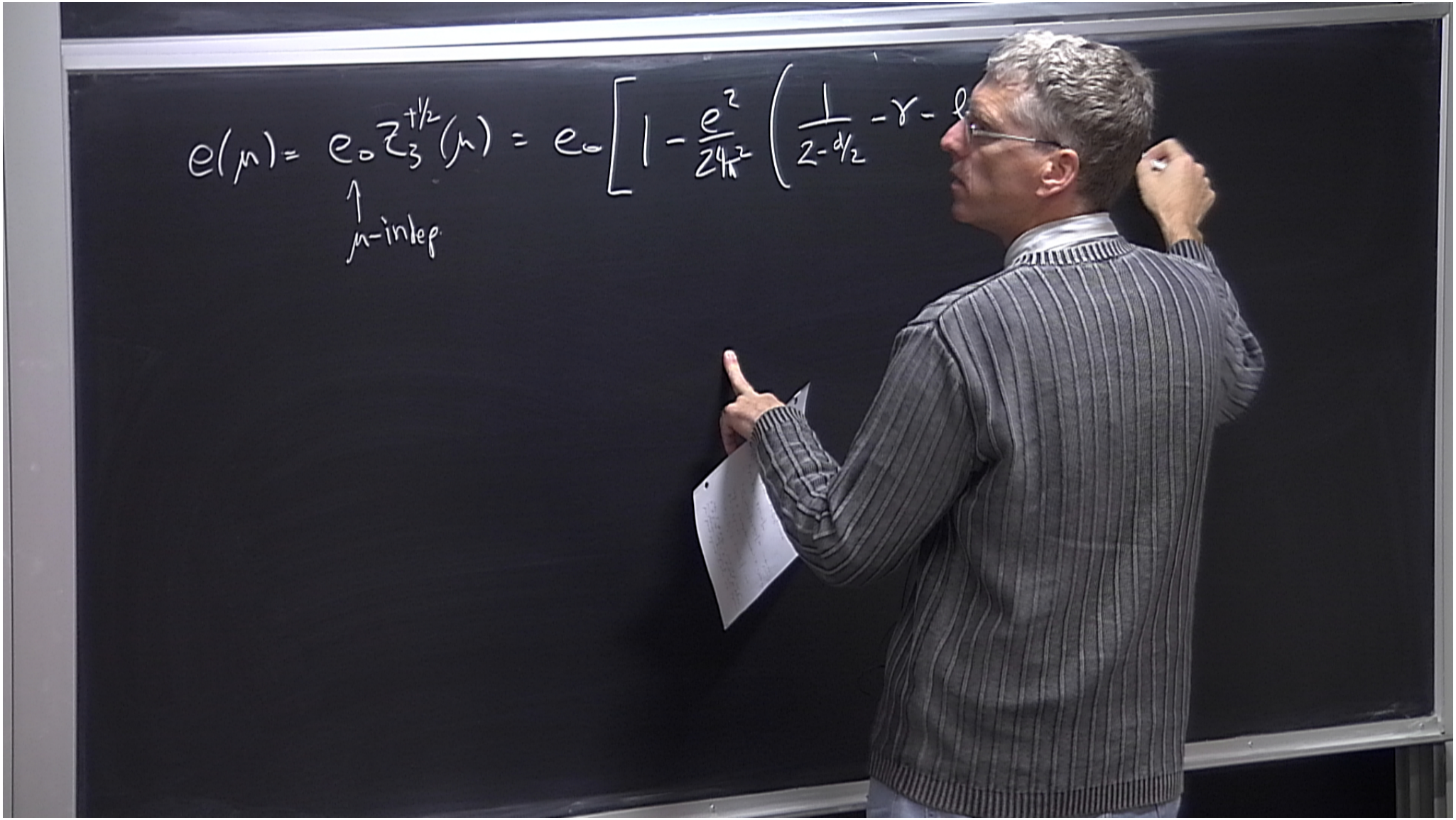
μ enters through regularization of divergences.

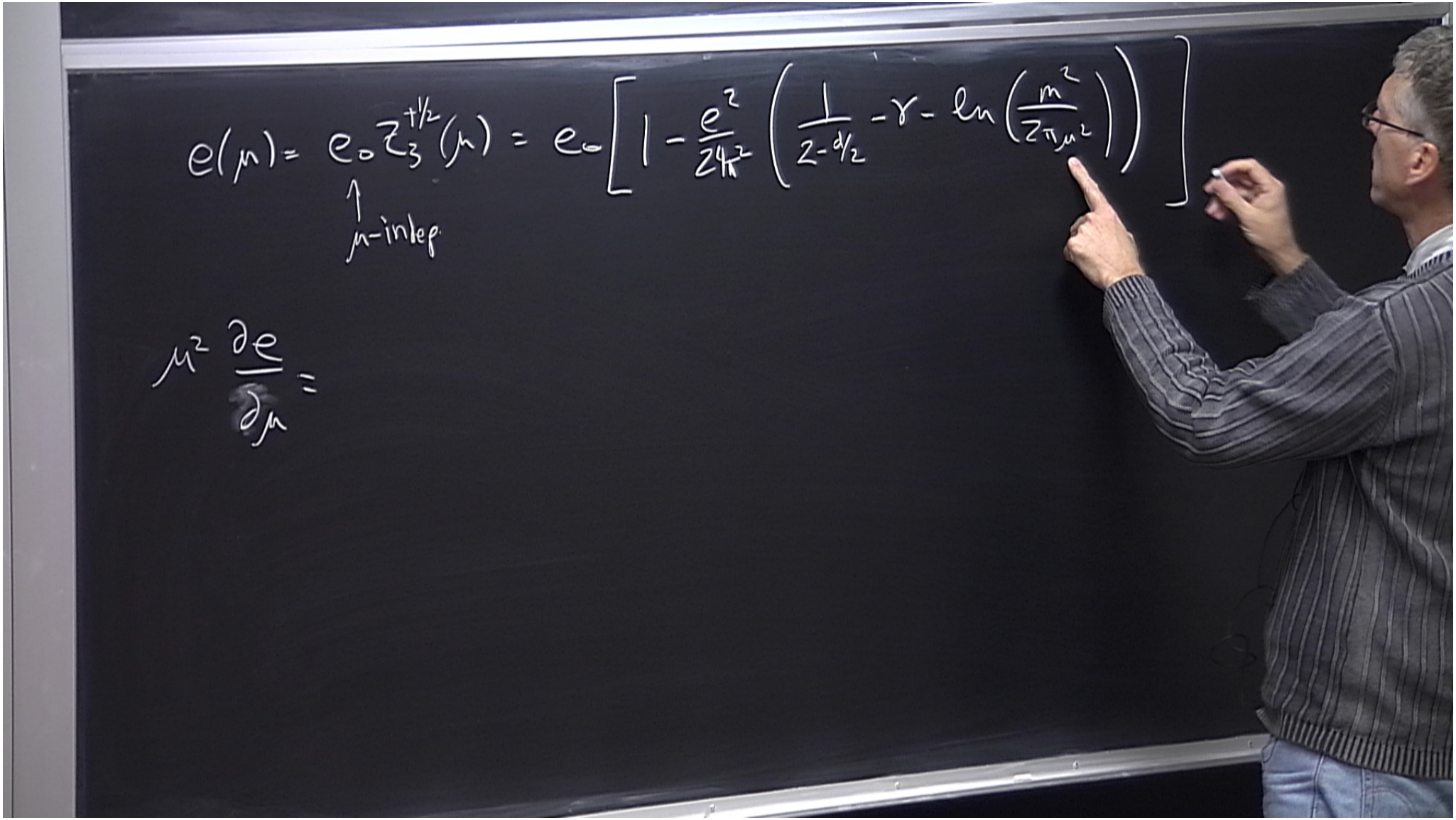
implicit μ-dependence in renormalized e.

These μ-dependences cancel. $\sigma = \frac{1}{E^2} f\left(\frac{E}{\mu}, e, \frac{\mu}{m}\right)$


$e_0 = Z_1 Z_2^{-1} Z_3^{-1/2} e$
$= 1$
$A_\mu^0 = Z_3^{1/2} A_\mu$
$\mathcal{L} = -\frac{1}{4} F_{0\mu\nu} F_0^{\mu\nu} - \bar{\psi}$
$+ ie_0 A_0)\psi_0$
$A_{0\mu} \to \frac{1}{e_0} A_0$
$\mathcal{L} =$
$- \bar{\psi}_0 (\partial + m_0 + i A_0)\psi_0$


e(μ) = e₀ Z₃^{+1/2}(μ)

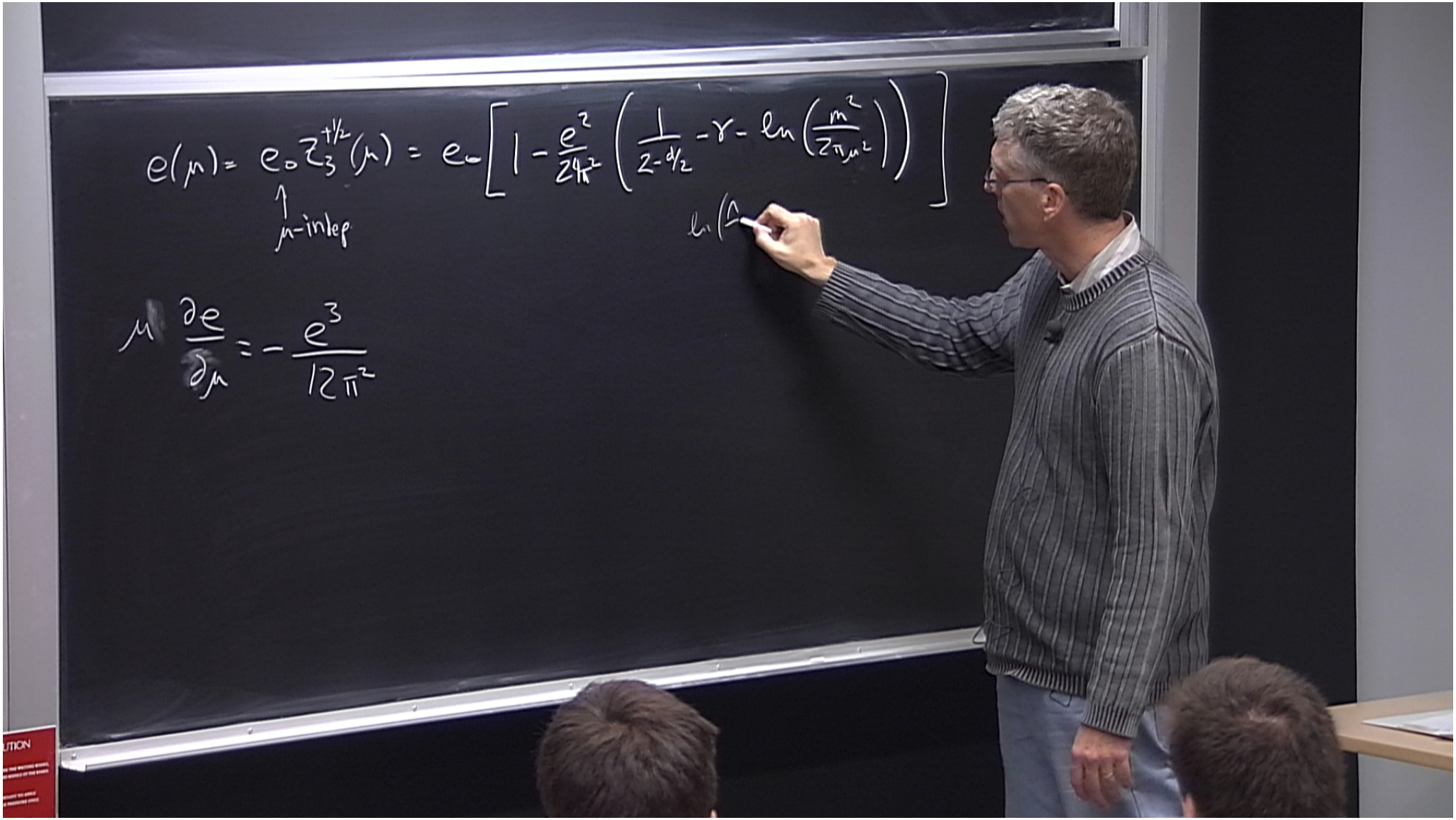

e(μ) = e₀ Z₃^{+1/2}(μ) = e₀ [1 − e²/24π² (1/2−d/2 − γ −
μ-indep

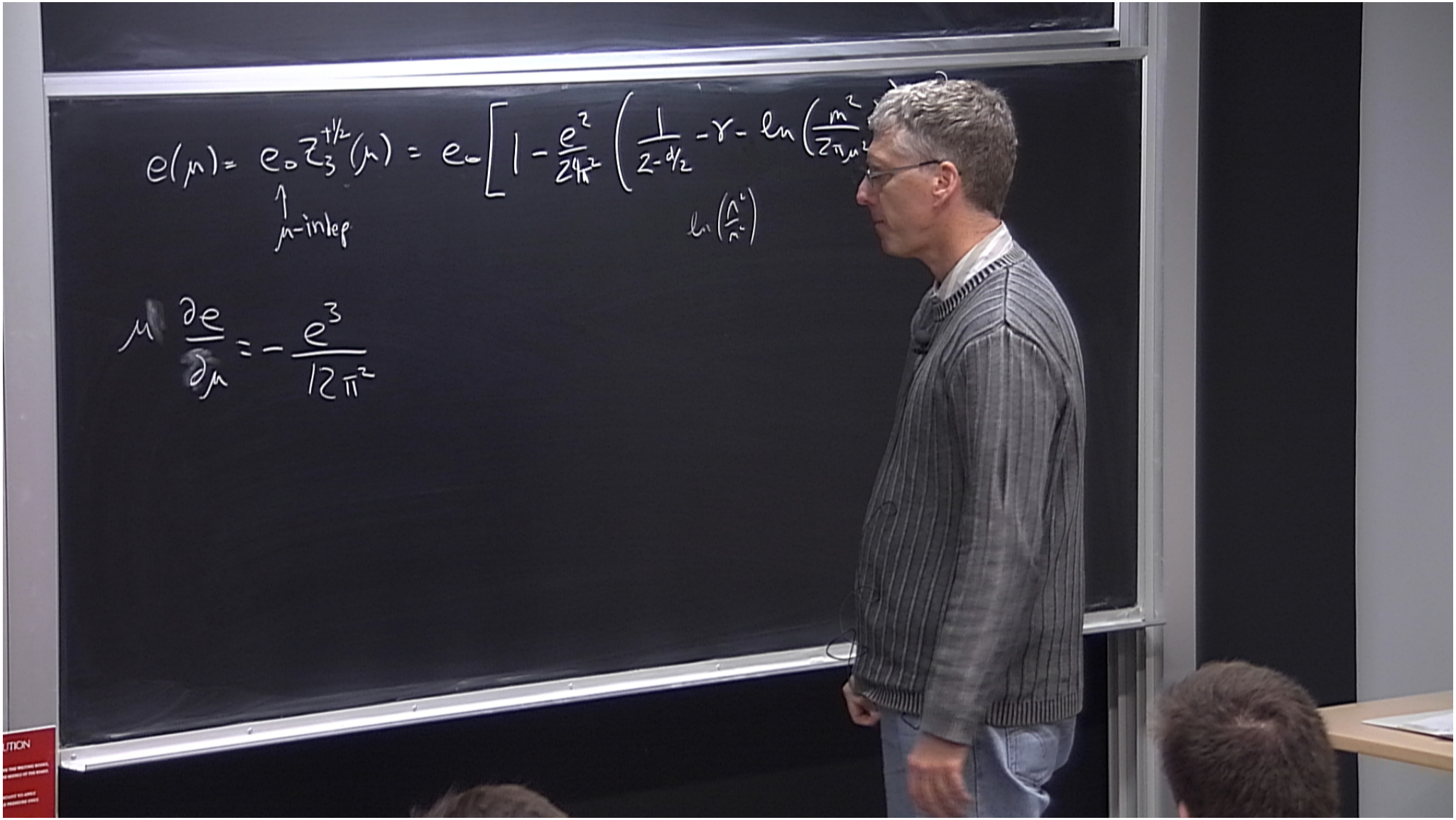

$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right)\right]$
μ-indep.
$\mu^2 \frac{\partial e}{\partial \mu} =$

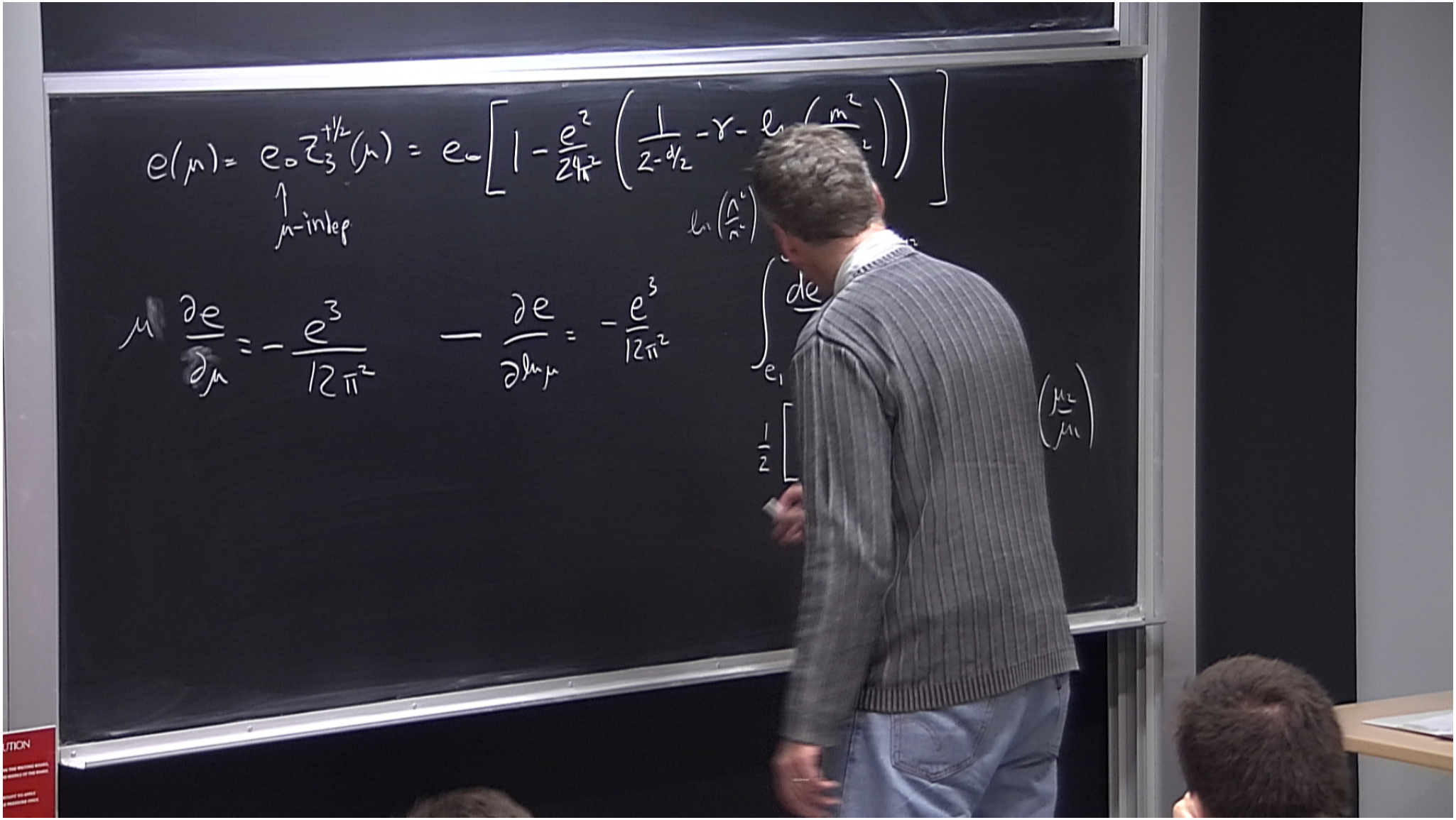
$$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right)\right]$$

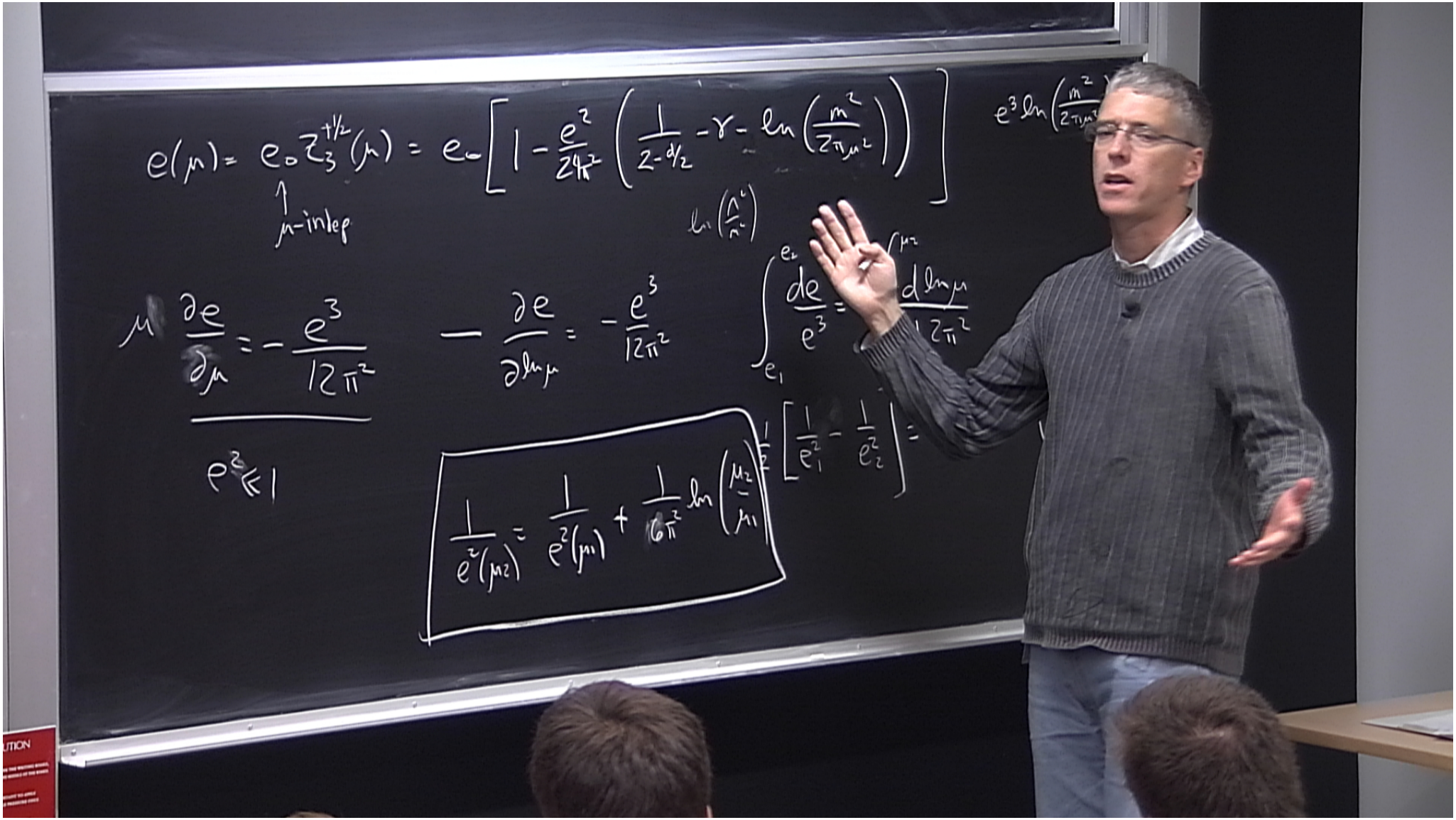
μ-inlep

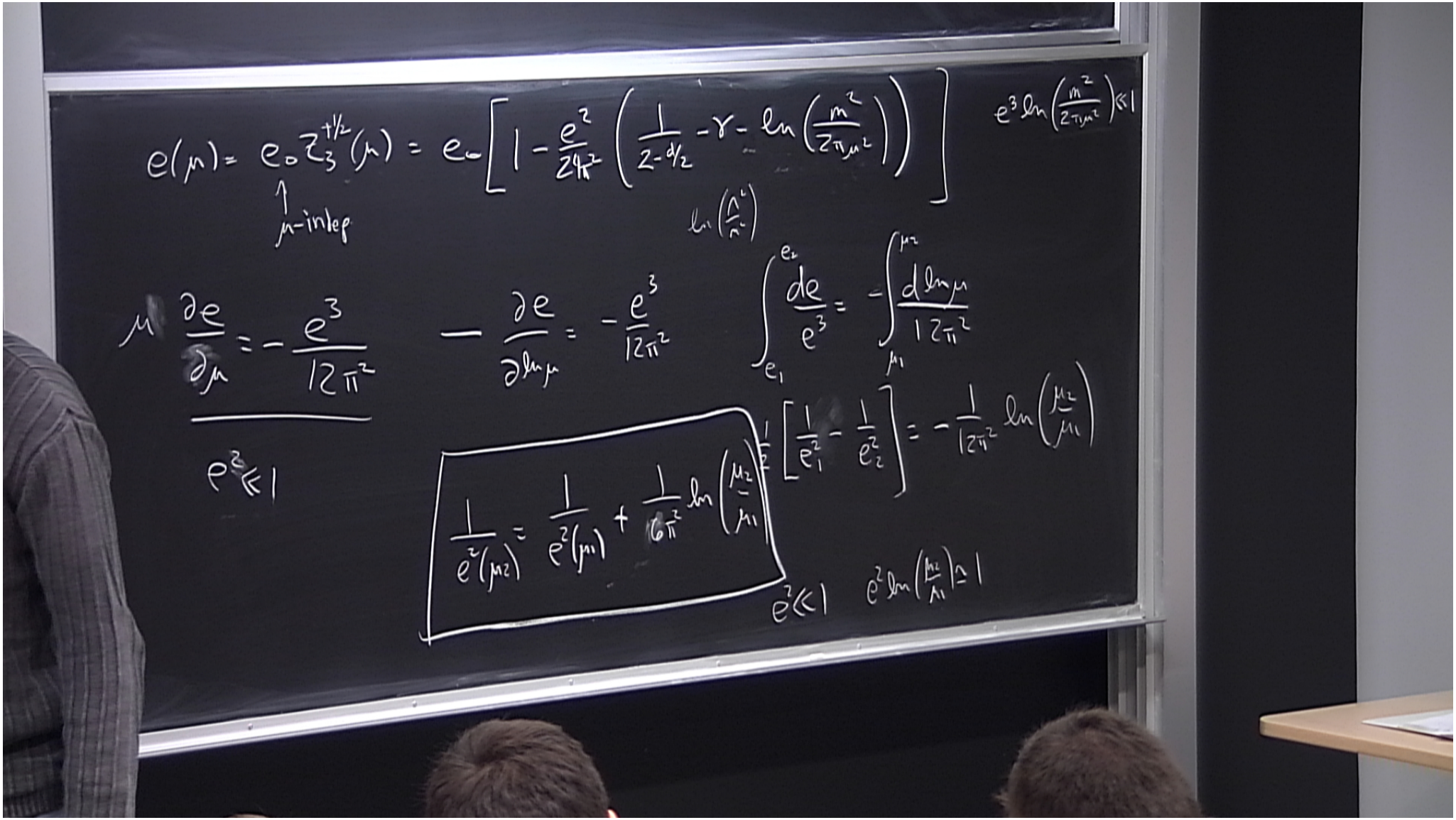
$$\mu\frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$$


$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{4\pi\mu^2}\right)\right)\right]$
μ-indep
$\ln(\Lambda$
$\mu\frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$


$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0 \left[1 - \frac{e^2}{24\pi^2} \left(\frac{1}{2-d/2} - \gamma - \ln \left(\frac{m^2}{4\pi\mu^2} \right) \right. \right.$
$\ln\left(\frac{\Lambda^2}{m^2}\right)$
μ-indep
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$


$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0 \left[1 - \frac{e^2}{24\pi^2} \left(\frac{1}{2-d/2} - \gamma - \ln \left(\frac{m^2}{\mu^2} \right) \right) \right]$
μ-indep
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
$-\frac{\partial e}{\partial \ln \mu} = -\frac{e^3}{12\pi^2}$
$\ln \left(\frac{\Lambda^2}{\mu^2} \right)$
$\int_{e_1}$
$\frac{1}{2}$
$\left(\frac{\mu_2}{\mu_1} \right)$


$e(\mu) = e_0 Z_3^{1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right)\right]$
μ-indep
$\mu\frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
$-\frac{\partial e}{\partial \ln\mu} = -\frac{e^3}{12\pi^2}$
$\int_{e_1}^{e_2} \frac{de}{e^3} = \int_{\mu_1}^{\mu_2} \frac{d\ln\mu}{12\pi^2}$
$e^2 \ll 1$
$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{6\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$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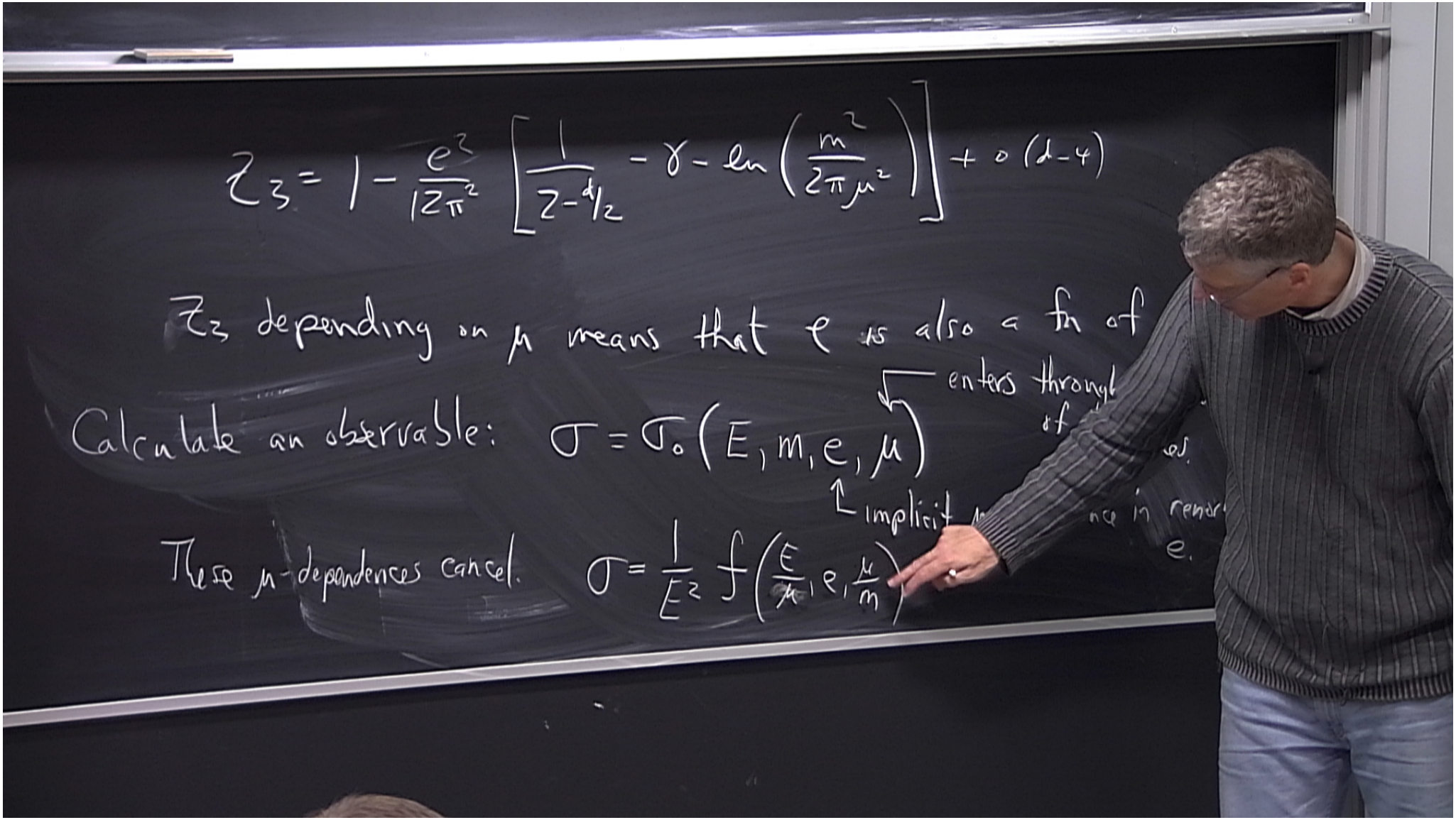
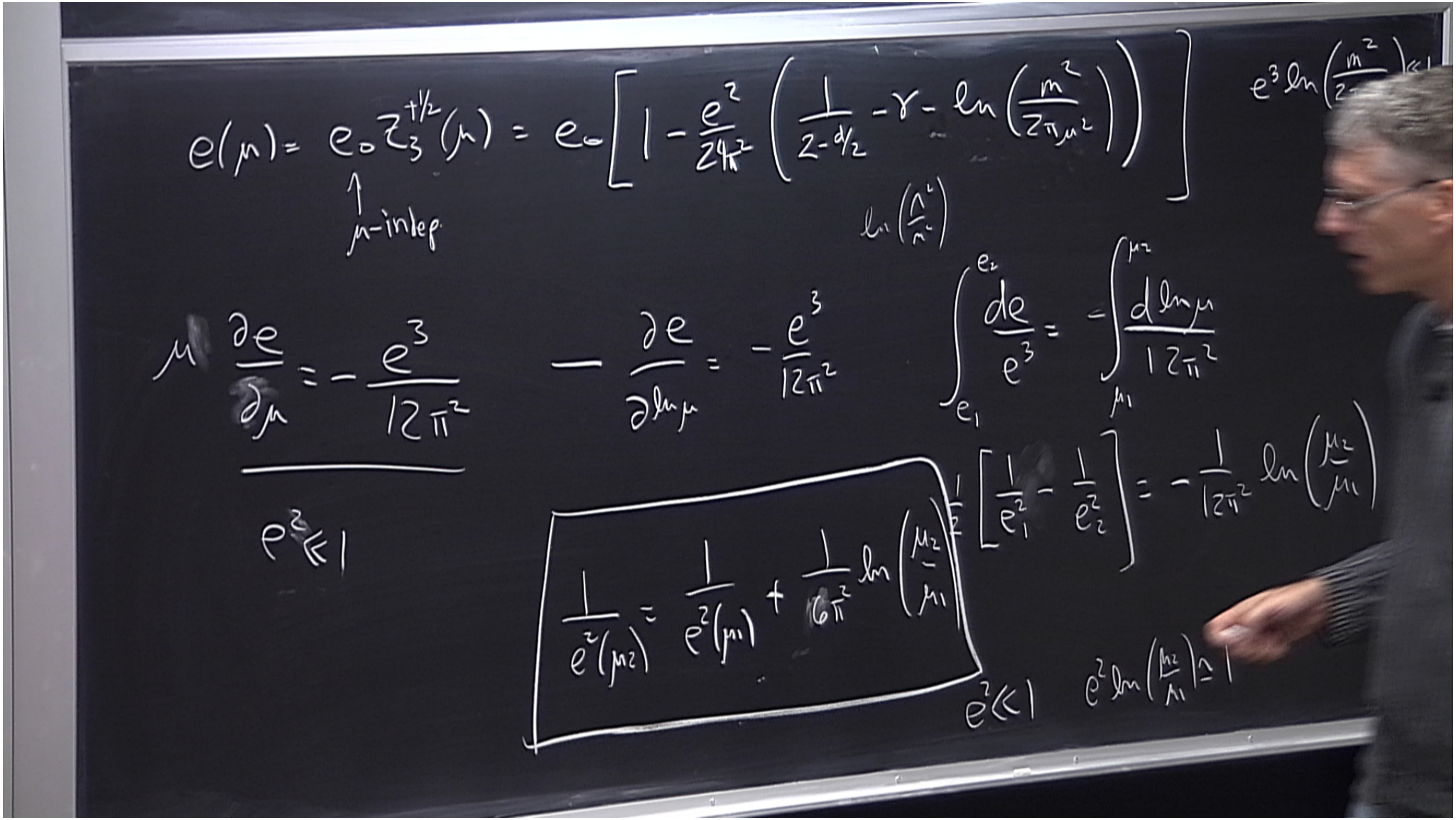
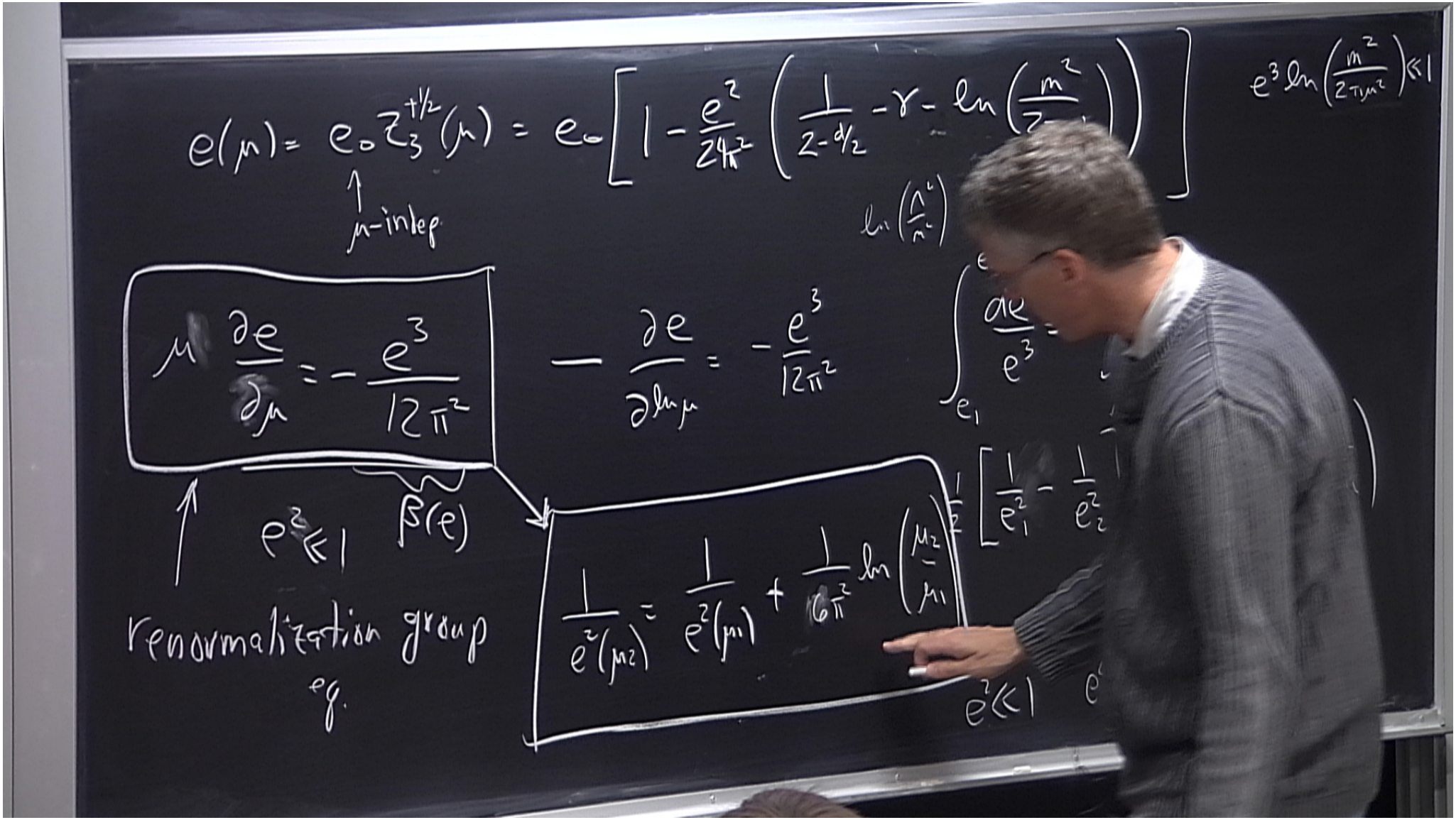
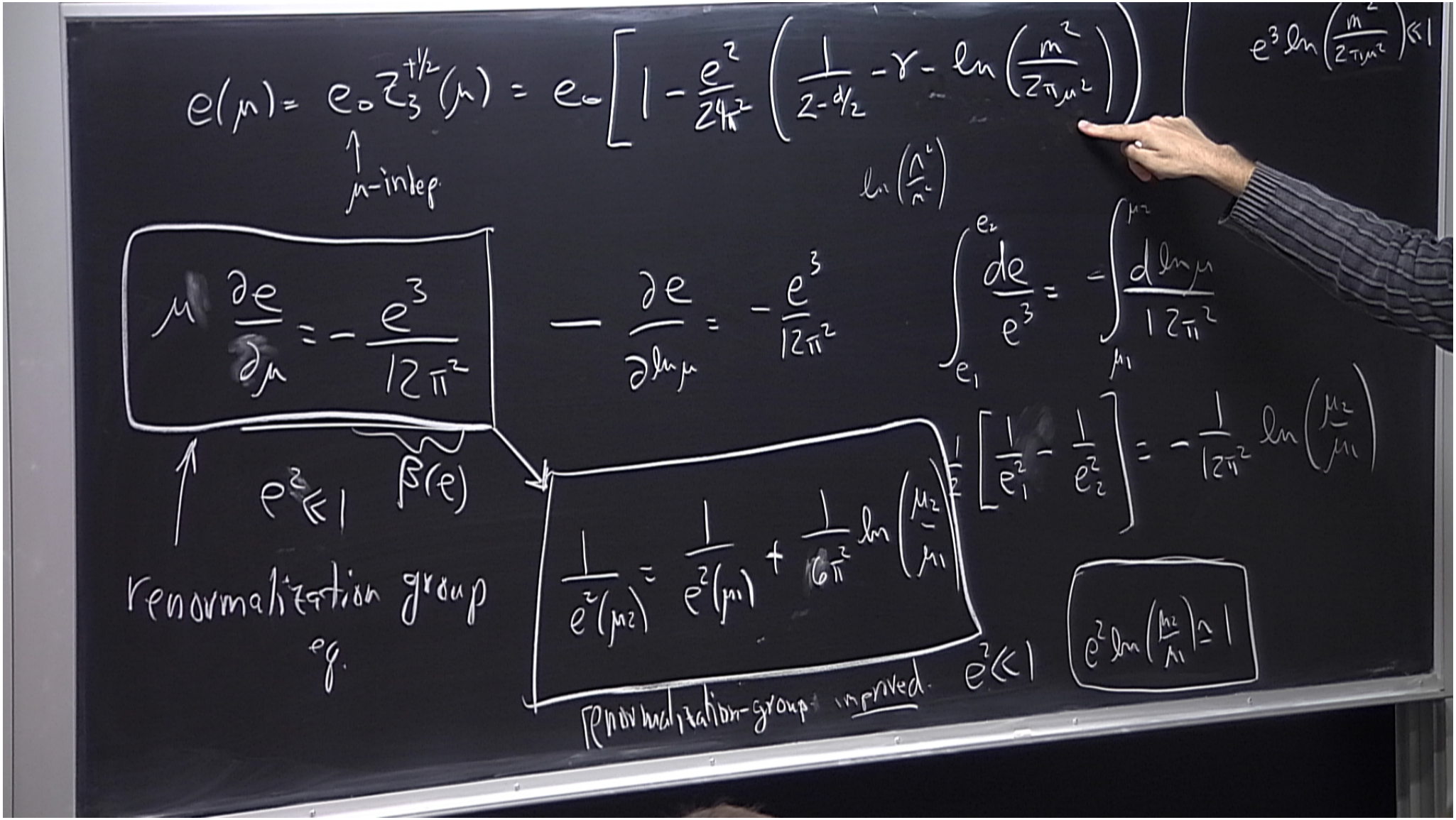
$e^3 \ln\left(\frac{m^2}{2\pi\mu^2}\right) \ll 1$
μ-indep
$\ln\left(\frac{\Lambda^2}{\mu^2}\right)$
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
$-\frac{\partial e}{\partial \ln\mu} = -\frac{e^3}{12\pi^2}$
$\int_{e_1}^{e_2} \frac{de}{e^3} = -\int_{\mu_1}^{\mu_2} \frac{d\ln\mu}{12\pi^2}$
$e^2 \ll 1$
$\frac{1}{2}\left[\frac{1}{e_1^2} - \frac{1}{e_2^2}\right] = -\frac{1}{12\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{6\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
$e^2 \ll 1$
$e^2 \ln\left(\frac{\mu_2}{\mu_1}\right) \simeq 1$


$Z_3 = 1 - \frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + o(d-4)$
Z_3 depending on μ means that e is also a fn of
enters through
Calculate an observable: $\sigma = \sigma_0(E, m, e, \mu)$
Implicit
These μ-dependences cancel. $\sigma = \frac{1}{E^2} f\left(\frac{E}{\mu}, e, \frac{\mu}{m}\right)$


$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right)\right]$
$e^3 \ln\left(\frac{m^2}{2\ldots}\right) \ll 1$
μ-indep.
$\ln\left(\frac{\Lambda^2}{m^2}\right)$
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
$-\frac{\partial e}{\partial \ln\mu} = -\frac{e^3}{12\pi^2}$
$\int_{e_1}^{e_2} \frac{de}{e^3} = -\int_{\mu_1}^{\mu_2} \frac{d\ln\mu}{12\pi^2}$
$e^2 \ll 1$
$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{16\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
$\frac{1}{2}\left[\frac{1}{e_1^2} - \frac{1}{e_2^2}\right] = -\frac{1}{12\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
$e^2 \ll 1$
$e^2 \ln\left(\frac{\mu_2}{\mu_1}\right) \gtrsim 1$


renormalization group
eg.


$e(\mu) = e_0 Z_3^{+1/2}(\mu) = e_0\left[1 - \frac{e^2}{24\pi^2}\left(\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right)\right.$
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μ-indep.
$\ln\left(\frac{\Lambda^2}{m^2}\right)$
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
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$e^2 \ll 1$
$\beta(e)$
renormalization group
eq.
$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{16\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
[Renormalization-group improved.
$e^2 \ll 1$
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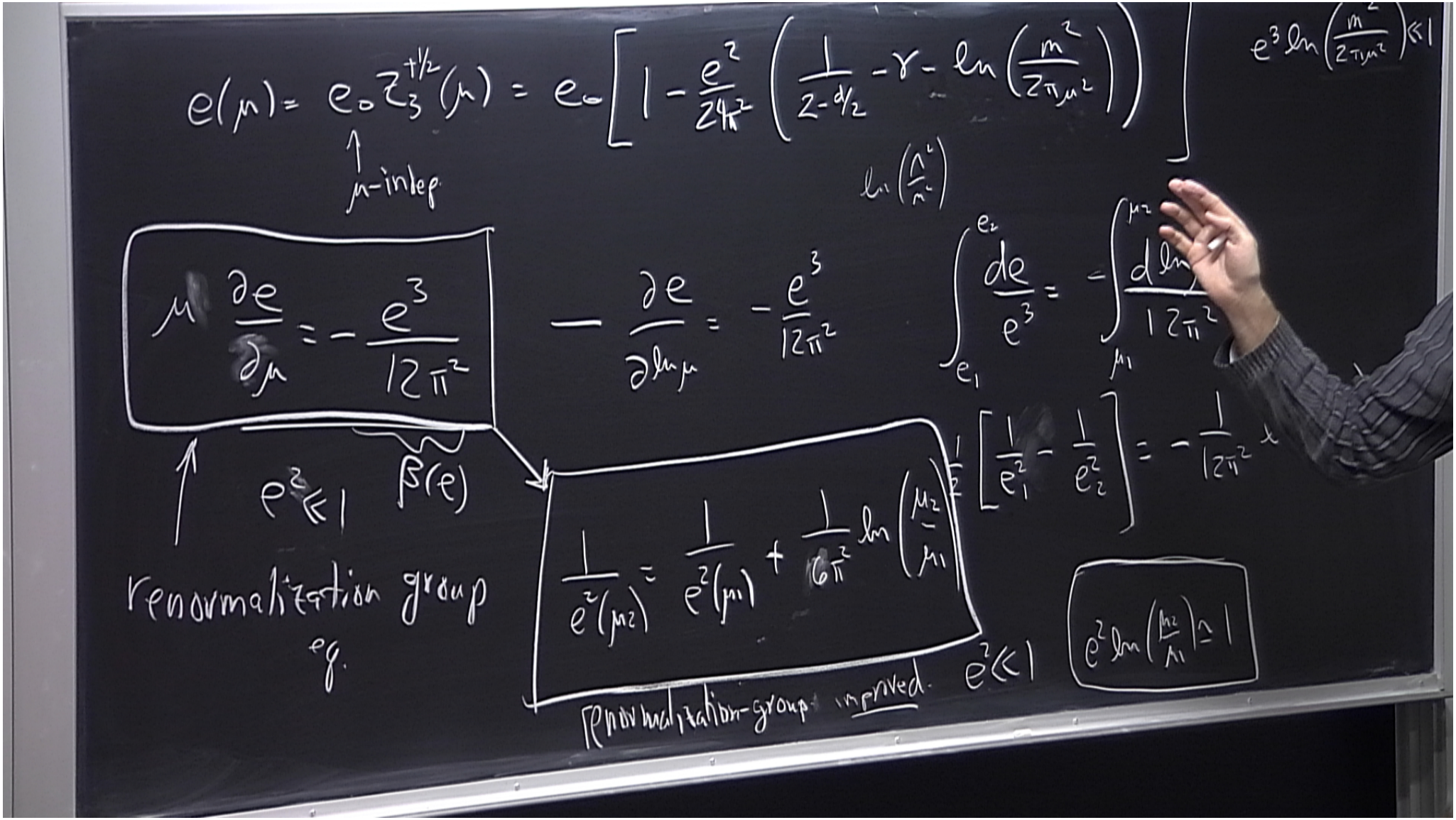
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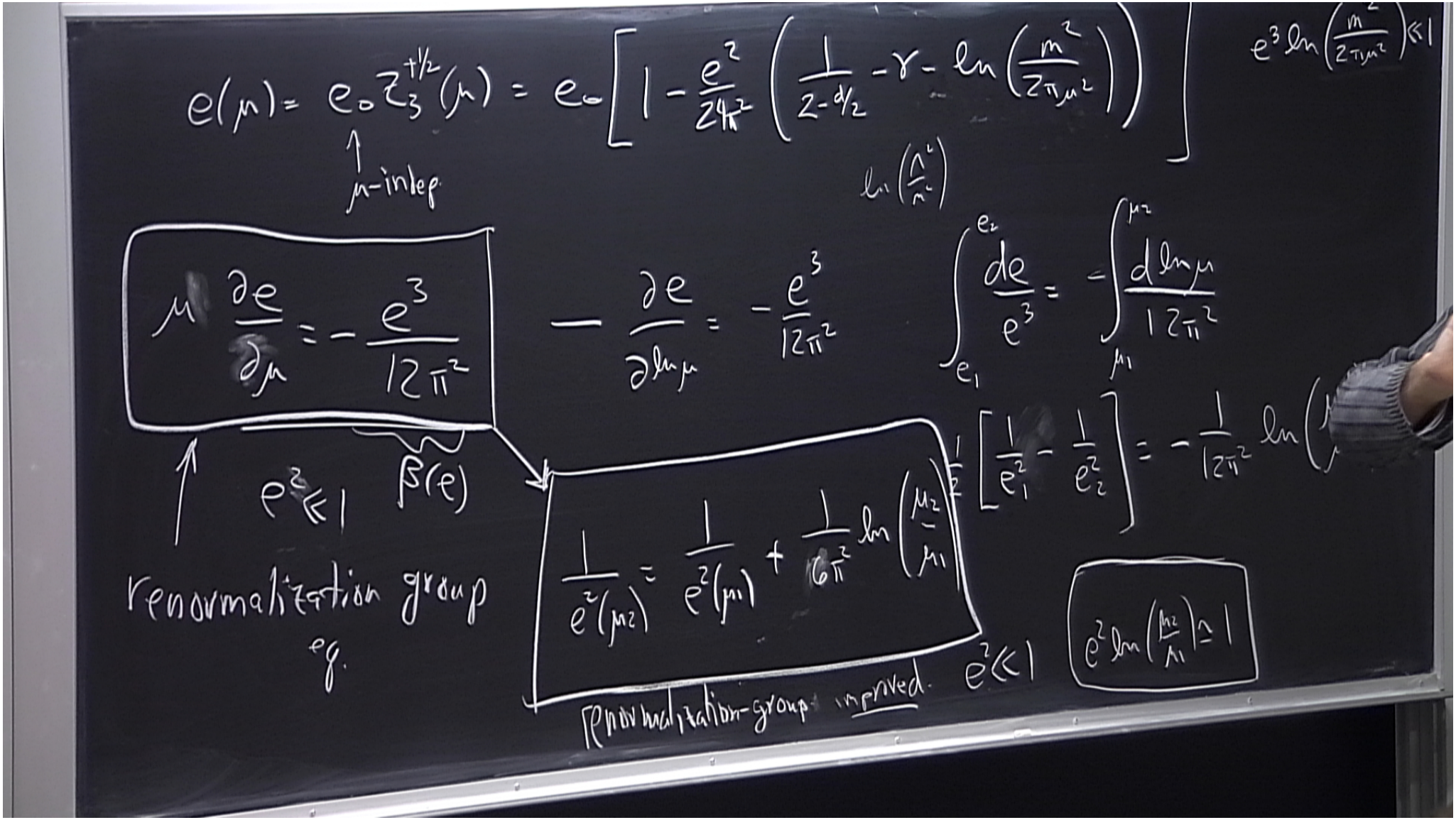
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$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{6\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$
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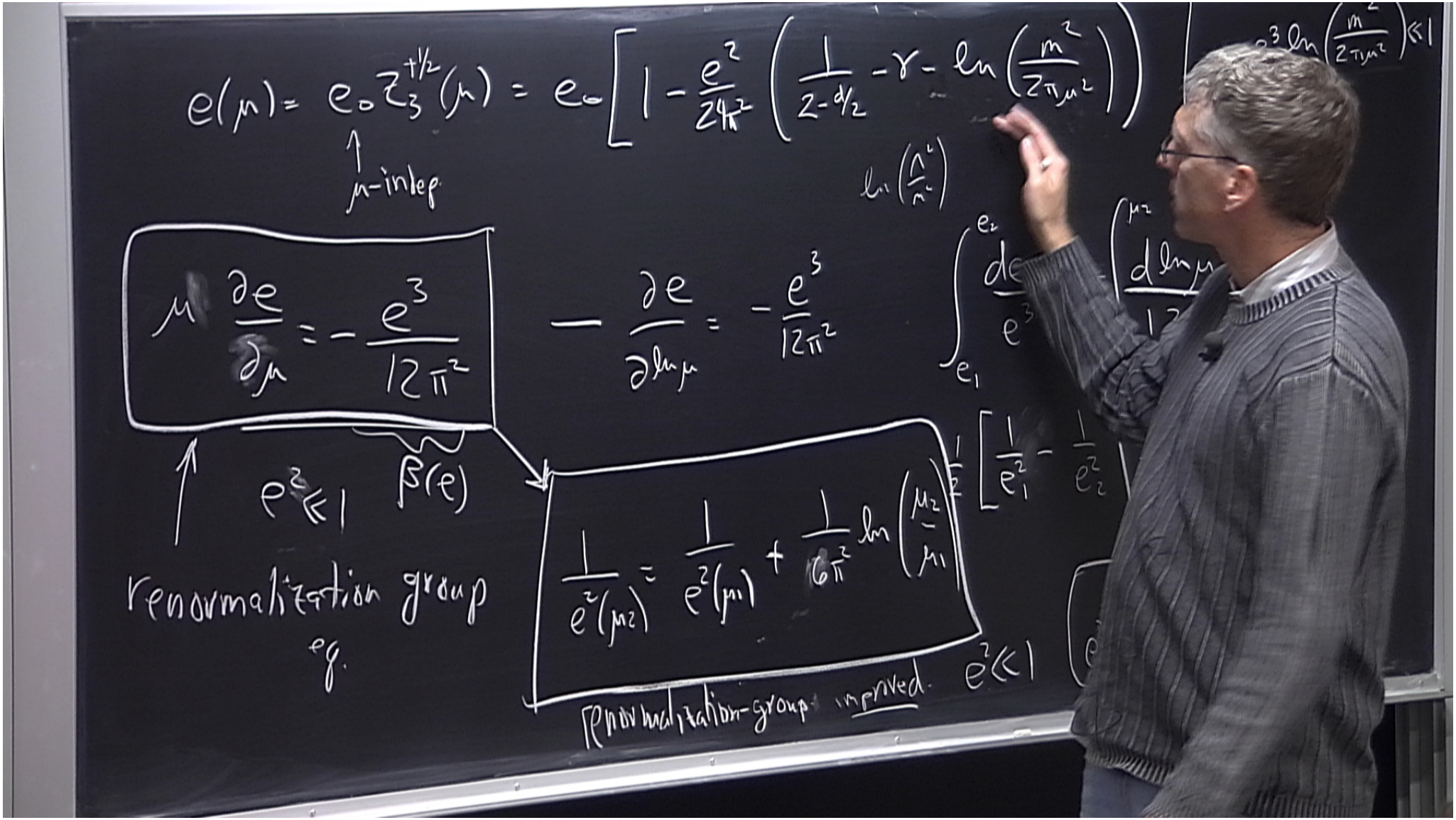
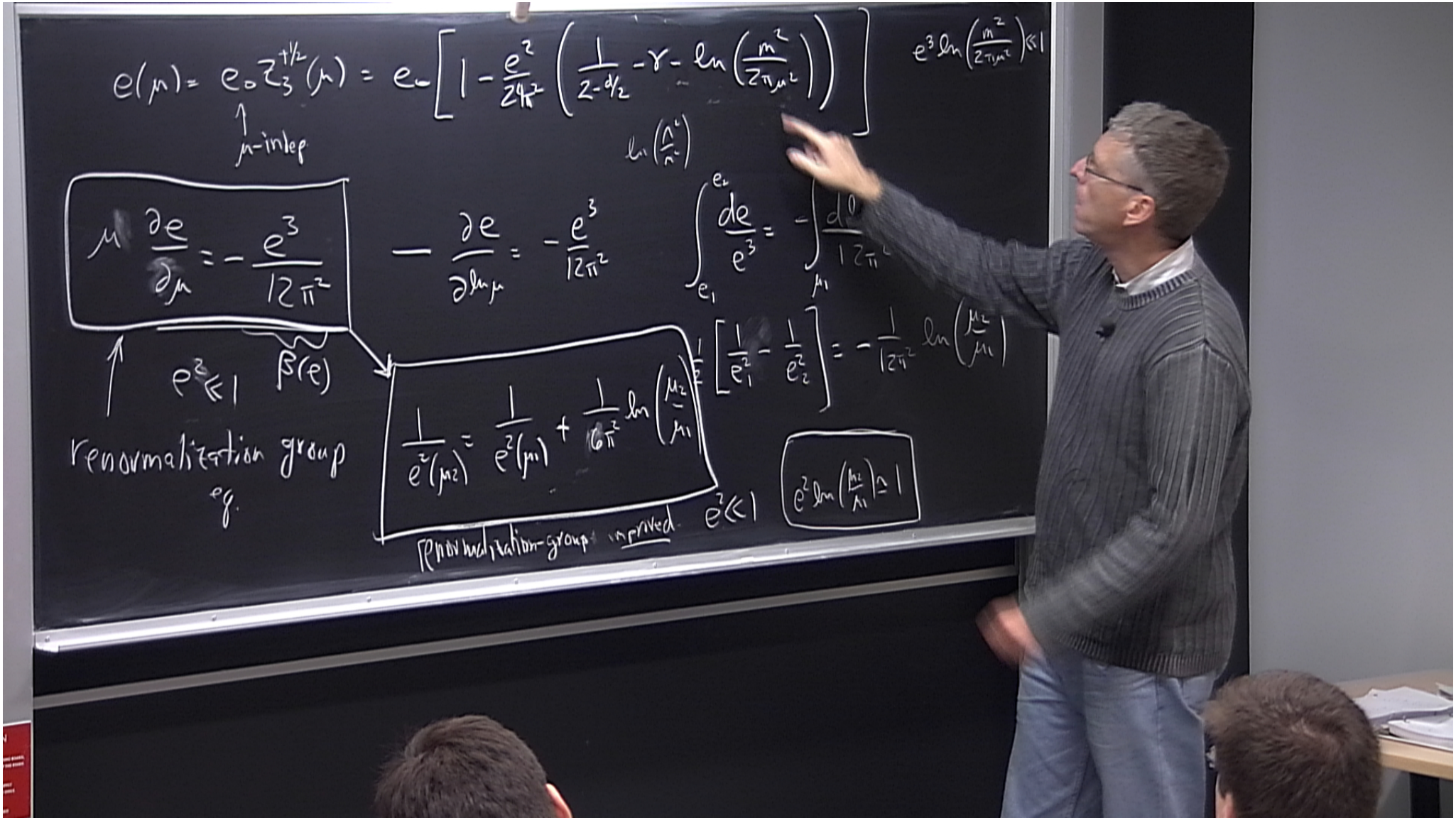
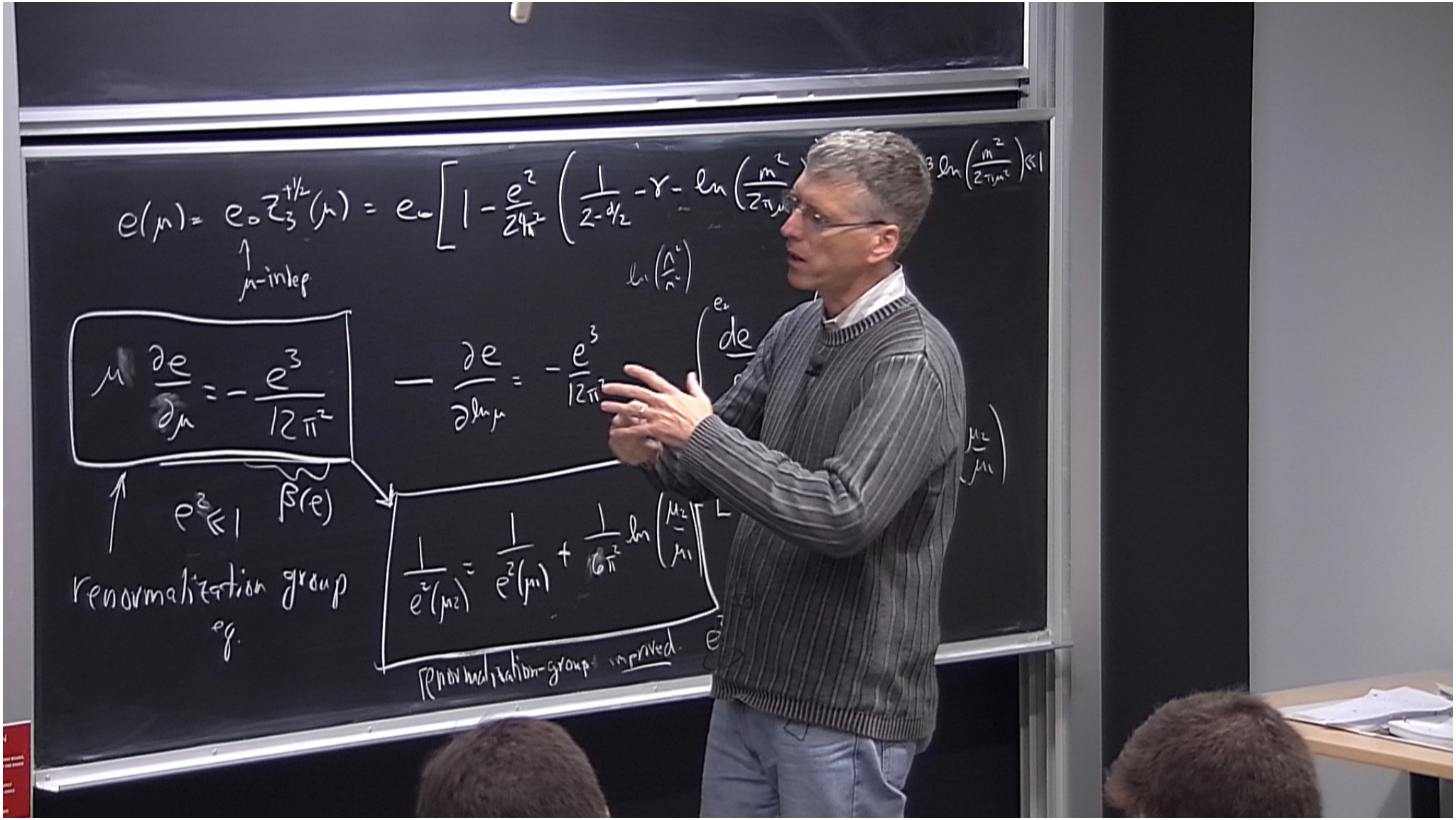

