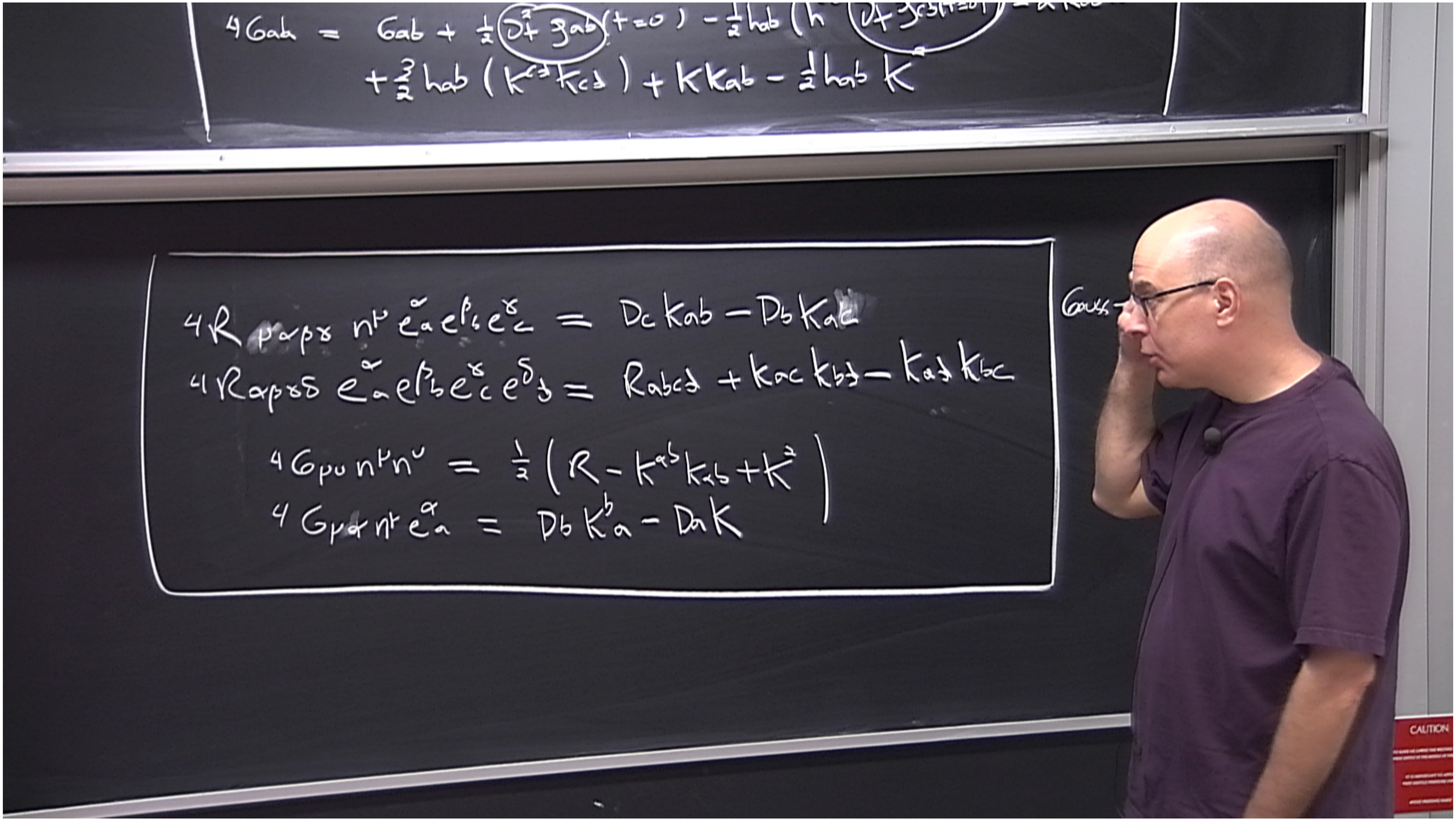
Title: Advanced General Relativity - Lecture 12

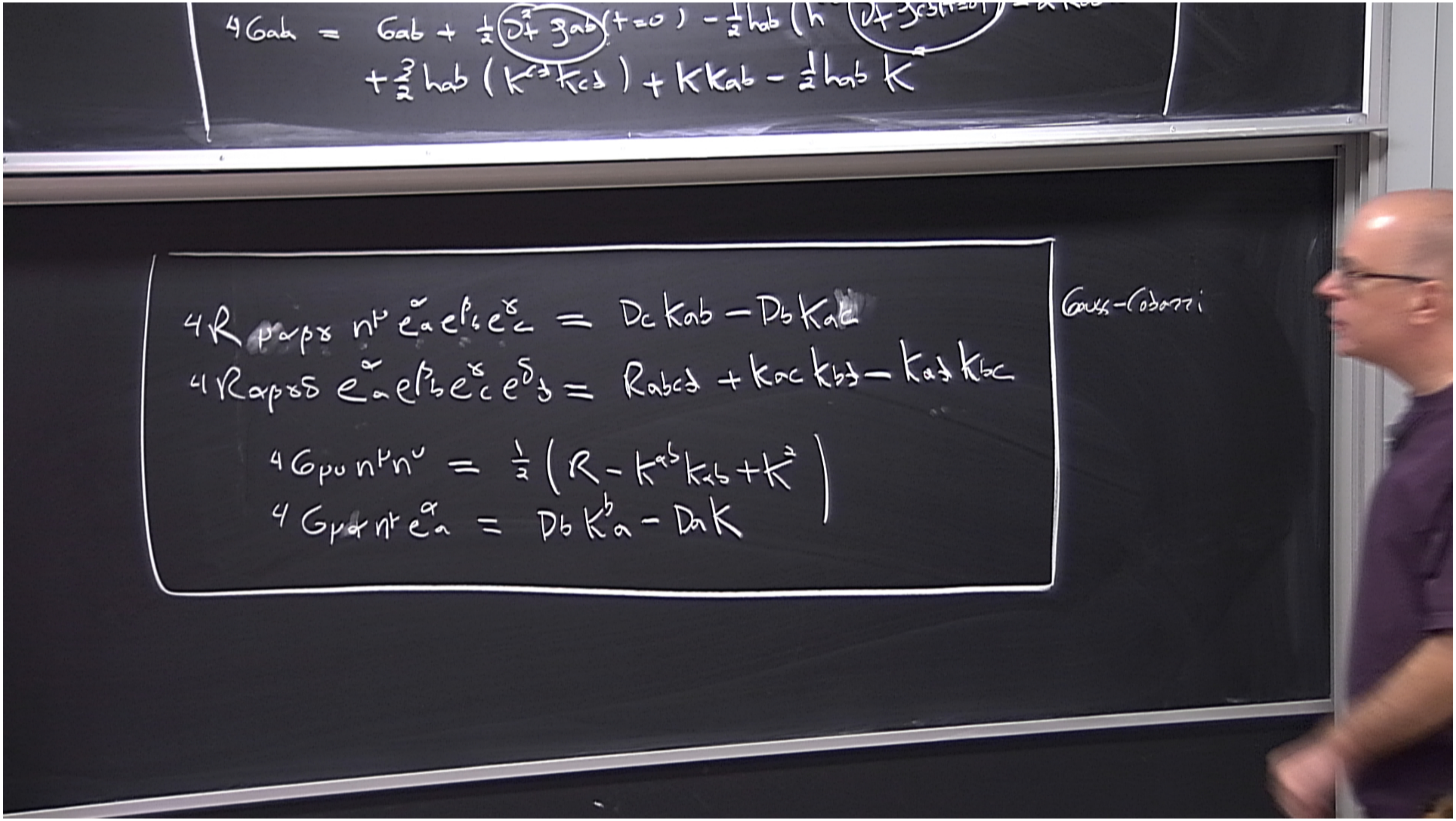
Date: Feb 15, 2012 03:30 PM

URL: http://pirsa.org/12020113

Abstract:



$^4G_{ab} = G_{ab} + \frac{1}{2}\partial_t^2 g_{ab}(t=0) - \frac{1}{2}h_{ab}(h^{cd}\partial_t g_{cd}(t=0))$
$+\frac{3}{2}h_{ab}(K^{cd}K_{cd}) + KK_{ab} - \frac{1}{2}h_{ab}K^2$
$^4R_{\mu\alpha\beta\gamma}n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_cK_{ab} - D_bK_{ac}$
$^4R_{\alpha\beta\gamma\delta}e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$
$^4G_{\mu\nu}n^\mu n^\nu = \frac{1}{2}(R - K^{ab}K_{ab} + K^2)$
$^4G_{\mu\alpha}n^\mu e^\alpha_a = D_bK^b_a - D_aK$
Gauss-


$+\frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} - \frac{1}{2} h_{ab} K^2$
$^4R_{\mu\alpha\beta\gamma} n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$
$^4R_{\alpha\beta\gamma\delta} e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$
$^4G_{\mu\nu} n^\mu n^\nu = \frac{1}{2}(R - K^{ab} K_{ab} + K^2)$
$^4G_{\mu\alpha} n^\mu e^\alpha_a = D_b K^b_a - D_a K$
Gauss-Codazzi

$$^4G_{ab} = G_{ab} + \frac{1}{2}\partial_t^2 g_{ab}(t=0) - \frac{1}{2}h_{ab}\left(h^{cd}\partial_t^2 g_{cd}(t=0)\right) - \ldots$$

$$+\frac{3}{2}h_{ab}\left(K^{cd}K_{cd}\right) + KK_{ab} - \frac{1}{2}h_{ab}K^2$$

$$^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$$

$$^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc}$$

Gauss-Codazzi

$$^4G_{\mu\nu}\, n^\mu n^\nu = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right)$$

$$^4G_{\mu\alpha}\, n^\mu e^\alpha_a = D_b K^b_a - D_a K$$

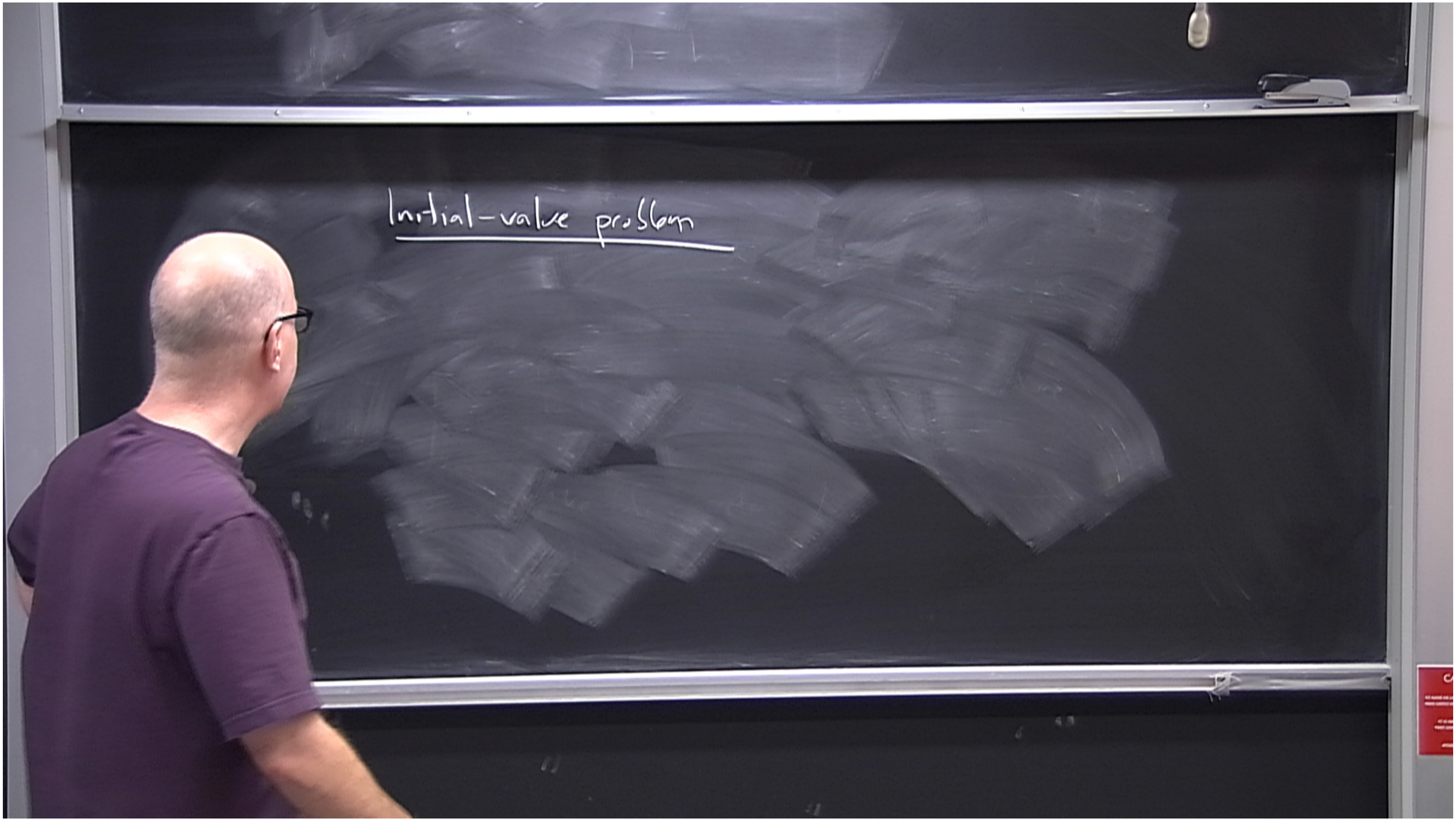
$$ {}^4G_{ab} = G_{ab} + \frac{1}{2}\partial_t^2 g_{ab}(t=0) - \frac{1}{2}h_{ab}(h^{cd}\partial_t^2 g_{cd}(t=0)) \ldots $$
$$ + \frac{3}{2}h_{ab}(K^{cd}K_{cd}) + K K_{ab} - \frac{1}{2}h_{ab}K^2 $$

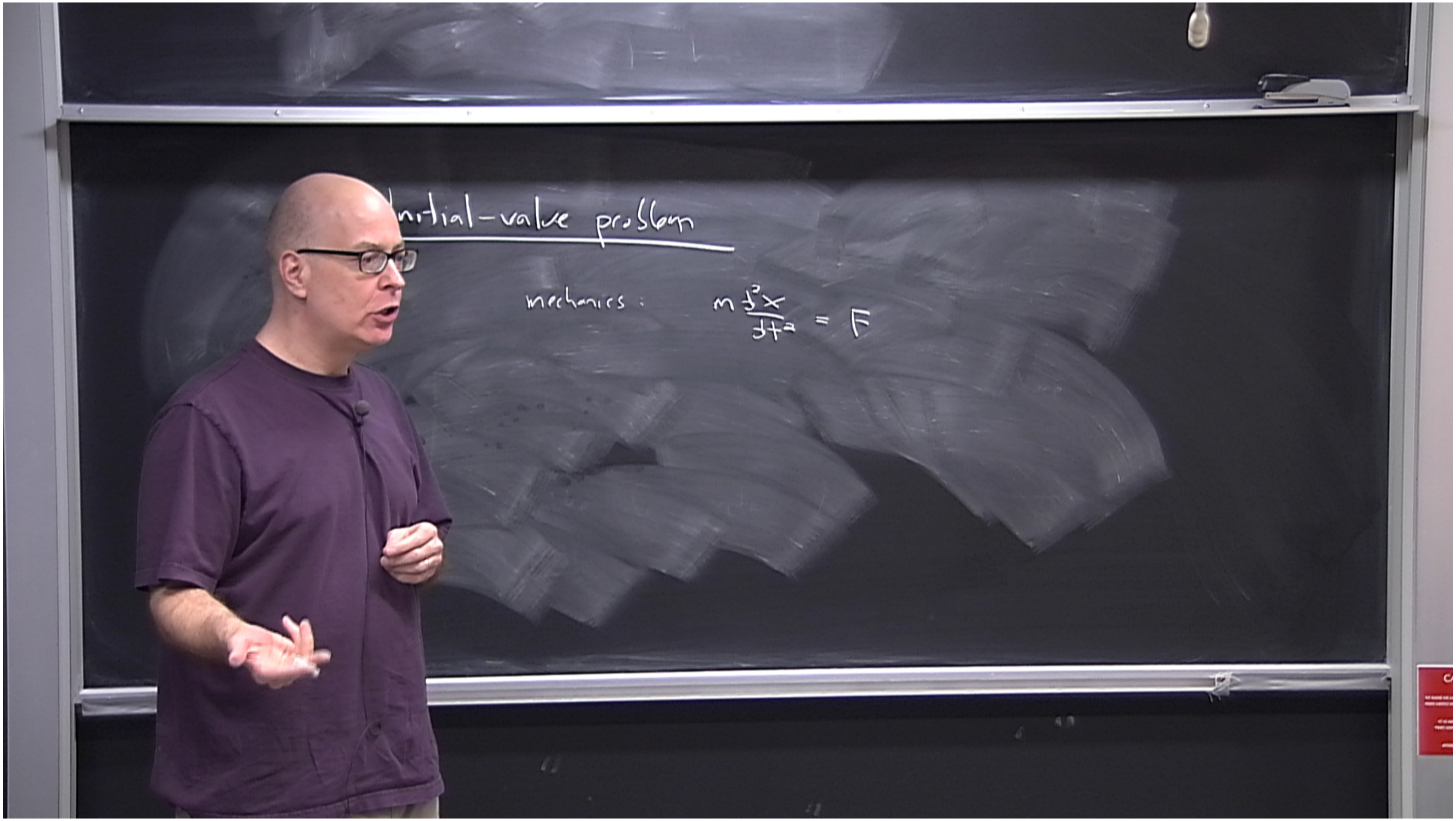
$$ {}^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac} $$
$$ {}^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac}K_{bd} - K_{ad}K_{bc} $$

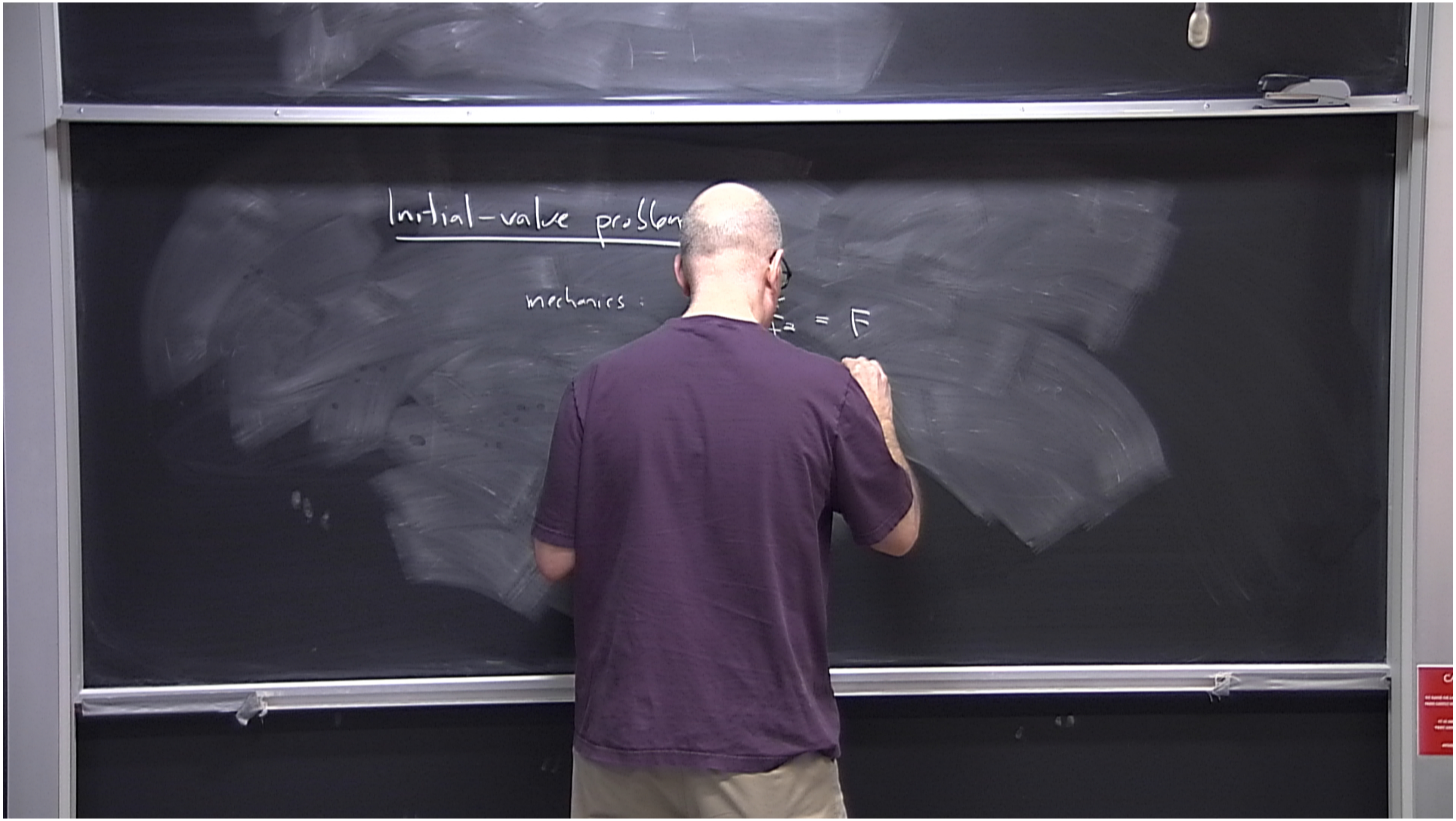
Gauss-Codazzi

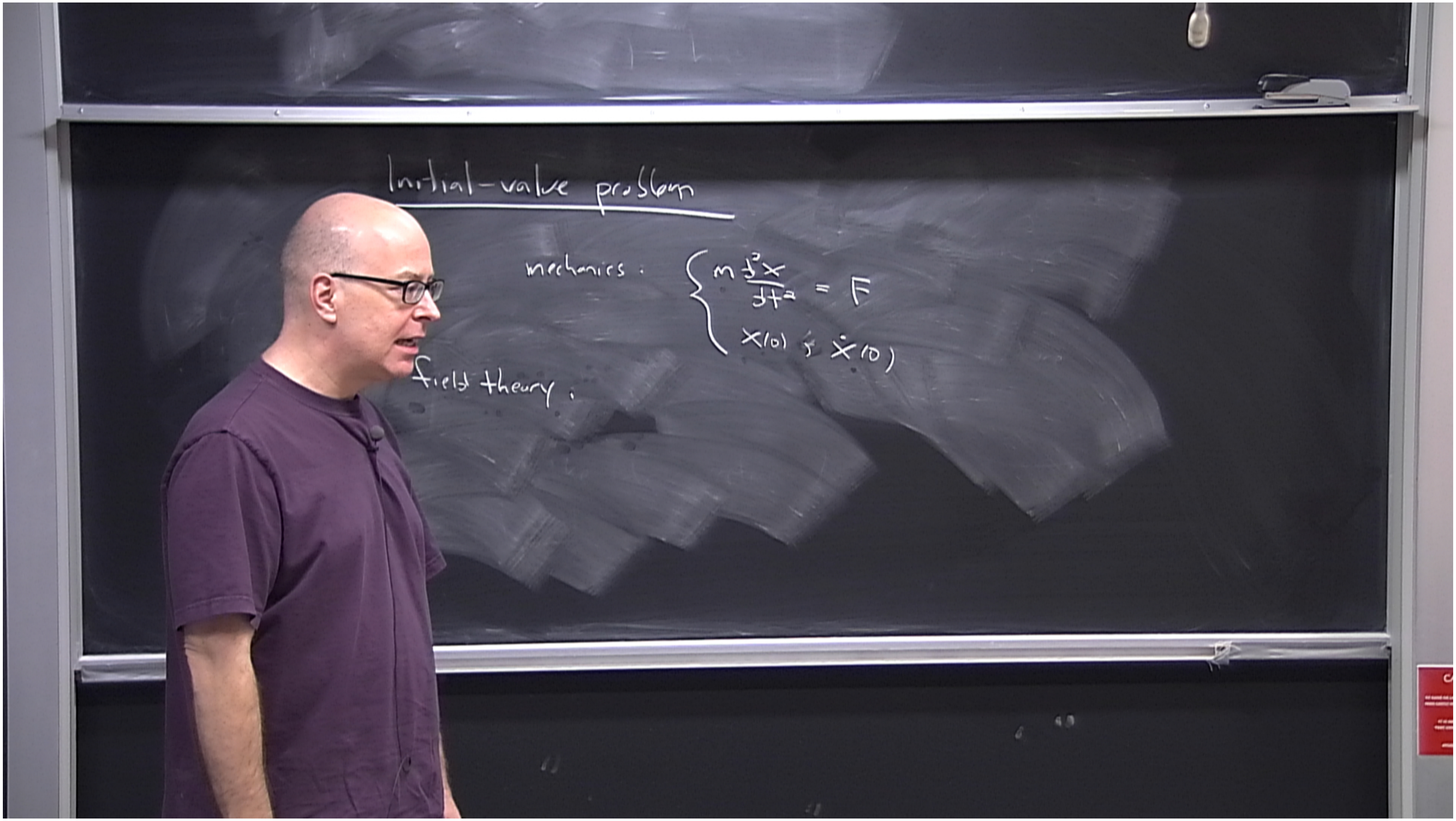
$$ {}^4G_{\mu\nu}\, n^\mu n^\nu = \frac{1}{2}\left(R - K^{ab}K_{ab} + K^2\right) $$
$$ {}^4G_{\mu\alpha}\, n^\mu e^\alpha_a = D_b K^b_a - D_a K $$

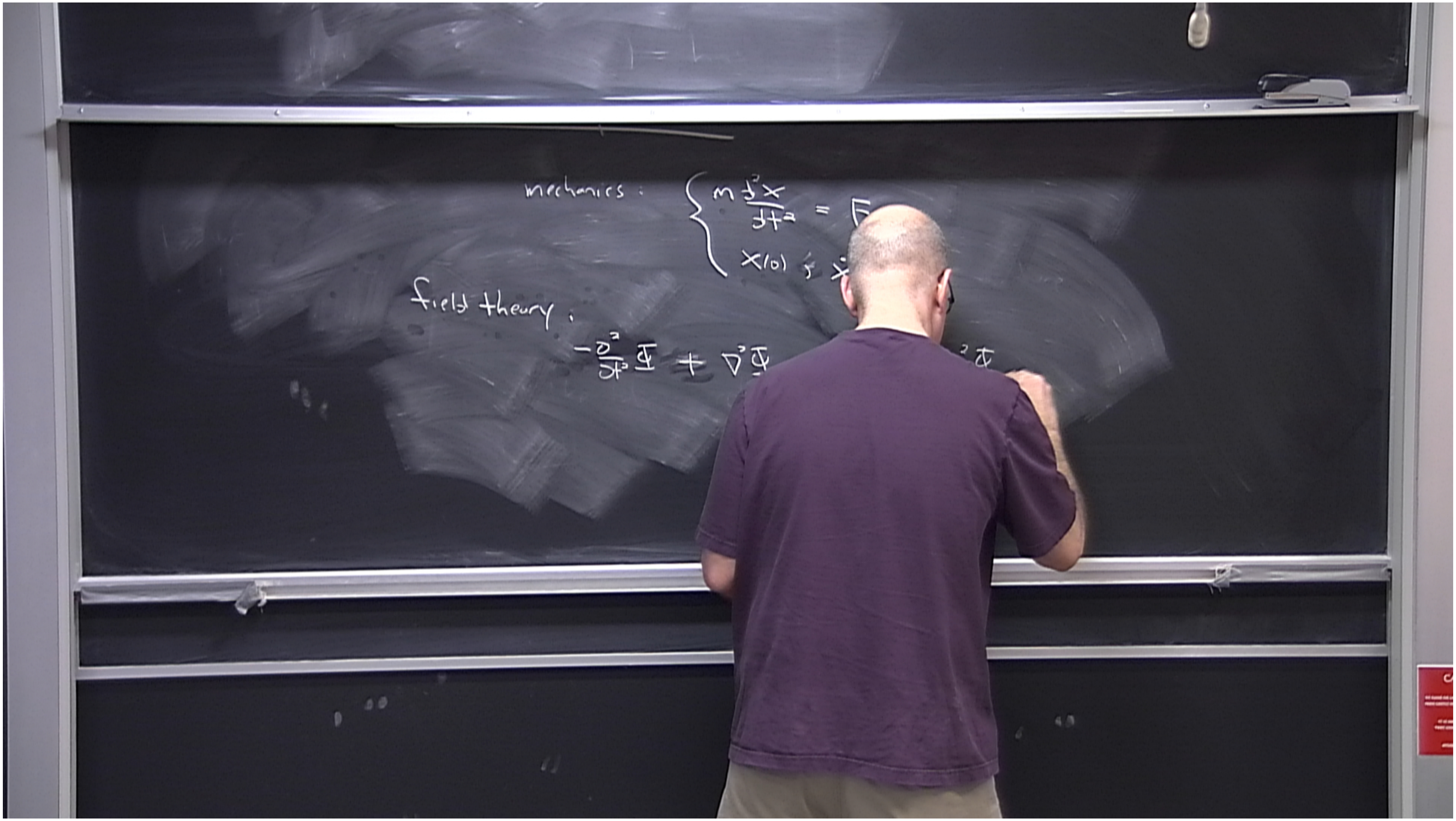
$$ h_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b $$
$$ K_{ab} = n_{(\alpha;\beta)}\, e^\alpha_a e^\beta_b $$

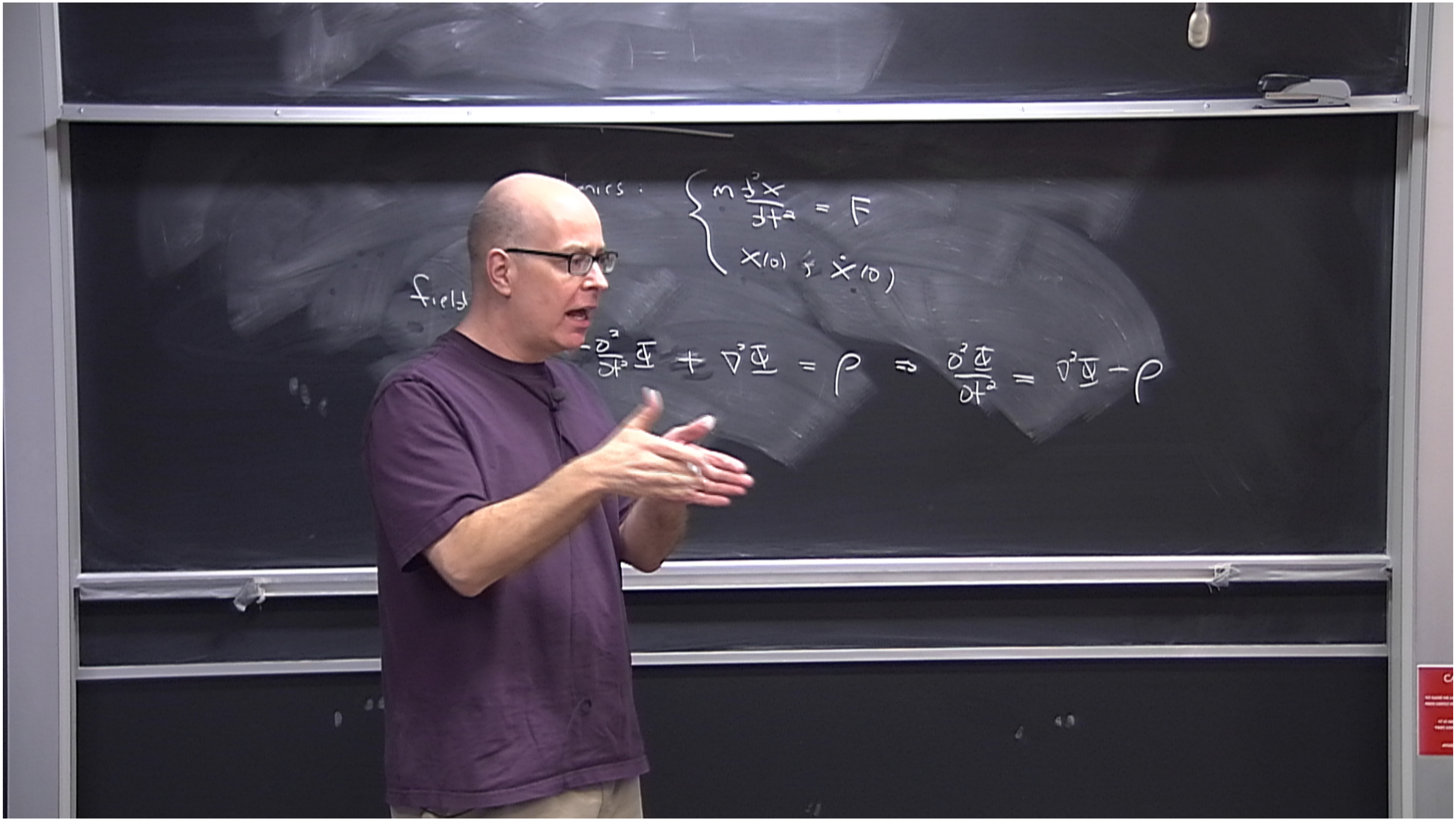

Initial-value problem

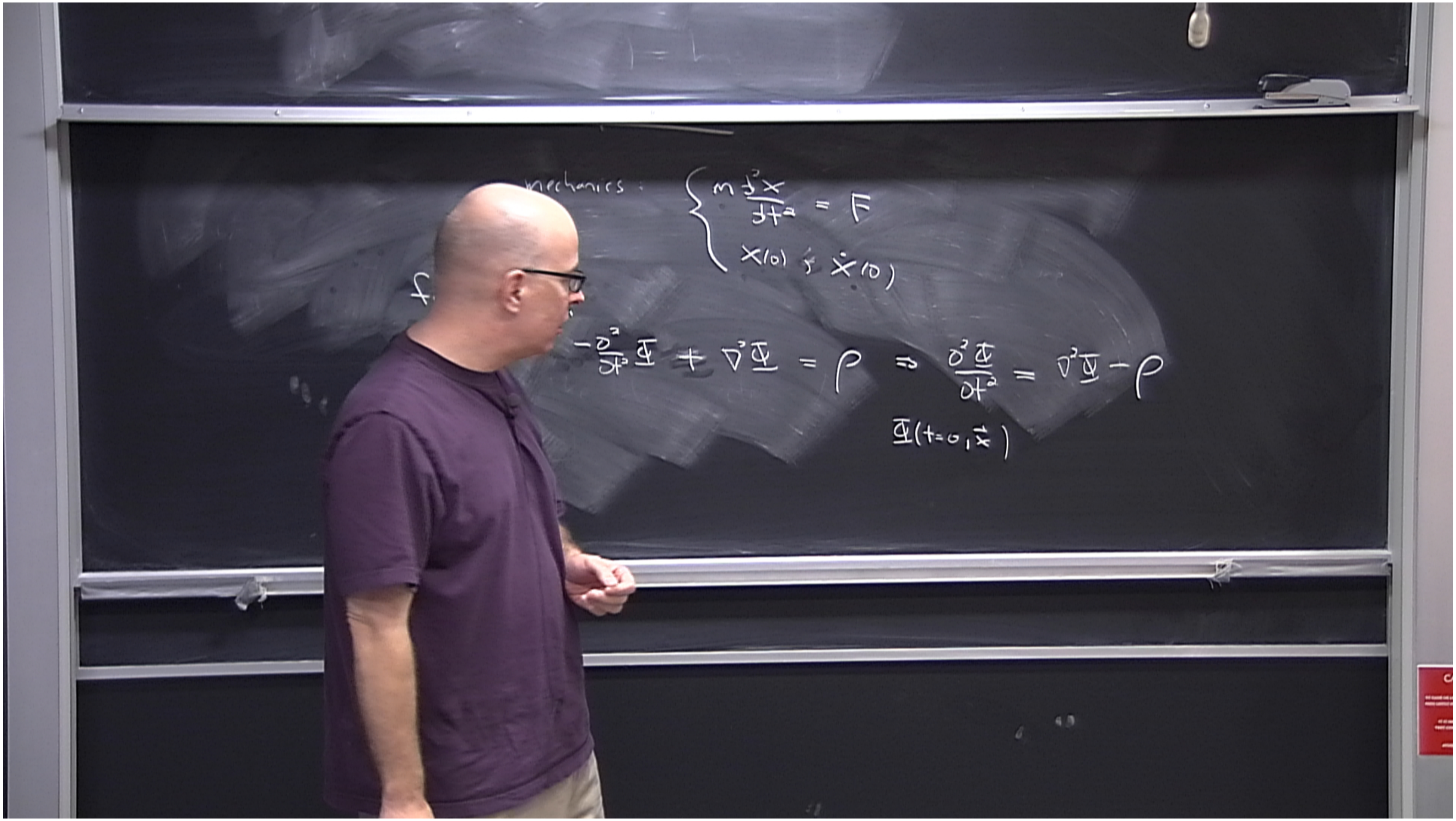

Initial-value problem
mechanics : $m\frac{d^2x}{dt^2} = F$

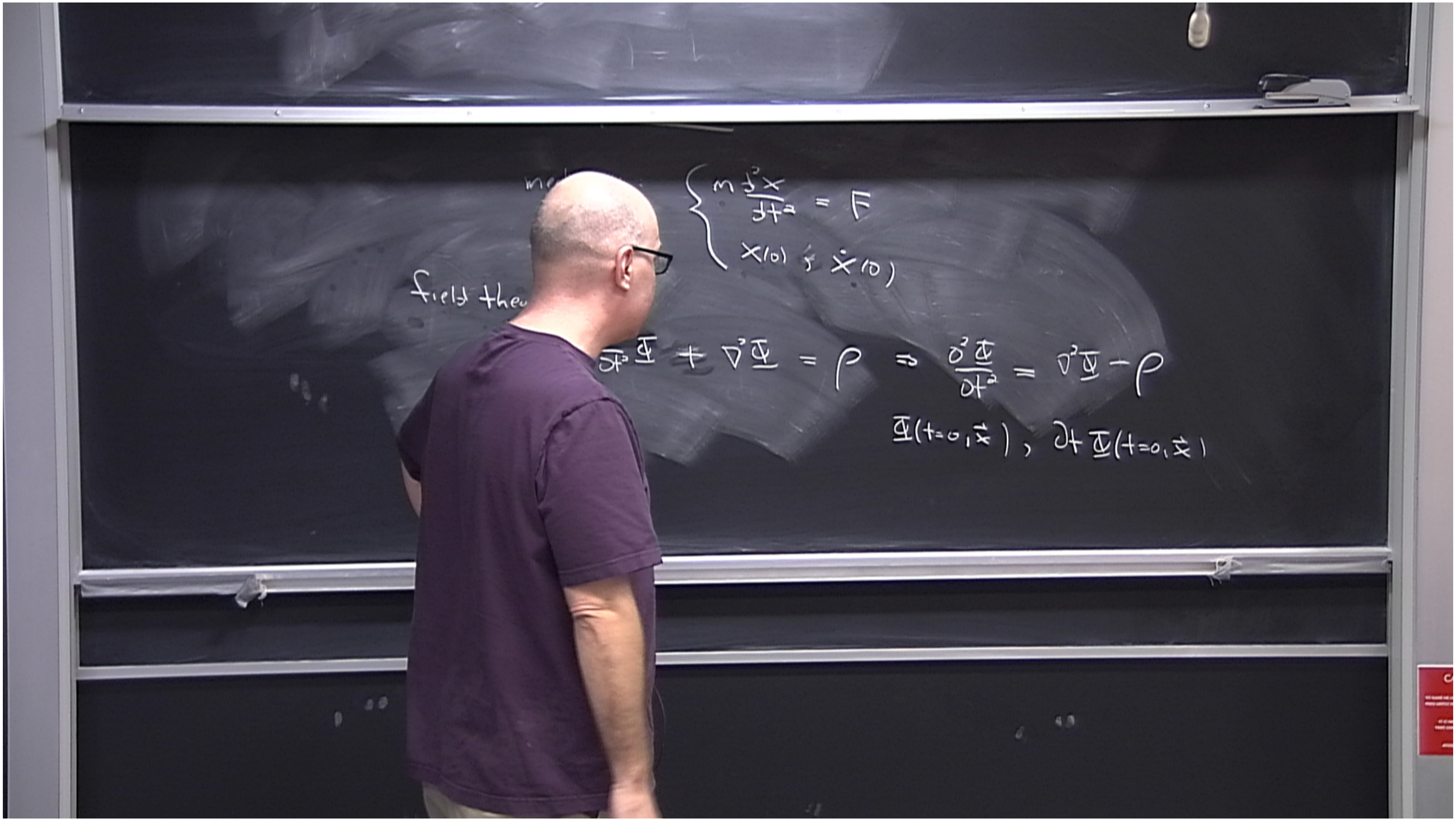

Initial-value problem
mechanics :
= F

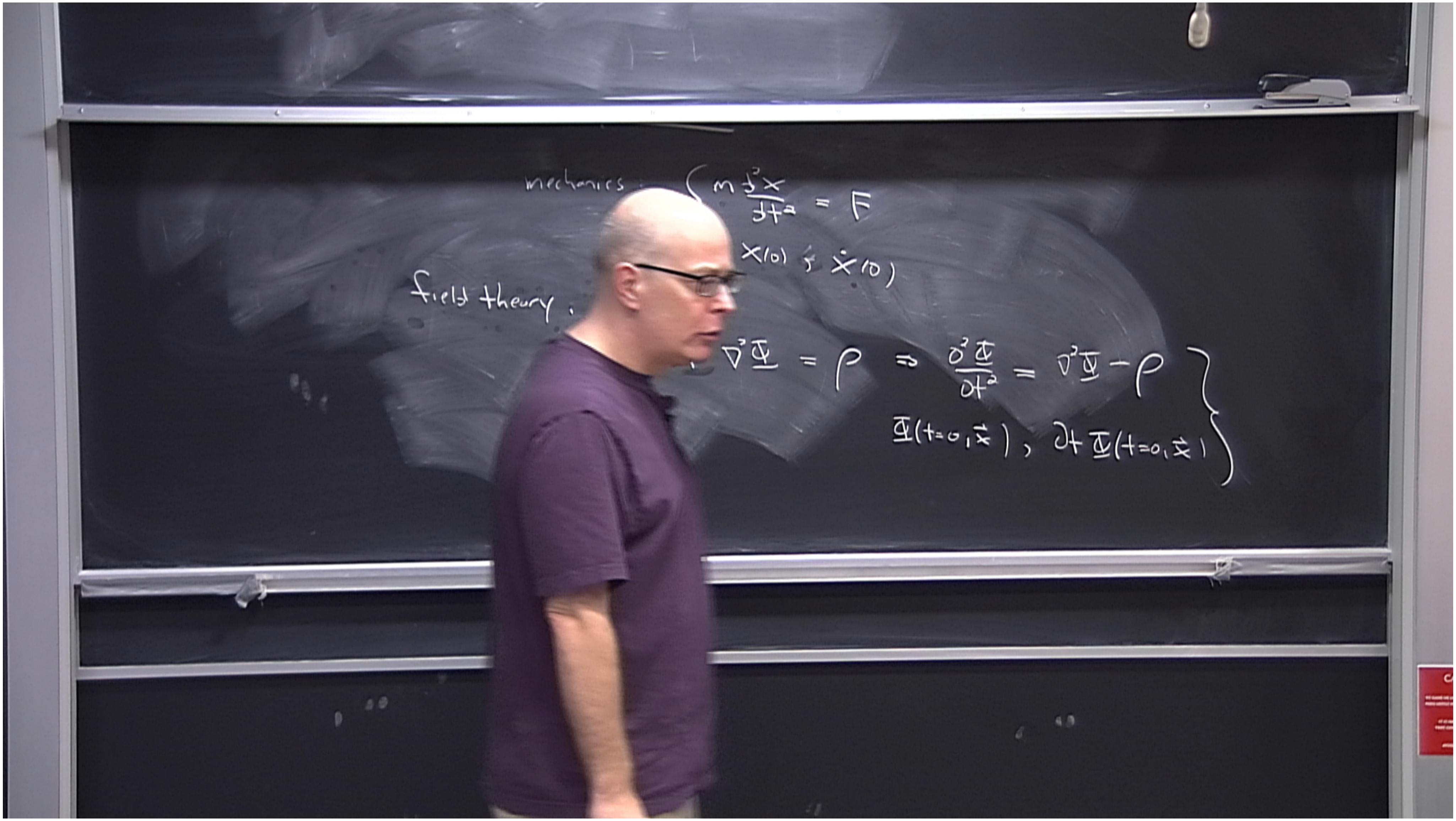

Initial-value problem
mechanics:
$m \frac{d^2x}{dt^2} = F$
$x(0)$; $\dot{x}(0)$
field theory:

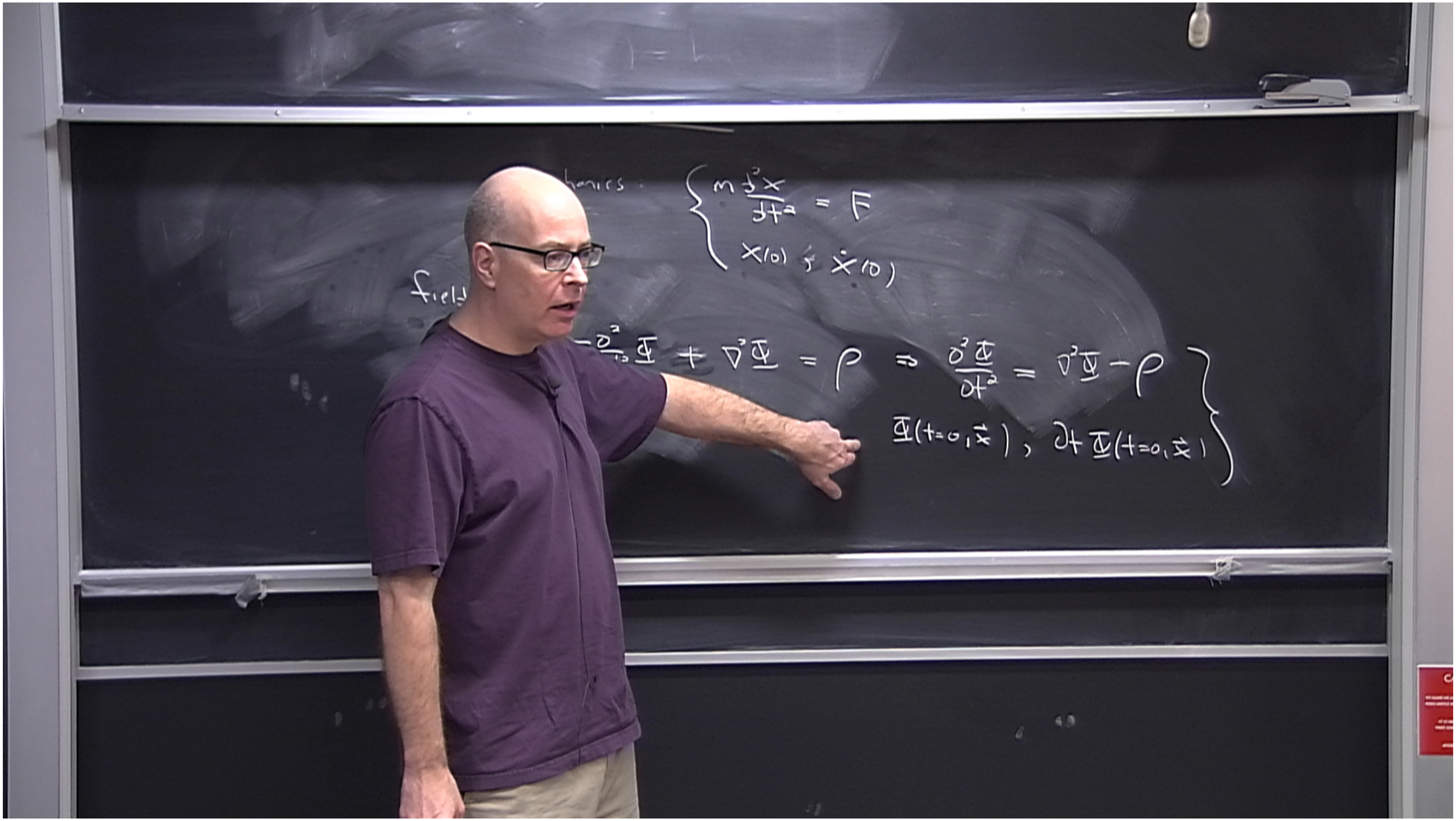

mechanics :
field theory :


m d²x/dt² = F
x(0) ; ẋ(0)
∂²Φ/∂t² = ∇²Φ − ρ

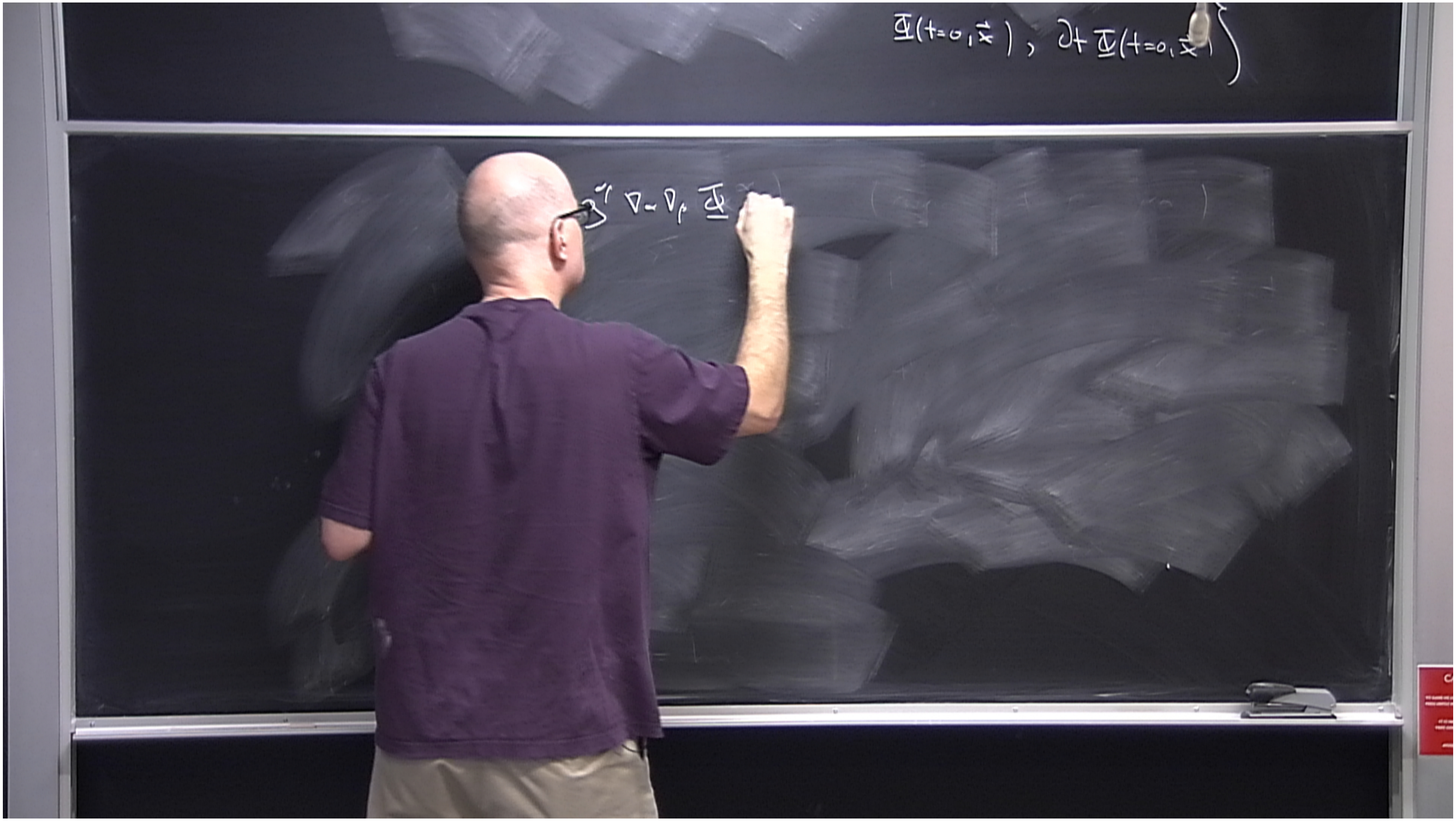

mechanics :
$m \frac{d^2x}{dt^2} = F$
$x(0)$; $\dot{x}(0)$
$-\frac{\partial^2}{\partial t^2}\Phi + \nabla^2\Phi = \rho \Rightarrow \frac{\partial^2\Phi}{\partial t^2} = \nabla^2\Phi - \rho$
$\Phi(t=0, \vec{x})$

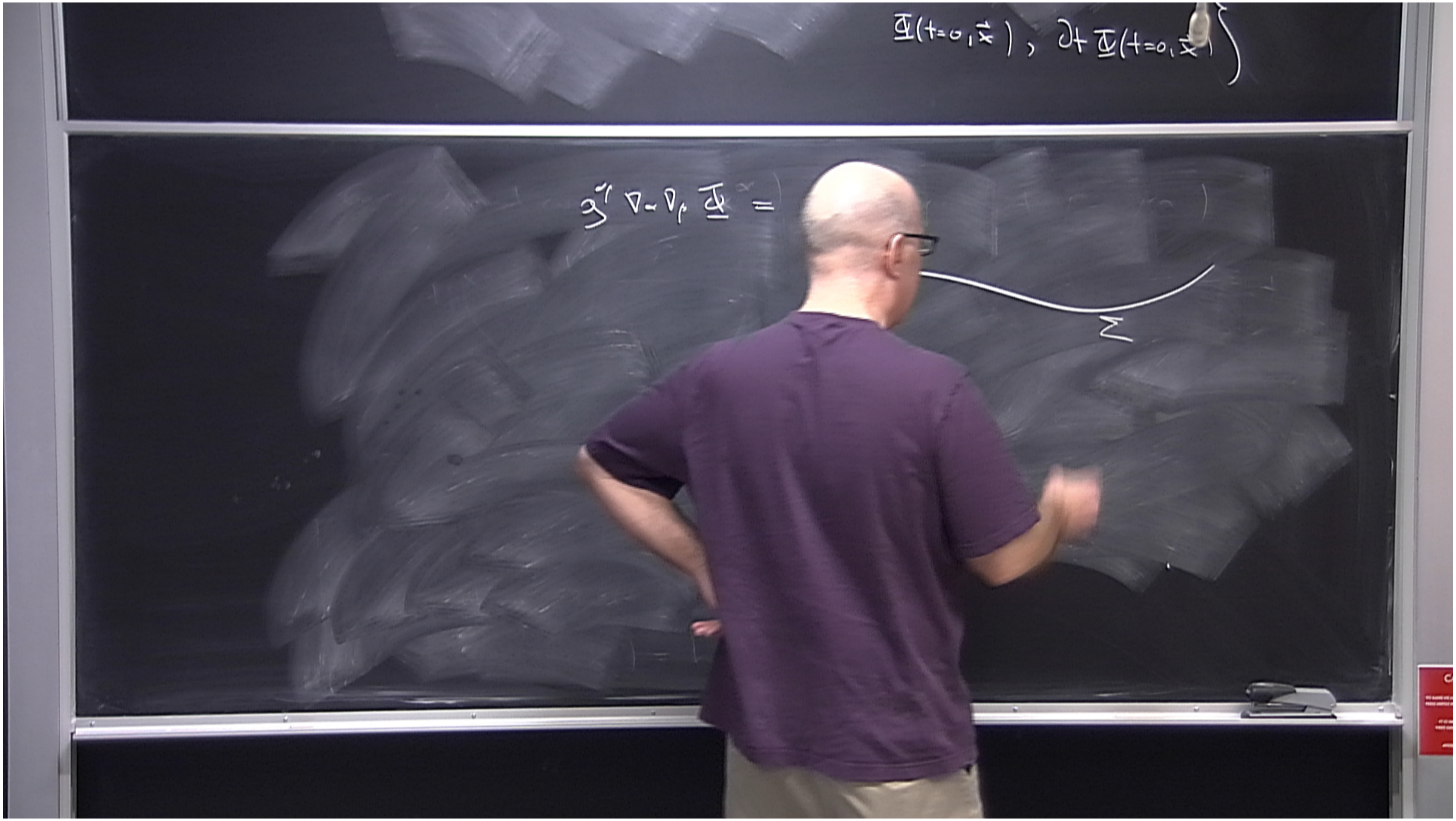

field theo
$m \frac{d^2x}{dt^2} = F$
$x(0)$; $\dot{x}(0)$
$+ \nabla^2\Phi = \rho \Rightarrow \frac{\partial^2\Phi}{\partial t^2} = \nabla^2\Phi - \rho$
$\Phi(t=0,\vec{x})$, $\partial_t\Phi(t=0,\vec{x})$

