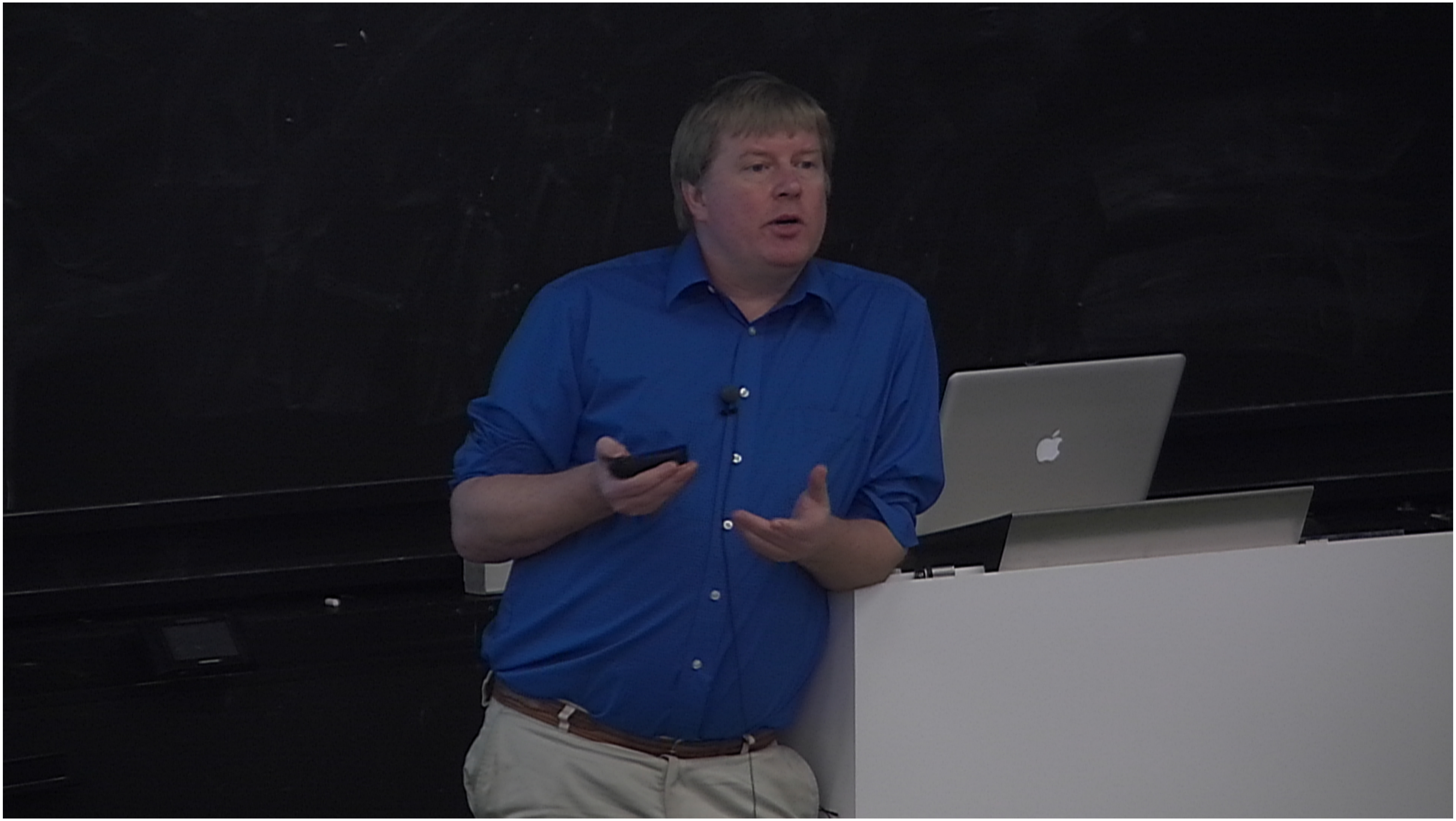
Title: Conformal Symmetry for General Black Holes

Date: Feb 09, 2012 02:00 PM

URL: http://pirsa.org/12020116

Abstract: The boundary conditions of general black holes in asymptotically flat spacetimes can be modified such that a conformal symmetry emerges. The black holes with asymptotic geometry modified in this manner satisfy the equations of motion of minimal supergravity in one dimension more. Their symmetry suggests that a dual conformal field theory description exists that can account for the black hole entropy even in the case of black holes that are far from extremality.