renormalization group
eg.
improved


renormalization group
eq.
improved
renormalization-group


renormalization group
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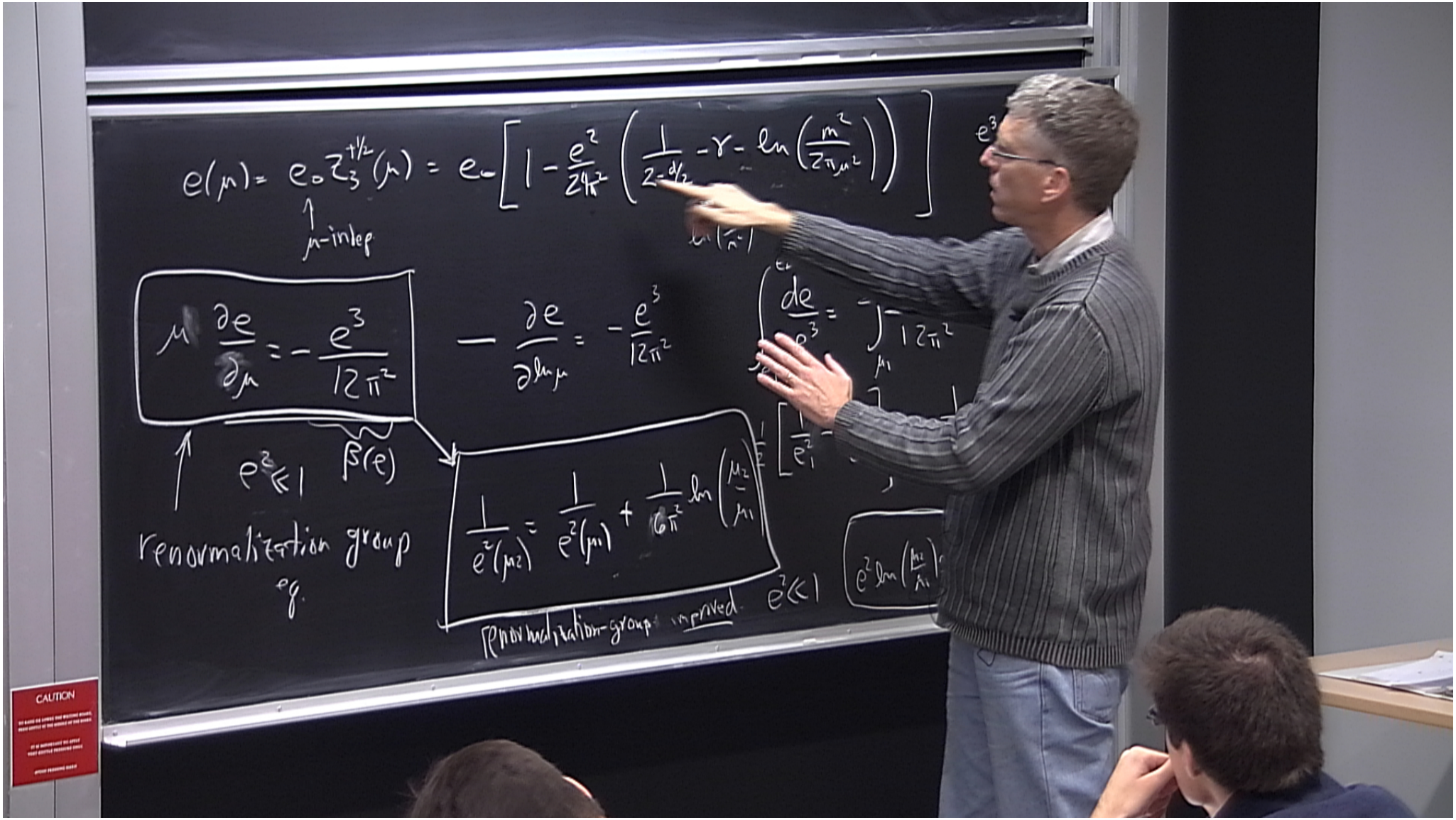

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μ-indep
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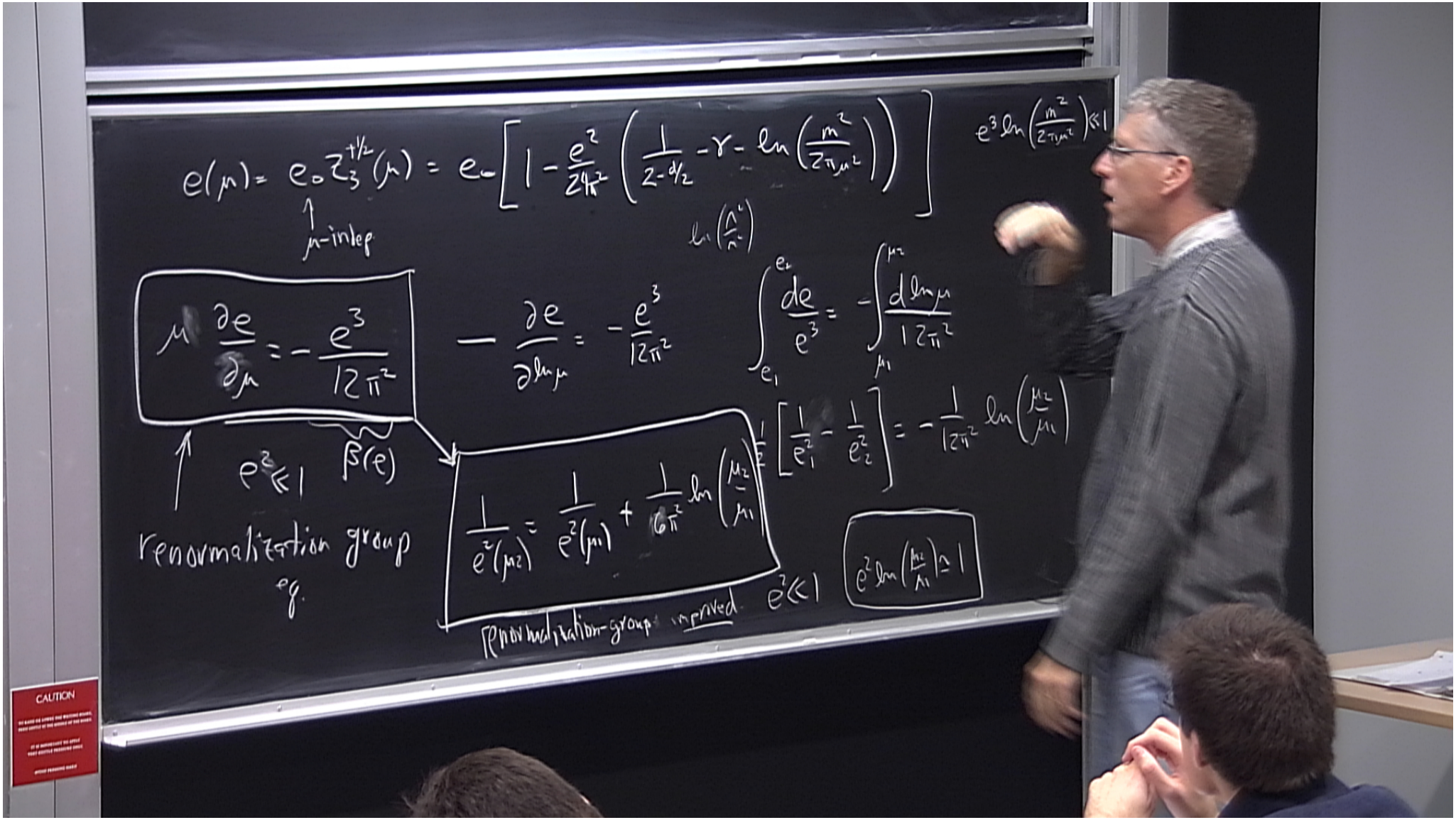
renormalization-group improved
$e^2 \ll 1$
CAUTION


renormalization group
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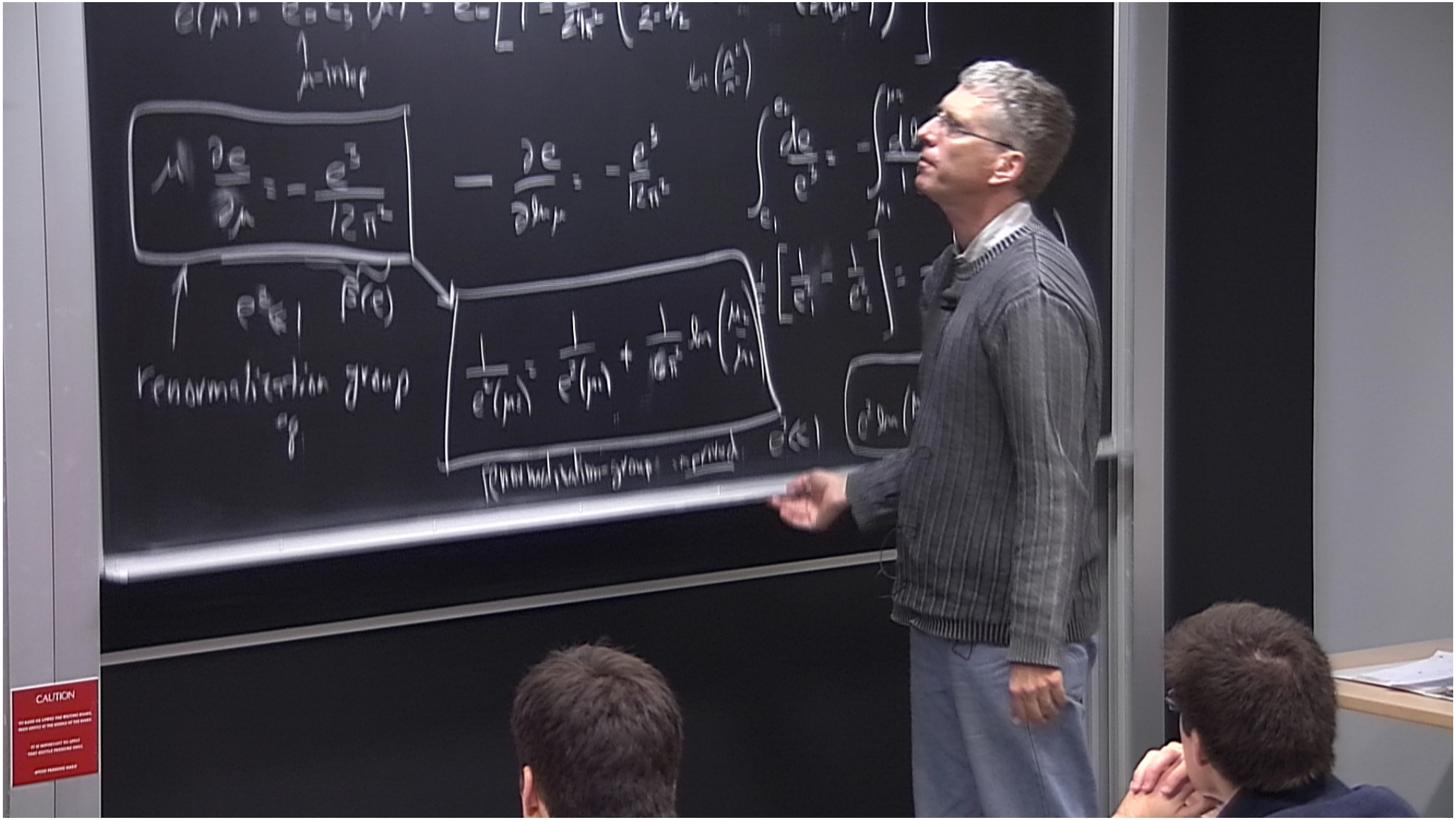
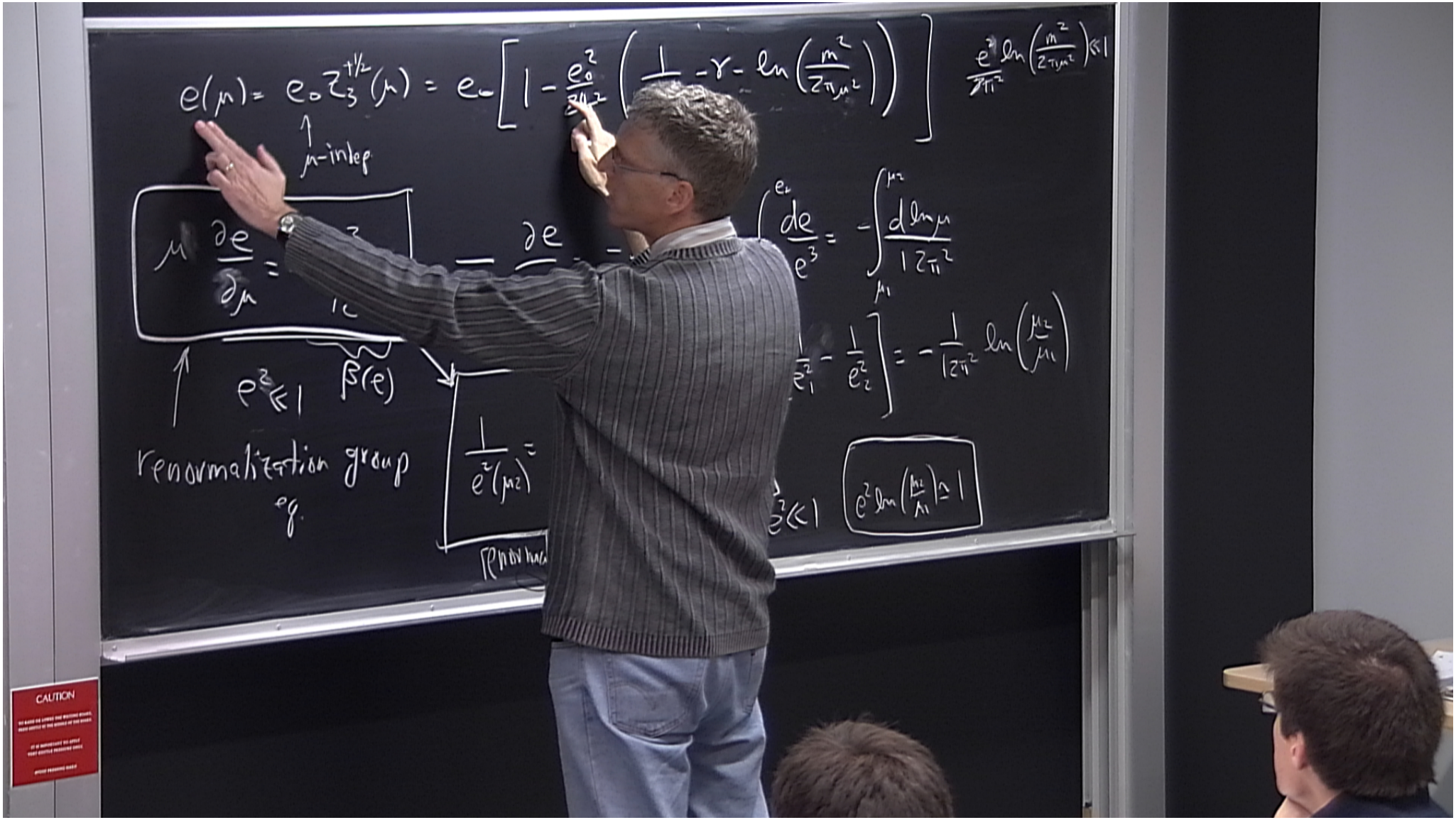
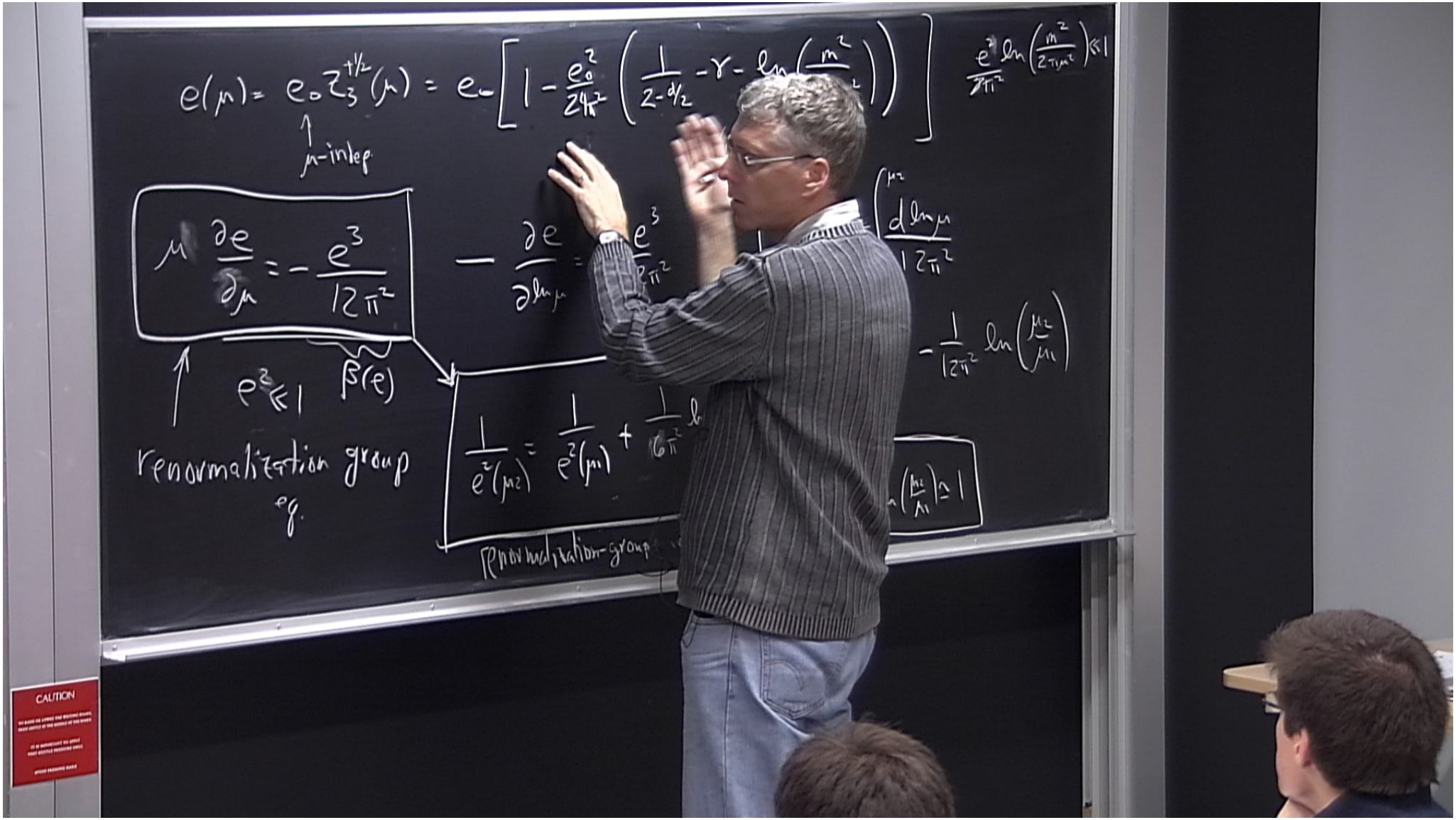
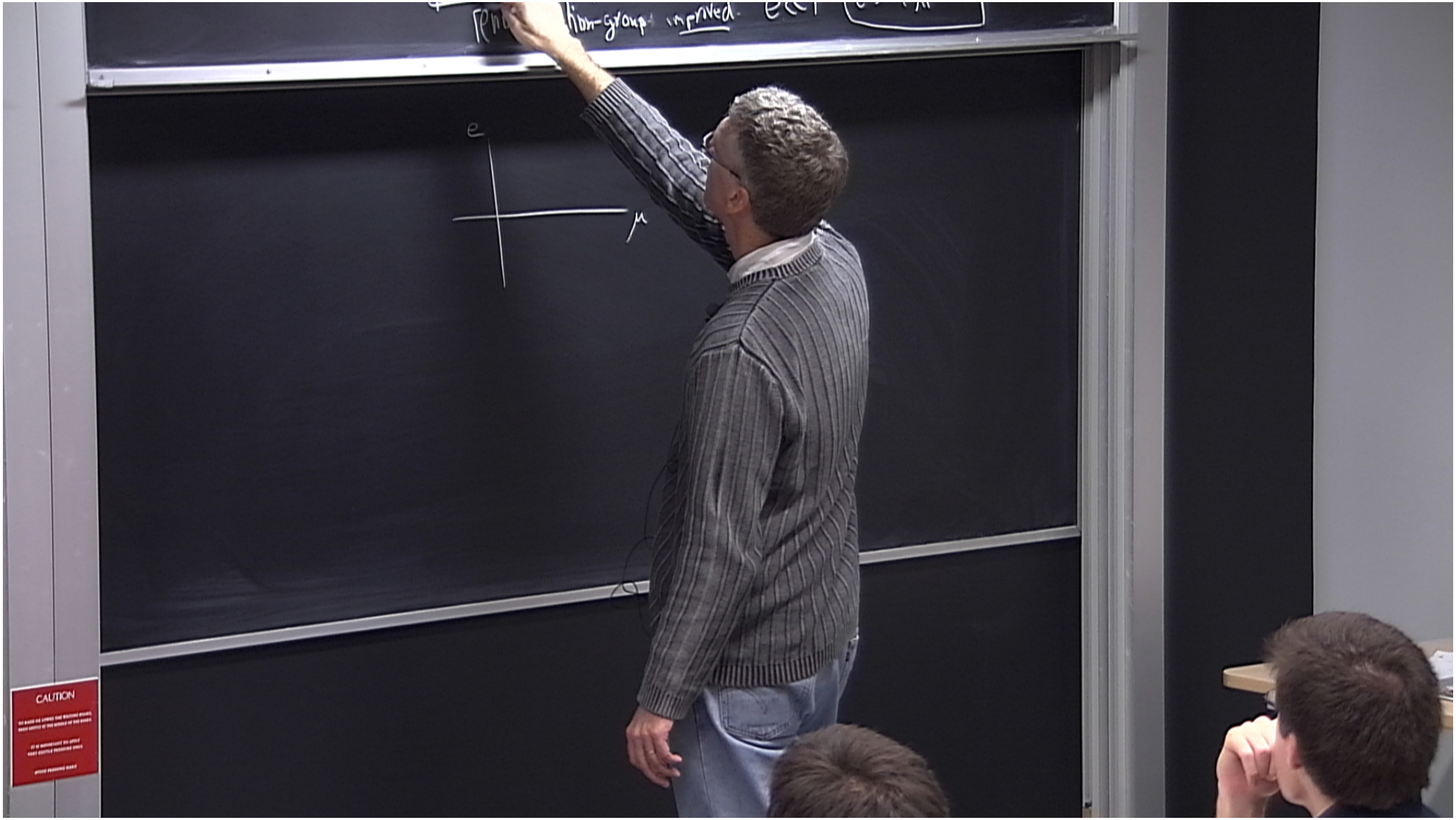
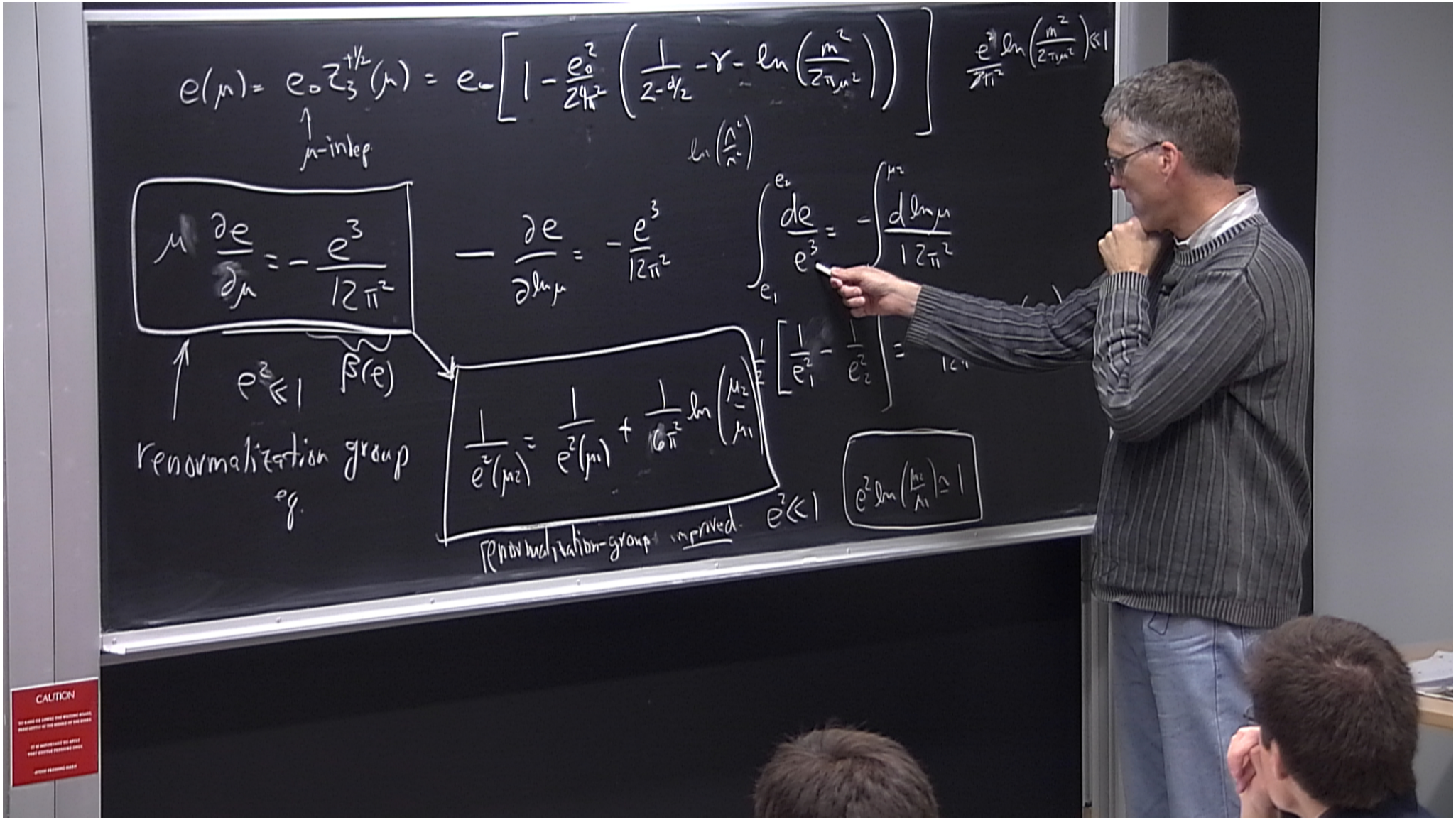

renormalization group
CAUTION


renormalization group
eg.


renormalization group
eg.
CAUTION


e
μ
CAUTION


renormalization group
eq.
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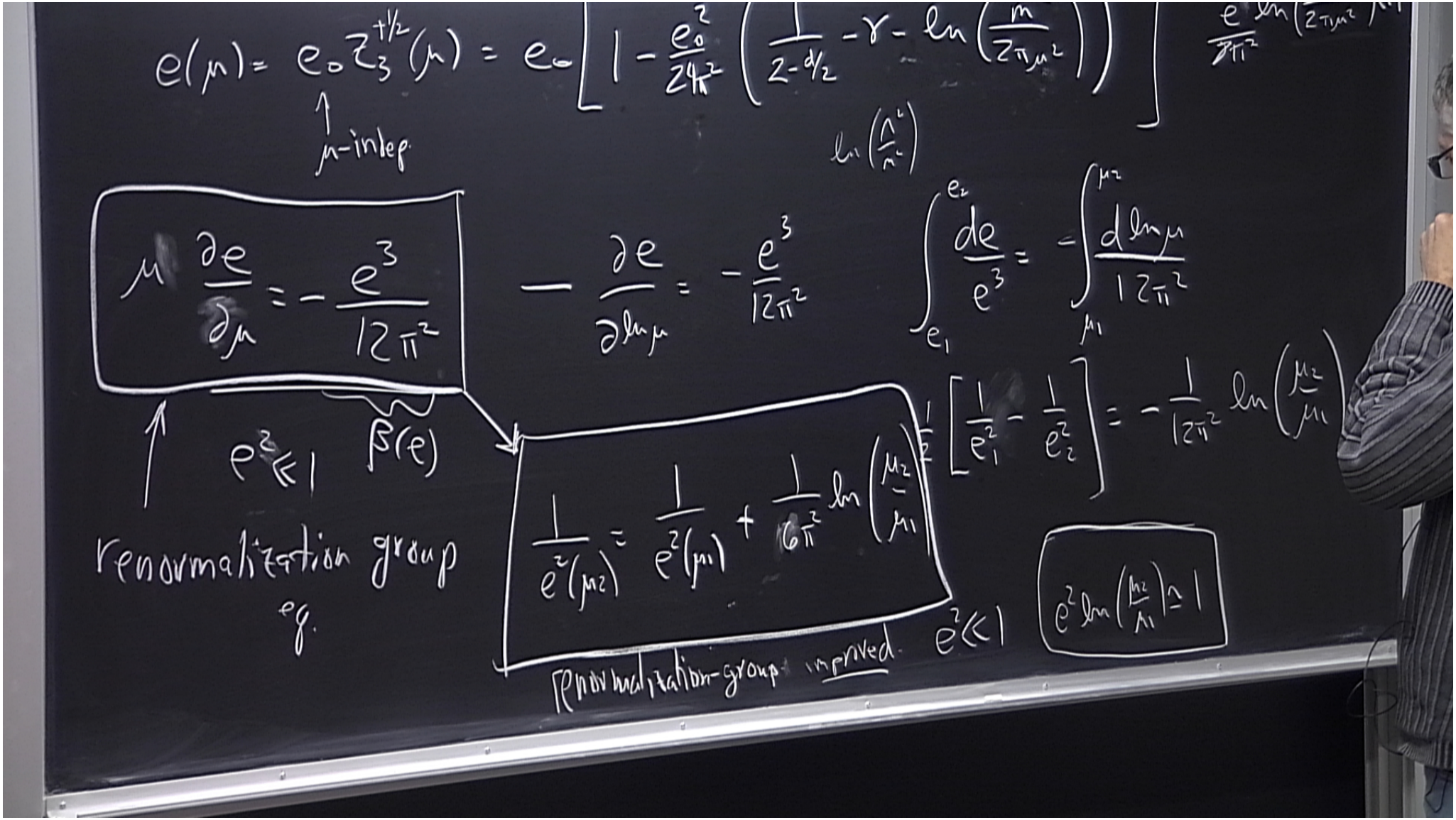
CAUTION


renormalization group
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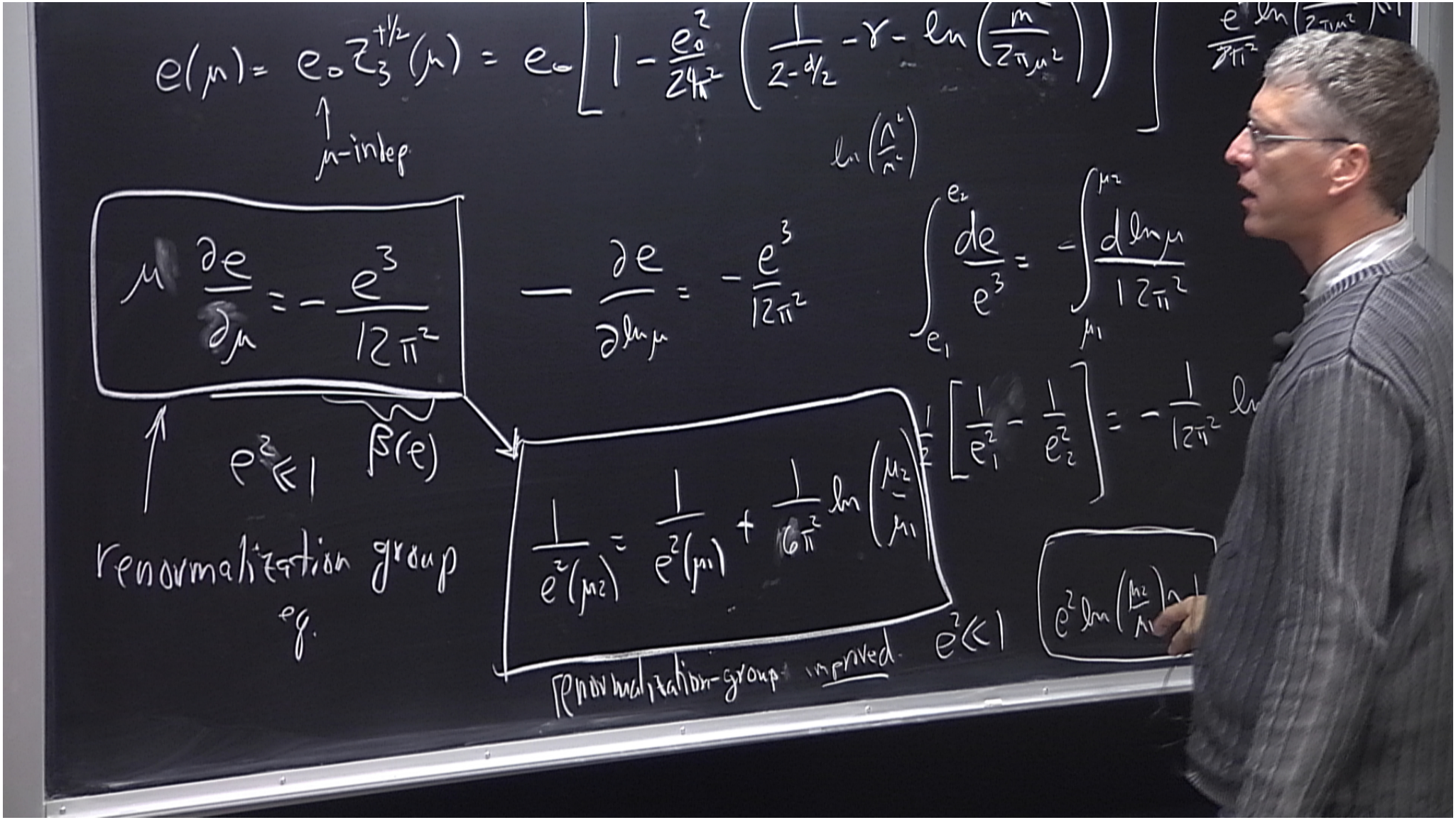
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μ-indep
$\ln\left(\frac{\Lambda^2}{m^2}\right)$
$\mu \frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}$
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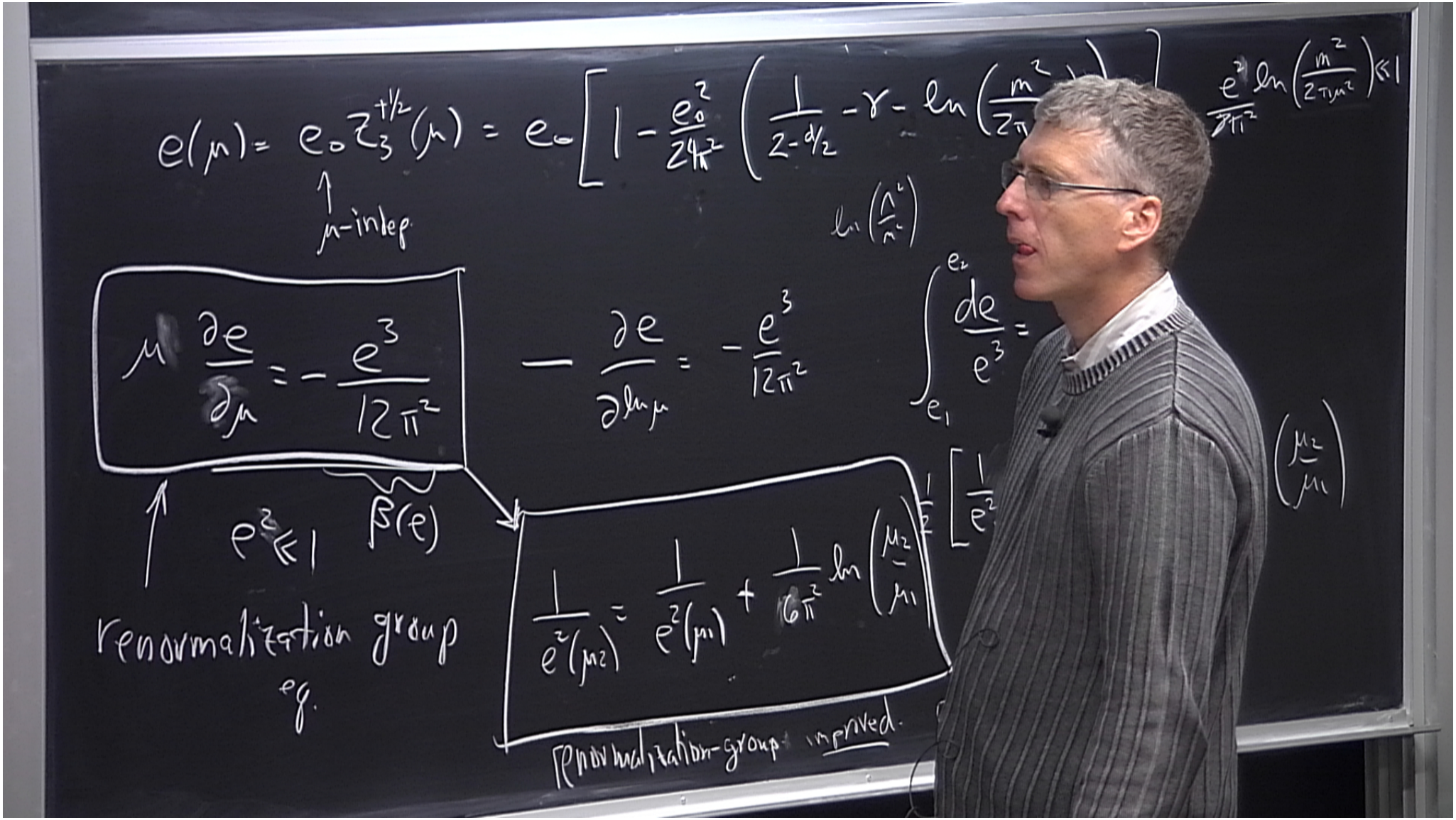
$\frac{1}{2}\left[\frac{1}{e_1^2} - \frac{1}{e_2^2}\right] = -\frac{1}{12\pi^2} \ln$
$e^2 \ll 1$
$\beta(e)$
renormalization group eq.
$\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{6\pi^2} \ln\left(\frac{\mu_2}{\mu_1}\right)$
renormalization-group improved
$e^2 \ll 1$
$e^2 \ln\left(\frac{\mu_2}{\mu_1}\right) \sim 1$


renormalization group
eg.
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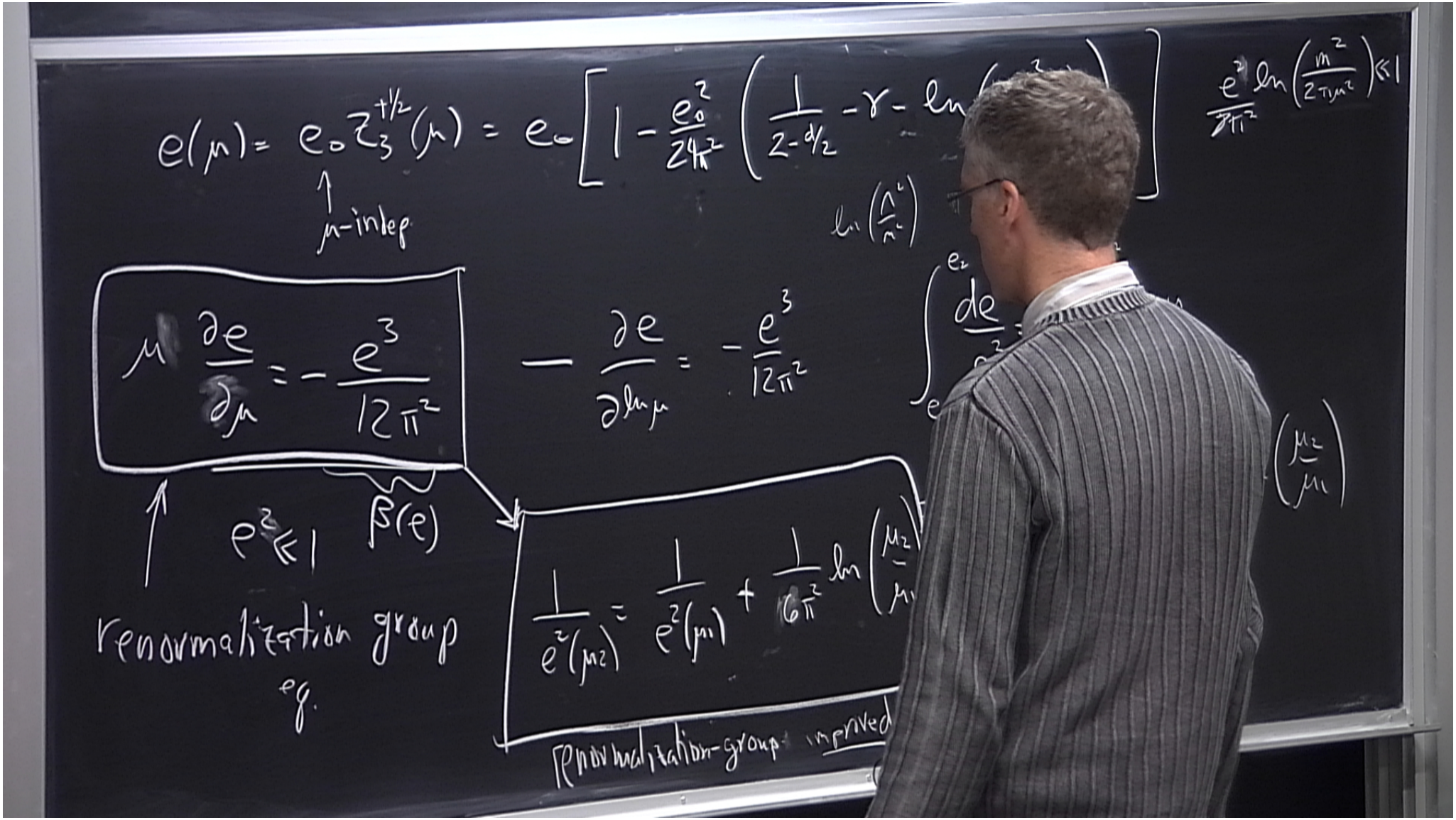
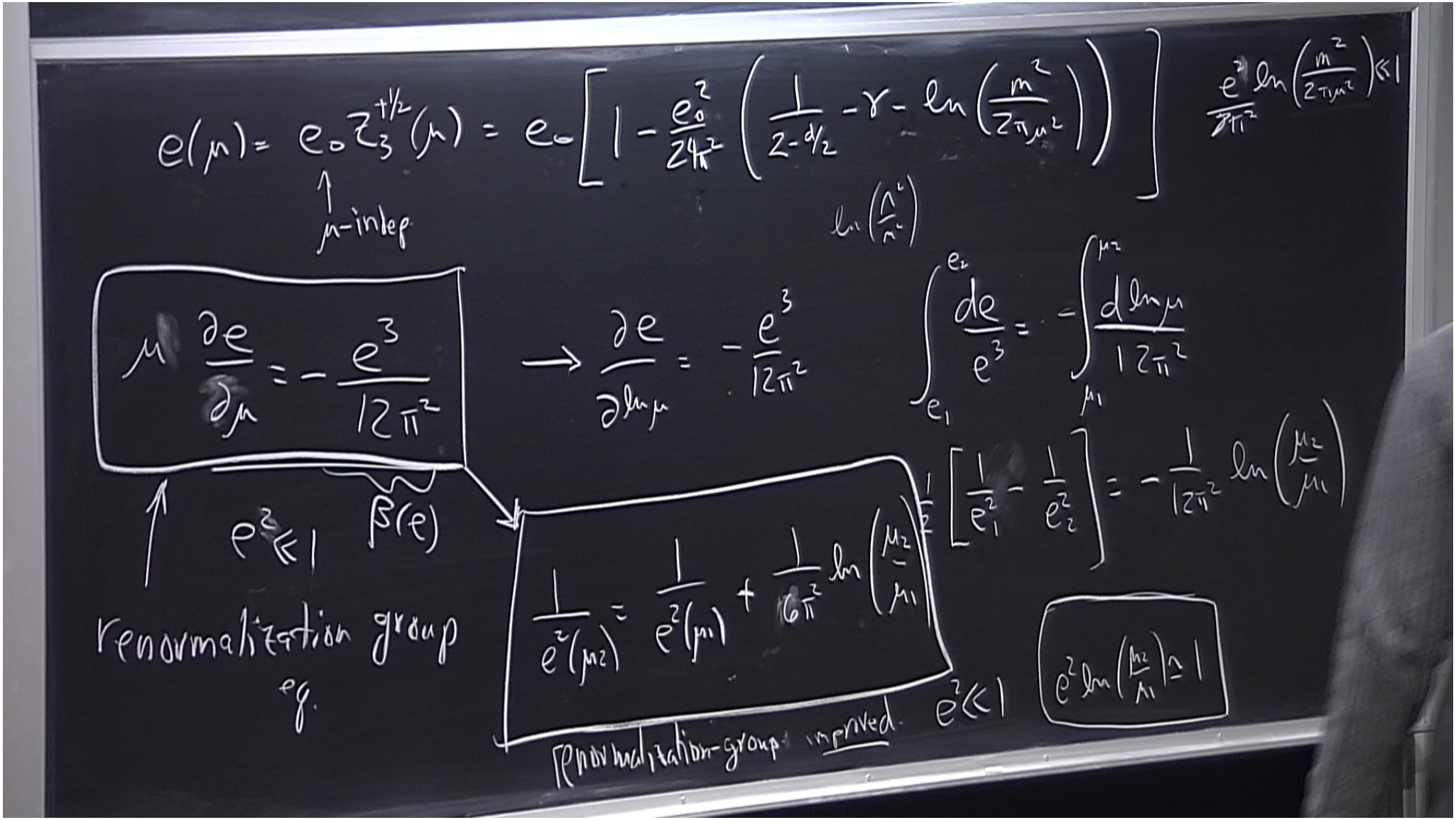

renormalization group
eg.