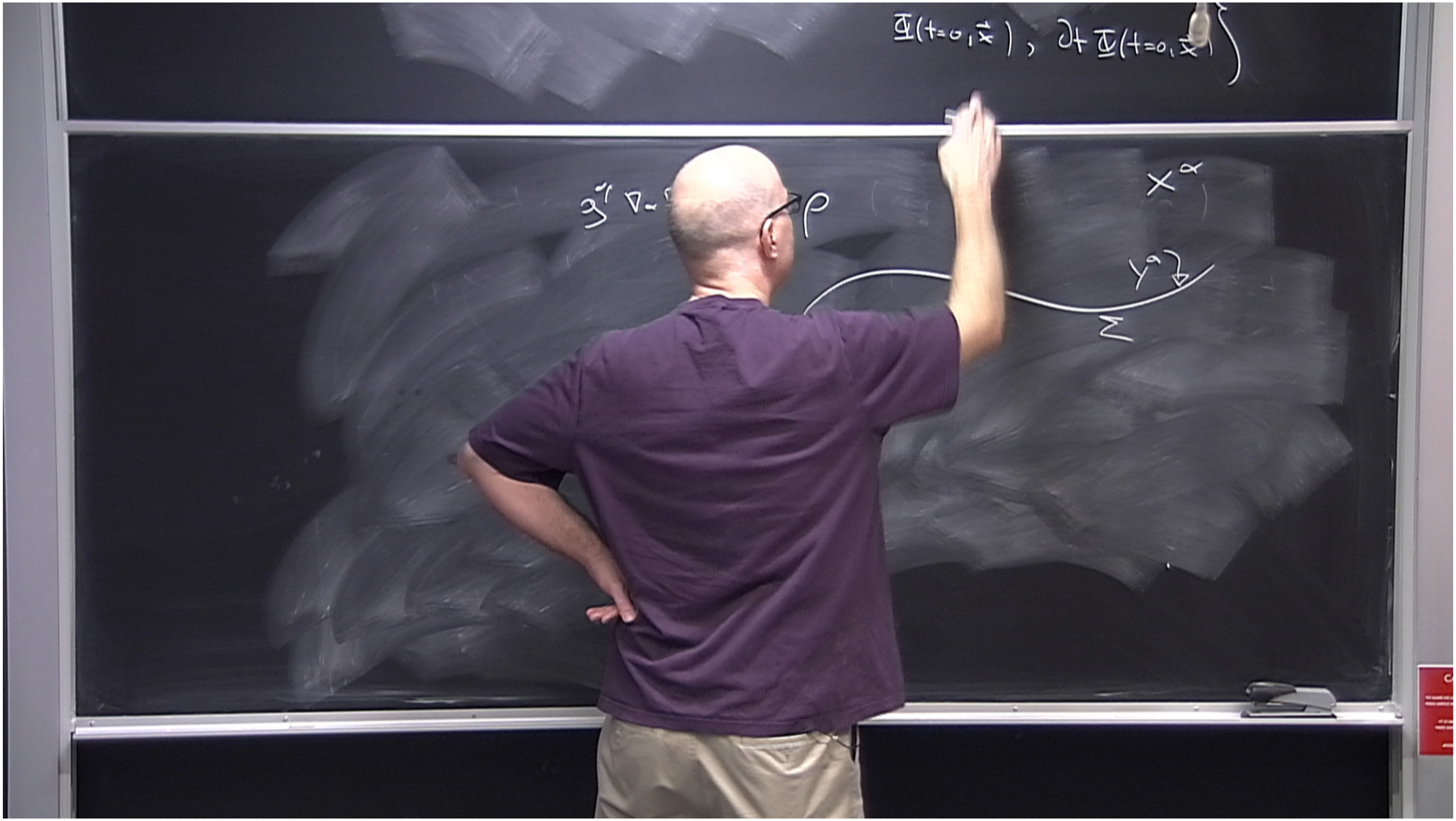

mechanics: m d²x/dt² = F
x(0) ; ẋ(0)
field theory:
∇²Φ = ρ ⇒ ∂²Φ/∂t² = ∇²Φ − ρ
Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)

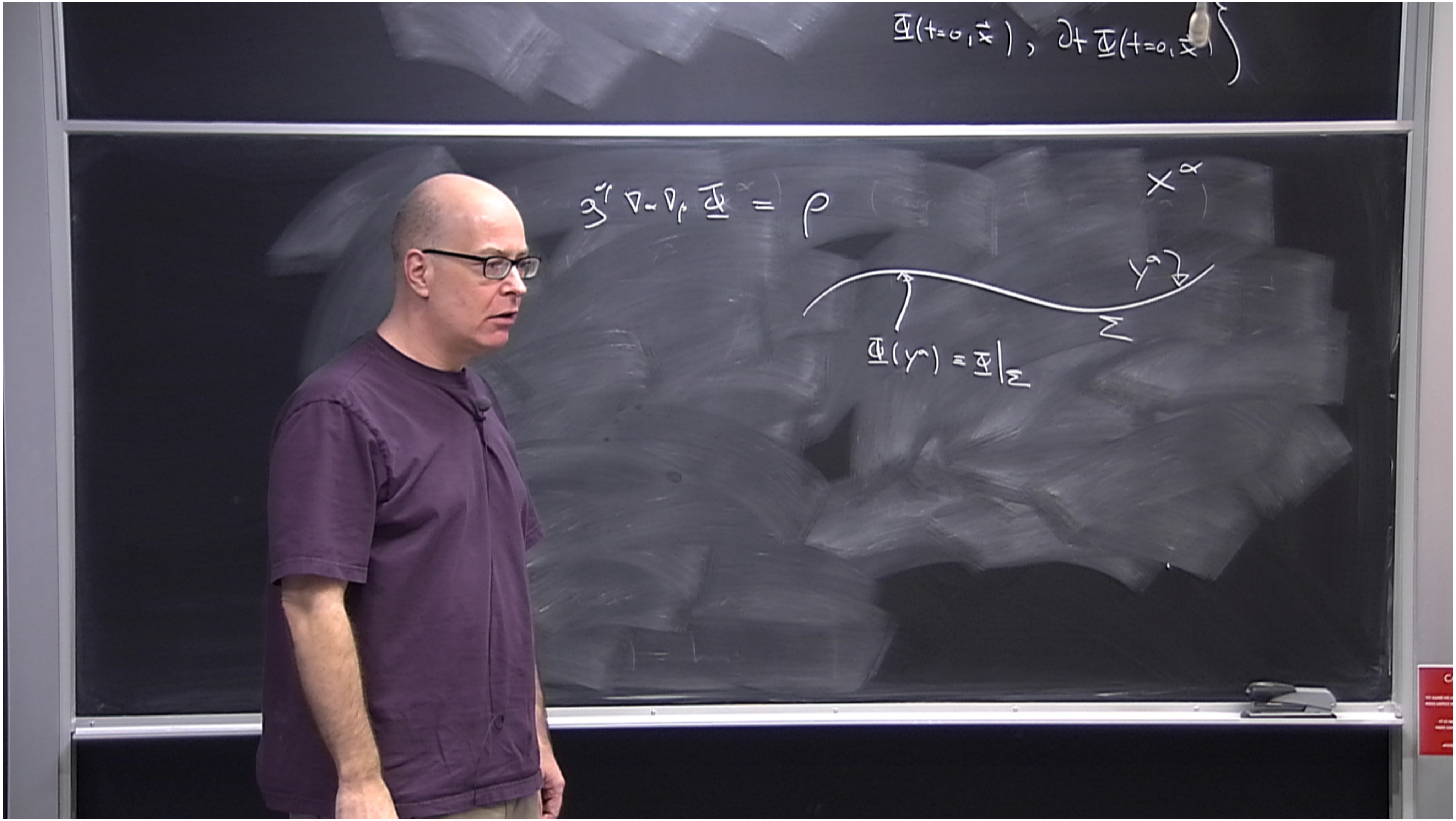


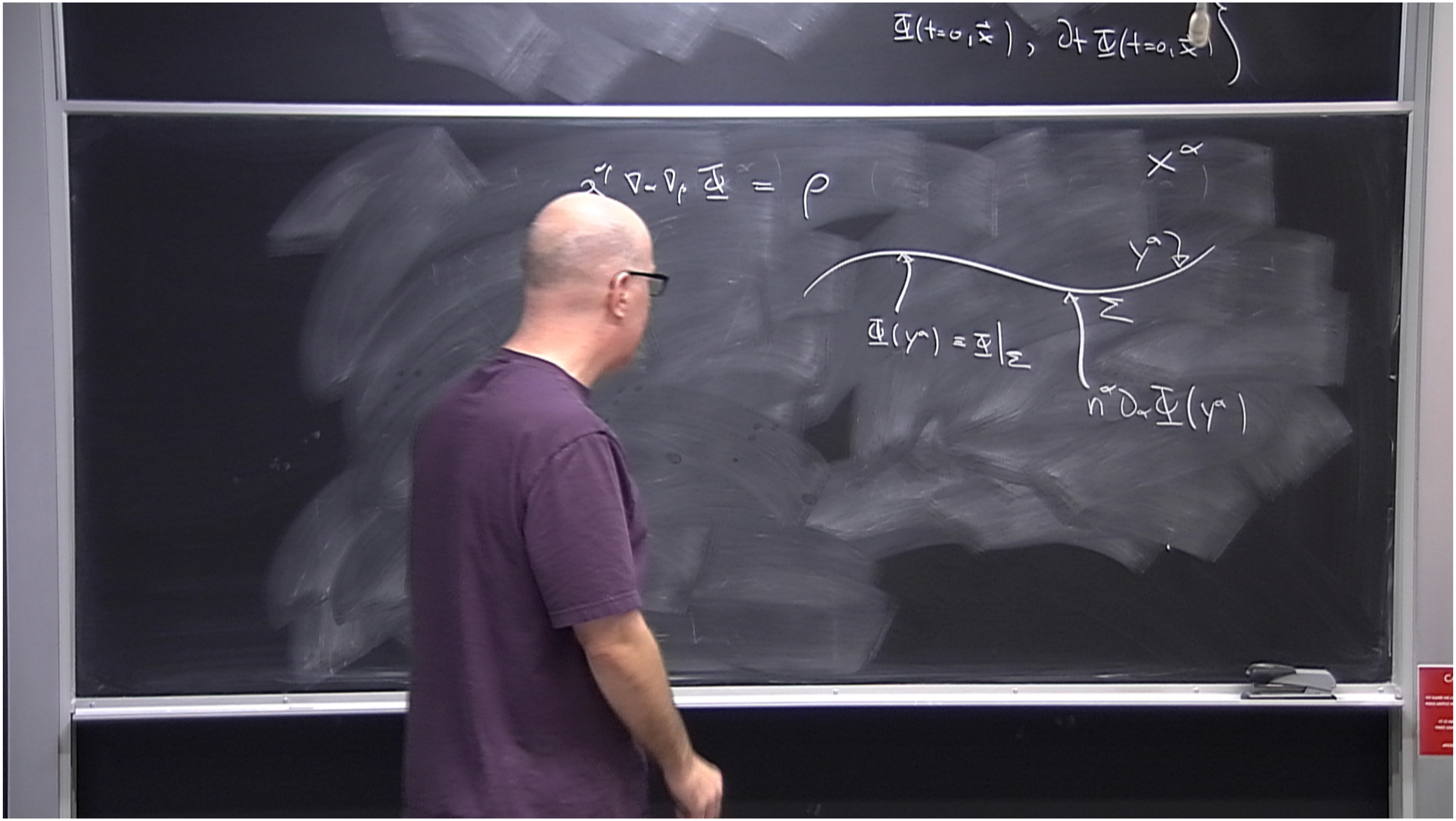
hanics :
$m \frac{d^2x}{dt^2} = F$
$x(0)$; $\dot{x}(0)$
field
$\frac{\partial^2 \Phi}{\partial t^2} = \nabla^2 \Phi - \rho$
$\Phi(t=0, \vec{x})$, $\partial_t \Phi(t=0, \vec{x})$

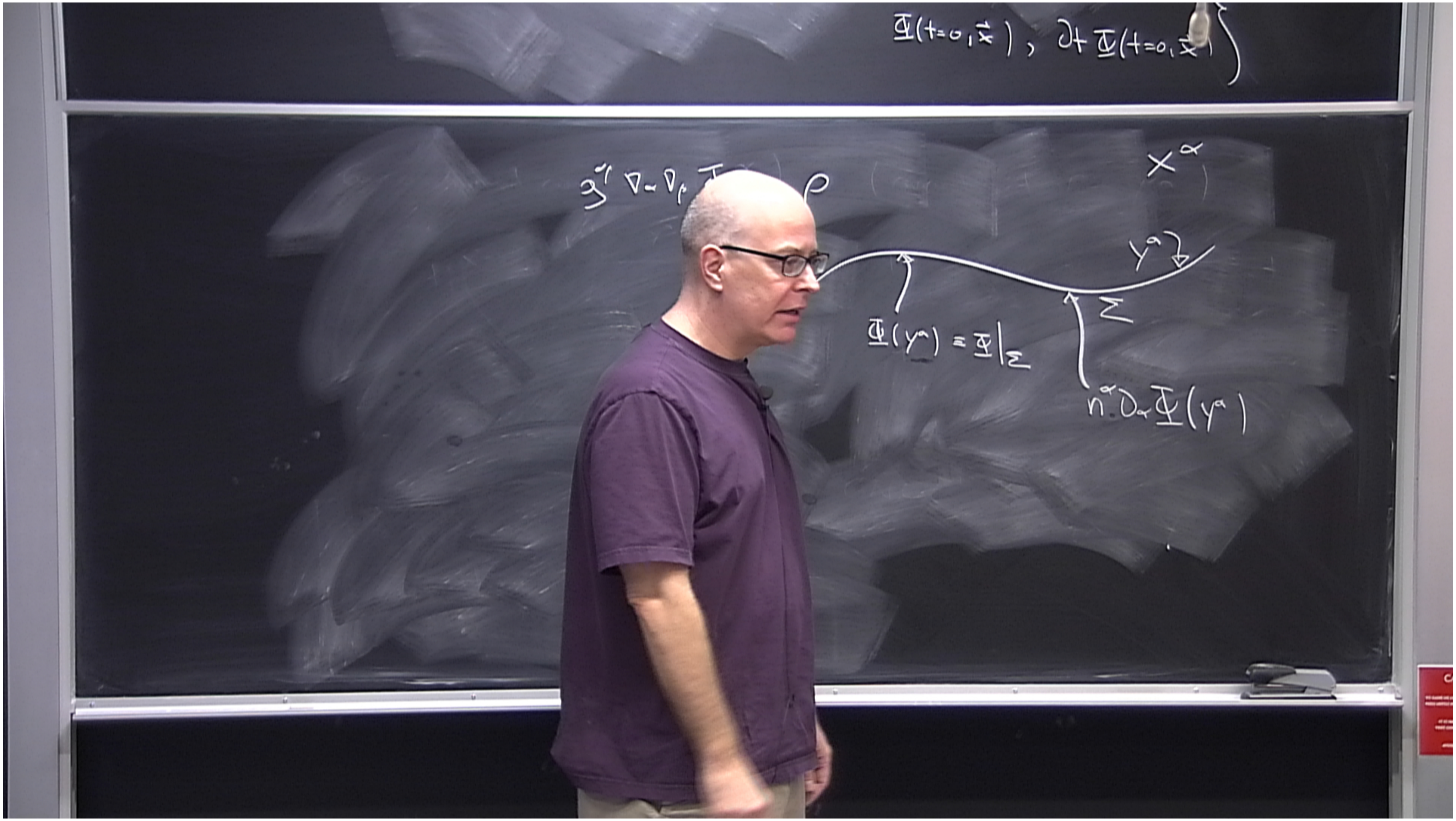

Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)}
g^{μν} ∇μ ∇ν Φ


$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$ }
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi =$
Σ

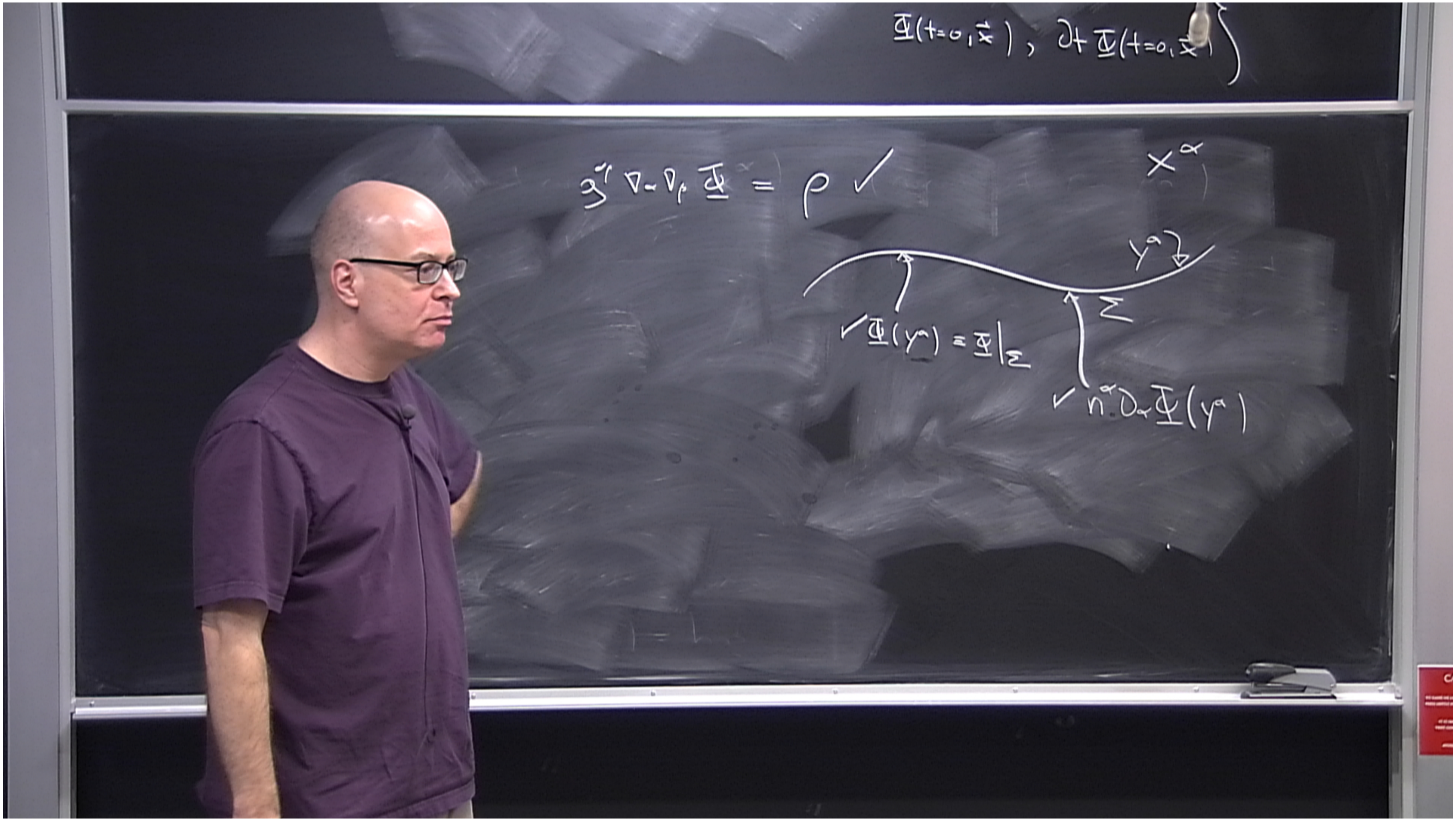

Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)
X^α
Σ

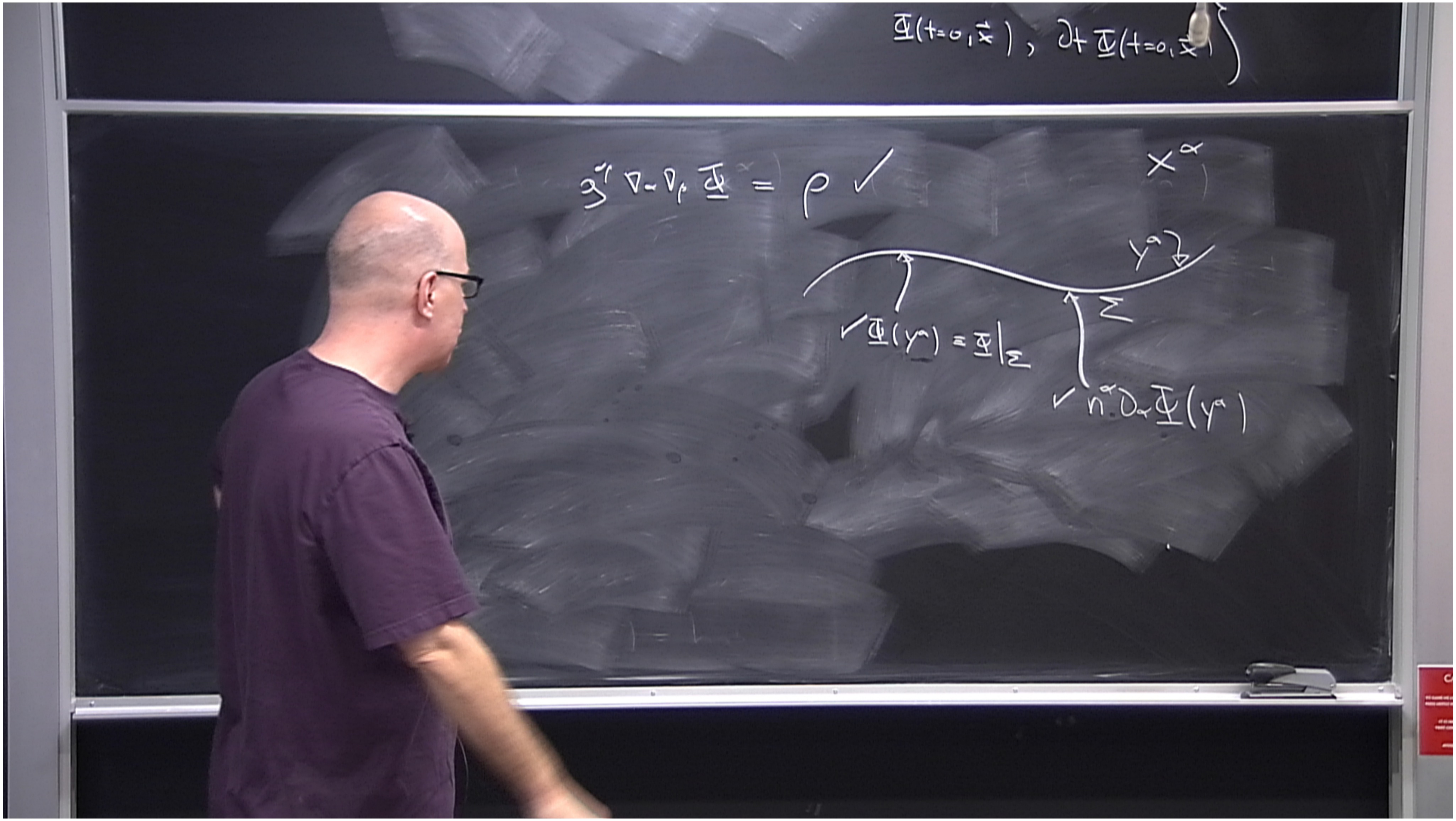

$\Phi(t=0,\vec{x}),\ \partial_t\Phi(t=0,\vec{x})\}$
$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = \rho$
x^α
y^a
Σ
$\Phi(y^a) \equiv \Phi|_\Sigma$

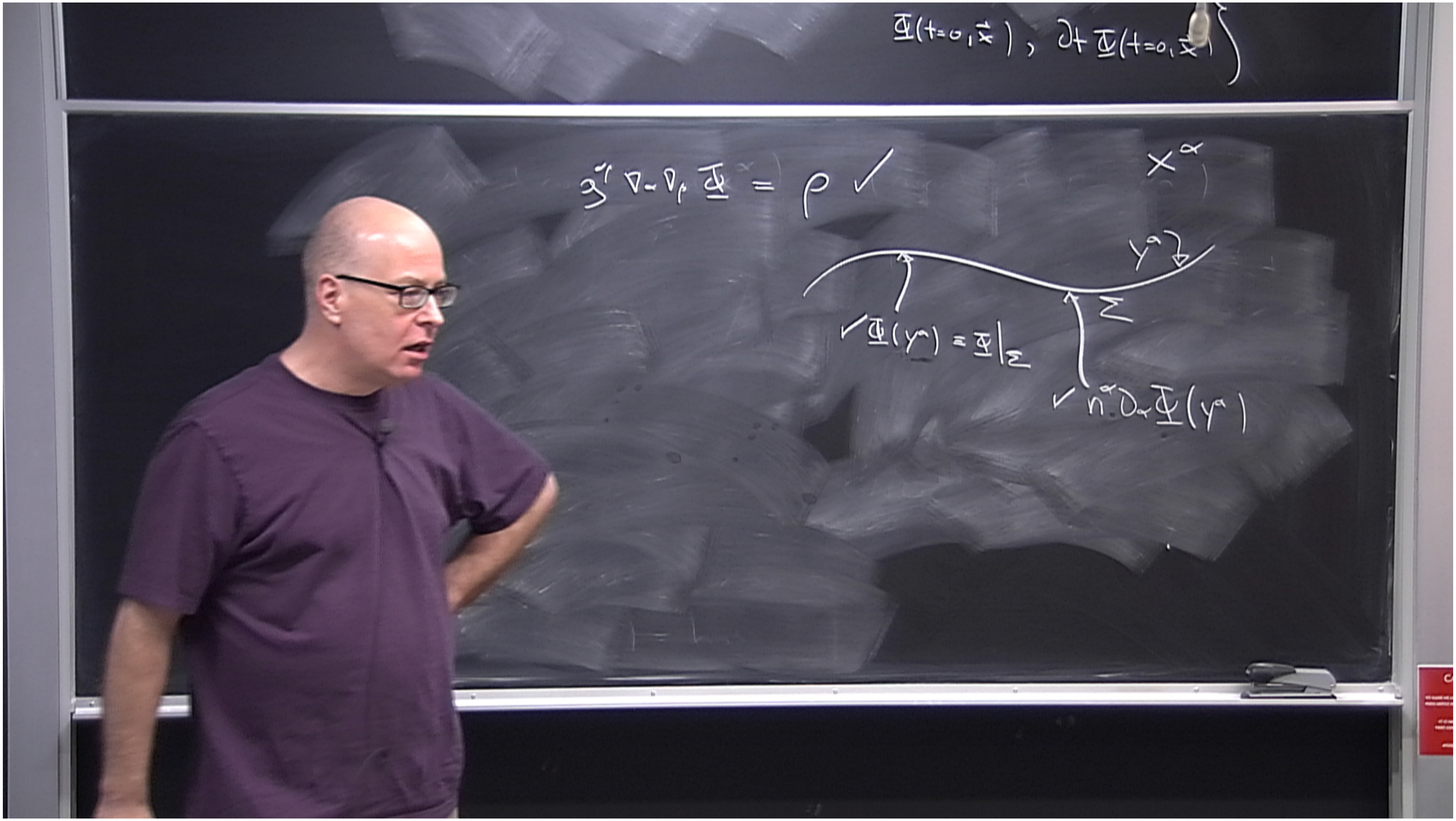

$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$ }
$\nabla_\alpha \nabla_\beta \Phi = \rho$
x^α
y^a
Σ
$\Phi(y^a) = \Phi|_\Sigma$
$n^\alpha \partial_\alpha \Phi(y^a)$

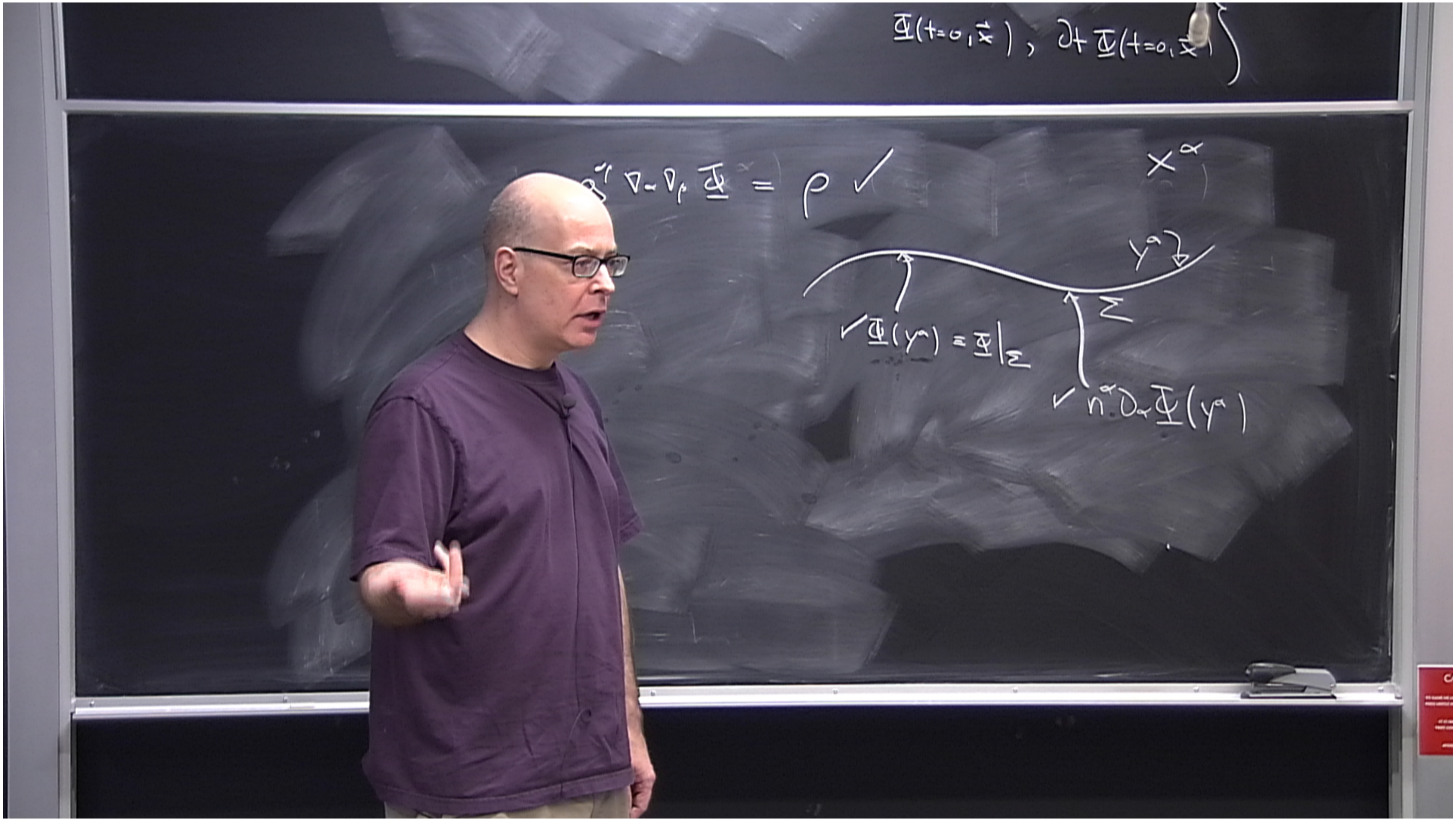


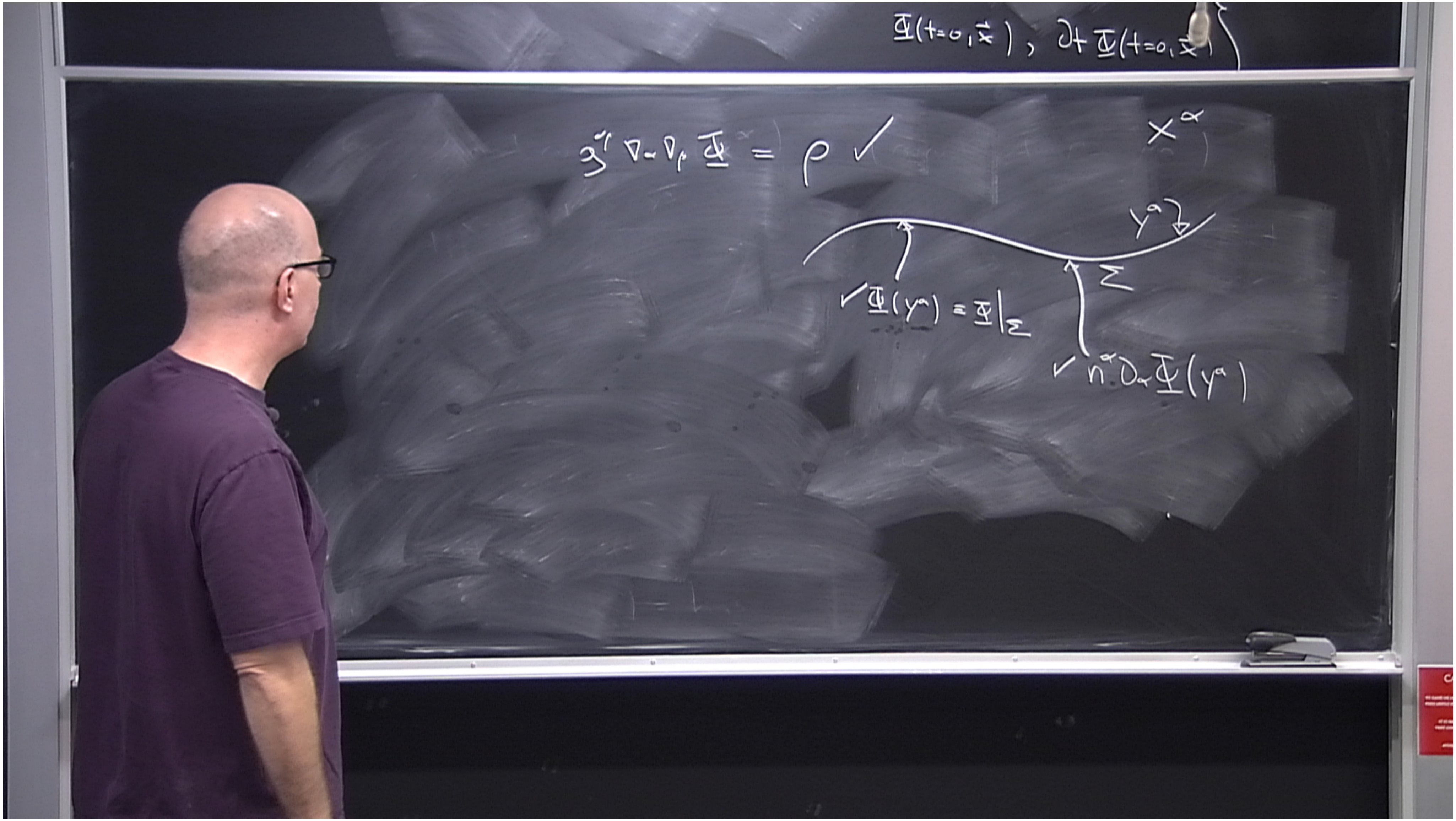
Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)
X^α
Y^a
Σ
Φ(y^a) = Φ|Σ
n^α ∂α Φ(y^a)


$\Phi(t=0,\vec{x}),\ \partial_t \Phi(t=0,\vec{x})\}$
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \partial_\alpha \Phi(y^a)$

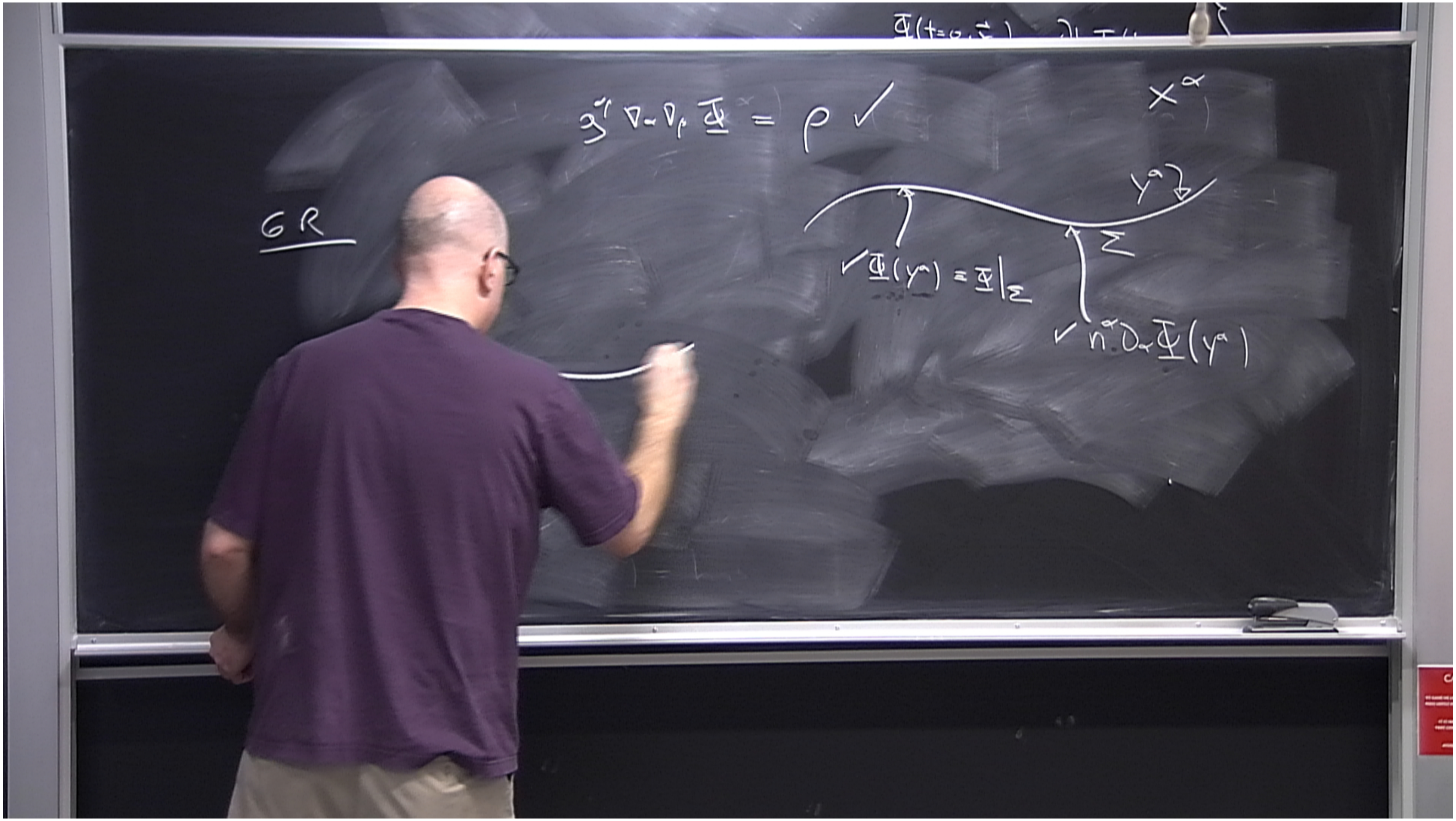

Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)
g^αβ ∇α ∇β Φ = ρ ✓
x^α
y^a
Σ
✓ Φ(y^a) = Φ|Σ
✓ n^α ∂α Φ(y^a)

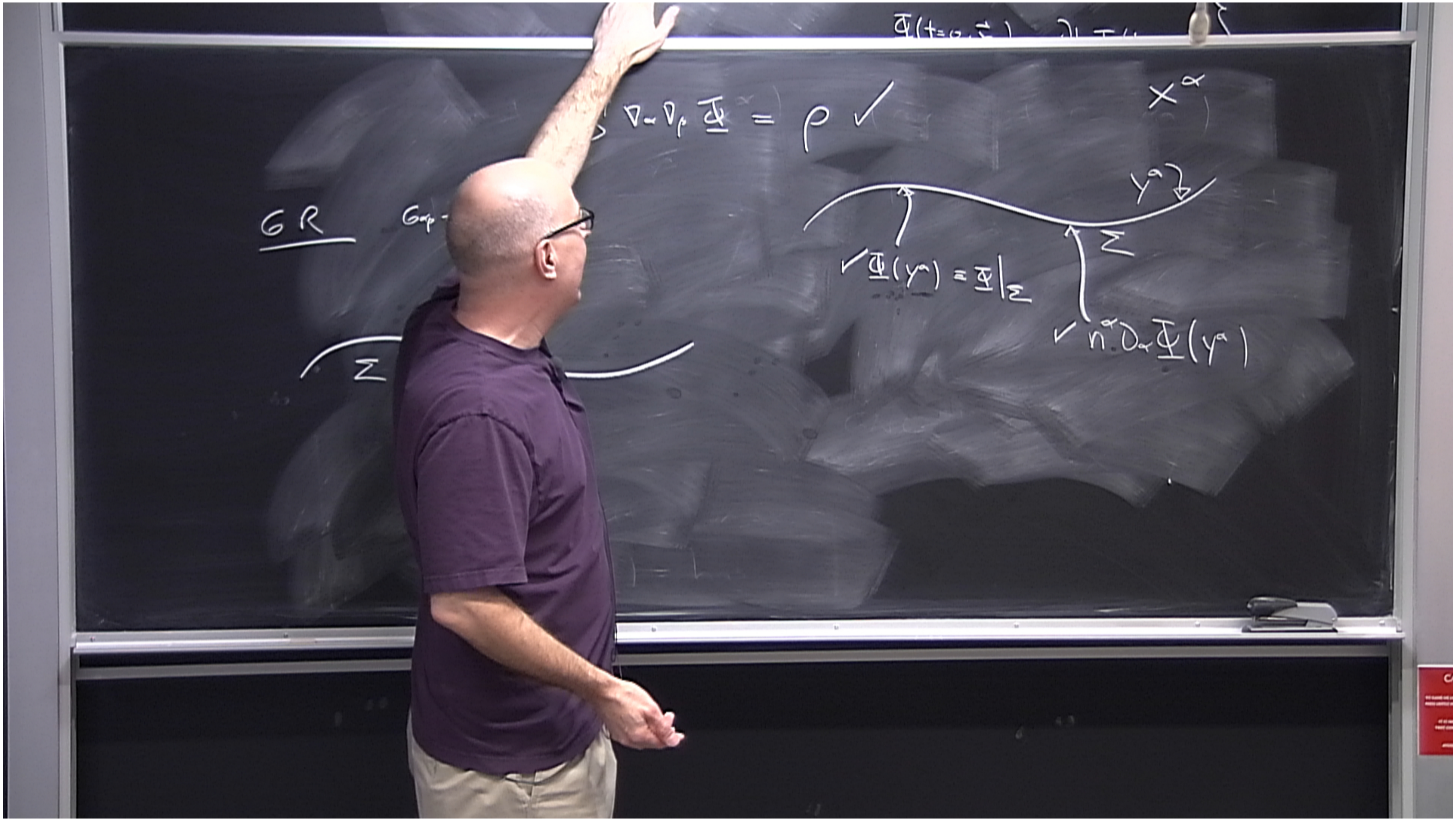

$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \partial_\alpha \Phi(y^a)$

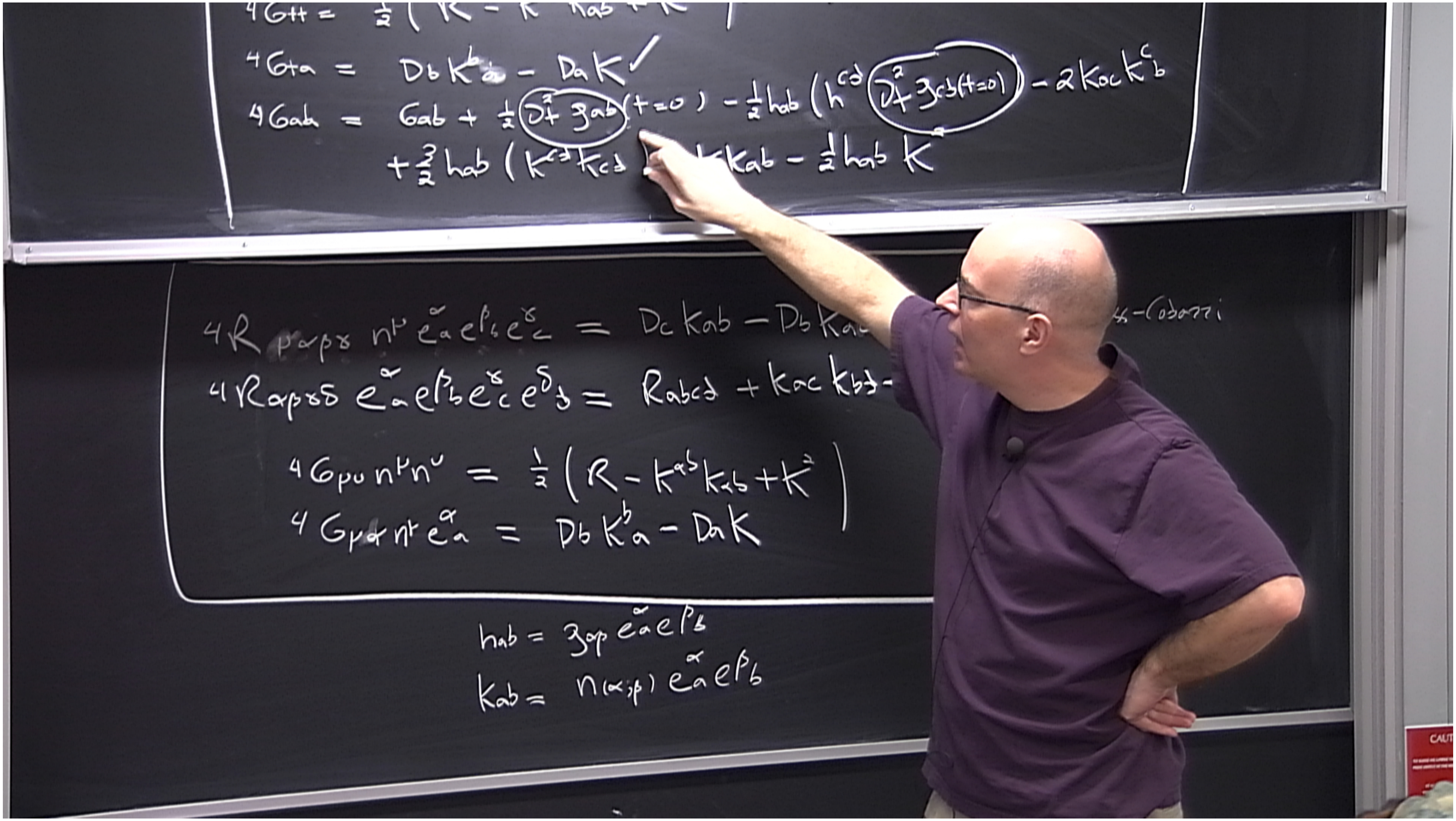

Φ(t=0, x), ∂t Φ(t=0, x)
= ρ ✓
x^α
y^a
Σ
✓ Φ(y^a) = Φ|Σ
✓ n^α ∂α Φ(y^a)

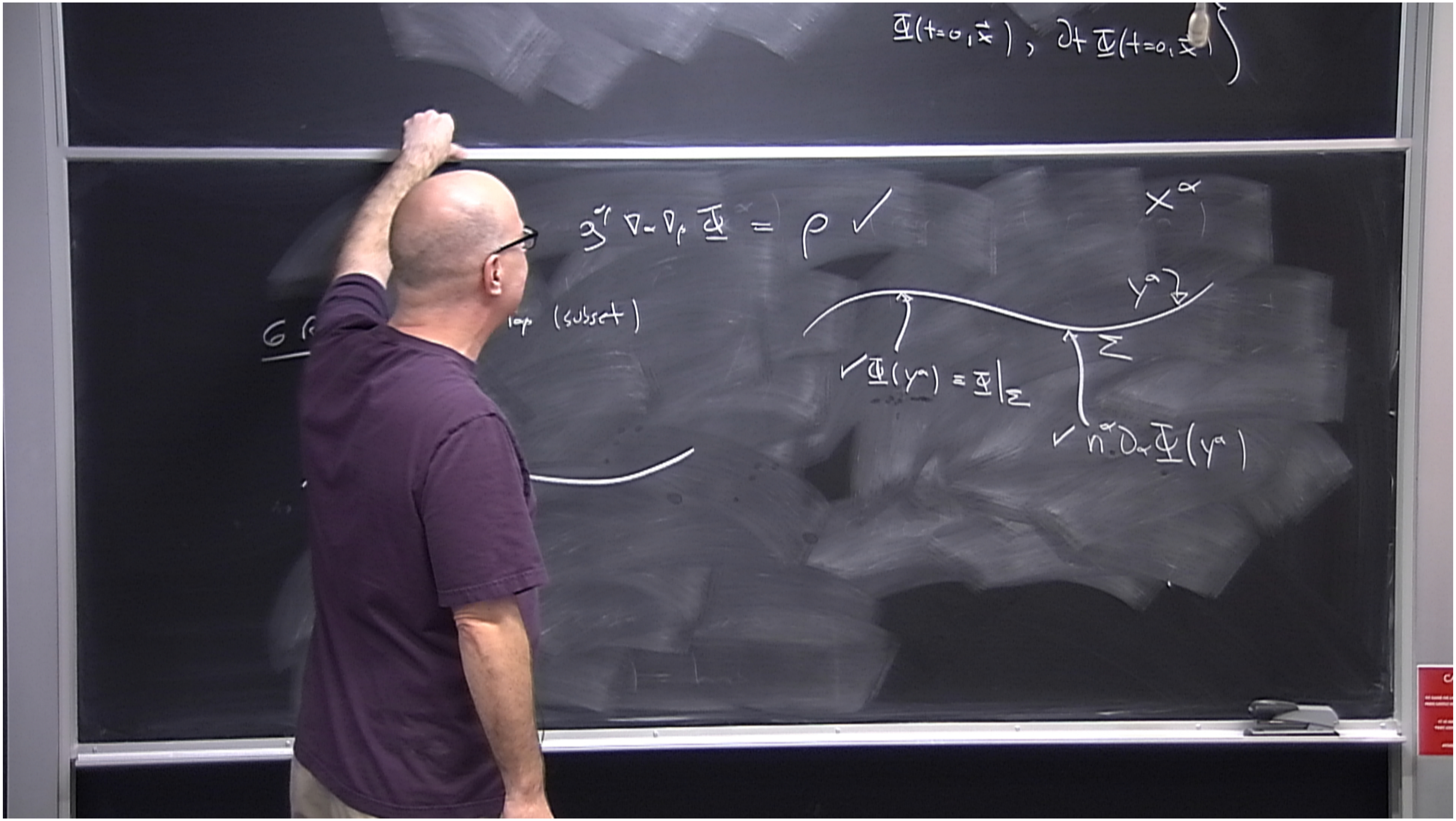


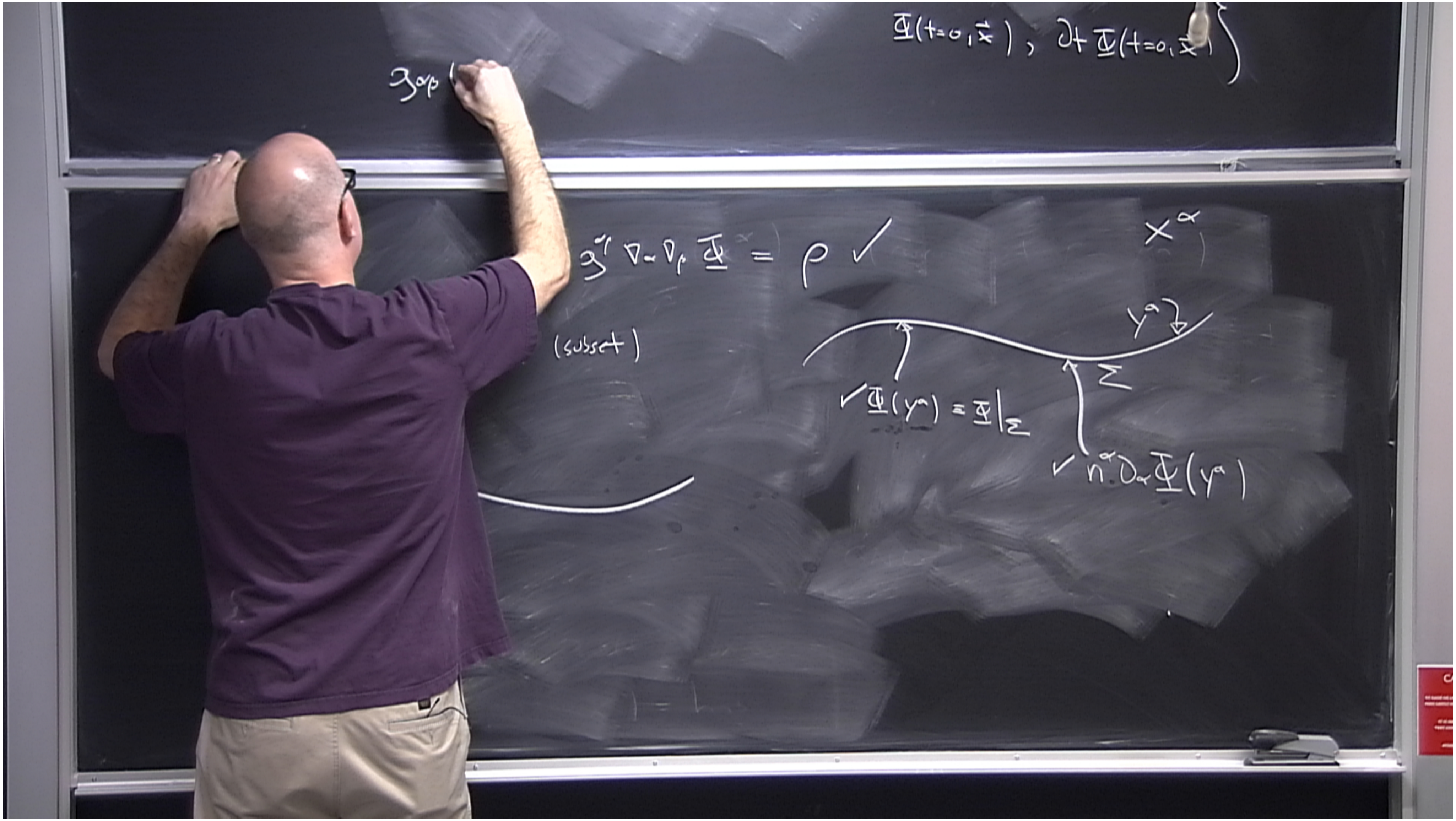
Φ(t=0, x⃗), ∂t Φ(t=0, x⃗)
g^αβ ∇α ∇β Φ = ρ ✓
x^α
y^a
Σ
✓ Φ(y^a) = Φ|Σ
✓ n^α ∂α Φ(y^a)