Outline

- ***The Big Picture***: phenomenology of black hole entropy.
- ***Black Hole Thermodynamics:*** a suggestive computation.
- ***Conformal Symmetry for General Black Holes***

  Isolating the near horizon geometry that exhibits conformal symmetry.
- Some references:

  FL, arXiv: 9702153.

  M. Cvetic and FL, arXiv: 9705192, 1106.3341, 1112.nnnn .

  A. Castro, A. Maloney, and A. Strominger, arXiv:1004.0996.

The Guiding Assumption

- Very general black holes (in $N \geq 2$ string theory) are usefully described as ***2D CFTs***.

- The guiding assumption is that such a theory applies to a class of black holes so large that ***special cases*** include:

  Kerr black holes (including ***Schwarzchild*** black holes, as the non-rotating limit).

  String theory black holes with many independent charges so that suitable "large charge" limits include ***AdS$_3$*** that are under good control and ***supersymmetric*** limits that are under even better control.

3

- ***The Goal (for now)***.

  Discovery of a precise *semi-classical* conformal symmetry.

  This amounts to a focus on large black holes and restriction to the asymptotic behavior of the entropy.

  This limit should suffice to specify the CFT central charge, the temperature, and account for the black hole entropy in the leading semi-classical approximation.

- ***The Goal (long term)***.

  Precise counting of black holes microstates.

  Outline: the two chiral sectors represent SUSY states (with 4 SUSY's) and non-BPS extremal states, respectively.

  A direct product (with zero-modes matched appropriat
  give all states.

Entropy Phenomenology (Kerr)

The Entropy of Kerr black holes

$$S = 2\pi \left(G_4 M^2 + \sqrt{G_4^2 M^4 - J^2} \right) \ .$$

The form of the entropy suggests an obvious chiral split

$$S = S_L + S_R$$

If this is the right interpretation we can define physical temperatures for left and right movers independently by the generalized first laws

$$dS_L = \beta_L dM \ \ , \quad dS_R = \beta_R dM \ .$$

(It follows that the Hawking temperature is $\beta_H = \beta_L + \beta_R$) .

5

Model for ***Angular Momentum:*** in the 2D CFT dual to 4D black holes only R-movers have the ability to carry J. The D=4 angular momentum is identified with an R-charge of a (0,4) SCFT in D=2.

5D Black holes have two angular momenta and are modelled by a (4,4) SCFT.

Following the suggestions we model for the *general* black hole entropy in terms of a 1D gas on a circle with radius $\mathcal{R}$:

$$S_{R,L} = \frac{\pi}{6} c T^{\rm phys}_{R,L} \times 2\pi\mathcal{R} \; .$$

The central charge and the radius $\mathcal{R}$ are not determined from the phenomenological model but their product is

$$c\mathcal{R} = 24 G_4^2 M^3 \; .$$

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The Inner Horizon

The split of the full entropy into two chiral parts is controlled geometrically by the area of the *inner* horizon

$$S_L - S_R = \frac{A_-}{4G_4} \,.$$

(This is for 4D, 5D and N=2 SUSY; in other cases a generalization applies).

Apparently there is a related quantization condition

$$\frac{1}{(8\pi G_4)^2} A_+ A_- = J^2 \,.$$

that can be interpreted as level matching of the 2D CFT dual to the general black holes.