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$\uparrow$ μ-indep

$\ln\left(\frac{\Lambda^2}{m^2}\right)$

$$\frac{e^2}{8\pi^2}\ln\left(\frac{m^2}{2\pi\mu^2}\right) \ll 1$$

$$\boxed{\mu\frac{\partial e}{\partial \mu} = -\frac{e^3}{12\pi^2}} \longrightarrow \frac{\partial e}{\partial \ln\mu} = -\frac{e^3}{12\pi^2}$$

$$\int_{e_1}^{e_2}\frac{de}{e^3} = -\int_{\mu_1}^{\mu_2}\frac{d\ln\mu}{12\pi^2}$$

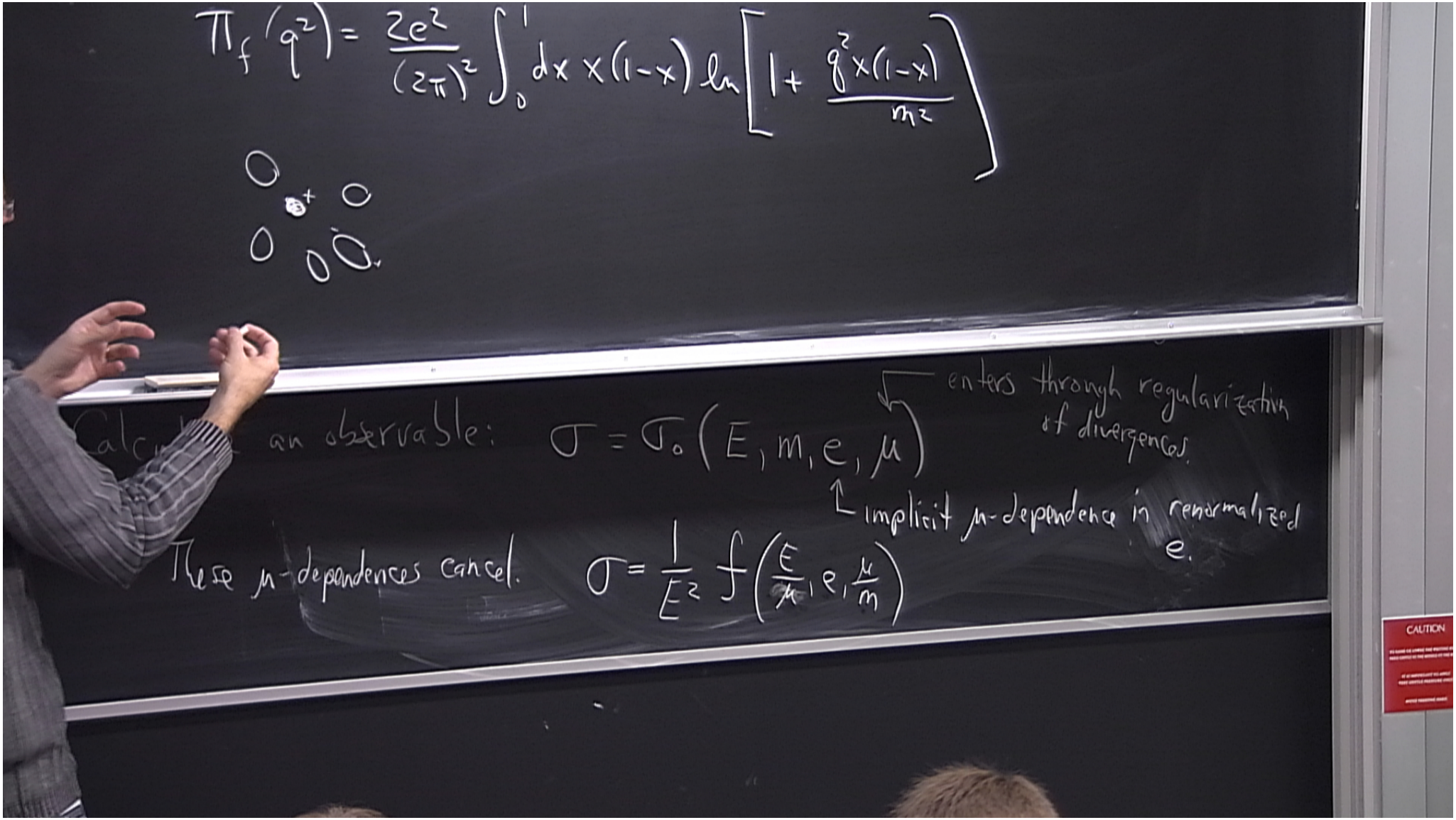
$$\frac{1}{2}\left[\frac{1}{e_1^2} - \frac{1}{e_2^2}\right] = -\frac{1}{12\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)$$

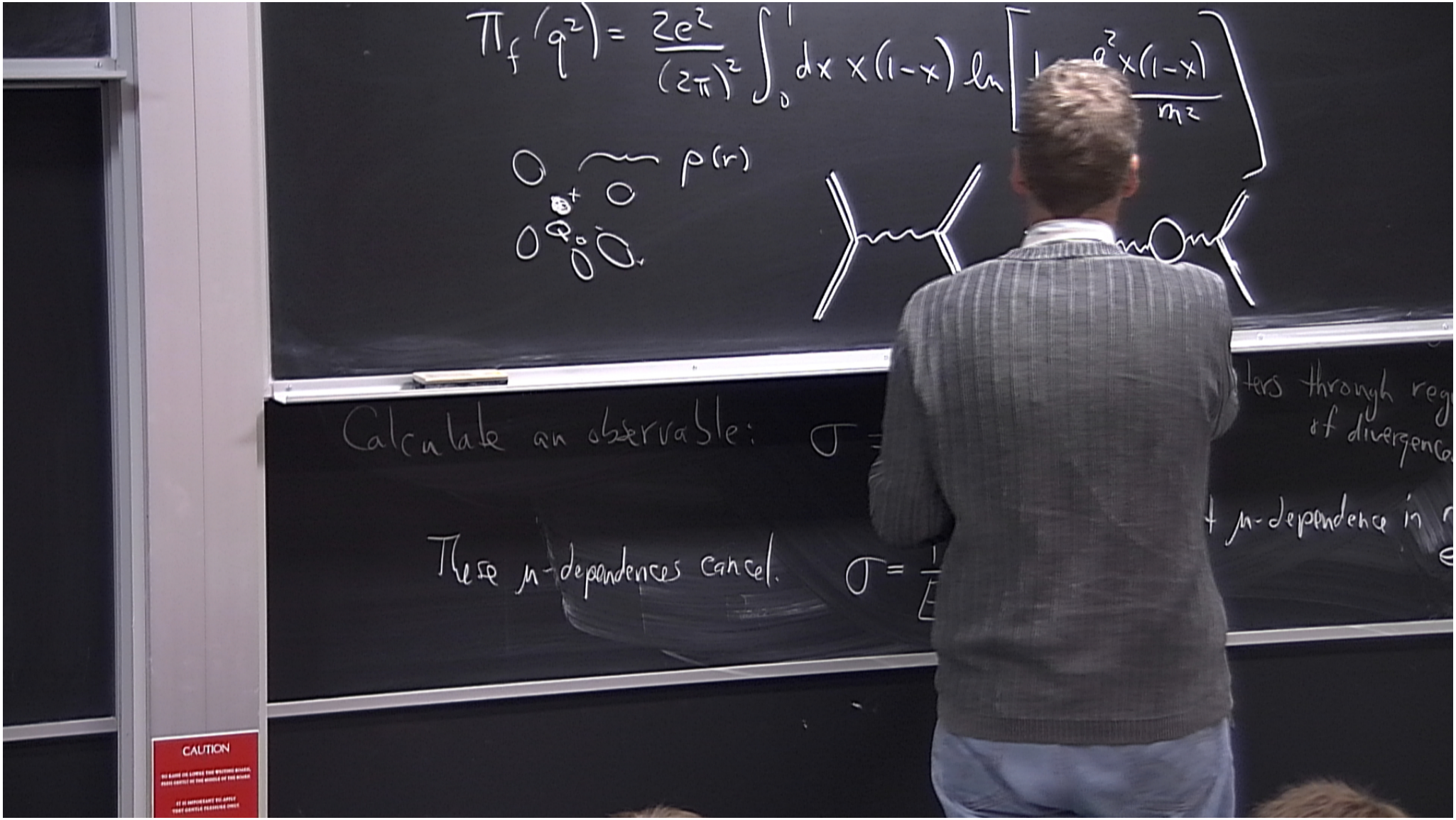
$e^2 \ll 1$ $\quad \beta(e)$

renormalization group eq.

$$\boxed{\frac{1}{e^2(\mu_2)} = \frac{1}{e^2(\mu_1)} + \frac{1}{6\pi^2}\ln\left(\frac{\mu_2}{\mu_1}\right)}$$

renormalization-group improved. $e^2 \ll 1$ $\quad \boxed{e^2\ln\left(\frac{\mu_2}{\mu_1}\right) \simeq 1}$


$\Pi_f(q^2) = \frac{2e^2}{(2\pi)^2}\int_0^1 dx\, x(1-x)\ln\left[1+\frac{q^2x(1-x)}{m^2}\right]$
Calculate an observable: $\sigma = \sigma_0(E, m, e, \mu)$
enters through regularization of divergences.
implicit μ-dependence in renormalized e.
These μ-dependences cancel. $\sigma = \frac{1}{E^2} f\left(\frac{E}{\mu}, e, \frac{\mu}{m}\right)$
CAUTION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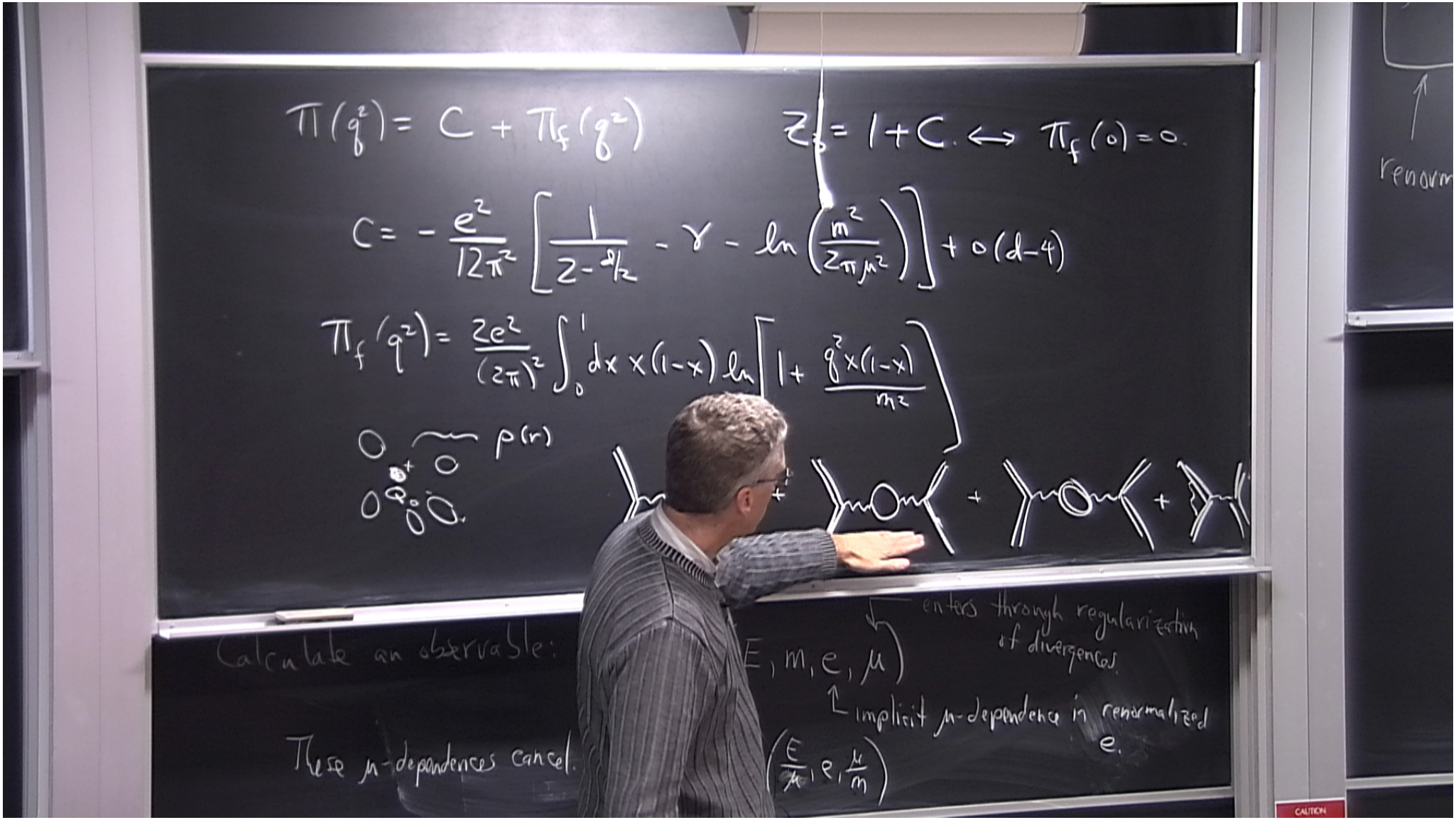
$\rho(r)$
Calculate an observable: $\sigma =$
These μ-dependences cancel. $\sigma =$
ters through reg
of divergence
μ-dependence in
CAUTION


$\Pi(q^2) = C + \Pi_f(q^2)$
$Z_3 = 1 + C \leftrightarrow \Pi_f(0) = 0.$
$C = -\frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + O(d-4)$
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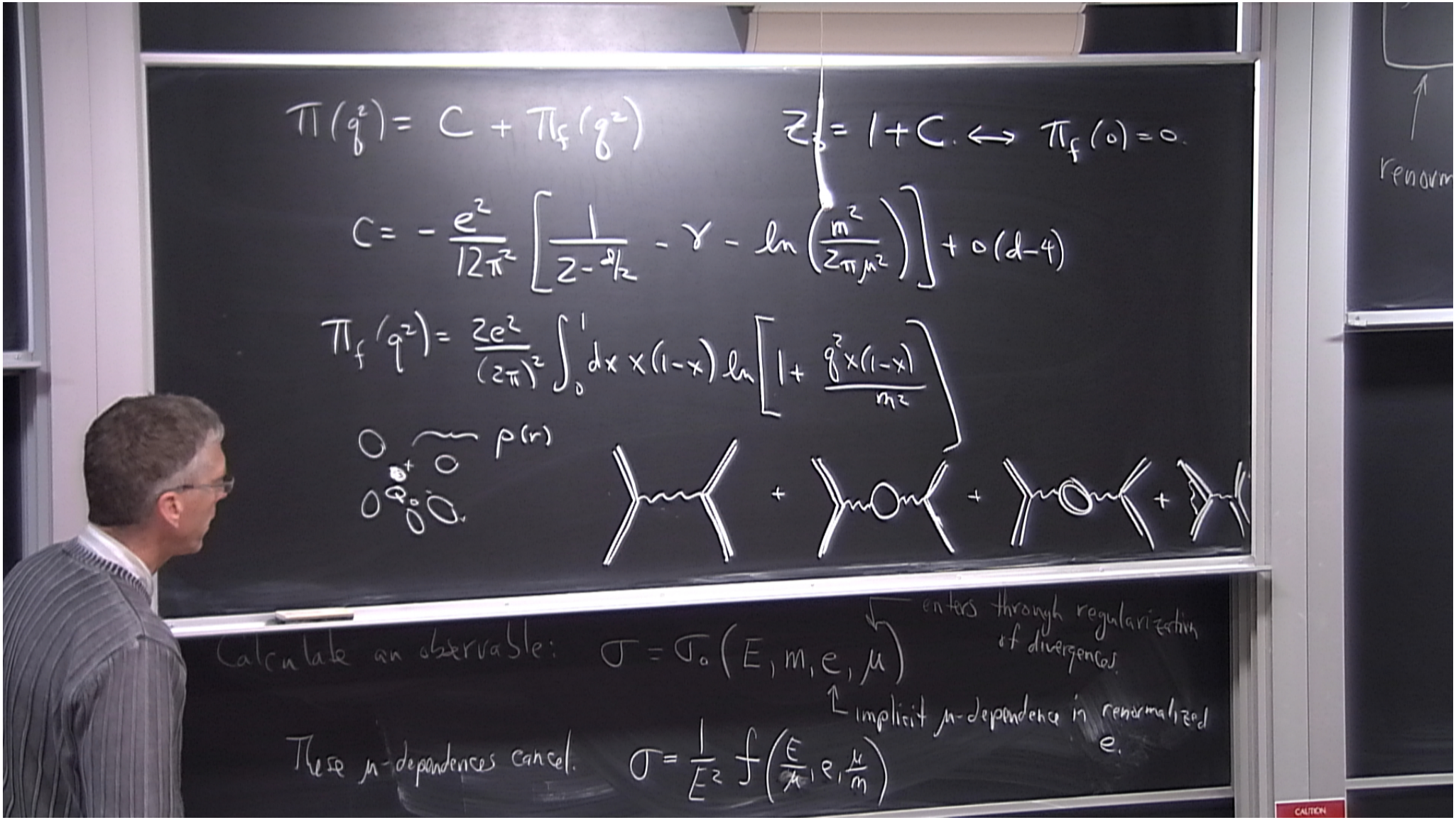
Calculate an observable:
$E, m, e, \mu)$
enters through regularization of divergences
implicit μ-dependence in renormalized e.
These μ-dependences cancel.
$\left(\frac{E}{\mu}, e, \frac{\mu}{m}\right)$
renorm
CAUTION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$\rho(r)$
+
+
+
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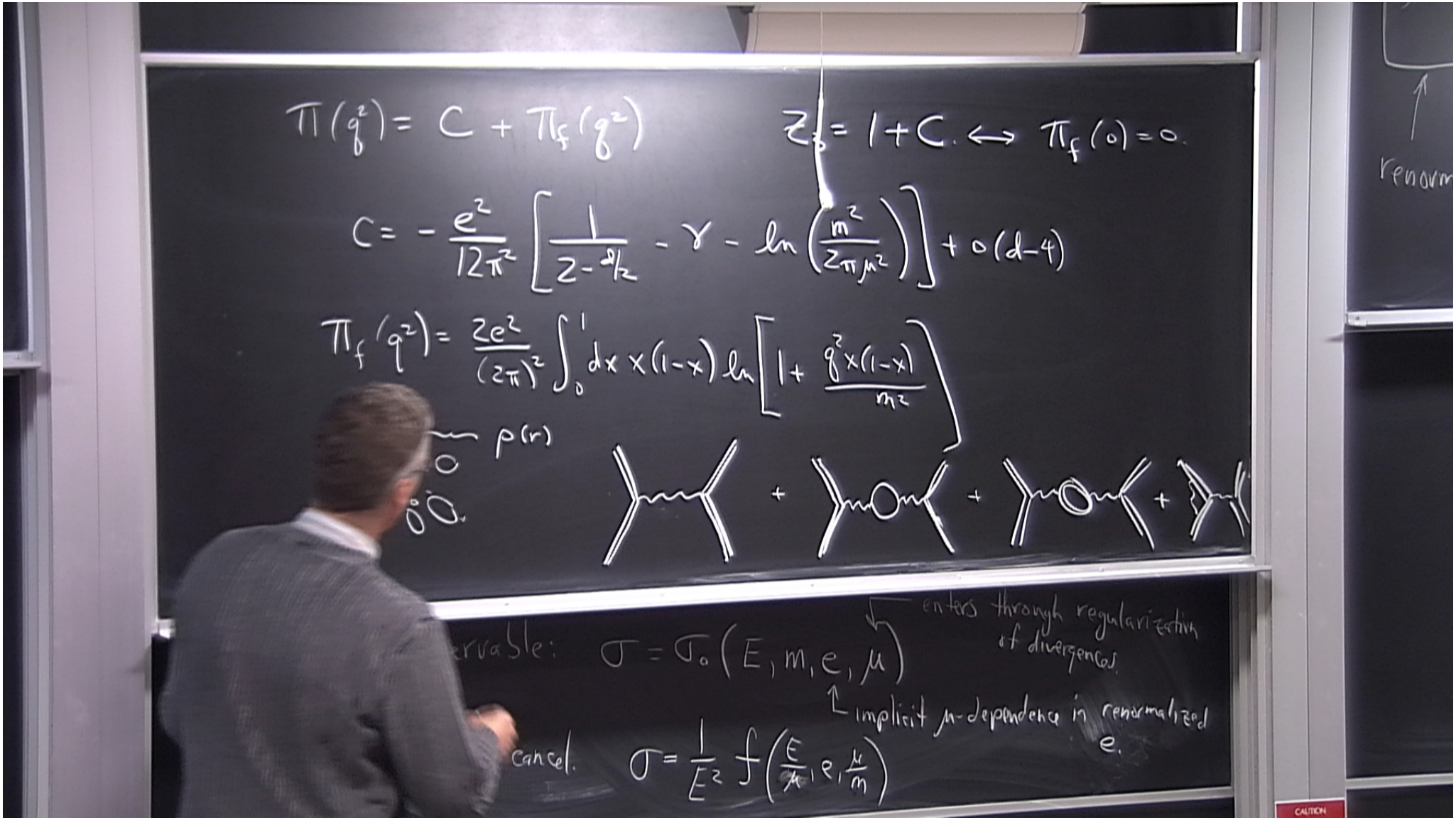
renorm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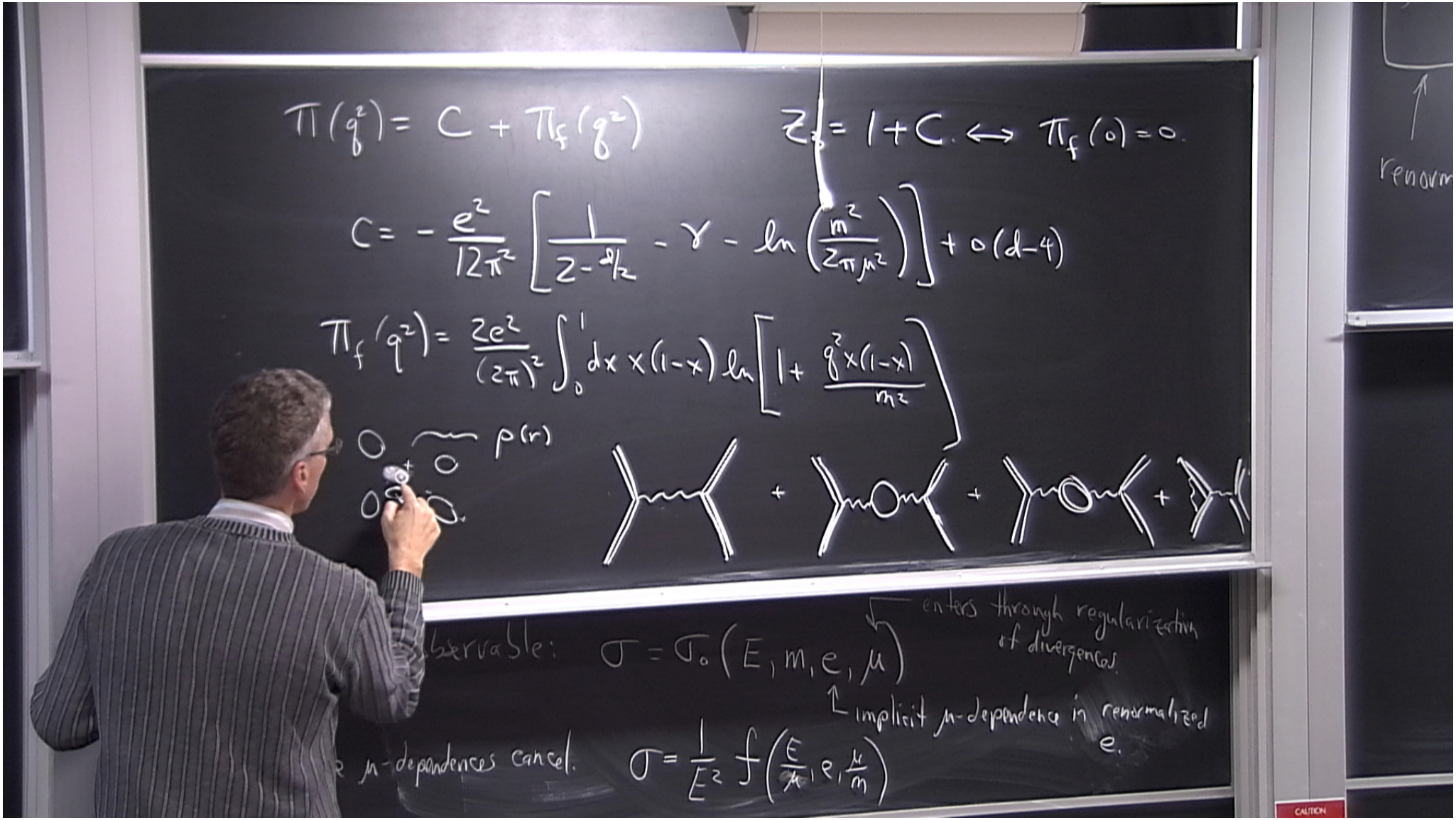
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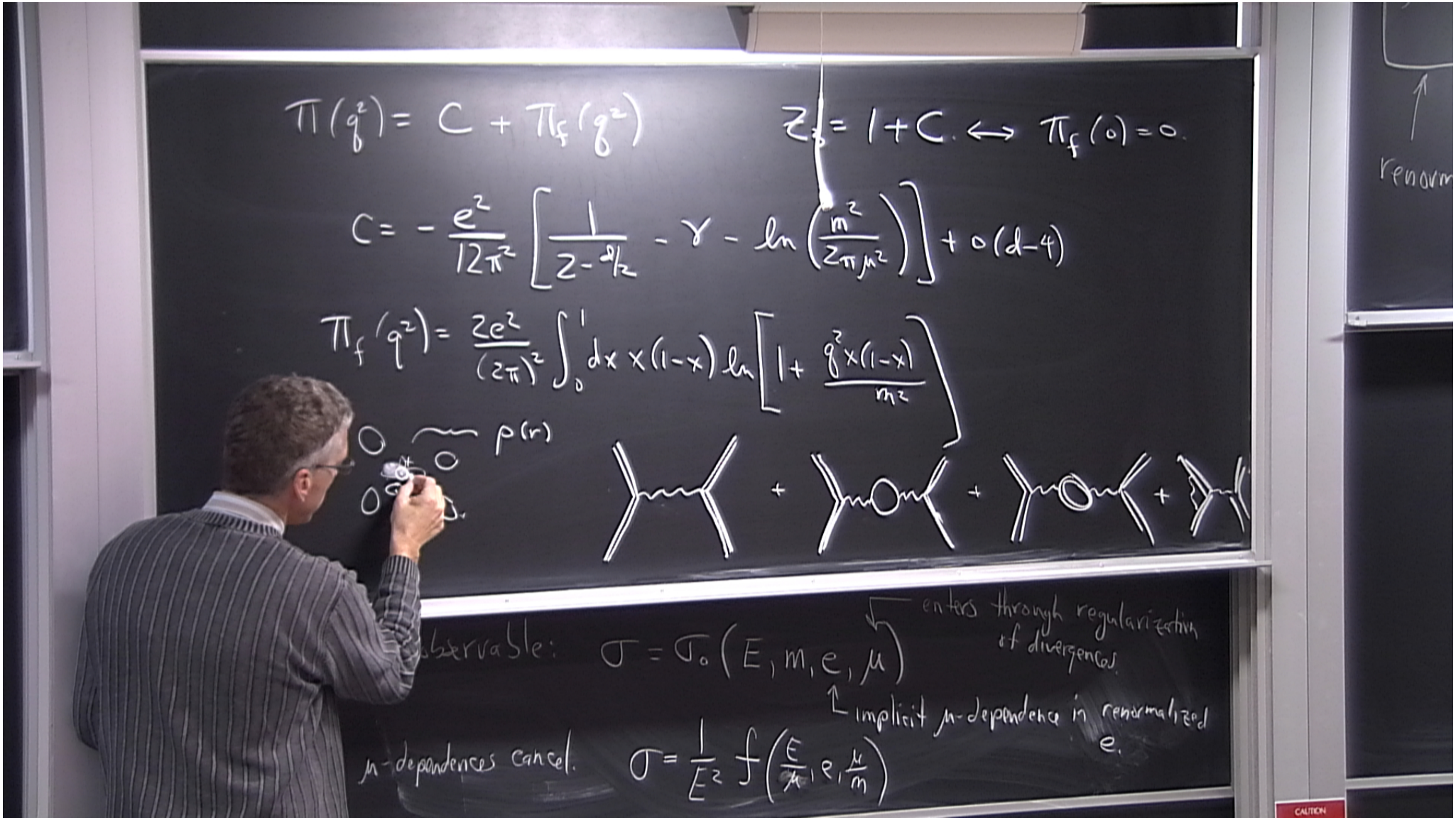
renorm
CAUTION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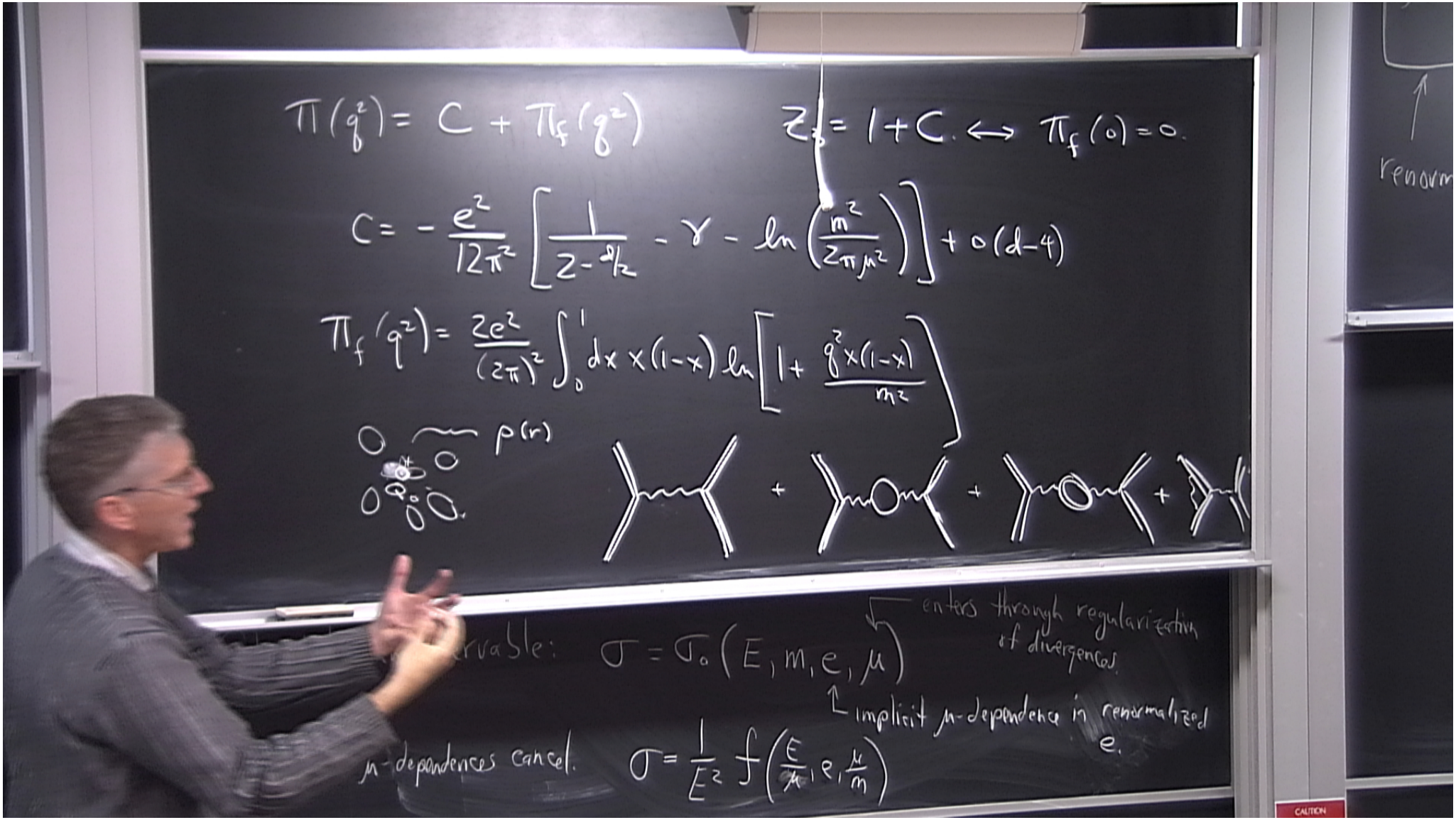
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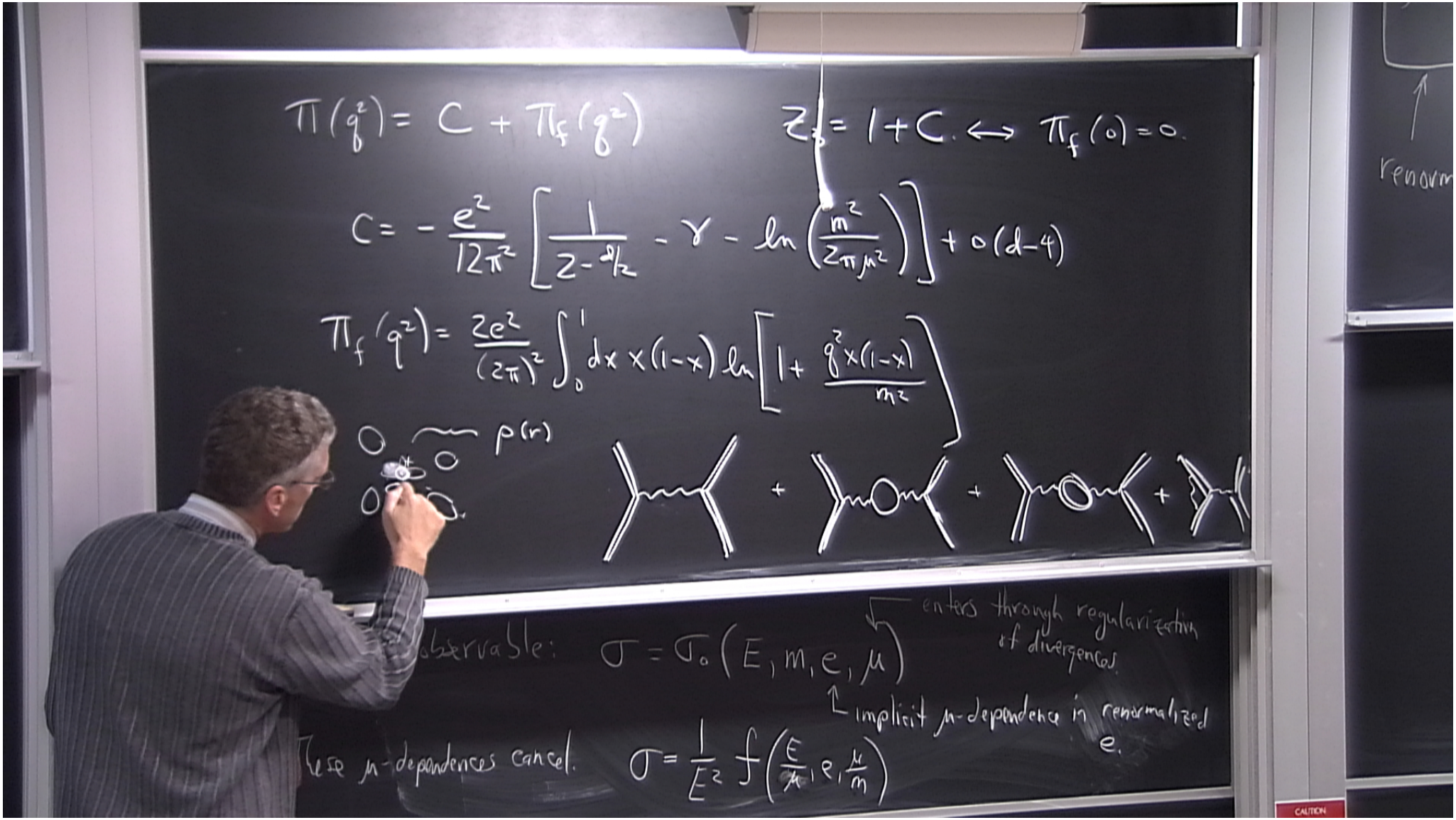
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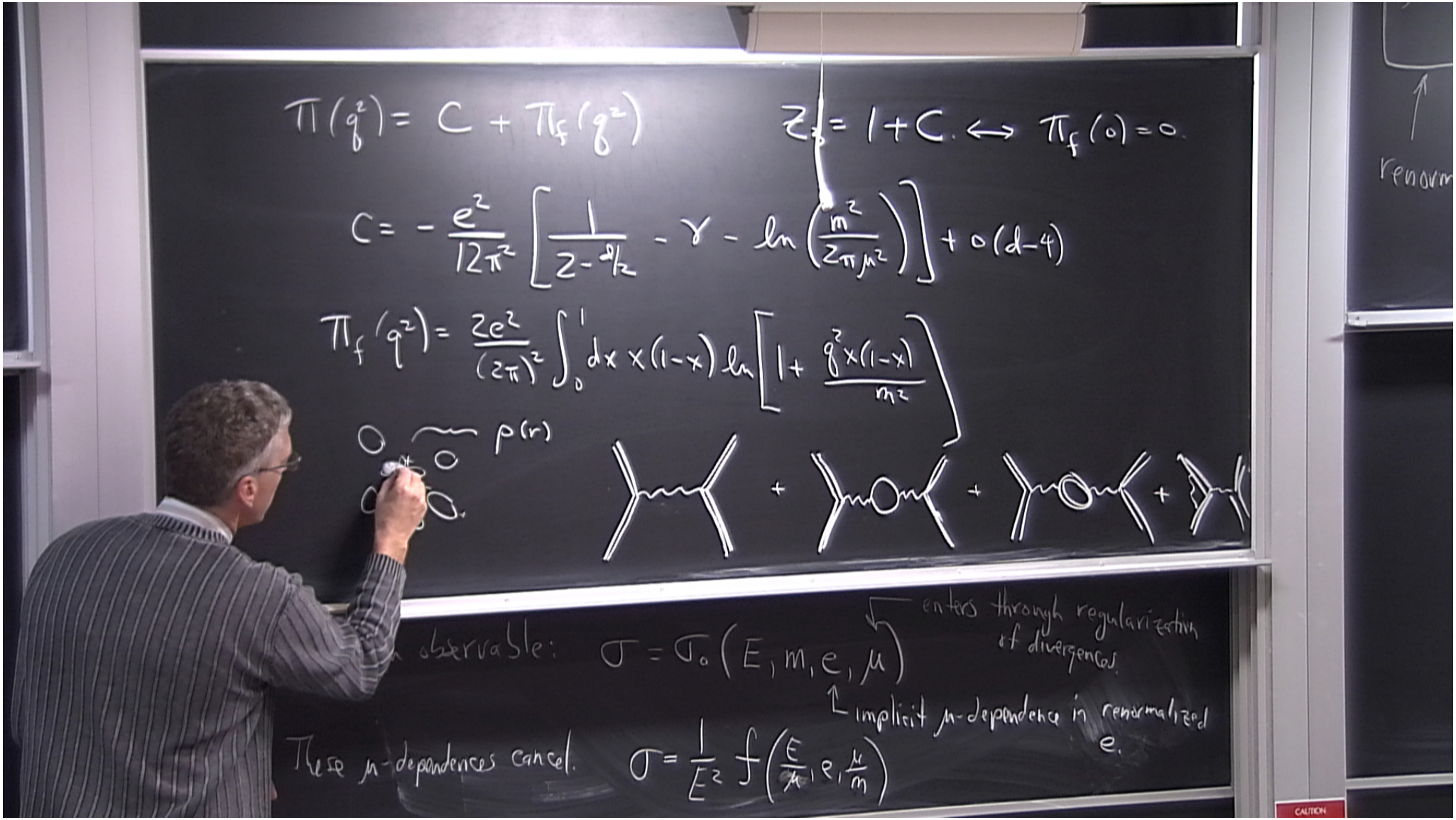
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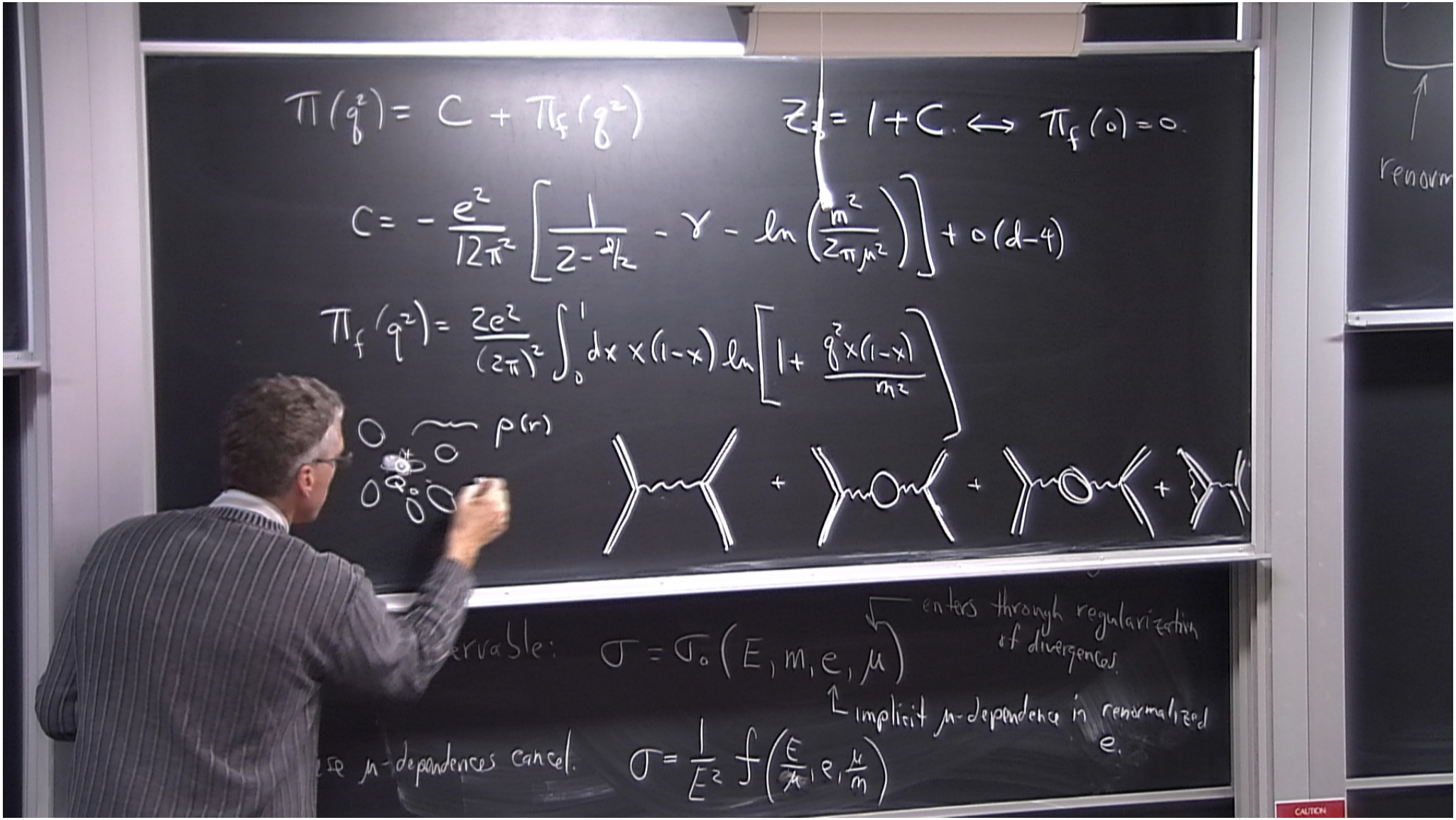
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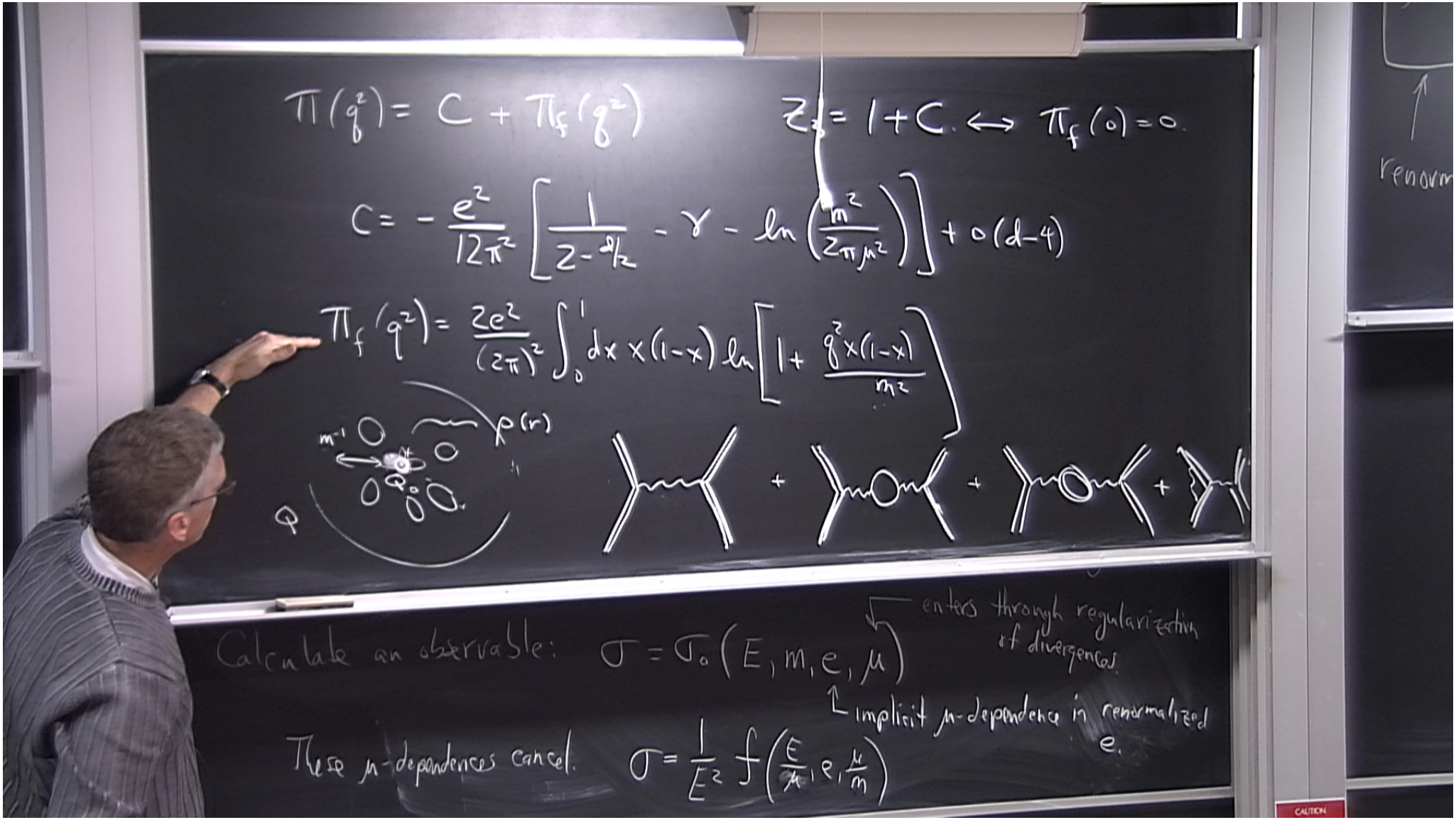
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q
+
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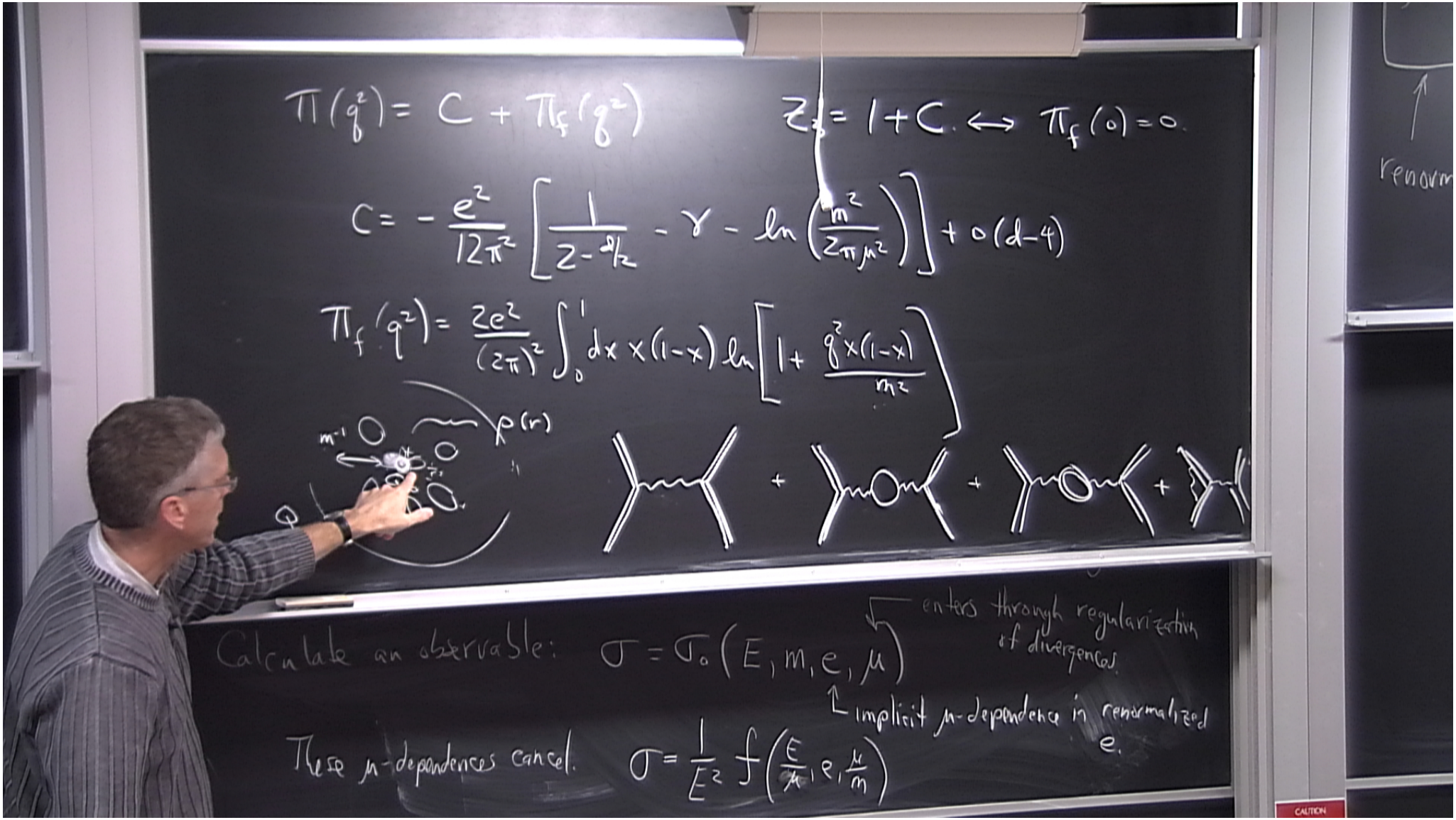
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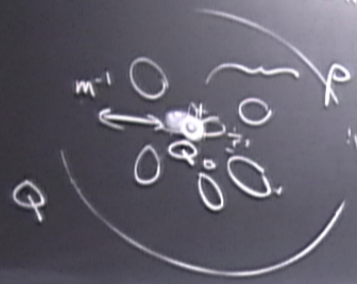

$$+ \quad + \quad +$$

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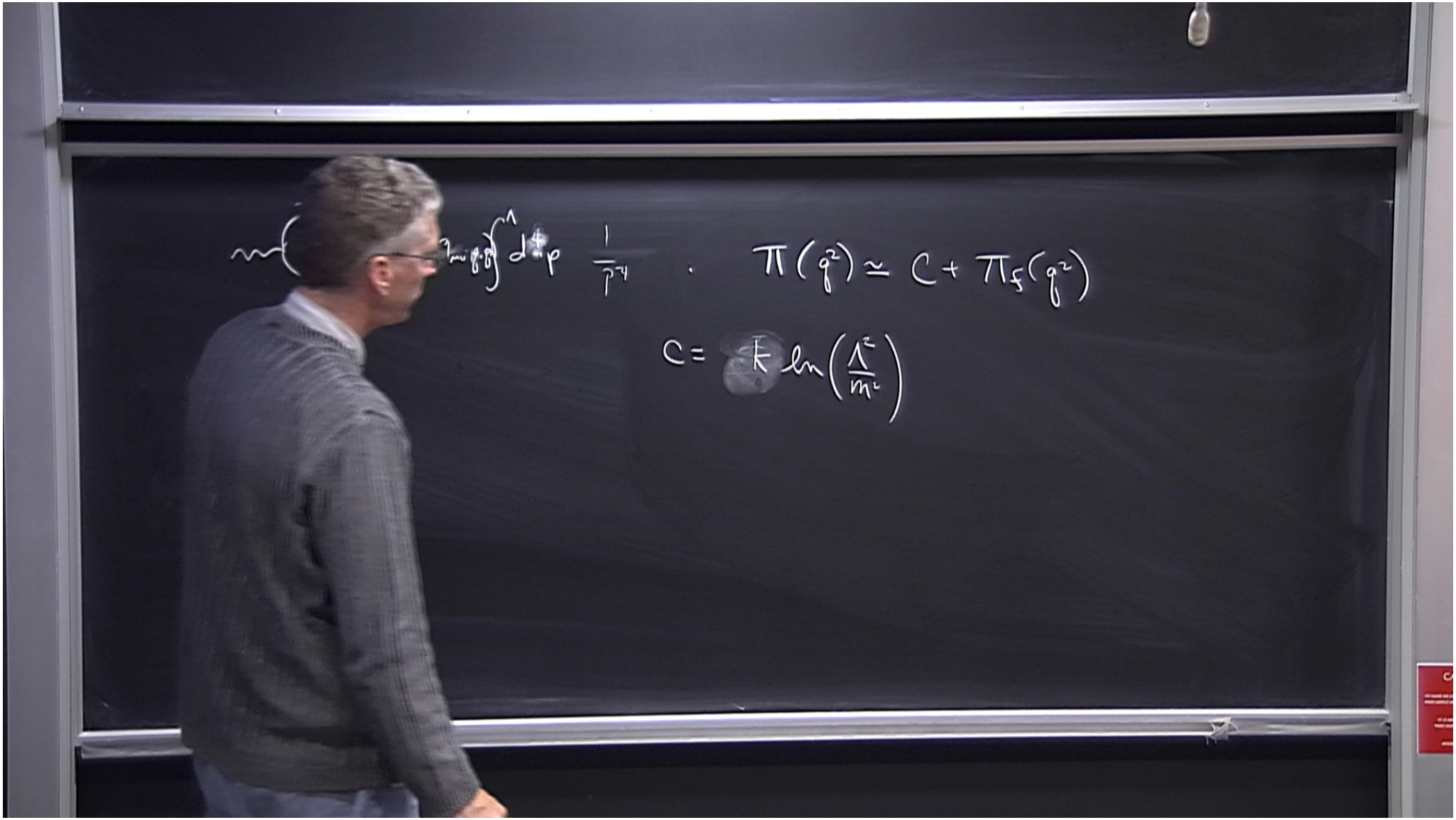
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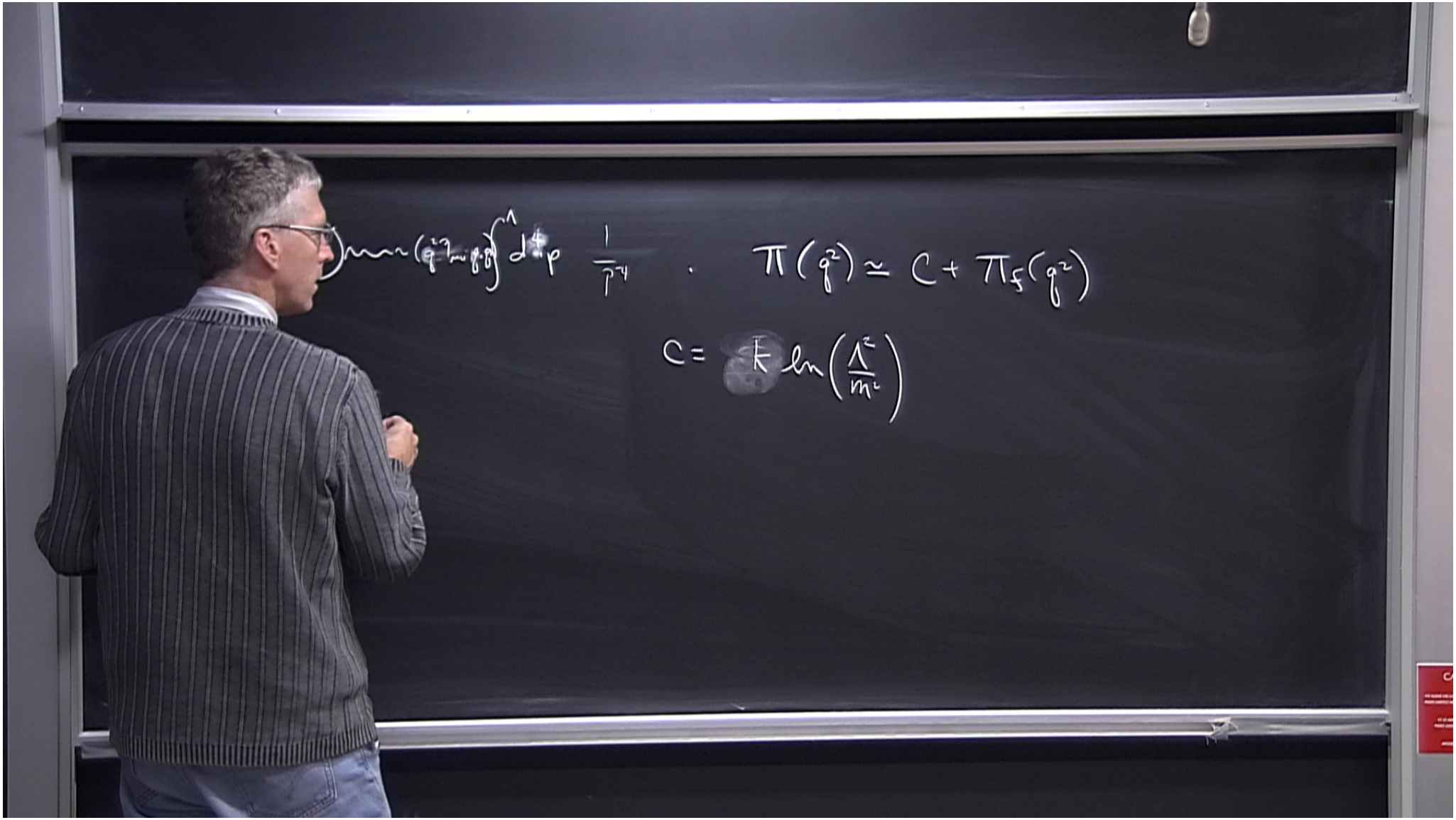
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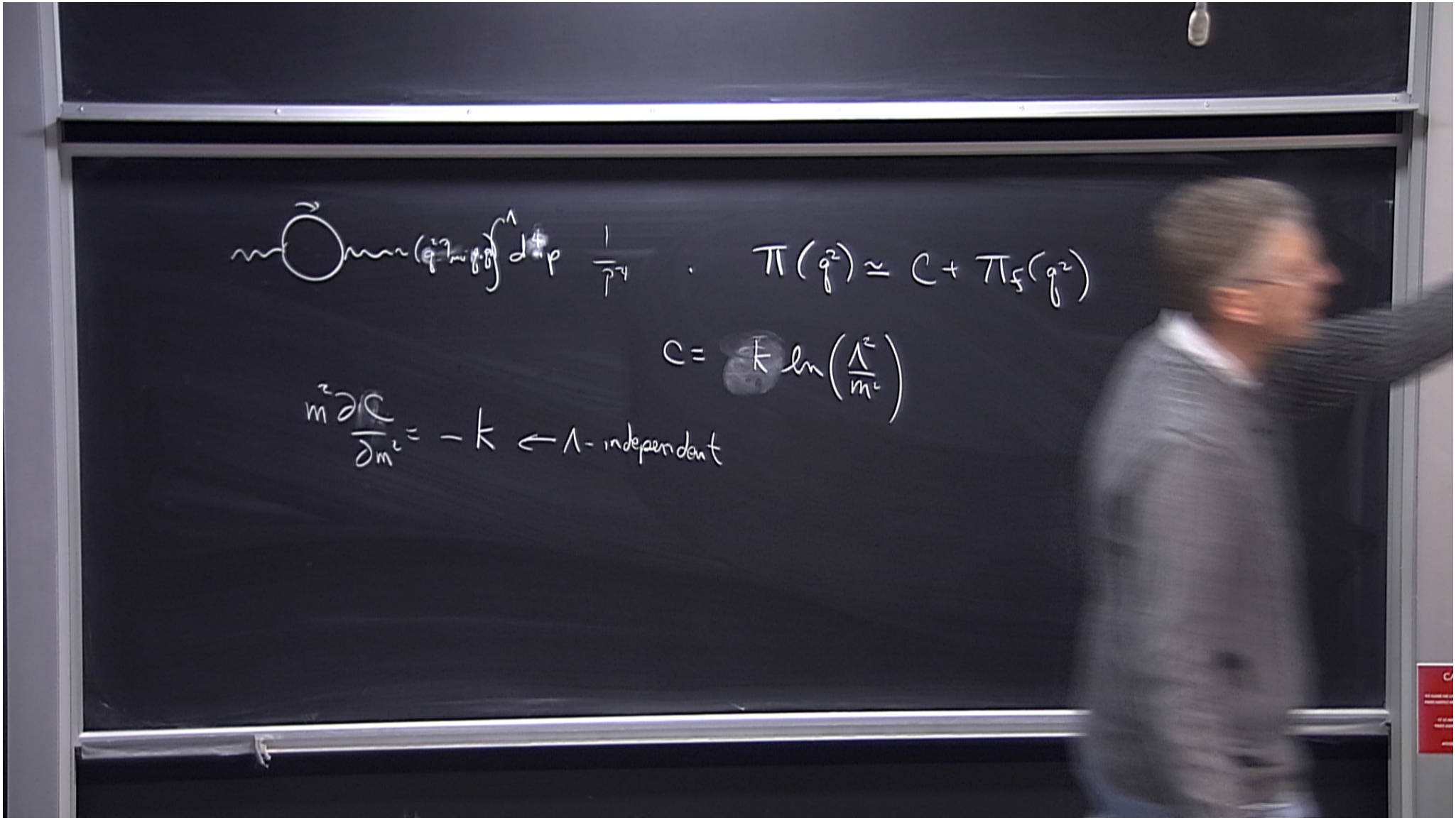
$\int^{\Lambda} d^4p \; \frac{1}{p^4} \cdot \Pi(q^2) \simeq C + \Pi_f(q^2)$
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$m^2 \frac{\partial C}{\partial m^2} = -k \leftarrow \Lambda$- independant

$$Z_3 = 1 - \frac{e^2}{12\pi^2}\underbrace{\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right]}_{C} + o(d-4)$$

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Calculate an observable: $\sigma = \sigma_0(E, m, e, \mu)$ ← enters through regularization of divergences.