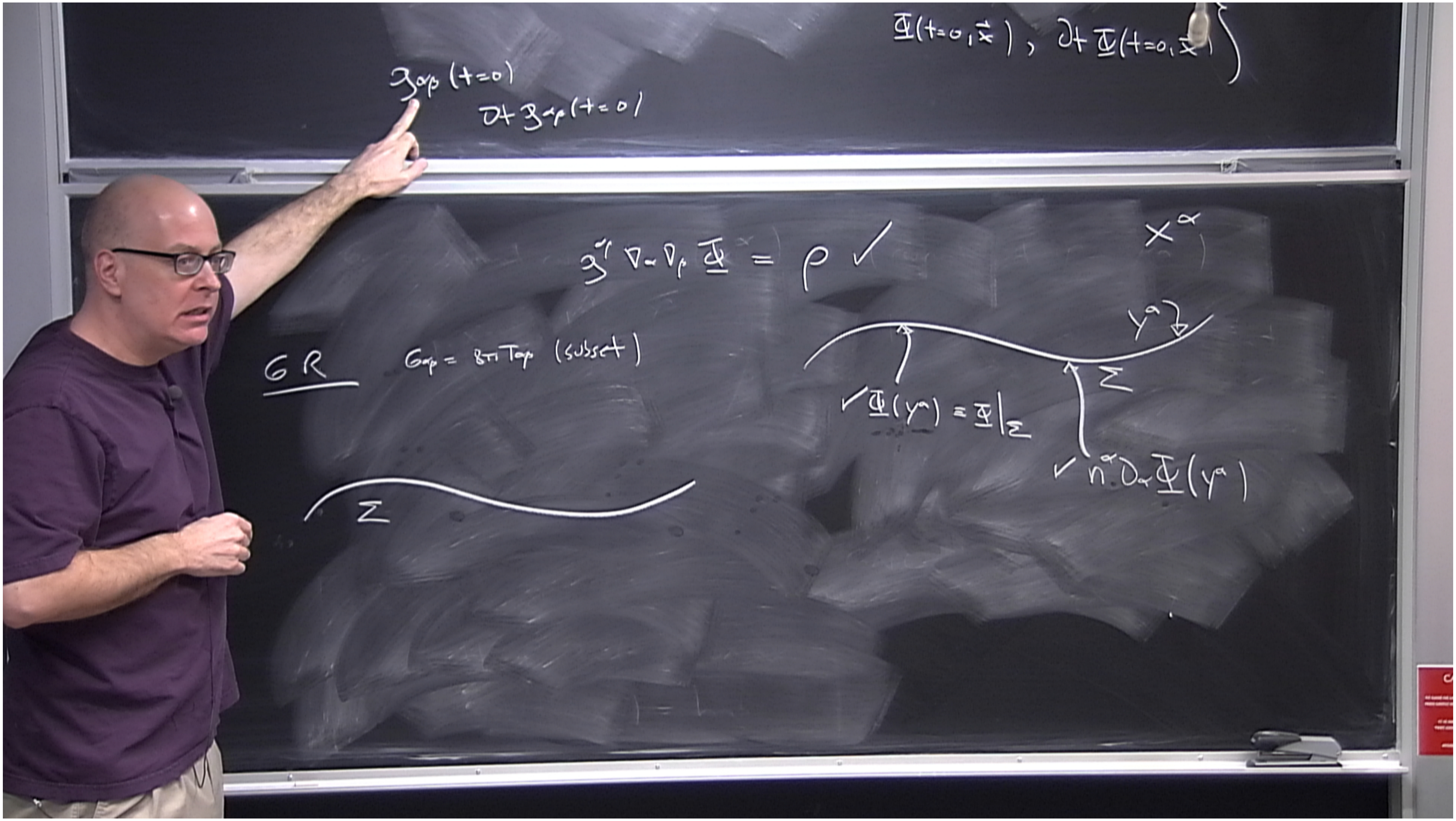

GR
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$

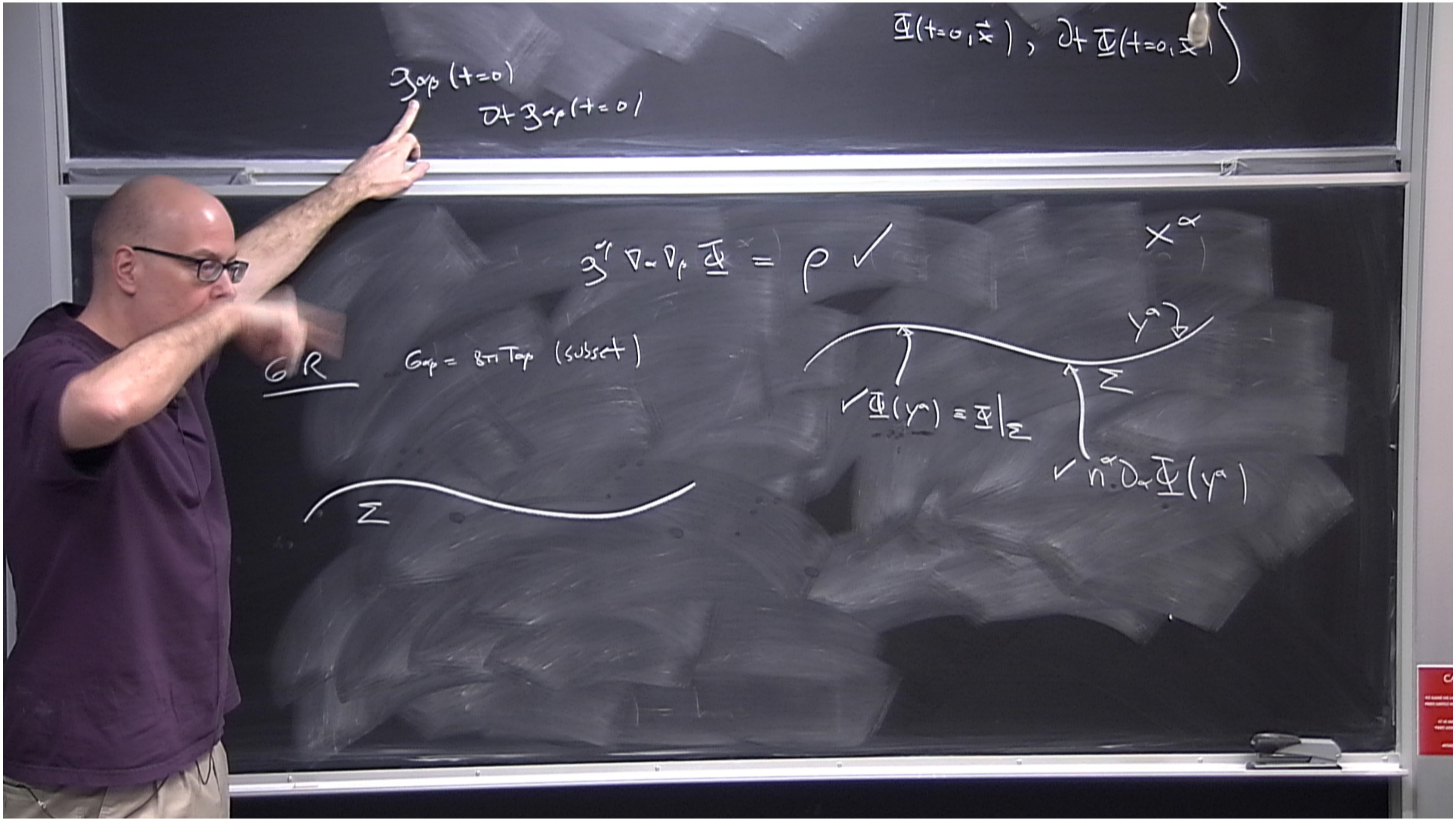

GR
$\nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$
Σ

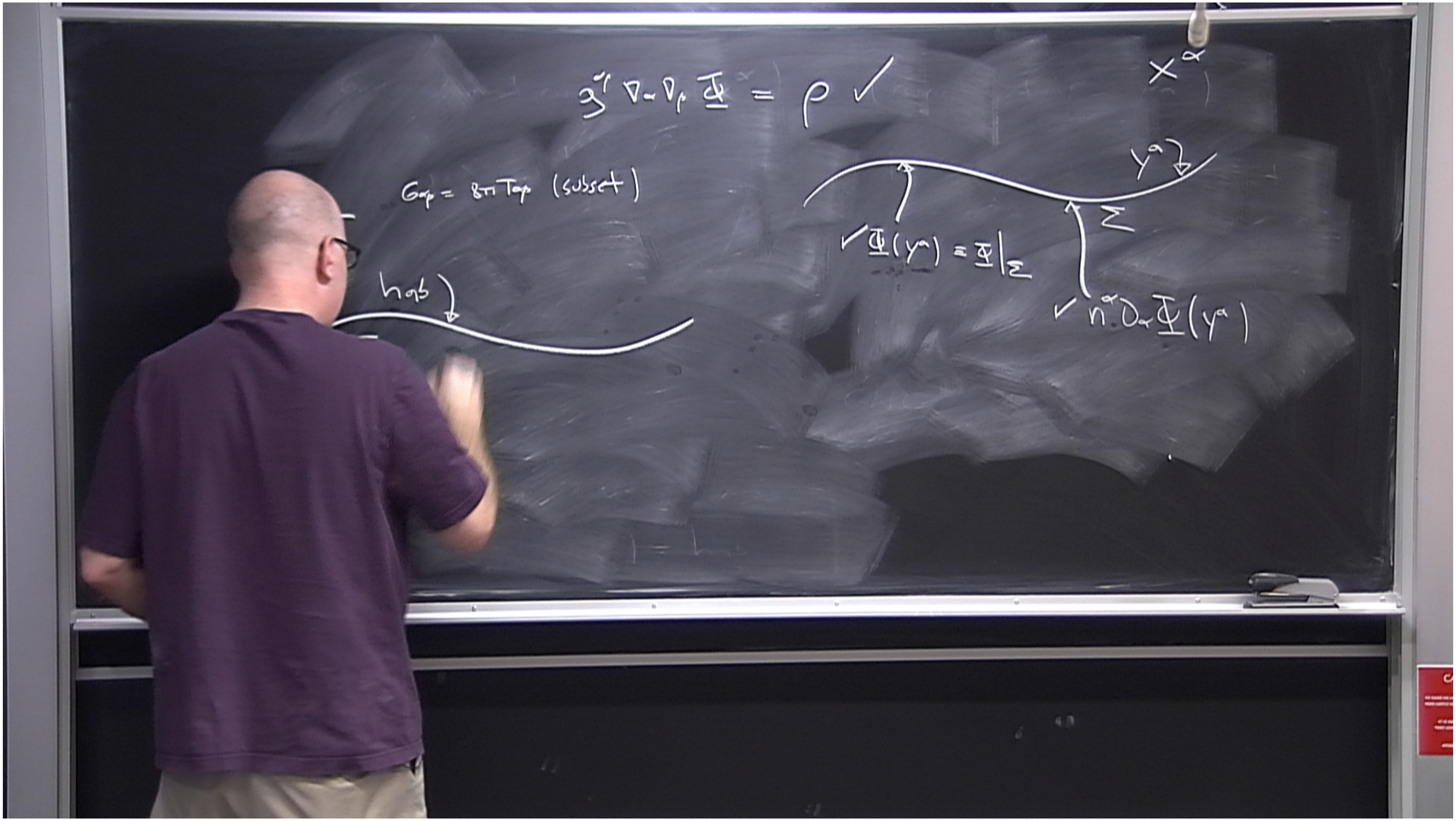

4 Gta = Db K^b_a − Da K ✓
4 Gab = Gab + ½ ∂t² gab (t=0) − ½ hab (h^cd ∂t² gcd(t=0)) − 2 Kac K^c_b
+ 3/2 hab (K^cd Kcd) ... K Kab − ½ hab K
4 Rμαβγ n^μ e^α_a e^β_b e^γ_c = Dc Kab − Db Kac
4 Rαβγδ e^α_a e^β_b e^γ_c e^δ_d = Rabcd + Kac Kbd −
4 Gμν n^μ n^ν = ½ (R − K^ab Kab + K²)
4 Gμα n^μ e^α_a = Db K^b_a − Da K
hab = gαβ e^α_a e^β_b
Kab = n(α;β) e^α_a e^β_b

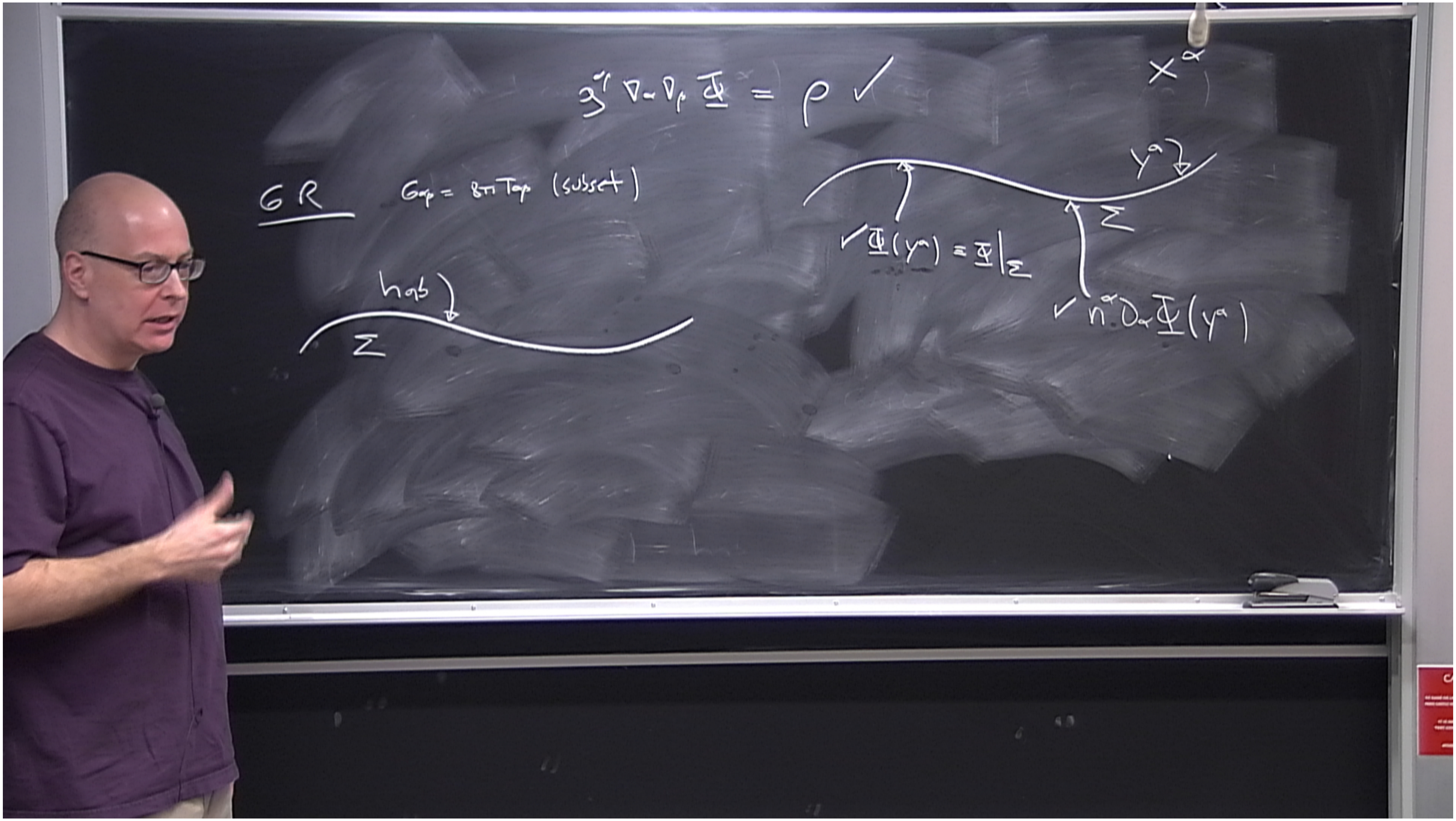

$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$
$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = \rho$ ✓
(subset)
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \partial_\alpha \Phi(y^a)$

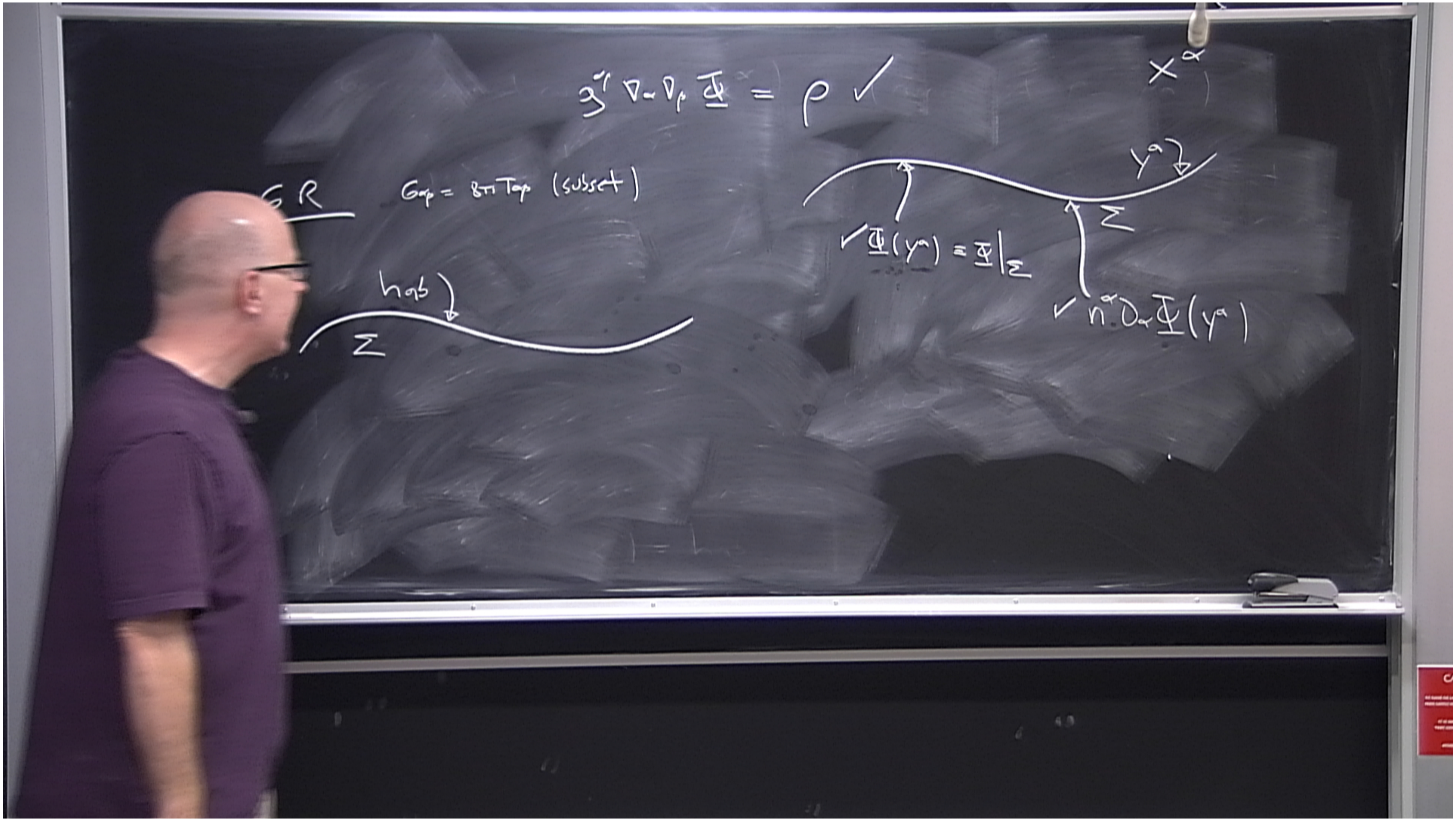

Φ(t=0, x), ∂t Φ(t=0, x) }
gαβ (
gαβ ∇α ∇β Φ = ρ ✓
xα
(subset)
ya
Σ
✓ Φ(ya) ≡ Φ|Σ
✓ nα ∇α Φ(ya)

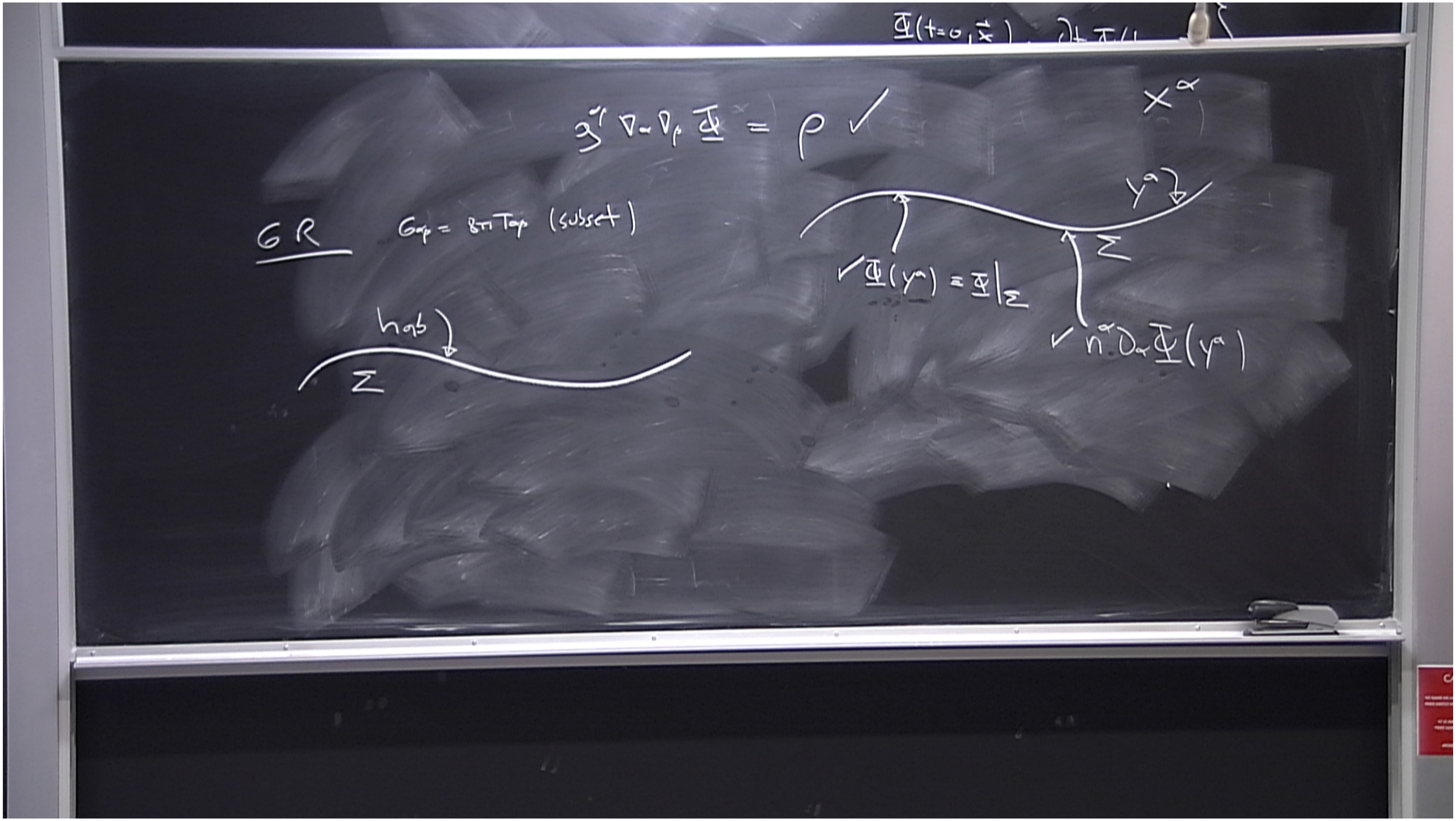

Φ(t=0, x), ∂t Φ(t=0, x)
gαβ (t=0)
∂t gαβ (t=0)
g^αβ ∇α ∇β Φ = ρ ✓
x^α
GR
Gαβ = 8π Tαβ (subset)
y^a
Σ
Φ(y^a) = Φ|Σ
n^α ∇α Φ(y^a)
Σ


$g_{\alpha\beta}(t=0)$
$\partial_t g_{\alpha\beta}(t=0)$
$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$
$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = \rho$ ✓
x^α
y^a
GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha\nabla_\alpha\Phi(y^a)$
Σ


$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
h_{ab}
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$


$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
h_{ab}
Σ
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$


$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$
GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
h_{ab}
Σ
x^α
y^a
Σ
$\Phi(y^a) = \Phi|_\Sigma$
$n^\alpha D_\alpha \Phi(y^a)$


$g^{\alpha\beta}\nabla_\alpha\nabla_\beta \Phi = \rho$ ✓
x^α
GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
h_{ab}
Σ
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$

$$+\frac{3}{2} h_{ab} (K^{cd} K_{cd}) + K K_{ab} - \frac{1}{2} h_{ab} K$$

$${}^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$$

Gauss-Codazzi

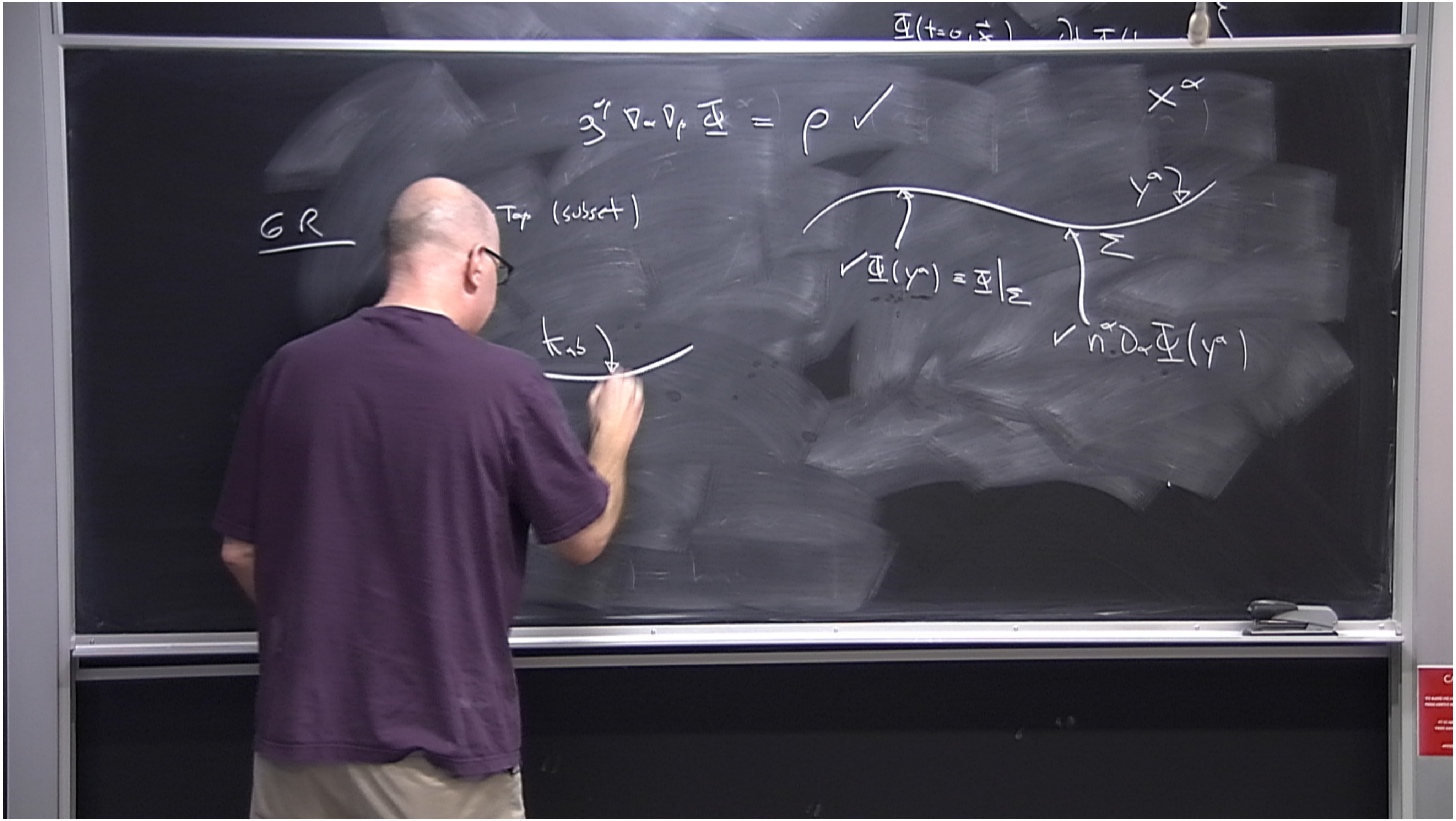
$${}^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac} K_{bd} - K_{ad} K_{bc}$$

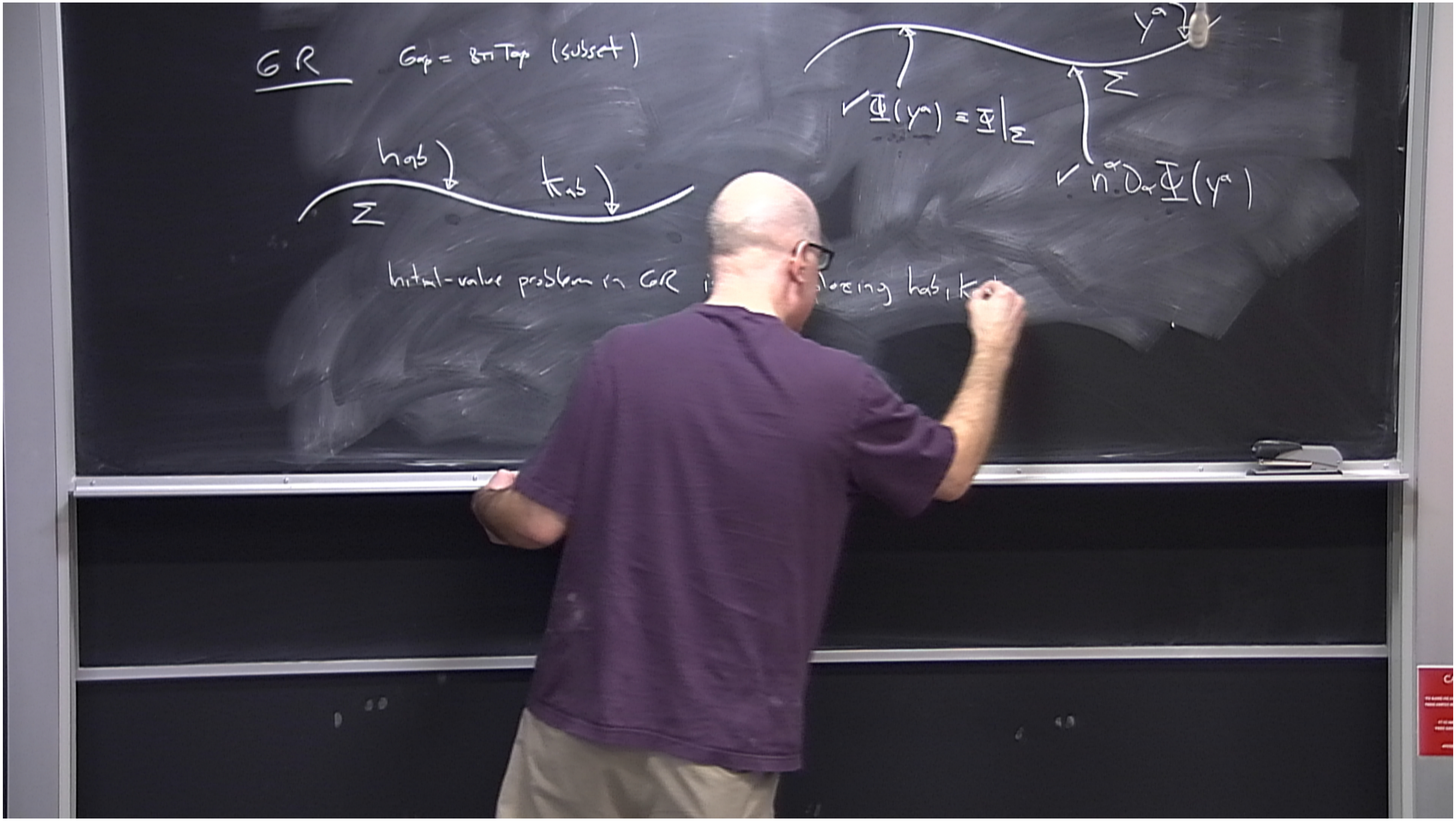
$${}^4G_{\mu\nu}\, n^\mu n^\nu = \frac{1}{2}\left(R - K^{ab} K_{ab} + K^2\right)$$

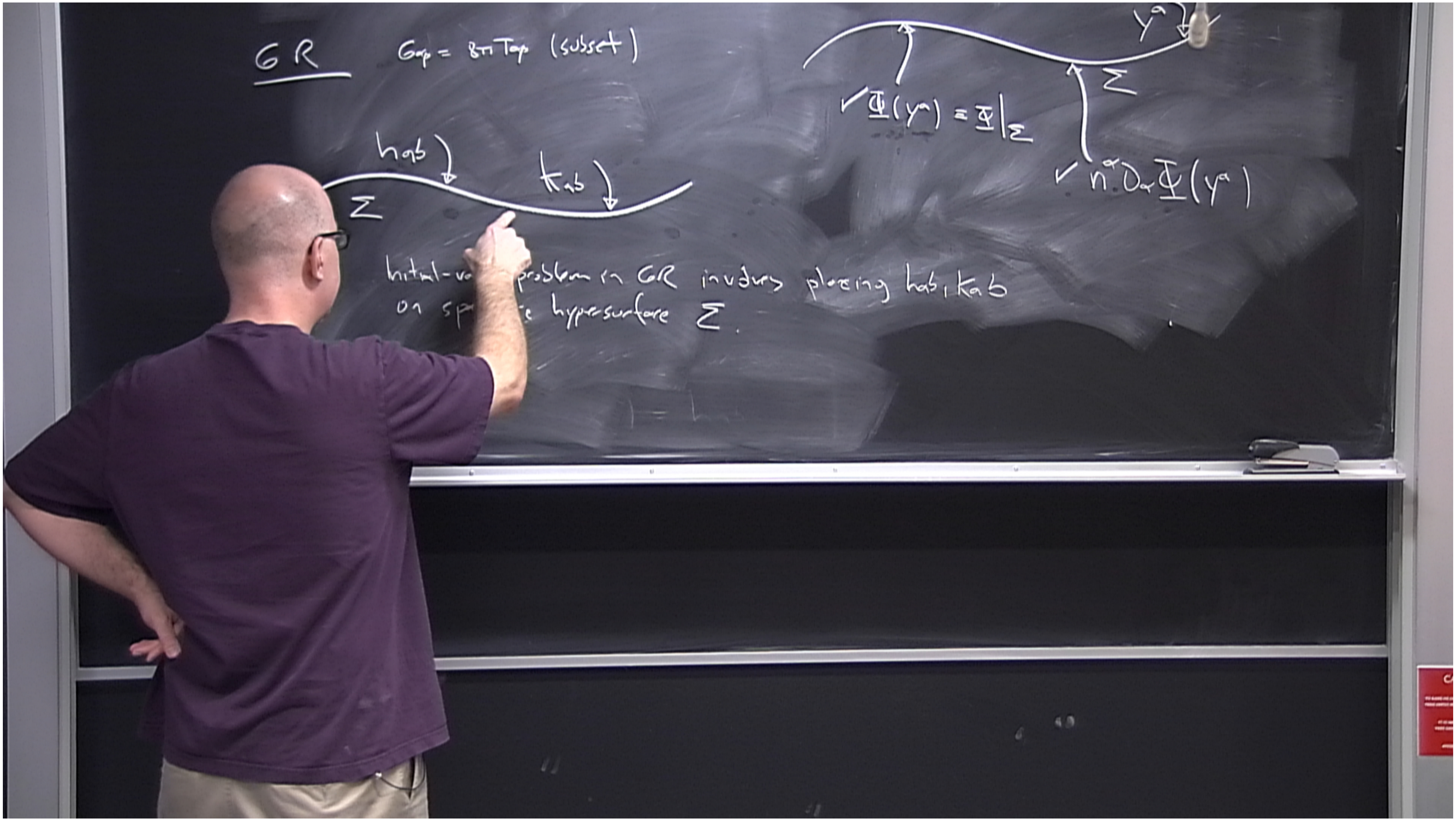
$${}^4G_{\mu\alpha}\, n^\mu e^\alpha_a = D_b K^b_a - D_a K$$

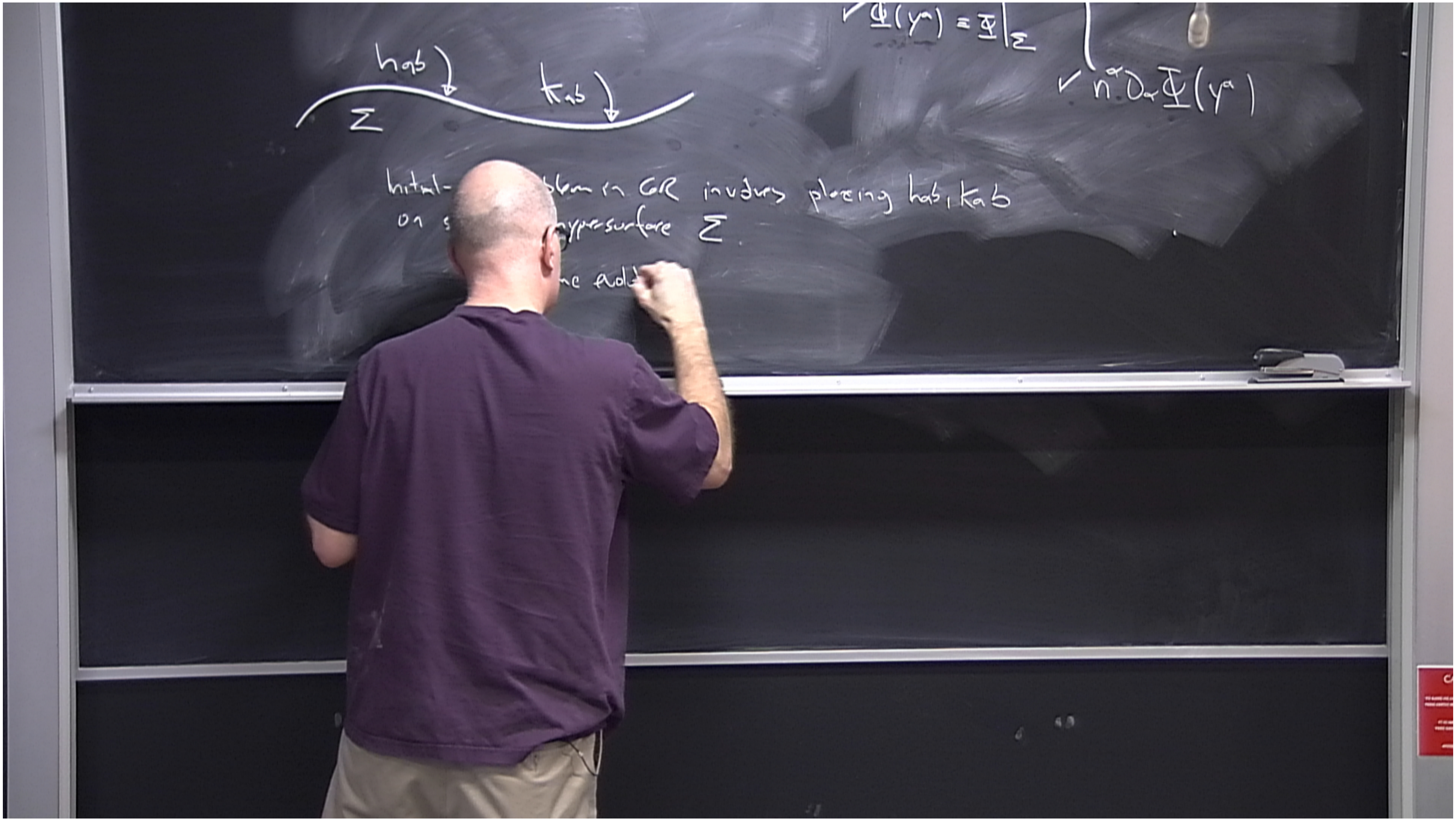
$$h_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b$$

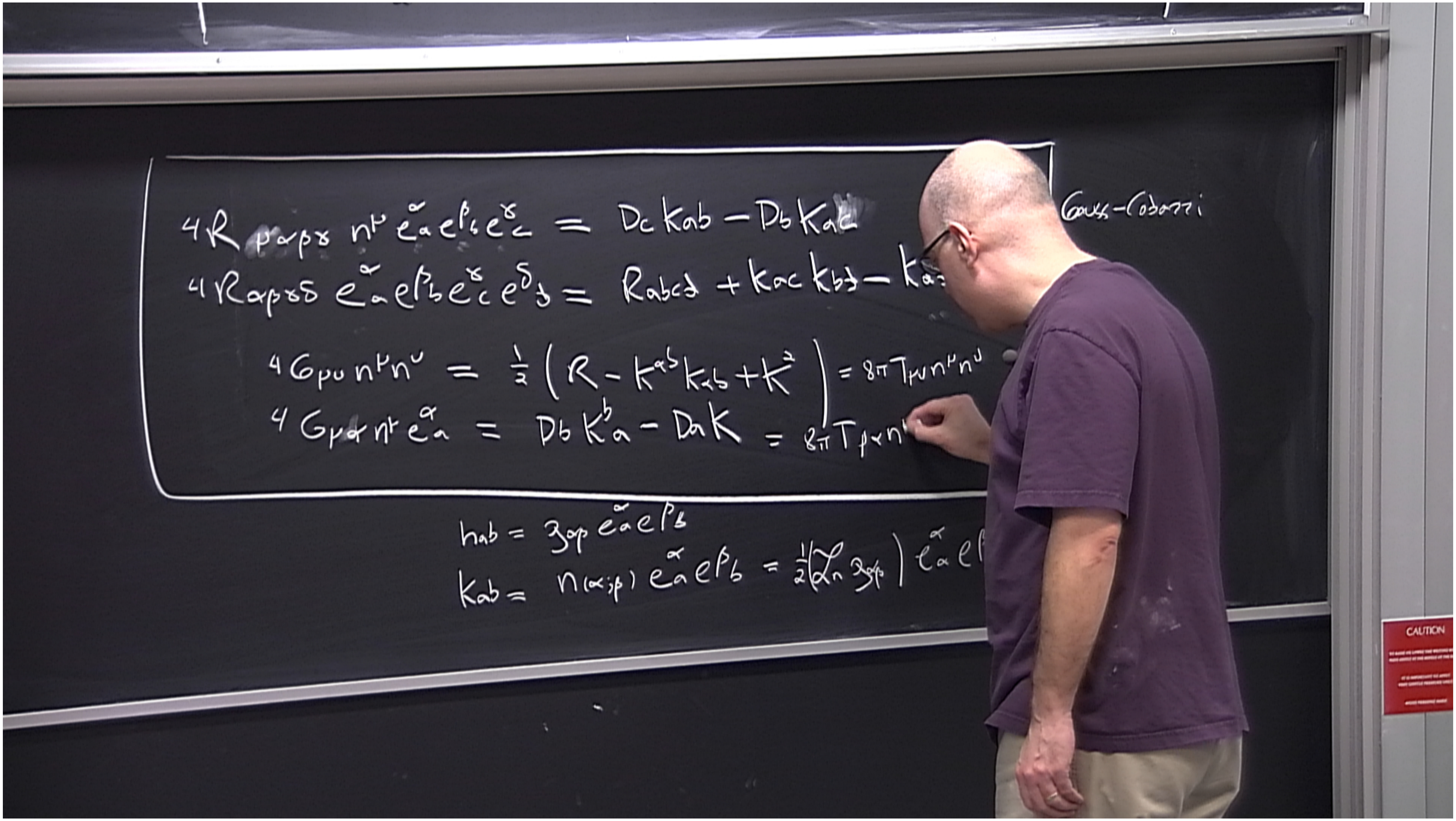
$$K_{ab} = n_{(\alpha;\beta)}\, e^\alpha_a e^\beta_b = \frac{1}{2}\left(\mathcal{L}_n g_{\alpha\beta}\right) e^\alpha_a e^\beta_b$$


GR
Top (subset)
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$
K_{ab}

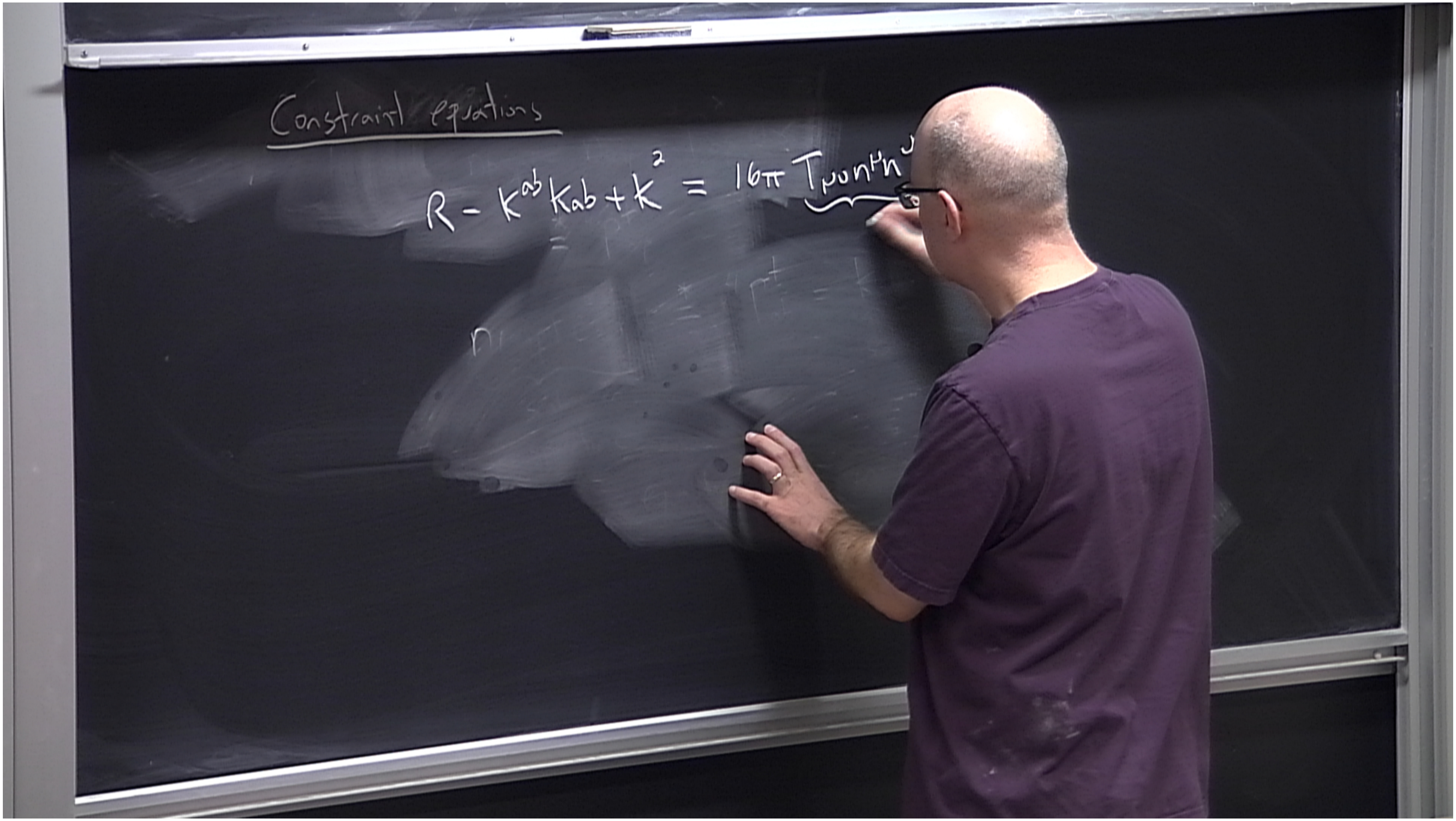

GR
Gαβ = 8π Tαβ (subset)
hab
Kab
Σ
Φ(ya) = Φ|Σ
nα ∂α Φ(ya)
ya
Σ

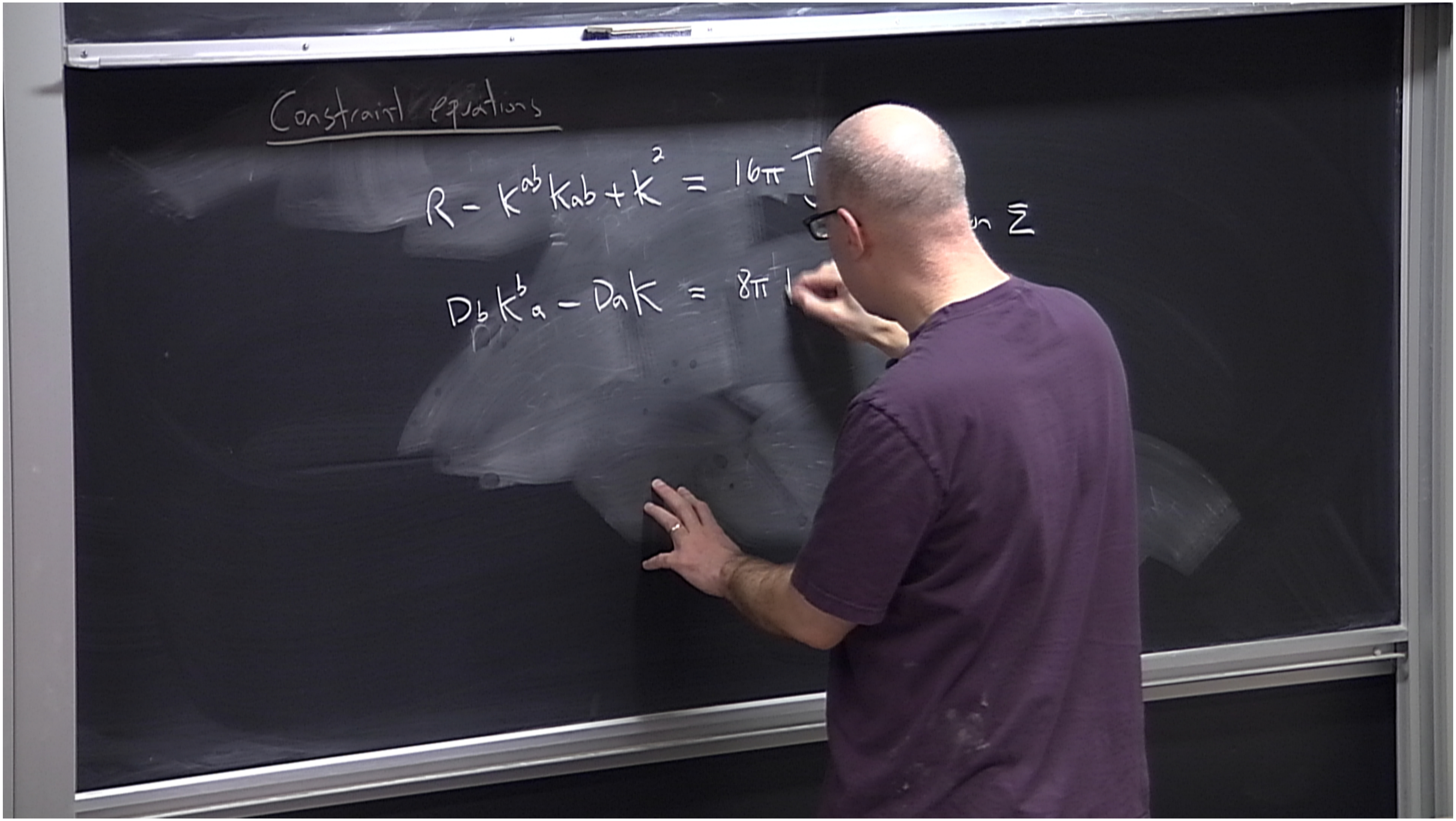

GR
Gαβ = 8π Tαβ (subset)
hab
kab
Σ
problem in GR involves placing hab, kab
hypersurface Σ.
y^a
Σ
Φ(y^a) = Φ|Σ
n^α Dα Φ(y^a)

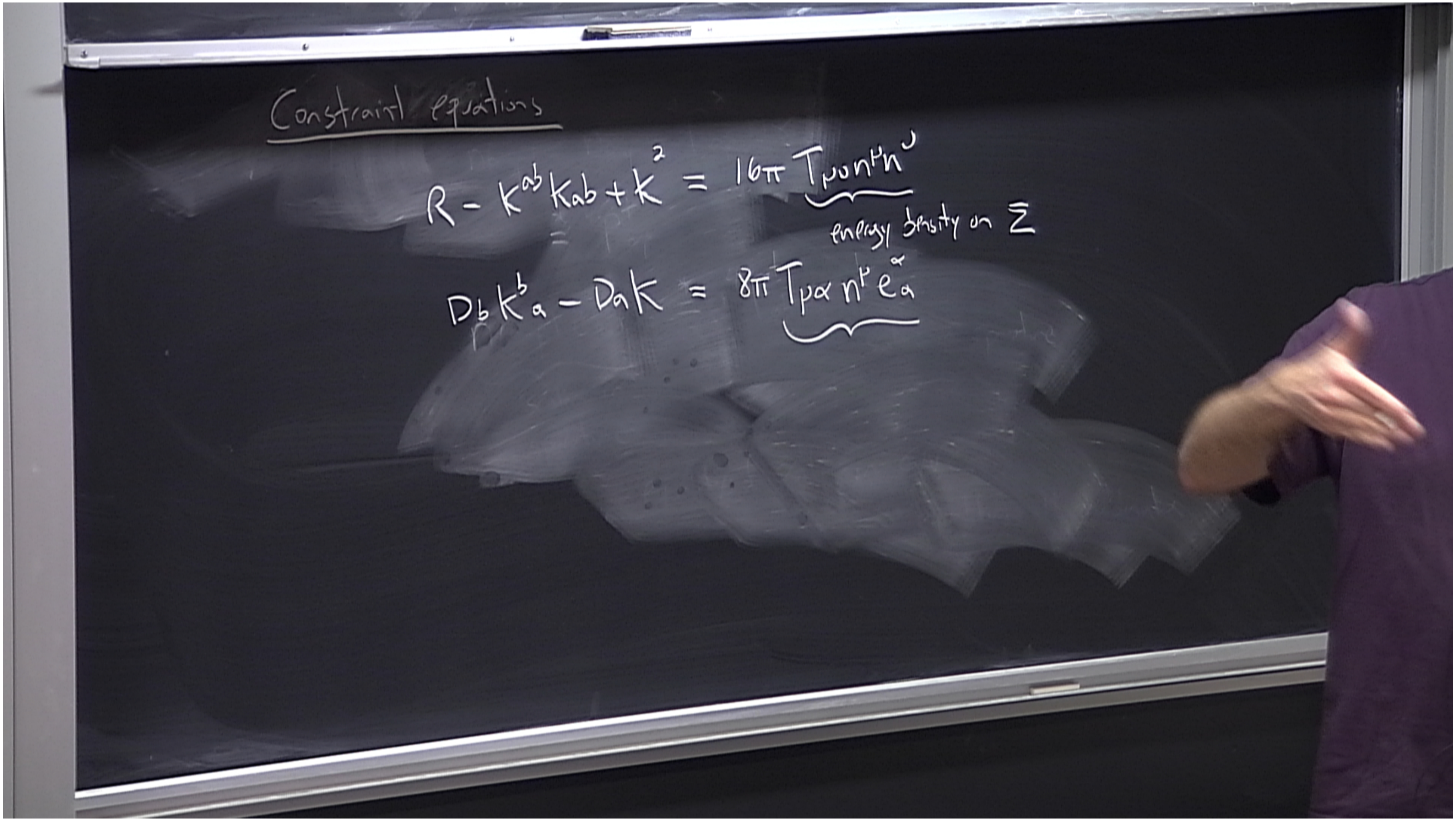

Φ(yᵃ) = Φ|Σ
hab
kab
Σ
nᵅ∂ᵅΦ(yᵃ)
in GR involves placing hab, kab
hypersurface Σ.