Parametric Charges

The addition of charges (in the context of $N = 4, 8$ SUGRA) involves the introduction of parametric mass and angular momentum m, a and parametric charges δ_i:

$$\begin{aligned} G_4 M &= \frac{1}{4} m \sum_{i=1}^{4} \cosh 2\delta_i \ , \\ G_4 Q_i &= \frac{1}{4} m \sinh \delta_i \ , \ (i = 1, 2, 3, 4) \ , \\ G_4 J &= ma(\Pi_c - \Pi_s) \ , \end{aligned}$$

where we employ the abbreviations

$$\Pi_c \equiv \prod_{i=1}^{4} \cosh \delta_i \ , \quad \Pi_s \equiv \prod_{i=1}^{4} \sinh \delta_i \ .$$

Entropy Revisited

The entropy is much more complicated after general charges have been added

$$S_L = \frac{2\pi m^2}{G_4}(\Pi_c + \Pi_s) \ ,$$

$$S_R = 2\pi\sqrt{\frac{m^4}{G_4^2}(\Pi_c - \Pi_s)^2 - J^2} \ .$$

But: the dependence on angular momentum can still be accounted for by the physical assumption that only right movers are able to carry angular momentum. We are on the right track!

Also: the quantization condition is preserved with

$$\frac{1}{(8\pi G_4)^2}A_+A_- = J^2 + J_4 \ .$$

10

The central charge is

$$c\mathcal{R} = \frac{24m^3}{G_4}(\Pi_c^2 - \Pi_s^2)$$

In this expression we can take the dilute gas limit. Then $c = 6n_1n_2n_3$ and $\mathcal{R}$ is an actual modulus, the size of the compactification scale that momentum excitations are distributed along. By assumption the additional compactification parameters (eg a CY radius) have smaller scale.

The Black Hole Geometry

The explicit geometry with all charges turned on

$$ds_4^2 = -\Delta^{-\frac{1}{2}} G(dt + \mathcal{A}_\phi d\phi)^2 + \Delta^{\frac{1}{2}} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X \sin^2\theta}{G} d\phi^2 \right) ,$$

where

$$\begin{aligned} X &= r^2 - 2mr + a^2 , \\ G &= r^2 - 2mr + a^2 \cos^2\theta , \\ \mathcal{A} &= \frac{2ma \sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi , \\ \Delta_0 &= \prod_{i=1}^{4} (r + 2m \sinh^2\delta_i) + 2a^2 \cos^2\theta [r^2 + mr \sum_i \sinh^2\delta_i \\ & \quad + 4m^2 (\Pi_c - \Pi_s)\Pi_s + 2m^2 \sum_{i<j<k} \sinh^2\delta_i \sinh^2\delta_j \sinh^2\delta_k + \frac{1}{2} a^2] . \end{aligned}$$

Abbreviation: $\mathcal{A}_{\rm red} = \frac{G}{a \sin^2\theta} \mathcal{A}_\phi$ depends on r alone.

The Laplacian

The entropy formula suggests a dual 2D CFT. But the corresponding features in the scalar wave equation are much more precise.

The general Laplacian:

$$\Delta_0^{-\frac{1}{2}}[\partial_r X \partial_r - \frac{1}{X}(\mathcal{A}_{\rm red}\partial_t - \partial_\phi)^2 + \frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G}\partial_t^2 + \frac{1}{\sin\theta}\partial_\theta \sin\theta \partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi^2]$$

Recall: Kerr black holes allow separation of variables. The addition of charges (largely) maintains this property:

$$\frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G} = r^2 + 2mr\left(1 + \sum_{i=1}^{4} s_i^2\right) + 8m^2(\Pi_c - \Pi_s)\Pi_s$$
$$-4m^2 \sum_{i<j<k} s_i^2 s_j^2 s_k^2 + a^2\cos^2\theta \, .$$

Hypergeometric Structure

The resulting radial equation is in the class with two regular singularities (the inner and outer horizons) and one irregular singularity (infinity).

The indices (exponents) of the regular singularities are the inverse temperatures (times $\frac{i}{2\pi}$).