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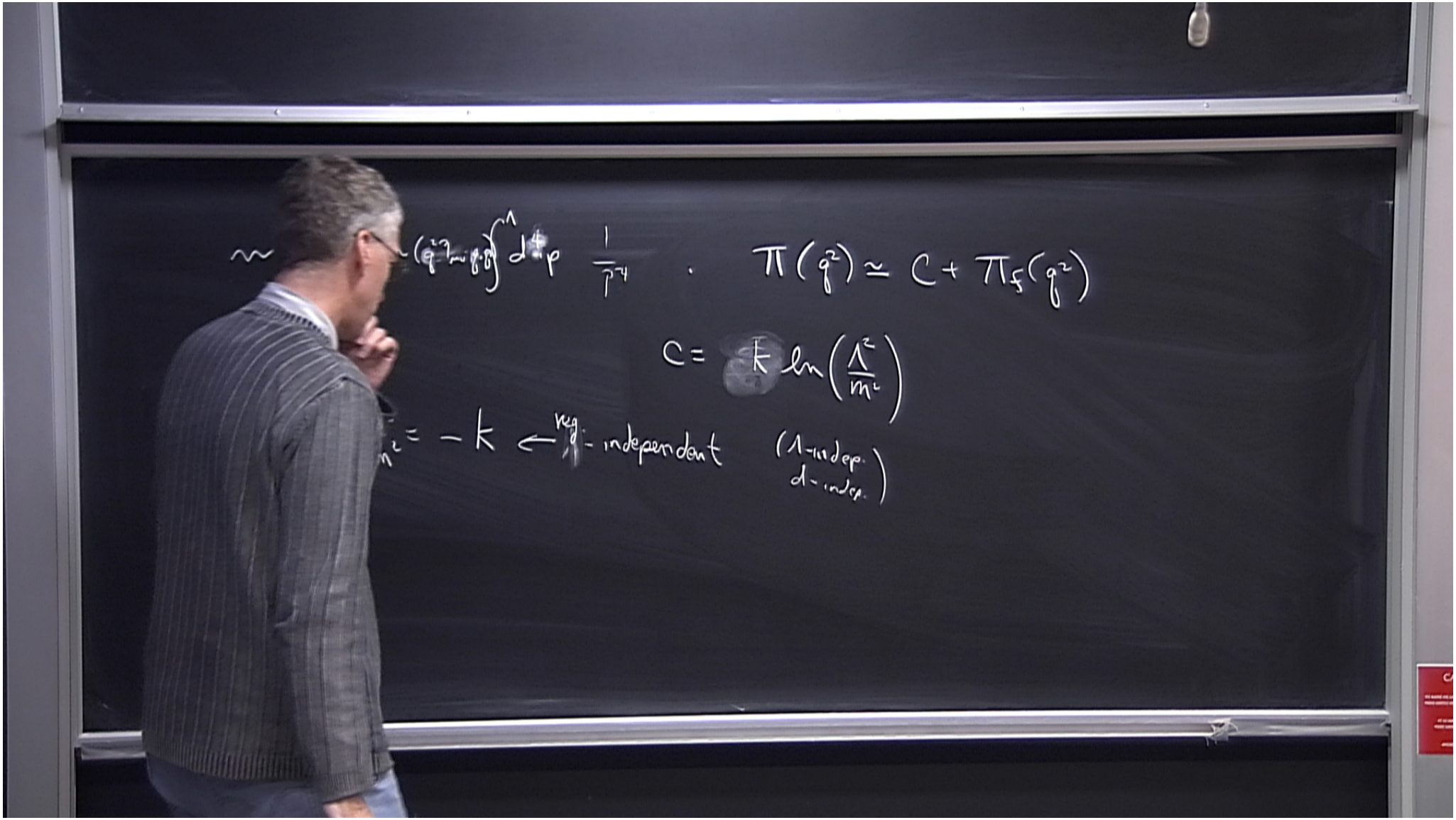
$\sim (q^2 \eta_{\mu\nu} - q_\mu q_\nu) \int^{\Lambda} d^4p \; \frac{1}{p^4} \cdot \quad \Pi(q^2) \simeq C + \Pi_f(q^2)$
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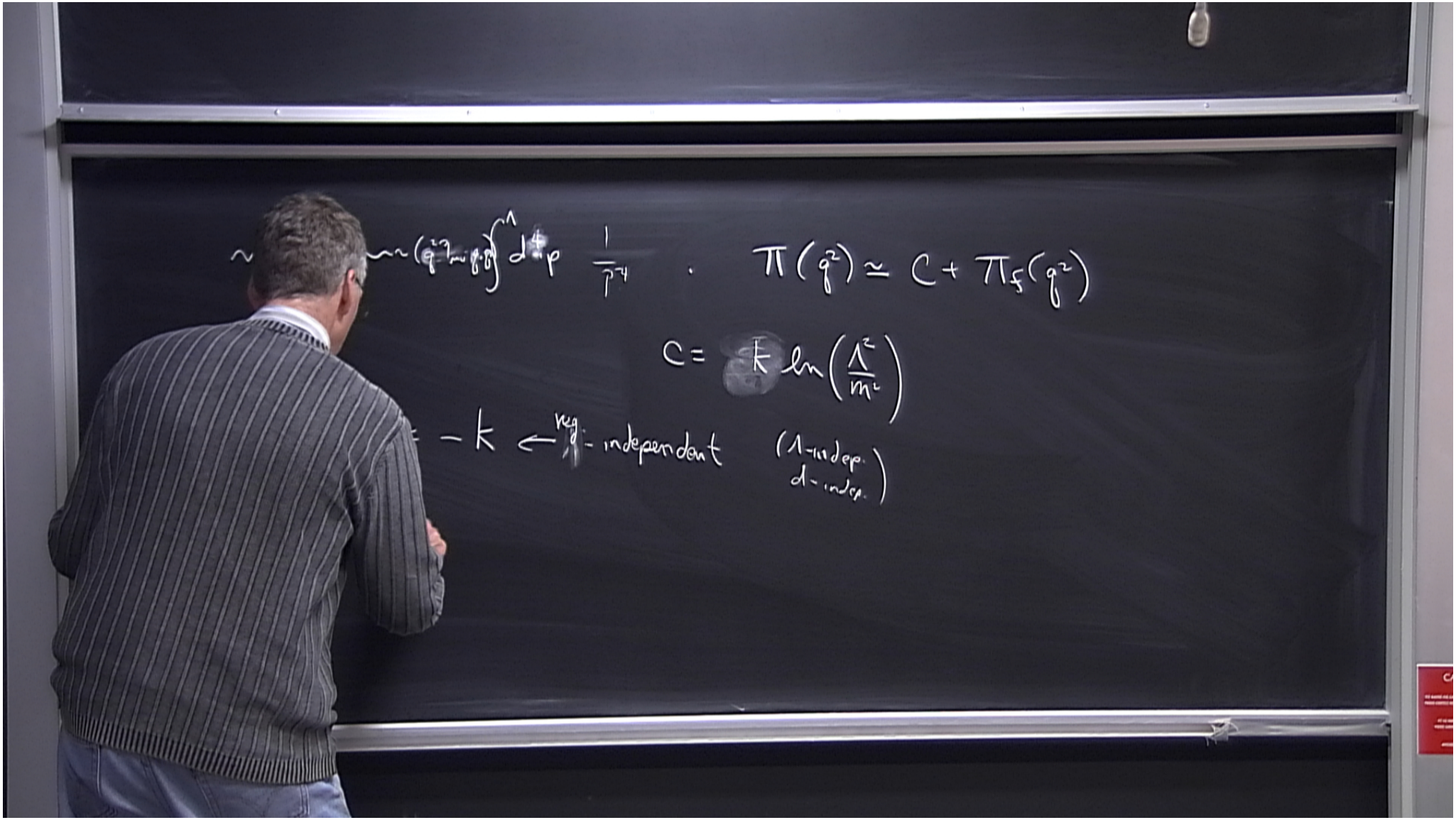
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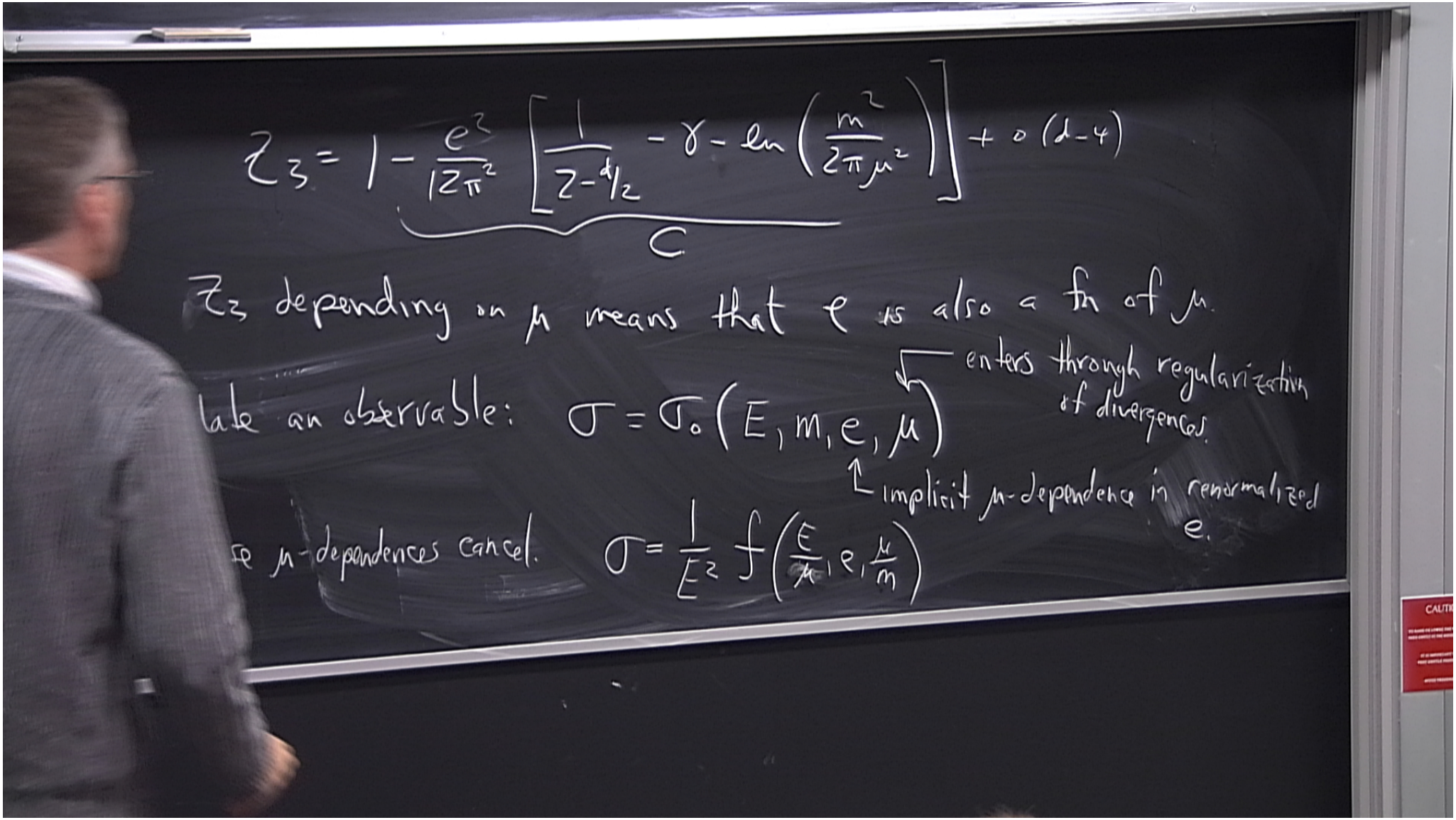
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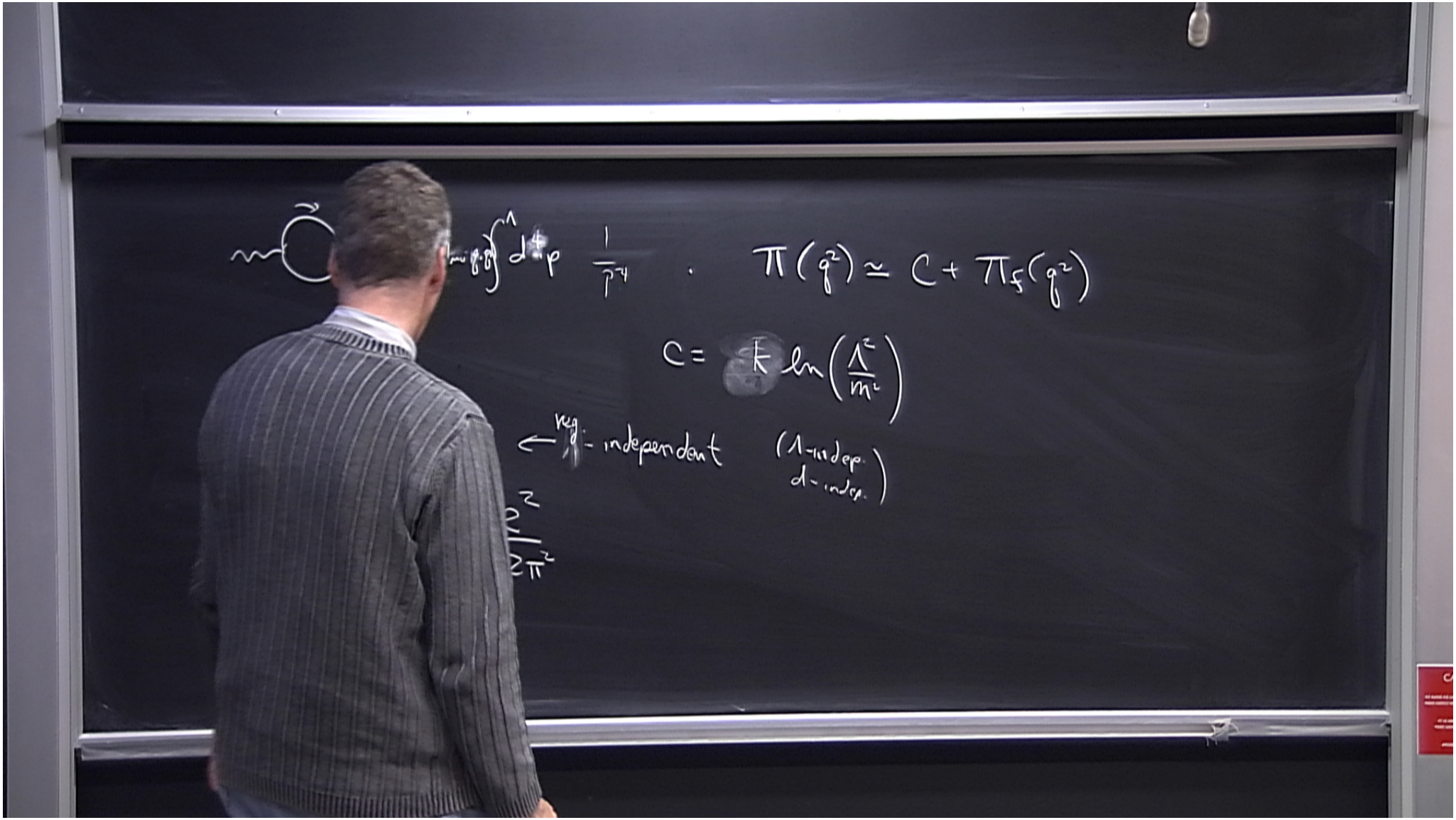
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reg. - independent
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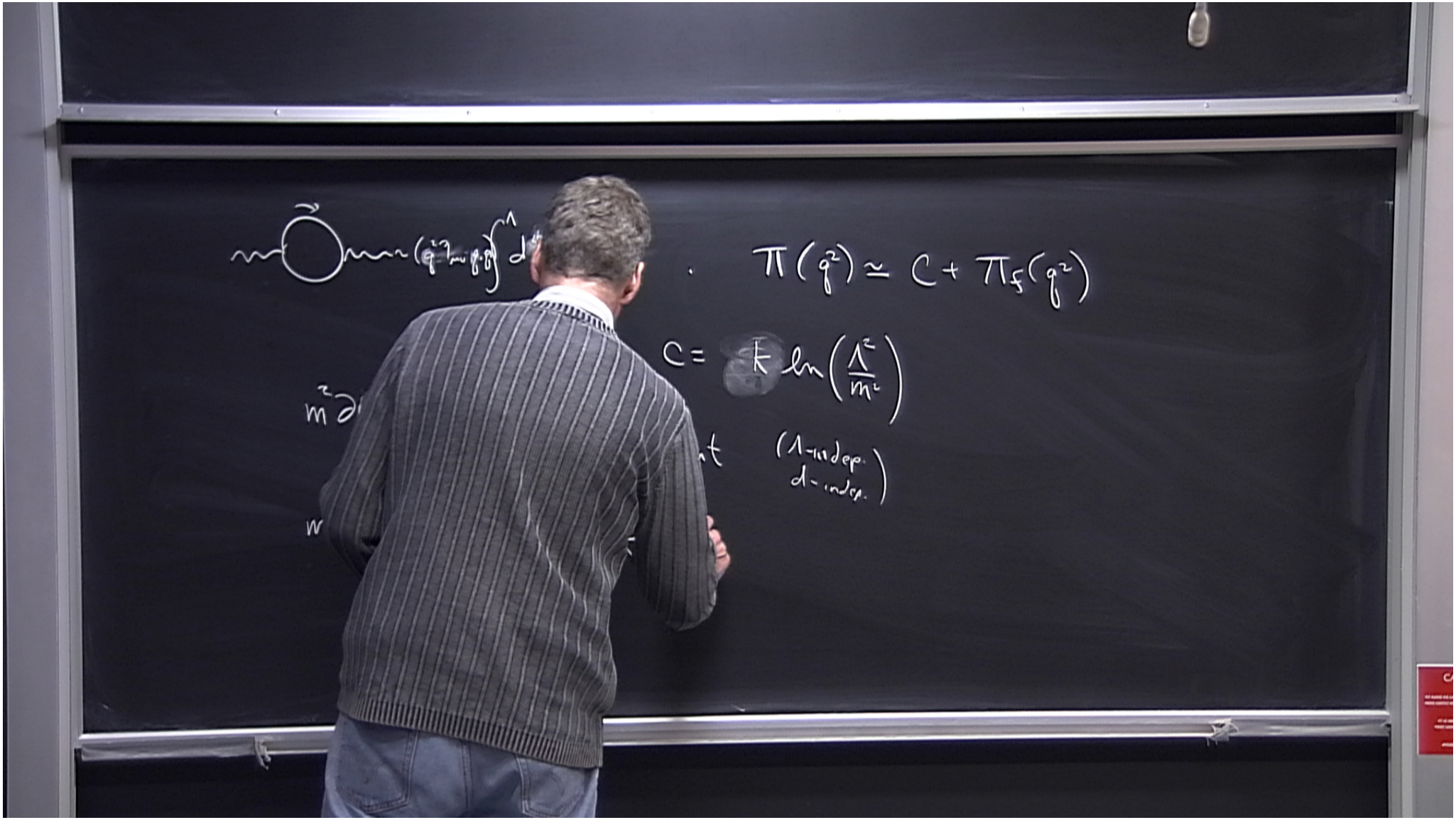
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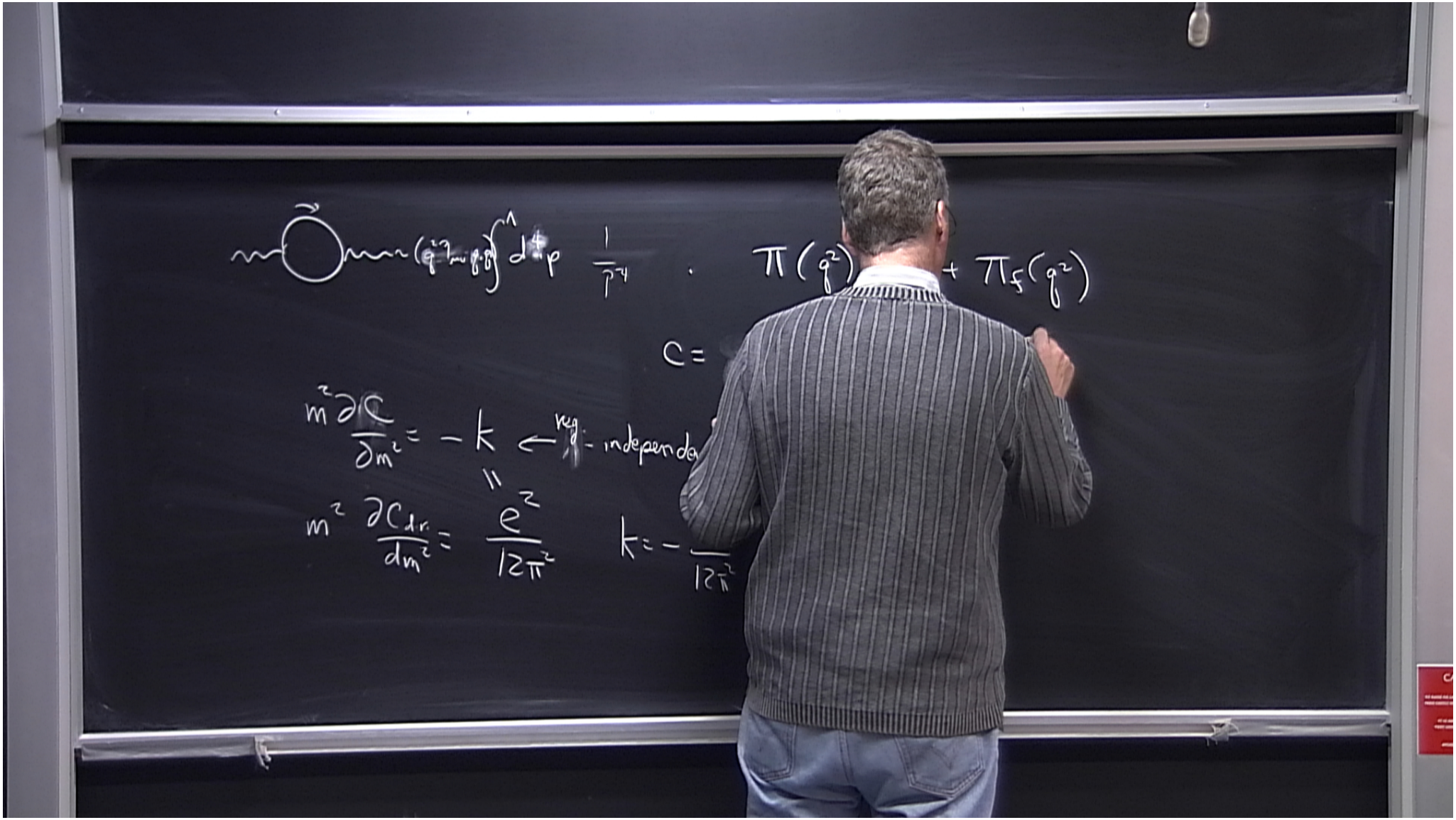

$\int d^4p \; \frac{1}{p^4} \cdot \Pi(q^2) + \Pi_5(q^2)$
$m^2 \frac{\partial C}{\partial m^2} = -k$
reg. independ
$m^2 \frac{\partial C_{div}}{dm^2} = \frac{e^2}{12\pi^2}$
$k = -\frac{}{12\pi^2}$



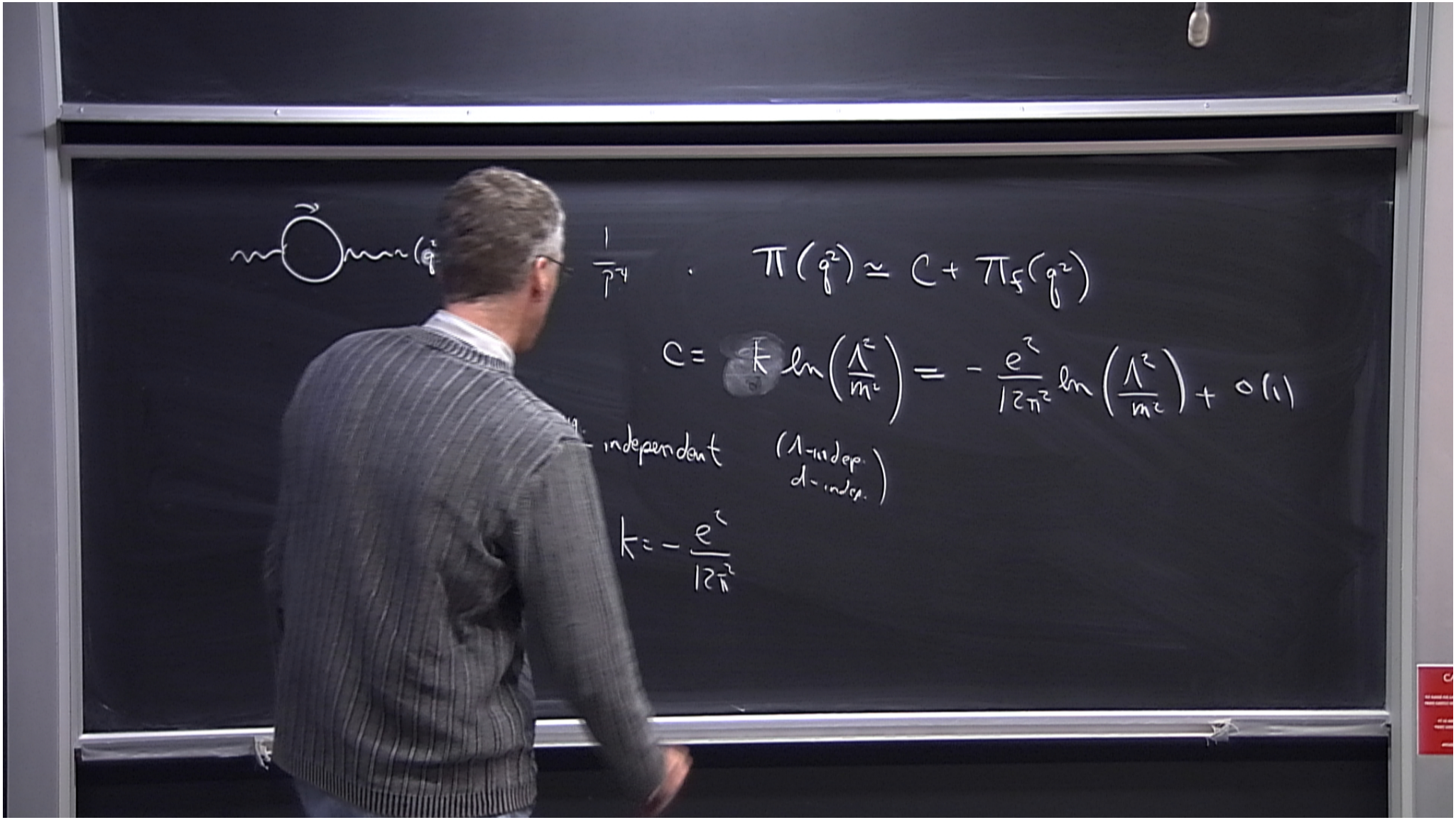
$$\int^{\Lambda} d^4p \; \frac{1}{p^4} \cdot \quad \Pi(q^2) \simeq C + \Pi_f(q^2)$$

$$C = k \ln\left(\frac{\Lambda^2}{m^2}\right) = -\frac{e^2}{12\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right) + O(1)$$

$$m^2 \frac{\partial C}{\partial m^2} = -k \leftarrow \text{reg.-independant} \quad (\Lambda\text{-ind. dep.}, \; d\text{-ind. dep.})$$

$$= \frac{e^2}{12\pi^2}$$

$$m^2 \frac{\partial C_{dr.}}{dm^2} = \frac{e^2}{12\pi^2} \qquad k = -\frac{e^2}{12\pi^2}$$


$\frac{1}{p^4}$. $\Pi(q^2) \simeq C + \Pi_f(q^2)$
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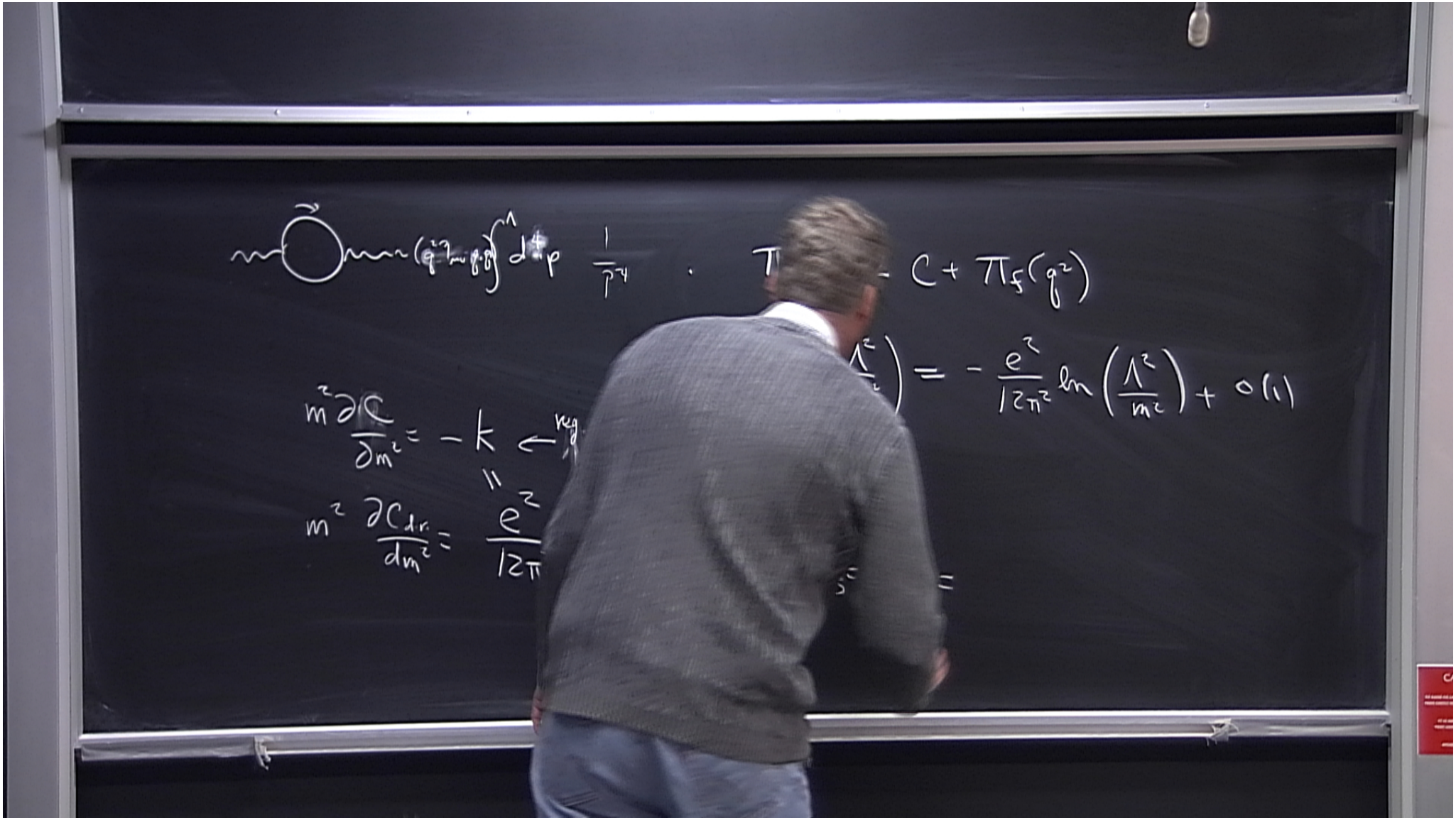
$\int^{\Lambda} d^4p \; \frac{1}{p^4}$
$+ C + \Pi_f(q^2)$
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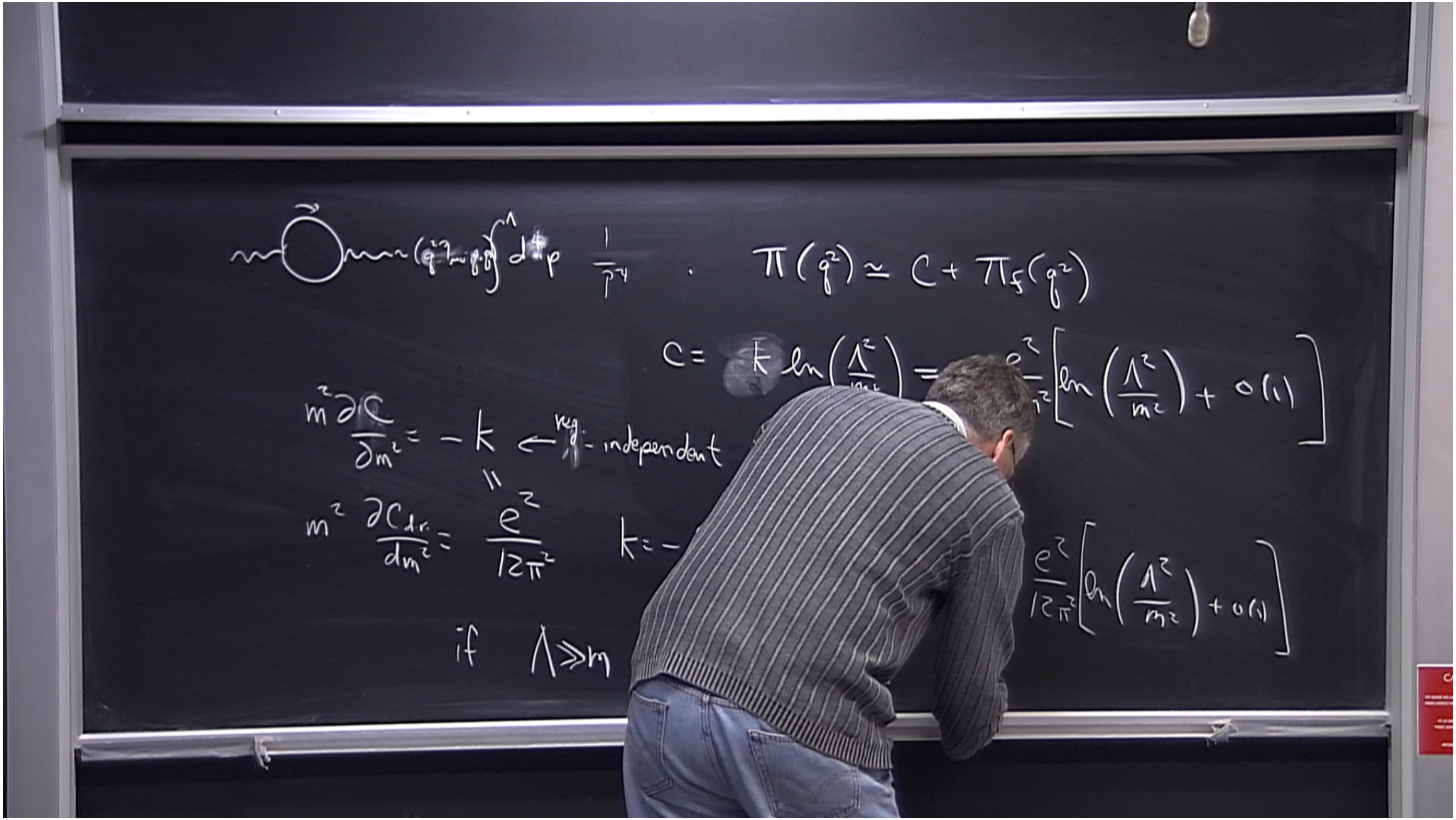
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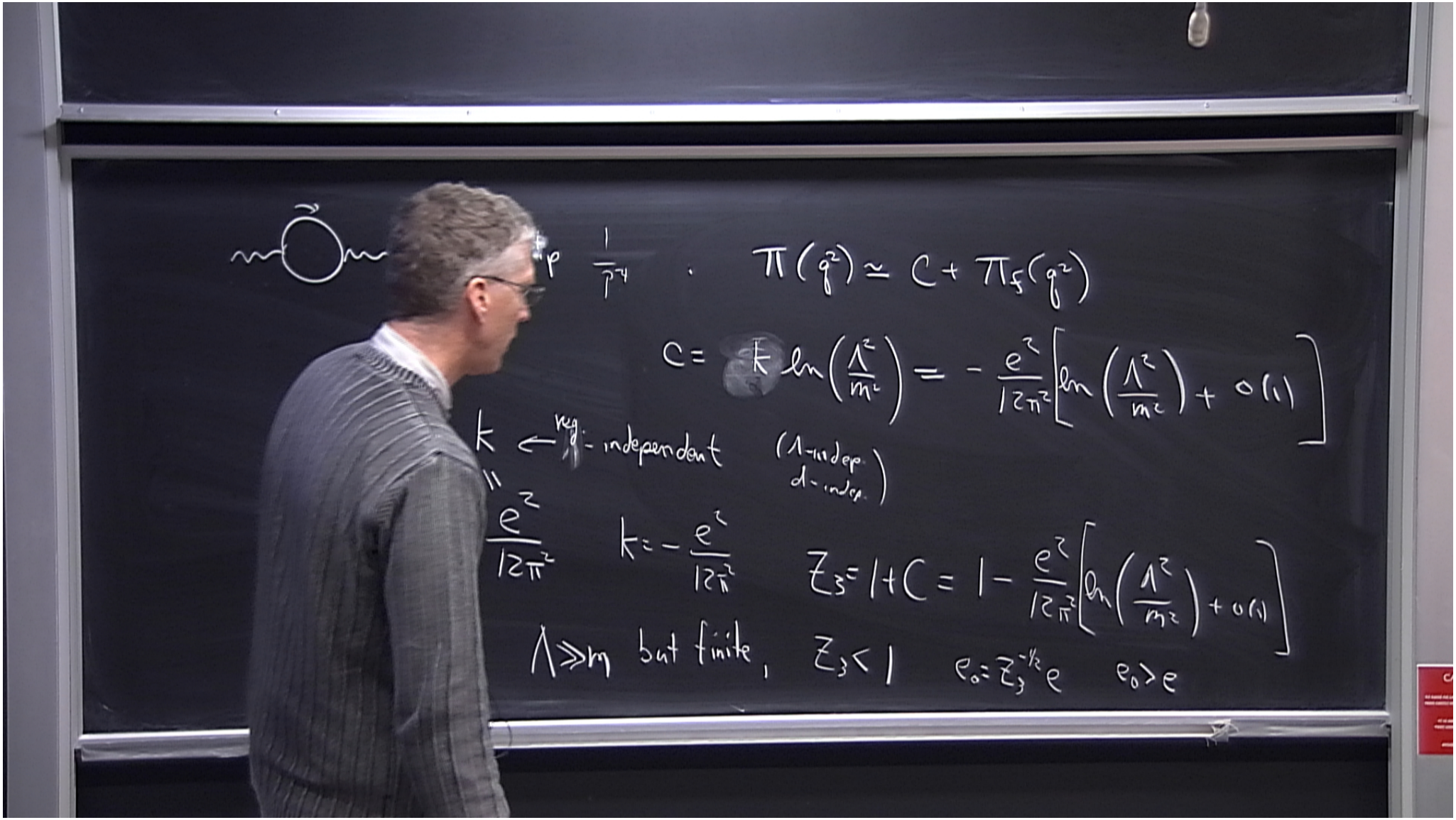
if $\Lambda \gg m$


p $\frac{1}{p^4}$. $\Pi(q^2) \simeq C + \Pi_f(q^2)$
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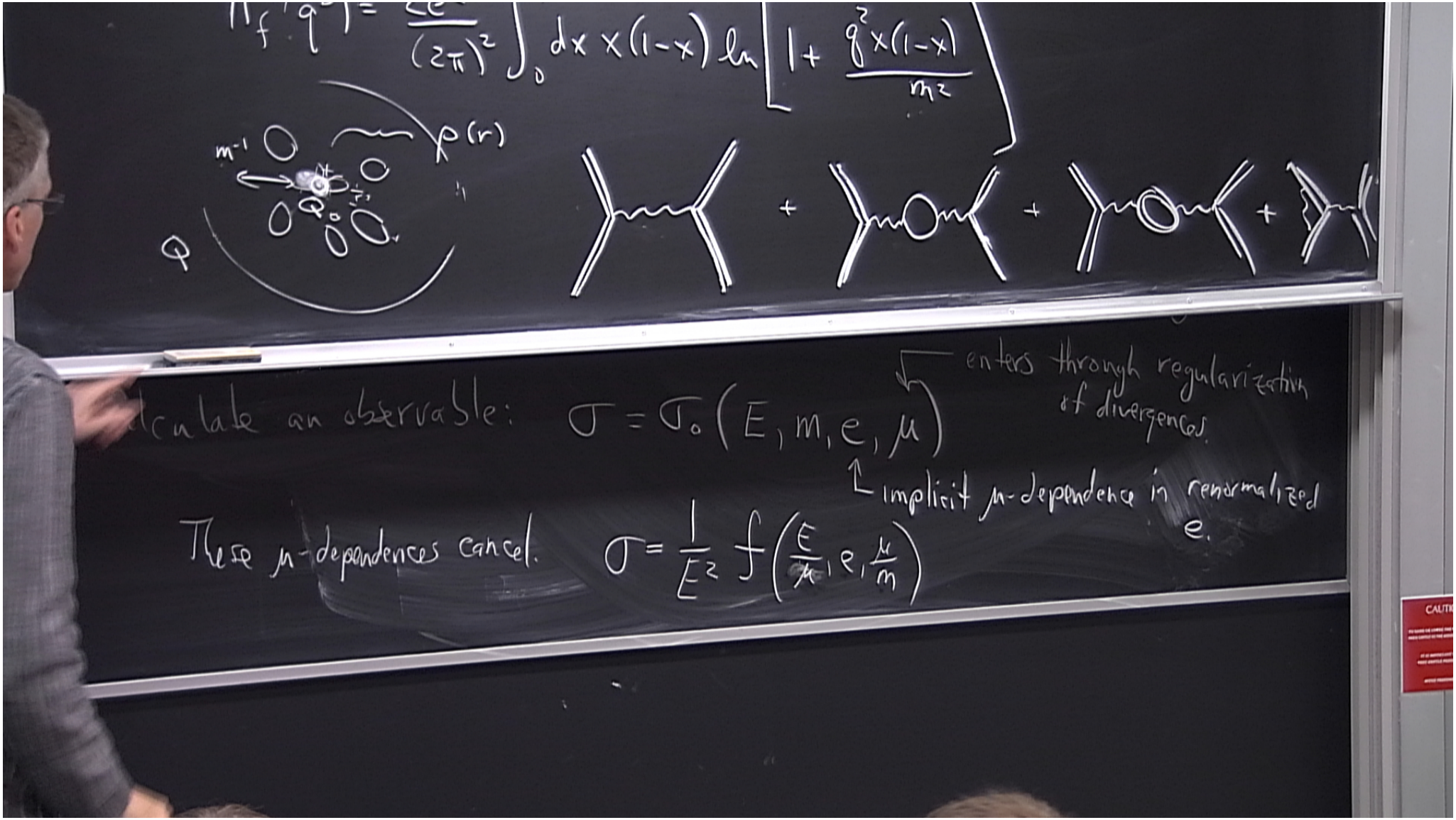
$k \leftarrow$ reg.-independant (Λ-indep. d-indep.)
$= \frac{e^2}{12\pi^2}$ $k = -\frac{e^2}{12\pi^2}$ $Z_3 = 1 + C = 1 - \frac{e^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) + O(1)\right]$
$\Lambda \gg m$ but finite, $Z_3 < 1$ $e_0 = Z_3^{-1/2} e$ $e_0 > e$


$\Pi_f(q^2) = \frac{2e^2}{(2\pi)^2} \int_0^1 dx\, x(1-x) \ln\left[1 + \frac{q^2 x(1-x)}{m^2}\right]$
m-1
ρ(r)
Q
+
+
+
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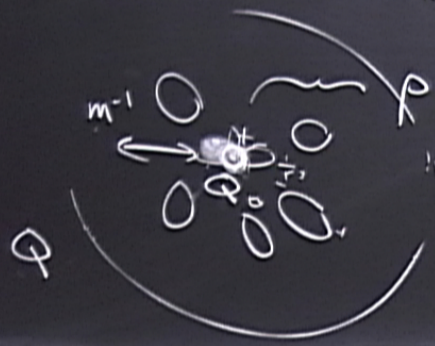

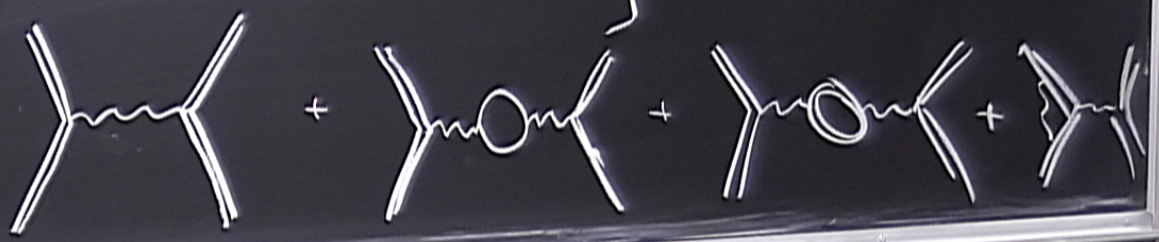

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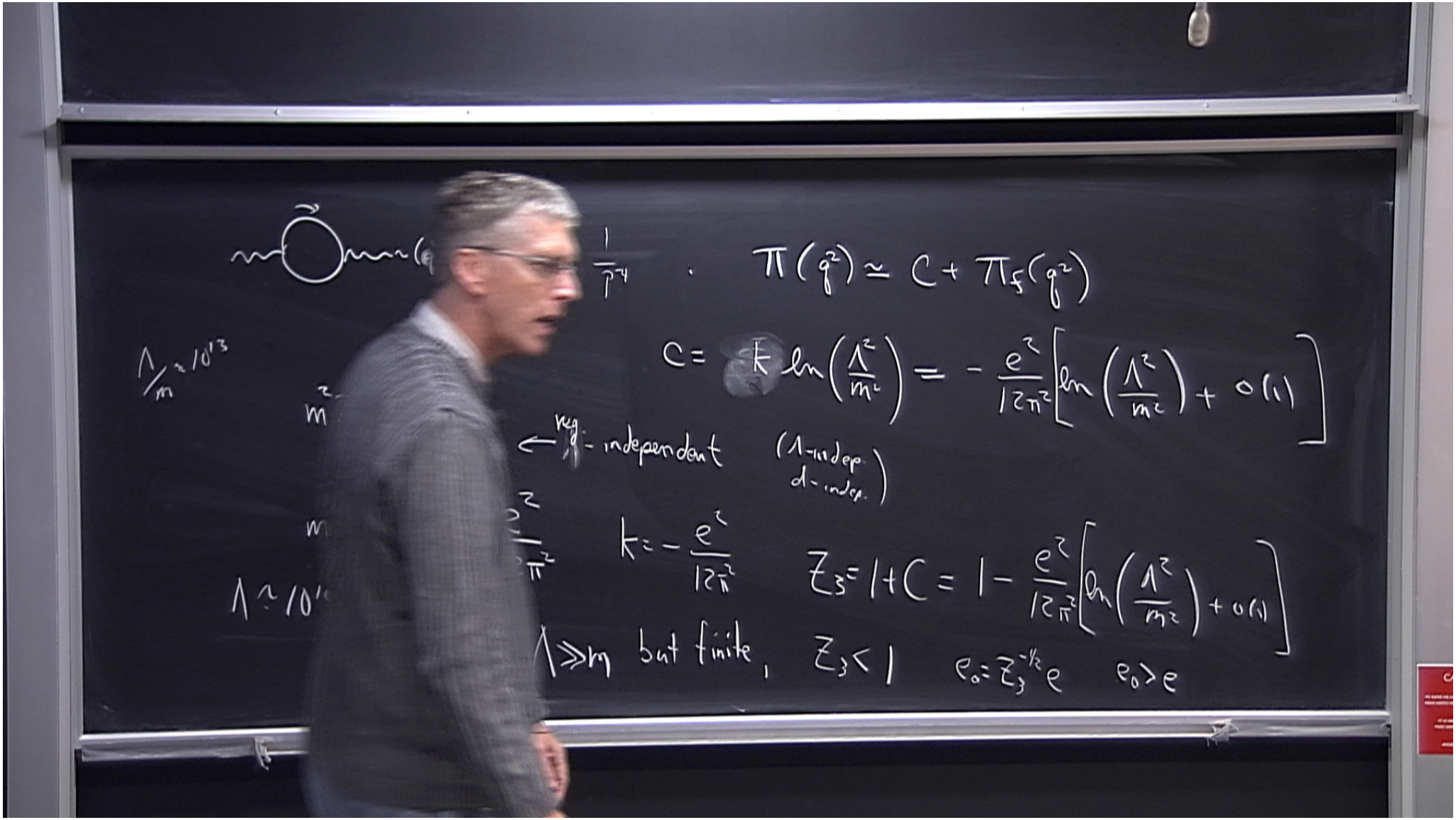
$\frac{1}{p^4}$
$\Pi(q^2) \simeq C + \Pi_f(q^2)$
$\Lambda/m \approx 10^{13}$
$C = k \ln\left(\frac{\Lambda^2}{m^2}\right) = -\frac{e^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) + o(1)\right]$
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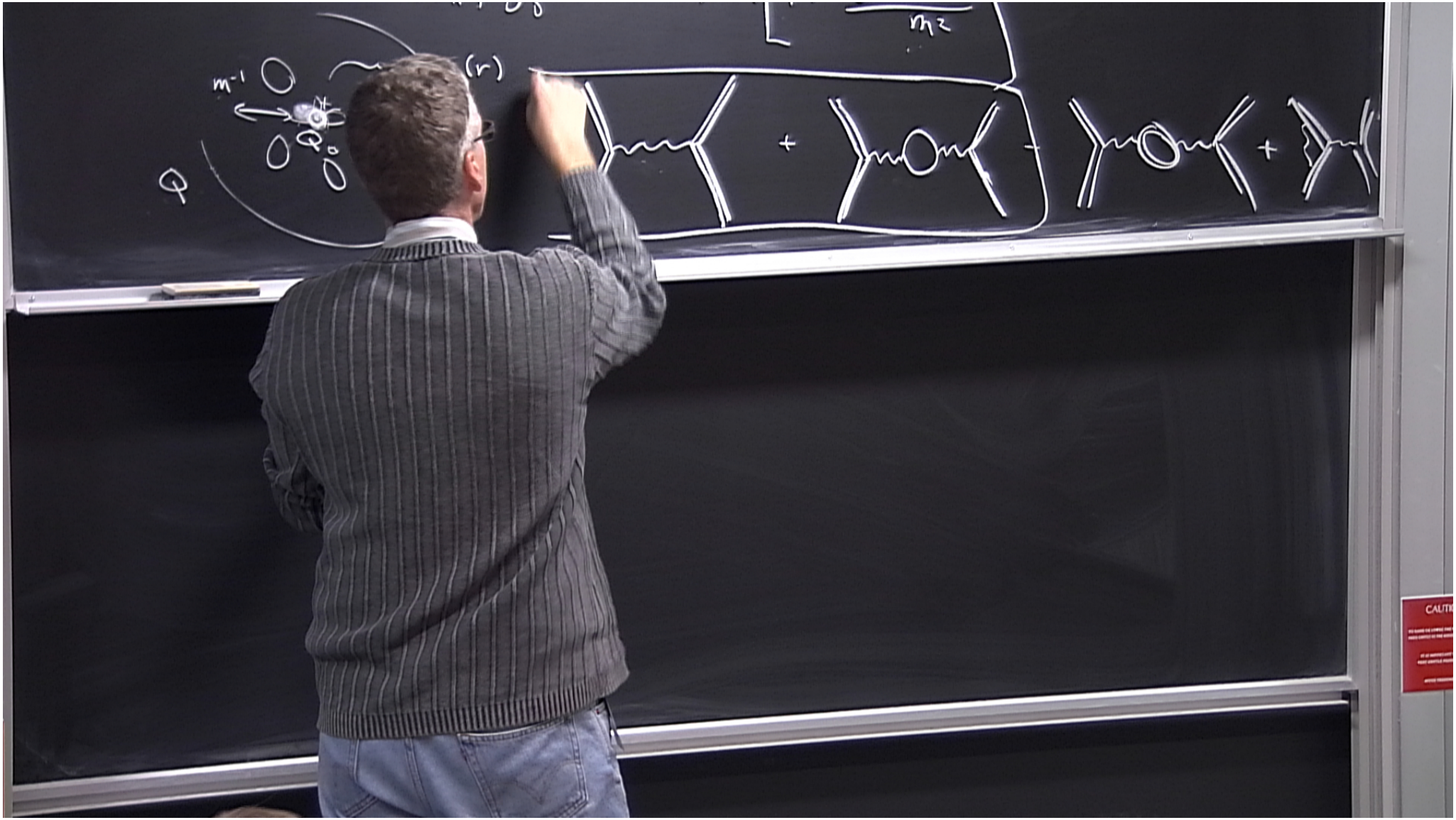
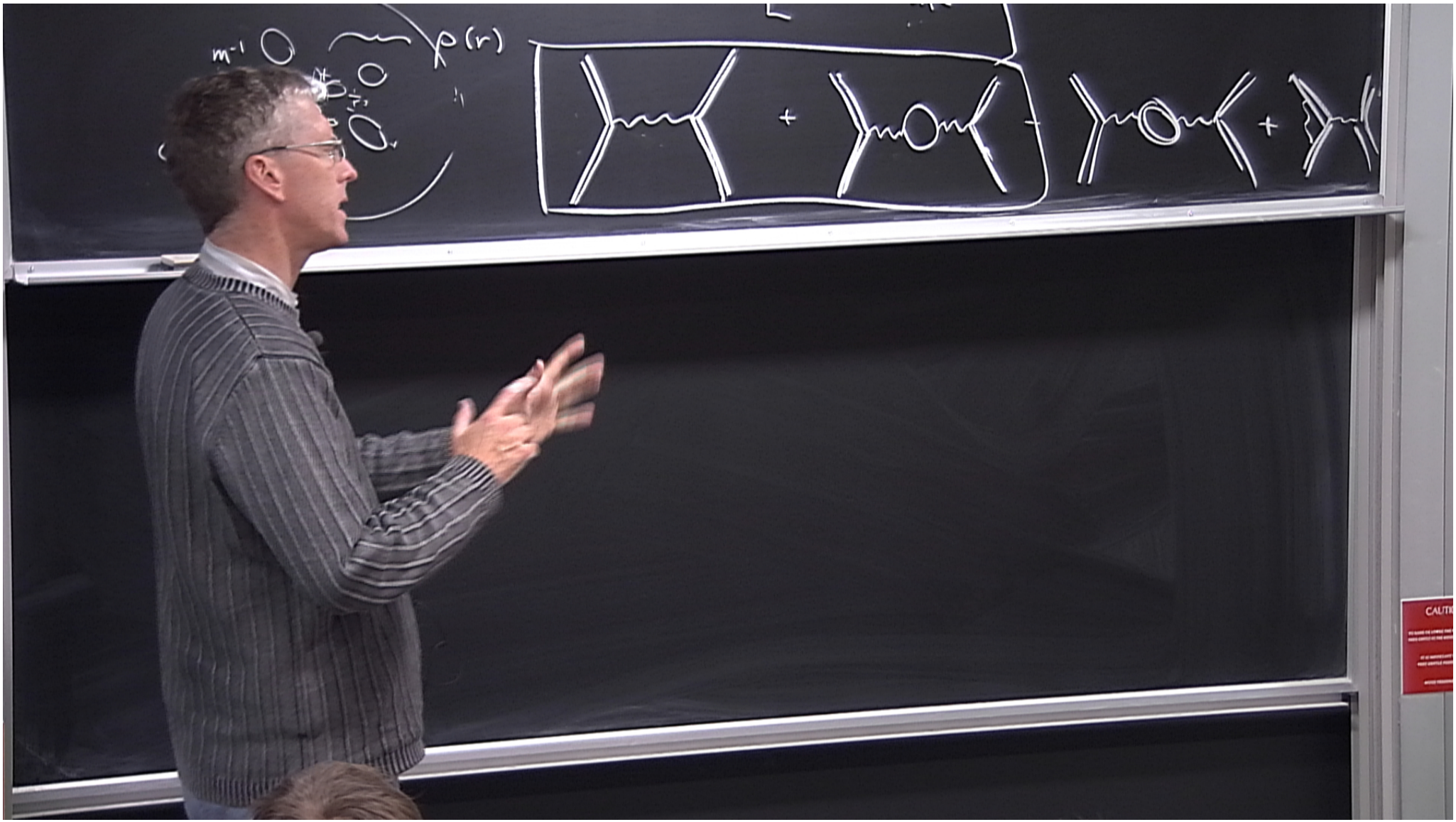
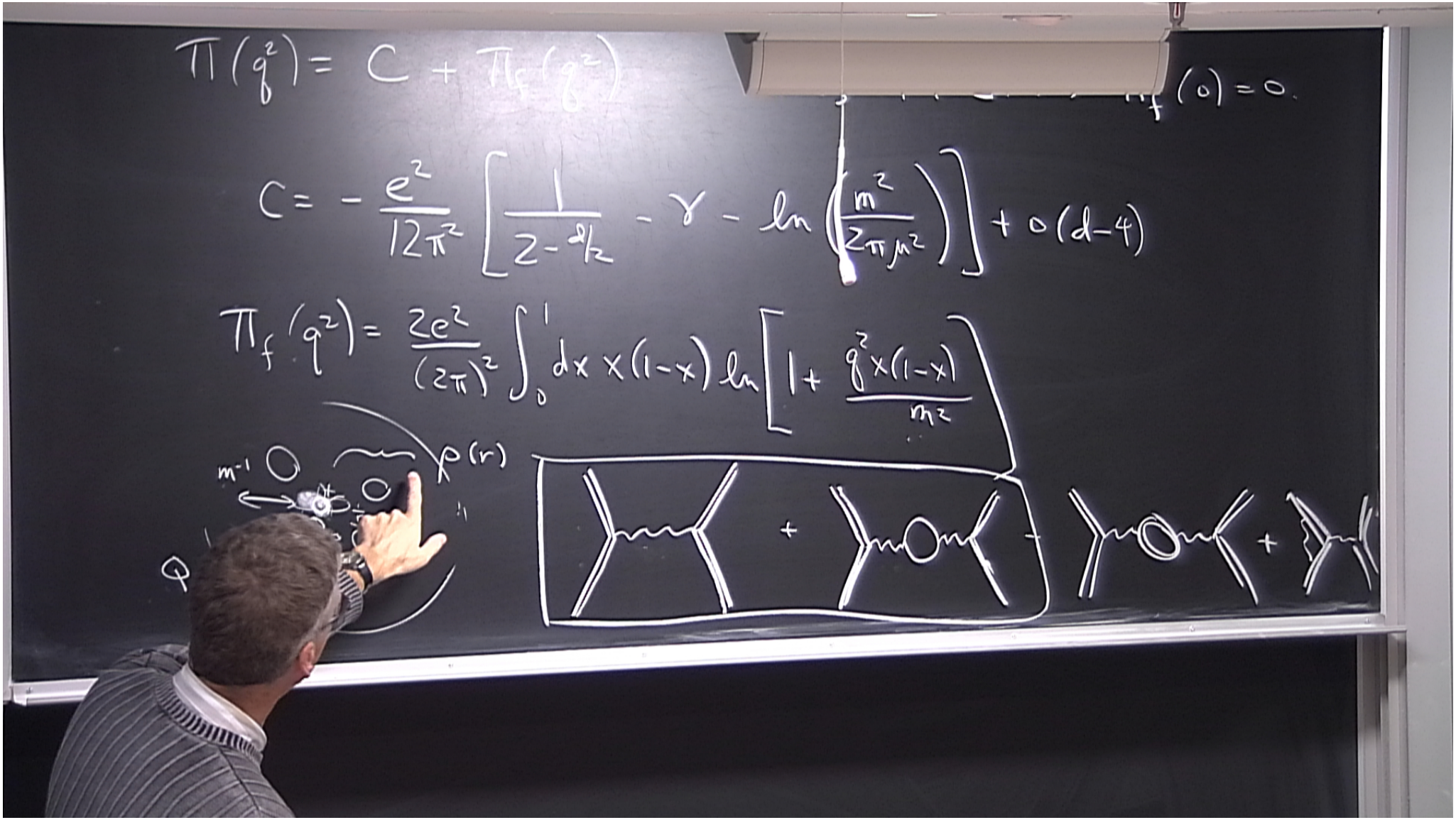
$e_0 = Z_3^{-1/2} e$
$e_0 > e$




m-1
(r)
Q
Q
m2
+
+
+


m-1
ρ(r)

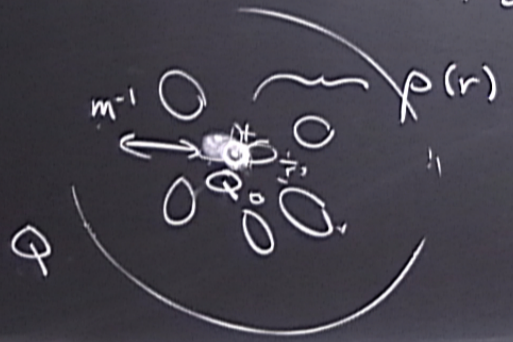

$\Pi(q^2) = C + \Pi_f(q^2)$
$\Pi_f(0) = 0.$
$C = -\frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + O(d-4)$
$\Pi_f(q^2) = \frac{2e^2}{(2\pi)^2}\int_0^1 dx\, x(1-x)\ln\left[1 + \frac{q^2 x(1-x)}{m^2}\right]$
m-1
p(r)
q
+
+
+