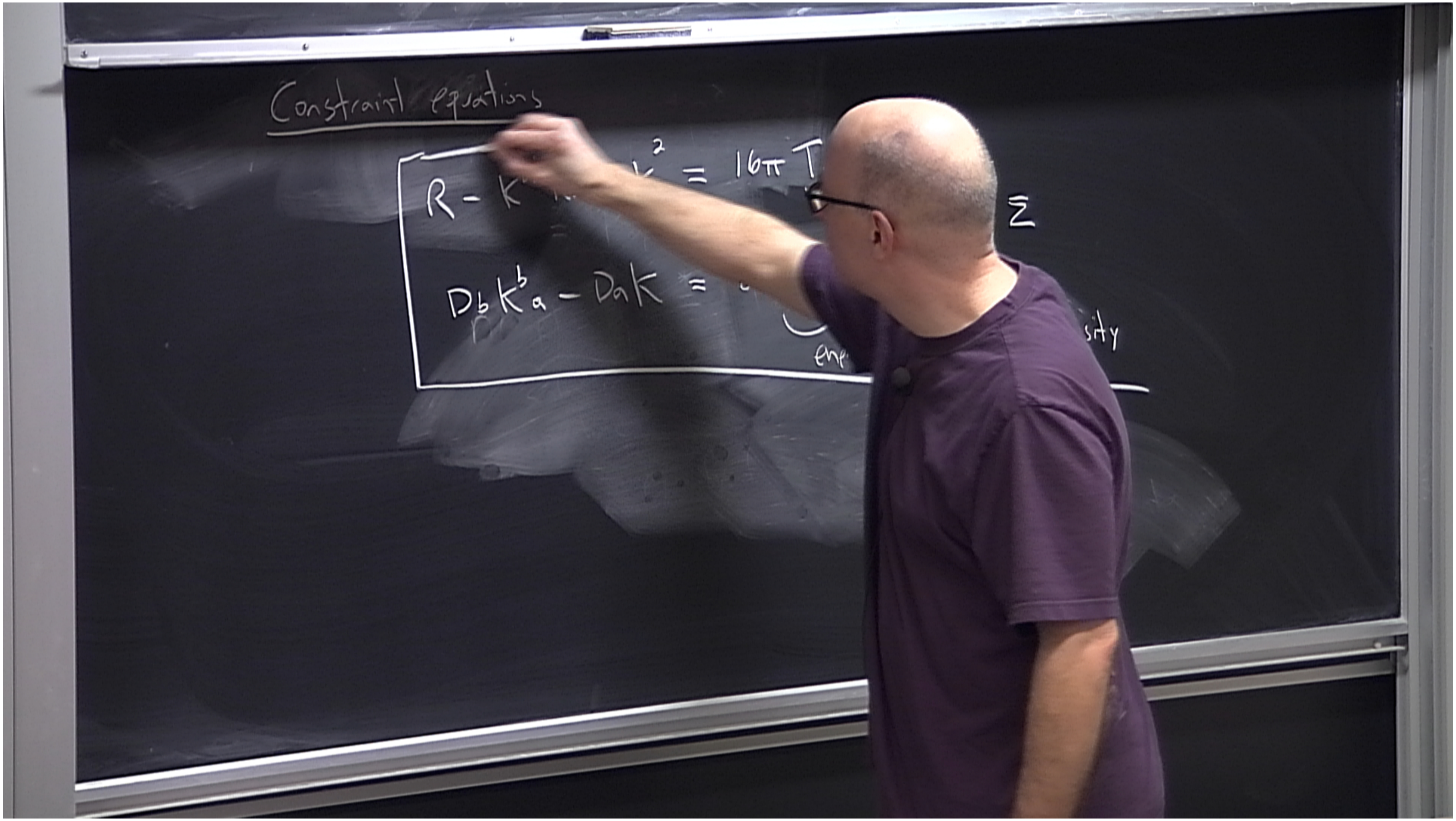


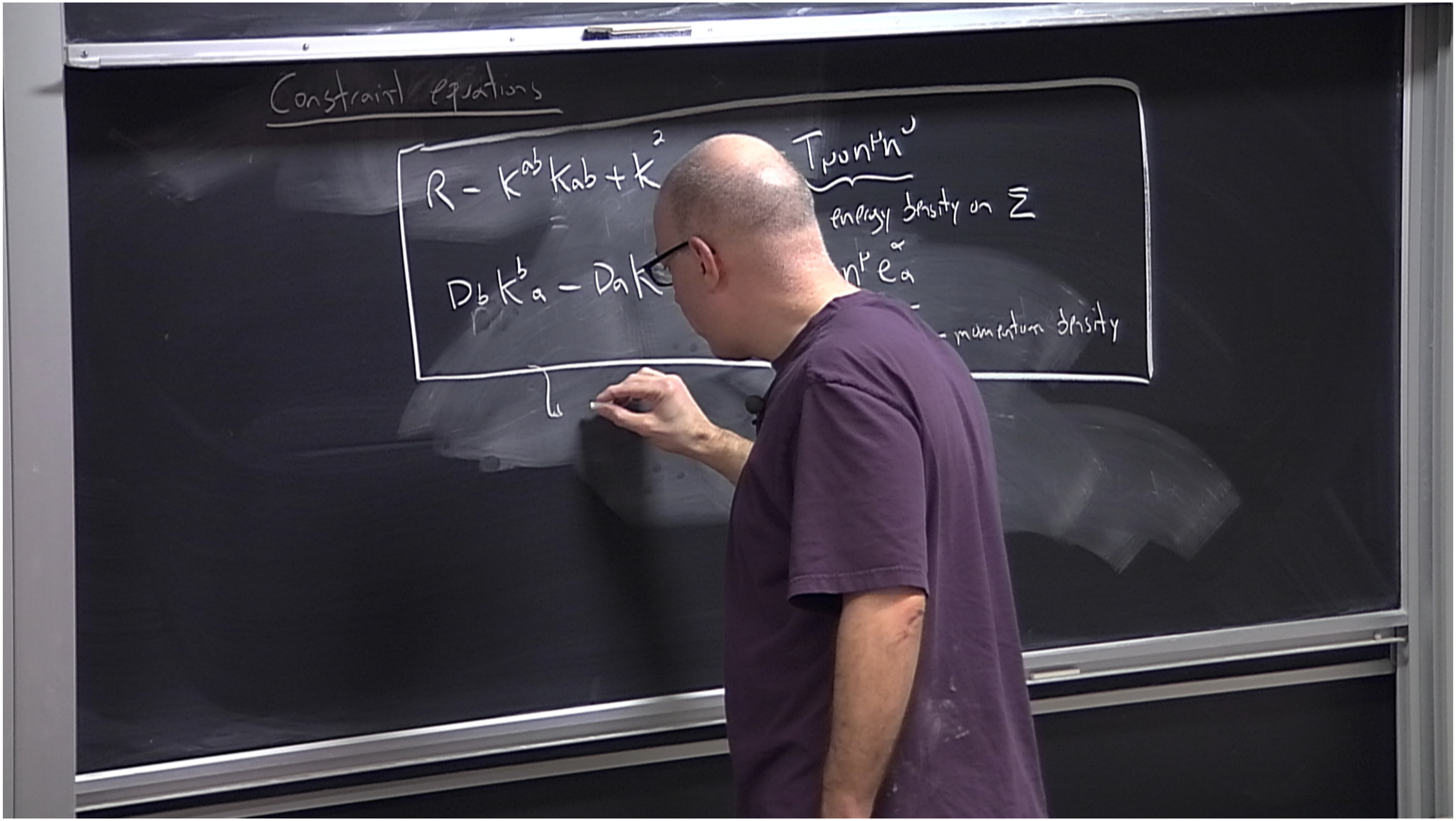
$^4R_{\mu\alpha\beta\gamma}\, n^\mu e^\alpha_a e^\beta_b e^\gamma_c = D_c K_{ab} - D_b K_{ac}$
$^4R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d = R_{abcd} + K_{ac} K_{bd} -$
Gauss-Codazzi
$^4G_{\mu\nu} n^\mu n^\nu = \frac{1}{2}\left(R - K^{ab} K_{ab} + K^2\right) = 8\pi T_{\mu\nu} n^\mu n^\nu$
$^4G_{\mu\alpha} n^\mu e^\alpha_a = D_b K^b_a - D_a K = 8\pi T_{\mu\alpha} n^\mu$
$h_{ab} = g_{\alpha\beta} e^\alpha_a e^\beta_b$
$K_{ab} = n_{(\alpha;\beta)} e^\alpha_a e^\beta_b = \frac{1}{2}(\mathcal{L}_n g_{\alpha\beta}) e^\alpha_a e^\beta$
CAUTION


Constraint equations
R - K^ab K_ab + K^2 = 16π T_μν n^μ n^ν


Constraint equations
R - K^ab Kab + K^2 = 16π T
n Σ
Db K^b a - Da K = 8π


Constraint equations
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu}n^\mu n^\nu$
energy density on Σ
$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$


Constraint equations
= 16π T
Db K^b_a − Da K =



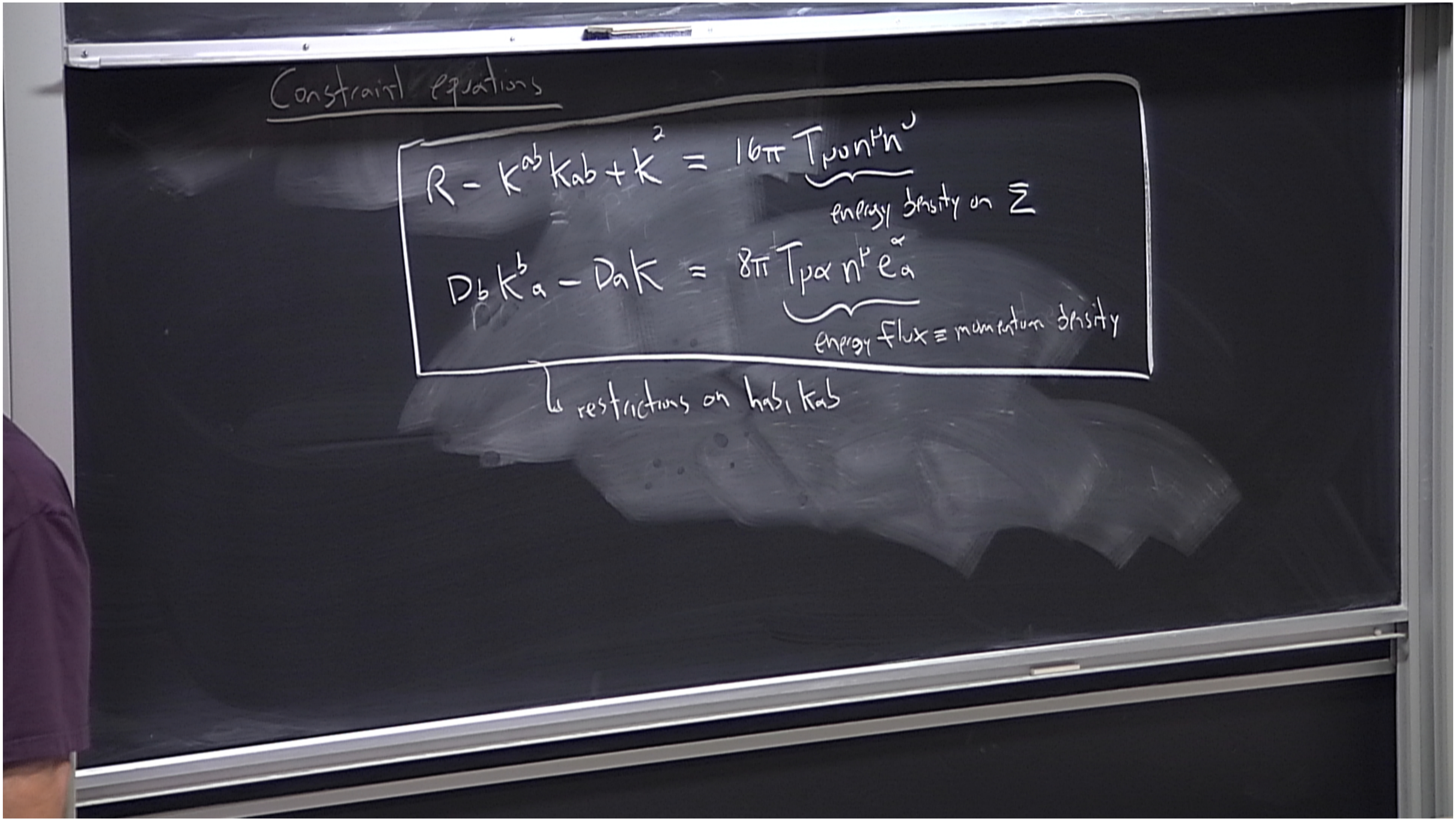
Constraint equations
energy density on Σ
momentum density

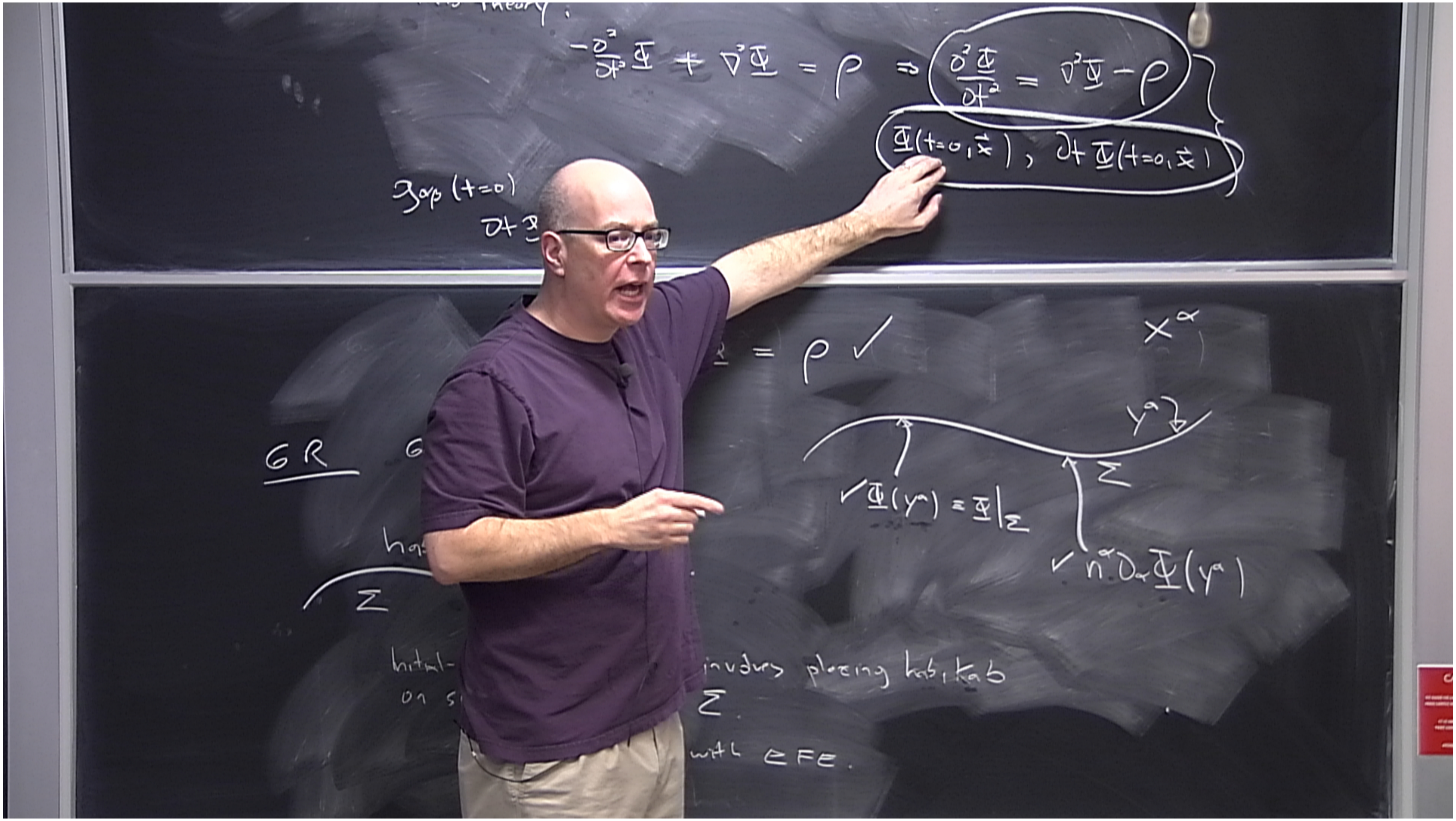
Constraint equations

$$R - K^{ab}K_{ab} + K^2 = 16\pi \, \underbrace{T_{\mu\nu} n^\mu n^\nu}_{\text{energy density on } \Sigma}$$

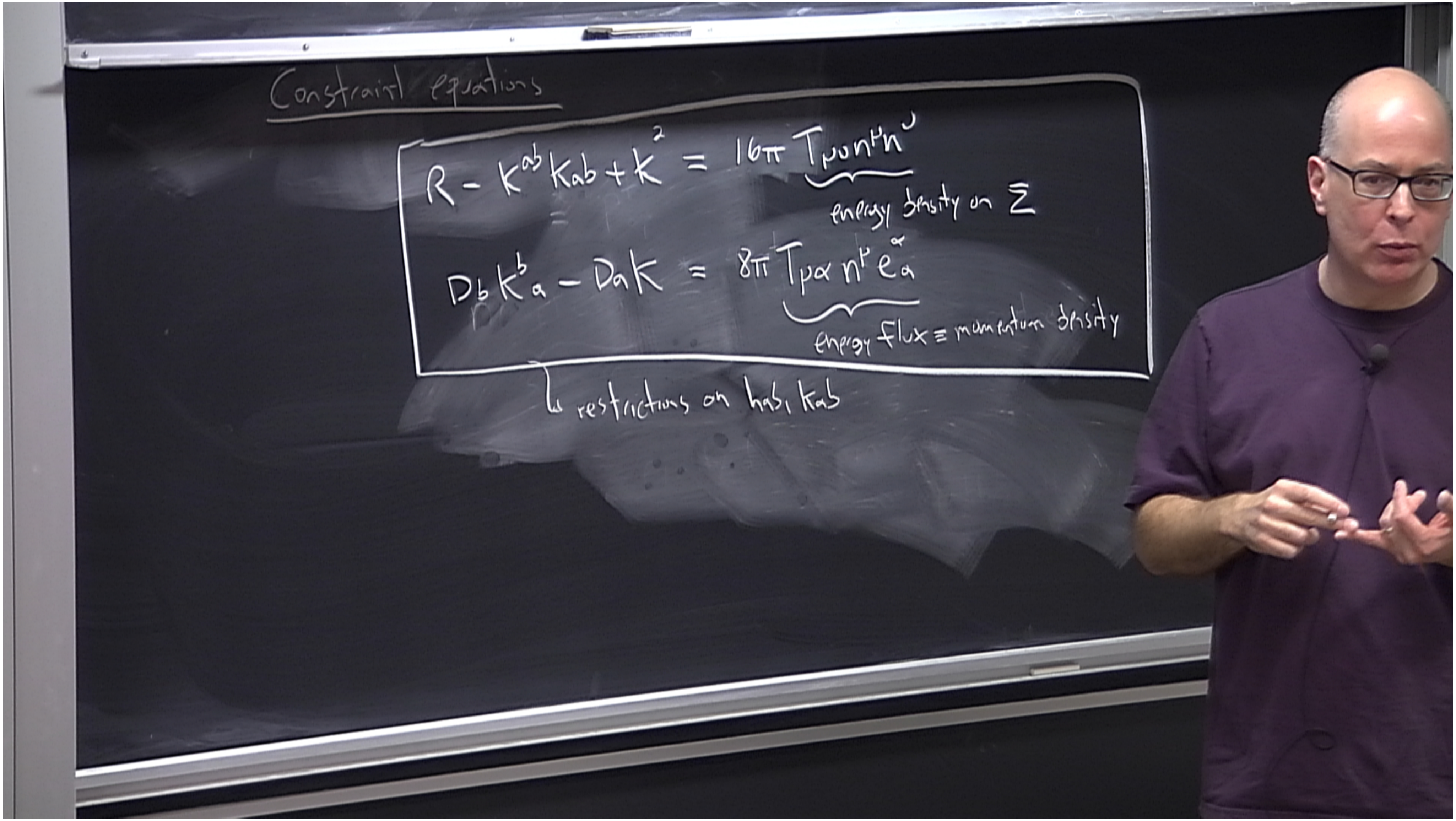
$$D_b K^b{}_a - D_a K = 8\pi \, \underbrace{T_{\mu\alpha} n^\mu e^\alpha_a}_{\text{energy flux} \equiv \text{momentum density}}$$

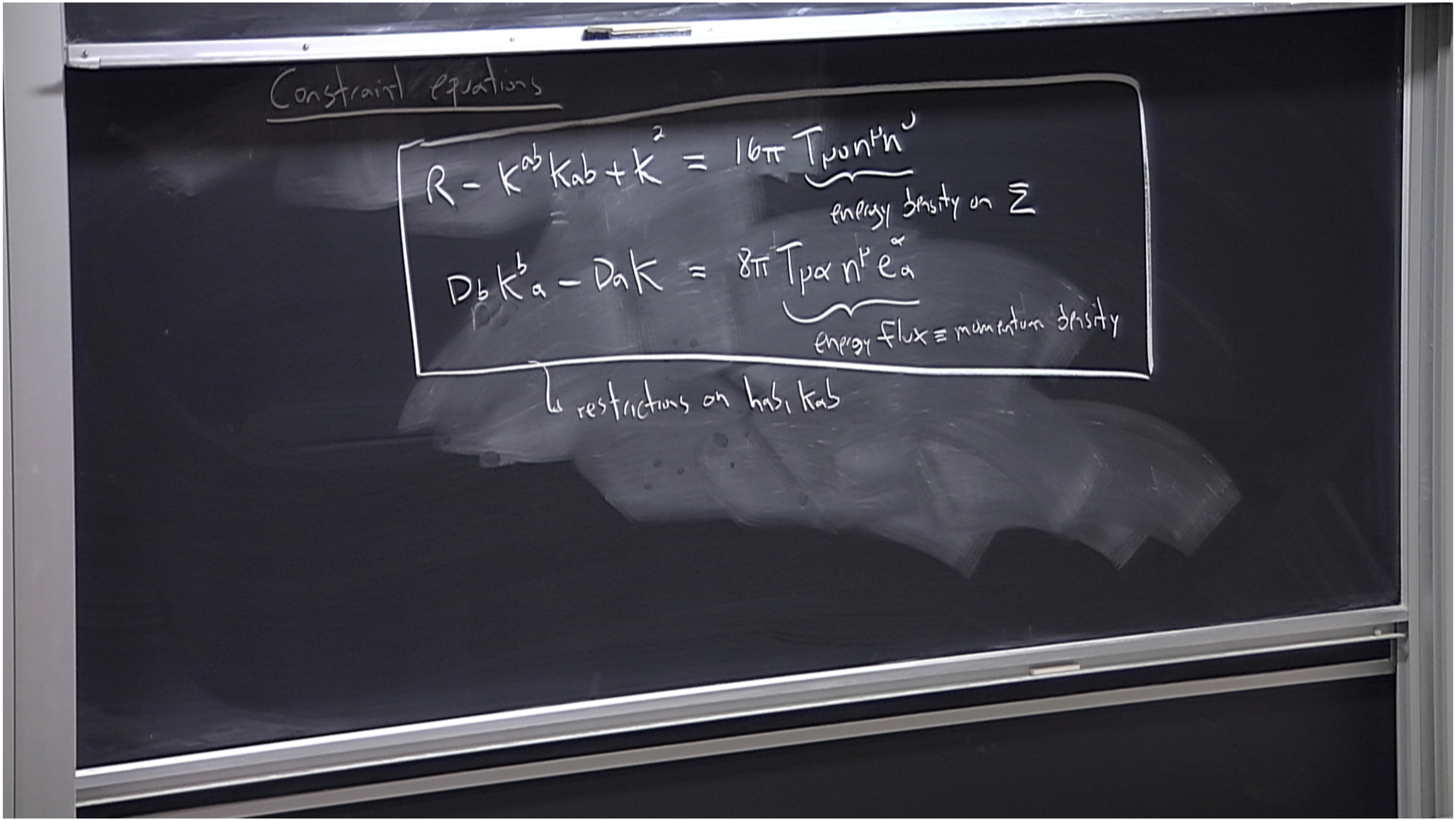
↳ restrictions on h_{ab}, K_{ab}

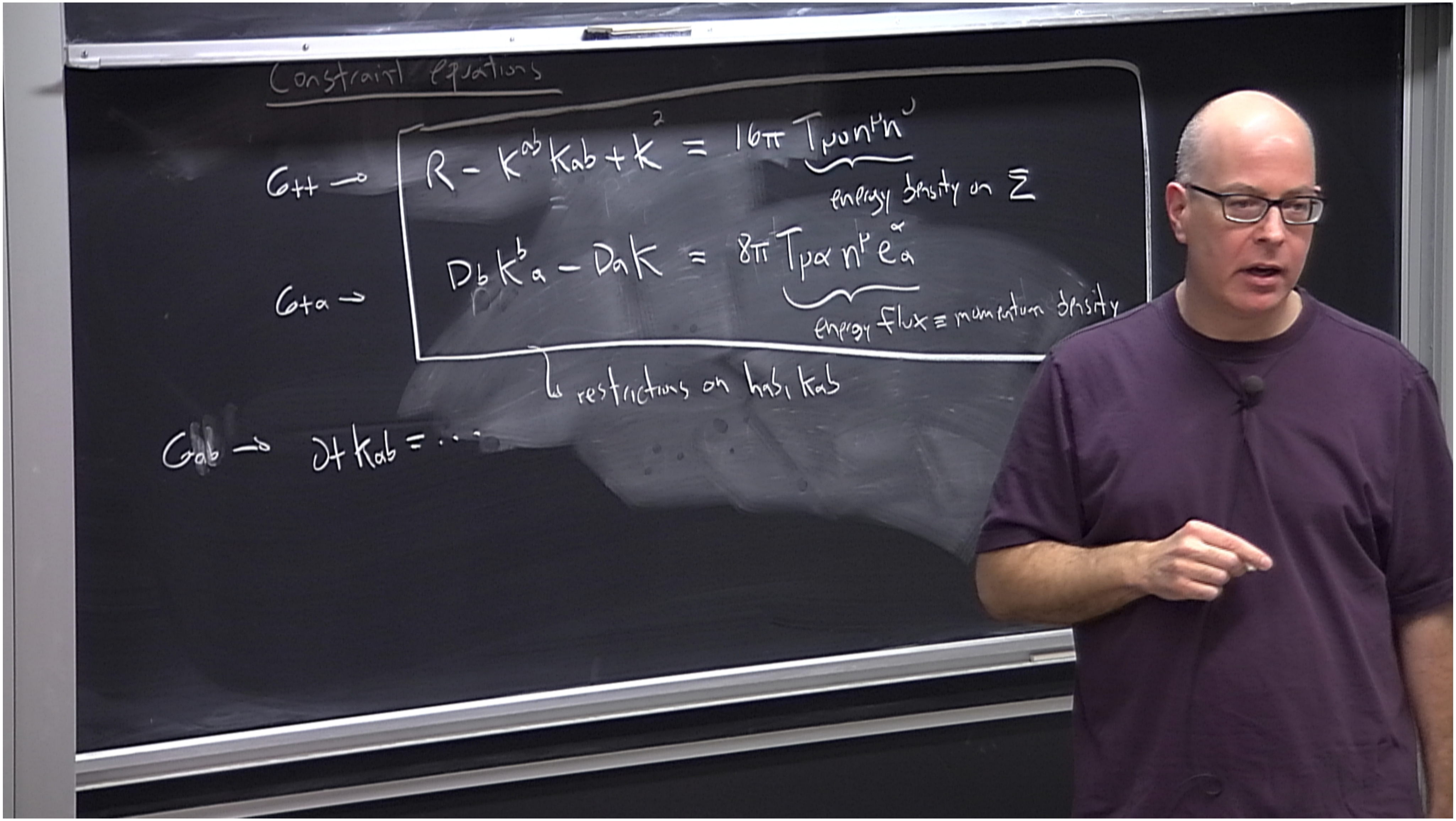

Constraint equations
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu}n^\mu n^\nu$
energy density on Σ
$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
energy flux ≡ momentum density
restrictions on h_{ab}, K_{ab}

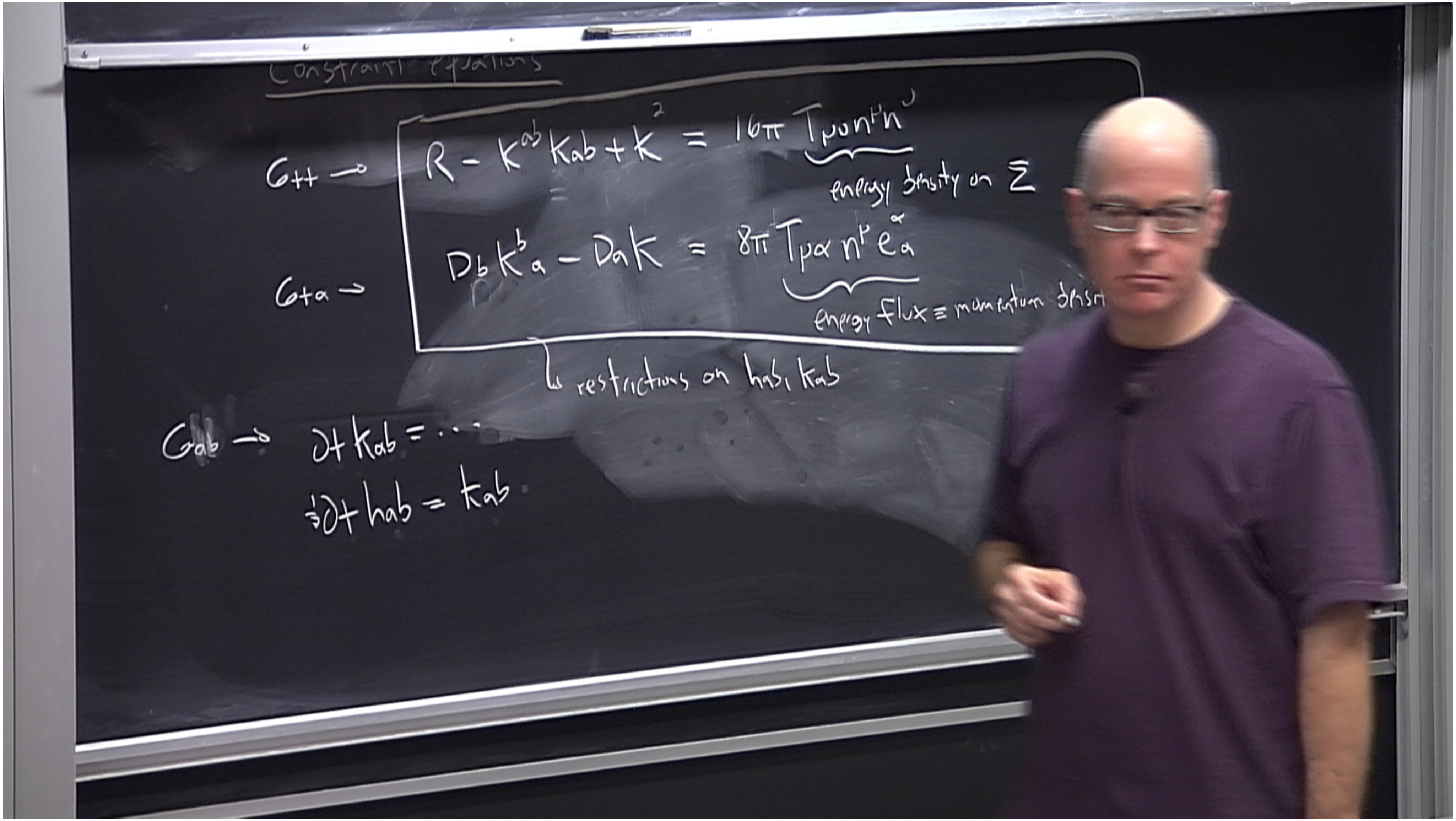


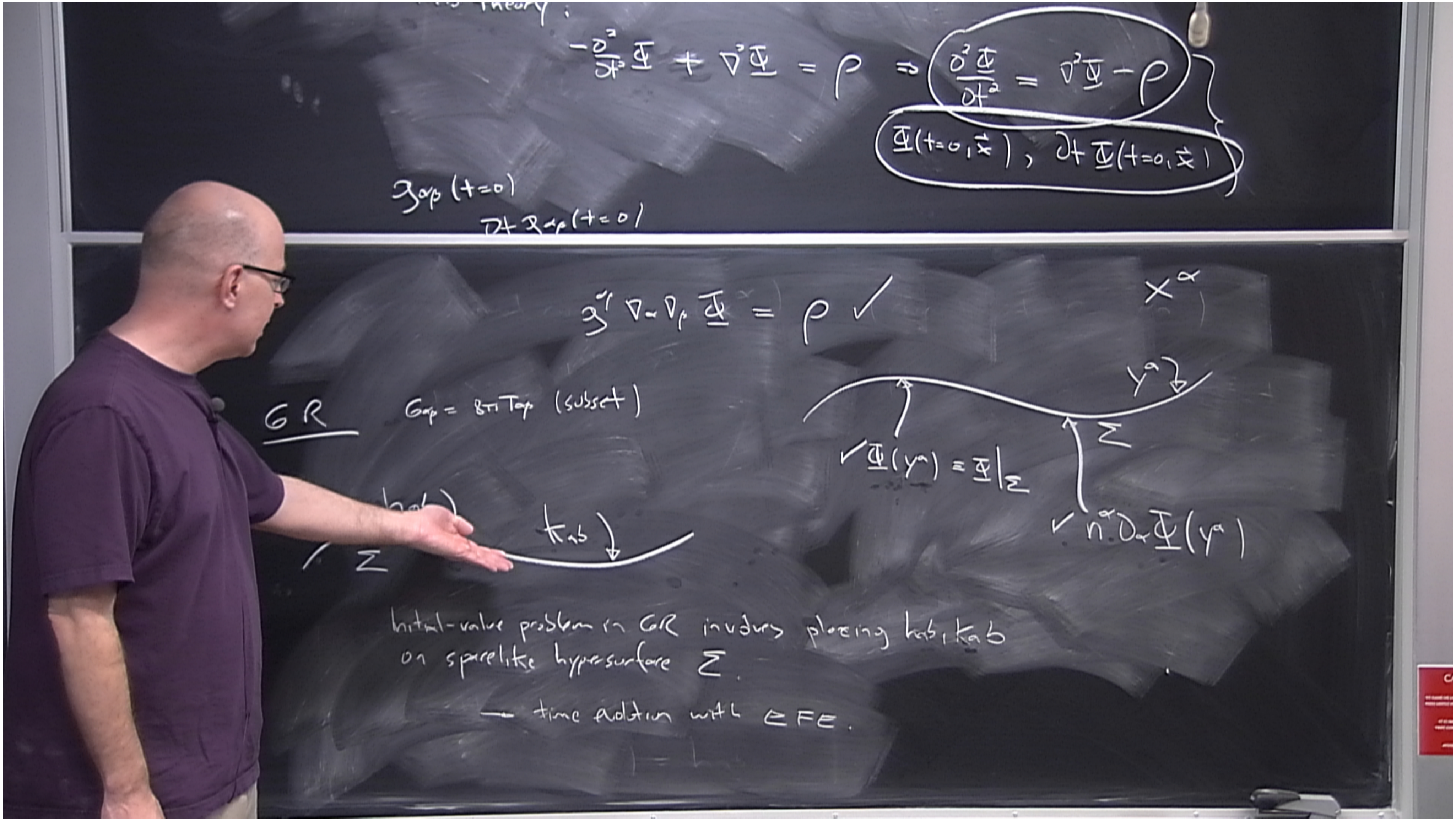
$-\frac{\partial^2}{\partial t^2}\Phi + \nabla^2\Phi = \rho \Rightarrow \frac{\partial^2\Phi}{\partial t^2} = \nabla^2\Phi - \rho$
$\Phi(t=0,\vec{x}),\ \partial_t\Phi(t=0,\vec{x})$
$g_{\alpha\beta}(t=0)$
$= \rho$ ✓
x^α
y^a
Σ
GR
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \partial_\alpha \Phi(y^a)$
Σ
involves placing h_{ab}, K_{ab}
Σ.
with EFE.

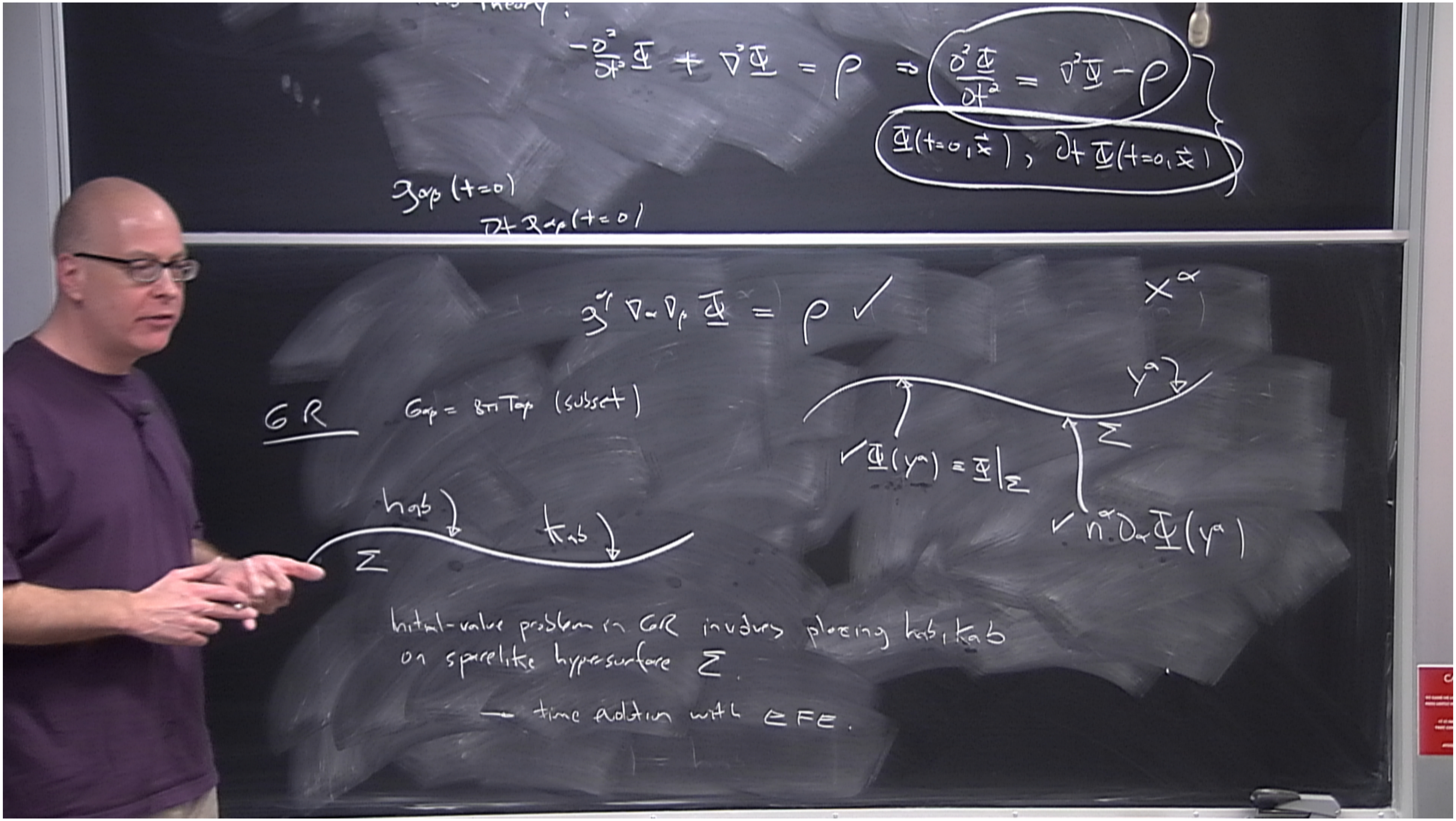

Constraint equations
$R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu}n^\mu n^\nu$
energy density on Σ
$D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
energy flux ≡ momentum density
restrictions on h_{ab}, K_{ab}

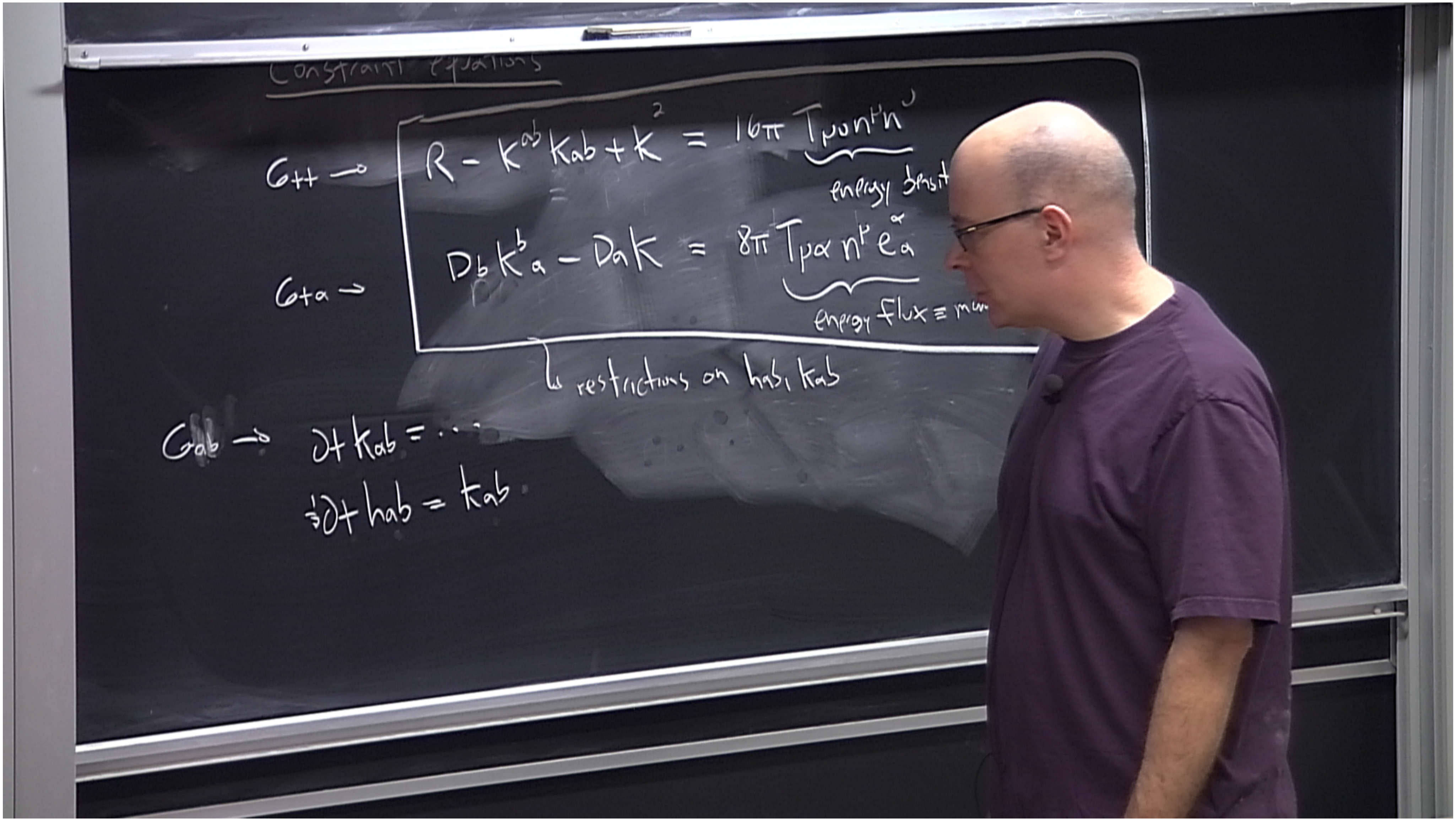

Constraint equations
R - K^ab Kab + K^2 = 16π Tμν n^μ n^ν
energy density on Σ
Db K^b a - Da K = 8π Tμα n^μ e^α a
energy flux ≡ momentum density
restrictions on hab, Kab


Constraint Equations
Gtt → R - K^ab Kab + K^2 = 16π Tμν n^μ n^ν
energy density on Σ
Gta → Db K^b a - Da K = 8π Tμα n^μ e^α a
energy flux ≡ momentum density
↳ restrictions on hab, Kab
Gab → ∂t Kab = . . .

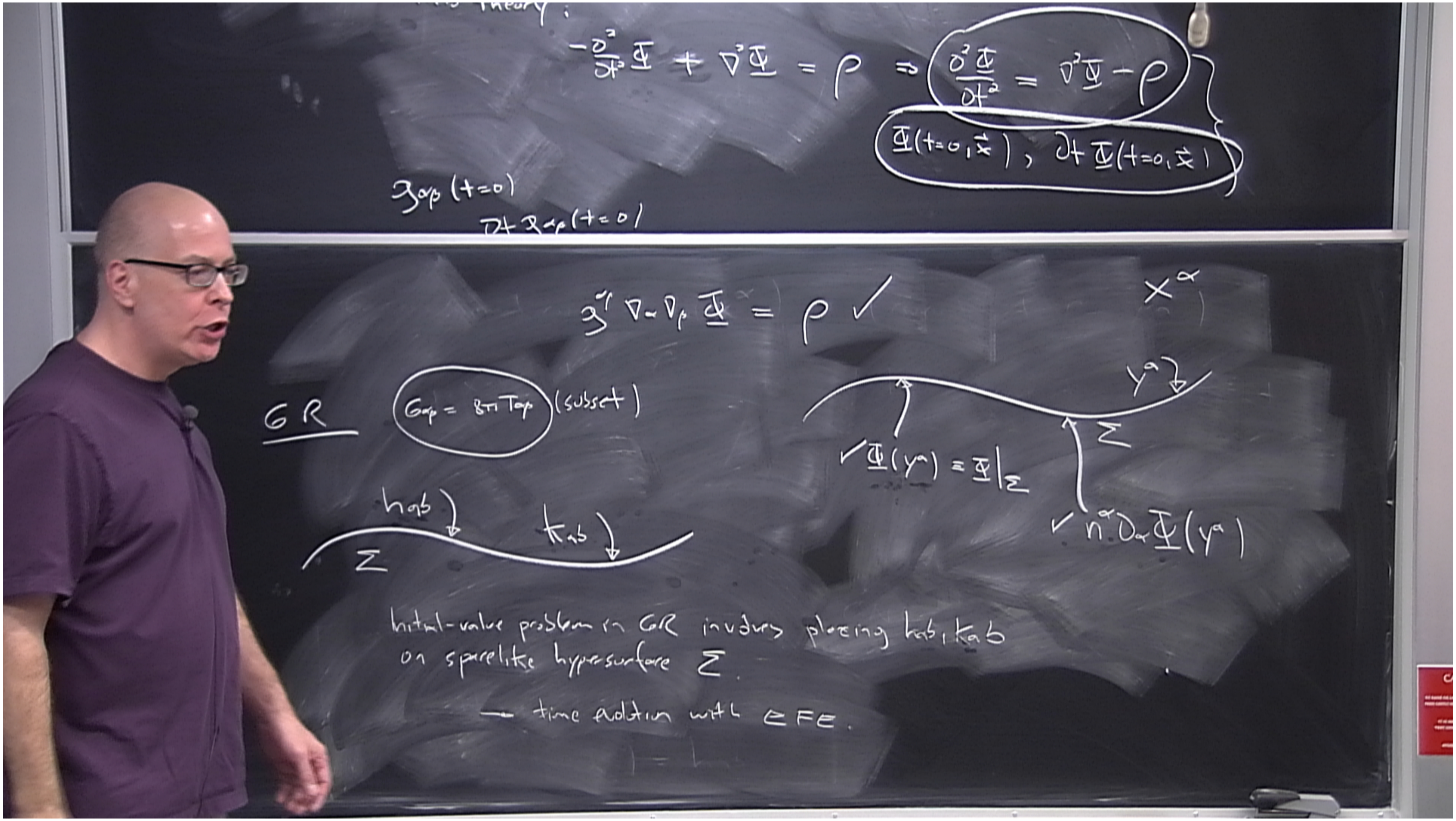

Constraint equations
G_tt → R − K^ab K_ab + K^2 = 16π T_μν n^μ n^ν
energy density on Σ
G_ta → D_b K^b_a − D_a K = 8π T_μα n^μ e^α_a
energy flux ≡ momentum density
restrictions on h_ab, K_ab
G_ab → ∂_t K_ab = …
½ ∂_t h_ab = K_ab


gαβ (t=0)
GR
Gαβ = 8π Tαβ (subset)
Σ
Kab
Φ(yᵃ) = Φ|Σ
nᵅ∇αΦ(yᵃ)
initial-value problem in GR involves placing hab, Kab
on spacelike hypersurface Σ.
→ time evolution with EFE.


GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha \nabla_\alpha \Phi(y^a)$
h_{ab}
k_{ab}
initial-value problem in GR involves placing h_{ab}, k_{ab}
on spacelike hypersurface Σ.
→ time evolution with EFE.
$g_{\alpha\beta}(t=0)$
$\partial_t g_{\alpha\beta}(t=0)$
$-\frac{\partial^2}{\partial t^2}\Phi + \nabla^2\Phi = \rho \Rightarrow \frac{\partial^2 \Phi}{\partial t^2} = \nabla^2\Phi - \rho$
$\Phi(t=0,\vec{x})$, $\partial_t \Phi(t=0,\vec{x})$



Constraint equations
Gtt → R - K^ab Kab + K^2 = 16π Tμν n^μ n^ν
energy density
Gta → Db K^b a - Da K = 8π Tμα n^μ e^α a
energy flux ≡ mom
↳ restrictions on hab, Kab
Gab → ∂t Kab = ...
½∂t hab = Kab



theory :
$-\frac{\partial^2}{\partial t^2}\Phi + \nabla^2\Phi = \rho \Rightarrow \frac{\partial^2\Phi}{\partial t^2} = \nabla^2\Phi - \rho$
$\Phi(t=0,\vec{x}),\ \partial_t\Phi(t=0,\vec{x})$
$g_{\alpha\beta}(t=0)$
$\partial_t g_{\alpha\beta}(t=0)$
$g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = \rho$ ✓
x^α
y^a
Σ
✓ $\Phi(y^a) = \Phi|_\Sigma$
✓ $n^\alpha\nabla_\alpha\Phi(y^a)$
GR
$G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (subset)
h_{ab}
K_{ab}
Σ
Initial-value problem in GR involves placing h_{ab}, K_{ab} on spacelike hypersurface Σ.
→ time evolution with EFE.