The singularity at infinity is irregular because $\Delta_0 \sim r^4$ in order to approach flat space.

If $\Delta_0 \sim r^2$ the singularity would be regular.

In this case there would be an $SL(2)$ symmetry that permutes the three singularities. This is desirable because it is an ingredient needed to identify a Virasoro algebra.

R-symmetry?

If the stronger condition $\Delta_0 \sim r$ (for large r) is satisfied, the angular Laplacian recovers spherical symmetry. This indicates an unbroken R-symmetry.

Additionally, in this case the indices of the regular singularity at infinity would be precisely $(l, -l-1)$, the well known conformal weights of (non-normalizable, normalizable) modes of a scalar field in AdS_3.

This stronger assumption is appropriate for near BPS black holes but apparently not for near-extreme Kerr.

15

The Physical Obstacle

Assumptions on Δ_0 can be justified physically by restricting to some region of space (like the near horizon region) or by focus on black holes with specific parameters.

In the wave equation we compare

$$-\frac{1}{X}(\mathcal{A}_{\rm red}\partial_t - \partial_\phi)^2 \quad \text{vs} \quad \frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G}\partial_t^2$$

Generically the first term dominates near the horizon and the second term dominates at large r.

Decoupling of near horizon geometry and asymptotic space: the terms are never simultaneously important so we can consistently ignore the second term.

Decoupling holds in various near extreme limits but *not* in general. It therefore fails for generic black holes where there is only one scale.

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16

Hidden Conformal Symmetry

Let us proceed (for now) by simply ignoring the offending terms in the wave equation.

Then we can construct the *normalized* $SL(2)^2$ generators:

$$L_0 = \frac{i}{2\pi}(\beta_H \Omega_H \partial_\phi - \beta_R \partial_t) = i\partial_{t_+} ,$$
$$\bar{L}_0 = \frac{i\beta_L}{2\pi}\partial_t = i\partial_{t_-} ,$$

where the "CFT-coordinates" are

$$t^+ = \frac{2\pi}{\beta_H \Omega_H}\phi \ , \quad t^- = \frac{2\pi}{\beta_L}(t - \frac{\beta_R}{\beta_H \Omega_H}\phi) \ .$$

The periodicity 2π of ϕ suggests

$$T_R^{\rm CFT} = \frac{1}{\beta_H \Omega} \ , \quad T_L^{\rm CFT} = \frac{\beta_R}{\beta_L}\frac{1}{\beta_H \Omega} \ .$$

The $T_{R,L}^{\mathrm{CFT}}$ inferred in this manner are proportional to the phenomenological $T_{R,L}^{\mathrm{phys}}$.

The proportionality constant determines the effective radius as

$$\mathcal{R} = \frac{\beta_R}{\beta_H \Omega_H} \ .$$

The phenomenological model gave $c\mathcal{R}$ so we can compute the central charge. The result obtained this way is

$$c = 12J \ .$$

This is the answer obtained for extreme Kerr black holes. The reasoning indicates that the extreme answer applies to all black holes!

Questions

Why ignore the terms that explicitly break $SL(2)^2$?

Is $SL(2)^2$ extended to a Virasoro algebra?

Is the reasoning limited to the wave equation?

Is the inverse temperature not the *imaginary* periodicity?

Why does the near extreme limit (where $\mathcal{R}$ is a modulus and $c = 6n_1n_2n_3$) not apply?

20

General Causal Structure

Recall the full metric

$$ds_4^2 = \Delta^{\frac{1}{2}} \left(\frac{dr^2}{X} + \frac{X \sin^2 \theta}{G} d\phi^2 \right) + \left[\Delta^{\frac{1}{2}} d\theta^2 - \Delta^{-\frac{1}{2}} G (dt + \mathcal{A}_\phi d\phi)^2 \right]$$

Assume $X = X(r)$, $G = X - a^2 \sin^2 \theta$
(and $\mathcal{A}_\phi$, Δ general functions of r, θ).