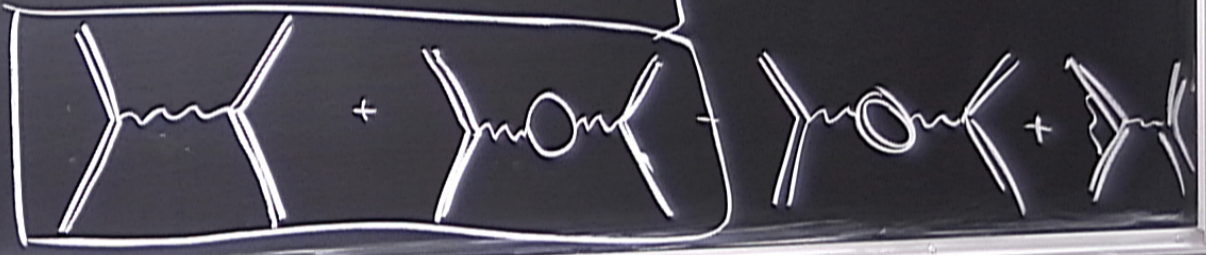
$$\Pi(q^2) = C + \Pi_f(q^2)$$

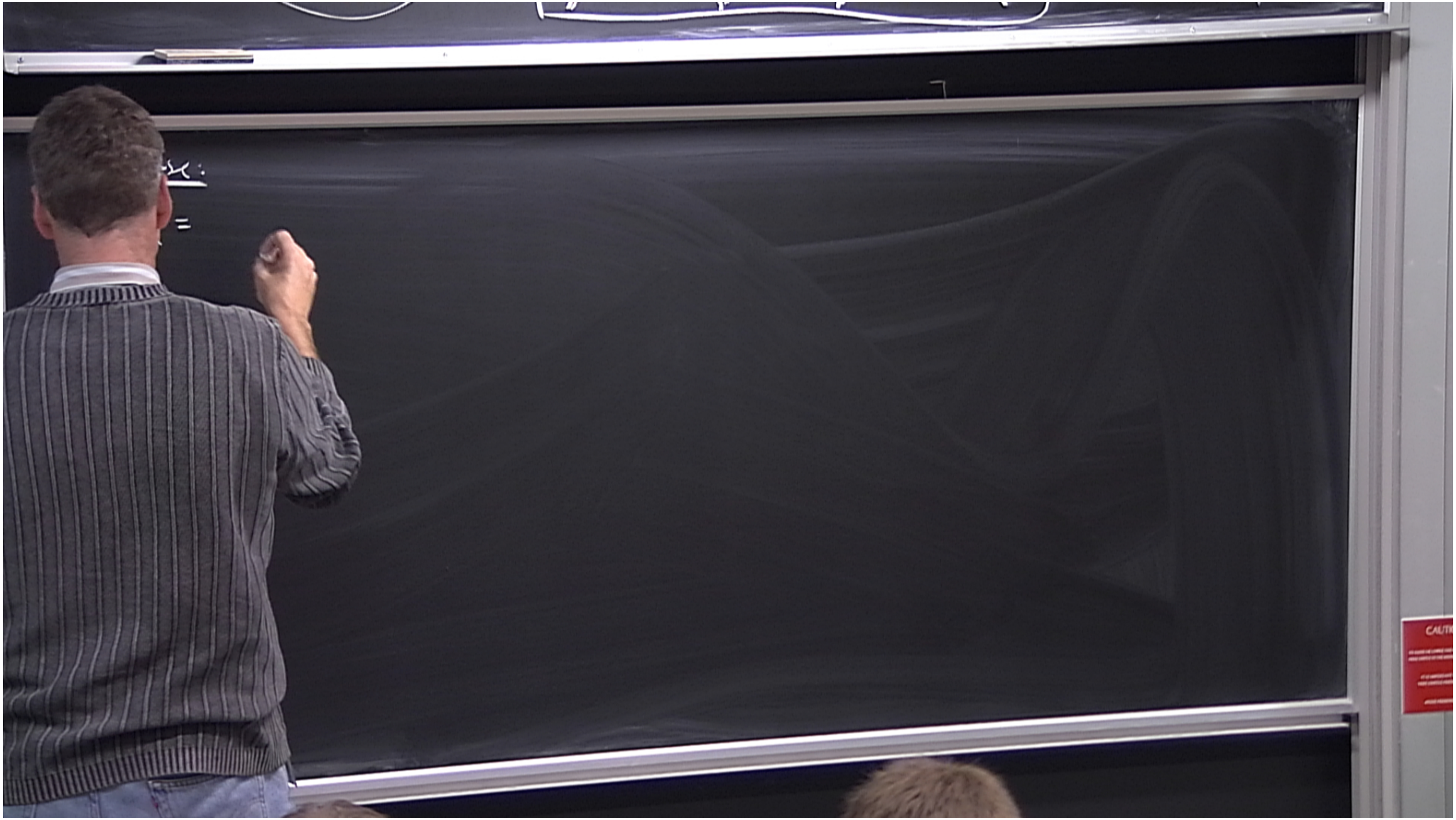
$$\Pi_f(0) = 0$$

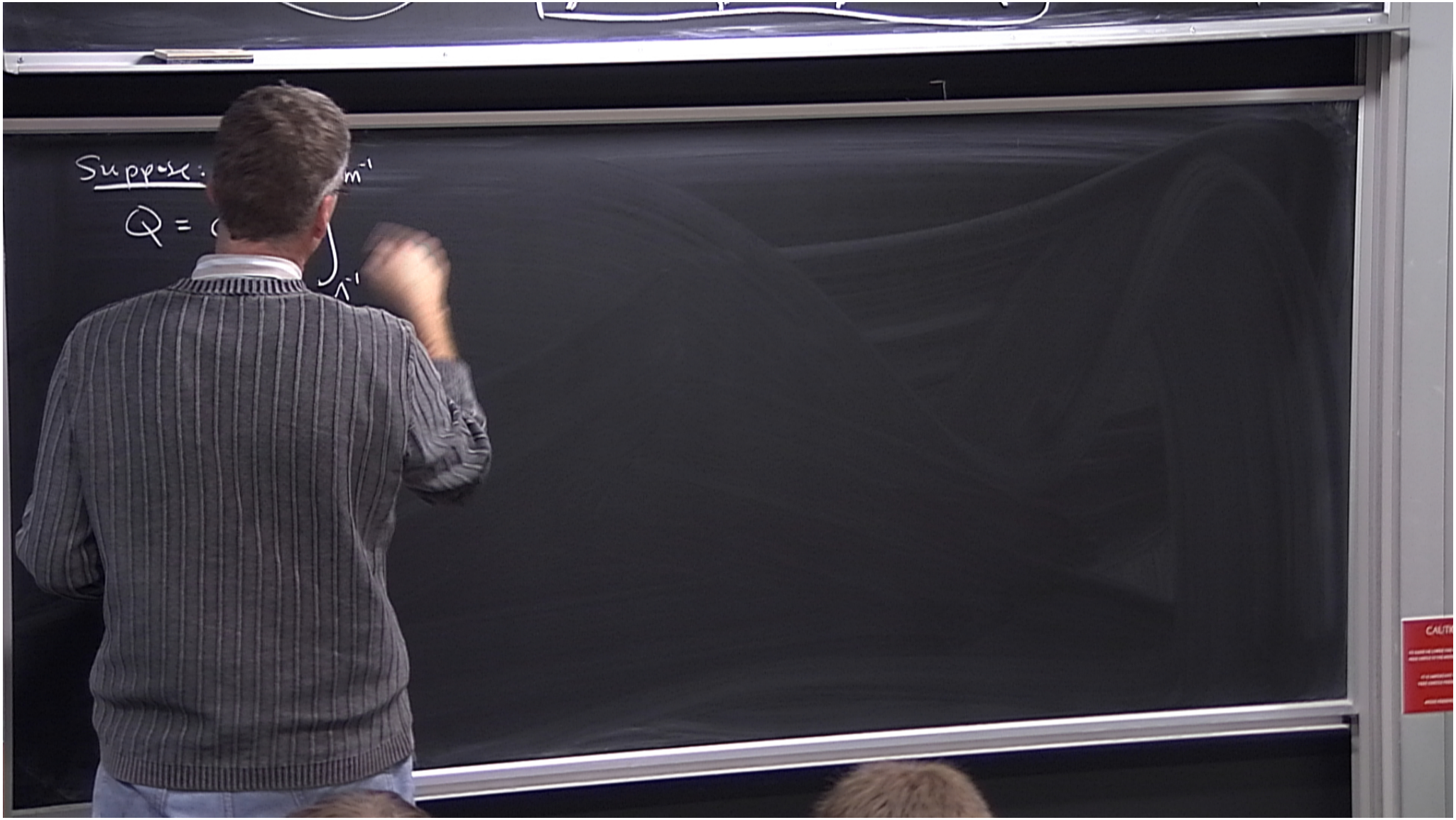
$$C = -\frac{e^2}{12\pi^2}\left[\frac{1}{2-d/2} - \gamma - \ln\left(\frac{m^2}{2\pi\mu^2}\right)\right] + O(d-4)$$

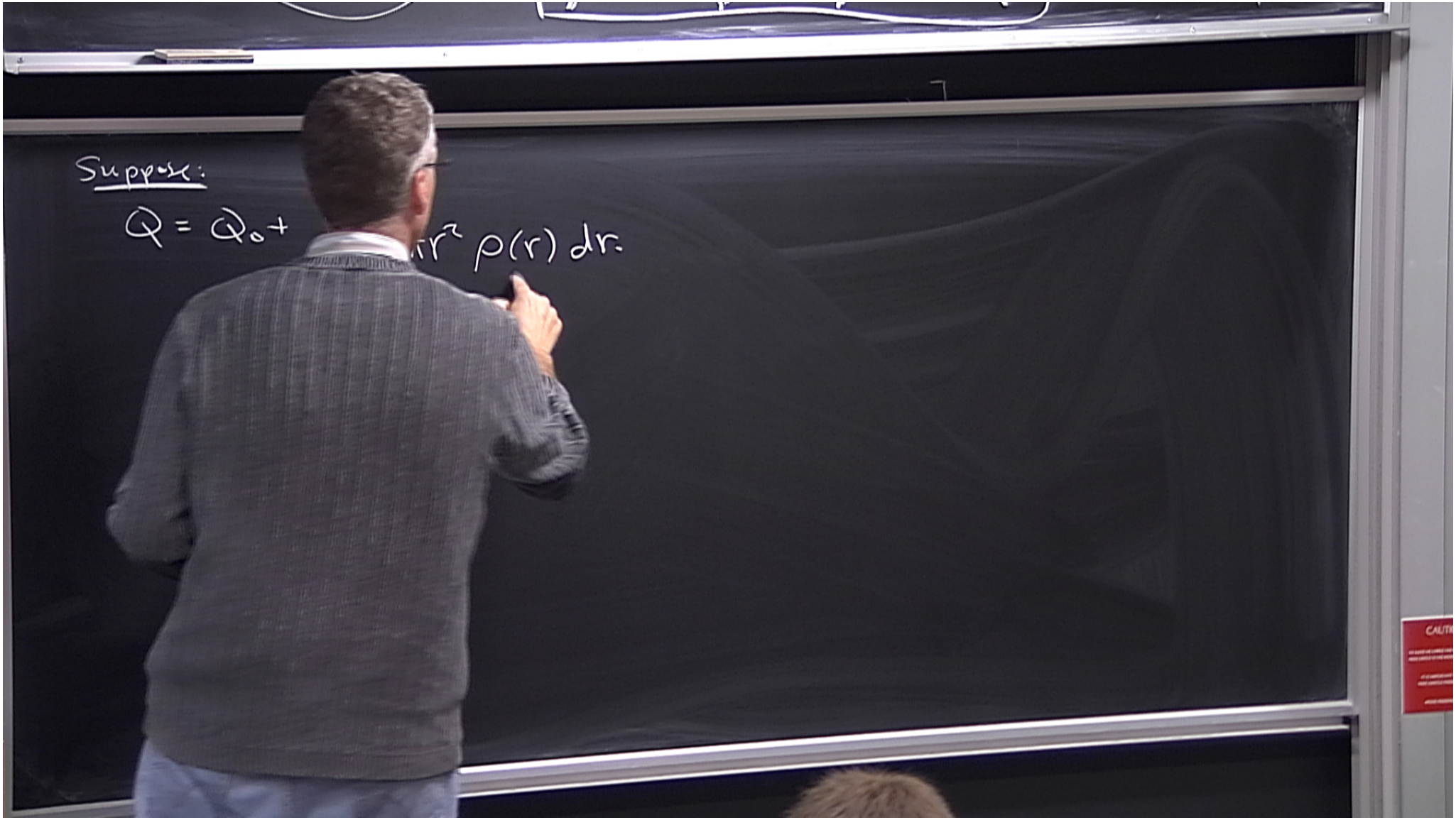
$$\Pi_f(q^2) = \frac{2e^2}{(2\pi)^2}\int_0^1 dx\, x(1-x)\ln\left[1 + \frac{q^2 x(1-x)}{m^2}\right]$$

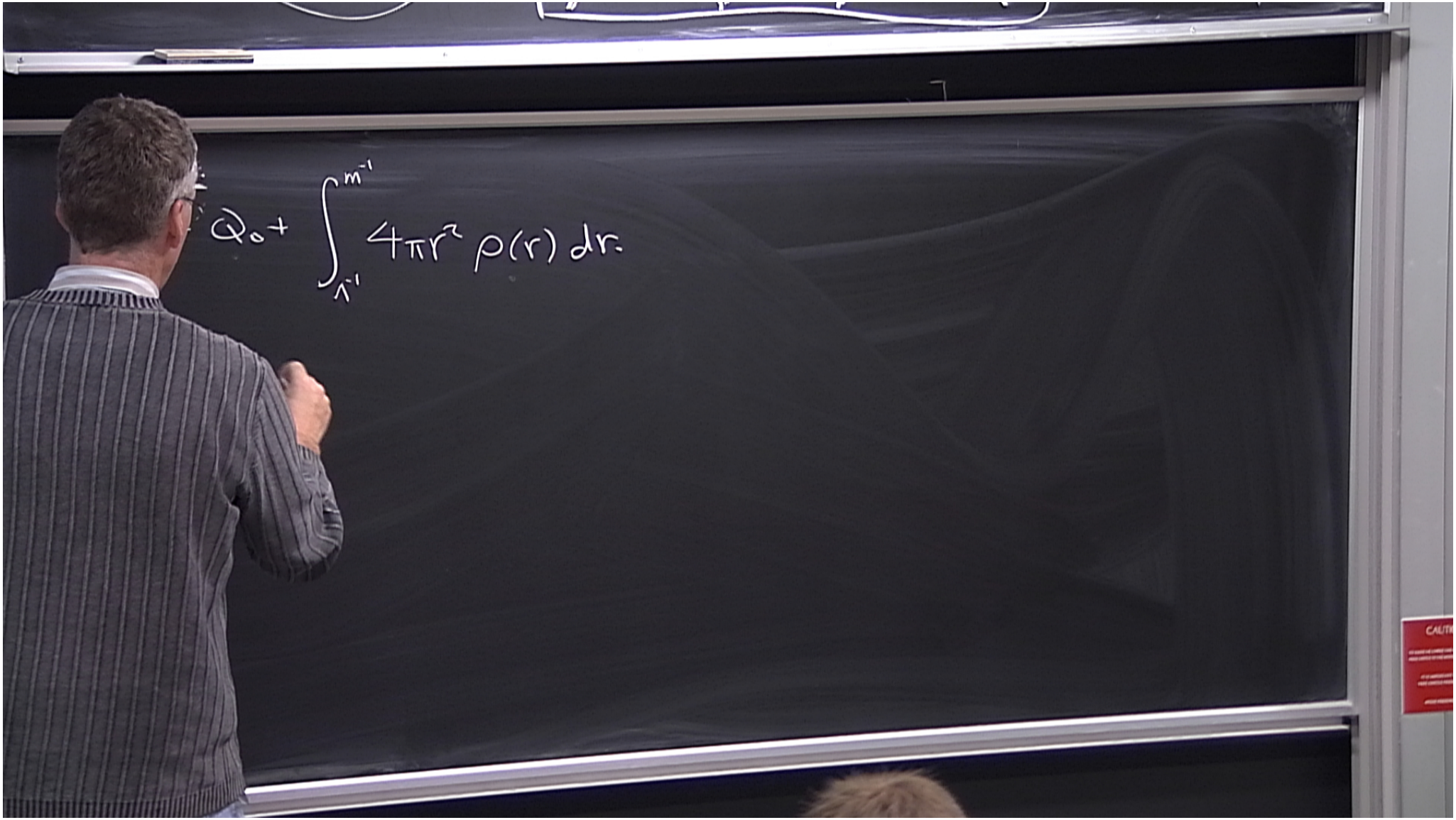


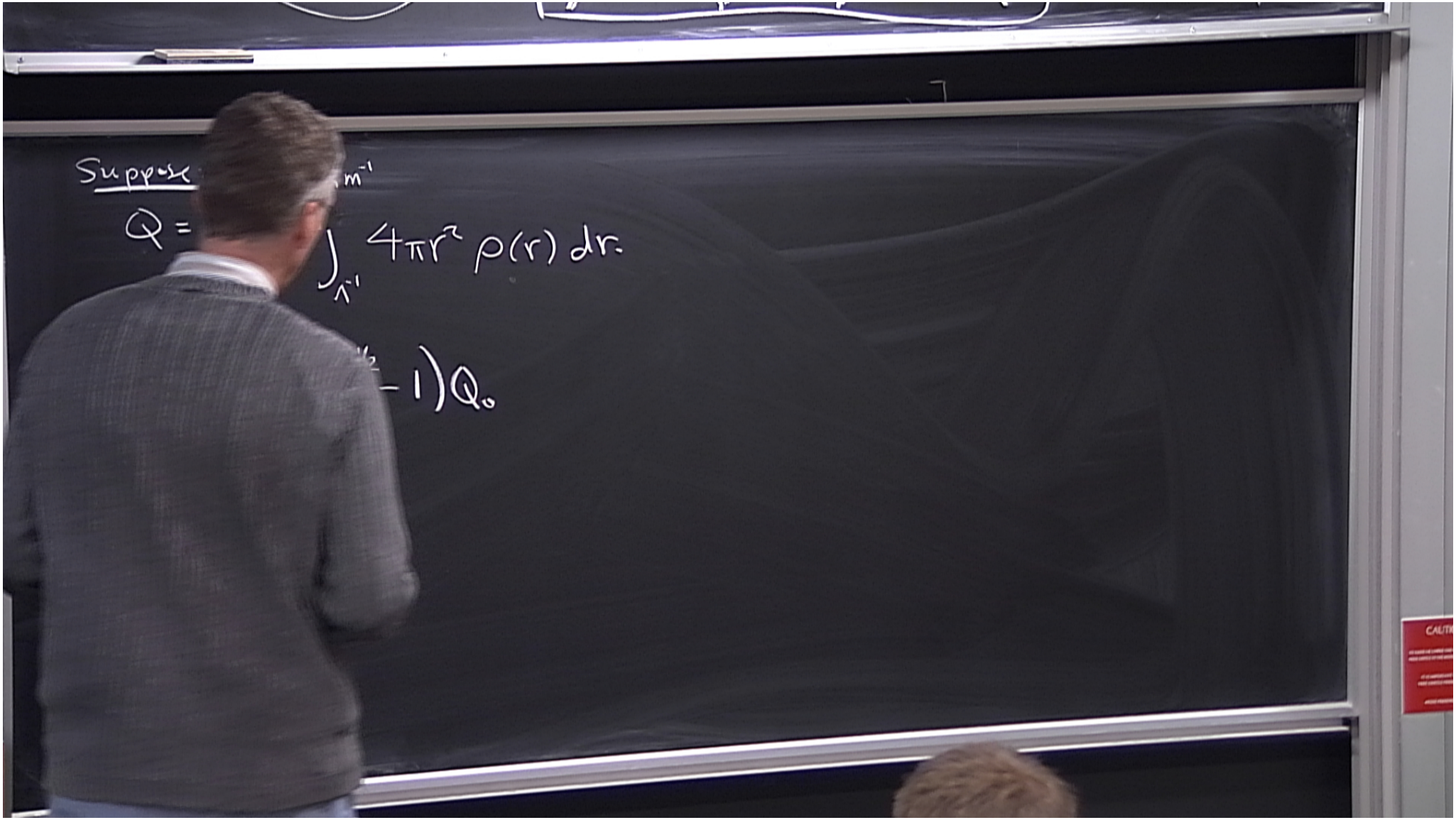


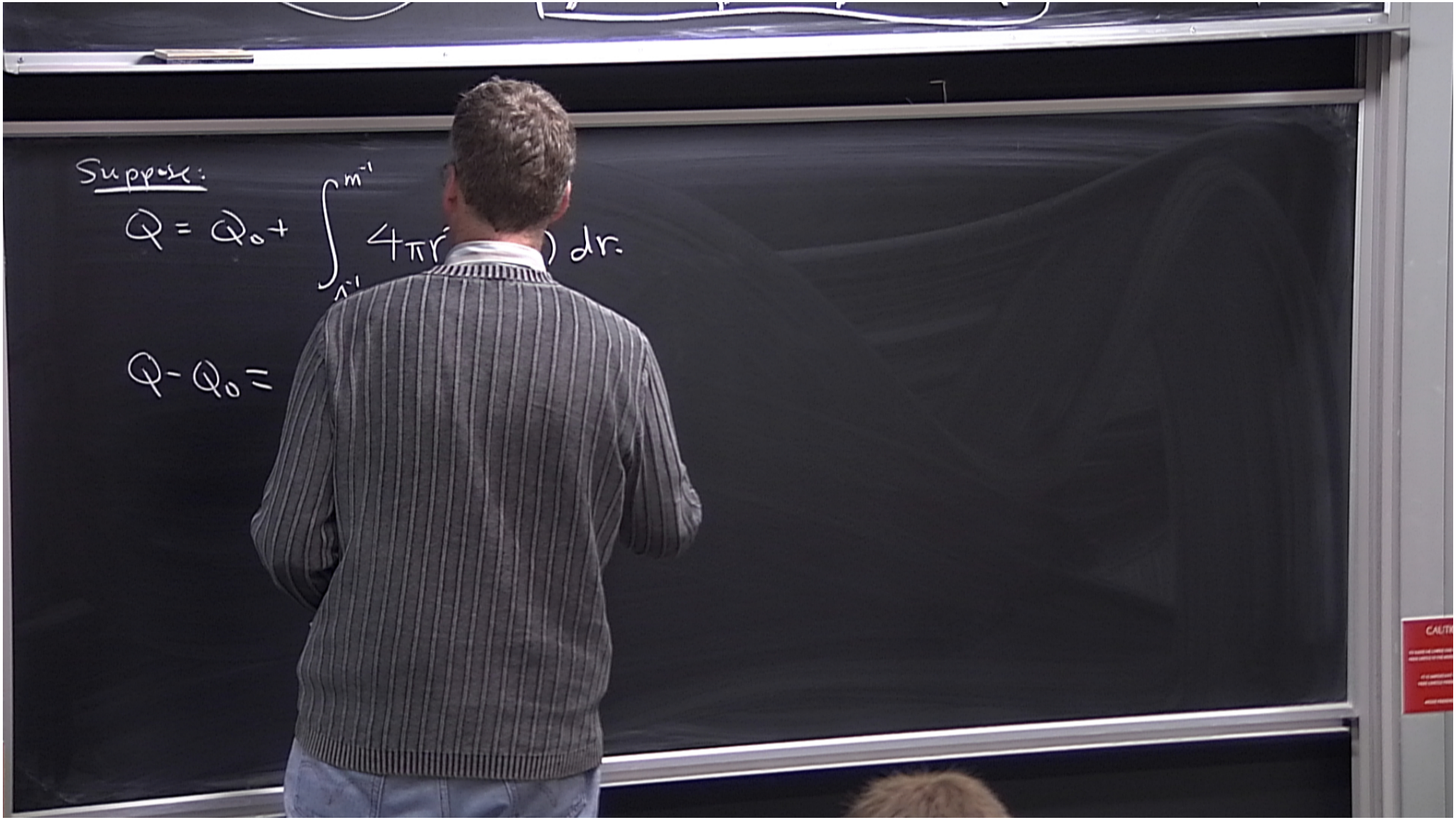


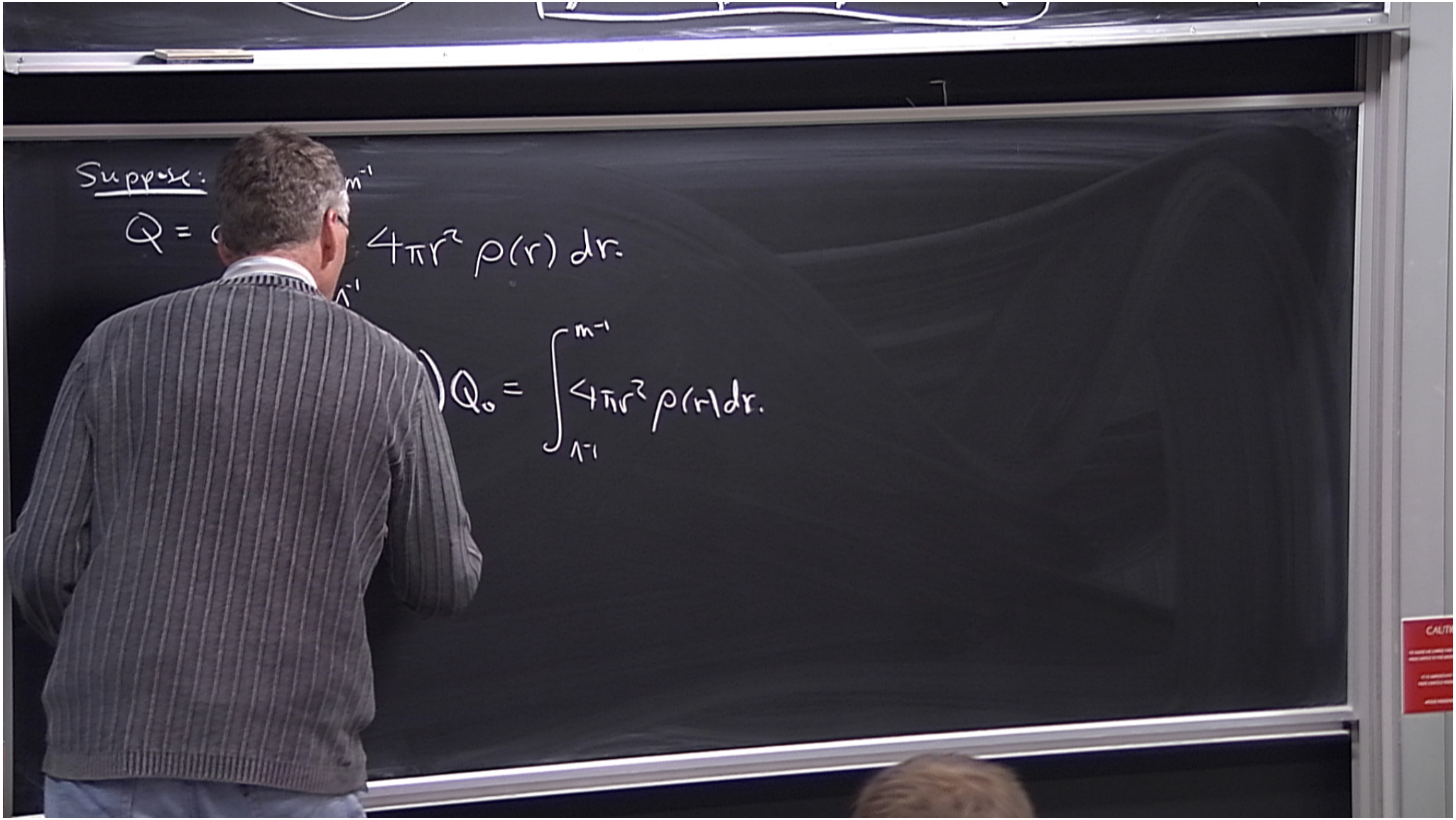

Suppose
m^{-1}
$Q = ($
$\int_{\lambda^{-1}}$

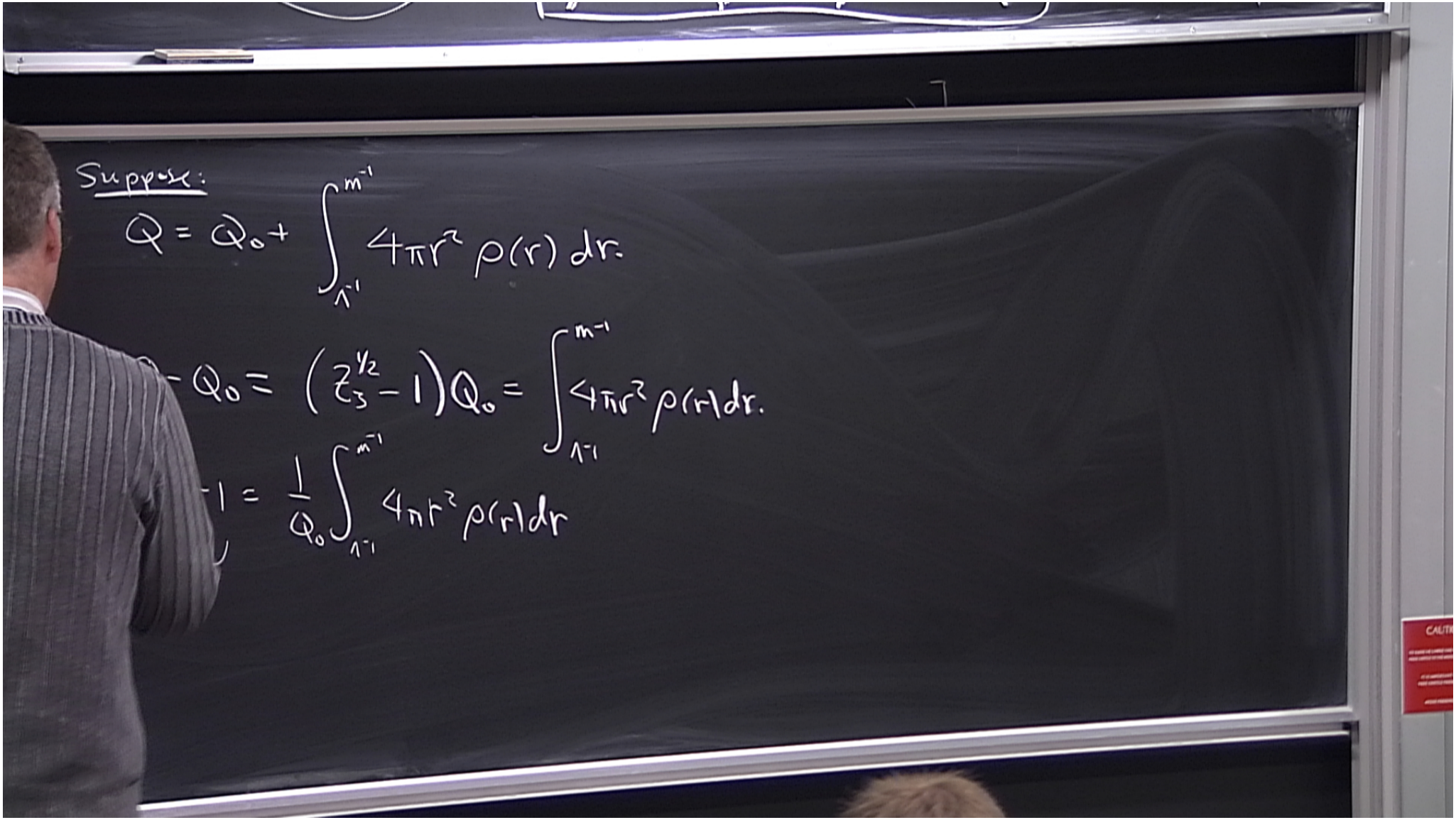

Suppose:
Q = Q₀ +
r² ρ(r) dr.

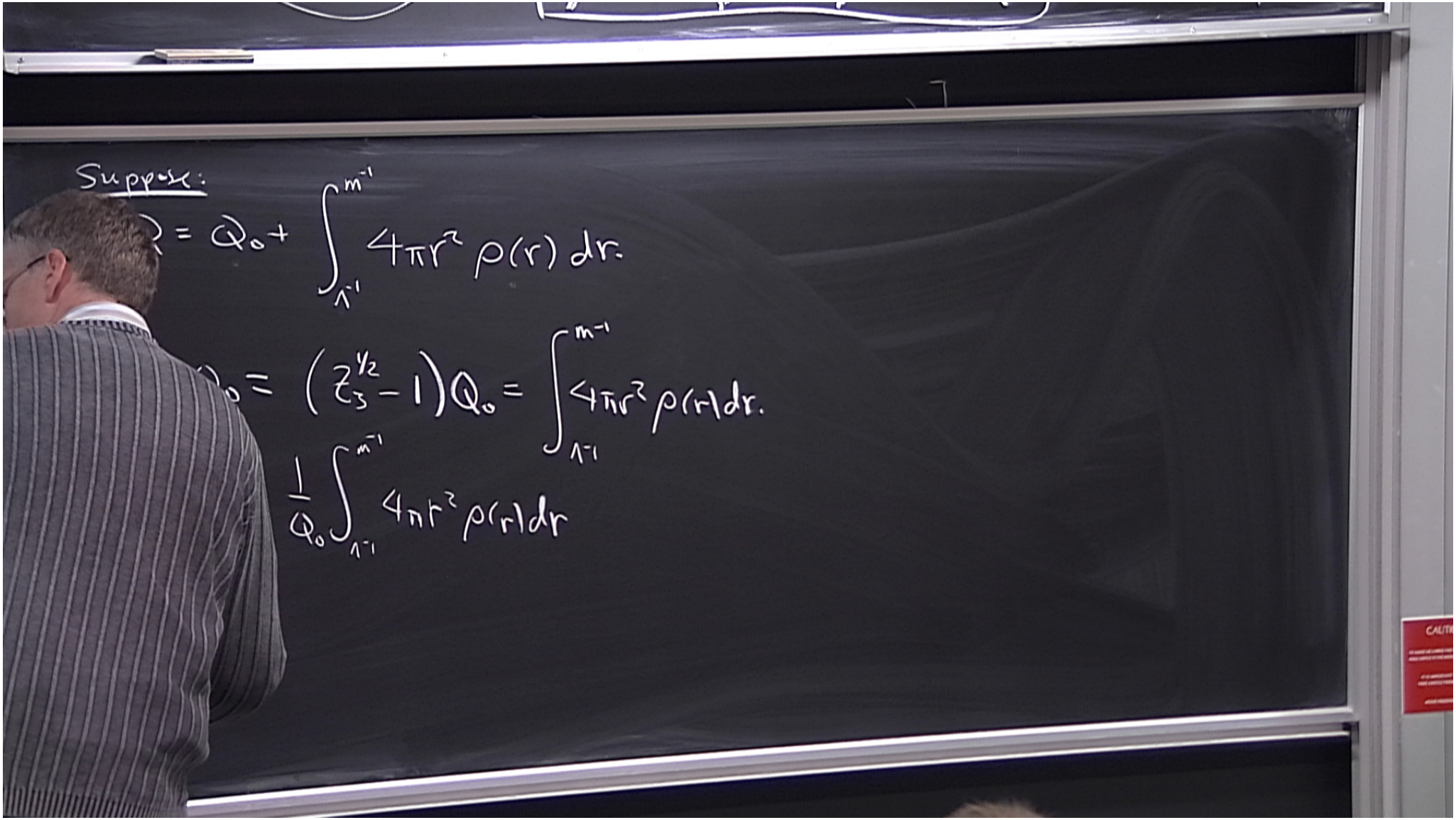

$Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$

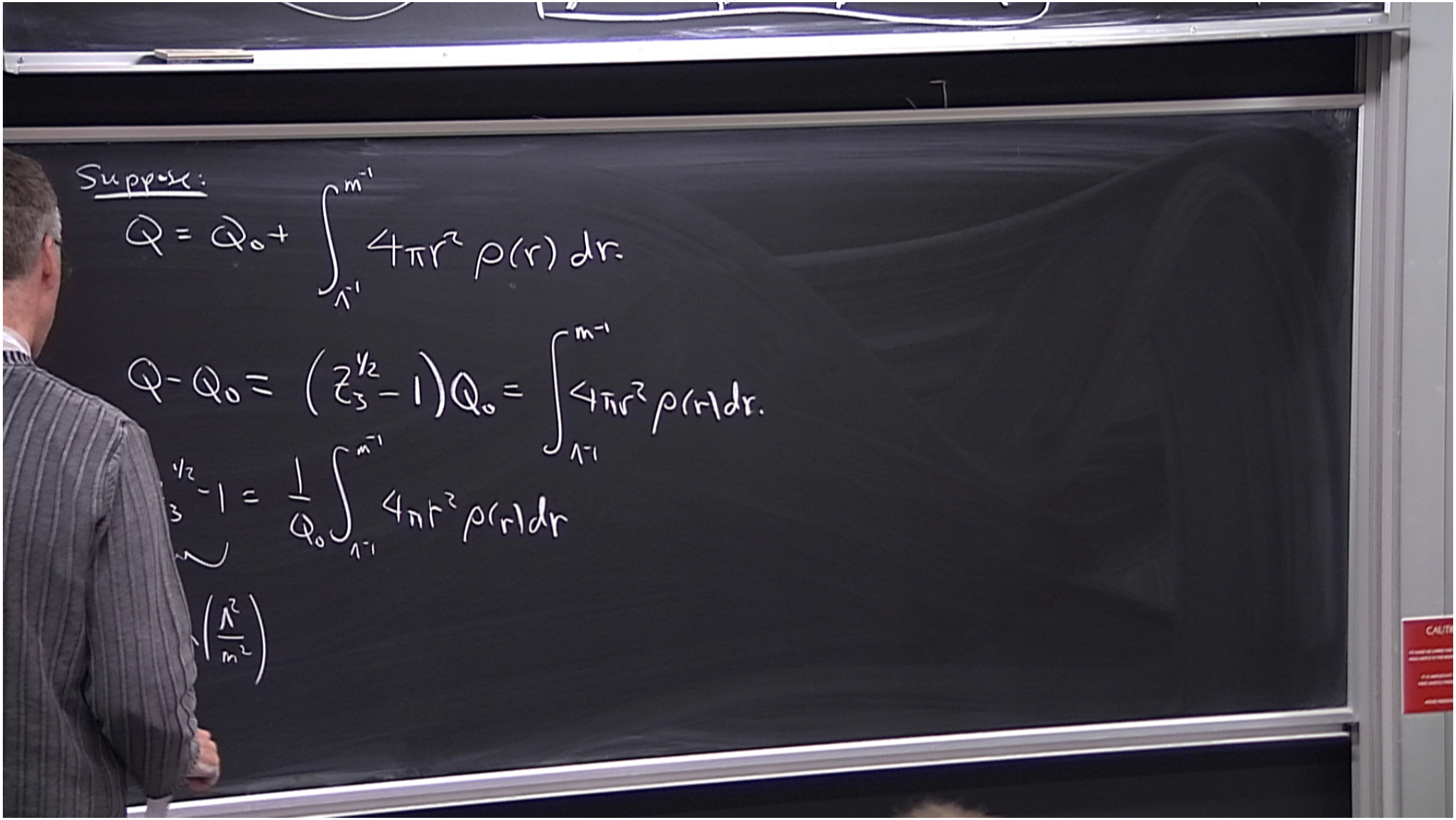

Suppose
m^{-1}
$Q = \int_{\lambda^{-1}} 4\pi r^2 \rho(r)\, dr.$
$-1)Q_0.$

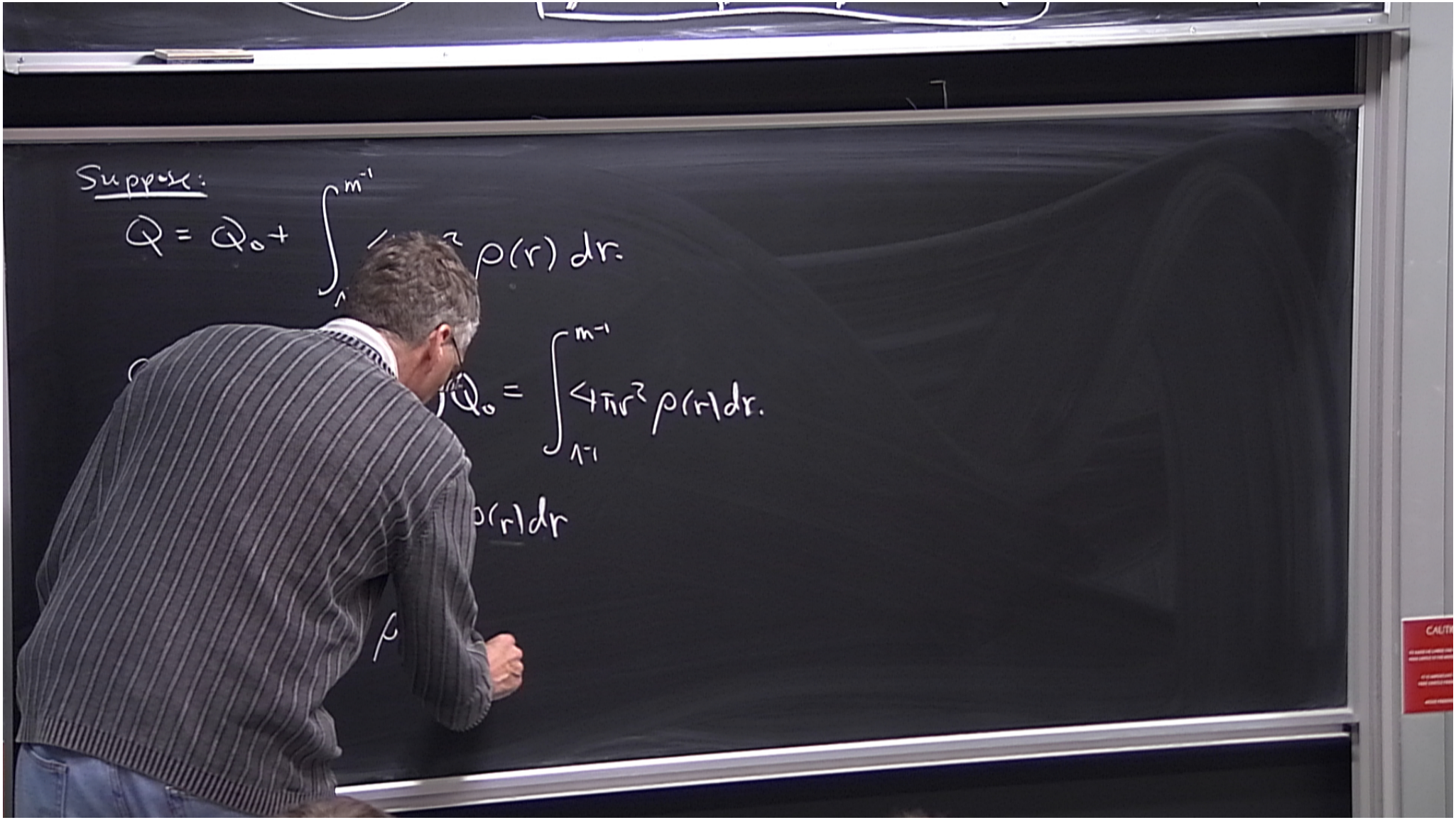

Suppose:
$Q = Q_0 + \int_{r-1}^{m-1} 4\pi r' \, dr.$
$Q - Q_0 =$

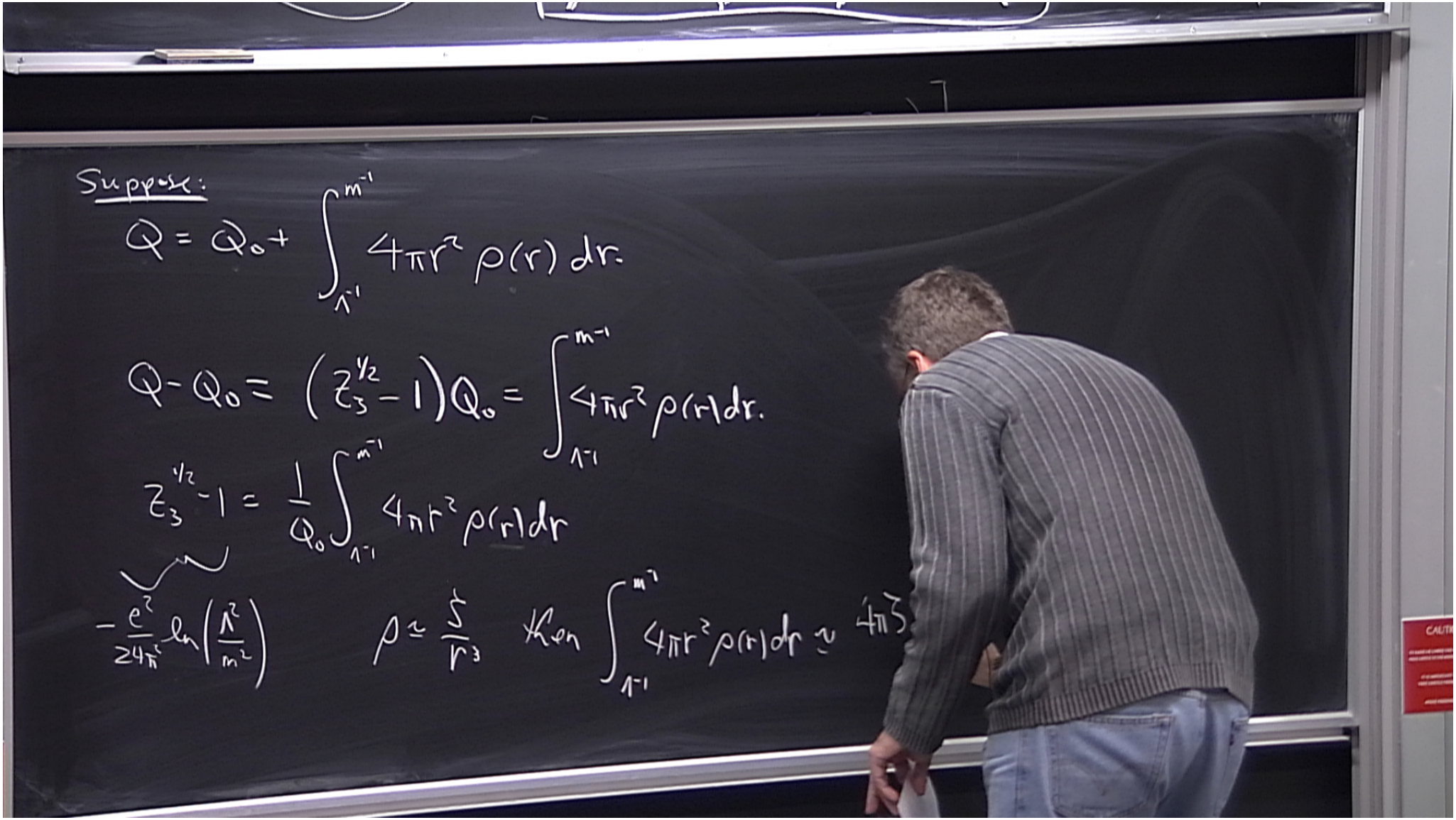

Suppose:
$4\pi r^2 \rho(r)\, dr.$
$Q_0 = \int_{1^{-1}}^{m-1} 4\pi r^2 \rho(r) dr.$

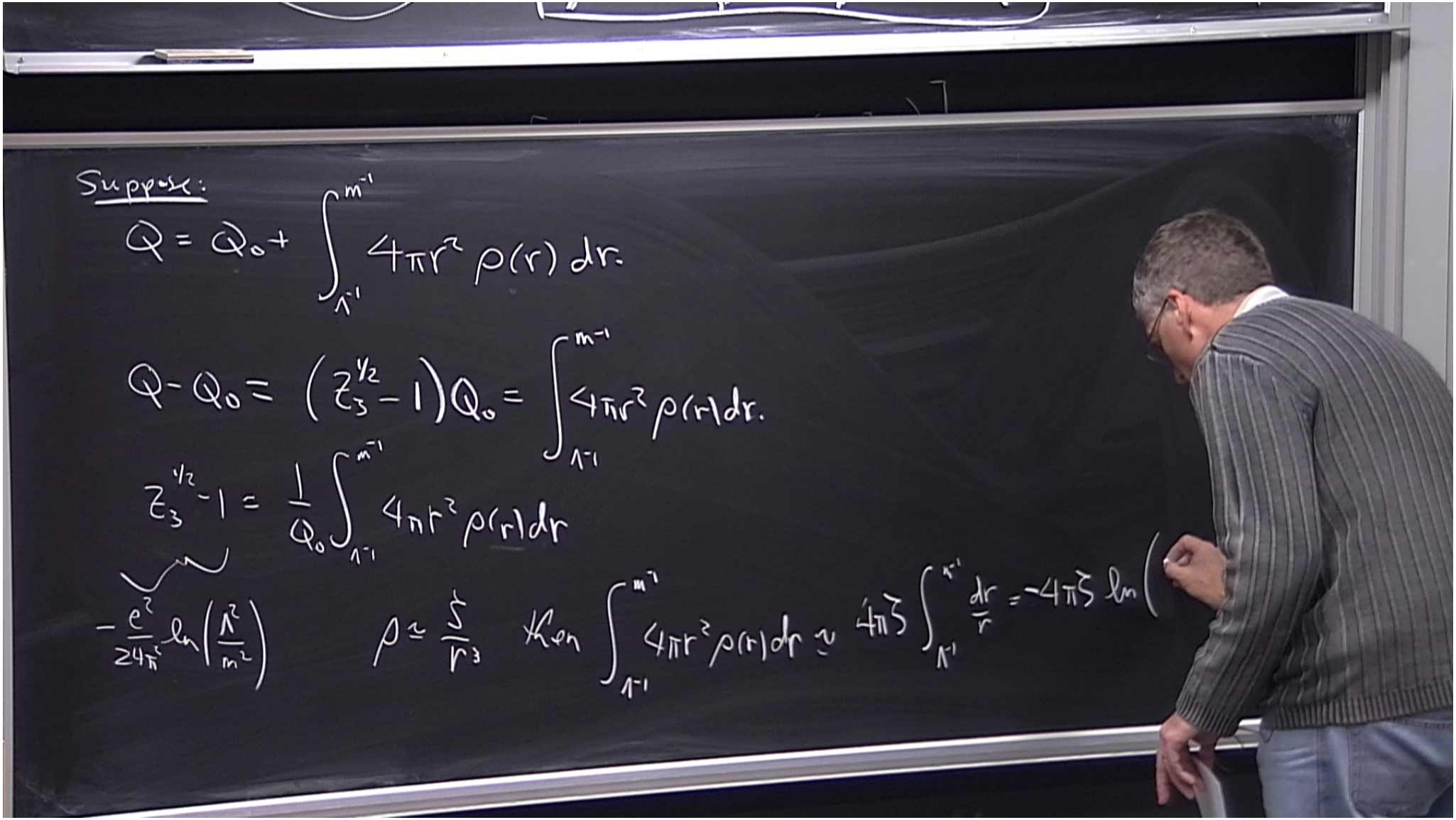

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = (Z_3^{1/2} - 1)Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$= \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$


Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$\frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$


Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = \left(Z_3^{1/2} - 1\right) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$\frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$


Suppose:
$Q = Q_0 + \int_{\Lambda}^{m^{-1}} \rho(r)\, dr.$
$Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$\rho(r) dr$


Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = \left(Z_3^{1/2} - 1\right) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{5}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi 5$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$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
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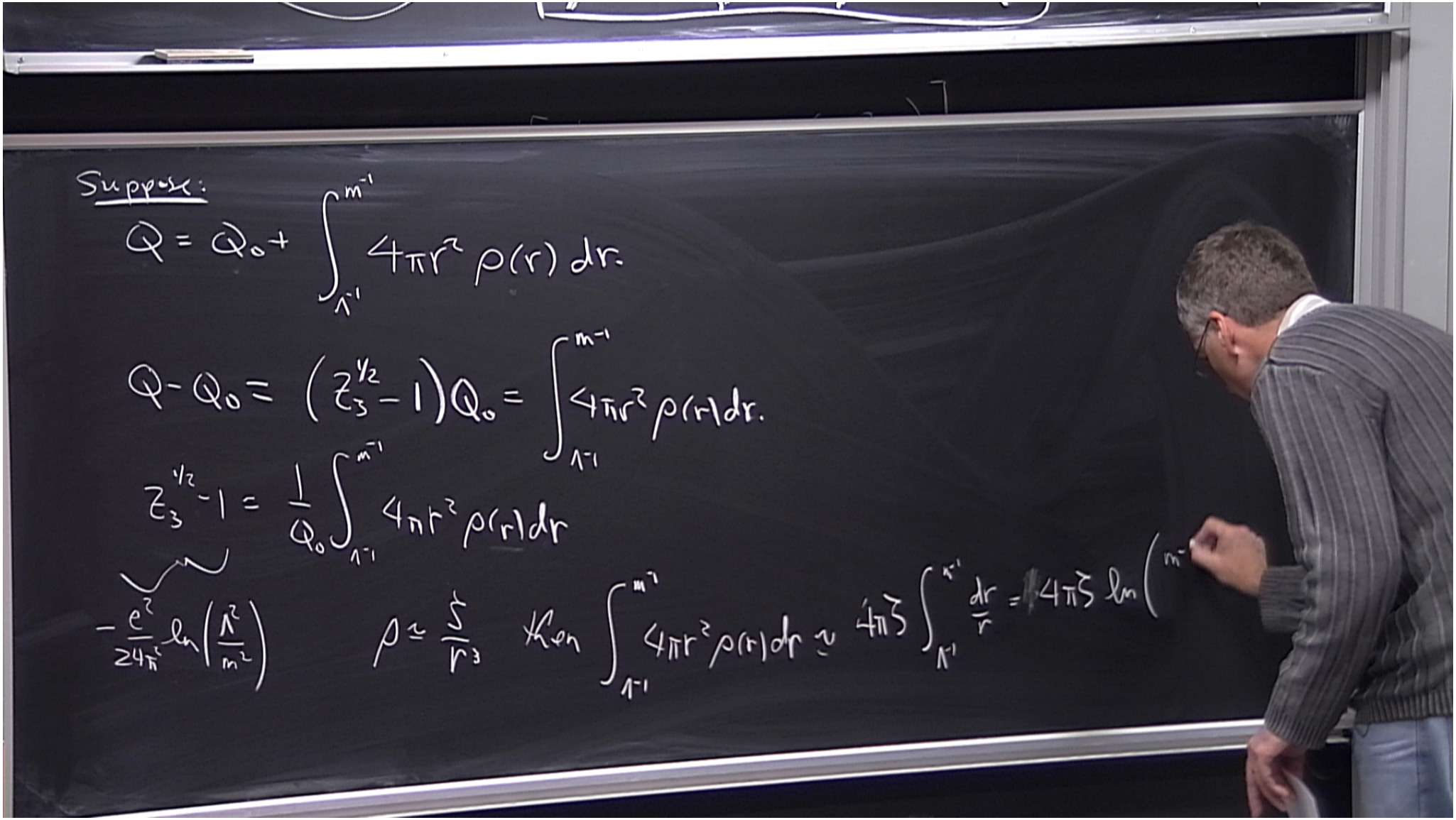
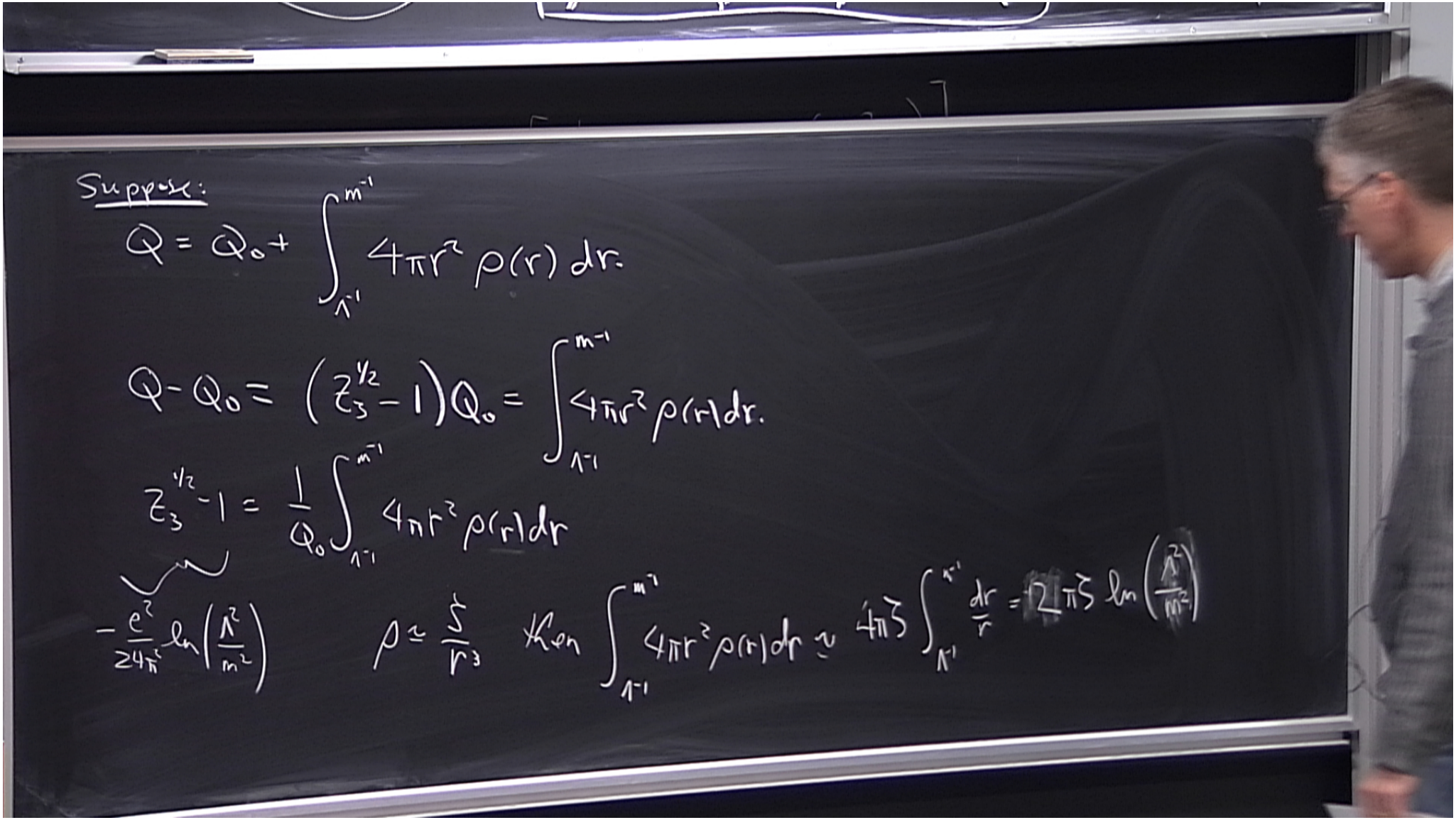
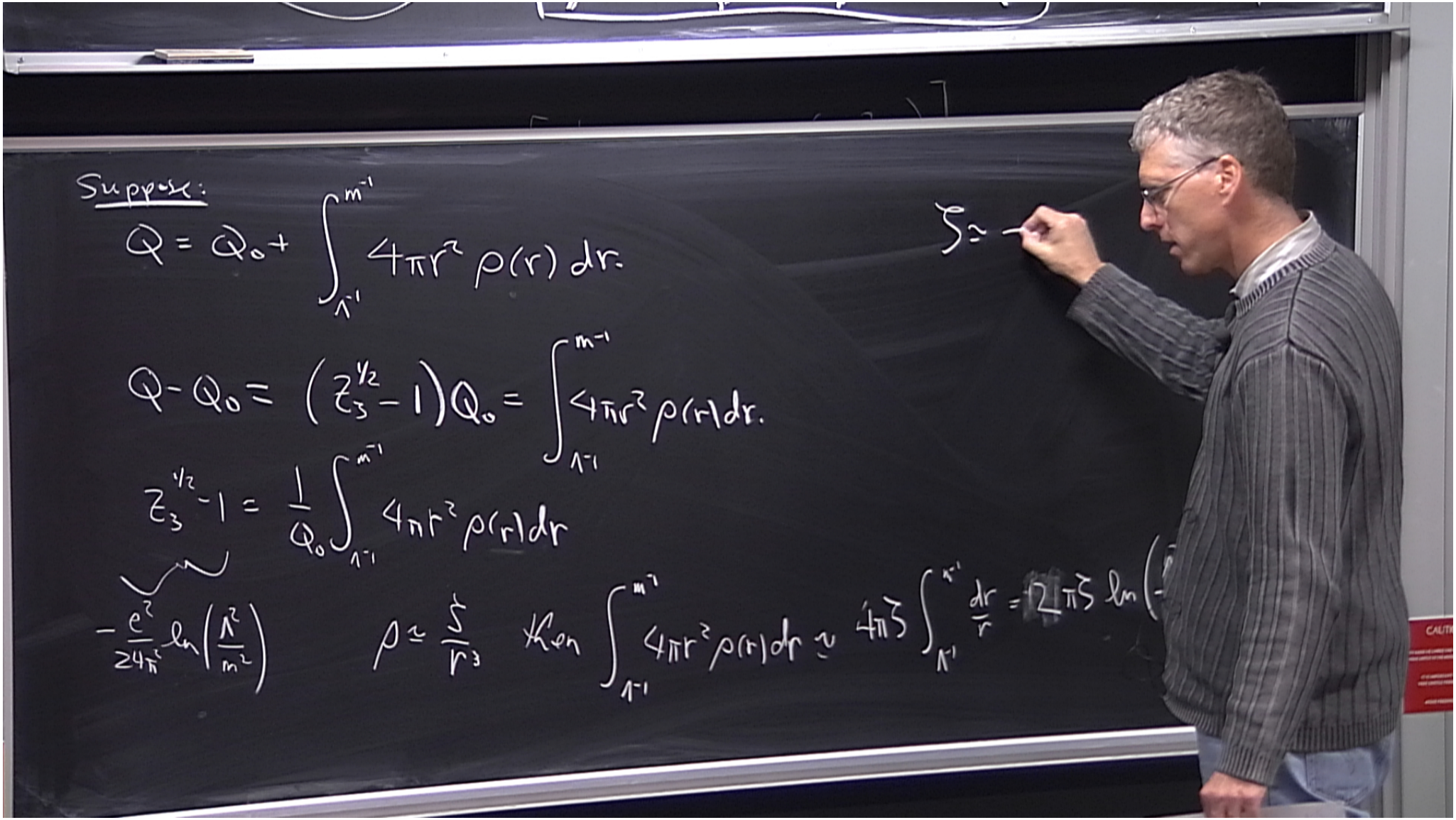
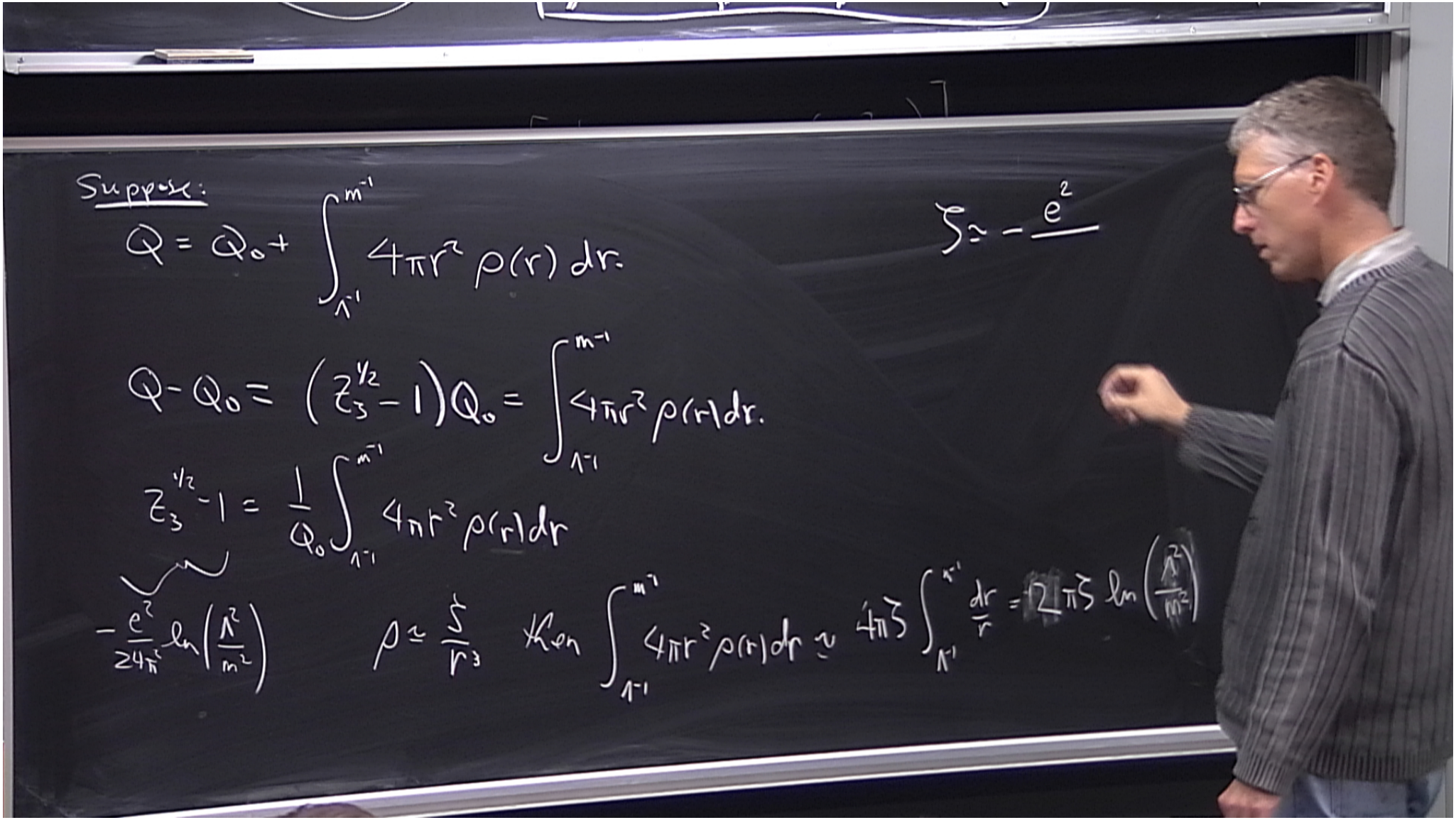
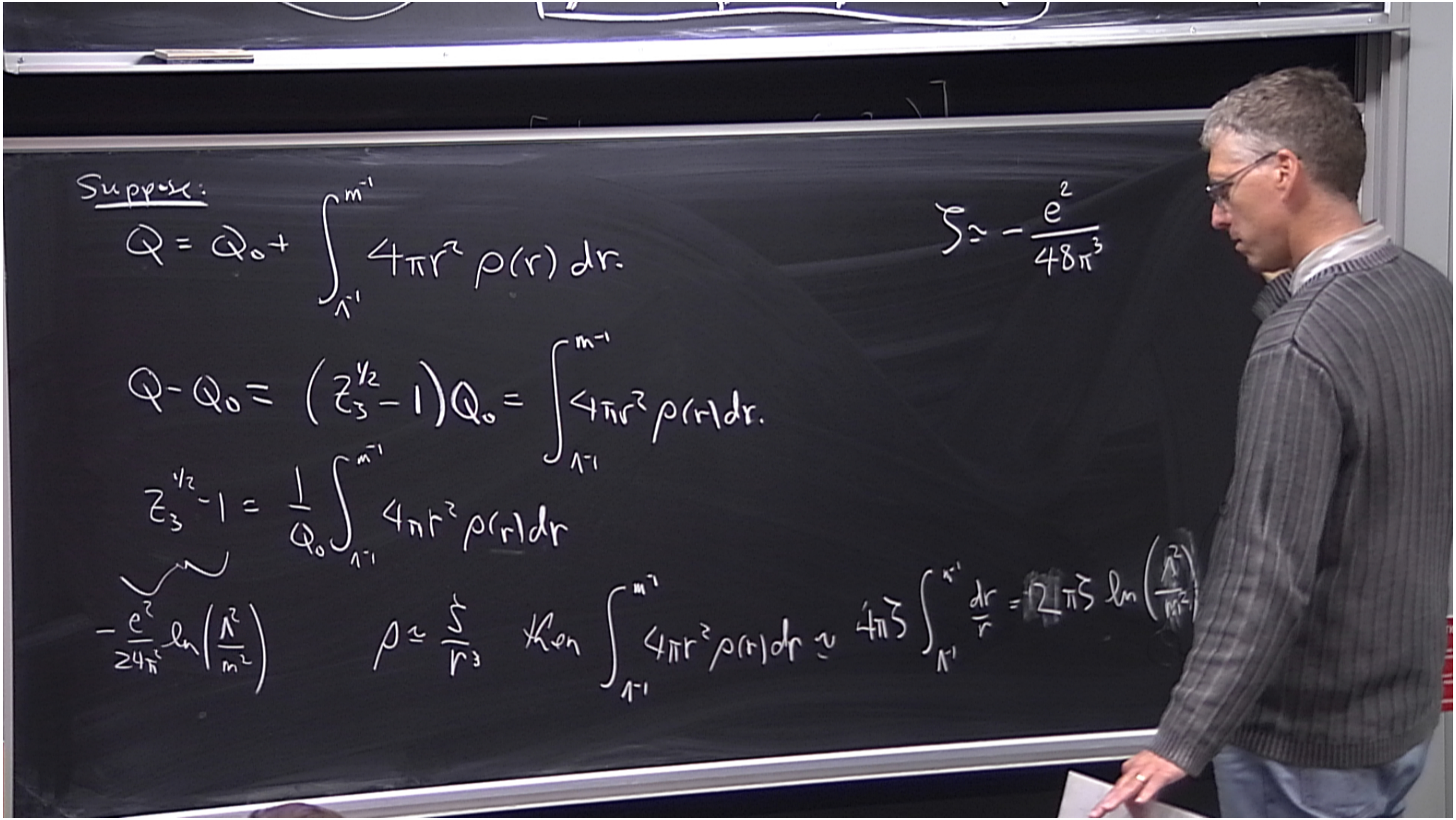
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{\zeta}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi\zeta \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = -4\pi\zeta \ln($