Constraint equations

$G_{tt} \to$ $$R - K^{ab}K_{ab} + K^2 = 16\pi \underbrace{T_{\mu\nu}n^\mu n^\nu}_{\text{energy density on } \Sigma}$$

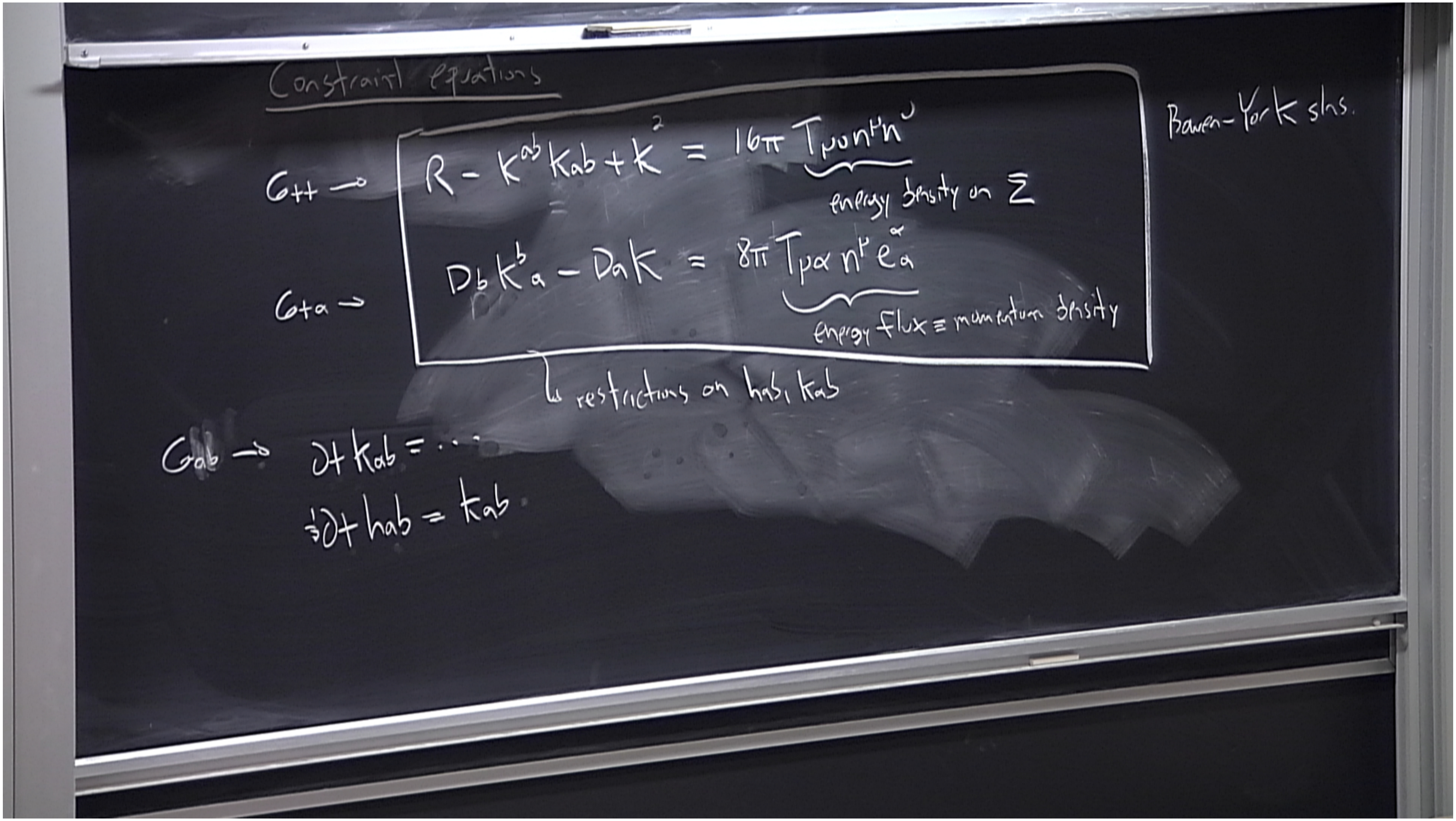
$G_{ta} \to$ $$D_b K^b{}_a - D_a K = 8\pi \underbrace{T_{\mu\alpha} n^\mu e^\alpha_a}_{\text{energy flux} \equiv \text{momentum density}}$$

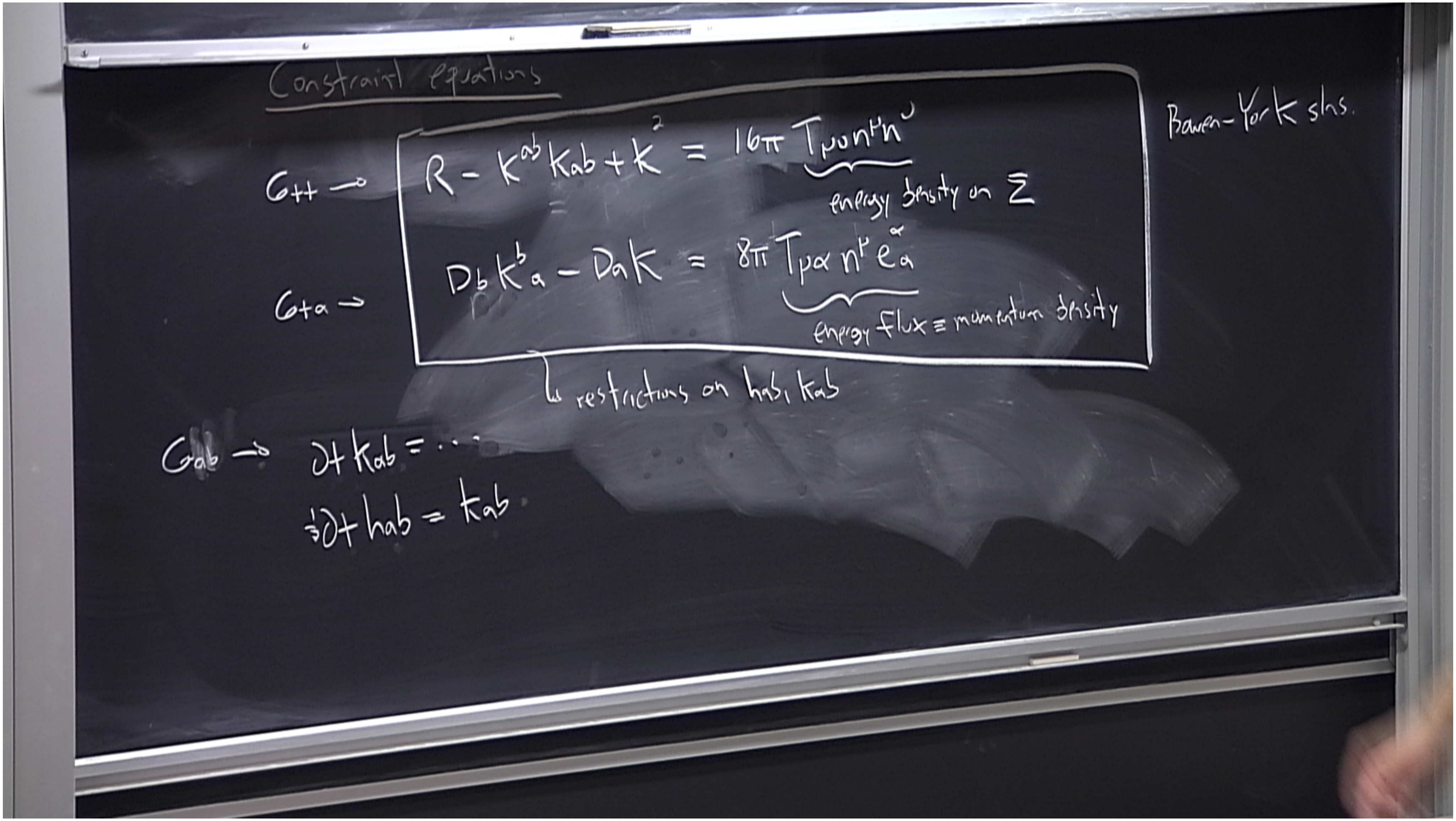
↳ restrictions on h_{ab}, K_{ab}

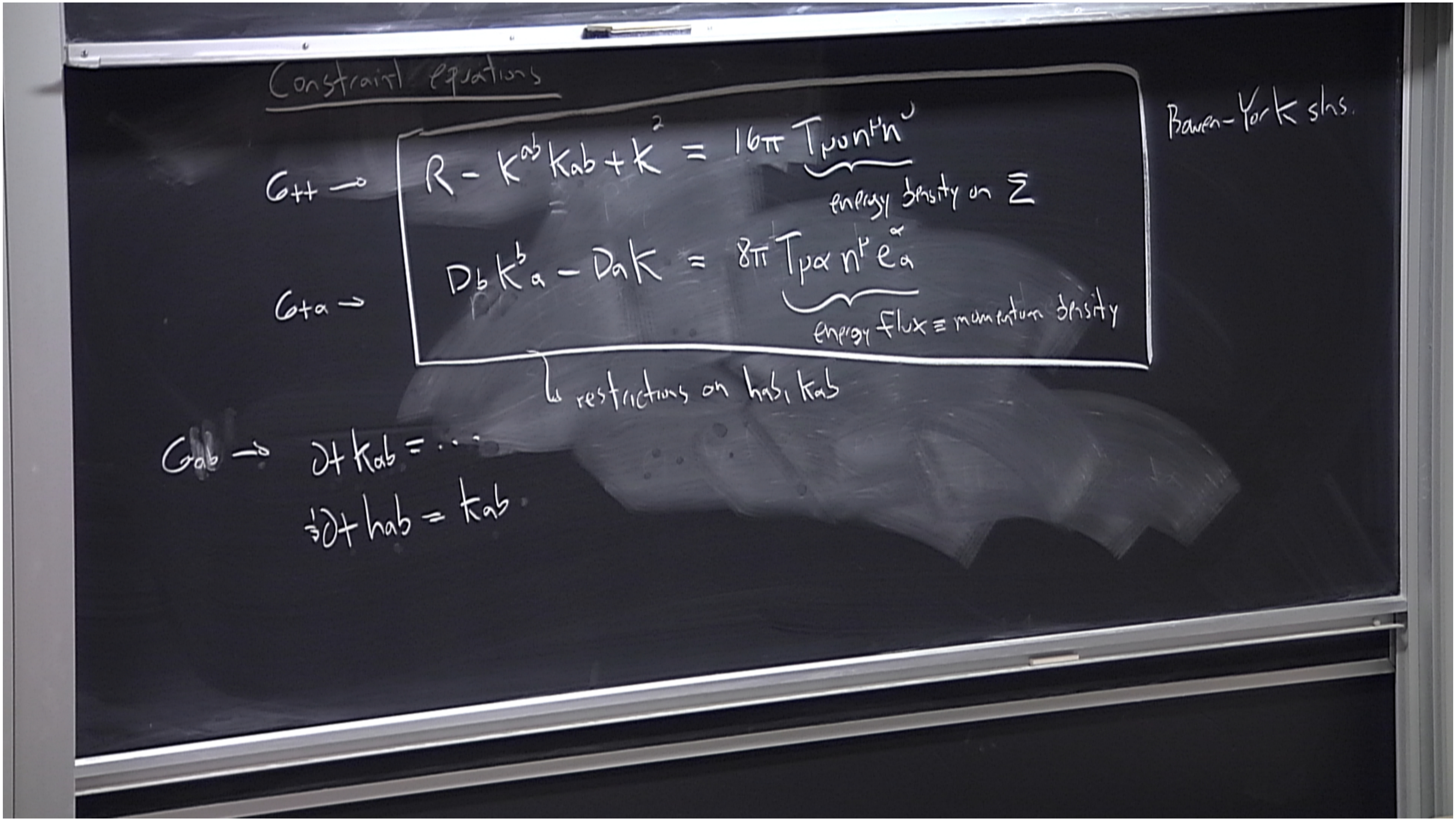
Bowen-York slns.

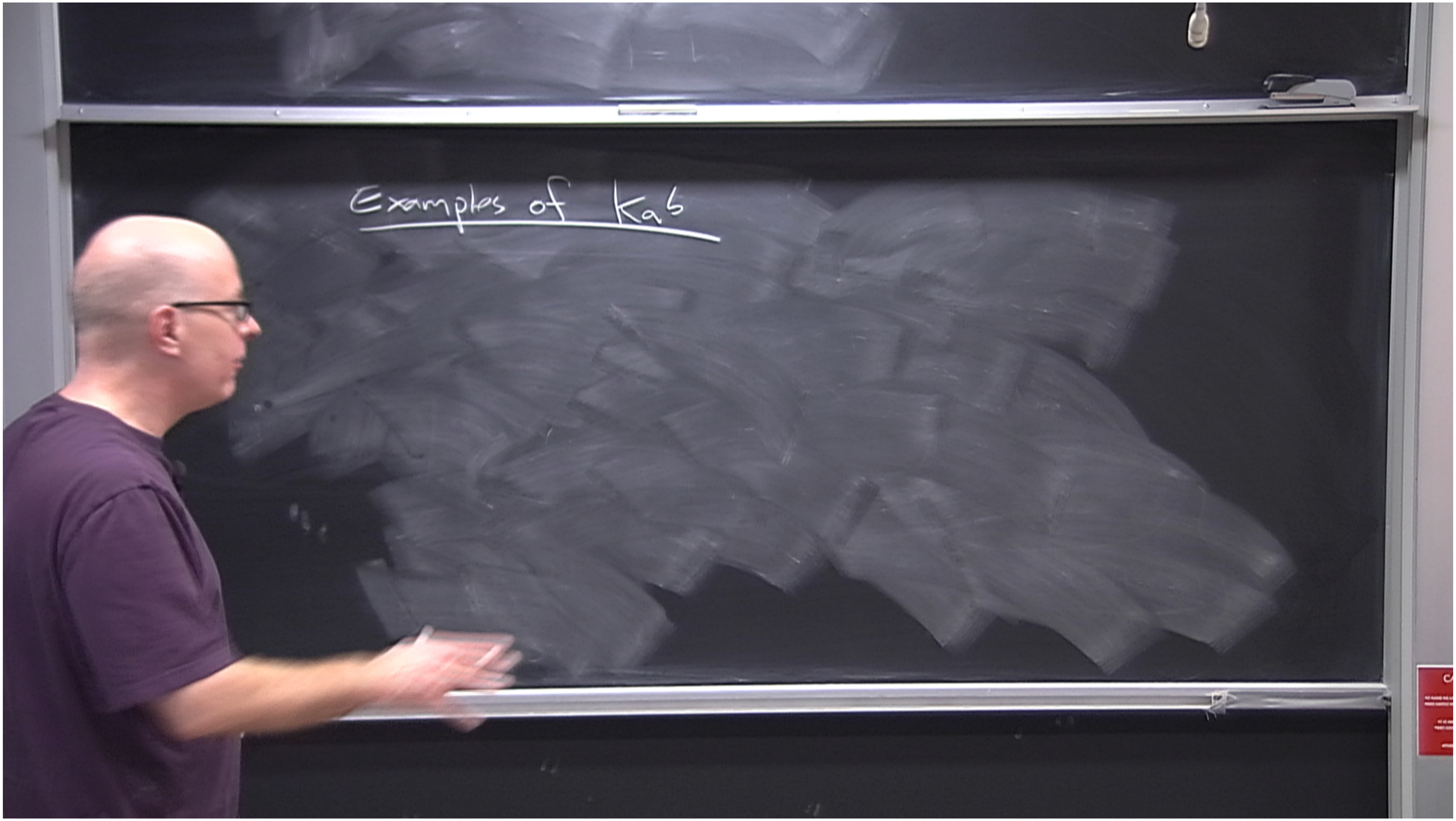
$G_{ab} \to$ $\partial_t K_{ab} = \ldots$

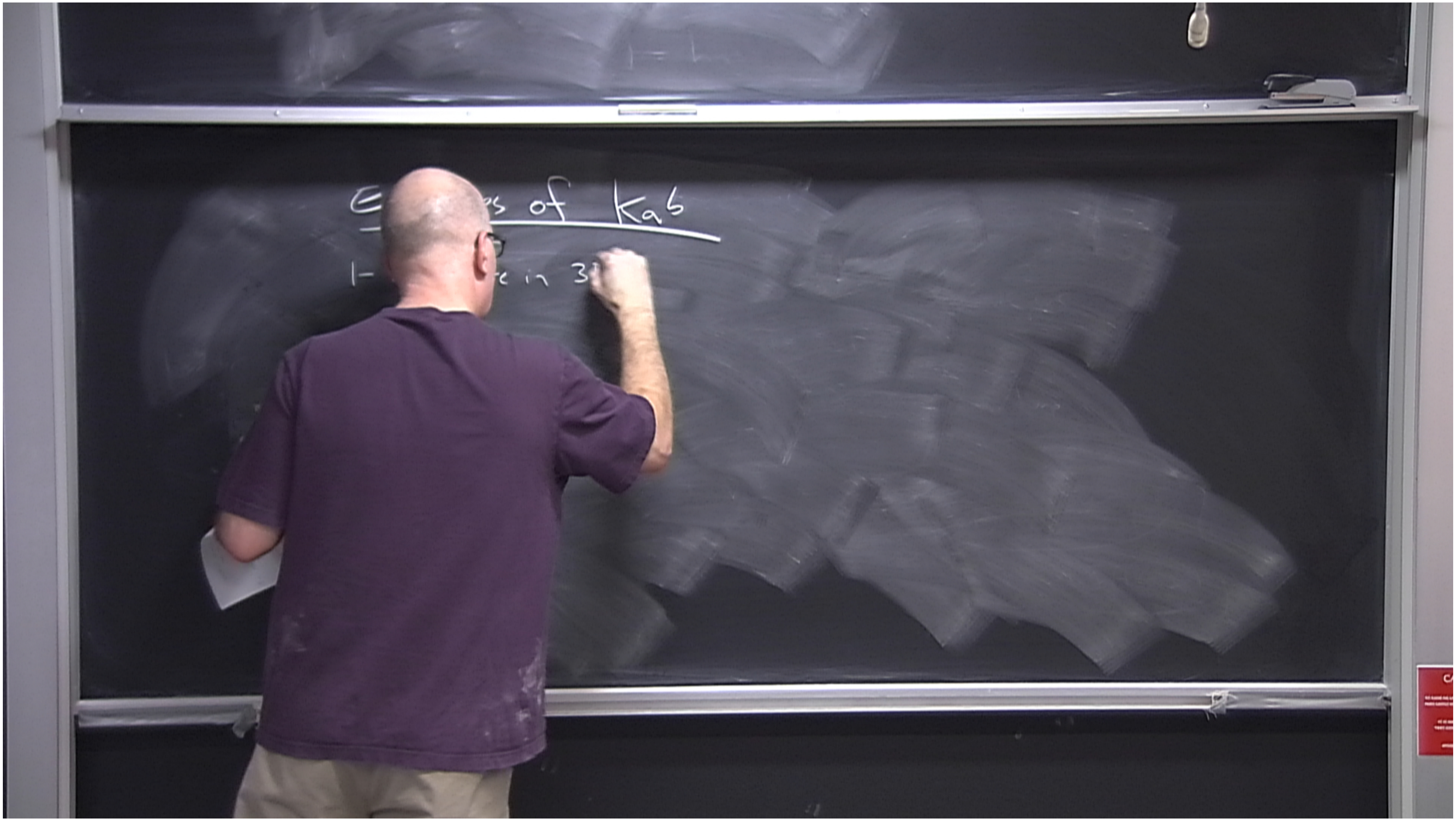
$\frac{1}{2}\partial_t h_{ab} = K_{ab}$

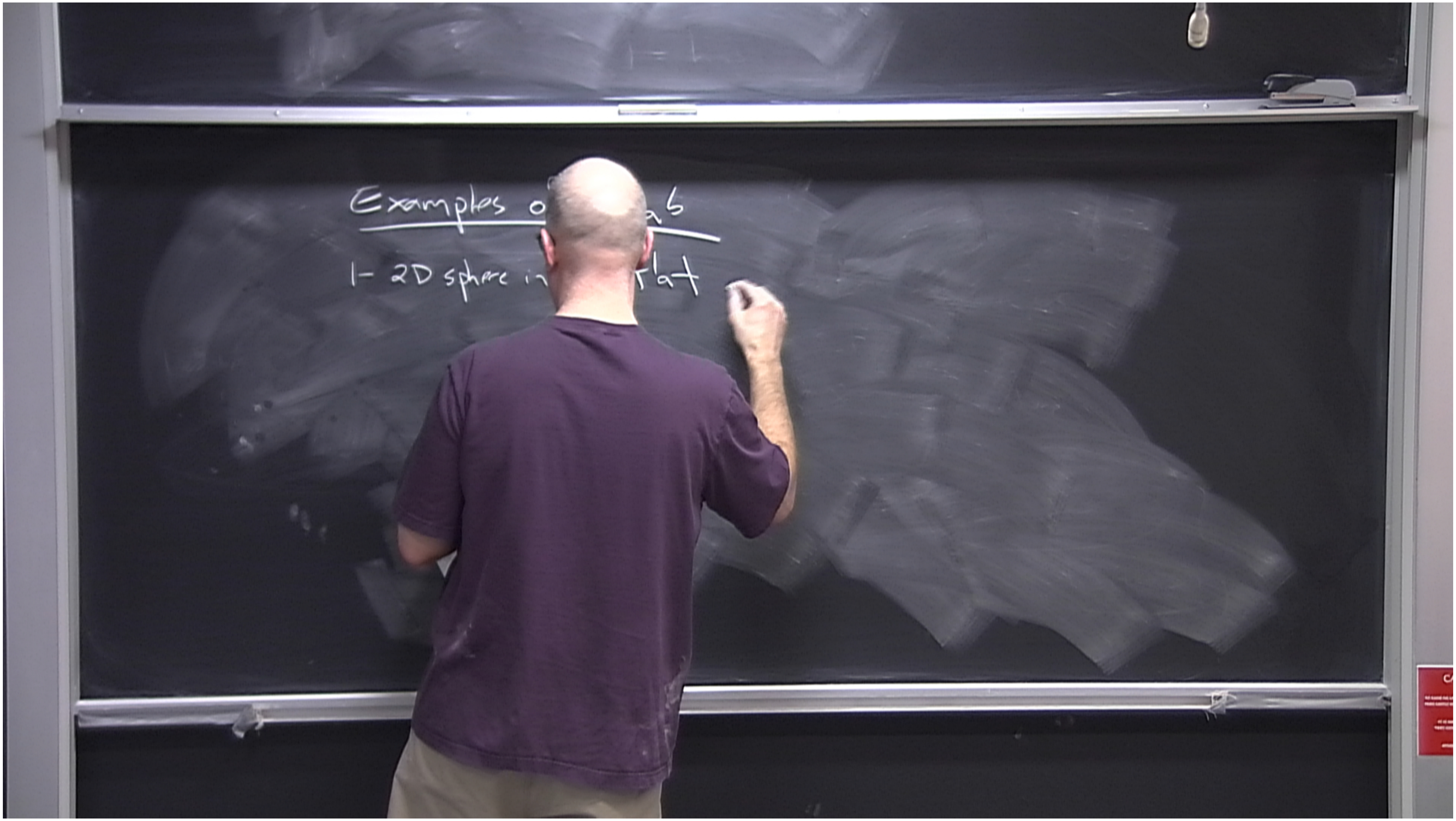

Constraint equations
Bowen-York slns.
$G_{tt} \rightarrow$ $R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu}n^\mu n^\nu$
energy density on Σ
$G_{ta} \rightarrow$ $D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
energy flux ≡ momentum density
restrictions on h_{ab}, K_{ab}
$G_{ab} \rightarrow$ $\partial_t K_{ab} = \dots$
$\partial_t h_{ab} = K_{ab}$

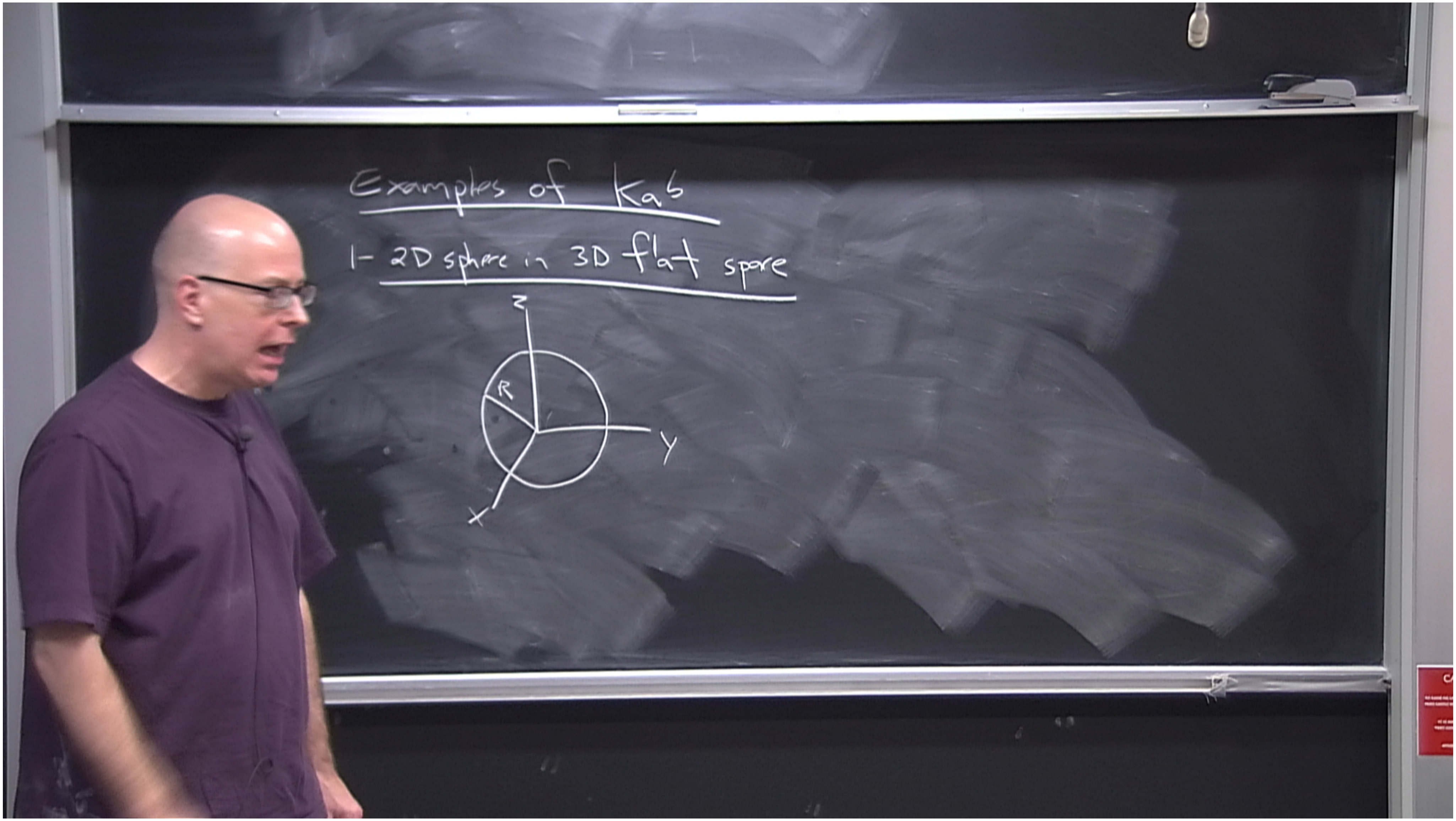

Constraint equations
$G_{tt} \to$ $R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
energy density on Σ
$G_{ta} \to$ $D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
energy flux ≡ momentum density
restrictions on h_{ab}, K_{ab}
Bowen-York sols.
$G_{ab} \to$ $\partial_t K_{ab} = \ldots$
$\frac{1}{2}\partial_t h_{ab} = K_{ab}$

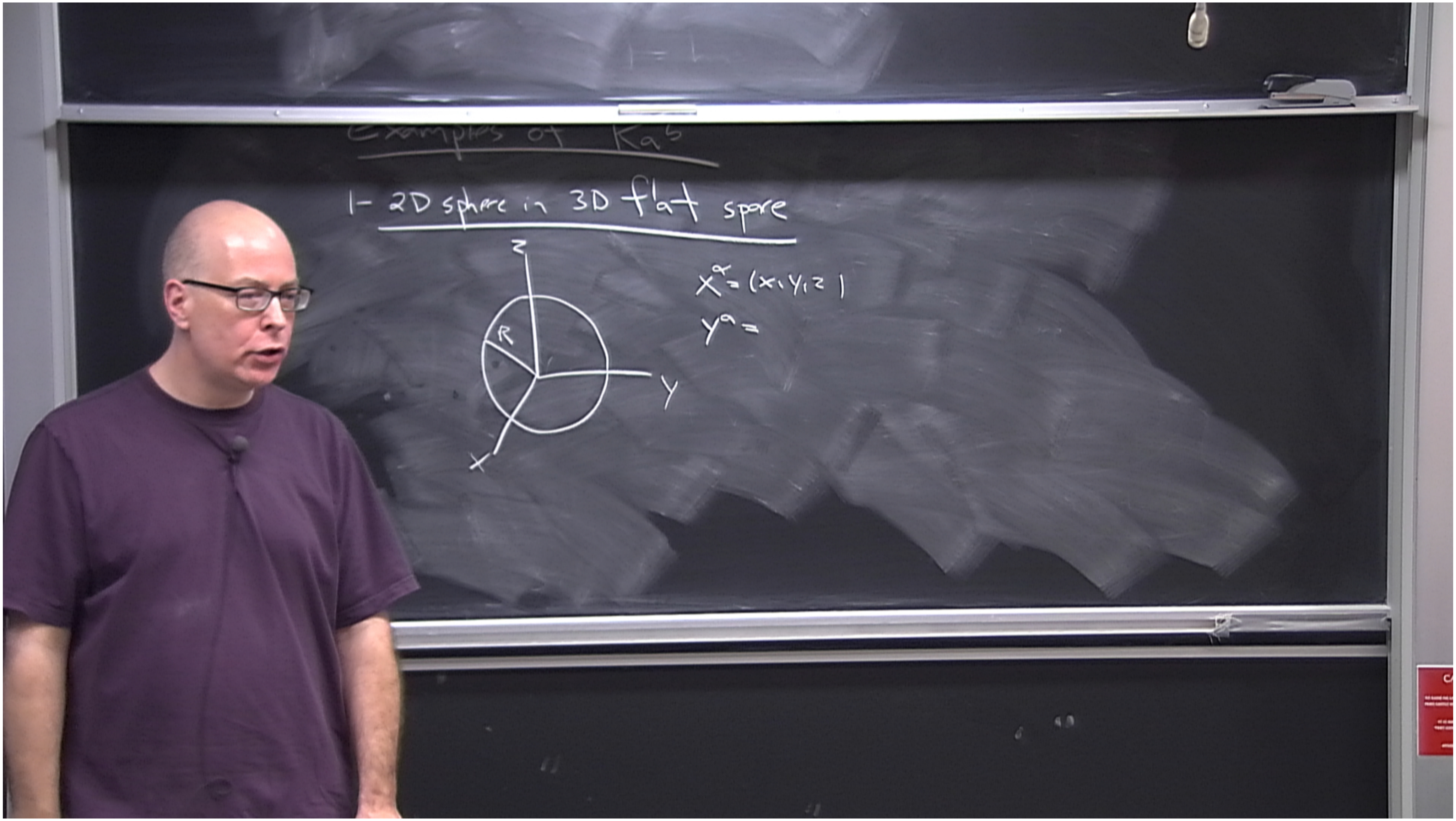

Constraint equations
$G_{tt} \rightarrow$ $R - K^{ab}K_{ab} + K^2 = 16\pi T_{\mu\nu} n^\mu n^\nu$
energy density on Σ
$G_{ta} \rightarrow$ $D_b K^b{}_a - D_a K = 8\pi T_{\mu\alpha} n^\mu e^\alpha_a$
energy flux ≡ momentum density
↳ restrictions on h_{ab}, K_{ab}
Bowen-York slns.
$G_{ab} \rightarrow$ $\partial_t K_{ab} = \ldots$
$\frac{1}{2}\partial_t h_{ab} = K_{ab}$

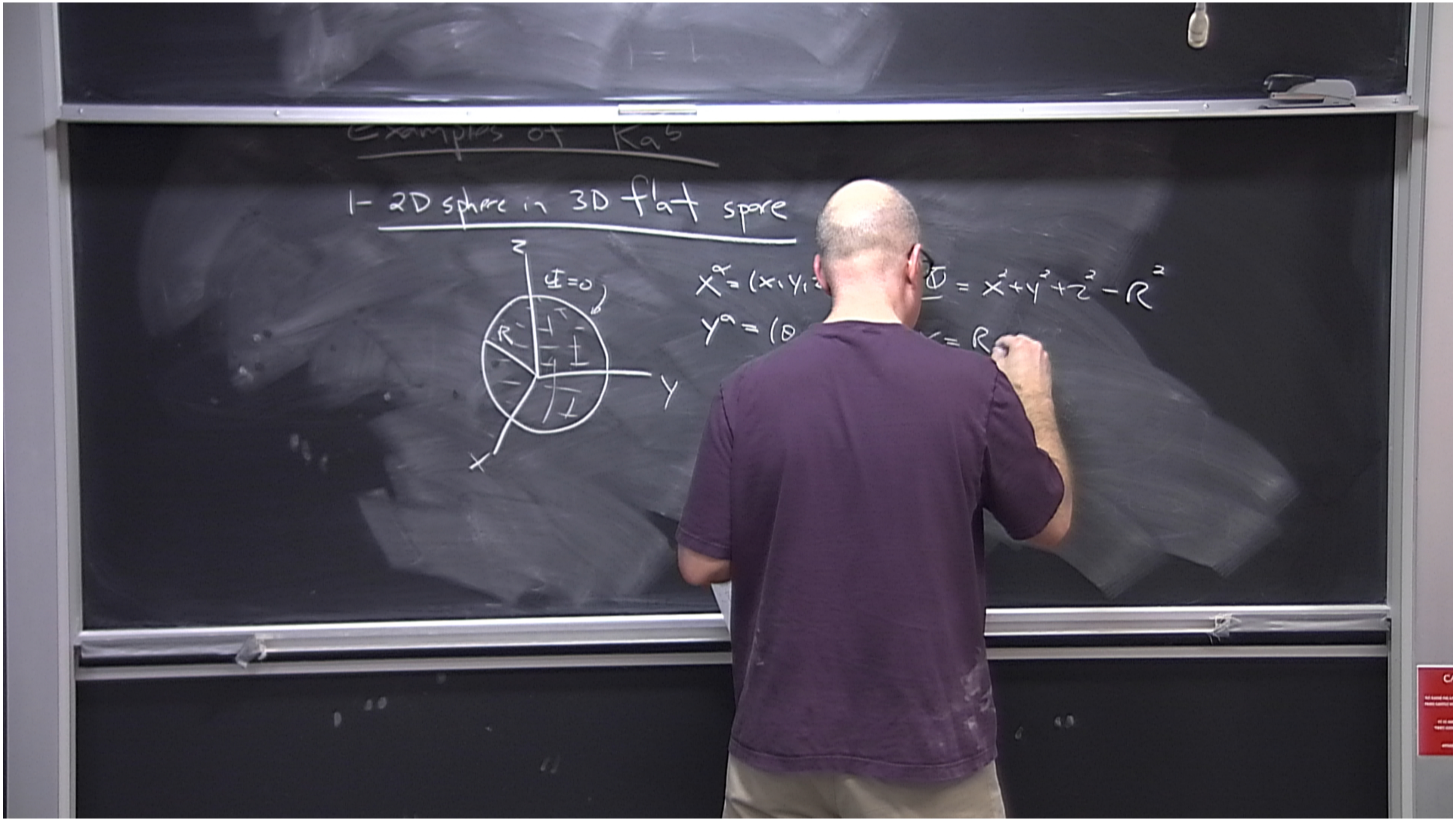

Examples of Kab

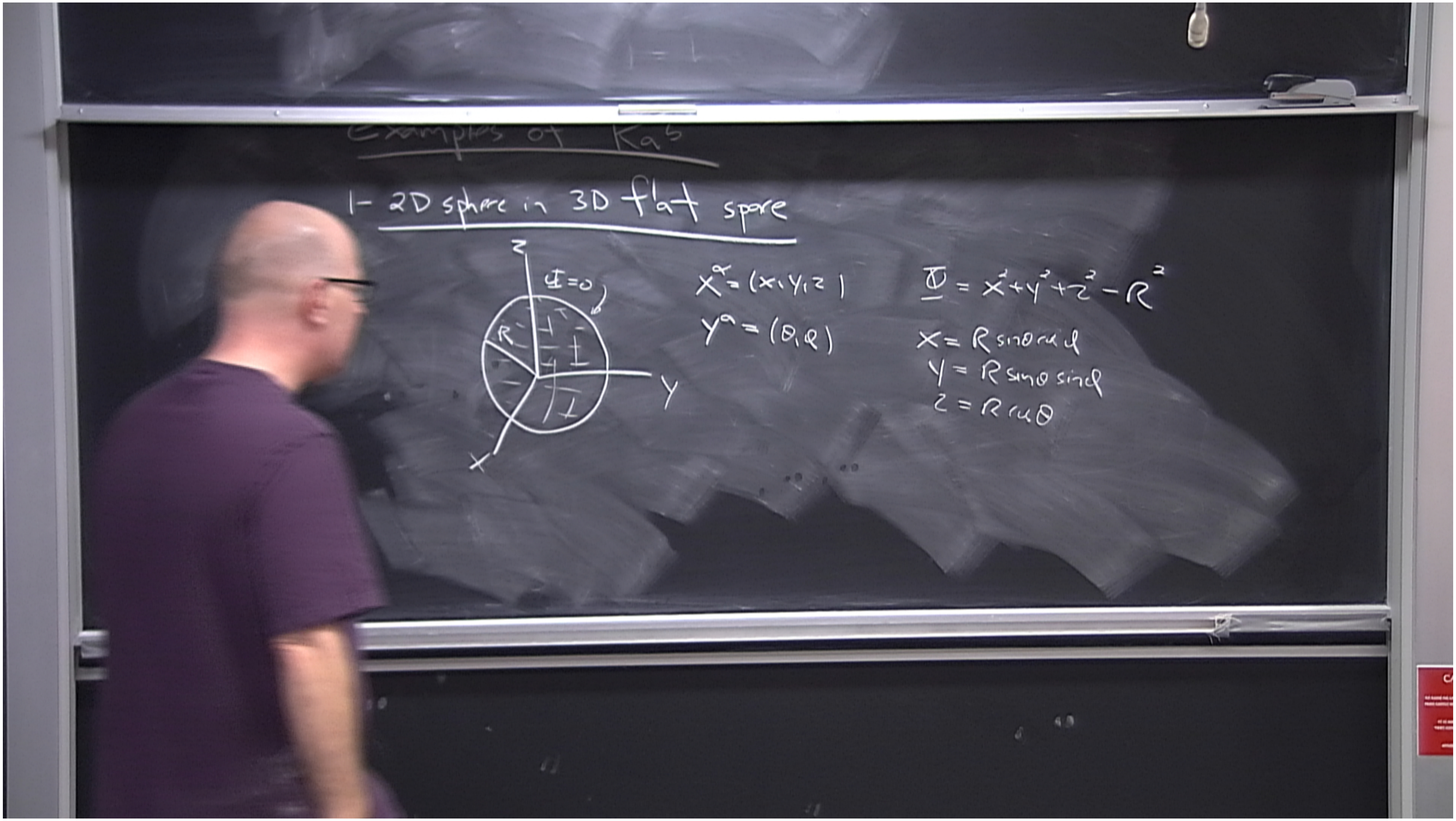


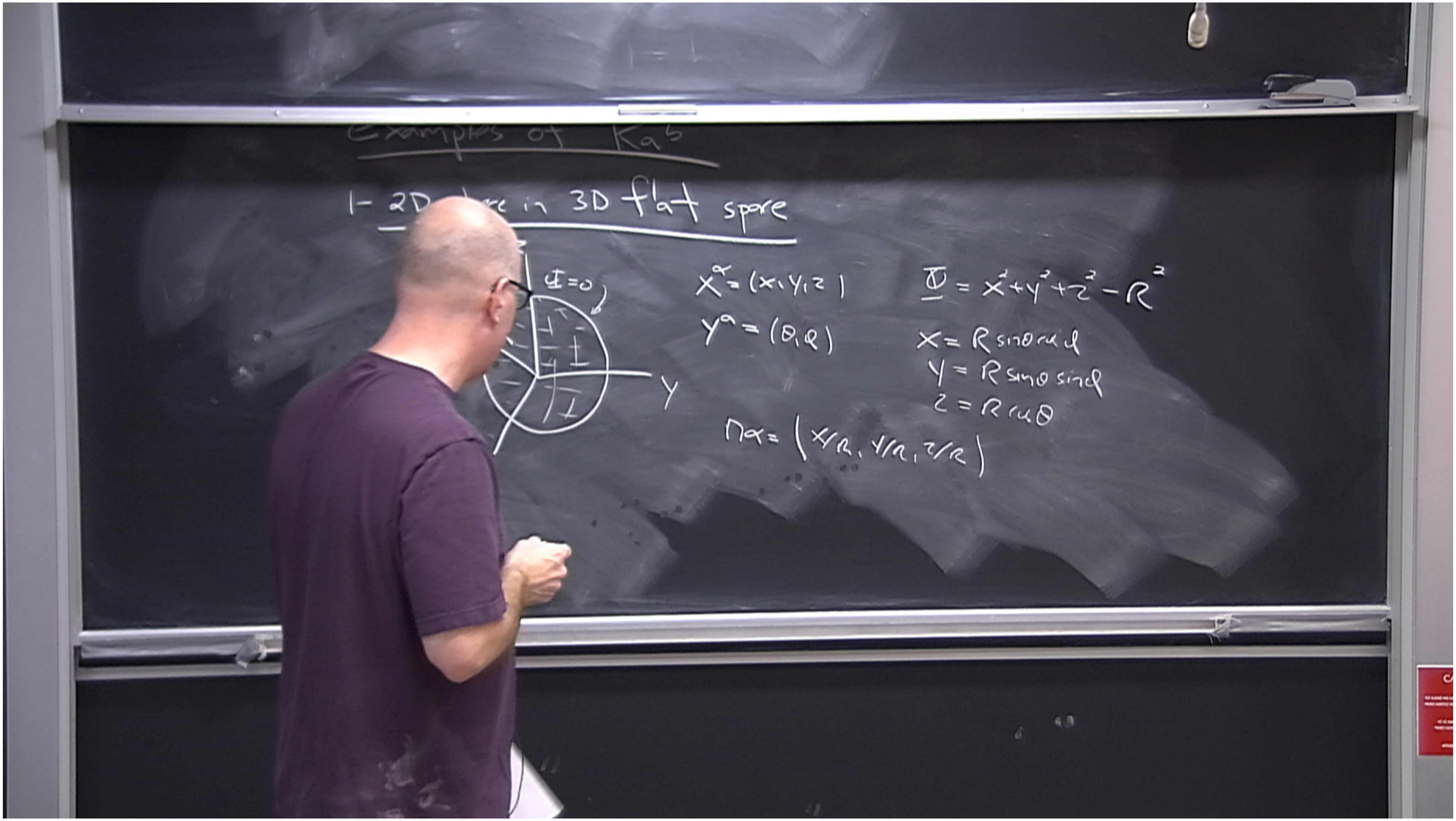

Examples o
1- 2D sphere i
flat


Examples of Kab
1- 2D sphere in 3D flat space
z
R
y
x

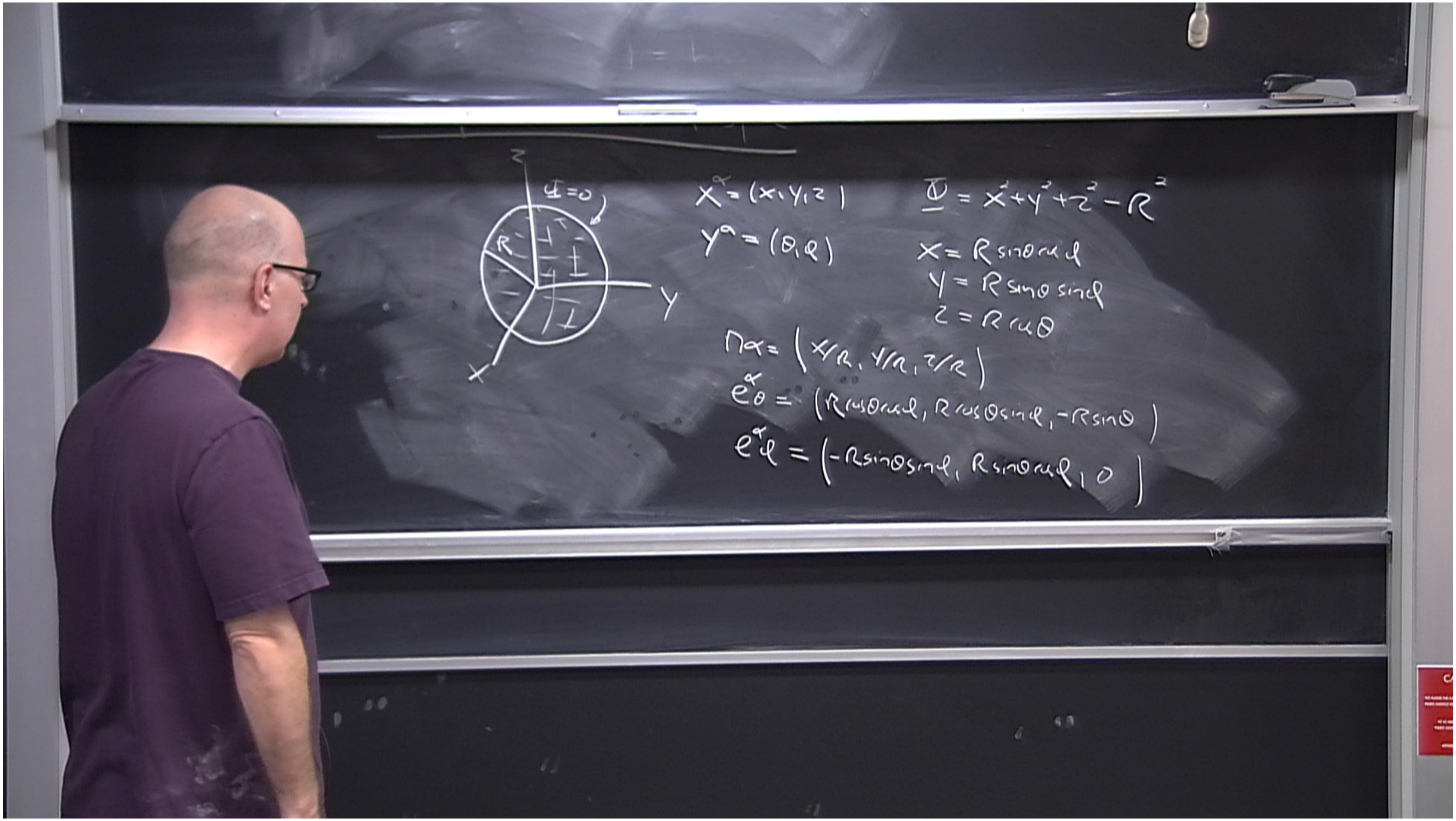

Examples of Ka's
1- 2D sphere in 3D flat space
$X^\alpha = (x, y, z)$
$y^a =$
z
R
y
x

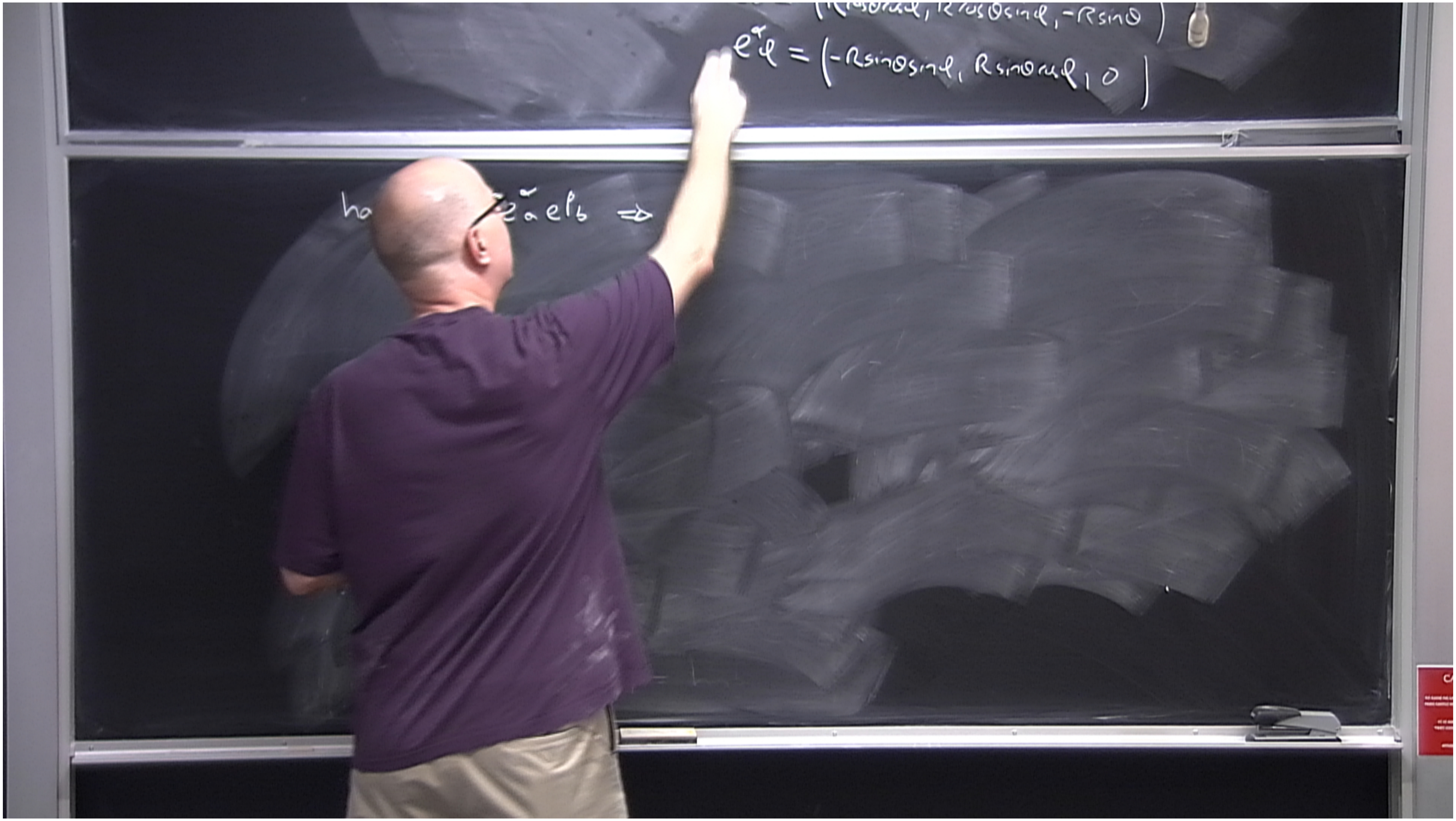

1- 2D sphere in 3D flat spore
Φ = x² + y² + z² − R²
z
y
x
R

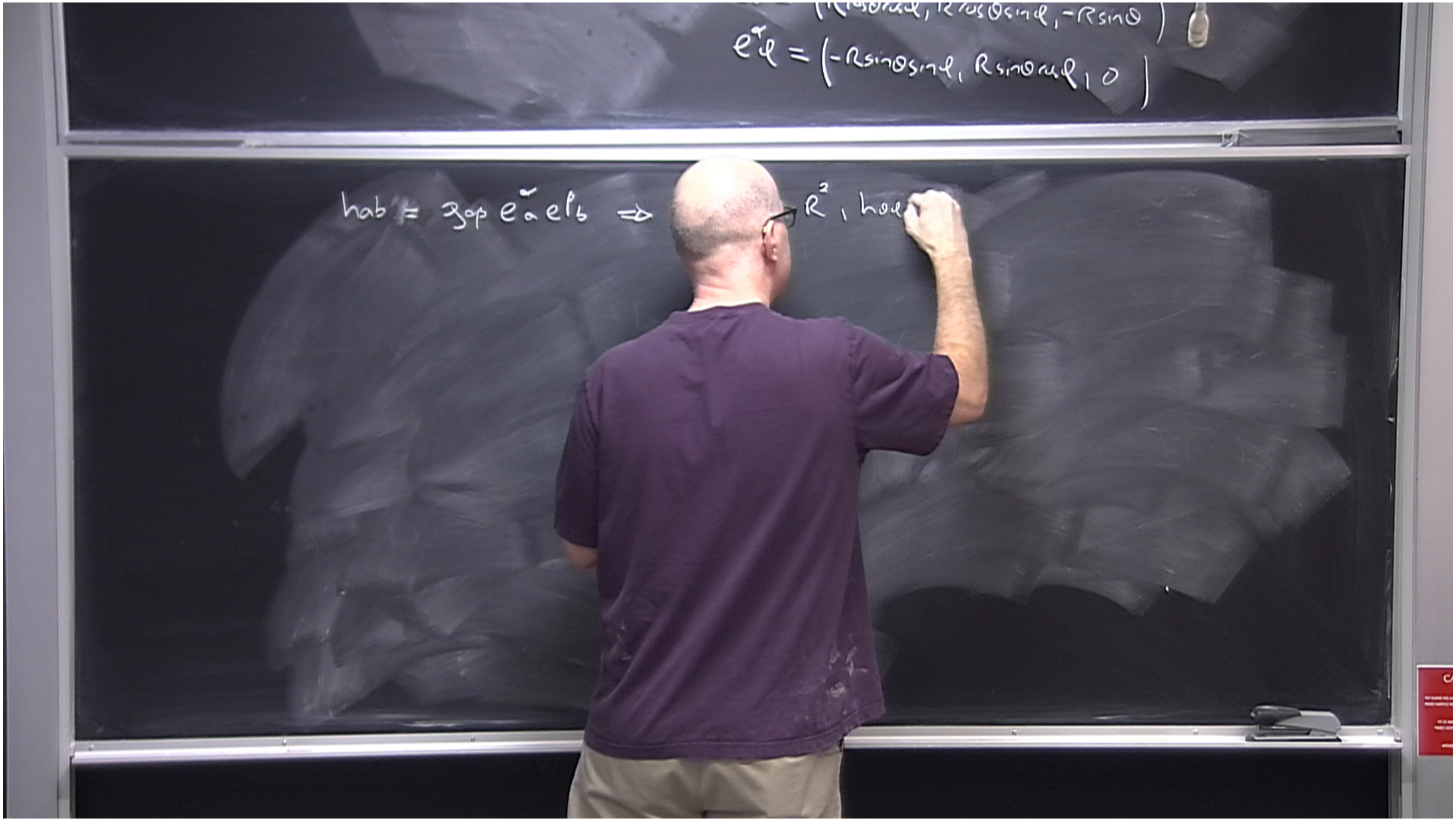

Examples of Kab
1- 2D sphere in 3D flat space
Z
Φ=0
R
Y
X
x^α = (x, y, z)
y^a = (θ, φ)
Φ = x² + y² + z² − R²
x = R sinθ cosφ
y = R sinθ sinφ
z = R cosθ