The ergosphere (inside static limit):

$$G = 0 \, .$$

The event horizon (t, ϕ subspace changes signature):

$$X = 0 \, .$$

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21

General Thermodynamics

The thermodynamics for the general black holes:

$$\Omega_H = \frac{a}{(\mathcal{A}_{\rm red})_{\rm hor}}\,,$$
$$\beta_H = \frac{4\pi(\mathcal{A}_{\rm red})_{\rm hor}}{(\partial_r X)_{\rm hor}}\,,$$
$$S = \frac{\pi(\mathcal{A}_{\rm red})_{\rm hor}}{G_4}\,.$$

Computations assume that $\mathcal{A}_{\rm red} \sim \frac{G}{a\sin^2\theta}\mathcal{A}_\phi$ depends on r only because then Ω_H is constant on the horizon. This assumption holds for explicit solutions.

A feature: the reduced angular potential $\mathcal{A}_{\rm red}$ is introduced for rotating black holes only but it is finite in the limit of vanishing rotation only.

The abstract expressions give the correct thermodynamics for the explicit solutions.

22

Interpretation of the Conformal Factor

A striking feature of the causal structure and thermodynamics computed with minimal assumptions: the conformal factor Δ does not appear.

Interpretation: Δ is a feature of the embedding into an ambient spacetime.

It can be adjusted to realize a “box” that provides an infrared regulator of the thermodynamics in general black hole spacetimes (which generically have negative specific heat).

Alternate Conformal Factors

Proposal: simply choose Δ so that $\Delta \sim r$ for large r.

Technical assumptions: separability survives and analyticity in r.

Then Δ is uniquely determined as

$$\Delta = (2m)^3(\Pi_c^2 - \Pi_s^2)r + (2m)^4\Pi_s^2 - (2m)^2(\Pi_c - \Pi_s)^2 a^2 \cos^2\theta$$

The geometry with this Δ has a Laplacian that is exactly the hypergeometric Laplacian.

Progress: now the analysis does not rely on the scalar field probe.

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Comments

Feature: the equations of motion is not satisfied for general Δ.

Interpretation: the sources supporting the modified Δ represents the physical matter that realizes the box enclosing the black hole.

In situations where the offending terms could be omitted for some reason (eg an approximation) the additional matter is negligible.

A 5D Lift

It is illuminating to lift the 4D geometry to a 5D geometry, via an auxiliary coordinate α:

$$\begin{aligned} ds_5^2 &= \Delta(d\alpha + \mathcal{B})^2 + \Delta^{-1/2} ds_4^2 \\ &= -\frac{X}{S}dt^2 + \frac{dr^2}{X} + \frac{S}{4m^2(\Pi_c - \Pi_s)^2}(d\alpha + \frac{\mathcal{A}_{\rm red}}{S}dt)^2 + d\Omega_2'^2 \ . \end{aligned}$$

The linearly shifted radial coordinate:

$$S = 8m^3[r(\Pi_c^2 - \Pi_s^2) - \frac{a^2}{2m}(\Pi_c - \Pi_s)^2 + 2m\Pi_s^2] \ .$$

In this form separability is manifest.

Bonus: massive scalar fields that couple minimally to the 5D geometry are separable.

Virasoro

The full 5D space is locally $\mathrm{AdS}_3 \times S^2$ so there is a $\mathrm{Virasoro}^2$ symmetry.

Deformations of the 4D spacetime are those that are independent of α so the full Virasoro is not realized on this space.

The parameter $\mathcal{R}$ is not determined by the procedure because the radius of α is arbitrary *a priori*.