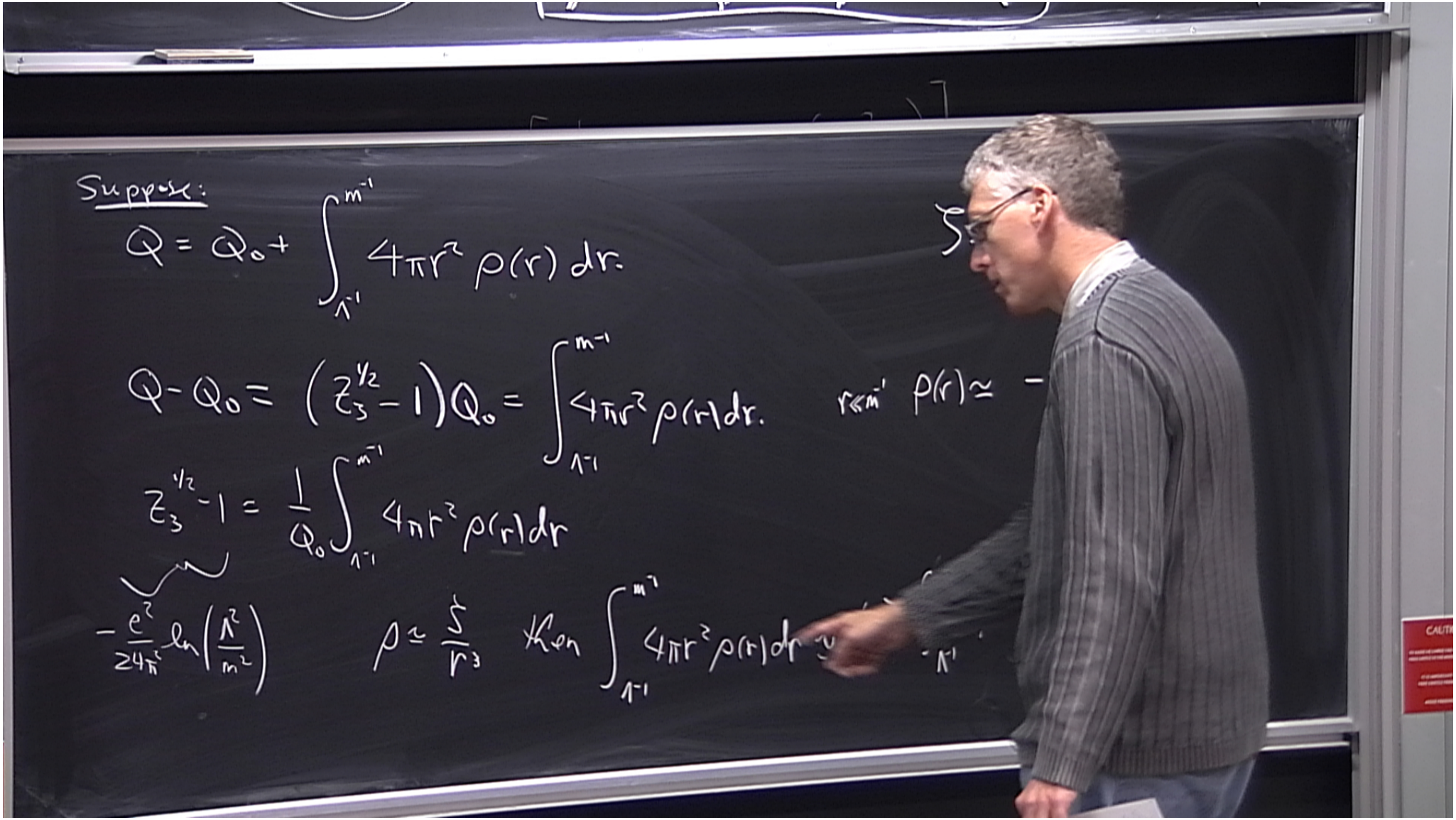

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{5}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi 5 \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = 4\pi 5 \ln\left(m^{-1}\right.$

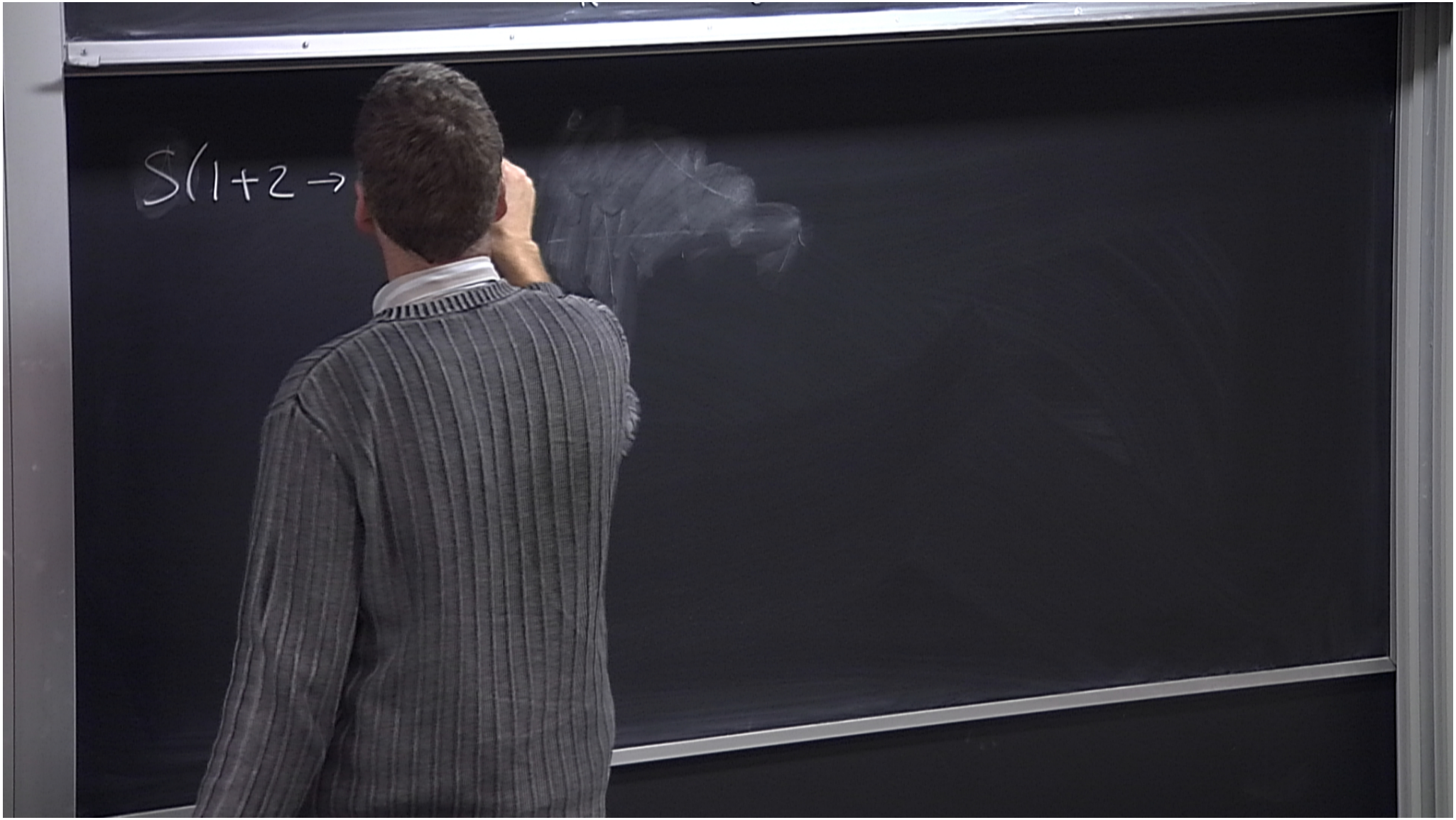

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{5}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi 5 \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = 12\pi 5 \ln\left(\frac{\Lambda^2}{m^2}\right)$

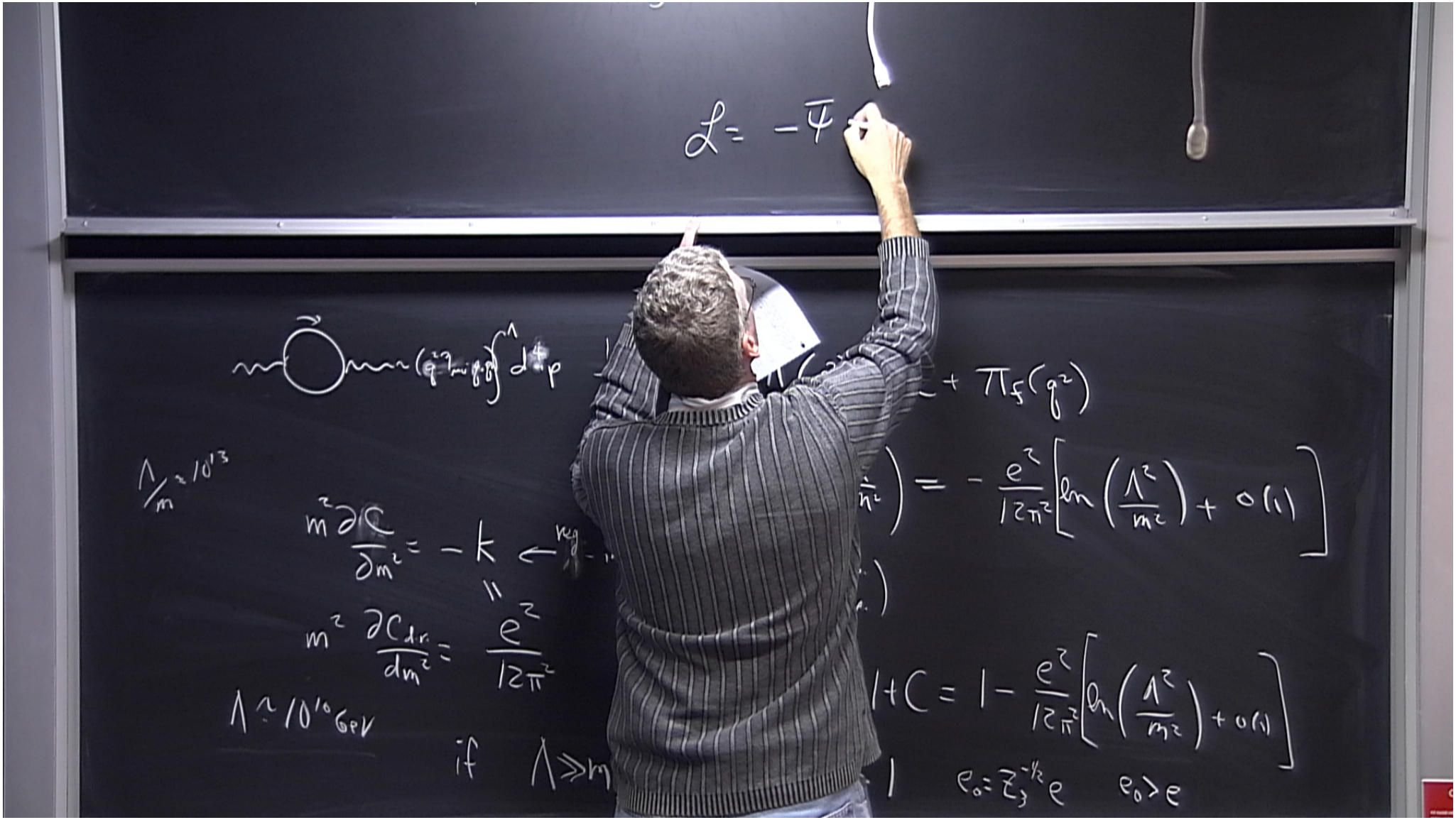

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{\zeta}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi\zeta \int_{\Lambda^{-1}}^{\kappa^{-1}} \frac{dr}{r} = 12\pi\zeta \ln\left(\frac{}{}\right.$

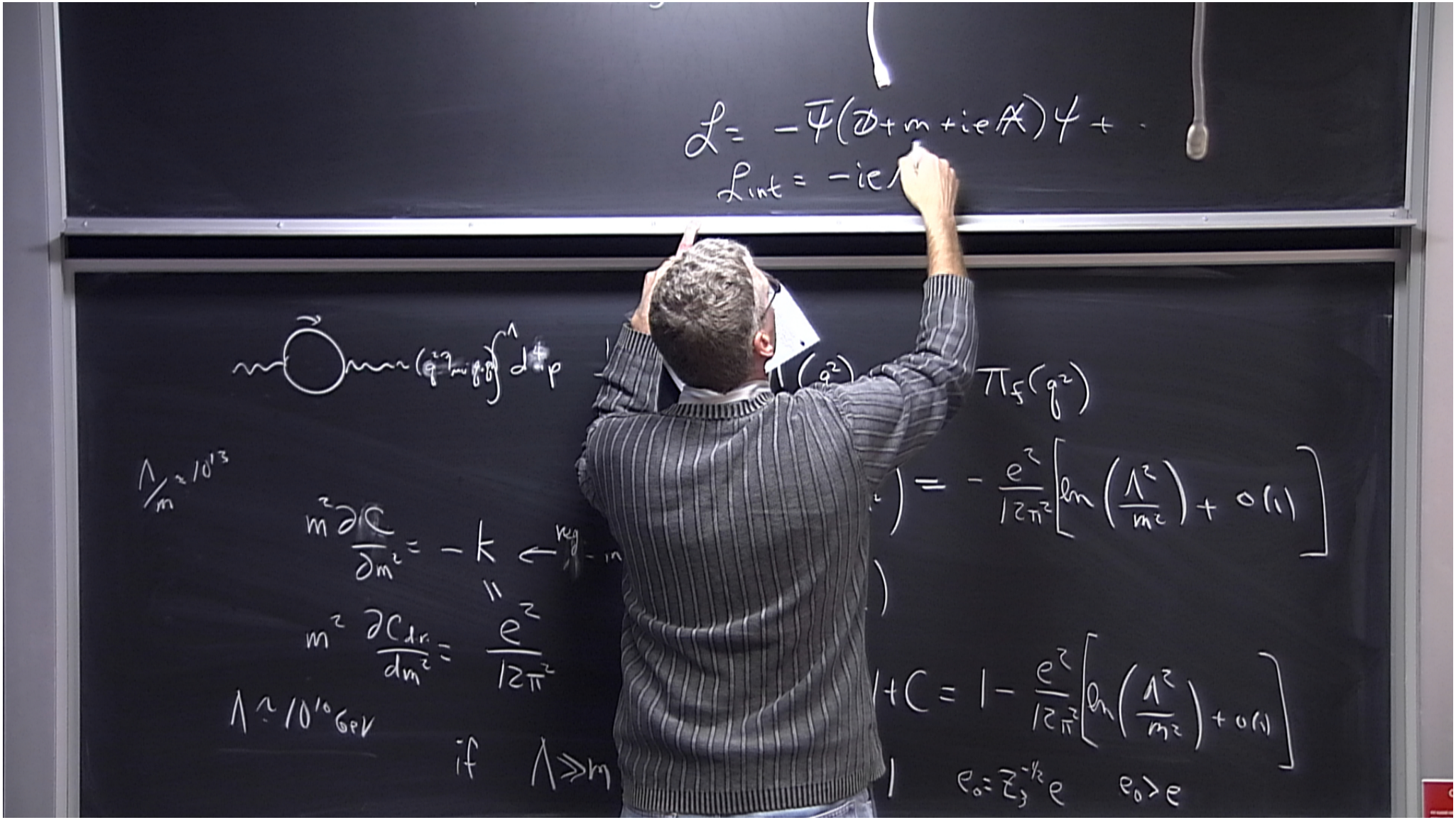

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$\zeta \simeq -\frac{e^2}{}$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{\zeta}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi\zeta \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = 12\pi\zeta \ln\left(\frac{\Lambda^2}{m^2}\right)$

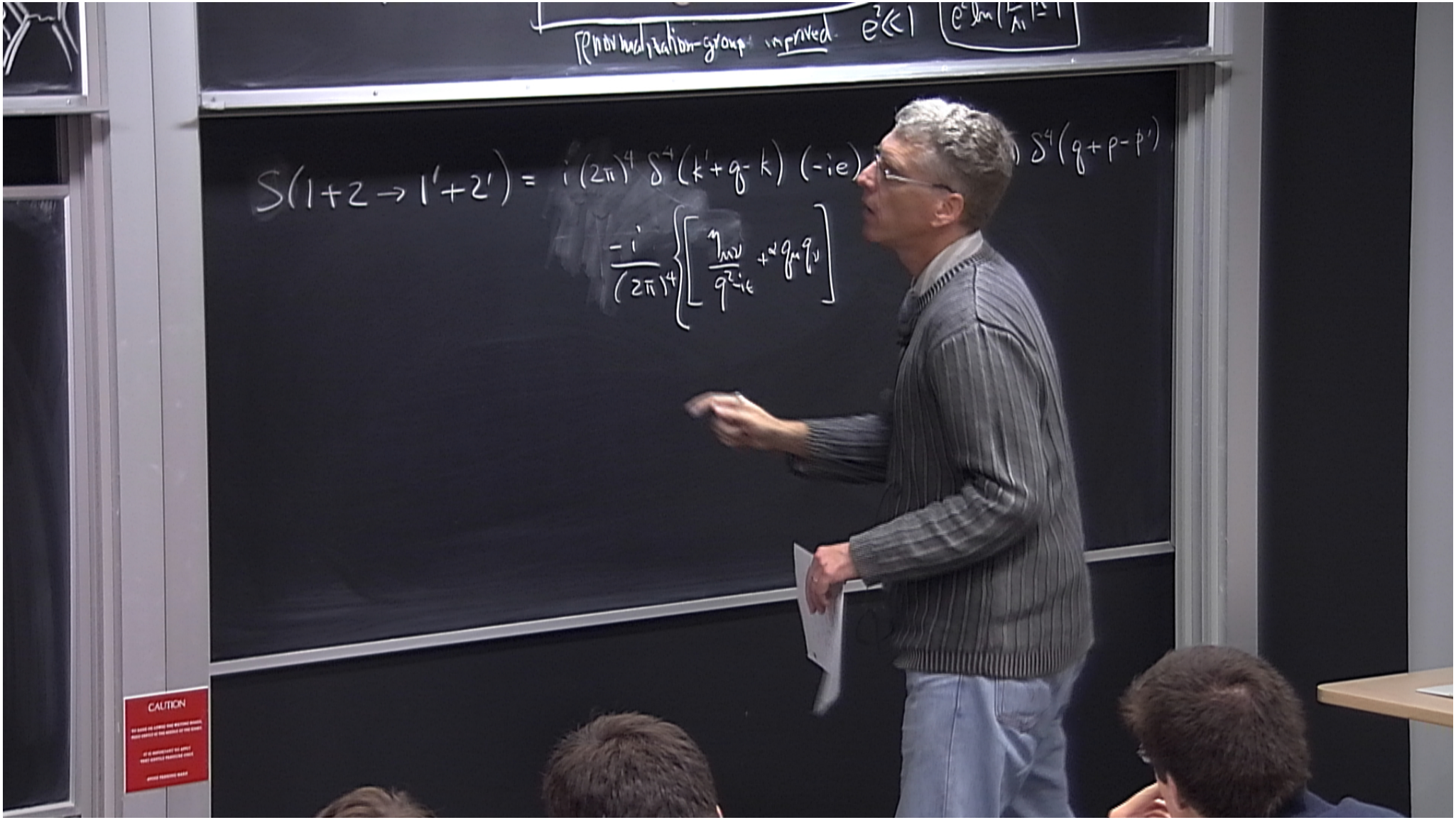

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$\zeta \simeq -\frac{e^2}{48\pi^3}$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{\zeta}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi\zeta \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = 12\pi\zeta \ln\left(\frac{\Lambda^2}{m^2}\right)$

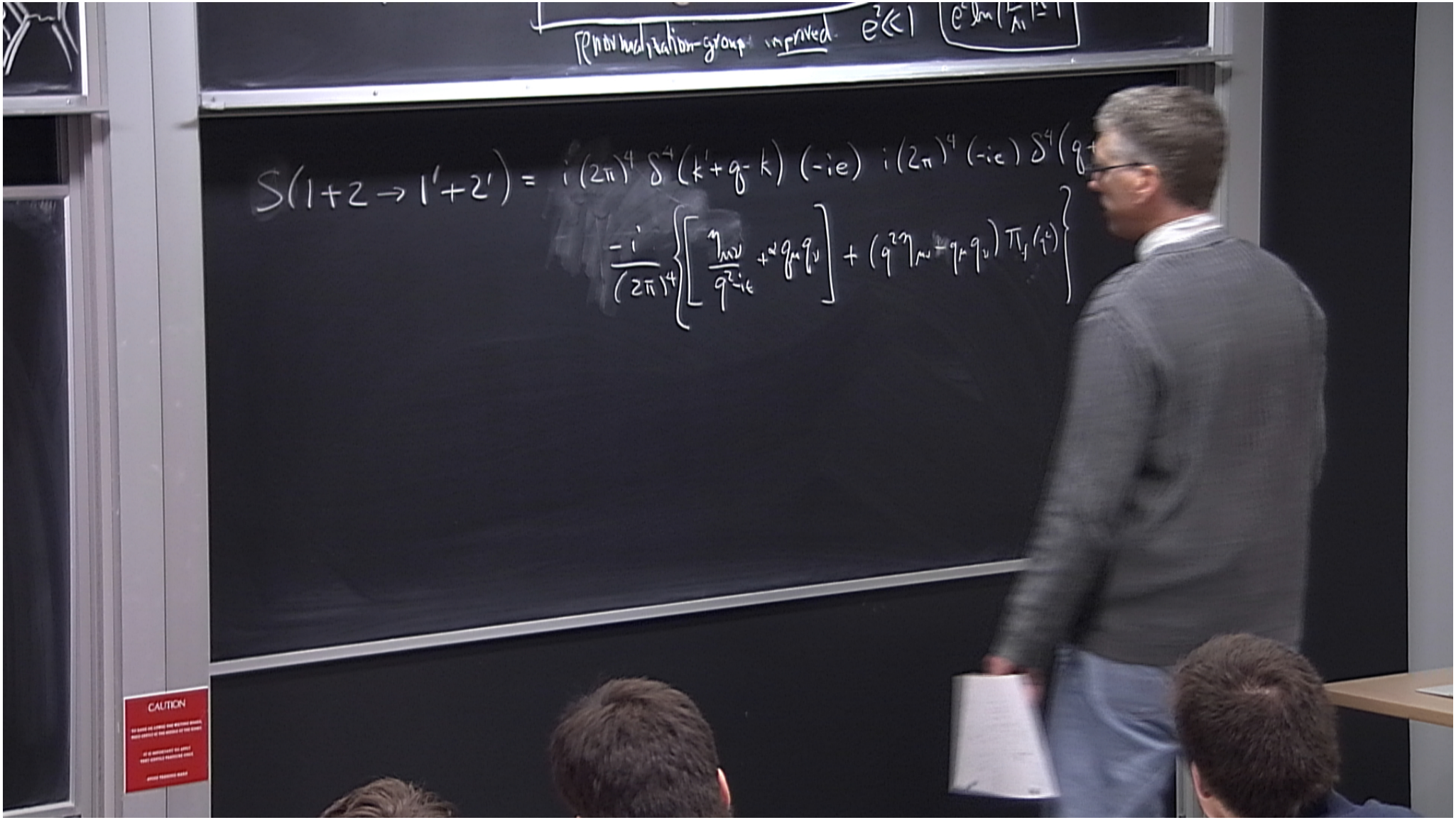

Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$r \ll m^{-1}$ $\rho(r) \simeq$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{5}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$

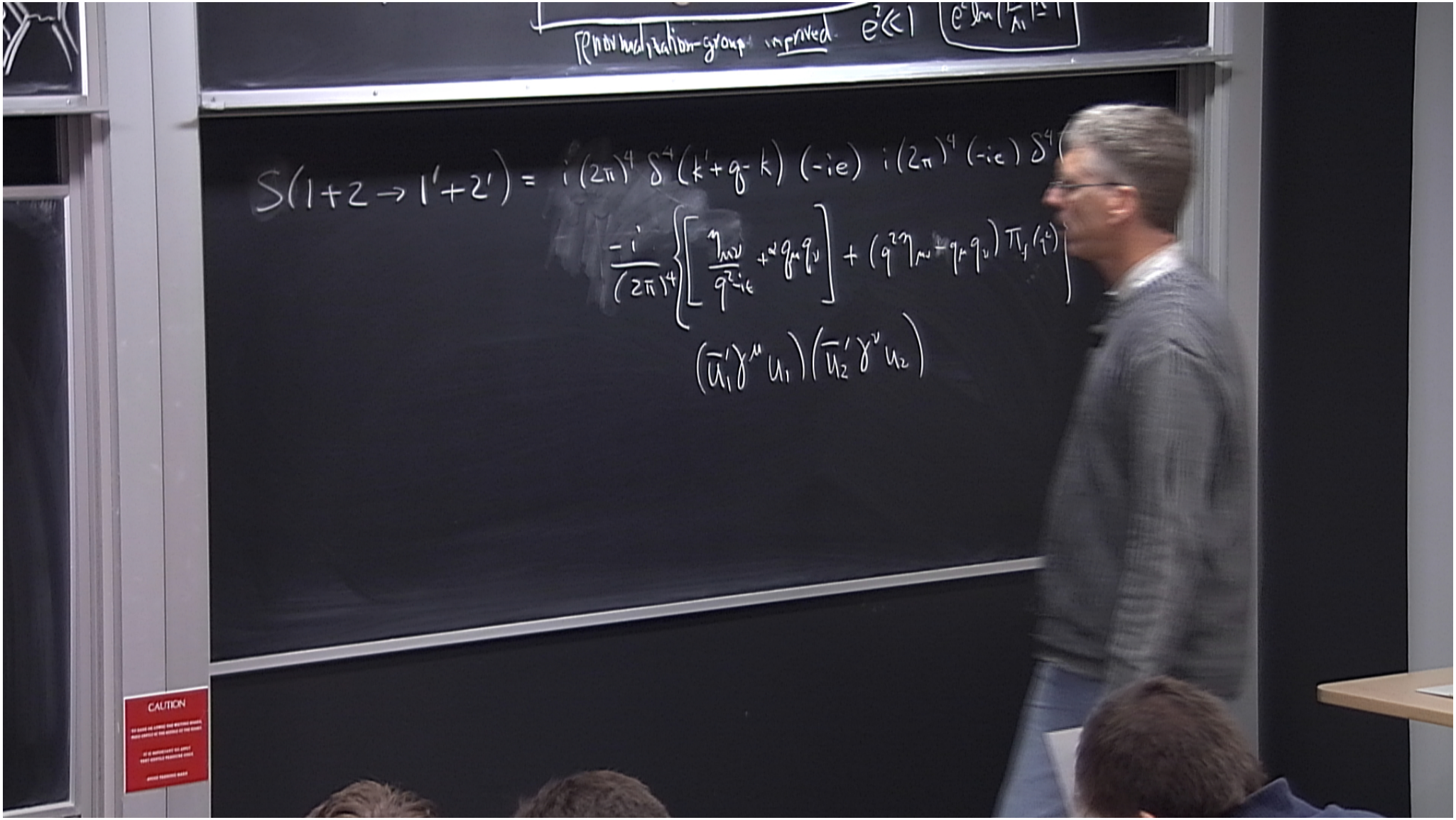

S(1+2→

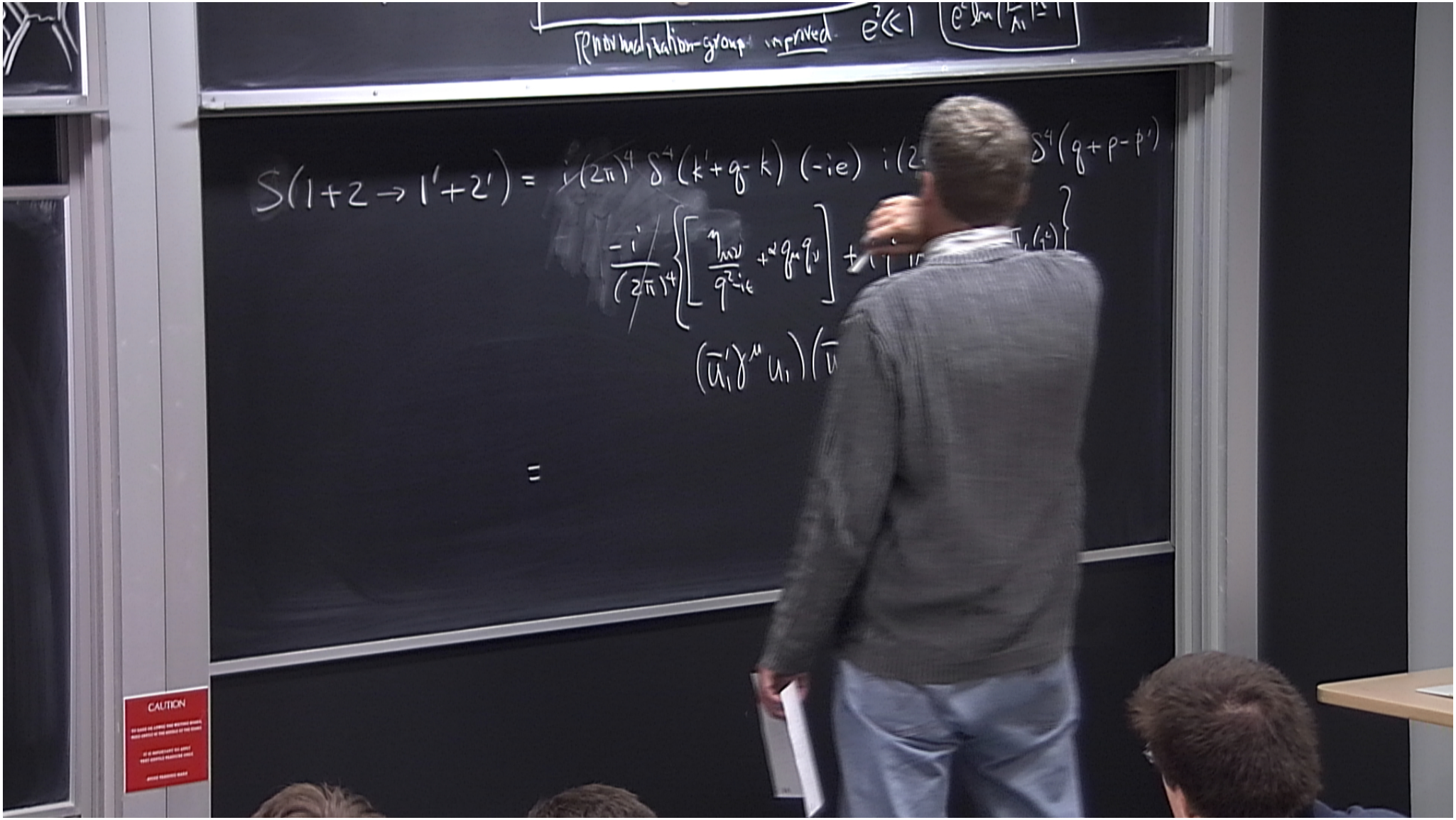

$\mathcal{L} = -\bar{\psi}$
$+ \Pi_f(q^2)$
$\frac{\Lambda}{m} \approx 10^{13}$
$m^2 \frac{\partial C}{\partial m^2} = -k \leftarrow$
$= \frac{e^2}{12\pi^2}$
$m^2 \frac{\partial C_{div}}{\partial m^2} = \frac{e^2}{12\pi^2}$
$\Lambda \simeq 10^{10}$ GeV
if $\Lambda \gg m$
$1 + C = 1 - \frac{e^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) + O(1)\right]$
$e_0 = Z_3^{-1/2} e$
$e_0 > e$

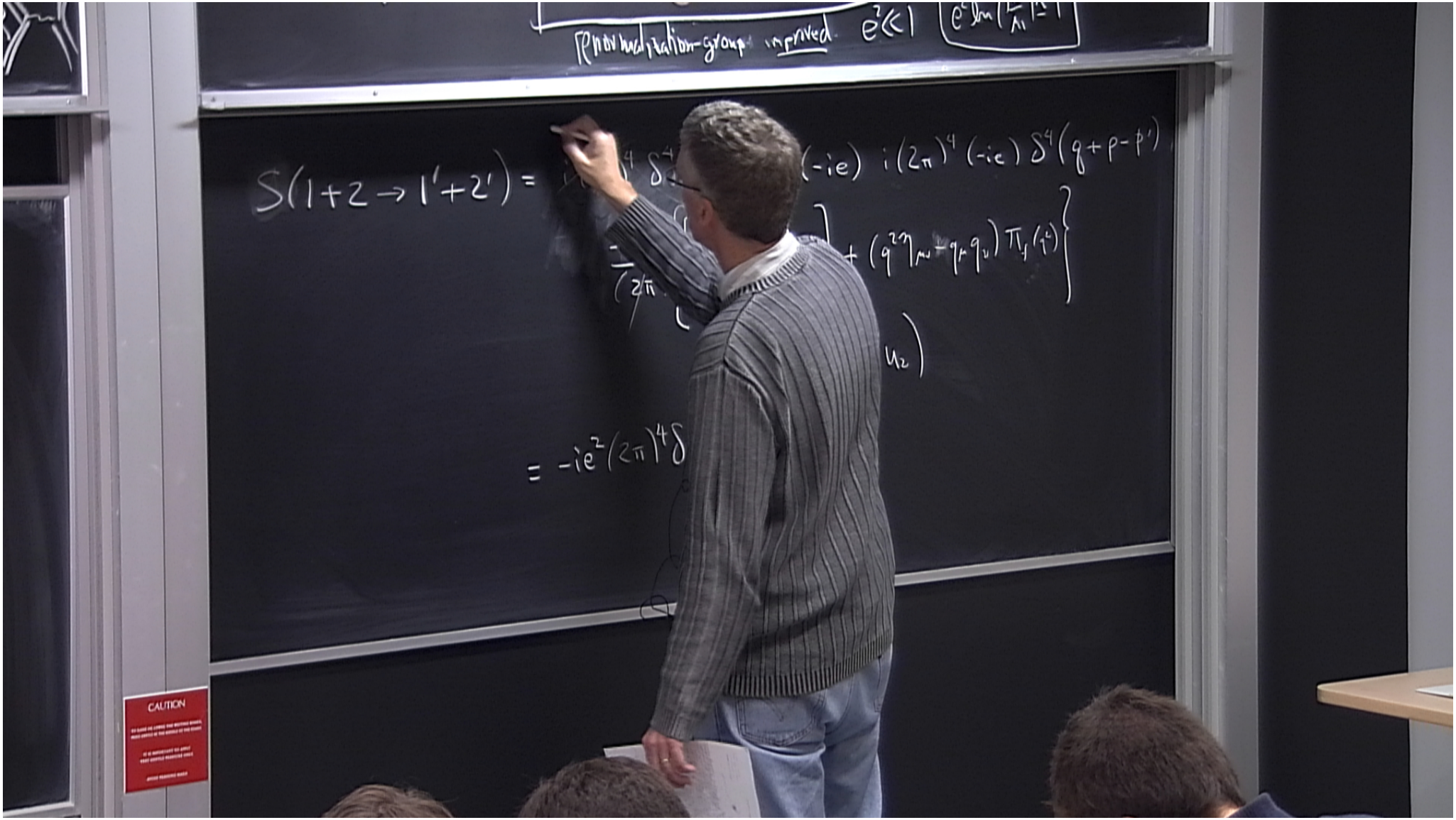

$\mathcal{L} = -\bar{\psi}(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie$
$\Pi_f(q^2)$
$= -\frac{e^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) + O(1)\right]$
$m^2\frac{\partial C}{\partial m^2} = -k \leftarrow$ reg
$= \frac{e^2}{12\pi^2}$
$m^2\frac{\partial C_{div}}{dm^2} = \frac{e^2}{12\pi^2}$
$\Lambda \sim 10^{10}$ GeV
if $\Lambda \gg m$
$+C = 1 - \frac{e^2}{12\pi^2}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) + O(1)\right]$
$e_0 = Z_3^{-1/2} e$ $e_0 > e$

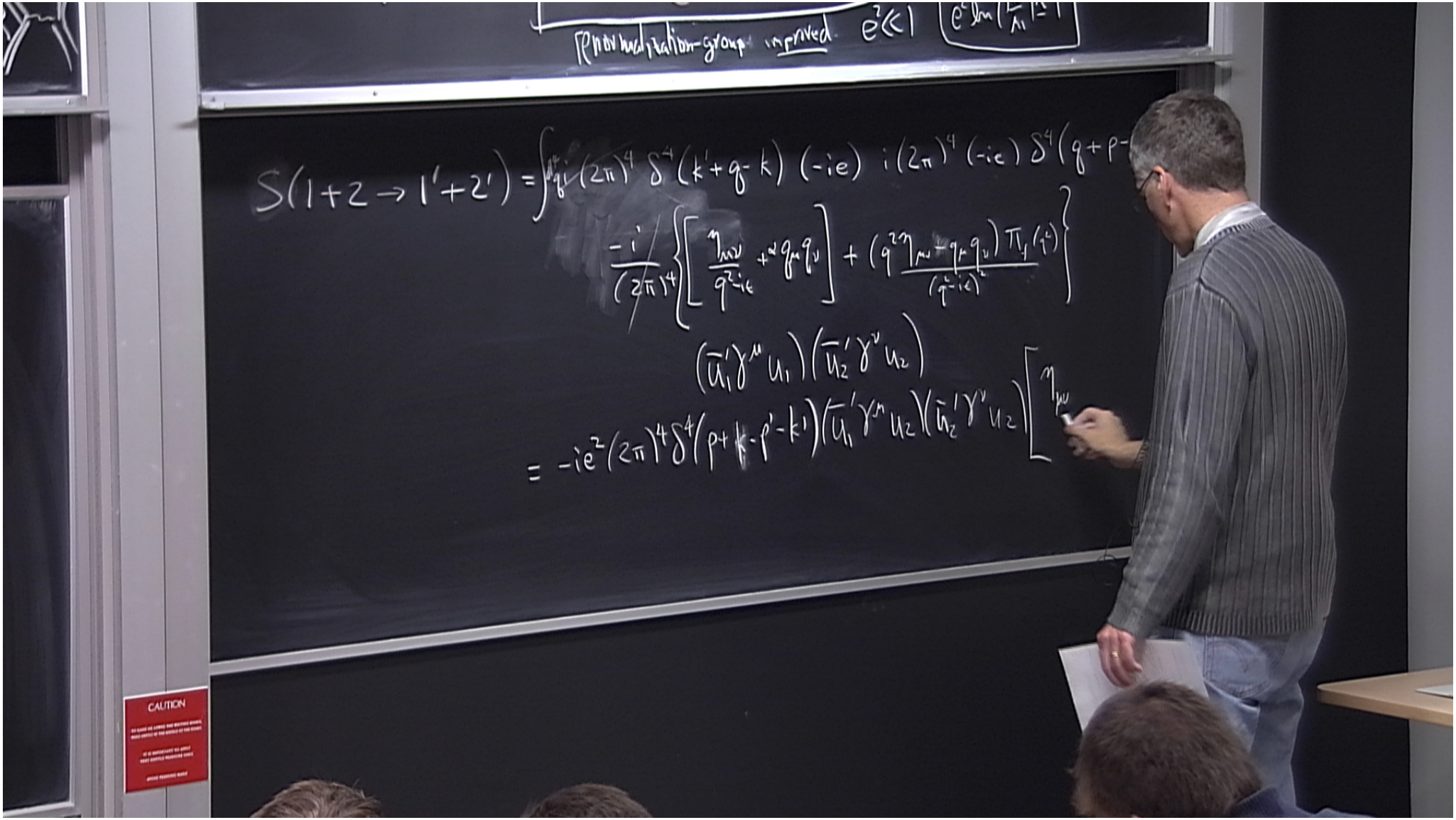

improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') = i(2\pi)^4 \delta^4(k'+q-k)(-ie)$
$\delta^4(q+p-p')$
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2+i\epsilon} + \alpha q_\mu q_\nu\right]\right.$
CAUTION

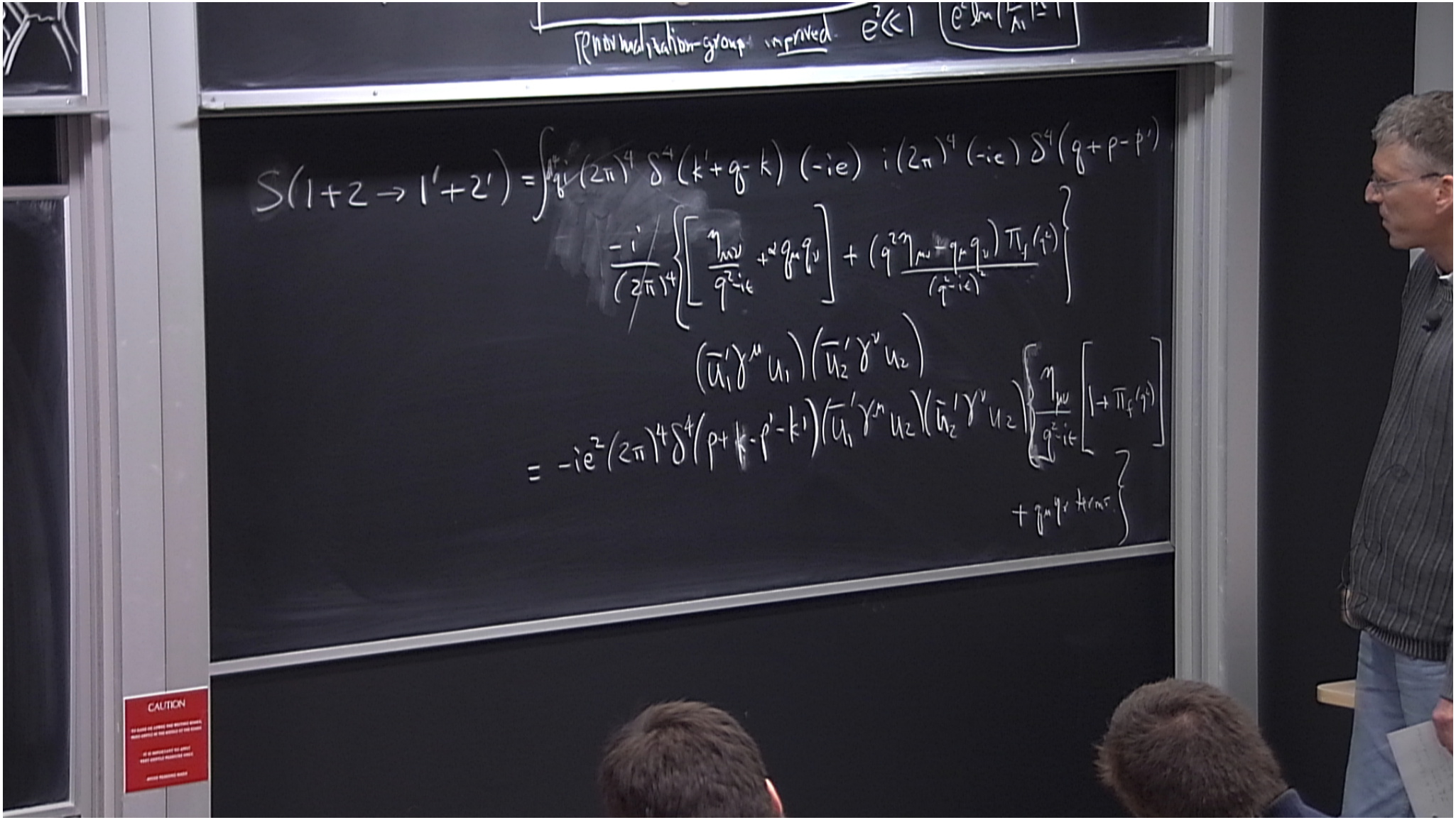

renormalization-group improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') = i(2\pi)^4 \delta^4(k'+q-k)(-ie)\; i(2\pi)^4(-ie)\delta^4(q+$
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha q_\mu q_\nu\right] + (q^2\eta_{\mu\nu} + q_\mu q_\nu)\Pi_f(q^2)\right\}$
CAUTION


renormalization-group improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') = i(2\pi)^4 \delta^4(k'+q-k)(-ie)\; i(2\pi)^4(-ie)\,\delta^4($
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2+i\epsilon} + \alpha q_\mu q_\nu\right] + (q^2\eta_{\mu\nu} + q_\mu q_\nu)\,\Pi_1(q^2)\right\}$
$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$
CAUTION