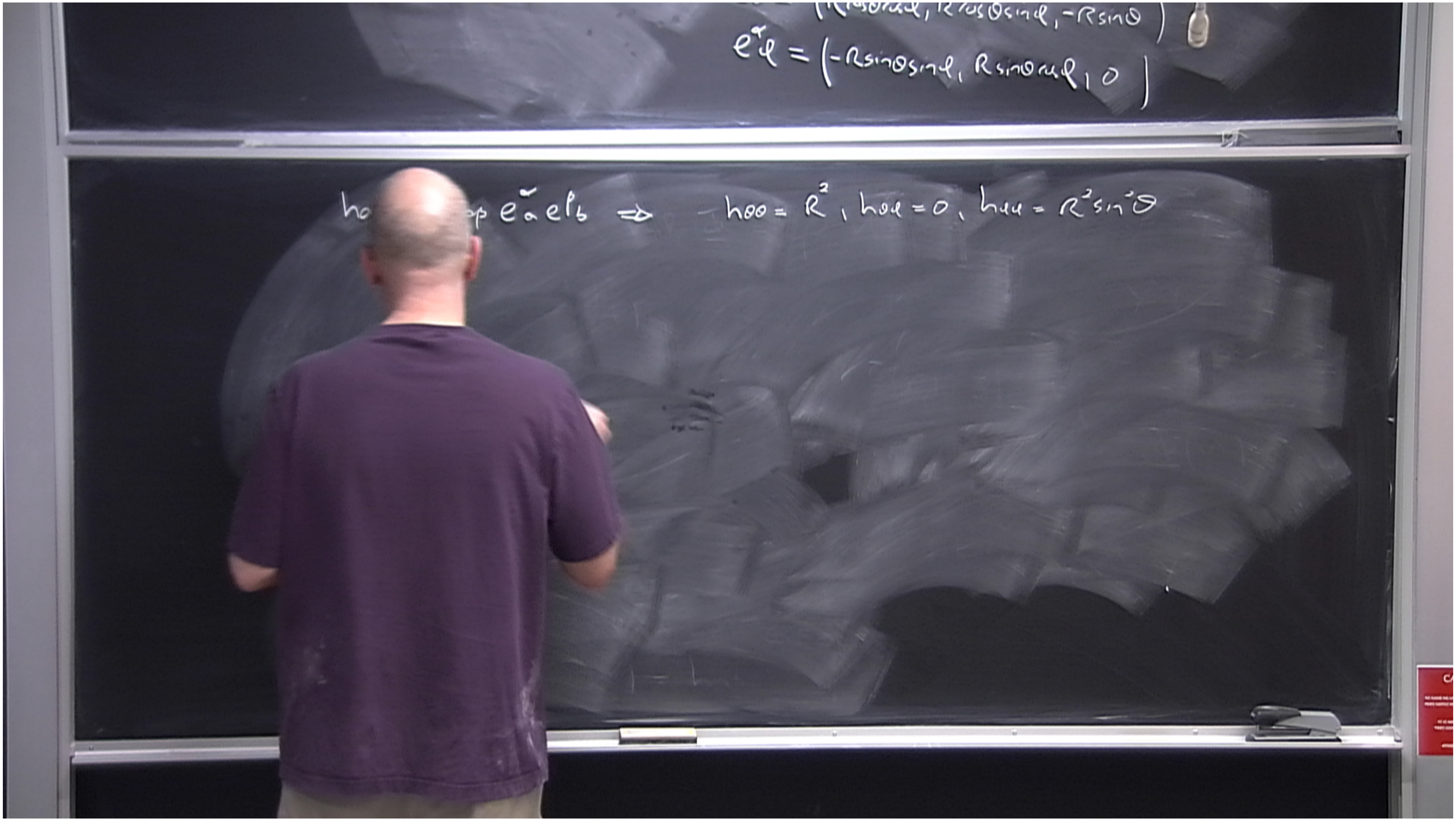


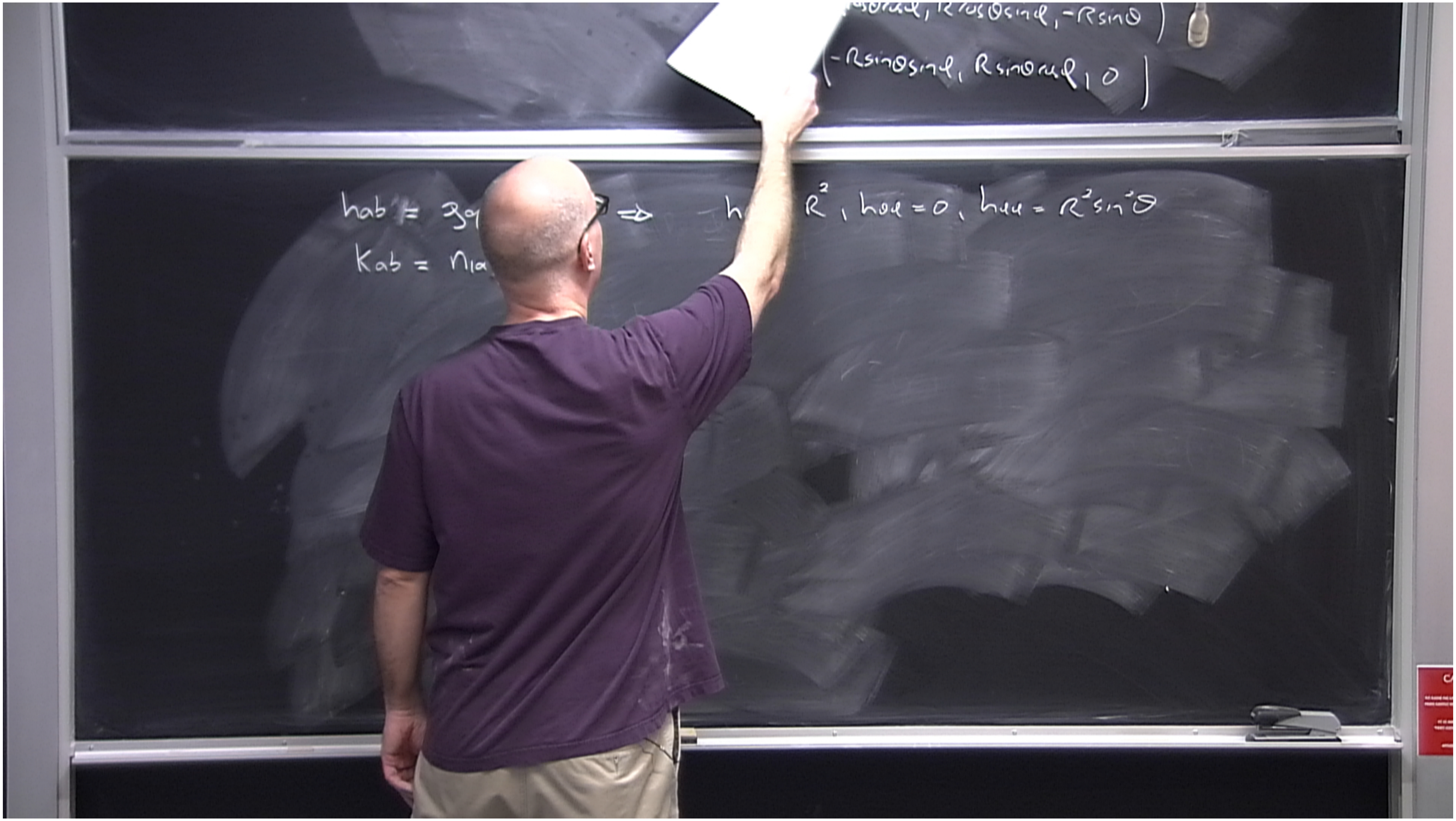
1- 2D … in 3D flat space
Φ = 0
y
X^α = (x, y, z)
y^a = (θ, φ)
Φ = x² + y² + z² − R²
x = R sinθ cosφ
y = R sinθ sinφ
z = R cosθ
n_α = (x/R, y/R, z/R)

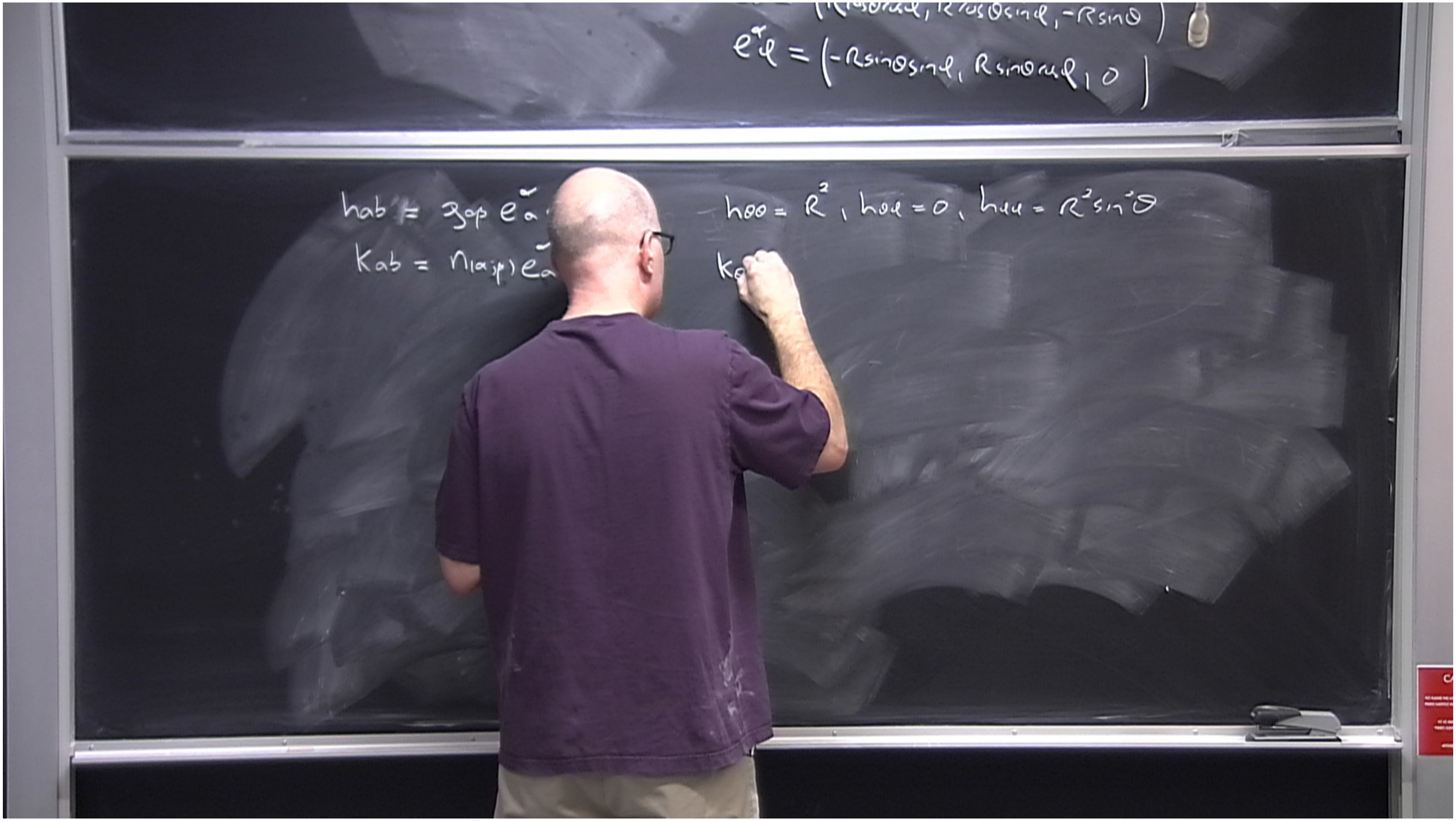

Z
Φ=0
R
Y
X
X^α = (x, y, z)
Y^a = (θ, φ)
Φ = x² + y² + z² − R²
x = R sinθ cosφ
y = R sinθ sinφ
z = R cosθ
n_α = (x/R, y/R, z/R)
e^α_θ = (R cosθ cosφ, R cosθ sinφ, −R sinθ)
e^α_φ = (−R sinθ sinφ, R sinθ cosφ, 0)

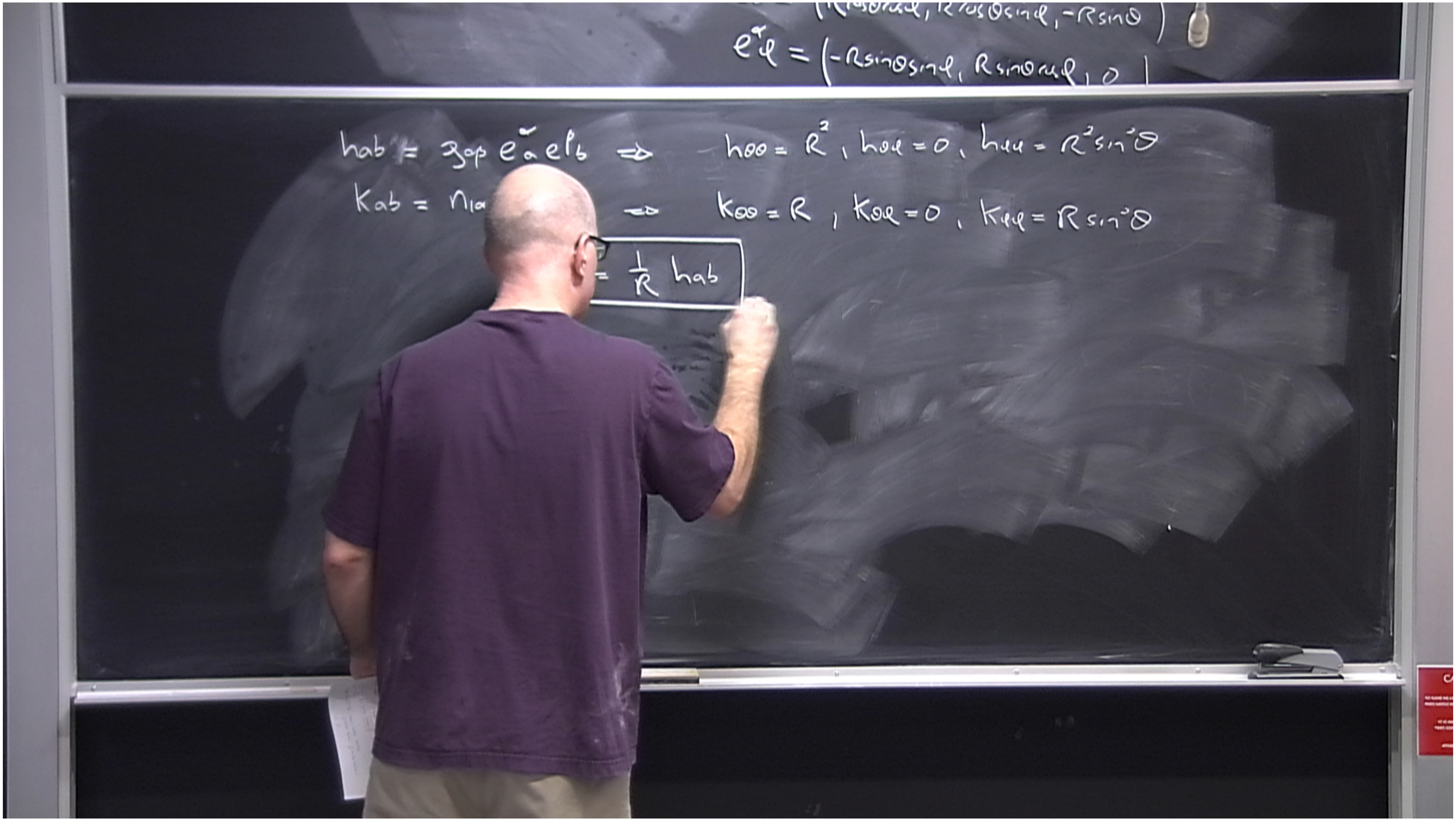

(R cosθ cosφ, R cosθ sinφ, -R sinθ)
e^φ_φ = (-R sinθ sinφ, R sinθ cosφ, 0)

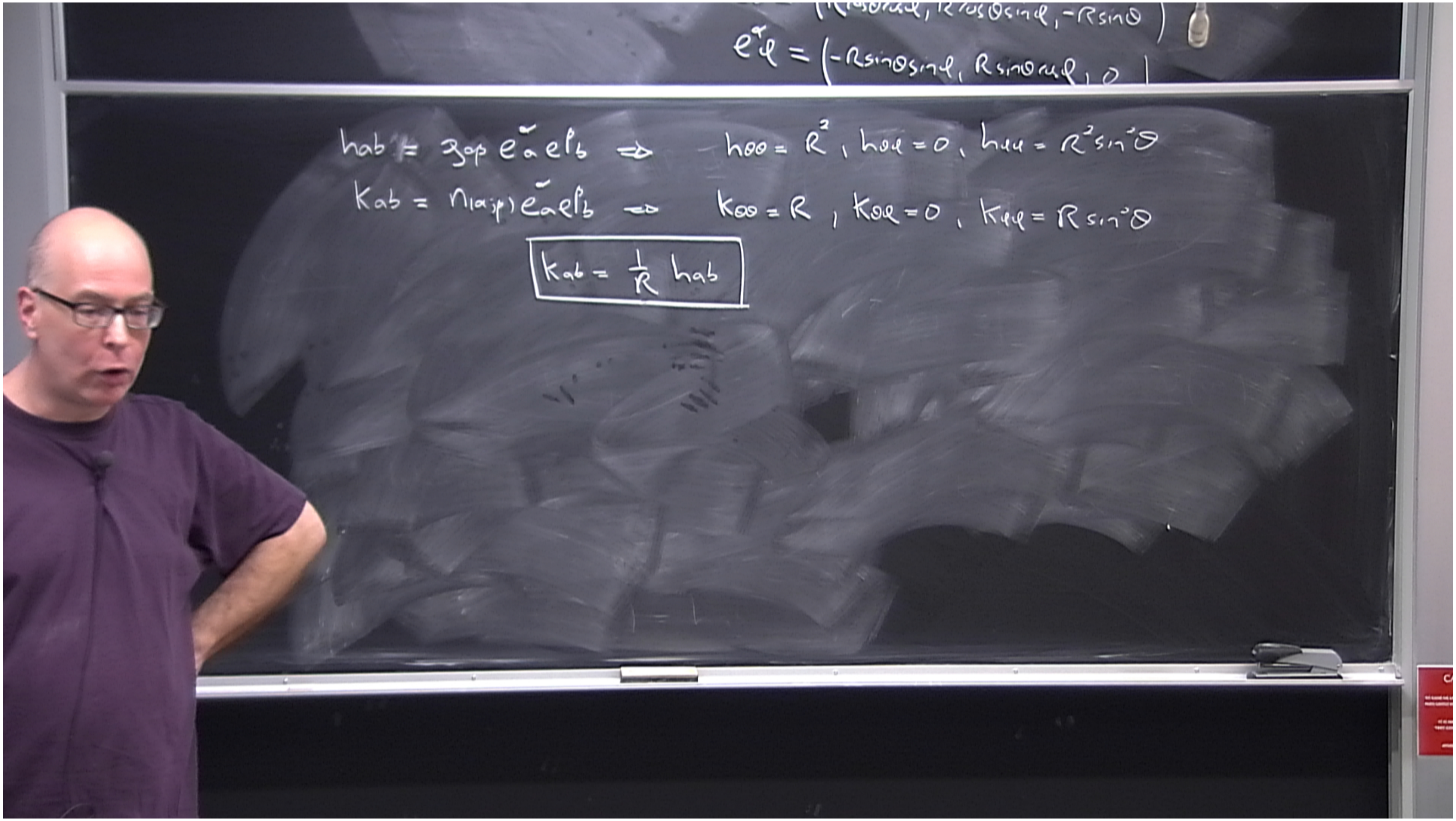

eφ = (-Rsinθsinφ, Rsinθcosφ, 0)
hab = gαβ eαa eβb ⇒
R², ho

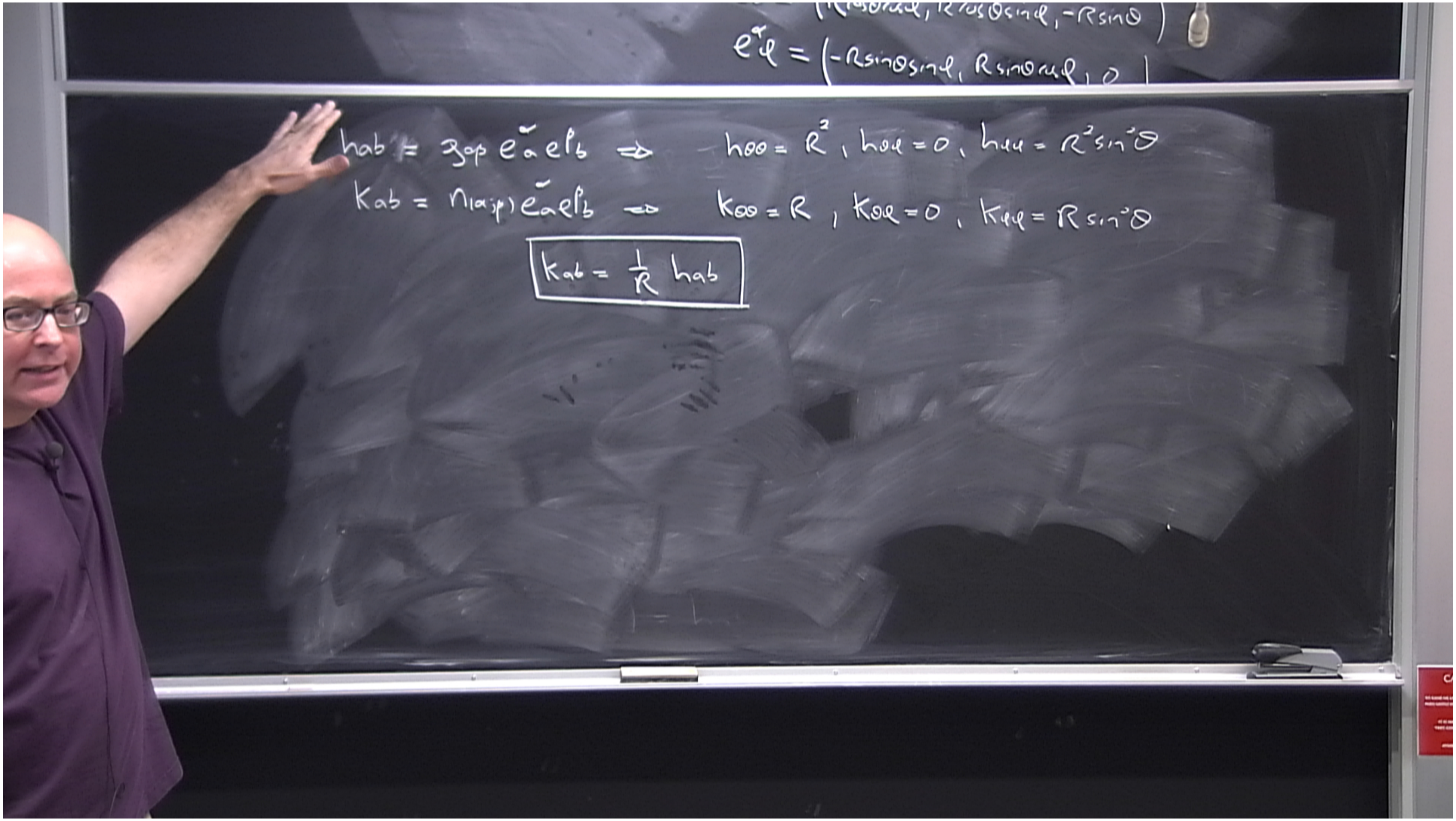

(R cosθ cosφ, R cosθ sinφ, -R sinθ)
e^φ_φ = (-R sinθ sinφ, R sinθ cosφ, 0)
hθθ = R², hθφ = 0, hφφ = R² sin²θ

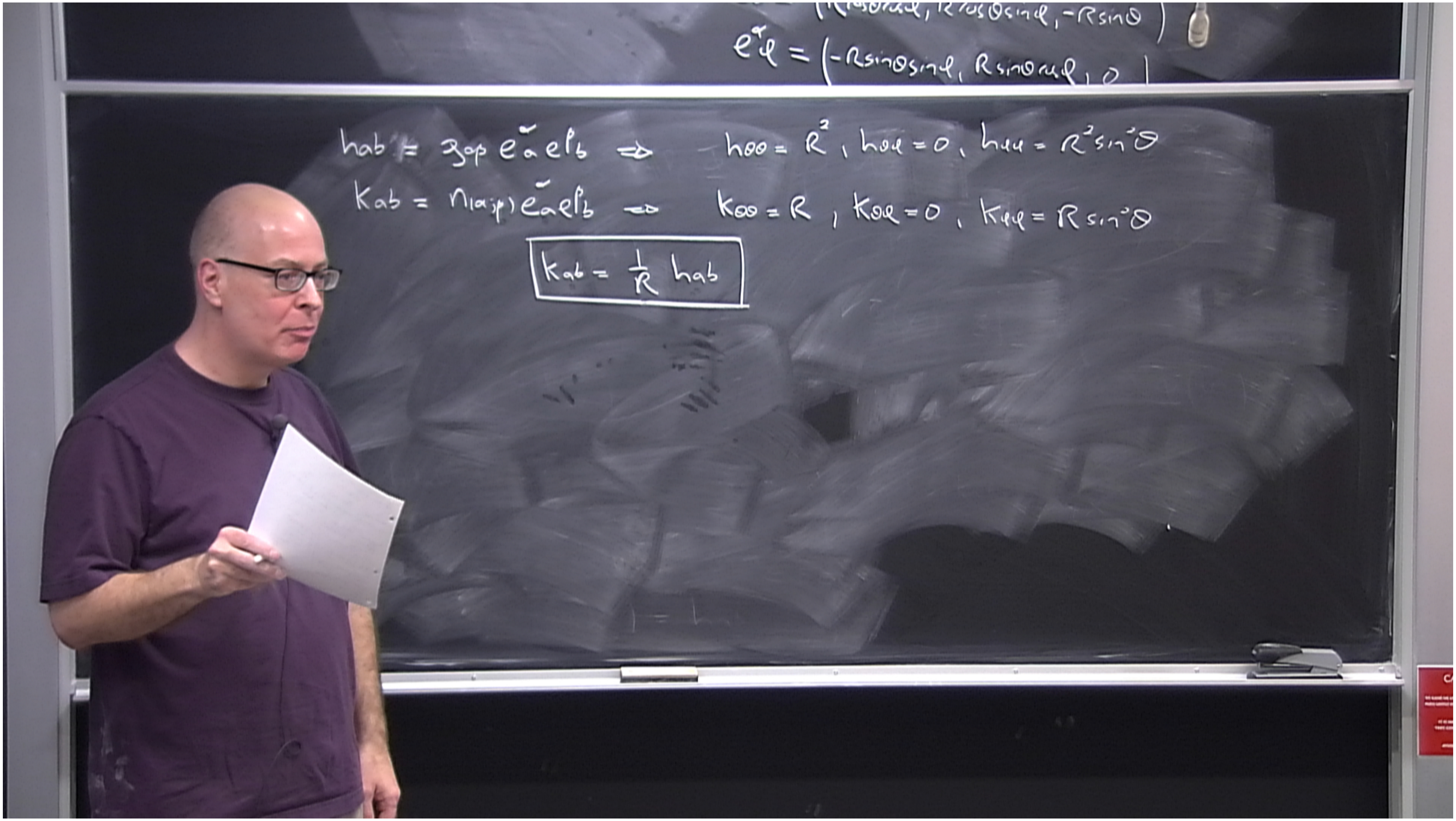


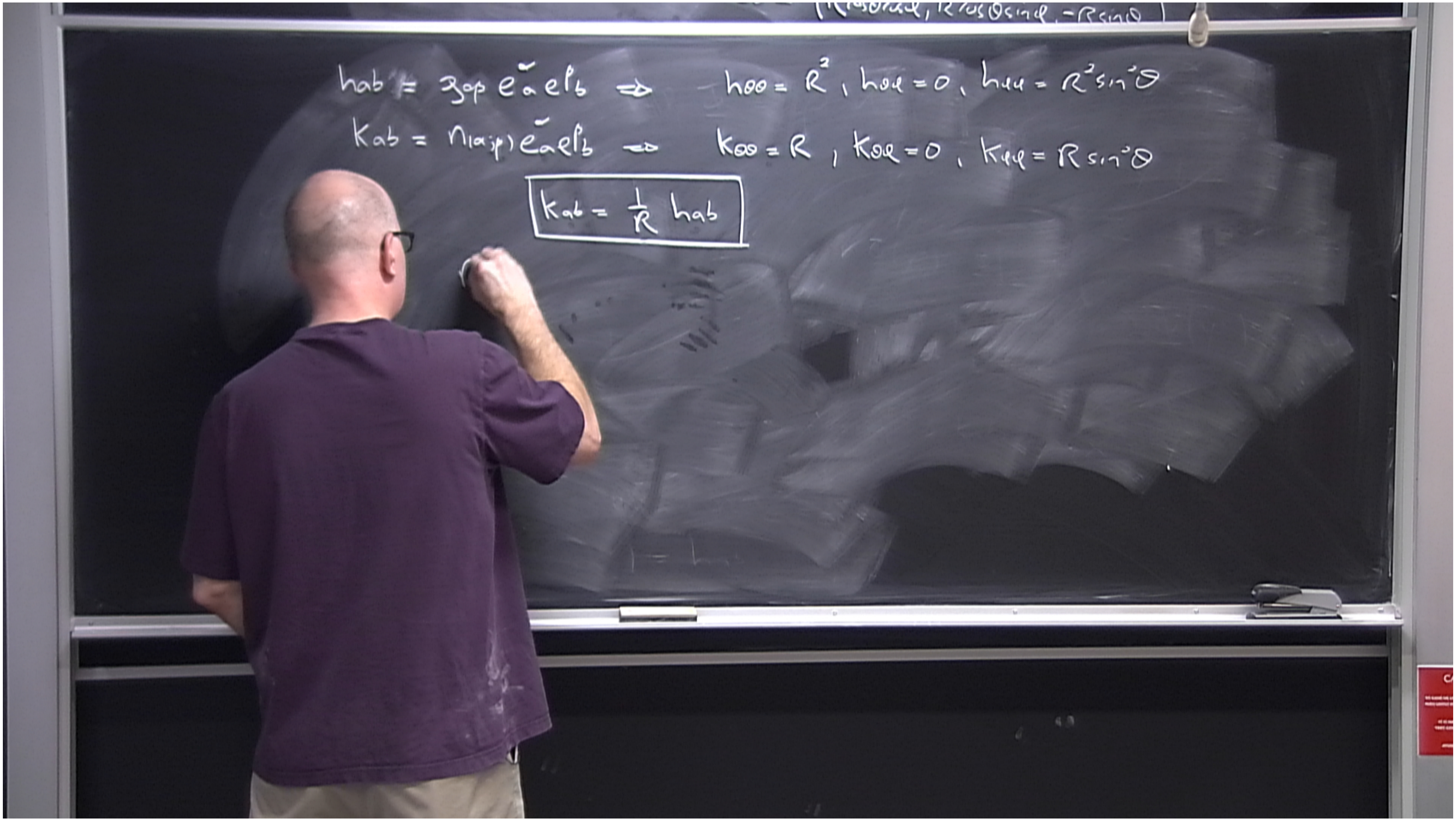

e^φ_φ = (-R sinθ sinφ, R sinθ cosφ, 0)
hab = gαβ e^α
Kab = n(α;β) e^α
hθθ = R², hθφ = 0, hφφ = R² sin²θ

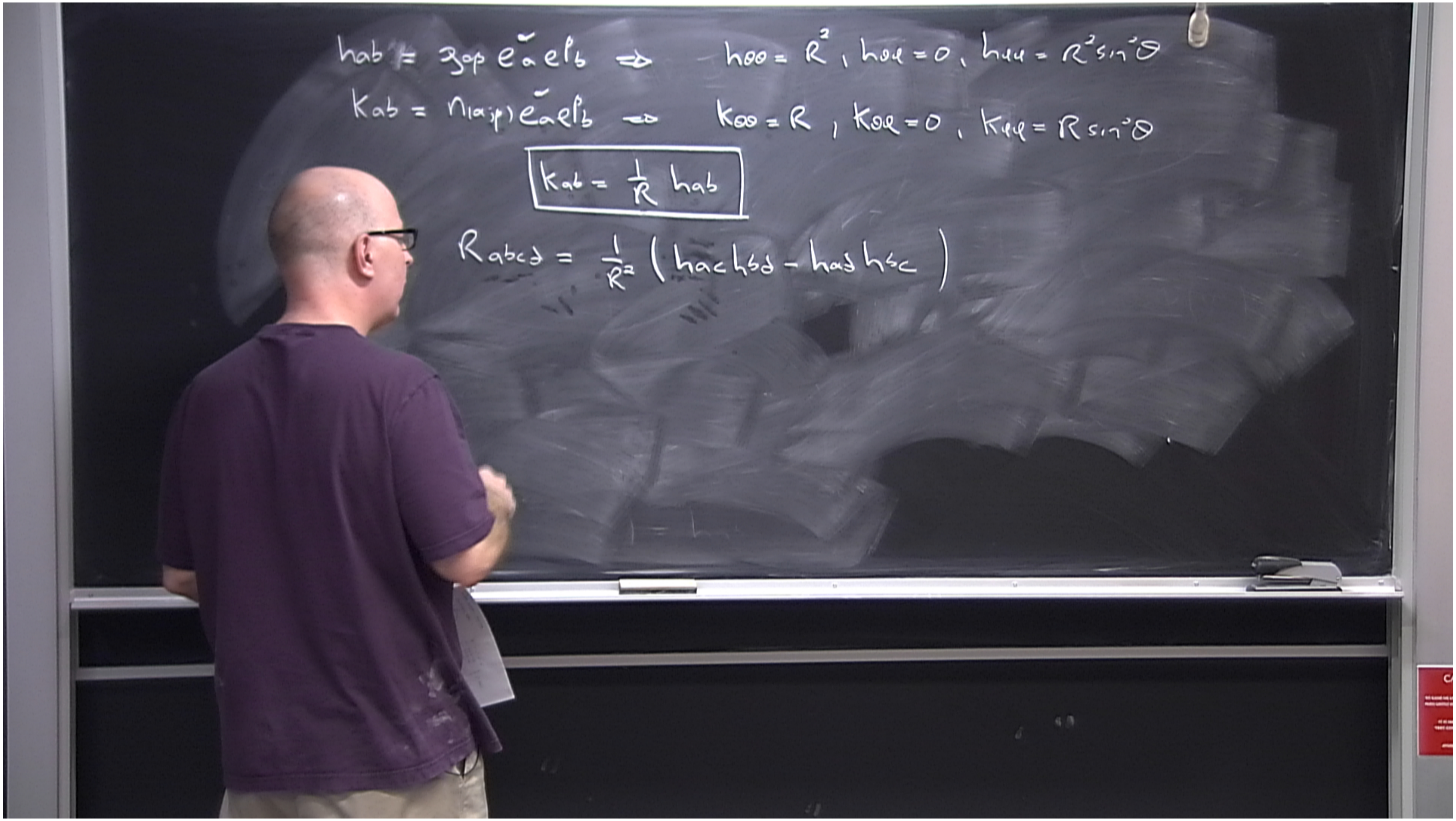

eᵅ_φ = (−R sinθ sinφ, R sinθ cosφ, 0)
h_ab = g_αβ eᵅ_a eᵝ_b ⇒ h_θθ = R², h_θφ = 0, h_φφ = R² sin²θ
K_ab = n ⇒ K_θθ = R, K_θφ = 0, K_φφ = R sin²θ
= 1/R h_ab

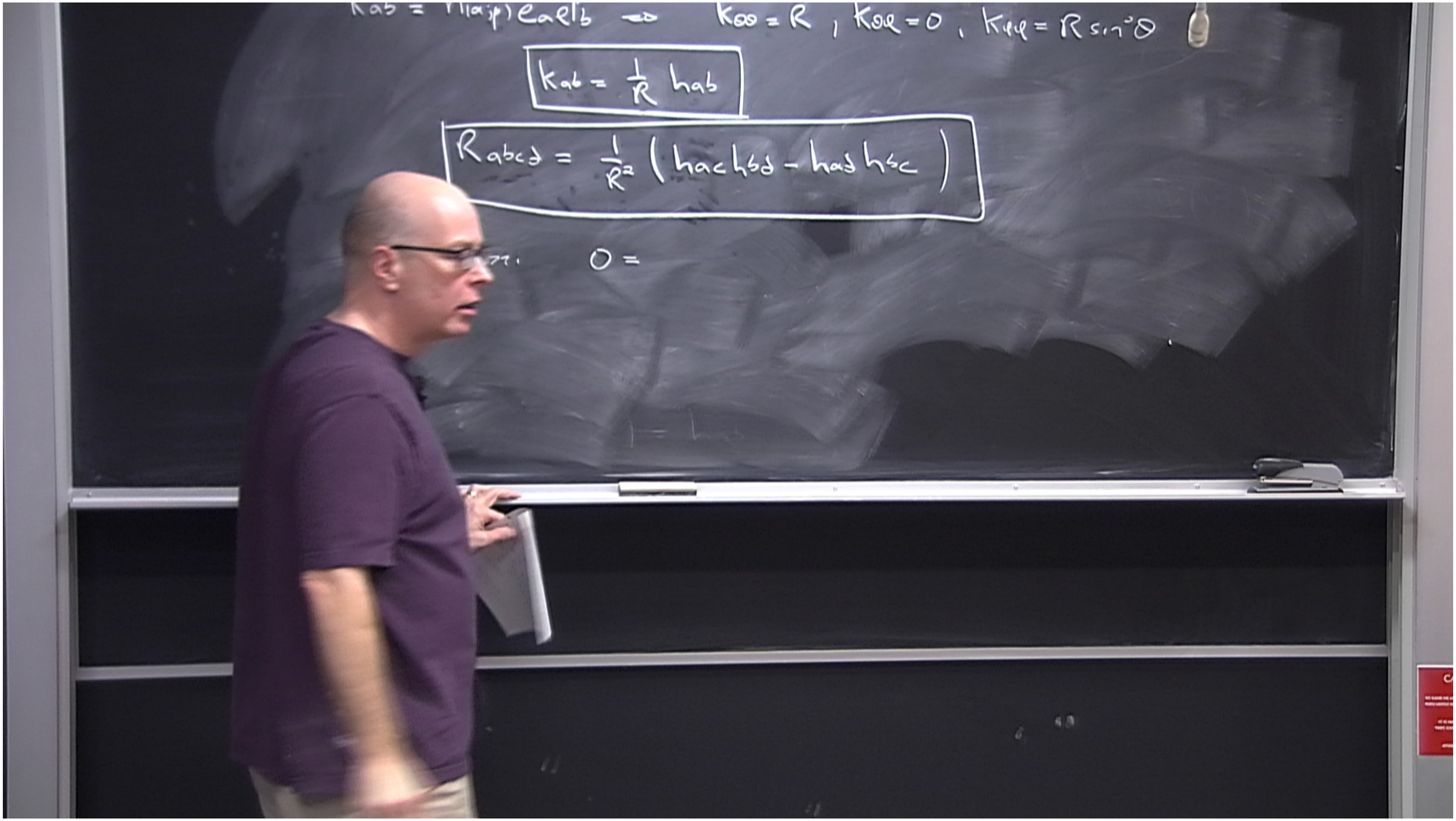

$e^{\alpha}_{\varphi} = (-R\sin\theta\sin\varphi, R\sin\theta\cos\varphi, 0)$
$h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow h_{\theta\theta} = R^2, h_{\theta\varphi} = 0, h_{\varphi\varphi} = R^2\sin^2\theta$
$K_{ab} = n_{(\alpha;\beta)} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow K_{\theta\theta} = R, K_{\theta\varphi} = 0, K_{\varphi\varphi} = R\sin^2\theta$
$K_{ab} = \frac{1}{R} h_{ab}$

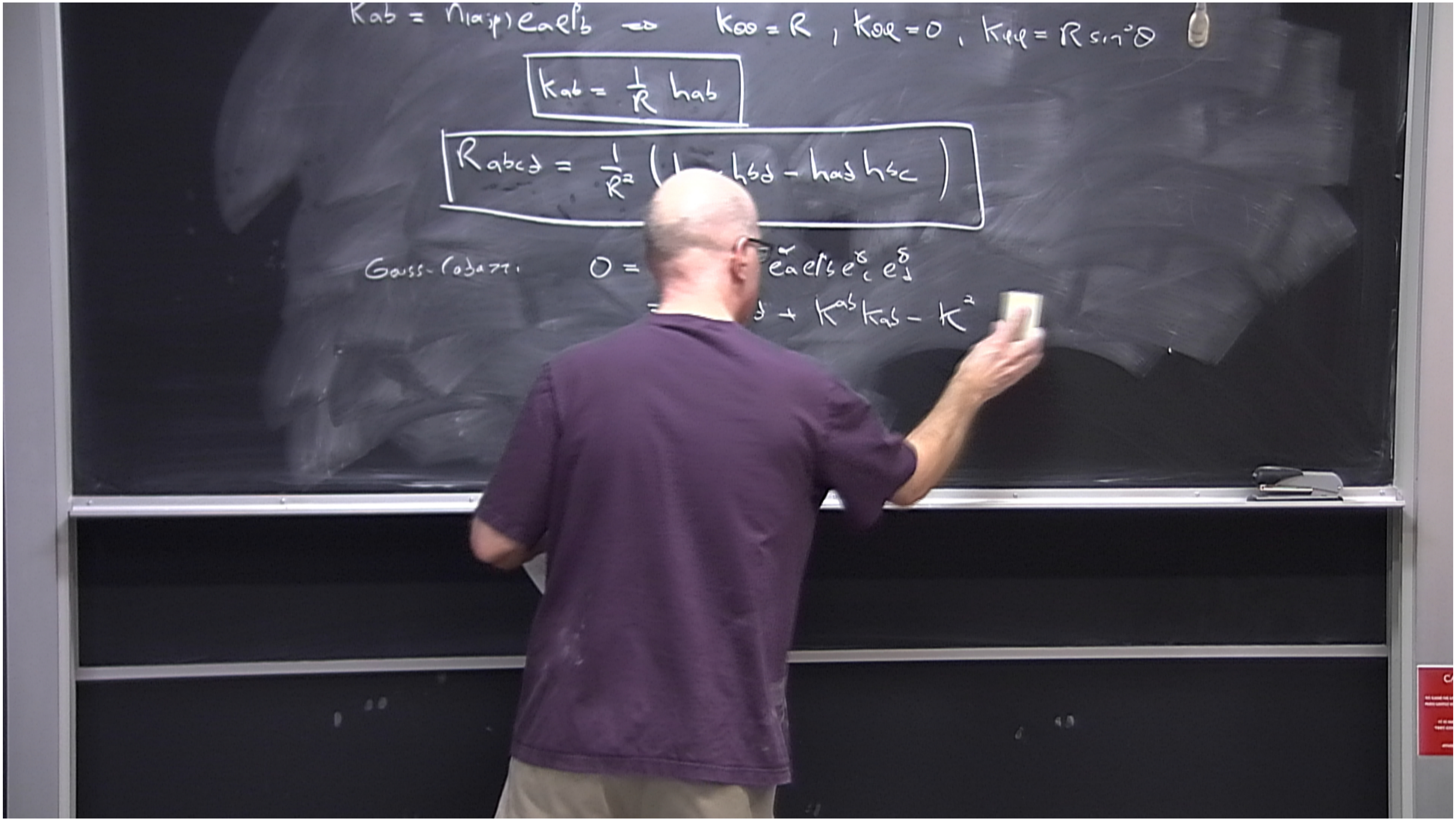

R cosθ cosφ, R cosθ sinφ, −R sinθ)
e^φ_φ = (−R sinθ sinφ, R sinθ cosφ, 0)
h_ab = g_αβ e^α_a e^β_b ⇒ h_θθ = R², h_θφ = 0, h_φφ = R² sin²θ
K_ab = n_(α;β) e^α_a e^β_b ⇒ K_θθ = R, K_θφ = 0, K_φφ = R sin²θ
K_ab = 1/R h_ab

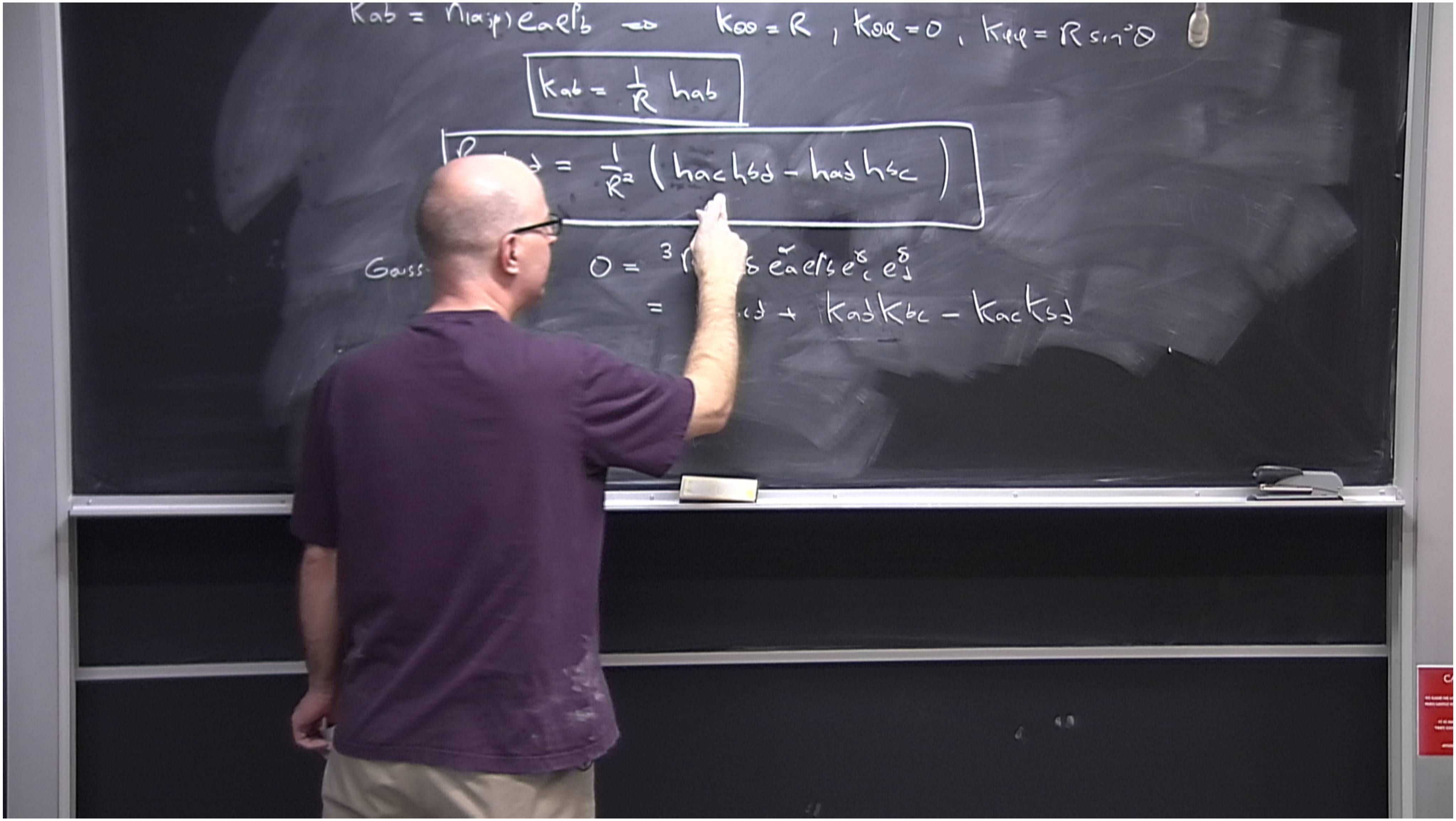

$e^{\alpha}_{\varphi} = (-R\sin\theta\sin\varphi, R\sin\theta\cos\varphi, 0)$
$h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow h_{\theta\theta} = R^2, h_{\theta\varphi} = 0, h_{\varphi\varphi} = R^2\sin^2\theta$
$K_{ab} = n_{(\alpha;\beta)} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow K_{\theta\theta} = R, K_{\theta\varphi} = 0, K_{\varphi\varphi} = R\sin^2\theta$
$K_{ab} = \frac{1}{R} h_{ab}$

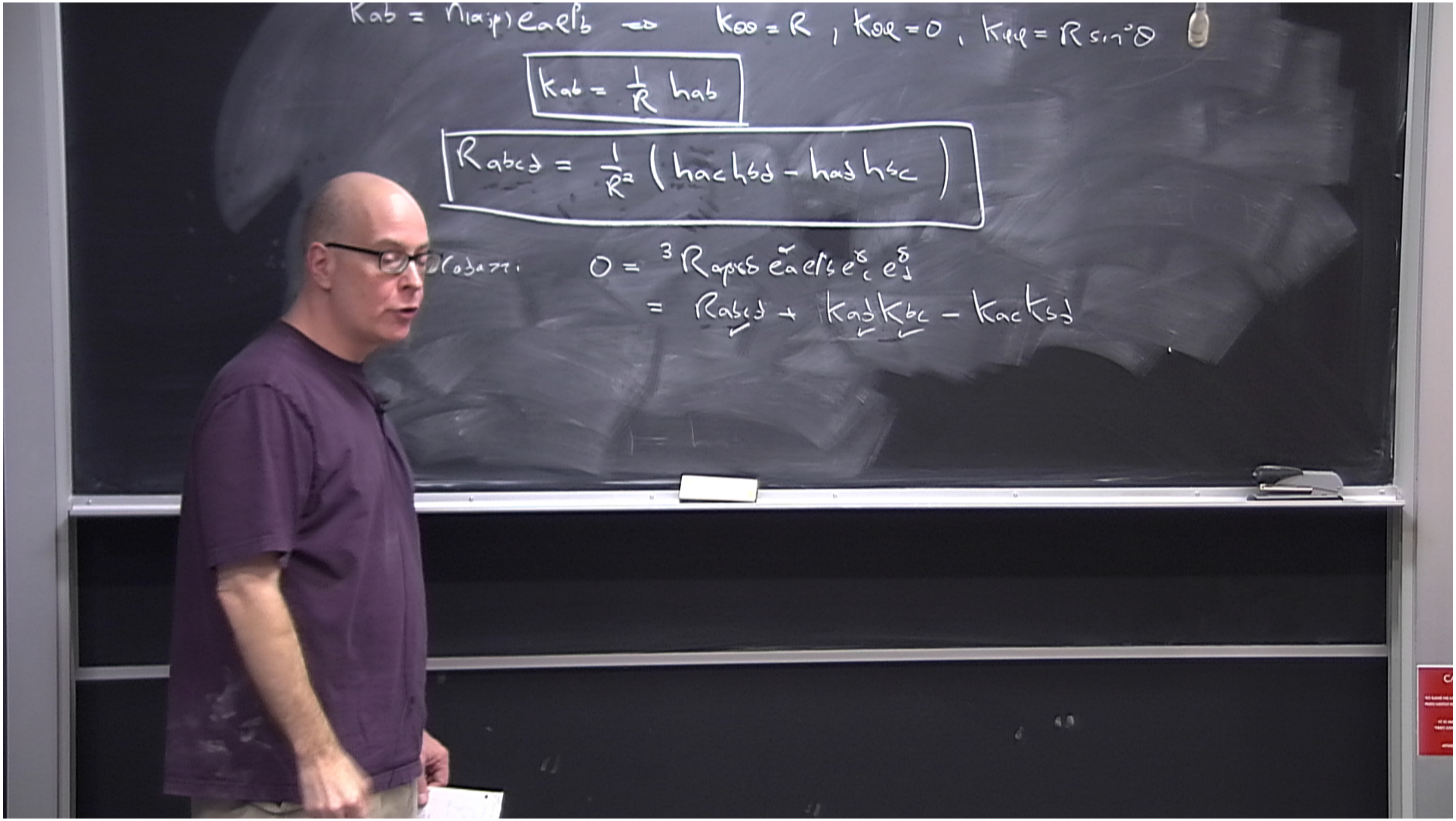

(R cosθ cosφ, R cosθ sinφ, −R sinθ)
hab = gαβ e^α_a e^β_b ⇒ hθθ = R², hθφ = 0, hφφ = R² sin²θ
Kab = n(α;β) e^α_a e^β_b ⇒ Kθθ = R, Kθφ = 0, Kφφ = R sin²θ
Kab = 1/R hab

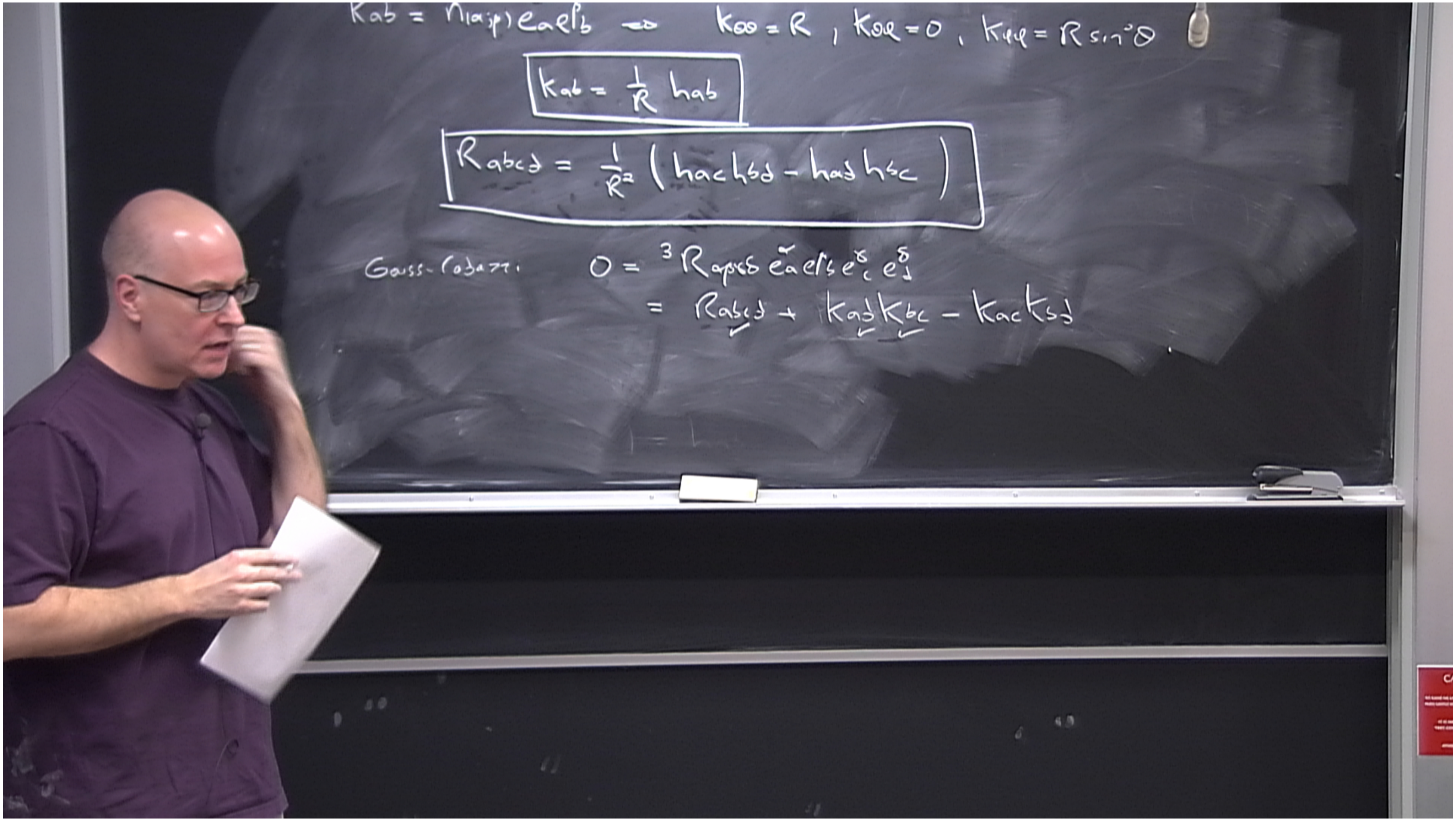

$h_{ab} = g_{\alpha\beta} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow h_{\theta\theta} = R^2, h_{\theta\varphi} = 0, h_{\varphi\varphi} = R^2 \sin^2\theta$
$K_{ab} = n_{(\alpha;\beta)} e^{\alpha}_{a} e^{\beta}_{b} \Rightarrow K_{\theta\theta} = R, K_{\theta\varphi} = 0, K_{\varphi\varphi} = R \sin^2\theta$
$K_{ab} = \frac{1}{R} h_{ab}$
$R_{abcd} = \frac{1}{R^2} (h_{ac} h_{bd} - h_{ad} h_{bc})$