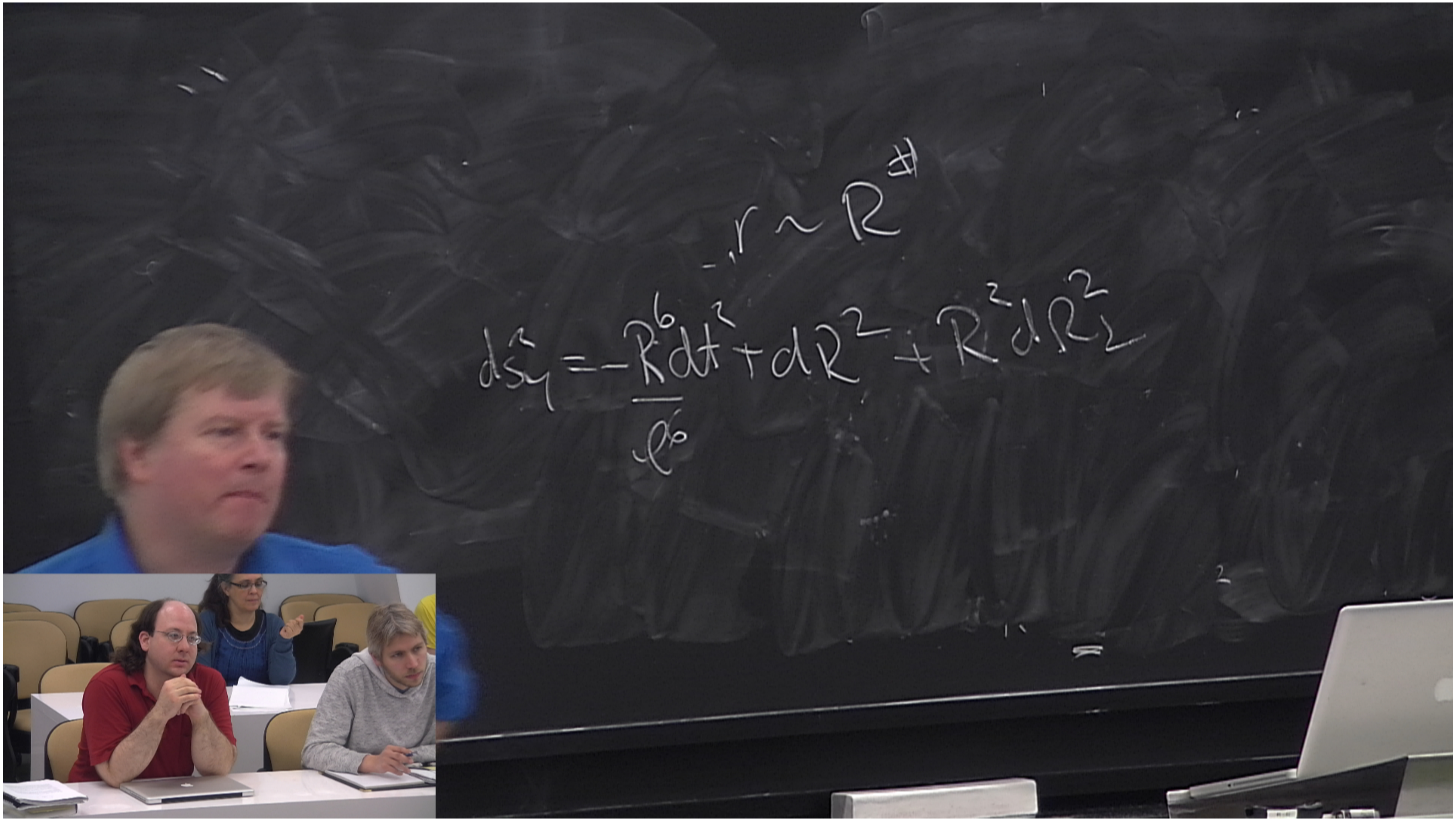
27

Summary

Progress towards conformal symmetry of general black holes:

- Black hole thermodynamics in a manner that exhibits independence of the conformal factor.
- A geometric construction that isolates the parts of the geometry that has $SL(2)^2$ symmetry.
- Rudiments of a full Virasoro symmetry even for very general black holes.

28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