Renormalization-group improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') = i(2\pi)^4 \delta^4(k'+q-k)(-ie)\, i(2$
$\delta^4(q+p-p')$
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha q_\mu q_\nu\right] +$
$(\bar{u}_1 \gamma^\mu u_1)(\bar{u}$
=
CAUTION


renormalization-group improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') =$
$(-ie)\ i(2\pi)^4(-ie)\,\delta^4(q+p-p')$
$+ (q^2\eta_{\mu\nu} - q_\mu q_\nu)\,\Pi_f(q^2)\Big\}$
$= -ie^2(2\pi)^4\delta$
CAUTION


renormalization-group improved
$e^2 \ll 1$
$S(1+2 \to 1'+2') = \int d^4q (2\pi)^4 \delta^4(k'+q-k)(-ie)\, i(2\pi)^4(-ie)\delta^4(q+p-$
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2+i\epsilon} + \alpha q_\mu q_\nu\right] + (q^2\eta_{\mu\nu} + q_\mu q_\nu)\frac{\Pi_1(q^2)}{(q^2+i\epsilon)^2}\right\}$
$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$
$= -ie^2(2\pi)^4\delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left[\eta_{\mu\nu}\right.$
CAUTION


renormalization-group improved
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$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha q_\mu q_\nu\right] + (q^2\eta_{\mu\nu} - q_\mu q_\nu)\frac{\Pi_f(q^2)}{(q^2-i\epsilon)^2}\right\}$
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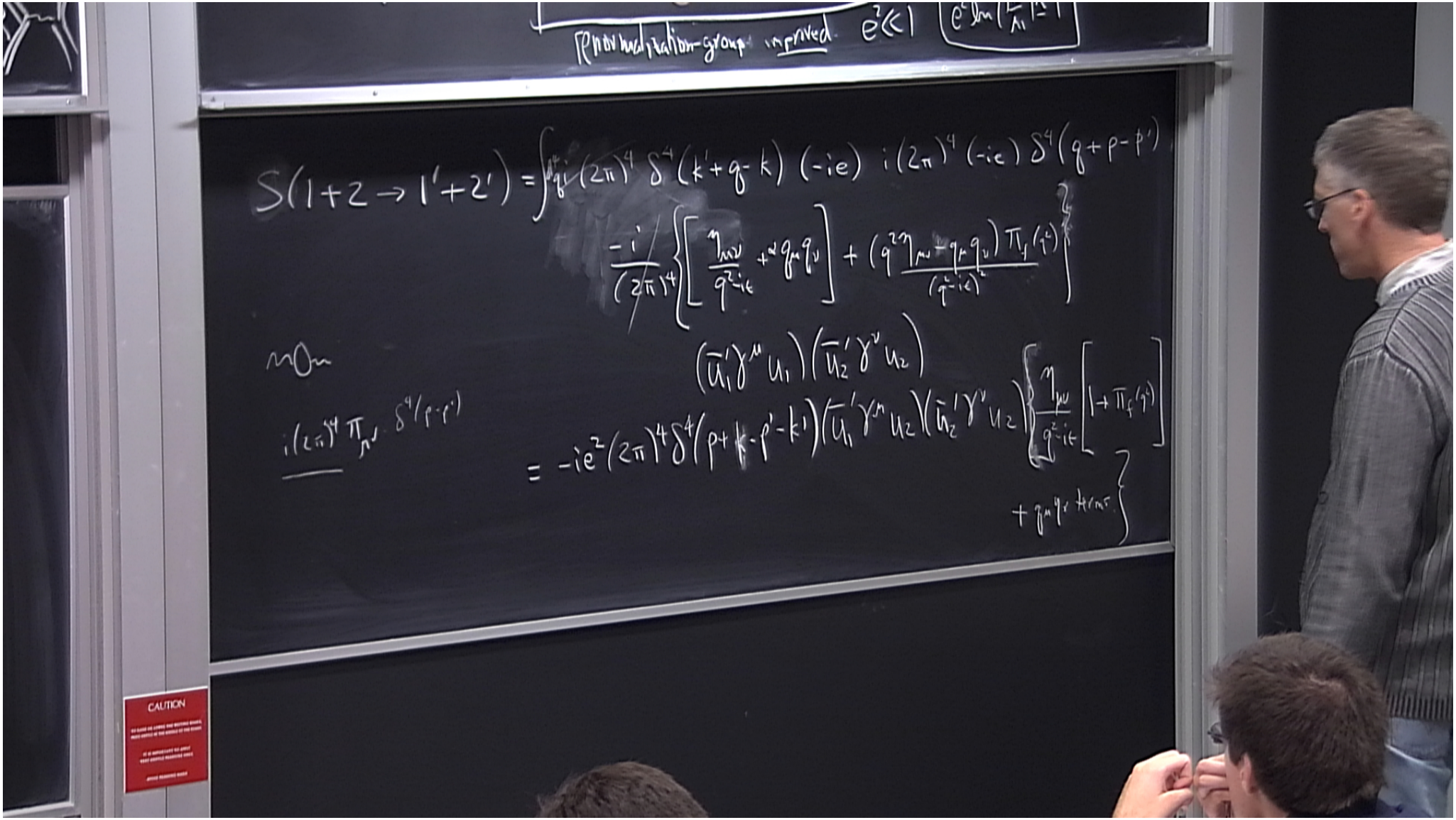
$= -ie^2(2\pi)^4\delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left\{\frac{\eta_{\mu\nu}}{q^2-i\epsilon}\left[1+\Pi_f(q^2)\right]\right.$
$\left. + q_\mu q_\nu \text{ terms}\right\}$
CAUTION


renormalization-group improved
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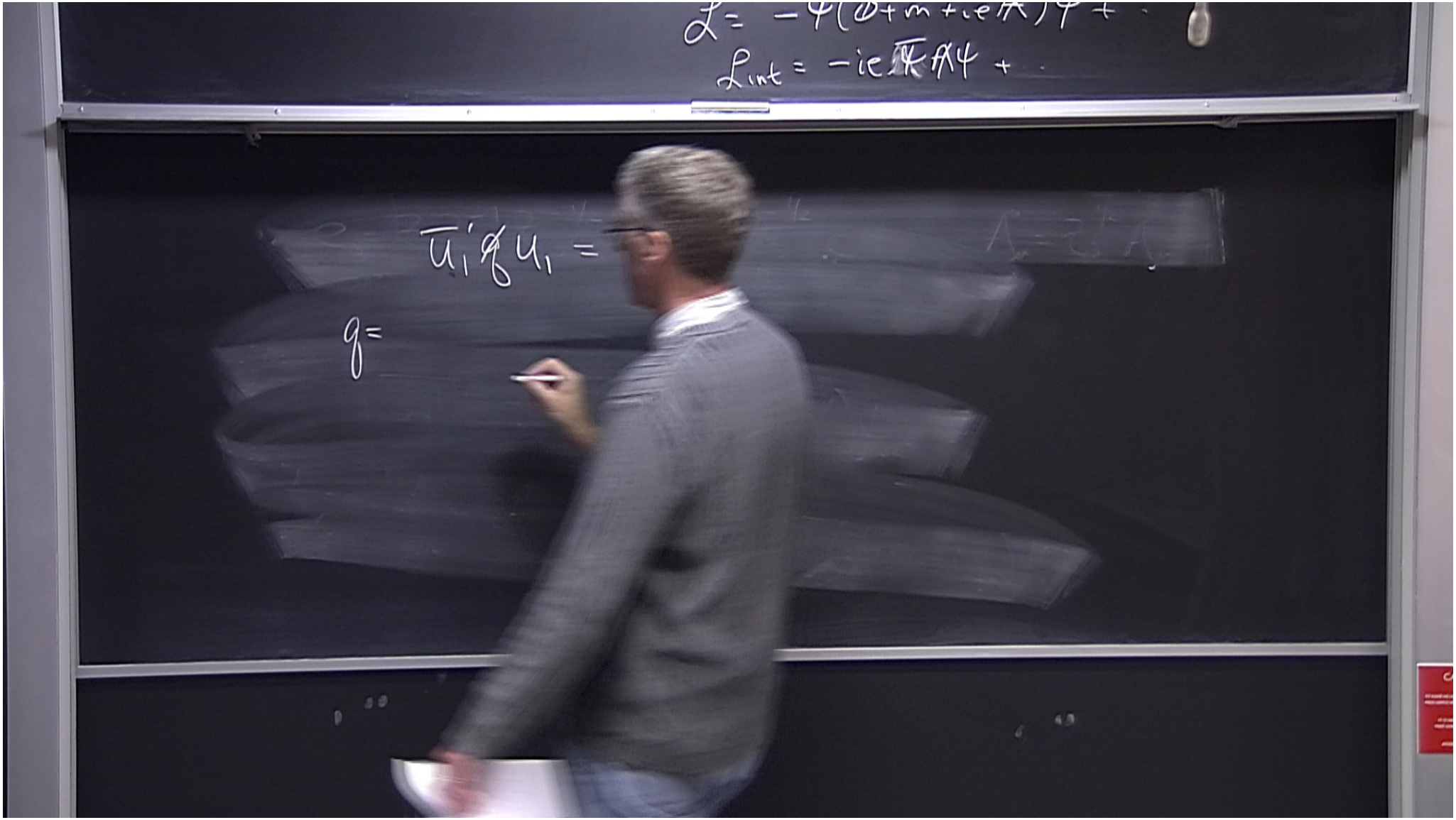
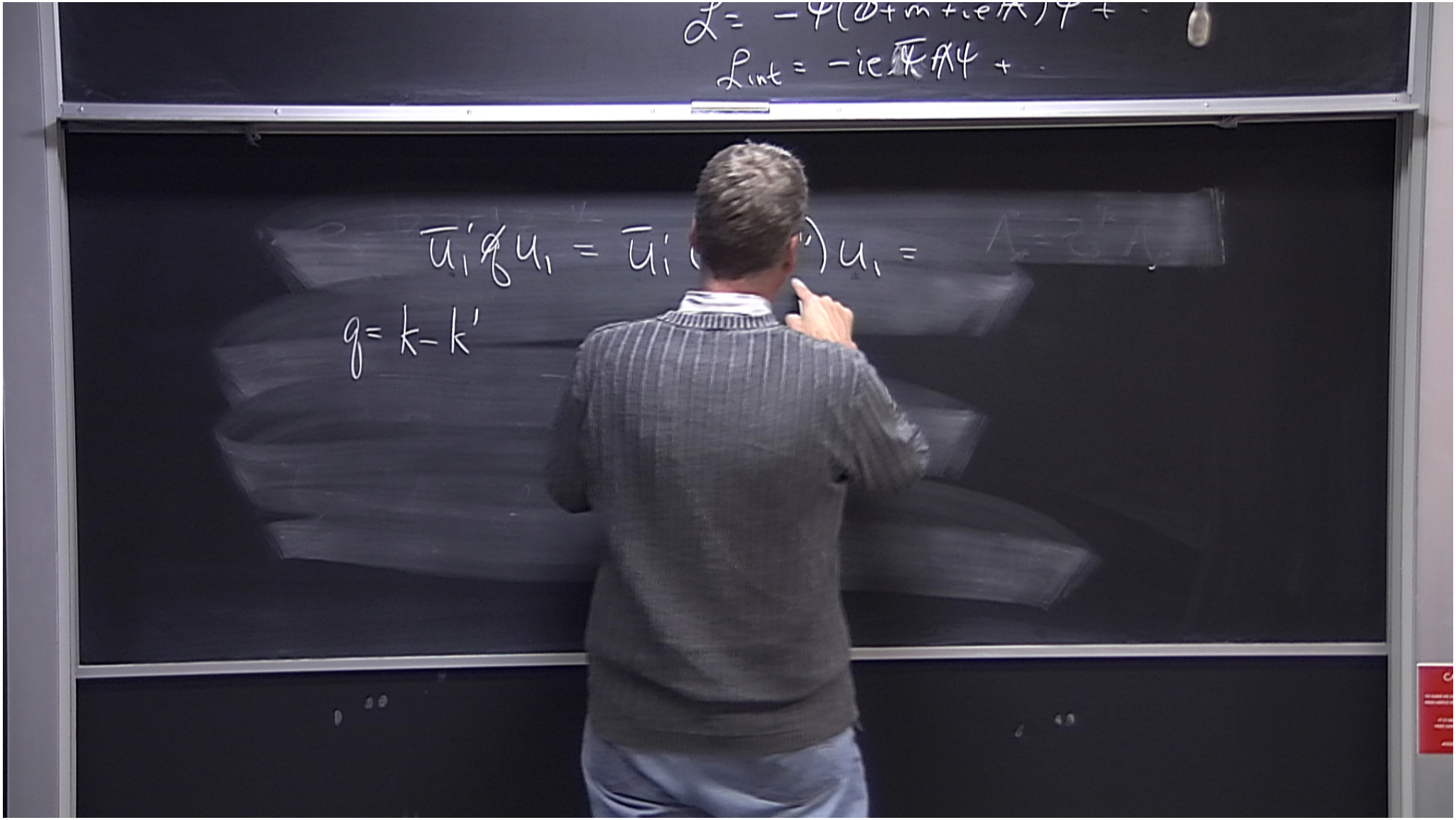
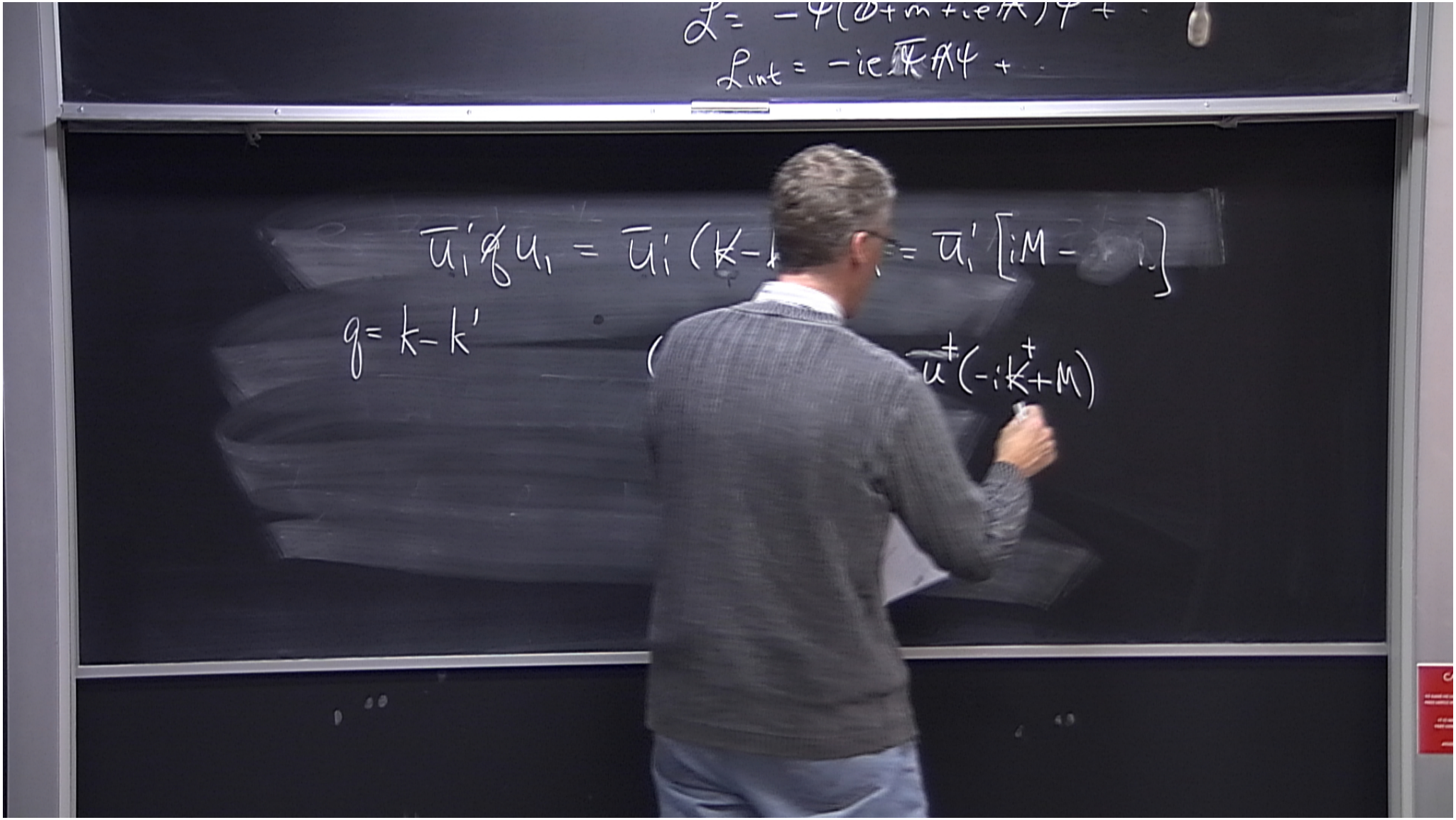
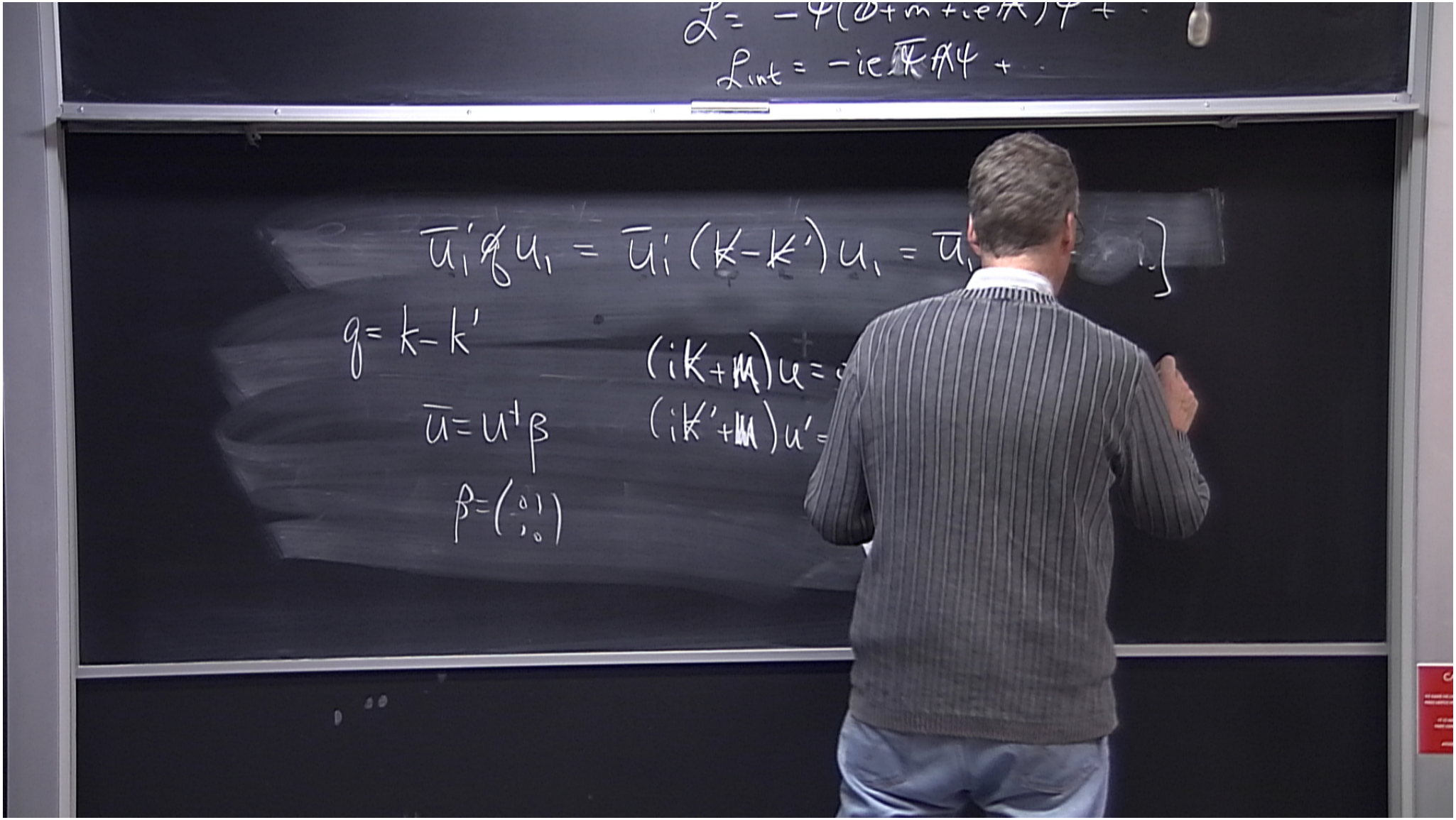
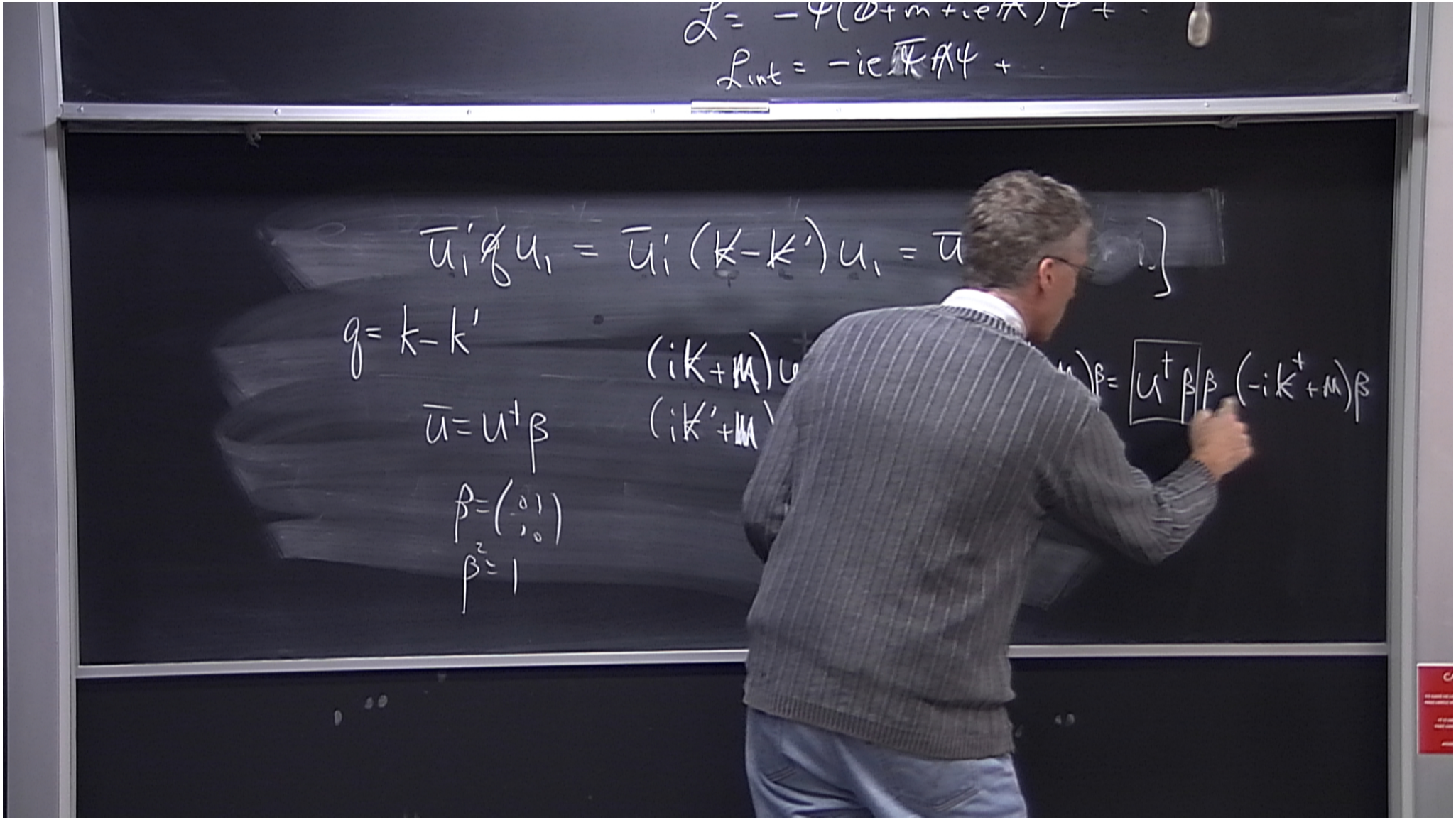
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha q_\mu q_\nu\right] + (q^2\eta_{\mu\nu} - q_\mu q_\nu)\frac{\Pi_f(q^2)}{(q^2-i\epsilon)^2}\right\}$
$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$
$i(2\pi)^4 \Pi_{\mu\nu}\, \delta^4(p-p')$
$= -ie^2(2\pi)^4\delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left\{\frac{\eta_{\mu\nu}}{q^2-i\epsilon}\left[1+\Pi_f(q^2)\right]\right.$
$\left. + q_\mu q_\nu \text{ terms}\right\}$
CAUTION

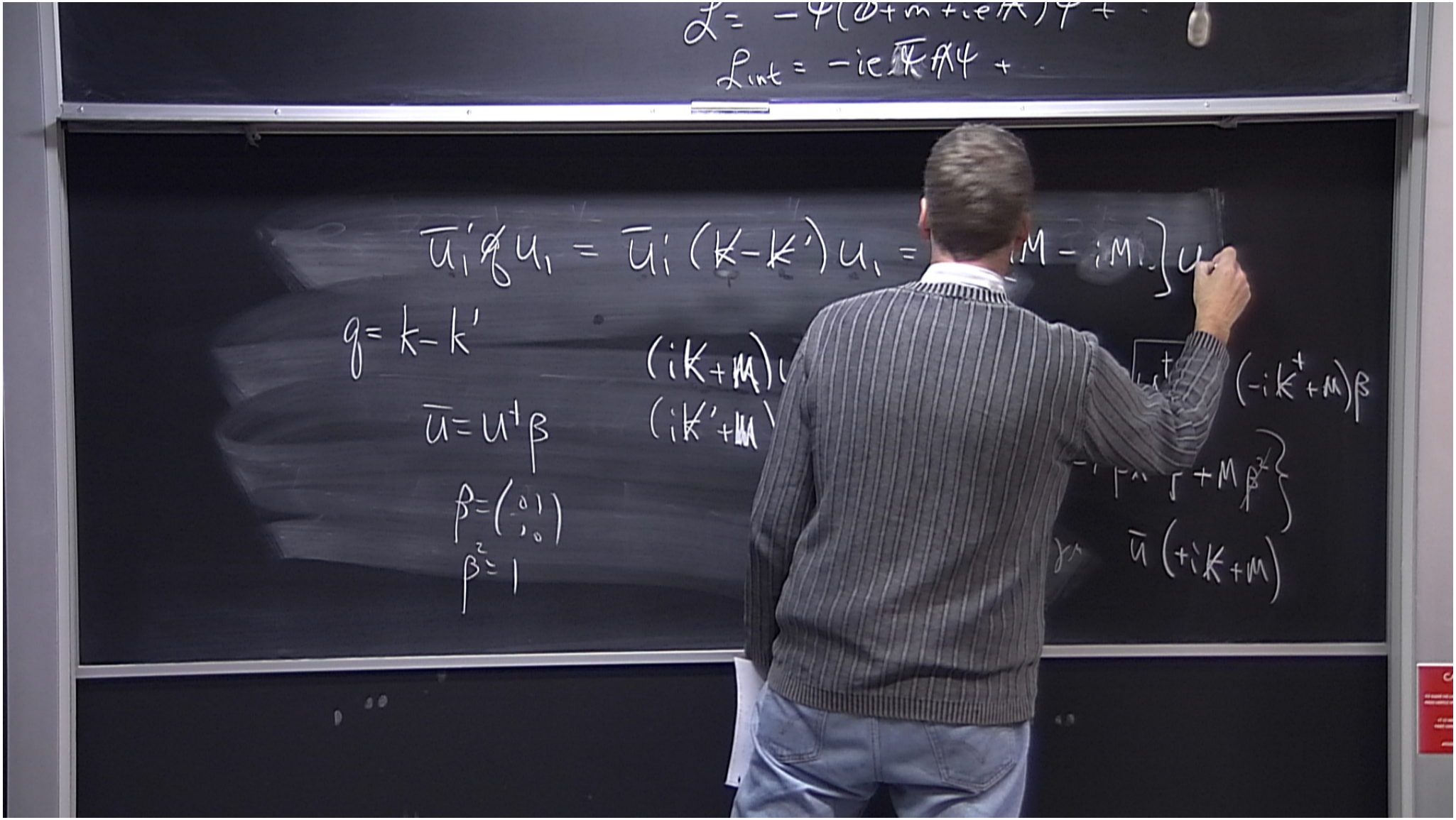

$\mathcal{L} = -\bar{\psi}(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 \not{q} u_1 =$
$q =$

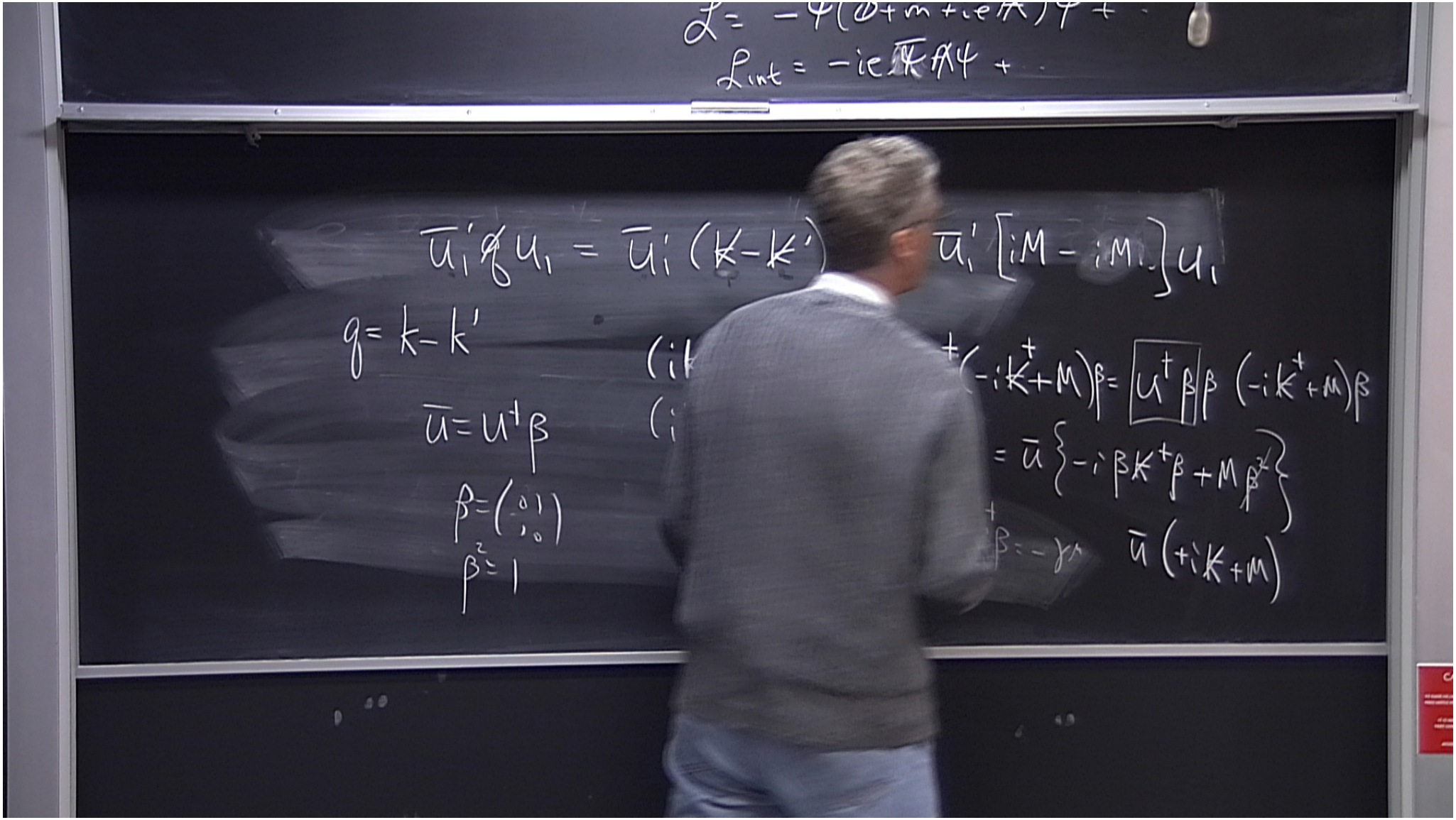

$\mathcal{L} = -\bar{\psi}(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 \not{q} u_1 = \bar{u}_1 ( ) u_1 =$
$q = k - k'$

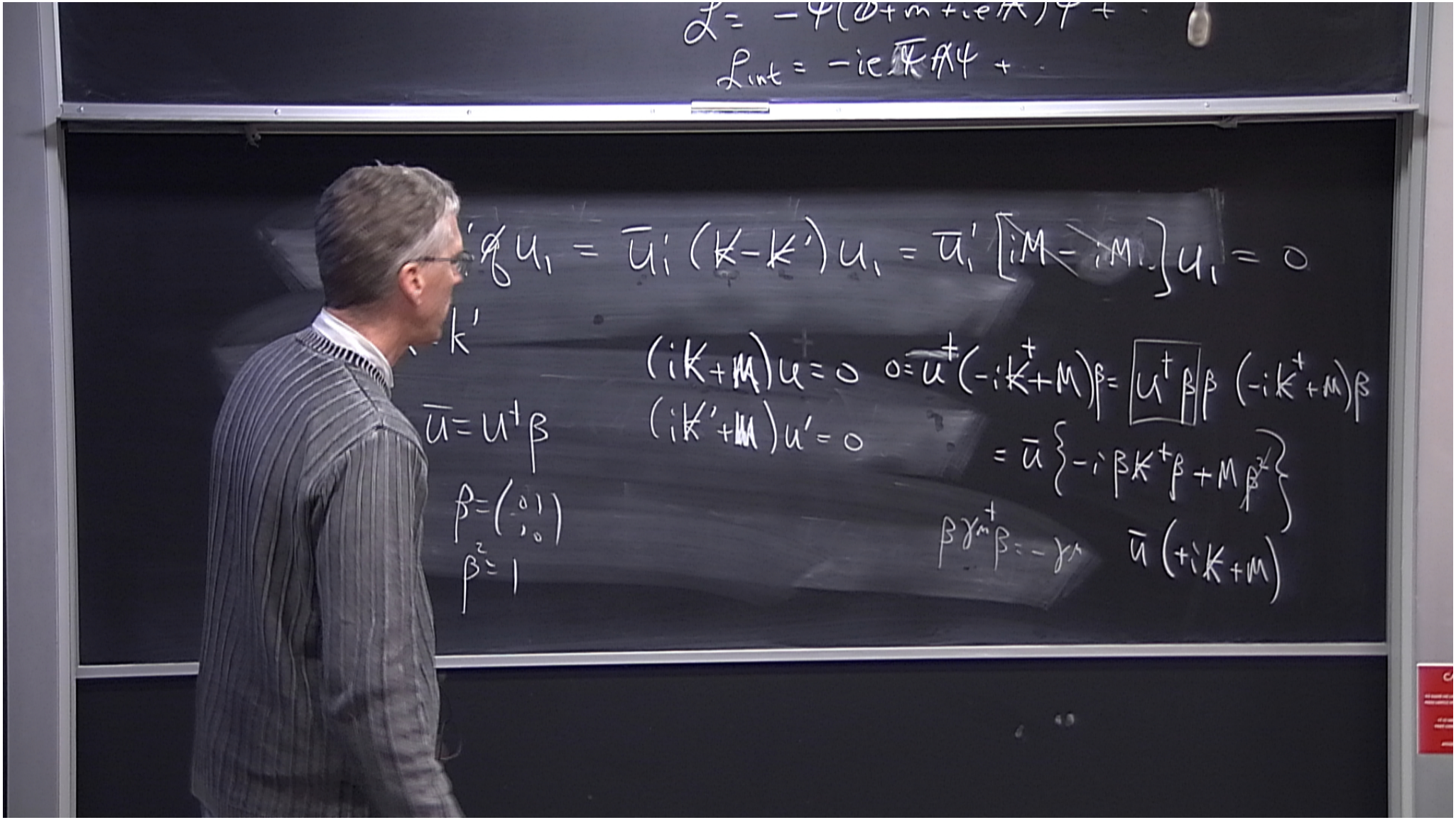

$\mathcal{L} = -\psi(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 \not{q} u_1 = \bar{u}_1 (\not{k} -$
$= \bar{u}_1 [iM -$
$q = k - k'$
$\bar{u}^+(-i\not{k}^+ + M)$


$\mathcal{L} = -\psi(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1' \not{q} u_1 = \bar{u}_1' (\not{k} - \not{k}') u_1 = \bar{u}_1'$
$q = k - k'$
$(i\not{k} + m) u =$
$\bar{u} = u^\dagger \beta$
$(i\not{k}' + m) u' =$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$


$\mathcal{L} = -\psi(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1' \not{q} u_1 = \bar{u}_1' (\not{k} - \not{k}') u_1 = \bar{u}$
$q = k - k'$
$\bar{u} = u^\dagger \beta$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\beta^2 = 1$
$(i\not{k} + M)u$
$(i\not{k}' + M)$
$M)\beta = \boxed{u^\dagger \beta}\beta\,(-i\not{k}^\dagger + M)\beta$


$\mathcal{L} = -\psi(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 \not{q} u_1 = \bar{u}_1 (\not{k} - \not{k}') u_1 =$
$q = k - k'$
$\bar{u} = u^\dagger \beta$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\beta^2 = 1$
$(i\not{k} + M)$
$(i\not{k}' + M)$
$(-i\not{k}^\dagger + M)\beta$
$\bar{u}(+i\not{k} + M)$


$\mathcal{L} = -\bar{\psi}(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 \not{q} u_1 = \bar{u}_1 (\not{k} - \not{k}')$
$\bar{u}_1 [iM - iM_1] u_1$
$q = k - k'$
$\bar{u} = u^\dagger \beta$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\beta^2 = 1$
$(-i\not{k}^\dagger + M)\beta = u^\dagger \beta \beta (-i\not{k}^\dagger + M)\beta$
$= \bar{u}\{-i\beta\not{k}^\dagger\beta + M\beta^2\}$
$\bar{u}(+i\not{k} + M)$


$\mathcal{L} = -\psi(\partial + m + ie\not{A})\psi +$
$\mathcal{L}_{int} = -ie\bar{\psi}\not{A}\psi +$
$\bar{u}_1 q u_1 = \bar{u}_1 (\not{k} - \not{k}') u_1 = \bar{u}_1 [iM - iM] u_1 = 0$
k'
$(iK + M)u = 0$
$(iK' + M)u' = 0$
$0 = u^\dagger (-iK^\dagger + M)\beta = u^\dagger \beta\beta (-iK^\dagger + M)\beta$
$= \bar{u}\{-i\beta K^\dagger \beta + M\beta^2\}$
$\bar{u} = u^\dagger \beta$
$\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\beta^2 = 1$
$\beta\gamma^{m\dagger}\beta = -\gamma^m$
$\bar{u}(+iK + M)$

$$S(1+2 \to 1'+2') = \int d^4q \, (2\pi)^4 \delta^4(k'+q-k)\,(-ie)\; i(2\pi)^4 (-ie)\,\delta^4(q+p-p')$$

$$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha \frac{q_\mu q_\nu}{q^2}\right] + \left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\frac{\Pi_f(q^2)}{(q^2-i\epsilon)^2}\right\}$$

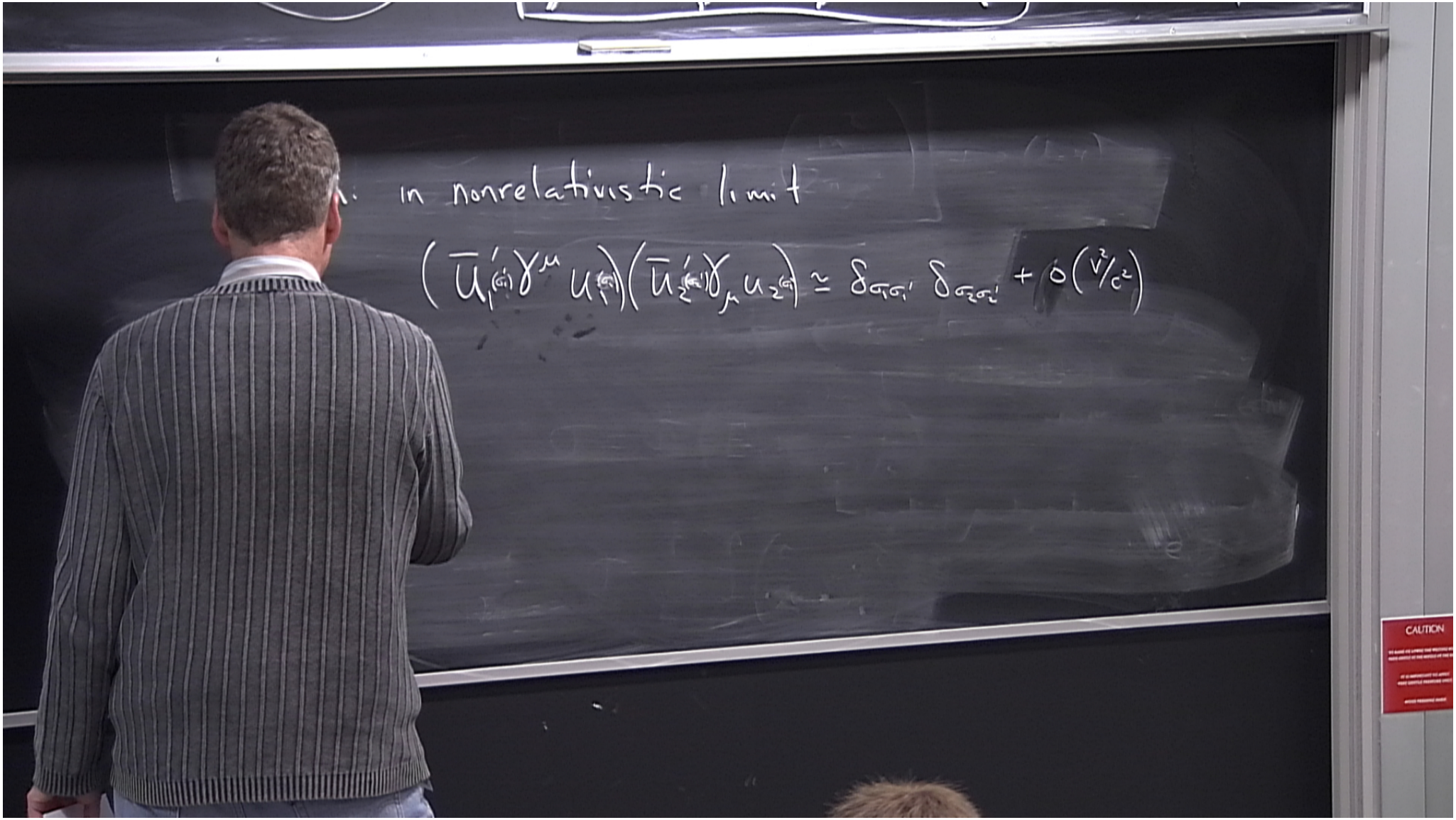
$$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$$

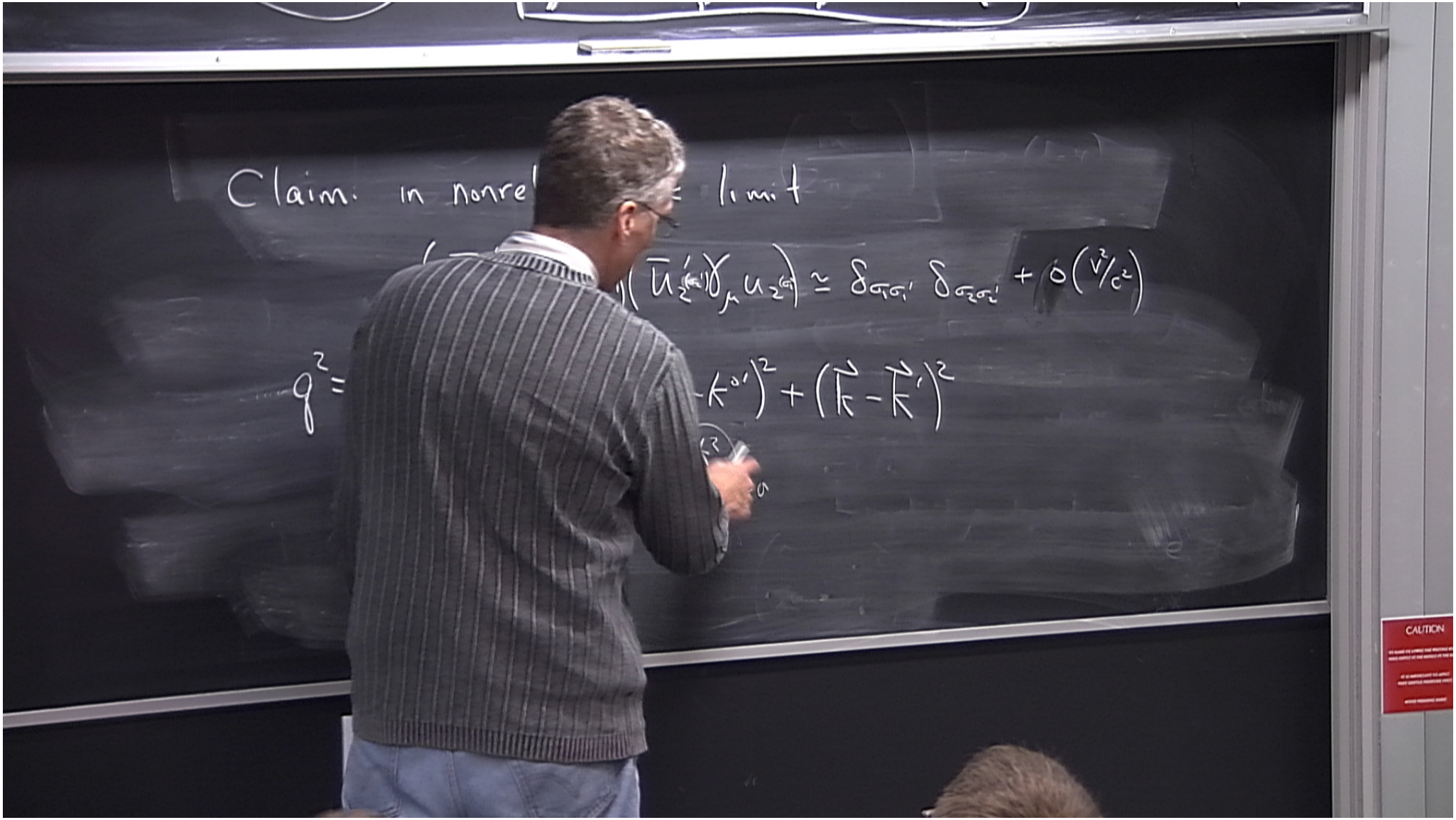
now

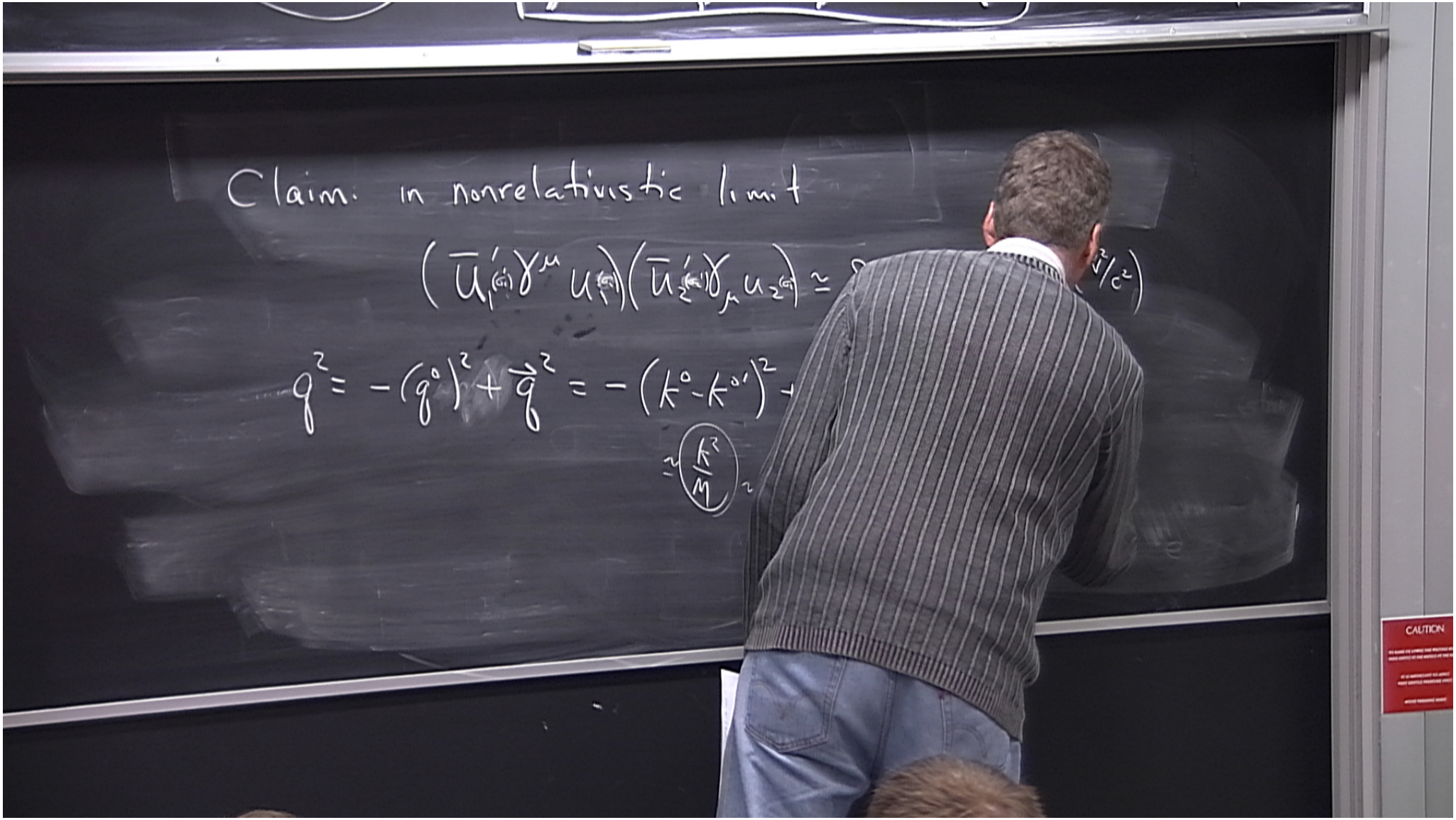
$$i(2\pi)^4 \Pi_{\mu\nu}\,\delta^4(p-p')$$

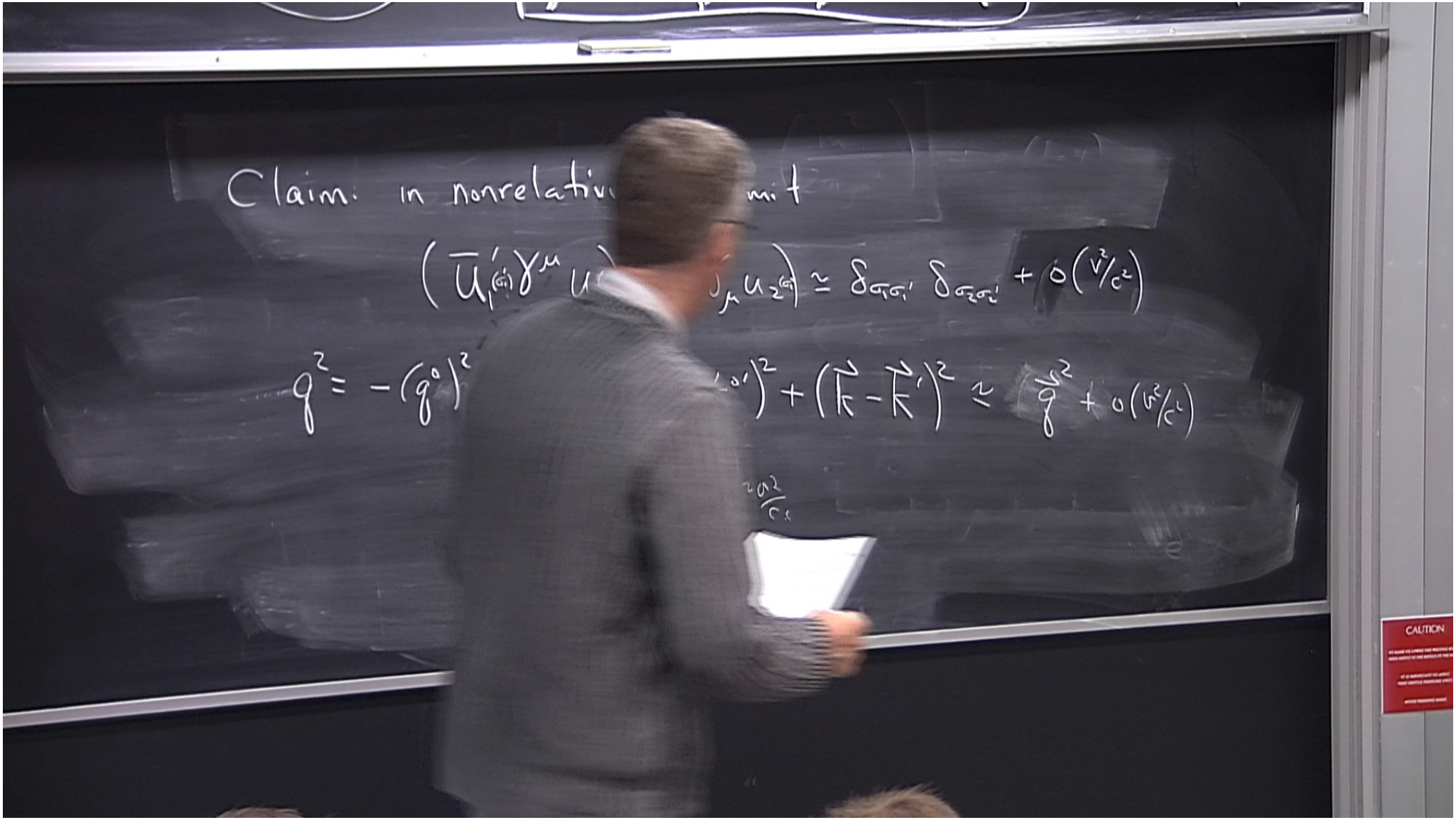
$$= -ie^2(2\pi)^4\delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left\{\frac{\eta_{\mu\nu}}{q^2-i\epsilon}\left[1+\Pi_f(q^2)\right]\right.$$

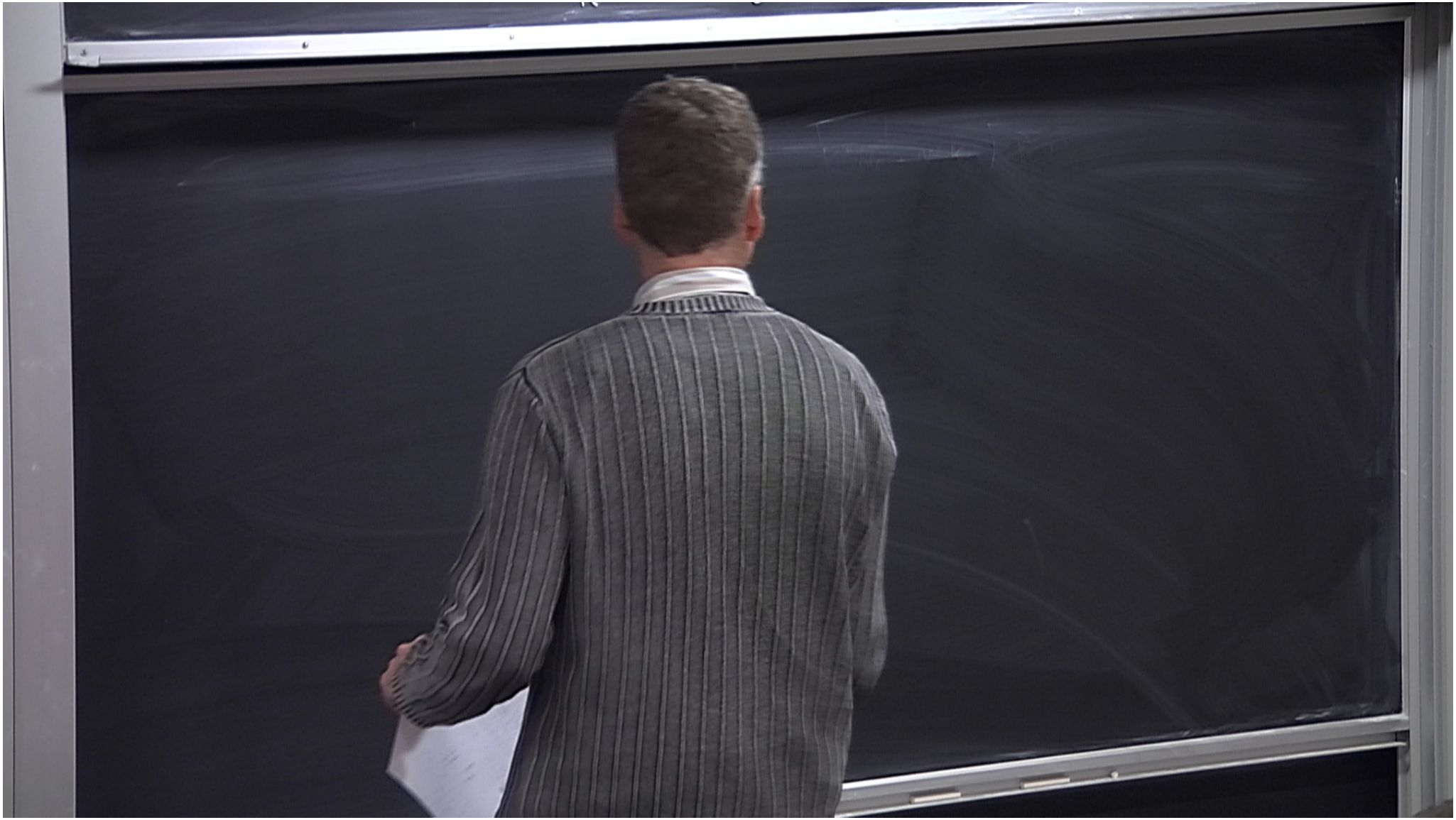
$$\left. + q_\mu q_\nu \text{ terms}\right\}$$

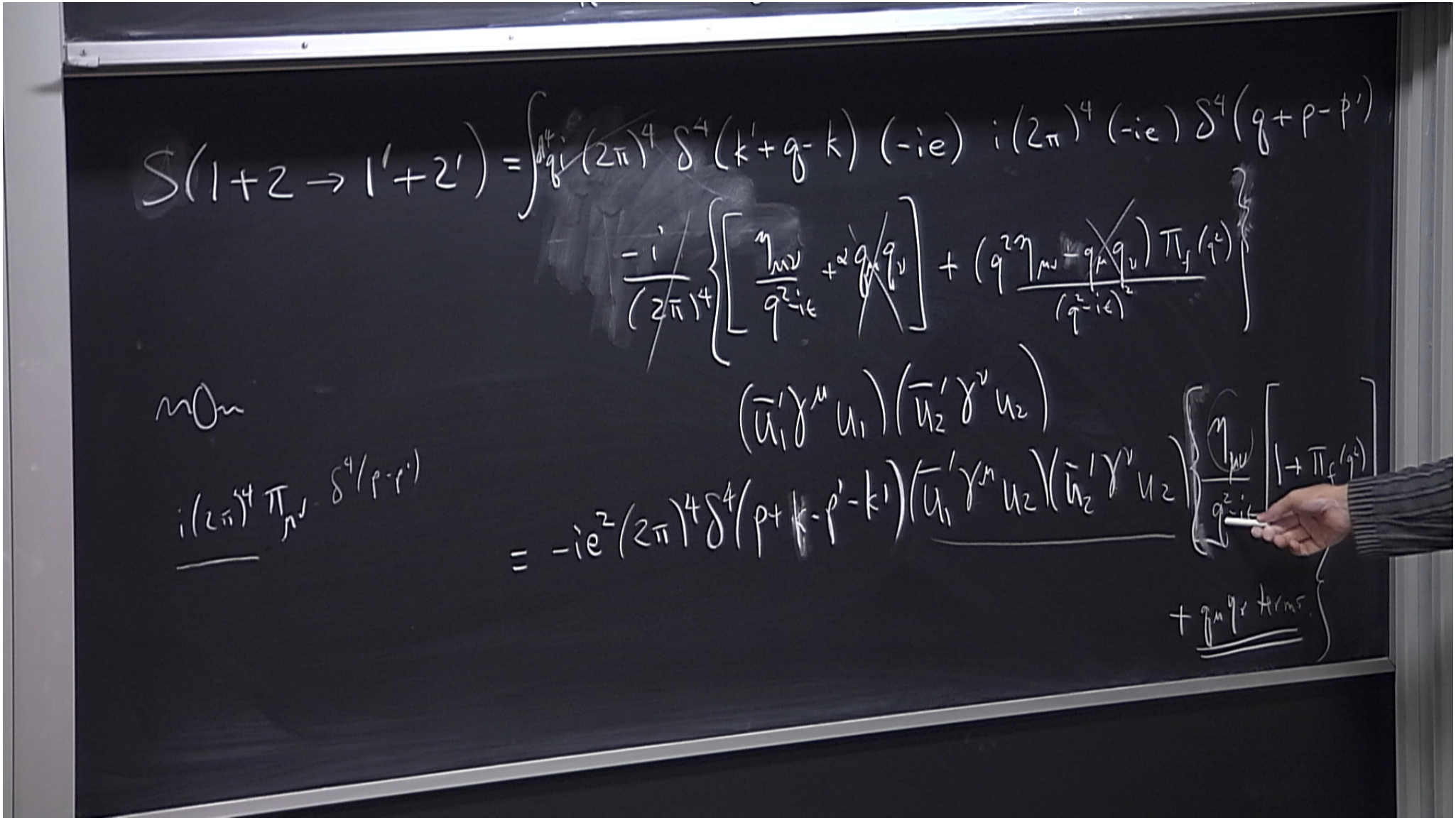

in nonrelativistic limit
$(\bar{u}_1' \gamma^\mu u_1)(\bar{u}_2' \gamma_\mu u_2) \simeq \delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'} + O(v^2/c^2)$
CAUTION

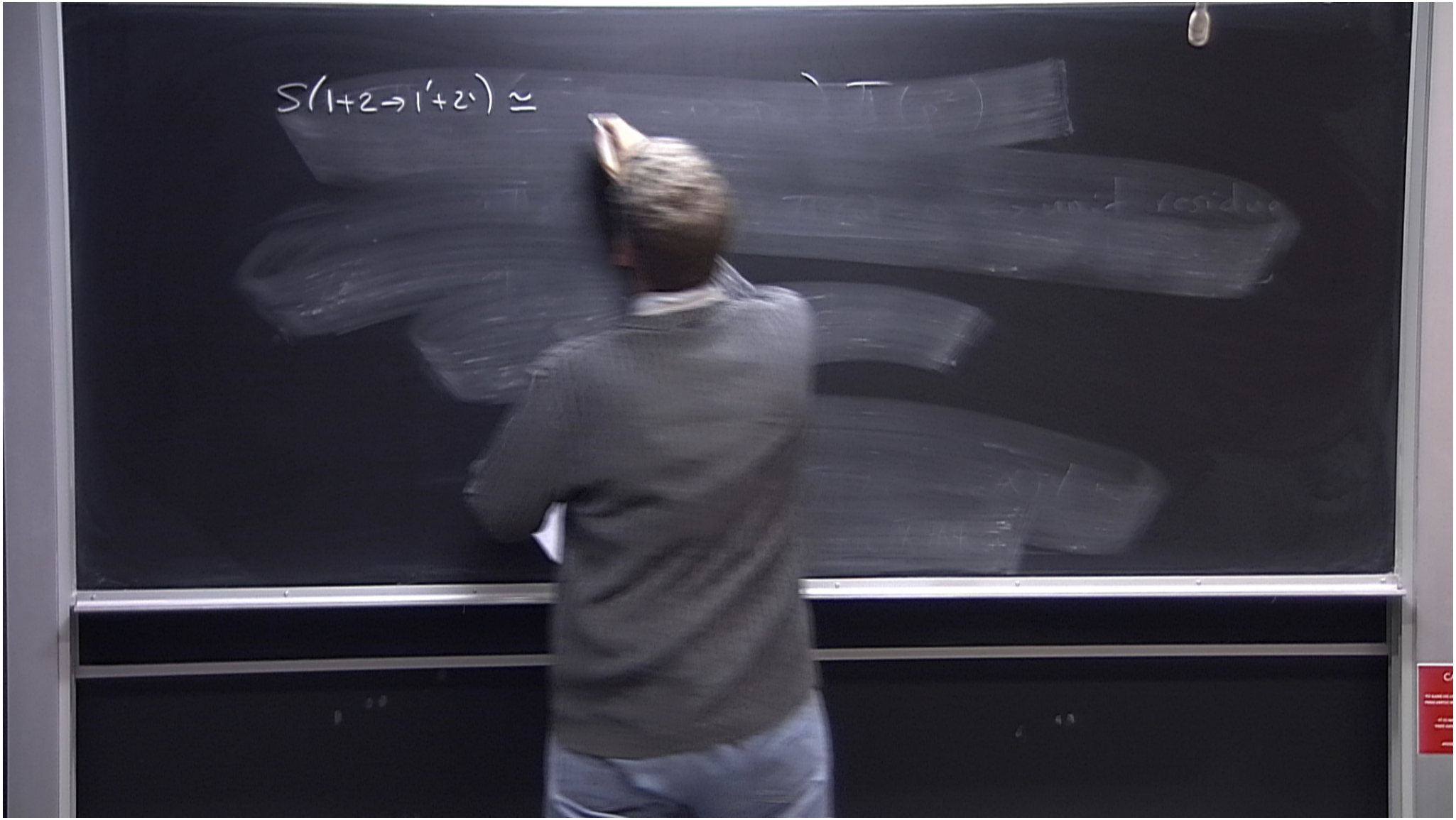

Claim: in nonre limit
$\bar{u}_{s'}(p')\gamma_\mu u_s(p) \simeq \delta_{\sigma\sigma'}\,\delta_{\sigma\sigma'} + O(v^2/c^2)$
$q^2 = (k^0 - k^{0\prime})^2 + (\vec{k} - \vec{k}')^2$
CAUTION