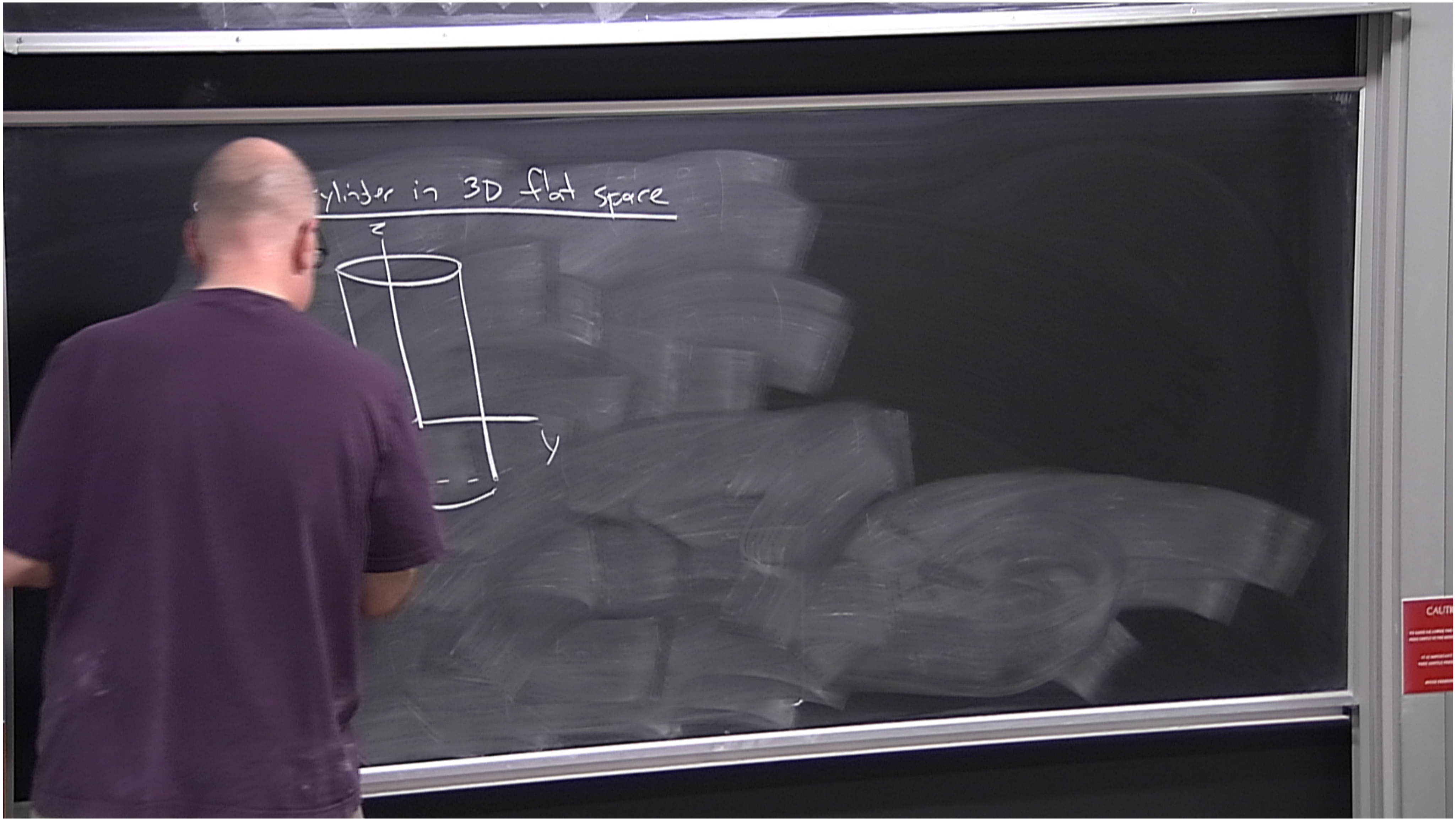

K_θθ = R , K_θφ = 0 , K_φφ = R sin²θ
K_ab = 1/R h_ab
R_abcd = 1/R² (h_ac h_bd − h_ad h_bc)
0 =

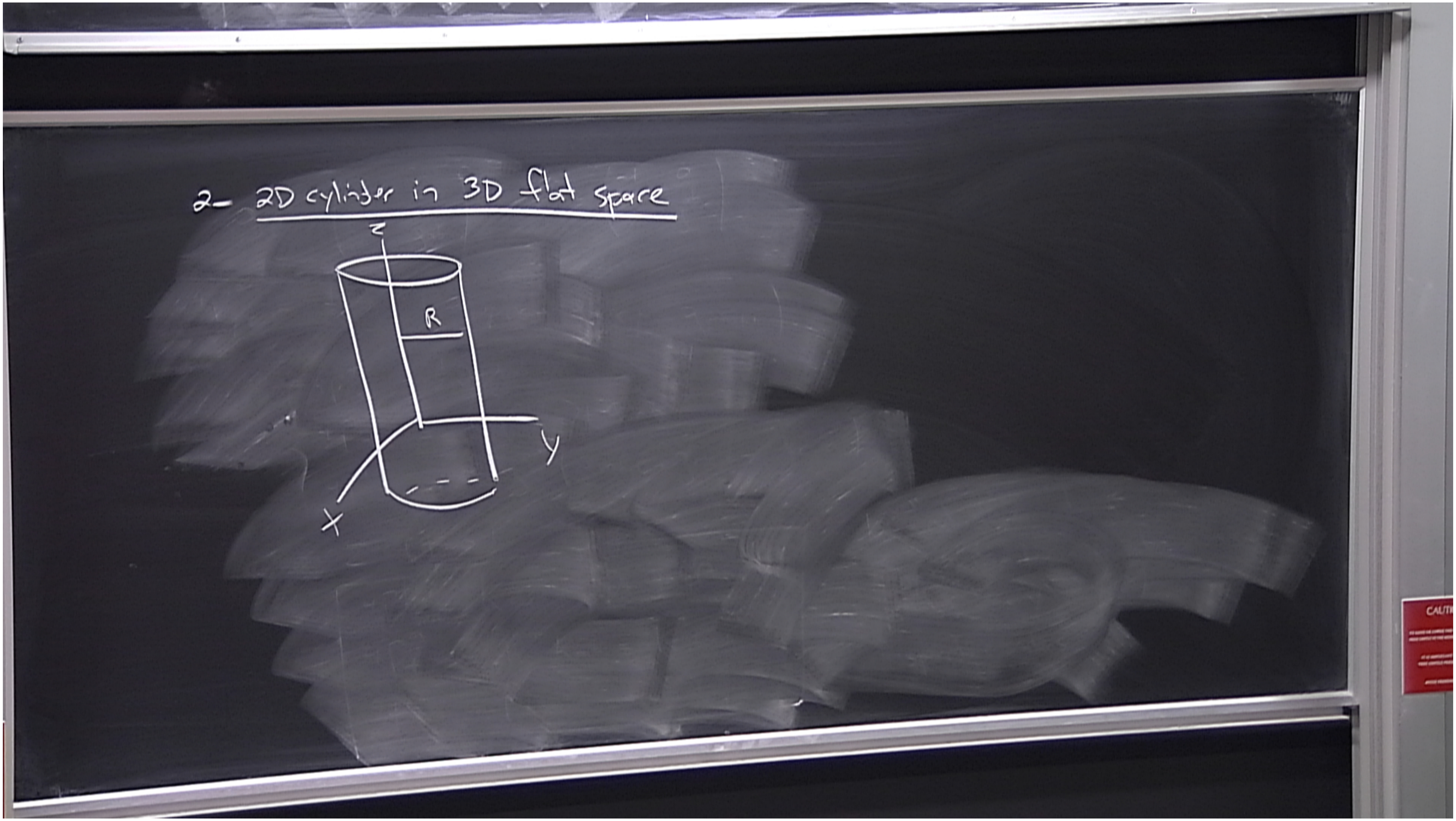

Kab = n(α;β) eαa eβb ⇒ Kθθ = R , Kθφ = 0 , Kφφ = R sin²θ
Kab = 1/R hab
Rabcd = 1/R² (hbd − had hbc)
Gauss-Codazzi
0 =
+ Kab Kab − K²

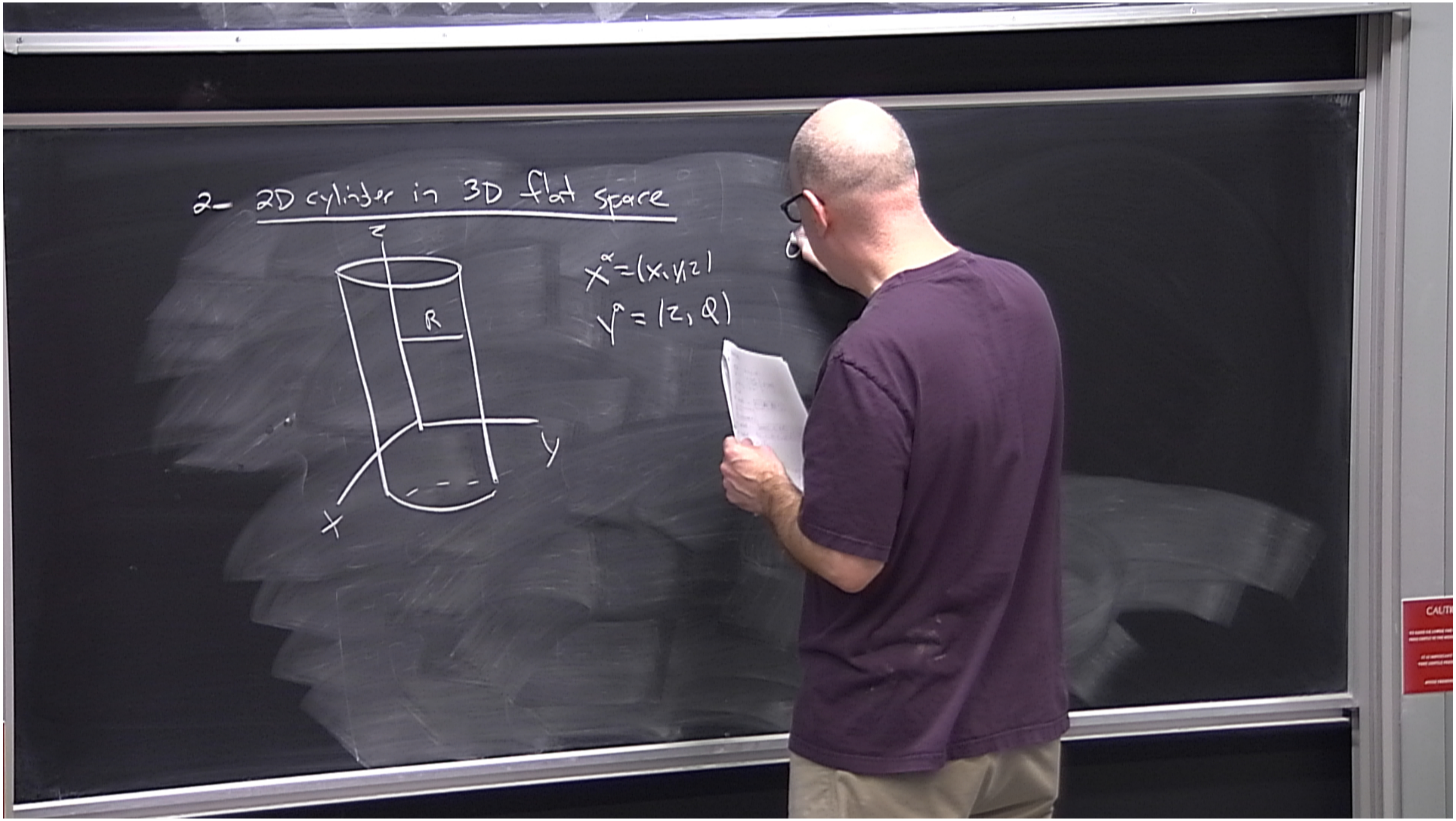

Kab = n(a;b) eaeb ⇒ Kθθ = R , Kθφ = 0 , Kφφ = R sin²θ
Kab = 1/R hab
= 1/R² (hac hbd − had hbc)
Gauss
0 =
= Kad Kbc − Kac Kbd

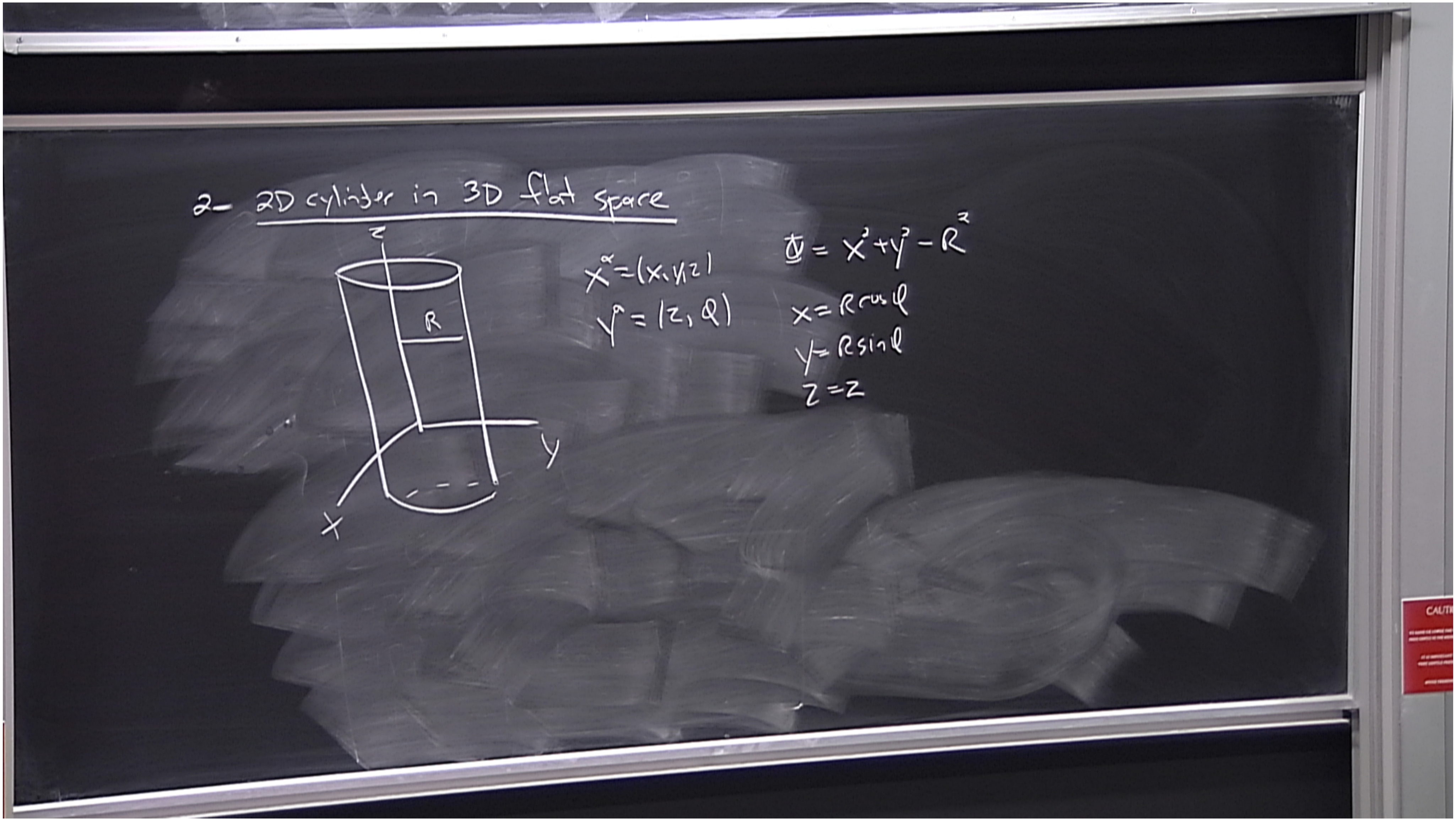

kab = 1/R hab
Rabcd = 1/R² (hac hbd − had hbc)

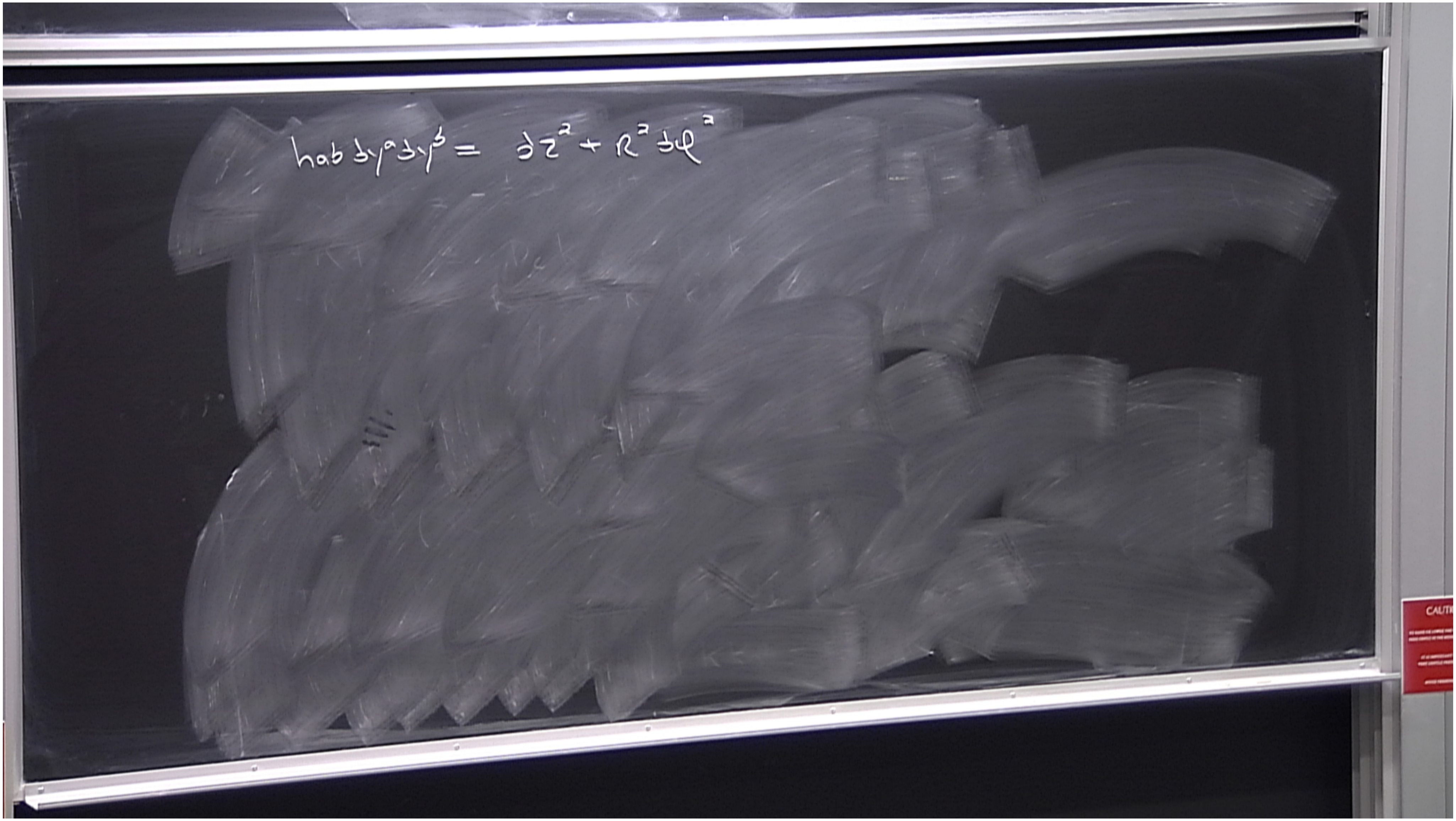

Kab = n(α;β) eαa eβb ⇒ Kθθ = R , Kθφ = 0 , Kφφ = R sin²θ
Kab = 1/R hab
Rabcd = 1/R² (hac hbd − had hbc)
Gauss-Codazzi: 0 = ³Rαβγδ eαa eβb eγc eδd
= Rabcd + Kad Kbc − Kac Kbd

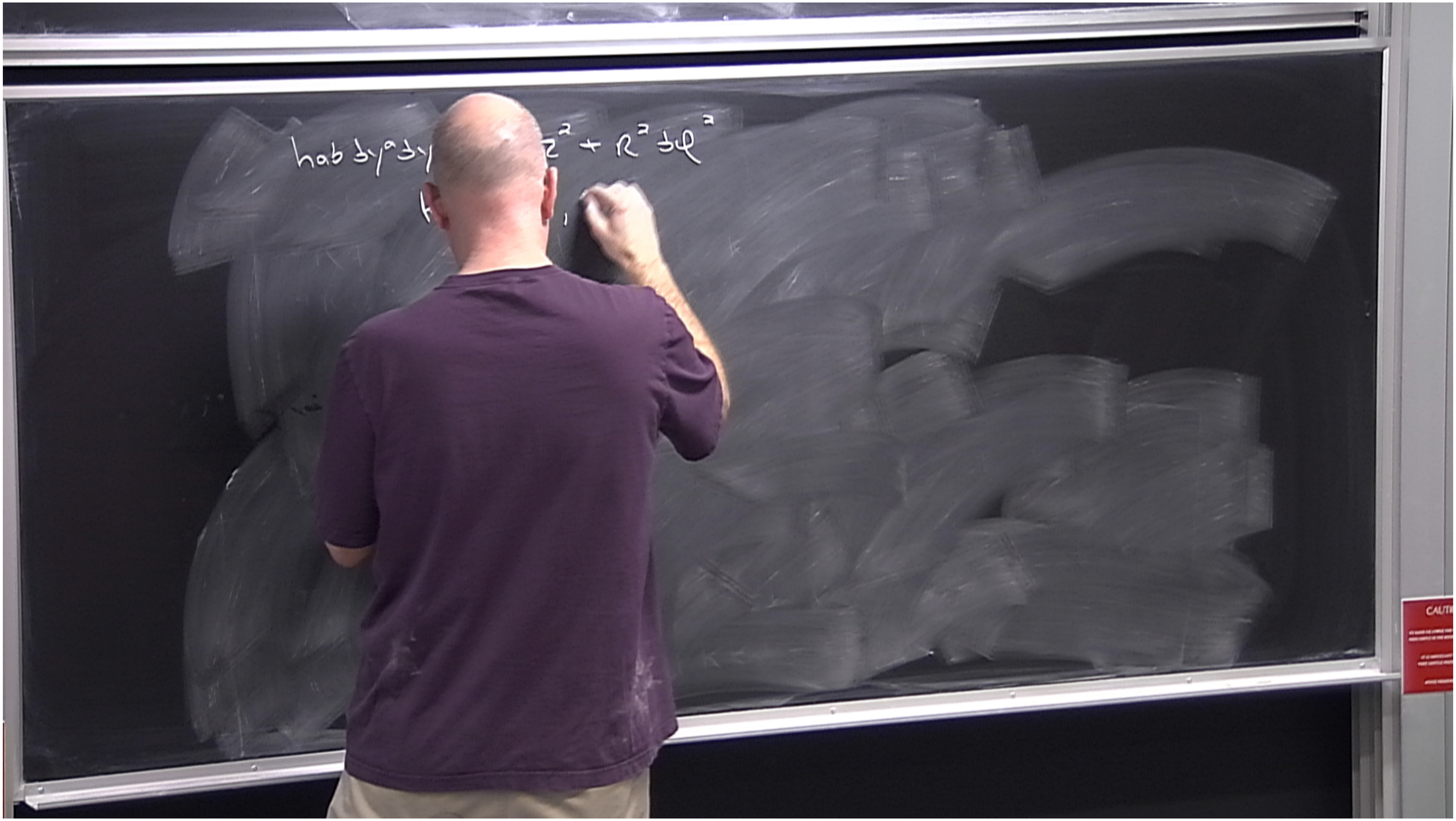

ylinder in 3D flat space
z
y

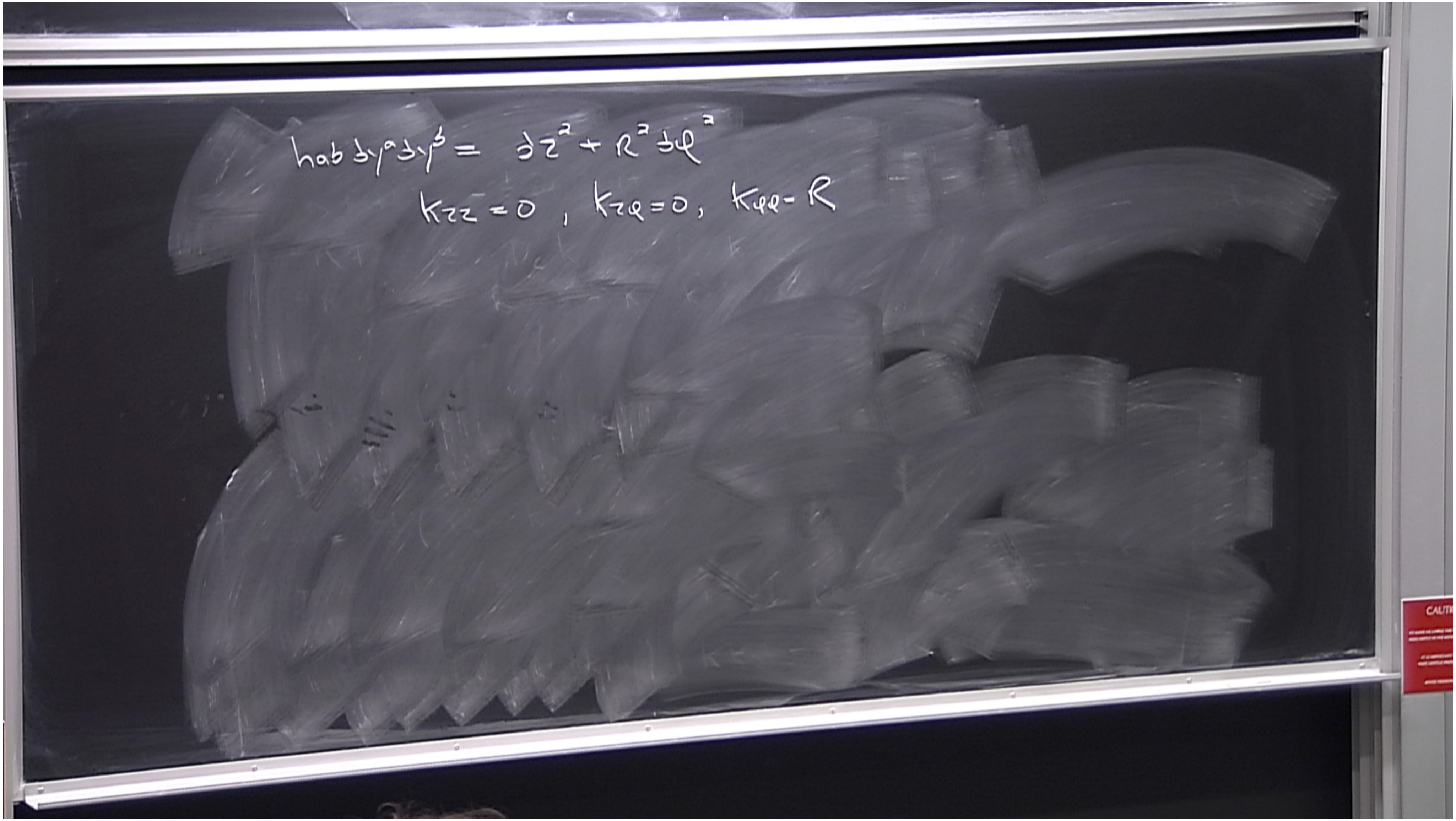

2– 2D cylinder in 3D flat space
z
R
y
x

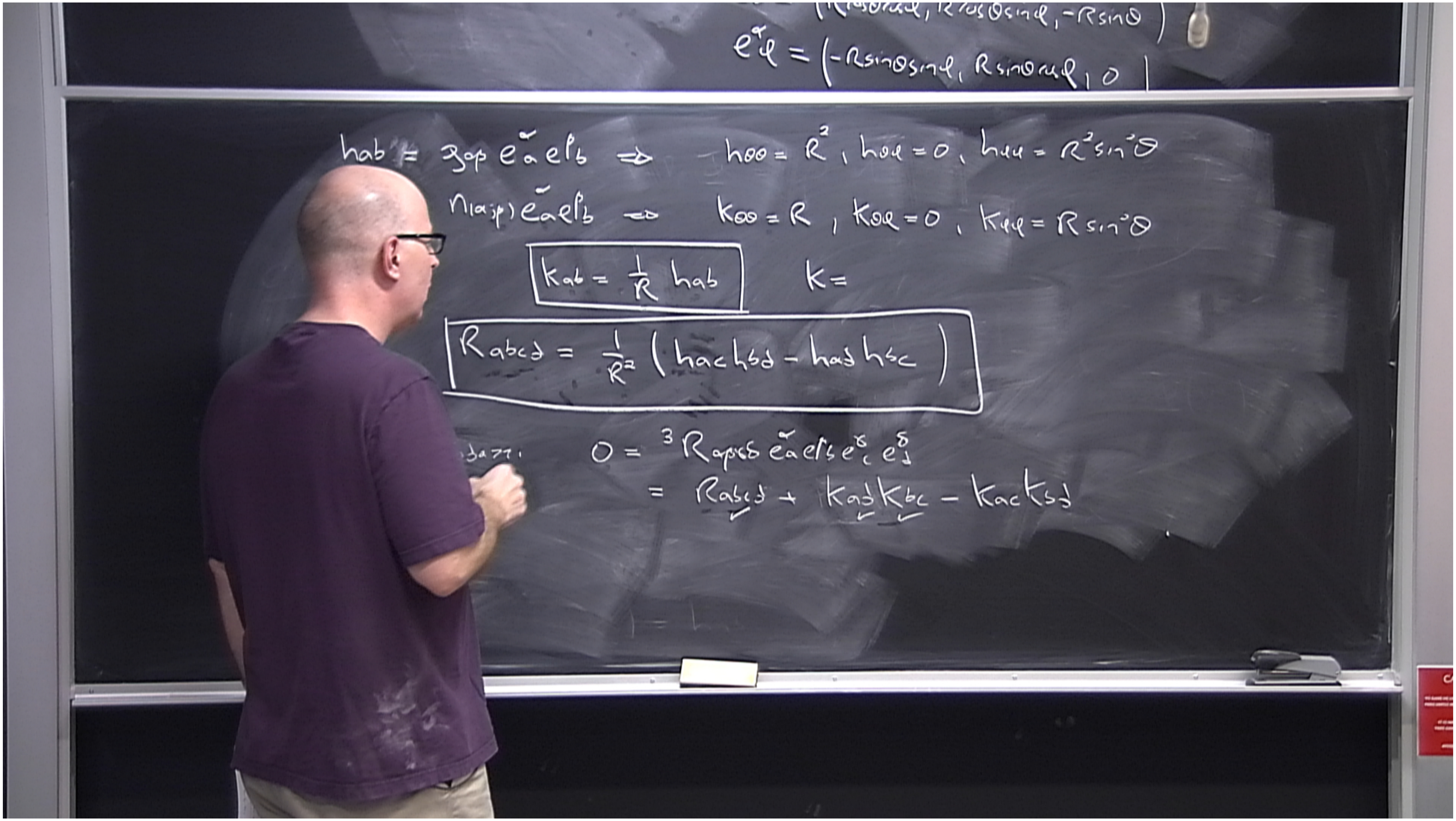

2 – 2D cylinder in 3D flat space
z
R
y
x
$x^\alpha = (x, y, z)$
$y^a = (z, \varphi)$
CAUTION

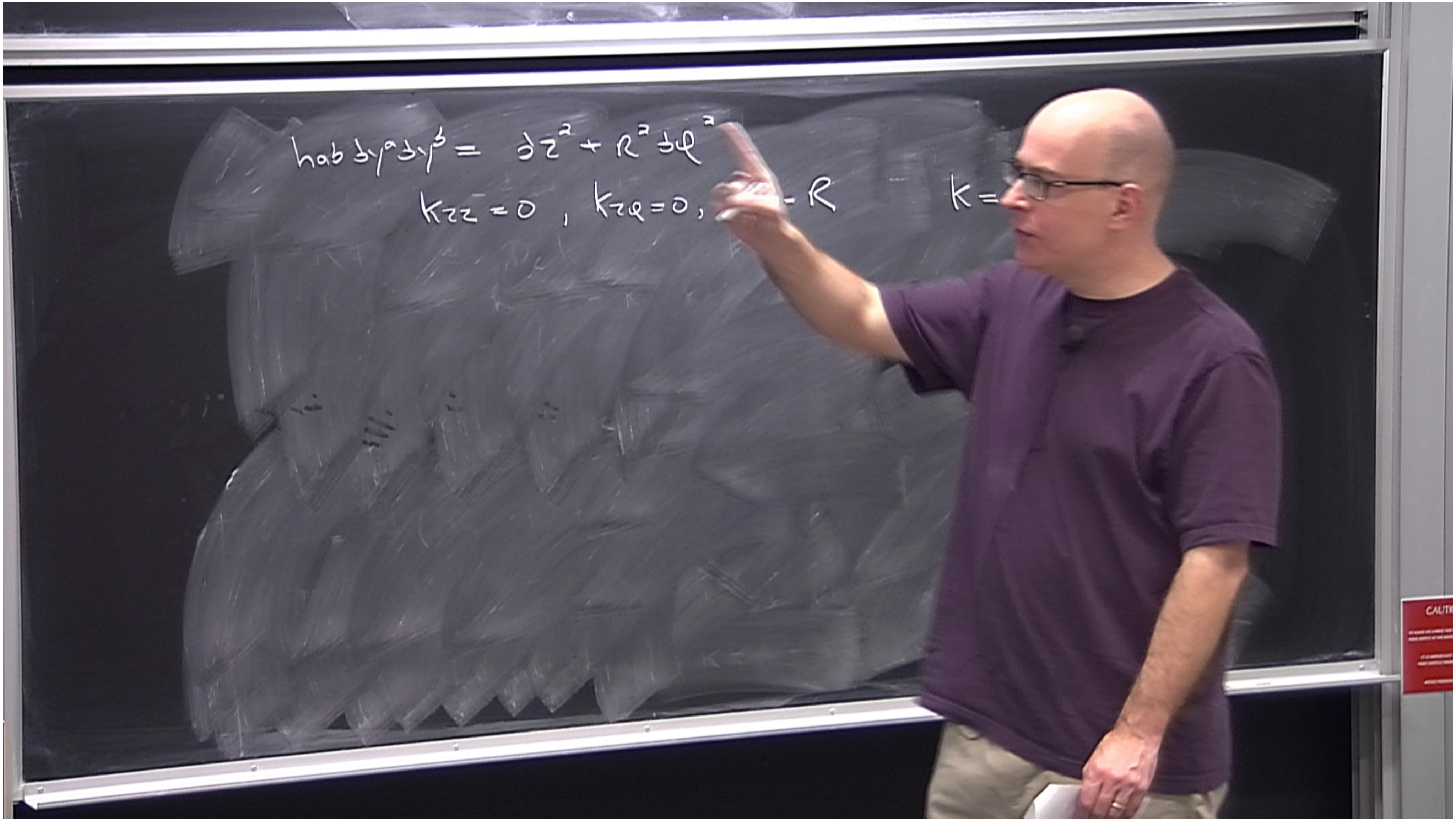

2- 2D cylinder in 3D flat space
z
R
y
x
x^α = (x, y, z)
y^a = (z, φ)
Φ = x² + y² − R²
x = R cos φ
y = R sin φ
z = z

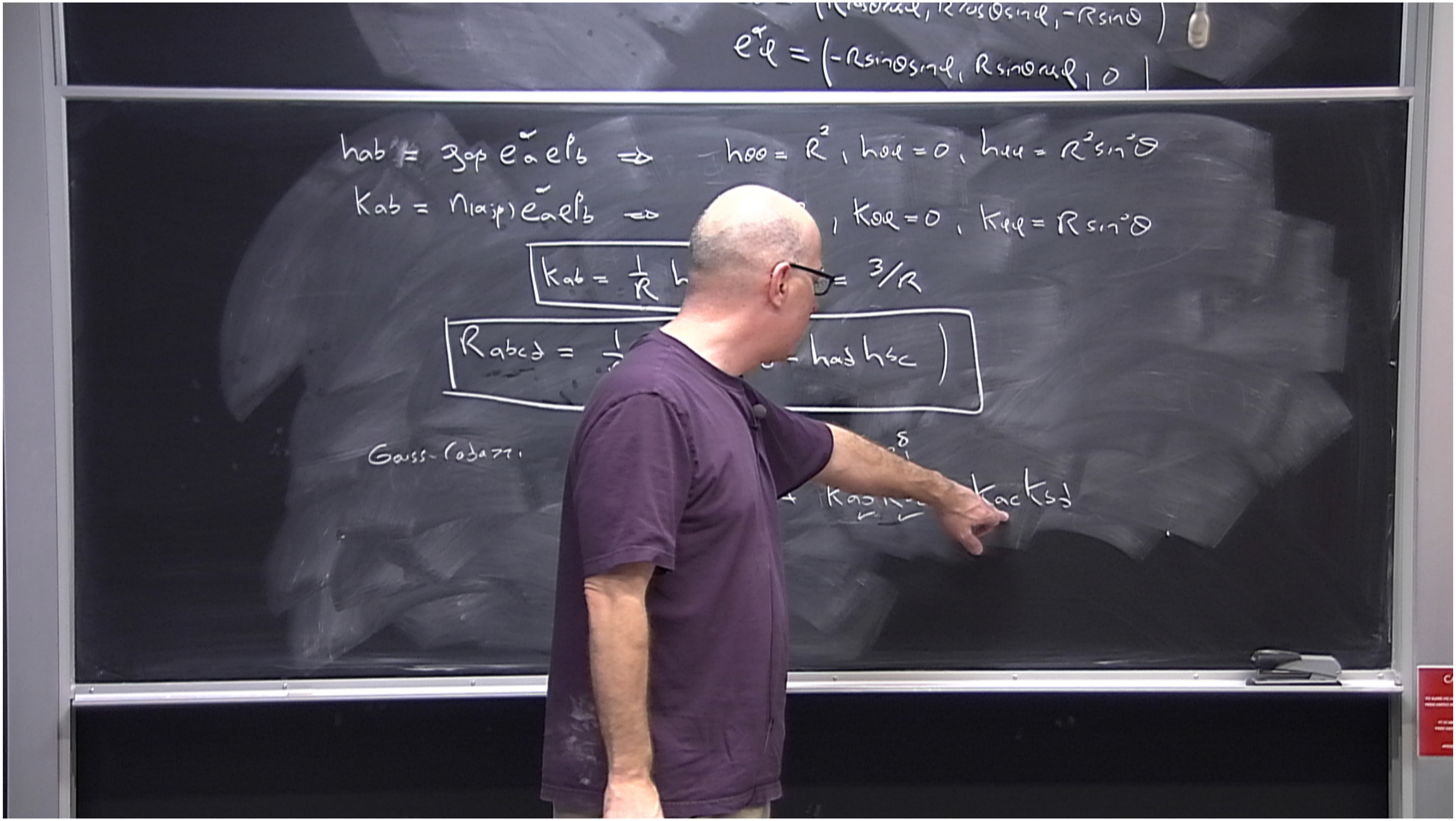

$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\varphi^2$



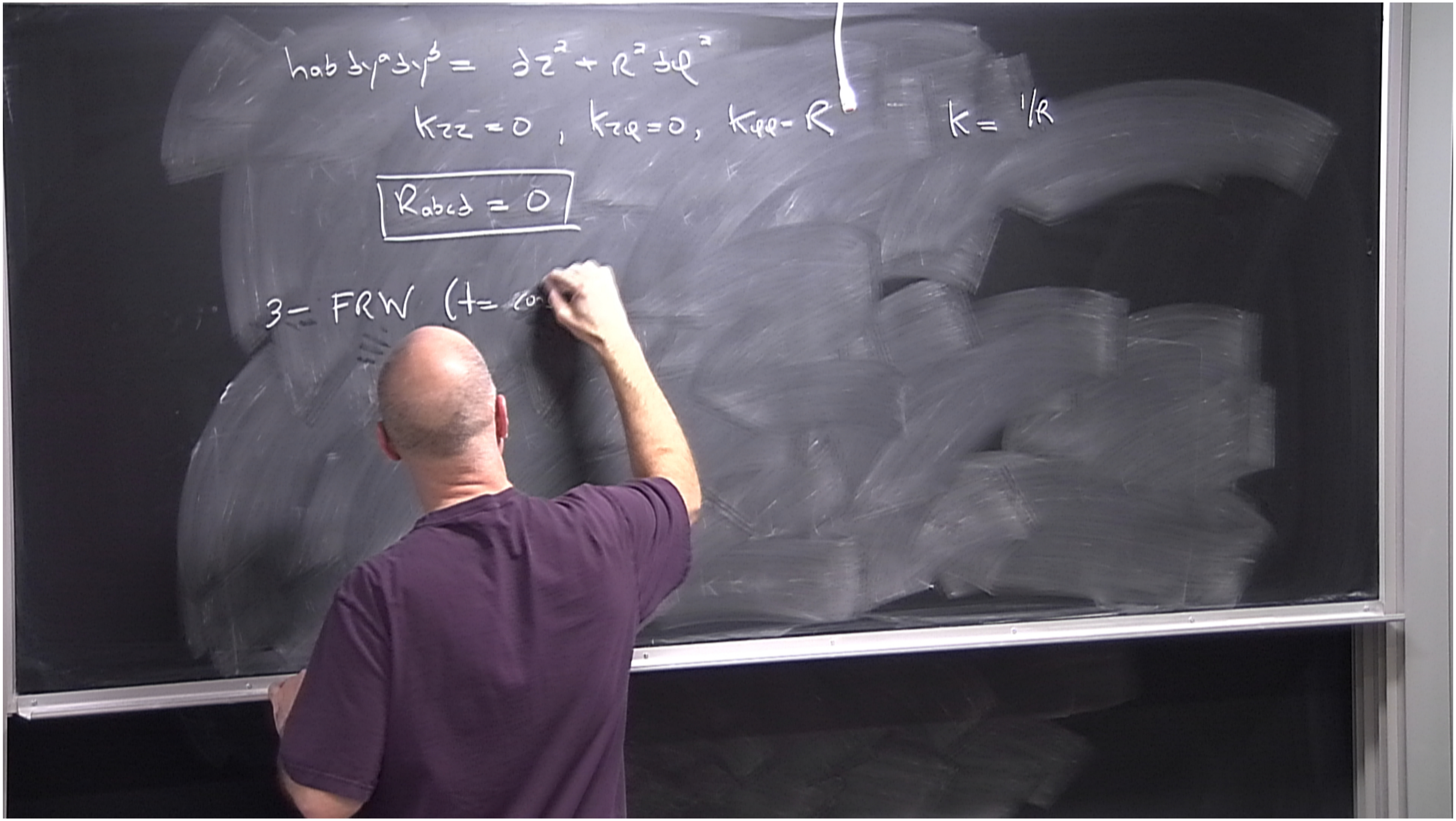

$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\phi^2$
$K_{zz} = 0$, $K_{z\phi} = 0$, $K_{\phi\phi} - R$


(R cosθ cosφ, R cosθ sinφ, −R sinθ)
e^α_φ = (−R sinθ sinφ, R sinθ cosφ, 0)
h_ab = g_αβ e^α_a e^β_b ⇒ h_θθ = R², h_θφ = 0, h_φφ = R² sin²θ
n_(α;β) e^α_a e^β_b ⇒ K_θθ = R, K_θφ = 0, K_φφ = R sin²θ
K_ab = 1/R h_ab
K =
R_abcd = 1/R² (h_ac h_bd − h_ad h_bc)
0 = ³R_αβγδ e^α_a e^β_b e^γ_c e^δ_d
= R_abcd + K_ad K_bc − K_ac K_bd


$h_{ab}\, dy^a dy^b = dz^2 + R^2 d\varphi^2$
$k_{zz} = 0$, $k_{z\varphi} = 0$, - R
k =



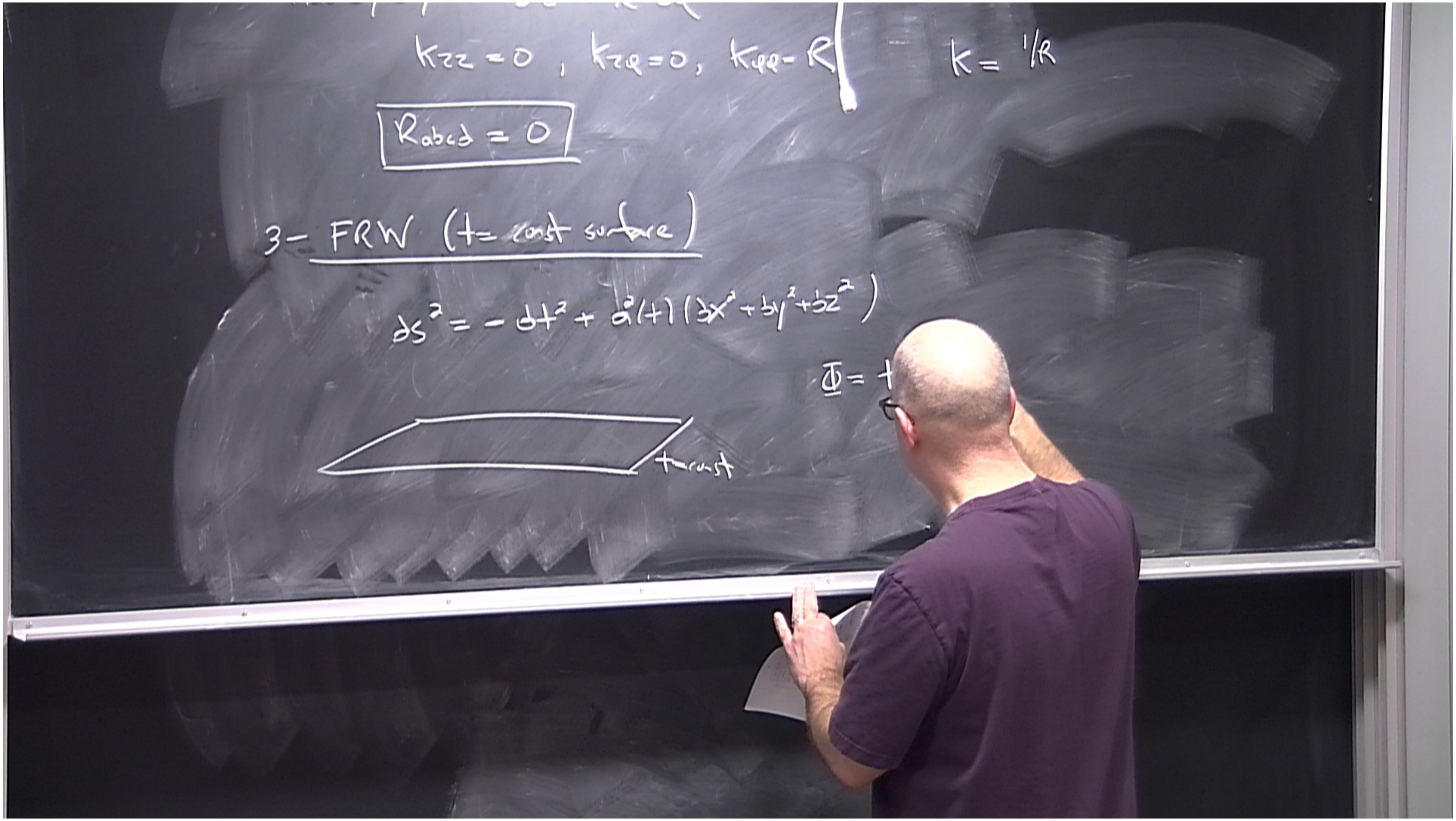
eαφ = (-R sinθ sinφ, R sinθ cosφ, 0)
hab = gαβ eαa eβb ⇒ hθθ = R², hθφ = 0, hφφ = R² sin²θ
Kab = n(α;β) eαa eβb ⇒ , Kθφ = 0, Kφφ = R sin²θ
Gauss-Codazzi

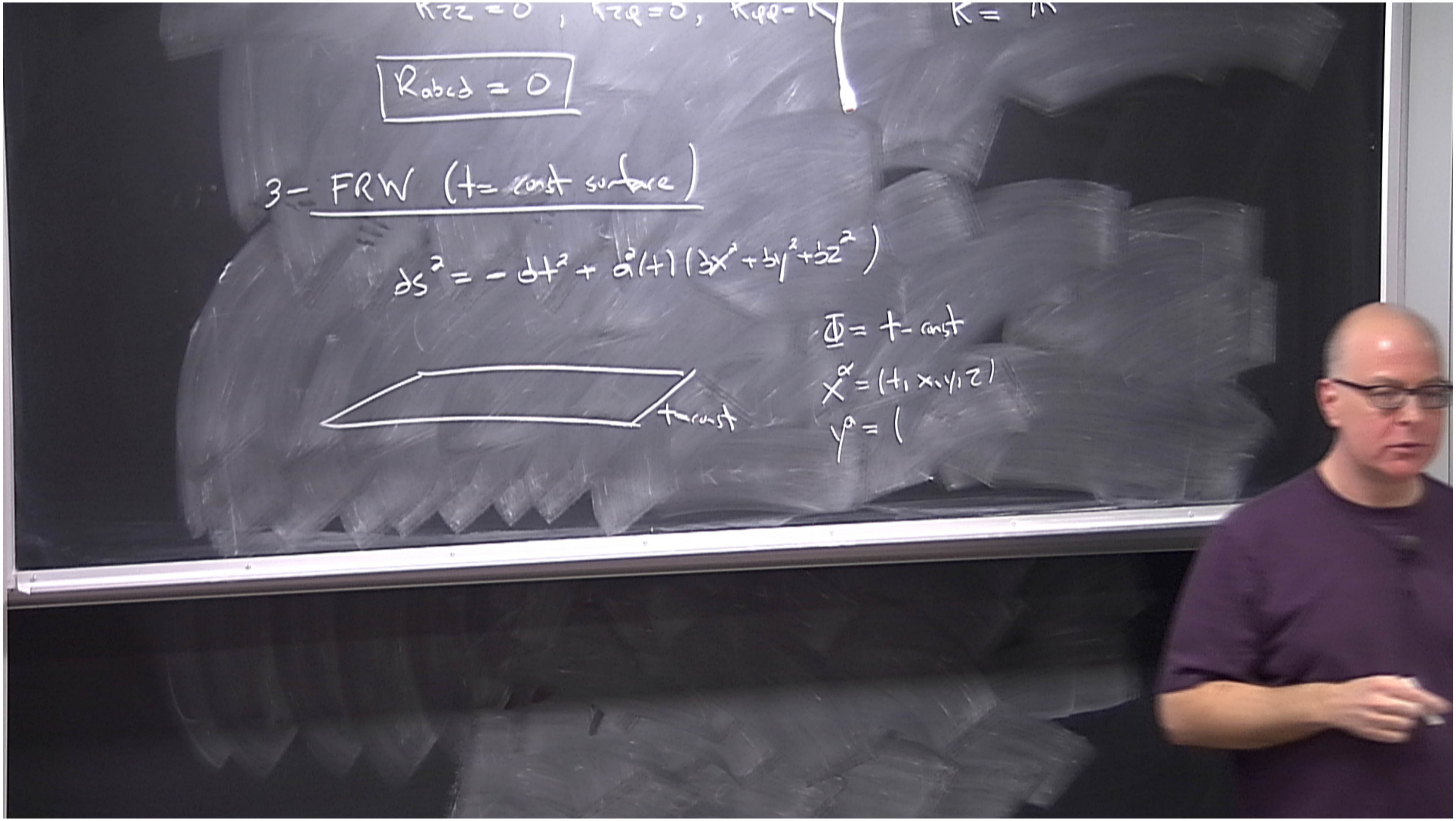

$h_{ab} dy^a dy^b = dz^2 + R^2 d\varphi^2$
$K_{zz} = 0$, $K_{z\varphi} = 0$, $K_{\varphi\varphi} = R$
$K = 1/R$
$R_{abcd} = 0$
3 – FRW (t= co

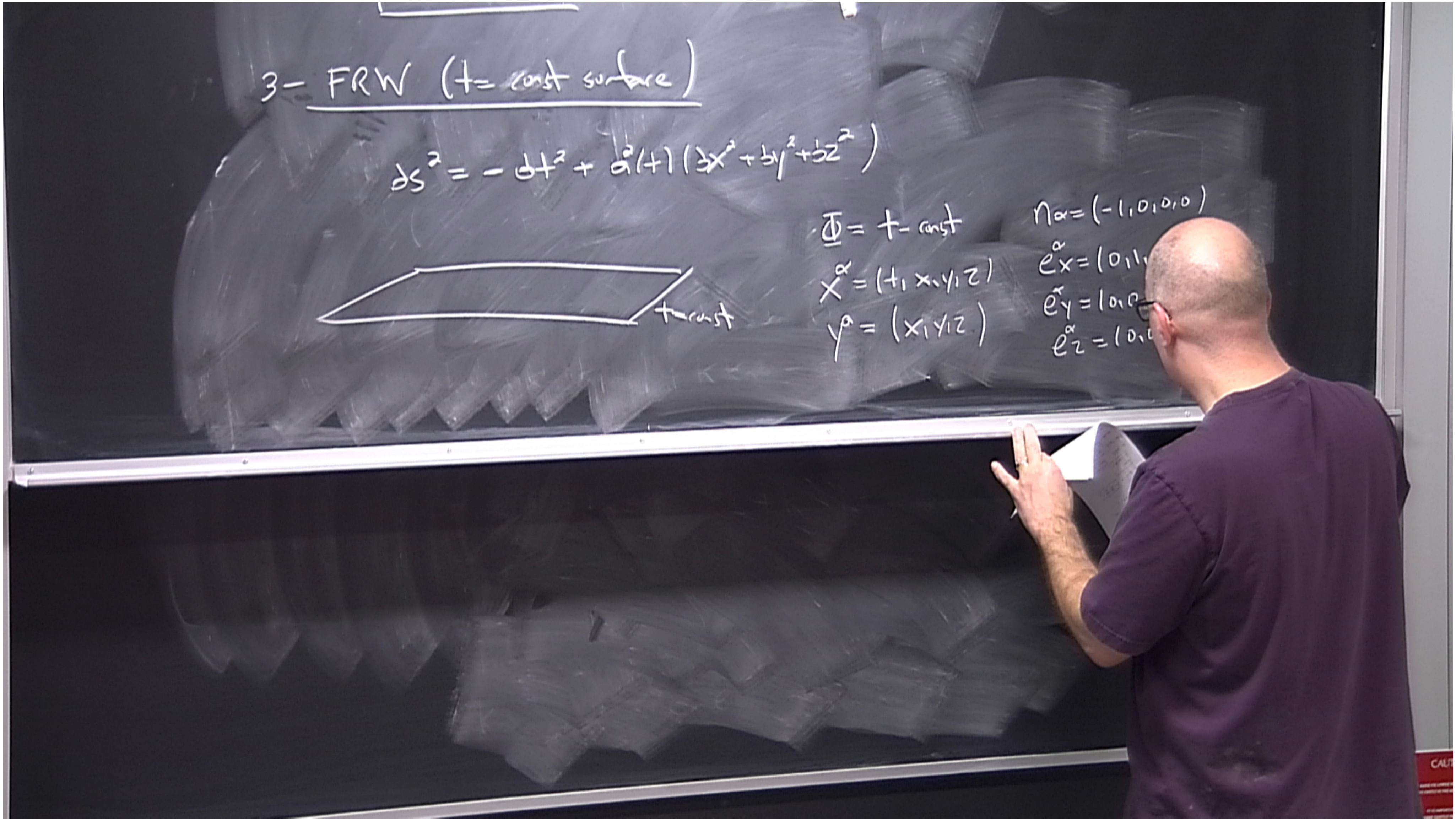
$$R_{abcd} = 0$$

3 – FRW (t = const surface)

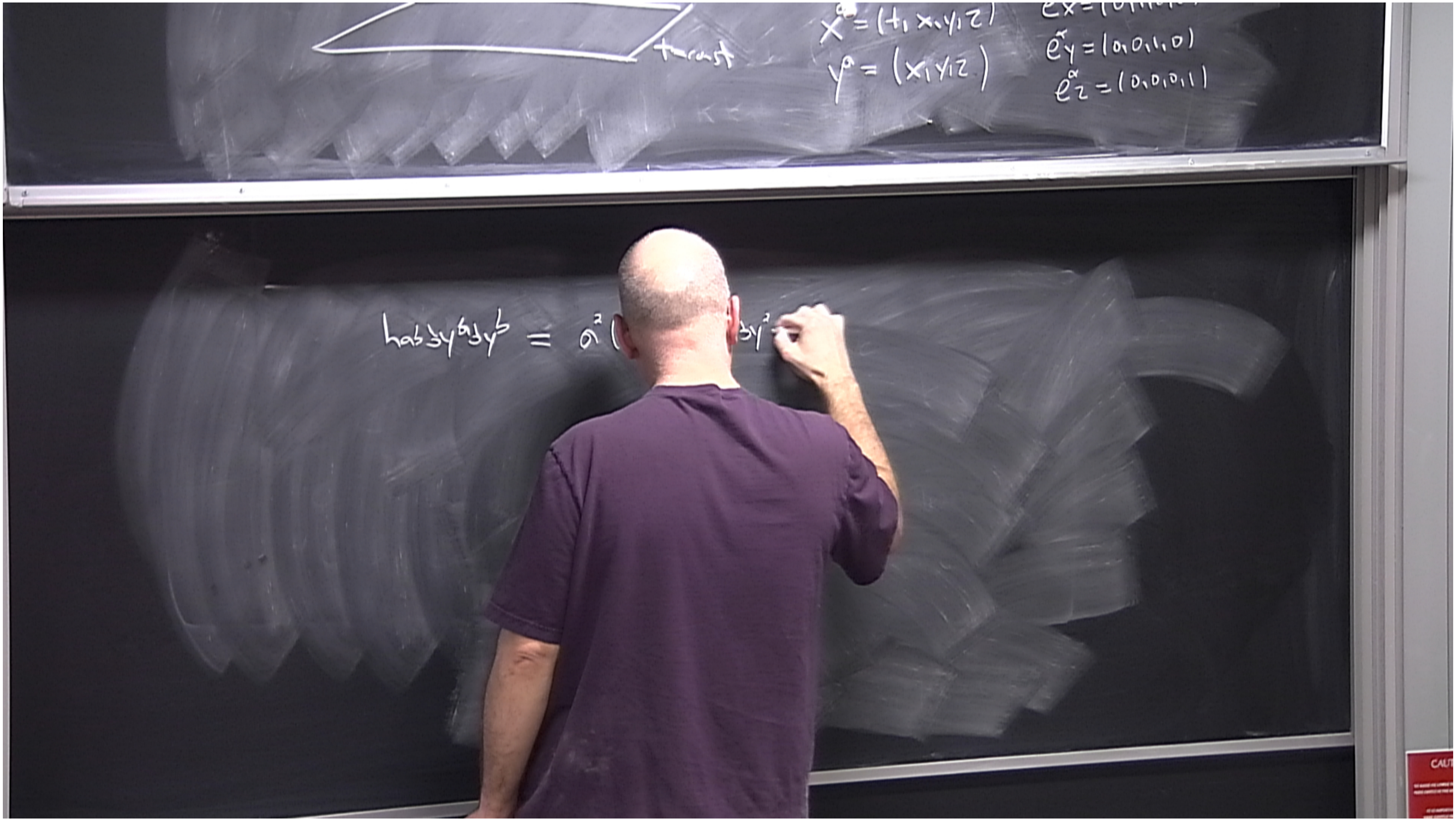
$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

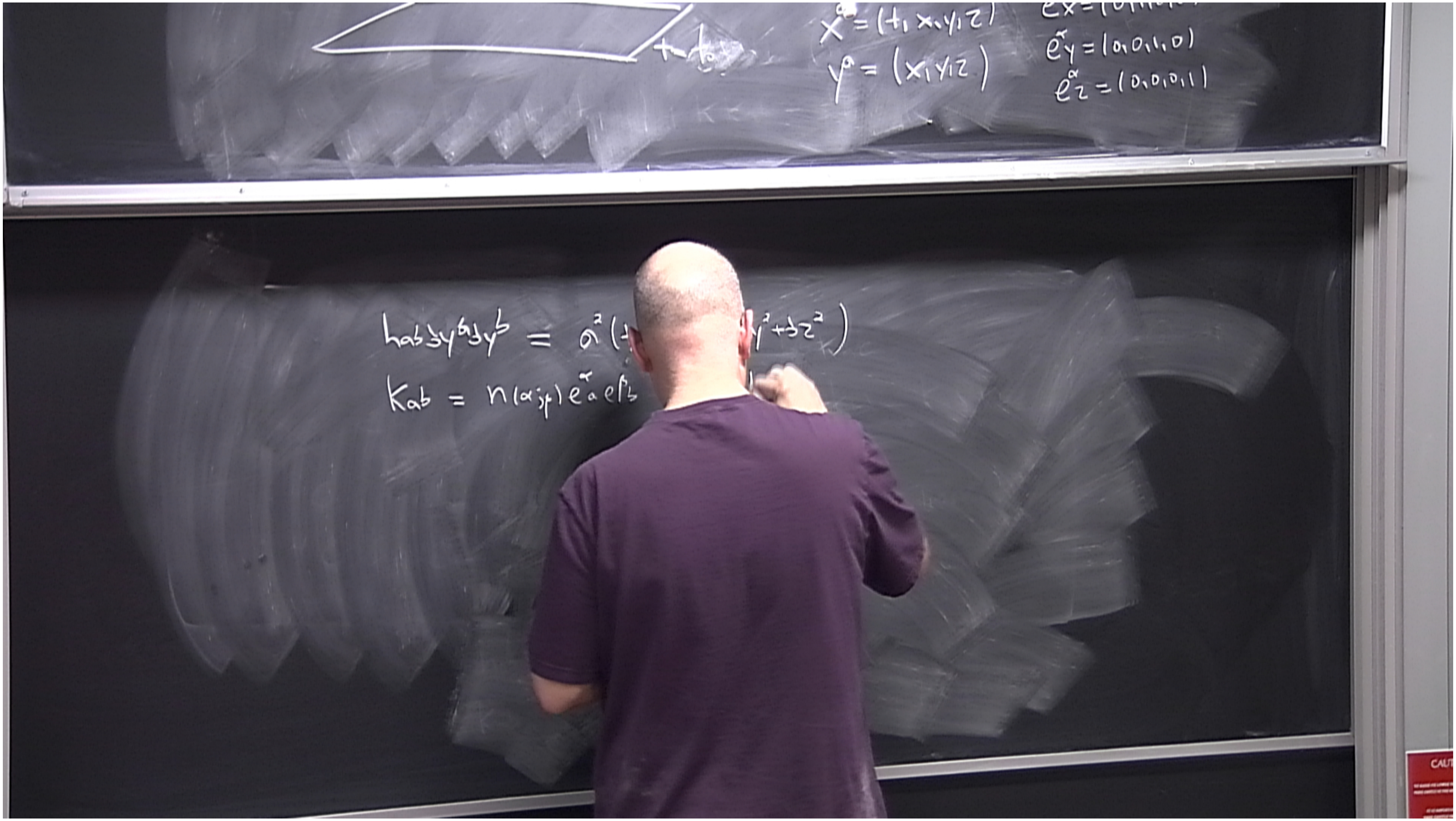

Kzz = 0 , Kzφ = 0, Kφφ = R
K = 1/R
Rabcd = 0
3 — FRW (t = const surface)
ds² = − dt² + a²(t) (dx² + dy² + dz²)
Φ = t
t = const

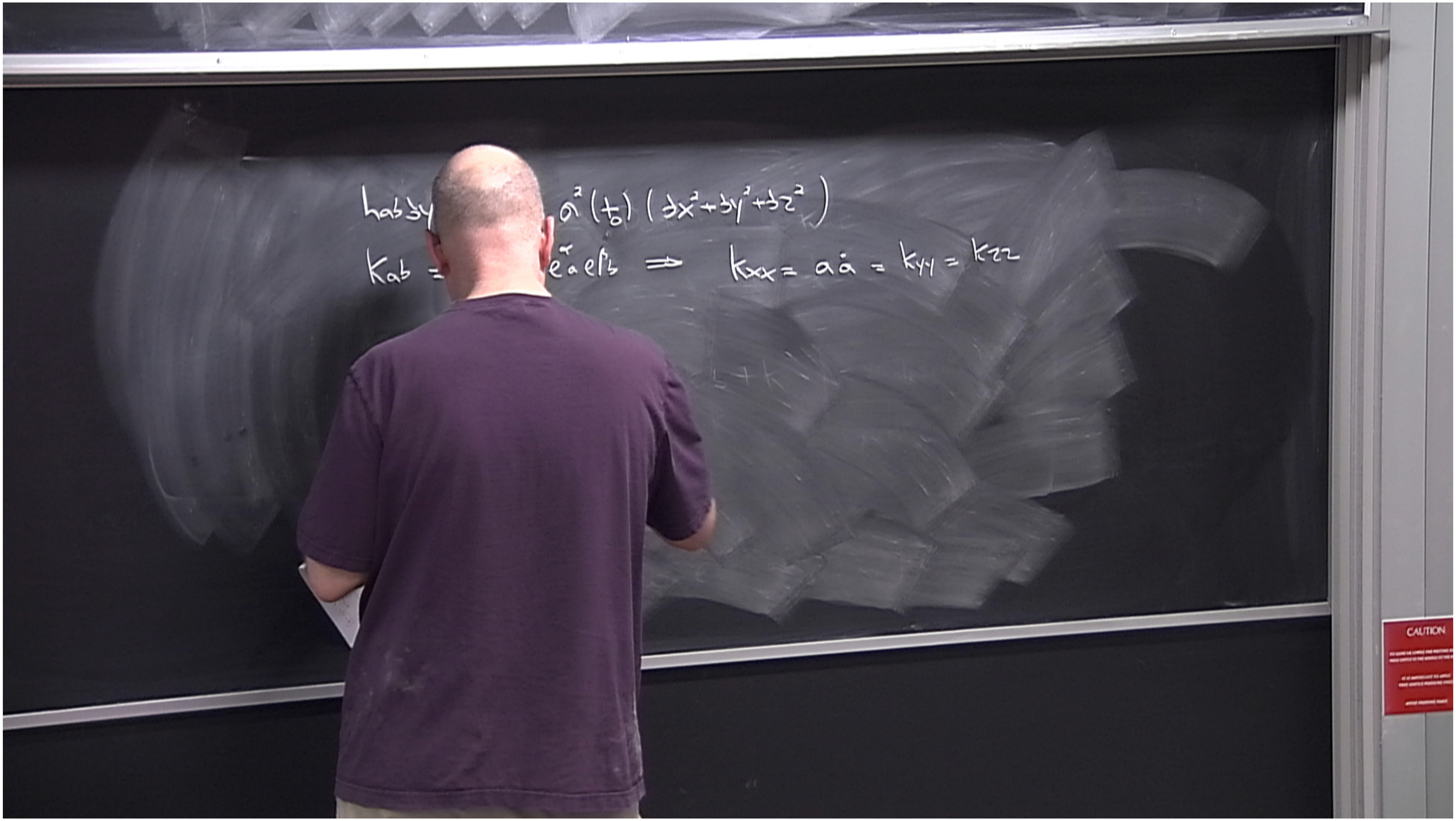

Rabcd = 0
3 – FRW (t = const surface)
ds² = – dt² + a²(t) (dx² + dy² + dz²)
Φ = t – const
x^α = (t, x, y, z)
y^a = (
t = const

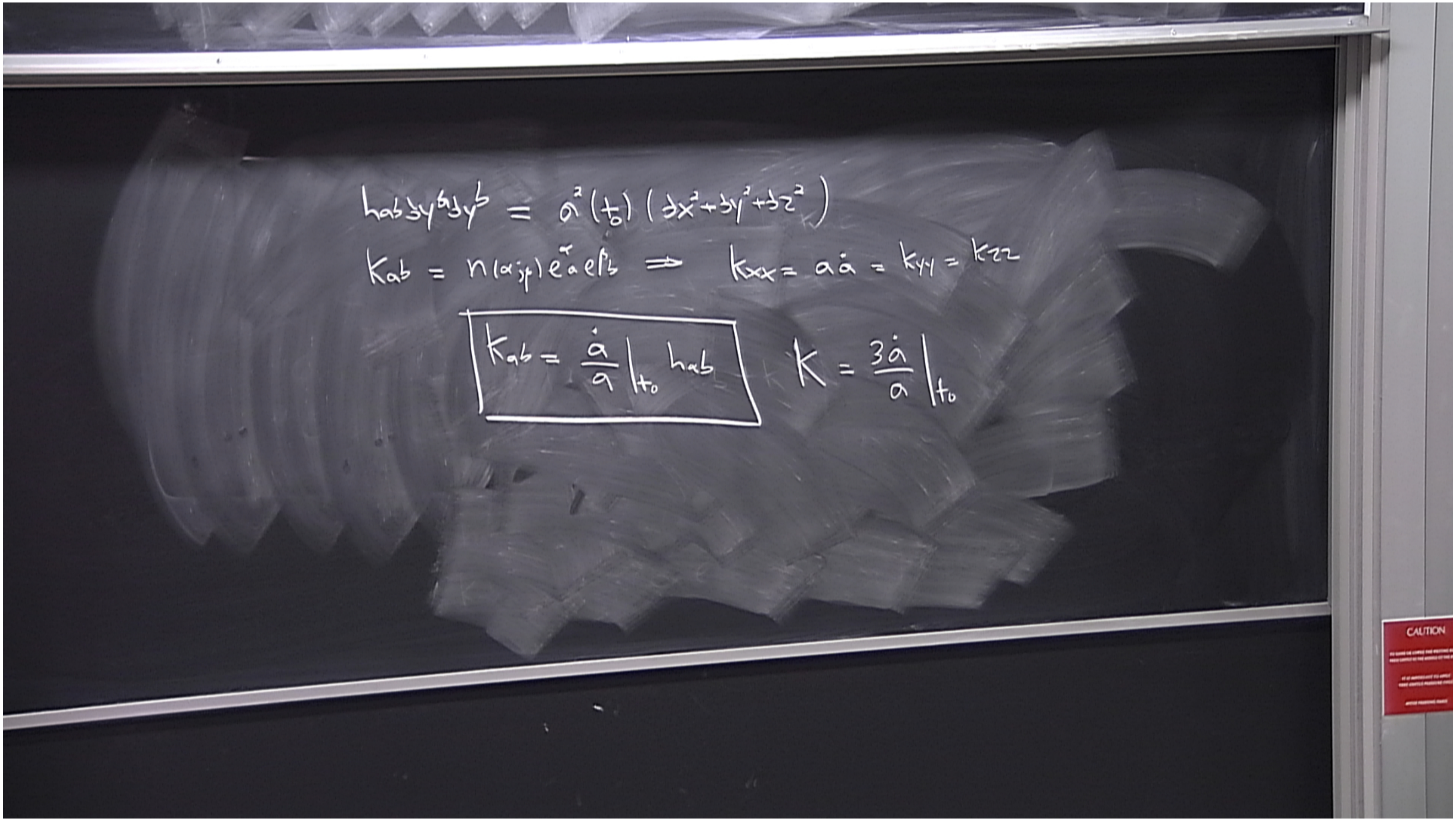


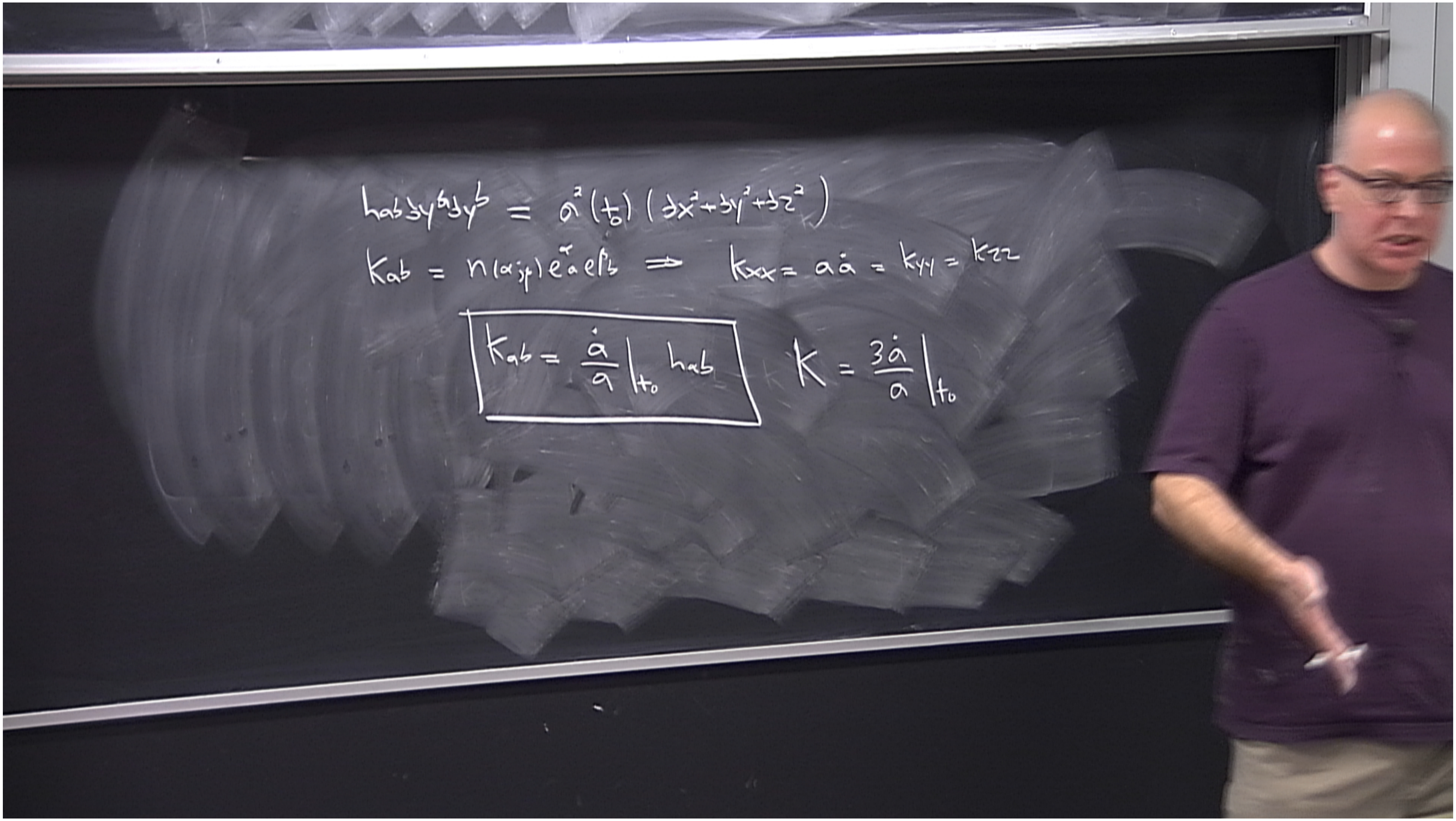
3– FRW (t= const surface)
ds² = – dt² + a²(t)(dx² + dy² + dz²)
Φ = t – const
x^α = (t, x, y, z)
y^a = (x, y, z)
n_α = (–1, 0, 0, 0)
t=const

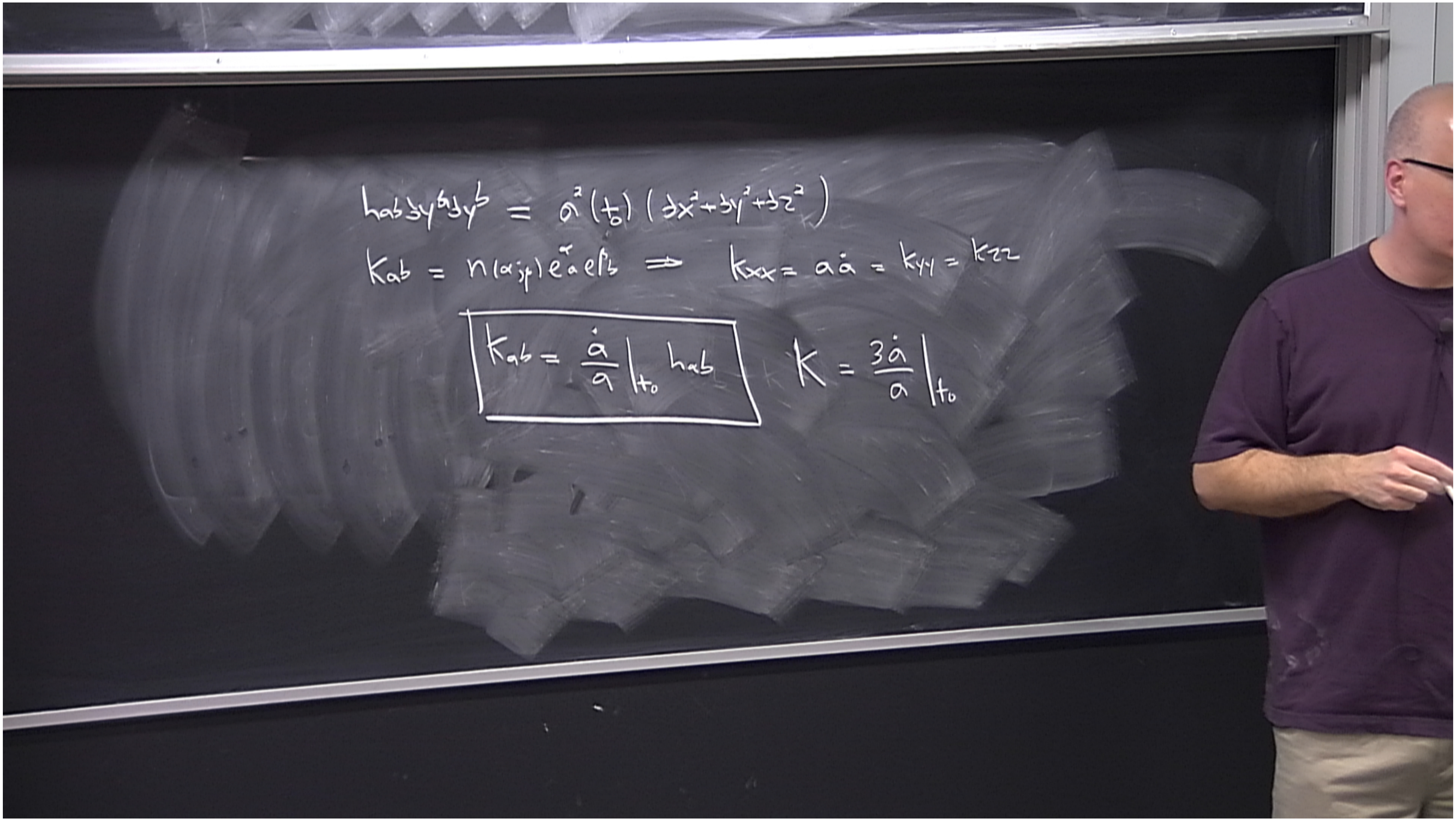

thrust
x^a = (t, x, y, z)
y^a = (x, y, z)
e_y^a = (0,0,1,0)
e_z^a = (0,0,0,1)


x^α = (t, x, y, z)
y^a = (x, y, z)
e^α_y = (0,0,1,0)
e^α_z = (0,0,0,1)
K_ab = n(α;β) e^α_a e^β_b


hab dy … a²(t₀) (dx² + dy² + dz²)
Kab = … eᵅa eᵝb ⇒ Kxx = a ȧ = Kyy = Kzz
CAUTION


$h_{ab}\,dy^a dy^b = a^2(t_0)\,(dx^2+dy^2+dz^2)$
$K_{ab} = n_{(\alpha;\beta)}\, e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$
$K_{ab} = \frac{\dot{a}}{a}\Big|_{t_0} h_{ab}$
$K = \frac{3\dot{a}}{a}\Big|_{t_0}$
CAUTION

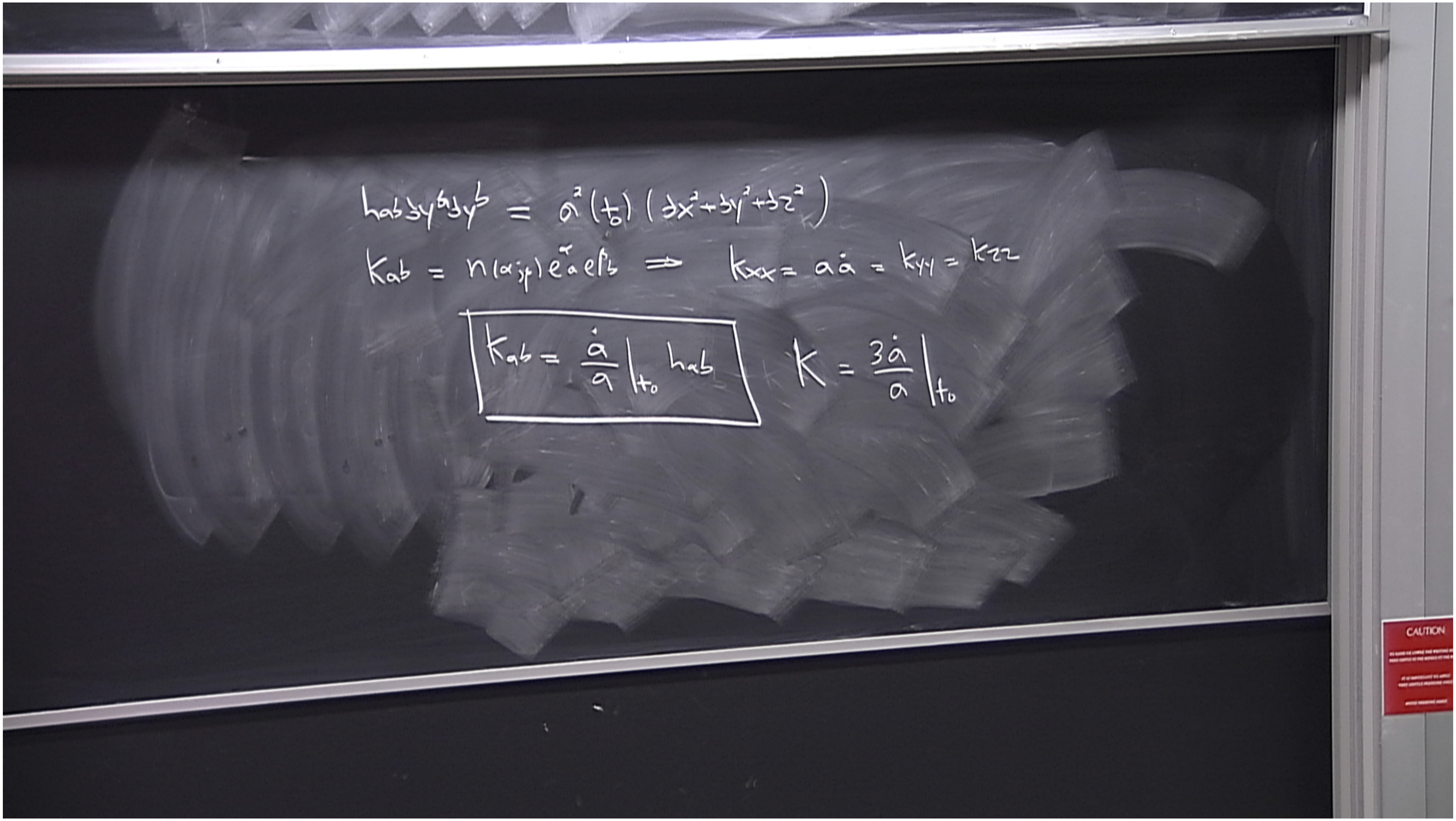

$h_{ab}\, dy^a dy^b = a^2(t_0)\,(dx^2 + dy^2 + dz^2)$
$K_{ab} = n(\alpha;\beta)\, e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$
$K_{ab} = \frac{\dot{a}}{a}\Big|_{t_0} h_{ab}$
$K = \frac{3\dot{a}}{a}\Big|_{t_0}$

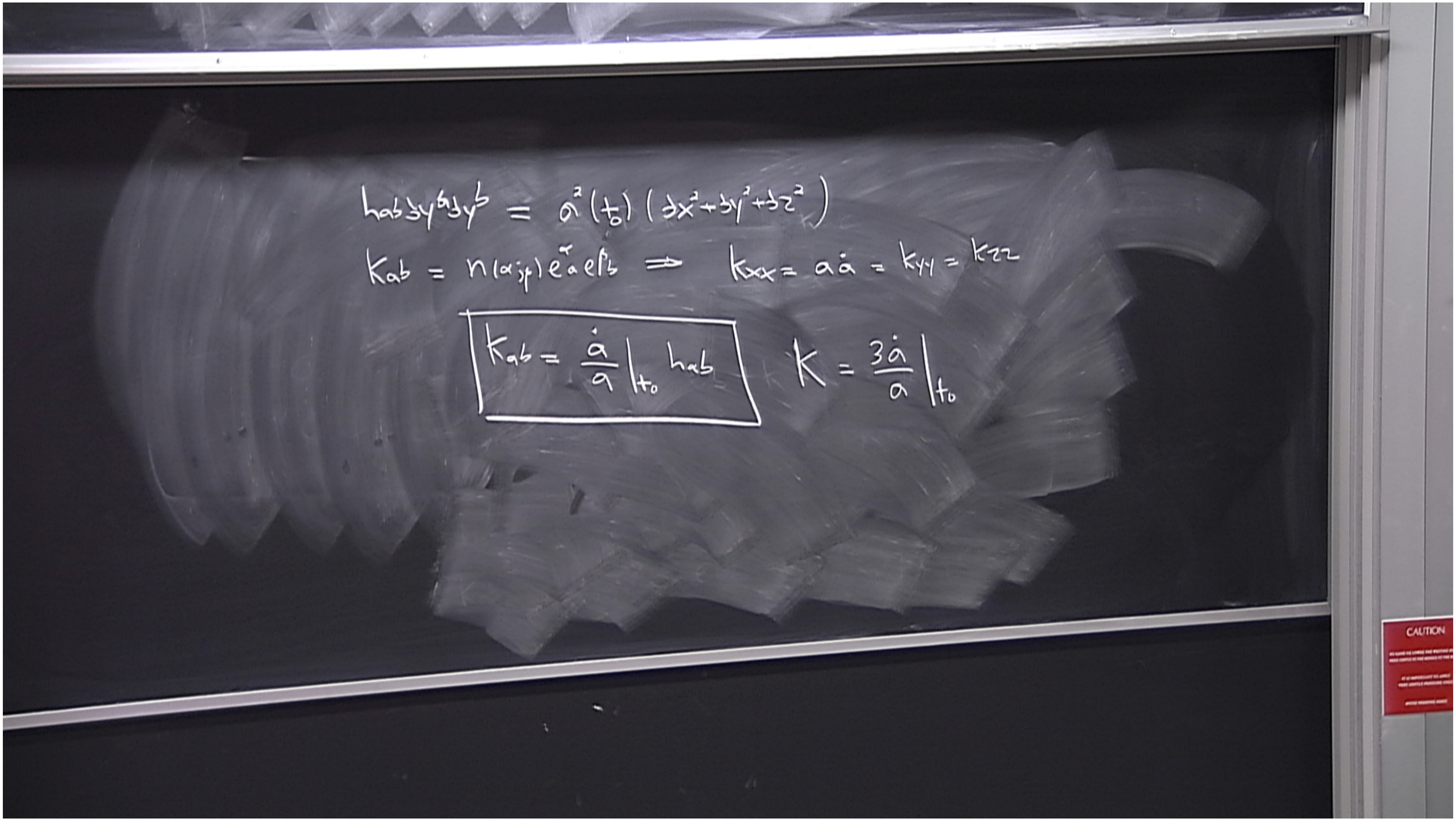

$h_{ab} \partial y^a \partial y^b = a^2(t_0)(\partial x^2 + \partial y^2 + \partial z^2)$
$K_{ab} = n(\alpha;\beta) e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$
$K_{ab} = \frac{\dot{a}}{a}\Big|_{t_0} h_{ab}$
$K = \frac{3\dot{a}}{a}\Big|_{t_0}$

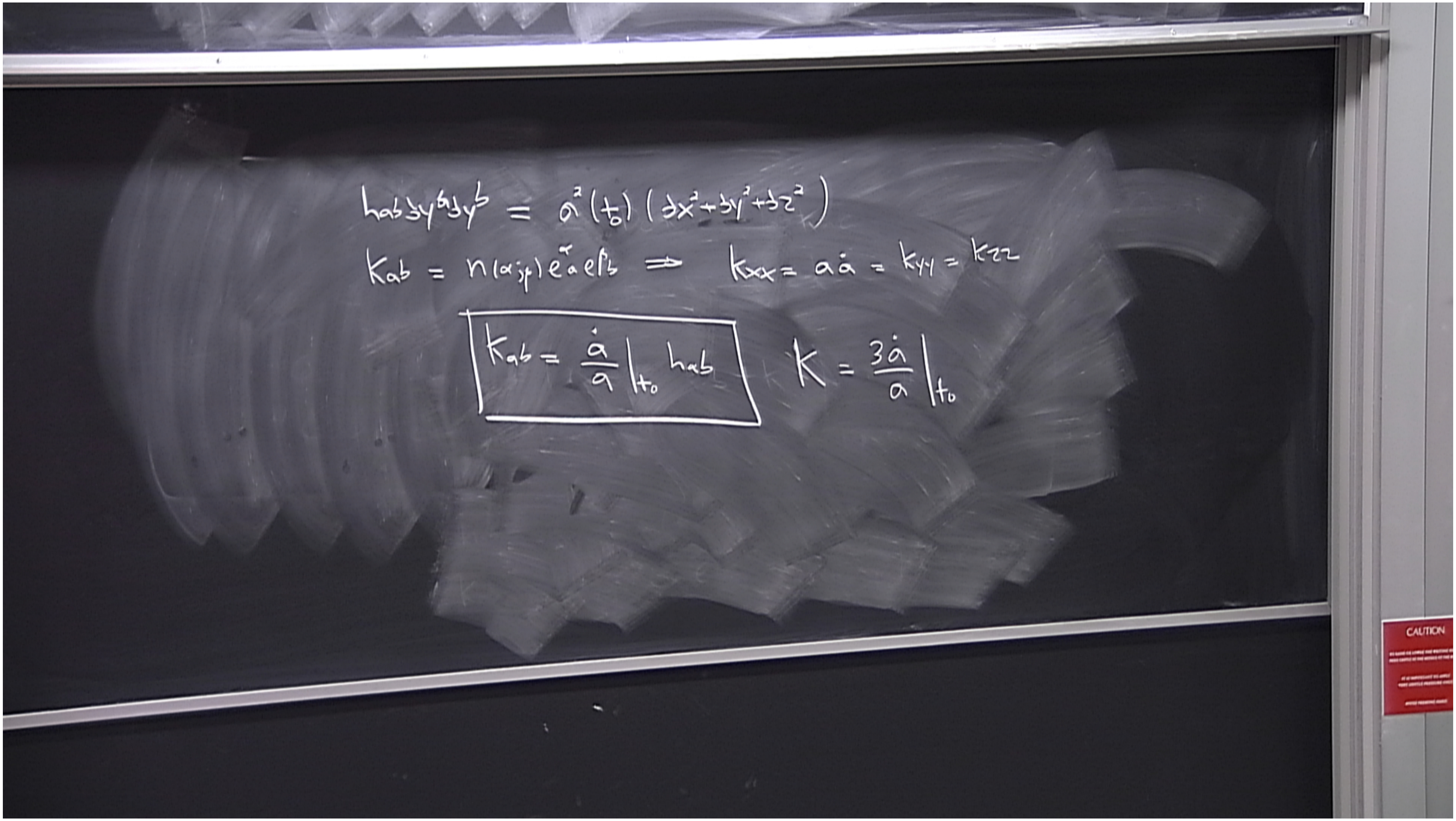
$$h_{ab}\, dy^a dy^b = a^2(t_0)\,(dx^2 + dy^2 + dz^2)$$

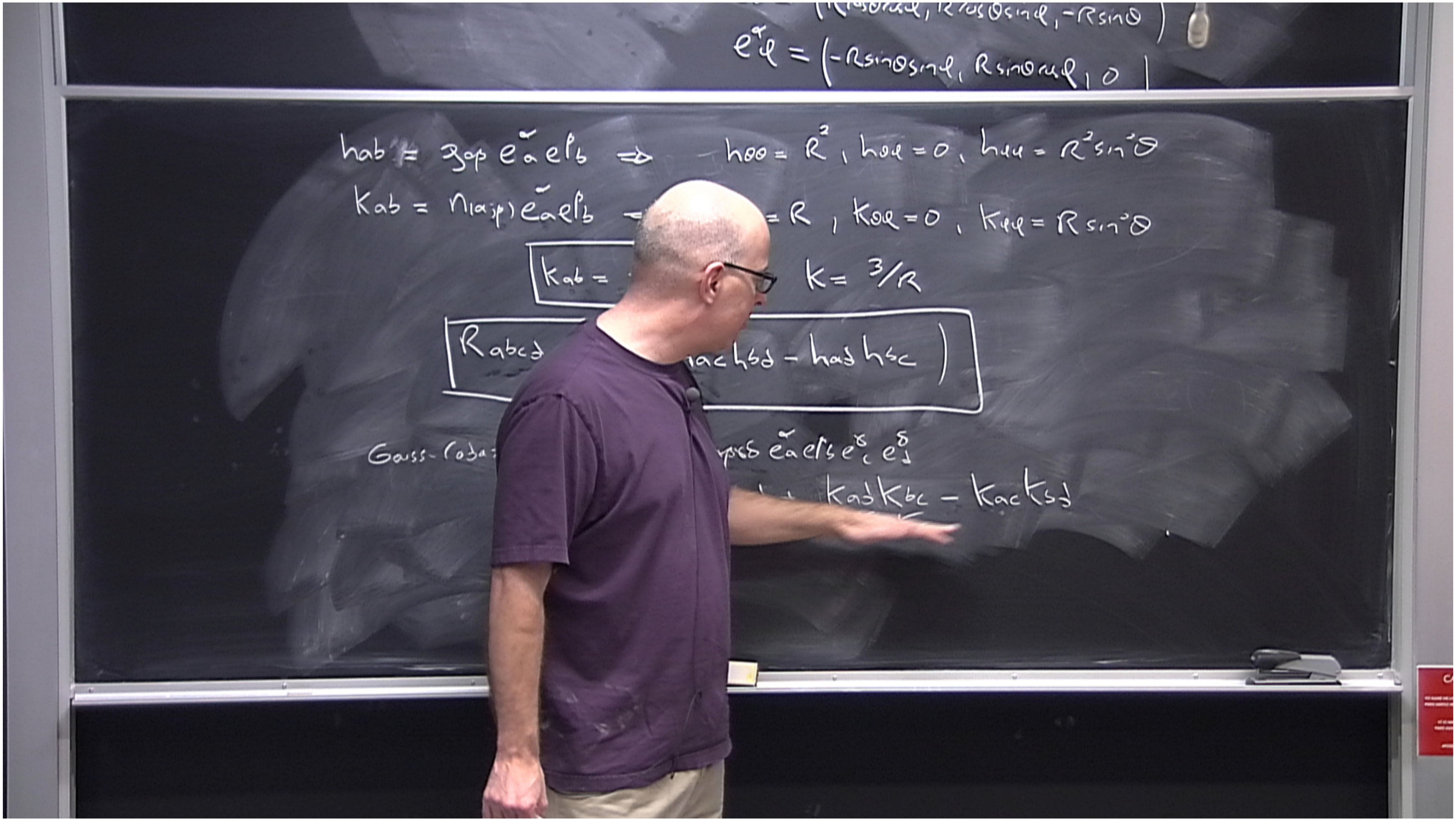
$$K_{ab} = n(\alpha;\beta)\, e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$$

$$\boxed{K_{ab} = \left.\frac{\dot{a}}{a}\right|_{t_0} h_{ab}} \qquad K = \left.\frac{3\dot{a}}{a}\right|_{t_0}$$


$h_{ab} \partial y^a \partial y^b = a^2(t_0)(\partial x^2 + \partial y^2 + \partial z^2)$
$K_{ab} = n(\alpha;\beta) e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$
$K_{ab} = \frac{\dot{a}}{a}\Big|_{t_0} h_{ab}$
$K = \frac{3\dot{a}}{a}\Big|_{t_0}$
CAUTION


$h_{ab}\,\partial y^a \partial y^b = a^2(t_0)\,(\partial x^2+\partial y^2+\partial z^2)$
$K_{ab} = n(\alpha;\beta)\, e^\alpha_a e^\beta_b \Rightarrow K_{xx} = a\dot{a} = K_{yy} = K_{zz}$
$K_{ab} = \frac{\dot{a}}{a}\Big|_{t_0} h_{ab}$
$K = \frac{3\dot{a}}{a}\Big|_{t_0}$
CAUTION


h_ab ∂y^a ∂y^b = a²(t_0) (∂x² + ∂y² + ∂z²)
K_ab = n(α;β) e^α_a e^β_b ⇒ K_xx = a ȧ = K_yy = K_zz
K_ab = ȧ/a |_t0 h_ab
K = 3ȧ/a |_t0
CAUTION



$$(R\cos\theta\cos\varphi,\ R\cos\theta\sin\varphi,\ -R\sin\theta)$$

$$e^\alpha_\varphi = (-R\sin\theta\sin\varphi,\ R\sin\theta\cos\varphi,\ 0)$$

$$h_{ab} = g_{\alpha\beta}\, e^\alpha_a e^\beta_b \Rightarrow h_{\theta\theta} = R^2,\ h_{\theta\varphi} = 0,\ h_{\varphi\varphi} = R^2\sin^2\theta$$

$$K_{ab} = n_{(\alpha;\beta)}\, e^\alpha_a e^\beta_b \Rightarrow K_{\theta\theta} = R,\ K_{\theta\varphi} = 0,\ K_{\varphi\varphi} = R\sin^2\theta$$

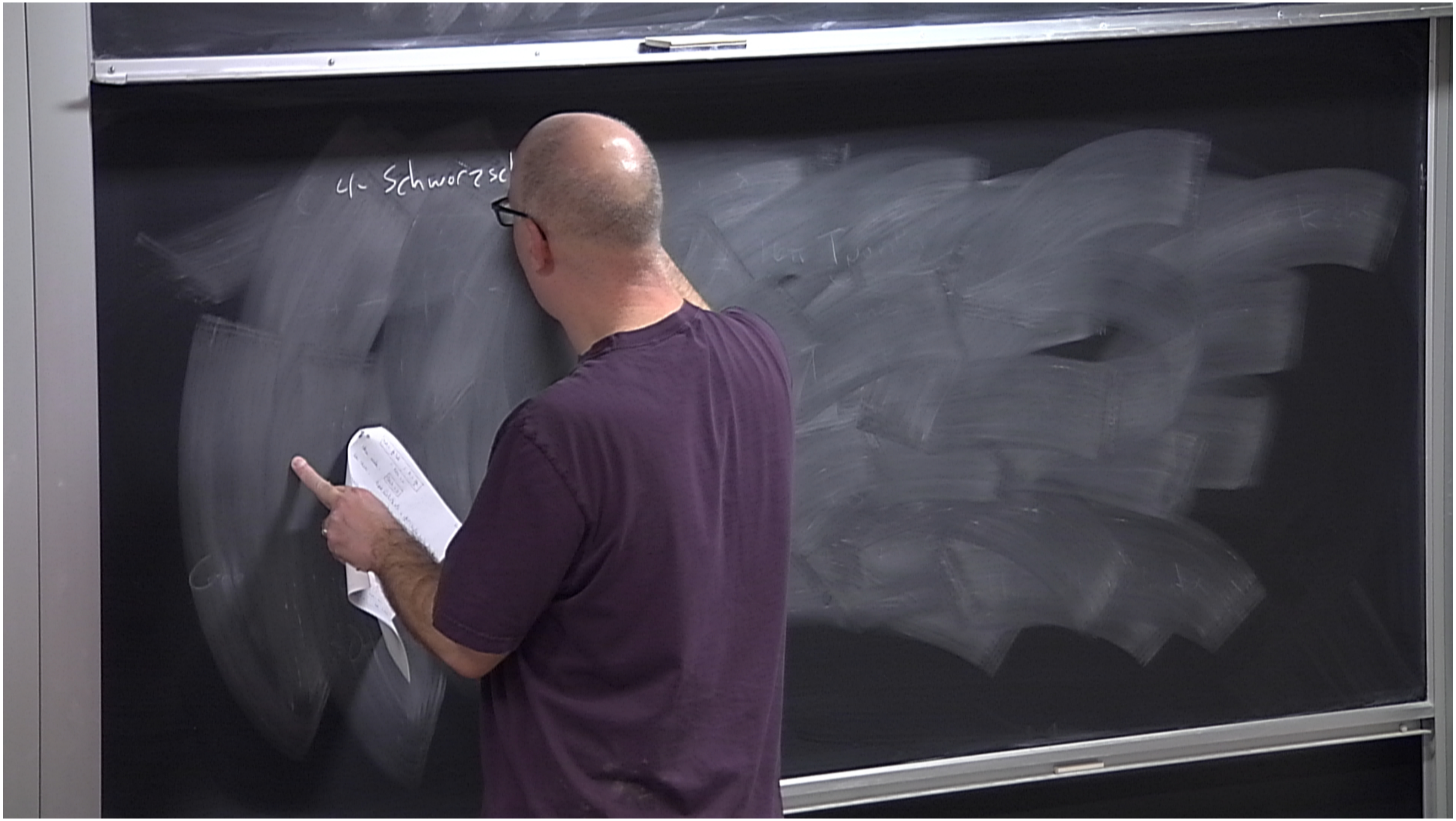
$$\boxed{K_{ab} = \frac{1}{R} h_{ab}} \qquad K = 3/R$$

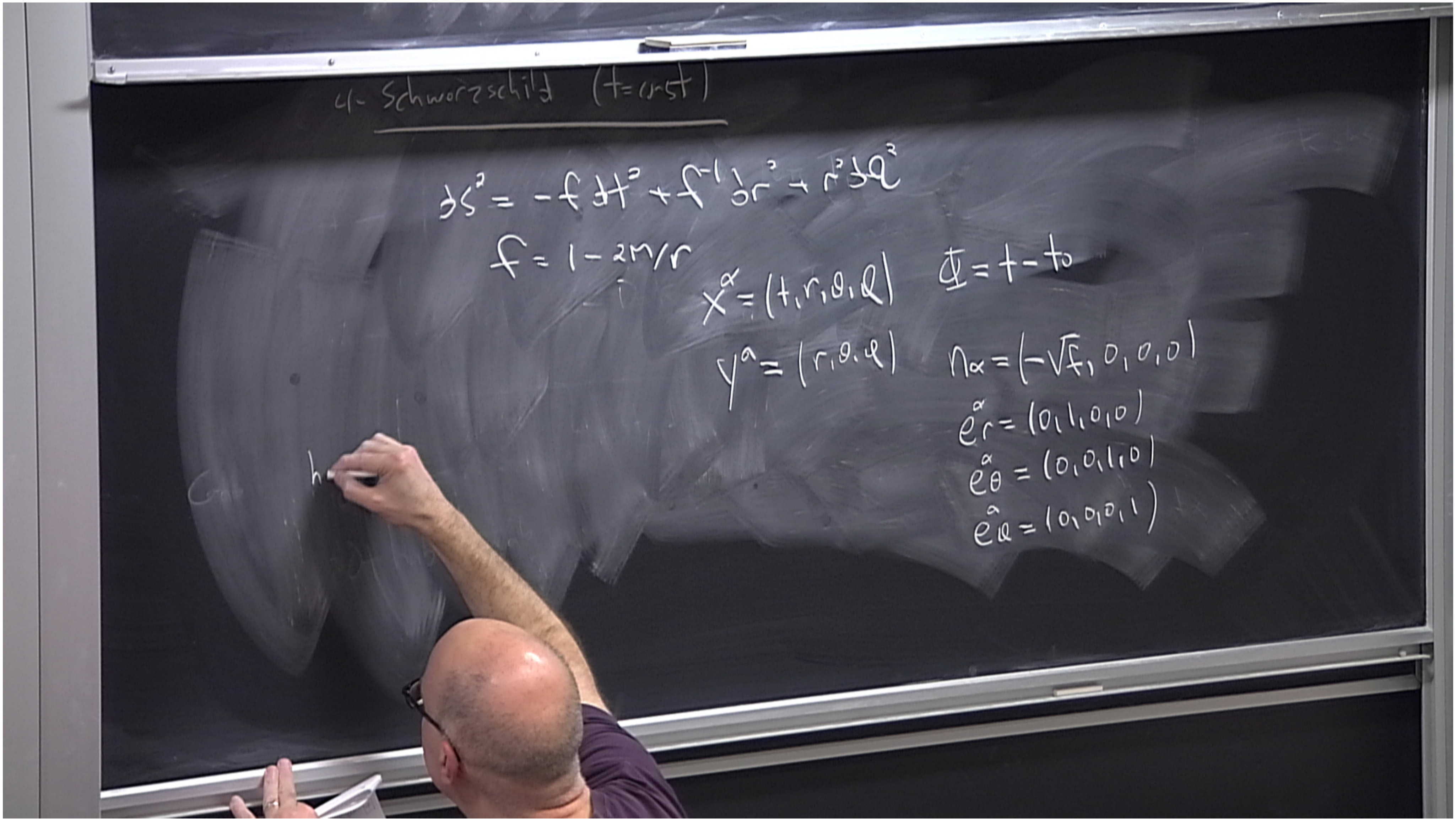
$$\boxed{R_{abcd} = \frac{1}{R^2}\left(h_{ac}h_{bd} - h_{ad}h_{bc}\right)}$$

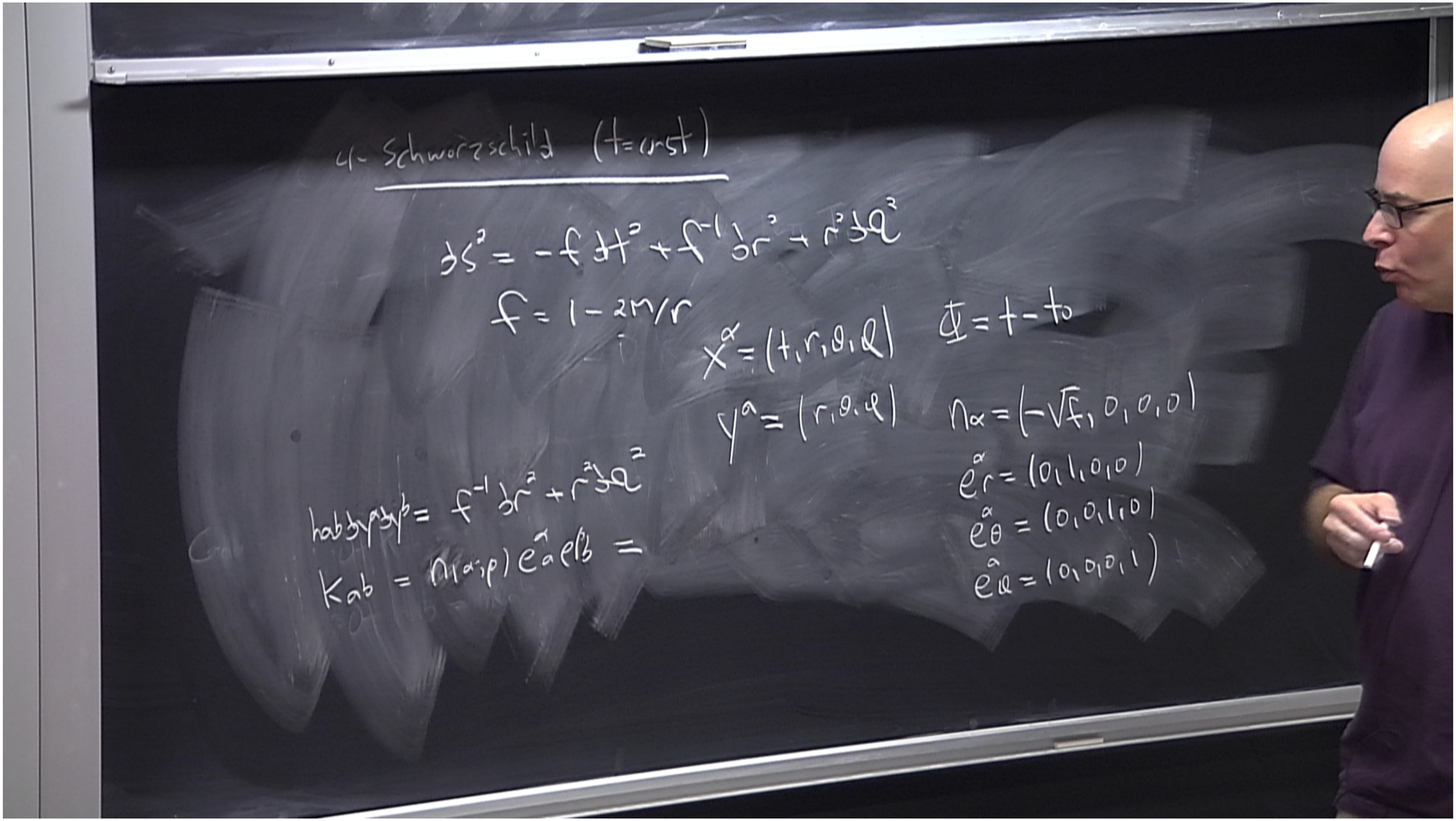
Gauss-Codazzi:

$$0 = {}^3R_{\alpha\beta\gamma\delta}\, e^\alpha_a e^\beta_b e^\gamma_c e^\delta_d$$

$$= R_{abcd} + K_{ad}K_{bc} - K_{ac}K_{bd}$$


4- Schwarzsc


4- Schwarzschild (t=const)
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$f = 1 - 2M/r$
$x^\alpha = (t, r, \theta, \varphi)$
$\Phi = t - t_0$
$y^a = (r, \theta, \varphi)$
$n_\alpha = (-\sqrt{f}, 0, 0, 0)$
$e^\alpha_r = (0, 1, 0, 0)$
$e^\alpha_\theta = (0, 0, 1, 0)$
$e^\alpha_\varphi = (0, 0, 0, 1)$


4- Schwarzschild (t=const)
$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega^2$
$f = 1 - 2M/r$
$x^\alpha = (t, r, \theta, \varphi)$
$\Phi = t - t_0$
$y^a = (r, \theta, \varphi)$
$n_\alpha = (-\sqrt{f}, 0, 0, 0)$
$e^\alpha_r = (0, 1, 0, 0)$
$e^\alpha_\theta = (0, 0, 1, 0)$
$e^\alpha_\varphi = (0, 0, 0, 1)$
$h_{ab}dy^a dy^b = f^{-1}dr^2 + r^2 d\Omega^2$
$K_{ab} = n_{(\alpha;\beta)} e^\alpha_a e^\beta_b =$