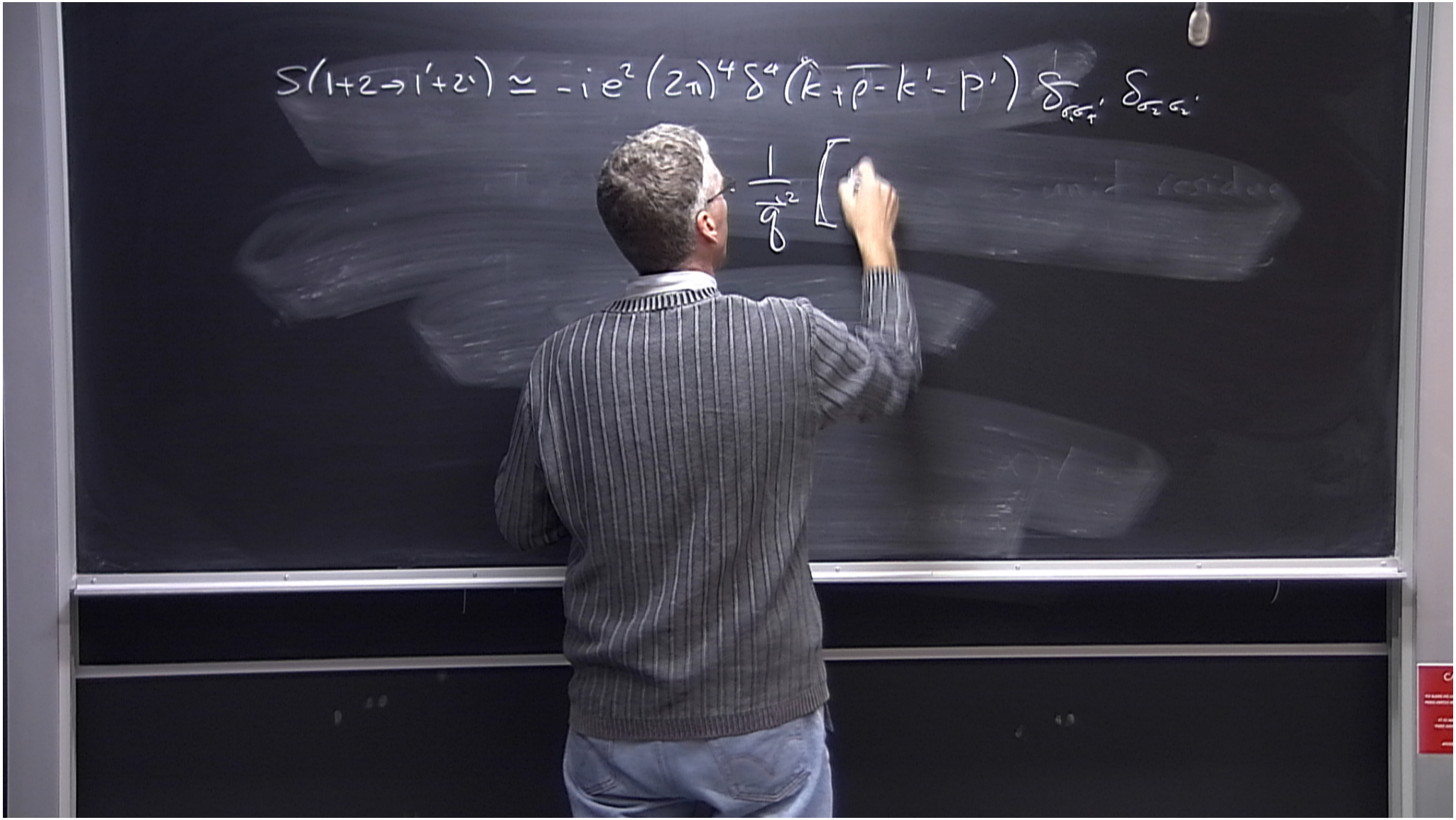

Claim: in nonrelativistic limit
$(\bar{u}_1' \gamma^\mu u_1)(\bar{u}_2' \gamma_\mu u_2) \simeq$
$q^2 = -(q^0)^2 + \vec{q}^2 = -(k^0 - k^{0\prime})^2 +$
$\simeq \left(\frac{\vec{k}^2}{M}\right)^2$
CAUTION

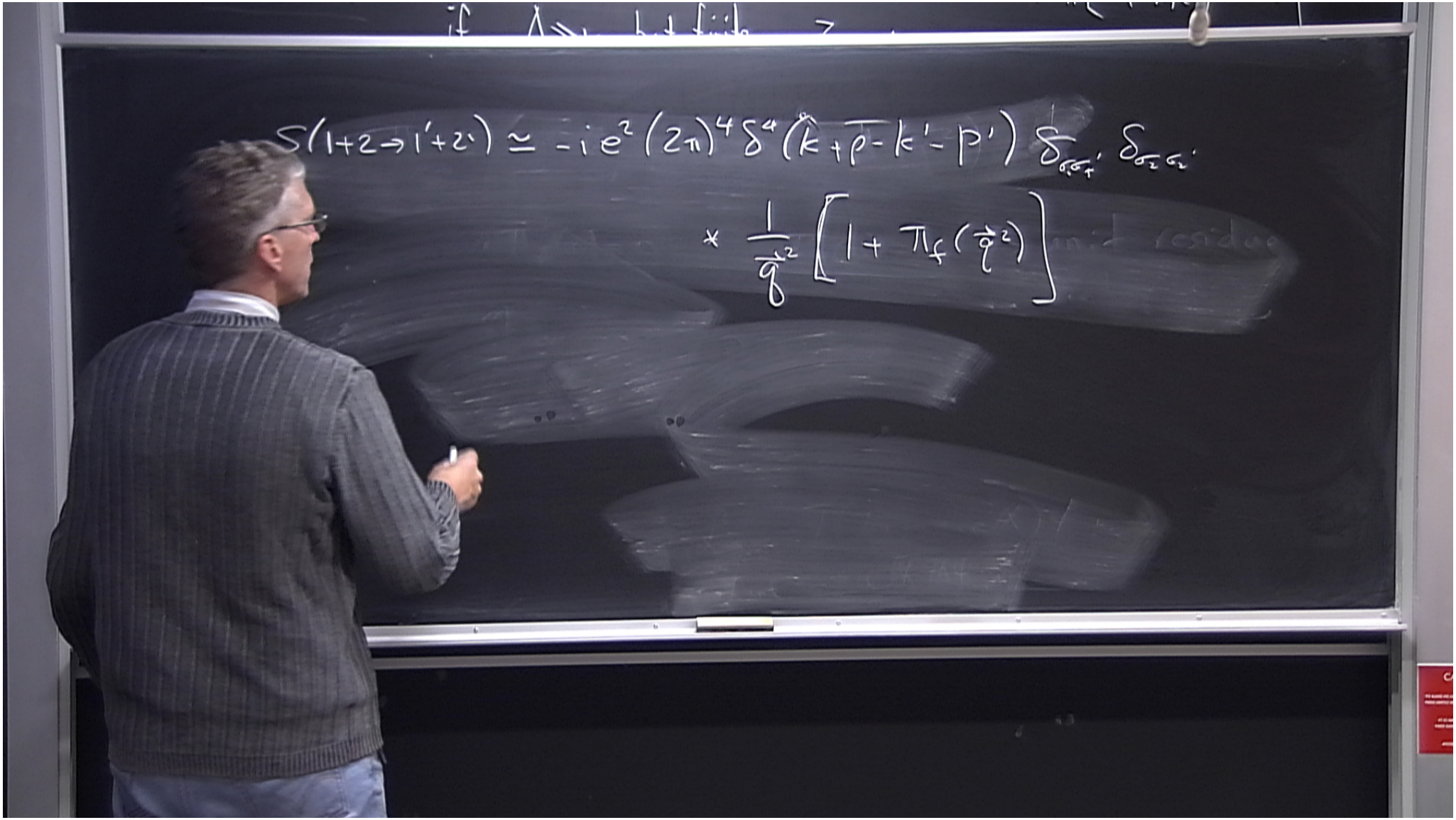

Claim: in nonrelativ
mit
$(\bar{u}'_{1(\sigma_1')}\gamma^\mu u$
$\partial_\mu u_{2(\sigma_2)}) \simeq \delta_{\sigma_1\sigma_1'}\delta_{\sigma_2\sigma_2'} + O(v^2/c^2)$
$q^2 = -(q^0)^2$
$)^2 + (\vec{k}-\vec{k}')^2 \simeq \vec{q}^2 + O(v^2/c^2)$
CAUTION

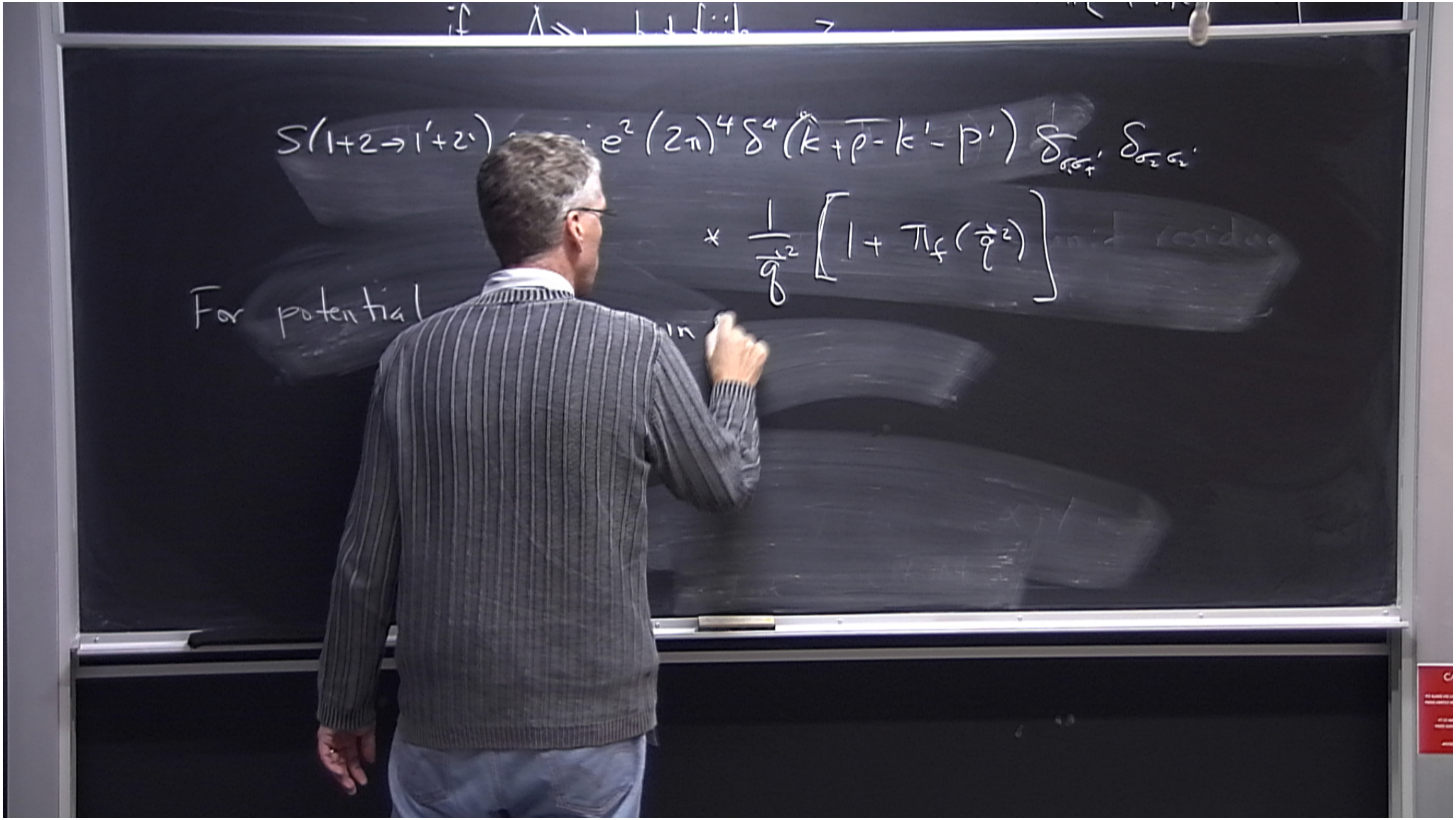


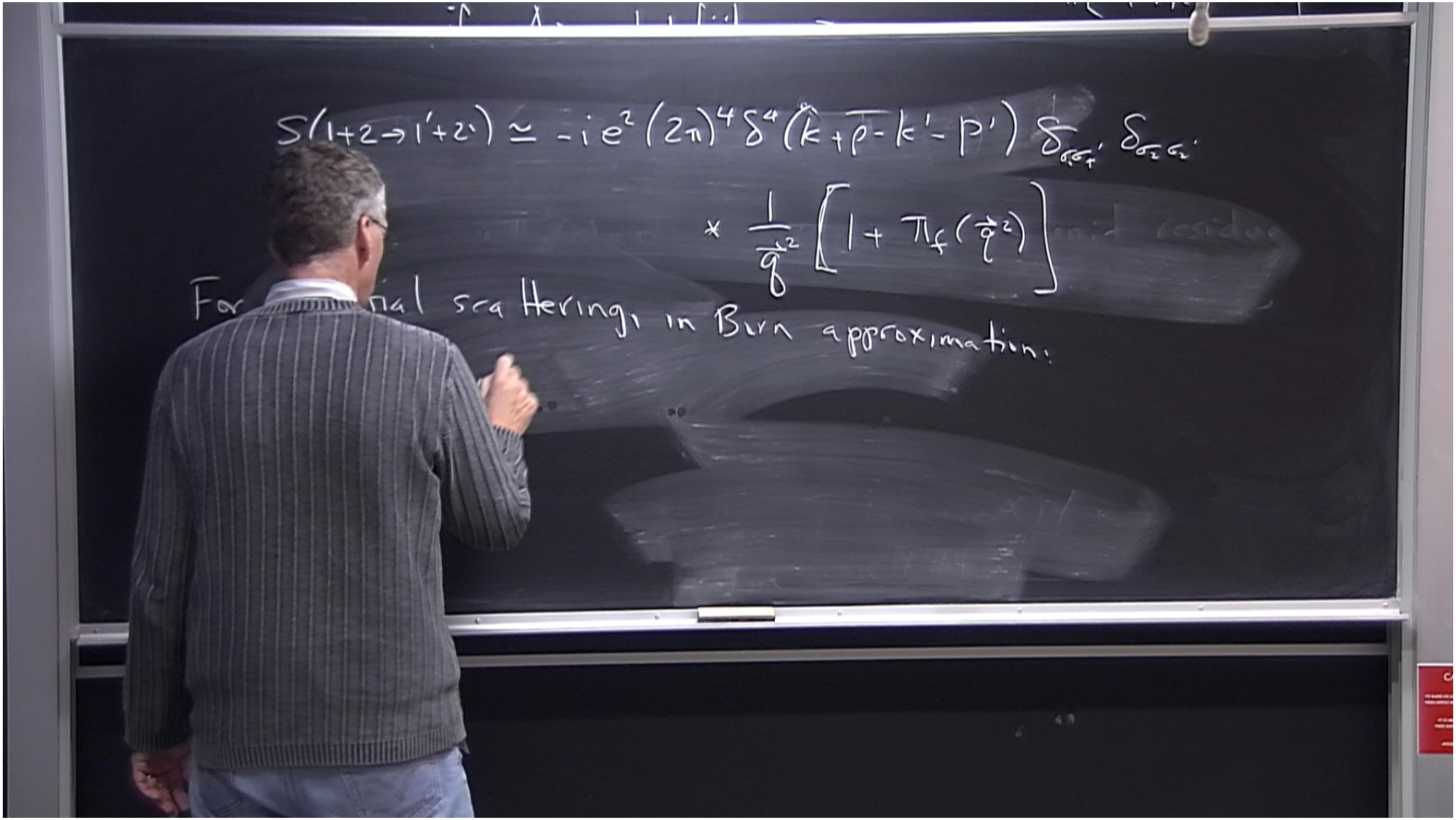

$S(1+2 \to 1'+2') = \int d^4q \, (2\pi)^4 \delta^4(k'+q-k)\,(-ie)\; i(2\pi)^4 (-ie)\,\delta^4(q+p-p')$
$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha \frac{q_\mu q_\nu}{q^2}\right] + (q^2\eta_{\mu\nu} - q_\mu q_\nu)\frac{\Pi_f(q^2)}{(q^2-i\epsilon)^2}\right\}$
$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$
$i(2\pi)^4 \Pi_{\mu\nu}\,\delta^4(p-p')$
$= -ie^2(2\pi)^4\delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left\{\frac{\eta_{\mu\nu}}{q^2-i\epsilon}\left[1+\Pi_f(q^2)\right]\right.$
$\left. + q_\mu q_\nu \text{ terms}\right\}$

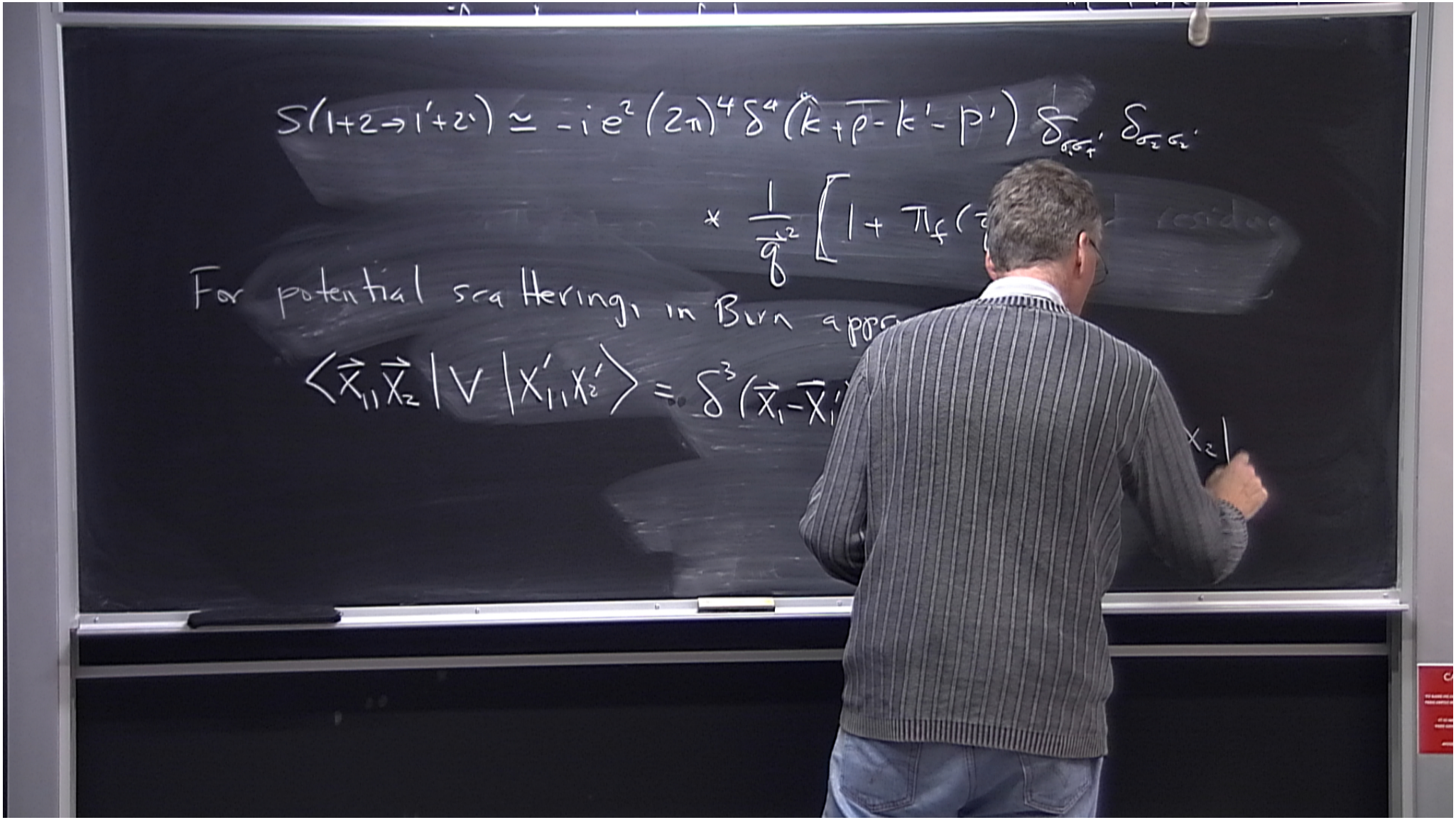

S(1+2→1'+2') ≃

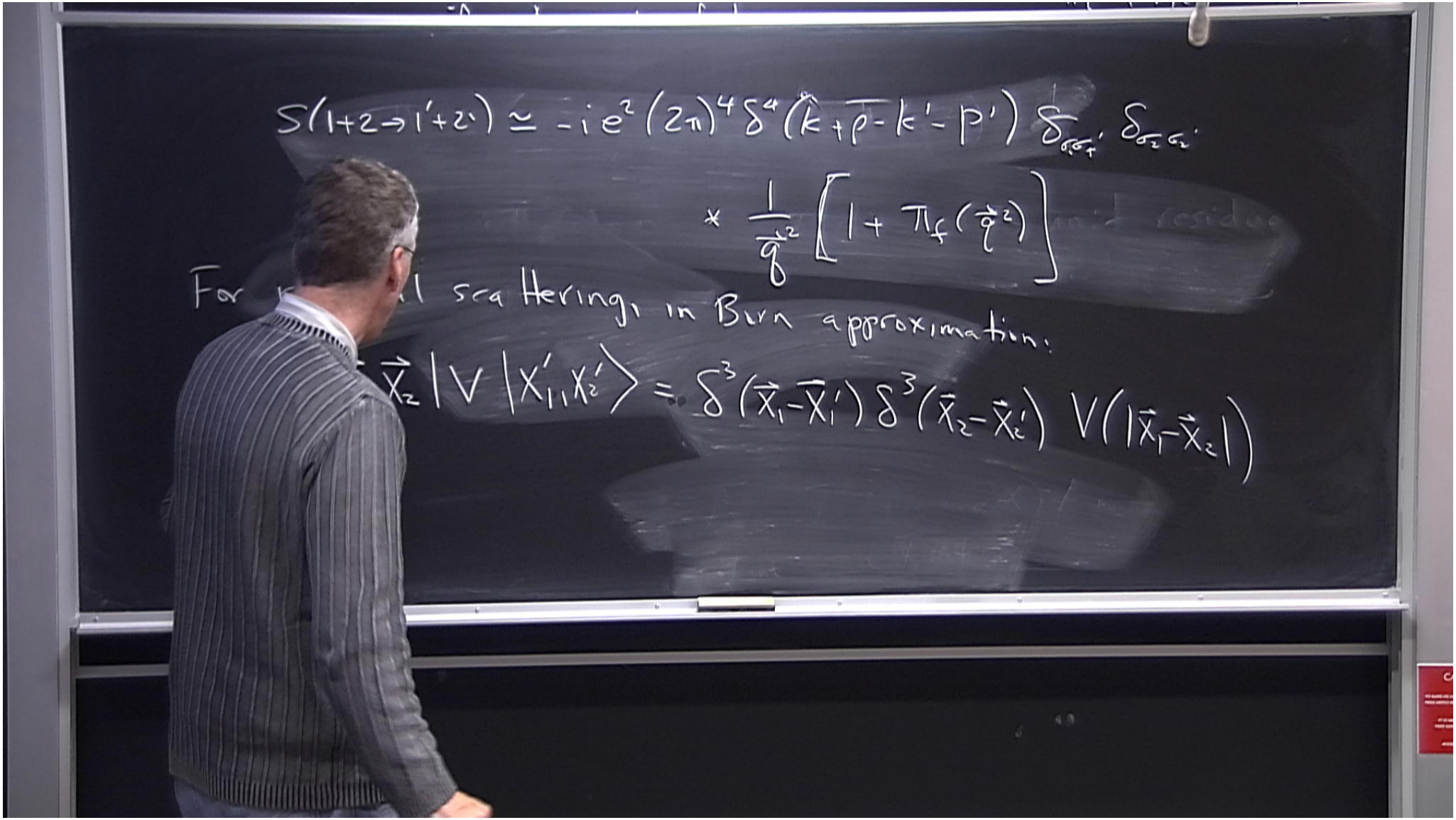

$S(1+2 \to 1'+2') \simeq -ie^2 (2\pi)^4 \delta^4(k+p-k'-p') \delta_{\sigma_1\sigma_1'} \delta_{\sigma_2\sigma_2'}$
$\frac{1}{q^2}$ [

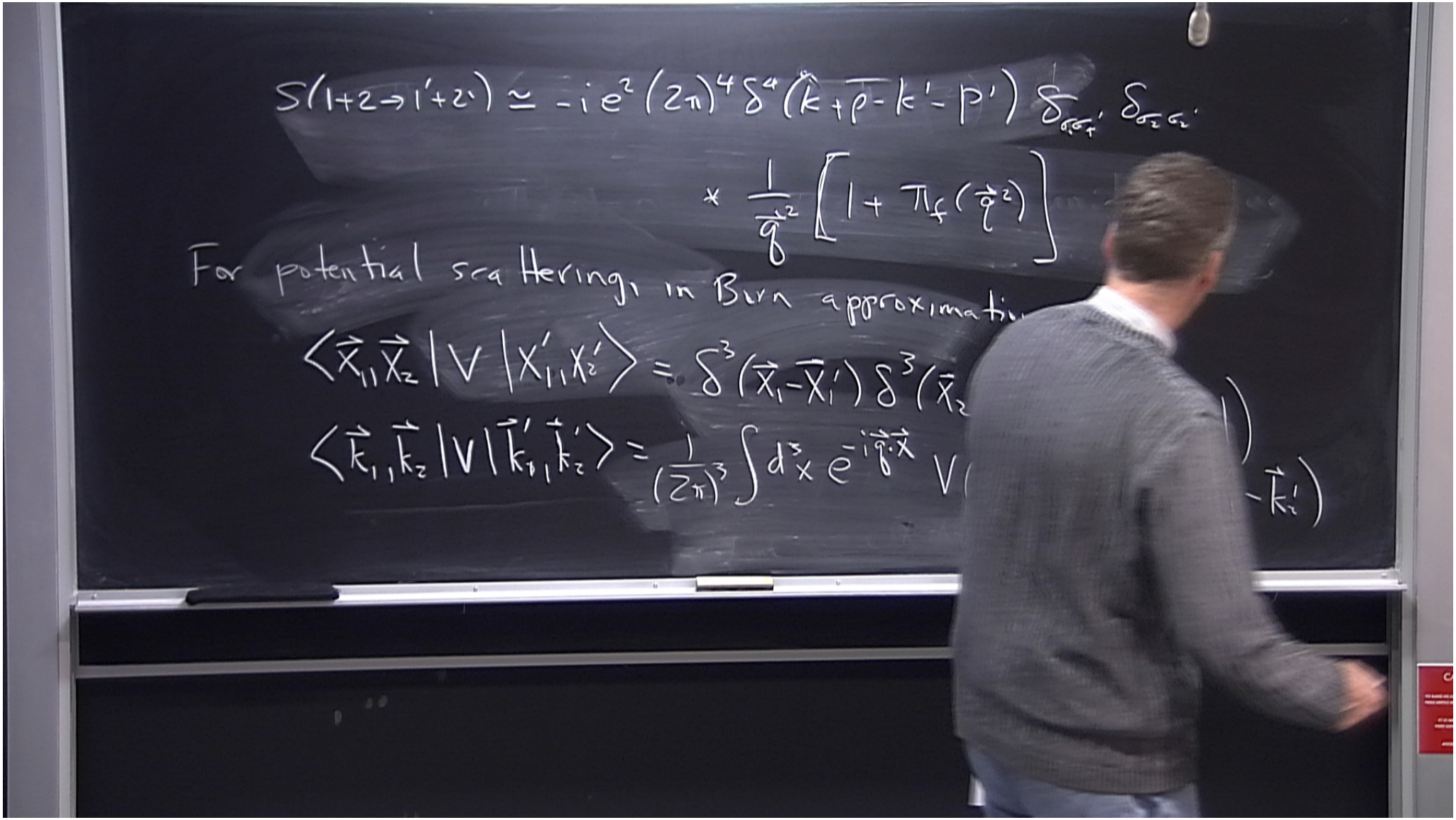

$S(1+2 \to 1'+2') \simeq -ie^2 (2\pi)^4 \delta^4(k+p-k'-p') \delta_{\sigma_1\sigma_1'} \delta_{\sigma_2\sigma_2'}$
$\times \frac{1}{\vec{q}^2} \left[1 + \Pi_f(\vec{q}^2) \right]$

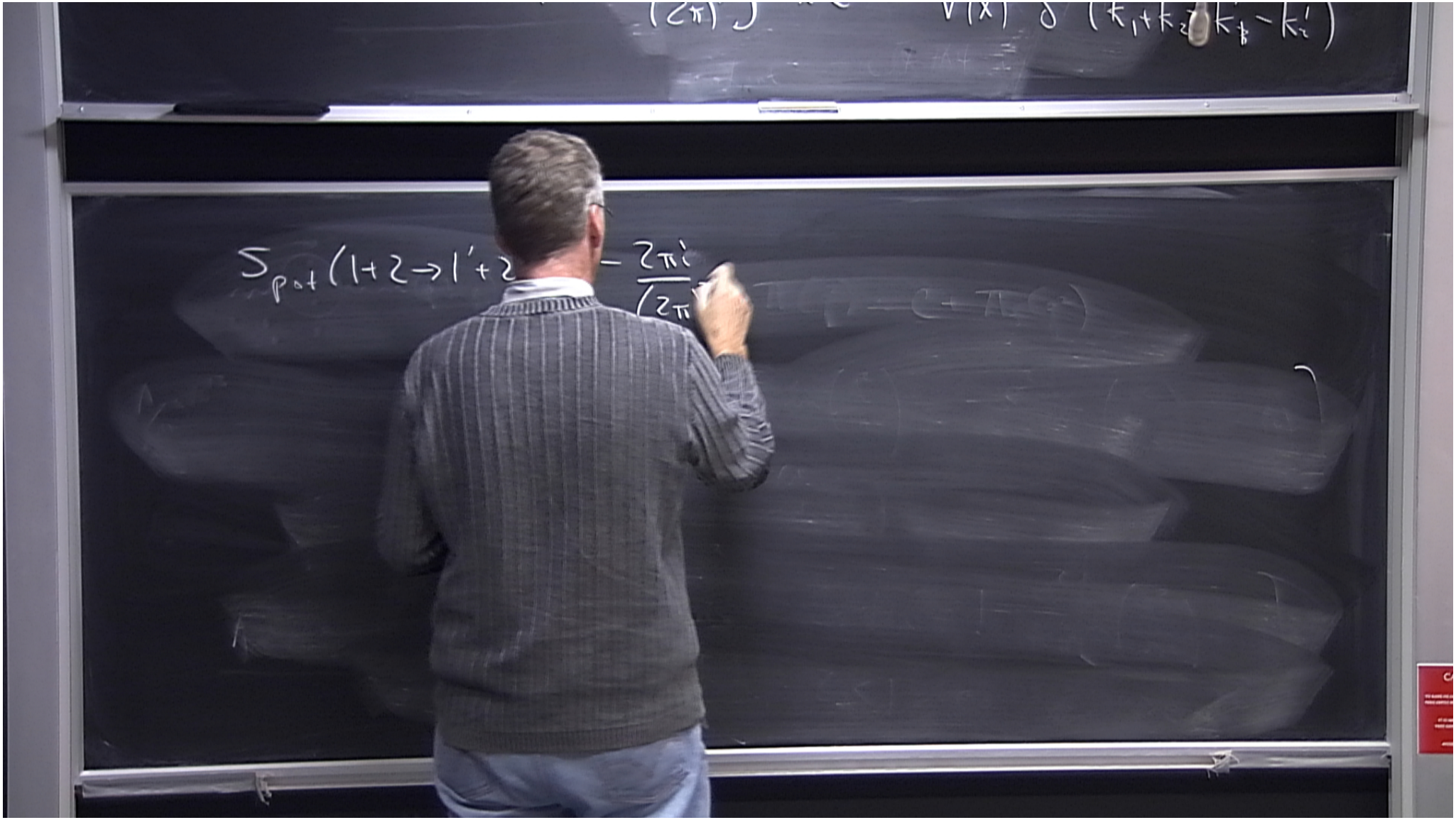

$S(1+2 \to 1'+2') \ldots i e^2 (2\pi)^4 \delta^4(k+p-k'-p') \delta_{\sigma\sigma'} \delta_{\sigma_2\sigma_2'}$
$\times \frac{1}{\vec{q}^2}\left[1 + \Pi_f(\vec{q}^2)\right]$
For potential

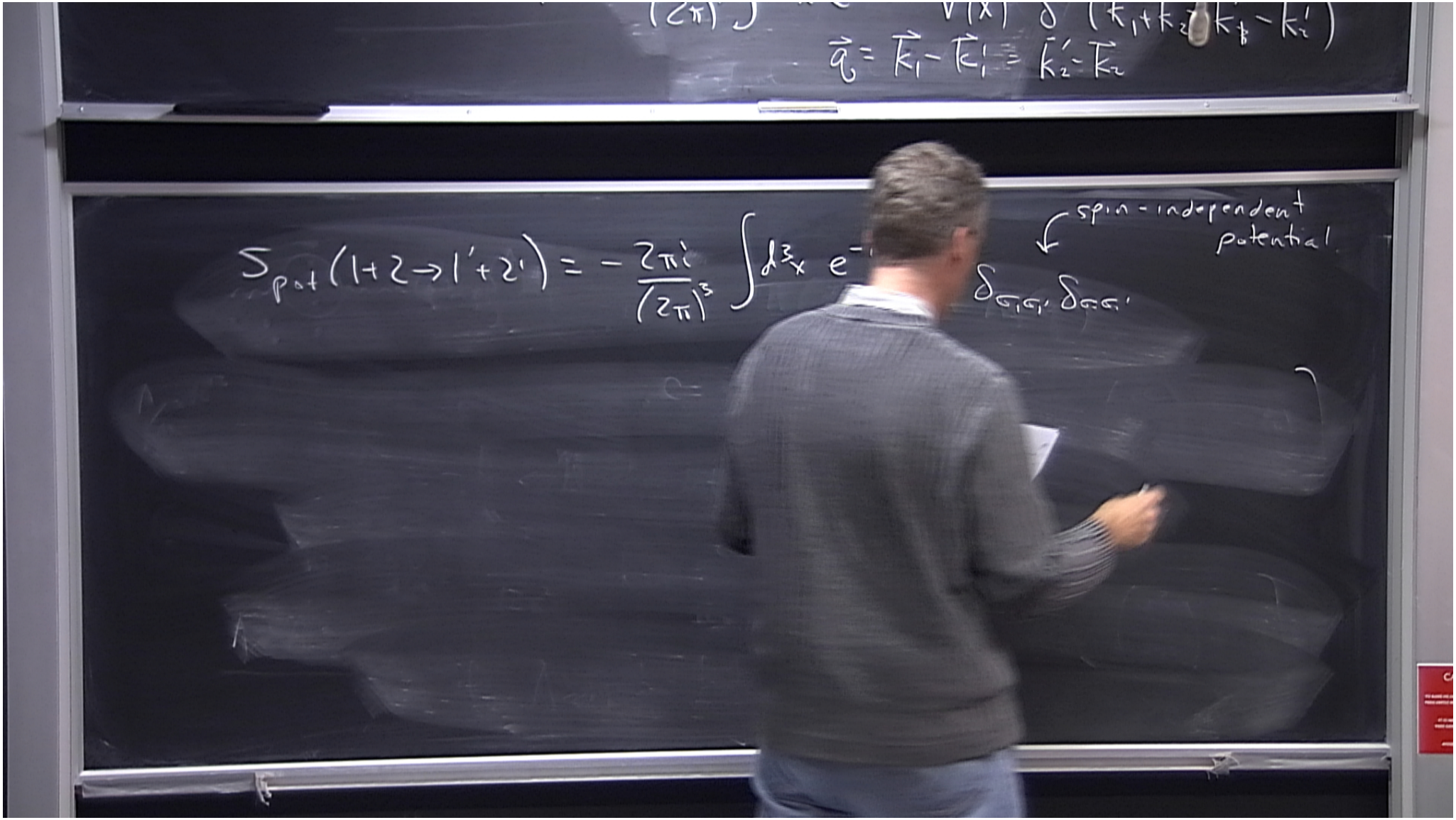

$S(1+2 \to 1'+2') \simeq -ie^2 (2\pi)^4 \delta^4(k+p-k'-p') \delta_{\sigma_1\sigma_1'} \delta_{\sigma_2\sigma_2'}$
$* \frac{1}{\vec{q}^2} \left[1 + \Pi_f(\vec{q}^2) \right]$
For nal scattering in Born approximation:


$S(1+2 \to 1'+2') \simeq -ie^2(2\pi)^4 \delta^4(k+p-k'-p')\,\delta_{\sigma_1\sigma_1'}\,\delta_{\sigma_2\sigma_2'}$
$\times \frac{1}{\vec{q}^2}\left[1+\Pi_f(\vec{q}\right.$
residue
For potential scattering, in Born appr
$\langle \vec{x}_1, \vec{x}_2 | V | x_1', x_2' \rangle = \delta^3(\vec{x}_1 - \vec{x}_1'$
$x_2|$


$S(1+2 \to 1'+2') \simeq -ie^2(2\pi)^4 \delta^4(k+p-k'-p')\,\delta_{\sigma_1\sigma_1'}\,\delta_{\sigma_2\sigma_2'}$
$\times \frac{1}{\vec{q}^2}\left[1+\Pi_f(\vec{q}^2)\right]$
For scattering in Born approximation:
$\langle \vec{x}_1, \vec{x}_2 | V | x_1', x_2' \rangle = \delta^3(\vec{x}_1-\vec{x}_1')\,\delta^3(\vec{x}_2-\vec{x}_2')\,V(|\vec{x}_1-\vec{x}_2|)$


$S(1+2\to 1'+2') \simeq -ie^2(2\pi)^4\delta^4(k+p-k'-p')\,\delta_{\sigma_1\sigma_1'}\,\delta_{\sigma_2\sigma_2'}$
$\times \frac{1}{\vec{q}^2}\left[1+\pi_f(\vec{q}^2)\right]$
For potential scattering in Born approximati
$\langle \vec{x}_1,\vec{x}_2|V|x_1',x_2'\rangle = \delta^3(\vec{x}_1-\vec{x}_1')\,\delta^3(\vec{x}_2$
$\langle \vec{k}_1,\vec{k}_2|V|\vec{k}_1',\vec{k}_2'\rangle = \frac{1}{(2\pi)^3}\int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\, V($
$-\vec{k}_2')$




$\vec{q} = \vec{k}_1 - \vec{k}_1' = \vec{k}_2' - \vec{k}_2$
$S_{pot}(1+2 \to 1'+2') = -\frac{2\pi i}{(2\pi)^3} \int d^3x \, e^{-}$
spin-independent potential
$\delta_{\sigma_1\sigma_1'} \delta_{\sigma_2\sigma_2'}$

$$\vec{q} = \vec{k}_1 - \vec{k}_1' = \vec{k}_2' - \vec{k}_2$$

spin-independent potential

$$S_{pot}(1+2 \to 1'+2') = -\frac{2\pi i}{(2\pi)^3} \int d^3x \, e^{-i\vec{q}\cdot\vec{x}} \, V(\vec{x}) \, \delta_{\sigma_1\sigma_1'} \delta_{\sigma_2\sigma_2'} \, \delta^4(k_1 + k_2 - k_1' - k_2')$$

equate S_{pot} with S:

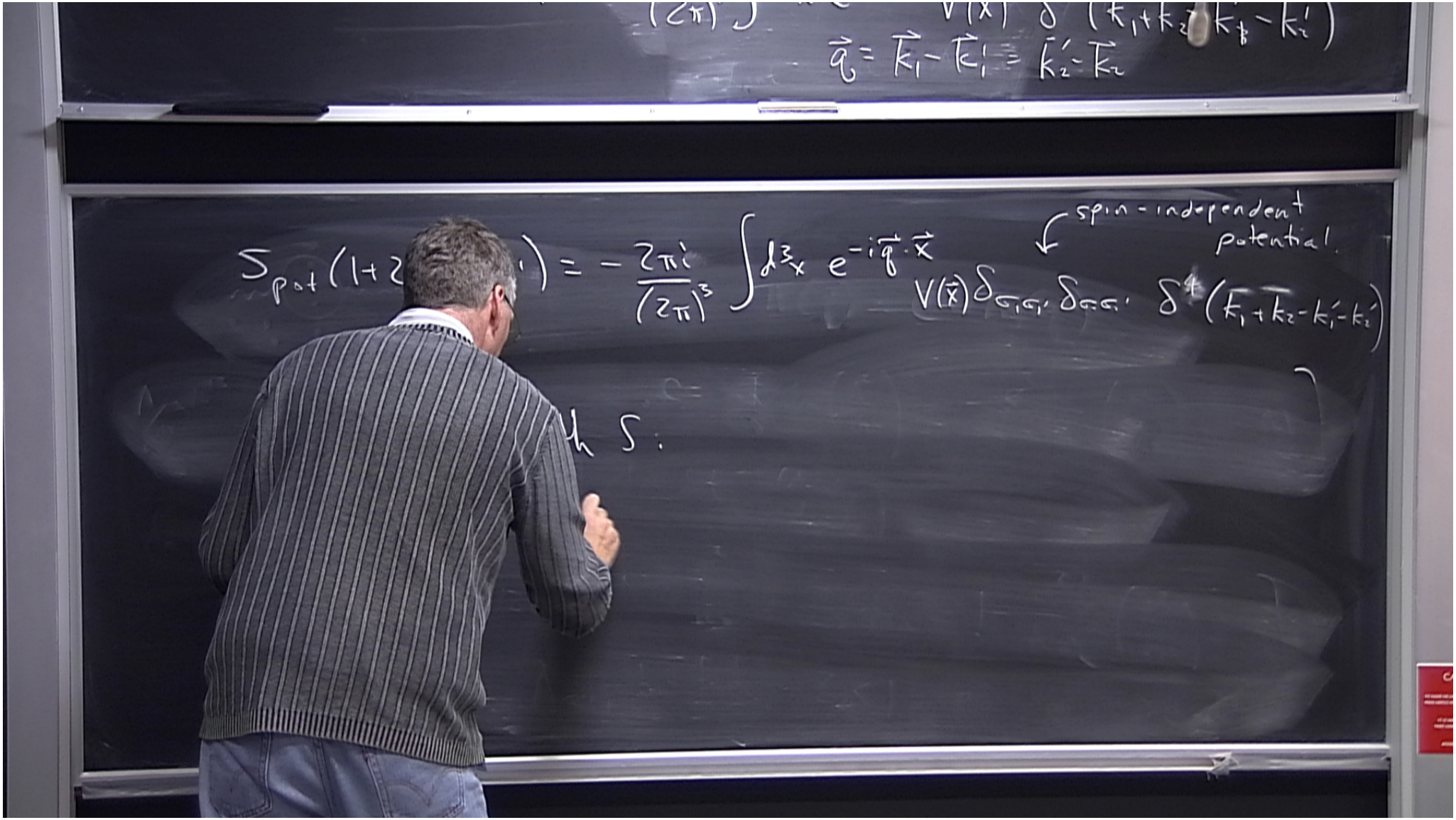
$$S(1+2 \to 1'+2') = \int d^4q \, (2\pi)^4 \delta^4(k'+q-k)\,(-ie)\; i(2\pi)^4 (-ie)\,\delta^4(q+p-p')$$

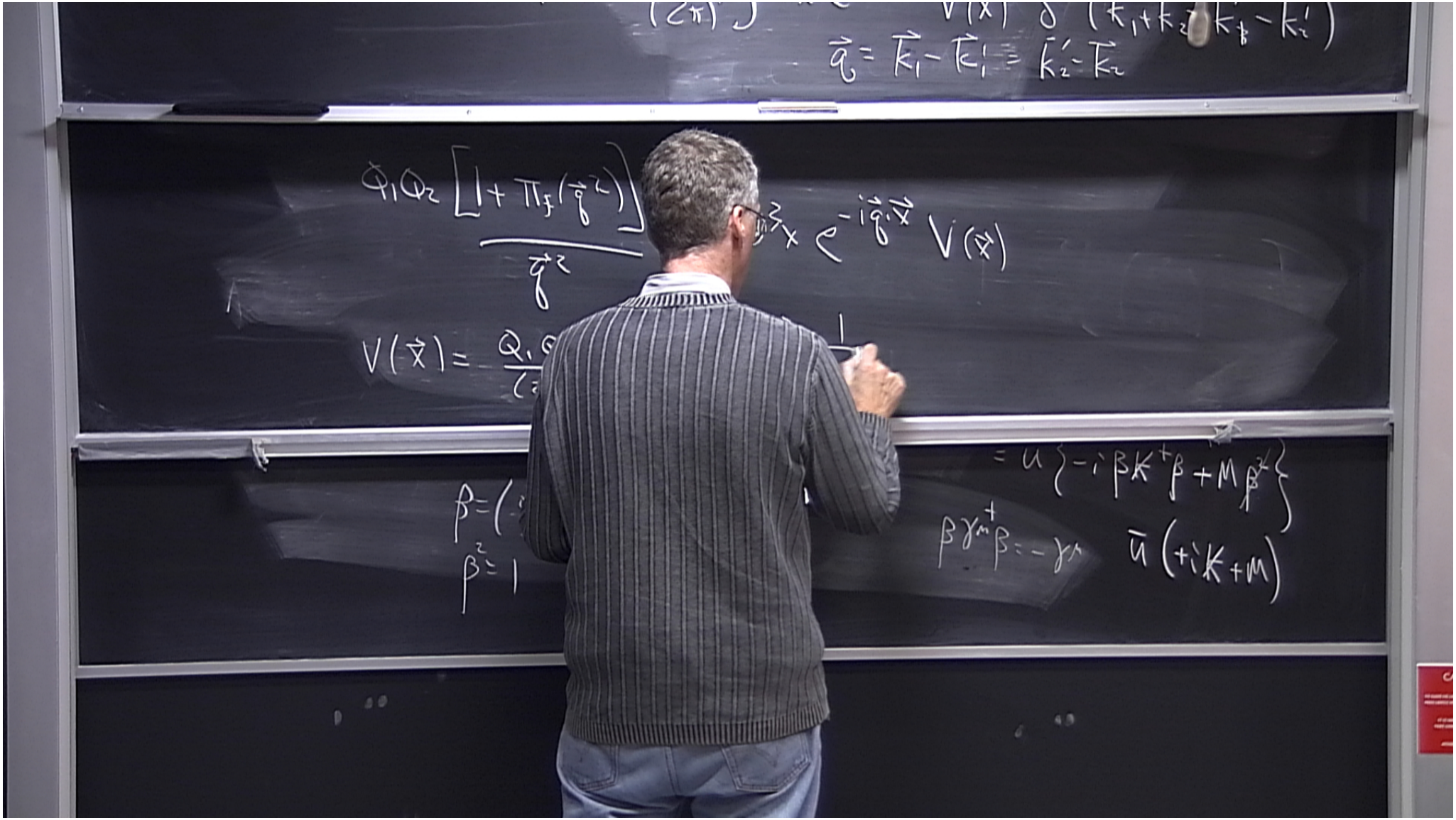
$$\frac{-i}{(2\pi)^4}\left\{\left[\frac{\eta_{\mu\nu}}{q^2-i\epsilon} + \alpha \frac{q_\mu q_\nu}{}\right] + \left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\frac{\Pi_f(q^2)}{(q^2-i\epsilon)^2}\right\}$$

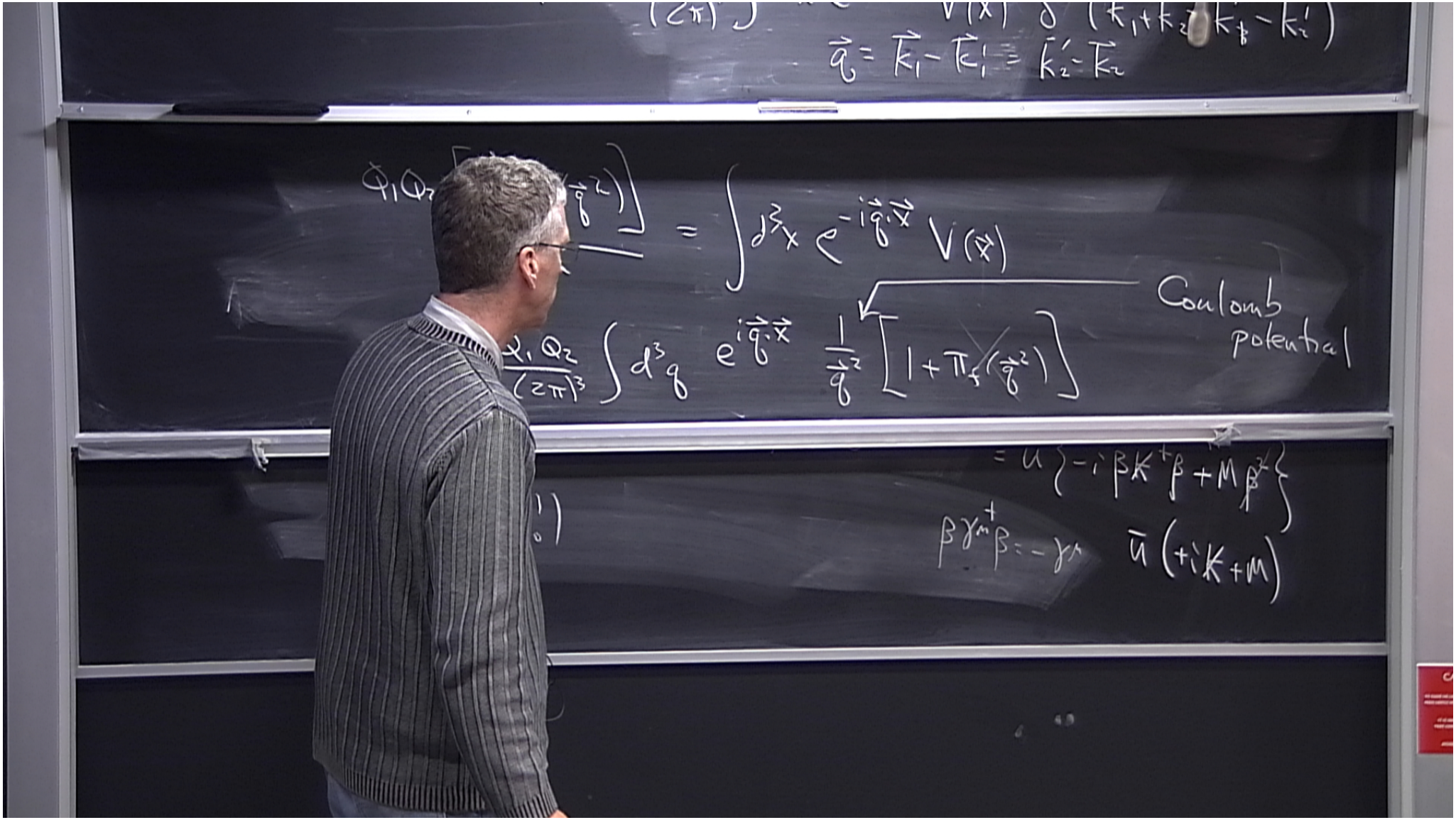
$$(\bar{u}_1'\gamma^\mu u_1)(\bar{u}_2'\gamma^\nu u_2)$$

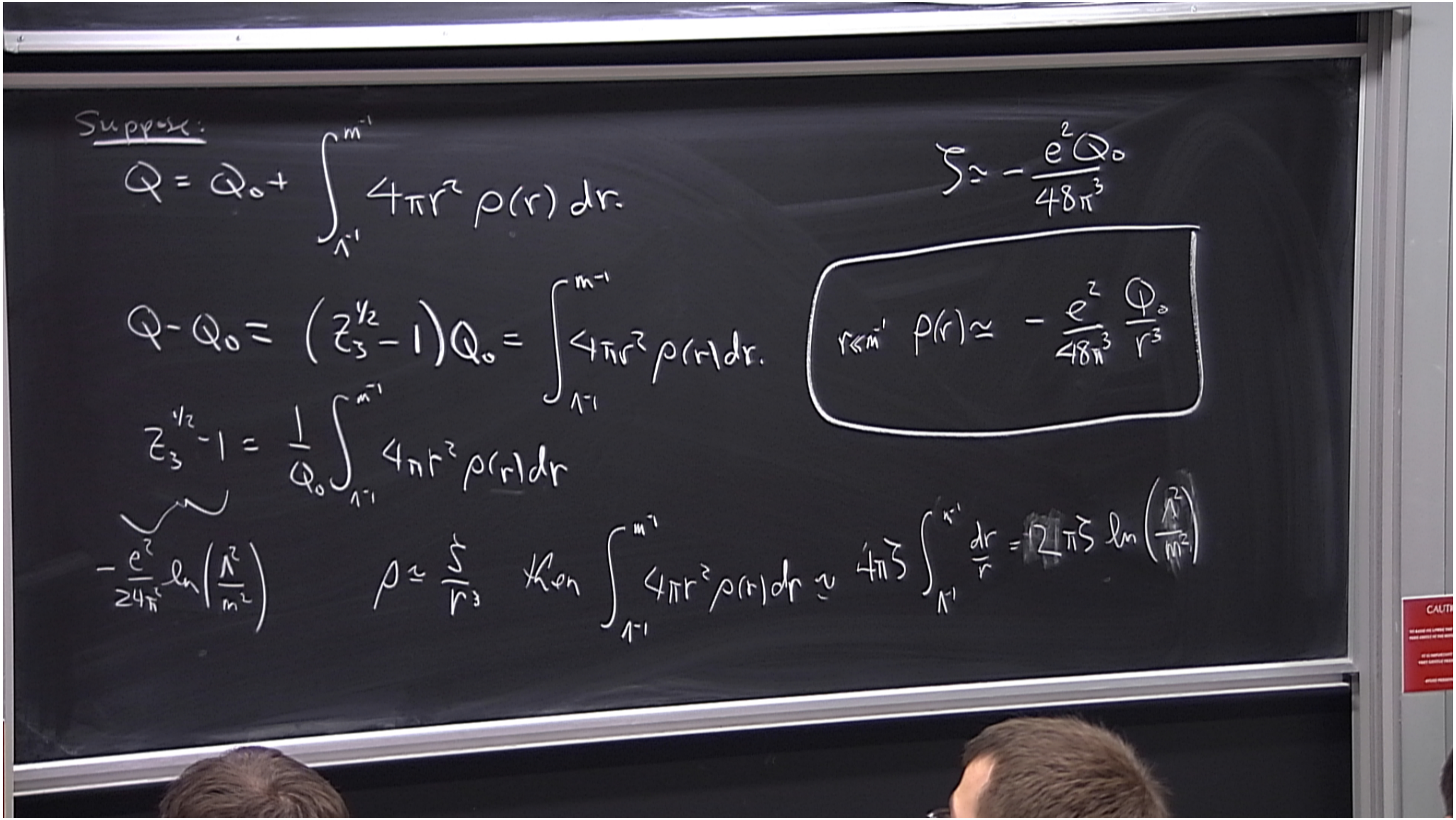
$$i(2\pi)^4 \Pi_{\mu\nu}\,\delta^4(p-p')$$

$$= -ie^2 Q_1 Q_2 (2\pi)^4 \delta^4(p+k-p'-k')(\bar{u}_1'\gamma^\mu u_2)(\bar{u}_2'\gamma^\nu u_2)\left\{\frac{\eta_{\mu\nu}}{q^2-i\epsilon}\left[1+\Pi_f(q^2)\right] + q_\mu q_\nu \text{ terms}\right\}$$


$\vec{q} = \vec{k}_1 - \vec{k}_1' = \vec{k}_2' - \vec{k}_2$
spin-independent potential
$V(\vec{x})\,\delta_{\sigma_1\sigma_1'}\,\delta_{\sigma_2\sigma_2'}\,\delta^4(k_1 + k_2 - k_1' - k_2')$


$\vec{q} = \vec{k}_1 - \vec{k}_1' = \vec{k}_2' - \vec{k}_2$
$Q_1 Q_2 \left[1 + \Pi_f(\vec{q}^2)\right] / \vec{q}^2$
$e^{-i\vec{q}\cdot\vec{x}} V(\vec{x})$
$V(\vec{x}) =$
$= u\{-i\beta K^+\beta + M\beta^2\}$
$\beta\gamma^{\mu\dagger}\beta = -\gamma^\mu$
$\bar{u}(+iK + M)$
$\beta^2 = 1$


$\vec{q} = \vec{k}_1 - \vec{k}_1' = \vec{k}_2' - \vec{k}_2$
$= \int d^3x\, e^{-i\vec{q}\cdot\vec{x}}\, V(\vec{x})$
Coulomb potential
$\frac{1}{\vec{q}^2}\left[1 + \pi_f(\vec{q}^2)\right]$
$\bar{u}\left(+i\not{K} + M\right)$


Suppose:
$Q = Q_0 + \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r)\, dr.$
$\zeta \simeq -\frac{e^2 Q_0}{48\pi^3}$
$Q - Q_0 = (Z_3^{1/2} - 1) Q_0 = \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr.$
$r \ll m^{-1}$ $\rho(r) \simeq -\frac{e^2}{48\pi^3} \frac{Q_0}{r^3}$
$Z_3^{1/2} - 1 = \frac{1}{Q_0} \int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr$
$-\frac{e^2}{24\pi^2} \ln\left(\frac{\Lambda^2}{m^2}\right)$
$\rho \simeq \frac{\zeta}{r^3}$ then $\int_{\Lambda^{-1}}^{m^{-1}} 4\pi r^2 \rho(r) dr \simeq 4\pi\zeta \int_{\Lambda^{-1}}^{m^{-1}} \frac{dr}{r} = 12\pi\zeta \ln\left(\frac{\Lambda^2}{m^2}\right)$